A GROUND-WATER INVENTORY OF THE WAIALUA BASAL-WATER BODY, ISLAND OF OAHU, HAWAII

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UNITS OF MEASUREMENT

English units have been used throughout in this report. Feet and day are the commonly used units for length and time, respectively. Volumes are stated in terms of millions of gallons, the unit commonly used in Hawaii. The following table converts measurements in the English system to the International System of Units (SI).

Multiply English units	By	To obtain SI units
acre	4047	square meter (m^2)
square foot (ft^2)	0.0929	square meter (m^2)
square inch (in^2)	0.0006452	square meter (m^2)
square mile (mi^2)	2.590	square kilometer (km^2)
foot (ft)	0.3048	meter (m)
inch (in)	25.4	millimeter (mm)
mile (mi)	1.609	kilometer (km)
yard (yd)	0.9144	meter (m)
acre-foot (acre-ft)	1233	cubic meter (m^3)
cubic foot (ft^3)	0.02832	cubic meter (m^3)
gallon (gal)	3.785	liter (L)
million gallons (10^6 gal or Mgal)	3785	cubic meters (m^3)
cubic foot per second-day [(ft^3/s)-d]	0.002447	cubic hectometer (hm^3)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
gallon per minute (gal/min)	0.06309	liter per second (L/s)
million gallons per day (10^6gal/d) or (Mgal/d)	0.04381	cubic meter per second (m^3/s)
foot per mile (ft/mi)	0.1894	meter per kilometer (m/km)
cubic foot per second per squre mile [(ft^3/s)/mi^2]	0.01093	cubic meter per second per square kilometer [(m^3/s)/km^2]
foot per day (ft/d)	0.3048	meter per day (m/d)
gallon per minute per foot [(gal/min)/ft]	0.207	liter per second per meter [(L/s)/m]
foot squared per day (ft^2/d)	0.0929	meter squared per day (m^2/d)

SYMBOLS

Symbol	Description
A	Area
A_{cs}	Area of control surface
A_i	Area of the interface
A_{wt}	Area of the water table
B	Ratio of million gallons of deep infiltration per inch of rainfall
CL	Caprock leakage
DI	Deep infiltration
I	Inflow
K	Hydraulic conductivity
ND	Inflow through the northern ground-water dam
O	Outflow
P	Pumpage
Q	Pumping rate
R	Rainfall
S_y	Specific yield
T	Transmissivity
V_f	A volume of basal water
V_b	Total basal storage
$\frac{d}{dt}$	Total derivative
$\frac{dV_f}{dt}$	Flux imbalance with respect to the basal-water body

Symbol	Description
f	Subscript to indicate fresh
g	Acceleration of gravity
h	Head
i	Interface
n_e	Effective porosity
p	Pressure
q_f	Flux per unit area
t	Time
r	Radial distance
s	Drawdown
v	Velocity
z	Elevation
γ	Specific weight
Δ	A change in
π	3.1416
$\frac{\partial}{\partial t}$	Partial-time derivative
$\frac{\partial}{\partial n}$	Partial-space derivative in the normal direction

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By R. H. Dale

ABSTRACT

The Waialua basal-water body underlies an area of about 18 square miles (46 square kilometers) on the north shore of the island of Oahu. The basal-water body is a body of fresh ground water that floats on saline ground water in a highly permeable and porous basaltic aquifer.

Inflow to the basal-water body is from the deep infiltration of applied irrigation water and from leakage through a low-permeability ground-water dam. Outflow from the basal-water body is from basal-water pumpage and leakage through low-permeability boundaries that separate the basal-water body from the ocean.

The basal-water flux, computed as either the sum of the inflow terms or the sum of the outflow terms, is about the same value. The basal-water flux is 55 million gallons per day (206,000 cubic meters per day), based on the sum of the outflow terms.

The effective porosity was computed at 0.09 by a time-series analysis of the covariations in deep infiltration, pumpage, and basal-water head. The volume of basal water in storage is estimated to be 1.4 x 10^{11} gallons (5.4 x 10^{8} cubic meters).

Pumpage from the basal-water body can be increased, but the amount of increase would depend on the proposed type of development. The most efficient development method is the skimming shaft. If shafts were used, an additional 15 million gallons per day (57,000 cubic meters per day) could be pumped on a sustained basis.

INTRODUCTION

The Waialua basal-water body underlies an area of about 18 mi^2 (46 km^2) on the north shore of the island of Oahu (fig. 1). About 30 Mgal/d ($113x10^3m^3/d$) is pumped for the irrigation of sugarcane, which is the principal use of the basal water.

Rosenau, Lubke, and Nakahara (1971) made estimates of the additional water that could be pumped from the basal-water body. However, a more recent study by Dale and Takasaki (1976) suggests that the earlier estimate was substantially in error. This report was written to resolve the discrepancy between the two estimates.

The Basal-Water Body

This report is primarily a quantitative analysis of the flow characteristics of the basal-water body. Most of the previous reports on the basal water of the Hawaiian Islands were descriptive. Older definitions of the hydrologic relationships that were satisfactory for the descriptive approach are unsatisfactory for the quantitative approach. In this section of the report, the basal-water features that are pertinent to the analysis are described, and terms are redefined, as necessary. For ground-water terms that do not require redefinition, the reader is referred to Lohman and others (1972).

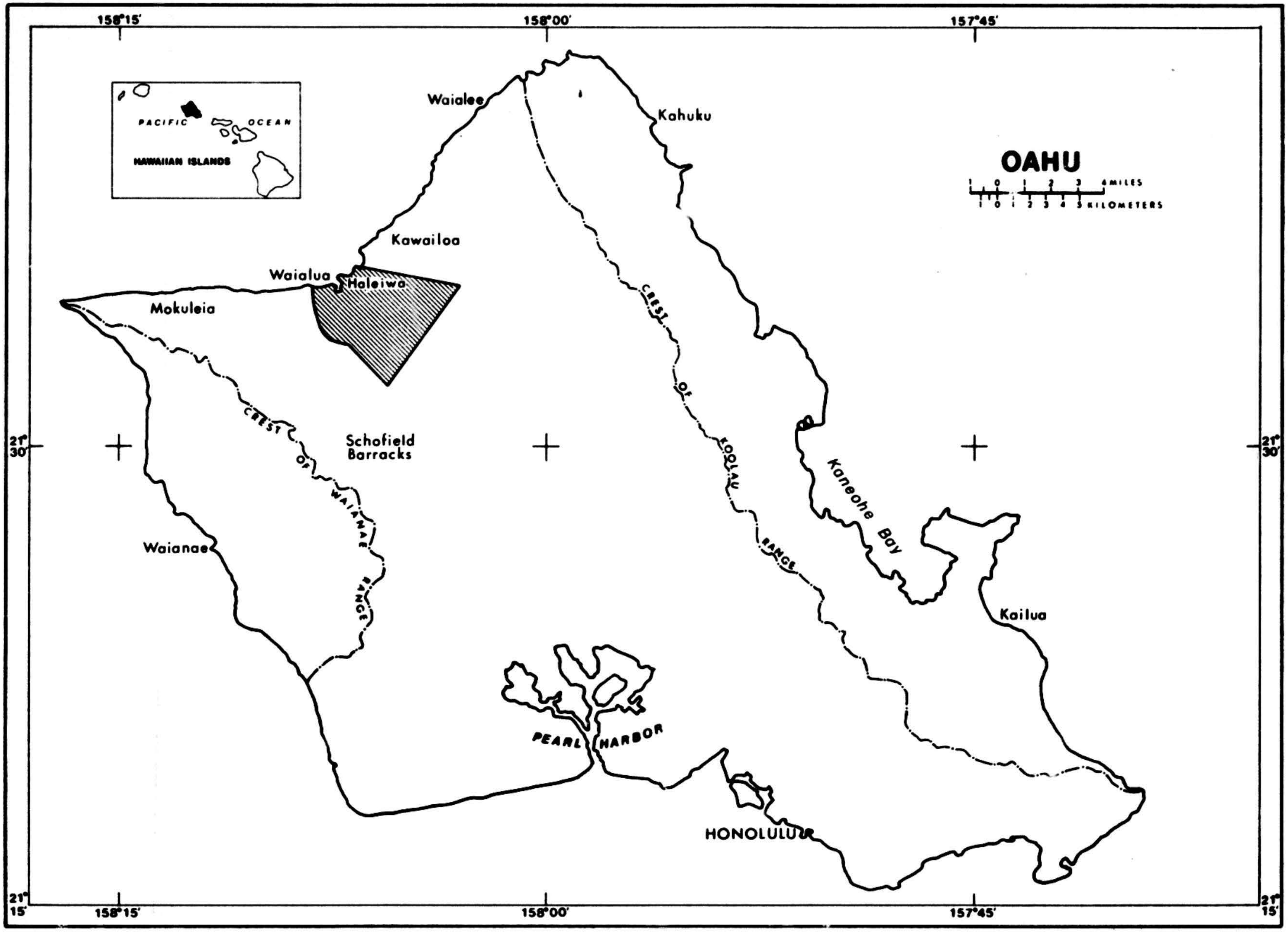

Figure 1. Index map of the Waialua area

The basal-water body is a body of fresh ground water that floats on saline ground water in a highly permeable, porous aquifer. In the above definition, the basal water and the saline ground water are defined on the basis of density, rather than by the concentration of dissolved constituents. The basal water has a density of 1.000 g/cm^3 (grams per cubic centimeter), and the saline ground water has a density of 1.025 g/cm^3.

There is not a sharp interface between the basal water and the saline ground water. Instead, there is a transition zone that separates the basal water from the saline ground water. The transition zone is a region that lies between the basal water of density 1.000 g/cm^3 and the saline ground water of density 1.025 g/cm^3. In most localities in the Hawaiian Islands, this transition zone is from 200 to 400 feet (60 to 120 meters) in thickness.

To simplify the analysis of the basal water, the transition zone is disregarded, and it is assumed that an interface separates the basal water from the saline ground water. The interface is a fictitious surface where the pressure at a given point on the basal-water side of the interface is exactly balanced on the saline ground-water side. If it is assumed, as it is for this report, that all flow lines are horizontal, then the pressure distribution is hydrostatic in a vertical column (fig. 2). In the basal water, the pressure increases linearly with depth to the top of the transition zone; in the transition zone, the increase is nonlinear; and in the underlying saline ground water, the pressure increase is again linear with depth (fig. 2). The intersection of the extended linear portions of the curve is a point on the theoretical interface, where the pressure balance would be exact if an interface actually existed.

In the simplest case, where the island is circular and of uniform permeability, the shape of the basal-water body is a double convex lens (fig. 3). The water table, which is the upper surface of the lens, is only slightly convex, and the water levels are near sea level. The interface, which is the lower surface of the lens, is strongly convex. For the density contrasts between the basal water and saline ground water, the position of the interface (i) below sea level is 40 times the altitude of the water table above sea level.

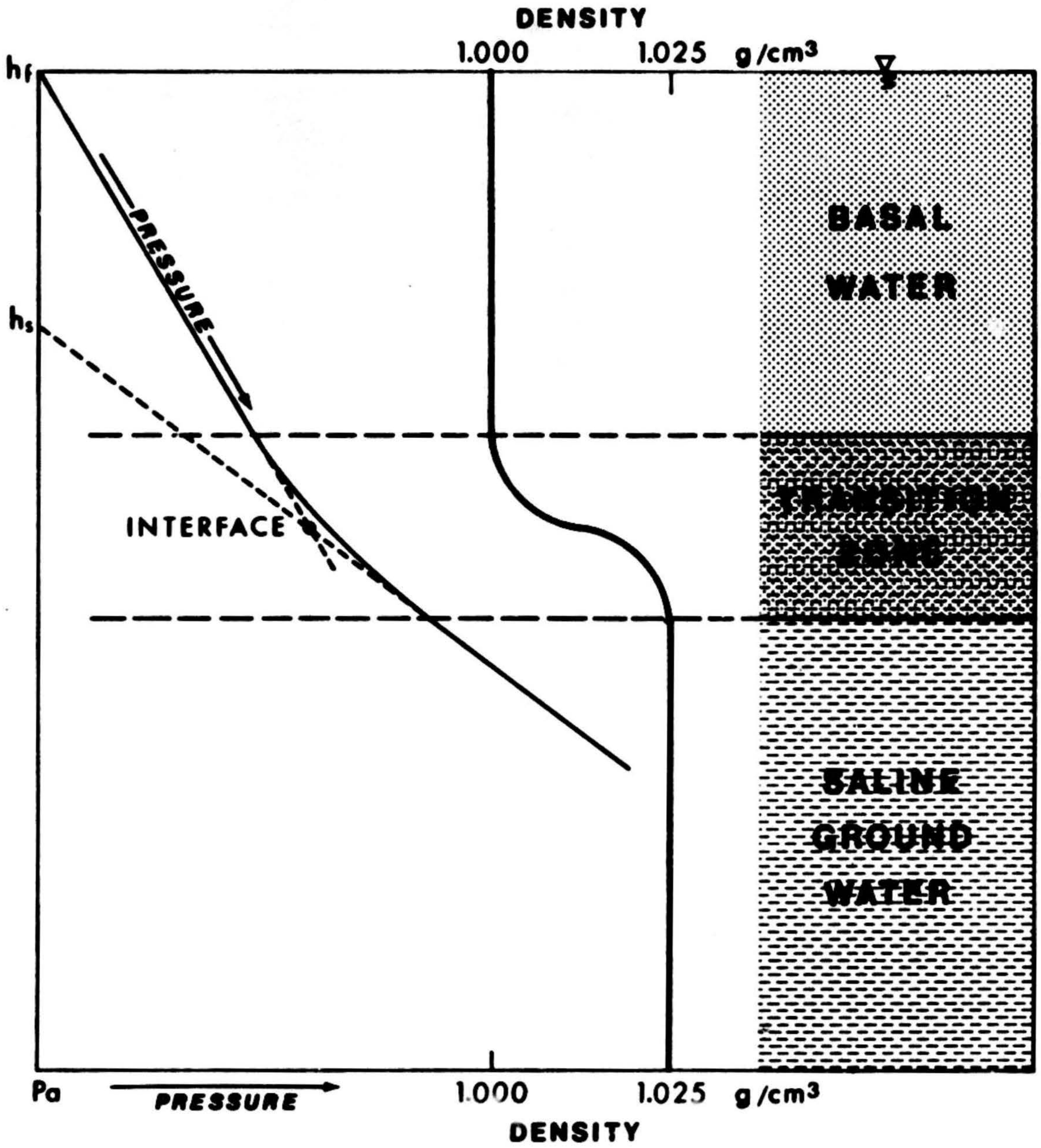

Figure 2. Pressure and density relationships for the basal water, transition zone, and saline ground water

Pressure increases linearly with depth from the water surface to the transition zone. In the transition zone, the pressure increase with depth is nonlinear. In the saline ground water, the pressure increase with depth is again linear. A point on the interface is determined by the intersection of projected linear parts of the curve.

Pa is atmospheric pressure.

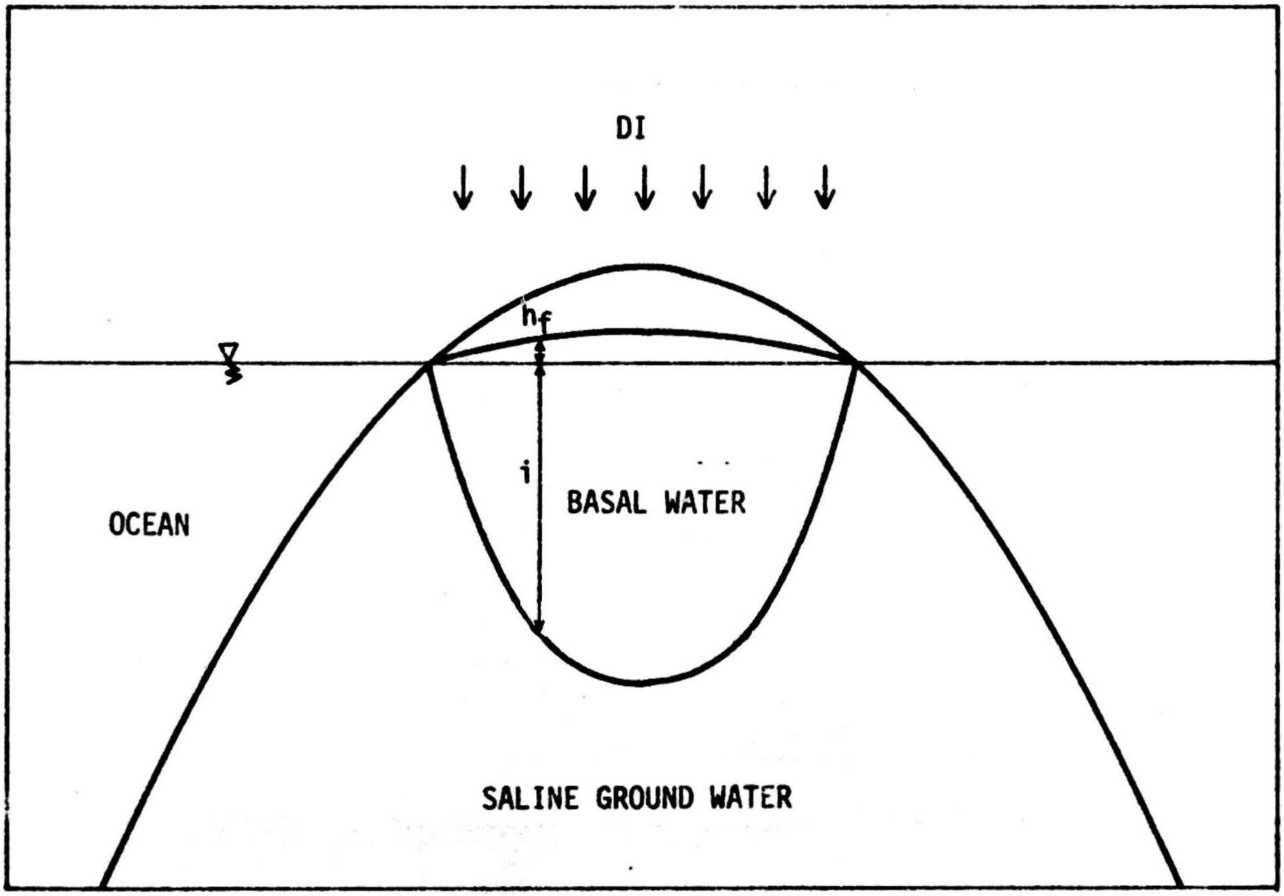

Figure 3. Schematic diagram of the Ghyben-Herzberg lens

Deep infiltration (DI) is uniform over the entire island. The depth of the interface (i) below sea level is 40 times the altitude of the water table (h_f) above sea level.

