UNITED STATES DEPARTMENT OF INTERIOR

GEOLOGICAL SURVEY

FIELD PROCEDURES AND DATA REDUCTION METHODS (WITH HEWLETT-PACKARD 97-67 PROGRAMS) FOR TOTAL FIELD RESISTIVITY SURVEYS

by

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Field Procedures and Data Reduction Methods

(with Hewlett-Packard-97-67 Programs[1/]) For Total Field

Resistivity Surveys

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I. CREW REQUIREMENTS AND BASIC EQUIPMENT

The following list of manpower and equipment is currently used by the U.S. Geological Survey and is recommended for the optimum operation of an efficient bipole-dipole total field resistivity survey.

(A) Crew

An optimum crew consists of five (5) persons. A party chief, an assistant party chief, an operator, and two field assistants.

(B) Vehicles

Three 3/4 ton, 4 x 4, vehicles are generally required. All of which should be equipped with winches.

(1) Transmitter truck, for carrying transmitter equipment, about 3 km of #8 current bipole cable, and a truck mounted or truck towed 30-40 kva generator. This vehicle must be equipped with a powerful radio transceiver.

(2) Receiver vehicle, generally a carryall type for carrying party chief and assistant party chief and the receiver equipment. This vehicle also must be equipped with a powerful radio transceiver.

[1/] Use of a specific brand name does not necessarily constitute endorsement of the product by the U.S. Geological Survey.

(3) Utility truck, for carrying two crew members, reels, porous pots, test equipment, tools, and so on. It is recommended that this vehicle be equipped with a precision odometer and an additional powerful radio transceiver.

(C) Transmitter Unit and Accessories

(1) The transmitter should be powered by a 30-40 kva generator (or a larger generator) and must be capable of pulsing a square wave current at peak to peak amplitudes of about 20 to 60 amperes, at about 400 volts, and preferably at several frequencies ranging from about 0.05 to about 0.5 Hz. A frequency of 0.1 Hz or less is used most often.

(2) Frequency control box (may be part of transmitter box).

(3) Dummy load.

(4) Bipole cable (2-4 km of #8 insulated cable).

(5) A cable "spitter" (generally operated with a hydraulic pump) to pick up the bipole cable and lay it in a "bird's nest" on the bed of the truck.

(6) HP-97 calculator (encased in a transparent plastic bag to protect it from dust), programs, charger, and data books.

(7) Invertor for charging calculator from vehicle battery if necessary.

(D) Receiver Unit and Accessories

(1) A potentiometric chart recorder operated with an invertor from the truck battery or a separate car battery (that

can be simultaneously charged with the truck battery). The recorder should have a sensitivity of as much as 0.1 mv full scale and various chart speeds including speeds of about 0.12 cm/sec and 0.06 cm/sec (1 inch/20 sec and 1 inch/40 sec).

(2) A battery powered self potential cancelling circuit (S.P. bucker), connected to recorder.

(3) A three-way switching box for successively connecting three pairs of potential electrodes (M and N, M and N′, N′ and N) to S.P. bucker.

(4) A three conductor cable with color coded terminals, for connecting potential electrodes to switching box.

(5) Premeasured coaxial cable on separate reels with fixed lengths ranging from about 30 meters to about 200 meters. (A length of 75 meters (250 feet) is most often used.) NOTE: Coaxial cables minimize wind noise.

(6) Several (4 to 6) copper-copper sulphate porous pots for potential electrodes and an adequate supply of copper sulphate crystals.

(7) Canteens for watering potential electrodes.

(8) Two azimuthal Brunton compasses.

(9) HP-67 calculator (placed in a transparent plastic bag to protect it from dust), programs, and charger. [OPTIONAL]

(10) Crystal clock for signal polarity determination. [OPTIONAL]

(11) Topographic map, scales, and data books.

(D) Radio Transceivers

(1) At least two powerful radio transceivers are required for communication between the transmitter vehicle and the receiver vehicle. A third radio transceiver installed in the utility vehicle is also valuable either as a spare, or for relaying messages in areas of difficult communication. Ninety (90) watt FM radio transceivers are recommended for most areas of mild topography, but single side band radio transceivers (40 watt) may be required for areas with rugged terrain.

(2) At least three portable radio transceivers (5 watts) (or two portable FM transceivers using the same frequency as the receiver-vehicle-mounted FM transceiver) are required for communication between party chief in receiver vehicle and crew members.

II. RECOMMENDED FIELD PROCEDURE

(A) Initial Preparations and Recommendations

(1) The party chief and his assistant should scout the area for determining the best locations for the current electrodes. Metal culverts and well casings, separated by a distance of 2-4 km, represent two of the best targets to be sought (for use as ready-made electrodes).

(2) For safety purposes: a) Do not locate a current electrode near a farm house or a school, unless that electrode can be guarded at all times when the current is pulsed into the ground. b) Do not operate equipment

during a thunder storm and disconnect all cables leading into the truck. c) Avoid laying the cable across major road crossing or animal pastures.

(3) If neither metal culverts nor well casings can be found at suitable sites, then buried sheets of aluminum foil or several connected long metallic rods can be used for current electrodes, preferably after soaking the ground with brine.

(4) Neither current electrode should be near a buried gas line, a long metal fence, or a grounded power line, otherwise one would have more than two "point sources", and unusually large signals will be recorded at stations which are located far away from the electrodes (A or B) but that are near these buried pipes or grounded poles.

(5) Having determined the best sites for the current electrodes, place "Danger-High Voltage" signs near one of the current electrodes, lay down the insulated current bipole cable (#8 gauge), and park the transmitter vehicle near the second current electrode.

(B) Preparations at Transmitter Site

(1) Upon connecting the bipole cable to the electrode, test the continuity of cable with an ohmmeter, using earth return. If no continuity, check connection at far end electrode.

(2) Connect the transmitter to the current electrodes and test the maximum (steady state) current that can be put

in the ground. A minimum peak to peak amplitude of greater than 20 amperes is desirable.

(3) Establish the polarity of the current electrodes (A electrode is +, B electrode is -). Ascertain that the same convention is adhered to throughout the survey in subsequent days.

(4) Set the two azimuthal Brunton compasses according to the magnetic declination of the survey area.

(5) Measure the length of the bipole with a precision odometer (if the bipole is placed along a straight flat road) or from the topographic map.

(6) Measure the angle of declination, β, that the bipole axis makes with the geographic North. The angle, β, should be measured in the clockwise direction from the geographic North to the negative electrode B (see figure 1), and must be expressed in degrees from 0° to 360^{0}.

(7) If a crystal clock is to be used, for establishing current signal polarity, synchronize clock with frequency control box.

