UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

DIGITAL MODEL OF THE ARIKAREE AQUIFER NEAR WHEATLAND, SOUTHEASTERN WYOMING

Open-File Report 77-676

Prepared in cooperation with the
WYOMING STATE ENGINEER

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Cheyenne, Wyoming

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For those readers who may prefer to use metric units, the conversion factors for English units used in this report are listed below.

Multiply English unit	By	To obtain metric unit
feet (ft)	0.3048	meters
miles (mi)	1.609	kilometers
acres	.004047	square kilometers
square miles (mi^2)	2.590	square kilometers
acre-feet (acre-ft)	.001233	cubic hectometers
feet per day (ft/day)	.3048	meters per day
cubic feet per second (ft^3/s)	.02832	cubic meters per second

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ABSTRACT

A digital model that mathematically simulates the flow of ground water, approximating the flow system as two-dimensional, has been applied to predict the long-term effects of irrigation and proposed industrial pumping from the unconfined Arikaree aquifer in a 400 square-mile area in southeastern Wyoming. Three cases that represent projected maximum, mean, and minimum combined irrigation and industrial ground-water withdrawals at annual rates of 16,176, 11,168, and 6,749 acre-feet, respectively, were considered. Water-level declines of more than 5 feet over areas of 124, 120, and 98 square miles and depletions in streamflow of 14.4, 8.9, and 7.2 ft^3/s from the Laramie and North Laramie Rivers were predicted to occur at the end of a 40-year simulation period for these maximum, mean, and minimum withdrawal rates, respectively. A steady-state flow system was approximately established at the end of the 40-year simulation period only for the case representing the minimum withdrawal rate. Sensitivity tests with respect to hydraulic conductivity and specific yield indicate that the transient simulations are little affected by uniform variations of magnitudes equal to the assumed limits of uncertainty associated with these parameters. A tenfold increase in the vertical hydraulic conductivity that was assumed for the streambeds results in smaller predicted drawdowns near the Laramie and North Laramie Rivers and a 36 percent increase in the predicted depletion in streamflow for the North Laramie River.

INTRODUCTION

The study involves a total area of about 400 mi^2 in central Platte County and extreme western Goshen County, Wyoming, as shown in figure 1. The area is bounded on the west by outcrops of rocks of pre-Tertiary age in the Laramie Mountains, on the north by a ground-water divide north of Cottonwood Creek, on the northeast by the North Platte River, and on the southeast by the Wheatland fault system. The southern boundary has been chosen arbitrarily and passes through the town of Wheatland, Wyoming, about 4 miles south of the confluence of the Laramie and North Laramie Rivers.

Ground water is presently being withdrawn from the Arikaree aquifer within the study area primarily for domestic, stock, and irrigation purposes. The domestic and stock wells are generally low-yield wells that tap only the upper portion of the aquifer; whereas, the irrigation wells are typically high-yield wells that tap most or all of the saturated thickness of the aquifer. As of December 1976, 25 irrigation wells had been completed in the Arikaree aquifer within the study area, and an additional 17 permits for irrigation wells had been granted by the Wyoming State Engineer (R. G. Stockdale, written commun., 1977). The total consumptive use of ground water from the Arikaree aquifer for irrigation in this area is estimated in this report to have been about 4,600 acre-feet in 1976.

The Missouri Basin Power Project plans to develop a well field within the study area at a site near the confluence of the Laramie and North Laramie Rivers. Water is to be withdrawn from the Arikaree aquifer to provide a supplemental water supply for the 1,500 megawatt, coal-fired Laramie River Station presently under construction northeast of Wheatland, Wyoming. The principal water supply for this power generation facility is to be provided by the Laramie River from storage in the proposed Grayrocks Reservoir. Ground water from the well field is to be utilized only during periods of insufficient surface-water supplies. Reservoir operation studies conducted on behalf of the Missouri Basin Power Project indicate that an average annual withdrawal of 1,450 acre-feet of ground water will be required during a projected 40-year lifetime of the facility (R. G. Stockdale, written commun., 1977).

Purpose and Scope

A digital ground-water flow model, based in part on the work of Lines (1976), has been developed for the Arikaree aquifer in the study area. The model mathematically simulates the ground-water flow system and is used to predict the long-term response of the aquifer to projected irrigation and industrial ground-water withdrawals. In particular, the effects of these withdrawals on water levels in the aquifer and on streamflow are assessed. Three alternatives representing maximum, minimum, and average projected ground-water withdrawal rates have been prepared by the Wyoming State Engineer (R. G. Stockdale, written commun., 1977) and are considered specifically in this study.

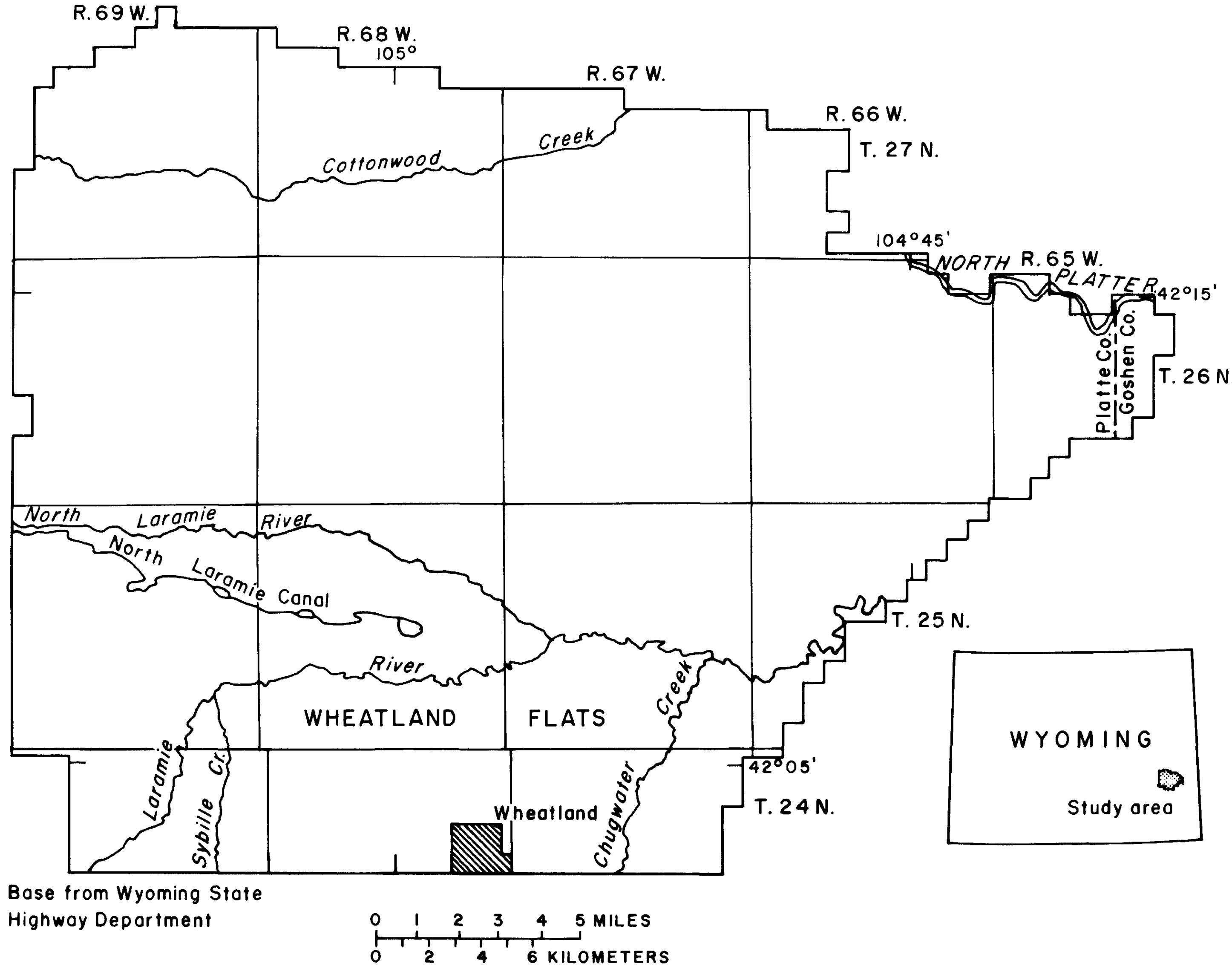


Figure 1.--Area described in this report.

The area of principal interest in this study is that portion of the study area that lies north of the Laramie River. (See fig. 1.) It is within this area that the irrigation and proposed industrial wells whose effects are assessed in this study are to be located. That portion of the study area that is bounded on the north and west by the Laramie River and on the south and east by the boundary of the study area is part of what is known as Wheatland Flats. In this area, imported surface water as well as ground-water from both the Arikaree aquifer and overlying terrace gravel deposits are applied for irrigation. Although basic data pertaining to the geology and hydrology of the Arikaree aquifer in Wheatland Flats are available (Weeks, 1964), these data were insufficient to achieve an accurate simulation of the ground-water flow system in this area with the digital model that was developed. Wheatland Flats was included within the modeled area principally to determine if the effects of pumping within the area of principal interest could be expected to propagate southward across the Laramie River. The simulation of the flow system that was developed in Wheatland Flats was sufficiently accurate for this purpose.

Previous Investigations

Previous investigations that consider all or part of the study area include the following: A hydrogeologic reconnaissance of the Glendo-Wendover area by Rapp and Babcock (1953), a report on the geology and ground-water resources of Goshen County by Rapp, Visher, and Littleton (1957), a report on the geology and ground-water resources of Platte County by Morris and Babcock (1960), a hydrologic investigation of the Wheatland Flats area by Weeks (1964), and a report describing the development of a digital ground-water flow model for that portion of the study area that is situated north of the Laramie River by Lines (1976). Much of the present study is based on the field data collected by Lines (1976).

Acknowledgments

This study was undertaken by the U.S. Geological Survey in cooperation with the Wyoming State Engineer. Water-level measurements were made by personnel from the office of the Wyoming State Engineer, and their assistance is greatly appreciated. Norman M. Denson of the U.S. Geological Survey kindly supplied his map and data pertaining to the bedrock geology of the area in advance of publication.

GEOLOGIC SETTING

Sedimentary rocks of Tertiary and Quaternary age are at the surface throughout most of the study area. These rocks are composed primarily of coarse to very fine-grained clastic debris that has been eroded principally from pre-Tertiary igneous, matamorphic, and sedimentary rocks in the Laramie Mountains on the western border of the area. Deposition of the Tertiary sediments began in Oligocene time and has continued with both major and minor interruptions to the present time. Because deposition was initiated on an erosional surface of considerable relief and has been interrupted by several erosional episodes, the aggregate thickness of the Tertiary and Quaternary sediments varies considerably throughout the study area. The bedrock geology of the study area is shown in figure 2.

Rocks of Tertiary Age

The rocks of Tertiary age that crop out in the study area include the White River Group of Oligocene age, the Arikaree Formation of Miocene age, and the Ogallala Formation of late Miocene age.

White River Group

The White River Group within Platte County has been divided into a lower unit, the Chadron Formation, and an upper unit, the Brule Formation (Morris and Babcock, 1960). Only the Brule Formation is exposed within the study area and, where exposed, it consists principally of white to buff blocky tuffaceous siltstone. The Brule Formation is coarse-grained in its upper part in the western half of the area and locally contains channel deposits of conglomerate and sandstone. Where it is predominantly siltstone, the Brule Formation is assumed to be impermeable and delimits the base of the Arikaree aquifer as defined in this report.

Arikaree Formation

The Arikaree Formation consists principally of poorly to moderately well-cemented, fine- to very fine-grained white to gray sandstone. A basal conglomeratic unit, consisting of loosely to well-cemented coarse to very coarse sandstone and conglomerate, rests uncomformably on the underlying Brule Formation throughout most of the area (Morris and Babcock, 1960). The Arikaree Formation ranges in thickness from 0 to about 700 feet within the study area (N. M. Denson, unpub. data).

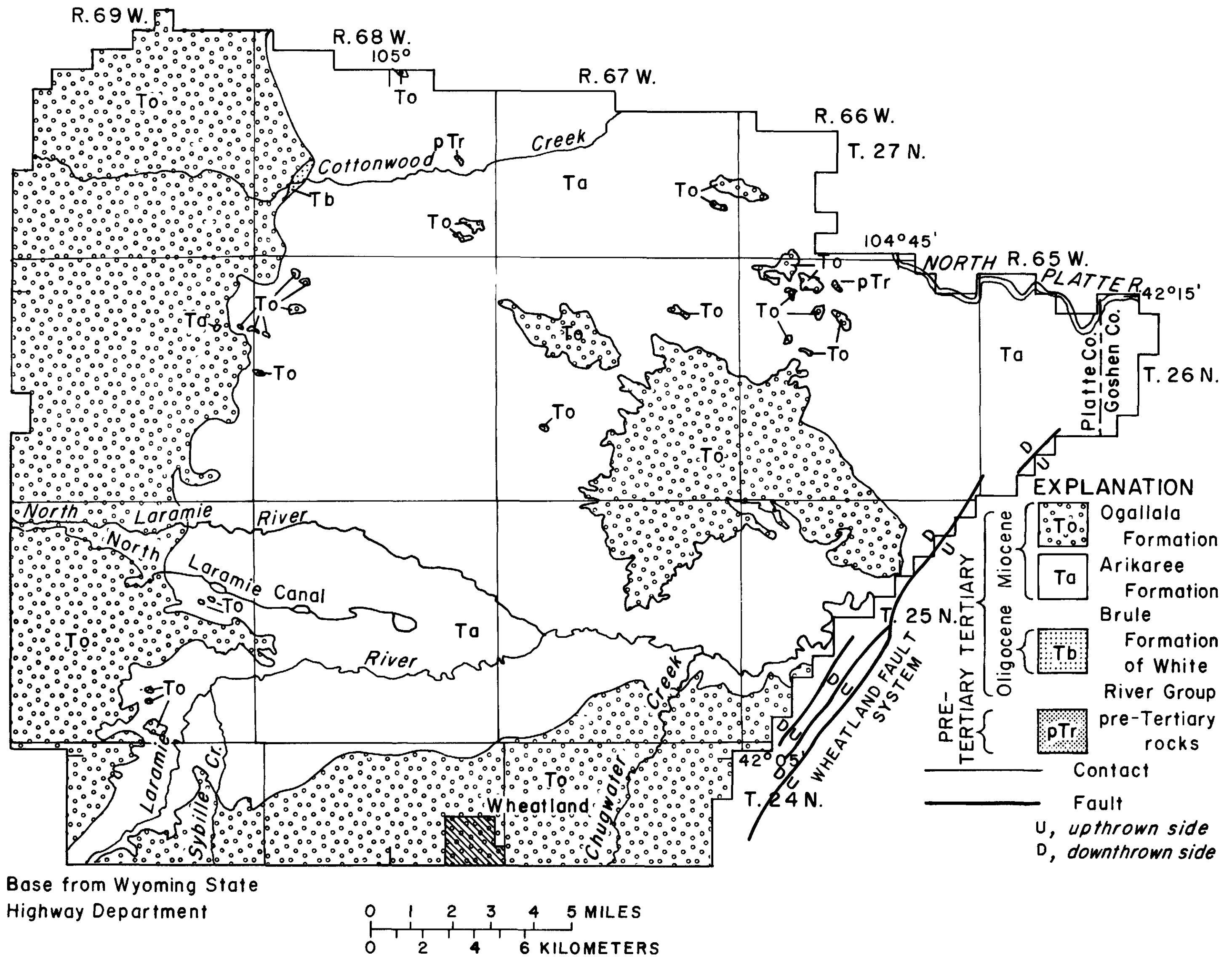


Figure 2.--Generalized distribution of the Tertiary formations (from N. M. Denson, unpub. data).

Ogallala Formation

The Ogallala Formation is exposed on upland surfaces within the study area and, where present, consists of loosely cemented calcareous claystone, siltstone, sandstone, and conglomerate.

Rocks of Quaternary Age

The deposits of Quaternary Age that are present in the study area include thick terrace-gravel deposits in the Wheatland Flats area south of the Laramie River, flood-plain alluvial deposits along the major stream valleys, and a veneer of slope wash on the upland surfaces. Seven Quaternary terraces, ranging from 5 to 85 feet thick and averaging 30 feet thick have been mapped in the Wheatland Flats area (Morris and Babcock, 1960). The terrace deposits are important locally as an unconfined aquifer that is perched above the underlying Arikaree aquifer (Weeks, 1964). The flood-plain deposits underlie the major stream valleys and, with the exception of the North Platte River valley, the thickness of these deposits within the study area is probably less than 25 feet (Morris and Babcock, 1960).

Structural Geology

The major post-Oligocene structural feature in the area is the Wheatland fault system that defines the southeastern boundary of the study area. The Wheatland fault system consists of a northeast-trending set of parallel faults that are upthrown on the southeast. Because the Arikaree Formation has been brought into contact with the assumed impermeable Brule Formation along much of the Wheatland fault system, it is assumed that the fault system acts as an impermeable barrier to the flow of ground water.

HYDROLOGIC PROPERTIES OF THE ARIKAREE AQUIFER

Definition of Arikaree Aquifer

The Arikaree aquifer is here defined to comprise, where present and saturated, the Arikaree Formation, the Ogallala Formation, and locally the upper part of the Brule Formation. In general, the siltstone of the Brule Formation defines the impermeable base of the aquifer; however, in the western half of the area, the upper part of the Brule Formation consists of coarser-grained material that is similar in its hydrologic properties to the overlying Arikaree Formation and is included as part of the Arikaree aquifer (Lines, 1976). The configuration of the base of the aquifer, which is adopted from Lines (1976), is shown in figure 3.

