THE HYDROLOGIC SETTING OF LAKES IN MINNESOTA AND ADJACENT STATES WITH EMPHASIS ON THE INTERACTION OF LAKES AND GROUND WATER

by

Thomas C. Winter

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ABSTRACT

The hydrologic settings of 150 randomly-selected lakes in the north-central United States were investigated by principal component analysis as a first attempt to develop a general classification of the hydrologic setting of lakes. Precipitation-evaporation balance and the water quality parameters have high factor loadings on the first principal component. Highest loadings on component 2 are for streamflow in and out of the lakes. Components 3 and 4 are characterized by geology and ground-water flow parameters. The drainage basin area/lake area ratio, the overland runoff parameter, has the highest loading on component 5.

The stability of the principal components were tested by randomly splitting the data and comparing a principal component analysis of each subsample. This showed the first two principal components to be the most stable. The components described by the ground-water parameters are less stable but there is justification for using them with caution.

Comparison of factor-score maps of each principal component with maps of the original parameters resulted in identifying index parameters to be used in developing a general hydrologic classification of lakes. The maps produced are intended to provide information on the areal distribution of the parameters related to the hydrologic setting of lakes in the north-central United States. In this respect it presents a much broader approach to classifying lakes hydrologically, than previous studies.

Of the parameters examined in this study, the distribution of dissolved solids of ground-water is most closely related to the distribution of lake types determined by other limnological typologies in the north-central United States.

Numerical-model simulations of vertical sections were run to define the principles of the interaction of lakes and ground water, and to evaluate parameters that control ground-water flow near lakes in order to assess whether those used in the classificatory system need modification.

This report demonstrates for the first time by analysis of ground-water flow in vertical section, that the existence, position, and head value at the stagnation point of the ground-water flow field, relative to the head represented by lake level, is the key to understanding the interaction of lakes and ground water. The stagnation point is the point of least head along the ground-water divide between flow cells beneath a lake. Therefore, if a stagnation point exists, the divide is continuous, the lake cannot leak, and it is the discharge point of ground-water flow systems. If there is no stagnation point, the ground-water divide is not continuous, and a lake can leak through part or all of its bed.

Factors that strongly influence the interaction of lakes and ground water are height of the water table on the downslope side of the lake relative to lake level, position and hydraulic conductivity of aquifers within the ground-water reservoir, ratio of horizontal to vertical hydraulic conductivity of the ground-water system, and lake depth.

The models are the first to show the detailed patterns of ground-water flow adjacent to lakes in a wide variety of hydrogeologic settings. They show that for most settings water does not move from higher lakes to lower lakes if a water-table mound exists between them.

The statistical and modeling studies indicate that the following are the most important parameters to be considered in classifying the hydrologic setting of lakes; dissolved solids concentration of ground-water, precipitation-evaporation balance, streamflow inlet and outlet, the ratio of drainage-basin area to lake area, and lake depth. Of the ground-water parameters, local relief is more important than regional slope and regional position. Drift texture and bedrock, measures of the hydraulic conductivity parameters, are very important.

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INTRODUCTION

Background of the study

Lakes are a valuable natural resource in many areas of the United States. In addition to being sources of water supply for many communities, they are the focus of recreational activity for many, and their aesthetic qualities are valued by all. That they are a desirable feature is evidenced by the large number of small reservoirs being constructed throughout much of the United States for a wide variety of purposes. Increasingly, lakes are being used as the focal point in planning urban developments.

Unfortunately, the popularity of lakes often leads to their deterioration. The increased input of nutrients through the activities of man causes the organisms in lakes to flourish at rates far in excess of natural conditions. This increased productivity in turn causes chemical changes in lakes that result in obnoxious odors, fish kills, and unsightly conditions in the lakes and along their shorelines.

Remedial action has been taken to alleviate lake problems in the past. These actions generally have been of two types: (1) treatment with chemicals to control aquatic organisms and (2) construction of diversions and outlet structures to control lake levels. The effects of these measures have ranged from beneficial to harmful.

In recent years, much money and research effort have been expended to understand the principles of lake chemistry, biology, and the nutrient balance of lakes. At the same time, comparatively little money and effort have been devoted to understanding the principles of lake hydrology or the water balance of lakes. With few exceptions, the relationship of groundwater to lakes has been a minor part of the hydrologic studies, and it remains the least studied and least understood aspect of lake hydrology.

Most studies have usually been prompted by a seriously deteriorated lake that was brought to the attention of a funding agency of government. These studies of individual or groups of lakes resulted in a great deal of information on those lakes, but the transfer value of the information obtained is limited, partly because it is not known how similar the studied lakes are to the multitude of other lakes that exist.

The number of problem lakes is sharply increasing. In order to evaluate lake problems and the effect of management projects, nutrient budgets and water budgets of lakes are needed. Only by thorough understanding of the interrelationships of lake hydrology, chemistry, and biology can progress be made in alleviating present lake problems and preventing more problems from arising.

In most studies of lake hydrology, atmospheric water (precipitation and evaporation) and surface-water interchange with lakes is measured. Ground water is generally calculated as the residual. This practice can lead to serious misunderstandings about the interaction of lakes and ground water, especially when the error of measurement inherent in precipitation and particularly evaporation is considered. Further, only through careful field techniques can streamflow measurement error be kept to relatively low values.

Few studies have been done where the ground-water relationship to a lake is of primary interest. Many studies of such interaction have been made by measuring the difference between lake and ground-water levels in one or a few wells near the lake shore. In only a few studies have sufficient numbers of test holes and wells been installed to adequately define the hydraulic properties and boundaries of geologic units and the shape and height of the water table in the vicinity of a lake.

Some misconception of the interaction of lakes and ground water has resulted from lack of understanding of ground-water flow, which leads to inadequate instrumentation and data collection. Poorly executed studies have led to polarization of ideas concerning the interaction of lakes and ground water, to the detriment of sound lake management. Some scientists and managers believe all lakes are discharge points of the ground-water system and therefore do not lose water through their bed. Others believe that water does move from the lake to the ground-water system; thus lakes are points of recharge. Some believe ground water flows in one side of lakes and out the other, a flow-through condition. Others believe all three situations exist.

Because of limitations in money, manpower, and time, it is impossible to study each problem lake in the necessary detail. Only by categorizing a system as complex as lakes, so that general relationships of groups of lakes can be seen, will it be possible to attack lake deterioration problems on a broad scale.

The purpose of classifying is to simplify the structure and relationship of objects so that general statements can be made about the classes of those objects (Sokal, 1974). It would be desirable, for example, to typify lakes according to lake-level fluctuation patterns or water chemistry. Similarly, it would be desirable if one could crudely estimate the water budget of a given type of lake simply by knowing something of its hydrologic setting. Classifications are useful also because they often lead to new hypotheses and stimulate further investigations into parameters that should be considered and into new methodologies.

Review of selected lake classifications

Most lake classifications are based on one or several chemical or biological parameters. A widely used classificatory system is that of the general trophic state of a lake, the two extremes being (1) oligotrophic, characterized by high transparency, low hypolimnetic oxygen deficit, and low productivity and (2) eutrophic, characterized by the opposite. Recently, much attention has been devoted to quantification of this classificatory system by developing a trophic level or productivity index (eg. Shannon and Brezonik, 1972; Hooper, 1969; Vallentyne, 1969; Tarapchak, 1973).

In the north-central United States, several workers have developed classificatory systems based on chemical and/or biological characteristics of lakes. Moyle (1956) showed a general gradation of some chemical and biological parameters across Minnesota. Sulfate, total alkalinity, total phosphorus, total nitrogen, and chloride all increase from northeast to southwest. This pattern is paralleled by the distribution of fish types and aquatic floras. On the basis of these data, Moyle recognized four general lake types that conform to geographic areas: (1) soft-water lakes in northeastern Minnesota, (2) productive, hard-water lakes in central Minnesota, (3) productive, saline lakes in southwestern Minnesota, and (4) highly saline prairie lakes.

Bright (1968) mapped the distribution of salinity and major ions in a sample of Minnesota's lakes and found three major types that correspond to major vegetation zones. These are the coniferous-deciduous forest of northeastern Minnesota, deciduous forest in central Minnesota, and the prairie of southwestern Minnesota.

Gorham (1971) used specific conductance and concentration of major ions to classify the lakes of Minnesota into four geographic groups that are similar to those of Moyle.

Tarapchak (1973) classified the lakes of Minnesota on the basis of major ion concentration and delineated groups nearly identical to Gorham's. In evaluating the use of eight biological trophic-state indicators and suites of phytoplankton as a basis of classification, however, Tarapchak found less distinct groups. The classificatory systems of Tarapchak, Moyle, Gorham,, and Bright are discussed in greater detail in a later section of this report.

A classification of lakes based on biota, food-source density, and productivity of lakes in Oregon is discussed by Donaldson (1969). Bortleson and others (1975) developed a classification of Washington's lakes on the basis of chemical and biological parameters. A separation of lakes in the semi-arid part of Kazakhstan (Russia) according to chemistry of the water is discussed by Forsh (1970). Shannon and Brezonik (1972) used principal component analysis to define a trophic state indicator that was then used to classify lakes in Florida.

Lake classifications based on basin origin and physiographic characteristics include the broad classification of Hutchinson (1957) and the classification of Minnesota's lakes by Zumberge (1952). Hutchinson lists 76 lake types grouped into eleven categories, such as those in tectonic basins, those associated with volcanic activity, landslides, glacial activity, etc. Zumberge's classification is similar in nature, but is restricted to Minnesota; thus, it is concerned largely with lakes of glacial origin and is much more detailed in this respect than the classification of Hutchinson.

Little work has been done on classifying lakes according to hydrologic characteristics. Generally lakes are separated into open or closed types depending on the presence or absence of surface-water outlets. A systematic classification of lakes (and reservoirs) based on components of the hydrologic budget is discussed by Bogoslovsky (1966). He has grouped the lakes of Russia into two principal types; those that have outflow predominant and those that have evaporation predominant. Subtypes within each major group are based on surface-water inflow and outflow and precipitation-evaporation relationships. Ground-water inflow and outflow are not directly considered. A classification also based on water budget components is presented by Szesztay (1974). Using precipitation, evaporation, and streamflow data, he classified lakes into one of nine combinations of "flow-controlled" and "climate-controlled" lakes. Szesztay discusses the utility of his classification in predicting water-level fluctuations.

Born and others (1974) have proposed a classification that includes the hydrogeologic aspects of lakes. They suggest two broad types of lakes--those dominated by ground-water and those by surface-water. Each group is subdivided according to the efficiency of the lake-water system; that is, high or low flushing rate for lakes dominated by surface-water, and high or low "Darcy characteristics" for lakes dominated by ground water. The term "Darcy characteristics" refers essentially to the hydraulic conductivity of the rocks underlying a lake basin.

Until recently, most classifications of objects were based on one or a few parameters. Shannon (1969) and Brezonik (1969) have criticized this simplistic approach to classifying lakes as being inadequate; this criticism is probably true of most other single-parameter classifications also. Classifications based on few parameters are usually optimal with respect to those parameters, but they are not likely to be of general use. It is generally true that simplistic classifications have to be modified as additional parameters are considered (Sokal, 1974). Conversely, classifications based on many parameters are more likely to be of widespread use but might not be totally satisfactory for any one purpose. To establish a workable classificatory system, generally many parameters must be considered. Once a workable classificatory system is developed, few indicator parameters are necessary to allocate objects to the proper taxa.

From the above discussion it is clear that a classification of lakes based on all factors related to lake hydrology has not been developed. Such a classification is badly needed if results of detailed studies, which are only going to become more expensive, are going to be applied with confidence to other lakes. It is evident also that the factors related to lake hydrology are not generally known or clearly understood. Much work needs to be done to identify and evaluate the relative importance of factors that control lake hydrology. This is particularly true of the factors that control the relationship of ground water to lakes.

Purpose and scope

The purpose of this report is to classify the lakes in the north-central United States according to their hydrologic setting and to examine especially the factors that control the interaction of lakes and ground water. The study uses the approach of developing a general classification. Many parameters related to lake hydrology are considered, with the goal of identifying independent parameters upon which to base the most parsimonious classificatory system. Parameters related to the interchange of lake water with atmospheric water, surface water, and ground water are analyzed by maps and multivariate statistical methods.

In an initial evaluation, the first major part of the report, generally accepted parameters that control ground-water flow are used. Because the detailed patterns of ground-water flow near lakes have never been examined, the second major part of the report deals with digital models of the interaction of lakes and ground water. This effort has two principal goals; 1) elucidation of the principles of the interaction of lakes and ground water and 2) evaluation of the ground-water parameters used in the initial classification for the specific purpose of classifying the hydrologic setting of lakes.

The statistical aspect of the study is based on a random sample of 150 natural lakes in the glaciated north-central United States. The area is bounded by the Missouri River on the west, Lake Michigan on the east, the Canadian border on the north, and central Iowa on the south. This region was selected for study because it has one of the nation's greatest concentrations of natural lakes in a wide variety of hydrologic settings. Physiographically, the area contains lakes ranging from those in hummocky, Precambrian crystalline terrain to lakes on glacial drift. The latter occur in a wide variety of drift types including outwash, sandy noncalcareous till, and silty calcareous till. Topographic settings range from hummocky end moraines to flat ground moraines, outwash plains, and river valleys. Some of the lakes are closely associated with Paleozoic and Mesozoic sedimentary rocks.

The region includes a wide variation in ground-water environments, especially with respect to ground-water flow systems and water quality. The flow systems range from those in fractured crystalline rocks to large regional systems associated with thick sections of unconsolidated drift and those characteristic of stratified sedimentary rock. The quality of ground water ranges from calcium magnesium bicarbonate in the east to calcium sulfate, sodium bicarbonate, sodium sulfate, and sodium chloride in the west.

The region has a marked climatic gradient characterized by low precipitation and high evaporation in the Dakotas to high precipitation and lesser evaporation in northeastern Minnesota. Within this climatic gradient is the line of equal annual precipitation and evaporation; west of the line evaporation exceeds precipitation, and east of it the opposite is true.

This is probably the only region of abundant natural lakes in the United States where the line of equal precipitation and evaporation transects a broad area.

The north-central states include several ecotones in close proximity, from prairie in the west to deciduous forest and coniferous forest in the east. Except for mountainous areas, the closeness of these ecotones is unique to this region.

The lakes themselves range from pure, oligotrophic lakes of low specific conductance in northeastern Minnesota to highly eutrophic lakes in Minnesota and the Dakotas. Lake-water chemistry ranges from soft water to bicarbonate hard water to saline water.

This wide variety of environments makes the north-central states particularly attractive for concentrated study because the region includes lake types found in many other parts of the country. Hydrologic characteristics of these lakes probably correspond to those in most glaciated and some unglaciated parts of the world; thus methodologies and results of studies here should have widespread transfer value.

CLASSIFICATION OF THE HYDROLOGIC SETTING OF LAKES IN THE NORTH-CENTRAL UNITED STATES

Physiographic Setting

Hypsography

The western part of the study area is characterized by broad uplands and lowlands separated by relatively steep hillsides (fig. 1). The highland along the western edge of the area immediately east of the Missouri River, commonly called the Coteau du Missouri, is greater than 1,800 ft (550 m) above mean sea level. In the northern part it rises to an altitude greater than 2,400 ft (730 m) above mean sea level. The small highland in north-central North Dakota, the Turtle Mountains, has an altitude similar to that of the Missouri Coteau. The broad lowland that includes Devils Lake in northeastern North Dakota has an altitude of 1,400 to 1,600 ft (430 to 490m) above mean sea level. The Coteau des Prairies, a broad upland in eastern South Dakota, has about the same altitude as the Missouri Coteau and is separated from it by the James River lowland, which lies about 1,200 to 1,400 ft (370 to 430 m) above mean sea level.

The highlands in Minnesota and Wisconsin are generally not so flat-topped as those in the Dakotas. They reach

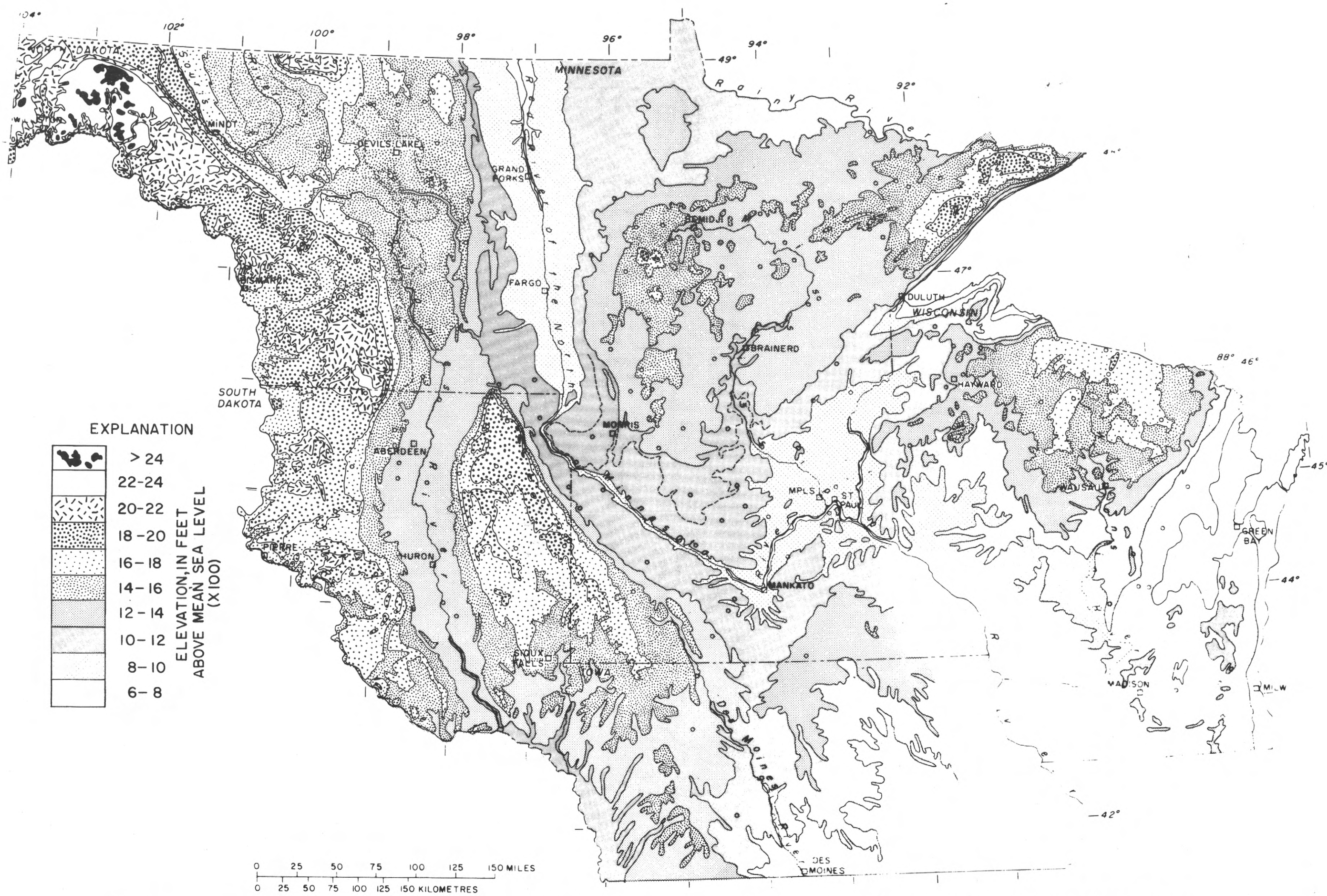


Figure 1.--Hypsography--topographic features of the north-central United States.

altitudes greater than 1,800 ft (550 m) locally in northwestern and northeastern Minnesota and in north-central Wisconsin. Much of central and north-central Minnesota consists of a broad upland that has an altitude of 1,200 to 1,400 ft (370 to 430 m) above mean sea level. The highland in Wisconsin slopes gradually in all directions to altitudes less than 800 ft (240 m) above mean sea level along Lakes Superior and Michigan and the Mississippi and St. Croix Rivers.

Bedrock geology

The pre-Pleistocene bedrock surface consists of Precambrian crystalline rocks in much of central and northern Minnesota (Sims, 1970) and northern Wisconsin (Holt and Skinner, 1973) (fig. 2). Paleozoic rocks occur to the south of these areas in Wisconsin and southeastern Minnesota. The stratigraphic section representative of the Paleozoic rocks in the southeastern part of the study area is shown in figure 3. A small area of Paleozoic rocks occurs in northwestern Minnesota. Paleozoic rocks also occur in the Dakotas, but they are covered by Cretaceous and Tertiary rocks. The bedrock

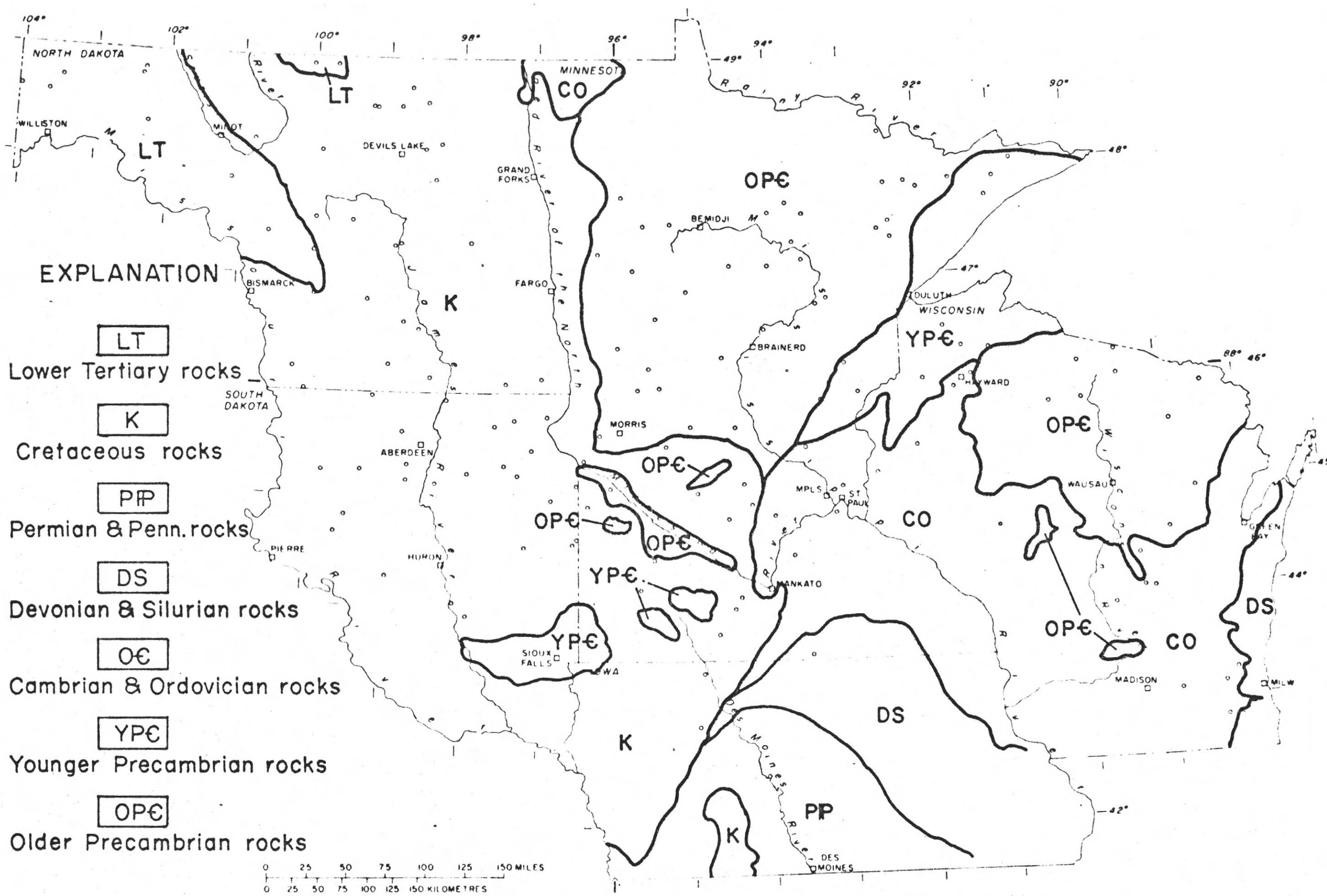
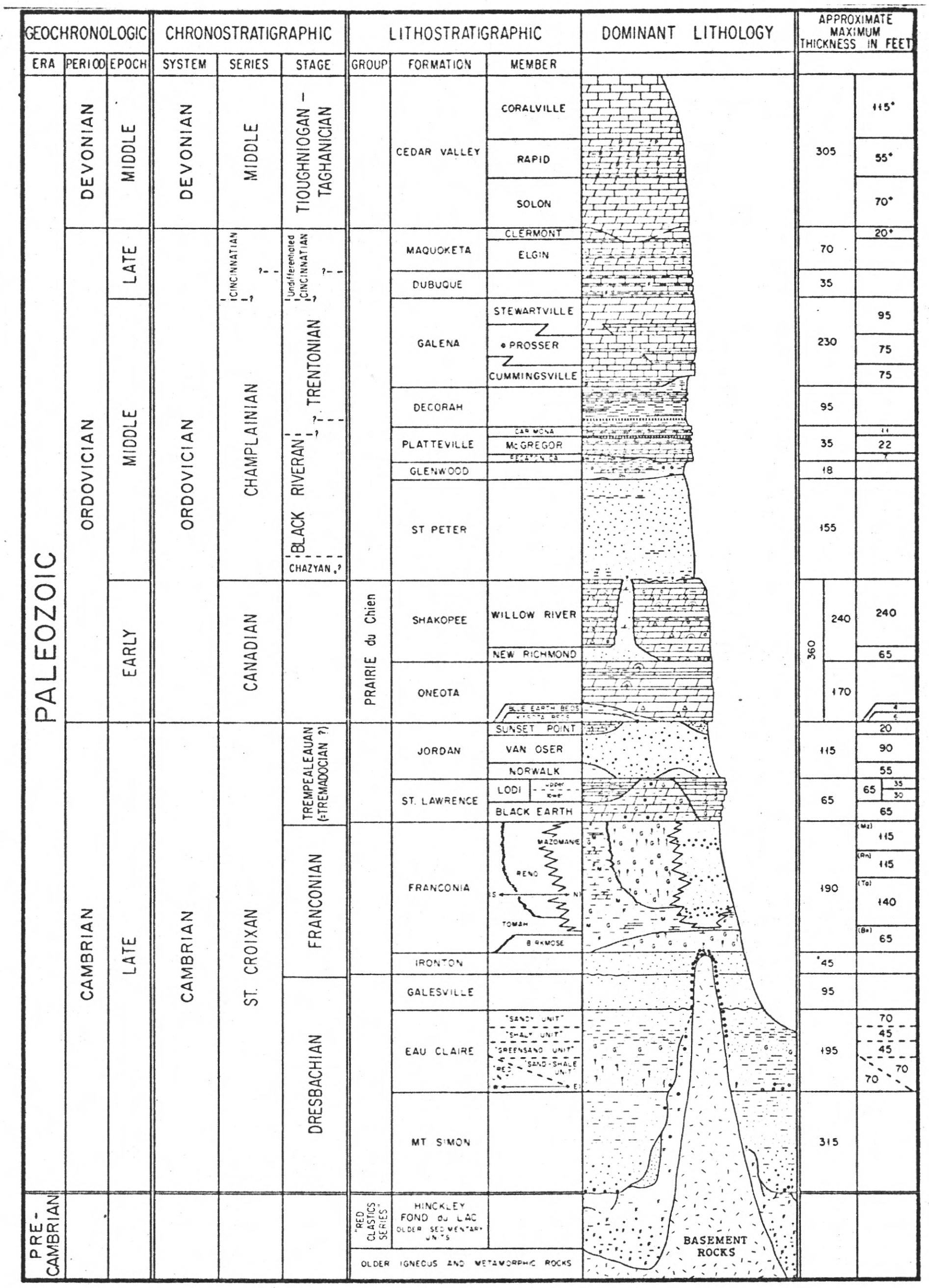


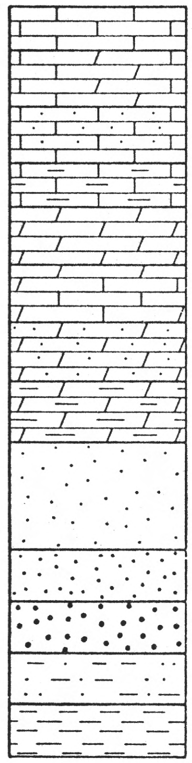
Figure 2.--Bedrock geology.



From: Austin, 1969

Figure 3.--Pre-Pleistocene rocks in the southeastern part of the area.

Explanation for Figure 3.



LIMESTONE
- dolomitic
- sandy
- shaly

DOLOMITE
- calcareous
- sandy
- shaly

SANDSTONE
- fine to very fine
- medium
- coarse
- shaly

SHALE

△	Chert
△	Oolitic chert
⊙	Oolites
⊥	Dolomitic
··	Silty
xxxxx	Bentonite
G	Glauconite
P	Pyrite
M	Mica
F	Feldspathic
Ph	Phosphate pellets
⌒	Stromatolites
ĭ	Worm bored
••	Conglomeratic
?	Questionable relationship

underlying drift in the Dakotas is largely Cretaceous shales and siltstones, but Tertiary sandstones, siltstones, and shales overlie Cretaceous rocks beneath the Missouri Coteau and the Turtle Mountains (Carlson, 1969; Flint, 1955). The stratigraphic section representative of the Dakotas is shown in figure 4.

The distribution of bedrock and its hydrologic properties is of interest to this study, because in areas where drift is thin ground-water flow systems associated with lakes very likely are partly controlled by bedrock.

Surficial geology

The surficial material in the study area is virtually all glacial drift. The end moraines, ground moraines, outwash plains and other glacial landforms (fig. 5) are the products of glacial ice lobes that moved into the area from the north, northwest, and northeast. In the western part of the area, glaciers moved down the lowlands occupied by the Souris, James, Red, Minnesota, and Des Moines River valleys. The Rainy lobe moved into northeastern Minnesota from the northeast. It was confined to the area west of the highland adjacent to Lake Superior. In the central and eastern part of the area, ice lobes moved out of the basins of Lake Superior, Lake Michigan, and Green Bay (fig. 6).

System	Group		Name of unit	Description	Thickness (feet)
Tertiary.			Ogallala formation(?)	Sandstone, quartzite, marl, and shale. High-level remnants only; sandstone and quartzite facies cap buttes.	Unknown
			Hell Creek formation.	Shale, siltstone, and sandstone, tough, light-gray to brownish; interbedded with carbonaceous shale and thin beds of lignite. Contains bones of terrestrial vertebrates. Forms steep buttes and mesas.	[1] 200
			Fox Hills sandstone.	Sandstone, medium-grained, poorly indurated, medium-gray, olive-gray, and greenish-gray hues, with subordinate gray siltstone and gray shale in lower part. Contains ferruginous and calcareous concretions. Weathers to orange sand. Irregular cementation results in lenticular weathered masses unrelated to stratification. Forms buttes and mesas with rounded profiles.	[1] 300
Cretaceous.	Montana group.	Pierre shale.	Elk Butte member.	Shale, blue-black, noncalcareous; with subordinate siltstone.	100 to 250
			Mobridge member.	Shale, dark blue-gray with bentonite beds, chalk, chalky shale, and sandy shale. Chalk beds are gray, weathering to yellowish and orange hues, and form cliffs.	[2] 100 to 300
			Virgin Creek member.	Shale, dark-gray, noncalcareous, siliceous in northern part of region. Thin bentonite beds occur in lower part of member, and scattered large concretions in upper part. Weathers to orange and brownish hues. Forms cliffs where siliceous.	240
			Verendrye member.	Shale, olive-gray, nonsiliceous, with abundant large, flattish, very dark purple concretions. Shale weathers to gumbo; concretions break down to small angular fragments.	180
			De Grey member.	Shale, noncalcareous, gray, with layers of bentonite, and (in southern part of region) conspicuous concretions of manganese-iron oxides and iron carbonate.	[1] 160
			Crow Creek member.	Shale and marl, calcareous, with a thin basal sandstone.	[1] 15
			Gregory member.	Shale, medium-gray to yellowish-gray, with concretions and calcareous layers. Includes discontinuous marl beds at base.	[1] 125
			Sharon Springs member.	Shale, thin-bedded, grayish-black, bituminous, weathering brownish black. Includes thin bentonite beds.	[1] 35
	Colorado group.	Niobrara formation	Smoky Hill chalk member.	Chalk, massive and shaly chalk, bluish-gray to dark-gray, weathering to yellowish orange and white.	150 to 300
			Fort Hays limestone member.	Chalk, massive and chalky shale; gray when fresh; weathering to white or yellowish orange.	
		Carlile shale	Codell sandstone member.	Sandstone, coarse, massive to crossbedded; pale-brown to brownish-gray.	[1] 100
			Unnamed unit.	Shale, bluish-gray to greenish-gray; with calcareous and ferruginous concretions. Fossiliferous.	200
			Greenhorn limestone.	Limestone, bluish-white, firm, thin-bedded, well-jointed, chalky, with layers of calcareous shale. Fossiliferous.	[1] 100
			Graneros shale.	Shale, dark-gray, with sandy beds near base. Locally contains pyrite.	[1] 100
			Dakota sandstone.	Sandstone, fine-grained, irregularly bedded; with siltstone, shale, and carbonaceous beds. Light olive gray when fresh; yellowish orange to light brown on weathered surfaces.	[1] 300
Pre-Cambrian.			Sioux quartzite.	Quartzite, mostly fine textured, but including conglomerate, siltstone, and shale. Light brownish gray to grayish red purple; less commonly gray, white, and orange. In places cut by basic igneous intrusions.	[3] 3,000
			Granite.	Granite, coarse-textured, dark-red, showing faint grain on weathered surfaces caused by subparallel orientation of minerals.	

[1] Maximum thickness.
[2] 100 feet in south; 300 in north.
[3] Minimum thickness.

From: Flint, 1955

Figure 4.--Pre-Pleistocene rocks in the western part of the area.

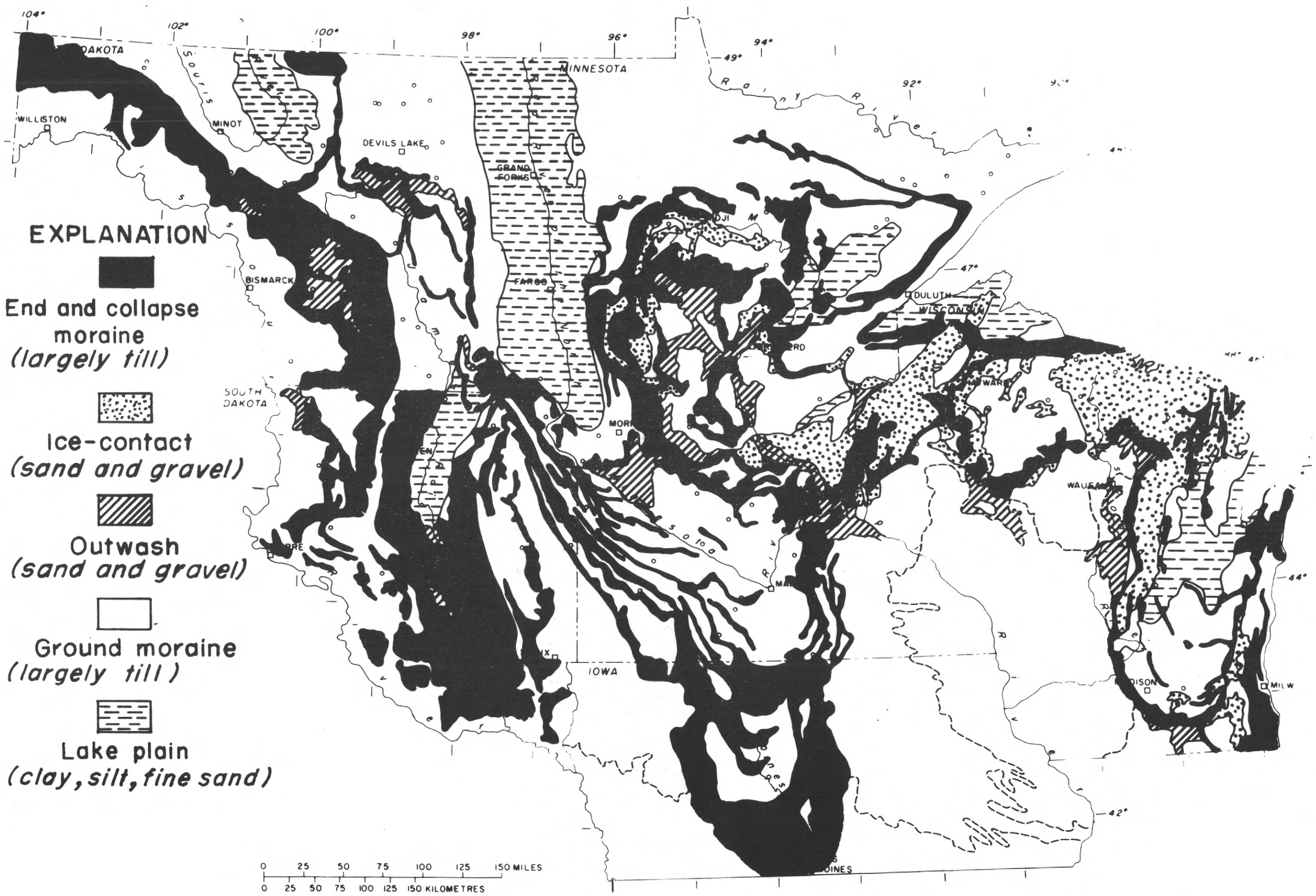


Figure 5.--Surficial geology.

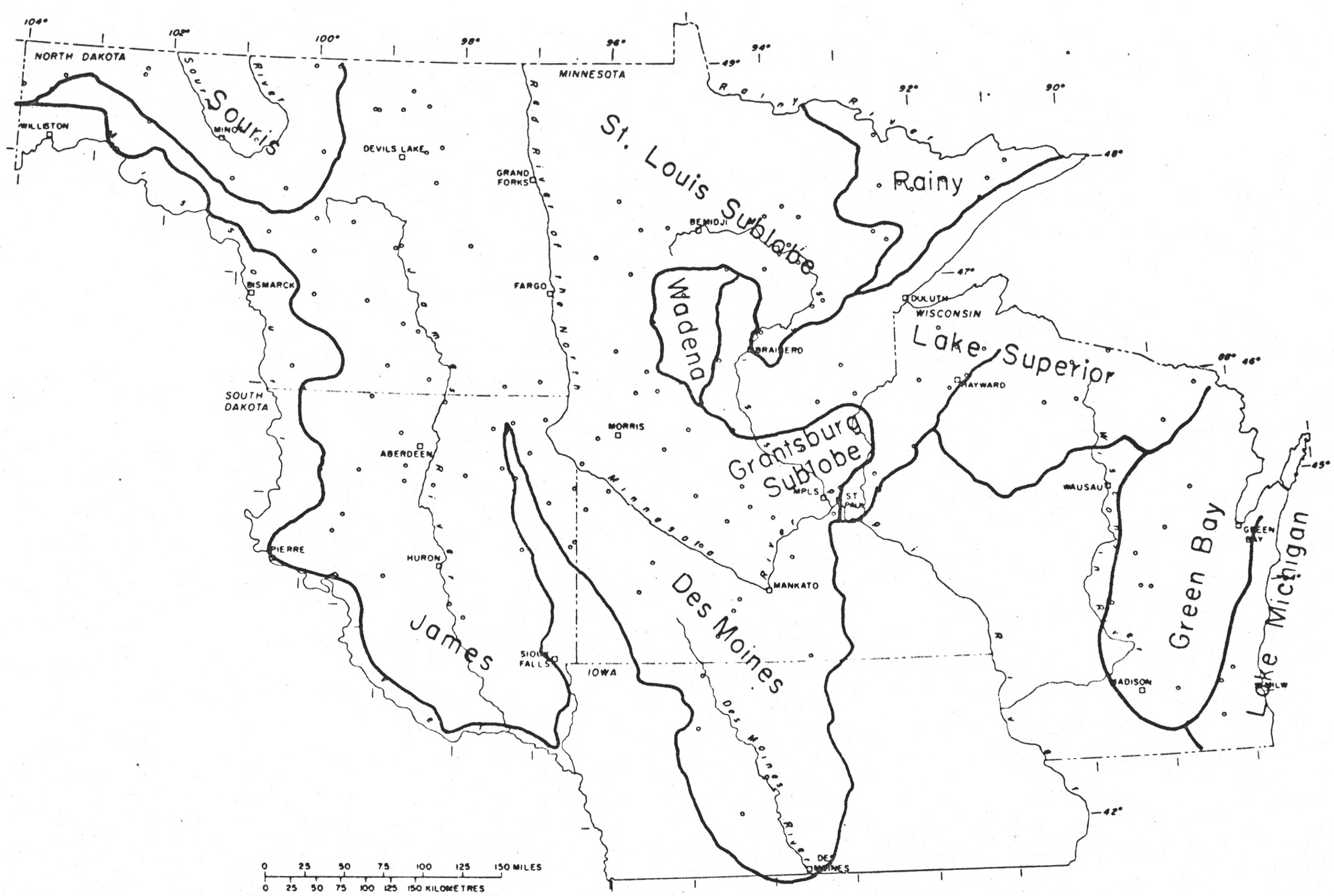


Figure 6.--Late-Pleistocene ice lobes.

Drift texture (particle-size characteristics of drift deposits) in the report area is related to the genesis of the various glacial deposits and landforms. Drift texture tends to reflect the bedrock over which the ice lobes moved. In the western part of the area, tills of the Souris, James, and Des Moines lobes are generally silty and clayey (till is a heterogeneous mixture of clay- through boulder-size materials). The clayey matrix of the till is derived from Cretaceous shales in a large area of that region. The Souris lobe is probably less clayey than the drift of the James and Des Moines lobes because it is closer to the Tertiary rocks.

Till of the Rainy, Superior, Green Bay, and Lake Michigan lobes generally is sandy and silty. The exception is the very clayey red till that rims the immediate shores of the Great Lakes.

Ice-contact deposits and outwash plains are stratified sand and gravel. The glacial lake plain areas are characterized by lacustrine deposits of fine sand, clay, and silt. Few existing lake basins are in glacial lake areas.

The main difference between the texture of end moraines and ground moraines is that end moraines commonly result from ice collapse and tend to have more sand and gravel ice-contact deposits incorporated within them than do ground moraines.

The mineralogy of drift is also closely related to the bedrock overridden by ice lobes. Drift mineralogy is of interest to this study because it is one of the primary controls of ground-water chemistry. Drift in the western part of the area has a high percentage of Cretaceous shales and siltstones. The drift is generally calcareous and contains pebbles of limestone and dolomite. Drift of the Superior Green Bay, Lake Michigan, and Rainy lobes is very weakly- or non-calcareous, and its mineralogy is dominated by granite and metasedimentary rocks that occur in that area. These drifts also contain volcanic rocks that are common in the Canadian Shield. Drift from the Superior lobe contains sandstone of Precambrian age that occurs in the Lake Superior basin. Green Bay and Lake Michigan lobe drift contains limestone and dolomite from the local Silurian and Devonian carbonates that occur on the east side of Wisconsin. For a detailed discussion of the glacial deposits and glacial history of the report area, regional summaries are given in Wright and Frey (1965).

Thickness of glacial drift is of interest in ground-water flow studies because, in areas of thin drift, ground water moves through bedrock even within local flow systems. The drift generally is thickest in areas of large end moraines, such as those on the Missouri Coteau in North Dakota, Coteau des Prairies in South Dakota and Minnesota, Alexandria moraine in Minnesota, and in northwestern Wisconsin. In these areas the drift is as much as 500 ft (150 m) thick, and areas of drift more than 400 ft (120 m) thick are common. Stringers of thick drift in North Dakota, Minnesota, and southeastern Wisconsin are areas where drift fills valleys in the bedrock.

A large area of North Dakota has drift less than 100 ft (30 m) thick (Bluemle, 1971), particularly in areas northwest, northeast, and east of Devils Lake. In Minnesota, the areas of drift less than 100 ft (30 m) thick are generally confined to the northeastern and southeastern part of the state (Tufford, 1966). Wisconsin also has extensive areas of drift less than 100 ft (30 m) thick (Trotta and Cotter, 1973), most notable is the area of the Green Bay lobe and some areas in the northern part of the state.

This, then, is the topographic and geologic framework that controls the occurrence and movement of surface water and ground water in the north-central United States.

Methods of Data Collection

Selection of lakes

Thousands of lakes occur in the north-central United States. Minnesota alone has more than 15,000 lakes greater than 10 acres (4 hectares) in area (Minnesota Section of

Waters staff, 1968). Because of their abundance, any work of regional scope must be based on a representative sample of lakes. Further, in order to analyze the data statistically, the sample must be selected on a random basis.

For this study, 150 lakes (fig. 7) were selected according to the following procedure.

(1) Townships were numbered that have three or more lakes on the 1:250,000 scale AMS (Army Map Service) sheets for the States of North and South Dakota, Minnesota, Wisconsin, and Iowa.

(2) A random-numbers table was used to select 150 townships.

(3) The lake nearest the center of the township that was at least 20 acres (8 hectares) in area and had topographic map (7½' or 15') coverage was selected for this study.

Selection and quantification of parameters

Because a general classification of the hydrologic setting of lakes has not been developed, and because all aspects of lake hydrology are not clearly understood, there is no precedent for the parameters that must be considered. The parameters selected for this initial study, therefore, are

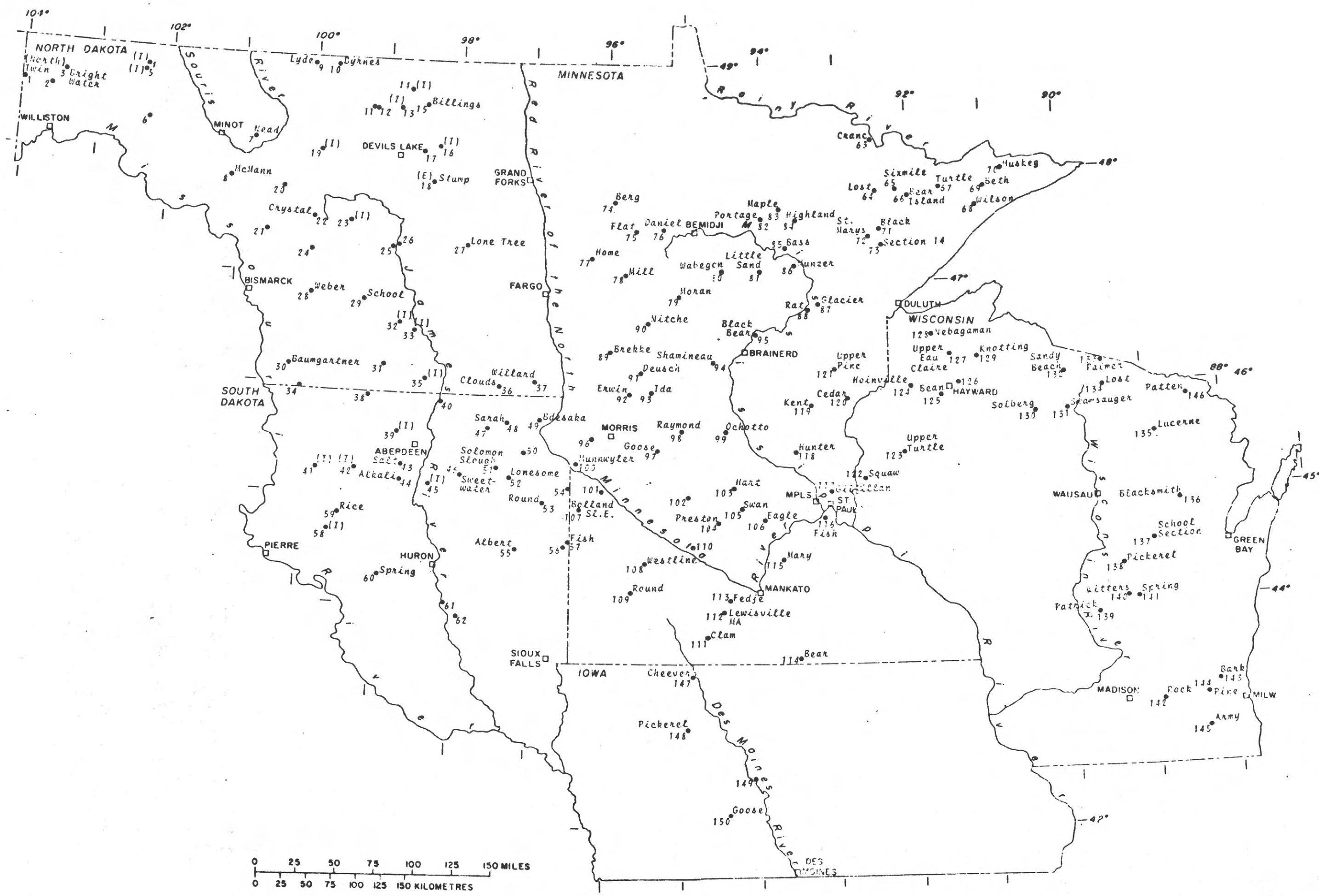


Figure 7.--Locations of lakes included in this study.

those that are believed by the author to control the atmospheric, surface, and ground-water interchange with a lake. Some types of water, such as atmospheric, can be measured, and values for precipitation and evaporation were taken from published maps. Other types, such as ground water, are little understood and unmapped, so parameters that control ground-water flow are used.

Initially, 17 parameters were selected as being related to lake hydrology (table 1). Some of these have more hydrologic meaning as differences or ratios, so the study is based on the following 13 parameters:

P-E	Precipitation-Evaporation (P-E) balance
IN	Streamflow in
OUT	Streamflow out
D/L	Ratio of drainage basin area to lake area
RS	Regional slope of the land surface
LR	Local relief of the lake's immediate drainage basin
RP	Relative altitude of the lake in the region
LF	Type of landform in which lake is situated
DM	Mineralogy of the drift
DT	Texture of the drift (drift hydraulic conductivity)
BR	Bedrock hydraulic conductivity
QTP	Ground-water quality type
QDS	Ground-water total dissolved solids

Table 1.--Basic data on 17 parameters related to the hydrologic setting of 150 lakes in North and South Dakota, Minnesota, Wisconsin, and Iowa.