(C) Preparations at Receiver Site

(1) Receiver stations should be set up at a distance of about 100 meters from the nearest buried pipe line, telephone cable (some old ones are embedded in a lead sheath), grounded power line poles, fences with metal posts, electric pumps, or any power line pole with a transformer box at the top of it. NOTE: a) An unusually large IP

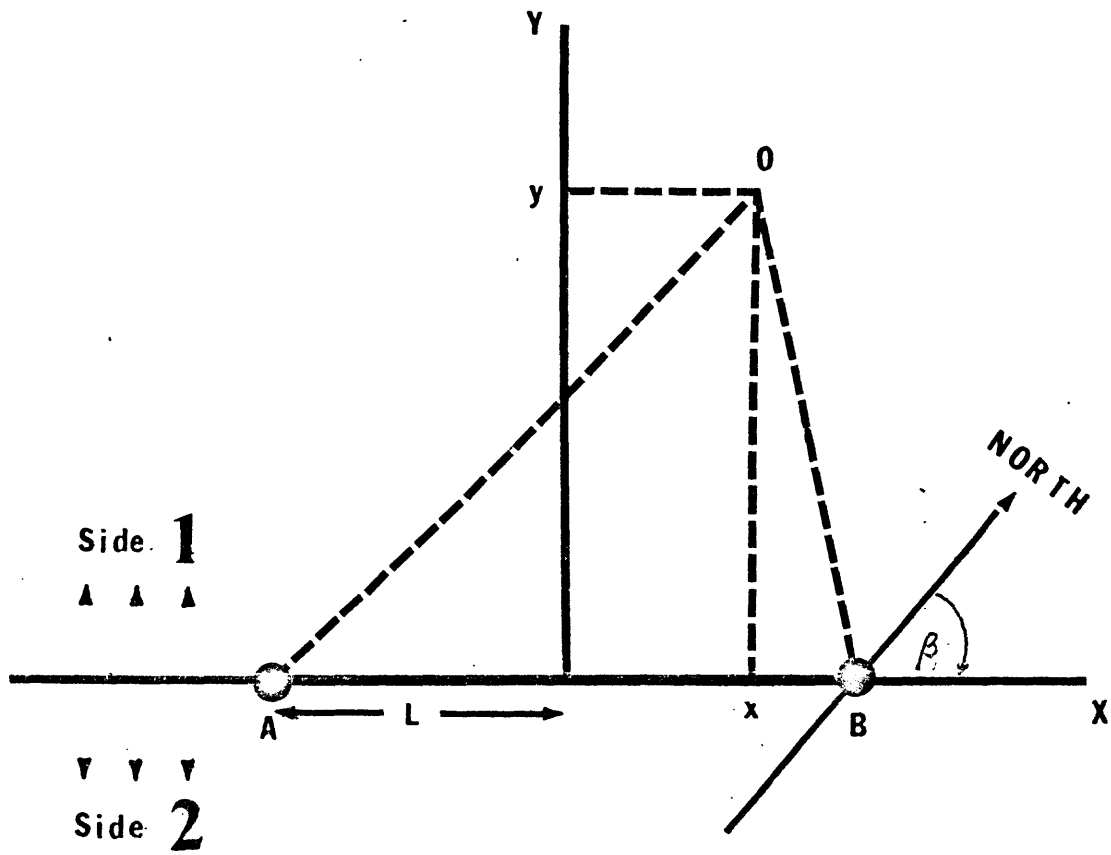

Figure 1.--Diagram for defining station coordinates (x, y, or AO, BO), side 1 and 2, and angle of declination β. A and B are current electrodes; O, station location; L, half length of current bipole.

(induced polarization) effect is observed when the station is located near a buried conductor. b) The station distance may have to be greater than 200 meters to eliminate the noise from transformers.

(2) Select two (nearly orthogonal) directions for setting up a left, and a right, measuring dipoles. The best choice would be one in which the hypotenuse of the (nearly right-angled) triangle would be approximately parallel (rather than at right angles) to the expected direction of the bipole primary electric field.

(3) Place three porous-pot electrodes (M, N, and N′) in the ground, one near the truck (M electrode) and one at each of the far ends of the measuring dipoles (see figure 2).

(4) Using the azimuthal Brunton compasses, the two field assistants should measure the azimuths (θ_L and θ_R) of the left and right dipoles. The measurements should be taken from the far ends of the dipoles (to avoid effects of the metallic body of the receiver truck) and the South seeking end (instead of the North seeking end) of the needle should be utilized in reading the angle. Thus the reported angles would be the azimuths of the measuring dipoles from the M electrode (negative or ground electrode nearest the truck) to the N and N′ electrodes, respectively.

(5) Using the color coded terminals of the cable from the switching box, ascertain that the connection of the M

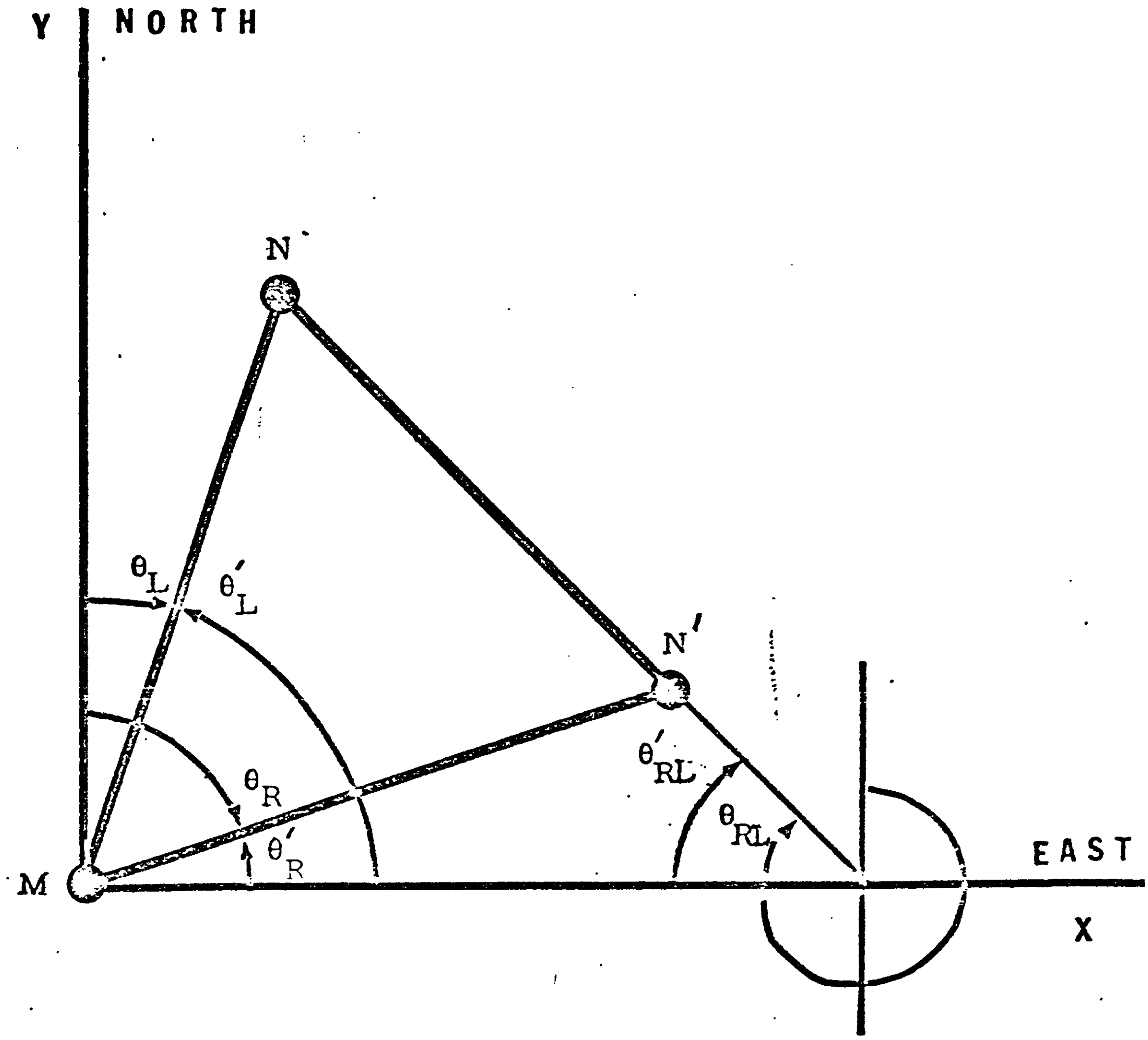

Figure 2.--Diagram for defining angles used in equations (1) through (13). M. N, and N' are potential electrodes.

electrode (electrode nearest the truck) to the chart recorder is such that it acts as a ground electrode while measuring the potential differences (ΔV_L and ΔV_R) for the left and right dipoles, respectively; and that the N′ electrode (electrode at the far end of the right dipole) acts as a ground electrode while measuring the potential difference ΔV_{RL} from N′ to N.