The theory, on which the lens shape is based, dates from the late 19th century, and is attributed to Badon-Ghyben (1888) and Herzberg (1901), and the lens is known as the Ghyben-Herzberg lens. The underlying theoretical assumptions for the formation of the lens-shaped body are:

(1) uniform ground-water recharge over the island;

(2) uniform permeability of the aquifer;

(3) essentially horizontal flow in the freshwater;

(4) a sharp interface between the basal water and the saline ground water; and

(5) a pressure balance at the interface between dynamic freshwater and static saline ground water.

The lens theory has proved satisfactory in establishing the maximum depth of wells near the interior of the island, but has not proved satisfactory at the ocean shore because the width of the gap through which the freshwater escapes to the ocean is reduced to zero in the Ghyben-Herzberg lens. This, of course, is a physical impossibility and, consequently, the observed conditions depart from the lens concept at the shore.

In the Waialua area, as with other locations in the Hawaiian Islands, coastal-plain deposits of low permeability directly overlie the permeable aquifer (fig. 4). These deposits, locally called caprock, form a leaky confining bed between the ground water underlying the island and the ocean. The caprock restricts the outflow of basal water to the ocean, and also restricts the flow of saltwater between the ocean and the saline ground water.

The term flow field is used in this report to define a region in space for which the ground-water flow is being considered. The freshwater flow field includes the basal-water body plus the region within the caprock that is filled with freshwater. The saline-water flow field includes the saline-water body plus the region within the caprock that is filled with saline water.

For a basal-water body with caprock, the lens shape within the aquifer is not apparent, because the caprock grossly changes the freshwater-head distribution. Most of the head loss in the freshwater flow field is within the caprock; consequently, the basal-water body is wedge or tabular shaped within the aquifer (fig. 4).

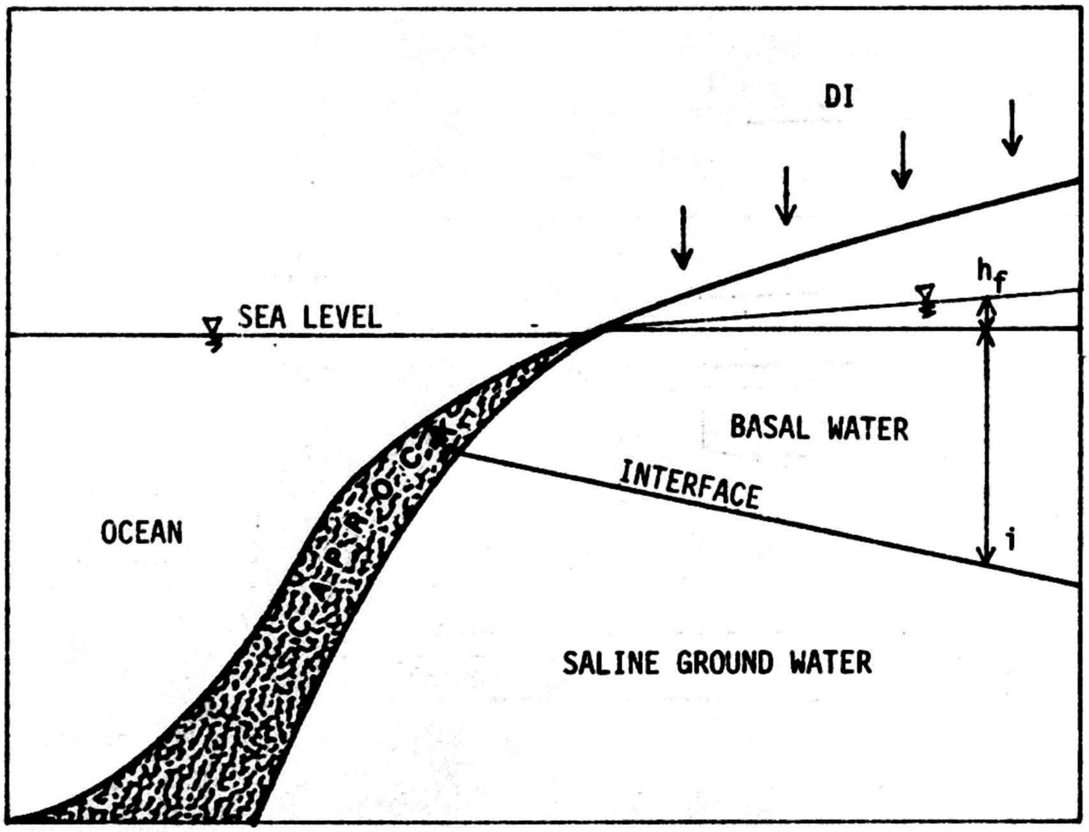

Figure 4. Schematic diagram of a basal aquifer with caprock

Most of the freshwater head is lost as the water moves through the caprock; consequently, the basal-water body is more wedge shaped, rather than lens shaped.

The basal-water body is influenced by both addition and subtraction of flux across the boundaries of the freshwater flow field. Flux is the volume rate of flow across a specified control surface. Inflow is flux across the flow field boundary directed into the flow field. Outflow is flux across the flow field boundary directed out of the flow field. Inflow consists of deep infiltration to the water table, and inflow from adjacent water bodies. Outflow consists of pumpage, and outflow to adjacent water bodies or the ocean.

Purpose and Scope

Effective and efficient management of a basal-water body requires knowledge of (1) basal-water flux; (2) basal-water storage; and (3) the equilibrium position of the interface for a given water-development plan. Basal-water flux and storage for the Waialua area are computed in this report. At this time, it is not possible to compute the interface position, because a basal-water development plan has not been proposed.

The work discussed in this report was begun in July 1974, in cooperation with the Board of Water Supply, City and County of Honolulu. Field work consisted mostly of running levels to measuring points of wells from a common sea-level datum so that detailed water-level contour maps could be constructed. In addition, the entire area was carefully canvassed to locate and measure all the points of natural ground-water discharge.

Several mathematical models were developed in the preparation of this report. Because of a deficiency of basic data, some of the models are based on inferred data. Thus, the degree to which the models reproduce the actual conditions in the Waialua area is unknown, and there may be a considerable departure between the actual and computed conditions. The major data deficiency is the complete lack of information on heads in the saline ground water underlying the basal-water body, making it impossible to measure the movement of the interface.

Well-Numbering System

The well-numbering system is a rectangular system based on latitude and longitude. The island of Oahu is subdivided into rectangles, measuring about a mile on each side, by the minute parallels and meridians. Each rectangle is uniquely located by a four-digit number consisting of minutes of latitude followed by minutes of longitude. Within each rectangle the wells are numbered serially, according to the date drilled. The well number consists of the four-digit number that identifies the rectangle, followed by a hyphen and the two-digit serial number.

Acknowledgments

The writer expresses his appreciation to the Honolulu Board of Water Supply and the Waialua Sugar Co., Inc., for their wholehearted cooperation and assistance.

GEOHYDROLOGIC FEATURES

This report is primarily a quantitative analysis of the basal-water flux and storage for the Waialua basal-water body. The geohydrologic features described in this report are the principal features that pertain exclusively to, and are necessary for the analysis. For an overview of the geohydrology of Oahu, the reader is referred to Stearns and Vaksvik (1935), Stearns (1966), Macdonald and Abbott (1970), and Rosenau, Lubke, and Nakahara (1971).

Geologic Features

The island of Oahu was built principally by the eruptions of two shield volcanoes. The Waianae volcano built the western part of the island, and the Koolau volcano built the eastern part. The contact between the two volcanoes forms the western boundary of the Waialua basal-water body (fig. 5).

For this report, the volcanic rocks are classified as either low-permeability rocks or high-permeability rocks. The low-permeability rocks consist of rocks associated with the caldera complex and dike-intruded lava flows. The high-permeability rocks consist of dike-free, thin-bedded basaltic lava flows. The low-permeability rocks are associated with the Waianae dike-impounded water body; the Koolau dike-impounded water body; and the northern ground-water dam (fig. 5). The high-permeability rocks are associated with the basal-water bodies and the Schofield high-level water body.

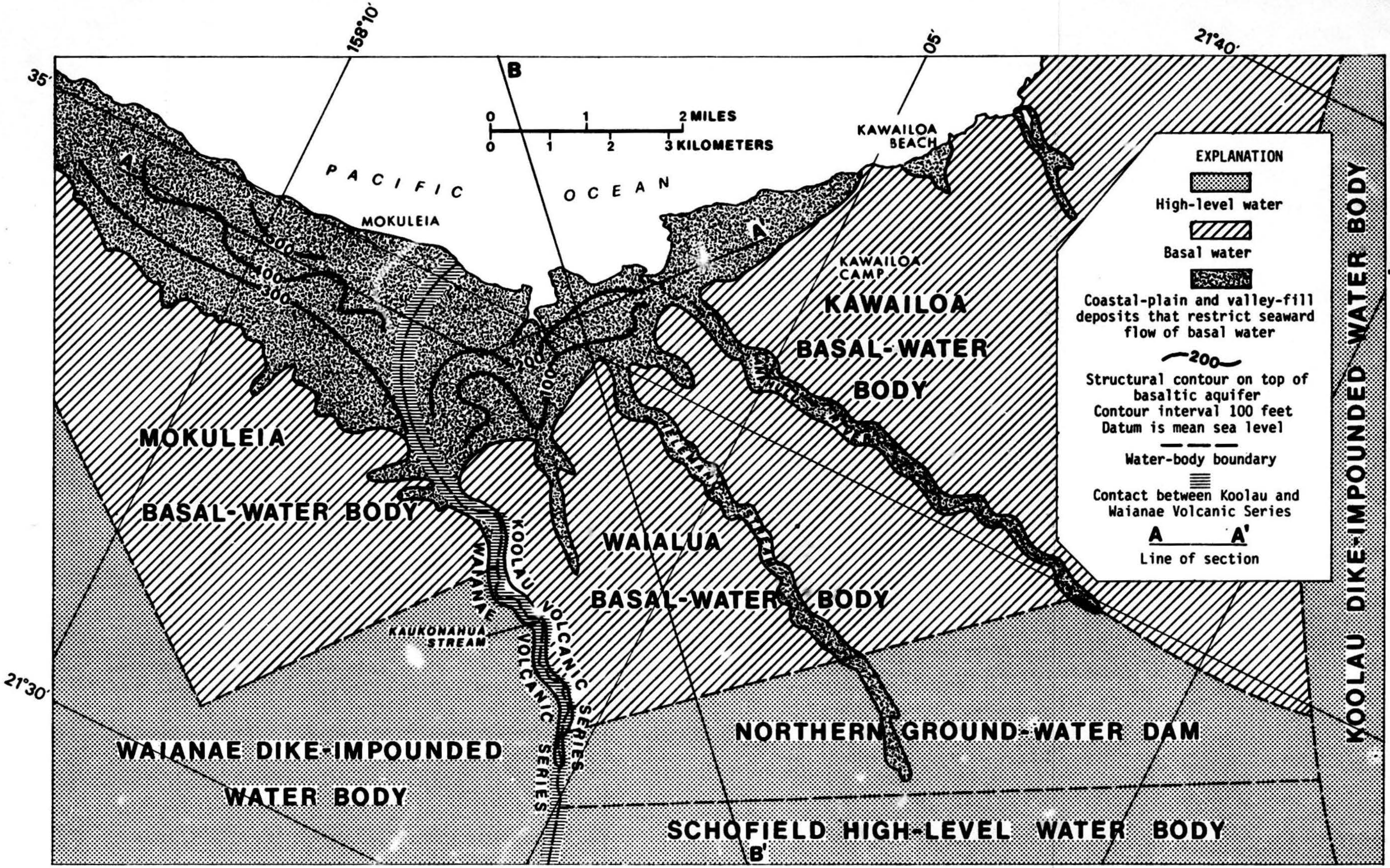

FIGURE 5. GEOHYDROLOGIC MAP OF NORTH-CENTRAL OAHU

The Waialua basal-water body is contained in the high-permeability basaltic lava flows of the Koolau Volcanic Series (Pliocene (?) and Pleistocene). These rocks, together with the high-permeability rocks of the Waianae Volcanic Series (Pliocene and Pleistocene (?)) are termed the basaltic aquifer. The basaltic aquifer consists of thin-bedded flows that contain numerous interflow clinker beds, which efficiently transmit the basal water.

The Waialua basal-water body is completely surrounded by low-permeability rocks. The low-permeability rock limits the inflow to the basal-water body and restricts the outflow to the ocean. The surrounding low-permeability rocks form four distinct boundaries for the basal-water body. The contact between the Koolau and Waianae volcanoes, and valley-fill deposits of the ancient Anahulu River, which are a part of the caprock, form two parallel boundaries that restrict flow in a direction parallel to the coast (figs. 6 and 7). The northern ground-water dam, named with respect to the Schofield high-level water body, restricts inflow from the Schofield high-level water body, and the caprock restricts the outflow to the ocean (figs.5 and 9). With the exception of the caprock, all of the boundaries can be considered as vertical boundaries. Structural contours on the top of the basaltic aquifer (fig. 5) indicate that the caprock-aquifer boundary dips at about 5 degrees.

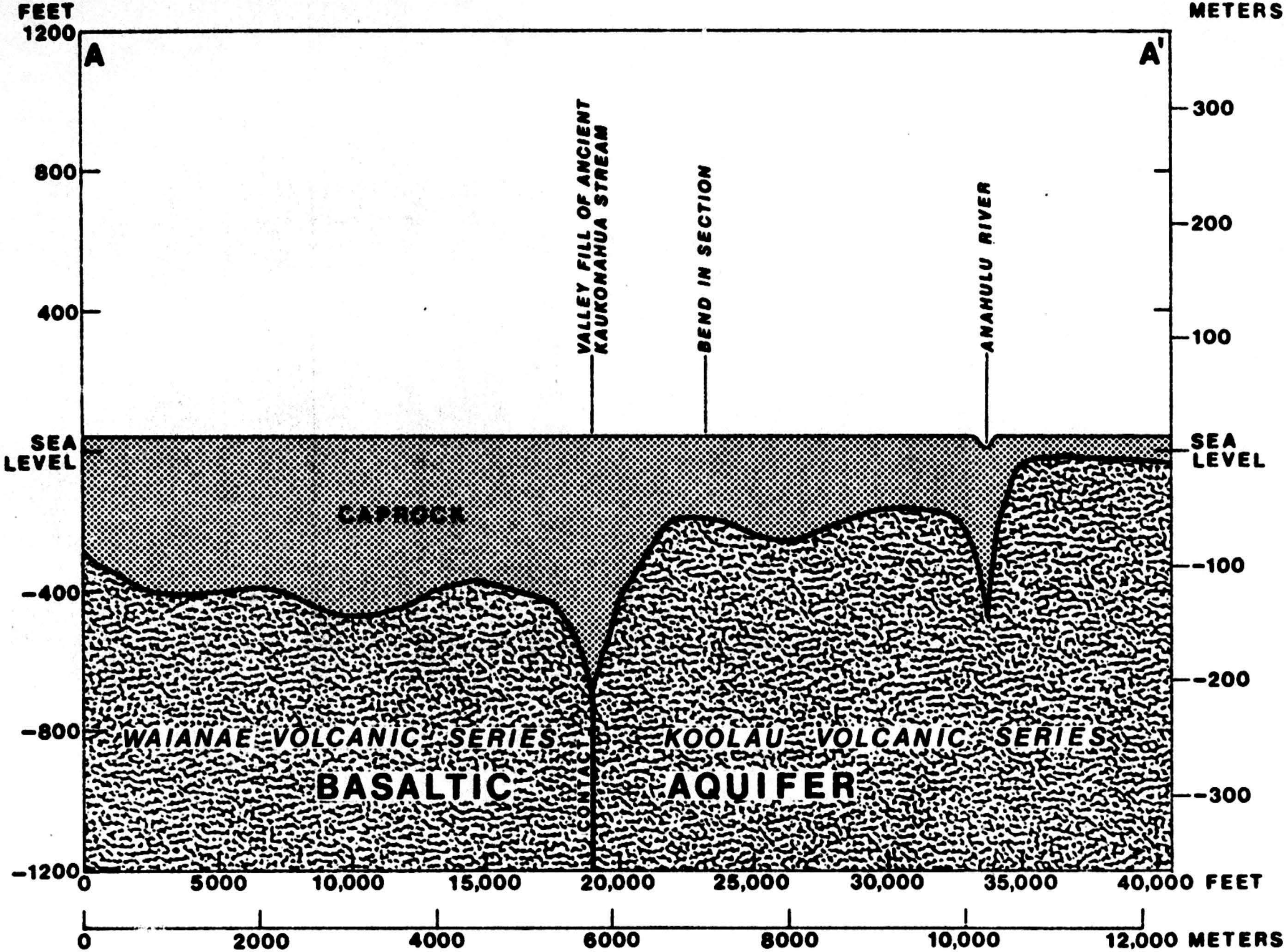

Figure 6. Geologic section A-A'