Lakes in North Dakota

	LAKE		PHYSIOGRAPHY			MORPHOMETRY			RUNOFF		GEOLOGY (Drift)			GEOLOGY (BR)	GW QUALITY		ATMOSPHERIC		
County	Name	Location	Regional slope (ft/100 ft)	Position in region	Local relief (x 10 ft)	Area of lake (mi^2)	Area of drainage basin (mi^2)	L/D ratio	Inlet	Outlet	Type of landform	Mineralogy	Particle size	Bedrock	Type	TDS (mg/l)	Precipitation	Evaporation	P-E Balance
Williams	Twin L. (North)	159.103. 8add	0.568	1	2.5	0.143	0.391	2.73	3	2	3	1	4	2	7	1025	14	36	-22
Williams	2	159.100.20dcc	.388	1	2.0	.125	.260	2.08	3	1	3	1	4	2	8	1400	14	35	-21
Divide	Bright Water	161. 98.31cbc	.306	1	4.8	.075	.346	4.61	1	1	1	1	2	2	8	6520	14	35	-21
Burke	4 (I)	162. 90.22bdd	.365	1	1.8	.067	.376	5.61	1	1	4	1	2	2	4	3750	15	34	-19
"	5 (I)	161. 90. 9ccd	.478	1	1.5	.128	.511	3.99	2	1	3	1	4	2	8	1960	15	34	-19
Mountrail	6	156. 90.20dcb	.126	3	9.8	.229	2.487	10.86	1	1	3	1	4	2	4	825	15	34	-19
McHenry	Nead (I)	155. 79.34bcd	.099	1	5.2	.116	.551	4.75	1	1	4	1	2	2	4	1100	16	32	-16
Ward	McMann	151. 82.36ccc	.326	3	4.0	.052	.347	6.67	1	1	1	1	2	2	1	300	16	33	-17
Rolette	Lyde	163. 73.24abb	.616	3	4.5	.329	1.025	3.12	1	2	1	1	2	2	4	1875	16	30	-14
"	Byrnes	163. 70.21cac	.440	3	4.2	.088	.315	3.58	1	1	1	1	2	2	4	1875	16	30	-14
Towner	11	159. 67.25ada	.095	1	1.0	.042	.087	2.07	1	1	4	3	2	2	1	1375	16	30	-14
"	12	159. 66.33cca	.071	1	1.5	.062	.223	3.60	1	1	2	3	4	2	1	1375	16	30	-14
Cavalier	13 (I)	159. 64.36ccb	.104	2	1.2	.047	.148	3.15	1	1	4	3	2	2	3	1650	17	29	-12
"	14 (I)	161. 62.31abc	.071	3	.5	.082	.292	3.56	1	1	4	3	2	2	8	1300	17	28	-11
"	Billings	159. 61.15aba	.080	3	1.2	.090	.345	3.83	2	2	3	3	4	2	1	375	17	28	-11
Ramsey	16 (I)	155. 60.36dab	.047	3	1.2	.047	.129	2.74	1	1	4	3	2	2	5	2000	18	29	-11
Ramsey	17	154. 61.20cbb	.057	2	2.2	.124	.402	3.24	1	1	4	3	2	2	8	2100	18	29	-11
Nelson	Stump (East)	151. 60.20cad	.203	1	11.0	4.300	9.012	2.10	2	1	5	3	1	2	1	675	18	29	-11
Pierce	19 (I)	154. 72.22bba	.047	3	4.2	.097	.277	2.86	1	1	4	1	2	3	7	600	16	31	-15
Sheridan	20	150. 76.21dda	.355	1	4.0	.043	.272	6.33	1	1	1	1	2	2	1	450	17	32	-15
"	21	145. 78.15adb	.156	3	7.2	.039	.260	6.67	1	1	1	2	2	2	1	950	16	33	-17
Wells	Crystal	147. 73.27ccb	.365	3	5.2	.064	.398	6.22	1	1	1	2	2	3	7	925	17	32	-15
"	23 (I)	146. 69. 4aca	.256	2	1.8	.275	1.906	6.93	1	1	3	2	4	1	4	850	17	32	-15
Kidder	24	143. 73. 7bdd	.232	3	5.8	.048	.194	4.04	1	1	1	2	2	3	3	1150	17	33	-16
Stutsman	25	144. 65.22adb	.501	2	.8	.073	.267	3.66	2	2	4	2	2	1	1	406	18	31	-13
"	26	144. 64. 8abb	.268	1	4.2	.047	.391	8.32	2	2	4	2	2	1	1	1075	18	31	-13
Steele	Lone Tree	144. 57.15cac	.308	3	2.8	.115	.334	2.90	2	1	1	4	2	1	3	2725	19	30	-11
Kidder	Weber	139. 74.25dab	.300	1	3.5	.265	1.471	5.55	1	1	4	2	2	1	3	1725	17	33	-16
Stutsman	School	138. 68.10dcb	.247	2	5.8	.414	1.368	3.30	1	1	1	2	2	1	3	925	18	32	-14
Emmons	Baumgartner	131. 76.16dab	.414	3	5.5	.242	2.234	9.23	2	1	4	2	2	3	1	1000	16	34	-18
Dickey	31	132. 66.33aac	.178	3	3.0	.094	.465	4.95	1	1	1	2	2	1	1	725	19	33	-14
La Moure	32 (I)	136. 64.20cdb	.445	1	1.8	.037	.622	16.81	1	1	3	2	2	1	3	900	19	32	-13
"	33 (I)	135. 63.11cbd	.341	1	2.5	.031	.384	12.39	1	1	3	2	2	1	3	1300	19	32	-13
Emmons	34	129. 75.35abd	.331	2	4.2	.194	.922	4.75	1	1	1	2	2	3	3	2350	16	34	-18
Dickey	35 (I)	130. 62.13bdc	.185	1	3.0	.075	.305	4.07	2	2	3	2	4	1	3	3250	20	32	-12
Sargent	Clouds	130. 54.32adc	.791	1	1.2	.161	.837	5.20	2	2	1	4	2	1	1	675	19	31	-12
Richland	Willard	130. 50.22bbd	.284	1	4.5	.495	1.064	2.15	1	1	5	4	4	1	4	850	20	31	-11

Table 1.--Continued-Lakes in South Dakota

	LAKE		PHYSIOGRAPHY			MORPHOMETRY			RUNOFF		GEOLOGY (Drift)			GEOLOGY (BR)	GW QUALITY		ATMOSPHERIC		
County	Name	Location	Regional slope (ft/100 ft)	Position in region	Local relief (x 10 ft)	Area of lake (mi^2)	Area of drainage basin (mi^2)	L/D ratio	Inlet	Outlet	Type of landform	Mineralogy	Particle size	Bedrock	Type	TDS (mg/l)	Precipitation	Evaporation	P-E Balance
McPherson	38	128. 69.15aad	.166	3	3.8	.492	1.201	2.44	2	1	1	2	2	3	1	625	18	34	-16
Edmunds	39 (I)	124. 66. 1aac	.142	1	.8	.053	.163	3.08	1	1	1	2	2	1	3	950	18	34	-16
Brown	40	128. 61.28ccc	.028	1	1.2	.055	.338	6.15	1	1	5	2	1	1	3	3250	19	32	-13
Potter	41 (I)	120. 76. 4bdc	.821	3	2.8	.034	.267	7.85	2	2	1	2	2	1	3	3200	16	35	-19
Faulk	42 (I)	120. 70. 4acd	.360	2	3.8	.100	1.106	11.06	1	1	1	2	2	1	1	525	17	35	-18
Brown	Salt	121. 65.20cac	.128	1	5.2	.120	.778	6.48	1	1	1	2	2	1	4	1600	18	34	-16
Spink	Alkali	119. 65. 5dac	.118	1	2.0	.162	.543	3.35	1	1	2	2	2	1	3	900	18	34	-16
"	45 (I)	119. 62.29cba	.114	1	.5	.033	1.267	38.89	1	1	5	2	1	1	3	3650	19	34	-15
Day	Sweetwater	120. 59.22bdc	0.492	3	5.8	0.328	1.190	3.63	1	1	3	2	4	1	1	800	19	33	-14
Marshall	47	125. 56.22cac	.521	3	5.8	.105	.716	6.82	1	1	4	2	2	1	1	675	19	32	-13
"	Sarah	126. 53.22bbc	.701	3	6.8	.123	.591	4.80	1	1	1	4	2	1	1	675	20	32	-12
Roberts	Bdesaka	126. 50.12dba	.331	1	9.2	.232	1.427	6.15	2	3	3	4	4	1	3	1000	20	31	-11
"	50	123. 51.20ddb	1.131	3	5.8	.047	.163	3.47	1	1	1	4	2	1	1	575	20	32	-12
Day	Solomon Slough	121. 55.27aca	.100	2	3.5	.143	1.395	9.76	2	2	1	2	2	1	3	1500	20	33	-13
Day and Grant	Lonesome	120. 53.13aab	.156	1	5.2	.333	1.917	5.76	2	2	1	2	2	1	3	1500	20	33	-13
Deuel	Round	117. 50. 4ddb	.715	3	3.2	1.440	2.314	1.61	2	2	3	4	4	1	3	1075	21	33	-12
Grant	54	119. 47.28acb	.369	1	1.5	.049	.151	3.08	1	1	1	4	2	1	3	1500	22	32	-10
Kingsbury	Albert	112. 53. 2dac	.175	1	5.0	5.703	8.602	1.51	2	1	4	2	2	1	3	975	21	34	-13
Deuel	56	113. 48.36bba	.639	3	3.8	.115	.345	3.00	1	1	4	4	2	1	1	350	22	33	-11
"	Fish	113. 47.16bba	.616	3	4.0	1.139	1.896	1.66	3	3	3	4	4	1	3	1475	22	33	-11
Hyde	58 (I)	114. 73.22ddb	.331	1	5.2	.305	1.416	4.64	2	1	3	2	4	1	8	1000	17	36	-19
Hyde	Rice	116. 72.26cda	.312	2	4.8	.313	1.597	5.10	2	2	1	2	2	1	8	1000	17	35	-18
Hand	Spring	109. 67. 5cca	.473	1	8.2	.866	3.386	3.91	2	2	3	2	4	1	3	1025	19	36	-17
Sanborn	61	106. 60. 6bba	.208	1	4.0	.047	.247	5.26	1	1	3	2	4	1	3	1875	20	36	-16
"	62	105. 59.14bac	.194	1	1.2	.040	.339	8.48	1	1	1	2	2	1	8	1775	21	36	-15

Table 1:--Continued-Lakes in Minnesota

	LAKE		PHYSIOGRAPHY			MORPHOMETRY			RUNOFF		GEOLOGY (Drift)			GEOLOGY (BR)	GW QUALITY		ATMOSPHERIC		
County	Name	Location	Regional slope (ft/100 ft)	Position in region	Local relief (x 10 ft)	Area of lake (mi^2)	Area of drainage basin (mi^2)	L/D ratio	Inlet	Outlet	Type of landform	Mineralogy	Particle size	Bedrock	Type	TDS (mg/l)	Precipitation	Evaporation	P-E Balance
St. Louis	Crane	67. 17.13bcd	.237	1	12.8	4.987	12.663	2.54	4	4	7	9	5	4	1	125	26	20	6
"	Lost	62. 16.29cba	.095	2	3.8	1.144	3.578	3.13	1	1	4	9	1	4	1	200	26	22	4
"	Sixmile	62. 14.21bdb	.142	3	6.5	.062	.240	3.87	1	2	7	9	3	4	1	475	27	22	5
"	Bear Island	61. 13.15bbb	.149	2	7.5	3.831	6.709	1.75	4	4	7	9	5	4	1	100	27	22	5
Lake	Turtle	62. 10.13cbd	.256	2	5.2	.521	1.192	2.29	3	1	7	9	5	4	1	175	27	21	6
"	Wilson	60. 6.22baa	1.465	3	13.5	1.007	2.778	2.76	3	3	7	10	3	4	1	300	28	22	6
Cook	Beth	62. 5.15abc	.275	3	14.8	.259	1.002	3.87	3	1	7	9	5	4	1	300	27	21	6
"	Muskeg	64. 3.21cdb	1.326	3	13.5	.051	.472	9.25	3	3	7	9	5	4	1	125	27	20	7
St. Louis	Black	58. 16.22abb	1.207	1	2.0	.073	.270	3.70	1	3	5	6	2	4	1	300	27	23	4
"	St. Marys	57. 17.16bbb	.663	1	2.0	.374	.647	1.73	1	1	5	6	2	3	1	150	27	23	4
"	Section 14	56. 16.14dcd	.170	1	2.0	.207	.536	2.59	1	1	4	6	2	4	1	100	28	23	5
Polk	Berg	149. 41.15bcd	.199	2	1.8	.096	.273	2.84	1	1	1	5	2	1	1	375	22	25	-3
Mahnomen	Flat	146. 39.16cdd	.223	3	4.2	.113	.352	3.12	1	1	1	5	2	1	1	425	23	25	-2
Clearwater	Daniel	146. 36. 4cab	.199	3	3.2	.086	.482	5.60	2	3	3	5	4	4	1	300	23	25	-2
Norman	Home	143. 44.13bdb	.421	2	3.8	.099	.591	5.97	1	1	4	5	2	1	1	450	22	27	-5
Becker	Mill	141. 40.16bcd	.298	3	5.8	.062	.288	4.65	1	1	1	5	2	1	1	400	23	27	-4
Hubbard	Moran	139. 35.14dbc	.316	1	3.8	.158	1.558	9.86	1	1	3	5	4	1	1	325	24	26	-2
Cass	Wabegon	142. 30.21ccd	.352	1	4.2	.066	.547	8.29	1	1	1	5	2	1	1	350	25	25	0
"	Little Sand	142. 26.28adc	.101	3	2.8	.627	1.458	2.33	1	1	4	5	2	3	1	320	25	25	0
Itasca	Portage	147. 26.10bab	.137	2	2.2	.098	.250	2.55	1	3	4	5	2	4	1	250	24	23	1
"	Maple	60. 27.25ccd	.099	3	6.8	.379	.770	2.03	1	3	1	5	2	4	1	250	24	23	1
"	Highland	59. 25.28acb	.071	3	4.2	.149	.411	2.76	1	1	2	5	4	4	1	150	25	23	2
"	Bass	56. 26.21ddb	.278	1	7.0	4.222	8.649	2.05	3	3	1	5	2	4	1	275	25	24	1
"	Munzer	54. 25.20dac	.450	1	6.5	.158	1.121	7.09	1	3	1	5	2	3	1	350	26	24	2
Aitkin	Glacier	50. 23.26acc	.076	1	4.5	.197	.790	4.01	1	1	1	8	2	3	1	270	27	25	2
"	Rat	49. 24.23acb	.090	1	4.0	.663	2.205	3.33	2	3	1	8	2	3	1	260	27	25	2
Otter Tail	Brekke	133. 42. 9dda	.170	3	4.0	.131	.418	3.19	1	1	1	5	2	4	1	350	22	29	-7
"	Nitche	136. 38. 9dda	.103	1	5.0	.116	.937	8.08	1	3	3	5	4	1	1	400	24	27	-3

Table 1.--Continued - Lakes in Minnesota

	LAKE		PHYSIOGRAPHY			MORPHOMETRY			RUNOFF		GEOLOGY (Drift)			GEOLOGY (BR)	GW QUALITY		ATMOSPHERIC		
County	Name	Location	Regional slope (ft/100 ft)	Position in region	Local relief (x 10 ft)	Area of lake (mi^2)	Area of drainage basin (mi^2)	L/D ratio	Inlet	Outlet	Type of landform	Mineralogy	Particle size	Bedrock	Type	TDS (mg/l)	Precipitation	Evaporation	P-E Balance
Otter Tail	Deusch	131. 39.14cdc	0.275	3	6.2	0.132	0.397	3.01	1	1	1	5	2	4	1	450	24	29	-5
Douglas	Erwin	129. 40.21add	.242	3	6.2	.162	.632	3.90	4	4	1	5	2	4	1	400	23	29	-6
"	Ida	129. 38.12cbc	.090	3	4.8	6.859	8.590	1.25	3	3	2	5	4	4	1	400	24	29	-5
Morrison	Shamineau	132. 31.19aaa	.189	2	9.8	2.163	3.130	1.45	1	1	3	10	4	4	1	275	26	28	-2
Crow Wing	Black Bear	47. 29.31cbc	.095	1	2.8	.334	.671	2.01	3	3	3	10	3	4	1	200	26	26	0
Stevens	96	124. 44.15acc	.052	3	.5	.054	.521	9.65	1	1	4	4	2	4	3	1400	22	31	-9
Pope	Goose	123. 37.21cdc	.246	3	4.8	.443	.926	2.09	1	1	1	5	2	1	1	400	24	30	-6
Stearns	Raymond	125. 35.13dca	.123	3	1.2	.095	.230	2.42	2	1	4	5	2	4	1	400	25	30	-5
"	Ochotto	125. 30.22cbc	.161	2	2.2	.060	.139	2.32	1	1	4	10	4	4	1	400	26	29	-3
Big Stone	Munnwyler	121. 46.12bab	.264	3	2.2	.047	.225	4.79	1	1	4	4	2	1	3	1120	22	32	-10
Lacqui Parle	101	119. 43.33bda	.246	1	2.8	.080	.317	3.96	1	1	4	4	2	1	3	2150	23	32	-9
Kandiyohi	102	118. 34.16ddd	.099	3	1.5	.131	.370	2.82	3	3	4	4	2	1	1	475	25	31	-6
Meeker	Hart	119. 29.21cbb	.142	2	5.2	.088	.345	3.92	1	1	1	7	2	1	1	425	26	31	-5
Renville	Preston	115. 31.11cbb	.043	2	2.0	1.018	1.716	1.69	2	2	4	4	2	4	1	625	26	31	-5
McLea'	Swan	117. 28.29cbc	.076	2	3.0	.501	.973	1.94	1	2	4	7	2	3	1	550	26	31	-5
Carver	Eagle	116. 26.34dac	.080	2	3.0	.268	1.027	3.83	3	1	4	7	2	3	1	475	27	31	-4
Lacqui Parle	Balland Sl.E.	117. 46.36dcd	.426	1	1.8	.072	.183	2.54	1	1	3	4	4	1	3	1650	23	33	-10
Redwood	Westline	111. 39.13dab	.142	1	1.0	.166	.543	3.27	2	1	4	4	2	1	3	750	24	33	-9
Murray	Round	108. 40.17ddd	.379	2	4.2	.252	.548	2.17	2	1	4	4	2	1	3	1600	24	34	-10
Renville	110	113. 33.31aba	.265	2	1.5	.097	.696	7.18	3	3	4	4	2	1	1	475	25	32	-7
Martin	Clam	103. 32.15bbb	.199	2	2.2	.108	.349	3.23	1	1	1	4	2	1	3	875	28	34	-6
Watonwan	Lewisville MA	106. 30.20cdb	.095	2	1.8	.046	.151	3.28	2	2	4	4	2	1	1	700	28	33	-5
"	Fedje	107. 30.13bbd	.104	1	1.8	.288	.531	1.84	1	1	4	4	2	1	1	400	28	33	-5
Freeborn	Bear	101. 22.17cda	.099	3	4.2	2.058	4.398	2.14	3	3	3	4	4	4	1	300	29	34	-5
Le Sueur	Mary	111. 24. 3acd	.175	2	2.8	.075	.198	2.64	2	2	4	4	2	3	1	300	28	32	-4
Dakota	Fish	27. 23.15cca	.852	3	6.8	.035	.214	6.11	1	1	1	10	3	3	1	250	28	31	-3
Ramsey	Gilfillan	30. 22.17cac	.132	3	3.2	.156	.319	2.04	1	3	4	7	2	3	1	250	28	30	-2
Sherburne	Hunter	34. 26.13dca	.294	3	4.0	.172	.854	4.97	1	1	1	7	2	3	1	250	27	29	-2
Kanabec	Kent	39. 24.21aab	.218	2	3.8	.052	.200	3.85	1	1	4	10	3	3	1	250	28	28	0
Pine	Cedar	40. 20.33bbb	.294	1	3.0	.112	.749	6.69	1	2	3	10	4	4	1	150	28	27	1
"	Upper Pine	43. 21.20ddd	.289	2	4.0	.351	1.022	2.91	3	4	3	10	3	3	1	200	28	27	1

Table 1.--Continued-Lakes in Wisconsin and Iowa

County	LAKE: Name	LAKE: Location	PHYSIOGRAPHY: Regional slope (ft/100 ft)	PHYSIOGRAPHY: Position in region	PHYSIOGRAPHY: Local relief (x 10 ft)	MORPHOMETRY: Area of lake (mi^2)	MORPHOMETRY: Area of drainage basin (mi^2)	MORPHOMETRY: L/D ratio	RUNOFF: Inlet	RUNOFF: Outlet	GEOLOGY (Drift): Type of landform	GEOLOGY (Drift): Mineralogy	GEOLOGY (Drift): Particle size	GEOLOGY (BR): Bedrock	GW QUALITY: Type	GW QUALITY: TDS (mg/l)	ATMOSPHERIC: Precipitation	ATMOSPHERIC: Evaporation	ATMOSPHERIC: P-E Balance
St. Croix	Squaw	31. 18. 8ddc	.355	2	4.0	.224	.559	2.50	1	1	1	10	4	3	1	300	28	30	-2
Barron	Upper Turtle	34. 14.21aaa	.365	2	7.0	.671	2.496	3.72	1	3	4	10	4	3	1	181	28	28	0
Washburn	Hoinville	41. 13.21bac	.104	1	6.0	.092	.336	3.65	1	1	2	10	4	4	1	106	29	27	2
"	Bean	40. 10.15dad	.289	1	3.5	.156	.647	4.15	3	3	1	10	3	3	1	108	30	27	3
Sawyer	126	41. 8 .10cab	0.223	3	5.2	0.397	0.789	1.99	1	3	1	10	3	4	1	127	31	26	5
Bayfield	Upper Eau Claire	44. 9 .10acc	.289	2	5.2	1.604	2.720	1.70	3	3	2	10	4	3	1	51	31	26	5
Douglas	Nebagamon	47.11 .35ddd	.647	3	7.2	1.523	3.868	2.54	3	3	1	10	3	3	1	130	30	25	5
Bayfield	Knotting	44. 6 .21ddb	.445	3	2.0	.117	.285	2.44	1	1	1	10	3	4	1	134	31	26	5
Price	Solberg	38. 1E.21ccc	.152	2	3.2	1.290	3.478	2.70	3	3	4	10	3	4	1	62	34	26	8
Oneida	Swamsauger	38. 4E.14bca	.118	3	2.8	.226	.419	1.85	3	3	2	10	4	4	1	80	32	26	6
Iron	Sandy Beach	42. 4E.22add	.133	3	1.5	.161	1.236	7.68	3	1	2	10	4	4	1	118	35	26	9
Vilas	Palmer	43. 8E.21abb	.118	3	4.8	1.022	1.820	1.78	3	4	1	10	4	4	1	111	34	25	9
"	Lost	40. 8E.10bdb	.260	3	6.0	.829	1.996	2.41	3	3	2	10	4	4	1	62	32	26	6
Forest	Lucerne	35.13E.10dbd	.294	3	8.2	1.629	3.466	2.13	2	3	4	10	3	4	1	208	30	26	4
Menominee	Blacksmith	28.16E.16dcc	.237	2	2.5	.151	.579	3.83	3	3	2	11	4	4	1	209	29	27	2
Waupaca	School Section	24.13E.16dcb	.185	2	3.8	.062	.386	6.23	2	3	1	11	2	4	1	860	30	28	2
Portage	Pickerel	21.10E. 5ccc	.402	3	7.2	.060	.513	8.55	1	1	2	11	4	3	1	148	31	28	3
Adams	Patrick	16. 7E. 9ada	.301	3	7.0	.058	.968	16.69	1	1	1	11	4	3	1	174	30	29	1
Waushara	Witters	18.10E.15bcd	.379	1	3.5	.067	.220	3.28	1	1	2	11	4	3	1	194	30	29	1
	Spring	18.11E.26baa	.298	1	3.5	.079	.361	4.57	3	3	5	11	4	3	1	194	30	28	2
Jefferson	Rock	7.13E.14bbd	.161	1	6.8	2.140	4.117	1.92	3	3	4	11	3	4	2	352	31	30	1
Washington	Bark	9.19E.26aba	.468	3	5.5	.097	.438	4.52	3	3	2	11	3	4	1	518	29	29	0
Waukesha	Pine	8.18E.32aaa	.197	3	7.5	1.116	2.640	2.37	1	1	2	11	4	4	1	478	30	29	1
Walworth	Army	4.18E.16bdb	.263	3	3.8	.123	.438	3.56	1	1	2	11	3	4	1	357	32	29	3
Florence	Patten	39.17E.18add	.275	3	5.5	.391	1.199	3.07	3	3	4	10	4	4	1	170	31	26	5
Emmet	Cheever	99.34 .20dca	.246	2	2.8	.174	.561	3.22	1	1	1	4	2	1	1	660	28	35	-7
Buena Vista	Pickerel	93.35 . labc	.095	2	2.5	.254	.488	1.92	1	2	1	4	2	1	1	775	29	36	-7
Webster	149	88.28 . 2ddb	.272	3	1.2	.014	.062	4.43	1	1	4	4	2	1	1	1550	33	36	-3
Greene	Goose	84.31 . 1ddd	.147	2	2.2	.640	1.301	2.03	1	1	1	4	2	1	1	825	30	37	-7

In order to analyze the above parameters statistically, it must be possible to obtain numerical values for each parameter for each lake. While this may appear to be obvious, measurement and statistical analysis of numbers has long been a problem in science (Stevens, 1968). For some of the parameters used in this study, the data are measurements of an object, feature, or quantity, such as P-E, D/L, RS, LR, and QDS. The values for these parameters are part of an interval or ratio scale, so they can be justly analyzed by most statistical methods (Stevens, 1946).

For another group of parameters, field measurements are not available. Therefore, numbers that reflect the relative order of magnitude of the feature were assigned from map analysis; IN, OUT, RP, DT, and BR. While not as desirable as actual field measurements, and some might consider the values to be from an ordinal scale, the values could also be from an interval scale, where the values do not have much accuracy. While the scaling of these parameters may not be the best possible, use of them is justified for the preliminary nature of this study.

The scaling for a third group of parameters--LF, DM, and QTP--presents somewhat of a problem. At first glance it might appear that the scaling for these parameters is nominal (arbitrary), thus justifying the use of only the simplest statistical description, such as mode. As will be pointed out in the discussion of each, however, the scaling is ordinal. Although use of ordinal-scaled parameters is not justified in advanced statistical methods according to Stevens (1946), there is some controversy over this matter (Stevens, 1968). The use of ordinal scales in factor analysis is common. Rummel (1970) indicates that use of such scales in factor analysis is justified, but interpretation of the results should be made with caution.

For the purpose of this study, where all possible variables should be examined in order to identify independent parameters upon which to base a classification, all 13 variables were included in the analysis. Further discussion of the three "questionable" variables, including an analysis in which they are excluded, is found in a later section of this report.

All data used in this study were taken from maps, reports, and office files. Field data were not collected in this phase because a classificatory system would seem to be most useful if lakes could be typified in a first approximation using information readily available to an office staff. The results and problems brought to light by this initial effort will lead to modification of the parameters selected, as evidenced by the second major part of the report, and field studies, which were started in the summer of 1975.

Atmospheric water parameters

Parameters related to atmospheric water interchange with a lake are precipitation and evaporation. The P-E balance map (fig. 8) was constructed by overlaying maps of P and E and using the intersection of isolines as control points. Precipitation (P), evaporation (E), and the balance of the two (P-E) at each lake are listed separately in table 1.

Surface water parameters

Parameters representing surface-water exchange with a lake include surface-water inlet, outlet, lake area (L), drainage-basin area (D), and the ratio D/L. The first two, inlet and outlet, are a measure of the streamflow exchange with a lake. Because stream discharge data were not available, numerical ranks, which give a relative measure of streamflow magnitude, were assigned from topographic map symbols as follows:

1 = no streamflow

2 = intermittent streamflow

3 = single solid line -- perennial streamflow

4 = double solid line -- large perennial streamflow

This scale is used for both the inlet and the outlet parameters.

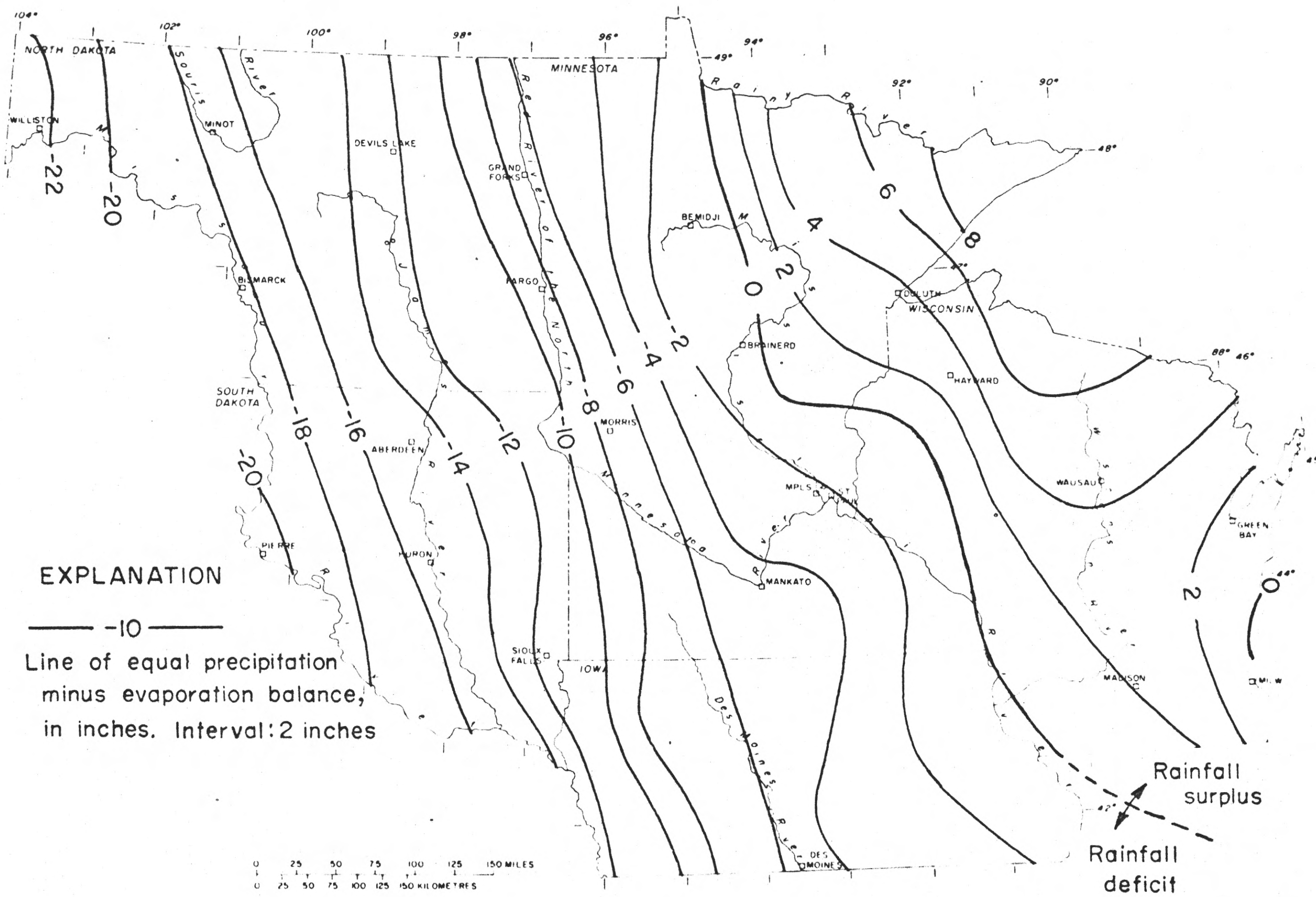


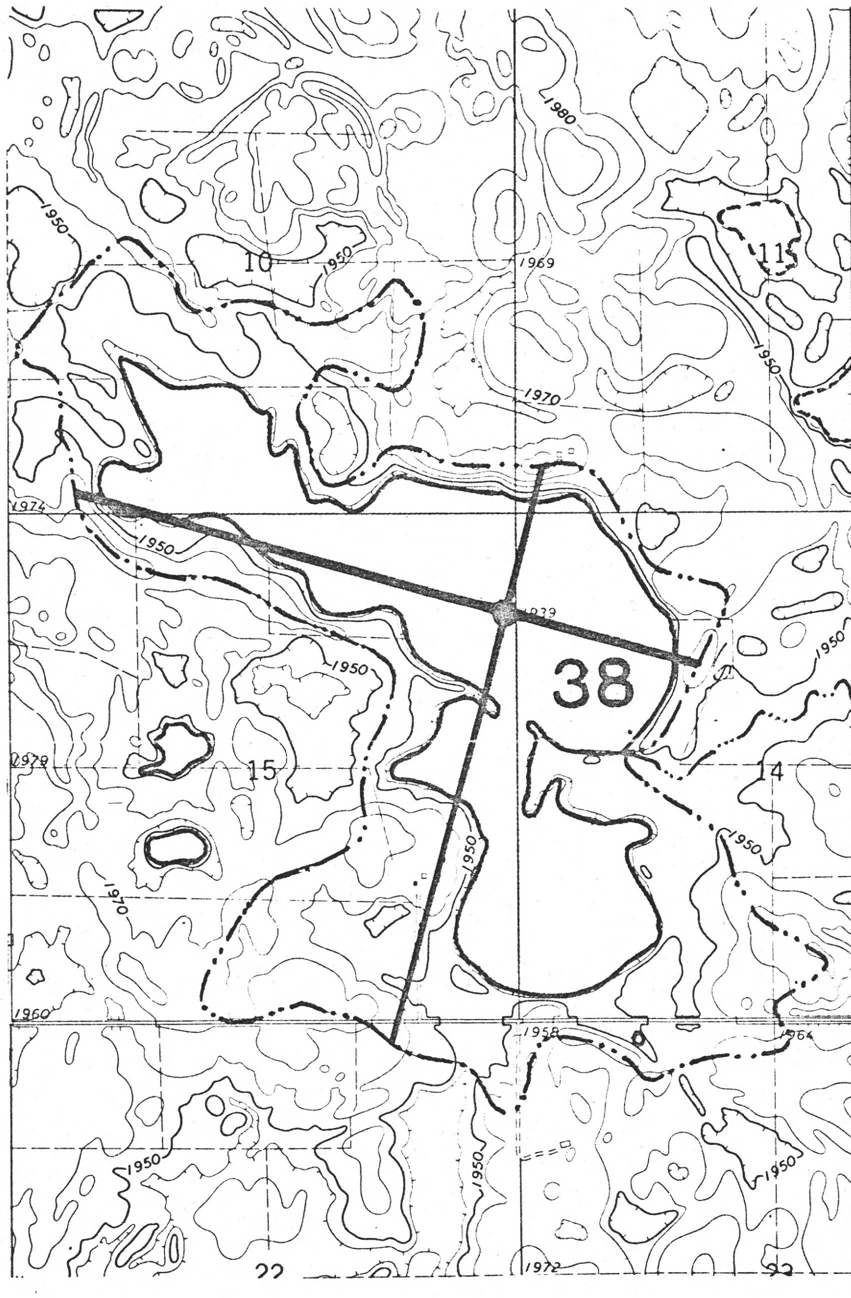
Figure 8.--Balance of precipitation minus evaporation.

The ratio of a lake's drainage basin area to surface area is used in this report as an approximation of overland-runoff potential. The drainage area of each lake, excluding the drainage areas of inlet streams, was outlined on a topographic map. (An example is given in fig. 9). The drainage area and lake area were then planimetered.

Geology and ground-water flow parameters

Parameters related to the ground-water interchange with a lake are those that control ground-water flow systems, including regional slope, position of the lake in the flow system, local relief, and the hydraulic conductivity of the rocks, represented by texture or particle size (Toth, 1963; Meyboom, 1963; Freeze, 1969b). Parameters related to ground-water chemistry, and presumably lake chemistry, are mineralogy of the drift, water-quality type, and dissolved solids content of ground water.

Regional slope was determined by calculating the maximum slope along a 20-mile (32-km) line that extended through the lake--10 miles (16 km) on either side of it. If the lake was near a watershed or river, the line of slope was extended 20 miles (32 km) from that boundary. If the distance from the major drainage divide to the major river was less than 20 miles (32 km), that distance was used to calculate



Base from U.S. Geological Survey Long Lake NW, So. Dak. 1:24,000

Figure 9.--Lake 38 and its drainage divide, and construction of lines for determining local relief.

the slope. To measure regional slope, a topographic map was compiled for the north-central states that has a scale of 1:1,000,000 and contour interval of 100 feet (30 m). This scale and contour interval were selected because they provide a good balance of acceptable detail for a regional parameter. A larger-scale map requires more generalized topography, and a smaller-scale map is too detailed and too big. Regional slope is recorded in feet per 100 feet.

Relative altitude (position) of the lake in the regional setting was selected as a parameter because it was assumed that the interaction of lakes and ground water may be partly dependent on it. For example, lakes that are high topographically presumably are more likely to lose water to the ground-water system than lakes that are low. Lakes in the lower part of a drainage basin are more likely to receive ground water from large ground-water flow systems. Position was determined from the regional topographic map mentioned above. A line was drawn along the maximum slope from the major watershed through the lake to the major drain (river). The difference in altitude between the highest and lowest points, divided by 3, separated the region into high, intermediate and low areas--a fine enough breakdown for this first approximation. A numerical rank was assigned to the lake's position within the region as follows:

3 = high = upper one-third of the major drainage basin

2 = intermediate = middle one-third of the major drainage basin

1 = low = lower one-third of the major drainage basin

Local relief of the water table in the immediate drainage basin of a lake provides a measure of the type and magnitude of local ground-water flow in the vicinity of a lake. In glacial terrain, relief of the water table generally is a subdued image of land-surface relief. Although this assumption is not always justified, for practical purposes it is an assumption that must often be made by hydrologists, and it has been defended by Freeze (1969b). Therefore, since water-table altitude data are not available, land-surface relief is used as a substitute that gives comparable relative results. To measure local relief the following method was used. (See fig. 9 as an example.) Using the drainage-basin outline, a line was drawn from the highest point on the divide through the center of the lake to the opposite divide. A second line, perpendicular to the first, was drawn from divide to divide. The total number of 10-foot contours intersected by these lines, divided by 4, the four lines transecting the drainage basin, is the measure of local relief used for this study.

Drift texture (particle size) is directly related to hydraulic conductivity. Because quantitative data on particle size are not available uniformly over the entire area, numerical ranks, reflecting increasing hydraulic conductivity, were assigned to major textural types as follows:

1 = clay and silt

2 = clayey and silty till

3 = sandy till

4 = sand and gravel

5 = fractures

In areas of thin drift, the hydraulic conductivity of bedrock must be considered because it can be a control on ground-water flow systems. As in drift, hydraulic conductivity and particle-size data are generally not available, so numerical ranks were assigned as follows:

1 = low hydraulic conductivity

2 = moderate hydraulic conductivity

3 = high hydraulic conductivity

4 = pipeline (fracture) hydraulic conductivity

Bedrock units that have low hydraulic conductivity are Pierre Shale in southern North Dakota and South Dakota; Niobrara, Carlisle, Greenhorn, Belle Fourche, Mowry, Newcastle, and Skull Creek units in North and South Dakota, and the Cretaceous units in western Minnesota and Iowa. Bedrock units that have moderate hydraulic conductivity are Pierre Shale in northern North Dakota and all Tertiary rocks in North and South Dakota. Bedrock units that have high hydraulic conductivity are Hell Creek and Fox Hills units in North and South Dakota, Paleozoic sandstones in Minnesota and Wisconsin, and Cretaceous rocks in the Iron Range area of Minnesota. Fracture-type hydraulic conductivity is characteristic of crystalline and carbonate rocks.

Landform was selected as a parameter because it is a readily available and therefore commonly used parameter in informal lake classifications. Because of its widespread use, it was included as a check of its quantitative utility. Numerical ranks were assigned to landform in a sequence that reflects the proximity of the landform to the ice lobe from which it originated. The sequence also reflects roughness of the terrain to a certain extent, although ice-contact deposits are often more hummocky than ground moraine.

1 = end and collapse moraine

2 = ground moraine

3 = ice-contact deposits

4 = outwash plain

5 = glacial-lake plain

6 = alluvial valley

7 = bedrock.

Ground-water quality parameters

The chemistry and mineralogy of drift, an important control on ground-water and lake-water chemistry, is only very generally known. Because of the lack of detailed information and because the gross mineralogy of drift sheets differ, numerical ranks were assigned to the various drift lobes (see fig. 6).

The sequence reflects decreasing degree of sulfate-carbonate content of the drift. Although there might be variations in the sequence as established, this characteristic of the drift decreases from west to east across the study area.

1 = Souris

2 = James

3 = James and Des Moines

4 = Des Moines

5 = Des Moines and Wadena

6 = Des Moines and Rainy

7 = Des Moines and Lake Superior

8 = Des Moines, Rainy, and Lake Superior

9 = Rainy

10 = Lake Superior

11 = Green Bay and Lake Michigan

Because lakes receive ground-water discharge, the chemical quality of ground water influences lake-water chemistry. The two parameters that best describe water quality in a general way are the major-ion type and total dissolved solids. The water analyses used are from wells tapping the upper part of the ground-water system near the study lakes. In most cases the numerical values recorded represent the most common type and the average dissolved solids of the samples collected within 6 miles of the lake. They are considered to be representative of the ground-water system near the lake.

The waters were typed according to their dominant cations and anions. The numerical ranks were assigned in an order that reflects the water types that might be expected as ground water moves from recharge to discharge areas. It follows to a certain degree the sequence of water types discussed by Schoeller (1959) and Chebotarev (1955).

1 = $CaMgHCO_3$

2 = $MgCaHCO_3$

3 = $CaMgSO_4$

4 = $MgCaSO_4$

5 = CaMgCl

6 = MgCaCl

7 = $NaHCO_3$

$8 = NaSO_4$

$9 = NaCl$

The data on ground-water quality were taken from Winter (1974) for Minnesota, Holt and Skinner (1973) for Wisconsin, county ground-water studies such as Huxel and Petri (1965), Hutchinson (1973), and Armstrong (1965) for parts of North Dakota, county and city studies such as Christenson (1962) for parts of South Dakota, and unpublished data from U.S. Geological Survey files for parts of North and South Dakota and Iowa.

Miscellaneous parameters

Parameters other than those discussed above might ultimately prove to be indicator parameters in a hydrologic classification of lakes. Lake depth is probably a significant parameter in the interaction of lakes and ground water, but the lack of data for this initial appraisal precluded its use. This is a first approximation using parameters that seem hydrologically reasonable and for which data could be obtained from published and unpublished sources. Field collection of data, such as lake levels and lake depth, and modeling of hydrologic phenomena, such as lake-ground water interaction, could modify the classificatory system developed here. The effect of digital modeling of the interaction of lakes and ground water on this initial classification is the subject of the next major section of this report.

Using these parameters as a basis for classifying the hydrologic setting of lakes, the problem now is to identify and evaluate those parameters that will lead to development of the best and simplest classification. The statistical analysis discussed in the remainder of this first major part of this report, and the evaluation of the parameters that control the interaction of lakes and ground water, the topic of the second major part of this report, are a first attempt to solve this problem.

Method of Analysis

Multivariate statistical methods

The strength of multivariate statistical methods is that many variables and samples can be analyzed simultaneously. Several statistical procedures are available to analyze large data sets; the most commonly used are cluster analysis, principal component analysis, and factor analysis.

Cluster analysis is an objective method of correlation analysis and has been used extensively in many disciplines (Sokal, 1974; Sneath and Sokal, 1973; Lance and Williams, 1967; Jardine and Sibson, 1971; and Parks, 1966). Using a symmetrical similarity matrix (e.g. correlation matrix), the analysis consists of comparing pair-by-pair relationships of variables or samples. The results are generally displayed in the form of a hierarchical tree-type diagram. The clusters can be formed from a number of relationships such as nearest neighbor, farthest neighbor, etc. (Wishart, 1969). Tarapchak (1973) used cluster analysis to classify Minnesota's lakes according to major cations and anions, trophic state indices, and biological characteristics.

Q-mode factor analysis, a technique similar to cluster analysis in that interest is on the samples rather than the variables, has been used for classificatory purposes in geologic studies (Imbrie, 1963; Imbrie and Purdy, 1962).

Principal component analysis is a method in which a correlation or covariance matrix is broken down into a set of orthogonal components equal in number to the number of variates. These components correspond to the eigenvalues and eigenvectors of the matrix. The eigenvalues are extracted in descending order of magnitude which is useful if only a few components are to be used to summarize the data. Although only a few components may be of interest to a study, all are needed to reproduce the correlations between variates exactly (Lawley and Maxwell, 1963).

Factor analysis is a useful multivariate statistical method because it objectively describes the variation in a mass of data with a minimum number of hypothetical factors. Factor analysis has been used to develop models that explain underlying cause-and-effect relationships. Another common usage is for developing empirical typologies that group interdependent variables into descriptive categories.

As stated by Rummel (1970), factor analysis focuses on related variation, that is, those phenomena that vary uniformly with each other and are thus linked into a pattern. In a data matrix, variables are column vectors, and cases or individuals are row vectors. Factor analysis determines the minimum number of independent coordinate axes (dimensions) necessary to define the configuration of vectors in space. In other words, it attempts to determine whether the same amount of variation in the data can be represented equally well by dimensions smaller in number than the number of columns or rows, whichever is less, in the data matrix.

For the problem of developing a hydrologic classification of lakes, two courses of action are available: (1) a classificatory system can be developed that is based on all the parameters discussed in the previous section of this report, which would undoubtedly result in a complex, inefficient, cumbersome system, or (2) a multivariate statistical technique can be used to identify independent, key indicator variables, thus arriving at a simpler classificatory system.

For this study, principal component analysis is used as an aid to arrive at a classification of the hydrologic setting of lakes. Cluster analysis and Q-mode analysis were not used because in these two methods interest is principally on classifying the samples themselves, whereas in this study interest is on analyzing variables for a more general classification. Factor analysis was not used because only a certain amount of the variance, the common variance, is considered in the analysis, and there has been considerable controversy over how to estimate this variance (Harman, 1967; Rummel, 1970; Matalas and Reiher, 1967). In the initial stages of this study, Q-mode and factor analysis were tried, but the results were less promising than in the principal component analysis.

Principal component analysis

Theoretical background

Principal component analysis is basically a study of variance. Because it is treated in a number of texts (e.g., Harman, 1967; Rummel, 1970) the following discussion is only a brief summary of the essentials needed to understand the principles of the method as applied to this study.

Each element in a data matrix has the general form x_{ij}, which identifies it as being in the ith row and the jth column. In matrix notation, the data matrix is written X_{nxm}, where n is the number of samples and m is the number of variables.

For many data matrices, the range of values between variables differs greatly. It is usually desirable to give each variable equal weight, so standard scores are calculated by

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, \tag{1}$$

where: s_j = the standard deviation of variable j.

$\bar{x}_j$ = the mean of variable j.

This results in the variable having a mean of 0 and standard deviation of 1. In matrix notation, Z_{nxm} is the matrix of standardized values.

Principal component analysis is usually performed on Gramian matrices; that is, those that are symmetrical and positive semi-definite. The matrix of correlation coefficients between all pairs of variables, designated R_{mxm}, is generally used. The product moment correlation coefficient between two variables, j and k, is

$$r_{jk} = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\left[\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2\right]\left[\sum_{i=1}^{n} (x_{ik} - \bar{x}_k)^2\right]}} \tag{2}$$

and for standard variables is

$$r_{jk} = \frac{\sum_{i=1}^{n} z_{ij}\, z_{ik}}{n} \qquad (3)$$

where: n = the number of samples.

Using standardized variables, the model for principal component analysis is:

$$z_j = a_{j1}f_1 + a_{j2}f_2 + a_{j3}f_3 + \cdots + a_{jq}f_q \qquad (4)$$

$(j = 1, 2, \cdots, m)$

more generally, $z_{ji} = \sum_{k=1}^{q} a_{jk}f_{ki}$ (5)

where: m = observed variables

q = new uncorrelated components, equal in number to m.

a = factor loadings

f = factor scores

In matrix notation:

$$Z = F\,A' \qquad (6)$$

where: A = matrix of factor loadings (a's) for each variable (row) on each new component (column)

F = matrix of factor scores (f's) for each case (row) on each new component (column)

This model simply states that each of the m observed variables is described linearly in terms of q new uncorrelated components $f_1, f_2, \cdots, f_q$.

Because the total variance is considered in principal component analysis, the dimensionality of the factored R matrix will usually be the same as the original R matrix. However, it is commonly found in interpreting the analysis that the first few principal components account for most of the variance in R, and the remainder of the components are attributed to minor factors and measurement error.

Principal components are simply the characteristic vectors, eigenvectors, of the matrix, R. Given R, the problem is to find a vector, E, such that

$$E\lambda = RE \tag{7}$$

or

$$RE - E\lambda = 0 \tag{8}$$

where: λ = eigenvalue

E = eigenvector, $\neq 0$

The eigenvalue is essentially a proportionality constant relating the eigenvector, E, to matrix R. Equation 8 can be rewritten as

$$(R - \lambda I)E = 0 \tag{9}$$

where: I = identity matrix

0 = vector of 0's

Equation 9 can be solved only if $(R - \lambda I)$ is singular; that is, only if the determinant of the matrix $(R - \lambda I) = 0$. The equation

$$\left| R - \lambda I \right| = 0 \tag{10}$$

is called the characteristic equation. The eigenvalues are the roots of this equation.

A number of methods of finding the eigenvalues, termed diagonalizing, of a real symmetric matrix have been developed. Methods commonly used are some modification of the Jacobi method (e.g. Greenstadt, 1967). This is an iterative technique whereby the off-diagonal elements of R are annihilated by successive orthogonal transformations. A diagonalization technique such as this has the effect of concentrating the variance within R into the principal diagonal, the eigenvalues.

Thus, the first eigenvalue accounts for the largest amount of variance, the second eigenvalue accounts for the largest amount of the remaining variance, and so forth until all the variance is accounted for.

Eigenvectors can be found by substitution of the eigenvalues into equation 9 and solving for the E's. Using the digital computer programs in the USGS STATPAC system, however, the eigenstructures are found simultaneously. The principal axes thus obtained are orthonormal eigenvectors of the matrix for which the similarity transformation is its eigenvalue matrix.

To relate the above discussion to factor analysis, the central equality of factor analysis is written as

$$R = U \Lambda U' \qquad (11)$$

where: R = matrix of correlation coefficients

U = matrix of eigenvectors

Λ = diagonal matrix of eigenvalues

Having this equality as a basis, the following relationships hold for relating the eigenstructures to the basic principal component model, $Z = FA'$:

$$Z'Z = R = AF'FA' = AA' \qquad (12)$$

$$AA' = R = U \Lambda U' \qquad (13)$$

$$A'A = \Lambda \qquad (14)$$

$$A = U \Lambda \qquad (15)$$

where: Z = matrix of original standardized variables (for this study)

R = matrix of correlation coefficients

A = matrix of factor loadings

Λ = diagonal matrix of eigenvalues

U = matrix of eigenvectors

The basic goal of principal component analysis is to find theoretical axes that account for the maximum of variance in a data set. To assure that the axes account for unrelated variation it is essential that they be orthogonal. In the derivation of factor loadings through the diagonalization techniques described, the eigenvectors are orthogonal. It has been found in many studies, however, that the components are easier to interpret if the axes are rotated. The commonly used technique to accomplish this is the VARIMAX procedure described by Kaiser (1959). In matrix notation the procedure is simply

$$B = AT \tag{16}$$

where: B = final factor matrix

A = initial factor matrix

T = orthogonal transformation matrix

This type of transformation assures that the orthogonality of the original factor matrix is preserved, and that the best possible fit to the rotated axes is attained.

Oblique rotation of factor axes has been used successfully by a number of workers (e.g. Imbrie and Van Andel, 1964), but it should be remembered in interpreting the results that independence of the factor axes is not preserved. Oblique rotation is useful if factor analysis is being used to investigate cause and effect relationships within the data.

Evaluation of parameters using principal component analysis

The power of component analysis in reducing the complexity of large data sets makes it particularly attractive for developing the simplest possible classificatory systems. It is especially helpful in a study such as the present one, in which data on 13 variables for 150 lakes in the north-central United States are analyzed, because many different types of data can be analyzed simultaneously. Various types of component analysis have been used to classify terrain (e.g. Cadigan and others, 1972), climate (e.g. Steiner, 1965), and environments (e.g. McCammon, 1968).

Because the model for principal component analysis is linear and many of the variables have highly skewed distributions, some of the data were transformed (table 2) so all the variables would have a linear, normal distribution. This was done in spite of the fact that transformations are not absolutely necessary unless statistical tests are used in the analysis. A principal component analysis was made

Table 2.--Transformations applied to the basic data of 13 parameters prior to principal component analysis.

Parameter	Transformation
Regional slope	log
Regional position	none
Local relief	log
L/D ratio	exponent
Inlet	log
Outlet	log
Landform	log
Drift mineralogy	log
Drift texture	log
Bedrock	none
GWQ type	add 1, log, log
GWQ total dissolved solids	log
Precipitation-evaporation balance	none

of the untransformed data set for comparison purposes. The results of the analyses of both the transformed and untransformed data were nearly identical. The classification discussed in the remainder of this report is based on the transformed data.

The first step in a principal component analysis is to derive the similarity matrix to be analyzed--in this study the matrix of correlation coefficients (table 3). This matrix often contains a great deal of information, and in this study suggests relationships that are similar to, but not as clearly emphasized as those resulting from the principal component analysis. Generally, the correlation coefficients listed in table 3 are low. The parameter that correlates highest with regional slope (RS) is local relief (LR), although the correlation coefficient is only 0.32212. Regional position (RP) the ratio of drainage basin area to lake area (D/L), landform (LF), and drift texture (DT) are not closely related to any other parameters. The correlation between inlet (IN) and outlet (OUT) is relatively high. The remainder of the parameters, drift mineralogy (DM), bedrock (BR), ground-water quality parameters (QDS and QTP) and the precipitation-evaporation balance (P-E) all correlate relatively high with one another. With this brief examination of the correlation matrix as a background, it is of interest now to proceed with the principal component analysis to see how some of the above

Table 3.--Matrix of correlation coefficients.

Parameter		REGSLOPE	REGPOSIT	LOCALREL	LD RATIO	INLET	OUTLET
REGSLOPE	(RS)	1.00000	0.05306	0.32212	-0.07483	0.08109	0.05659
REGPOSIT	(RP)	0.05306	1.00000	0.20063	-0.10029	0.03773	0.04889
LOCALREL	(LR)	0.32212	0.20063	1.00000	-0.23798	0.17961	0.23524
LD RATIO	(D/L)	-0.07483	-0.10029	-0.23798	1.00000	-0.06377	-0.06244
INLET	(IN)	0.08109	0.03773	0.17961	-0.06377	1.00000	0.66113
OUTLET	(OUT)	0.05659	0.04889	0.23524	-0.06244	0.66113	1.00000
LANDFORM	(LF)	-0.12570	-0.14694	-0.11866	0.09890	0.22204	0.10961
DFT MNRL	(DM)	0.00153	0.18720	0.24131	-0.08351	0.25180	0.38491
DFT TEXR	(DT)	0.18877	0.06941	0.32876	-0.21252	0.36687	0.30881
BEDROCK	(BR)	-0.13361	0.28495	0.28208	-0.08873	0.29948	0.38144
GWQ TYPE	(QTP)	-0.01921	-0.27476	-0.24286	0.10996	-0.13844	-0.28224
GWQ TDS	(QDS)	-0.03755	-0.20895	-0.33317	0.16397	-0.29265	-0.41013
PE BAL	(P-E)	-0.03813	0.18561	0.25613	-0.08730	0.27049	0.44147

	LANDFORM	DFT MNRL	DFT TEXR	BEDROCK	GWQ TYPE	GWQ TDS	PE BAL
(RS)	-0.12570	0.00153	0.18877	-0.13361	-0.01921	-0.03755	-0.03813
(RP)	-0.14694	0.18720	0.06941	0.28495	-0.27476	-0.20895	0.18561
(LR)	-0.11866	0.24131	0.32876	0.28208	-0.24286	-0.33317	0.25613
(D/L)	0.09890	-0.08351	-0.21252	-0.08873	0.10996	0.16397	-0.08730
(IN)	0.22204	0.25180	0.36687	0.29948	-0.13844	-0.29265	0.27049
(OUT)	0.10961	0.38491	0.30881	0.38144	-0.28224	-0.41013	0.44147
(LF)	1.00000	0.09812	0.20077	0.13447	0.02400	-0.04647	0.13876
(DM)	0.09812	1.00000	0.36323	0.60216	-0.63518	-0.74745	0.91134
(DT)	0.20077	0.36323	1.00000	0.33152	-0.13827	-0.40192	0.33277
(BR)	0.13447	0.60216	0.33152	1.00000	-0.47317	-0.63970	0.67835
(QTP)	0.02400	-0.63518	-0.13827	-0.47317	1.00000	0.74107	-0.66563
(QDS)	-0.04647	-0.74745	-0.40192	-0.63970	0.74107	1.00000	-0.83048
(P-E)	0.13876	0.91134	0.33277	0.67835	-0.66563	-0.83048	1.00000

relationships are clarified and other more subtle relationships emerge.

Relationships of the parameters to the component axes

The eigenvalues (sum of squares of each column) of the correlation matrix and the cumulative proportion of total variance they account for are shown in table 4. In the unrotated factor matrix (table 4), each column vector is a principal axis, or eigenvector. The values for each variable are factor loadings, which are a measure of the variance each variable contributes to each of the principal axes.

Loadings of certain variables dominate each principal component and often suggest a descriptive label for that component. For the first principal component, column 1, the highest loadings are for P-E balance (P-E), ground-water dissolved solids (QDS) (negatively), drift mineralogy (DM), bedrock (BR), and ground-water quality type (QTP) (negatively). These are a combination of chemical parameters (QDS, QTP, and DM), atmospheric water (P-E), and bedrock hydraulic conductivity (BR). The second principal component is dominated by regional slope (RS) and local relief (LR). Both are controls on ground-water flow, thus this component could be considered a ground-water flow component. The parameters that load high on the third component are inlet, landform, and outlet. The

Table 4.--Principal component analysis: unrotated matrix of factor loadings.