III. DATA ACQUISITION PROCEDURE

(1) Upon arrival at a receiver station, the party chief shall call the operator (at the transmitter) to: (a) inform him of their arrival at the site, (b) receive, record, and plot data that the operator would have computed for the previous station, and (c) inform the operator that they should be ready to receive the pulsed signal in 3-5 minutes.

(2) While the assistant party chief and the two field assistants are setting up the measuring dipoles, the party chief shall plot the location of the new station, measure and record its coordinates (x, y or AO, BO; see figure 1 and read caution on the use of AO and BO in the next section), record the azimuth angles of the left and right dipoles (θ_L and θ_R; see figure 2) when they are transmitted to him by the two field assistants, and prepare the recorder.

(3) The assistant party chief shall be responsible for directing the two field assistants, preparing the electrode nearest the truck, and ascertaining the proper connection of the three color coded terminals to the proper electrodes.

(4) When the operator announces the amount of current in amperes, this will signify that a steady current (having the announced peak to peak amplitude) is being pulsed in the ground.

(5) Three potential differences ΔV_L, ΔV_R, and ΔV_{RL} are recorded successively. For each set of measurements, the party chief will request the operator to announce the "sign" of the pulsed current (or he will use the flashing light on a crystal clock in areas of difficult radio communication) to determine the polarity of the received signal. The operator should announce at least one "Plus" followed by one "Minus", for each "sign" request from the party chief.

(6) The announced sign that coincides with the movement of the recorder pen to the right, is the required voltage sign.

(7) Depending on the signal to noise ratio, a set of three to eight pulses may be measured for each ΔV. The average of each set of measurements is then recorded.

(8) As soon as the three potential differences are measured the party chief instructs the operator to "shut down" the transmitter. This will also serve as a signal to the crew that the measurements have been completed, and to pick up the receiver array.

(9) Before leaving the site, or while moving to the next station, the party chief transmits the following two sets of data to the operator (see figures 1 and 2 for definitions): (a) Station number, left angle (θ_L), Right angle (θ_R), Left magnitude (ΔV_L), Right magnitude (ΔV_R), and Right-Left

magnitude (ΔV_{RL}); (b) The coordinates (AO and BO; or x and y) in miles, the bipole side 1 or 2 (if AO and BO are used), the current (I) in amperes, and the length of the measuring dipoles (MN = MN′) in feet. All transmitted data must be confirmed by the operator.

(10) By the time the party chief and the crew arrive at the next station, the operator would have used the two HP 97-67 programs (given in the appendix section) to compute the magnitude and direction of the measured total potential difference (ΔV and ψ), and the simple total field apparent resistivity ($\bar{\rho}_{|E|}$) and other parameters of interest as explained in the next section.

IV. DATA REDUCTION AND PROGRAM DESCRIPTIONS

Two programs for HP 97-67 programmable calculators are given in the Appendix. The first program (Program I) is for the calculation of the average magnitude and direction of the total potential difference from three horizontal components, whereas the second program (Program II) is for the computation of the simple-total-field apparent resistivity and other parameters of interest.

(A) Input to Program I

The following five (5) input parameters are required:

θ_L = azimuth of left dipole (MN) in degrees [Store in register 1]

θ_R = azimuth of right dipole (MN′) in degrees [Store in register 2]

ΔV_L = potential difference in millivolts for left dipole [Store in register 3]

ΔV_R = potential difference in millivolts for right dipole [Store in register 4]

ΔV_{RL} = potential difference in millivolts for right-left dipole [Store in register 5]

NOTE:

(1) If one of the ΔV's could not be measured store 0.00 in appropriate register.

(2) Use the value of ΔV_{RL} as measured without normalization to the length of MN = MN'.

(3) It is not required to measure the azimuth, θ_{RL}, of N'N dipole.

(4) Proper algebraic sign for measured ΔV values must be used.

(5) For ideal data, the value of $(\Delta V_L - \Delta V_R - \Delta V_{RL})$ is equal to zero.

(6) The lengths of MN and MN' are assumed to be equal for the use of this program.

To run Program I, after storing the above five parameters, PRESS A.

(B) Output of Program I

Three values (ψ_1, ψ_2, ψ_3) for the azimuth ψ of the measured "electric field" and their average (ψ_{AV}) are given. These are followed by three values $(\Delta V_1, \Delta V_2, \Delta V_3)$ for the measured total potential difference and their average ΔV_{AV}. The output parameters:

ψ_1, ΔV_1 are calculated from θ_L, ΔV_L and θ_R, ΔV_R,

ψ_2, ΔV_2 are calculated from θ_L, ΔV_L and θ_{RL}, $\Delta V_{RL(N)}$,

ψ_3, ΔV_3 are calculated from θ_R, ΔV_R and θ_{RL}, $\Delta V_{RL(N)}$,

where θ_{RL} is a calculated azimuth of the dipole N′N, and $\Delta V_{RL(N)}$ is the potential difference, between the N′ and N electrodes, normalized to the dipole length MN, with MN = MN′. The scatter in the values of ψ_1, ψ_2, ψ_3 (which is always accompanied by a scatter in the values of ΔV_1, ΔV_2, ΔV_3) is an index of the accuracy of the measurements.

NOTE:

> If only two components of ΔV are measured and the third one was too small or too noisy to measure (and therefore 0.000 was stored in the appropriate register) then no average for ψ or ΔV will appear in the output. Instead, one value for the appropriate angle (ψ_1, ψ_2, or ψ_3) and a corresponding total potential difference (ΔV_1, ΔV_2, or ΔV_3) will be printed. The zeros appearing for the other values of ψ and ΔV do not signify zero values but rather lack of information. Thus, the quality of data obtained with two components only cannot be properly checked.