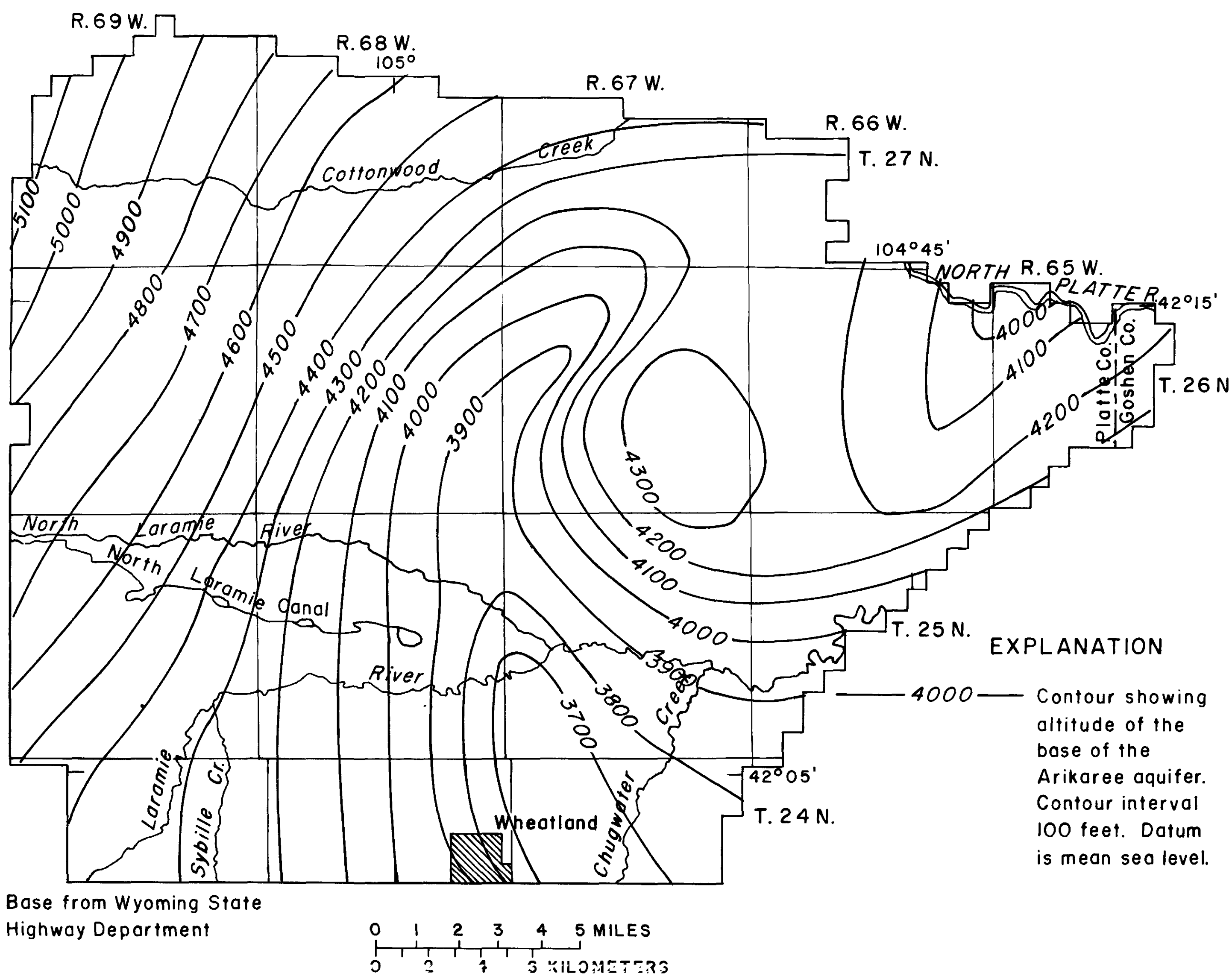


Figure 3.--Configuration of the base of the Arikaree aquifer (modified from Lines, 1976).

The aquifer is considered to be wholly unconfined although locally it may exhibit semiconfined properties as a result of variations in the vertical component of hydraulic conductivity. As defined, the Arikaree aquifer excludes the Quaternary terrace and flood-plain deposits as well as the confined aquifer in the lower part of the White River Group that reportedly yields water to three wells in the northern part of the area (Lines, 1976).

Potentiometric Surface

The flow of ground water through permeable rocks, or an aquifer, occurs in response to decreasing hydraulic head with distance along the path of flow. The hydraulic head, or simply head, at any point in an aquifer is the height measured relative to an arbitrary datum plane, such as sea level, to which water will rise in a tightly cased well open only to that point in the aquifer. The rate at which head changes with distance in a specified direction is termed the hydraulic gradient in that direction.

In general, the flow of water in an aquifer is three-dimensional; that is, both horizontal and vertical components of flow will be present. In considering regional flow systems, it is frequently adequate to neglect vertical flow and to adopt a two-dimensional approximation. In the case of an unconfined aquifer, the two-dimensional approximation is adequate wherever the gradient of the water table is small (Jacob, 1963). Large water-table gradients usually occur only in the immediate vicinity of discharging wells, and the two-dimensional approximation is generally considered to be valid in unconfined aquifers at distances from pumping wells greater than 1.5 times the prepumping saturated thickness at the well sites (Hantush, 1964). Since the maximum and average prepumping saturated thickness of the Arikaree aquifer in the study area is about 1,000 and 300 feet, respectively, it is assumed that the two-dimensional aquifer approximation is appropriate for the application of the digital model for which a minimum finite-difference grid spacing of 2,640 feet was used.

In the two-dimensional approximation it is assumed that the flow of water through the aquifer can be described quantitatively by a potentiometric surface, the altitude of which is obtained by averaging the hydraulic head over the saturated thickness of the aquifer. In practice, the potentiometric surface is approximated by the measured altitudes of water levels in wells tapping the aquifer and by the altitude of streams that are in direct hydrologic connection with the aquifer. The potentiometric surface for the Arikaree aquifer in the study area is shown in figure 4. The configuration of this surface north of the Laramie River is taken directly from Lines (1976) and represents the surface that was defined by water-level measurements made in September 1973. The configuration in the Wheatland Flats area south of the Laramie River is adapted from Weeks (1964).

Saturated Thickness

The saturated thickness of the Arikaree aquifer in the study area was calculated as the difference in altitude between the potentiometric surface shown in figure 4 and the base of the aquifer shown in figure 3. The saturated thickness ranges from 0 feet at points near the western and northeastern boundaries of the study area to nearly 1,000 feet near the Wheatland fault system along the southeastern boundary. The saturated thickness is less than 300 feet in more than half of the study area.

Hydraulic Conductivity

The hydraulic conductivity is a parameter that measures the ability of an aquifer to transmit water. The hydraulic conductivity may be defined as the volume of water transmitted per unit time through unit cross-sectional area under unit hydraulic gradient at a point in the aquifer.

Hydraulic conductivity is generally a function of three-dimensional location within the aquifer. In the two-dimensional aquifer approximation, the hydraulic conductivity is taken to be the mean value obtained by averaging the hydraulic conductivity over the saturated thickness of the aquifer. Mean values of hydraulic conductivity for the Arikaree aquifer that have been obtained from analyses of the aquifer in the study area and in other areas of southeastern Wyoming range from 0.17 to 60.0 ft/d (M. C. Crist, oral commun., 1977).

A parameter that is useful in expressing the ability of an aquifer to transmit water in the two-dimensional aquifer approximation is the transmissivity. The transmissivity at a point is the product of the mean hydraulic conductivity, as defined above, and the saturated thickness at that point. Transmissivity is thus the volume of water transmitted per unit time under unit hydraulic gradient through a vertical section of the aquifer of unit width and of height equal to the saturated thickness.

Specific Yield

The specific yield is the volume of water per unit volume of saturated aquifer that can be released from storage by gravity drainage. Specific yield is thus a dimensionless parameter that measures the amount of water that is taken into or released from storage in an unconfined aquifer as a result of a rise or fall of the potentiometric surface. A specific yield of 0.12 was estimated by Lines (1976) for the Arikaree aquifer in the study area and by Weeks (1964) for the Arikaree aquifer in Wheatland Flats. The value of 0.12 for the specific yield is thus adopted for the Arikaree aquifer in this study.

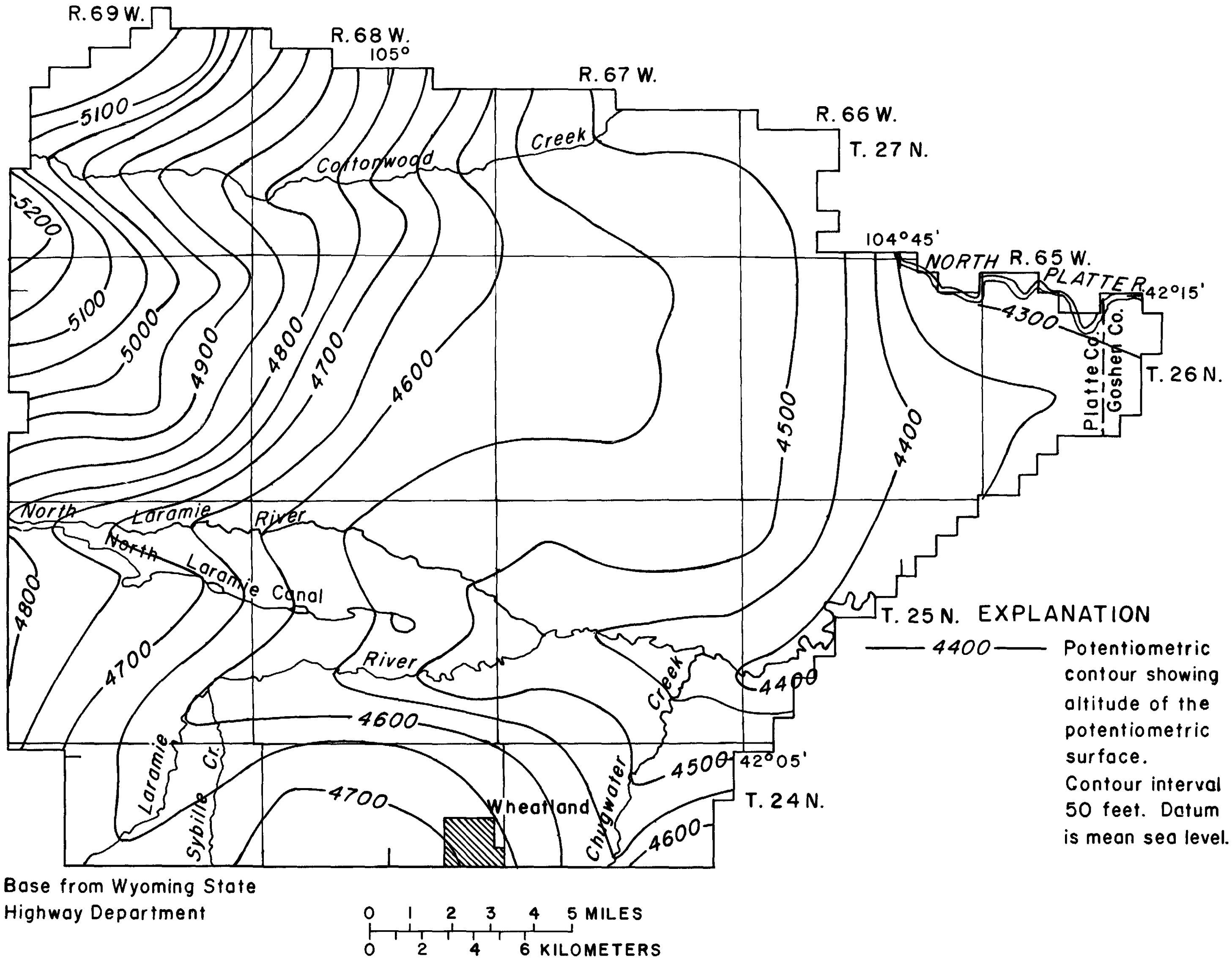


Figure 4.--Configuration of the potentiometric surface for the Arikaree aquifer (modified from Weeks, 1964 and Lines, 1976).

Recharge and Discharge

Water enters an aquifer system in areas of recharge and leaves the system in areas of discharge. In the two-dimensional aquifer approximation, the flow of water within the system is controlled by the potentiometric surface, whose configuration is determined both by the hydrologic properties of the aquifer and by the rates and distribution of recharge and discharge. If the recharge and discharge rates are maintained at nearly constant values over a sufficiently long period of time, a steady-state flow system will evolve in which the long-term average rate of discharge from the aquifer will equal the long-term average rate of recharge to the aquifer, and the potentiometric surface will assume a configuration appropriate to the maintenance of the average flow rates. Once steady-state flow is established, the potentiometric surface will, however, generally fluctuate about its steady-state configuration in response to short-term and seasonal variations in the rates and distribution of recharge and discharge.

The long-term hydrologic operation of an aquifer system will generally be one of steady-state flow unless the system is subject to large stresses, such as intensive agricultural, industrial, or municipal pumping, that cannot be readily accommodated within a stable steady-state flow regimen. It is presumed that the Arikaree aquifer within the study area has established a system of steady-state flow that incorporates the natural inflows and outflows of water to the system as well as the present level of irrigation development within the area. The infiltration of surface water that is applied for irrigation in Wheatland Flats and near the North Laramie Canal provides seasonal recharge to the aquifer. Surface-water irrigation commenced in the area in 1883, and sufficient time has presumably elapsed to allow the system to adjust to this source of recharge. The withdrawal of ground water for irrigation has been more recently imposed on the system, and it is assumed that these withdrawals presently constitute a minor perturbation on the steady-state flow system.

It is assumed that the configuration of the potentiometric surface shown in figure 3 and the measurements and estimates of discharge and recharge reported by Lines (1976) and Weeks (1964) are representative of an existing system of steady-state flow within the Arikaree aquifer of the study area. Under this assumption, the steady-state rates of discharge from the aquifer and recharge to the aquifer are as described below.

Discharge

Through direct measurement and by estimation, Lines (1976) concluded that the net rate of discharge from the Arikaree aquifer within that portion of the study area lying north of the Laramie River is about 22 ft^3/s. Weeks (1964) indicates that the net rate of discharge from the Arikaree within that portion of Wheatland Flats that is included in the present study is about 30 ft^3/s. Discharge from the aquifer occurs, in order of importance, as leakage to streams, underflow and seepage across the boundaries of the area, and evapotranspiration.

There are six perennial streams within the study area that are in apparent hydrologic connection with the Arikaree aquifer. These are the Laramie River, the North Laramie River, Sybille Creek, Chugwater Creek, Cottonwood Creek and the North Platte River. (See fig. 1.) Streamflow measurements conducted along the reaches of these streams by Weeks (1964), Lines (1976), and as part of the present study indicate that, although individual reaches may exhibit either gains or losses in surface flow, each stream shows a net increase in flow upon crossing the study area. The streams are incised into rocks of the Arikaree aquifer, and the net increase in streamflow is presumably due to upward leakage from the aquifer through intervening flood-plain alluvial deposits to the streams. Individual gaining or losing reaches occur because of the variable ability of the flood-plain alluvium to store and transmit water and because of varying head relationships between the alluvium and the underlying Arikaree aquifer.

The rate of ground-water discharge to the streams can be expected to change with time in response to changes in the configuration of the potentiometric surface that are produced by variations in the rate and areal distribution of recharge. Consequently, a measured gain (or loss) in streamflow must be referred to the configuration of the potentiometric surface that existed at the time of measurement. It is here assumed that the net gain in streamflow within the study area north of the Laramie River, as determined from streamflow measurements made in November 1973 and February 1974 (Lines, 1976), is consistent with the configuration of the potentiometric surface shown in figure 4, which is based on water-level measurements that were made in September 1973 (Lines, 1976). The configuration of the potentiometric surface within the Wheatlands Flats area is adapted from Weeks (1964), and it is assumed that the rate of ground-water discharge to Chugwater Creek, Sybille Creek, and the Laramie River upstream from its confluence with Sybille Creek (fig. 1) is that reported by Weeks (1964). The assumed net rates of gain for the streams within the study area are summarized in table 1.

The outflow of ground water from the study area occurring as subsurface flow, or underflow, across the areal boundaries is generally insignificant. The potentiometric surface shown in figure 4 indicates, however, that there is water movement towards the escarpment that defines the northeastern boundary along Guernsey Reservoir and the southeastern boundary along the Wheatland fault system. On these boundaries water is apparently discharged as seepage that is consumed by evapotranspiration. The net rate of discharge cannot be measured, but the prevailing hydraulic gradients and the transmissivity values obtained from steady-state calibration of the digital simulation model imply discharge rates of 1.0 ft^3/s and 1.5 ft^3/s along the northeastern and southeastern boundaries, respectively. These rates are consistent with estimates made by Lines (1976).

Table 1.--Assumed steady-state ground-water discharge and recharge rates for the Arikaree aquifer within the study area

Discharge rates

	Discharge rate (ft^3/s)
1. Discharge to streams:	
Laramie River	15.0
North Laramie River	6.0
Sybille Creek	6.0
Chugwater Creek	14.0
Cottonwood Creek	5.0
North Platte River	3.0
2. Underflow	3.1
3. Evapotranspiration	0.0
Total	52.1

Recharge rates

	Recharge rate (ft^3/s)
1. Precipitation	31.7
2. Underflow	17.5
3. North Laramie Canal	2.9
Total	52.1

Because the saturated zone of the Arikaree aquifer is generally at depths greater than 50 feet below land surface throughout the study area, evapotranspiration directly from the aquifer is considered to be negligible. During the summer months hydrophytes and phreatophytes extract water directly from the saturated zone of the flood-plain deposits in the stream valleys. This evapotranspiration loss will be replenished either by leakage from the streams or by increased upward leakage from the underlying Arikaree aquifer. In the latter case there will be, in effect, a net indirect evapotranspiration loss from the Arikaree aquifer. Data are not available for the study area to assess quantitatively the rate associated with this contribution to evapotranspiration in the stream valleys. It is assumed, however, that the net rate of indirect evapotranspiration discharge from the Arikaree aquifer is negligible in comparison to the other discharge process acting within the study area. The assumed steady-state rates of ground-water discharge from the Arikaree aquifer within the study area are summarized in table 1.