Parameter		Component 1	2	3	4	5	6
REGSLOPE	(RS)	0.06606	0.70764	-0.04272	0.23268	-0.50603	0.10588
REGPOSIT	(RP)	0.30169	0.19238	0.43240	0.21513	0.49094	0.57782
LOCALREL	(LR)	0.44650	0.61897	0.07449	-0.00177	-0.07472	0.09696
LD RATIO	(D/L)	-0.20454	-0.45921	-0.10786	0.65131	-0.35889	0.31115
INLET	(IN)	0.48403	0.12727	-0.65979	0.26618	0.28549	-0.09018
OUTLET	(OUT)	0.60397	0.07182	-0.48242	0.34586	0.23466	-0.26333
LANDFORM	(LF)	0.13289	-0.38551	-0.57203	-0.33210	-0.15765	0.43594
DFT MNRL	(DM)	0.85550	-0.20032	0.14093	-0.03303	-0.19107	-0.05106
DFT TEXR	(DT)	0.52627	0.31571	-0.37396	-0.33605	-0.11159	0.24312
BEDROCK	(BR)	0.76813	-0.18417	0.05797	-0.04499	0.17457	0.16290
GWQ TYPE	(QTP)	-0.72965	0.13295	-0.37606	-0.08463	0.10794	0.11691
GWQ TDS	(QDS)	-0.88903	0.07681	-0.16485	0.04201	0.12674	0.09278
PE BAL	(P-E)	0.89719	-0.24169	0.12765	-0.02739	-0.14952	-0.06446
Sum of squares		4.67983	1.55745	1.54920	0.95112	0.92240	0.83350
Percent of variance		36.0	12.0	11.9	7.3	7.1	6.4
(cumulative)		(36.0)	(48.0)	(59.9)	(67.2)	(74.3)	(80.7)

		7	8	9	10	11	12	13
	(RS)	-0.33262	0.02282	-0.22437	-0.09573	0.03091	0.00413	0.00268
	(RP)	-0.20780	0.06791	0.03123	0.11625	0.00916	-0.03820	0.00051
	(LR)	0.43616	-0.42529	0.10149	0.10957	-0.05947	0.00445	-0.00076
	(D/L)	0.26278	0.07896	0.10133	-0.01364	-0.00007	-0.00912	0.00008
	(IN)	-0.10987	0.00407	0.06560	-0.12251	-0.34924	0.01363	0.01214
	(OUT)	-0.02991	-0.04756	-0.03218	0.16134	0.35050	-0.00224	-0.01705
	(LF)	-0.23446	-0.34858	0.00970	0.04234	0.04954	-0.01815	-0.01307
	(DM)	-0.03306	0.11850	-0.11940	0.28006	-0.15112	0.13566	-0.13736
	(DT)	0.17280	0.46017	0.21173	-0.02052	0.09131	0.05808	0.02131
	(BR)	0.25132	-0.02995	-0.35648	-0.33263	0.06981	0.08250	-0.01420
	(QTP)	0.23475	0.15722	-0.34136	0.19006	-0.07352	-0.19420	-0.02073
	(QDS)	0.02852	-0.02864	-0.11358	0.13055	-0.00387	0.33655	0.07248
	(P-E)	-0.01448	0.02880	-0.13187	0.17525	-0.06350	-0.05342	0.19248
Sum of squares		0.63129	0.56909	0.41030	0.35108	0.29735	0.18452	0.06287
Percent of variance		4.9	4.4	3.1	2.7	2.3	1.4	0.5
(cumulative)		(85.6)	(90.0)	(93.1)	(95.8)	(98.1)	(99.5)	(100.0)

fourth component is dominated by the ratio of drainage basin area to lake area, the overland-runoff parameter. By the fifth axis, parameters that load high, for example regional slope, begin to repeat their high loadings on previous axes.

In deciding how many components to use in a study, a trade-off must be made between simplicity (number of components) and the amount of variance one wishes to account for. The unrotated factor-loadings matrix (table 4) shows that the first three components account for 60 percent of the variance. The fourth component adds an additional 7 percent. Eight components are needed to reach the 90 percent level. Repetition of high loadings of certain variables on components beginning with the fifth, however, suggest that only four or five components are going to be of use in the study.

A clearer picture of factor loadings on component axes is sometimes obtained by using a rotation technique such as VARIMAX. It is usually not necessary to rotate all the axes; again, a decision must be made weighing complexity and variance explained. A helpful guide in making this decision is to plot the eigenvalues as in figure 10. The first three eigenvalues are clearly separated from the others and a lesser break occurs between the sixth and seventh. It is of interest, then, to compare the results of 3-, 4-, 5-, and 6-axis VARIMAX rotations.

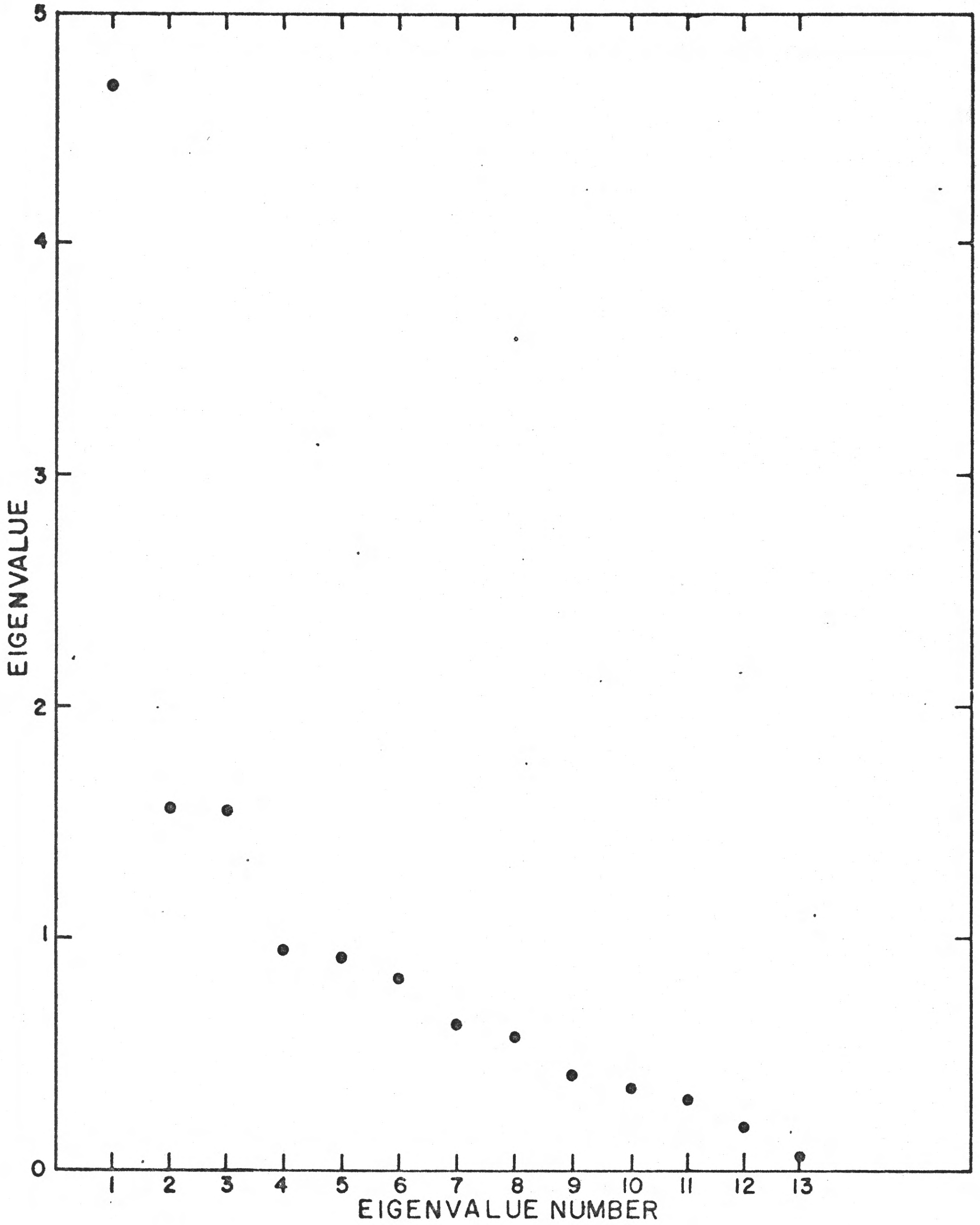


Figure 10.--Relative magnitude of eigenvalues of principal component analysis.

Rotation of the first three principal components (table 5), which account for 60 percent of the total variance, results in a slight rearrangement and simplification of important factor loadings compared to the unrotated matrix (table 4). Factor loadings on the first principal component are dominated by P-E, QDS, DM, and QTP. The value for bedrock is reduced compared to the unrotated matrix, although it is still rather high. The rotated first principal axis could now be considered the "atmospheric water-chemistry" component, a slight simplification of the unrotated first component. The second principal component (table 5) is characterized by high loadings on streamflow inlet (IN) and outlet (OUT), and can be considered a "streamflow" component. Drift texture (DT) and landform (LF) have moderately-high loadings on this axis. The highest loadings on the third component are local relief (LR) and regional slope (RS), both controls on ground-water flow; thus, this axis can be considered a "ground-water flow" component.

If the first four principal components are rotated, the pattern of factor loadings (table 6) for the first two remain the same as in the 3-axis rotation. The third component differs in that landform has the highest loading and the ground-water flow parameters are secondary in the 4-axis rotation, whereas loadings on regional slope and local relief are highest in the 3-axis rotation. The highest factor loadings on the

Table 5.--Principal component analysis: factor loadings of 3-axis VARIMAX rotation.

		Component		
Parameter		1	2	3
REGSLOPE	(RS)	-0.16292	-0.11915	0.68279
REGPOSIT	(RP)	0.37679	0.24719	0.33455
LOCALREL	(LR)	0.24110	-0.16887	0.70809
LD RATIO	(D/L)	-0.08849	0.02390	-0.50591
INLET	(IN)	0.12985	-0.81052	0.10956
OUTLET	(OUT)	0.31977	-0.69751	0.11762
LANDFORM	(LF)	0.00472	-0.54382	-0.44467
DFT MNRL	(DM)	0.86102	-0.22231	0.03298
DFT TEXR	(DT)	0.22374	-0.58473	0.35286
BEDROCK	(BR)	0.74764	-0.26112	0.01263
GWQ TYPE	(QTP)	-0.82305	-0.03830	-0.11233
GWQ TDS	(QDS)	-0.86405	0.22419	-0.16313
PE BAL	(P-E)	0.90425	-0.24897	0.00063
Sum of squares		3.94565	2.11683	1.72401
Percent of variance		30.4	16.3	13.3
(cumulative)		(30.4)	(46.7)	(60.0)

Table 6.--Principal component analysis: factor loadings of 4-axis VARIMAX rotation.

Parameter		Component 1	2	3	4
REGSLOPE	(RS)	-0.19973	-0.32436	0.59025	-0.25999
REGPOSIT	(RP)	0.34397	0.08132	0.48590	0.01569
LOCALREL	(LR)	0.20861	-0.27149	0.50070	-0.46916
LD RATIO	(D/L)	-0.07505	-0.15562	-0.06339	0.80913
INLET	(IN)	0.14900	-0.85190	-0.07970	-0.04864
OUTLET	(OUT)	0.33211	-0.78132	0.02187	0.03153
LANDFORM	(LF)	0.05625	-0.27404	-0.71661	-0.10950
DFT MNRL	(DM)	0.86629	-0.17763	-0.01754	-0.10313
DFT TEXR	(DT)	0.23317	-0.44606	-0.10068	-0.60492
BEDROCK	(BR)	0.75601	-0.20581	-0.06157	-0.10788
GWQ TYPE	(QTP)	-0.81180	0.00645	-0.19896	0.00135
GWQ TDS	(QDS)	-0.86252	0.19854	-0.07844	0.18898
PE BAL	(P-E)	0.91207	-0.19741	-0.04701	-0.08575
Sum of squares		3.95424	1.97235	1.42176	1.38926
Percent of variance		30.4	15.2	10.9	10.7
(cumulative)		(30.4)	(45.6)	(56.5)	(67.2)

fourth principal component are D/L ratio and drift texture--two parameters that are not clearly related to each other.

A 5-axis rotation (table 7) results in a clearer picture of variance compared to the others. The first two components are similar to those in the 3- and 4-axis rotations. The third component is clearly characterized by high loadings on regional slope and local relief, both of which are ground-water-flow parameters. The fourth component is dominated by high loadings on landform and regional position, which are clearly related, and can be considered a component of geology and ground-water. The D/L ratio, the "overland runoff" component, alone has the highest loading on the fifth component.

The 6-axis rotation (table 8) is similar to the 5-axis rotation. The difference is that the fourth component is characterized by high loadings on landform and moderately high loadings on drift texture. A high loading on regional position shifts from the fourth in the 5-axis rotation to the sixth component in the 6-axis rotation.

From the above analysis it is obvious that interpretation of the principal components becomes easier as more components are considered. Whereas compound descriptive labels describe the 3-axis VARIMAX components, single and hydrologically meaningful labels describe the 5-axis components. When six or more components are considered, some factor loadings become redundant.

Table 7.--Principal component analysis: factor loadings of 5-axis VARIMAX rotation.

Parameter		Component 1	2	3	4	5
REGSLOPE	(RS)	-0.06448	-0.02731	0.90046	0.02416	0.02884
REGPOSIT	(RP)	0.22817	-0.12632	-0.06879	0.70911	-0.16344
LOCALREL	(LR)	0.25041	-0.17675	0.57524	0.21395	-0.35072
LD RATIO	(D/L)	-0.03161	-0.03691	-0.02954	-0.12144	0.89408
INLET	(IN)	0.11101	-0.90050	0.06010	-0.08767	-0.06036
OUTLET	(OUT)	0.29463	-0.82693	0.07589	0.02007	0.02242
LANDFORM	(LF)	0.10095	-0.21862	-0.24186	-0.71373	-0.05242
DFT MNRL	(DM)	0.89576	-0.13494	0.06039	-0.06178	-0.03749
DFT TEXR	(DT)	0.29324	-0.34530	0.31503	-0.33273	-0.47612
BEDROCK	(BR)	0.71121	-0.30549	-0.15586	0.08125	-0.17255
GWQ TYPE	(QTP)	-0.81526	0.00475	-0.06592	-0.20229	-0.01926
GWQ TDS	(QDS)	-0.88321	0.17027	-0.11907	-0.02400	0.13216
PE BAL	(P-E)	0.93124	-0.17548	0.01087	-0.05647	-0.03808
Sum of squares		3.93537	1.88239	1.36103	1.24724	1.23398
Percent of variance		30.3	14.5	10.5	9.6	9.5
(cumulative)		(30.3)	(44.8)	(55.3)	(64.9)	(74.4)

Table 8.--Principal component analysis: factor loadings of 6-axis VARIMAX rotation.

Parameter		Component 1	2	3	4	5	6
REGSLOPE	(RS)	-0.06591	-0.01240	0.89926	-0.06724	0.08428	-0.05968
REGPOSIT	(RP)	0.16141	0.00988	0.07057	-0.08720	-0.01947	0.94713
LOCALREL	(LR)	0.23877	-0.14378	0.62502	-0.02437	-0.28649	0.22808
LD RATIO	(D/L)	-0.05851	0.01383	-0.05448	0.05414	0.95097	-0.01751
INLET	(IN)	0.10550	-0.88871	0.07977	0.19177	-0.03197	0.03459
OUTLET	(OUT)	0.31156	-0.86322	0.06006	-0.00963	-0.01241	-0.02345
LANDFORM	(LF)	0.03497	-0.07961	-0.16551	0.86865	0.13970	-0.09826
DFT MNRL	(DM)	0.89272	-0.12809	0.06634	0.11774	-0.01635	0.01323
DFT TEXR	(DT)	0.25130	-0.24879	0.39938	0.55779	-0.32060	0.04996
BEDROCK	(BR)	0.67905	-0.24040	-0.08935	0.21351	-0.09227	0.32459
GWQ TYPE	(QTP)	-0.82609	0.02845	-0.05634	0.15969	0.00937	-0.10947
GWQ TDS	(QDS)	-0.88493	0.17248	-0.12804	-0.05280	0.12183	-0.05481
PE BAL	(P-E)	0.92836	-0.16974	0.01749	0.12217	-0.02007	0.02502
Sum of squares		3.84886	1.75789	1.43639	1.22086	1.14150	1.08799
Percent of variance		29.6	13.5	11.0	9.4	8.8	8.4
(cumulative)		(29.6)	(43.1)	(54.1)	(63.5)	(72.3)	(80.7

To summarize briefly the hydrological meaning of the principal component analysis, the 5-axis VARIMAX rotation results can be interpreted as follows: (1) the first component is characterized by atmospheric water and chemical parameters, (2) the second by "streamflow" parameters, (3) the third and fourth components by "ground-water flow" parameters, and (4) the fifth by the "overland runoff" parameter.

Although the loadings on the five rotated components have the most meaning hydrologically, it is necessary to test their statistical stability. In lieu of testing by means of repeating the study with a new set of samples, the test used herein consists of randomly splitting the 150-sample set and comparing the results of analysis of each subsample. The results of a 5-axis VARIMAX rotation of each subsample are given in tables 9 and 10.

The same parameters load high on the first two components in both tables, and they duplicate the results of the analysis of the 150-sample set. In table 9, the D/L ratio loads high on the third component and in table 10 it has the highest loading on the fifth component. Landform loads high on the fourth component in table 9, but regional position loads highest on the fourth component in table 10. Regional slope loads highest on the fifth component in table 9, and regional slope and local relief have the highest loadings on the third com-

Table 9.--Principal component analysis of half of the sample lakes--Group A: factor loadings of 5-axis VARIMAX rotation.

		Component				
Parameter		1	2	3	4	5
REGSLOPE	(RS)	-0.17941	0.09018	-0.04400	-0.02260	0.84845
REGPOSIT	(RP)	0.51726	0.01849	-0.08227	-0.26808	0.42582
LOCALREL	(LR)	0.22585	0.03712	-0.61964	0.12111	0.46563
LD RATIO	(D/L)	-0.06250	-0.05651	0.87807	0.20134	0.07423
INLET	(IN)	0.13035	0.90026	-0.05239	0.15894	0.10679
OUTLET	(OUT)	0.29749	0.84364	-0.08148	0.01736	0.00207
LANDFORM	(LF)	0.07677	0.10920	0.11784	0.89143	-0.08320
DFT MNRL	(DM)	0.89004	0.17523	-0.05528	0.07583	0.03778
DFT TEXR	(DT)	0.16301	0.40385	-0.52865	0.50261	0.25282
BEDROCK	(BR)	0.72729	0.13145	-0.18553	0.23113	0.04476
GWQ TYPE	(QTP)	-0.85729	-0.04874	-0.03288	0.14910	0.14797
GWQ TDS	(QDS)	-0.84913	-0.24120	0.20493	-0.09663	0.05162
PE BAL	(P-E)	0.93559	0.16782	-0.05518	0.09924	-0.01417
Sum of squares		4.14503	1.84695	1.55001	1.30101	1.23395
Percent of variance		31.9	14.2	11.9	10.0	9.5
(cumulative)		(31.9)	(46.1)	(58.0)	(68.0)	(77.5)

Table 10.--Principal component analysis of half of the sample lakes--Group B: factor loadings of 5-axis VARIMAX rotation.

Parameter		Component 1	2	3	4	5
REGSLOPE	(RS)	0.02998	0.03502	0.82638	0.14629	0.17271
REGPOSIT	(RP)	0.08509	-0.03310	0.06943	-0.84247	-0.07799
LOCALREL	(LR)	0.26134	0.21075	0.71194	-0.13856	-0.13916
LD RATIO	(D/L)	-0.07190	-0.04579	0.01753	0.07060	0.96951
INLET	(IN)	0.11139	0.92220	0.02177	0.05450	-0.05926
OUTLET	(OUT)	0.31942	0.80674	0.08785	0.05589	-0.00107
LANDFORM	(LF)	0.07903	0.25049	-0.50900	0.37900	0.09369
DFT MNRL	(DM)	0.89813	0.08100	-0.01682	0.09260	-0.04785
DFT TEXR	(DT)	0.48889	0.12093	0.21391	0.36804	-0.16755
BEDROCK	(BR)	0.74787	0.30355	-0.17660	-0.25009	-0.06051
GWQ TYPE	(QTP)	-0.69752	-0.09736	-0.30255	0.27910	0.08359
GWQ TDS	(QDS)	-0.87738	-0.13550	-0.24104	0.04463	-0.02440
PE BAL	(P-E)	0.92367	0.16401	-0.00706	0.11778	-0.00791
Sum of squares		3.91785	1.78094	1.68908	1.20541	1.04918
Percent of variance		30.1	13.7	13.0	9.3	8.1
(cumulative)		(30.1)	(43.8)	(56.8)	(66.1)	(74.2)

ponent in table 10. The results of the subsample listed in table 10 are very similar to the results of the 150-sample set.

From the above comparison, it is clear that the first two components are statistically stable. Although it does not load high on the same axes in both subsamples, the D/L ratio emerges as an independent parameter in both subsamples, therefore can be considered an independent parameter for purposes of this report. The geology and ground water parameters in one subsample generally duplicate the 150-sample set, but they are less definitive in the other, thus these parameters should be used with caution. This result concerning the ground water parameters suggests that perhaps other parameters would be more appropriate for classifying the hydrologic setting of lakes. The following major part of this report, in which numerical simulation is used to evaluate the factors that control the interaction of lakes and ground water, will help resolve this problem. It will be shown however, that data on other appropriate parameters are not readily available. Because of this, and because the ground-water parameters chosen previously are not entirely unstable (emerge as independent in the analyses of the 150-sample set and one subsample--table 10) they will be used in this first attempt to classify the hydrologic setting of lakes. The lesser stability of the ground-water parameters, however, must be kept in mind in the following discussion.

Comparison of maps showing the areal variation of each principal component with maps of the related parameters

Analysis of factor loadings and variance results in identification of independent theoretical factor axes in a data set. Factor loadings show the clustering of variables around the theoretical axes and the amount of variance explained by those axes. Factor scores relate individual samples to the hypothetical axes. By mapping factor scores, the areal variation in each theoretical factor axis can be shown. Factor scores were calculated from (Kaiser, 1962):

$$f = A'R^{-1}\bar{z} \tag{17}$$

where: A = matrix of factor loadings

R = matrix of correlation coefficients

$\bar{z}$ = vector of sample standard scores

Comparison of factor-score maps with maps of the original parameters is helpful in determining index parameters upon which to base the simplest classificatory system. Principal component analysis has proven useful in this study in identifying independent theoretical factor axes that have hydrologic meaning. Each component, however, is identified by a group of parameters that must all be used to closely reproduce the results. One of the goals of this study is to identify a minimum number of parameters that give essentially the same pattern of variation as the associated theoretical component. The following discussion examines each of the five principal components relative to the 13 original parameters.

It will be noticed that some maps show contours and others show point data. This reflects the judgement of the author concerning the validity of regionalizing point data. If a parameter or statistical value showed reasonable general regional patterns, a contour map was drawn, but if local variability was so great that regional patterns would be misleading, point data were used. The symbols on the point data maps were carefully designed to facilitate recognition of areal trends and areas of similarity.

Component 1

Excluding the three water-quality parameters, the remaining parameters in this study relate to lake hydrology, or lake-water budgets. The four original parameters that have high loadings on the first component are P-E balance, drift mineralogy, ground-water dissolved solids and ground-water quality type. P-E balance is the only lake hydrology parameter that loads high on the first principal component, so it must be considered in a hydrologic classification.

With respect to water quality, it is desirable to select as few parameters as possible to develop an adequate and simple classification relating lakes to ground-water chemistry. It should be noted that all the parameters that load high on component 1 have gradational patterns across the region and all influence the areal pattern of factor scores. There is fair agreement, especially in the western part of the area, between the factor-score map of component 1 (fig. 11) and the ground-water total dissolved solids map (fig. 12). Areal patterns are more important in the comparison than the actual factor-score values. Areas of low dissolved solids are similar to the high minus factor-score values in parts of the Missouri Coteau and the Coteau des Prairies. In northern Minnesota, the similarity in the two maps is shown by the eastern extensions of the 0.80 isopleth (fig. 11) and the 250 mg/l dissolved solids line (fig. 12); both resemble the shape of the St. Louis sublobe (fig. 6). In southern Minnesota, the 1.00 isopleth is similar to the 250 mg/l dissolved solids line; both resemble the Grantsburg sublobe (fig. 6).

The ground-water quality types (fig. 13) also agree to a certain extent with the factor-score map of component 1, but this agreement is confined almost entirely to the Dakotas. Nearly all of Minnesota, Wisconsin, and Iowa are of one water-quality type, suggesting this parameter is of little use in classifying lakes in the north-central United States.

The factor-score map of component 1 is useful because it points out the similarity of certain areas that are not usually related. The area in northeastern Minnesota, for example, is

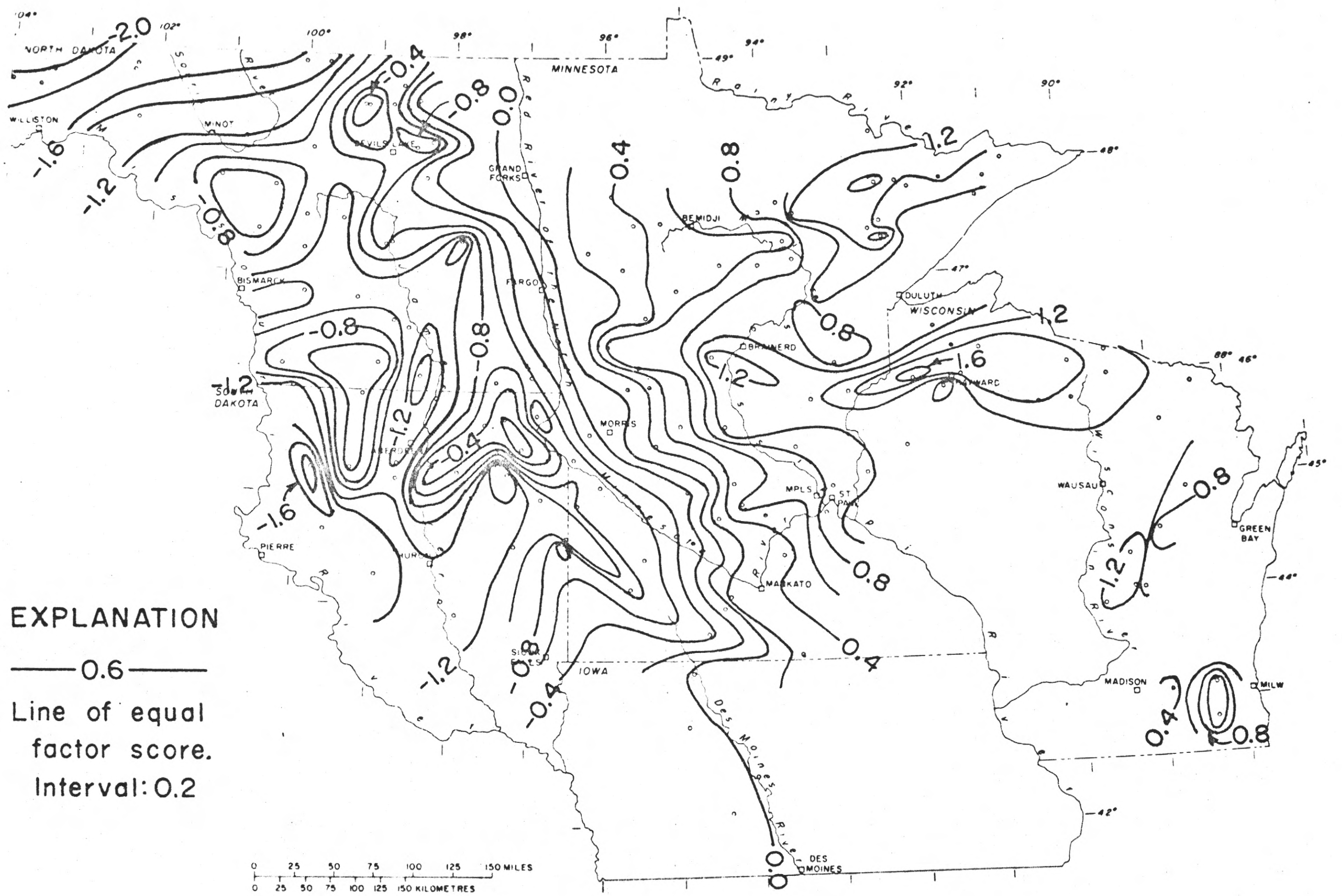


Figure 11.--Factor scores of component 1 of principal component analysis.

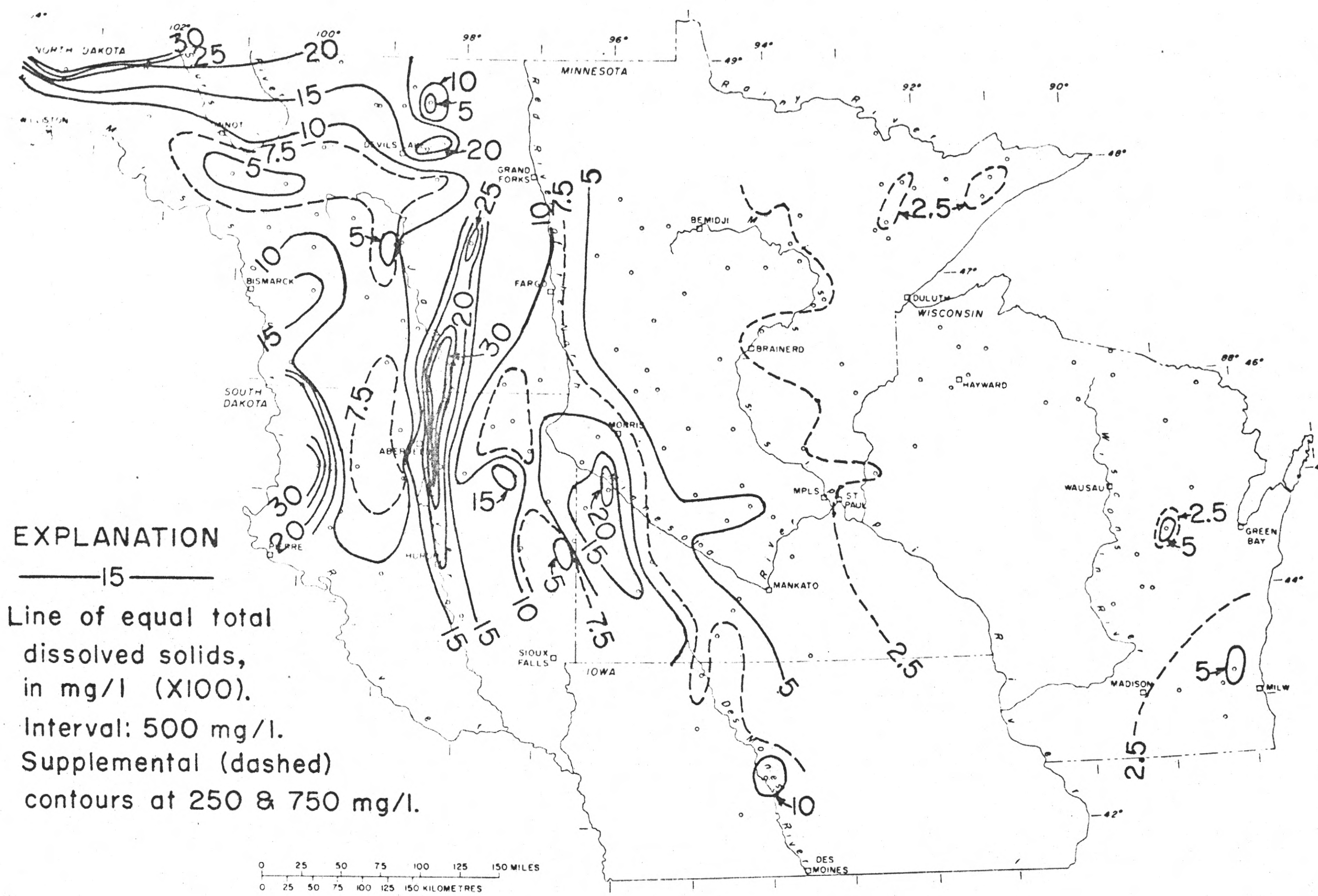


Figure 12.--Total dissolved solids content of ground water in the upper part of the ground-water system.

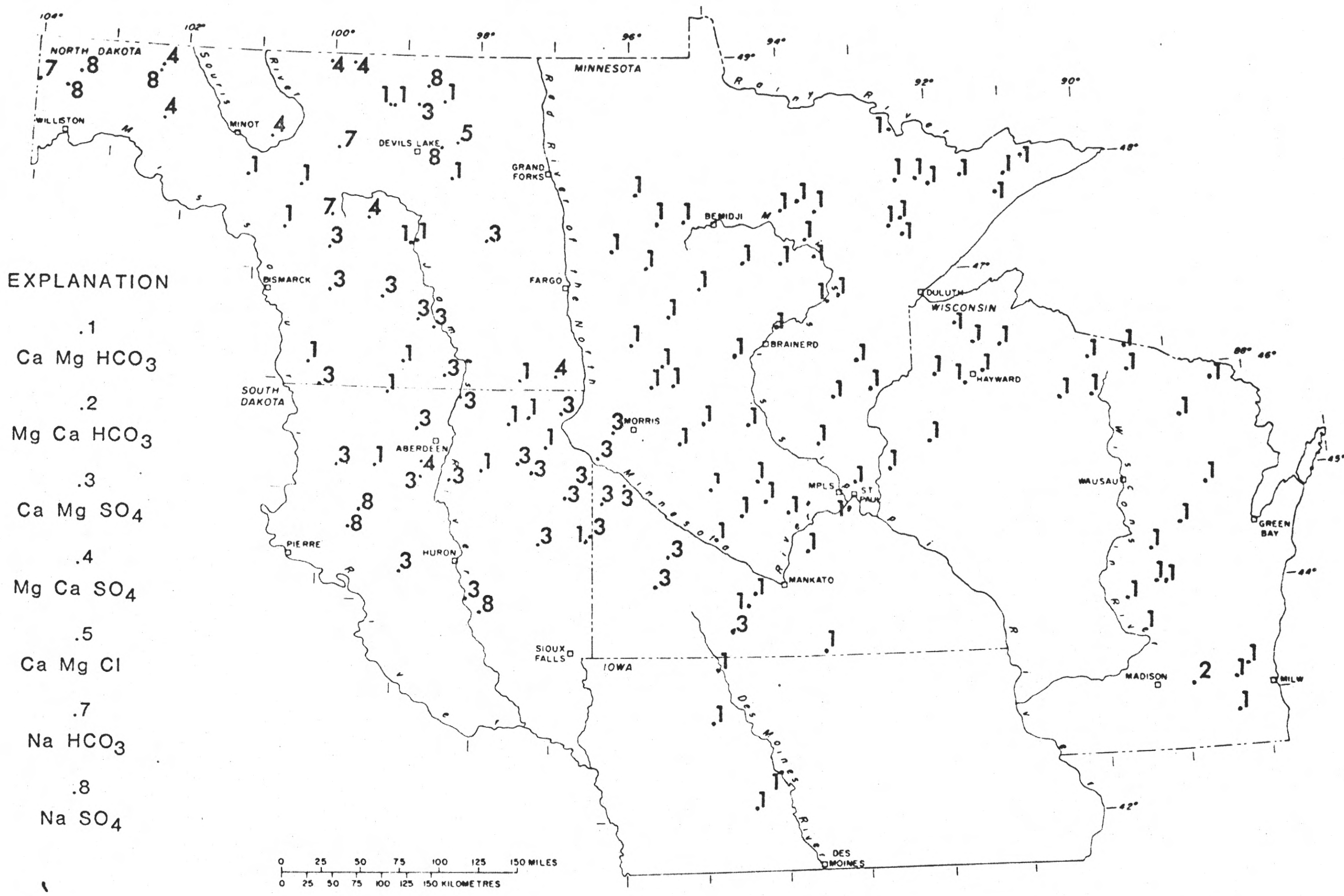


Figure 13.--Water-quality type of ground water in the upper part of the ground-water system.

similar to part of northwestern Wisconsin and a small area in central Wisconsin. In the Dakotas, the areas of factor scores between -1.20 and -1.60 are confined largely to the Souris River basin, the low area east of Bismarck, the James River lowland, and parts of the lower elevations of the Missouri Coteau.

Most classifications of lakes based on chemical and biological parameters in the north-central States have tried to relate lake types to glacial geology. There is some justification for this because ground-water chemistry is related to drift chemistry. The hydrodynamics of the ground-water system, however, are such that hydrochemical characteristics related to certain geologic units are modified by ground-water flow. (Discussed in more detail in the second major part of this report.) Thus, ground-water chemistry is more closely related to lake chemistry than are geologic unit boundaries.

In summary, dissolved solids is the best parameter to use as a measure of ground-water quality for this classificatory system because: (1) it has more areal variability and it is easier to measure than water-quality type, where a complete water analysis is needed, and (2) it is easier to measure and has more hydrogeologic meaning than drift mineralogy.

Bedrock loads moderately high on factor one. Bedrock in this report is a hydraulic conductivity parameter and could be important to lake ground-water interaction in areas of thin drift.

Component 2

Component two is characterized by high loadings on inlet and outlet, and is the surface-water relationship factor. A plot of factor scores (fig. 14) shows four groups that have the areal distribution shown in figure 15. Values greater than 0.1 are common in the Dakotas and are scattered throughout Minnesota; very few occur in Wisconsin. In contrast, the values of less than -0.75 are common in Wisconsin and parts of Minnesota. To compare the factor scores to inlet and outlet, figure 16 shows the distribution of the inlet and outlet characteristics of the lakes used in this study. Comparison of the factor-score map (fig. 15) to the parameter map (fig. 16) reveals similar patterns. Lakes that have no inlet or outlet have scores greater than 0.1, and those that have perennial inlets and outlets have the lowest factor scores.

For the most theoretically correct hydrologic classification, both inlet and outlet should be used. For the sake of simplicity in classifying, however, it would be desirable to use only one of the parameters. Outlet is commonly used in classifying lakes to determine whether a lake is open or closed. Hydrologically, outlet is an important parameter

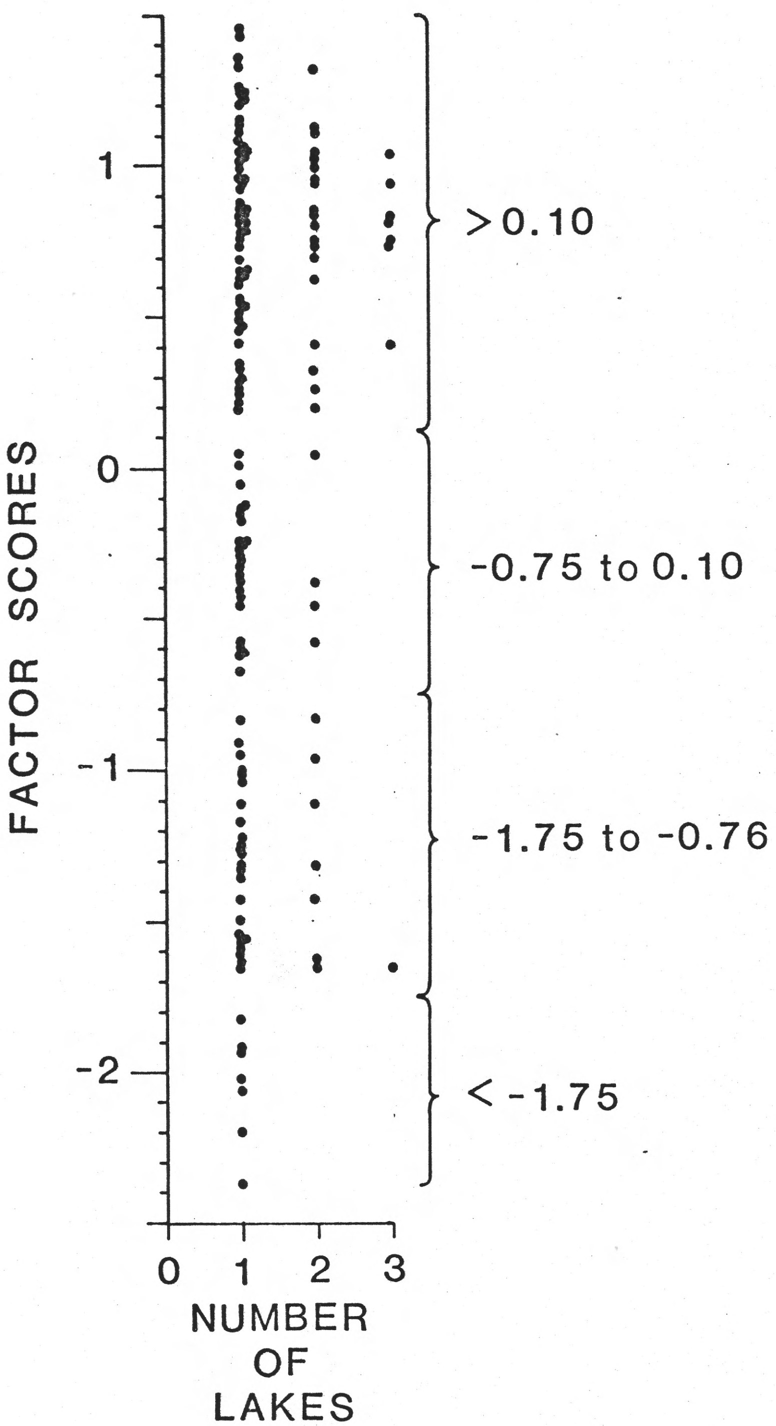


Figure 14.--Relative magnitude of factor scores of component 2.

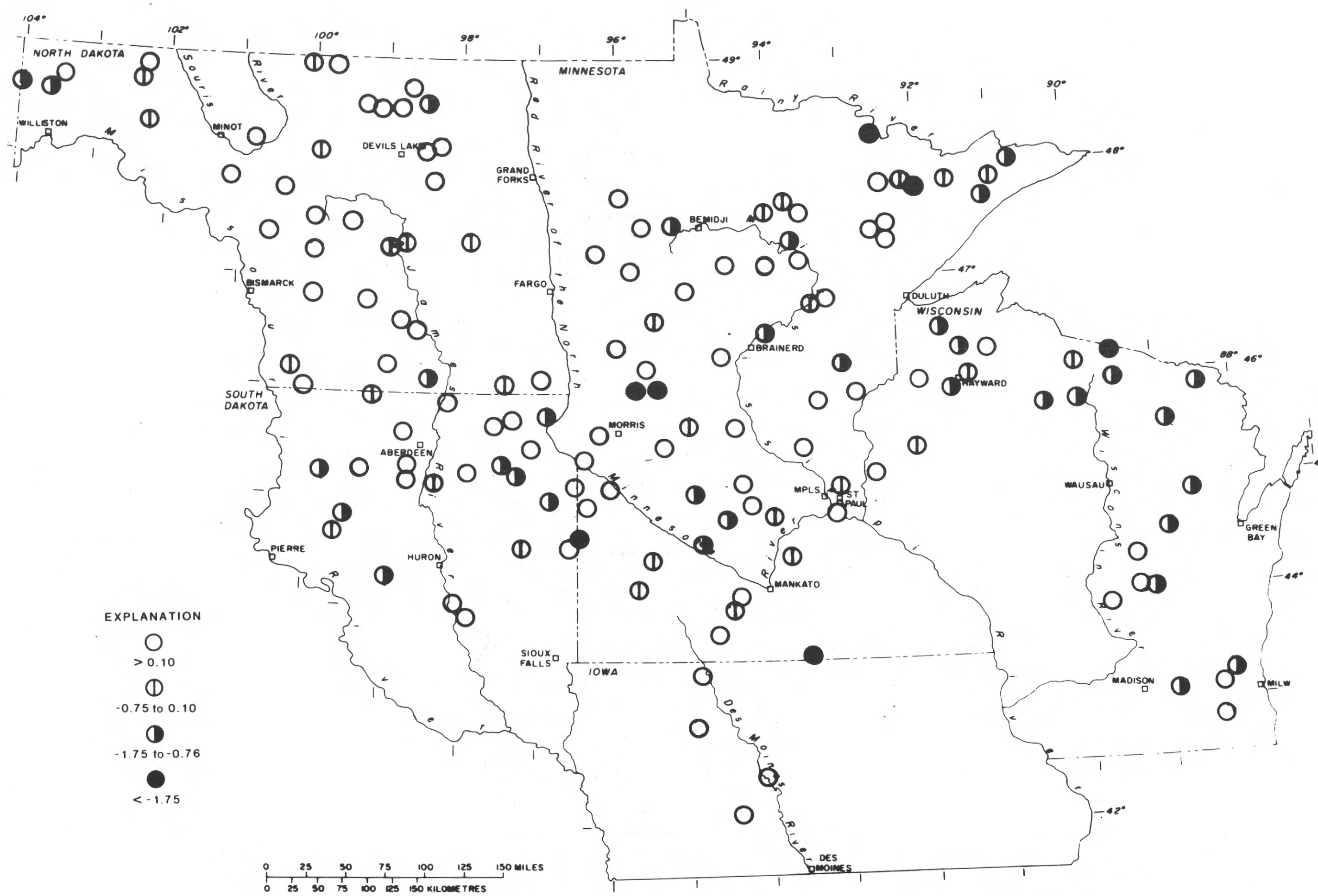


Figure 15.--Factor scores of component 2.

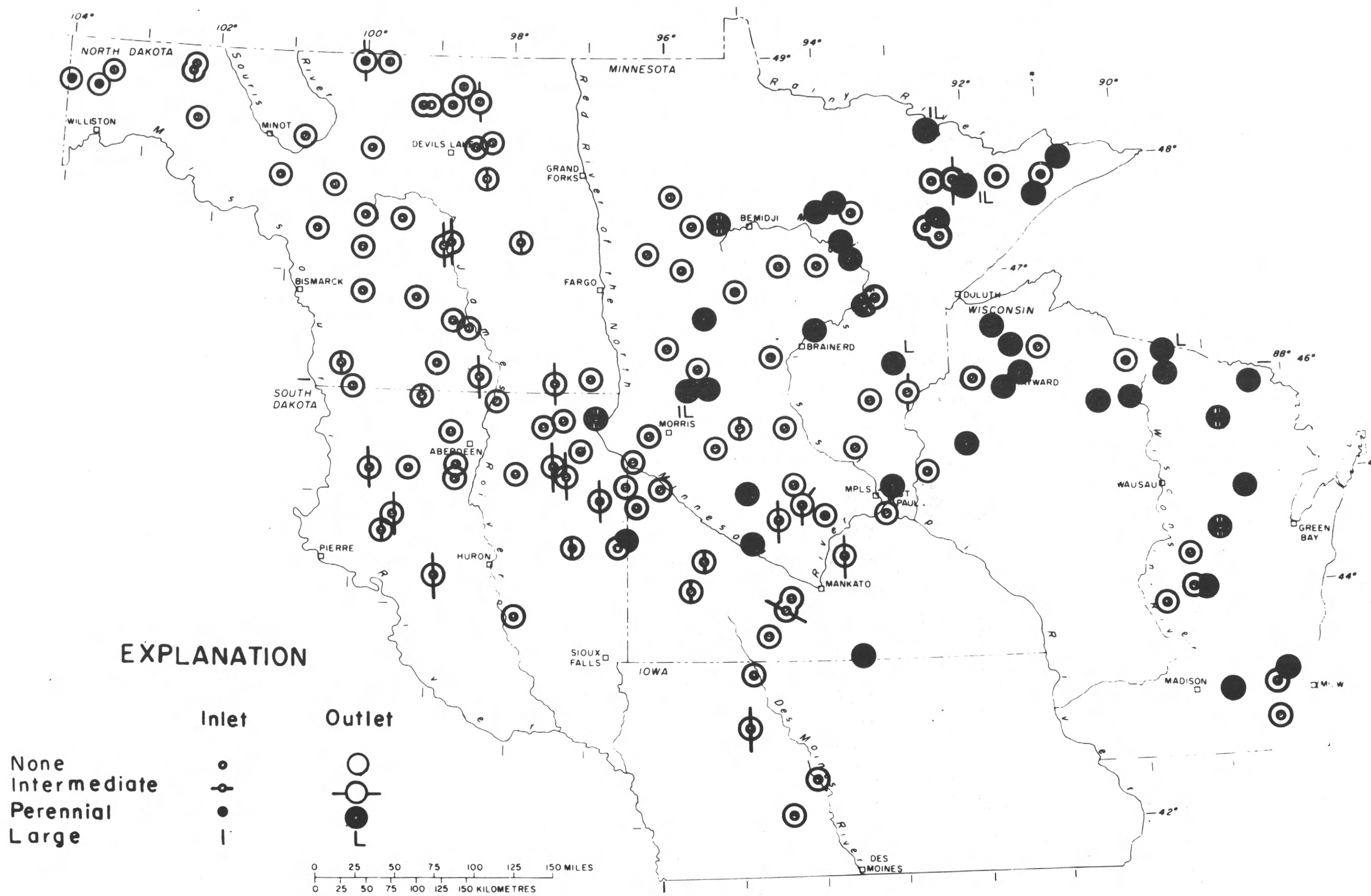


Figure 16.--Inlet and outlet characteristics of the study lakes.

because it indicates how effective the lake is as a sink. An argument can be made for choosing inlet, however, because it is an indicator of the amount of water and other materials brought into a lake from beyond its immediate drainage basin. If a single streamflow parameter had to be chosen for a classification, however, outlet is relatively more important for most purposes.

It is of interest to examine components 1 and 2 together because their stability makes them the most reliable of those used to classify lakes. Factor scores for component one versus component two (fig. 17) were plotted to identify groups of samples that have similar scores. Selected groups were then mapped to determine their areal distribution (fig. 18). It must be remembered in analyzing these figures that the water quality parameters weigh heavily in the definition of component 1.

The scores representing sample lakes that tend to be closed and evaporation exceeds precipitation are concentrated in the Dakotas and southwestern Minnesota. Most of the lakes in the northeastern half of Minnesota and all of Wisconsin tend to have precipitation exceed evaporation. Of these, open lakes are common in northern Wisconsin, and east-central Minnesota and adjacent Wisconsin have a concentration of closed lakes. Parameters that load high on components 1 and 2 are especially

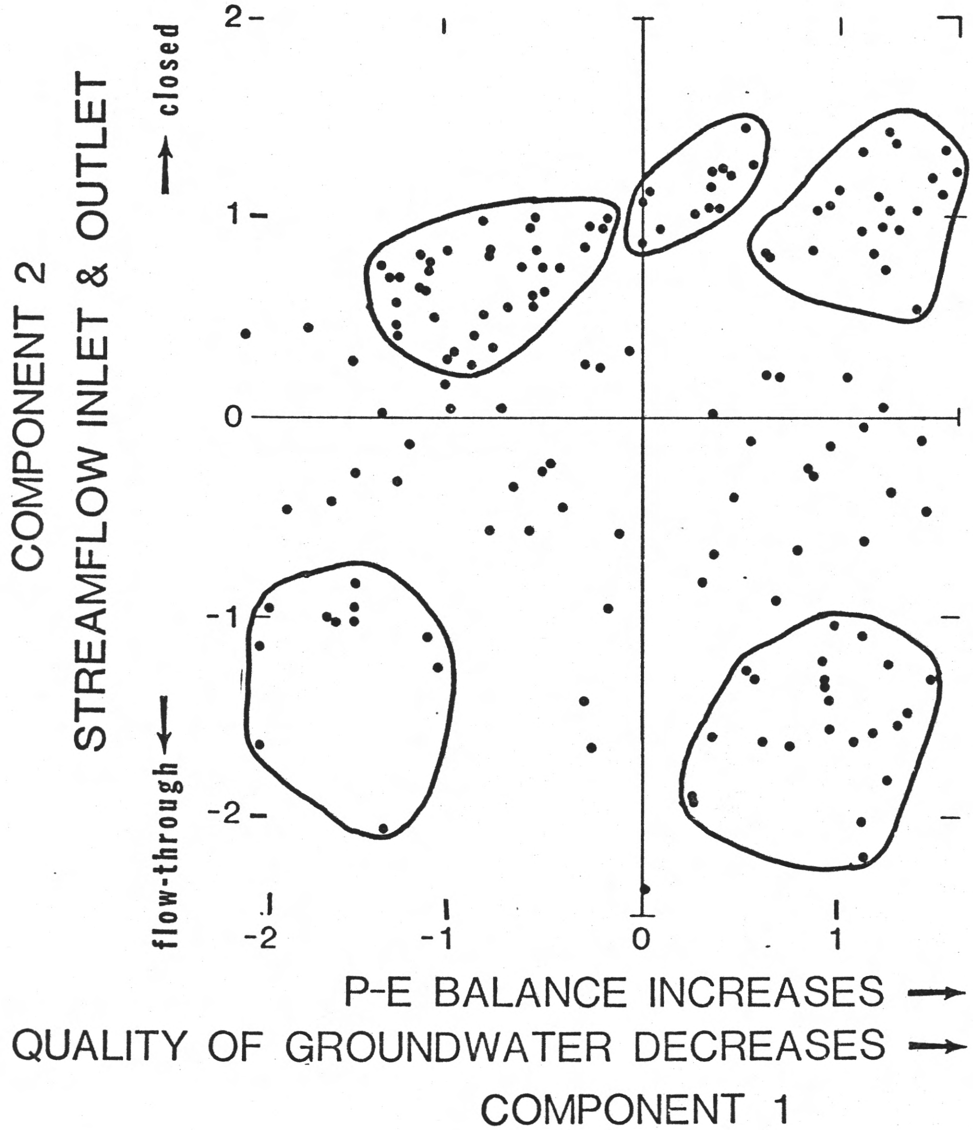


Figure 17.--Factor scores of components 1 vs. 2.