(C) Examples for Program I

(1) Example for high quality data from three components:

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
$\theta_L = 269^o$	$\psi_1 = -81.840^o$	
$\theta_R = 2^o$	$\psi_2 = -80.628^o$	-81.840 *** -80.628 ***
$\Delta V_L = +0.46$ mv	$\psi_3 = -81.705^o$	-81.705 ***
$\Delta V_R = +0.05$ mv	$\psi_{AV} = -81.391^o$	-81.391 ***
$\Delta V_{RL} = +0.4$ mv	$\Delta V_1 = 0.466$ mv	0.466 *** 0.468 ***
	$\Delta V_2 = 0.468$ mv	0.456 ***
	$\Delta V_3 = 0.456$ mv	0.463 ***
	$\Delta V_{AV} = 0.463$ mv	

(2) Example of high (?) quality data from two components:

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
$\theta_L = 93^o$	ψ_1 = -----	
$\theta_R = 181^o$	ψ_2 = -----	0.000 *** 0.000 ***
ΔV_L = ----- (noisy)	$\psi_3 = 48.234^o$	48.234 *** 0.000 ***
$\Delta V_R = +1.1$ mv	ΔV_1 = -----	0.000 *** -1.620 ***
$\Delta V_{RL} = -2.25$ mv	ΔV_2 = -----	
	$\Delta V_3 = -1.620$ mv	

(3) Example of low quality data from three components:

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
$\theta_L = 272^\circ$	$\psi_1 = -5.887^\circ$	-5.887 ***
$\theta_R = 10^\circ$	$\psi_2 = -8.897^\circ$	-8.897 ***
$\Delta V_L = -0.1$ mv	$\psi_3 = -19.819^\circ$	-19.819 ***
$\Delta V_R = -0.70$ mv	$\psi_{AV} = -11.534^\circ$	-11.534 ***
$\Delta V_{RL} = +0.4$ mv	$\Delta V_1 = -0.728$ mv	-0.728 ***
	$\Delta V_2 = -0.528$ mv	-0.528 ***
	$\Delta V_3 = -0.807$ mv	-0.807 ***
	$\Delta V_{AV} = -0.688$ mv	-0.688 ***

(4) Example of high quality data (same as example 1) but with ΔV_L and ΔV_R interchanged:

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
$\theta_L = 269^\circ$	$\psi_1 = -7.150^\circ$	-7.150 ***
$\theta_R = 2^\circ$	$\psi_2 = +7.189^\circ$	7.189 ***
$\Delta V_L = +0.05$ mv	$\psi_3 = -60.541^\circ$	-60.541 ***
$\Delta V_R = +0.46$ mv	$\psi_{AV} = -20.167^\circ$	-20.167 ***
$V_{RL} = +0.4$ mv	$\Delta V_1 = +0.466$ mv	0.466 ***
	$\Delta V_2 = -0.351$ mv	-0.351 ***
	$\Delta V_3 = +0.998$ mv	0.998 ***
	$\Delta V_{AV} = 0.371$ mv	0.371 ***

NOTE: Large scatter in output values signify error in input or in measurements.

(D) <u>Input to Program II</u>

There are two modes (mode A or mode B) for inputing data in Program II which depend on whether the station coordinates are defined in terms of x and y (in miles) or in terms of AO and BO (in miles). See figure 1.

CAUTION: If the y-coordinate of a station is small with respect to its x-coordinate, DO NOT use the program in mode B (with input values of AO and BO), as "Error" may appear in calculator window or large errors in the computed values of y and of $\psi_o(N)$ [see definition for $\psi_o(N)$ in next section] may be obtained as a result of small errors in the measured values of AO and BO. Therefore, for $y << x$, measure x and y and run the program in mode A as described below.

(1) In mode A (x and y coordinates are used), the following eight (8) input parameters are required:

x = x-coordinate of station in miles [Store in Register 1]

y = y-coordinate of station in miles [Store in Register 2]

L = AB/2 = Half length of current bipole in miles = constant [Store in Register 3]

I = peak to peak current amplitude (in amperes) [Store in Register A]

ΔV = Average total potential difference obtained from Program I (in millivolts) [Store in Register B]

ψ = Average azimuth of total electric field (in degrees) from program I [Store in Register C]

MN = MN′ = length of potential dipoles (in feet) [Store in Register D]

β = angle of declination of bipole axis, measured in degrees clockwise from geographic North to electrode B (see figure 1). [Store in Register E]

To run Program II in mode A, after storing the above eight parameters, PRESS A.

2) In mode B (AO and BO coordinates are used), the following nine (9) input parameters are required:

AO = distance (in miles) from positive current electrode to station. [Store in Register 1]

BO = distance (in miles) from negative current electrode to station. [Store in Register 2]

L = as defined in mode A above. [Store in Register 3]

Side 1 or 2 = side 1 for positive values of y and side 2 for negative values of y (see figure 1). [Store 1 or 2 in Register 4]

I, ΔV, ψ, MN, and β = as defined in mode A above. [Store in Registers A, B, C, D, and E, respectively]

To run Program II in mode B, after storing the above nine parameters, PRESS B.

(E) Output of Program II

Nine output parameters (to either input (A) or (B) are given in the following order:

x = same as defined in input (A).

y = same as defined in input (A).

AO = same as defined in input (B).

BO = same as defined in input (B).

$\psi_o(N)$ = azimuth (0-360°) of primary electric field vector measured clockwise from geographic North.

$\psi(N)$ = azimuth (0-360°) of measured electric field vector measured clockwise from geographic North.

$\bar{\rho}_{|E|}$ = simple total field apparent resistivity.

$\bar{\rho}_{E_o}$ = primary field apparent resistivity.

$\bar{\rho}_{E}$ = complete total field apparent resistivity.

(F) Examples for Program II

(1) Example for mode (A):

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
x = -2.67 miles	x = -2.670 miles	-2.670 ***
y = 7.00 miles	y = 7.000 miles	7.000 ***
L = 0.981 miles	AO = 7.201 miles	7.201 ***
I = 24 amps	BO = 7.895 miles	7.895 ***
ΔV = 0.178 mv	$\psi_o(N) = 302.634^o$	302.634 ***
$\psi = -47.8^o$	$\psi(N) = 312.200^o$	312.200 ***
MN = 250 ft	$\bar{\rho}_{\lvert E \rvert}$ = 294.952 ohm-m	294.952 ***
$\beta = 0.000^o$	$\bar{\rho}_{E_o}$ = 290.851 ohm-m	290.851 ***
	$\bar{\rho}_E$ = 299.111 ohm-m	299.111 ***

(2) Example for mode (A) with erroneous data (same as example 1, but with erroneous algebraic sign for x):

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
x = 2.67 miles	x = 2.67 miles	2.670 ***
y = 7.00 miles	y = 7.000 miles	7.000 ***
L = 0.981 miles	AO = 7.895 miles	7.895 ***
I = 24 amps	BO = 7.201 miles	7.201 ***
ΔV = 0.178 mv	$\psi_o(N) = 57.366^o$*	57.366 ***
$\psi = -47.8^o$	$\psi(N) = 312.2^o$	312.200 ***
MN = 250 ft	$\bar{\rho}_{\lvert E \rvert}$ = 294.95 ohm-m*	294.952 ***
$\beta = 0.00^o$	$\bar{\rho}_{E_o}$ = -77.165 ohm-m*	-77.165 ***
	$\bar{\rho}_E$ = -1127.415 ohm-m*	-1127.415 ***

*NOTE: large deviation of $\psi_o(N)$ from $\psi(N)$ (which results in a correspondingly large deviation in the values of $\bar{\rho}_{|E|}$, $\bar{\rho}_{E_o}$, and $\bar{\rho}_E$) is more often caused by errors in inputting or measuring the data than by the effect of lateral heterogeneities in the ground.

(3) Example for mode (B)

INPUT	OUTPUT	SAMPLE OUTPUT FROM HP 97
AO = 6.65 miles*	x = -5.040 miles	-5.040 ***
BO = 8 miles*	y = 5.267 miles	5.267 ***
L = 0.981 miles	AO = 6.650 miles	6.650 ***
side = 1*	BO = 8.000 miles	8.000 ***
I = 24 amps	$\psi_o(N) = 255.0^o$	255.000 ***
ΔV = 0.276 mv	$\psi(N) = 284.6^o$	284.600 ***
$\psi = -75.4^o$	$\bar{\rho}_{\|E\|}$ = 311.172 ohm-m	311.172 ***
MN = 250 feet	$\bar{\rho}_{E_o}$ = 270.561 ohm-m	270.561 ***
β = 0.00	$\bar{\rho}_E$ = 357.879 ohm-m	357.879 ***

*NOTE: Values of AO, BO, and side must be re-stored in Registers 1, 2, and 4, respectively, for every re-run of a problem.