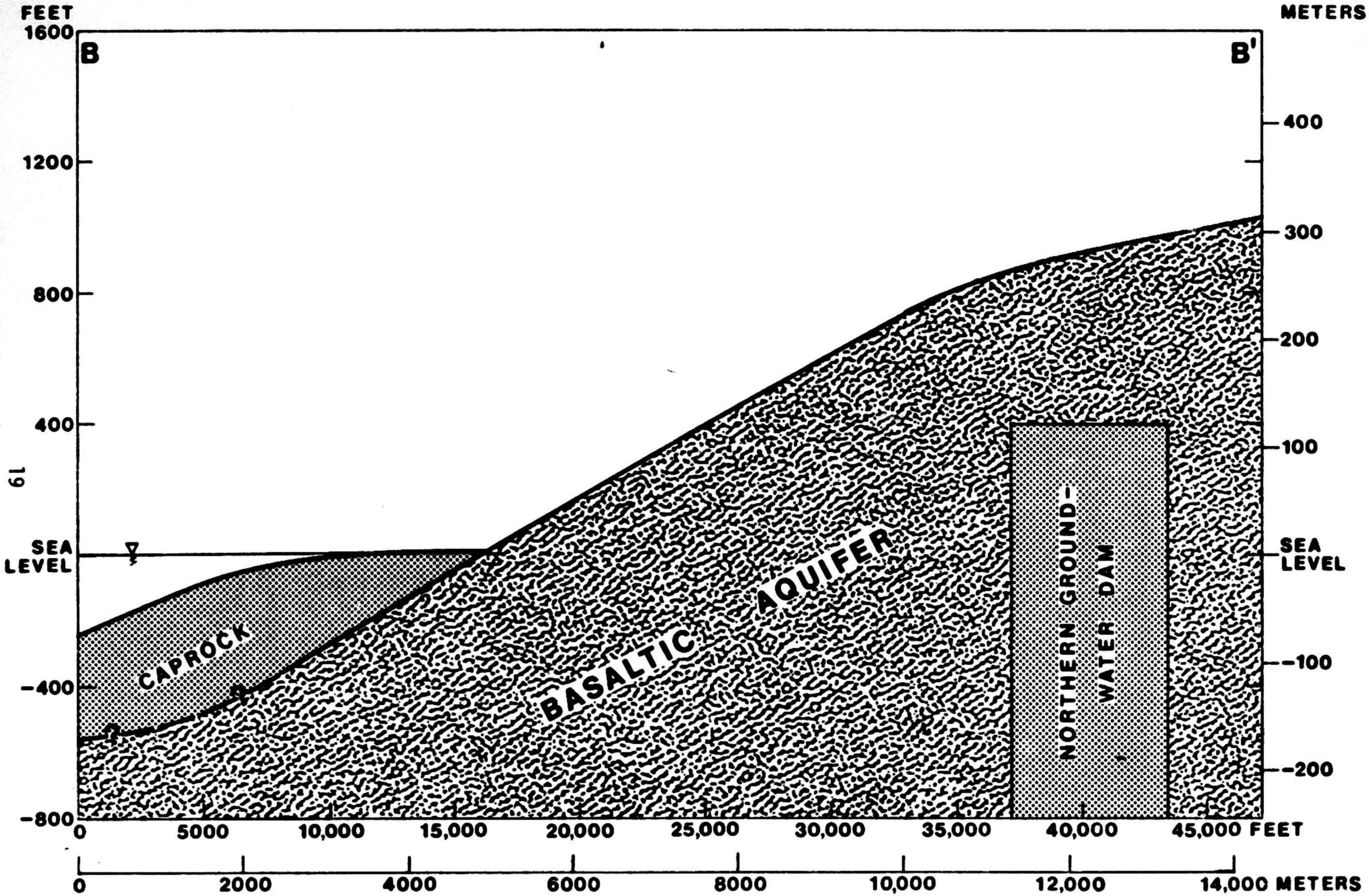

Figure 7. Geologic section B'-B'

Hydrologic Features

The Waialua basal-water body is a body of fresh ground water in the high-permeability basaltic aquifer. Lateral flow is restricted by the surrounding low-permeability deposits. The boundaries of the water body are shown in section view in figures 8 and 9, as well as the relationship of the Waialua basal-water body with adjacent water bodies. The areal basal-water head for the Waialua basal-water body is shown on figure 10.

The limits of the flow field under consideration for the Waialua basal-water body are mapped on figures 8 and 9. These boundaries are placed at either the water table, the interface, or the contact between the basaltic aquifer and the low-permeability rocks.

Of the low-permeability boundaries that surround the Waialua basal-water body, the northern ground-water dam is considered as an inflow boundary; the caprock and deep valley fill of the Anahulu River are considered as outflow boundaries; and the contact between the Waianae and Koolau volcanoes is considered as a no-flow boundary.

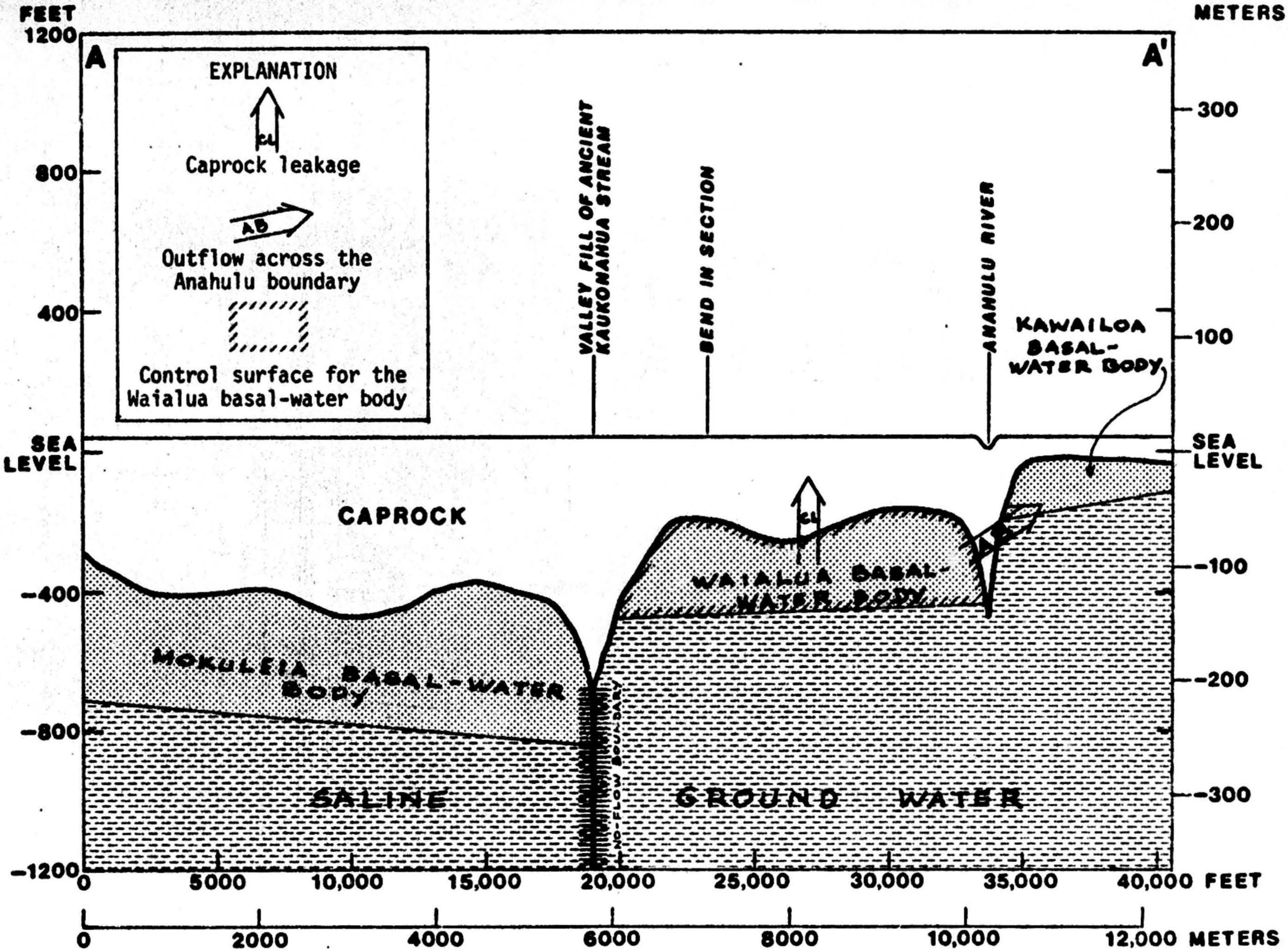

Figure 8. Hydrologic section A-A'

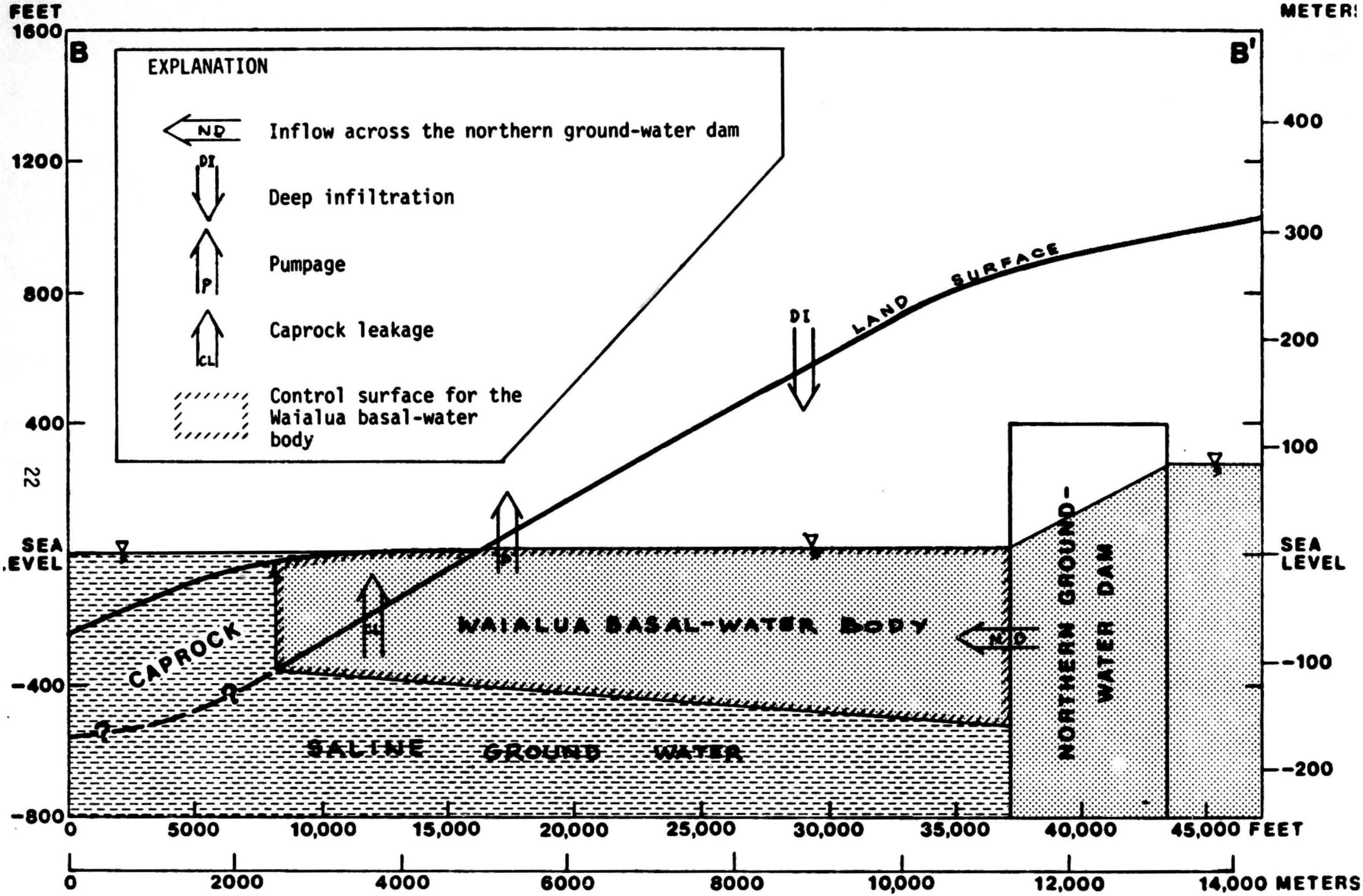

Figure 9. Hydrologic section B-B'

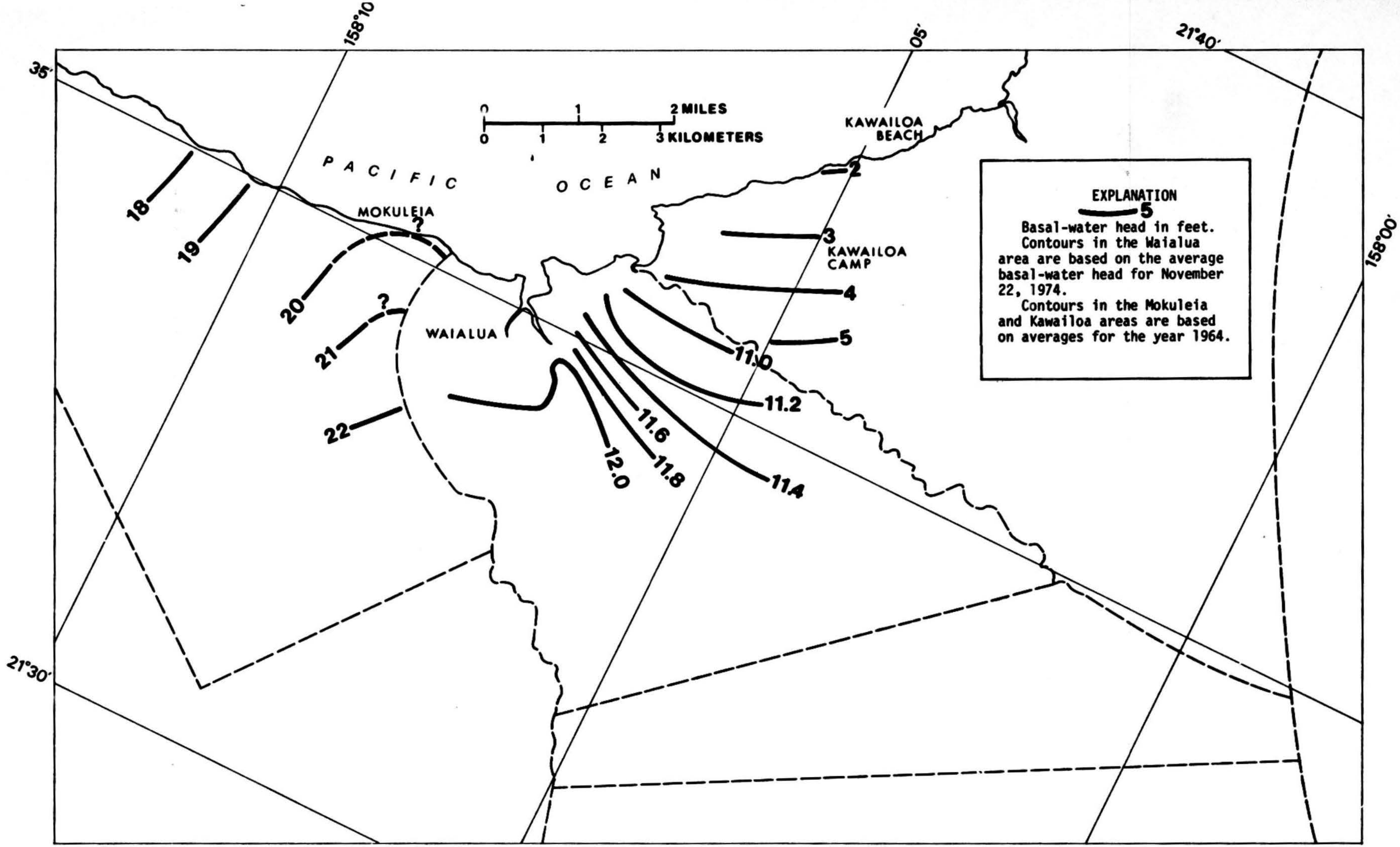

FIGURE 10. BASAL-WATER-HEAD MAP OF NORTH-CENTRAL OAHU

The components of inflow to and outflow from the Waialua basal-water body are shown schematically on figures 8 and 9, using nomenclature that follows. Inflow to the Waialua basal-water body is from two sources: (1) inflow from the Schofield water body, through the northern ground-water dam (ND); and (2) inflow from deep infiltration of rainfall and applied irrigation water (DI).

Outflow from the Waialua basal-water body is considered as: (1) pumpage (P); (2) caprock leakage (CL); and (3) lateral flow through the valley-fill deposits of the ancient Anahulu River to the Kawailoa basal-water body (AB).