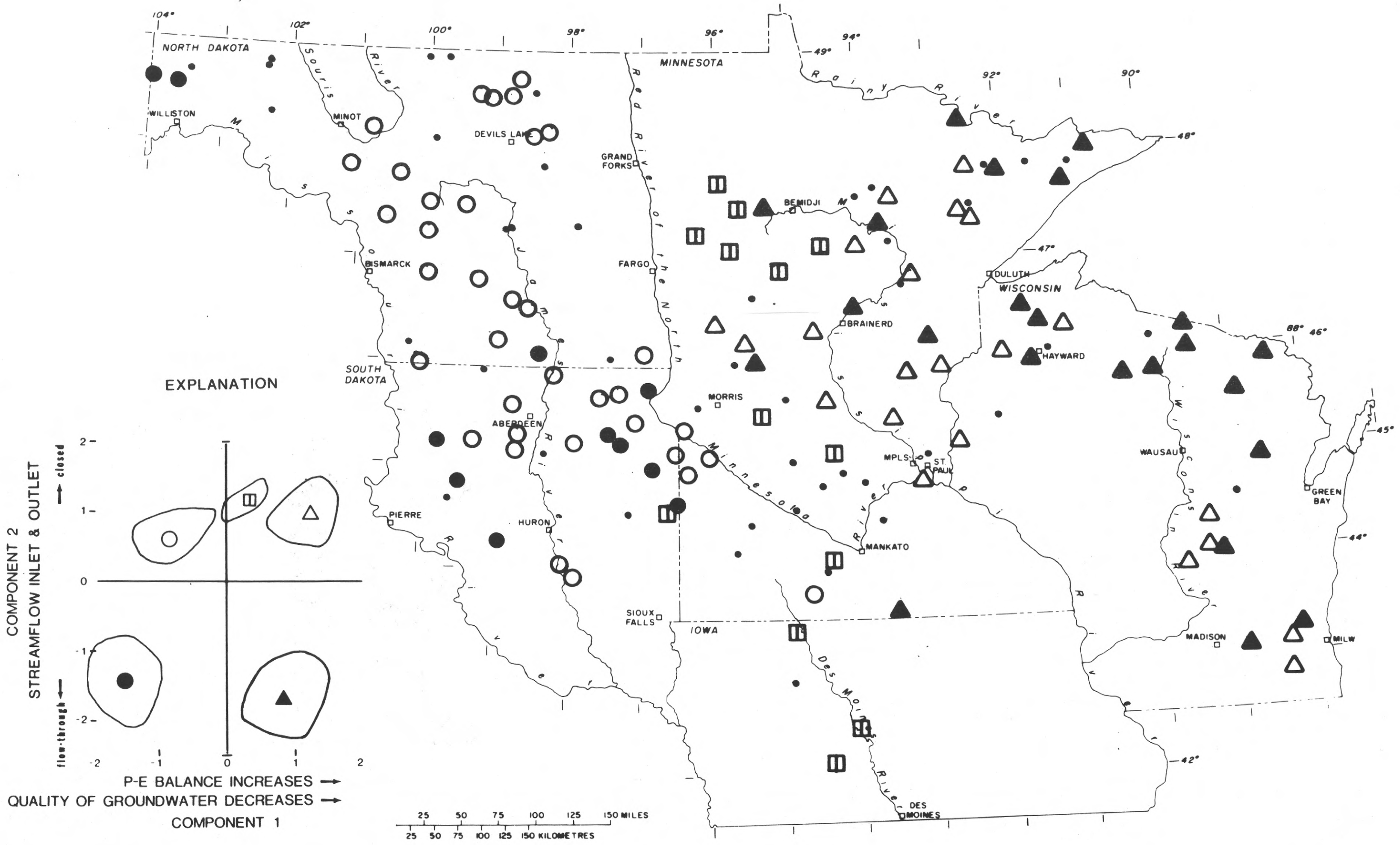


Figure 18.--Areal distribution of selected groups of factors 1 and 2 combined.

interesting because of their interrelationship. Most closed lakes are in the region where evaporation considerably exceeds precipitation and the dissolved solids content of ground water is high. This should lead to buildup of salts in those lakes that receive ground-water inflow. Thus the highly variable input of ground water, in both quantity and quality (discussed in the second major part of this report), combined with the P-E balance and surface water characteristics of lakes, could explain much of the high variability in lake water quality in many parts of the Dakotas and western Minnesota.

Components 3 and 4

Components 3 and 4 are characterized by high loadings of ground-water flow parameters. Regional slope and local relief load high on component 3, and landform and regional position load high on component 4.

To facilitate comparison with factor-score maps, lakes that have similar regional slope and local relief characteristics were determined from a graph (fig. 19), and their areal distribution mapped (fig. 20). The group of lakes north of Devils Lake, North Dakota, those in the James River lowland, and many lakes in the Minnesota River lowland have low slope and low local relief. The majority of lakes on the Missouri Coteau have moderate slope and moderate relief, as do many lakes on

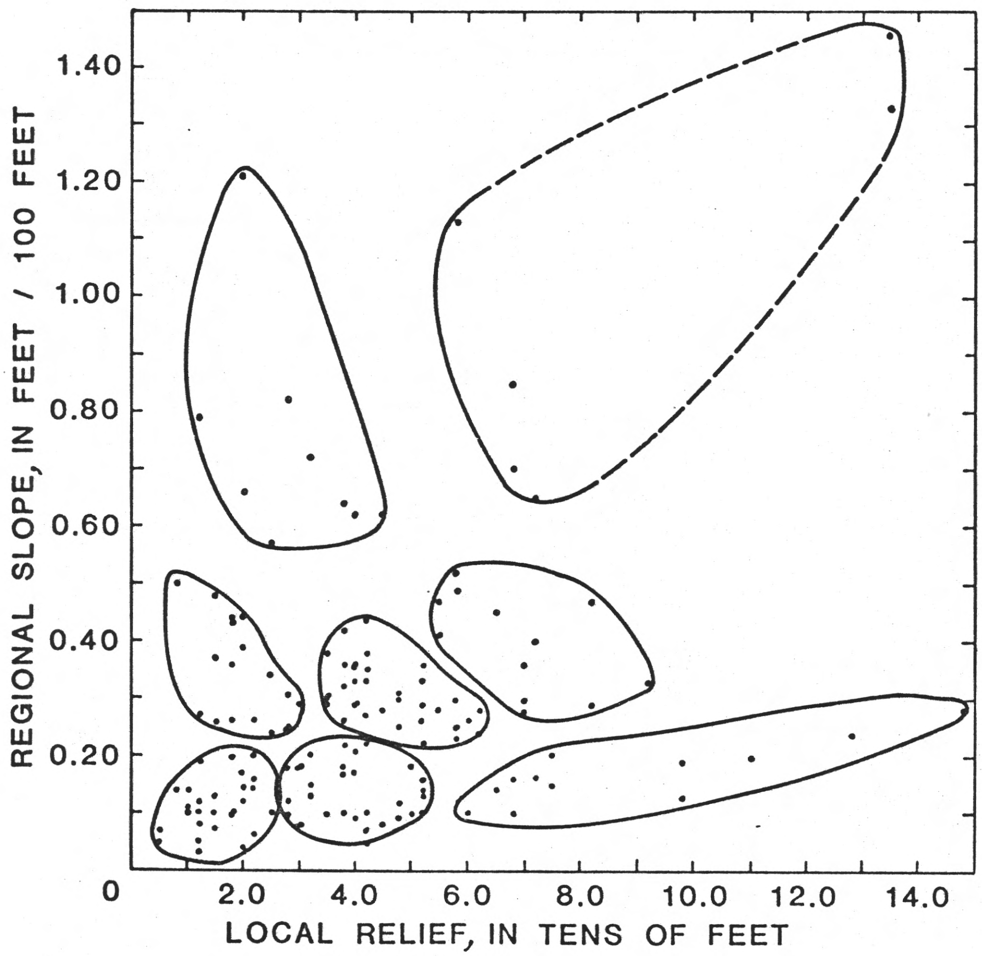


Figure 19.--Delineation of groups of various combinations of regional slope and local relief.

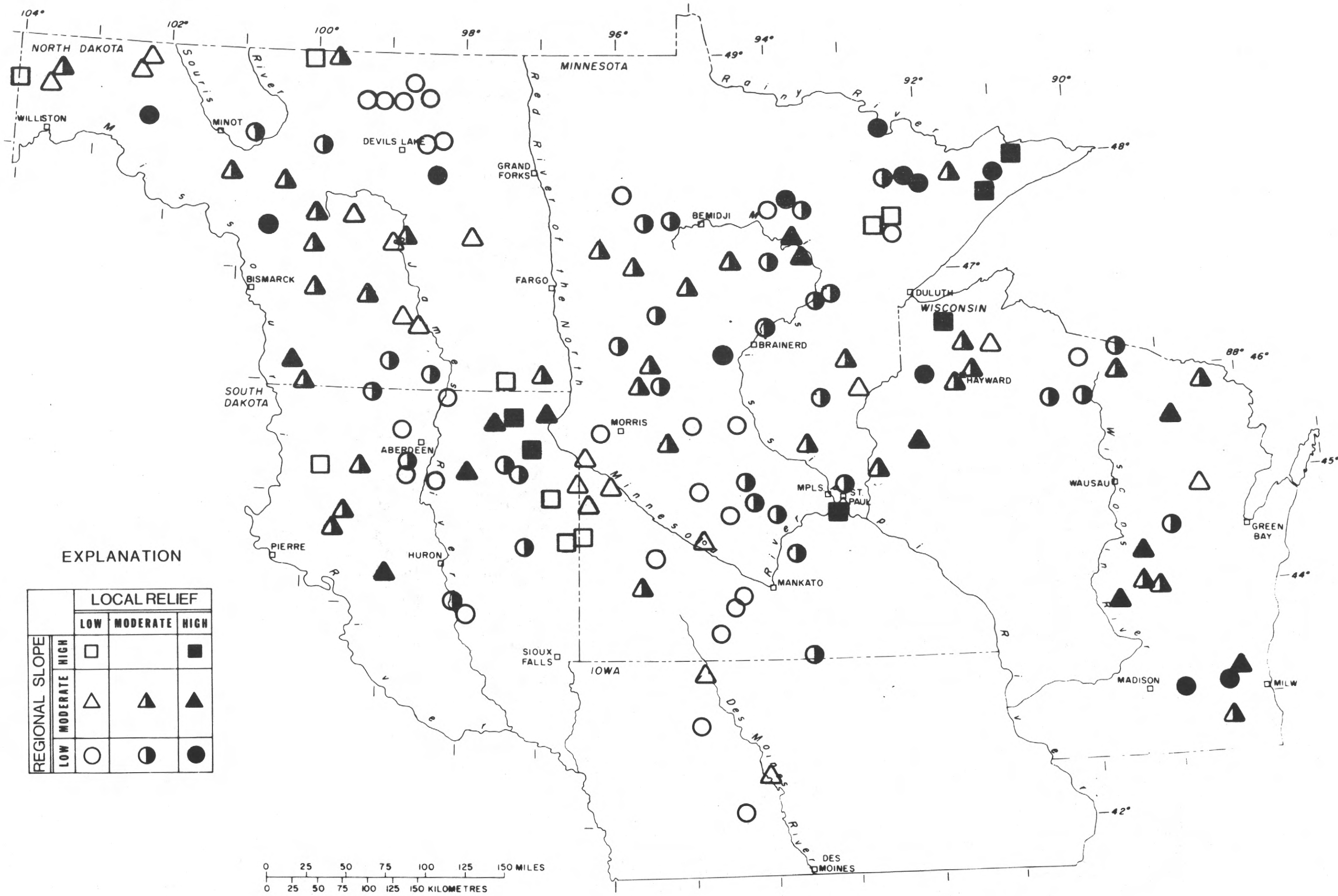


Figure 20.--Characteristics of regional slope and local relief near the study lakes.

the Alexandria moraine and in the morainal area of northern Wisconsin. Lakes characterized by high slope and high local relief occur along the east side of the Coteau des Prairies and in the Canadian Shield area of northeastern Minnesota.

For comparison with figure 20, groups of lakes that have similar factor scores were determined from a graph (fig. 21) and mapped (fig. 22). Lakes that have high factor scores are those that have high slope and high local relief. In contrast, those with negative values are those that have low slope and low local relief. There is generally good agreement between the patterns shown on the factor-score map (fig. 22) and those on the original data map (fig. 20).

Because component 4 is also characterized by parameters related to ground-water flow, the scores for component 4 must be examined before further discussion of component 3. The areal distribution of scores for component 4 (fig. 23) is closely related to the hypsographic map (fig. 1). Because there is some relationship between the distribution of landforms and regional position, the map also resembles the landform map (fig. 5). The factor-score map (fig. 23) shows high values in the upland areas of the Missouri Coteau, Turtle Mountains, and Coteau de Prairies in the Dakotas, the Alexandria Moraine, part of the Itasca area, and part of the Grantsberg sublobe area in Minnesota, and the morainal area in northern Wisconsin.

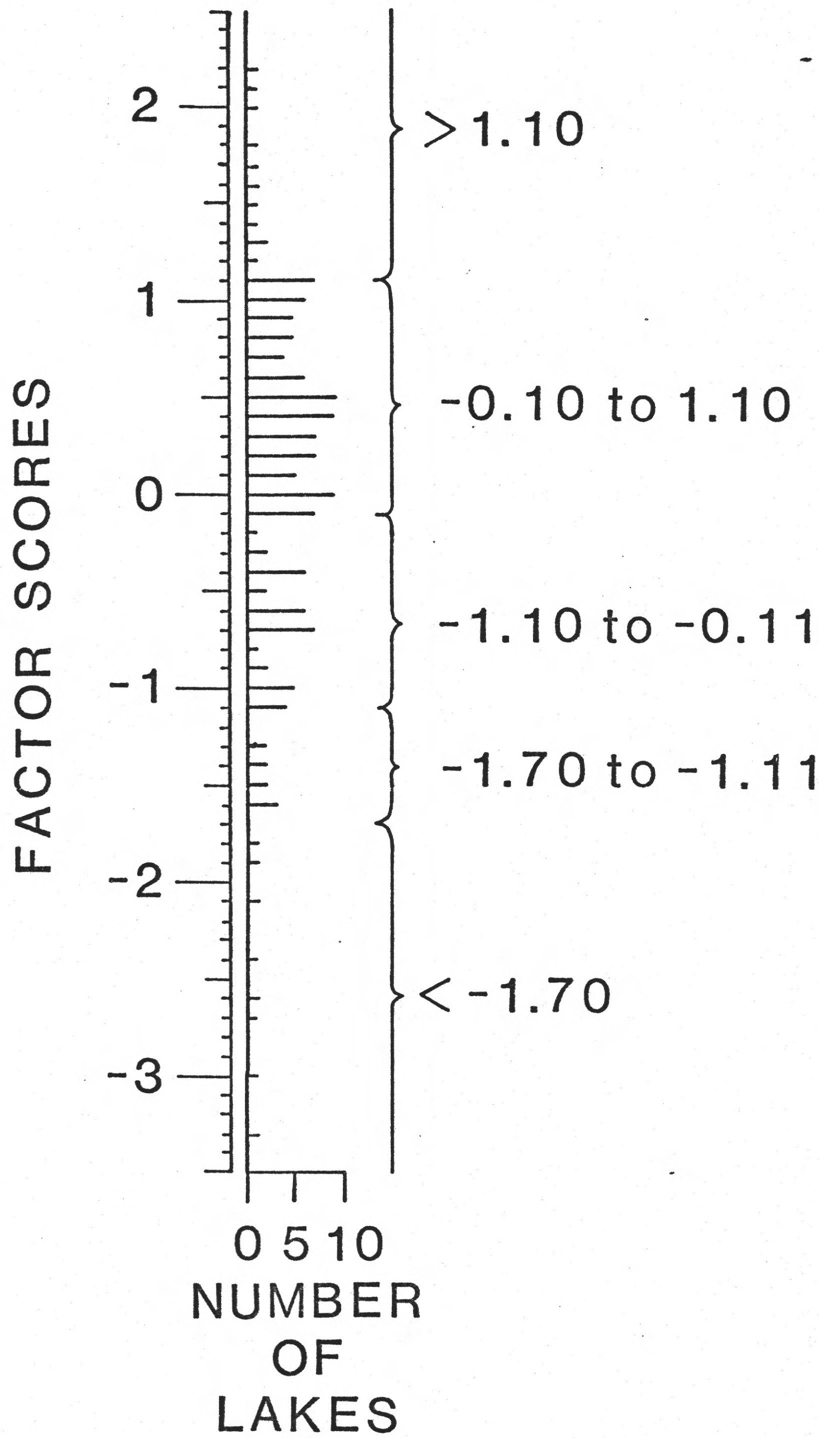


Figure 21.--Relative magnitude of factor scores of component 3.

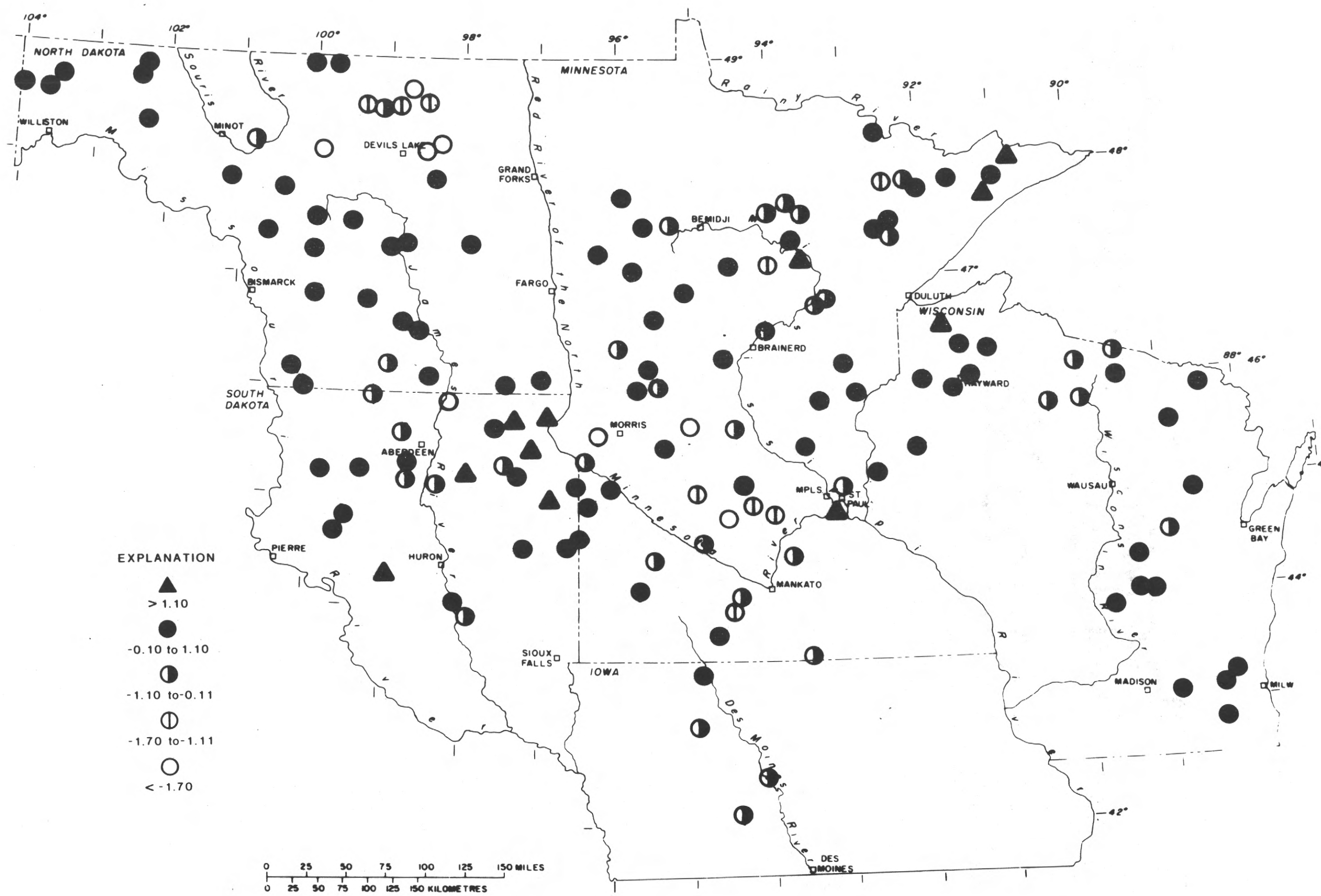


Figure 22.--Factor scores of component 3.

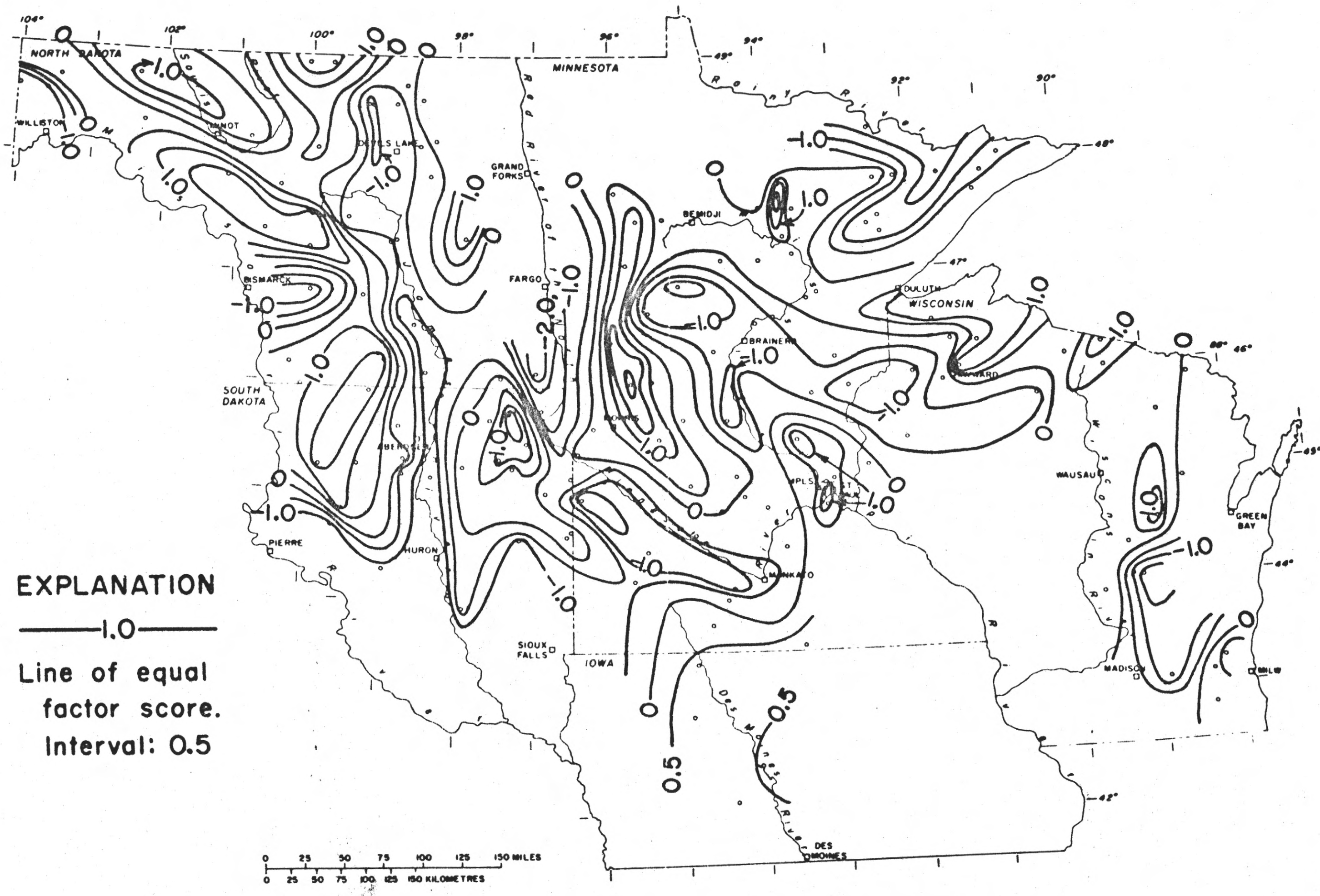


Figure 23.--Factor scores of component 4.

Because components 3 and 4 are both characterized by ground-water flow parameters, it is of interest to examine them together. Clusters of factor scores for groups of components 3 and 4 combined (fig. 24) were delineated, then plotted on a map (fig. 25). The lakes north of Devils Lake in North Dakota are characterized by low regional slope, low local relief and have moderate regional position. A number of lakes on the Missouri Coteau, especially in North Dakota, and on the Alexandria moraine in Minnesota are characterized by moderate regional slope and local relief, and high regional position. Most lakes in northwestern North Dakota occur low topographically but have high regional slope and high local relief. Most lakes in the James River lowland are characterized by moderate regional slope and local relief, but have low regional position. Many lakes in the Minnesota River lowland in Minnesota, particularly those west of Minneapolis, are similar to those north of Devils Lake, North Dakota. Many lakes in Wisconsin are characterized by a moderate regional slope and local relief, although they tend toward the high side, and moderate regional position. Lakes on the east side of the Coteau de Prairies are characterized by high regional slope and local relief and moderate to high regional position.

Because studies of lake-ground water interaction are few, and little is known of the parameters that control the relationship it is difficult to single out an index parameter that

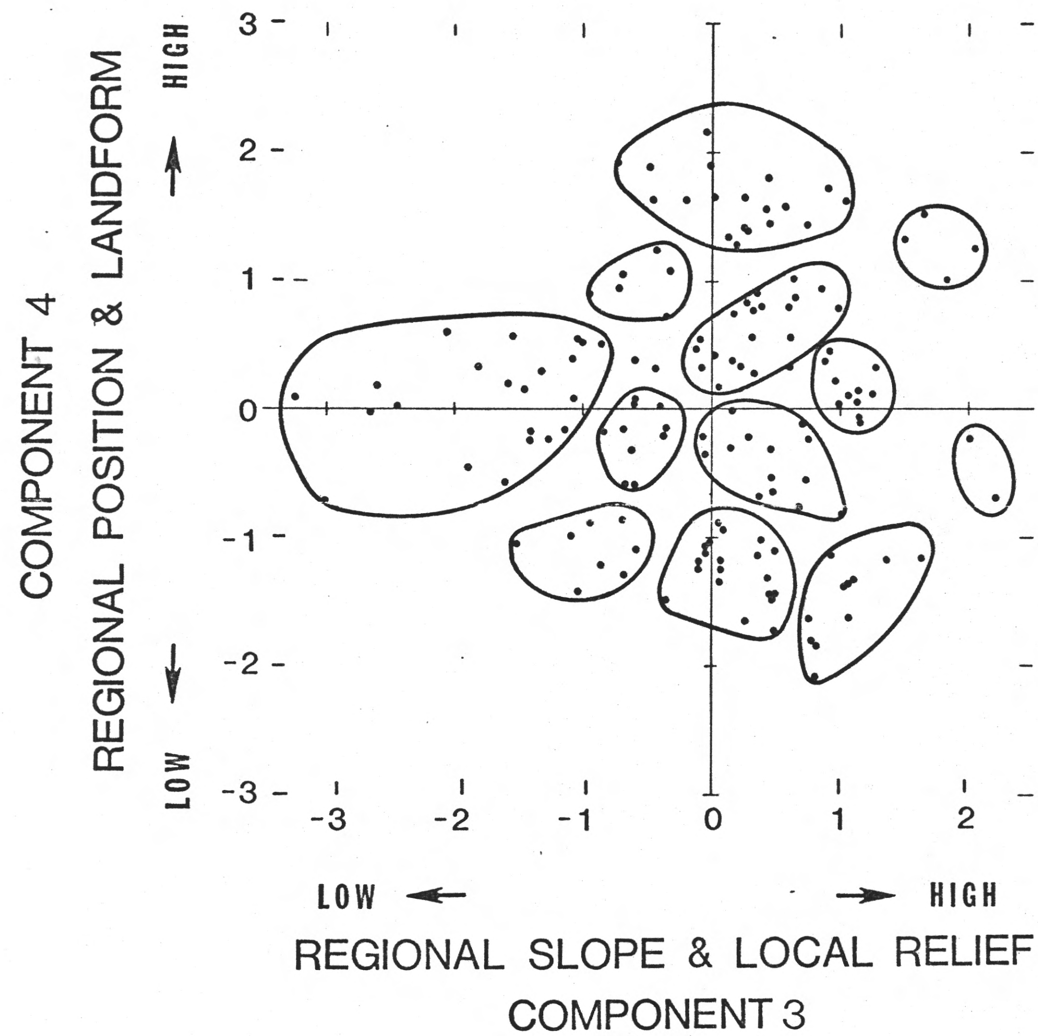


Figure 24.--Factor scores of components 3 vs. 4.

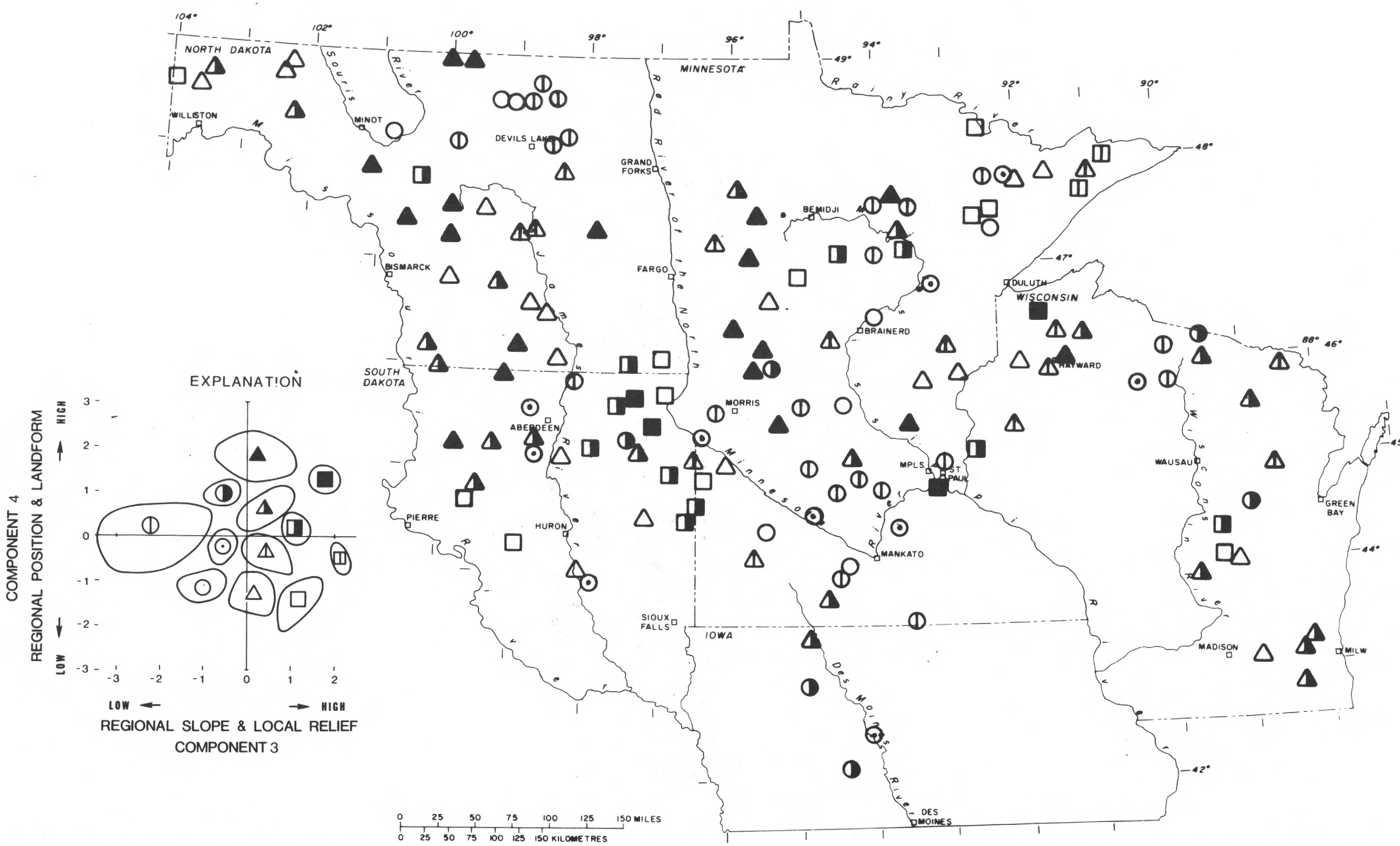


Figure 25.--Areal distribution of various groups of factors 3 and 4 combined--a measure related to regional slope, local relief, and regional position of lakes.

can be used to characterize or classify lakes with regard to their relationship to ground water. This difficulty points to the need for ground-water flow modeling in the vicinity of lakes and subsequent sensitivity analysis of the parameters that control ground-water flow to determine the key parameters that control that flow. In the interim, until such modeling work is done, it seems that regional position is a better quantitative hydrological parameter than landform because, as just one example, end moraines, which are generally considered to be topographic highs, often occur low in the regional setting. For the present then, regional position, regional slope, and local relief must all be considered in a hydrologic classification of lakes.

Component 5

Component 5, characterized by the ratio of a lake's drainage-basin area to its surface area (D/L), is a measure of overland-runoff potential. Drift texture loads moderately high on this axis and is discussed below.

Groupings of D/L ratio data were determined from a graph (fig. 26) and mapped areally (fig. 27). Many lakes in North Dakota, particularly on the Coteau du Missouri, and in South Dakota have greater D/L ratios than other lakes in the study area. Some of the lowest values of D/L ratio are in the lakes

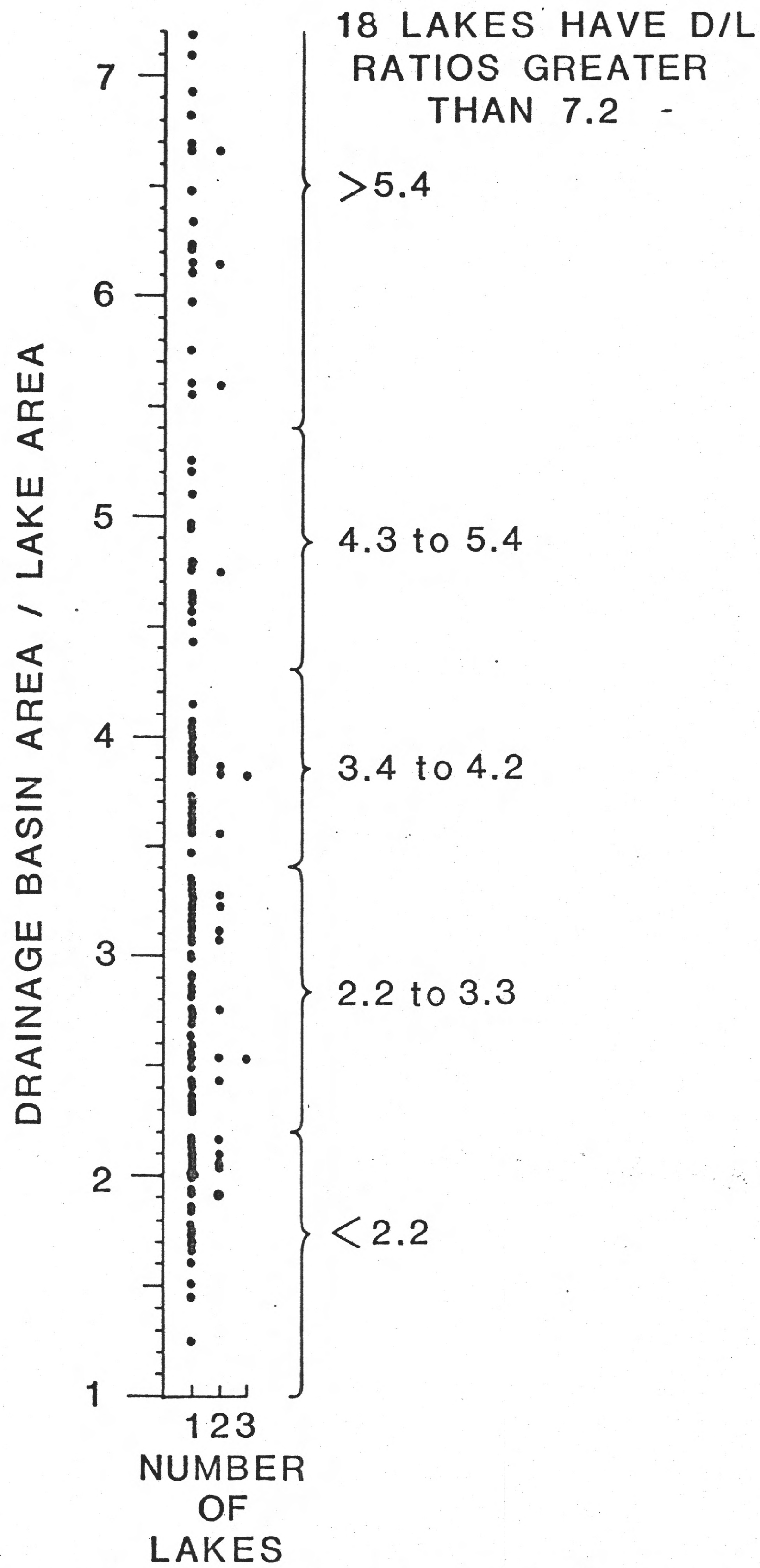


Figure 26.--Relative magnitude of the drainage-basin area/lake area ratio.

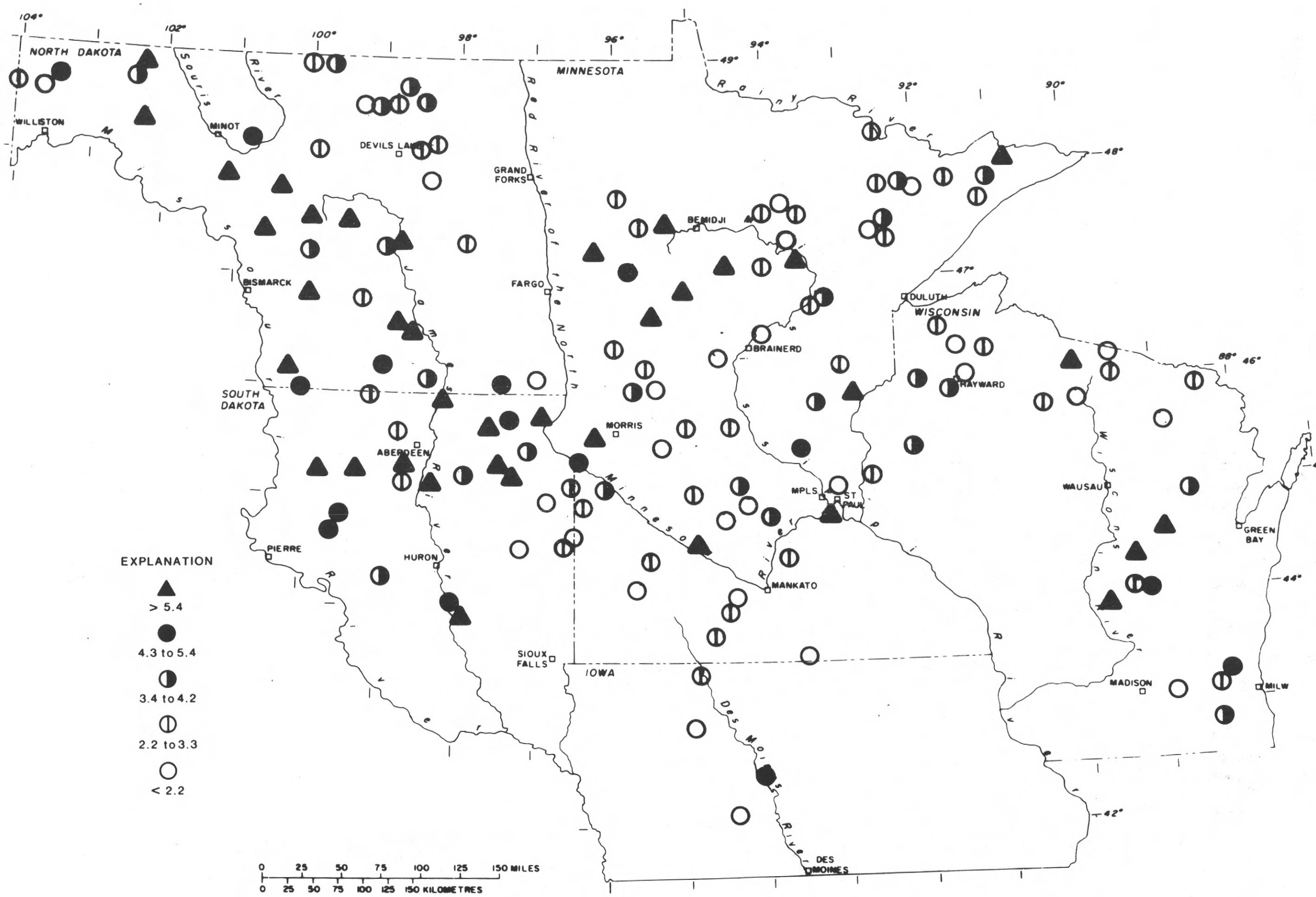


Figure 27.--Characteristics of drainage-basin area/lake area ratio of the study lakes.

of northeastern North Dakota, many in southern Minnesota, northern Iowa, and northern Wisconsin. The map of factor scores (fig. 28) is somewhat less clear than the D/L ratio map. The reason for this is because drift texture has some influence on the factor-score map. It is clear, however, that the D/L ratio must be one of the index parameters.

To analyze the combined surface-water components, 2 and 5, a plot of factor scores for components 2 versus 5 (fig. 29) is used to identify groups, which are mapped in figure 30. Lakes that that have little to no streamflow interchange and larger D/L ratios are common in North Dakota, central Minnesota, and southeastern Wisconsin. Lakes that have larger streamflow interchange and smaller D/L ratios are common along the Mississippi River in north-central and in south-central Minnesota, as well as northwestern and eastern Wisconsin. Most of the sample lakes that have largest streamflow interchange and larger D/L ratios are in northeastern Minnesota and north-central Wisconsin.

Drift texture, from a hydrologic point of view, is one of the most important parameters that control ground-water flow. It is the only principal hydrologic parameter that does not load high onto one of the principal components. This study is concerned basically with the variance of the parameters and, although drift texture is a key hydrologic parameter, its variance

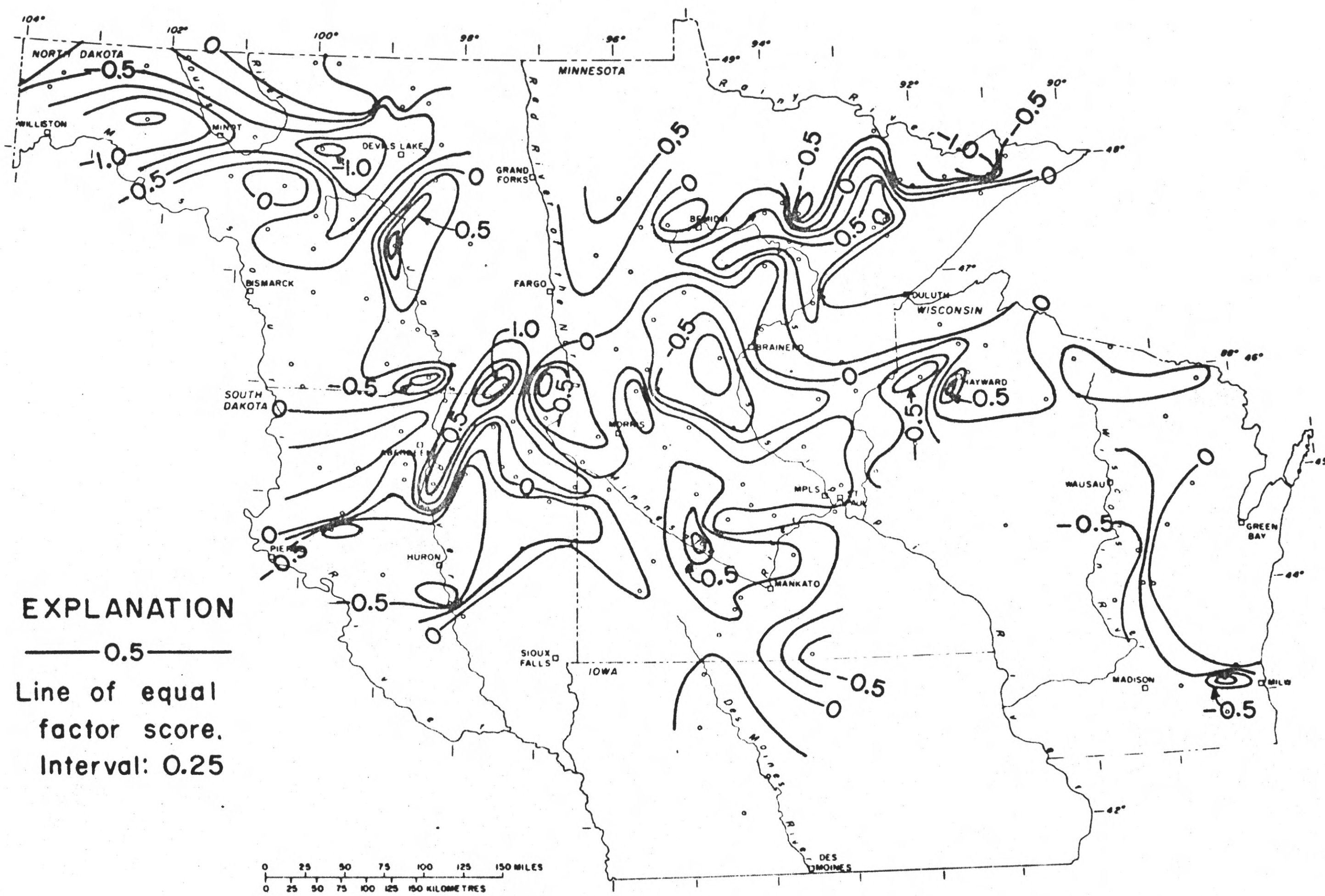


Figure 28.--Factor scores of component 5.

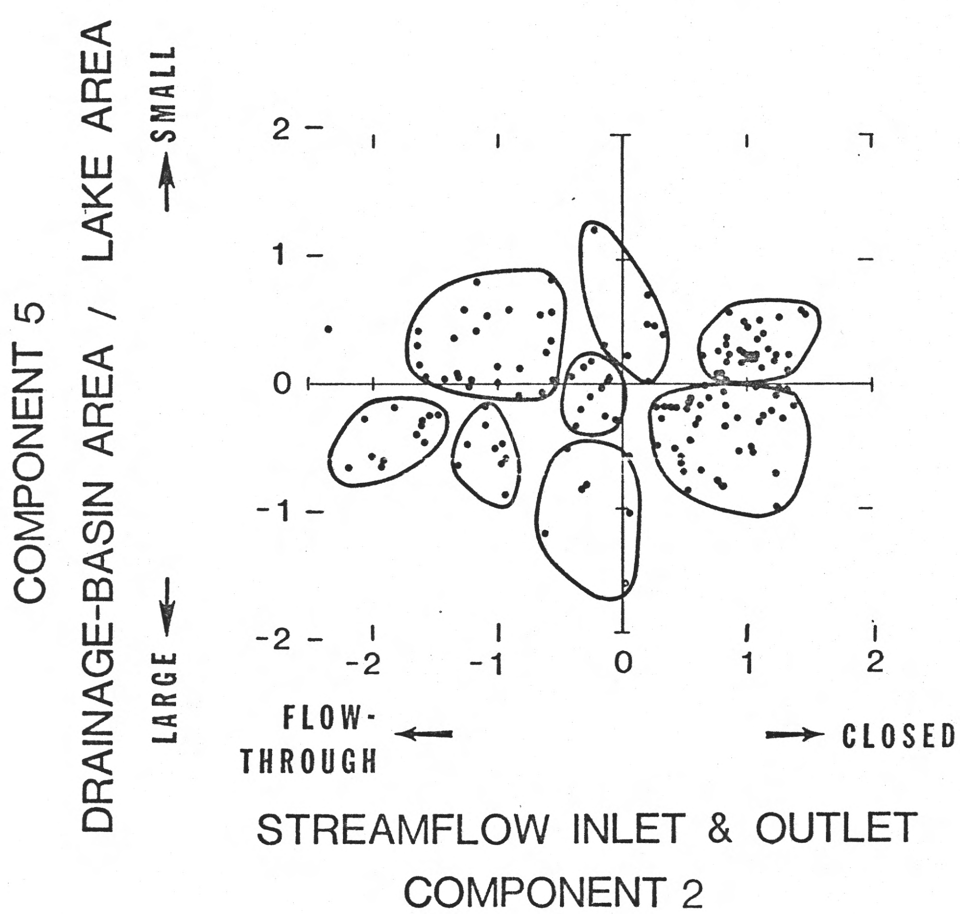


Figure 29.--Factor scores of components 2 vs. 5.

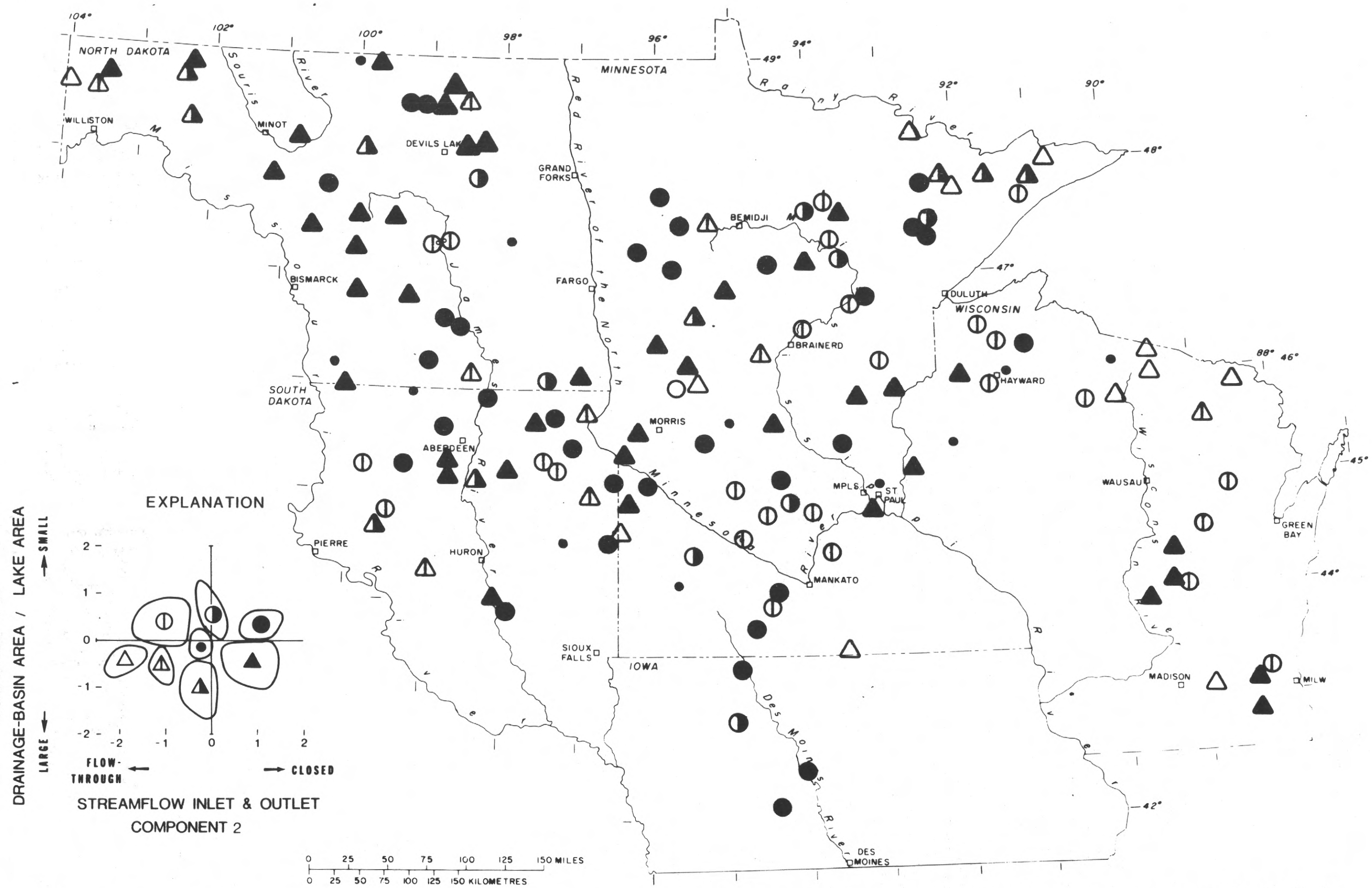


Figure 30.--Areal distribution of various groups of factors 2 and 5 combined--a measure related to streamflow and overland runoff relationships to the study lakes.

distribution is not as important from a statistical point of view. It must nevertheless be part of the classificatory system because of its hydrologic importance. The areal distribution of drift texture is shown on the map of glacial deposits (fig. 5).

Principal component analysis of selected parameters

From the above discussion, the parameters ground-water quality type, drift mineralogy, landform, bedrock hydraulic conductivity, and probably surface-water inlet are the lesser significant parameters. In the discussion of "selection and quantification of parameters" earlier in the report, scaling of the first three (QTP, DM, LF) presented somewhat of a problem. Because of the scaling problem and because they do not seem to be significant in the development of a hydrologic classification of lakes, a principal component analysis was made of the data set with the three "less certain" parameters removed.

To assess the effect of removing these parameters, results of the 5-axis VARIMAX rotation for both the 13-parameter analysis (table 7) and the 10-parameter analysis (table 11) are compared. There is little difference in the results of the two analyses, the most obvious being the reversal of parameters that have high loadings on components 4 and 5. Regional position loads high on component 4 in the 13-parameter analysis (table 7) and it loads high on component 5 in the 10-parameter

Table 11.--Principal component analysis of selected parameters: factor loadings of 5-axis VARIMAX rotation.