(V) THEORY

(A) Equations for Program I

(1) Computation of the angle θ_{RL}:

As shown in figure (2), the slope of the line N′N (for MN = MN′) is given by:

$$\text{slope} = \frac{\sin\theta'_L - \sin\theta'_R}{\cos\theta'_R - \cos\theta'_L} = \frac{\cos\theta_L - \cos\theta_R}{\sin\theta_R - \sin\theta_L}, \qquad (1)$$

and therefore the angle

$$\theta'_{RL} = \tan^{-1}\left\{\frac{\sin\theta'_L - \sin\theta'_R}{\cos\theta'_R - \cos\theta'_L}\right\} = \tan^{-1}\left\{\frac{\cos\theta_L - \cos\theta_R}{\sin\theta_R - \sin\theta_L}\right\}. \quad (2)$$

The angle, θ_{RL} which the dipole N′N forms with the geographic North (in a clockwise direction) is related to θ' by

$$\theta_{RL} = \alpha + \theta'_{RL} \quad (3)$$

where α is a multiple of $\frac{\pi}{2}$ and its value depends on the quadrants in which MN and MN′ are located.

(2) Normalization of ΔV_{RL}:

Inasmuch as the potential difference ΔV_{RL} is measured between the ends of the dipole N′N which in general is larger in length than MN, the value of ΔV_{RL} must be normalized to the common dipole length of MN (with MN = MN′). First, we obtain the length of N′N, using the law of cosines with MN = MN′, from

$$N'N = MN\sqrt{2(1 - \cos(\theta_L - \theta_R)}, \quad (4)$$

then, the normalized potential difference, ΔV_{RLN}, can be calculated from the equation

$$\Delta V_{RLN} = \Delta V_{RL} \times \frac{MN}{N'N} \quad (5)$$

or

$$\Delta V_{RLN} = \Delta V_{RL} \,/\, \sqrt{2(1 - \cos(\theta_L - \theta_R)}, \quad (6)$$

which indicates that the actual length of MN or N′N are not required for the computation of ΔV_{RLN} (provided that MN = MN′).

(3) Determination of ΔV and ψ from two or three components:

For a given total field of magnitude ΔV and direction ψ, the left and right components, ΔV_L and ΔV_R, are given by (see figure 3):

$$\Delta V_L = \Delta V \cos(\theta_L - \psi), \quad (7)$$

$$\Delta V_R = \Delta V \cos(\theta_R - \psi). \quad (8)$$

Using the relation

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta,$$

we can write

$$\Delta V_L = \Delta V(\cos\theta_L \cos\psi + \sin\theta_L \sin\psi), \quad (10)$$

$$\Delta V_R = \Delta V (\cos\theta_R \cos\psi + \sin\theta_R \sin\psi). \quad (11)$$

Solving equations (10) and (11) simultaneously, for ψ and ΔV, we get

$$\psi = \tan^{-1}\left\{\frac{\Delta V_L \cos\theta_R - \Delta V_R \cos\theta_L}{\Delta V_R \sin\theta_L - \Delta V_L \sin\theta_R}\right\}, \quad (12)$$

$$\Delta V = \Delta V_R/\cos(\theta_R - \psi) . \quad (13)$$

Similarly we can solve for ψ and ΔV from ΔV_L, θ_L and ΔV_{RLN}, θ_{RL}, or from ΔV_R, θ_R and ΔV_{RLN}, θ_{RL}. Such solutions result in values of ψ_1, ψ_2, ψ_3, and ΔV_1, ΔV_2, ΔV_3, which then can be averaged to obtain ψ_{AV} and ΔV_{AV}.

(B) <u>Equations for Program II</u>

(1) Equations relating AO, BO, x, y, and L:

It can be shown (see figure 1) that:

$$AO = \sqrt{(x + L)^2 + y^2} , \quad (14)$$

$$BO = \sqrt{(x - L)^2 + y^2} , \quad (15)$$

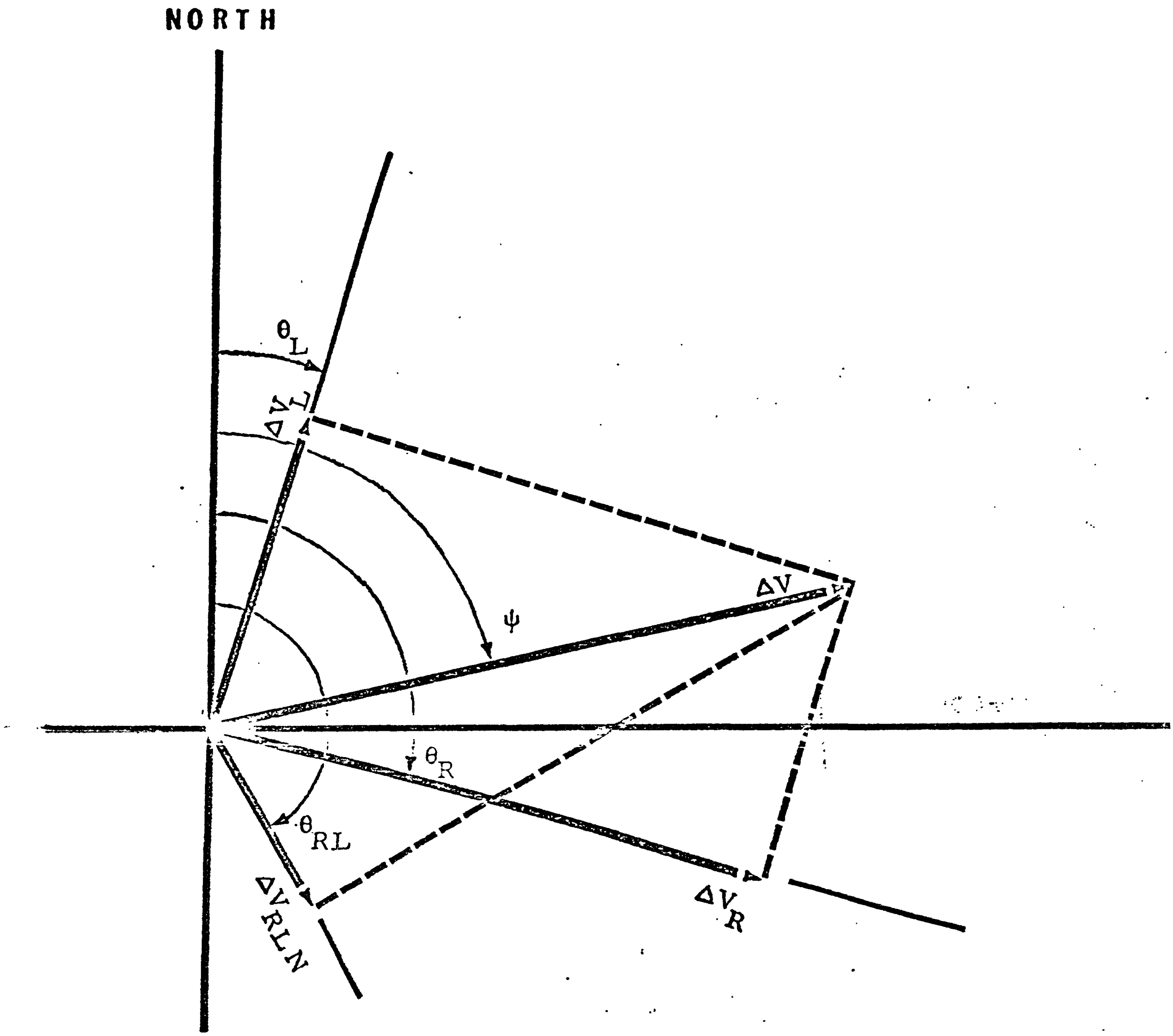

Figure 3.--Diagram showing relation between total field vector and its components.