Ground water that is withdrawn from the Arikaree aquifer for irrigation, stock, and domestic use is not included in the assumed steady-state discharge rates. These rates refer specifically to the flow system that existed in 1973 when the potentiometric surface was measured and the streamflow measurements were made by Lines (1976). Lines (1976) estimated that about 2,900 acre-feet of water was consumptively withdrawn for irrigation in the area of principal interest in 1973. Consumptive withdrawal here denotes the total withdrawal minus any return flow to the aquifer. This rate of consumptive withdrawal is equivalent to an average discharge rate of 4 ft^3/s from the aquifer. It is presumed that most of this water was derived from storage within the aquifer and that the withdrawal of this water had negligible effect on the steady-state flow system.

Irrigation wells that tap the Arikaree aquifer have also been developed within that portion of the Wheatland Flats area that is included in the present study. These wells are primarily supplemental water supplies to lands receiving surface water for irrigation. The rate at which ground water is being withdrawn for irrigation in this area is not known. However, since the present study is concerned with assessing the effects to be expected from development that is to occur north of the Laramie River, the existing irrigation development within Wheatland Flats can be largely ignored. If the effects of the development considered in the present study are found to extend into Wheatland Flats, there would be need to consider these effects in conjunction with the present and expected future irrigation development in Wheatland Flats.

Ground-water withdrawals for purposes other than irrigation are considered to be negligible within the present study area. It is not known how many stock and domestic wells tap the Arikaree aquifer but Weeks (1964) estimated that each farm within the Wheatland Flats area consumes an average of only 0.3 acre-feet of water annually.

Recharge

The major sources of recharge to the Arikaree aquifer within the study area include direct infiltration of precipitation, leakage from the North Laramie Canal, infiltration of surface water that is applied for irrigation in the Wheatland Flats area, and underflow and infiltration of water along the southern and western boundaries of the area. Because it is generally not possible to measure the recharge rates directly, it is here assumed, in accordance with the hypothesis that a system of steady-state flow exists within the aquifer, that the net rate of recharge is equal to the net rate of discharge as determined principally from streamflow measurements.

The measurements reported by Lines (1976) indicate that the net gain in streamflow by streams within the area of principal interest average 0.06 ft^3/s per mi^2 of drainage area. It is assumed that this component of the net discharge to streams arises solely from water that is directly infiltrated from precipitation falling on the respective drainage areas. The net rate of recharge from this source for the total study area amounts to 24 ft^3/s, which is equal to 6.5 percent of the average annual precipitation of 1.04 feet at Wheatland, Wyoming. Because information on the area distribution is lacking, it is assumed that recharge from precipitation occurs at a uniform rate throughout the study area.

Although the net gain in streamflow within the northern portion of the study area averages 0.06 ft^3/s per mi^2, Lines (1976) reported an average gain of 0.08 ft^3/s per mi^2 of drainage area for the upper reaches of the Laramie and North Laramie Rivers. He attributed this excess rate of gain to water that is recharged to the Arikaree aquifer by leakage from the North Laramie Canal. The North Laramie Canal (fig. 1) is an irrigation diversion that conveys water from the North Laramie River through a series of small reservoirs to irrigated cropland lying between the Laramie and North Laramie Rivers. Stream discharge measurements made along the canal in May 1974 (Lines, 1976) indicate that there is significant water loss along the canal. The average annual diversion from the North Laramie River through the canal is about 10,000 acre-feet, and Lines (1976) estimated the net annual recharge to the aquifer from the canal was about 2,100 acre-feet. It is found that this rate of leakage and ultimate recharge is consistent with the configuration of the potentiometric surface shown in figure 4 and the development of the steady-state model, which is described later in this report.

On the basis of measured gains in streamflow along the Laramie River, Sybille Creek, and Chugwater Creek, Weeks (1964) conluded that 38 percent of the surface water that was applied for irrigation in the Wheatland Flats area during the 1960 irrigation season was recharged to the Arikaree aquifer in the area and was ultimately discharged to streams. In order to achieve the gains in streamflow for Chugwater

Creek, Sybille Creek, and the Laramie River indicated in table 1, it was necessary to adopt an average rate of recharge to the Arikaree aquifer within the Wheatland Flats area of 0.21 ft^3/s per mi^2 of area. This rate of recharge is considerably greater than that assigned to precipitation, and it is assumed that it can be attributed to direct infiltration of surface water that is applied for irrigation within the area.

Water enters the study area as underflow along the southern boundary and as underflow and infiltration of minor surface streams rising in the Laramie Mountains along the western boundary. The corresponding recharge rates along these boundaries cannot be measured directly, but the net inflow rate of 17.5 ft^3/s along the boundaries was computed during steady-state calibration of the digital model.

The assumed steady-state rates of recharge to the Arikaree aquifer within the study area are summarized in table 1.

DEVELOPMENT OF DIGITAL MODEL

Digital ground-water flow models are developed in order to describe and simulate mathematically the flow of ground water within aquifer systems. In order to describe accurately the details of flow within a particular aquifer system, a model must be provided with a complete set of hydrologic property data, appropriate boundary conditions, and a valid concept of the prevailing flow system. Models are capable of treating hydrologic properties and complex boundary conditions that vary both in space and time. If properly constructed, a model will respond to changing internal and external flow conditions analogously to the real system that it is intended to represent. The utility of a model is twofold: (1) By assessing the accuracy with which it represents the observable flow in a particular system, it permits the testing and refining of both the hydrologic data and the conceptual model for the hydrologic operation of the system, and (2) it permits the prediction of the future response of the system to hypothetical imposed stresses such as pumping and changing climatic conditions. The predictive capability of a model is especially useful to water managers who must regulate the development of ground-water supplies and who need to know the impact of present and future development on these supplies.

Theoretical Basis

The mathematical theory underlying the development of a digital model to describe ground-water flow is founded on two fundamental hydrologic principles. The first is Darcy's law, which states, in its simplest form, that the volume of water flowing per unit time through unit cross-sectional area of the aquifer is proportional to the prevailing hydraulic gradient. The second principle is the conservation of mass, which states that, because water can neither be created nor destroyed, the rate at which water enters a particular volume of the aquifer must equal the sum of the rates at which water leaves the volume and at which water is taken into storage within the volume.

If these two principles are stated mathematically and combined, there results a second-order partial differential equation that describes the flow of ground water at a point within the aquifer. The solution of this flow equation yields the distribution of head within the aquifer as a function of location and time. In general, the head distribution will be three dimensional; however, the flow equation can be averaged over the saturated thickness of the aquifer to yield a flow equation in the two-dimensional aquifer approximation. The solution of the two-dimensional flow equation gives the vertically averaged hydraulic head as a function of time and two-dimensional location within the plane of the aquifer.

The two-dimensional aquifer approximation is usually employed in the digital simulation modeling of regional aquifer systems because (1) there is rarely field data available pertaining to the vertical components of ground-water flow or the vertical distribution of aquifer properties; (2) high-yield wells, such as irrigation wells, are usually open to all or most of the saturated thickness of the aquifer and effectively average the flow and aquifer properties over the vertical interval to which they are open; and (3) in reducing the dimensionality of the flow equation by one, considerable simplification in the mathematical complexity of the problem is achieved. The resulting two-dimensional ground-water flow equation may be written as

$$\frac{\partial}{\partial x}\left(Kb\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(Kb\frac{\partial h}{\partial y}\right) = S_y\frac{\partial h}{\partial t} + W \qquad (1)$$

where

$b = b(x,y,t)$ is the saturated thickness [L],
$h = h(x,y,t)$ is the hydraulic head [L],
$K = K(x,y)$ is the hydraulic conductivity [LT^{-1}],
S_y = the specific yield [dimensionless],
t = time [T],
$W = W(x,y,t)$ is a source-sink term [LT^{-1}], and
x,y = spatial variables in the plane of the aquifer [L].

In the above equation, the variables h and K represent averages taken over the saturated thickness of the aquifer. The specific yield is here treated as a constant that applies to the aquifer as a whole; although generally, it is a function of the spatial variables x and y. The spatial variation of the specific yield is usually sufficiently small, however, that it can be safely neglected without impairing the accuracy of the solution to equation 1. The source-sink term W incorporates all of the vertical inflows and outflows to the aquifer including recharge by precipitation, discharge to streams, and discharge from wells. The equation is nonlinear because the saturated thickness depends upon the value of head for an unconfined aquifer.

In order to obtain a solution of equation 1 that is unique and that pertains to a particular aquifer problem, an appropriate set of boundary conditions must be provided. Because equation 1 is a parabolic second-order partial differential equation, the boundary conditions consist of a set of initial conditions and either of two types of conditions on the spatial boundaries. The initial conditions involve specifying the distribution of head at a particular instant in time, which is usually taken as the starting time for the simulation. The conditions on the spatial boundaries involve specifying, as function of time, either the value of head on the boundaries or the rate at which water crosses the boundaries.

Once the initial conditions, boundary conditions, and hydrologic property data are specified, equation 1 can be solved for the head distribution as a function of the spatial variables and time. In most practical problems, it is necessary to employ numerical rather than analytic methods of solution. The availability of high-speed digital computers makes it practical to solve equation 1 numerically for highly complex aquifer systems. The generation of the numerical solution of equation 1 for a particular aquifer system is the essence of a digital ground-water flow model.

Numerical Procedures

A modified version of the computer code described by Trescott, Pinder, and Larson (1976) is employed in the present study to construct a two-dimensional flow model of the Arikaree aquifer. In this approach the ground-water flow equation is solved by finite-difference techniques. The area to be modeled is divided into a network, or grid, of rectangular cells as shown for the study area in figure 5. The flow equation is approximated at the center, or node, of each cell at a fixed point in time by an implicit finite-difference equation involving a centered difference scheme for the spatial variables and a backwards difference for the time variable. The resulting set of algebraic finite-difference equations are solved iteratively by the strongly implicit procedure (Stone, 1968) for the values of head at each node at the given time. The solution is carried forward in time in discrete time steps that in most problems can be increased geometrically to speed the simulation over a specified time interval.

These numerical procedures permit the treatment of nonhomogeneous and, to a limited extent, anisotropic aquifer problems. A nonhomogeneous aquifer is one in which the hydrologic properties of the aquifer, such as the hydraulic conductivity, are regarded as variable with respect to location within the aquifer. An anisotropic aquifer is one in which the hydraulic conductivity varies with direction. In the present application, the Arikaree aquifer is assumed (1) to be isotropic, since there are no data on the possible directional dependence of the hydraulic conductivity; (2) nonhomogeneous with respect to hydraulic conductivity; and (3) homogeneous with respect to specific yield.

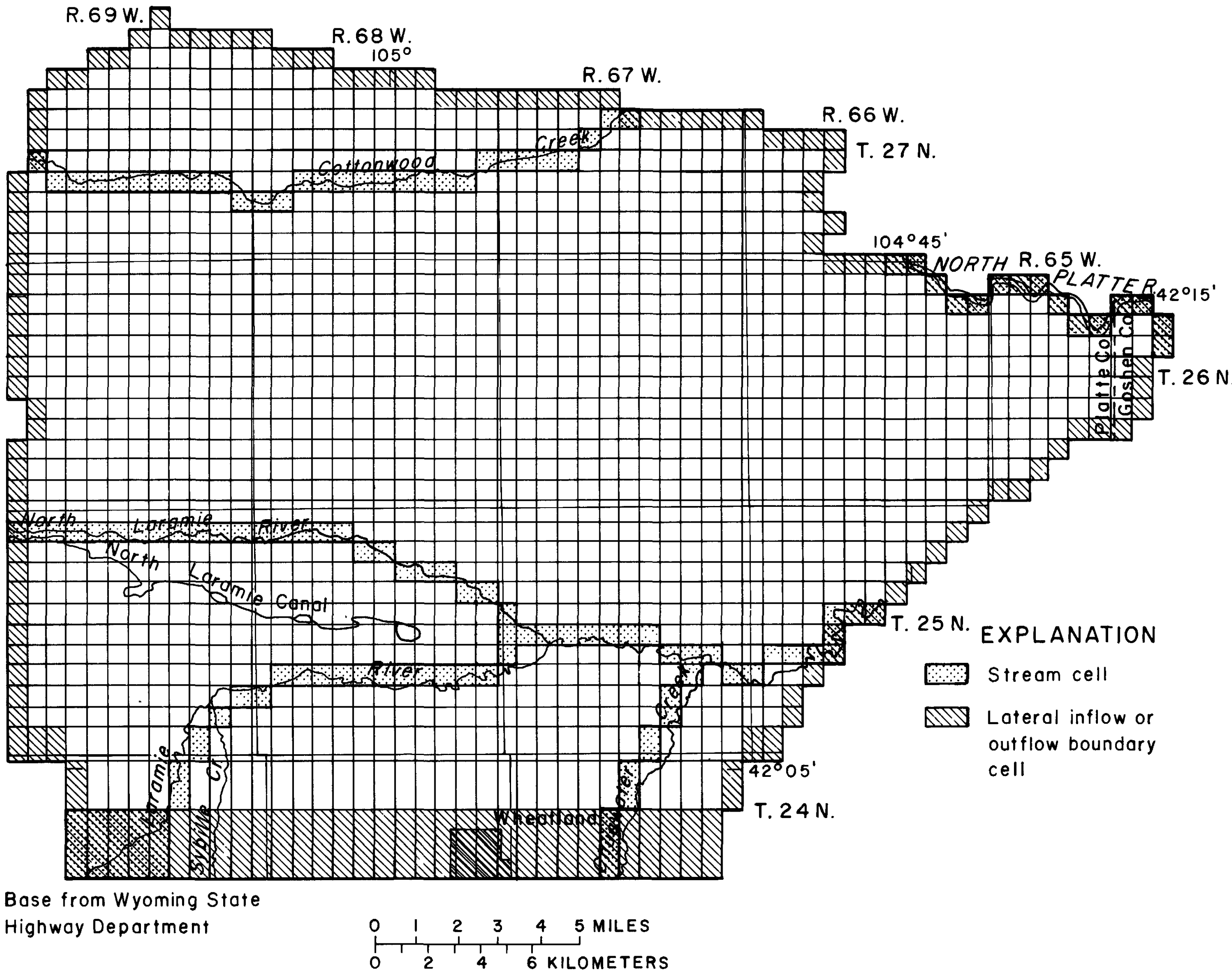


Figure 5.--Finite-difference grid employed in the digital model.

Boundary Conditions

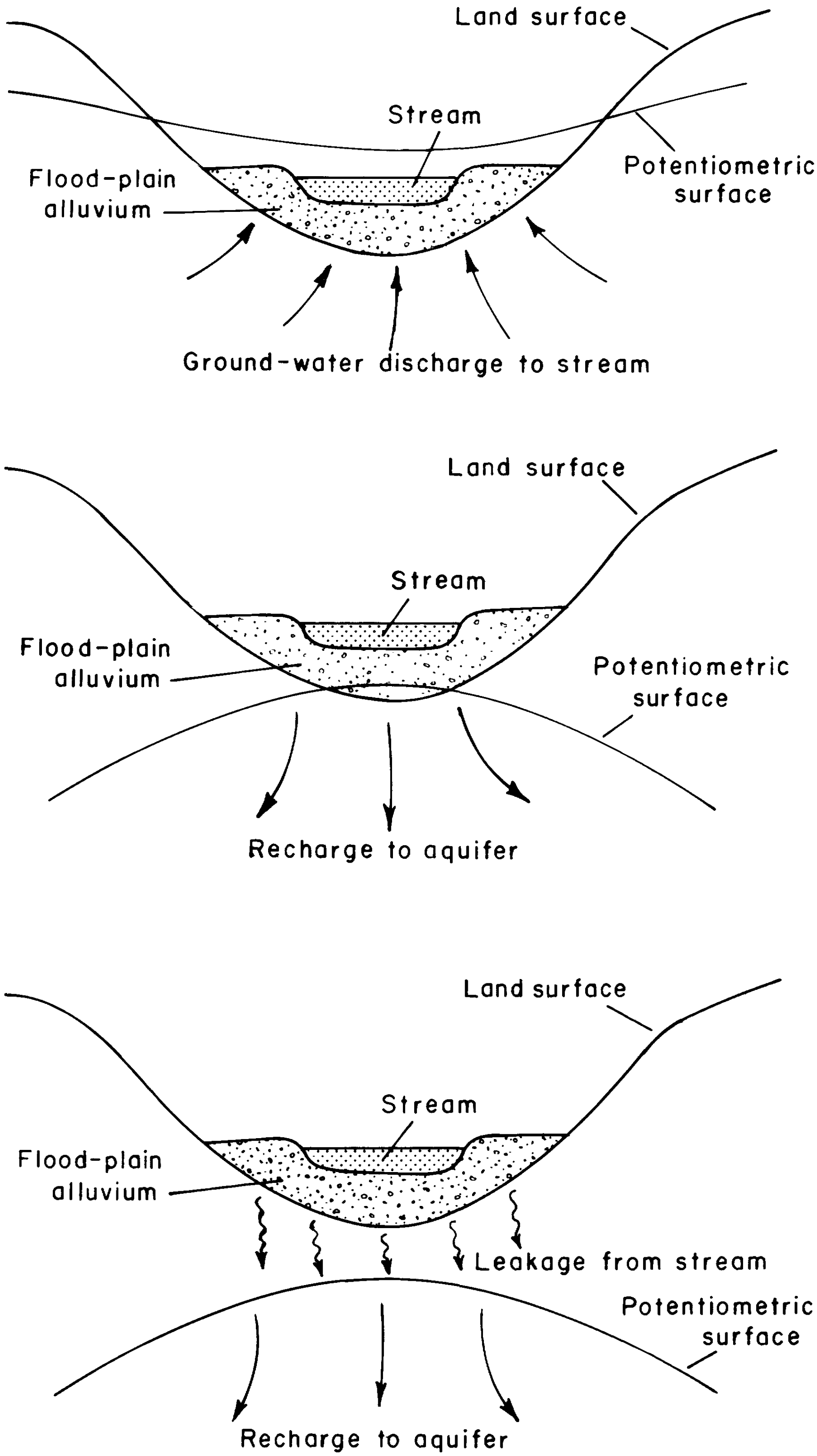
The computer code has provision for applying three types of spatial boundary conditions. These include constant head boundaries, wherein the value of the head on the boundary is maintained at its initial value throughout the simulation; zero-flux boundaries across which there is no flow into or out of the model area; and discharge boundaries, which are simulated by the placement of discharging or recharging wells along the boundary. The computer code automatically places a zero-flux boundary on the perimeter of the modeled area as a computational convenience; consequently, if no other boundary condition is specified, a zero-flux boundary will automatically be assigned.