Parameter		Component 1	2	3	4	5
REGSLOPE	(RS)	-0.13914	0.03292	0.89278	0.07242	-0.00619
REGPOSIT	(RP)	0.15311	0.00584	0.08429	-0.04871	0.95721
LOCALREL	(LR)	0.29174	0.09049	0.65461	-0.24619	0.18833
LD RATIO	(D/L)	-0.03920	-0.01315	-0.06533	0.96194	-0.05937
INLET	(IN)	0.11803	0.91887	0.07118	-0.04304	0.01629
OUTLET	(OUT)	0.30475	0.83504	0.05935	0.00606	0.01200
DFT TEXR	(DT)	0.38122	0.33971	0.38820	-0.31902	-0.16068
BEDROCK	(BR)	0.80262	0.21101	-0.06540	-0.04969	0.22906
GWQ TDS	(QDS)	-0.88798	-0.15666	-0.14203	0.08926	-0.03815
PE BAL	(P-E)	0.91274	0.16482	0.03245	0.00327	0.03127
Sum of squares		2.64738	1.76273	1.42173	1.10766	1.03642
Percent of variance		26.4	17.6	14.2	11.1	10.4
(cumulative)		(26.4)	(44.0)	(58.2)	(69.3)	(79.7)

analysis (table 11). The reverse is true of D/L ratio. The difference in variance between components 4 and 5 in both analyses is slight and the reversal of positions is not unusual in this type of statistical analysis. Furthermore, it has no effect whatsoever on the goal of the study--to identify independent parameters. To check the effect of removing the 3 parameters, factor scores of components 3 versus 5, the ground-water flow components, were plotted to identify groupings of lakes and then mapped (fig. 31) as in figures 24 and 25. Although there are slight differences in the maps, the results are sufficiently similar to justify removal of the "less certain" parameters. Because their presence or absence does not significantly affect the results of the analysis, the 13-parameter analysis results are considered valid.

Parameters to be considered in classifying the hydrologic setting of lakes

In summary, a preliminary list of parameters to be considered in a hydrologic classification of lakes in the north-central United States are listed in table 12.

Ideally, it would be desirable to present a single map showing lake types for the north-central United States. This is not done because the map obviously would be either very complex or very generalized and in either case could be somewhat

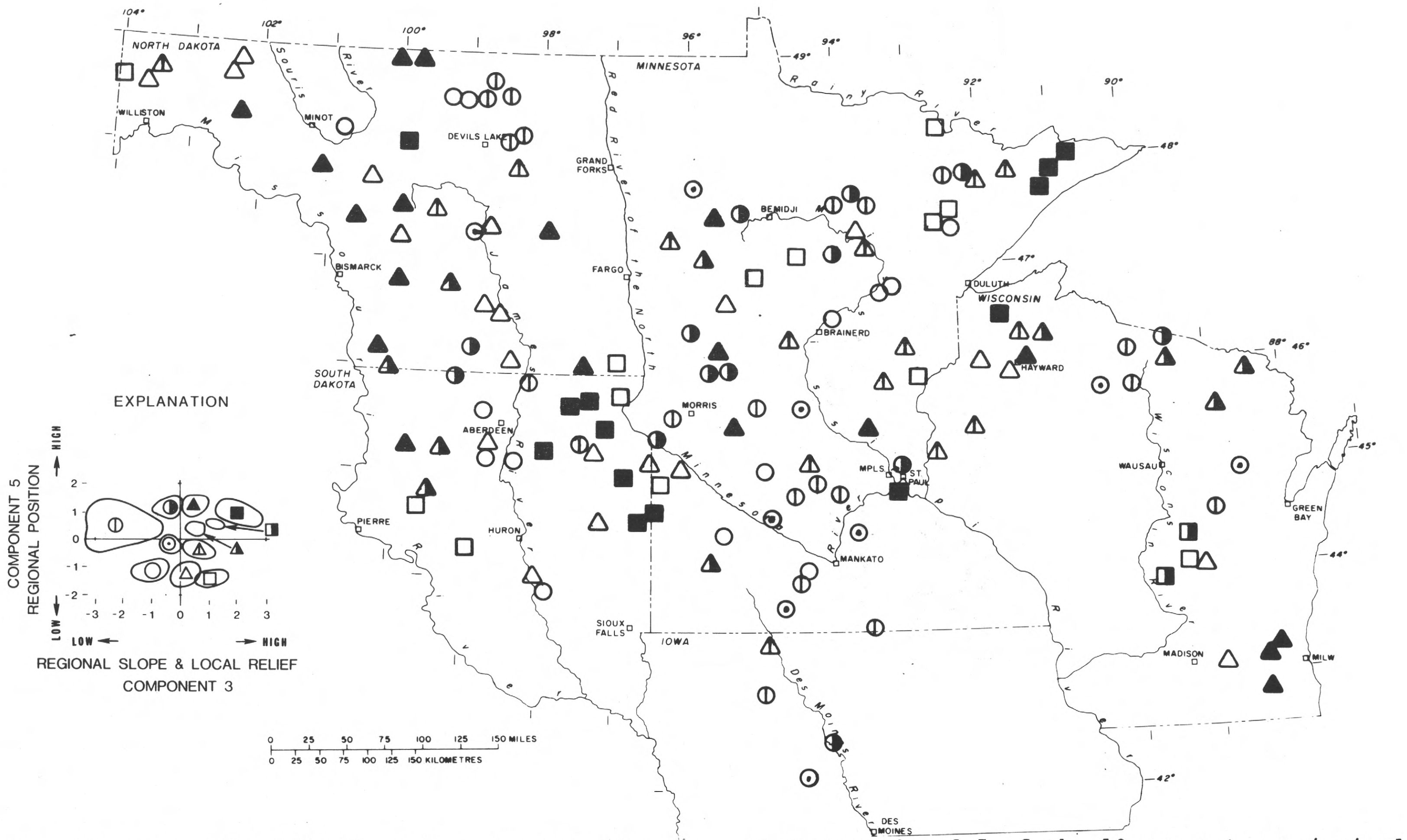


Figure 31.--Areal distribution of various combinations of factors 3 and 5 of the 10-parameter principal component analysis--a measure related to ground-water relationship to the study lakes (compare with figure 26).

Table 12.--Preliminary list of index parameters that should be considered in classifying the hydrologic setting of lakes.

Parameter	Interchange
Water-quality parameter	
Total dissolved solids of ground water	
Lake hydrology parameters	atmospheric water interchange
Precipitation-evaporation balance	atmospheric water interchange
Outlet	surface-water interchange
(Inlet)	surface-water interchange
Drainage basin area - lake area ratio	surface-water interchange
Regional position	ground-water interchange
Regional slope	ground-water interchange
Local relief	ground-water interchange
(Drift texture)	

Parameters in parentheses are of lesser hydrologic or statistical significance.

misleading. It must be emphasized that lakes of different types occur in close proximity in many parts of the north-central United States. This study identifies and maps the areal distribution of the independent variables that must be considered in order to classify lakes hydrologically.

To refine the classificatory system proposed in this report, data on dependent variables are needed that were not available for this study. A classificatory system of lake hydrology would be most commonly used to categorize or predict certain lake-chemistry, lake-level, or lake-water-budget characteristics. If data on these parameters were available, the parameters identified in this study as being important in classifying lakes could be related to the dependent parameters through multiple regression or some similar analysis. This type of analysis would identify those parameters most closely related to the dependent parameter of interest, probably simplifying the classification considerably. It is conceivable that the classification ultimately developed may have several variations depending on the dependent parameter of interest.

That this classification can be carried no further at this point is clear. As stated in the first part of this report, two choices of approach were open to this study. Lakes that have dependent-variable-type data could have been chosen as samples upon which to base a hydrologic classification, but at the expense of statistical requirements such as randomness.

The alternate choice was to satisfy the statistical requirement of using random samples, at the expense of having to work with lakes that might have no data. The second was chosen because it provides a more sound, scientific base for long-term studies.

As expected, only a few of the lakes chosen for this study had chemical, biological, or water-level data, and none had water-budget data. As a beginning step in refining the classification, collection of lake chemistry and lake-level data on selected lakes was begun in 1975 (lake-level gages were established on about 30 lakes). Analysis of these data is beyond the scope of this report because several years of record are needed to even minimally define patterns of lake-level fluctuation. For the present, however, the classification consists of identification and maps of independent parameters which describe the hydrologic setting of lakes in the north-central United States.

Comparison of the hydrologic classification with other classificatory systems

Detailed comparison of the hydrologic classification proposed in this report and the limnological classifications of lakes in the north-central United States of Tarapchak (1973), Bright (1968), Gorham (1971), and Moyle (1956), is beyond the scope of this report. Such an effort would require collection of hydrologic data, as in this report, for the lakes included in their reports, then using some multivariate statistical technique, such as multiple regression analysis to facilitate the comparison. Some general statements, however, are of interest.

The classifications of the authors mentioned above all have one pattern in common. Lakes generally increase in salinity from low values in northeastern Minnesota to high values in southwestern Minnesota and eastern South Dakota. In addition, the water-quality type of lakes in the northeast part of the region is calcium bicarbonate, whereas sulfate and chloride are dominant anions in some of the lakes in the southwest part of the area. There is a general tendency for all the above authors to relate these differences to surficial geology, although Bright (1968) recognized the importance of ground-water quality and flow in interpreting his data.

The use of surficial geology by the above workers to explain the distribution of lake types is expected because it is reasonable and geologic information is readily available. However, ground-water chemistry is clearly a better parameter, because it not only reflects geology but the modification of geological boundaries by ground-water flow (Maclay and Winter, 1967; Winter, 1974). The ground-water flow parameters discussed in this report are "built in" to the ground-water dissolved solids parameter.

By having considerably more data than was available to Bright, it was possible to construct the map of dissolved solids content of ground water for this report (fig. 12), which resembles the patterns of lake-water chemistry shown in the works of Tarapchak, Gorham, Bright, and Moyle. The close interaction of ground water with lakes is demonstrated in the next chapter of this report.

The atmospheric and surface water parameters are probably related to the lake types of the above authors, but the relative influence of those parameters must await further data collection and statistical studies.

NUMERICAL SIMULATION ANALYSIS OF GROUND-WATER FLOW NEAR LAKES

Most studies of the interaction of lakes and ground water have been in response to a need for water budget information for a particular lake or groups of lakes. Some studies have not used observation-well instrumentation to define the ground-water flow systems, but instead assumed certain hydrogeologic conditions. Generally those that used wells either used too few wells, that is they assumed a simple relationship between lakes and ground water that could be determined by one or two wells near the shore line, or they used the saturation approach, that is, as many as 50 wells were installed within the immediate drainage basin of the lake. Some studies assumed that definition of the water table near a lake was sufficient to understand the interactions.

The relation of ground water to prairie potholes in North Dakota was studied by Eisenlohr and others (1972). Although in the early stages of this study the ground-water component was calculated as a residual, a seepage measuring device was subsequently developed. In latter stages of the study, wells were placed near some potholes to determine the relationship of ground-water levels to pothole water levels (Sloan, 1972).

The ground water relation to small lakes in Minnesota was studied by Manson and others (1968) and Allred and others

(1971) by placing one or several wells near the lake shores. The study approach was similar to that of Eisenlohr and others (1972). The general conclusion of these studies was that most of the lakes studied had a net loss to ground water.

In a study of Lake Sallie, Minnesota (McBride, 1969), the ground-water component of the lake-water budget was a primary interest. Wells were placed within the entire drainage basin of Lake Sallie to define lateral ground-water movement. At some sites several closely spaced wells were completed at different depths to define the vertical component of ground-water flow. Quantitative flow nets were then constructed to calculate the quantities of ground water moving into and, in one small area during part of the year, out of Lake Sallie. McBride used a digital modeling technique to define the vertical distribution of hydraulic head (defined later) in the ground water system near a lake. His models are of the flow within the immediate drainage basin of the lake itself--from the local divide to the midpoint of the lake (McBride and Pfannkuch, 1975).

Studies of the interaction between lakes and ground water similar to those conducted by McBride (1969), although not using simulation modeling, were done in Wisconsin by Hackbarth (1968), Hennings (1974), and Possin (1973). The lakes in these studies are the flow-through type--ground water enters one side of the lakes, and the lakes leak to the ground-water system on the other side.

Meyboom (1967, 1966) studied ground-water flow systems in the vicinities of lakes and potholes in the prairie provinces of Canada. Meyboom's work is some of the first to examine the problem of determining ground-water flow systems and the strength and position of ground-water divides or absence of such divides beneath lakes. Studies by Freeze (1969a, 1969b), although of ground-water flow systems within large drainage basins, consider the relationship of those systems to lakes in the prairie provinces of Canada. The scale of the hydrologic sections, however, is such that the detailed flow patterns by the lakes can not be defined.

Williams (1968) examined ground-water flow systems near small depressions in vertical section and the relationship of those flow systems to wetlands in northern Illinois. The techniques of investigation and results of the study were similar to the work of Meyboom. Studies similar to and using the techniques of Meyboom were also done in East Germany by Schumann (1973).

A general overview of studies of the interaction of lakes and ground water, including a literature review, is given by Born and others (1974). They also discuss and present conceptual diagrams of many variations of ground-water flow systems near lakes.

The relationship of ground water to large lakes and reservoirs has been studied by several Russian hydrogeologists.

Zektzer (1973) discussed the role of ground-water flow in water and salt balances of Lake Baikal and the Caspian Sea. Zektzer and Kudelin (1966) discussed the methods of determining ground-water flow to lakes with special reference to Lake Ladoga, USSR. Payne (1970) used radioactive tracer techniques to study the ground-water aspect of the water balance of Lake Challa, Africa. Haefeli (1972) has calculated the ground water inflow into Lake Ontario from the Canadian side. The study made use of the cross-sectional digital ground-water flow model of Freeze (1969b). Van Everdingen (1972), as part of an intensive study of the Lake Diefenbaker, Saskatchewan, site, constructed a series of piezometer nests (closely-spaced group of wells, each well completed at a different depth) before the dam was constructed. The changes in the potentiometric level of different aquifers in the ground-water system were observed as the lake filled. The study showed reversals of flow within some of the aquifer zones as a result of the creation of Lake Diefenbaker. A similar study is under way at the Kendrid Lake site, North Dakota, by Downey and Paulson (1974).

None of the above studies have used simulation modeling to examine the detailed patterns of ground-water flow on both sides and beneath lakes for a wide variety of hydrogeologic settings.

The general principles underlying the interaction of lakes and ground water need to be defined to solve even such basic problems as determining the optimum number and placement of observation wells needed to define ground-water flow systems near lakes.

This second major section of the report examines ground-water flow systems near lakes through digital models simulating ground-water flow in vertical sections. Use of such models makes it possible to examine the general principles of the interaction of lakes and ground water. By varying shape and altitude of the water table in relation to lake levels, lake depth, sediment distribution, size and position of aquifers within the ground-water system, the ratio of horizontal to vertical hydraulic conductivity, the ratio of hydraulic conductivity of aquifers to that of the general ground-water matrix, and depth of the ground-water flow system, it is possible to locate the position and strength (explained later) of the divide that exists under a lake or the percentage of the lake bottom that loses water to the ground water system, if there is no continuous divide. Although the cross sections are hypothetical, the values of the parameters listed above are realistic and representative of physiographic and hydrogeologic conditions in the glacial terrain of the north-central United States. Lakes in this region are of primary interest in this study, but the models used to define the interaction of lakes and ground water and the general principles evolved from this study should have general application.

Ground-water flow systems

Theoretical background

Physical and mathematical concepts basic to ground-water flow model

Certain physical and mathematical concepts, basic to understanding the movement of ground water through porous media, must be understood before the relationship of ground water to lakes can be examined. These concepts are hydraulic potential, hydraulic head, Darcy's Law, equation of continuity, Laplace's equation, and boundary conditions.

The first two concepts deal with the driving force for movement of ground water. The ground-water system is a field which, by definition, is a region at every point of which there corresponds a value of a physical quantity. In the case of ground-water flow fields, the physical quantity of interest is hydraulic potential and, according to Hubbert (1940), must be amenable to measurement at every point in the field and have properties such that flow is always from regions of high to regions of low values. Hubbert showed that hydraulic potential was related to elevation, pressure, and velocity along a given flow line of a fluid, and could be determined as a generalization of Bernoulli's Theorem. Because ground-water flow velocities are low, Bernoulli's Theorem for fluids is (Hubbert, 1940, p. 802)

$$\Phi = gz + \frac{p-p_o}{\rho} \qquad (18)$$

where: Φ = hydraulic potential at any point P in the field

g = acceleration due to gravity

z = elevation of P above a standard horizontal datum

p = pressure at point P

p_o = atmospheric pressure

ρ = density of water

This relationship reduces to

$$\Phi = gh \qquad (19)$$

where: h = height above the standard horizontal datum

after applying

$$p = \rho g(h-z) + p_o \qquad (20)$$

to equation 18 (Hubbert, 1940, p. 793).

Hydraulic head is defined as the quantity

$$h = \frac{\phi}{g}. \qquad (21)$$

Since hydraulic head (h) equals hydraulic potential (ϕ) divided by a constant, it is also a potential quantity and obeys the laws of potential theory. This is an important relationship for ground-water studies because ground-water flow fields can be determined simply from values of hydraulic head which are obtained from measuring water levels in piezometers or wells. In cross section, the elevation of the water level above any arbitrary datum, such as mean sea level, is plotted at the elevation of the piezometer or well opening and the isopotential lines are drawn. In isotropic media, ground-water flow lines are drawn perpendicular to the isopotential lines.

The driving force discussed above is made use of in Darcy's equation, a linear force-flux relation which states that the saturated flow of water through a column of soil is directly proportional to the head difference and inversely proportional to the length of the column. For flow in the x-direction of a homogeneous isotropic system.

$$q_x = -K\frac{\delta h}{\delta x} \qquad (22)$$

where: q_x = specific discharge in $x$direction

K = hydraulic conductivity

h = hydraulic head

x = distance in the direction of flow

$\frac{\delta h}{\delta x}$ = gradient of head

For a two-coordinate system such as vertical sections, a similar expression is written for the z-direction. If the two-coordinate system is anisotropic, hydraulic conductivity must be written $K(x, z)$.

Although Darcy's Law does not hold for large head gradients, the velocities of ground-water flow are sufficiently low for the relationship to apply to nearly all field situations. Also, throughout the report, the viscosity and temperature are assumed to remain constant so that hydraulic conductivity is a property of the medium only.

A force-flux law such as Darcy's equation is only one of three basic elements needed to set up a general distributed-parameter field equation for ground-water flow. The other two elements are a conservation of mass statement and a thermodynamic equation of state. The first of these two elements is discussed below, but the other need not be considered for this study because the ground-water system is assumed to be at steady state.

The equation of continuity expresses the law of conservation of matter. It is derived from the fact that the change in mass stored in a small, elemental, rectangular parallelepiped equals the difference between the mass entering and the mass leaving. For flow of incompressible fluid through rigid porous media the equation has the form (Domenico, 1972).

$$\frac{\delta q_x}{\delta x} + \frac{\delta q_y}{\delta y} + \frac{\delta q_z}{\delta z} = 0 \qquad (23)$$

where: q_x, q_y, q_z = specific discharge in the three coordinate directions

Combining Darcy's Law (22) with the continuity equation (23) for a three-dimensional system results in

$$\frac{\delta}{\delta x}(K\frac{\delta h}{\delta x}) + \frac{\delta}{\delta y}(K\frac{\delta h}{\delta y}) + \frac{\delta}{\delta z}(K\frac{\delta h}{\delta z}) = 0, \qquad (24)$$

which is a combination of the three basic elements of a general distributed-parameter field equation of flow, with the third part handled by the steady-state assumption.

For homogeneous media equation 24 reduces to the Laplace equation

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta y^2} + \frac{\delta^2 h}{\delta z^2} = 0, \tag{25}$$

and for nonhomogeneous media it has the form

$$\frac{\delta}{\delta x}\left[K(x,y,z)\frac{\delta h}{\delta x}\right] + \frac{\delta}{\delta y}\left[K(x,y,z)\frac{\delta h}{\delta y}\right] + \frac{\delta}{\delta z}\left[K(x,y,z)\frac{\delta h}{\delta z}\right] = 0. \tag{26}$$

This study is concerned only with two-dimensional, cross-sectional systems, thus only the x-and z-directions are of interest (the y-dimension is large) and equation (26) becomes

$$\frac{\delta}{\delta x}\left[K(x,z)\frac{\delta h}{\delta x}\right] + \frac{\delta}{\delta z}\left[K(x,z)\frac{\delta h}{\delta z}\right] = 0. \tag{27}$$

Equation (27) is the basic equation of ground-water flow that is used in this thesis for defining the head distribution within the ground-water system, and the relationship of that head to lake levels, under steady-state conditions.

In order to solve equation (27), it is necessary to define mathematically the boundary conditions for the region of interest. A general model for the type of flow section of interest in this report is shown in figure 32. This figure represents an x-z coordinate system that has the origin in the lower left corner and for any point (P) in the system there is a value of hydraulic head. Because there is no flow across the vertical boundaries projected down from the major divide and the major drain, the head gradient is zero ($\frac{\delta h}{\delta x} = 0$) along the two sides of the diagram. It is also assumed that the base of the system is impermeable, thus $\frac{\delta h}{\delta z} = 0$. The pressure along the water table is atmospheric and, because of steady-state conditions, head (h) is a function of x. The hydraulic conductivity (K) values are different for each geologic unit within the system. For this study, the size and position of zones of high hydraulic conductivity (termed aquifers for convenience) are varied, and the degree of anisotropy is varied.

Numerical solutions of the ground-water flow equation

Recent development of numerical methods as approximate solutions to the equation of ground-water flow (Remson and others, 1971; Trescott and Pinder, 1975; Prickett and Lonnquist, 1971: Freeze, 1969b) has made it possible to simulate complex ground-water flow systems. Although a variety of numerical techniques are being developed, the finite-difference method has been well documented and has been used successfully in a number of studies. The finite-difference models used herein have

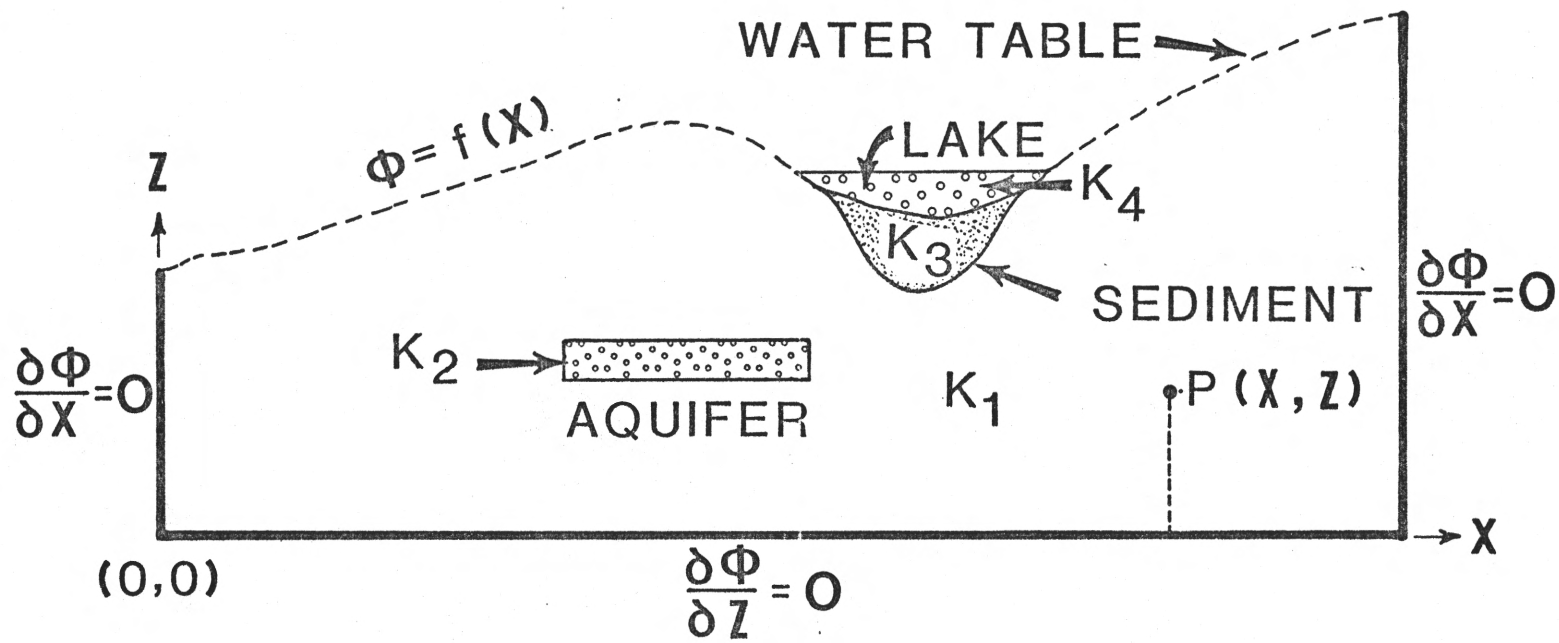


Figure 32.--Physical meaning of mathematical expressions for boundaries of hydrologic-section models.

been developed by the U.S. Geological Survey over a number of years (Pinder and Bredehoeft, 1968; Pinder, 1970; Trescott, 1973; and Trescott and Pinder, 1975).

A ground-water flow field is a continuum of points with assignable hydraulic head. To solve the ground-water flow equation by finite-difference methods, it is necessary to replace this continuum of points by a finite set of points arranged in a grid. A linear, finite-difference equation, an algebraic expression, is then written for each grid point in the field, replacing the continuous derivatives of equation (27). The value of head (h) calculated at each point is considered to be representative for that small area of the field around the point. The solution of the ground-water flow equation for the entire field of interest requires simultaneous solution of all the finite-difference equations written for the grid. Thus, the solution of head (h) for a field containing n nodes requires simultaneous solution of n linear finite-difference equations. For the simulations of this study, the number of nodes ranged from less than 1,000 to 1,800.

The finite-difference equation that approximates the ground-water flow equation (27) for this report, that is, vertical cross sections and steady-state conditions has the form

$$\frac{1}{\Delta x_j}\left\{\left[K_{xx}(i,j+\tfrac{1}{2})\frac{(h_{i,j+1}-h_{i,j})}{\Delta x_{j+\frac{1}{2}}}\right]-\left[K_{xx}(i,j-\tfrac{1}{2})\frac{(h_{i,j}-h_{i,j-1})}{\Delta x_{j-\frac{1}{2}}}\right]\right\}$$

$$+\frac{1}{\Delta z_i}\left\{\left[K_{zz}(i+\tfrac{1}{2},j)\frac{(h_{i+1,j}-h_{i,j})}{\Delta z_{i+\frac{1}{2}}}\right]-\left[K_{zz}(i-\tfrac{1}{2},j)\frac{(h_{i,j}-h_{i-1,j})}{\Delta z_{i-\frac{1}{2}}}\right]\right\}=0 \qquad (28)$$

where:

Δx_j = space increment in the x-direction for column j

Δz_i = space increment in the z-direction for row i

i = index in the z-direction

j = index in the x-direction

h = hydraulic head

K_{xx} = hydraulic conductivity in the x-direction

K_{zz} = hydraulic conductivity in the z-direction

To make solution of equation 28 more amenable to the numerical method used, the notation can be simplified to the form

$$F_{i,j}(h_{i,j+1} - h_{i,j}) - D_{i,j}(h_{i,j} - h_{i,j-1}) + H_{i,j}(h_{i+1,j} - h_{i,j})$$

$$-B_{i,j}(h_{i,j} - h_{i-1,j}) = 0 \qquad (29)$$

where

$$F_{i,j} = \left[\frac{2K_{xx[i,j]} K_{xx[i,j+1]}}{K_{xx[i,j]} \Delta x_{j+1} + K_{xx[i,j+1]} \Delta x_j} \right] / \Delta x_j \qquad (29a)$$

$$D_{i,j} = \left[\frac{2K_{xx[i,j]} K_{xx[i,j-1]}}{K_{xx[i,j]} \Delta x_{j-1} + K_{xx[i,j-1]} \Delta x_j} \right] / \Delta x_j \qquad (29b)$$

$$H_{i,j} = \left[\frac{2K_{zz[i+1,j]} K_{zz[i,j]}}{K_{zz[i,j]} \Delta z_{i+1} + K_{zz[i+1,j]} \Delta z_i} \right] / \Delta z_i \qquad (29c)$$

$$B_{i,j} = \left[\frac{2K_{zz[i,j]} K_{zz[i-1,j]}}{K_{zz[i,j]} \Delta z_{i-1} + K_{zz[i-1,j]} \Delta z_i} \right] / \Delta z_i \qquad (29d)$$

To better understand the meaning of the solution scheme of equation (29), it is convenient to refer to a stencil, a diagram depicting the pattern of points and the appropriate numerical coefficients involved in the difference operator (fig. 33a). For solving a problem, it is necessary to use many variations of the stencil shown in figure 33a and the corresponding difference operator, depending on the position of

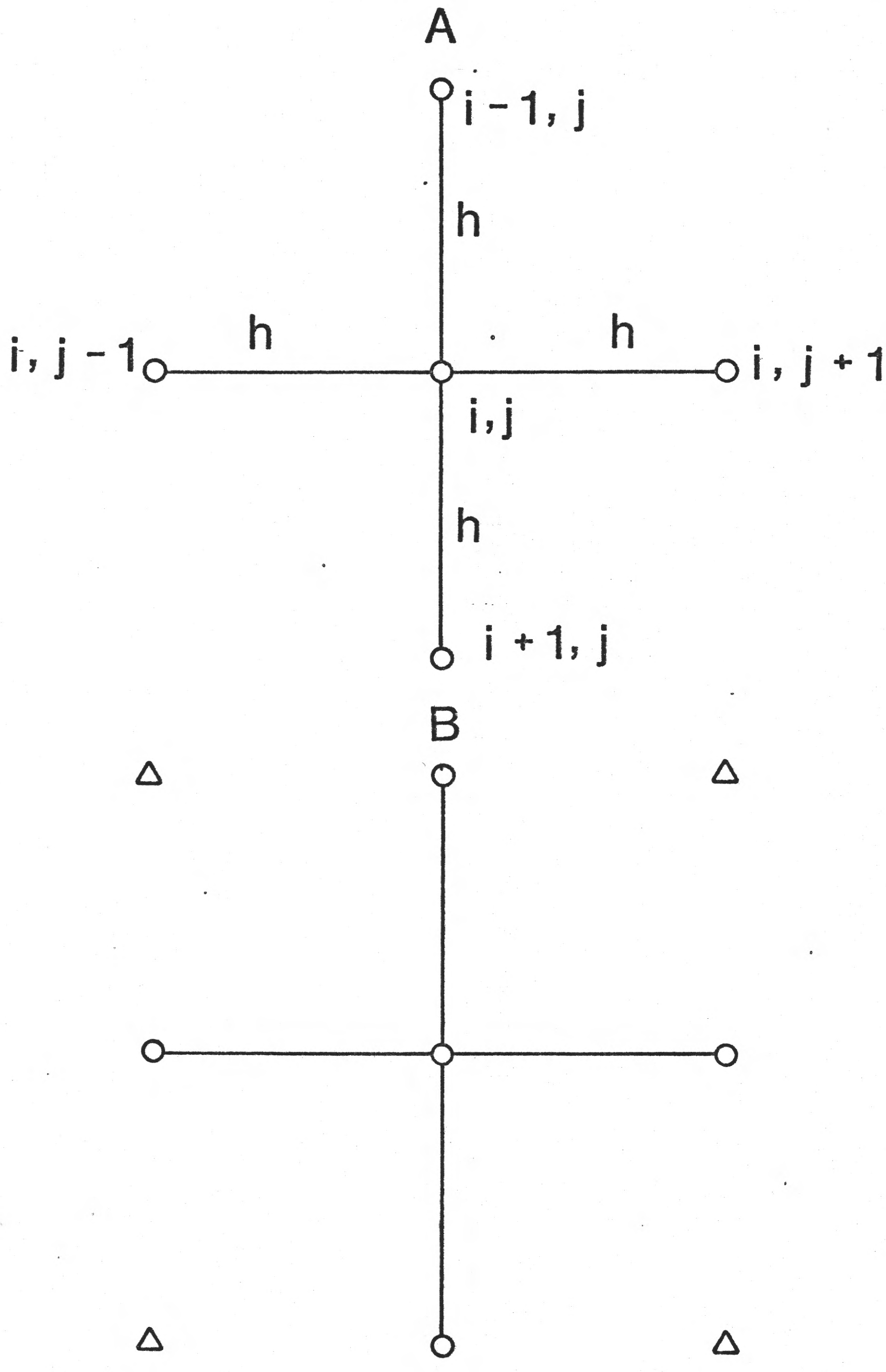


Figure 33.--Finite-difference stencils.

the point in the grid system. Freeze (1969b) discusses the many variations of stencils and corresponding variations in the ground-water flow equation for nodes along boundaries of no flow, constant head, and changes in grid mesh. The models used in this study, however, handle variable grid spacing and no-flow boundaries automatically.

Equation (29) may be rearranged and expressed as

$$Bh_{i-1}+Dh_{j-1}+Eh+Fh_{j+1}+Hh_{i+1} = 0 \tag{30}$$

where

E = -(B+D+F+H)

B,D,F,H = coefficients calculated in equations 29 a-d

h = hydraulic head

or in matrix form as

$$\bar{\bar{A}}\bar{h} = 0 \tag{31}$$

where

$\bar{\bar{A}}$ = matrix of coefficients

$\bar{h}$ = vector of hydraulic head

A number of algorithms are available for solving linear finite-difference equations using matrix algebra. Two commonly used in ground-water flow problems are the ADI (alternating direction implicit) and SOR (successive overrelaxation) methods. A third, SIP (strongly implicit procedure), is being used increasingly because of certain advantages over the other two, such as increased efficiency. Detailed development of these iterative methods is readily available in the hydrogeologic literature (Remson and others, 1971, Trescott and Pinder, 1975, and Cooley, 1974, for all three methods; and Freeze, 1969b, for SOR); therefore, the following is only a brief discussion of some of their distinguishing characteristics.

All iterative techniques for solving linear finite-difference equations are concerned with convergence. That is, the goal of the solution technique is to calculate the head distribution for all nodes in the region of interest (an iteration) repeatedly until the change in head at all nodes from one iteration to the succeeding iteration is less than some pre-set value.

The successive overrelaxation method (SOR) improves head values one row (or column) at a time. Convergence is slow using a formula such as the line Gauss-Seidel formula, so an acceleration parameter is used that increases the rate of convergence by calculating an extrapolated value of head (overrelaxation). In spite of the acceleration parameter, in addition to applying a two-dimensional correction (Aziz and Settari, 1972), Trescott and Pinder (1975) found that SOR was generally inefficient for a cross-sectional problem if the direction of solution is not chosen properly. Although Freeze (1969b) used SOR successfully, many of his cross-sectional simulations required up to several hundred iterations. SOR was not used in this study, but this brief discussion is included here because the work of Freeze is referred to extensively in this report.

The alternating-direction implicit method (ADI) is characterized by two sets of equations, which are modifications of equation (29). Each iteration consists of solving an equation for head along rows, then solving another for head down columns. This alternating-direction procedure continues until convergence. It has been found that a series of iteration parameters greatly speeds convergence. Rather than use a single acceleration parameter as in the SOR method, the series of parameters is used in a cyclic manner, beginning with the minimum. The iteration parameters are calculated from

$$M_\ell = \omega_\ell (B+D+F+H) \qquad (32)$$

where

ω = acceleration parameter

B, D, F, and H = coefficients calculated in equations 29 a-d.

The minimum acceleration parameter is defined by

$$\omega_{min} = \text{Min} \sum_{i}^{N_z} \sum_{j}^{N_x} \left[\frac{\pi^2}{2N_x^2} \frac{1}{1 + \left(\frac{K_{zz[i,j]}(\Delta x_j)^2}{K_{xx[i,j]}(\Delta z_i)^2} \right)} , \right.$$

$$\left. \frac{\pi^2}{2N_z^2} \frac{1}{1 + \left(\frac{K_{xx[i,j]}(\Delta z_i)^2}{K_{zz[i,j]}(\Delta x_j)^2} \right)} \right] \qquad (33)$$

and the maximum by

$$\omega_{max} = \begin{cases} 1 & [K_{xx} \approx K_{zz}] \\ 2 & [K_{xx} >> K_{zz} \text{ or } K_{zz} >> K_{xx}] \end{cases} \qquad (34)$$

Depending on the problem, generally between 5 and 10 iteration parameters are used.

Stone (1968) devised the strongly implicit procedure (SIP) as a means of solving equations of the form of equation (31) more efficiently than either SOR or ADI, for most problems. He states that ADI is more implicit than SOR because it is more closely related to direct methods of solving the equations, such as Gaussian elimination. The SIP is even more closely related to direct methods, thus the term "strongly implicit."

The procedure consists of adding a matrix (B) to (A) such that equation (31) becomes

$$(A+B)\bar{h} = O+\bar{\bar{B}}\bar{h} \qquad (35)$$

which leads to the iteration scheme

$$(A+B)\bar{h}^{n} = O+\bar{\bar{B}}\bar{h}^{n-1}$$

The coefficient matrix (A) is constructed from the five nodes shown in figure 33a and has the schematic form shown in figure 34a. Matrix (B) is formed by considering the diagonal nodes shown as triangles in figure 33b two at a time, which modifies matrix (A) to the form shown in figure 34b. This new matrix (B) is much more easily factored into sparse lower and upper triangular matrices than is matrix (A). The nodes represented by triangles in figure 33b are considered alternately in the solution process, that is, the lower left and upper right are considered together, then the upper left and lower right. The details of developing SIP are given in Remson and others (1971).

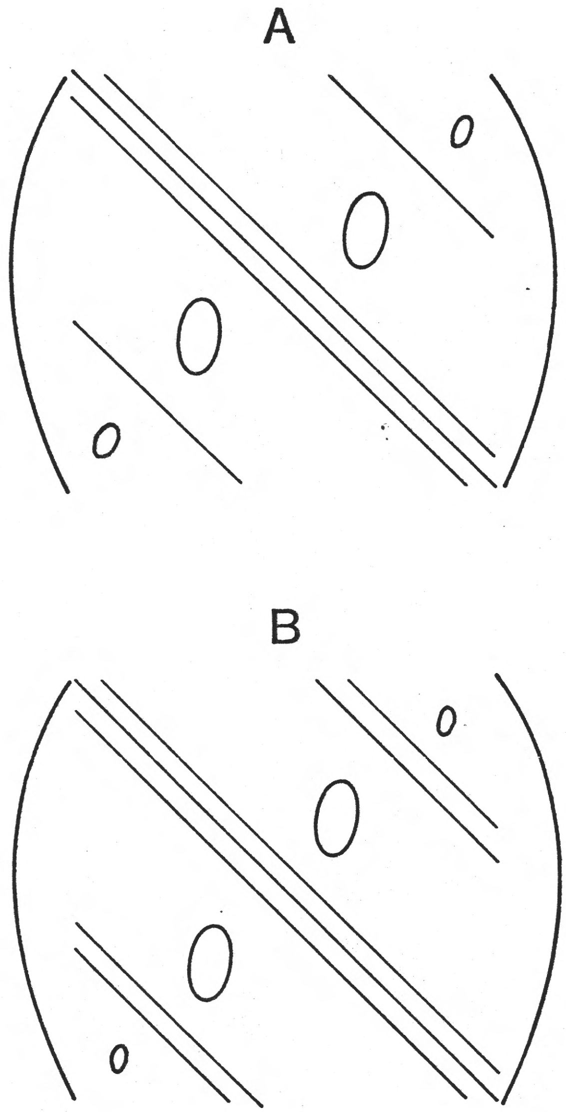


Figure 34.--Generalized form of coefficient matrix of ground-water flow equation:

(a) Form used in most solution techniques such as ADI.

(b) Form used in SIP.

Iteration parameters are used in the SIP procedure and equations for their computation, depending on the problem, are given in Stone (1968). In the cross-sectional problem of this study, ten iteration parameters were used.

The computer programs used for this study were developed by Trescott and Pinder (1975). The alternating-direction-implicit method (ADI) was used to calculate the distribution of hydraulic potential within some of the cross-sectional simulations but, as more complex models were considered, it was necessary to turn to SIP. As an example of SIP's greater efficiency, some checkruns, where both numerical techniques were used on the same section, ADI did not converge after 99 iterations, whereas SIP needed as few as 16. It was found that SIP adequately handled all conceivable simulations except one, where extremely high hydraulic conductivity contrasts existed in close proximity, both in the vertical and horizontal directions.

Models of Hydrologic Sections

Only two comprehensive studies have been published on modeling ground-water flow in vertical section (Toth, 1963; Freeze, 1969b). Subsequent work based on the report by Freeze has been done by Haefeli (1972) and by Freeze (1969a). These studies have concentrated on ground-water flow patterns in vertical section in large basins and, although lakes occur along some of the sections modeled, the scale of the sections were such that detailed flow patterns in the immediate vicinity of lakes could not be shown.

Toth's work is based on an analytical solution to the ground-water flow equation; whereas, the work of Freeze is based on a numerical solution. Freeze (1969b) discusses the advantages and disadvantages of each approach. In the analytical-solution approach Toth had to make some rigid simplifying assumptions: The field had to be approximated by a rectangle. The ratio of horizontal to vertical hydraulic conductivity must be 1; that is, the porous media must be isotropic. The water table was simulated by superimposing a sine curve on a low regional slope. The analytical approach requires that equations be set up for each individual case considered according to the boundary configurations. There is no *general* analytical solution to the ground-water flow equation when the boundaries of the system are changed.

The advantages of the numerical solution as outlined by Freeze (1969b), on the other hand, are as follows: All of the restrictions of the analytical solution are removed by the numerical solution. The true shape of the water table can be approximated; that is, it is not restricted to the equation for a sine curve. Equation 27 can be handled in the numerical solution; that is, anisotropic and heterogeneous conditions can be simulated. The numerical solution is general; only one mathematical derivation and one computer program need be written that can handle any water table and geologic configuration.

Both basic studies by Toth and Freeze assume no-flow boundaries at the base of the system and no-flow boundaries beneath the regional topographic highs and lows. In addition, the water table is at steady state.

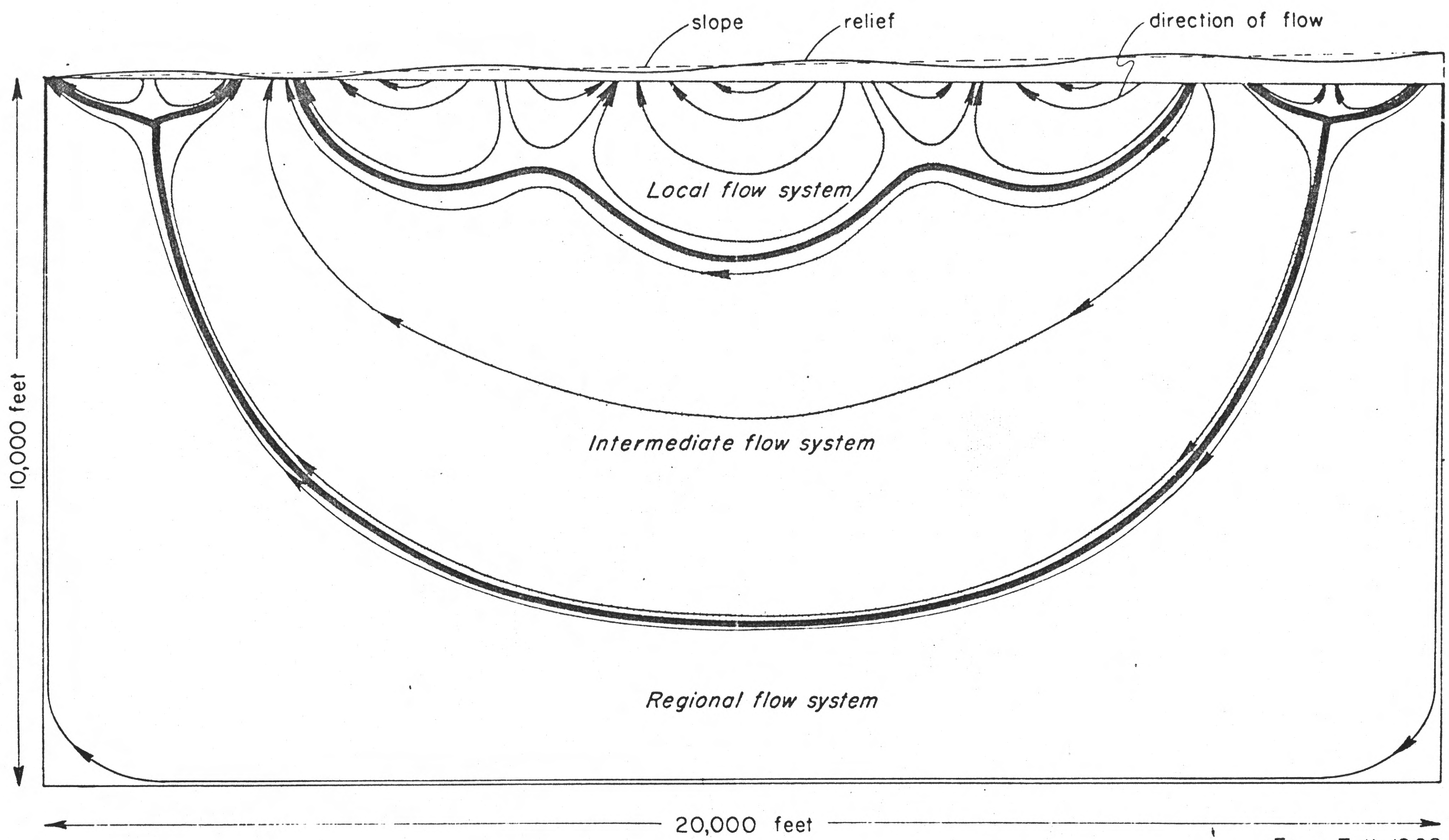
An example of one of Toth's cross-sectional ground-water flow simulations is shown in figure 35. The simulation is of a thick ground water system that has low regional slope and low local relief. Although Toth presents many such diagrams showing the effects on ground-water flow of many combinations of slope, relief and system thickness, this diagram best depicts his delineation and definition of local, intermediate, and regional flow systems.

An example of the flexibility of numerical models in closely simulating field conditions is shown in figure 36 (Freeze, 1969b). The diagram shows a variable water table in different parts of the section, anisotropic media, and zones of different hydraulic conductivities.

Digital models of ground-water flow near lakes

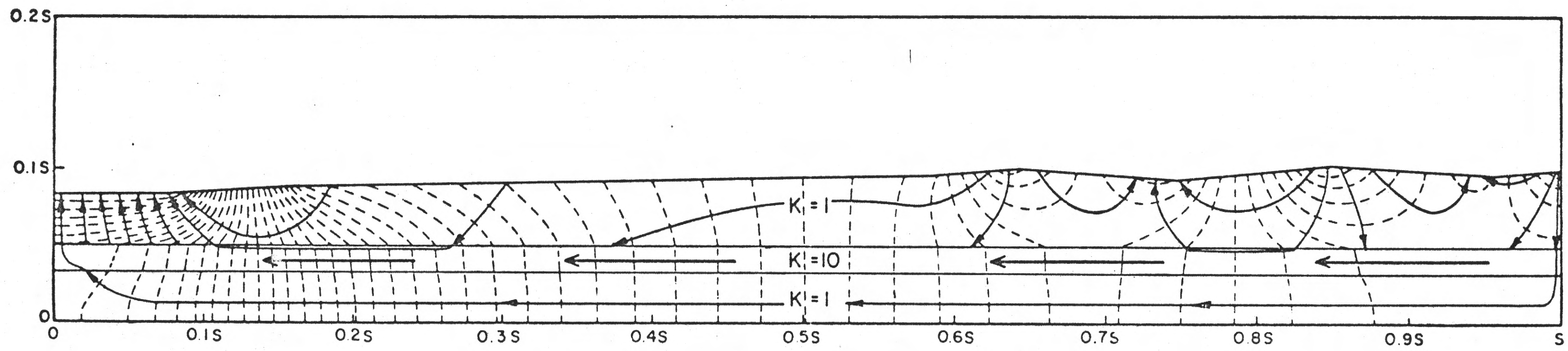
Practical Considerations and Assumptions of this Study

The assumptions on boundary conditions for this study relate closely to those of Freeze (1969b); that is, (1) the base of the system is considered to be impermeable. (2) The vertical no-flow boundaries at each end of the section are considered to be con-



From Toth,1962

Figure 35.--Local, intermediate, and regional ground-water flow systems determined from an analytical solution to the ground-water flow equation.



From: Freeze, 1969b

Figure 36.--Ground-water flow systems determined from a numerical solution to the ground-water flow equation.

trolled by the major topographic high and the major drain on the ground-water system. (3) The upper boundary of the flow system is the water table and it is at steady state. In the model used for this study, the steady-state water table is specified by assigning infinite storage to the water-table nodes. Freeze discusses thoroughly the justification for these boundary conditions and particularly the assumption that vertical no-flow boundaries exist at the two end positions. As he points out, many workers have found that vertical no-flow boundaries do not occur beneath every topographic high. This is undoubtedly true, but the assumption that they do exist beneath the major high and low is considered to be valid.

Although the sections discussed in this report are hypothetical, they closely approximate field conditions for lakes in the glaciated north-central United States. The variations in height of the water table relative to lake level are reasonable for this region. Although the hydraulic conductivities that have been assigned to the geologic matrix in general are representative of silty till in this region, in the simulations of this study, the relative values of hydraulic conductivity are the controlling factor, not the actual values. The simulated sections apply equally to geologic settings that have the given relative hydraulic conductivities, whether the setting is sand within till or gravel within sand. The hydraulic conductivities assigned to the zones of higher relative hydraulic conductivity vary from 100 to 1,000 times greater than the surrounding geologic materials. There are, of course, some zones within the glacial drift that have hydraulic conductivity greater than 1,000 times that of the surrounding materials, but the values chosen are representative of a great many geologic settings. For convenience in this report, the geologic matrix is referred to as "till", and the zones of relatively higher hydraulic conductivity are referred to as "aquifers".

The hydraulic conductivity assigned to the lake sediments are as low as can be assigned with the computer program used. In most lakes, the littoral zone is free of fine grained sediment; therefore, in the models that contain lake sediments, the sediments purposely were not extended to the shore line.

The lake water was simulated by assigning very high hydraulic conductivity values to the nodes within lakes. This proved to adequately simulate lakes because the calculated head values were identical to the initial head values within each lake simulated in this study.

The least well known parameter needed in the ground-water flow model is the ratio of horizontal to vertical hydraulic conductivity (K_h/K_v). This ratio has generally not been determined in glacial terrain and it is a point around which much of the discussion of the following section of this report centers. It will be shown that it is critical to the interaction of lakes and ground water. If the ratio is less than 100, lakes rarely lose water to the ground-water system under the conditions simulated in the models and if the ratio is greater than 1,000, lakes lose water under many conditions. If the ratio is between 100 and 1,000, other parameters that control ground-water flow become more important in the relationship of ground water to lakes.

Weeks (1969) measured K_h/K_v within a single outwash sand and gravel aquifer in Wisconsin and found the ratio to be not more than 20. Vecchioli and others (1974) calculated K_h/K_v for part of the drift section of Long Island, New York as approximately 500. Bennett and Giusti (1971), in using electric-analog techniques to study ground-water flow in the coastal plain of Puerto Rico, show that the ratio had to be 1,000 for the simulations to match field data. The importance of this parameter in ground-water flow modeling has been recognized because it is the topic of recent papers by Freeze (1972) and Gillham and Farvolden (1974). Freeze concludes that K_h/K_v values of 100 or larger are not uncommon, and in fact were necessary to correlate simulations with field measurements in his study of the Old Wives Lake basin in Saskatchewan (Freeze, 1969a). Gillham and Farvolden (1974) tested the sensitivity of the ratio and found it to be particularly important in areas of recharge and discharge.

Considering the above studies, the models discussed in the following section of this report use both 100 and 1,000 as lower and upper examples.

Ground-water Flow Diagrams

Much of the remainder of this report is a discussion of ground water flow systems near lakes, which are best illustrated by diagrams showing ground-water flow in cross-section. Understanding of the important features of this type of diagram is basic to the discussion of the simulation results. A ground-water flow diagram shows the distribution of hydraulic head within the ground water system (figure 37). After the head is calculated for each node, lines of equal head, equipotential lines, are drawn. Ground-water flow lines are drawn perpendicular to the equipotential lines, if the media is isotropic. Equipotential lines are the projections of water-table contours into the subsurface.

To give the most accurate picture of ground-water flow systems, and to estimate relative quantities of ground water moving through various parts of the ground-water reservoir, a flow net should be drawn. Construction of a quantitative flow net in a medium that is isotropic ($K_h/K_v = 1$), requires that flow lines and equipotential lines be orthogonal such that curvilinear squares are formed (Harr, 1962).