Basal-water storage is computed as the volume that is saturated with basal water times the effective porosity. With the exception of the caprock-aquifer boundary, the volume is tabular with nearly vertical sides (figs. 8 and 9). Changes in storage are a result of nearly vertical movement of either the water table or the interface. The water table has an area of about 16.1 mi^2 (42 km^2), and the interface has an area of about 18 mi^2 (46 km^2).

BASAL-WATER FLUX

Throughout the past 50 years, the basal-water body has been under almost steady-state conditions. For steady-state conditions, the mass-balance equation is

$$I - O = 0 \tag{1}$$

where I = inflow

O = outflow.

Equation 1 may be expanded in terms of the previously described components of inflow and outflow to yield

$$\overline{DI} + \overline{ND} - \overline{P} - \overline{CL} - \overline{AB} = 0 \tag{2}$$

where

DI = deep infiltration,

ND = flux across the northern ground-water dam,

P = pumpage,

CL = caprock leakage,

AB = flux across the Anahulu boundary, and

the bars above the term indicate long-term mean values.

The basal-water flux may be computed as either the sum of the inflow terms ($\overline{DI} + \overline{ND}$) or the sum of the outflow terms ($\overline{P} + \overline{CL} + \overline{AB}$). For this report, both sums are computed.

The Inflow Terms

The inflow terms consist of deep infiltration directly on the basal-water area and inflow from the Schofield high-level water body. The deep infiltration is primarily from irrigation water that has been applied in excess of plant requirements. The annual rainfall generally does not exceed the annual evaporation; therefore, it might be inferred that the deep infiltration of rainfall on nonirrigated land is trivial. Contributions from this source are not considered in this analysis for that reason, and may well be the source of the imbalance of calculated inflow and outflow discussed later on page 43.

The inflow from the Schofield high-level water body has been computed in an earlier report (Dale and Takasaki, 1976).

Deep Infiltration

Deep infiltration is related primarily to application of irrigation water in excess of the evapotranspiration rate of sugarcane. Deep infiltration is computed as agricultural water supply minus potential evapotranspiration. The water supply for agriculture consists of (1) rainfall directly on the sugarcane; (2) surface water, diverted by a system of small dams and ditches; and (3) pumped basal water.

Prior to 1974, the Waialua Agricultural Co. had a little less than 15.8 mi^2 (41 km^2) of sugarcane under irrigation (fig. 11). After 1974 the area was increased, but this report considers pre-1974 conditions only. The irrigated area overlies the three basal-water bodies of north-central Oahu, and there is no accounting of the allocation of the water supply for each separate basal area. Therefore, deep infiltration is computed for the total irrigated area, and the deep infiltration to the Waialua basal-water body is determined on a prorated basis.

The agricultural water supply consists of rainfall on the irrigated area, gross surface-water diversions, and basal-water pumpage. Rainfall on the irrigated area (fig. 12) ranges from 30 to 50 inches per year (760 to 1270 mm/yr)(fig. 12), and averages 35 inches per year (890 mm/yr). This average rainfall amounts to 26 Mgal/d (98×10^3 m^3/d) on the irrigated area. The gross surface-water diversions are estimated at 57 Mgal/d (216×10^3 m^3/d), which is all of the water that was diverted into reservoirs for the period 1960-64 (Rosenau and others, 1971). The losses that occur in transporting the water from the reservoirs to the fields have not been subtracted, because these losses represent gains to the basal-water bodies. The total pumpage from the three basal-water bodies averaged 43 Mgal/d (163×10^3 m^3/d) for the years 1951-70. The agricultural water supply, which is the sum of rainfall, surface water, and ground water, averages 126 Mgal/d (477×10^3 m^3/d) (table 1).

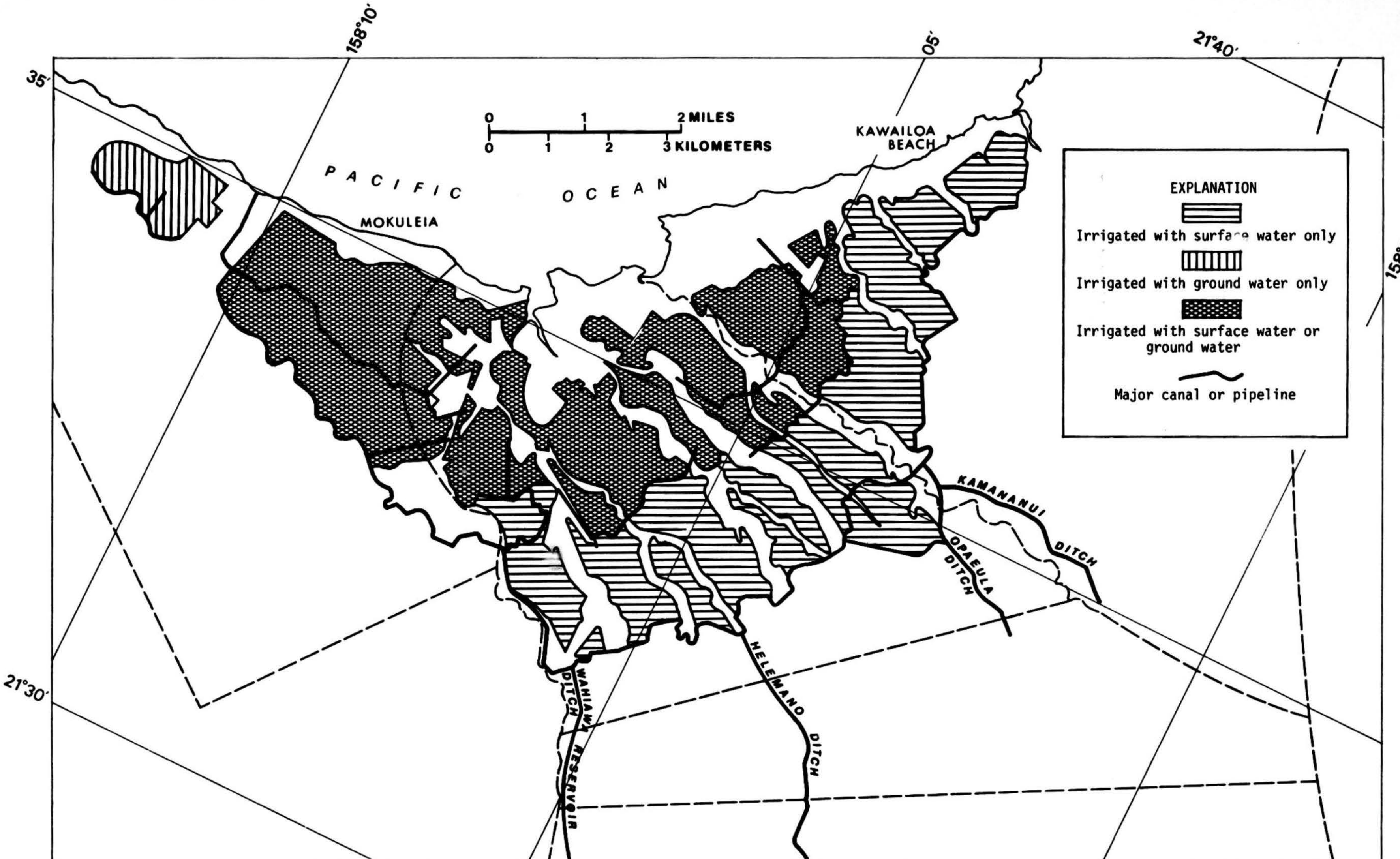

FIGURE 11. IRRIGATED-AREA MAP OF NORTH-CENTRAL OAHU

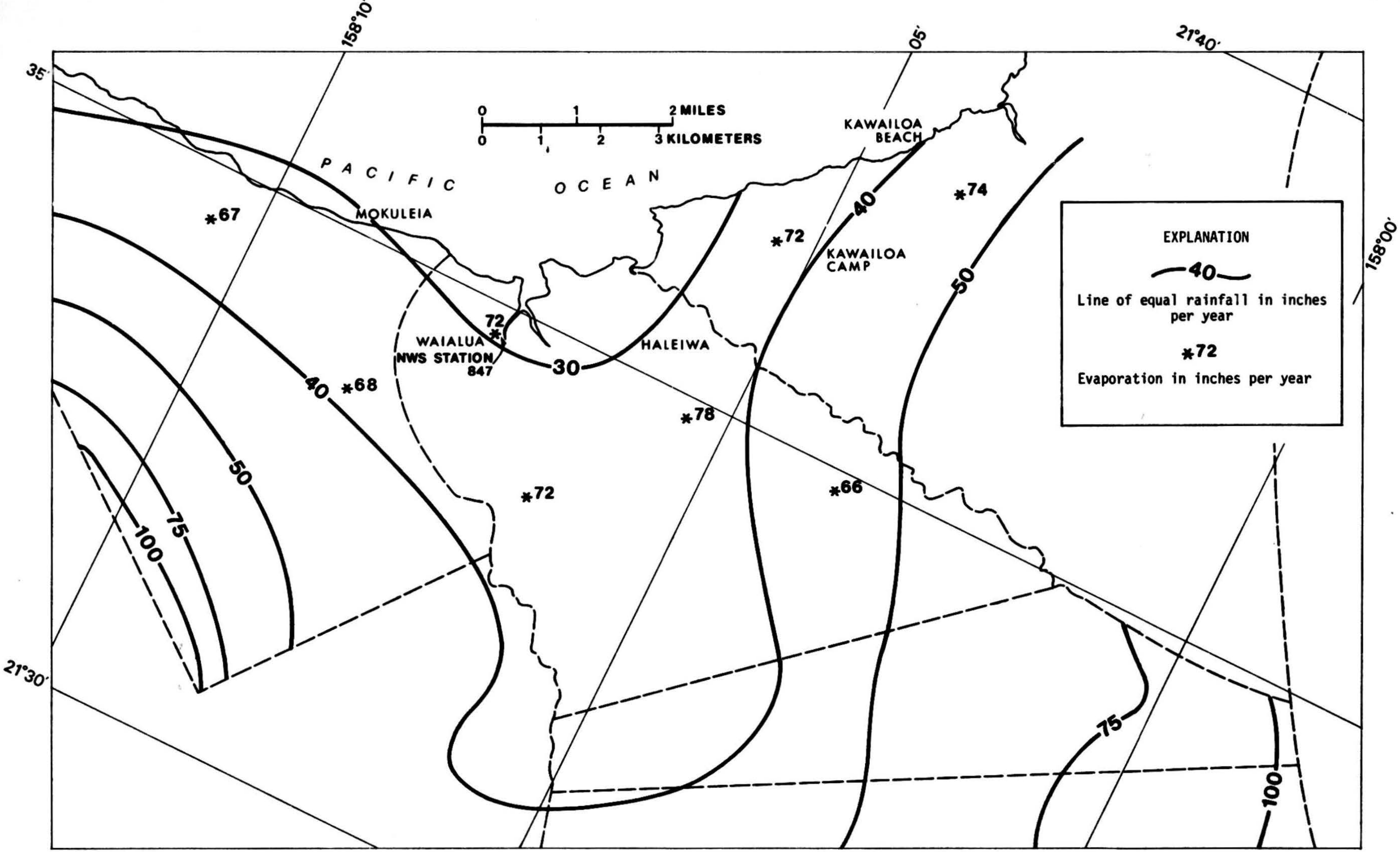

FIGURE 12. CLIMATOLOGIC MAP OF NORTH-CENTRAL OAHU

Evapotranspiration from the irrigated area is based on the relationship between transpiration of sugarcane and evaporation from a class A pan. Ekern (1970) has shown that the transpiration by sugarcane is essentially equal to pan evaporation after the sugarcane is 4 months old. During the first 3 months, the transpiration to pan-evaporation ratio ranges between 0.4 and 0.8. The growing period, which is 20 months, is long compared to the 3 months during which the ratio is less than one. To simplify the calculations, it was assumed average ratio of transpiration to pan evaporation equaled (.4 + .6 + .8 + 17 (1.))/20 or 0.94 over the entire period. The average evaporation rate is 6 feet per year (1.83 m/yr), based on the 8 evaporation stations within the area (fig. 12). Thus, the 15.8 mi^2 ($41\ km^2$) of sugarcane transpires 18,600 Mgal/yr ($70 \times 10^6 m^3/yr$) 51 Mgal/d ($192 \times 10^3\ m^3/d$).

Deep infiltration is equal to the agricultural water supply minus evapotranspiration. For the entire irrigated area, deep infiltration averages 75 Mgal/d ($284 \times 10^3\ m^3/d$). The deep infiltration to the Waialua basal-water body averages 35 Mgal/d ($132 \times 10^3\ m^3/d$), based on a prorating of the irrigated area overlying the Waialua basal-water body to the total irrigated area (table 1).

Table 1. Agricultural water use, in million gallons per day, for the total irrigated area

Agricultural water supply		Agricultural water disposal	
Rainfall	26	Evapotranspiration	51
Surface water	57	Deep infiltration	1/ 75
Ground water	43		
Total	126	Total	126

1/ The Waialua basal-water body underlies 47 percent of the total irrigated area. By prorating according to this percentage, deep infiltration to the Waialua basal-water body is estimated as 35 Mgal/d ($132 \times 10^3 m^3/d$).

Inflow Across the Northern Ground-Water Dam

Inflow across the northern ground-water dam has been estimated at 18 Mgal/d (68 x 10^3 m^3/d) (Dale and Takasaki, 1976). This estimate is a residual quantity based on an analysis of the flux through the Schofield water body, and the estimate is subject to substantial error. This estimate is the poorest of any of the terms in equation 2, and any lack of balance between inflow flux and outflow flux should be attributed to this estimate.

Sum of the Inflow Terms

The basal-water flux computed as the sum of the inflow terms is 53 Mgal/d (201 x 10^3 m^3/d) (table 3). The prorated deep infiltration has been computed as 35 Mgal/d (132 x 10^3 m^3/d), and the inflow across the northern ground-water dam has been estimated as 18 Mgal/d (68 x 10^3 m^3/d). The value for deep infiltration is reasonably close to values computed elsewhere in the Hawaiian Islands. However, there is no available check for the inflow across the northern ground-water dam.

The Outflow Terms

The outflow terms consist of basal-water pumpage, caprock leakage, and outflow across the Anahulu boundary.

Pumpage

The mean annual pumpage for the period 1928-73 was 13,100 Mgal/yr ($50 \times 10^6 m^3$/yr), which is equivalent to a mean daily rate of 36 Mgal/d (136×10^3 m^3/d). Annual pumpage ranged between 12,400 Mgal/yr ($47 \times 10^6 m^3$/yr) and 13,700 Mgal/yr ($52 \times 10^6 m^3$/yr), the range being 10 percent of the mean.

The location of the pumping plants and the installed pumping capacity are shown on figure 13. With the exception of the skimming shaft (3404-02), each pumping plant consists of a series of wells that are connected to a common header pipe. One or more pumps pump from the common header pipe. To simplify the well numbering of the system of wells, the pumping plant is labeled with the well number that corresponds to the well with the lowest digit suffix.

Caprock Leakage

Caprock leakage occurs in the low-lying areas where the water table intersects the land surface. In the Waialua and Kawailoa areas, the caprock leakage is of the form of diffuse seeps, discrete springs, or perennial flow in the lower reaches of the streams.

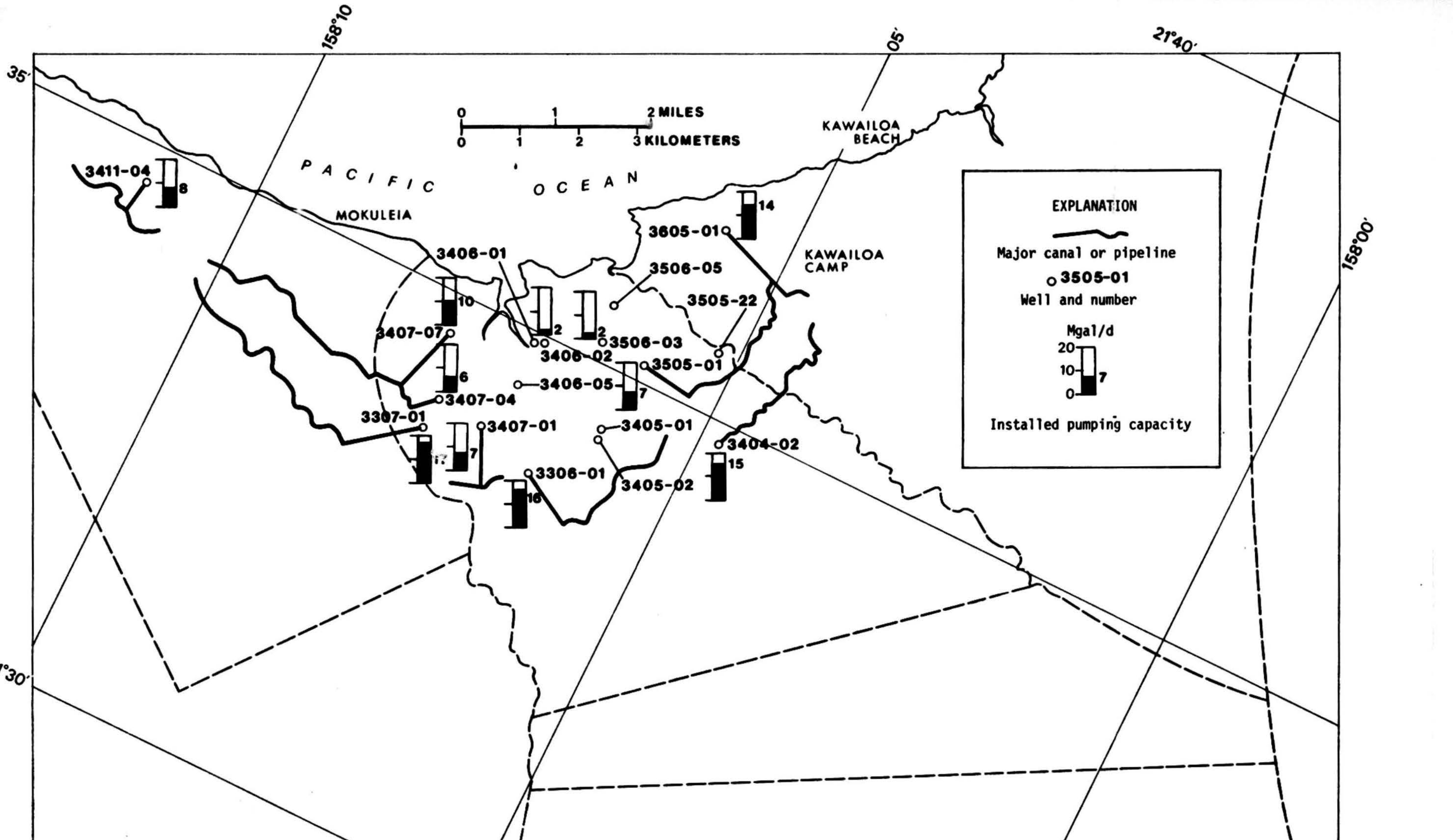

FIGURE 13. MAP OF NORTH-CENTRAL OAHU SHOWING INSTALLED PUMPING CAPACITY

In north-central Oahu, there are five marsh areas that are sustained by the seeps and springs. These marshes are numbered from north to south (fig. 14), with marsh 1 being in the Kawailoa area, and marshes 2 through 5 being in the Waialua area. With the exception of the large spring at Kawailoa beach that discharges into the ocean, all of the discrete spring flow is tributary to the marsh areas; therefore, the caprock leakage is cataloged according to the marsh areas.