$$x = \frac{AO^2 - BO^2}{4L}, \tag{16}$$

$$y = \pm\sqrt{AO^2 - \left(\frac{AO^2 - BO^2}{4L} + L\right)^2}. \tag{17}$$

Thus, if x and y are given, then we compute AO and BO from (14) and (15), whereas if AO and BO are given then we compute x and y from (16) and (17).

(2) Computation of $\psi_o(N)$

The angle, ψ_o, of the bipole primary electric field is computed from (Zohdy, 1973; Zohdy and Stanley, 1974)

$$\psi_o = \tan^{-1}\left[\frac{\frac{y}{AO^3} - \frac{y}{BO^3}}{\frac{x+L}{AO^3} - \frac{x-L}{BO^3}}\right] \tag{18}$$

This angle is measured positive in the <u>counterclockwise</u> direction. The angle $\psi_o(N)$ (which is the azimuthal value of ψ_o measured positive, from 0^o to 360^o, in a <u>clockwise</u> direction with respect to geographic North) is calculated from a knowledge of the quadrant in which the measuring station is located and from the angle of declination, β, of the bipole axis with respect to north. The proper manipulations for evaluating $\psi_o(N)$ will not be discussed here. The reason for evaluating $\psi_o(N)$ is to compare it directly with $\psi(N)$ which is also referred to the geographic North.

(3) Computation of $\psi(N)$

The algebraic signs of ψ and ΔV which are obtained from program I are used to evaluate $\psi(N)$, which expresses the value of ψ from 0° to 360° in the clockwise direction from geographic North. Thus for

$$\begin{aligned}
&+\psi,\ +\Delta V; \quad \psi(N) = \psi \\
&+\psi,\ -\Delta V; \quad \psi(N) = \psi + 180^{\circ} \\
&-\psi,\ +\Delta V; \quad \psi(N) = 360 + (-\psi) \\
&-\psi,\ -\Delta V; \quad \psi(N) = 180 + (-\psi)
\end{aligned} \tag{19}$$

(4) Computation of apparent resistivities:

The simple total field, $\bar{\rho}_{|E|}$, the primary field, $\bar{\rho}_{E_o}$, and the complete total field, $\bar{\rho}_E$, apparent resistivities are calculated from the following equations (Zohdy, 1973; Zohdy and Stanley, 1974; Zohdy, 1978):

$$\bar{\rho}_{|E|} = \frac{2\pi}{MN\left\{\left(\frac{x+L}{AO^3} - \frac{x-L}{BO^3}\right)^2 + \left(\frac{y}{AO^3} - \frac{y}{BO^3}\right)^2\right\}^{1/2}} \cdot \frac{\Delta V}{I} \tag{20}$$

$$\bar{\rho}_{E_o} = \bar{\rho}_{|E|} \cdot \cos\delta \ , \tag{21}$$

$$\bar{\rho}_{E} = \bar{\rho}_{|E|} / \cos\delta \ , \tag{22}$$

where $\delta = \psi(N) - \psi_o(N)$ = angle of rotation.

VI. APPENDIX

A summary of user instructions and program listings for Programs I and II are given on the following pages. Note that the given key codes are for the HP 97. The key entries of SPC (space) may either be deleted or replaced by PAUSE for HP 67 users.

User Instructions

PROGRAM I

Azimuth (ψ) and Magnitude (ΔV) of Total Field

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
0	Load sides 1 and 2				
	INPUT				
1	Store θ_L in Register 1	degrees	STO	1	
2	Store θ_R in Register 2	degrees	STO	2	
3	Store ΔV in Register 3	mv	STO	3	
4	Store ΔV_R in Register 4	mv	STO	4	
5	Store ΔV_{RL} in Register 5	mv	STO	5	
6	Press A				
	Repeat steps 1 through 6 for each station				
	NOTE: (1) Store observed (unnormalized) value				
	of ΔV_{RL} in Register 5				
	(2) If ΔV_L, ΔV_R or ΔV_{RL} $\approx$ 0; or if any				
	one of these values was not measured				
	press (CLx) to obtain zero and store				
	in corresponding register.				
	OUTPUT				
	ψ_1 = azimuth from left and right dipoles				degrees
	ψ_2 = azimuth from left and right left dipoles				degrees
	ψ_3 = azimuth from right and right left dipoles				degrees
	ψ_{AV} = average azimuth				degrees
	ΔV_1 = magnitude from left and right dipoles				mv
	ΔV_2 = magnitude from left and right left dipoles				mv
	ΔV_3 = magnitude from right and right left dipoles				mv
	ΔV_{AV} = average magnitude				mv
	NOTE: No average is given if only two components				
	are used. Printed zeros designate "no				
	data" not zero values.				

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Store 180° in R_E, and add 0.01 to θ_L to avoid division by zero in equation (2) in case $\theta_L = 45°$ and $\theta_R = 135°$ or $\theta_L = 225°$ and $\theta_R = 315°$.
002	1	01	
003	8	08	
004	0	00	
005	STOE	35 15	
006	RCL1	36 01	
007	.	-62	
008	0	00	
009	1	01	
010	+	-55	
011	STO1	35 01	
012	COS	42	Compute θ'_{RL} from equation (2)
013	RCL2	36 02	
014	COS	42	
015	-	-45	
016	RCL2	36 02	
017	SIN	41	
018	RCL1	36 01	
019	SIN	41	
020	-	-45	
021	÷	-24	
022	TAN⁻¹	16 43	
023	2	02	Compute θ_{RL} from equation (3)
024	7	07	
025	0	00	
026	+	-55	
027	RCL2	36 02	
028	RCLE	36 15	
029	+	-55	
030	X⇄Y	-41	
031	X>Y?	16-34	
032	GTO3	22 03	
033	RCLE	36 15	
034	-	-45	
035	*LBL3	21 03	
036	STO6	35 06	Compute ΔV_{RLN} from equation (6)
037	RCL1	36 01	
038	RCL2	36 02	
039	-	-45	
040	COS	42	
041	CHS	-22	
042	1	01	
043	+	-55	
044	2	02	
045	x	-35	
046	√X	54	
047	1/X	52	
048	RCL5	36 05	
049	x	-35	
050	STO7	35 07	
051	RCL2	36 02	Check if ΔV_L or $\Delta V_R = 0$, if not
052	COS	42	
053	RCL3	36 03	
054	X=0?	16-43	
055	GTO5	22 05	
056	x	-35	
057	RCL1	36 01	compute ψ_1 from equation (12) using ΔV_L and ΔV_R. Otherwise store zero in R_8.
058	COS	42	
059	RCL4	36 04	
060	X=0?	16-43	
061	GTO5	22 05	
062	x	-35	
063	CHS	-22	
064	+	-55	
065	RCL1	36 01	
066	SIN	41	
067	RCL4	36 04	
068	x	-35	
069	RCL2	36 02	
070	SIN	41	
071	RCL3	36 03	
072	x	-35	
073	CHS	-22	
074	+	-55	
075	÷	-24	
076	TAN⁻¹	16 43	
077	*LBL5	21 05	
078	STO8	35 08	
079	PRTX	-14	
080	RCL6	36 06	Check if ΔV_L or $\Delta V_{RLN} = 0$, if not compute ψ_2 from equation (12) using ΔV_L and ΔV_{RLN}. Otherwise store zero in R_9
081	COS	42	
082	RCL3	36 03	
083	X=0?	16-43	
084	GTO6	22 06	
085	x	-35	
086	RCL1	36 01	
087	COS	42	
088	RCL7	36 07	
089	X=0?	16-43	
090	GTO6	22 06	
091	x	-35	
092	CHS	-22	
093	+	-55	
094	RCL1	36 01	
095	SIN	41	
096	RCL7	36 07	
097	X=0?	16-43	
098	GTO6	22 06	
099	x	-35	
100	RCL6	36 06	
101	SIN	41	
102	RCL3	36 03	
103	x	-35	
104	CHS	-22	
105	+	-55	
106	÷	-24	
107	TAN⁻¹	16 43	
108	*LBL6	21 06	
109	STO9	35 09	
110	PRTX	-14	
111	RCL6	36 06	
112	COS	42	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	θ_L	θ_R	ΔV_L	ΔV_R	ΔV_{RL}	θ_{RL}	ΔV_{RLN}		