In order to improve the simulation of underflow into and out of the study area, a method was developed and incorporated into the computer code to permit the automatic calculation and placement of lateral inflow or outflow boundaries where desired. The initial head and transmissivity distributions in the vicinity of a designated discharge boundary cell and the initial leakage and recharge rates for the cell are used to calculate a net inflow or outflow rate. This net rate is assumed to be the requisite initial rate of underflow across the boundary and is imposed on the boundary cell subsequently throughout the simulation. In order to allow for changing saturated thickness with time in unconfined aquifer problems, the rate is multiplied by the ratio of the transmissivity at the boundary cell for the current time step to the initial transmissivity at the cell. Numerical experiments have been performed to test this boundary-condition method and indicate that it serves as a reasonable approximation as long as changes in saturated thickness at the boundary do not exceed 50 percent of the initial saturated thickness at the boundary. Because the boundaries of the study area were sufficiently distant from the discharging wells included in this study, no changes in saturated thickness were predicted to occur at these boundaries during the transient simulations.

Streamflow Accounting Procedure

The original version of the computer code has been further modified for the present study by the addition of a streamflow accounting procedure that allows for an improved treatment of ground-water and surface-water interactions. A somewhat idealized concept of the manner in which streams may interact with an underlying unconfined aquifer is shown schematically in figure 6.

Figure 6a shows a gaining stream that is connected to and receives water from the aquifer by upward leakage through the streambed. The potentiometric surface in the vicinity of the stream is higher than the head in the stream and associated alluvium. The stream is a discharge point for the aquifer, and the rate of leakage from the aquifer to the stream is controlled by the vertical hydraulic conductivity and thickness of the streambed and the difference in head between the aquifer and the stream.



a. Stream and alluvium are in hydraulic connection with the aquifer. Potentiometric surface, defined as average of hydraulic head over saturated thickness of aquifer, is higher than head in stream and alluvium. Aquifer discharges ground water to stream.

b. Stream and alluvium are in hydraulic connection with the aquifer. Potentiometric surface in aquifer is lower than head in stream. Stream leaks water to aquifer.

c. Stream and alluvium are disconnected from the aquifer. Potentiometric surface in aquifer is below streambed. Stream leaks water to aquifer through intervening unsaturated zone.

Figure 6.--Idealized interaction between a stream and an underlying unconfined aquifer.

In figure 6b the stream and associated alluvium are again connected to the aquifer, but the head in the aquifer is less than the head in the stream. The stream loses water to the aquifer and thus provides a source of recharge to the aquifer. As in the previous case, the rate of leakage is controlled by the vertical hydraulic conductivity and thickness of the streambed and the difference in head between the stream and the aquifer.

Figure 6c depicts a losing stream that is disconnected from the aquifer. The stream loses water to the aquifer by downward leakage through the streambed into the unsaturated zone that separates the stream from the aquifer. The rate of leakage from the stream is controlled by the vertical hydraulic conductivity of the streambed and the head in the stream but is independent of the head in the aquifer. Leakage from the stream is again a source of recharge to the aquifer.

Any of these three hypothetical cases may occur within a given reach of a real stream. In addition, transitions between these three cases may occur with time at a given point as a result of changing water-level conditions within the aquifer. For example, a transition from case a to case c in figure 6 may occur as a result of drawdowns beneath the stream produced by intensive pumping nearby. That this does in fact occur has been documented in many cases, for example, along a reach of the Arkansas River in Colorado by Moore and Jenkins (1966) and for the Little Plover River in Wisconsin by Weeks, Ericson, and Holt (1965).

The streamflow accounting procedure allows for any of the above three cases to operate within designated stream cells of the finite-difference grid. (See fig. 5.) Transitions between the cases are allowed along a given stream reach in both space and time. Thus, streams are allowed either to gain or to lose water along a given reach, and they may disconnect from and reconnect to the aquifer in response to computed changes of the potentiometric surface near the stream. The input data that are required include the values of head in the streams and the vertical hydraulic conductivity of the streambed for the stream cells.

The net leakage from a point on a stream cannot exceed the net flow in the stream at that point; consequently, provision has been made to maintain a cumulative total of the flow in the streams by summing the input flow with the downstream gains and losses for each stream cell. The input flow is the streamflow at the upstream entry point of the stream on the model boundary and must be provided at the beginning of the simulation. The streamflow summation progresses downstream from stream cell to stream cell for each stream, and tributary inflows and irrigation diversions at each stream cell are added and subtracted, respectively. Leakage from the stream to the aquifer in a given stream cell is not permitted to exceed the calculated flow in the stream at that cell.

Because streamflow measurements indicated that each of the streams included in the model presently gain water from the aquifer, the heads of the streams were initially set at values 1 foot lower than the heads in the aquifer for each of the stream cells. The heads in the streams were then held constant throughout both the steady-state and transient simulations. The vertical hydraulic conductivity of the streambed in each stream cell was calculated from Darcy's law using the initial assumed steady-state leakage rate from the aquifer to the stream and assigning a value of 1 foot for the thickness of the streambed confining layer.

Hydraulic connection between a stream and the underlying aquifer was considered to be broken whenever the head in the aquifer in a stream cell was lower than the altitude of the streambed in that cell. As long as flow was present in the streams, it was assumed that the depth of water in the streams was uniformly 1 foot, a value which is typical of the depth of water in streams within the study area. When hydraulic connection was broken in a stream cell, water leaked from the stream to the aquifer in that cell at a rate determined by the depth of water in the stream and the vertical hydraulic conductivity of the streambed. Because the head difference between the stream and the aquifer and the depth of water in the streams were both initialized at 1 foot, the rate of leakage from the streams to the aquifer in each stream cell was restricted to be no greater than the initial rate of gain of the stream in that cell. These resultant maximum rates of leakage from the streams are comparable in magnitude to the estimated rate of leakage from the North Laramie Canal within the stream cells assigned to the canal. Thus, to the extent that the North Laramie Canal and the streams can be assumed to behave similarly with regard to vertical leakage, the maximum leakage rates assigned to the stream cells are judged to be reasonable limiting values.

STEADY-STATE CALIBRATION

The development of a predictive ground-water flow model for the study area here commences with the simulation of the steady-state flow system that is presumed to operate presently within the aquifer. The steady-state simulation not only serves as the initial point for subsequent transient simulations but also permits refinement of the hydrologic data that are input to the model. If a well-defined set of boundary conditions and an adequate set of hydrologic data were available for the study area, the construction of the steady-state simulation would be straightforward. Unfortunately, an adequate set of hydrologic data is lacking, and it becomes necessary to utilize the steady-state simulation, in effect, to generate the missing data. The largely trial-and-error process by which this is accomplished is here termed steady-state calibration.

The hydrologic data that are input to both the steady-state and transient simulations are as follows:

1. The steady-state potentiometric surface.

   Values are taken directly from figure 4, but are replaced by the calculated steady-state heads for the transient simulation. (See p. 27.)

2. The base of the aquifer.

   Values are taken directly from figure 3.

3. The parameters that pertain to the leakage of water to or from streams.

   The difference in hydraulic head between the aquifer and the streams as well as the thickness and hydraulic conductivity of the streambed are assigned values to insure that the aquifer discharges water to the streams at the rates listed in table 1.

4. The rate and distribution of recharge by precipitation.

   As was discussed on page 16, a uniformly distributed rate of 0.06 ft^3 per mi^2 has been adopted for the rate of recharge by the direct infiltration of precipitation within the study area except in the Wheatland Flats area where this rate has been increased to 0.21 ft^3/s per mi^2.

5. The rates of underflow into and out of the study area.

   These rates are generated automatically by the model using the discharge boundary computational scheme described on page 21.

6. The parameters for the streamflow accounting procedure.

   The streams that are included in the steady-state and transient simulations are those listed in table 1 except that, because of their geographic proximity, Sybille Creek is combined with the upper reach of the Laramie River. In addition, the North Laramie Canal is treated as though it were a stream that is perched above the water table in accordance with the depiction of figure 6c and loses water to the aquifer at an average rate of 2.9 ft^3/s. The data that are supplied to the streamflow accounting procedure described on page 21 include values for the discharge in the streams at their upstream entry point into the study area and streambed altitudes along the stream reaches within the study area. The former were estimated from available streamflow records, and the latter were obtained from topographic maps of the area under the assumption that the streams were uniformly 1-foot deep.

7. Specific yield.

The specific yield for the steady-state simulation is set equal to zero; for the transient simulations, a value of 0.12 was adopted. (See p. 10.)

8. The finite-difference grid.

Although not strictly part of the hydrologic data set, the finite-difference grid must nevertheless be designed in accordance with the hydrologic complexity of the area to be modeled. The finite-difference grid adopted for the present study area is shown in figure 5. The grid is aligned north-south and east-west, and the spacing between nodes is uniformly 0.5 mile except for cells near the southern boundary where the spacing in the north-south direction is increased geometrically by a factor of 1.5.

In addition to the above data the distribution of hydraulic conductivity within the study area must be provided as input to both the steady-state and transient simulations. Since the model is developed under the two-dimensional aquifer approximation, reference is here made specifically to values of hydraulic conductivity that have been averaged over the saturated thickness of the aquifer. Direct information on the distribution of vertically averaged hydraulic conductivity (or, equivalently, the transmissivity distribution) is lacking over most of the study area. Such data can be obtained, in principle, by imposing a stress, such as a discharging well, on the aquifer and monitoring the magnitude and rate at which the effects of the stress propagate through the aquifer. Such aquifer tests are frequently conducted to determine aquifer properties, but few have been conducted for the Arikaree aquifer within the study area.

If it is assumed that the steady-state potentiometric surface is known and that all of the hydrologic data, other than the hydraulic-conductivity distribution, are accurately determined, then the steady-state simulation can be utilized to calculate the requisite hydraulic-conductivity distribution. (In accordance with the two-dimensional aquifer approximation, the distribution of a parameter, such as head or hydraulic conductivity, here refers to the lateral distribution of the vertically averaged values of that parameter.) The resultant hydraulic-conductivity distribution is that which yields an internally consistent steady-state simulation in which the calculated head distribution is coincident with the measured steady-state potentiometric surface. The calibration of the steady-state model with respect to hydraulic conductivity proceeds iteratively by repeatedly adjusting the hydraulic-conductivity distribution and calculating a new steady-state head distribution until the agreement between the calculated steady-state head distribution and the measured potentiometric surface is judged to be satisfactory. This calibration procedure is frequently a time-consuming process since each new adjustment to the hydraulic-conductivity distribution is made by trial-and-error. The precision with which the calibration process can be performed is limited by the accuracy with which the remaining hydrologic data as well as the hydrologic operation of the system are known.

A measure of the agreement between the calculated and measured head distribution is provided by what shall here be designated the root-mean-square (or rms) deviation. The rms deviation is defined analogously to the standard deviation of statistics. Let N be the total number of nodes in the finite-difference grid (in the present study N = 1558) and let i be an index by which the nodes are numbered sequentially from 1 to N. Let h_i and h_i^0 denote the calculated and the measured values of head, respectively, at node i. The rms deviation r is then defined as

$$r = [\frac{1}{N} \sum_{i=1}^{N} (h_i - h_i^0)^2]^{\frac{1}{2}} \tag{2}$$

The rms deviation is thus a measure of the mean departure of the calculated head distribution from the measured head distribution.

The calibration of the steady-state model for the Arikaree aquifer within the study area yielded a calculated steady-state head distribution that departs from the measured potentiometric surface with an rms deviation of 6.45 ft and a maximum departure of ±20.0 feet. This degree of fit between the calculated and observed steady-state head distribution is judged to be satisfactory. The hydraulic-conductivity distribution that yielded this calculated head distribution is adopted for subsequent use in the transient simulations. In addition, the calculated steady-state head distribution, which by way of the calibrated steady-state model is consistent both with the generated hydraulic-conductivity distribution and the remaining hydrologic parameters, is adopted in place of the measured potentiometric surface as the appropriate initial head distribution for the transient simulations.

In order to test the hypothesis that the present flow system within the Arikaree aquifer is one of steady-state flow, a transient, that is, time-dependent, simulation of the period 1969 to 1977 was made in which historical pumpage data were approximated and an account was made of variations in the rate of recharge from precipitation.

The first irrigation well was drilled within the area of principal interest in the fall of 1964. It was not until 1969, however, that significant ground-water utilization for irrigation began. It is assumed that prior to 1969 the single existing irrigation well had negligible effect on the aquifer and that any disruption of an established steady-state flow system by irrigation pumpage would have occurred after 1969. The transient simulation was begun in 1969 and continued through the spring of 1977. Each year of the simulation was divided into two seasons. An irrigation season of 182 days represented essentially the months April through September during which time the irrigation wells consumptively withdrew 1.4 feet of water for each acre assigned to the well. A nonirrigation season of 183 days represented essentially the months October through March during which time the irrigation wells were not pumped. The number of wells and the total amount of ground water consumptively withdrawn were increased each irrigation season in accordance with the data shown in figure 7, which were obtained from the records of the Wyoming State Engineer.

The net rate of recharge from precipitation was adjusted annually in order to assess the effects of such variations on the assumed steady-state flow system. The adjustments in recharge were referenced to the period April 1973 through March 1974 during which time the altitude of the potentiometric surface was measured (September 1973, fig. 4) and the streamflow measurements were made (November 1973 and February 1974, table 1). The adjusted annual recharge rates were calculated by multiplying the assumed annual recharge rate of 0.06 ft^3/s per mi^2 for the reference period by the ratio of the annual precipitation for the corresponding periods from April 1969 through March 1977 to the annual precipitation for the reference period. (See p. 25.) Monthly precipitation data for Wheatland, Wyo., were used to generate these recharge rates, which are also shown in figure 7.

The results of the 8-year transient simulation indicate that the climatic variations of recharge and the historical pumpage for irrigation have had little discernible effect on either the water levels or the streamflows in the area of principal interest. A net decline in water levels of 5 to 10 feet is indicated but is restricted to the immediate vicinities of the irrigation wells. Water-level measurements made in the area in March 1977, by personnel of the Wyoming State Engineer's office, indicate that changes in water levels of this magnitude have occurred in the area. Unfortunately, these data are too scanty to provide an adequate test of the transient operation of the model. The results of this transient simulation and the 1977 water-level measurements are, however, consistent with the hypothesis that the large-scale hydrologic operation of the aquifer system is presently one of steady-state flow. This steady-state flow system constitutes the initial state for the long-term predictive simulations that are described below.

TRANSIENT SIMULATIONS

Three long-term transient simulations have been undertaken to assess the combined effects of proposed industrial and expected irrigation pumpage from wells that tap or are planned to tap the Arikaree aquifer within the area of principal interest. These predictive simulations are based on pumpage data that were supplied by the Wyoming State Engineer for three pumping scenarios that represent, in essence, maximum,

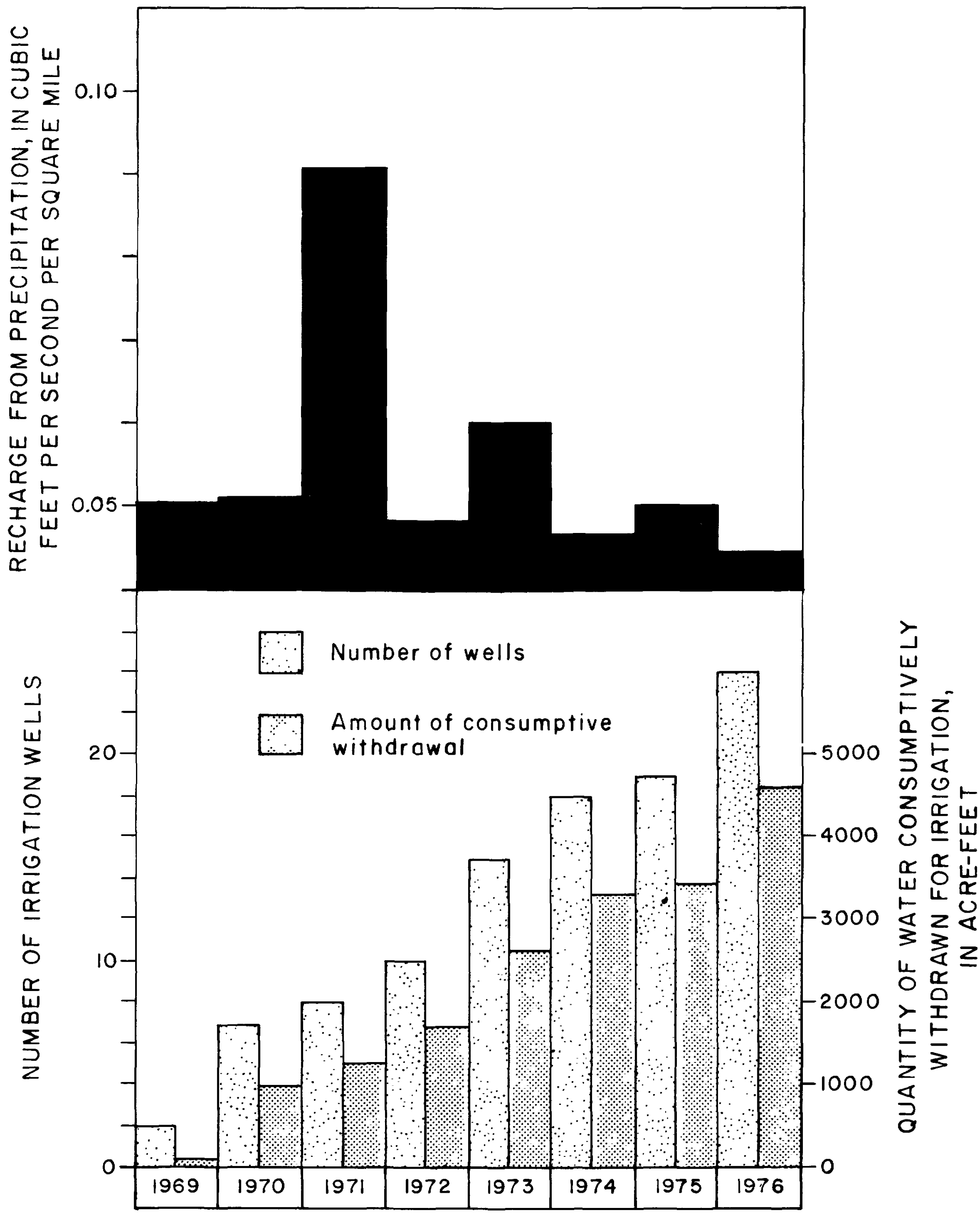


Figure 7.--Number of irrigation wells, consumptive withdrawal for irrigation, and the assumed rate of recharge from precipitation during the period 1969 to 1976.

mean, and minimum proposed ground-water utilization in the area. These three scenarios shall hereafter be designated case I, case II, and case III, respectively. The net pumpage rates appropriate to each of these cases are as follows:

Case 1.--It is assumed that all of the 42 irrigation wells presently under permit by the State of Wyoming are in production and that each well consumptively applies 1.4 feet of ground water annually to each acre assigned to the well, including all enlargements to the assigned acreages. The net annual rate of ground-water consumption for irrigation in this case is 10,066 acre-feet. The annual rate of ground-water withdrawal for industrial use is set at 6,110 acre-feet, which is the maximum rate of annual withdrawal that is projected to be required by the Missouri Basin Power Project.