The caprock leakage to the marshes was computed as being equal to the evaporation from an equivalent open body of water plus all the channel flow from the marsh to the ocean. Seepage into streambeds, which is a small component of the coastal-plain leakage, was measured directly. The summation of the caprock leakage for the Waialua area is 9.71 Mgal/d (36.8 x 10^3 m^3/d). For future computational purposes, this figure is rounded to 10 Mgal/d (38 x 10^3 m^3/d) (table 2).

Outflow Across the Anahulu Boundary

The deep valley fill of the ancient Anahulu River forms a leaky boundary that impedes outflow from the Waialua basal-water body to the Kawailoa basal-water body. Data is not available to compute the flux in the low-permeability deposits; therefore, a vertical control surface for the computation is placed just upgradient from the valley fill. For this location of the control surface, the volumetric flux includes the outflow to marsh 2; channel seepage to the Anahulu River (fig. 14); and the outflow across the Anahulu boundary.

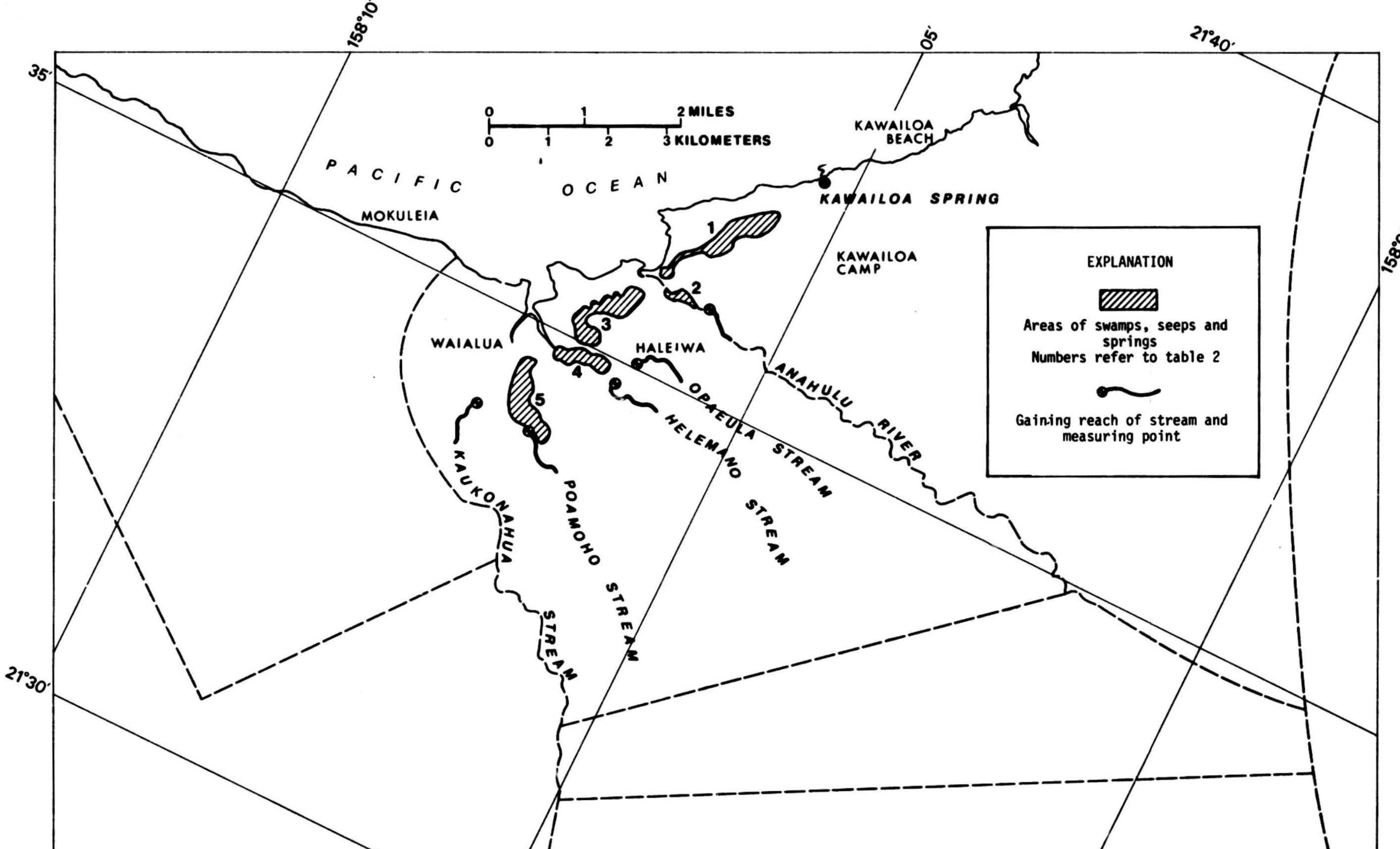

FIGURE 14. MAP OF NORTH-CENTRAL OAHU SHOWING AREAS OF CAPROCK LEAKAGE

Table 2. Caprock leakage

Location of leakage as shown in figure 14	Discharge rate (Mgal/d) Waialua area	Kawailoa area
Marshes and springs:		
1	---	2.13
2	2.56	---
3	3.12	---
4	2.01	---
5	1.32	---
Kawailoa Spring	---	0.22
Channel seepage		
Anahulu River	0.10	---
Opaeula Stream	0.04	---
Helemano Stream	0.04	---
Poamoho Stream	0.04	---
Kaukonahua Stream	0.48	---
Total	9.71	2.35

The volumetric flux across the control surface is given by

$$Q = \iint_A q_f \, dA , \tag{3}$$

where

Q = volumetric flux across the boundary ,

A = the area of the control surface , and

q_f = the flux computed normal to A.

Equation 3 can be expressed as

$$Q = - 7.48 \times K \iint_A \left(\frac{\partial h_f}{\partial n}\right) dA , \tag{4}$$

where

K = hydraulic conductivity in feet per day, and

$\frac{\partial h_f}{\partial n}$ = the component of hydraulic gradient measured normal to the control surface.

The flux across the control surface is computed directly from equation 4. The application of the equation requires numerical estimates for hydraulic conductivity, the hydraulic gradient, and a description of the control surface.

Hydraulic conductivity

For steady radial flow, transmissivity is expressed as

$$T = -\frac{2.30Q\log_{10}(r_2/r_1)}{2\pi(s_1-s_2)}, \qquad (5)$$

where

T = transmissivity in feet^2 per day,

Q = pumping rate in feet^3 per day,

r_1 = radial distance to observation point 1,

r_2 = radial distance to observation point 2,

s_1 = drawdown at point 1, and

s_2 = drawdown at point 2.

For steady-state conditions, the interface can be considered as a stationary, impermeable boundary located at an altitude of $-40h_f$ with respect to sea level. Thus, the effective thickness of the aquifer is $-41h_f$, even though the permeable basalt extends to a greater depth. For this particular condition, the hydraulic conductivity may be expressed as

$$K = \frac{T}{41h_f} \;. \qquad (6)$$

It should be emphasized that the value of transmissivity is not for the entire depth of the aquifer but that it is limited to the thickness of the basal-water flow field.

For steady-state conditions, and recognizing that transmissivity is restricted to depth of the freshwater flow field rather than the depth of the aquifer, hydraulic conductivity may be expressed as

$$K = -\frac{2.30\ Q \log_{10}(r_2/r_1)}{41\ h_f\ 2\pi\ (s_1-s_2)} \quad . \qquad (7)$$

The computation of the hydraulic conductivity of the basaltic aquifer is based on equation 7 and data collected during the last 2 months of 1974 (fig. 15).

On November 18, all the irrigation pumps in the Waialua area were turned off because of heavy rains during the preceding week. All the irrigation wells remained off until December 2. On December 2, shaft 3404-02 started pumping 6.95 Mgal/d (26 x 10^3 m^3/d) and pumped continuously at this rate until December 12.

Steady-state drawdown was reached by December 7, 1974 (fig. 10). This may be difficult to see from figure 15 because of the water-level oscillations that were caused by the variation in mean daily ocean level. The variations in ocean level cause all of the water levels to rise or fall each day and produce fluctuations that could be confused with the effects of pumpage.

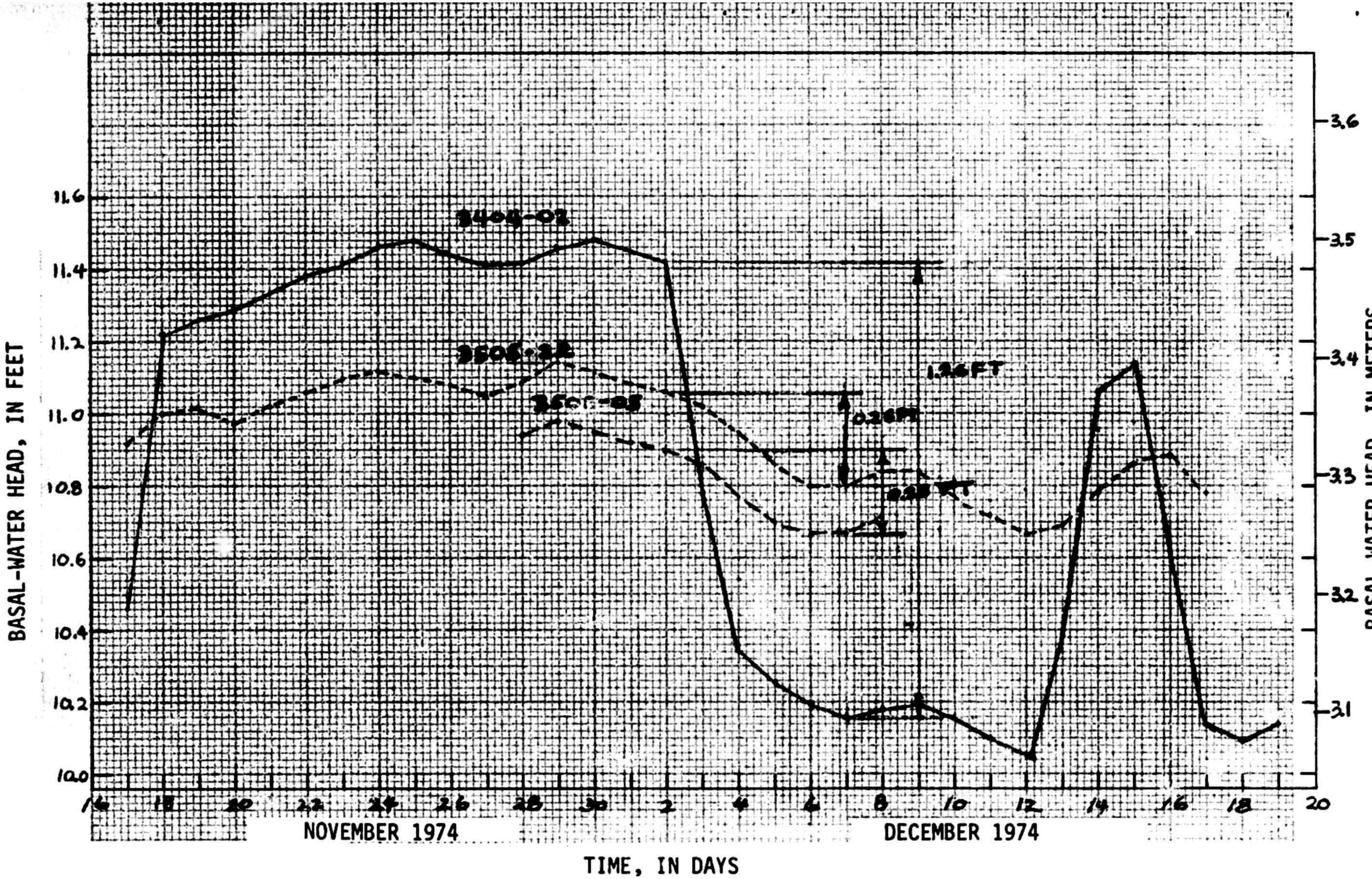

Figure 15. Hydrographs for shaft 3404-02 and wells 3505-22 and 3406-05

Because of the effect of the oscillations of the ocean level, there is some uncertainty as to the accuracy of the drawdown at distant wells. Therefore, point 1 is measured at the shaft and point 2 is the average of wells 3505-22 and 3506-05 (fig. 16). There is a complication of a horizontal shaft for the observation point, because it is difficult to determine an effective well radius. The shaft is about 6 feet x 6 feet (1.8 m x 1.8 m) in cross section and is about 600 feet (180 m) long. For this calculation, it is assumed that the shaft is equivalent to a 10-foot (3 m) radius well that fully penetrates the freshwater flow field.

Based on the above simplications and assumptions, the hydraulic conductivity is calculated to be 1600 ft/d (490 m/d) (fig. 16).

Hydraulic gradient

The hydraulic gradient measured normal to the control surface is 1/14,000. This value was measured from the basal-water head map (fig. 10).

The control surface

The control surface for the computation is placed just upgradient from the Anahulu boundary. It extends 30,000 feet (9,000 m) in width, from the shoreline to the ground-water dam. It extends 450 feet (140 m) in depth, from the water table to the interface. Thus, the area of the control surface is $14x10^6 ft^2$ ($1.3x10^6$ m^2).

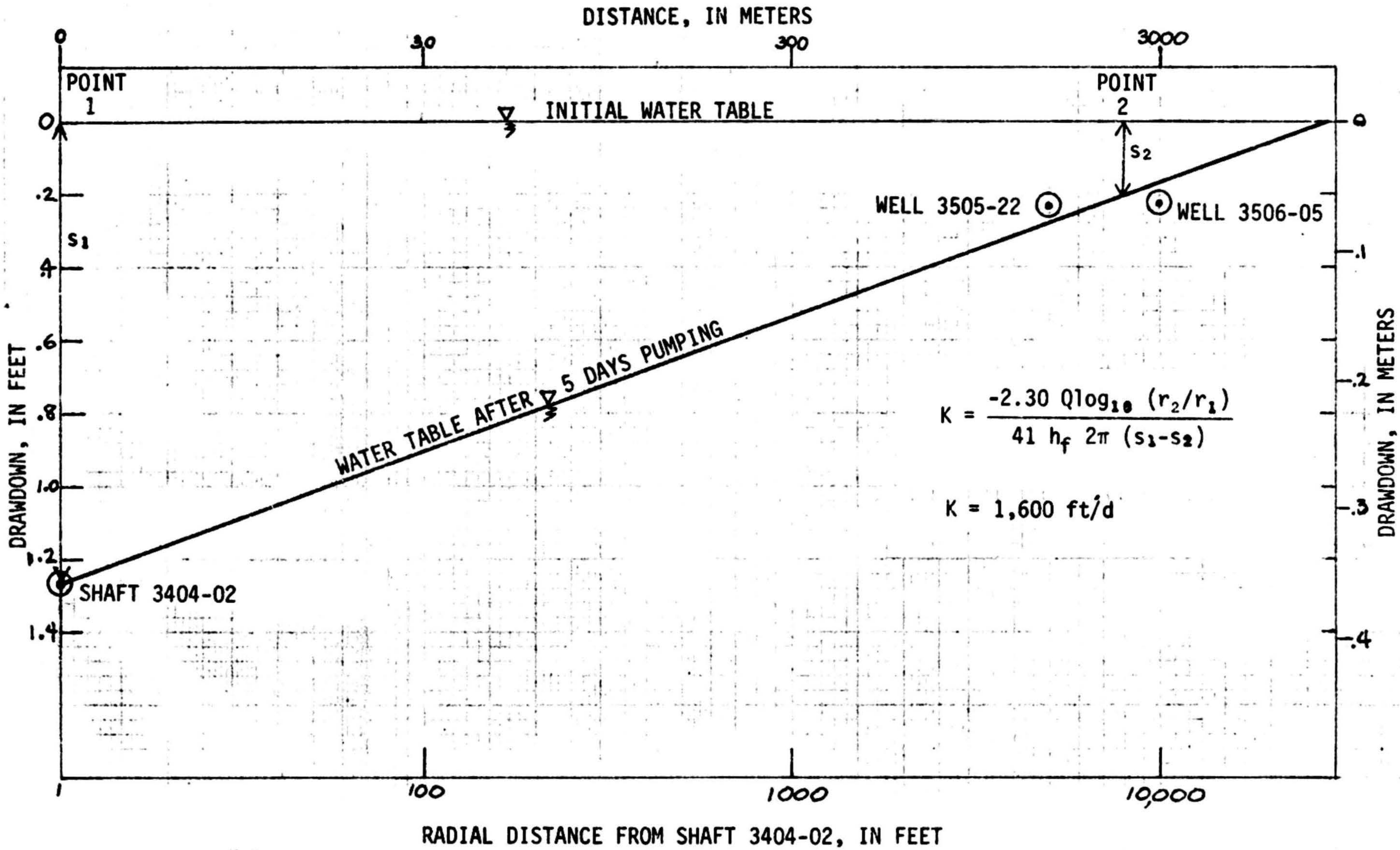

Figure 16. Steady-state drawdown cone for shaft 3404-02. Pumping rate 6.95 Mgal/d ($26 \times 10^3\ m^3/d$).