If the medium is anisotropic ($K_h/K_v > 1$) the squares are deformed according to the degree of anisotropy. In this study, K_h/K_v is either 100 or 1,000. To compensate for the anisotropy, equation 27 can be transformed to the LaPlace equation by

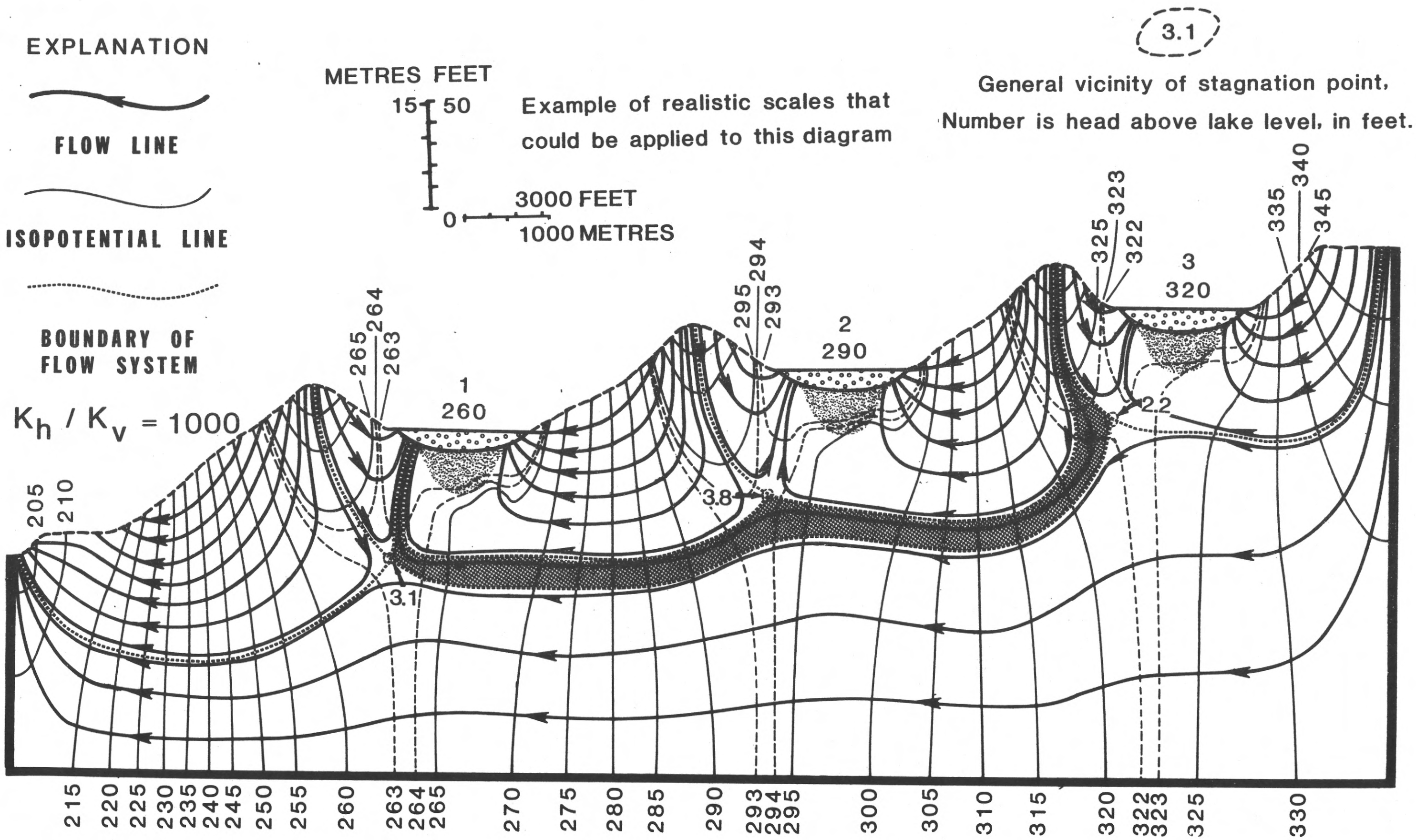


Figure 37.--A quasi-quantitative flow net of ground-water flow near lakes in a multiple-lakes system that does not contain aquifers.

a transformation of coordinates. The scale factor used to do this is the square root of the ratio K_h/K_v (Harr, 1962, p. 29).

A requirement for constructing precise flow nets is that the section have no vertical exaggeration (Van Everdingen, 1963). Vertical exaggeration is usually used for convenience in illustrating important details of geology and ground-water movement that would be lost if the true scales were used. The sections discussed in this study have a vertical exaggeration of 80:1. Because the graphical correction of Van Everdingen (1963) was not applied to the sections in this study, they cannot be used as exact quantitative diagrams. The diagrams of Van Everdingen (1963) provide good examples of the effect of the correction on the ground-water flow fields. The effect basically is that the flow systems are much more rectangular and the local systems extend deeper into the ground-water reservoir than in flow sections not corrected for vertical exaggeration. Although the flow nets drawn for this study are not quantitatively precise, they do show relative proportions of flow in different parts of the ground-water system.

The ground-water system illustrated in figure 37 consists of several flow systems of different magnitude. The upper part of the ground-water system consists largely of local flow systems where water moves from high points on the water table (water table mounds) to adjacent lowlands occupied by lakes. Regional ground-water flow occurs deep in the system. Recharge to the regional flow system occurs at the major drainage divide and discharge from this system is to the major drain (stream). A zone of intermediate flow is recharged at the water-table mound between lakes 2 and 3 and is discharged into lake 1. It should be noted, but will be discussed in more detail later, that much more ground water moves through local flow systems than the deep regional system as shown by the closer spacing of the flow lines in the former. A thorough discussion of similar type ground-water flow diagrams is given by Toth (1963), Meyboom (1966, 1967), and Freeze (1969b).

Of special interest to this study are the lines, hereafter referred to as divides, separating the several types of flow systems. In following the line dividing the zone of local flow from zones of larger-magnitude flow (by lake 3, fig. 37, for example), it should be noticed that there is a point on the divide at which the head is a minimum compared to every other point along the divide. This point of minimum head occurs beneath the shore line on the downslope side of the lake. In the case illustrated, the hydraulic head at this point is 2.2 feet (0.7m) higher than the water-level altitude of lake 3. The hydraulic head everywhere else along the divide is greater than 2.2 feet (0.7m) above the lake-level altitude. Thus, under the given conditions, it is impossible for water to move from the lake to the ground-water system because the hydraulic gradient is upward toward the lake bottom.

This point of minimum head is the stagnation point commonly referred to in the ground-water flow literature (e.g. Harr, 1962). It is a point in the flow field at which vectors of flow are equal in opposite directions and therefore cancel. A value of head exists at the stagnation point and it is this value of head relative to the head represented by lake level that is of prime interest in understanding the interaction of lakes and ground water. If a stagnation point exists that has a head greater than that of lake level, a continuous ground-water divide exists beneath the lake making it impossible for water to move against the hydraulic gradient from the lake to the ground-water system. The position of the stagnation point is determined by the distribution of head within the ground-water system. Its relative position in the flow field, therefore, is not affected by the problem of vertical exaggeration. This was checked by running one of the sections using smaller vertical exaggeration.

The stagnation point is a point of diversion of ground-water flow paths (fig. 37). Water moving downward from the water-table mound on the downslope side of the lake is diverted upward toward the lake. A small amount of water that moves beneath the lake from the upslope side is diverted upward toward the lake on the downslope side. Water moving in the local or intermediate flow systems of the lakes downslope is diverted downslope and water moving in the regional flow system is diverted deeper into the ground-water reservoir.

Much of the remainder of the report is concerned with the effect of varying the height of the water table in relation to lake level, K_h/K_v, K_{aq}/K_t (ratio of the hydraulic conductivity of the aquifer to that of the surrounding till), position and size of aquifers, and lake depth, on the position and head value of the stagnation point relative to lake level.

To illustrate the effect of varying several parameters on the ground-water flow systems in general and the stagnation point in particular, figures 38 and 37 can be compared.

The presence of aquifers of limited extent, which have hydraulic conductivity 1,000 times greater than the till and are at an intermediate depth within the ground-water system, change the configuration of the local flow systems. The aquifers also decrease the head at the stagnation points relative to the level of each lake and move the points closer to the bottom of lakes 1 and 2. In addition to the limited aquifer beneath the water-table mound between lakes 2 and 3, the height of the water table relative to the level of lake 3 was decreased on both sides of the lake. These two changes result in reduced local ground-water flow into one-half of lake 3 on the upslope side, a small amount of ground-water flow into the littoral zone of the lake on the downslope side, and loss of lake water to the ground-water system through most of the downslope half of the lake bottom.

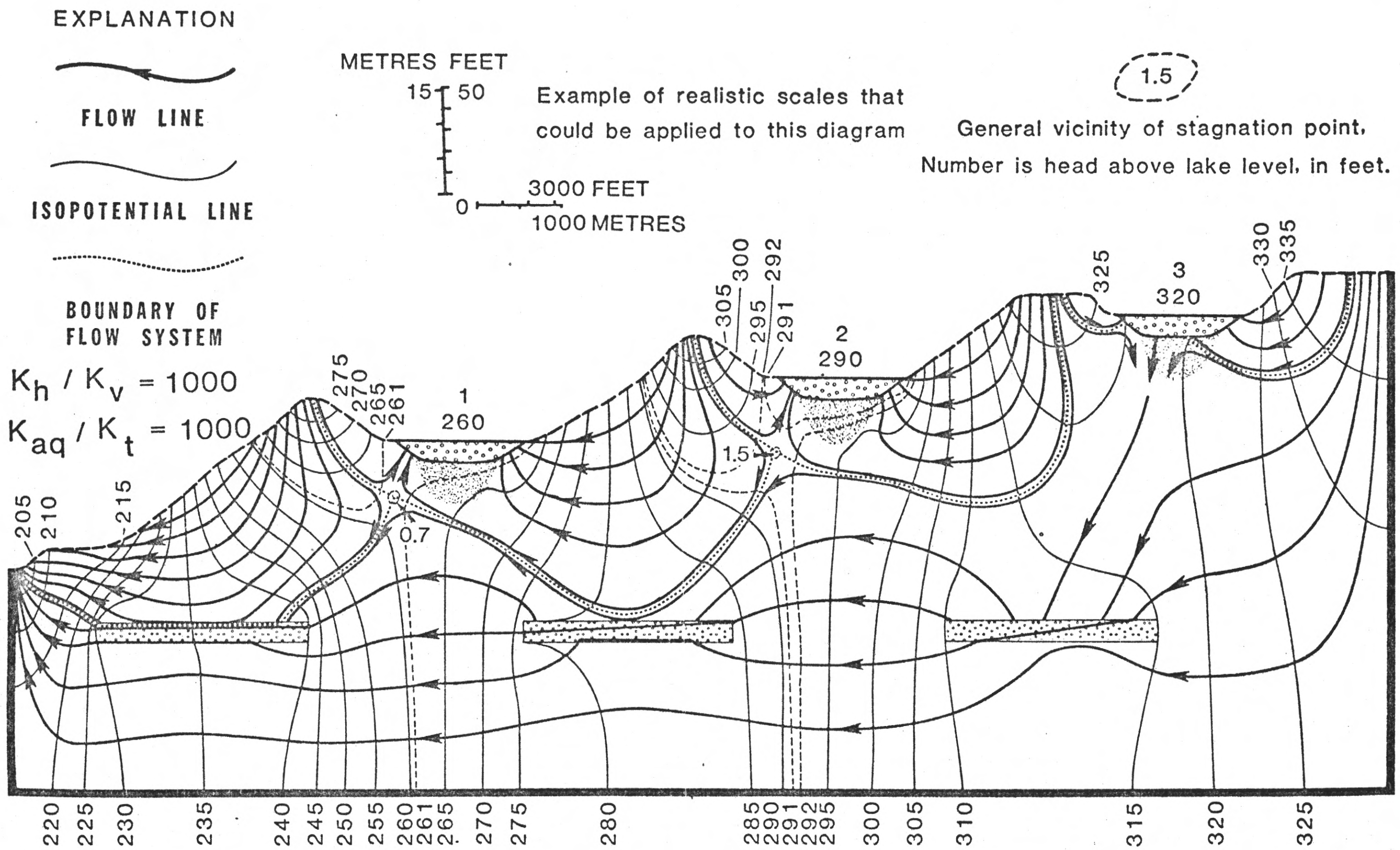


Figure 38.--A quasi-quantitative flow net of ground-water flow near lakes in a multiple-lakes system that contains aquifers.

To appreciate fully the discussion in the following part of the report, the diversion of ground-water flow lines caused by the presence of limited aquifers should be noted (fig. 38). It should be noted also that an intermediate flow system was sketched in on figure 37, but not on figure 38. This was done for comparison purposes. An intermediate flow system could have been included on figure 38 or deleted from figure 37. The intermediate system was included on figure 37 to illustrate where such systems can occur, where they are recharged, and where they discharge. The intermediate system could be recharged at any of the upper three water table mounds or discharged into any of the lower lakes or the major drain. Such systems have importance especially in explaining water chemistry differences in lakes occurring at different altitudes in a region.

Ground-water chemistry is controlled partly by the length of time a particle of water is in contact with minerals in the geologic framework through which the water flows. Thus, the longer the flow path, or the lower the hydraulic conductivity, the longer the residence time, and the more solution that will take place. For lakes situated on geologic materials of similar hydraulic conductivity, those that receive ground water from intermediate or large local ground-water flow systems are more likely to contain more, and a wider variety of, dissolved minerals than lakes that receive ground water from small local flow systems. It will be seen in this chapter that the size of local flow systems alone can vary widely. Therefore, the large variation in lake-water chemistry that is so frequently noted in lakes in close proximity in many parts of the north-central United States is not at all surprising. On the contrary, after becoming aware of the factors that control the interaction of lakes and ground water discussed in this thesis, and realizing that virtually every lake has its unique ground water contribution, a wide variety in the chemistry of lakes should be expected.

Because the stagnation point is the key to understanding the relation of ground-water to lakes and it is determined by the distribution of head, the remainder of the illustrations in the report are contoured computer printouts that show only isopotential lines. The isopleth interval is variable; only enough lines were drawn to depict the general distribution of hydraulic head, the specific location and head of the stagnation point, or the percent of lake bottom through which the lake loses water to the ground-water system. Figure 37, a specially constructed flow net, can be compared with figure 39, a contoured computer printout, to aid in understanding the remainder of the illustrations in this report. Both diagrams are from the same data set.

The following sections of the report discuss two general settings. In the first; one lake is situated within the side of

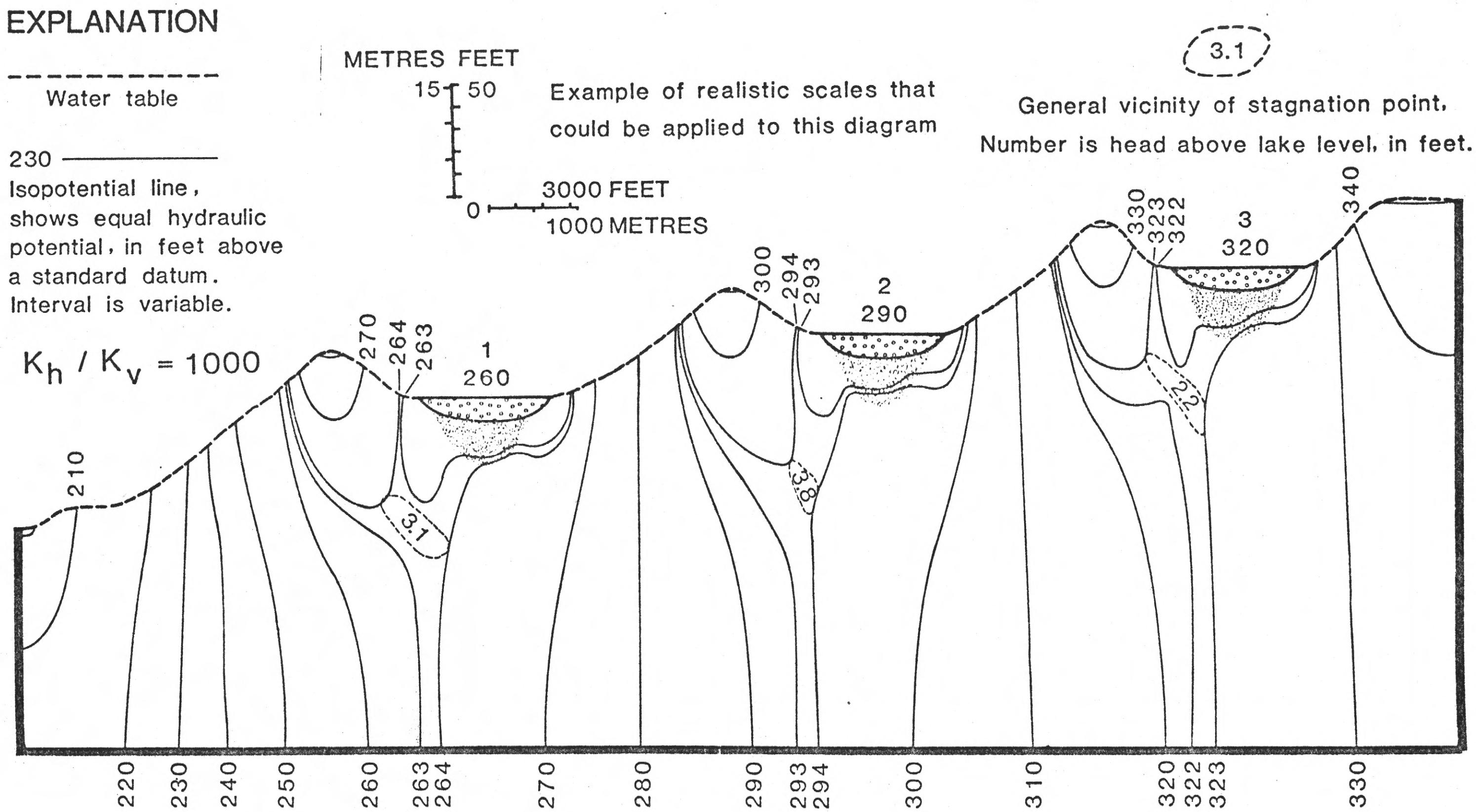


Figure 39.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that does not contain aquifers.

a drainage basin. The interaction of lakes and ground water is thoroughly studied for this type of setting by examining all possible combinations of the parameters mentioned previously. In the second general setting, three lakes are situated within the side of a drainage basin to examine the relationship of the lakes to each other and to compare the results to the one-lake setting.

To allow for the most universal application of this study, the diagrams of vertical ground-water flow are discussed in dimensionless terms--they can represent a system of any size. But because lakes in the glacial terrain of the north-central United States are of primary interest in this study, the dimensions expected to be of most general interest are also given.

One-Lake System

The approximate dimensions, relative to total length and thickness of the ground-water system, of features in the one-lake system are given below and in figure 40. The dimensions in terms of english and metric units shown in parentheses are included as a realistic example.

The thickness of the system ranges from 0.35T (70 ft; 21m) at the lower end of the basin to T (as much as 200 ft; 61m) at the upper end. The length of the system (L) is approximately 6 miles (9km). The height of the water table above lake level is

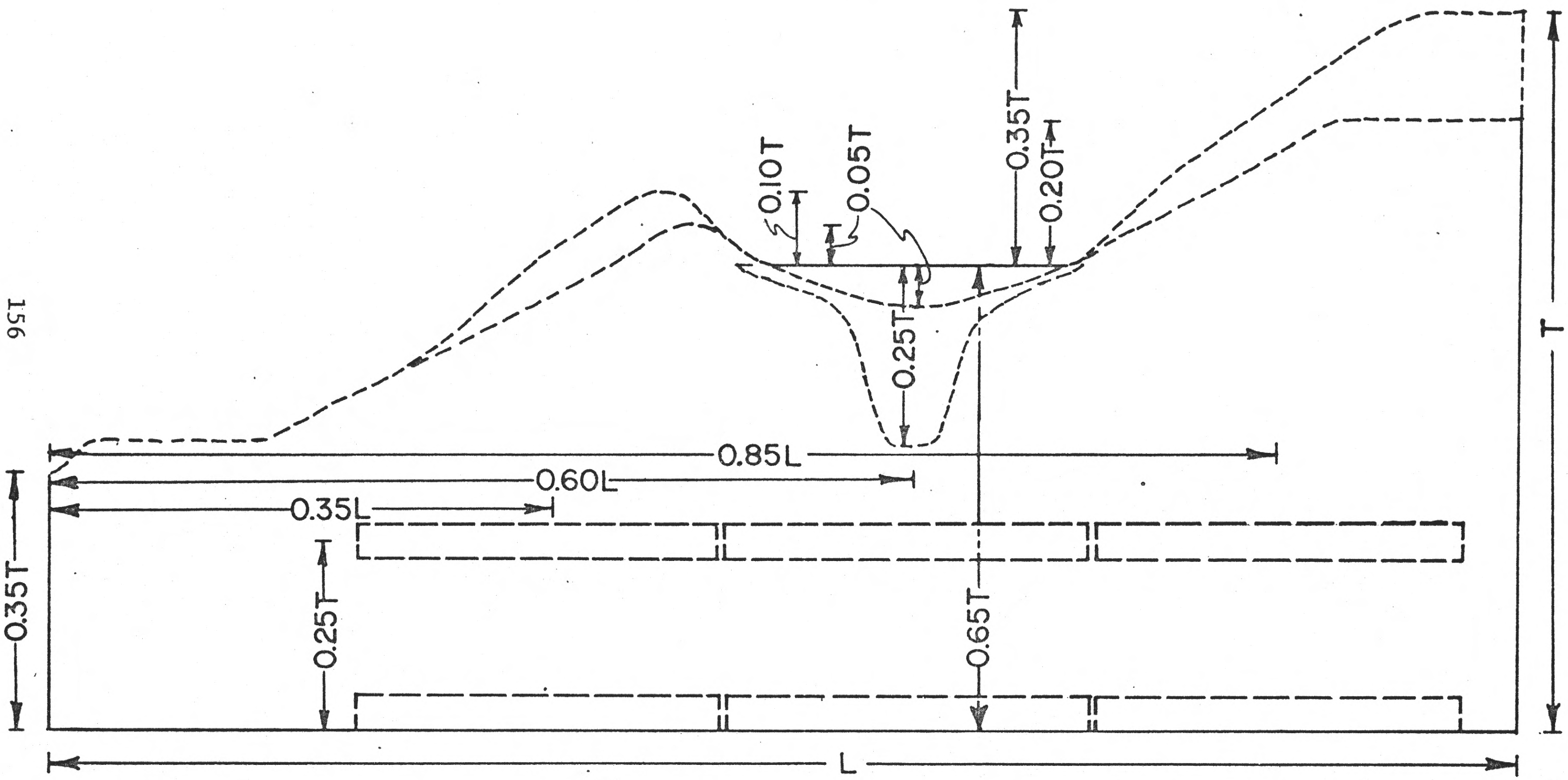


Figure 40.--Dimensions of features included in the diagrams of the one-lake system.

varied between 0.20T and 0.35T (40 and 70 ft; 12 and 21m) on the upslope side of the lake and between 0.05T and 0.10T (10 and 20 ft; 3 and 6m) on the downslope side. Lake depths simulated are 0.05T and 0.25T (10 and 50 ft; 3 and 15m). Values of K_h/K_v (horizontal to vertical hydraulic-conductivity ratio) used in the simulations are 100 and 1,000. Simulated aquifers vary from full aquifers (0.05TxL) that extend the entire length of the basin to aquifers of limited extent (0.05T x 0.25L) that occur upslope from the lake (center point at 0.85L), beneath the lake (center point at 0.60L), and downslope (center point at 0.35L) from the lake. The aquifers are placed both at the base and at an intermediate level (center point at 0.25T above the base of the system) of the ground-water reservoir. The lake surface is at 0.65T above the base and the midpoint is about 0.60L from the downslope end of the section for all simulations of the one-lake system. The system simulated can be thought of as a ground-water reservoir that has a datum of 100 feet (30m). The values of head are in feet relative to that datum, thus the lake surface has an altitude of 230 feet (70m), the higher water table at the upslope end of the section 300 feet (100m), and the water table at the downslope end 170 feet (52 m).

A large number of simulations could be run varying each of these parameters. Generally, the parameters are varied between two values so the general direction of change of the stagnation point could be determined. If a change in a parameter lowers the head at the stagnation point relative to lake level, an even greater change of that parameter in the same direction would lower the head at the point even more.

The pattern of ground-water flow near a lake that has a water table 0.35T higher than lake level on the upslope side and 0.1T higher on the downslope side, has no sediments, and K_h/K_v = 1,000, is shown in figure 41. A ground-water divide occurs deep beneath the lake that has a head at the stagnation point 0.009T (1.8 ft; 0.5m) higher than lake level. The position of the stagnation point is approximately 0.2T above the base of the system. In contrast, if lake sediments are simulated, and all other parameters are held constant, the ground-water divide shifts slightly upward, and the head at the stagnation point increases to 0.02T (4.2 ft; 1.3m) higher than lake level (fig. 42). Thus, it is evident that the presence of lake sediments has a significant effect on the position and head of the stagnation point. Because lakes without sediments are virtually unknown, much of the remainder of this report is concerned with lakes that have a sediment layer.

The effect on the interaction of lakes and ground water of lowering the water table 0.05T (10 ft; 3m) on the downslope side and holding all other parameters as in figure 42, is shown in figure 43. The head at the stagnation point drops to 0.007T (1.4 ft; 0.4m) above lake level and the point moves upward even closer to the bottom of the lake. Again, holding all

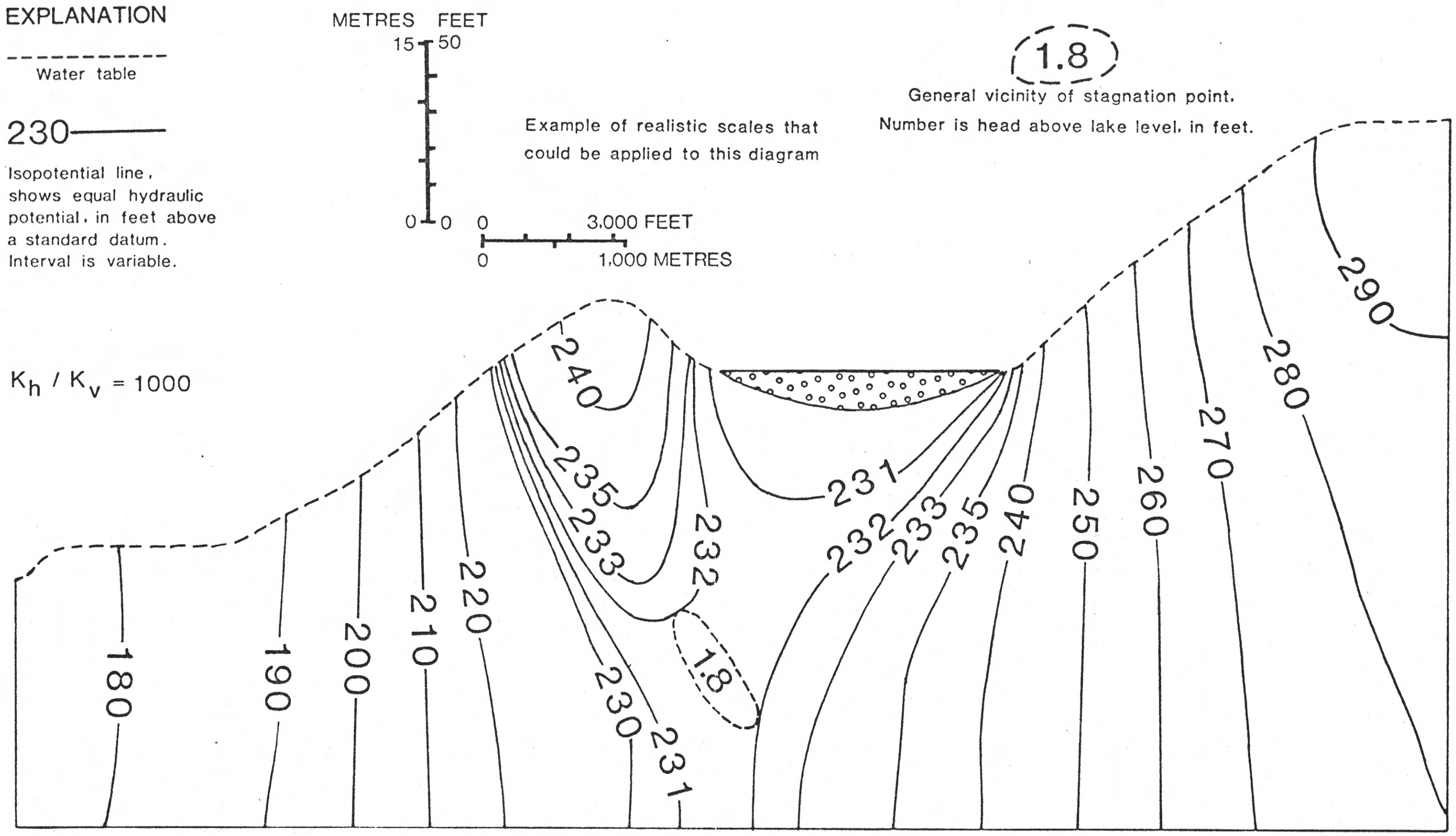


Figure 41.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that does not contain aquifers or lake sediments.

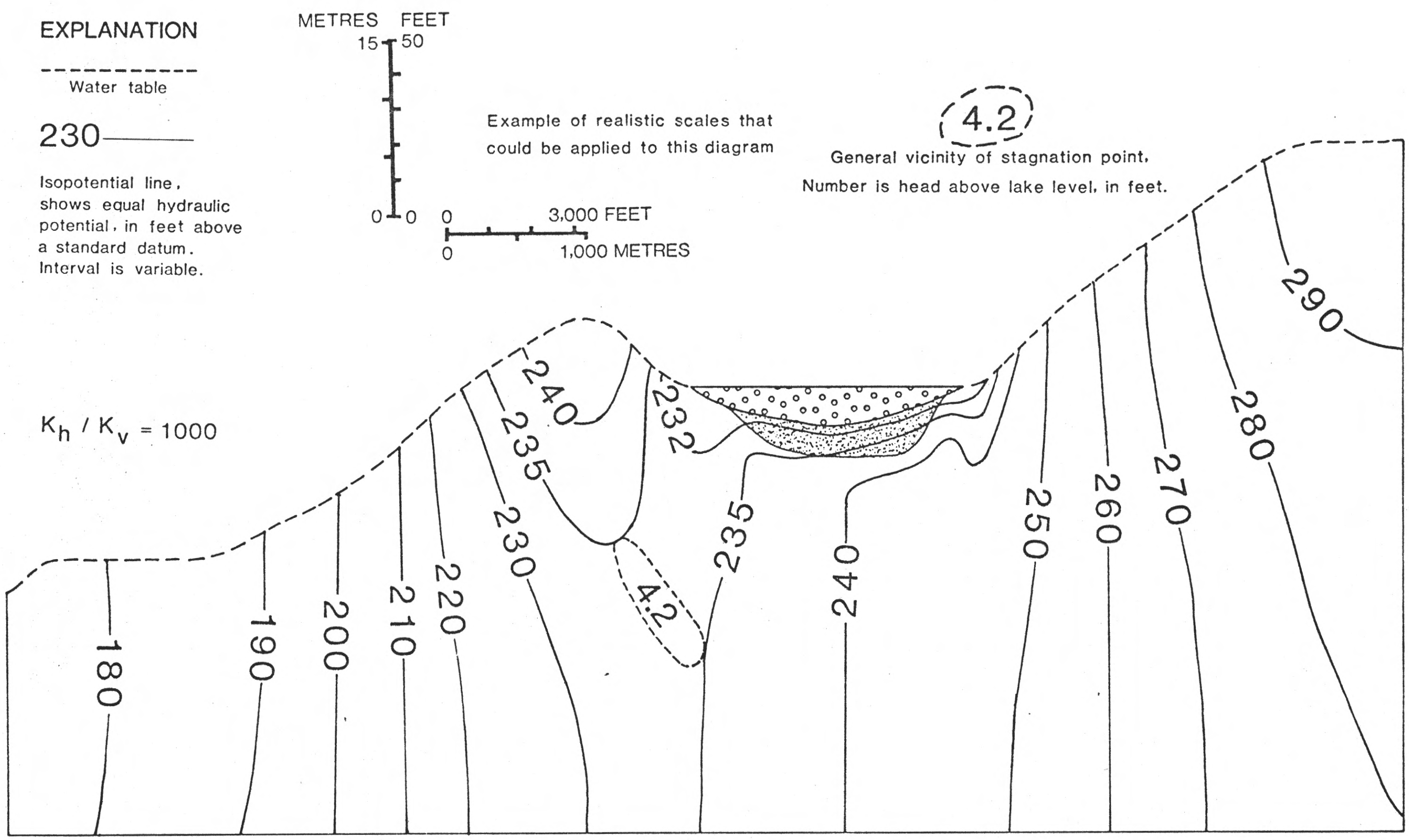


Figure 42.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that contains lake sediments but does not contain aquifers.

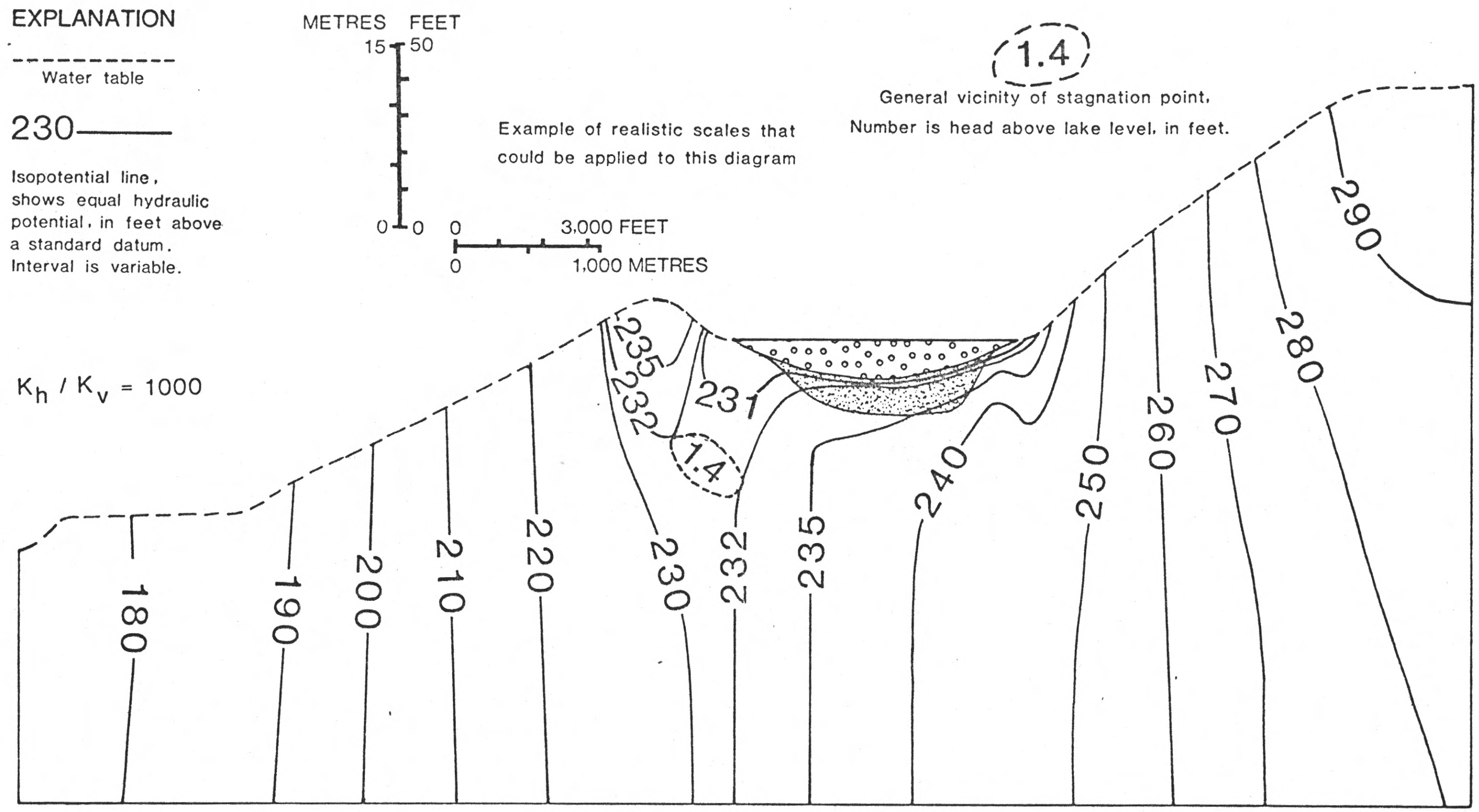


Figure 43.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has a low water-table mound on the downslope side of the lake and does not contain aquifers.

parameters as in figure 43. but lowering the water table on the upslope side of the lake to 0.20T (40 ft; 12m) above lake level, decreases the head at the stagnation point to 0.004T (0.9 ft; 0.3m) above lake level (fig. 44). The stagnation point remains at about the same position as in figure 43. This suggests that the height of the water table relative to lake level on the upslope side of a lake has less influence on the stagnation point of the ground-water divide beneath a lake than that on the downslope side.

The presence of extensive aquifers within the ground-water reservoir has a significant effect on the interaction of lakes and ground water as seen in figure 45. Given the same setting as in figure 42, but simulating an aquifer at the base of the ground-water system that extends the full length of the basin and has $K_{aq}/K_t = 100$, decreases the head at the stagnation point from 0.02T (4.2 ft; 1.3m) (figure 42) to 0.001T (0.2 ft; 0.1m) higher than lake level. Again, holding all parameters constant, but increasing K_{aq}/K_t to 1,000 causes the lake to lose water to the ground-water system through its entire bed (fig. 46).

The effect on the interaction of lakes and ground water of raising the extensive aquifer vertically to the 0.25T (intermediate) level in the ground-water system, where the setting is otherwise similar to figure 45, is shown in figure 47. The weak ground-water divide (head at the stagnation point is

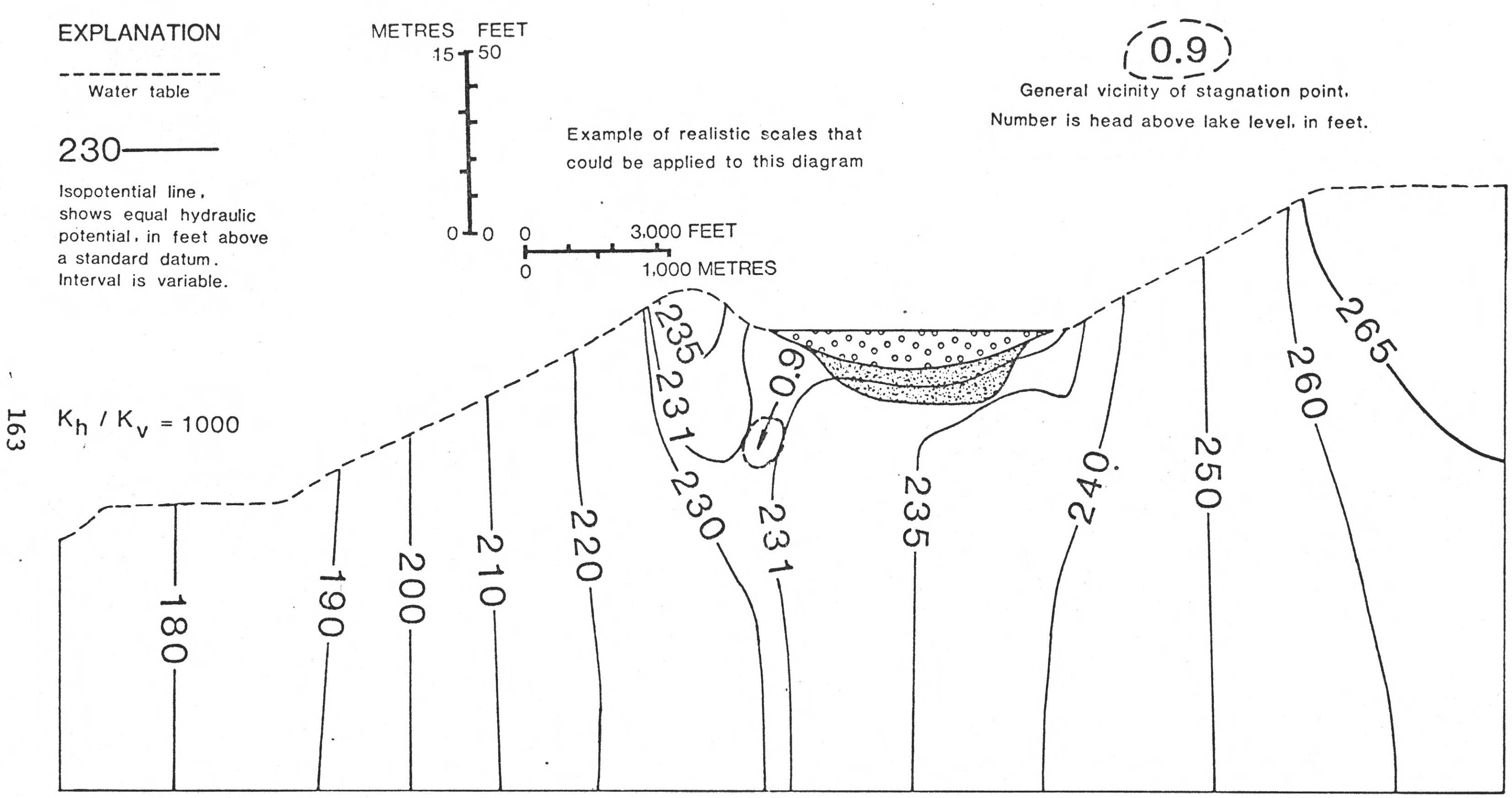


Figure 44.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has low water-table mounds on both sides of the lake and does not contain aquifers.

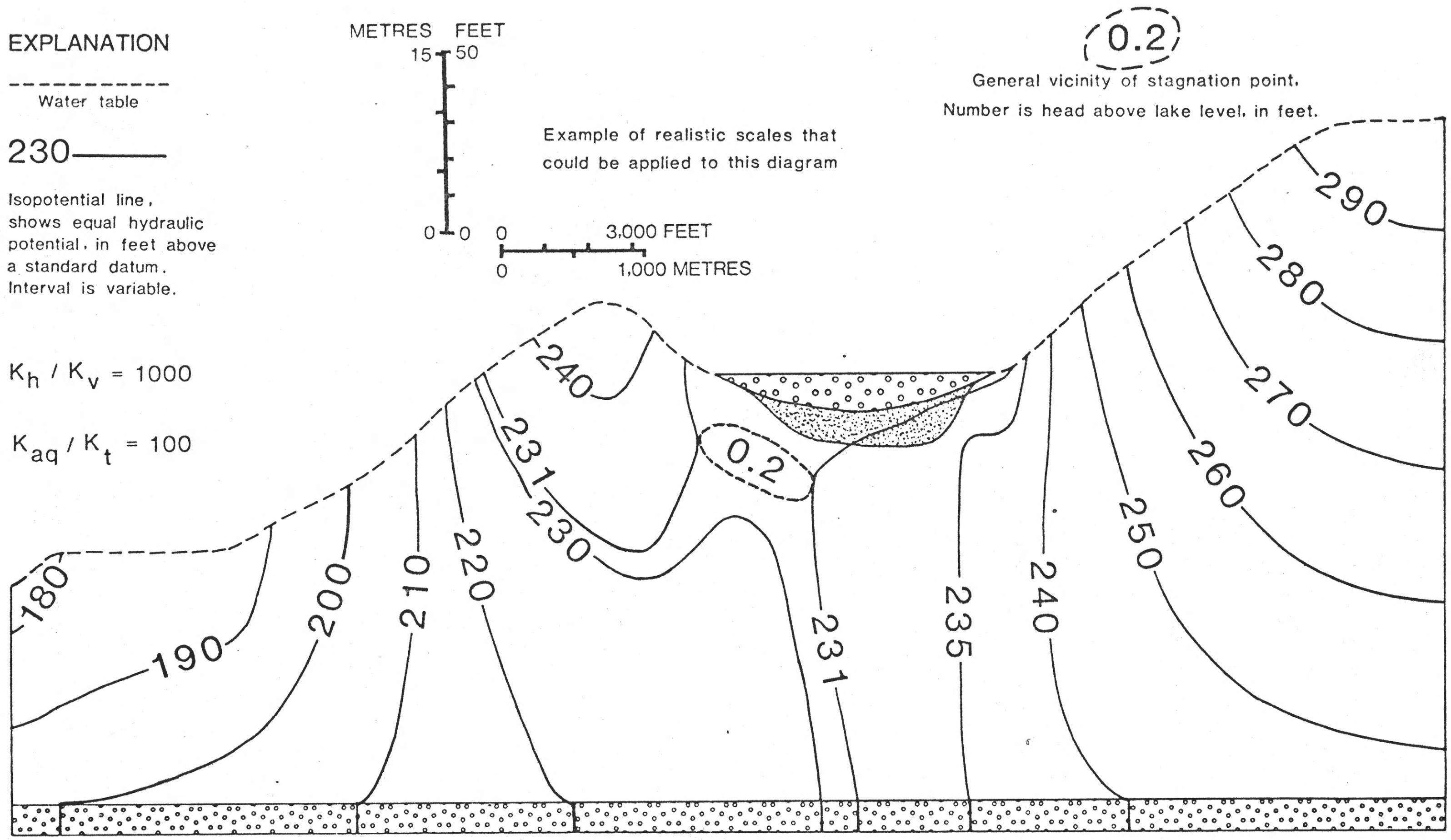


Figure 45.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an extensive aquifer of relatively low hydraulic conductivity at the base of the system.

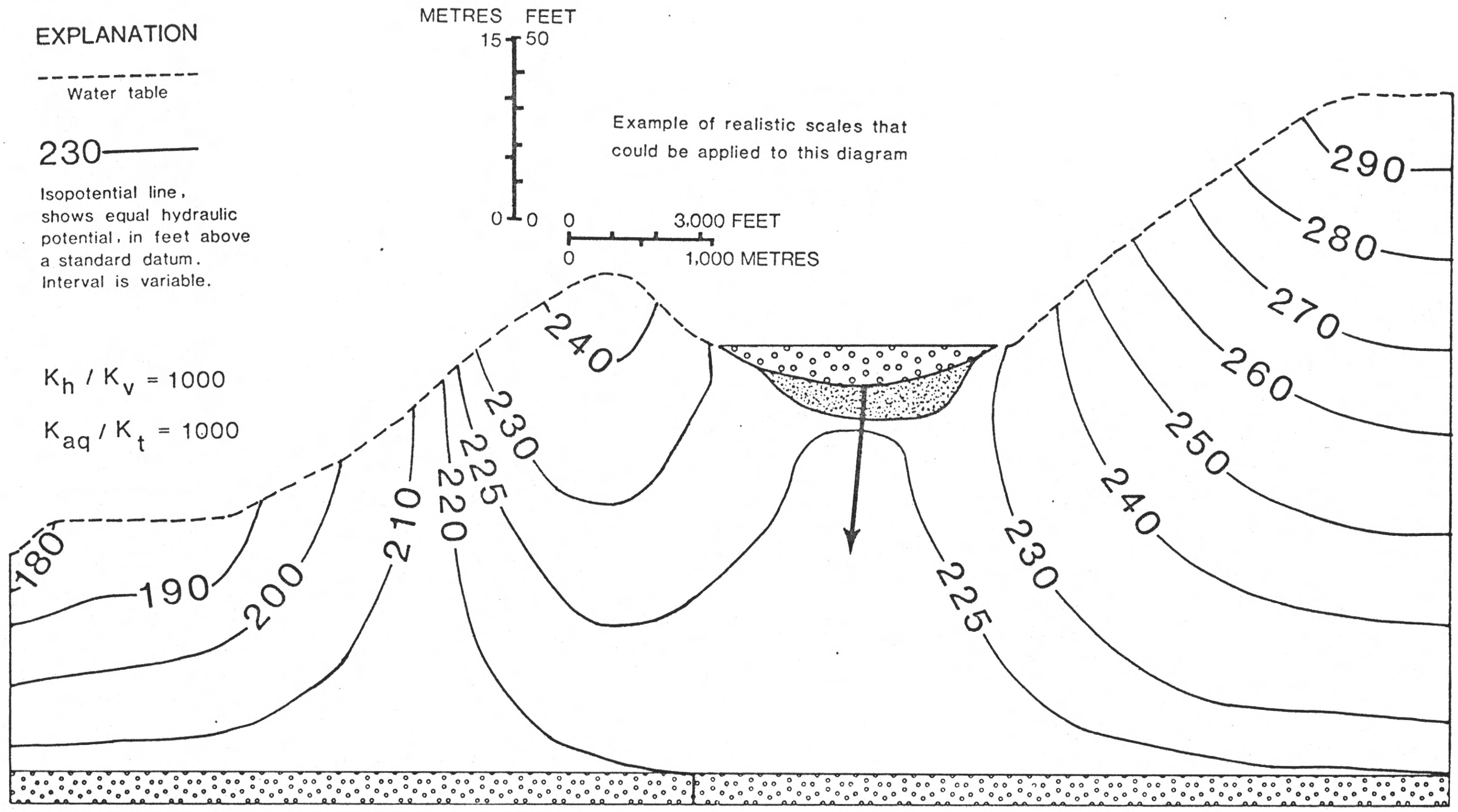


Figure 46.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an extensive aquifer of relatively high hydraulic conductivity at the base of the system.

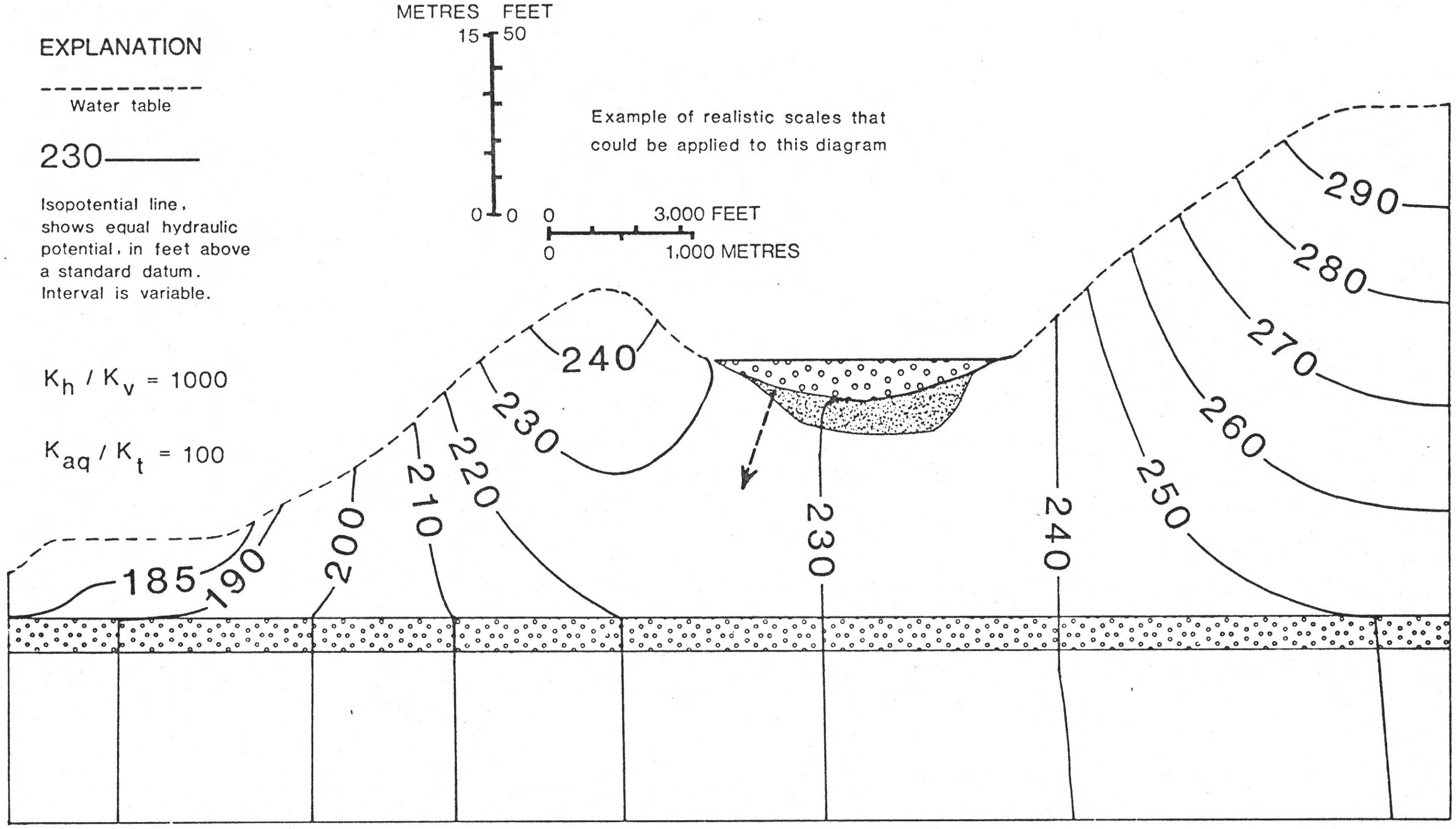


Figure 47.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an extensive aquifer of relatively low hydraulic conductivity in the middle of the system.

0.001T (0.2 ft; 0.1m) higher than lake level) in the former is obliterated and the lake loses water through three-eighths of its bed.

The position of smaller aquifers of limited extent within the ground-water reservoir also has varying influences on the interaction of lakes and ground water. All parameters other than the location of the limited aquifer at the base of the ground-water system are held constant in figures 48, 49, and 50. In the simulation that has a limited basal aquifer upslope from the lake (0.85L) (figure 48), the head at the stagnation point is 0.006T (1.3 ft; 0.4m) above lake level. With the limited basal aquifer beneath the lake (0.60L) (figure 49), the head at the stagnation point is 0.009T (1.8 ft; 0.5m) above lake level. The limited basal aquifer underlying the water-table mound on the downslope side of the lake (0.35L) causes the lake to lose water through three-eighths of its bed (figure 50).