Case II.--It is again assumed that the 42 irrigation wells presently under permit are in production. The pumping rates are taken to be the same as in case I, except that those irrigation wells that supply water to lands having both surface-water and ground-water rights are assumed to pump at one-half the case I rates. The net annual rate of consumption of ground water for irrigation in this case is 9,718 acre-feet. The annual industrial consumption of ground water is set at 1,450 acre-feet, which is the annual average withdrawal rate that is projected to be required by the Missouri Basin Power Project.

Case III.--Only those 26 irrigation wells whose ground-water rights are senior to the rights of the industrial wells are assumed to be in production. Each well is assumed to withdraw ground water at the same rate as in case I, except that all enlargements to the originally assigned acreages are excluded. The total annual consumption of ground-water for irrigation in this case is 5,299 acre-feet. Industrial withdrawals are set at the projected annual average rate of 1,450 acre-feet.

Each transient simulation for the above cases was executed for a 40-year simulation period. Each year of the simulation period was divided into a 182-day irrigation season, during which the irrigation wells were assumed to be pumping at constant discharge rates, and a 183-day nonirrigation season, during which the irrigation wells were not pumped. The industrial pumpage was distributed equally among six designated industrial wells. During the case I simulation, each of the industrial wells was pumped at a constant discharge rate of 1.41 ft^3/s throughout the simulation period; during the case II and case III simulations, the industrial wells were assumed to be pumping only during the irrigation season at a rate of 0.67 ft^3/s per well. The locations of the irrigation and proposed industrial wells were provided by the Wyoming State Engineer (R. G. Stockdale, written commun., 1977) and are shown in figure 8.

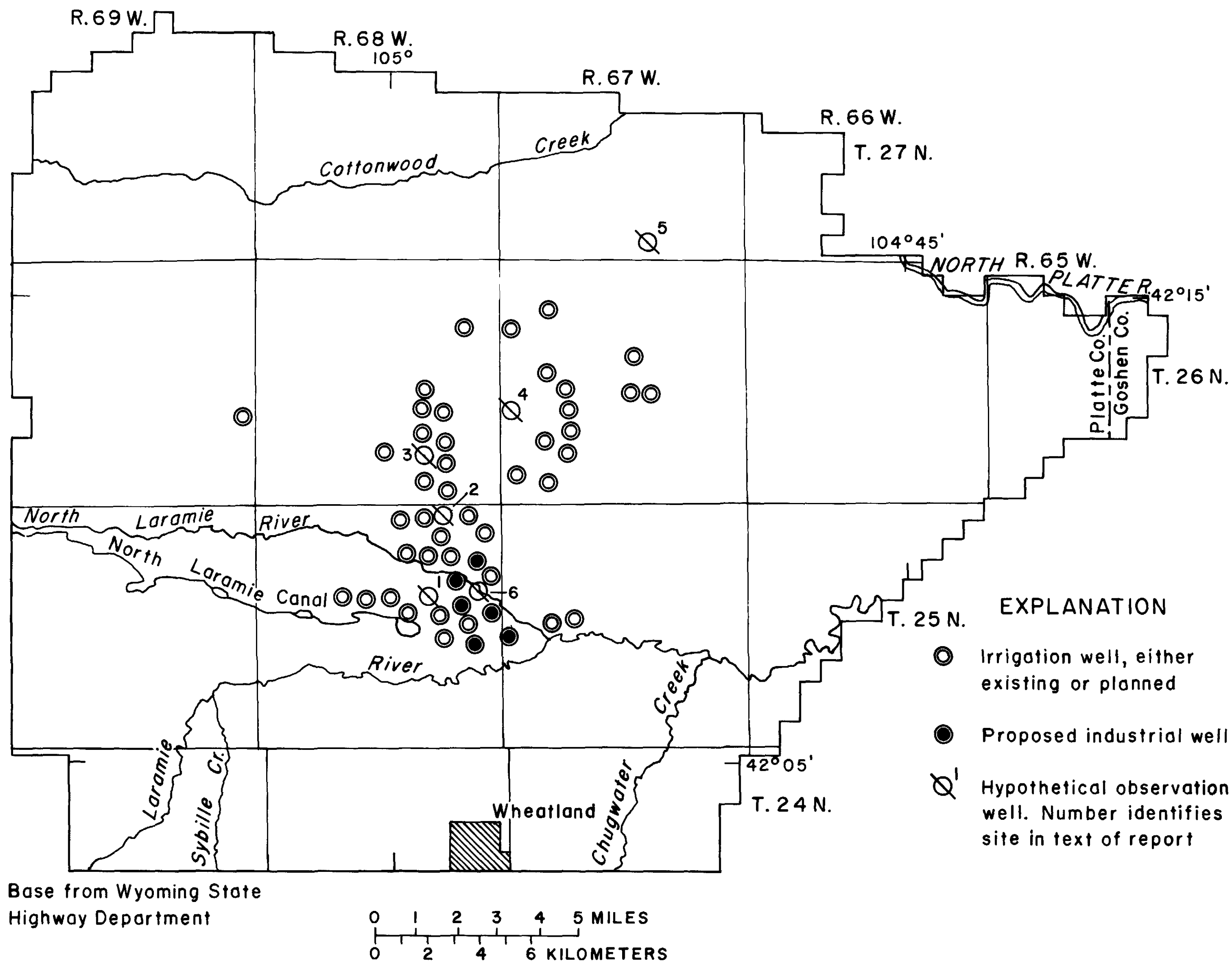


Figure 8.--Locations of the irrigation, industrial, and hypothetical observation wells included in the transient simulations.

The results of these transient simulations are shown in figures 9 through 19. Figures 9, 10, 11 show the distribution of drawdown that is predicted for the study area at the end of the 40-year simulation period for cases I, II, and III, respectively. Figures 12 through 17 show plots of drawdown versus time that are predicted for six locations shown in figure 8. Figures 18 and 19 show the rates of stream depletion that are predicted for the Laramie and North Laramie Rivers, respectively.

The areal distribution of drawdown at the end of the simulation period is shown in figures 9, 10, and 11. The resultant cone of depression that is produced collectively by the pumping wells is here defined to be that area in which the predicted drawdowns exceed 5 feet. The resultant cones of depression at the end of the simulation period for cases I, II, and III involve areas of 124, 120, and 98 mi^2, respectively.

Wells that are located within the resultant cone of depression can be expected to experience reduced yields and increased lift requirements as a consequence of the drawdown caused by the combined irrigation and industrial pumping. Some shallow wells that do not fully penetrate the aquifer may become nonproductive. The effect will be greatest on shallow wells located near the pumping irrigation and industrial wells, in the vicinity of which the drawdowns will be greater than the average values indicated in figures 9, 10, and 11. The yields of the fully penetrating irrigation and industrial wells will decrease with increasing drawdown because of the greater pumping lift that will be required and because of decreasing saturated thickness, and hence transmissivity, in the well vicinities.

Figures 12 through 17 show predicted water-level declines for the 40-year simulation period at six sites located as shown in figure 8. Site 1 is located between the Laramie and North Laramie Rivers; sites 2, 3, and 4 are near the centers of the resultant cones of depression produced by cases I, II, and III, respectively; site 5 is near the periphery of the resultant cone of depression for each of the cases; and site 6 is in the bed of the North Laramie River. These plots approximate hydrographs that would be obtained from hypothetical observation wells located at the corresponding sites.

The sawtooth pattern exhibited by these predicted water-level declines results from dividing each year of the simulation period into an irrigation and nonirrigation season. During each nonirrigation season, the water levels partially recover from the effects produced by pumping during the previous irrigation season. The recovery is generally less in the case I simulation because the industrial wells were assumed to be pumping at constant rates throughout the year. The general trend in all three cases is one of continually declining water levels at gradually decreasing rates of decline. The water levels will asymptotically approach values appropriate to the establishment of a steady-state flow system. Only for case III, however, is a new steady-state system approximately established by the end of the 40-year simulation period.

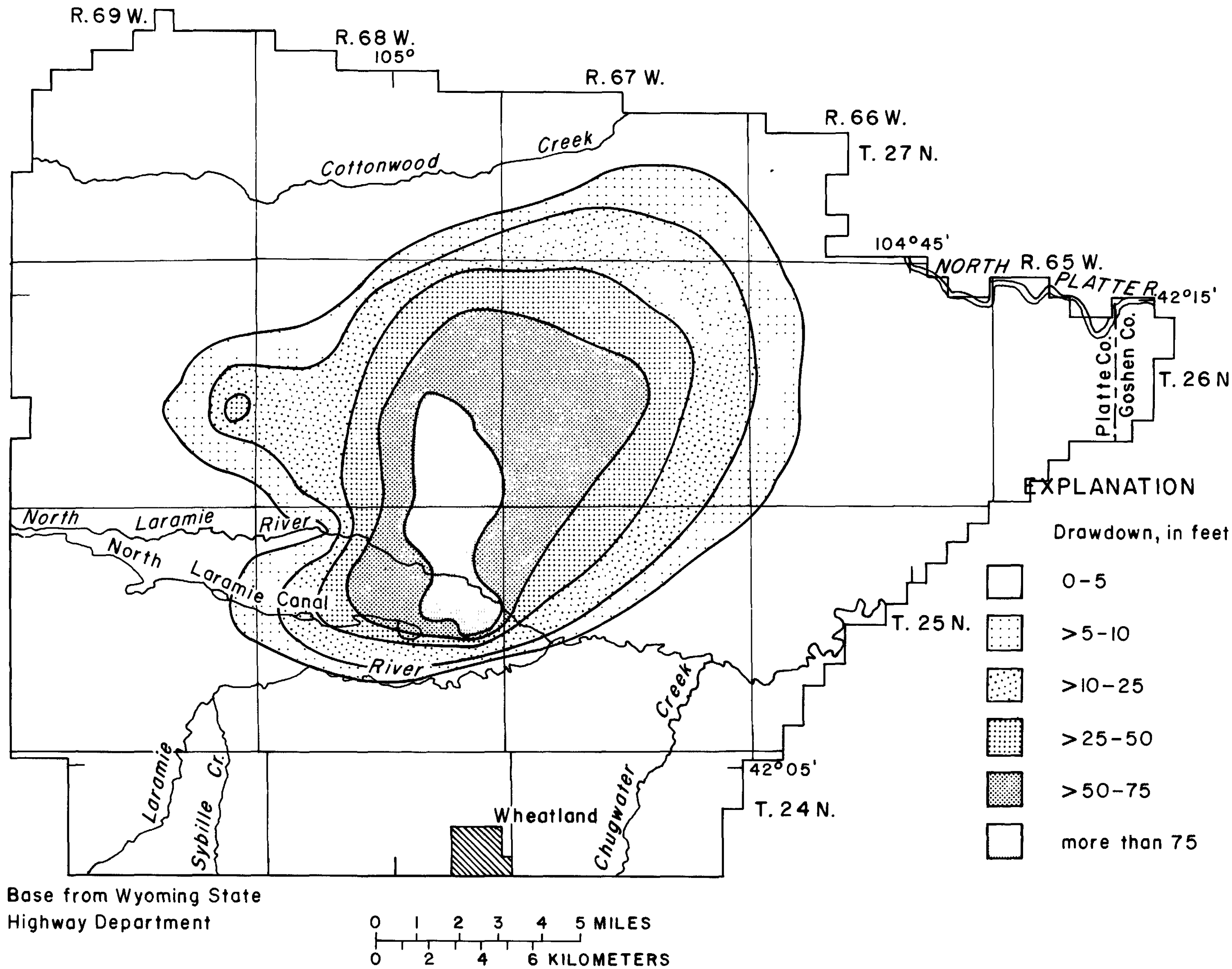


Figure 9.--Drawdown distribution for case I at the end of the 40-year simulation period.

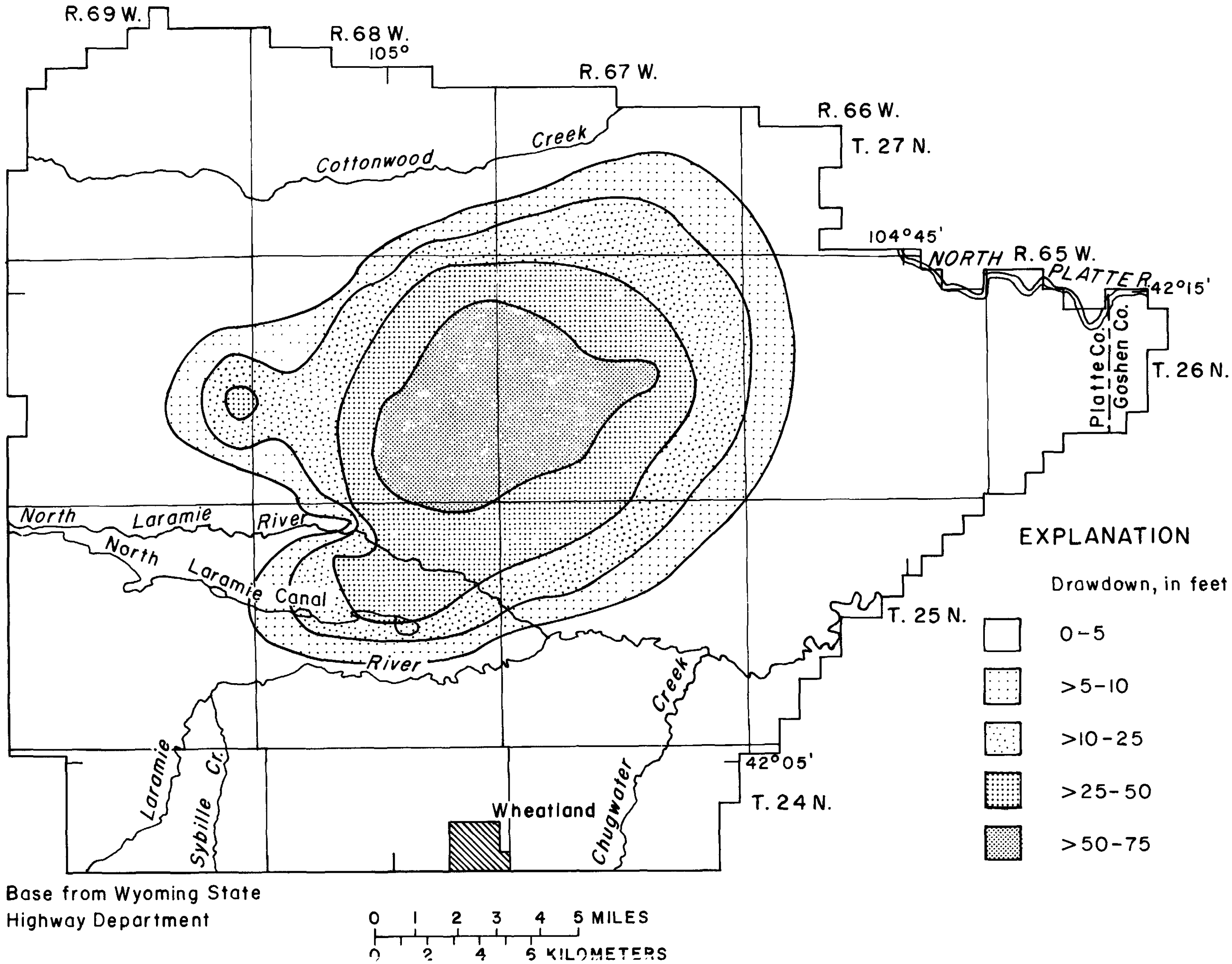


Figure 10.--Drawdown distribution for case II at the end of the 40-year simulation period.