Flux

The flux through the control surface located upstream from the Anahulu boundary is 12 Mgal/d (45 x 10^3 m^3/d), based on equation 4 and the estimates of hydraulic conductivity, hydraulic gradient, and cross-sectional area. The outflow across the Anahulu boundary is the computed amount of flux across the control surface minus caprock leakage from marsh 2 and seepage into the Anahulu River. Thus, the outflow across the Anahulu boundary is 9 Mgal/d (34 x 10^3 m^3/d).

Sum of the Outflow Terms

The basal-water flux computed as the sum of the outflow terms is 55 Mgal/d (208 x 10^3 m^3/d) (table 3). Both the pumpage and caprock leakage are measured quantities, and, therefore, they are considerably more accurate than all other terms of the flux equation.

Comparison of Inflow and Outflow Terms

The sum of the inflow terms is equal to 53 Mgal/d (201 x 10^3 m^3/d), and the sum of the outflow terms is 55 Mgal/d (208 x 10^3 m^3/d) (table 3). The difference between the two sums is relatively small. The difference does not indicate that water is being removed from storage on a long-term basis. Rather, the difference is the result of errors in the estimate. As noted earlier, the assumption of no deep infiltration in nonirrigated areas may be responsible for a portion of this difference.

Table 3. Terms of the mass-balance equation

Terms	Rate (Mgal/d)
Inflow	
Deep infiltration	35
Inflow across the northern ground-water dam	18
Inflow flux	53
Outflow	
Pumpage	36
Caprock leakage	10
Outflow across the Anahulu boundary	9
Outflow flux	55

The outflow terms are much more accurate than the inflow terms. Pumpage is a measured quantity; caprock leakage is in part measured directly and in part estimated; and outflow across the Anahulu boundary is calculated in a direct manner. In contrast, both inflow terms have been estimated as residual quantities that remain after other elements of the hydrologic budget have been satisfied.

In view of the inaccuracies in computing the inflow terms, it seems reasonable to assume that the basal-water flux is 55 Mgal/d (208×10^6 m^3/d), the sum of the outflow terms.

BASAL-WATER STORAGE

The Concept of Basal-Water Storage

The volume of basal water stored changes in response to a movement of the water table, a movement of the interface between saltwater and freshwater, or a simultaneous movement of both the water table and the interface. A storage change that results from movement of the water table is defined as a change in top storage (Wentworth, 1942), because the storage change takes place at the top of the basal-water body. Similarly, a storage change that results from movement of the interface is defined as a change in bottom storage, because the storage change takes place at the bottom of the basal-water body.

Both the water table and the interface move in response to changes in recharge to or discharge from the basal-water body. The water table moves by either saturating or draining void spaces filled with air, whereas the interface moves as a result of the displacement of saline ground water. The water-table movement can be rapid, because there is little resistance to the saturation or drainage of the voids. However, the displacement of the interface is slow, because, in order to be displaced, the saline ground water must move through several miles of the basaltic aquifer (fig. 9) under weak hydraulic gradients.

In an analysis of basal-water storage, it is necessary to separate the effects of changes in top storage from those of bottom storage. Immediately following a change in ground-water withdrawal rates, the change in top storage is dominant, because of the ease with which the voids can be drained or saturated. However, over the long term the change in bottom storage becomes dominant, because it must be 40 times the top-storage change as the system moves completely from one steady-state configuration to another.

In the Waialua area, direct measurements of interface movement have not been made, because this would require deep wells to monitor the saline ground-water head. A few deep wells have been drilled elsewhere on Oahu, but none have been drilled through the Waialua basal-water body.

In the mathematical analysis, calculations are based on the inferred movement of the interace, rather than on measured movement. It is clearly based on data elsewhere on Oahu, that shortly after a pumping-rate change, the associated storage change is by water-table movement or from top storage. Over a long period following a pumping-rate change, the associated storage change is principally by interface movement or from bottom storage. The problem is to deduce, "What is a short period of time, and what is a long period of time?"

The Mathematics of Basal-Water Storage

Head is defined at a point in the flow field as

$$h = \frac{p}{\gamma} + \frac{v^2}{2g} + z, \tag{8}$$

where

h = head,

p = pressure,

γ = specific weight,

v = velocity,

g = acceleration of gravity, and

z = elevation of the point with respect to an arbitrary datum.

The term $v^2/2g$ is eliminated from ground-water problems, because ground-water velocities are so small that the value for this term is trivial. Therefore, head is indicated by

$$h = \frac{p}{\gamma} + z. \tag{9}$$

By separating the ground water into two flow fields, one fresh with the specific weight of γ_f and the other saline with a specific weight of γ_s, it is possible to express the fluid flow for each flow field separately and independently. The driving force for the freshwater flow field is

$$h_f = \frac{p}{\gamma_f} + z, \tag{10}$$

where the subscript f indicates fresh basal water. The driving force for the saline-water flow field is

$$h_s = \frac{p}{\gamma_s} + z, \tag{11}$$

where the subscript s indicates saline water.

The interface position at a geographic point can be computed if both the freshwater head and the saline-water head are known. It has been assumed that the freshwater is floating on the saline ground water, which implies buoyant equilibrium. Therefore, at any point on the interface, the upward directed pressure in the saline water must be exactly balanced by the downward directed pressure in the freshwater. Equating equations 10 and 11 in terms of equal pressure, and recognizing that z equals i at the interface, yields

$$i = - h_f \left(\frac{\gamma_f}{\gamma_s - \gamma_f}\right) + h_s \left(\frac{\gamma_s}{\gamma_s - \gamma_f}\right). \qquad (12)$$

Inserting the commonly used values for γ_f and γ_s yields

$$i = -40\ h_f + 41\ h_s\ . \qquad (13)$$

Taking the time derivative of equation 12 yields

$$\frac{\partial i}{\partial t} = - 40 \frac{\partial h_f}{\partial t} + 41 \frac{\partial h_s}{\partial t}\ . \qquad (14)$$

The change in bottom storage is given by

$$7.48 \times n_e \iint_{A_i} \left(-\frac{\partial i}{\partial t}\right) dA_i,$$

where

n_e = effective porosity, and

A_i = area of the interface in the Waialua area.

The change in top storage is given by

$$7.48 \times S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt},$$

where

S_y = specific yield, and

A_{wt} = area of the water table in the Waialua area.

On the long-term, basal-water inflow and outflow fluxes have been balanced, but on the short-term, the fluxes are not necessarily in balance. The flux imbalance is equal to the summation of the change in top and bottom storage, and is expressed as

$$\frac{dV_f}{dt} = 7.48 \times S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt} + 7.48 \times n_e \iint_{A_i} \left(-\frac{\partial i}{\partial t}\right) dA_i, \quad (15)$$

where

$\frac{dV_f}{dt}$ = the flux imbalance for the basal-water body.

Substituting the right-hand side of equation 14 into the last term of equation 15, yields

$$\frac{dV_f}{dt} = 7.48 \times S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt} + 7.48 \times n_e \iint_{A_i} \left(40\frac{\partial h_f}{\partial t} - 41\frac{\partial h_s}{\partial t}\right) dA_i. \quad (16)$$

For the saline-water flow field the volumetric-flux imbalance is equal to the saline-water flux across the caprock-aquifer boundary, and is equal to the change in bottom storage. This relationship is given by

$$\iint_{A_{cs}} \left(\frac{\partial h_s}{\partial n}\right) dA_{cs} = 7.48 \times n_e \iint_{A_i} \left(\frac{\partial I}{\partial t}\right) dA_i, \qquad (17)$$

where

$\iint_{A_{cs}} \left(\frac{\partial h_s}{\partial n}\right) dA_{cs}$ = the flux of saline water through the caprock.

When the basal-water body changes from a steady-state condition to an imbalanced flux condition, the flux imbalance is initially equal to the change in top storage. The reason for this is that the interface cannot move, because there is no gradient across the caprock (equation 17). Since $\partial i/\partial t$ is initially equal to zero, for early-time conditions

$$\frac{\partial h_s}{\partial t} = \frac{40}{41}\frac{\partial h_f}{\partial t} . \tag{18}$$

Thus, for early-time conditions, the saline-water head changes almost at the same rate as the freshwater head. This change in the saline-water head produces the gradient across the caprock, and sets the saline-water flow field in motion. The gradient across the caprock continues to increase with time, until a steady saline-water flux through the caprock occurs. For late-time conditions, when the steady flux occurs,

$$\frac{\partial h_s}{\partial t} = 0 . \tag{19}$$

Even without historical records of either saline-water head or interface position, it is possible to make basal-water storage calculations by using either early data or late data of flux imbalance and freshwater head. For the early-time

$$\frac{dV_f}{dt} = 7.48 \times S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt} \,, \tag{20}$$

and for the late-time

$$\frac{dV_f}{dt} = 7.48 \times S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt} + 7.48 \times n_e \iint_{A_i} \left(40\frac{\partial h_f}{\partial t}\right) dA_i \quad . \tag{21}$$

The early-time equation is from equation 15, with the bottom-storage term set equal to zero, the late-time equation is from equation 16, with $\partial h_s/\partial t$ set equal to zero.

Evaluation of Storage Parameters

The two storage parameters to be evaluated are specific yield and effective porosity. Specific yield may be evaluated by utilizing early-time data in a transposed form of equation 20. Effective porosity may be evaluated by utilizing late-time data in a simplified and transposed form of equation 21.

Specific Yield

If only the early data are used following a flux imbalance when the change in bottom storage is essentially zero, then

$$\frac{dV_w}{dt} = 7.48\ S_y \iint_{A_{wt}} \left(\frac{\partial h_f}{\partial t}\right) dA_{wt}\ . \tag{22}$$

Or, for a finite period of time,

$$\Delta V_w = 7.48 \times S_y \iint_{A_{wt}} \Delta h_f\ dA_{wt}. \tag{22a}$$

The specific yield may be solved for directly by transposing equation 22a to

$$S_y = \frac{\Delta V_w}{7.48 \times \iint_{A_{wt}} \Delta h_f\ dA_{wt}}, \tag{23}$$

and applying the drawdown data for shaft 3404-02 after 5 days pumping (fig. 15).

The flux imbalance for the 5-day period is equal to the volume pumped by shaft 3404-02. The pumping rate was 6.95 Mgal/d (26×10^3 m^3/d); therefore, for 5 days ΔV_w would be equal to 35 Mgal (130×10^3 m^3).

The integral term, $7.48 \times \iint_{A_{wt}} \Delta h_f \, d\, A_{wt}$, was evaluated as 340 Mgal (1300×10^3 m^3) by planimetering the drawdown cone of figure 17. The cone was estimated to have spread between the Anahulu River and the adjacent wedge of valley-fill alluvium. In the other direction, the cone was estimated to have spread from the caprock to the northern ground-water dam.

The specific yield, S_y, is 0.10, determined as a ratio of the volume pumped, ΔV_w, to the volume of the drawdown cone, $7.48 \times \iint_{A_{wt}} \Delta h_f \, d\, A_{wt}$ (equation 23). This value for specific yield is approximately of the same magnitude as unpublished determinations of specific yield computations for the Honolulu area. The value is somewhat larger than the value of 0.064, as determined for the Schofield ground-water body (Dale and Takasaki, 1976, p. 41). In general, the value appears reasonable, and the fact that it is somewhat larger may be due to one or more of the following:

1. Some water may have been obtained from the bounding valley-fill deposits (transient leakage).

2. Five days may be too long a period for early time, in that some change in bottom storage may already be occurring.

3. Some water may be obtained from internal storage in the aquifer (due to the compressibilities of both water and the aquifer skeleton).

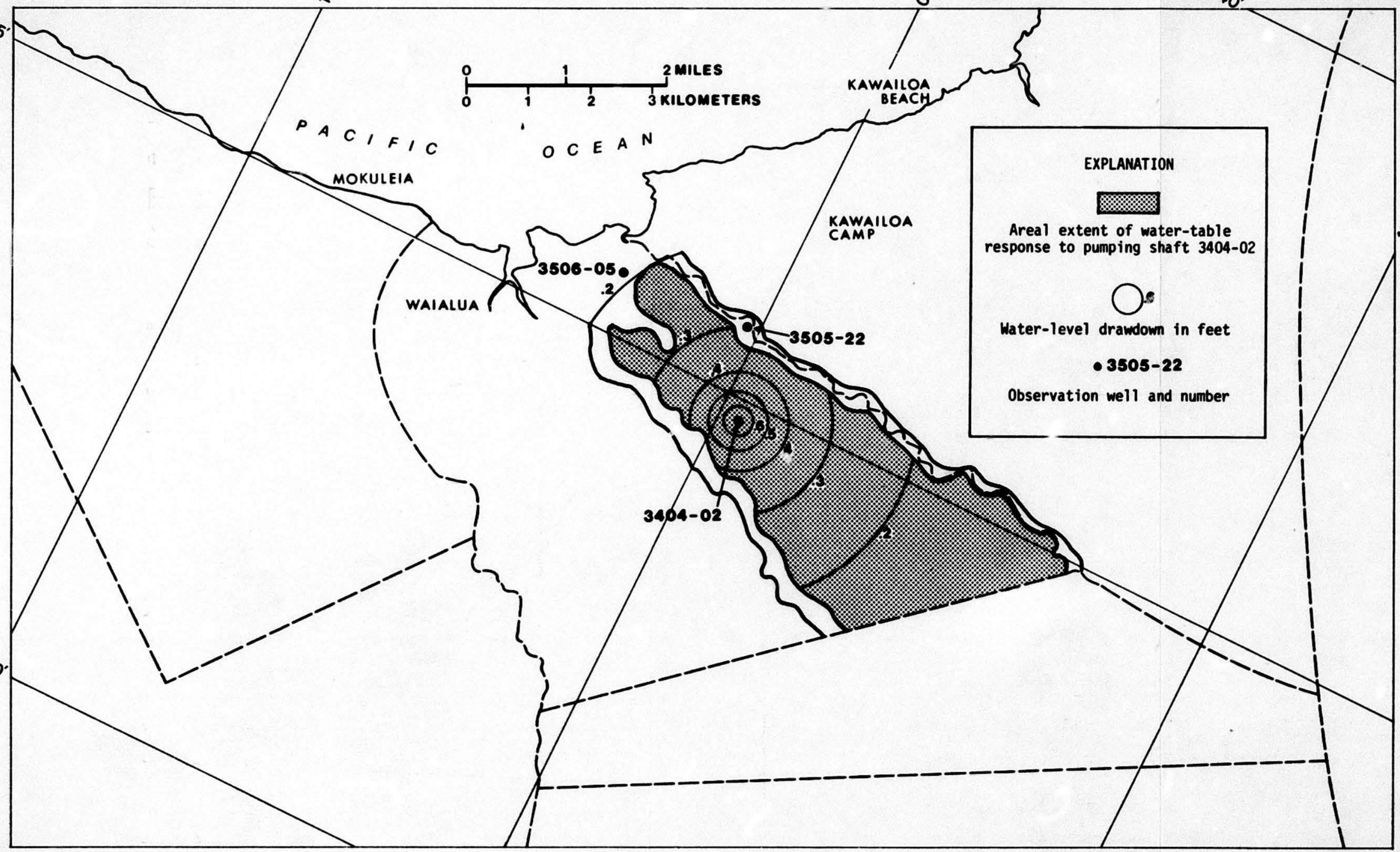

FIGURE 17. MAP OF NORTH-CENTRAL OAHU SHOWING DRAWDOWN CAUSED BY PUMPING SHAFT 3404-02 FOR 5 DAYS AT 6.95 MGAL/DAY

Effective Porosity

If only the late-time data are used following a flux imbalance when the saline-water flux is at a steady rate, then equation 21 is applicable. By assuming that the water-table area is equal to the interface area, and assuming that the specific yield is equal to the effective porosity, equation 21 may be simplified to

$$\frac{dV_f}{dt} = 7.48 \times 41\ n_e \iint_{A_i} \left(\frac{\partial h_f}{\partial t}\right) dA_i . \tag{24}$$

Note that errors in this assumption are not especially critical because the bottom-storage term is on the order of 40 times larger than the top-storage term.