Similar patterns of ground-water flow near lakes hold for settings that have aquifers at the same position laterally but moved vertically to 0.25T above the base of the ground-water system. One difference is that, if a limited aquifer directly beneath a lake is raised vertically, it tends to increase the head at the stagnation point. In a section simulating this situation, the head at the stagnation point increases from 0.009T (1.8 ft; 0.5m) (figure 49) to 0.012T (2.5 ft; 0.8m) higher than lake level with the aquifer at the higher position. If the

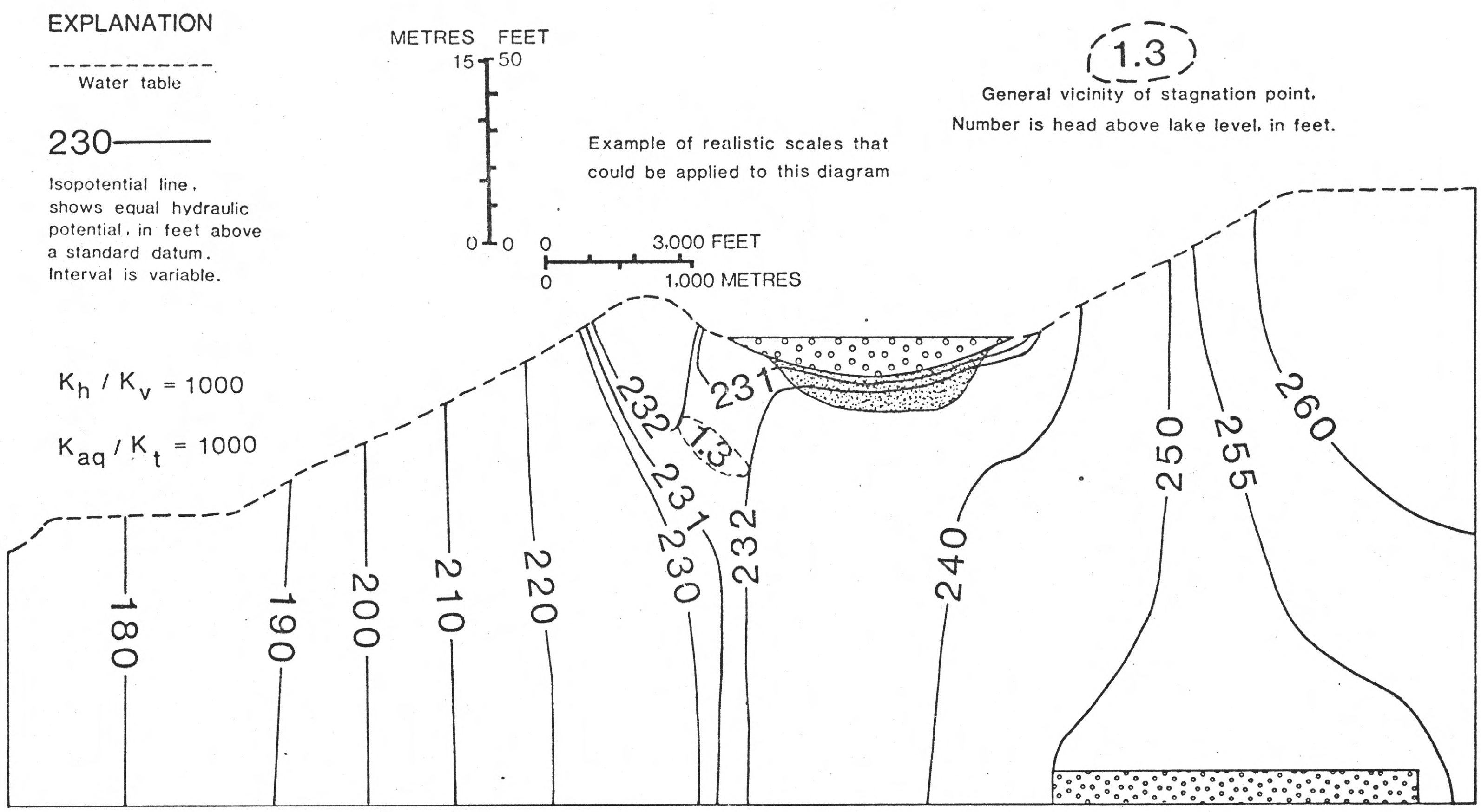


Figure 48.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an aquifer of limited extent upslope from the lake at the base of the system.

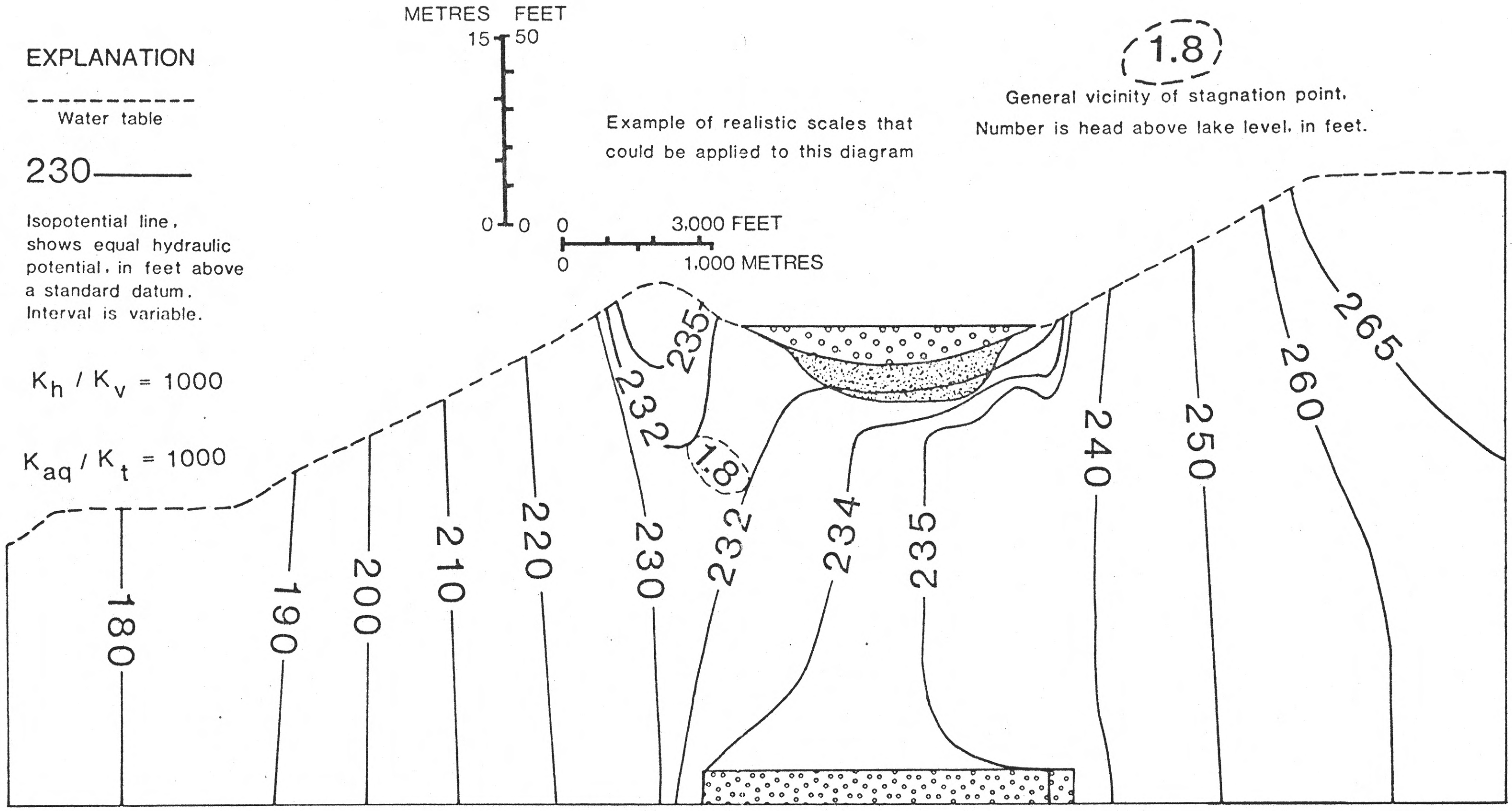


Figure 49.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an aquifer of limited extent beneath the lake at the base of the system.

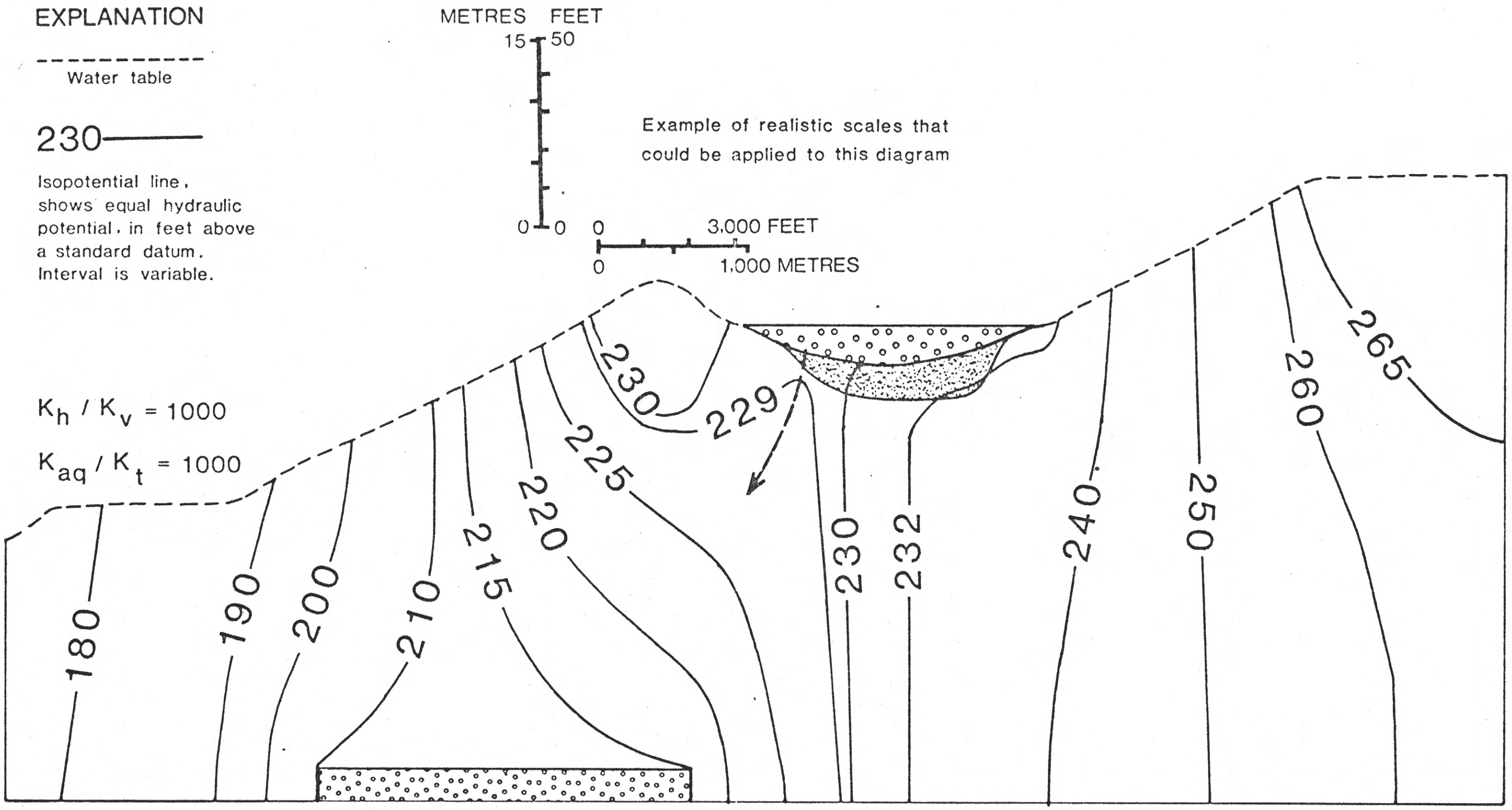


Figure 50.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has an aquifer of limited extent downslope from the lake at the base of the system.

limited aquifer is raised to directly under, and in contact with, the lake sediments, the head at the stagnation point increases to 0.016T (3.3 ft; 1.0m) higher than the lake level. If the aquifer is at an intermediate level but downslope from the lake (0.25T, 0.35L), a situation parallel to that shown in figure 50, the percentage of lake bottom through which water moves to the ground-water system increases from three-eighths to four-eighths.

These simulations suggest that the position of limited aquifers beneath or upslope from a lake has little influence on the interaction of lakes and ground water. But if the limited aquifer underlies the water-table mound on the downslope side of the lake, there is a strong tendency for the head at the stagnation point to be only slightly above lake level, if at all; in the latter case the lake loses water through part of its bed.

The effect of lake depth on the interaction of lakes and ground water can be seen by comparing figure 51 with figure 45. All parameters in these two simulations, except depth, are identical. The small difference between lake level and head at the stagnation point associated with the shallow lake (fig. 45) is eradicated in the deep-lake setting, and the lake loses water through about six tenths of its bed. This example is one of the more dramatic in comparing ground-water flow near shallow and deep lakes. In some simulations the difference is not as

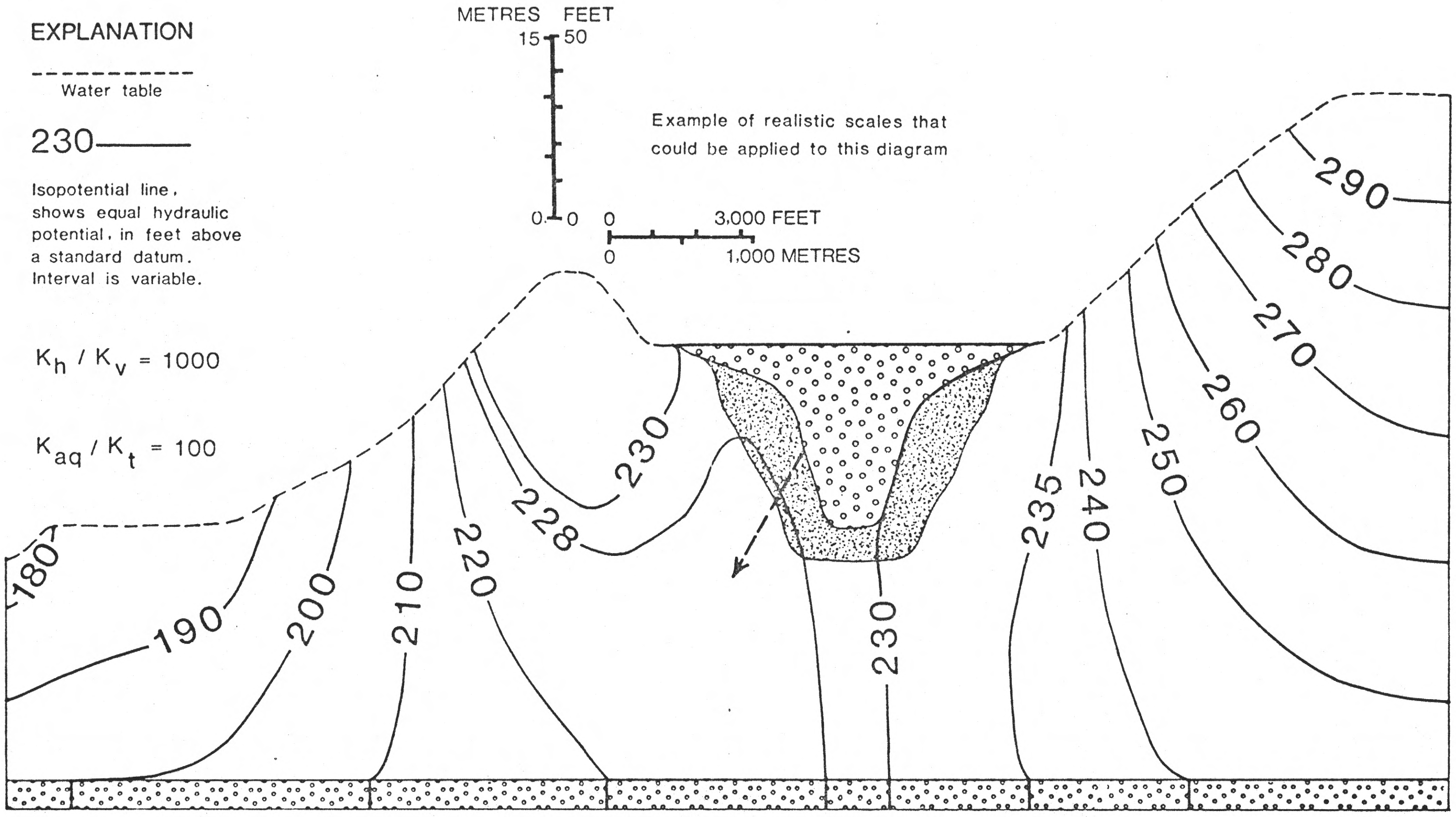


Figure 51.--Distribution of hydraulic head in the ground-water reservoir of a one-lake system that has a deep lake and an extensive aquifer of relatively low hydraulic conductivity at the base of the system.

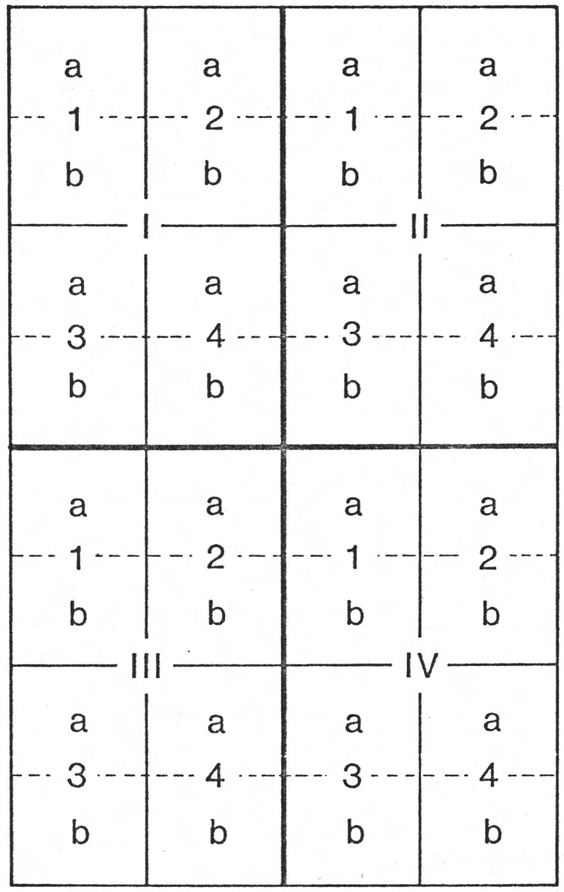
great, but in all cases the difference between lake level and the head at the stagnation point for deep lakes is less, or they have more tendency to lose water, than shallow lakes in equivalent settings.

It is evident that a large number of simulations would be needed to examine all possible combinations of the variables considered in this study. To keep the analysis as simple as possible, yet show the underlying relationships of the interaction of lakes and ground water, a parallel series of lake-ground water settings are compared and summarized in table 13. At first glance the table might seem complex, but with a little study it is not so formidable. It should be realized that many variables are summarized on this one table. It seemed a better choice to be able to compare all possible combinations of lake settings on a single table than to have a larger number of simpler tables that would have to be cross-referenced.

The table consists of four quadrants, each of which is subdivided into four subquadrants. Each subquadrant has an upper and lower part. For ease of discussion, the different parts of the table are refered to as shown in figure 52. The major quadrants are the combinations of hydraulic conductivity (K_h/K_v and K_{aq}/K_t) used in the study. In quadrant I, both ratios are 100 and in quadrant IV, both are 1,000. Within the subquadrants are combinations of the height of the water table above lake level for both sides of the lake. In subquadrant I-1 the

Table 13.--Results of digital simulations of the one-lake system: comparison of the effects of all combinations of the parameters that control ground-water flow on lake-ground water interaction.

K_{aq}/K_t	K_h/K_v = 10^2								K_h/K_v = 10^3							
10^2	0.35T, 0.10T				0.35T, 0.05T				0.35T, 0.10T				0.35T, 0.05T			
	[13)[1]	10[2]-FB[3]	(4)[4] (0.1)[5]	D[6]]	24)	10-FB	(1.3)	D	21)	10-FB	(0.2)	D	25)	10-FB	(4/8)	L
	34)	FM	(5.0)	D	36)	FM	(1.7)	D	35)	FM	(3/8)	L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
		PBD		D		PBD		D	40)	PBD	(0.8)	D	43)	PBD	(1/8)	L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
		PMD		D	56)	PMD	(0.1)	D	55)	PMD	(1/8)	L		PMD		L
	[16)	FM	(2) (0.7)	D					[14)	FB	(6.8)	L]				
	15)	FM	(4) (⬭)	D/L]												
		50-FB		D	66)	50-FB	(0.7)	D	65)	50-FB	(6/10)	L	67)	50-FB	(6/10)	L
		FM		D	70)	FM	(0.8)	D		FM		L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
		PBD		D	76)	PBD	(3/10)	L	74)	PBD	(4/10)	L		PBD		L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
	79)	PMD	(RF 0.5)	D		PMD		L		PMD		L		PMD		L
	0.20T, 0.10T				0.20T, 0.05T				0.20T, 0.10T				0.20T, 0.05T			
	28)	10-FB	(2.8)	D	31)	10-FB	(0.5	D	29)	10-FB	(5/8)	L	32)	10-FB	(7/8)	L
		FM		D	37)	FM	(0.6)	D		FM		L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
	44)	PBD	(RF 0.5)	D	48)	PBD	(⬭)	D/L	45)	PBD	(0.2)	D		PBD		L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
		PMD		D	58)	PMD	(⬭)	D/L	57)	PMD	(2/8)	L		PMD		L
	68)	50-FB	(1.3)	D	69)	50-FB	(0.1)	D		50-FB		L		50-FB		L
	71)	FM	(2.1)	D	72)	FM	(0.4)	D		FM		L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
	77)	PBD	(0.3)	D		PBD		L		PBD		L		PBD		L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
	80)	PMD	(RF 0.6)	D	81)	PMD	(2/10)	L		PMD		L		PMD		L
10^3	0.35T, 0.10T				0.35T, 0.05T				0.35T, 0.10T				0.35T, 0.05T			
	22)	10-FB	(2/8)	L	26)	10-FB	(3/8)	L	23)	10-FB	(8/8)	L	27)	10-FB	(8/8)	L
	20)	FM	(7/8)	L		FM		L	19)	FM(2)	(8/8)	L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
	41)	PBD	(0.1)	D		PBD		L	42)	PBD	(0.4)	D		PBD		L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
		PMD		L		PMD		L		PMD		L		PMD		L
	[17)	FM	(4) (8/8)	L					[19a)	FM(4)	(8/8)	L]				
	18)	FM	(2) (7/8)	L]												
		50-FB		L		50-FB		L		50-FB		L		50-FB		L
		FM		L		FM		L		FM		L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D		PBB		D
	75)	PBD	(5/10)	L		PBD		L		PBD		L		PBD		L
		PMU		D		PMU		D		PMU		D		PMU		D
		PMB		D		PMB		D		PMB		D		PMB		D
		PMD		L		PMD		L		PMD		L		PMD		L
	0.20T, 0.10T				0.20T, 0.05T				0.20T, 0.10T				0.20T, 0.05T			
	30)	10-FB	(6/8)	L	33)	10-FB	(7/8)	L		10-FB		L		10-FB		L
		FM		L		FM		L		FM		L		FM		L
		PBU		D		PBU		D		PBU		D	38)	PBU	(1.3)	D
		PBB		D		PBB		D		PBB		D	39)	PBB	(1.8)	D
	46)	PBD	(⬭)	D/L	49)	PBD	(2/8)	L	47)	PBD	(2/8)	L	50)	PBD	(3/8)	L
		PMU		D		PMU		D		PMU		D	52)	PMU	(1.4)	D
		PMB		D		PMB		D		PMB		D	53)	PMB	(2.5)	D
		PMD		L		PMD		L		PMD		L	59	PMD	(4/8)	L
													[54)	PMB	(3.3)	D
													51)	PBU & PBB	(2/8)	L]
		50-FB		L		50-FB		L		50-FB		L		50-FB		L
		FM		L		FM		L		FM		L		FM		L
		PBU		D		PBU		D		PBU		D		PBU		D
		PBB		D		PBB		D		PBB		D	73)	PBB	(1.6)	D
		PBD		L		PBD		L		PBD		L		PBD		L
		PMU		D		PMU		D		PMU		D	78)	PMU	(1.2)	D
		PMB		D		PMB		D		PMB		D		PMB		D
		PMD		L		PMD		L		PMD		L		PMD		L



Footnotes to table 13

0.35T, 0.10T: Height of water table above lake level on upslope and downslope sides, respectively

1 = simulation number--keyed to table 14
2 = lake depth. In each cell, entries above dashed line are shallow lakes (0.05T) and those below it are deep lakes (0.35T)
3 = Aquifer position (see text for explanation)
4 = thickness of aquifer if greater than 0.05T; 2 = 0.10T, 4 = 0.20T
5 = Height of head of stagnation point above lake level, if decimal (in feet), proportion of lake bed that leaks, if fraction
6 = D = divide exists beneath lake
    L = lake loses water through its bed

Figure 52.--Key to table 13.

water table is 0.35T (70 ft; 21m) higher on the upslope side and 0.10T (20 ft; 6m) higher on the downslope side. In subquadrant I-4, the water table on the upslope side is 0.20T (40 ft; 12m) higher than lake level and 0.05T (10 ft; 3m) higher on the downslope side. Within each of the subquadrants, the upper part (a) refers to shallow lakes and the lower part (b), refers to deep lakes. Within each part (a and b), the position and size of the aquifers are listed in a parallel fashion as follows:

FB = Full (extensive) aquifer at the base of the ground-water system

FM = Full (extensive) aquifer at an intermediate level (0.25T) of the ground-water system

PBU = Partial (limited extent) aquifer at the base of the ground-water system upslope from the lake (0.85L)

PBB = Partial (limited extent) aquifer at the base of the ground-water system beneath the lake (0.60L)

PBD = Partial (limited extent) aquifer at the base of the ground-water system downslope from the lake (0.35L)

PMU = Partial (limited extent) aquifer at an intermediate level of the ground-water system upslope from the lake (0.25T, 0.85L)

PMB = Partial (limited extent) aquifer at an intermediate level of the ground-water system beneath the lake (0.25T, 0.60L)

PMD = Partial (limited extent) aquifer at an intermediate level of the ground-water system downslope from the lake (0.25T, 0.35L)

The simulations shown in brackets in table 13 are included for comparison purposes. They are generally of settings similar to one in the subquadrant. The footnotes which explain these special settings are listed on figure 52.

The simulations of settings where aquifers are not present are included in table 14, which is a general summary of the one-lake simulations run for this study.

The parallel construction of table 13 makes it relatively easy to compare the effect of changes in the various parameters on the interaction of lakes and ground water. If interest is in the effect of different hydraulic conductivity ratios for given water-table combinations, aquifer size and position, and lake depth, that setting is found in equivalent positions in the major quadrants. For example, if one wishes to compare settings where K_{aq}/K_t is 100 to another where it is 1,000, and the setting has a water table 0.35T (70 ft; 21m) higher than lake level on the upslope side and 0.05T (10 ft; 3m) higher on the downslope side, K_h/K_v is 1,000, and the lake is 0.05T (10 ft; 3m) deep, subquadrant II-2a can be compared with subquadrant IV-2a for whatever aquifer situation is of interest. If interest is in comparison of different water table configurations for given hydraulic conductivity ratios, this can

Table 14.--Summary of simulations of one-lake system.

Number	Model	Depth		Sediments		K_h/K_v				K_{aq}/K_t		Aquifer: None	Aquifer: Full: Base	Aquifer: Full: Middle	Aquifer: Partial: Base: Upslope	Aquifer: Partial: Base: Beneath	Aquifer: Partial: Base: Downslope	Aquifer: Partial: 0.25T: Upslope	Aquifer: Partial: 0.25T: Beneath	Aquifer: Partial: 0.25T: Downslope	Height of water table above lake (relative to total system thickness): Upslope	Side	Down-slope	slope Side	Results: Head at stagnation point above lake level	Results: Percent of bed that loses water.
		0.05T	0.25T	Y	N	1	10^2	10^3	10^4	10^2	10^3										0.35	0.20	0.10	0.05		
1	3-11:20.71	X			X	X						X									X		X		RF(20)	
2	3-11:17.24	X			X		X					X									X		X		RF(12)	
3	3-12: 3.80	X			X			X				X									X		X		1.8	
4	3-13:14.95	X			X				X			X									X		X			5/8
5	3-14: 9.31	X			X		X					X									X			X	RF(4)	
6	3-14:20.74	X			X			X				X									X			X	0.2	
7	3-24: 4.11	X			X			X				X										X	X		1.4	
8	3-21: 8.58	X			X		X					X										X		X	RF(4)	
9	3-21: 7.43	X			X			X				X										X		X	0.1	
10	3-27: 8.99	X		X				X				X									X		X		4.2	
11	3-28: 4.71	X		X				X				X									X			X	1.4	
12	4-1 : 9.16	X		X				X				X										X		X	0.9	
13	3-17:23.54	X			X		X			X			X[1]								X		X		0.1	
14	3-18:24.67	X			X			X		X			X[2]								X		X			6/8
15	3-17:22.66	X			X		X			X				X[1]							X		X			
16	3-18: 5.60	X			X		X			X				X[2]							X		X		0.7	
17	3-17: 8.17	X			X		X				X			X[1]							X		X			8/8
18	3-18:23.72	X			X		X				X			X[2]							X		X			7/8
19	3-14:10.57	X			X			X			X			X[1]							X		X			8/8
20	3-19:23.22	X		X			X				X			X[2]							X		X			7/8
21	3-28:10.08	X		X				X		X			X								X		X		0.2	
22	3-19:25.64	X		X			X				X		X								X		X			2/8
23	3-31: 9.19	X		X				X			X		X								X		X			8/8
24	4-11: 7.31	X		X			X			X			X								X			X	1.3	
25	3-28: 9.55	X		X				X		X			X								X			X		4/8
26	4-11: 3.37	X		X			X				X		X								X			X		3/8
27	3-28: 9.51	X		X				X			X		X								X			X		8/8
28	4-10: 3.79	X		X			X			X			X									X	X		2.8	
29	3-31: 5.28	X		X				X		X			X									X	X			5/8
30	4-10: 6.05	X		X			X				X		X									X	X			6/8
31	8-4 :16.83	X		X			X			X			X									X		X	0.5	
32	4-1 : 9.73	X		X				X		X			X									X		X		7/8
33	4-14: 3.41	X		X			X				X		X									X		X		7/8
34	8-4 :16.88	X		X			X			X				X							X		X		5.0	
35	8-4 : 4.11	X		X				X		X				X							X		X			3/8
36	8-5 :17.21	X		X			X			X				X							X			X	1.7	
37	8-5 :16.60	X		X			X			X				X								X		X	0.6	
38	4-4 : 9.43	X		X				X			X				X							X		X	1.3	
39	4-1 :12.80	X		X				X			X					X						X		X	1.8	
40	4-4 : 8.02	X		X				X		X							X				X		X		0.8	
41	8-6 :16.17	X		X			X				X						X				X		X		0.1	
42	8-5 :20.62	X		X				X			X						X				X		X		0.4	
43	8-5 :18.13	X		X				X		X							X				X			X		1/8
44	4-3 :11.40	X		X			X			X							X					X	X		RF(0.5)	
45	4-4 : 4.46	X		X				X		X							X					X	X		0.2	
46	4-3 : 9.54	X		X			X				X						X					X	X			
47	4-4 :10.74	X		X				X			X						X					X	X			2/8
48	4-3 :10.65	X		X			X			X							X					X		X		
49	4-3 : 5.08	X		X			X				X						X					X		X		2/8
50	4-3 :10.15	X		X				X			X						X					X		X		3/8
51	4-7 :10.38	X		X				X			X				X		X					X		X		2/8
52	8-4 :16.90	X		X				X			X							X				X		X	1.4	
53	4-8 : 4.98	X		X				X			X								X			X		X	2.5	
54	4-9 : 4.78	X		X				X			X								X[3]			X		X	3.3	
55	4-4 : 8.96	X		X				X		X										X	X		X			1/8
56	8-5 :17.07	X		X			X			X										X	X			X	0.1	
57	4-4 : 8.00	X		X				X		X										X		X	X			2/8
58	8-4 :16.87	X		X			X			X										X		X		X		
59	4-2 :11.88	X		X				X			X									X		X		X		4/8
60	3-24: 7.79		X		X			X				X									X			X		
61	3-24: 7.95		X		X			X				X										X	X		0.9	
62	3-22: 8.99		X		X		X					X										X		X	RF(4)	
63	3-27: 4.18		X		X			X				X										X		X		
64	8-7 :16.73		X	X				X				X										X		X	0.5	
65	8-7 : 4.70		X	X				X		X			X								X		X			6/10
66	8-6 :15.40		X	X			X			X			X								X			X	0.7	
67	8-6 :19.32		X	X				X		X			X								X			X		6/10
68	8-7 :15.83		X	X			X			X			X									X	X		1.3	
69	8-7 :15.36		X	X			X			X			X									X		X	0.1	
70	8-6 :15.61		X	X			X			X				X							X			X	0.8	
71	8-7 :15.30		X	X			X			X				X								X	X		2.1	
72	8-7 :14.90		X	X			X			X				X								X		X	0.4	
73	8-7 :21.19		X	X				X			X						X					X		X	1.6	
74	8-6 :16.11		X	X				X		X							X				X		X			4/10
75	8-7 :15.98		X	X			X				X						X				X		X			5/10
76	8-6 :16.16		X	X			X			X							X				X			X		3/10
77	8-7 :16.50		X	X			X			X							X					X	X		0.3	
78	8-7 :21.18		X	X				X			X							X				X		X	1.2	
79	8-6 :17.23		X	X			X			X										X	X		X		RF(0.5)	
80	8-7 :17.45		X	X			X			X										X		X	X		RF(0.6)	
81	8-7 :16.30		X	X			X			X										X		X		X		2/10

1/ Aquifer thickness is 0.20T
2/ Aquifer is 0.10T
3/ Aquifer is in contact with base of sediments

be done within a major quadrant.

The arrangement of table 13 is such that there is a general trend for the difference between lake level and head at the stagnation point to become less and for the potential for water loss from the lake to increase as one moves from quadrant I to quadrant IV. An example of this can be seen by comparing the PBD setting for shallow lakes in each subquadrant 3. In subquadrant I-3, a ground-water divide exists that extends to the base of the ground-water reservoir (RF 0.5). In sub-quadrant II-3 the ground-water divide weakens to where the head at the stagnation point is 0.001T (0.2 ft; 0.1m) higher than lake level. In subquadrant III-3 the ground-water divide is at the threshold of being obliterated and the lake losing water. And in subquadrant IV-3, where the hydraulic conductivity ratios are highest, the lake loses water through two-eighths of its bed.

Knowing the effect of a certain direction of change of a given parameter on the head at the stagnation point makes simulation of every possible combination of parameters unnecessary. If it can be shown that a ground-water divide exists beneath a lake in a setting that has greatest potential for water loss (e.g. PBU in subquadrant IV-4) then an even stronger divide will exist in settings where this potential is less. Conversely, if a lake loses water in a setting that has greatest potential for a divide to exist (e.g. PBD in sub-quadrant I-2b) then it should lose even more water in settings where the potential for water loss is greater. In these cases, one simulation is sufficient to determine the relationship for eight others. In some situations, the simulation may have even greater implications because if a shallow lake loses water, for a given setting, a deep lake in a similar setting will lose even more water.

Some general relationships concerning the one-lake system can be deduced from table 13. The following changes in parameters tend to weaken the difference in head between the lake and the stagnation point beneath a lake or tend to cause a lake to lose water: (1) lowering the water table on either side of a lake (lowering it on the downslope side has a greater effect than lowering it on the upslope side), (2) increasing K_h/K_v, (3) increasing the hydraulic conductivity of aquifers, which has a slightly greater effect than increasing K_h/K_v, (4) increasing the depth of a lake, (5) raising aquifers from the base to an intermediate level in the ground-water system. There is one exception to this statement. If an extensive aquifer that has low K_h/K_v and low K_{aq}/K_t (quadrant I, table 13) is raised from the base to an intermediate level, the head at the stagnation point increases slightly.

Aquifers that extend the full length of the ground-water basin have a greater effect on the interaction of lakes and ground water than limited aquifers. If no aquifer is present, a lake generally will not lose water even if K_h/K_v is 1,000 (table

14). Limited aquifers upslope and beneath a lake have little effect on the ground-water divide beneath a lake. Under the conditions simulated in this study, aquifers in these positions will not cause the divide to weaken regardless of the hydraulic conductivity of the medium. Limited aquifers downslope from a lake, whether at the base or at an intermediate level of the ground-water system, have a significant effect on the interaction of lakes and ground water because under many conditions of high K_h/K_v and K_{aq}/K_t and low water-table configurations, a lake will have a weak ground-water divide or will lose water. In every simulation run for this study the stagnation point associated with the ground-water divide is always under the downslope littoral or shoreline zone of a lake. This has particular significance when designing an observation-well program for studying the interaction of lakes and ground water because determining the position and head, or absence, of the stagnation point is the key to the relationship.

Multiple-lakes system

In much of the glaciated north-central United States, lakes occur at different altitudes within major drainage basins. It is a common belief that water moves successively from higher lakes to lower lakes through the ground-water system within a major drainage basin--a situation where all the lakes would be the flow-through type. Digital simulations were run of a number of variations in the hydrogeologic setting of three lakes along a regional slope from a major divide to a major drain. The vertical exaggeration of the sections are the same as those in the one-lake system (80X).

Moderate regional slope and local relief

The length and thickness dimensions of the multiple-lakes system are different from those of the one-lake system, thus the proportions of the features are different. As before, the dimensions in parentheses are included as examples of realistic field situations.

A three-lake simulation set will be discussed later in which the models are of a system that has a water table with very low regional slope, a physiographic condition common in many parts of the north-central United States. (See fig. 1).

One type of lake-ground water setting not simulated in this study is that of a straight-line, sloping water table between adjacent lakes. As will be apparent in the following sections of this thesis the height of the water table mound on the downslope side of a lake relative to lake level is an important control on the interaction of lakes and ground water. Lakes in a setting of a straight-line water table simply would gain ground-water inflow on one side and lose water out the other.

The thickness of the ground-water reservoir of the three-lake system ranges from T (as much as 250 ft; 76m) on the upslope end of the section to 0.40T (100 ft; 33m) on the downslope end. The height of the water table above lake level on the regional upslope sides of the two lower lakes is 0.20T (50 ft; 15m) and on the downslope side of all three lakes is 0.08T (20 ft; 6m). The highest lake in the section has the water table 0.12T (30 ft; 9m) above lake level on the upslope side. The difference between the level of the highest lake and the elevation of the water table on the upslope side of that lake was purposely made less than the two lower lakes because it is believed that lakes that occur high topographically are more likely to have lower water tables adjacent to them than the lower lakes. The water-table altitudes were varied in some of the simulations. The dimensions of the shallow lakes are about 0.1L wide and 0.20T deep. The aquifers are about 0.04T thick and those of limited areal extent are 0.15L long. As before, the datum for the head values is 100.

The positions of the three lakes are as follows: the surface of lake 1 is about 0.65T above the base and 0.32L from the left side of the section, lake 2 is at 0.75T and 0.60L, and lake 3 is at 0.88T and 0.85L. The aquifers are positioned at the base and, if at an intermediate level, at 0.30T above the base. The lateral position of the limited aquifers varies as noted in the text.

A simulation of a three-lake system in which each lake is shallow, is underlain by sediment, has a 0.08T (20 ft; 6m) water-table mound on the downslope side of each, K_h/K_v of the ground-water system is 1,000, and it contains no aquifers, is shown in figure 39. The position and head of the stagnation point on the ground-water divide beneath each lake is shown as in the discussion of the one-lake simulations. Under the given conditions, each lake gains ground-water discharge from its own local ground-water flow system, which is completely isolated from the others.

For the three-lake system, comparison of all possible combinations of different water-table altitudes, lake depths, hydraulic conductivity ratios, and size and position of aquifers, as was done for the one-lake system, would lead to a far more complex summary table. The basic goal of simulating the multiple-lake system is to determine if the lakes have interrelationships with each other that would not be evident in simulations of the one-lake system. The results of the multiple-lake simulations in which the water table has moderate regional slope and local relief are summarized in table 15.

The effect on the interaction of lakes and ground water of an extensive aquifer at the base of the system can be seen by comparing row 1 with rows 2 and 3 of table 15. If an extensive aquifer at the base of the system has hydraulic conductivity 1,000 times greater than till and the K_h/K_v of the ground-water system is 100, the difference in head between the lake and the stagnation point beneath lake 2 is very small. In

Table 15.--Summary of simulations of multiple-lakes system--water table of system has moderate regional slope and local relief.

Number	Model	Lake depth: Shallow	Lake depth: Deep	Kh/Kv: 10^2	Kh/Kv: 10^3	Kaq/K_t: 10^2	Kaq/K_t: 10^3	Aquifer position: None	Aquifer position: Full: Base	Aquifer position: Full: 0.30T	Aquifer position: Partial: Base: 0.15L	Partial: Base: 0.45L	Partial: Base: 0.75L	Partial: 0.30T: 0.15L	Partial: 0.30T: 0.45L	Partial: 0.30T: 0.75L	Height of water table above lake level (relative to total system thickness): Lake 1: Upslope: 0.20	Lake 1: Upslope: 0.16	Lake 1: Upslope: 0.12	Lake 1: Upslope: 0.08	Lake 1: down-slope: 0.08	Lake 1: down-slope: 0.04	Lake 2: Upslope: 0.20	Lake 2: Upslope: 0.16	Lake 2: Upslope: 0.12	Lake 2: Upslope: 0.08	Lake 2: down-slope: 0.08	Lake 2: down-slope: 0.04	Lake 3: Upslope: 0.20	Lake 3: Upslope: 0.16	Lake 3: Upslope: 0.12	Lake 3: Upslope: 0.08	Lake 3: down-slope: 0.08	Lake 3: down-slope: 0.04	Results: Lake 1: Height of head of stagnation point above lake level (in feet)	Results: Lake 1: Percent of bed that loses water	Results: Lake 2: Height of head of stagnation point above lake level (in feet)	Results: Lake 2: Percent of bed that loses water	Results: Lake 3: Height of head of stagnation point above lake level (in feet)	Results: Lake 3: Percent of bed that loses water
1	4-22:28.67	X			X			X									X				X		X				X				X		X		3.1		3.8		2.2	
2	4-23: 5.70	X		X			X		X								X				X		X				X				X		X		3.4		0.2			6/6
3	4-25:14.96	X			X		X		X								X				X		X				X				X		X		8.3			5/6		6/6
4	4-24: 5.74	X		X			X						1				X				X		X				X				X		X		RF(8.9)		RF(6.4)		RF(0.8)	
5	4-25:10.08	X		X			X					X	X				X				X		X				X				X		X		RF(6.7)		1.4		0.9	
6	4-24:10.95	X			X		X					X	X				X				X		X				X				X		X		4.2		2.8		1.1	
7	5-1:21.20	X			X		X					X	X				X				X			X			X					X		X	4.0		2.7		0.3	
8	5-2:10.85	X			X		X				X	X	X				X				X			X			X					X		X	1.7		2.5		0.3	
9	5-22:20-95	X			X		X				XS	XR	X				X				X		X				X				X		X		1.0		3.4		0.9	
10	5-16:6.77	X		X			X				XS	XR	X					X				X		X				X				X		X	1.0		3.2		0.2	
11	5-14:10.51	X			X		X				XS	XR	X					X				X		X				X				X		X	0.6		2.7		0.1	
12	4-25:20.58	X			X		X								X	X	X				X		X				X				X		X		4.4		1.8		0.1	
13	5-7:19.57	X			X		X								X	X	X				X			X			X					X		X	4.5		1.7			3/6
14	5-5:4.99	X			X		X							X	X	X	X				X			X			X					X		X	0.7		1.5			3/6
15	5-8:11.22	X			X		X							XS	X	X	X				X			X			X					X		X	0.1		1.5			3/6
16	5-12:5.60	X			X		X							XS	X	X		X				X		X				X				X		X			1.1			3/6
17	5-22:18.29	X			X		X							XS	XR	X	X				X		X				X				X		X			2/6	2.4			
18	5-15:6.70	X		X			X							XS	XR	X		X				X		X				X				X		X	1.3		3.4			2/6
19	5-13:10.17	X			X		X							XS	XR	X		X				X		X				X				X		X		2/6	1.8			3/6
20	5-19:7.43		X		X			X									X				X		X				X				X		X		2.2		2.7		0.8	
21	5-22:11.13		X	X			X				XS	XR	X				X				X		X				X				X		X		1.6		4.2(RF)		0.1	
22	5-21:13.49		X		X		X				XS	XR	X				X				X		X				X				X		X			2/8	2.1			3/8
23	5-28:8.05		X	X			X							XS	XR	X	X				X		X				X				X		X		2.0		RF(0.1)			3/8
24	5-23:26.79		X		X		X							XS	XR	X	X				X		X				X				X		X			4/8	1.0			5/8

1 = Partial aquifer at base upslope from lake 3
XR = Aquifer shifted from 0.45L to 0.53L
XS = Aquifer shifted from 0.15L to 0.22L

this simulation, there is very little change in the head at the stagnation point for lake 1 compared to the simulation listed in row 1, but there is water loss through the bottom of lake 3. If K_h/K_v is increased to 1,000 (row 3, table 15), holding all other parameters as in row 2 of table 15, a very strong divide is established beneath lake 1, and lakes 2 and 3 lose water through most, or all, of their bed (fig. 53).

Further evidence that the lakes are independent of each other can be seen by comparing rows 6 and 7 of table 15. In these simulations, K_h/K_v and K_{aq}/K_t are both 1,000 and two limited aquifers are at the base of the ground-water system at the 0.45L and 0.75L positions. The only difference between the two is that the water-table mound between lakes 2 and 3 was decreased to only 0.04T (10 ft; 3m) above the level of lake 3, and the upslope side of lake 3 was decreased to 0.08T (20 ft; 6m) higher than lake level for the latter simulation (row 7). The effect of this change in the water-table near lake 3 on lakes 1 and 2 is very little; whereas, the head at the stagnation point beneath lake 3, decreases 0.003T (0.8 ft; 0.2m). If a limited aquifer is added at the 0.15L position, and all other parameters are as in the simulation listed in row 7 (table 15), the head at the stagnation point on the ground-water divide beneath lake 1 decreases considerably, and there is little effect on lakes 2 and 3 (compare rows 8 and 7).

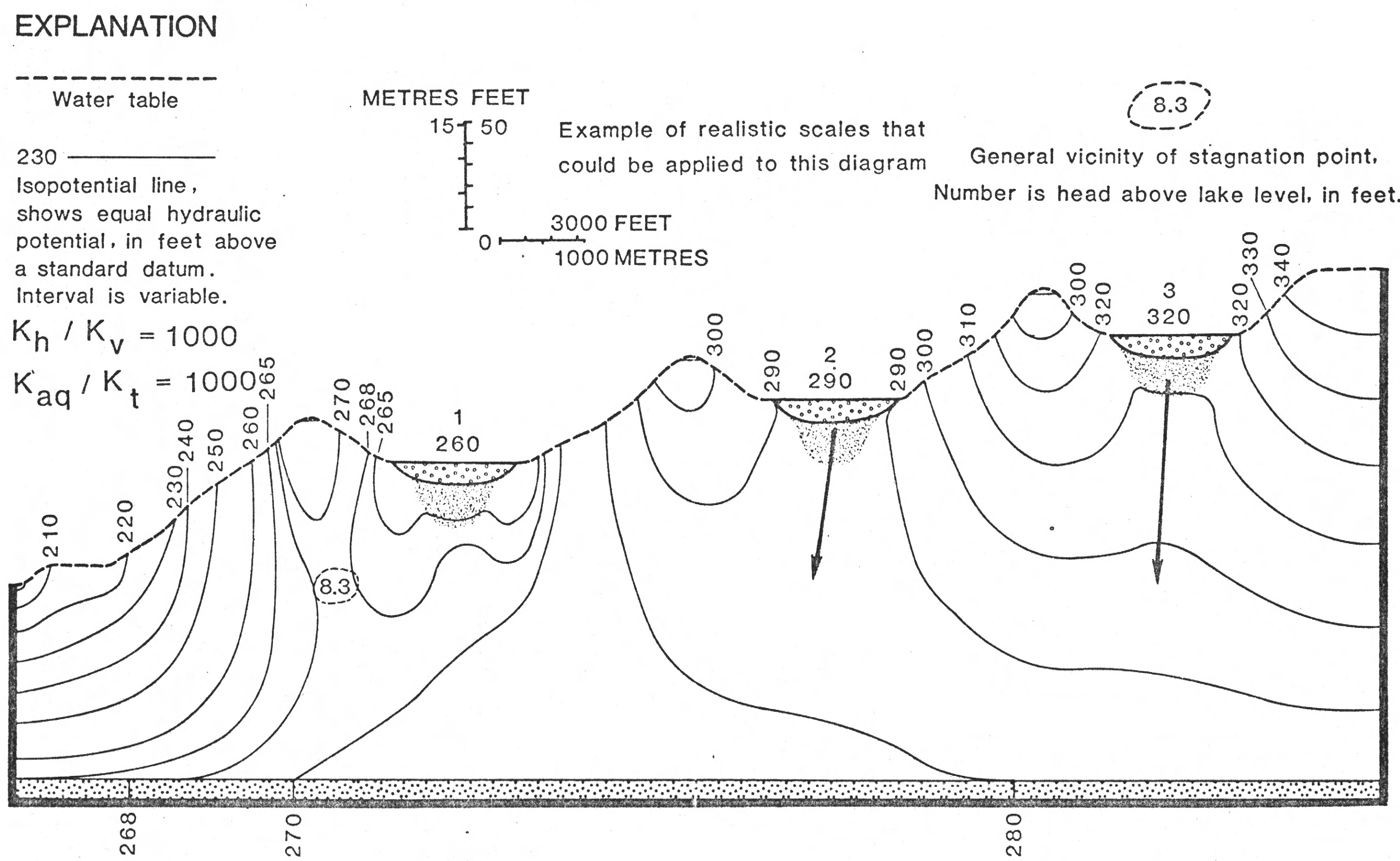


Figure 53.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has an extensive aquifer of relatively high hydraulic conductivity at the base of the system.

The effect on the interaction of lakes and ground water of moving the position of limited aquifers laterally, can be seen by comparing rows 14 and 15 of table 15. In the simulation listed in row 14, the aquifer at the 0.15L position is downslope from the edge of lake 1 (figure 38). In the simulation listed in row 15 (fig. 54), this aquifer is shifted slightly in the upslope direction (to 0.22L), so that the upslope part of the aquifer is partially beneath the downslope side of lake 1. All other parameters for these two simulations are held constant. This change lowers the head at the stagnation point near lake 1 by 0.002T (0.6 ft; 0.2m). Thus, it is evident that the interaction of lakes and ground water is sensitive to the lateral position of limited aquifers downslope from a lake. The maximum effect is felt if the upslope part of an aquifer is beneath the downslope side of a lake, and the aquifer underlies the water-table mound downslope from a lake.

The effect on the interaction of lakes and ground water of moving the limited aquifers vertically from the base of the system to a middle position is similar to the one-lake situation; that is, the head at the stagnation point is lowered. (Compare rows 6 and 12, and rows 11 and 19). (See figures 55 and 56.) Note that the middle aquifer in these two simulations is shifted to the right, from the 0.45L to about the 0.53L position, compared to the other simulations discussed to this point. This was done for most of the multiple-lakes simulations because of

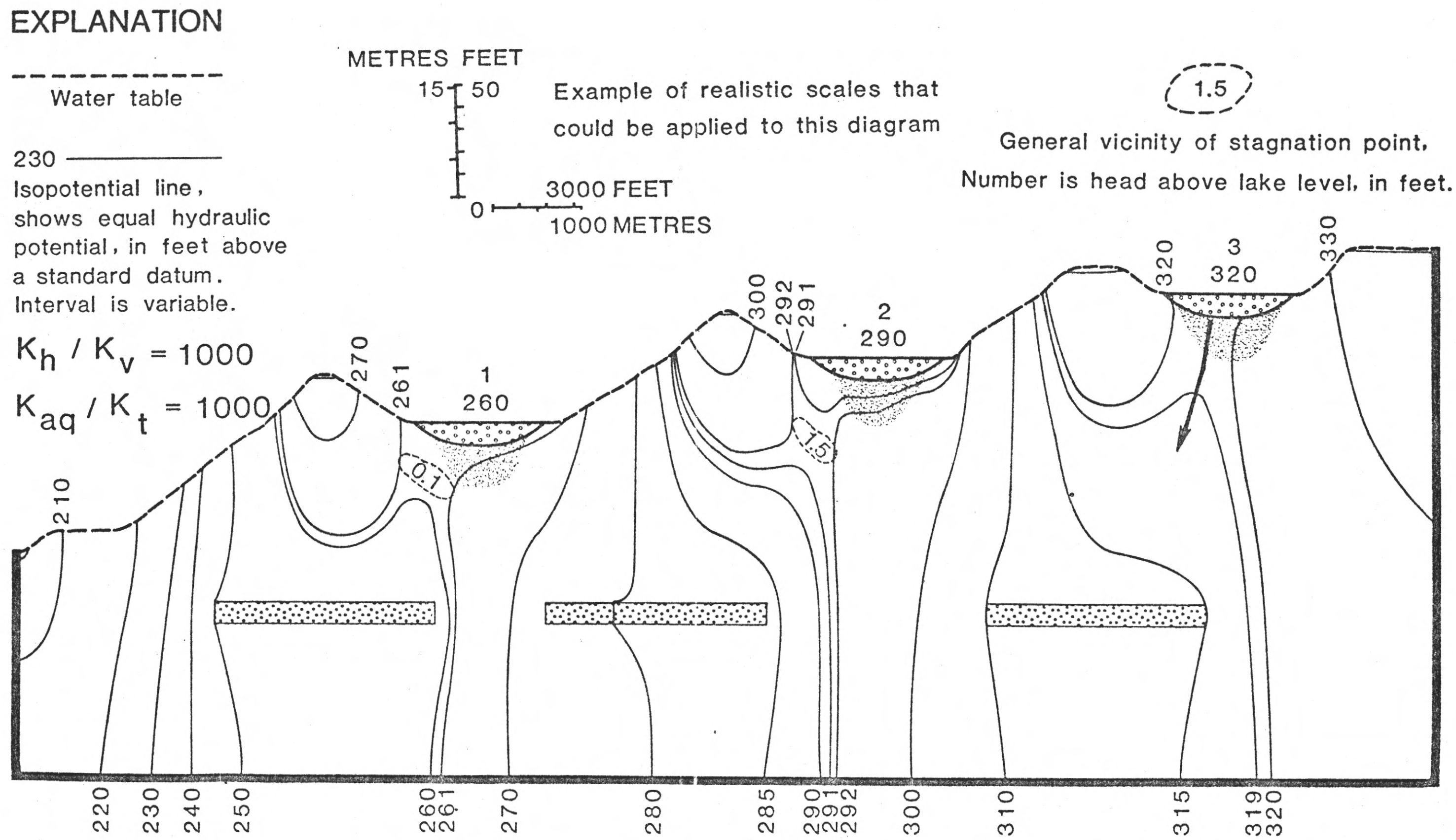


Figure 54.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has various height water-table mounds and aquifers of limited extent in the middle of the system.