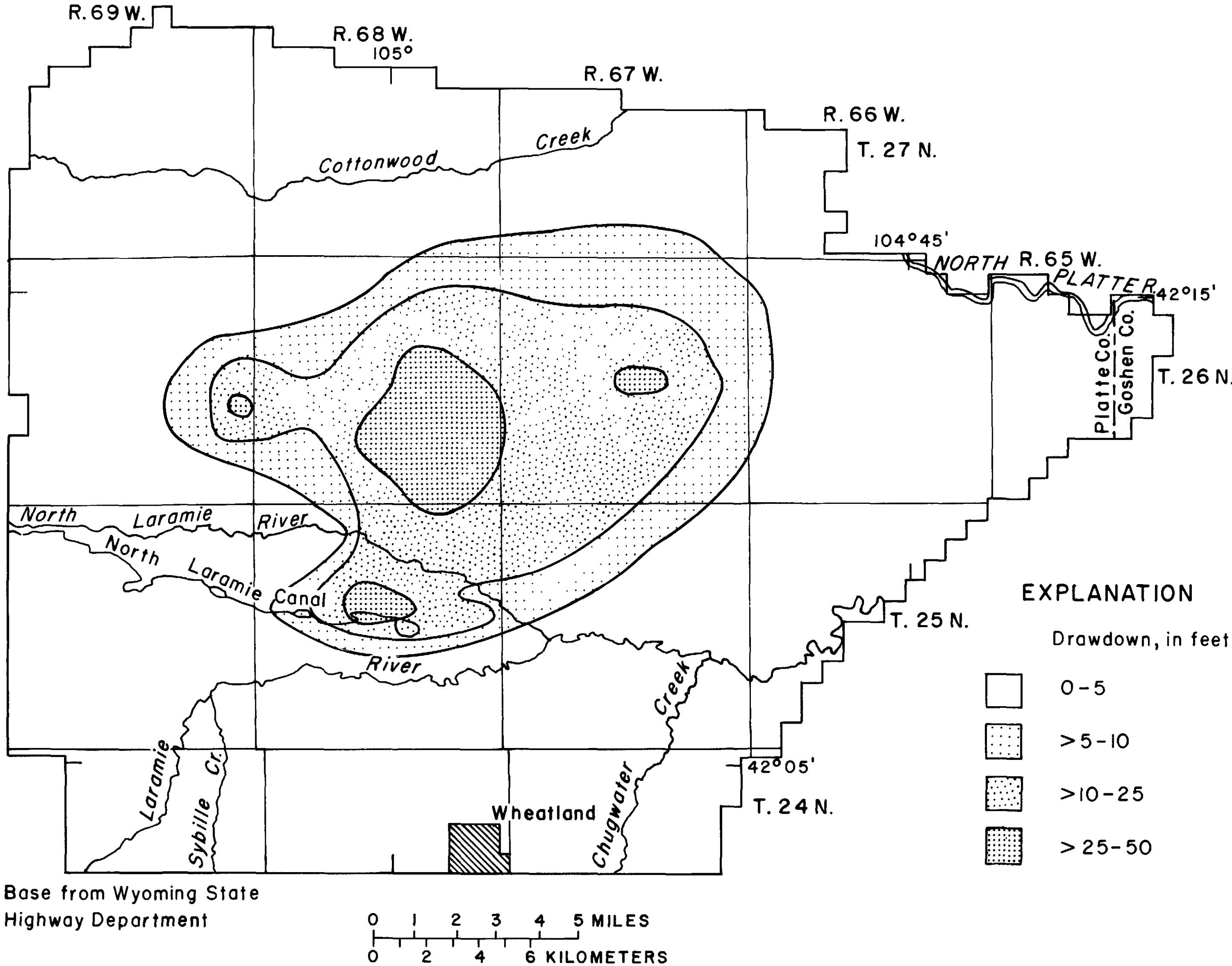


Figure 11.--Drawdown distribution for case III at the end of the 40-year simulation period.

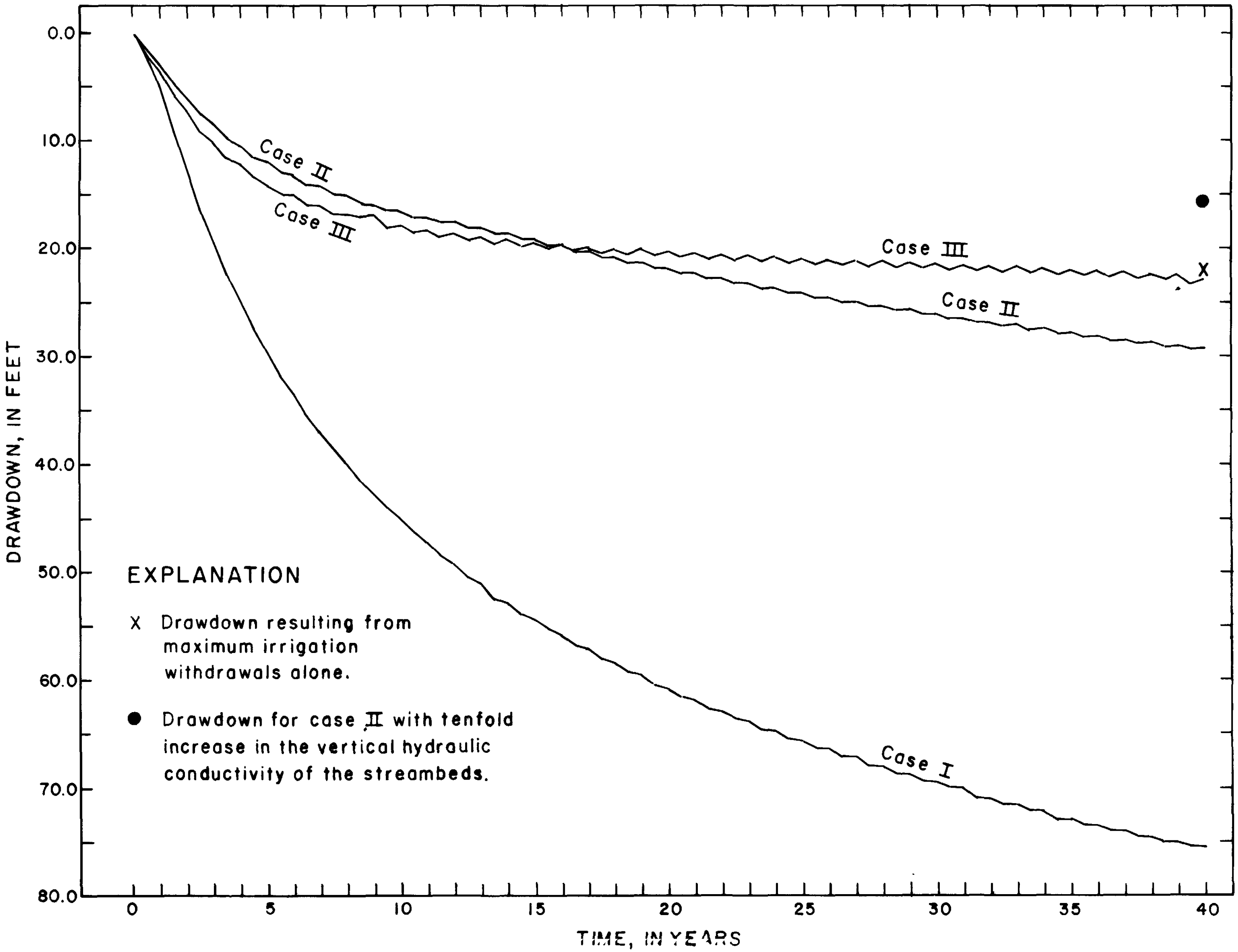


Figure 12.--Predicted drawdown versus time for a hypothetical observation well located at site 1 in figure 8.

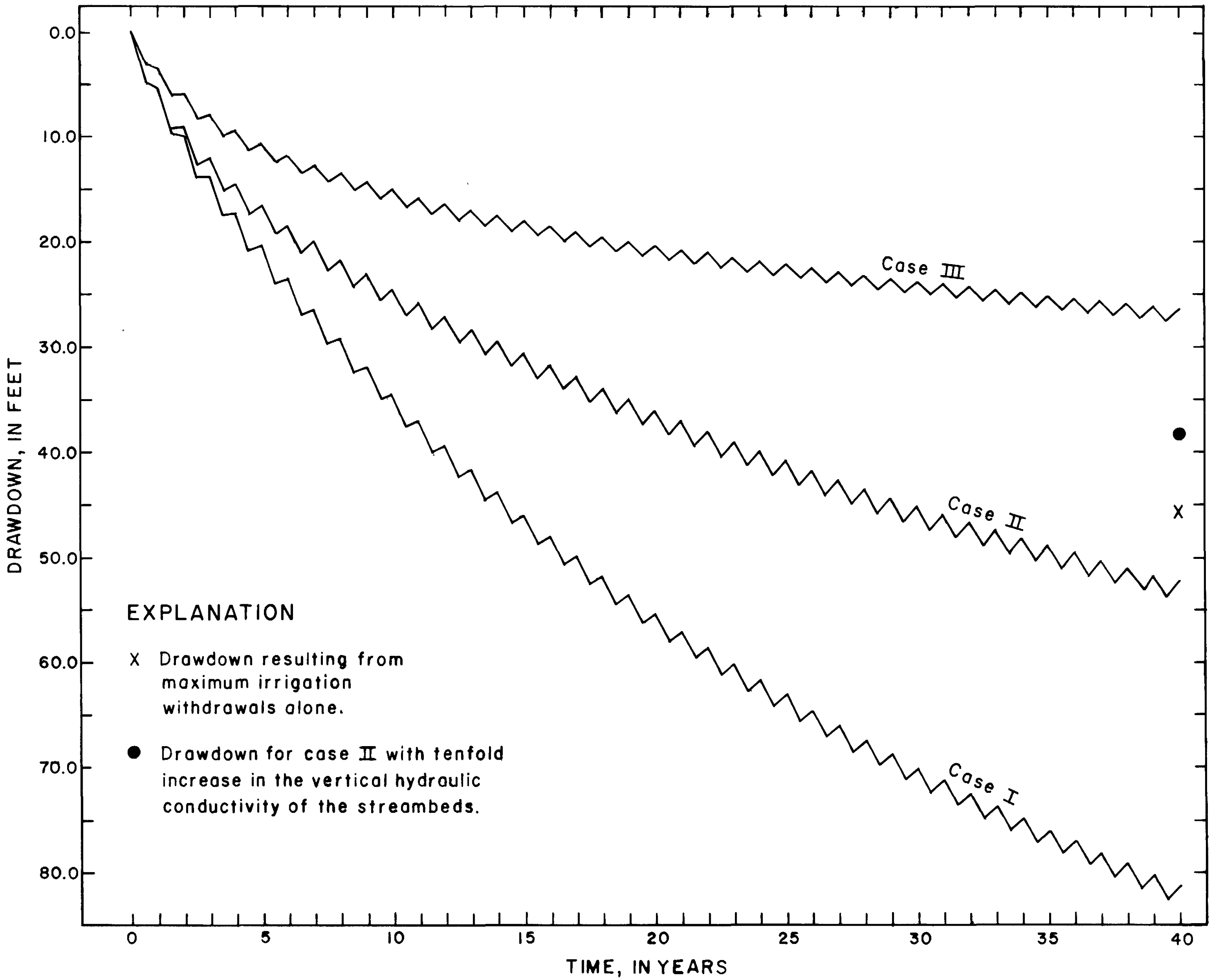


Figure 13.--Predicted drawdown versus time for a hypothetical observation well located at site 2 in figure 8.

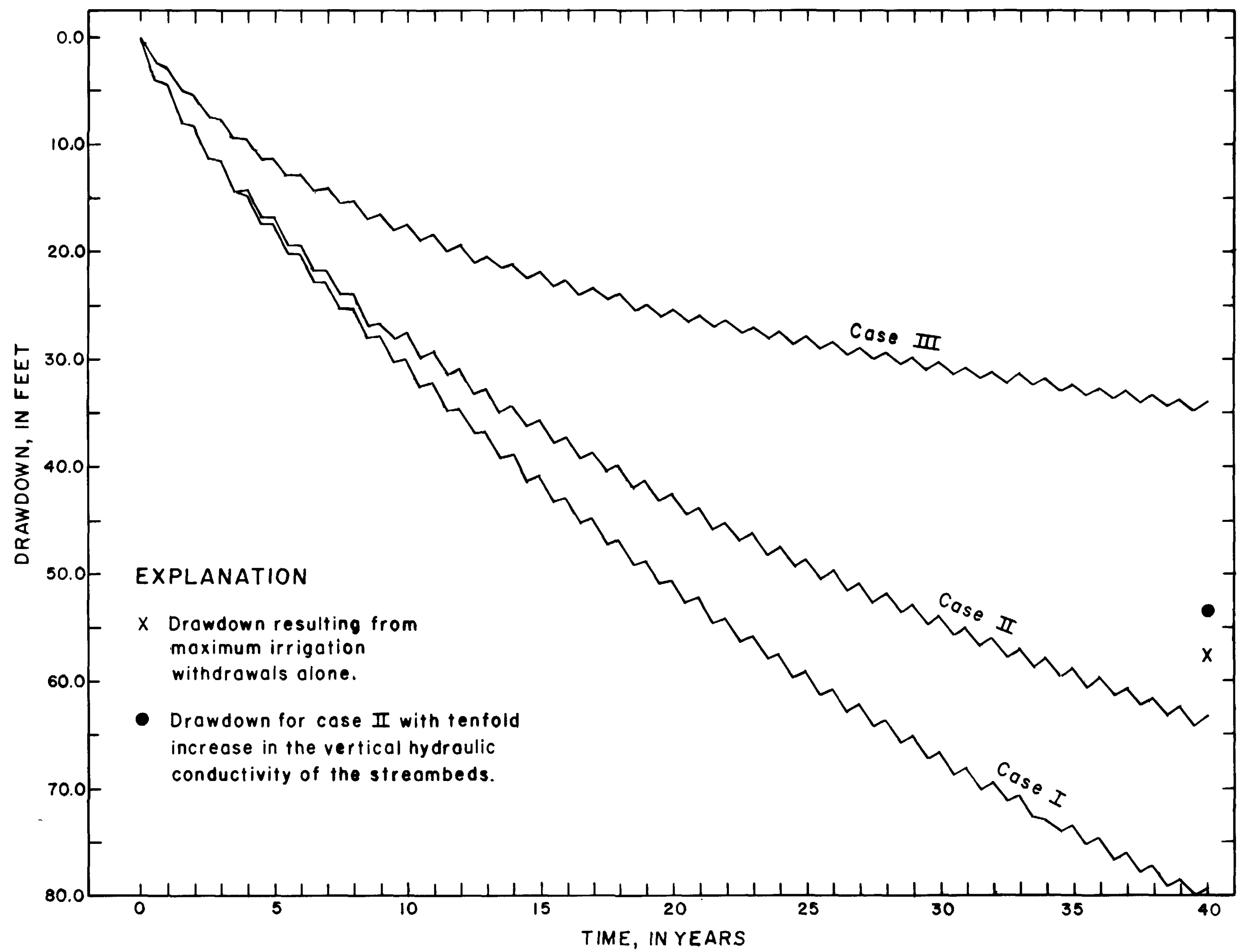


Figure 14.--Predicted drawdown versus time for a hypothetical observation well located at site 3 in figure 8.

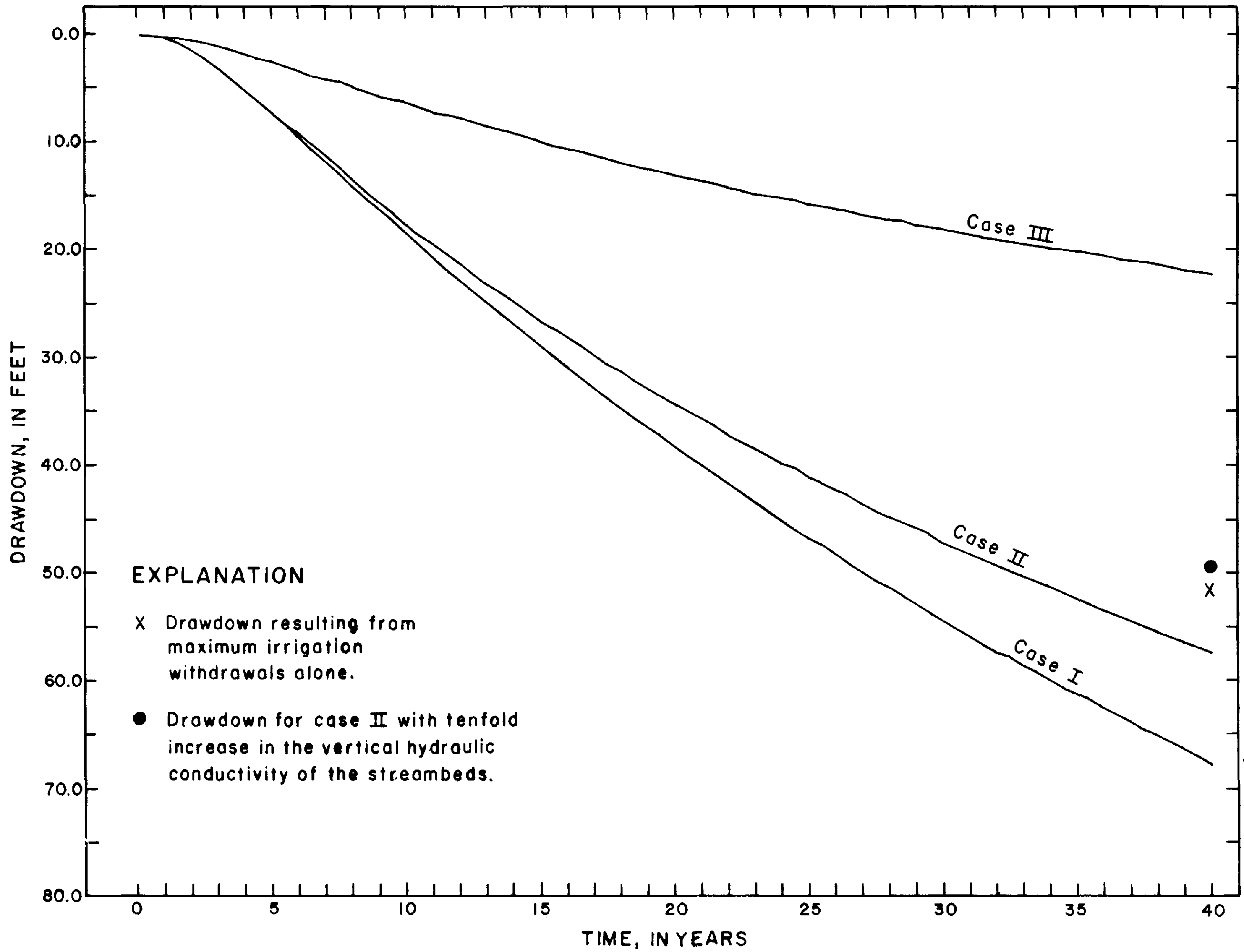


Figure 15.--Predicted drawdown versus time for a hypothetical observation well located at site 4 in figure 8.