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCL4	36 04	check if ΔV_R or $\Delta V_{RLN} = 0$, if not compute ψ_3 from equation (12) using ΔV_R and ΔV_{RLN}. Otherwise store zero in R_0.
114	X=0?	16-43	
115	GTO7	22 07	
116	x	-35	
117	RCL2	36 02	
118	COS	42	
119	RCL7	36 07	
120	X=0?	16-43	
121	GTO7	22 07	
122	x	-35	
123	CHS	-22	
124	+	-55	
125	RCL2	36 02	
126	SIN	41	
127	RCL7	36 07	
128	x	-35	
129	RCL6	36 06	
130	SIN	41	
131	RCL4	36 04	
132	x	-35	
133	CHS	-22	
134	+	-55	
135	÷	-24	
136	TAN-1	16 43	
137	*LBL7	21 07	
138	STO0	35 00	
139	PRTX	-14	
140	RCL8	36 08	check if R_8, R_9, or $R_0 = 0$, if not compute ψ_{AV}.
141	X=0?	16-43	
142	GTO8	22 08	
143	RCL9	36 09	
144	X=0?	16-43	
145	GTO8	22 08	
146	+	-55	
147	RCL0	36 00	
148	X=0?	16-43	
149	GTO8	22 08	
150	+	-55	
151	3	03	
152	÷	-24	
153	SPC	16-11	
154	PRTX	-14	
155	SPC	16-11	
156	*LBL8	21 08	check if $R_8 = 0$, if not compute ΔV_1 from equation (13). Otherwise store zero in R_A
157	RCL1	36 01	
158	RCL8	36 08	
159	X=0?	16-43	
160	GTOa	22 16 11	
161	-	-45	
162	COS	42	
163	1/X	52	
164	RCL3	36 03	
165	x	-35	
166	*LBLa	21 16 11	
167	PRTX	-14	
168	STOA	35 11	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	RCL1	36 01	check if $R_9 = 0$, if not compute ΔV_2 from equation (13). Otherwise, store zero in R_B.
170	RCL9	36 09	
171	X=0?	16-43	
172	GTOD	22 14	
173	-	-45	
174	COS	42	
175	1/X	52	
176	RCL3	36 03	
177	x	-35	
178	*LBLD	21 14	
179	PRTX	-14	
180	STOB	35 12	
181	RCL2	36 02	check if $R_0 = 0$, if not compute ΔV_3 from equation (13). Otherwise, print zero
182	RCL0	36 00	
183	X=0?	16-43	
184	GTOE	22 15	
185	-	-45	
186	COS	42	
187	1/X	52	
188	RCL4	36 04	
189	x	-35	
190	*LBLE	21 15	
191	PRTX	-14	
192	X=0?	16-43	check if ΔV_3, R_A, or $R_B = 0$; if not compute ΔV_{AV}. Otherwise restore θ_L to original value and return.
193	GTO9	22 09	
194	RCLA	36 11	
195	X=0?	16-43	
196	GTO9	22 09	
197	+	-55	
198	RCLB	36 12	
199	X=0?	16-43	
200	GTO9	22 09	
201	+	-55	
202	3	03	
203	÷	-24	
204	SPC	16-11	
205	PRTX	-14	
206	*LBL9	21 09	
207	RCL1	36 01	
208	.	-62	
209	0	00	
210	1	01	
211	-	-45	
212	STO1	35 01	
213	CLX	-51	
214	RTN	24	
215	R/S	51	
220			

LABELS					FLAGS
A	B	C	D	E	0
a	b	c	d	e	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS (ON OFF)	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n____

User Instructions

PROGRAM II
Total Field Azimuths & Apparent Resistivities

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
	USE EITHER INPUT (A)				
1	Store x in Register 1	miles	STO	1	
2	Store y in Register 2	miles	STO	2	
3	Store L in Register 3	miles	STO	3	
4	Store I in Register A	Amps	STO	A	
5	Store ΔV in Register B	mvolts	STO	B	
6	Store ψ in Register C	degrees	STO	C	
7	Store MN in Register D	feet	STO	D	
8	Store β in Register E	degrees	STO	E	
9	PRESS A		A		
	OR USE INPUT (B)				
1	Store AO in Register 1	miles	STO	1	
2	Store BO in Register 2	miles	STO	2	
3	Store L in Register 3	miles	STO	3	
4	Store 1 or 2 in Register 4*	----	STO	4	
5	Store I in Register A	amps	STO	A	
6	Store ΔV in Register B	mvolts	STO	B	
7	Store ψ in Register C	degrees	STO	C	
8	Store MN in Register D	feet	STO	D	
9	Store β in Register E	degrees	STO	E	
10	PRESS B		B		
	*Note: side "1" is for y +				
	side "2" is for y -				
	OUTPUT FOR INPUT (A) OR (B)				
	x				miles
	y				miles
	AO				miles
	BO				miles
	ψ_o(N) (0-360^o CLOCKWISE from North)				degrees
	ψ(N) (0-360^o CLOCKWISE from North)				degrees
	$\bar{\rho}_{\lvert E \rvert}$				ohm-m
	$\bar{\rho}_{E_o}$				ohm-m
	$\bar{\rho}_E$				ohm-m