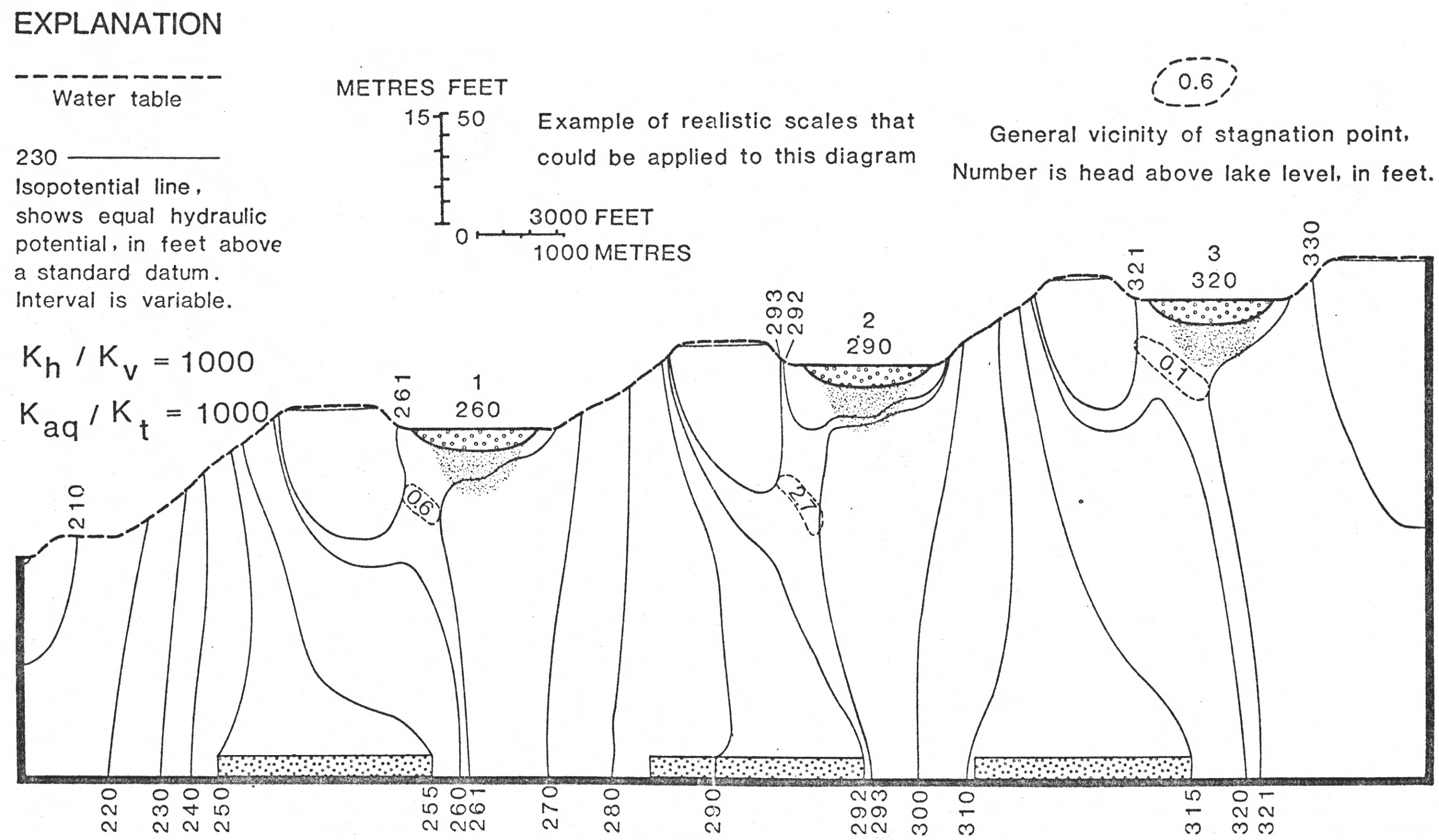


Figure 55.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has low water-table mounds and aquifers of limited extent at the base of the system.

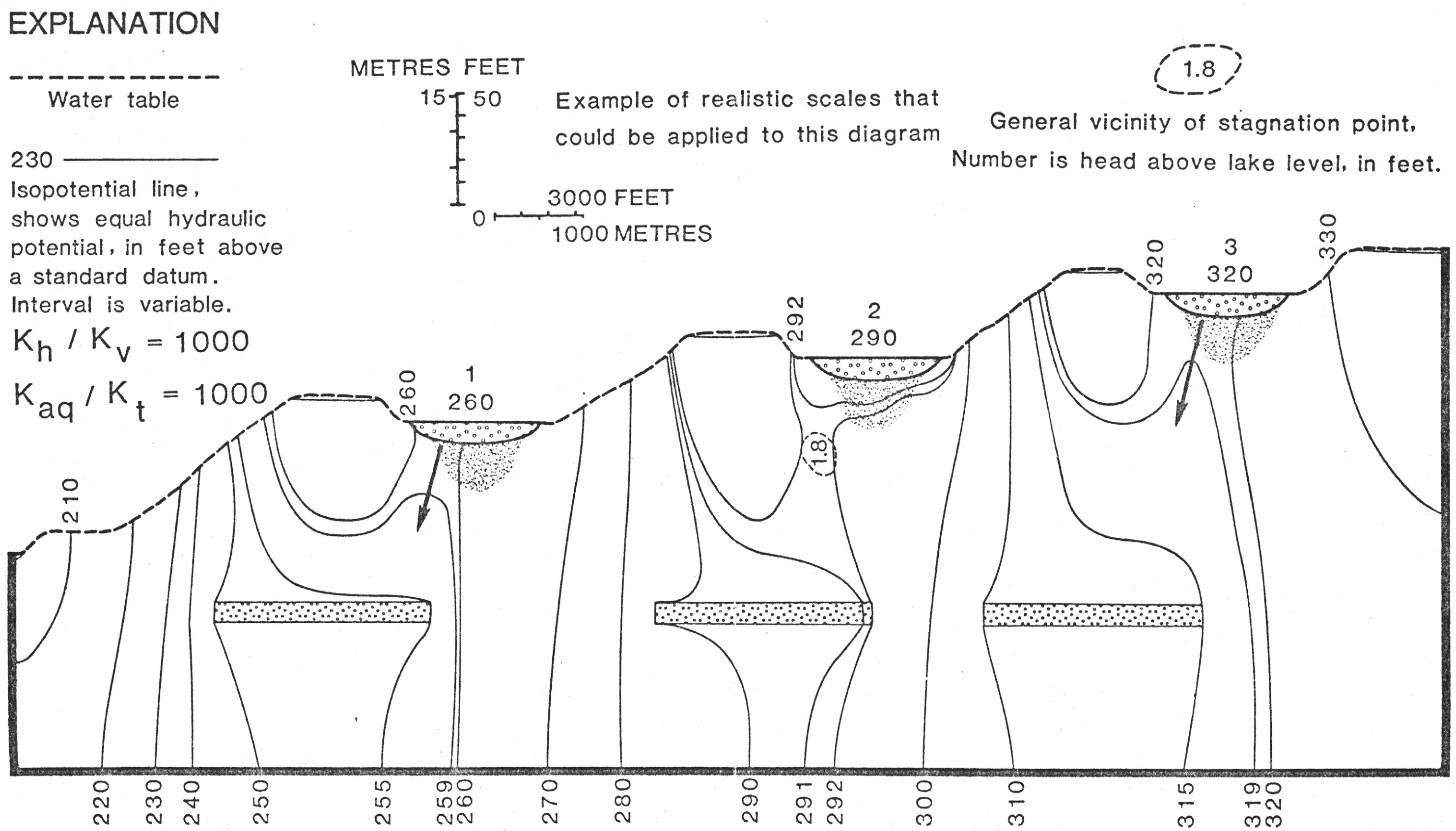


Figure 56.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has low water-table mounds and aquifers of limited extent in the middle of the system.

the findings mentioned in the previous paragraph.

The effect of deep lakes compared to shallow lakes is also similar to the simulations of the one-lake system. Comparing rows 9 and 22 of table 15, in which all parameters are similar except lake depth, shows the relatively greater influence on the ground-water flow systems near deep lakes compared to shallow lakes. The ground-water divide beneath lake 1 in the simulation of shallow lakes (fig. 57) has a head at the stagnation point 0.004T (1 ft; 0.3m) greater than lake level; whereas, in the simulation of deep lakes (fig. 58), the lake loses water through two-eighths of its bed. Beneath lake 2, the head at the stagnation point decreases from 0.014T (3.4 ft; 1.0m) greater than lake level in the shallow-lake simulation to 0.008T (2.1 ft; 0.6m) in the deep-lake simulation. Beneath lake 3, the difference in head between the lake and the stagnation point of 0.004T (0.9 ft; 0.3m) for the shallow lake, can be compared to the deep lake which loses water through three-eighths of its bed.

In most of the three-lake simulations discussed thus far, K_{aq}/K_t and K_h/K_v are 1,000. These values were chosen because, as shown in the one-lake system simulations, they tend to create conditions for maximum water loss from the lake to the ground-water system. It is of interest, therefore, to compare rows 21 and 22 (table 15). The simulation summarized in row 21 has K_h/K_v = 100; whereas, in the latter this ratio is 1,000. The stagnation points occur very deep in the system in the former and, in fact,

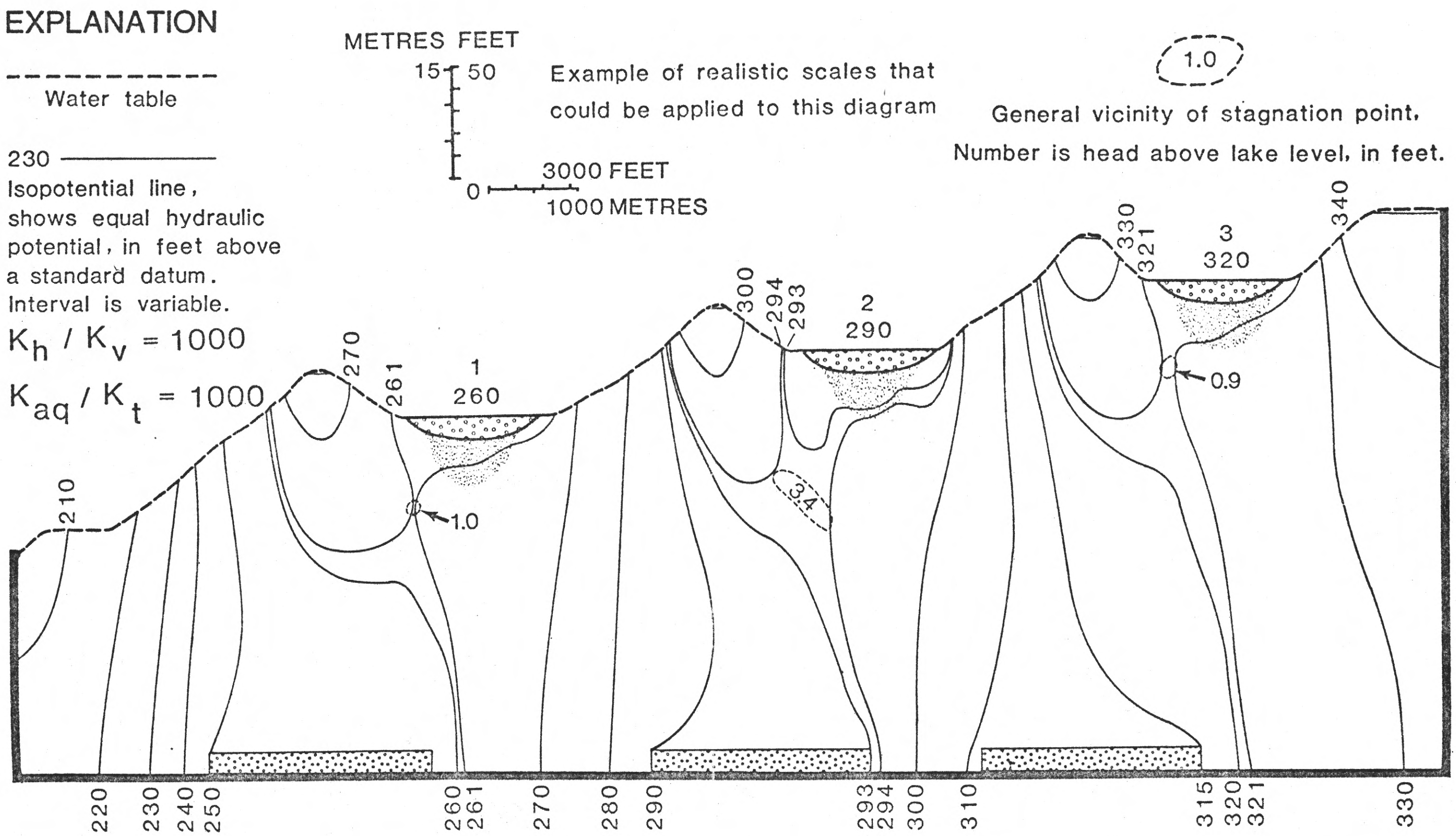


Figure 57.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has high water-table mounds and aquifers of limited extent at the base of the system.

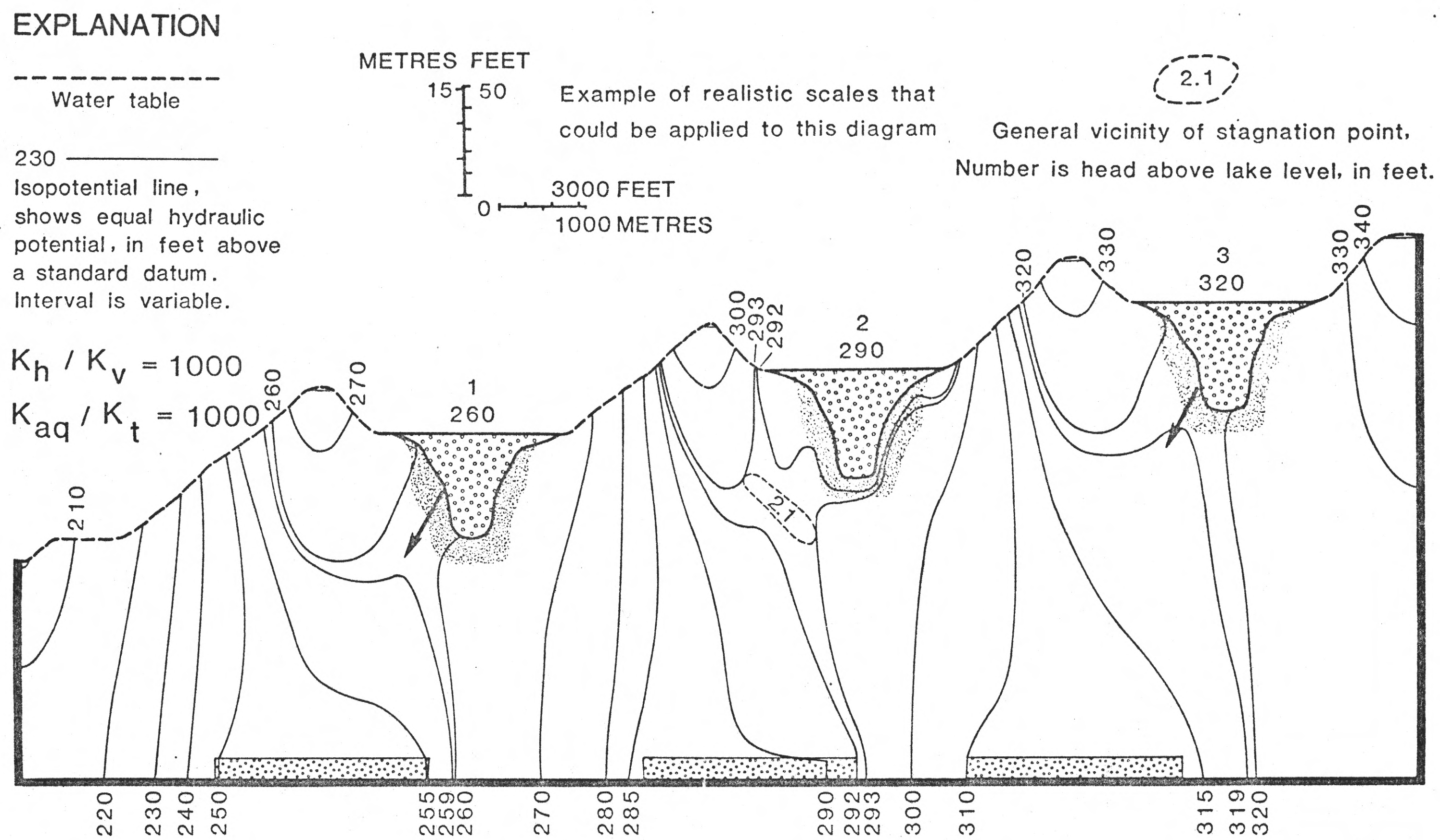


Figure 58.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has deep lakes, high water-table mounds, and aquifers of limited extent at the base of the system.

the local flow cells around each of the lakes extend nearly the full thickness of the ground-water system and little ground water flows past the middle lake.

It is evident from the above discussion that, in most cases, lakes in a multiple-lakes system act essentially independently. Changing the height of a water-table mound or the position of aquifers by one lake and not by the others has a considerable effect on that one lake that is similar to the equivalent one-lake simulation. The change may have a minor effect on an adjacent lake--changing the head at the stagnation point, for example, by perhaps 0.0004T to 0.0008T (0.1 to 0.2 ft; 0.03 to 0.06m).

The only situation where a considerable effect is felt on the lakes is in the case of extensive aquifers within the ground-water system, and where K_h/K_v and K_{aq}/K_t are large (row 3 of table 15) (fig. 53). In this case, and there are similar cases in the one-lake settings, there is massive downward movement of water in the upper region of the ground-water basin and massive upward movement in the lower region. The great difference in head between the lake and the stagnation point associated with the lowest lake seems to be established because of the large influence of the water-table mound on the downslope side of that lake. If the water-table mound were not there, there would be massive upward movement on the lower side of the diagram and the hinge line (the point at the water table separating flow away from the water-table from flow toward the water table) would be near the middle of the diagram. Apparently the mound is sufficiently strong to cause downward movement that will not be overcome, but instead, will be responsible for a strong ground-water divide that has a head at the stagnation point of very large magnitude. It is interesting to note in this case that the stagnation point is not beneath the lake. Rather, it is downslope from the lake and is nearly directly beneath the highest part of the water-table mound.

Low regional slope and local relief

A series of simulations were run of a multiple-lakes system in which regional slope and local relief were kept to a minimum (summarized in table 16). The length and thickness dimensions of the multiple-lakes system that has low regional slope and local relief are unlike those for the system of moderate slope and relief, thus the proportions of the features again are different. The thickness of the system ranges from T(160 ft; 49m) on the upslope end to 0.60T (100 ft; 30m) on the downslope end of the section.

The water table on the upslope side of all the lakes is 0.13T (20 ft; 6m) higher than the adjacent lake level, and on the downslope side is 0.06T (10 ft; 3m) higher. The difference in altitude between the lakes is 0.06T (10 ft; 3m). Only shallow lakes (0.06T; 10 ft; 3m) are simulated. The aquifers

Table 16.--Summary of simulations of multiple-lakes system--water table of system has low regional slope and local relief.

Number	Model	Lake depth: Shallow	Lake depth: Deep	Kh/Kv: 10^2	Kh/Kv: 10^3	Kaq/K_t: 10^2	Kaq/K_t: 10^3	Aquifer position: None	Aquifer position: Full: Base	Aquifer position: Partial: Base: 0.35L	Partial: Base: 0.55L	Partial: Base: 0.82L	Partial: 0.45T: 0.35L	Partial: 0.45T: 0.55L	Partial: 0.45T: 0.82L	Height of water table above lake level (relative to total system thickness): Lake 1: Upslope 0.13	Lake 1: down-slope 0.06	Lake 2: Upslope 0.13	Lake 2: down-slope 0.06	Lake 3: Upslope 0.13	Lake 3: down-slope 0.06	Results: Lake 1: Height of head of stagnation point above lake level (in feet)	Lake 1: Percent of bed that loses water	Lake 2: Height of head of stagnation point above lake level (in feet)	Lake 2: Percent of bed that loses water	Lake 3: Height of head of stagnation point above lake level (in feet)	Lake 3: Percent of bed that loses water
25	6-4:5.31[2]	X			X			X								X	X	X	X	X	X	3.0		3.0		3.0	
26	6-17:9.36[3]	X			X			X								X	X	X	X	X	X	3.0		3.0		3.0	
27	6-5:7.56	X				X			X							X	X	X	X	X	X	2.7		1.9		0.4	
28	6-9:2.38	X		X			X		X							X	X	X	X	X	X	2.8		1.8			
29	6-11:5.14	X			X		X		X							X	X	X	X	X	X	3.9			7/8		8/8
30	6-11:6.92	X			X		X			X	X	X				X	X	X	X	X	X	1.6		1.9		1.7	
31	6-16:25.29	X		X			X						X	X	X	X	X	X	X	X	X	2.1		2.2		RF(2.1)	
32	6-16:26.10	X			X		X						X	X	X	X	X	X	X	X	X	0.4		0.8		0.7	
33	6-18:6.49[4]	X			X			X								X	X	X	X	X	X	RF(4)		RF(3)		RF(1)	

2 = K system = 0.2 ft/day
3 = K system = 200.0 ft/day
4 = Shallow ground-water system

are 0.06T (10 ft; 3m) thick, and the limited aquifers are 0.13L long.

A ground-water system that has no aquifers is shown in figure 59. Ground-water divides of equal size and strength (the difference in head between the lake and the stagnation point is 0.02T (3.0 ft; 0.9m) by each lake) occur beneath each lake. It should be noted that simulations of this system of lower slope and relief differ somewhat from the other three-lake system discussed previously in that the water table by the highest lake is the same as by the other lakes.

Simulations that have extensive aquifers at the base of the system, K_{aq}/K_t = 100, and K_h/K_v = 1,000, show no water loss from any of the lakes for any of the simulations. This is true also if the hydraulic conductivity ratios are reversed; that is, K_{aq}/K_t = 1,000 and K_h/K_v = 100. In simulations that have both ratios at 1,000, a ground-water flow pattern similar to the other three-lake system occurs--there is water loss through the entire bed of the higher two lakes and a divide established beneath the lowest lake.

In simulations that have limited aquifers beneath the water-table mounds between the lakes, and the up-slope edge of the aquifers are partly beneath the lake, a ground-water divide occurs beneath the lake, although the difference in head between the lake and the stagnation point of each is rather small (fig.

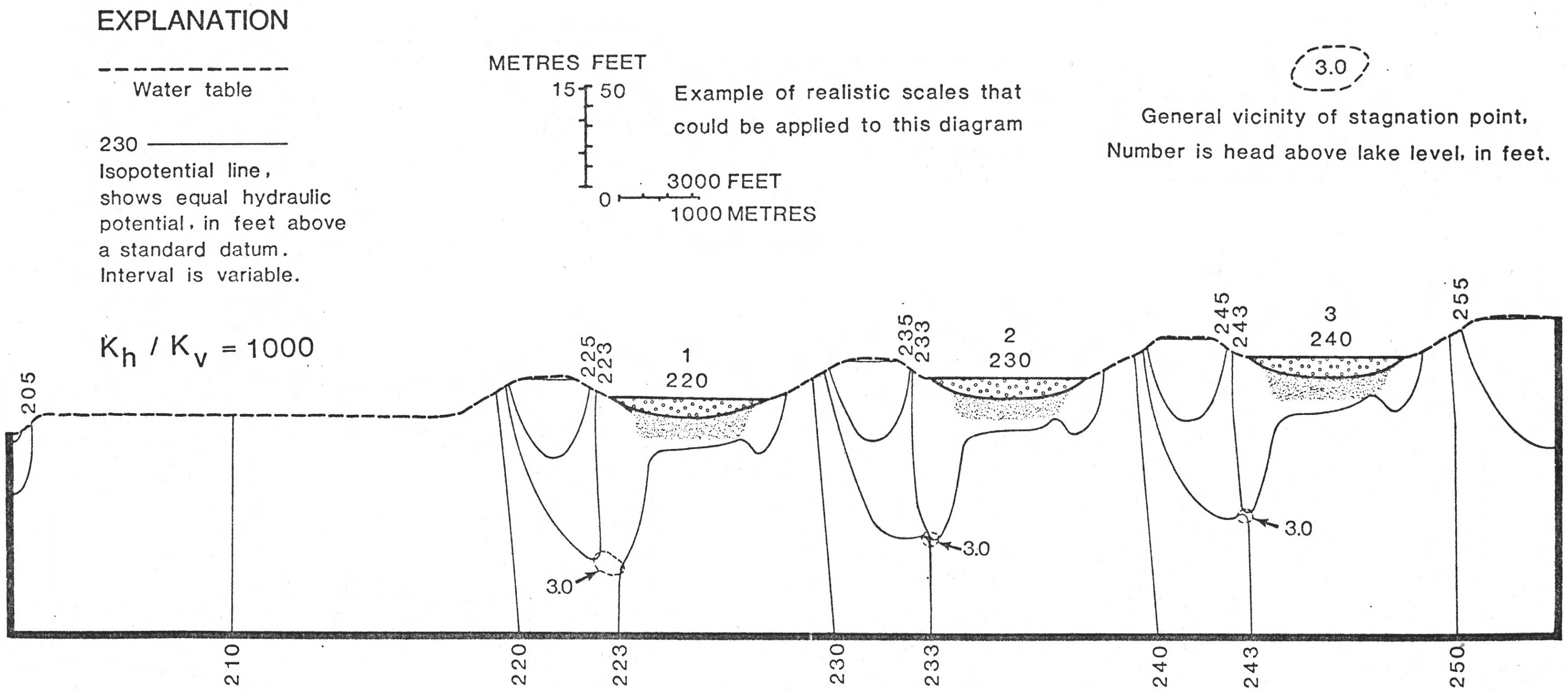


Figure 59.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has low regional slope and does not contain aquifers.

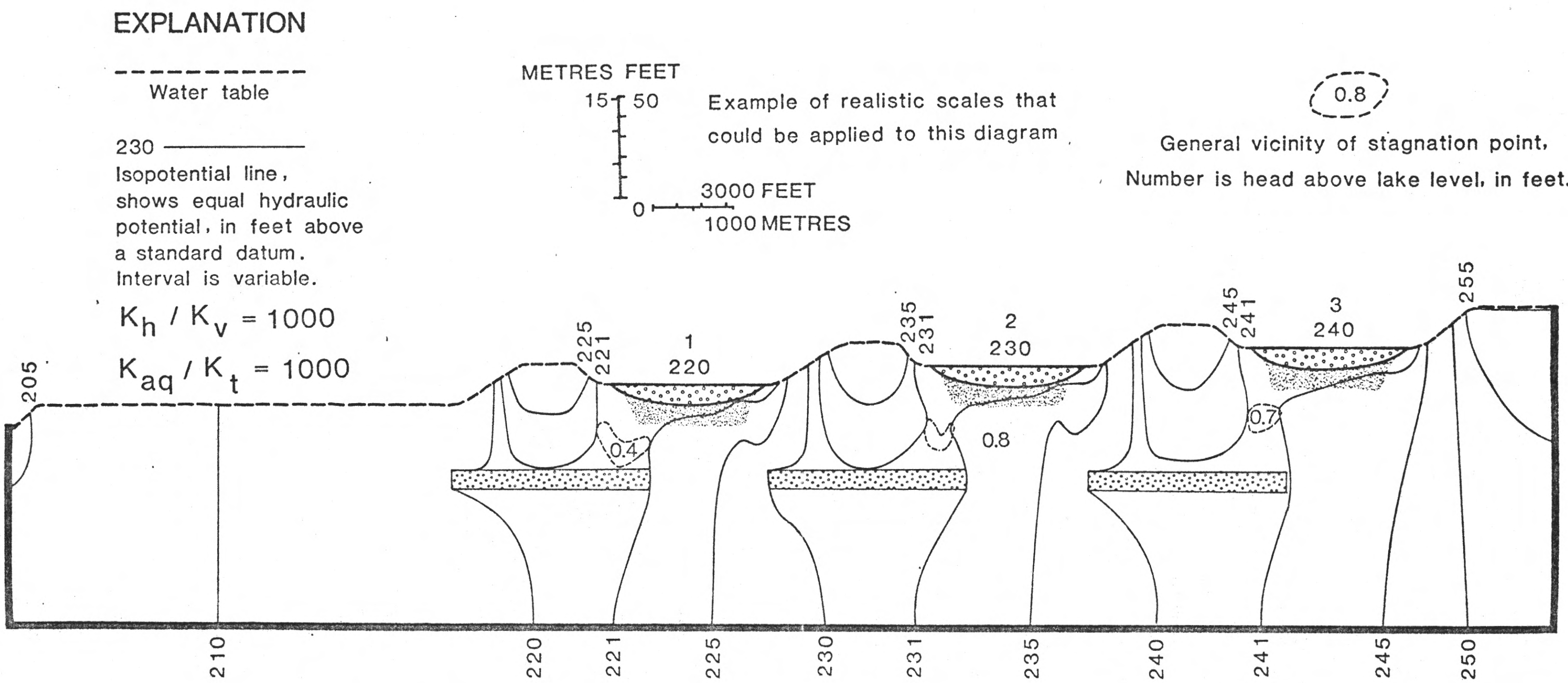


Figure 60.--Distribution of hydraulic head in the ground-water reservoir of a multiple-lakes system that has low regional slope and aquifers of limited extent in the middle of the system.

60). This result holds even when the limited aquifer is moved to about 0.13T (20 ft; 6m) below the bed of the lower lake.

In the three-lake system that has low regional slope and local relief, it appears that the ground-water divides tend to be slightly stronger in most cases than where a higher regional slope is simulated. The slope of the water table from the lowermost water-table mound to the left edge of the hydrologic sections in the simulation of moderate regional slope, shows a drop of several tens of feet. The water-table slope in this same part of the section, in the simulations of low regional slope, is essentially flat, resulting in very slow ground-water movement. This very flat water table near the valley of the major drain on the system acts as a "check valve" on the entire system and tends to increase the tendency of the lakes to have stronger local ground-water flow systems than in conditions of a higher-sloping water table.

In none of the simulations does water move from one lake to the next downslope if there is a water-table mound between the two lakes. Even if an upper lake loses water to the ground-water system, the movement generally is deep to the system and the water moves beneath the local flow systems associated with lower lakes. Thus, even in settings where lakes occur at various altitudes down a sloping valley side, each of the lakes, for most settings, acts as an independent entity and has little relationship to lakes either upslope or downslope from it.

This conclusion gives increased importance to the summary table (table 13) of the one-lake system.

The effect of reservoir thickness on ground-water flow systems is extensively discussed by Toth (1963), and to a lesser extent by Freeze (1969b). The general effect of a thin ground-water reservoir is that regional flow does not occur for many settings, especially if regional slope is low. To check the effect of a thin ground-water system on the interaction of lakes and ground water a setting was simulated that was 100 feet (30m) thick on the upper end of the section and 50 feet (15m) thick on the lower end. The difference between the section and figure 59 is that the local flow systems associated with each lake extend to the base of the ground-water reservoir, and regional flow does not occur.

Quantitative aspects of ground-water interchange with lakes

Most of the discussion in this section of the report is concerned with the qualitative aspects of the relationship of ground water to lakes. Attention is focused on factors that control the presence and strength, or the absence of a ground--water divide beneath a lake. The importance of the position and head relative to lake level of the stagnation point of a ground-water divide, if present, is stressed. Not only is the basic presence or absence of a divide of interest to those concerned with lake hydrology, but the quantities of water involved are also of great interest.

The basic concept of a flow net is presented in the discussion of ground-water flow diagrams. It is pointed out in that section that figures 37 and 38 are not quantitative flow nets in the precise meaning of the term, because the ground-water reservoirs they illustrate are considerably anisotropic and vertical exaggeration is great.

A quantitative flow net can be used to calculate ground-water discharge through an isotropic medium using the following form of Darcy's Law (Freeze, 1969b):

$$Q = K \frac{\Delta\phi}{\Delta_s} \Delta m\text{-}w \qquad (36)$$

where Q = discharge through a segment of the flow net,

K = hydraulic conductivity,

$\Delta\phi$ = drop in hydraulic head between equipotential surfaces,

Δ_s = length of flow path in the segment of the flow net,

Δ_m = width of the segment of the flow net perpendicular to the flow direction,

w = thickness of the flow system perpendicular to the plane of the diagram.

Taking a unit thickness of the system (w=1), and considering that for the squares of the net $\Delta s = \Delta m$, equation 36 reduces to

$$Q = K\Delta\phi. \qquad (37)$$

Useful properties of a flow net are that discharge in each flow channel remains constant throughout its length, and equal quantities of water move through each flow channel. In spite of the fact that figures 37 and 38 cannot be used to accurately calculate discharge, they are useful in estimating relative quantities because the *proportions* of flow between the various flow systems remain constant.

Referring to figure 37, it is shown by relative spacing of stream lines that much more water moves through the local flow systems than the intermediate or regional systems. The difference in quantities of flow between the various flow systems is not as great in the situation represented by figure 38, partly because of the relatively weak local flow near, and the leakage from, lake 3.

The relationship of ground water to lake 3 of figure 38 is of interest when considering quantities of flow into and out of the lake. Despite the fact that the lake loses water through at least half of its bed, the flow net indicates more ground-water gain into the lake than loss out of it. Not only is the loss relatively low when considering only the hydraulic potential distribution, but most of the outflow must be through the sediments, which is true of nearly all situations where lakes lose water to the ground-water system. From equation 37, it is clear that Q must be rather low if both K and $\Delta\phi$ are low. This pattern of inflow in the littoral zone of a lake, even on the downslope side, and loss, if present, through the lake sediments, is a common phenomenon in nearly all the simulations in which loss occurs.

McBride (1969) found that the inflow to Lake Sallie, Minnesota, is greatest in the shallows near the shoreline and decreases logarithmically toward the deeper part of the lake. He later showed, through a modeling approach (McBride and Pfannkuch, 1975), that this is a general phenomenon. It can also be corroborated by a detailed flow net analysis.

The results of this study and those of McBride are important to the calculation of water budgets for lakes. Because even if a lake loses water through much of its bed, the quantity of ground-water gain in the littoral zone may be considerably greater than loss through the sediments of the lake.

The other important factor in calculating the quantities of ground-water interchange with a lake is the hydraulic conductivity (K) of the ground-water system. Specific values of hydraulic conductivity have been mentioned little in the discussion up to this point, because in the models all K values are relative. If hydraulic conductivity is changed from 0.2 ft/day (0.06 m/day) to 200.0 ft/day (61 m/day), but all the other K values in each simulation are changed accordingly, the results are identical. (Compare rows 25 and 26 of table 15.)

If interest is in quantities of ground-water discharge, for use in determining water budgets for example, specific values of K are extremely important (see equation 37). Obviously, far more ground water will interact with a lake situated in highly conductive rock material such as sand and gravel than a lake situated in silty till, provided the hydraulic gradients ($\Delta\phi$) are similar.

Areal and temporal variations

The hydrologic sections discussed in this report represent a two-dimensional vertical view of the ground-water system. By carefully selecting the line of section on a map, the general qualitative relationship of lakes and ground water can be adequately determined. To assure areal coverage of the lake's entire ground-water basin, however, the most correct analysis of the entire drainage basin of a lake would have to consider radial flow. Perhaps the best approach would be to use 3-dimensional modeling techniques which have recently received much research attention and are becoming available for general use. Until 3-dimensional models of ground water flow near lakes are operational, an alternate approach to providing areal coverage is to run 2-dimensional models of selected lines of section. (The lines of section must be chosen so they radiate from a lake and are representative of a pie-shaped part of the ground-water system near a lake.)

Because the ground-water system is dynamic, a steady-state hydrologic section can be representative of only a single point in time. As lake and water-table altitudes change, so must a different hydrologic section be prepared to represent the new conditions. If a lake is in a setting of widely fluctuating water-table and lake levels, especially if the changes are out of phase with one another, or are much greater in one than the other, a great many sections might be needed. Not only would flow nets be needed to calculate the changed rates of ground-water discharge but the basic relationship of a losing versus a gaining lake might be changed. An example of this is given by Meyboom (1967), who found that some lakes in the prairie provinces of Canada are underlain by ground-water divides for part of the year (spring and summer), but lose water to the ground-water system for the remainder of the year (fall and winter). Depending on the accuracy needed for calculation of ground-water discharge, sections would have to be drawn to reflect the changing conditions.

There are probably many lakes in hydrogeologic settings where the trends in lake levels and water-table levels are essentially parallel for most of the year (Winter and Pfannkuch, 1976; McBride, 1969). In such situations, hydrologic sections would be needed for one or only a few times during the year. Freeze (1969b) argues, in considering large basins, that the effect of small-scale cycles of wetting and drying during the year can be approximated by a steady-state average.

The most likely situations where hydrologic sections would be needed for more than one or two times during a year would be if: (1) a large rise in lake level would not be accompanied by a similar rise in the water table; for example, a flood wave passing through a lake that has streamflow in and out, or a large drop in lake level caused by release of water through a dam; (2) a large drop in the water table caused by discharge of ground water by phreatophytes, or dissipation of water-table mounds because of lack of rainfall. Both latter conditions had a bearing on the results of Meyboom's study.

Implications for field studies of lake-ground water interaction

It was pointed out in the introductory paragraphs of this section of the report that many different approaches to studying the relationship of ground water to lakes have been tried. Some workers have thought that a single piezometer (observation well) beside a lake would adequately define the relationship, while others placed large numbers of piezometers near a lake. Knowledge of the possible variations of ground-water flow systems near lakes, as demonstrated in the simulations of this study, suggest placement of piezometers that would optimize information on the interaction of lakes and ground water using a minimum number of piezometers.

Rather than place piezometers within the drainage basin of a lake, most effort should be on finding the maximum altitude of the water-table along the areal ground-water divide around a lake. For flow net analysis it is necessary to find this water-table-mound maximum and to place a piezometer on it for each segment of the lake's drainage basin where different ground-water flow patterns are suspected. As seen from the simulations, one of the most important locations is the height of the water table above lake level on the downslope side of a lake.

Finding the maximum height of a water-table mound can be a difficult chore. It has been demonstrated a number of times in the field that water-table mounds do not directly underlie topographic highs. Topography is not shown on the hydrologic sections--one can visualize topographic highs anywhere along the sections. In addition, anyone with field experience knows that accessibility of roads and land owner's permission is an important constraint in locating drilling sites and observation wells. It is easily demonstrated in any of the simulations of a three-lake system (fig. 37, for example) that a piezometer placed closer to lake 2 than lake 3 along the water table between lakes 2 and 3 would record water-table altitudes lower than the level of lake 3. One would conclude in this case that, since the water table at that point is at an intermediate altitude between the two lakes, water flows from lake 3 to lake 2; an erroneous conclusion reached in many studies of lake-ground water interaction in which few wells are placed and the principles of ground-water flow near lakes are not understood.

In addition to locating key points along the water-table divide around a lake, it is important to know the location and head relative to lake level of the stagnation point on the ground-water divide beneath the lake, if such a point exists. This is best accomplished by placing a nest of closely-spaced piezometers, each completed at a different depth, in the most appropriate location. As suggested by the simulations of this study, the most appropriate location to look for the stagnation point is beneath the shore line on the downslope side of the lake.

Data on the other factors that control ground-water flow (hydraulic conductivity of the various geologic units, defining hydrogeologic boundaries, location of aquifers) are not easily obtained, but there are accepted methods for doing so.

Field determination of the critical K_h/K_v ratio, on the other hand, is a serious problem. Although it has not been field tested, the method of determining K_h/K_v from measured head values proposed by Gillham and Farvolden (1974) may prove useful.

EVALUATION THROUGH DIGITAL MODELING OF THE GROUND-WATER PARAMETERS USED TO CLASSIFY THE HYDROLOGIC SETTING OF LAKES

Parameters closely related to ground-water flow that were identified as being important in developing a hydrologic classification of lakes (the first major part of this report) are regional slope, local relief, regional position, and drift texture (a measure of hydraulic conductivity). A main purpose of this report is to identify and evaluate the factors that control the interaction of lakes and ground water, 1) to better understand the phenomenon and 2) to modify the classification by identifying ground-water parameters of most importance, and new parameters that should be included.

Parameters found to have various degrees of control on the interaction of lakes and ground water are, 1) height of the water table relative to lake level, 2) position of the lake in the regional setting, 3) size and position of aquifers within the ground-water reservoir, 4) hydraulic conductivity of the geologic units relative to each other, and, for quantitative estimates of ground-water discharge, actual values of hydraulic conductivity, 5) thickness of the ground-water reservoir, 6) the ratio of horizontal to vertical hydraulic conductivity, and 7) lake depth.

The height of the water table relative to lake level is related to the parameters regional slope and local relief of the first part of this thesis. The simulations of the second part of this report show a strong influence of local relief of the water table adjacent to a lake on the interaction of lakes and ground water. The relative height of the water table on the downslope side of a lake is particularly important. The effect of regional slope on the interaction of lakes and ground water can be assessed by comparing simulations discussed in "multiple-lakes system" of this thesis. Regional slope has a lesser effect on the interaction of lakes and ground water than local relief, but the simulation study indicates it should be retained in the classification.

The relative altitudinal position of a lake is of questionable importance after assessing the simulation results. If the assumption holds that the height of the water table relative to lake level is lower in the higher parts of drainage basins, as simulated in the three-lakes system--moderate regional slope and local relief, this parameter should be retained in the classification. If, on the other hand, there is no difference, as simulated in the three-lake system--low regional slope and local relief, this parameter is not needed in the classification. Until field studies are done to check the assumption, the parameter should be considered in classifying the hydrologic setting of lakes, but with the reservation mentioned above.

The size and position of aquifers within the ground-water reservoir, the definition of the geometry and boundaries of the geologic framework, is important to the interaction of lakes and ground water. It is a parameter that is not usually obtainable from map and office data. Because extensive field work beyond the scope of this report is necessary to define the geologic framework, a parameter describing the distribution of aquifers is not included in the classification as it now exists.

The hydraulic conductivity of bedrock has some importance here, however, because of the great effect of extensive aquifers on the interaction of lakes and ground water. Where bedrock of high hydraulic conductivity underlies drift, the assumption of a no-flow boundary at the base of the ground-water reservoir is not valid. In such regions, therefore, the simulations of lake-ground water interaction that include extensive aquifers at the base of the ground-water system, are applicable. Thus, the bedrock parameter should be retained in the classification of the hydrologic setting of lakes.

The hydraulic conductivity of the geologic units in the ground-water reservoir is a key control on the interaction of lakes and ground water, therefore it must be considered in classifying the hydrologic setting of lakes. The parameter, as scaled in the first part of the report, is extremely generalized. Much field work must be done to obtain realistic K values near lakes of interest and to refine the scaling for drift texture used in the first part of this report.

Thickness of the ground-water reservoir can be an important factor in the interaction of lakes and ground water if the situation is such that local flow systems extend to the base of the reservoir. In such settings the addition of water, and especially the influence of highly mineralized ground water from intermediate and regional flow systems, obviously could not affect lakes in the lower parts of drainage basins. In spite of this, drift thickness is not considered critical enough to include in the classificatory system.

The ratio of horizontal to vertical hydraulic conductivity is one of the most important controls on the interaction of lakes and ground water. Realistic values, however, are virtually unavailable and methods of obtaining them are not well developed. Thus, at this time, the K_h/K_v ratio cannot realistically be one of the parameters used to classify the hydrologic setting of lakes.

Lake depth is shown in this thesis to greatly influence the interaction of lakes and ground water. Lake-depth data are not readily available, but unlike some of the hydrologic parameters just discussed, they can be obtained relatively easily. Thus, lake depth data were collected in the summer of 1975 so they could be included in this report as far as feasible.

It was not possible to obtain depth data for all 150 lakes used in this study. Therefore the ideal analysis, that is including depth in the analysis to assess its statistical relationship to the other parameters, could not be made.

Missing data can be estimated statistically if a close functional relationship can be established between the variable of interest and another variable. Of the 13 variables analyzed in the first part of this report, local relief should be the most closely related to lake depth. A plot of the two variables, and calculation of a correlation coefficient, showed no relationship. Until complete coverage of lake depth data can be obtained and included in a statistical analysis, lake depth must be treated as an independent parameter in the classification of the hydrologic setting of lakes.

Lake depth data were obtained for 138 lakes. Of these, two are greater than 100 feet (30m) deep, eleven are between 50 and 100 feet (15 and 30m) deep, forty-two are between 10 and 49 feet (3 and 15m) deep, and the remainder are 10 feet (3m) deep or less. The distribution of lake depth is shown in figure 61. Of the sample lakes, the Dakotas contain only two greater than 10 feet (3m) deep, and Wisconsin contains only one less than 10 feet (3m) deep.

A relationship that has been assumed is that deeper lakes are more likely to occur in areas of end and collapse moraine than in less hummocky terrain. Comparison of figures 61 and 5, however, shows little relationship between lake depth and landform.

The evaluation, based on digital simulations of ground-

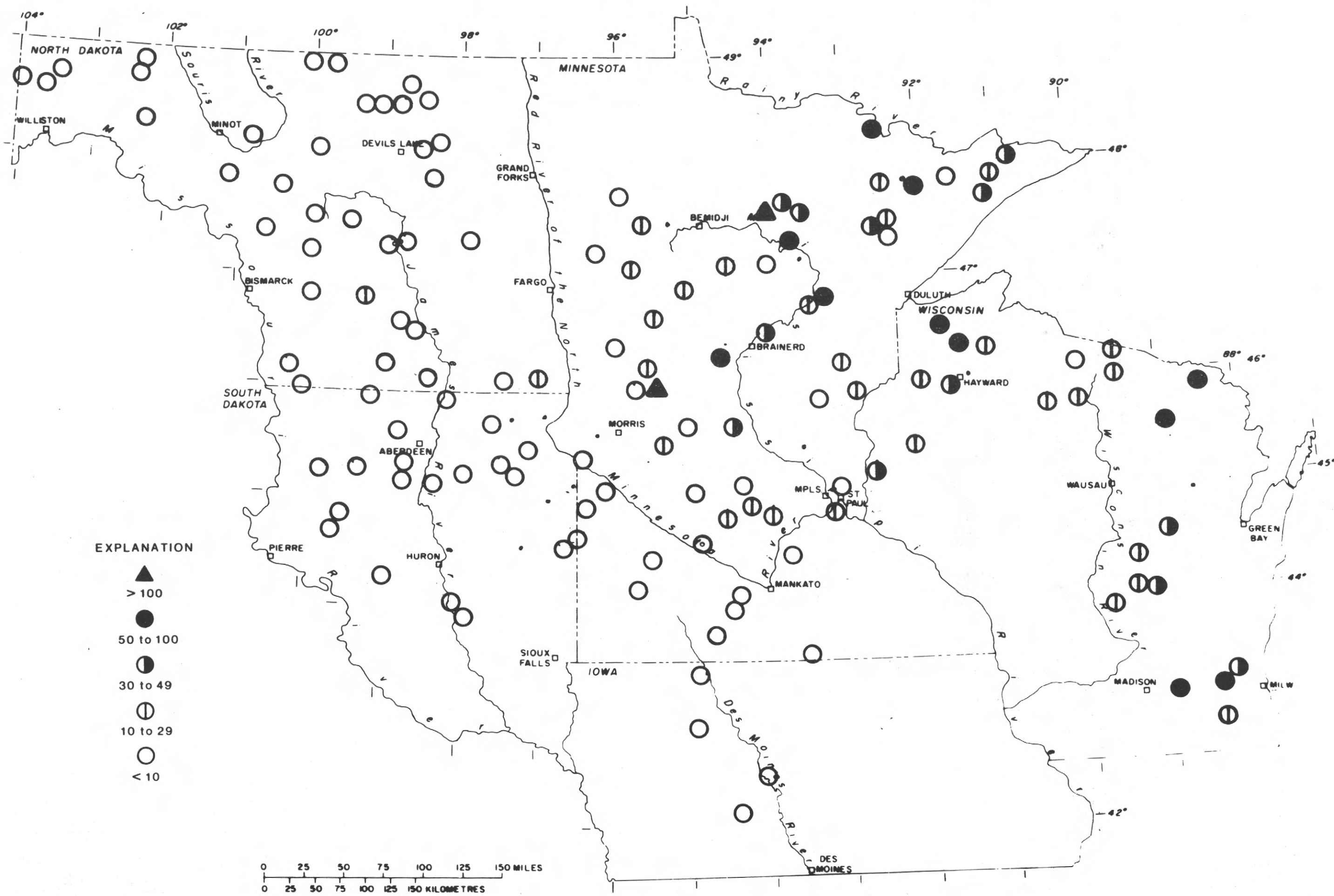


Figure 61.--Depths of the lakes included in this study.

water flow near lakes, on the classification of the hydrologic setting of lakes presented in the first part of this report, can be summarized as follows. Of the ground-water-flow parameters, local relief has more significance than regional slope and regional position. For the simplest classification, the latter two can probably be ignored, or at least be given less weight than local relief. Drift texture and bedrock; that is, distribution of geologic units and their associated hydraulic conductivity values and ratios, are very important, and attempts should be made to refine these parameters. Lake depth is an important parameter in classifying the hydrologic setting of lakes.

SUMMARY

Lake hydrology consists of study of the interchange of a lake with atmospheric, surface, and ground water. Data on the hydrologic setting of 150 randomly-selected lakes in the north-central United States were analyzed by principal component analysis.

In this study, parameters, the original variables, that load high on each of the principal components, the calculated statistical axes, are statistically independent and are clearly related to each other. Precipitation-evaporation balance (atmospheric water) and the water quality parameters have high factor loadings on the first principal component. Highest loadings on the second principal component are for streamflow in and out of the lakes. Principal components 3 and 4 are characterized by geology and ground-water flow parameters. The drainage basin area/lake area ratio, the overland runoff parameter, has the highest loading on the fifth principal component.

The stability of the principal components was tested by randomly splitting the data and comparing a principal component analysis of each subsample. This showed the first two principal components to be the most stable. The components described by the ground-water parameters are less stable but there is justification for using them with caution.

Comparison of factor score maps of each principal component with maps of the original parameters makes it possible to identify the index parameters that can be used in a classificatory system. A re-analysis to check the justification of deleting parameters that are of less statistical importance or have questionable scaling (ground-water quality type, drift mineralogy, and landform) shows no change in the results because of their removal.

This study is a first attempt to identify and map the areal distribution of independent parameters that can be used in developing a general hydrologic classification of lakes. Because of lack of data of dependent variables that might be of interest, the relative importance of the parameters identified could not be determined for specific purposes. The maps produced are intended to provide information on the areal distribution of the parameters related to the hydrologic setting of lakes in the north-central United States. In this respect it presents a much broader approach to classifying lakes hydrologically, than previous studies.

Of the parameters examined in this study, the distribution of dissolved solids in ground-water is most closely related to the distribution of lake types determined by other limnological typologies in the north-central United States. This suggests that ground-water quality is more useful than geology in explaining the distribution of lake-chemistry types.

The relationship of ground water to lakes is the least well known aspect of lake hydrology. Disregarding the instability of the principal components that are related to the ground-water parameters, the statistical studies discussed in the first major chapter have not reduced significantly the number of ground-water parameters that must be considered in classifying the hydrologic setting of lakes, partly because the principles of the interaction of lakes and ground water are so poorly understood. The second major chapter, which discusses digital model simulations of ground-water flow near lakes, is a first step toward a general theoretical understanding of the interaction of lakes and ground water. Only by clear understanding of this relationship would it be possible to identify the controlling ground-water parameters and thus refine the classification discussed in the first major chapter.

Numerical-model simulations were run to define the principles of the interaction of lakes and ground water, and to evaluate parameters that control ground-water flow near lakes in order to assess whether those used in the classifactory system need modification.

The numerical models, using the alternating-direction-implicit (ADI) and strongly implicit procedure (SIP) methods to solve the ground-water flow equation for vertical sections, were used to simulate one-lake and multiple-lakes systems.

This report demonstrates for the first time by analysis of ground-water flow in vertical sections, that the existence, position, and head value at the stagnation point of the ground-water flow field, relative to the head represented by lake level, is the key to understanding the interaction of lakes and ground water. The stagnation point is the point of least head along the divide between ground water flow cells beneath a lake. Therefore, if a stagnation point exists, the divide is continuous, the lake cannot leak, and it is the discharge point of ground-water flow systems. If there is no stagnation point, the ground water divide related to the lake is not continuous, and the lake can leak through part or all of its bed.

Factors that strongly influence the interaction of lakes and ground water are height of the water table on the downslope side of the lake relative to lake level, position and hydraulic conductivity of aquifers within the ground-water reservoir, ratio of horizontal to vertical hydraulic conductivity of the ground-water system, and lake depth. Of lesser significance to lake-ground water interaction are height of the water table on the upslope side of a lake relative to lake level, regional slope, ground-water reservoir thickness, and presence of lake sediments.

The models are believed to be the first to show the detailed patterns of ground-water flow beside and beneath lakes in a wide variety of hydrogeologic settings. They are unique also because they are the first to show ground-water flow patterns beneath and between a series of lakes, each at a different altitude, along a valley side. A significant finding is that, for most settings, water does not move from higher lakes, through the ground, to lower lakes if a water-table mound exists between the lakes. This can happen only if one presumes a straight-line water table profile connects the lakes.

In field studies of the interaction of lakes and ground water it is especially important to define the geologic framework including the hydraulic conductivities of the geologic units--the boundaries of the system, the ratio of horizontal to vertical hydraulic conductivity, the height of the ground-water watershed relative to lake level, and the strength (head at the stagnation point relative to that represented by lake level) and position of the stagnation point.

From the statistical studies of the first part of this report and the modeling studies of the second part, the following are the most important parameters to be considered in classifying the hydrologic setting of lakes. The water quality, atmospheric, and surface-water parameters are the same as those listed in table 12. They are ground-water dissolved solids, precipitation-evaporation balance, streamflow inlet and outlet, and the ratio of drainage-basin area to lake area. Of the ground-water parameters, local relief is more important than regional slope and regional position. Drift texture and bedrock, measures of the hydraulic conductivity parameters, are very important. Lake depth must be an index parameter in classifying the hydrologic setting of lakes.

The classification developed in this report is a first attempt to classify the hydrologic setting of lakes. Although independent parameters are identified, and maps of their areal distribution should be useful to those interested in lake typology, the classification would benefit greatly if better values (field and laboratory measurement) of stream discharge, drift mineralogy, and drift and bedrock texture (hydraulic conductivity) were used rather than the ranking values used for this report.

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