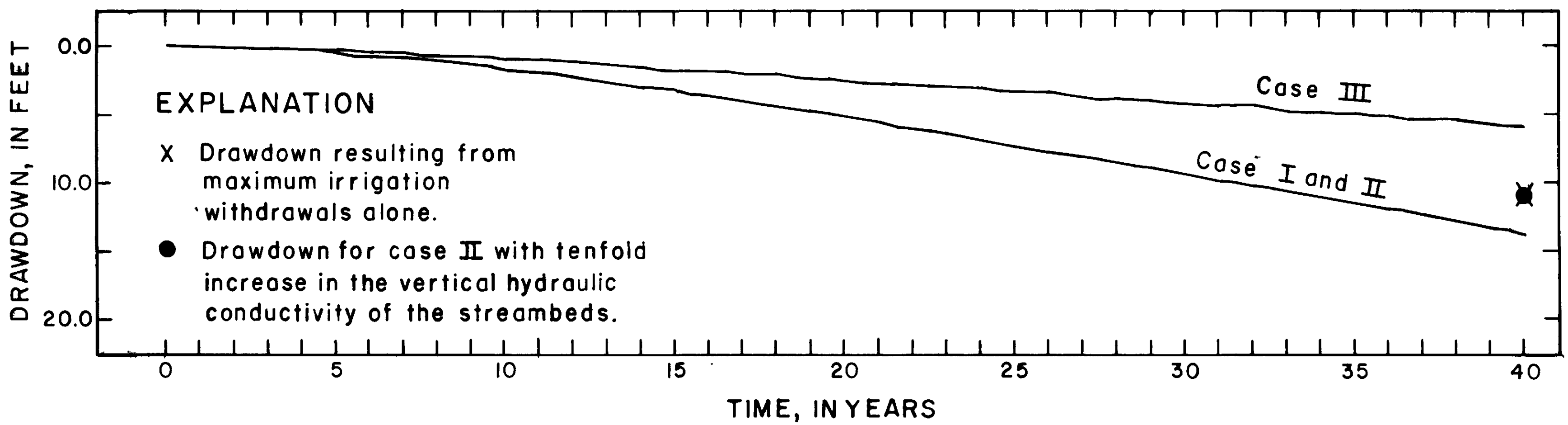


Figure 16.--Predicted drawdown versus time for a hypothetical observation well located at site 5 in figure 8.

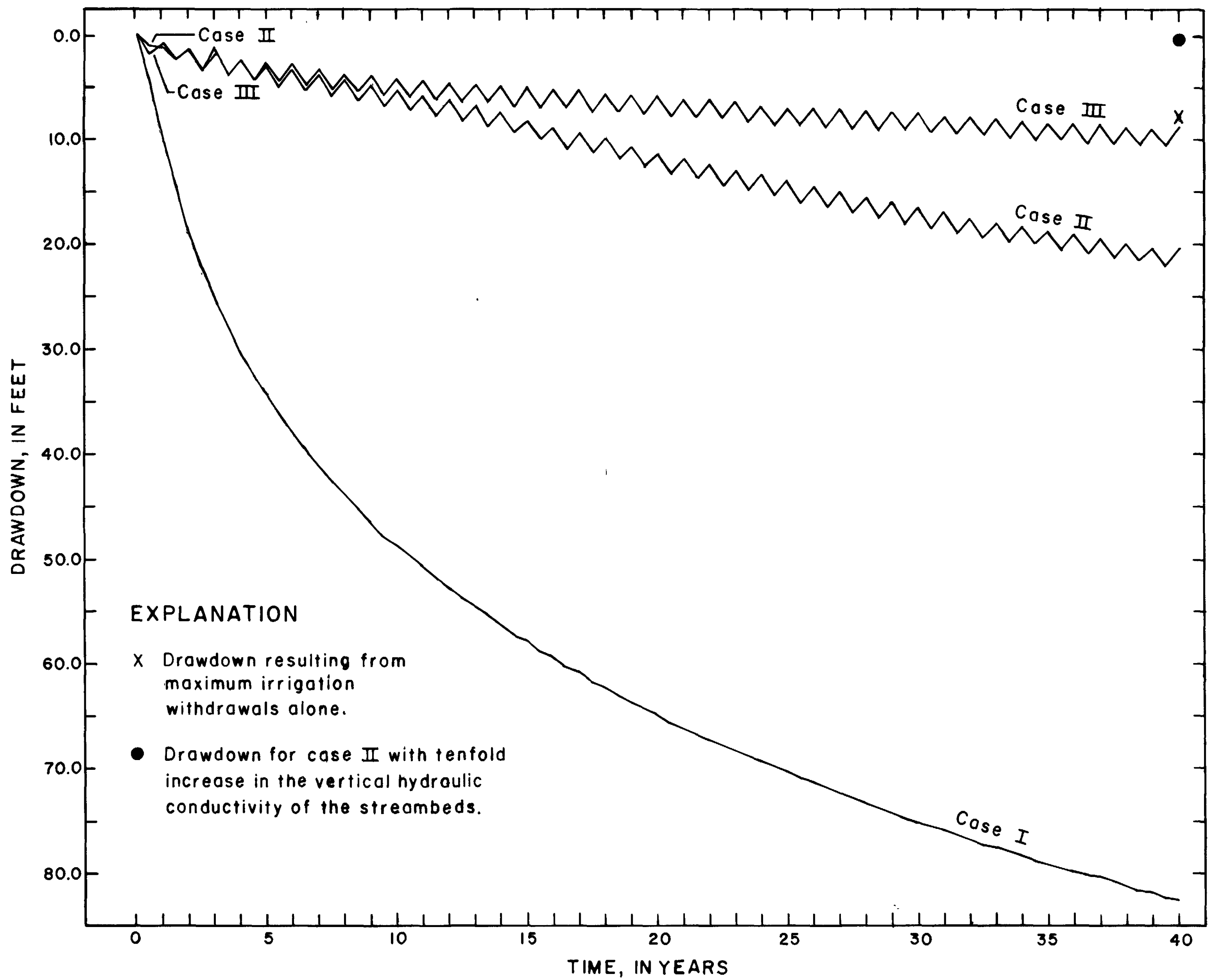


Figure 17.--Predicted drawdown versus time for a hypothetical observation well located in the bed of the North Laramie River, site 6 in figure 8.

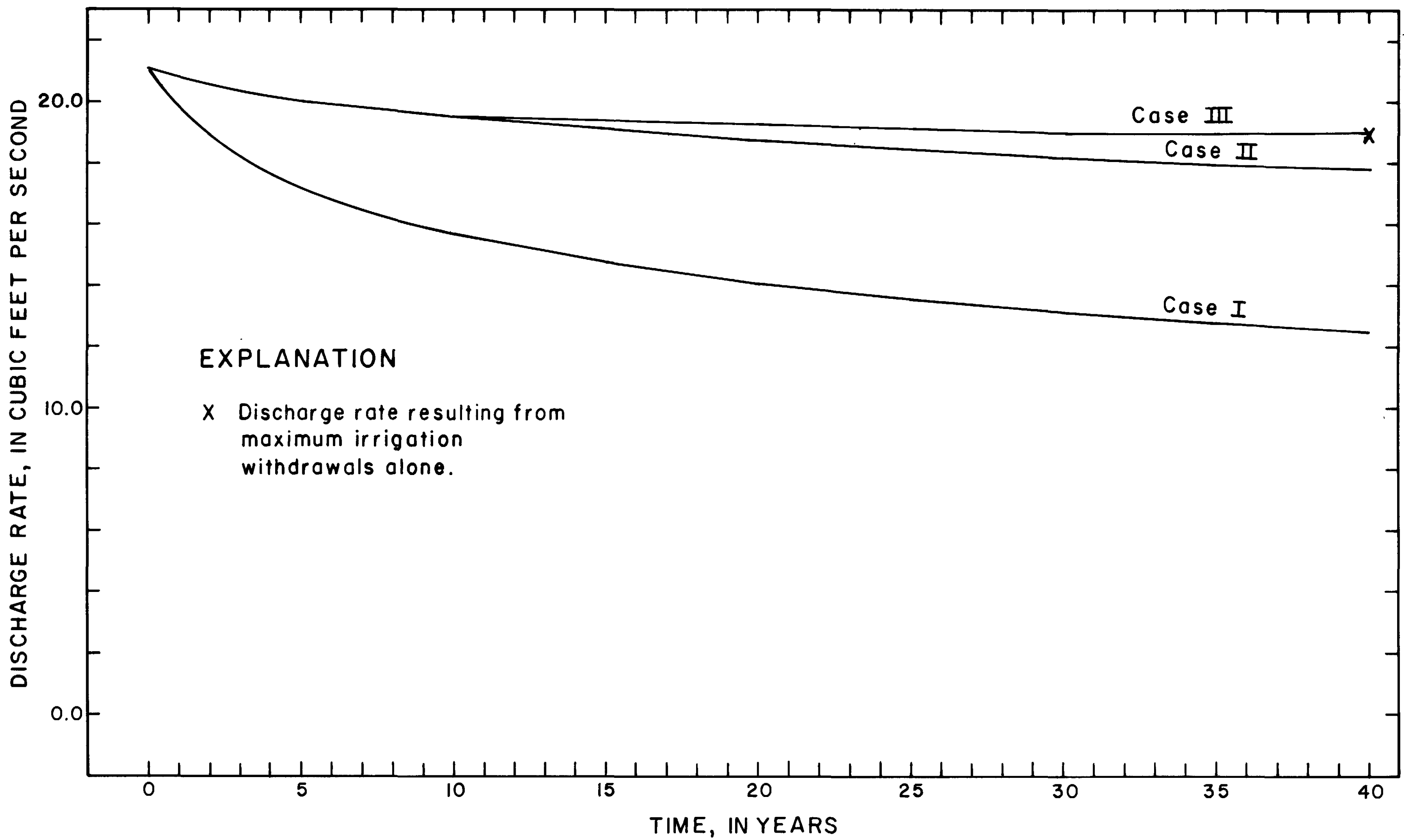


Figure 18.--Predicted decrease in ground-water discharge from the Arikaree aquifer to the Laramie River during the 40-year simulation period by cases I, II, and III pumping rates.

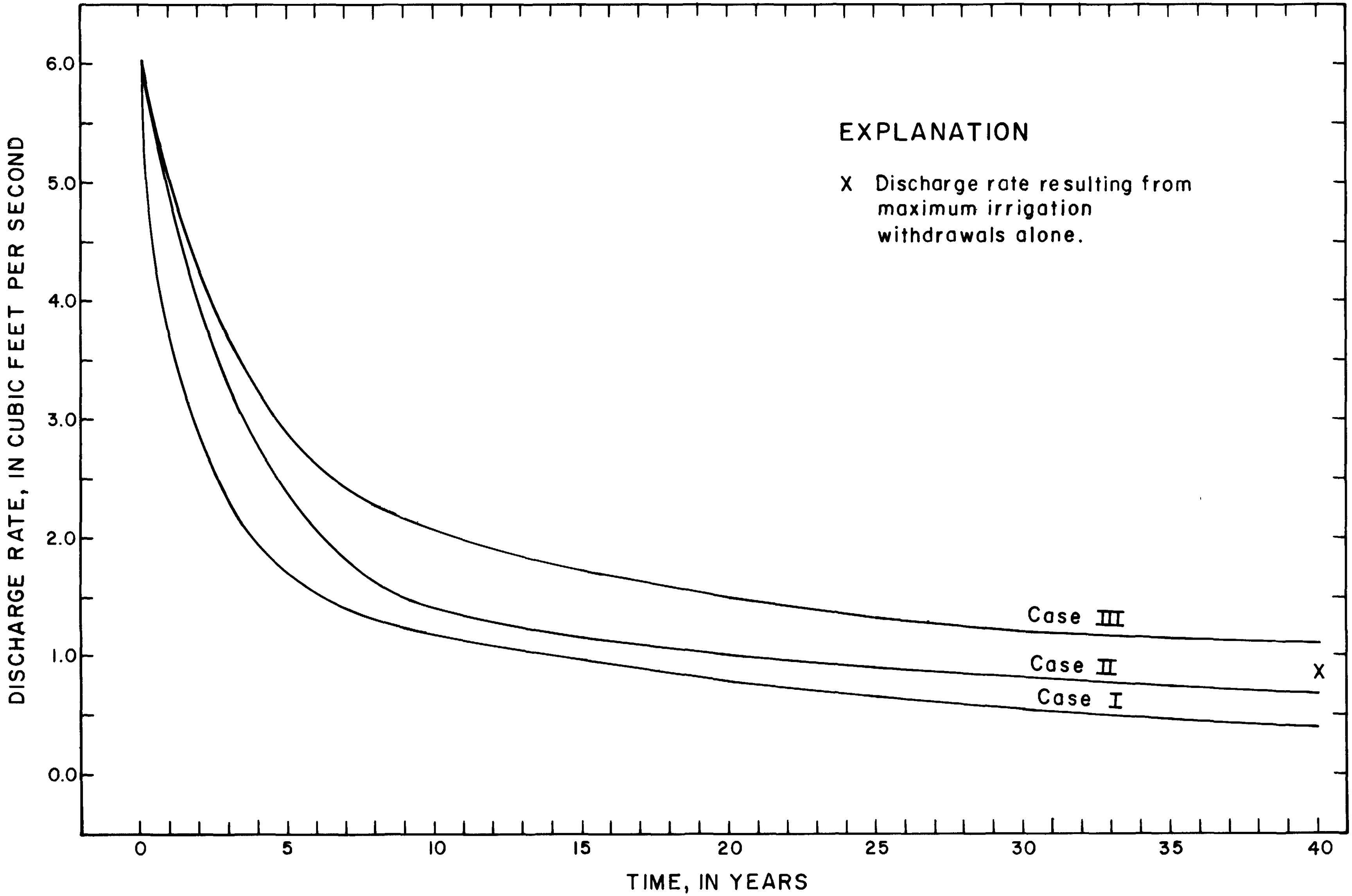


Figure 19.--Predicted decrease in ground-water discharge from the Arikaree aquifer to the North Laramie River during the 40-year simulation period for cases I, II, and III pumping rates.

Although the net pumping rate of case II is greater than that for case III, the drawdown at sites 1 and 6 are initially greater for case III than for case II (figs. 12 and 17). This occurs because these sites are near irrigation wells whose pumping rates are reduced in the case II simulation by the assumed availability of surface-water supplies for irrigation.

Site 5 (fig. 16) is near the edge of the resultant cone of depression for all three cases. Essentially no effect is produced at this site for case III, and small but nearly identical effects are produced at this site for cases I and II. It is apparent that for cases I and II the resultant cone of depression is continuing to enlarge.

Stream depletions are predicted to occur along reaches of both the Laramie and North Laramie Rivers. At the beginning of each transient simulation, when pumping from the irrigation and industrial wells first occurs, the water that is discharged by the wells is derived wholly from ground water that is held in storage within the aquifer. As pumping proceeds and cones of depression coalesce about the pumping wells, water that under initial steady-state conditions was being discharged from the aquifer to the streams will be intercepted and diverted to the wells. As pumping continues, the potentiometric surface in the vicinity of the streams is drawn below the head in the streams and water is diverted from surface flow to the aquifer. Eventually, the potentiometric surface may be lowered sufficiently beneath the streams to break hydraulic connection between the streams and the aquifer, and the streams will leak water to the aquifer at a rate governed by stream stage and the vertical hydraulic conductivity of the streambed. In other words, a transition from the depiction of figure 6a through 6b to that of 6c can be expected to occur.

The extent of the stream reach that is affected by pumping will depend upon the net pumping rate, the availability of surface flow in the streams, and the rate at which water leaks into the aquifer from the streams. The rate of leakage from streams to the aquifer will presumably be limited by the depth of water in the streams and the hydraulic conductivity of the streambeds. In the present application, the limiting rates are coincidently equal to the initial steady-state rates of gain by the streams. The reaches of the Laramie and North Laramie Rivers that are shown to be affected by pumping in figures 9, 10, and 11 reflect these limiting leakage rates.

Figure 17 shows the drawdown that would occur in a hypothetical observation well located in the streambed of the North Laramie River. Hydraulic connection between the stream and aquifer is broken in each of the three pumping cases, although it is only in the case I simulation that the drawdown beneath the stream is appreciable. The graph of drawdown versus time for case III clearly shows the trend toward the establishment of a steady-state flow system near this observation point.

The predicted stream depletion rates for the Laramie and North Laramie Rivers are reflected in a decreasing rate of ground-water discharge to these streams from the aquifer. This is shown in figures 18 and 19, which display the net rate of ground-water discharge from the aquifer to the Laramie and North Laramie Rivers, respectively, over the simulation period for each of the three pumping cases. The stream depletions at the end of each 40-year simulation are listed in table 2. In all three cases, the water that is lost from these streams is less than the net gain in streamflow from ground-water discharge that occurs within the study area upstream from the reaches affected by pumping. For the North Laramie River, however, the downstream losses at the case I and case II pumping rates essentially equal the upstream gains within the study area, and further stream depletions will occur only at the expense of surface flow originating upstream from the study area. The lower case III pumping rates produce considerably less effect on the streams, and a steady-state flow system, for which the pumpage is supplied by intercepted ground-water discharge to the Laramie and North Laramie Rivers, is approached at the end of the simulation period.