Effective porosity may be solved for directly by transposing equation 24 to

$$n_e = \frac{\dfrac{dV_f}{dt}}{7.48 \iint_{A_i} \left(41 \dfrac{\partial h_f}{\partial t}\right) dA_i}, \tag{25}$$

and applying data related to the annual variation in ground-water pumpage. Before the equation can be properly used, it is necessary to determine how much time must elapse before late time is reached.

The problem of determining when late time begins may be solved by comparing seasonal basal-water-level fluctuations against the seasonal pumping pattern. Basal-water pumpage in the Waialua area follows a seasonal pattern. During the months of November through March, rainfall and streamflow generally satisfy the demand for irrigation water, and there is little basal water pumped during this period.

From April through October, the demand for irrigation water is large, and basal water is used to supplement the rainfall and streamflow.

In 1969, heavy pumping for irrigation started on April 10 and continued through November 10 (fig. 18e). The weekly pumping pattern during this period was about 5 days of pumping during week days and 2 days of no pumping during the weekend. This weekly periodic pumping causes the hydrograph to look like a comb during the pumping period (fig. 18a). The basal-water-head trend, both during the pumping and nonpumping seasons, follows the ocean-level trend (fig. 18b). The daily mean ocean level was subtracted from the basal-water head to remove the effects of the change in ocean level. This graph, called the adjusted basal-water head, is the light line-weight graph of figure 18c. The final step in data reduction was to take a 7-day moving average of the adjusted basal-water head to remove the effects of the weekly pumping pattern. This graph is the heavy line-weight graph of figure 18c.

The 7-day moving average of the adjusted basal-water head can be approximated by a transient drawdown curve that lasts about 5 weeks, which is followed by a linear decline that continues for the remainder of the pumping season (fig. 18e). It is apparent that the transient drawdown occurs during the time that the saline-water flow field is being energized, and that the linear portion of the generalized fresh-water-head record corresponds to late time.

The linear slope is related to a flux imbalance that includes a variation in both pumpage and deep infiltration from long-term mean values. The variation of deep infiltration from the long-term mean is unknown, and, therefore, it is not possible to solve directly for effective porosity using the data from figure 18.

The value of figure 18 is that it clearly shows that early time is on the order of 3 or 4 days, and that late time occurs after about 5 weeks following the flux imbalance.

In the earlier part of this report (p. 25), it was assumed that, on the long-term average, the basal-water flux was in equilibrium, and the basal-water body was being operated under steady-state conditions. Under the steady-state assumption, the mass-balance equation for the Waialua basal-water body was expressed as

$$\overline{DI} + \overline{ND} - \overline{P} - \overline{CL} - \overline{AB} = 0. \qquad (2)$$

In this section of the report, the basal-water body is considered to depart slightly from the steady-state conditions. Small annual perturbations in deep infiltration, pumpage, and ground-water storage are used to solve for the effective porosity. The analysis utilizes time-series techniques. The time step of one year was selected for this analysis, because certain time-dependent features average out over the calendar year. For any calendar year, the mass-balance equation may be expressed as

$$DI\ (t) + \overline{ND} - P\ (t) - \overline{CL} - \overline{AB} = 7.48 \times n_e \iint_{A_i} (41\frac{\partial h_f}{\partial t})\ dA_i, \qquad (26)$$

where

$DI\ (t)$ = annual deep infiltration,

$P\ (t)$ = annual ground-water pumpage, and

$7.48 \times n_e \iint_{A_i} (41\frac{\partial h_f}{\partial t})\ dA_i$ = change in basal-water storage.

It is assumed in equation 26 that the fluxes across the three boundaries are constant, because the year-to-year changes in head are small, both in the basal-water body and in the adjacent high-level water bodies.

Subtracting equation 2 from equation 26 yields

$$\{DI\ (t) - \overline{DI}\} - \{P\ (t) - \overline{P}\} = 7.48 \times n_e \iint_{A_i} (41 \frac{\partial h_f}{\partial t})\ dA_i, \qquad (27)$$

where the bracketed terms are the annual deviations from the long-term mean. In equation 27, the deviation of pumpage from 1928 to 1973, the annual change in head, and the area of the interface are known. The deviation of deep infiltration from 1928 to 1973 and the effective porosity are the unknown quantities.

In order to solve equation 27, using time-series techniques, the deviation of deep infiltration must be estimated. Deep infiltration is computed as

$$DI\ (t) = R\ (t) + SW\ (t) + \alpha P(t) - ET\ (t), \qquad (27a)$$

where

DI (t) = deep infiltration,

R (t) = rainfall,

SW (t) = surface-water diversions for irrigation,

$\alpha P(t)$ = that part of the ground-water pumpage applied to the agricultural lands in the Waialua area,

ET (t) = evapotranspiration, and

the units are expressed in millions of gallons per year.

Expanding equation 27a in terms of the deviation yields

$$\{DI\ (t) - \overline{DI}\} = \{R(t) - \overline{R}\} + \{SW\ (t) - \overline{SW}\}$$
$$+ \alpha\{P\ (t) - \overline{P}\} - \{ET\ (t) - \overline{ET}\}. \qquad (27b)$$

The control surface for equations 27a and 27b is at the land surface and not at the basal-water body (fig. 9). As the water filters through the unsaturated zone, the uneven seasonal rate of deep infiltration is smoothed. Consequently, only the major variations in the deep infiltration rate can be detected by the changes in basal-water head. The major variations are the rainfall and streamflow terms and the minor variations are the pumpage and evapotranspiration terms.

If it is assumed that the two minor terms cancel; that is, if

$$\alpha\{P\ (t) - \overline{P}\} = \{ET\ (t) - \overline{ET}\}, \tag{27c}$$

then

$$\{DI\ (t) - \overline{DI}\} = \{R\ (t) - \overline{R}\} + \{SW\ (t) - \overline{SW}\}. \tag{27d}$$

The rainfall and the surface-water terms of equation 27d may be combined, because of conditions in the Waialua area. The drainage basins are small, less than 50 square miles (130 km^2), and because the basins are small, the streamflow is directly related to the rainfall. About 90 percent of the streamflow is diverted for irrigation. Consequently, the surface-water supply and the rainfall have identical trends. Therefore, equation 27d may be expressed as

$$\{DI\ (t) - \overline{DI}\} = B \cdot \{R_{888}\ (t) - \overline{R_{888}}\}, \qquad (27e)$$

where

B = an unknown relating coefficient, and

R_{888} (t) = the rainfall at National Weather Service gage 888 (Helemano Intake).

In equation 27e, B has the dimensions of millions of gallons of deep infiltration, resulting from direct rainfall and stream diversions, per inch of rain at gage 888. R_{888} (t) has the dimensions of inches of rain per year.

Substituting equation 27e into equation 27 yields

$$B\ \{R_{888}\ (t) - \overline{R_{888}}\} - \{P\ (t) - \overline{P}\} = 7.48 \times n_e \iint_{A_i} (41 \frac{\partial h_f}{\partial t})\ dA. \qquad (27f)$$

The integral of equation 27f must be evaluated before the time-series techniques can be used. Because the area of the interface is small (500 million square feet), and because the aquifer is very permeable, the water-level changes from year-to-year are essentially uniform over the entire basal-water body. Therefore, a single well (well 3406-01) was used to estimate the average change in head over the entire water body, making it possible to express the entire right-hand side of equation 27 as:

$$7.48 \times n_e \iint_{A_i} \left(41 \frac{\partial h_f}{\partial t}\right) dA_i = 153{,}000\ n_e\ \{\Delta h_f\ (t)\}, \qquad (28)$$

where the annual basal-water head change, $\Delta h_f\ (t)$, is based on water-level measurements made in well 3406-01 during the last week of December of each year (fig. 19). Substituting equation 28 into equation 27e, and transposing yields,

$$B\ \{R_{888}\ (t) - \overline{R_{888}}\} - \{P\ (t) - \overline{P}\} - 153{,}000\ n_e\ \{\Delta h_f\ (t)\} \approx 0. \qquad (29)$$

The lefthand side of equation 29 is indicated as being approximately equal to zero because there are undoubtedly small errors in the underlying assumptions on which the equation is based, as well as measurement errors. Therefore, to be exact, equation 29 must be expressed as

$$B\ \{R_{888}\ (t) - \overline{R_{888}}\} - \{P\ (t) - \overline{P}\} - 153{,}000\ n_e\ \{\Delta h_f\ (t)\} = \varepsilon, \qquad (29a)$$

where ε = the error term.

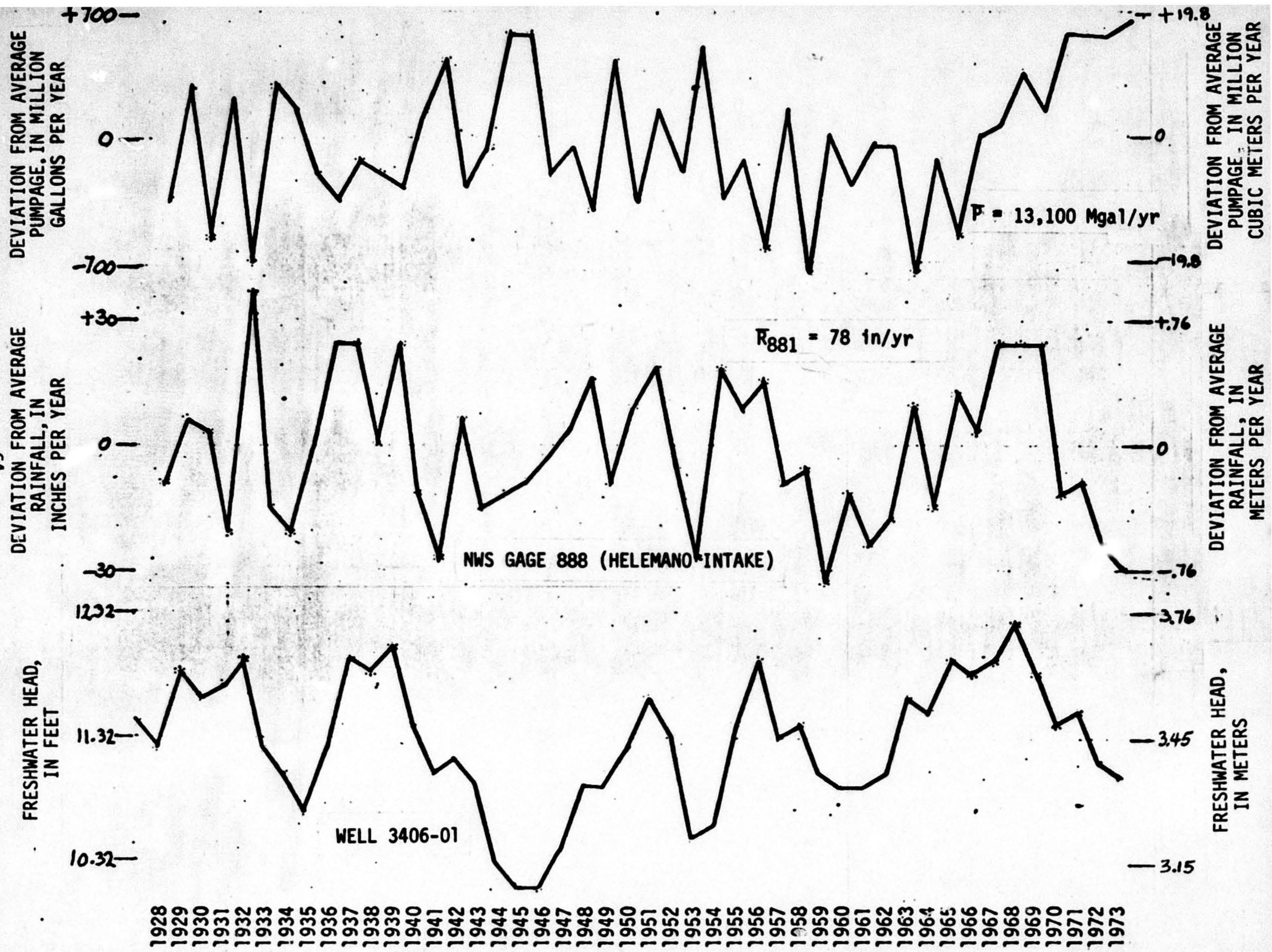

Fig. 19. Deviation from average pumpage and rainfall and basal-water head, 1928-73

Equation 29a was solved using minimization techniques similar to those used by Hamming (1971, p. 221-245). The procedure basically is one of searching for values of B and n_e that will minimize the sum of ε^2. The computed effective porosity was 0.09, which compares favorably with the value computed for specific yield. The coefficient B was computed as 121 Mgal/in ($1.8 \times 10^6 m^3/cm$).

The equation

$$\frac{\Delta h_f}{\Delta t} = \frac{-\{P(t) - \overline{P}\} + 121\ \{R_{888}(t) - \overline{R_{888}}\}}{153000(0.09)} \tag{30}$$

was used to generate a computed freshwater-head record (fig. 20). The fit between the historical and the computed hydrographs is very good.

The time-series solution for effective porosity appears reasonable in view of the good comparison between the storage-parameter estimates, and between the computed and historic hydrographs.

The value of freshwater head ranged between 10.12 feet (3.08 m) and 12.27 feet (3.73 m) during the 46-year period. Thus, based on equation 12, the interface was ranging between 405 feet (123 m) and 490 feet (149 m) below sea level. The value of effective porosity is based on this interval. However, since the value of effective porosity for this interval is about the same as the specific yield, which is measured at the water table, it will be considered that the effective porosity has a uniform value of 0.09 in the interval from the water table to 490 feet (149 m) below sea level.

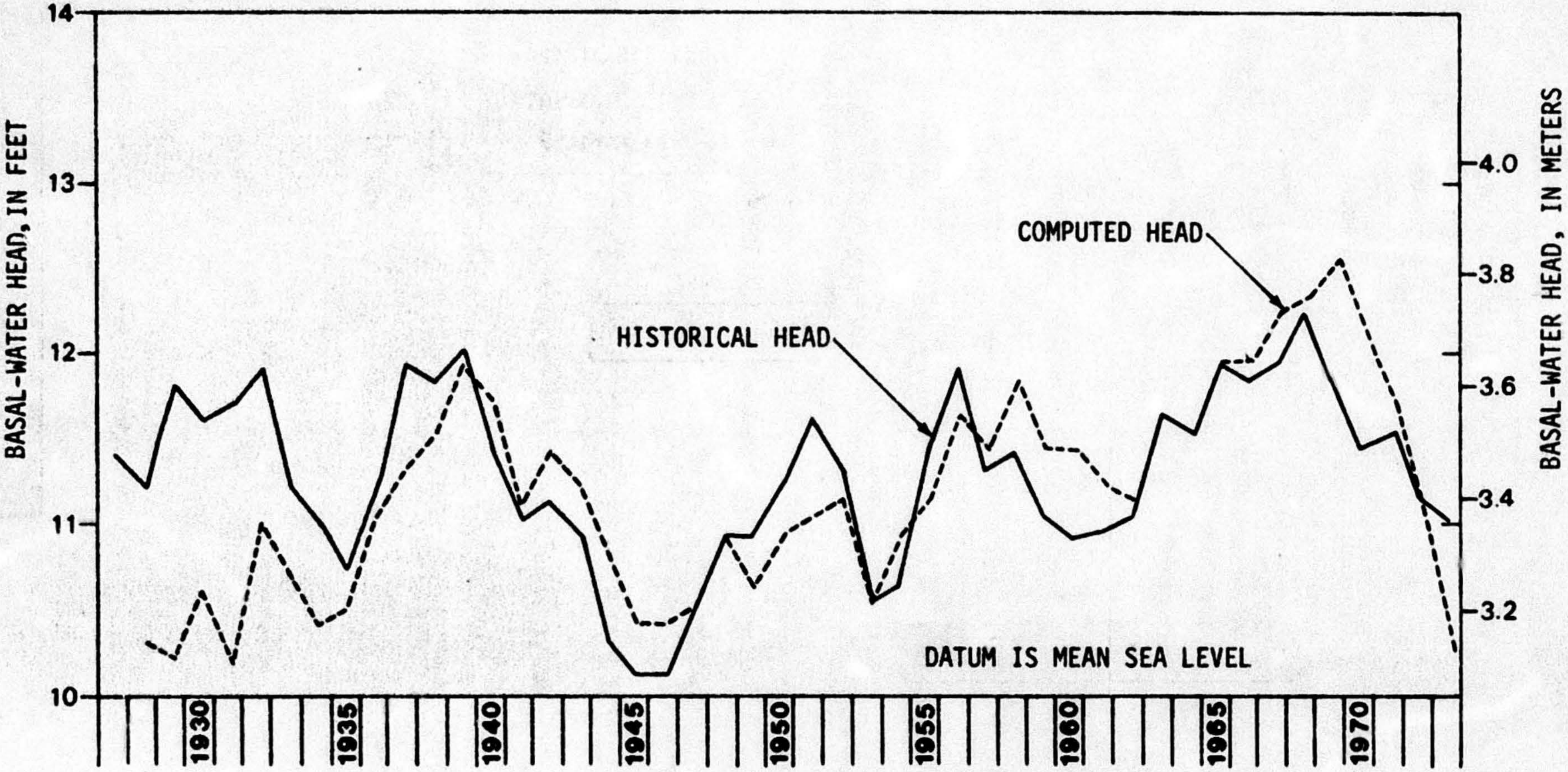

Figure 20. Computed and historical basal-water hydrographs for well 3406-01

The Volume of Basal-Water Storage

The volume of water in basal-water storage is equal to the effective porosity times the total volume occupied by the basal-water body. The effective porosity is 0.09 as determined by the preceding calculations. The volume occupied by the basal-water body is the volume contained between the water table and the interface and bounded laterally by the coastal plain and valley-fill deposits and the northern ground-water dam. The average water-table altitude has been about 11 feet (3.3 m) above sea level, and the average interface has been about 440 feet (130 m) below sea level. Thus, on the average, the thickness of the basal-water body has been 450 feet (140 m). The areal extent is either 16.1 mi^2 (41 km^2) or 17.9 mi^2(46 km^2), depending on whether the water-table area or the interface area is measured. For this calculation, assume that the areal extent is 17.0 mi^2 (44 km^2). Thus, the volume occupied by the basal-water body is 1.6×10^{12} gallons (6×10^9 m^3). The volume of basal water in storage is 0.09 times the volume occupied by the basal-water body, and is equal to 1.4×10^{11} gallons (5.4×10^8 m^3).