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Compute AO and BO from equation (14) and (15) also compute AO^3 and BO^3.
002	RCL1	36 01	x
003	PRTX	-14	Print x
004	RCL3	36 03	L
005	+	-55	
006	STO4	35 04	x+L
007	X^2	53	$(x+L)^2$
008	RCL2	36 02	y
009	PRTX	-14	Print y
010	SPC	16-11	
011	X^2	53	y^2
012	+	-55	$y^2+(x+L)^2$
013	√X	54	AO
014	PRTX	-14	Print AO
015	3	03	
016	Y^x	31	
017	STO5	35 05	AO^3
018	RCL2	36 02	y
019	X^2	53	y^2
020	RCL1	36 01	x
021	RCL3	36 03	L
022	-	-45	
023	STO6	35 06	x−L
024	X^2	53	
025	+	-55	
026	√X	54	
027	PRTX	-14	Print BO
028	SPC	16-11	
029	3	03	
030	Y^x	31	
031	STO7	35 07	BO^3
032	1/X	52	Compute $\bar{P}_{IE1}$ from equation (20)
033	CHS	-22	
034	RCL5	35 05	
035	1/X	52	
036	+	-55	
037	RCL2	36 02	
038	X	-35	
039	STO8	35 08	$y\left(\frac{1}{AO^3}-\frac{1}{BO^3}\right)$
040	X^2	53	
041	STO8	35 08	
042	RCL4	36 04	
043	RCL5	36 05	
044	÷	-24	$\frac{x+L}{AO^3}$
045	RCL6	36 06	
046	RCL7	36 07	
047	÷	-24	$\frac{x-L}{BO^3}$
048	-	-45	
049	STO9	35 09	$\frac{x+L}{AO^3}-\frac{x-L}{BO^3}$
050	X^2	53	
051	RCL8	36 08	
052	+	-55	
053	√X	54	
054	1/X	52	
055	2	02	
056	X	-35	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	Pi	16-24	
058	X	-35	
059	8	08	
060	4	04	
061	9	09	
062	7	07	
063	.	-62	
064	3	03	
065	3	03	
066	6	06	
067	X	-35	
068	RCLD	36 14	MN
069	÷	-24	
070	RCLA	36 11	I
071	÷	-24	
072	RCLB	36 12	ΔV
073	ABS	16 31	
074	X	-35	
075	P⇄S	16-51	
076	STO2	35 02	$\bar{P}_{IE1}$
077	P⇄S	16-51	
078	RCL8	36 08	compute ψ_0 from equation (18)
079	RCL9	36 09	
080	÷	-24	
081	TAN⁻¹	16 43	
082	CHS	-22	Compute ψ_0 (N) i.e. ψ_0 with respect to geographic North.
083	STOI	35 46	$-\psi_0$
084	1	01	
085	8	08	
086	0	00	
087	STO0	35 00	180
088	2	02	
089	X	-35	
090	STO9	35 09	360
091	RCL1	36 01	x
092	RCL2	36 02	y
093	X	-35	xy
094	0	00	
095	X≤Y?	16-35	
096	GTO1	22 01	
097	0	00	
098	RCLI	36 46	
099	X≤Y?	16-35	
100	GTO3	22 03	
101	*LBL4	21 04	
102	RCL0	36 00	
103	+	-55	
104	GTO2	22 02	
105	*LBL3	21 03	
106	RCL9	36 09	
107	+	-55	
108	*LBL2	21 02	
109	RCLE	36 15	β
110	+	-55	
111	RCL9	36 09	
112	X⇄Y	-41	

REGISTERS

0	1	2	3	4	5	6	7	8	9
- ,	x or AO	y or BO	L	x+L; or side 1 or 2	AO^3	x−L	BO^3	$\left(\frac{y}{AO^3}-\frac{y}{BO^3}\right)^2$	
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
I	ΔV	ψ	MN	β	

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	X≤Y?	16-35	
114	GTO5	22 05	
115	X⇄Y	-41	
116	-	-45	
117	*LBL5	21 05	
118	STOI	35 46	
119	PRTX	-14	
120	GTOb	22 16 12	
121	*LBL1	21 01	
122	RCL1	36 01	
123	X=0?	16-43	
124	GTO2	22 02	station on equatorial axis
125	RCL2	36 02	
126	X=0?	16-43	
127	GTO7	22 07	station on polar axis
128	RCLI	36 46	
129	X>0?	16-44	
130	GTO2	22 02	
131	GTO4	22 04	
132	*LBL7	21 07	
133	RCL3	36 03	
134	RCL1	36 01	
135	ABS	16 31	
136	X>Y?	16-34	
137	GTOc	22 16 13	
138	RCLI	36 46	
139	GTO2	22 02	
140	*LBLc	21 16 13	
141	RCLI	36 46	
142	GTO4	22 04	
143	*LBLb	21 16 12	Compute ψ(N) i.e. ψ with respect to geographic North
144	RCLB	36 12	
145	X>0?	16-44	
146	GTO8	22 08	
147	RCLC	36 13	
148	RCL0	36 00	
149	+	-55	
150	PRTX	-14	
151	SPC	16-11	
152	GTO9	22 09	
153	*LBL8	21 08	
154	RCLC	36 13	
155	X<0?	16-45	
156	GTOa	22 16 11	
157	PRTX	-14	
158	SPC	16-11	
159	GTO9	22 09	
160	*LBLa	21 16 11	
161	RCL9	36 09	
162	+	-55	
163	PRTX	-14	
164	SPC	16-11	
165	*LBL9	21 09	Compute cos δ, $\delta = \psi(N) - \psi_0'(N)$
166	RCLI	36 46	
167	-	-45	
168	COS	42	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	P⇄S	16-51	
170	STO1	35 01	
171	RCL2	36 02	$\overline{P}_{E1}$, $\overline{P}_{E_0}$, $\overline{P}_E$. Compute $\overline{P}_{E_0}$ and $\overline{P}_E$ and print $\overline{P}_{E1}$, $\overline{P}_{E_0}$ and $\overline{P}_E$
172	PRTX	-14	
173	×	-35	
174	PRTX	-14	
175	RCL2	36 02	
176	RCL1	36 01	
177	÷	-24	
178	PRTX	-14	
179	P⇄S	16-51	
180	RTN	24	
181	*LBLB	21 12	Compute x and y from equations (16) and (17) using A0, B0, and L.
182	RCL1	36 01	
183	X²	53	
184	RCL2	36 02	
185	X²	53	
186	-	-45	
187	RCL3	36 03	
188	÷	-24	
189	4	04	
190	÷	-24	
191	STO5	35 05	
192	RCL1	36 01	
193	X²	53	
194	RCL5	36 05	
195	RCL3	36 03	
196	+	-55	
197	X²	53	
198	-	-45	
199	√X	54	
200	STO6	35 06	
201	RCL4	36 04	
202	1	01	
203	X=Y?	16-33	
204	GTO6	22 06	
205	RCL6	36 06	
206	CHS	-22	
207	STO6	35 06	
208	*LBL6	21 06	
209	RCL5	36 05	
210	STO1	35 01	
211	RCL6	36 06	
212	STO2	35 02	
213	GTOA	22 11	
214	R/S	51	
220			

LABELS

A	B	C	D	E
I	ΔV			
a	b	c	d	e
0	1	2	3	4
5	6	7	8	9

FLAGS: 0, 1, 2, 3

SET STATUS

FLAGS (ON / OFF)	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n ___

References

Zohdy, A. A. R., 1973, Total field resistivity mapping [abstract]: Geophysics, v. 36, p. 1231.

Zohdy, A. A. R. and W. D. Stanley, 1974, A computer program for the calculation of bipole-dipole apparent resistivity maps over horizontally stratified media: NTIS (Natl. Tech. Inform. Serv., PB-232 727, Springfield, Va. 22151.

Zohdy, A. A. R., 1978, (in press), Total field resistivity mapping and sounding over horizontally layered media: Geophysics, v. 43, p. (?).