The southward migration of the resultant cone of depression is effectively arrested at the Laramie River. Consequently, little or no effect is predicted to occur within the Wheatland Flats area as a result of pumping within the area of principal interest. This result is due to the apparent high rate of ground-water discharge, approximately 12 ft^3/s (Weeks, 1964; Lines, 1976), from the Arikaree aquifer in the Wheatland Flats area to the Laramie River. Significant water-level declines in the Wheatland Flats area would not be expected to occur until this rate of discharge from the aquifer to the Laramie River was reduced appreciably.

In order to separate the effects produced by the industrial wells from those produced by the irrigation wells, a 40-year simulation was undertaken without the industrial wells and with the irrigation wells pumping at the annual average case I rates. The resultant drawdowns at the end of the 40-year simulation period at the hypothetical observation well sites shown in figure 4 are shown by crosses in figures 12 through 17. The corresponding stream depletions for the Laramie and North Laramie Rivers at the end of the simulation period are shown also by crosses in figures 18 and 19. The net effect for this case of maximum projected ground-water withdrawal for irrigation alone is to produce water-level declines that are generally less than those produced under case II conditions with the industrial wells. The effect on the North Laramie River is approximately the same as occurs under case I and case II conditions; that is, the net steady-state discharge from the aquifer to the stream is intercepted by the wells. The effect on the Laramie River is slightly less than that occurring under case II conditions. These results indicate that industrial pumpage at the projected annual average rate of 1,450 acre-feet, which constitutes 13 percent of the total case II pumping rate, would produce little additional effects to those that are to be expected from maximum irrigation pumping. Sustained industrial pumping at the maximum projected annual rate of 6,110 acre-feet would

Table 2.--Predicted rates of stream depletion from the Laramie and North Laramie Rivers at the end of the 40-year transient simulation period

Stream	Stream-depletion rate (ft^3/s)		
	Case I	Case II	Case III
Laramie River	8.7	3.4	2.3
North Laramie River	5.7	5.5	4.9
Total	14.4	8.9	7.2

produce substantially greater effects on both water levels and streamflow as is indicated by the case I simulation. Because equation 1 is nonlinear for unconfined aquifer problems, the effects produced by the industrial wells are not simply additive to those produced by the irrigation wells alone.

ACCURACY OF THE SIMULATION

A digital model is an idealized mathematical approximation to a real hydrologic system, and because it is an approximation, it has limited capability to simulate in detail the behavior of the hydrologic system that it is intended to represent. The departure of the simulated behavior from the actual behavior is here termed the error of the simulation. Error is introduced by the numerical procedures, specifically the finite-difference techniques, that are employed to solve the flow equation (equation 1) and by error associated with the input hydrologic field data. The error arising from the numerical procedures can be controlled through the design of the finite-difference grid, the choice of the forward time step during transient simulations, and the specification of the accuracy with which the finite-difference equations are solved. Error is contributed by the field data through (1) limitations in the accuracy with which measureable quantities, such as the rate of discharge of water from the aquifer to streams, can be determined, and (2) through uncertainties associated with the indirect determination of properties, such as the hydraulic conductivity distribution. Given a sufficient number of measurements distributed over a sufficiently long period of time, the errors in the field data can be treated statistically, and their net effect on the simulation can be assessed and minimized.

An indication of the effect of uncertainties in the input hydrologic data is obtained by determining the sensitivity of the model to these data. Such sensitivity tests are conducted by varying the data by a fixed amount and assessing the consequent effect on the results of the simulation. A set of four transient simulations, in which the hydraulic conductivity was increased uniformly by 10 percent and 25 percent and the specific yield was increased by 25 percent and 50 percent, were undertaken using the case II pumping rates. These parameters were chosen because they enter directly into the flow equation (equation 1) and because they were both determined by indirect procedures with which considerable uncertainty may be anticipated. The effects of these variations on predicted water-level declines and stream depletion rates are assessed at the end of the 40-year simulation period. The results of these sensitivity tests are summarized in table 3, which lists the rms deviation (equation 2) between the drawdown distributions that are calculated with and without variation of the indicated parameter, the predicted drawdowns at the six hypothetical observation well sites shown in figure 8, and the predicted stream depletion rates.

Table 3.--Sensitivity-test results on drawdowns and stream-depletion rates for the Laramie and North Laramie Rivers at the end of 40-year simulations using case II pumping rates

		Simulation with unaltered hydrologic parameter data	Hydraulic conductivity increased by 10 percent	Hydraulic conductivity increased by 25 percent	Specific yield increased by 25 percent	Specific yield increased by 50 percent
Rms deviation (ft)			1.6	3.8	2.3	4.1
Drawdown (ft)	Site 1	26.0	25.0	23.6	23.8	22.0
	2	49.3	47.4	44.9	44.2	40.3
	3	59.4	57.8	55.9	52.8	47.8
	4	52.7	52.4	52.2	45.3	39.7
	5	11.0	13.0	15.7	8.2	6.4
	6	18.7	17.6	16.4	15.8	13.6
Stream-depletion rate (ft^3/s)	Laramie River	3.0	2.4	2.7	2.7	2.5
	North Laramie River	5.2	5.1	5.0	5.1	5.0

The rms deviation is a measure of the average change that is produced in the calculated drawdown distribution by the indicated variations in the hydraulic conductivity and the specific yield. The values of the rms deviation, together with the drawdowns listed in table 3, indicate that the predicted water-level declines are relatively insensitive to variations of the magnitudes indicated for these two parameters. A uniform increase in the hydraulic conductivity causes the resultant cone of depression to enlarge more rapidly, as is evident by the increased drawdown at observation well site 5 for the higher hydraulic conductivity values. The predicted drawdowns at sites near the center of the cone of depression correspondingly decrease. Increases in the specific yield produce uniformly smaller drawdowns because more water is available from storage at the higher values and a smaller decrease in saturated thickness is required to release a given quantity of water from storage. The precise value of the specific yield is most important early in simulation period when most of the water being discharged by the wells is derived from storage.

The effect of variations in the hydraulic conductivity on the predicted stream-depletion rates is difficult to assess because changes in the hydraulic-conductivity distribution will change the calculated steady-state configuration and, in particular, the calculated steady-state rate of discharge from the aquifer to the streams. The stream-depletion rates listed in table 3 for the 10-percent and 25-percent increases in hydraulic conductivity include the effects that are produced on the steady-state simulation as well as those produced on the transient simulation. The net effect of uniformly increasing the hydraulic conductivity is initially to cause the release of more water from storage within the aquifer and to reduce the stream depletion rates. This is reflected in the lower stream depletion rates for the Laramie River listed in table 3.

Increases in specific yield, as indicated in table 3, produce decreases in the stream-depletion rates. Because more water is available from storage at the higher specific yield values, less water is required from the streams during the simulation period in order to supply the discharging wells. This effect is most apparent on the stream-depletion rate for the Laramie River. Because of its proximity to the area of expected maximum ground-water withdrawals, the stream-depletion rate at the end of the simulation period for the North Laramie River is largely unaffected by the indicated variations in either the specific yield or the hydraulic conductivity.

Although the results of the transient simulations indicate that uniform increases in the hydraulic conductivity distribution of 10 and 25 percent produce little effect on the long-term results, such is not the case for the steady-state simulation. If the hydraulic conductivity is increased uniformly by 10 and 25 percent, the calculated steady-state head distribution departs from the measured potentiometric surface with an rms deviation of 9.2 and 15.6 feet, respectively, and a maximum departure of 32 and 47 feet, respectively. These departures are larger than those that would be acceptable through the steady-state calibration process. It is concluded that variations of 10 and 25 percent are indicative of the maximum uncertainty to be associated with the hydraulic conductivity distribution that was determined through steady-state calibration. This calibration process, however, is founded on data, such as gains in streamflow, the rate and distribution of recharge from precipitation, and a potentiometric surface constructed from water-level measurements in widely-spaced wells, with which an unknown amount of uncertainty is associated. In order to test the sensitivity of the model to these data, it would be necessary to repeat the steady-state calibration process and generate a new hydraulic conductivity distribution for each parameter that is varied. Because steady-state calibration is such a time consuming process, it was judged impractical to proceed with this extension of the sensitivity test analysis. A better approach would be to collect streamflow and water-level data for the aquifer at selected intervals over a sufficiently long period of time that statistical limits of uncertainty can be determined for these data and the effects of these limits on the simulations then assessed.

One of the least known and most important parameters is the vertical hydraulic conductivity of the streambeds. This parameter, together with its distribution along the stream reaches and the stage in the streams, controls the movement of water between the streams and the aquifer. Values of the streambed conductivity for streams in the study area were calculated from the assumed steady-state rates at which ground water was discharged to the streams from the aquifer under the assumption that the head difference between the streams and the aquifer was 1 foot. By setting this head difference to 0.1 foot, the conductivity values can be increased by a factor of 10 while maintaining the same steady-state ground-water discharge rates. The limiting rates of leakage of water from the streams to the aquifer are then also increased by the factor 10. The effects of this tenfold increase in the vertical hydraulic conductivity of the streambeds has been assessed under transient conditions using the average annual case II pumping rates. The resulting drawdown distribution at the end of the 40-year simulation period is shown in figure 20, and the computed drawdowns at the six hypothetical observation well sites are indicated in figures 12 through 17 by closed circles. Because more water is available from the streams in this case, the resultant cone of depression is decreased from 120 to 102 mi^2, and drawdowns near the streams (fig. 17) are significantly reduced. The area near the center of the cone of depression in which drawdowns exceed 50 feet is reduced from 21 to 8 mi^2. The rate of stream depletion in the North Laramie River is increased from 5.5 to 7.5 ft^3/s, and in the Laramie River it is decreased from 3.0 to 2.0 ft^3/s. At the end of the 40-year simulation period, 64 percent of the net pumping rate of 14.9 ft^3/s is derived from streamflow compared to 57 percent for the case II simulation at the original values of streambed conductivity.

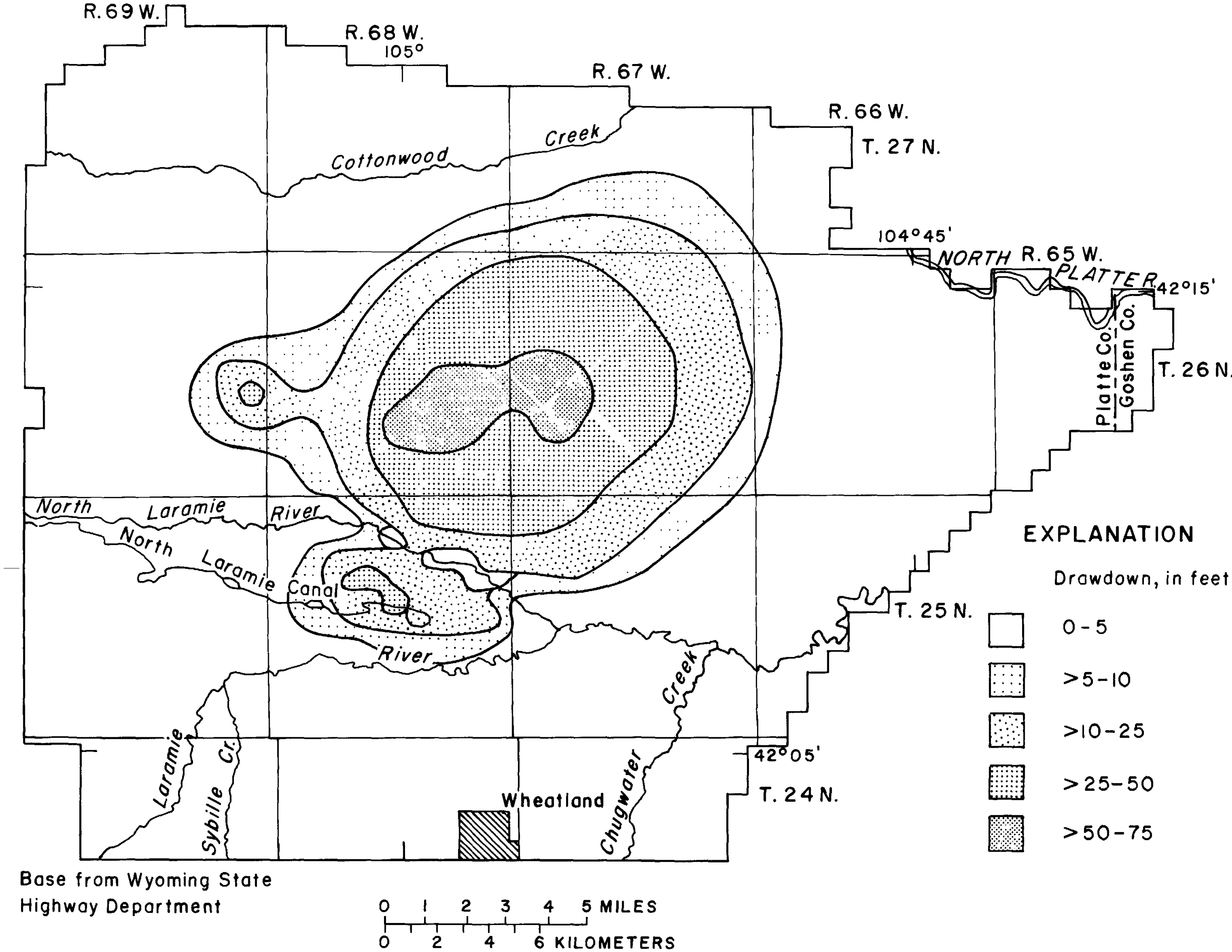


Figure 20.--Drawdown distribution for case II at the end of the 40-year simulation period with the vertical hydraulic conductivity of the streambeds increased by a factor of 10.

The most important effect of the particular values that are assumed for the vertical hydraulic conductivity of the streambeds is to determine the rate at which water can leak to the aquifer from the streams whenever the head in the aquifer falls below the head in the overlying streams. The corresponding stream-depletion rates are in turn determined by these leakage rates. The values that were assumed for the streambed hydraulic conductivities for streams in the study area yield maximum rates of leakage from the streams that are comparable to the average rate of leakage from the North Laramie Canal as estimated by Lines (1976). The values that were assumed for the vertical hydraulic conductivity of the streambeds ranged from 0.00006 to 0.013 ft/d, which are somewhat smaller than the range of values from 0.008 to 0.064 ft/d that Weeks (1964) estimated for the vertical hydraulic conductivity of the Arikaree aquifer in Wheatland Flats. If these values are representative of existing conditions, then presumably silt and clay in the flood-plain alluvium act to seal the aquifer from the streams. It is possible to test this hypothesis and to determine values for the vertical hydraulic conductivity of the streambeds by conducting aquifer tests near the streams as described, for example, by Weeks, Ericson, and Holt (1965).

The adequacy of the model can be judged ultimately only by the accuracy with which it reproduces the observed response of the aquifer to an imposed stress, such as pumping, of known magnitude. This can be accomplished for the present model by collecting pumpage data and monitoring water levels and streamflows within the study area as development proceeds. The model can then be updated and refined to provide a means by which the effects of present and future development can be more accurately predicted and the degree of accuracy can be assessed. Unfortunately, the present level of ground-water development has produced insufficient change, as indicated by the water-level and streamflow measurements made in 1973 and 1977, to provide a definitive test of the model.

SUMMARY AND CONCLUSIONS

A digital model that employs finite-difference techniques to solve numerically the equation of ground-water flow, approximating the flow system as two dimensional, was developed for the unconfined Arikaree aquifer near the confluence of the Laramie and North Laramie Rivers near Wheatland, Wyoming. The model is based on hydrologic field data collected by Lines (1976) in the area north of the Laramie River and by Weeks (1964) in the Wheatland Flats area.

Steady-state calibration of the model with respect to the hydraulic-conductivity distribution was performed under the assumption that steady-state flow presently exists within the aquifer. The hydraulic-conductivity distribution for the aquifer, as generated through the steady-state calibration process, was employed in the subsequent transient simulations. A transient simulation of the period 1969 to 1977 yielded results that were consistent with the steady-state flow hypothesis and the results of water-level and streamflow measurements that were made during the winter of 1977.

The steady-state model provided the initial conditions for three transient simulations that were undertaken to assess the long-term effects of maximum, mean, and minimum proposed levels of ground-water utilization for irrigation and industry within the area. Water-level declines of more than 5 feet over areas of 124 and 120 mi^2 and of more than 50 feet over areas of 38 and 21 mi^2 are predicted for the maximum and mean levels of development, respectively, at the end of a 40-year simulation period. In addition, depletion in streamflow of 6,400 and 2,500 acre-feet per year for the Laramie River and of 4,100 and 4,000 acre-feet per year from the North Laramie River are predicted to result from these maximum and mean levels of development, respectively. At the proposed minimum level of development average water-level declines are predicted to be less than 50 feet throughout the study area and to be greater than 5 feet over an area of 98 mi^2. Depletion in streamflow of 1,700 and 3,500 acre-feet per year are predicted for the Laramie and North Laramie Rivers, respectively, in this case. A system of steady-state flow is approximately established at the end of the 40-year simulation period only for this case of minimum development.

The model is based on hydrologic field data whose overall accuracy cannot be readily assessed at present. Sensitivity tests conducted on the transient simulations indicate that, although the details of the transient simulations may be affected by the precise values of the hydrologic properties, the ultimate consequences of the proposed levels of development would remain the same. Initially, most of the water that would be discharged from the aquifer by the pumping wells would be derived from water held in storage within the aquifer, and the withdrawal of this water would be accompanied by the development and growth of coalescing cones of depression around the pumping wells. As the resultant cone of depression migrates along reaches of the Laramie and North Laramie Rivers, an increasing fraction of the water discharged by the wells would be derived from streamflow, either from the direct infiltration of surface flow to the aquifer or by the interception of ground water that would otherwise have discharged from the aquifer to the streams. Eventually most or all of the water that is discharged by the wells may be supplied by water lost from the streams, and a new system of steady-state flow would be established.

In order to improve the accuracy with which the model can predict the details of the future effects of ground-water withdrawal in the area, it will be necessary to monitor both water levels and streamflows as development proceeds and to use these data to refine and update the model.

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