The basal water in storage depends on the average freshwater head, where the average is taken over a period of time and over the entire basal-water body. The volume is expressed as

$$V_b = 7.48 \times 41\ h_f \times A \times n_e , \tag{31}$$

where A = average areal extent of the basal-water body, and

V_b = volume of the basal water in storage.

Assuming A = 17.0 mi^2 (44 km^2) and n_e = 0.09, then

$$V_b = (13.1 \times 10^9)\ h_f . \tag{32}$$

Usable basal-water storage depends on the thickness of the transition zone and on the depth that wells or shafts penetrate below sea level.

The graph of the basal water in storage as related to average prevailing freshwater head (fig. 21) is based strictly on interface theory. Assuming that the domestic quality water lies 100 feet (30 m) above the interface, 2.5 feet (0.8 m) of head correction must be allowed to compensate for the thickness of the transition zone. Thus, from figure 21, 32 x 10^9 gallons (1.24 x 10^8 m^3) of basal water is in storage that is of too poor a quality for domestic use.

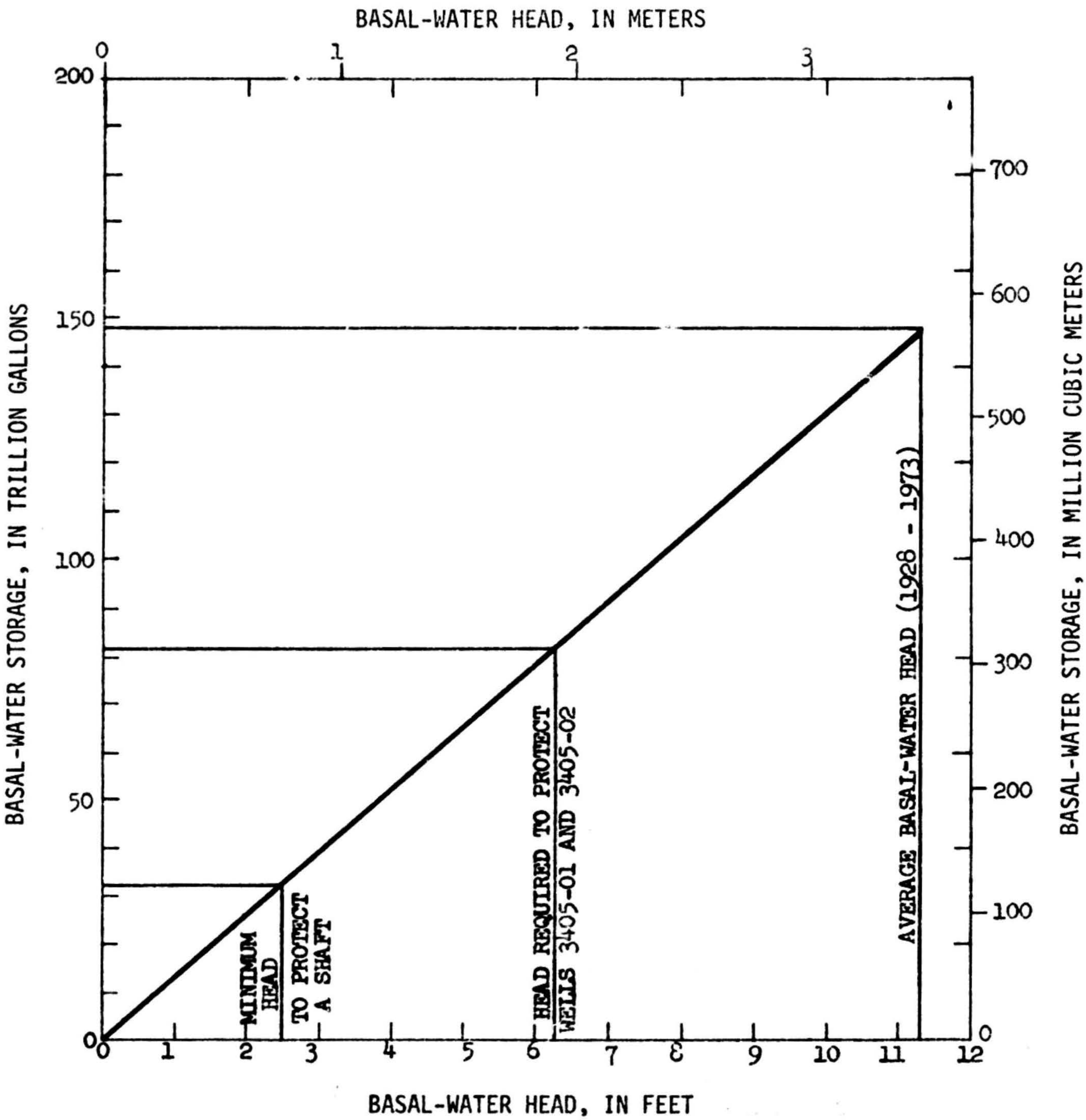

Figure 21. Graph showing the relationship between basal-water storage and basal-water head

The remaining basal-water storage is useful ground-water storage only if it were developed by a skimming shaft that does not penetrate below sea level. In order to protect deep wells from saline ground water, the computed interface must lie 100 feet (30 m) below the bottom of the well. Honolulu Board of Water Supply wells 3405-01 and 3405-02 are drilled to depths of 141 and 146 feet (43 and 44.5 m) below sea level and if they are to be protected from the saline ground water, the interface must be stabilized about 250 feet (76 m) below sea level. This will require that 6.25 feet (1.9 m) of freshwater head be maintained to protect the wells.

THE EFFECTS OF INCREASING PUMPAGE

Based on the outflow terms, the basal-water flux averages 55 Mgal/d (206 x 10^3 m^3/d) and present pumpage averages 36 Mgal/d (136 x 10^3 m^3/d). Therefore, there is 19 Mgal/d (71 x 10^3 m^3/d) that is presently being discharged from the basal-water body as either coastal-plain leakage or flux across the Anahulu boundary that could be pumped from the basal-water body. The 19 Mgal/d (71 x 10^3 m^3/d) is a theoretical upper limit that, in practice, cannot be attained, because a certain amount of natural coastal-plain leakage is required to maintain a basal-water body.

An increase in pumpage causes a decrease in freshwater head, which causes a decrease in both top and bottom storage. The decrease in storage continues until the coastal-plain leakage plus the flux across the Anahulu boundary is reduced by an amount equal to the increase in pumpage. In the limit, the basal-water body would eventually shrink to zero thickness by the combination of interface and water-table movement, if the additional 19 Mgal/d (71 x 10^3 m^3/d) were pumped.

In the management of a basal-water body, it is the position of the interface that is critical. The interface must never rise any higher than about 100 feet (30 m) below the bottom of the well, or else saline water will be pumped by the well. In addition, the interface must be, on the average, maintained at some distance more than 100 feet (30 m) below the bottom of the well in order to maintain usable ground-water storage.

The amount of allowable additional pumpage must be based on whether a well or group of wells is to be protected against the salt-water intrusion that occurs when the interface rises. As determined previously, an average freshwater head of 6.25 feet (1.9 m) would have to be maintained in the Honolulu Board of Water Supply wells 3405-01 and 3405-02. This makes no allowance for usable basal-water storage.

As a first approximation, the outflow across the Anahulu boundary and the caprock leakage (basal-water leakage) can both be considered as a linear function of the Waialua basal-water head. The graph of basal-water leakage, as related to basal-water head (fig. 22), is based on the points for zero leakage at zero head and for 19 Mgal/d ($70 \times 10^3 m^3/d$) leakage at 11.3 feet (3.44 m) of head.

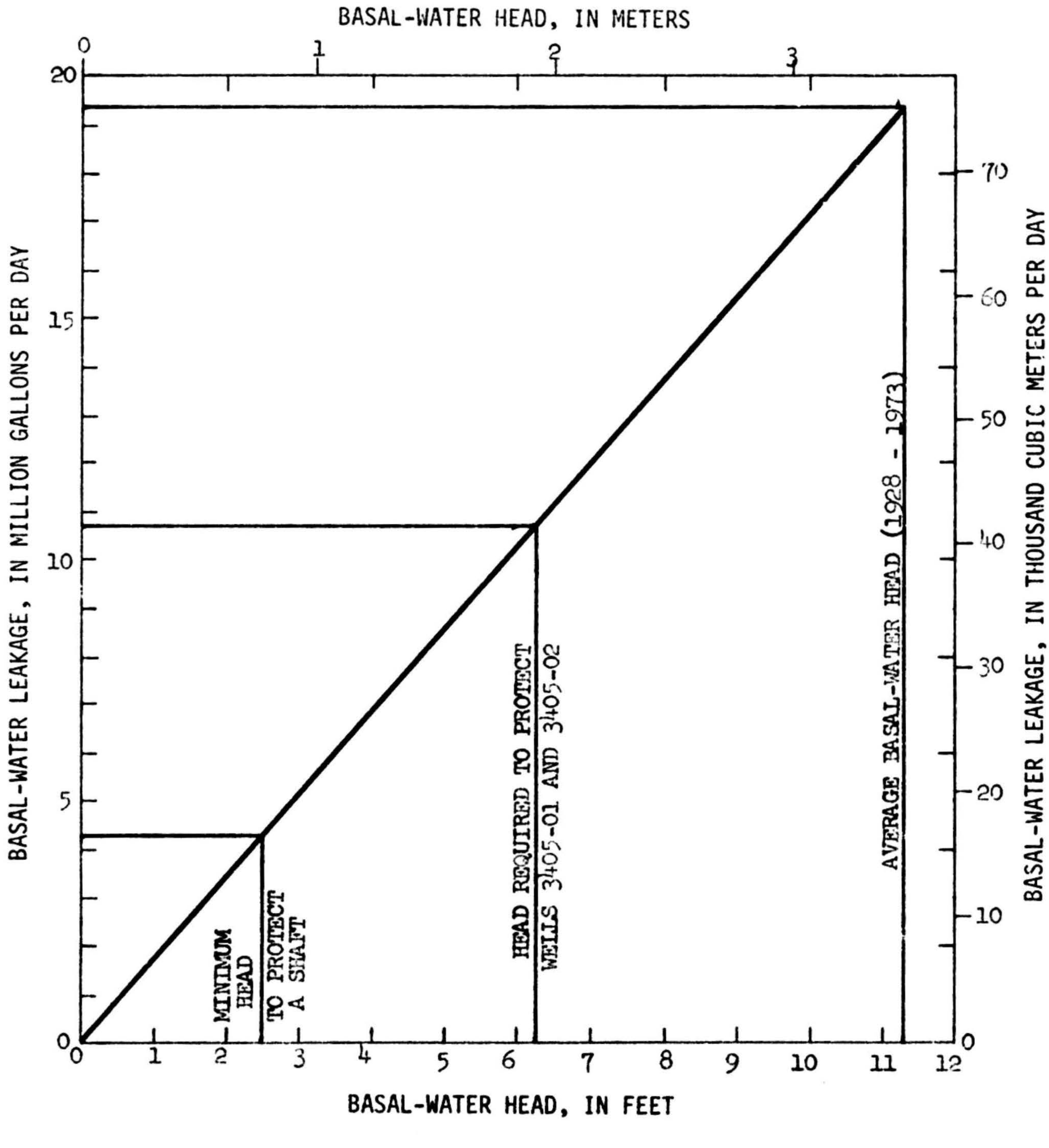

Figure 22. Graph showing the relationship between basal-water leakage and basal-water head

The amount of the basal-water leakage that can be captured depends on how much the basal-water head can be reduced. If a shaft were used to develop the water, 4 Mgal/d (16 x 10^3 m^3/d) of basal water would be lost, because 2.5 feet (0.76 m) of head must be maintained to allow for the transition-zone thickness. The difference between the 19 Mgal/d (70 x 10^3 m^3/d) and 4.3 Mgal/d (16 x 10^3 m^3/d) could be pumped on a continuous basis by using a shaft. Thus, by using a shaft 15 Mgal/d (57 x 10^3 m^3/d) could be pumped on a continuous basis. If wells 3405-01 and 3405-02 are to be protected, then the basal-water head can be reduced to 6.25 feet (1.9 m) with a resulting increase in sustained pumpage rate of 8 Mgal/d (30 x 10^3 m^3/d).

In addition to the sustained increase in pumpage that occurs with a reduction in basal-water head, a large transient supply is available as the basal-water body is reduced in size. This transient supply results from mining of basal water primarily held in bottom storage.

In shifting from an average basal-water head of 11.3 feet (3.4 m) to an average basal-water head of 2.5 feet (0.76 m), 32 x 10^9 gallons (1.2 x 10^8 m^3) of basal water would be mined from storage. This water can be mined at any desired rate, but once it is mined, the basal-water pumpage must be reduced to a lower rate.

Care should be exercised in the basin development not to install excess pumping capacity during the transient period, because once the transient supply has been mined, the pumpage must be reduced to the sustainable rate.

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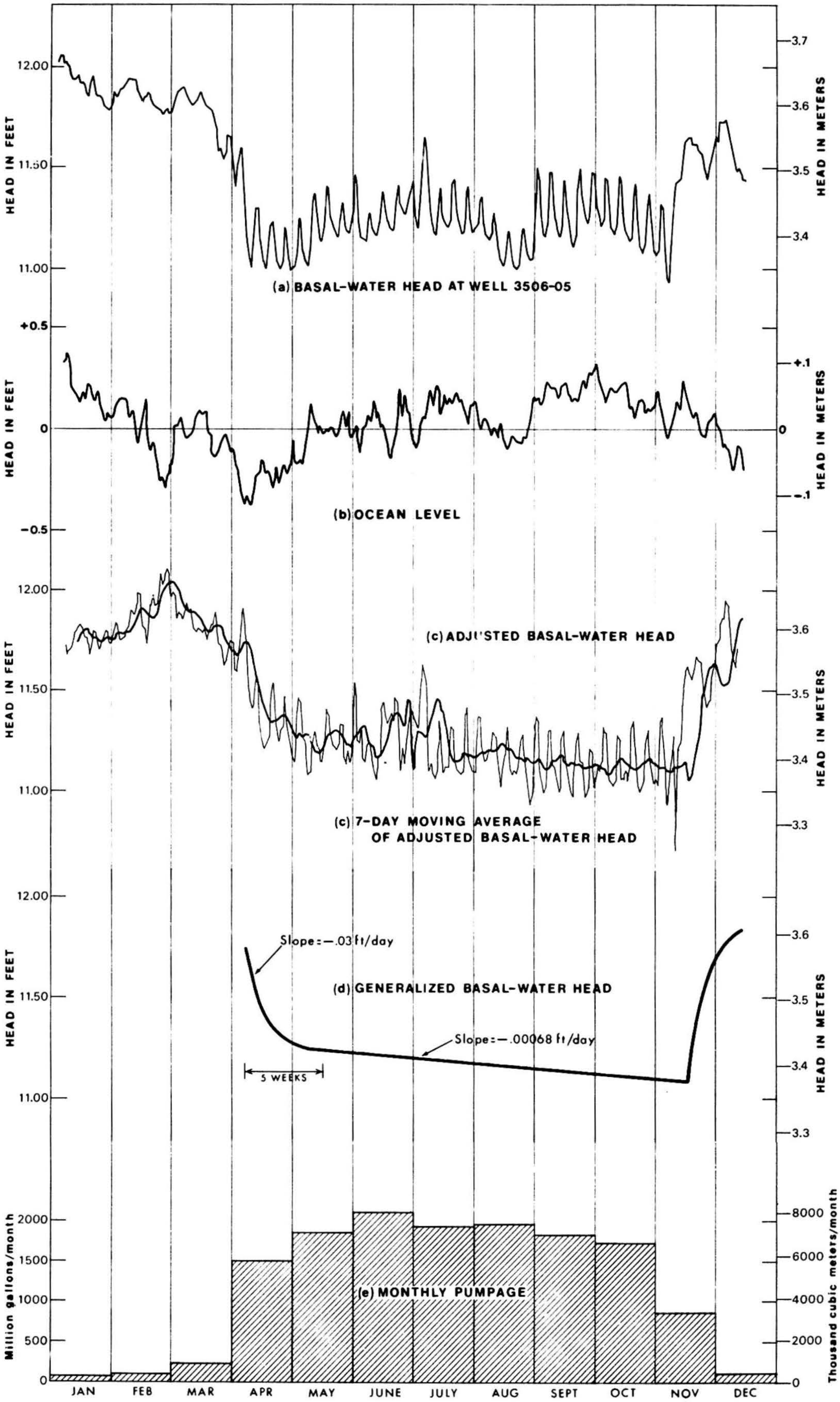

Figure 18. Reduction of basal-water head at well 3506-05 for 1969