RISK ANALYSES FOR A WATER-SUPPLY SYSTEM— OCCOQUAN RESERVOIR, FAIRFAX AND PRINCE WILLIAM COUNTIES, VIRGINIA

U.S. GEOLOGICAL SURVEY
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By ROBERT M. HIRSCH

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Factors for converting U.S. customary units to SI metric units

SI (International System of Units) is a modernized metric system of measurement. An asterisk after the last digit of the factor indicates that the conversion factor is exact and that all subsequent digits are zero, all other conversion factors have been rounded to four significant digits.

To convert from:	To:	Multiply by:
inch (in)	millimeter (mm)	25.4*
foot (ft)	meter (m)	0.3048
mile (mi)	kilometer (km)	1.609
acre	meter^2 (m^2)	4,047
mile^2 (m^2)	kilometer^2 (km^2)	2.590
gallon (gal)	meter^3 (m^3)	.003785
million gallons per day (Mgal/d)	meter^3 per second (m^3/s)	.04381

RISK ANALYSES FOR A WATER-SUPPLY SYSTEM--OCCOQUAN RESERVOIR, FAIRFAX AND PRINCE WILLIAM COUNTIES, VIRGINIA

by

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ABSTRACT

This report demonstrates two techniques for evaluating risks in the operation of a water-supply system. Both rely on reconstructed historical streamflow data to develop estimates of the probabilities of certain specific events occurring in the future. These techniques are applied to the Occoquan Reservoir which was experiencing an unprecedented low level of storage in the autumn of 1977. The two techniques are used respectively to evaluate the overall adequacy of the existing reservoir and to evaluate the risks in the 1977 crisis.

In the first technique, the general risk analysis model (GRAM), simulations of the reservoir's contents are carried out under a set of assumptions about withdrawal rates and emergency procedures. The results of the GRAM simulation for the Occoquan Reservoir are in the form of estimates of the probabilities that in any year certain emergency procedures will have to be invoked. These estimates are given for a range of rates of withdrawals and for four different stages of emergency actions. Also given are the estimated probabilities of entering emergency conditions in one year given that an emergency had occurred in the previous year. Due to the year-to-year persistence of low flows, these latter (conditional) probabilities are higher than the former (marginal) probabilities.

The second technique used is position analysis. In this procedure probability distributions of future storages are estimated under existing storage conditions and an assumed rate of withdrawal from the reservoir. The position analysis which was initialized at the October 1, 1977, conditions indicates that the probability of entering a Stage III emergency (the prohibition of all uses of water non-essential to life, health, and safety) in the autumn of 1977 or winter of 1978 was 10 percent at that time. However, with a reduction in water use by 8 million gallons per day, this probability would have fallen to 4 percent.

If a long reconstructed historical streamflow record is available to a water supply agency, then the agency will have the capability to undertake its own risk analyses. It can carry out comparisons of alternative operating policies by using techniques such as GRAM. It can also evaluate short-term risk of different plans of operation during crisis situations by using position analysis.

INTRODUCTION

The purpose of this report is to demonstrate the use of historical hydrologic data in simulation models which evaluate the risks of alternative patterns of operation for a water supply reservoir. The techniques are applied to the Occoquan Reservoir, which is the primary source of water for the Fairfax County Water Authority. It supplies water to parts of Fairfax County, Prince William County, and to the City of Alexandria, Virginia.

This report provides an example of how some newly developed analytical tools may be used. Although applied in an emergency situation in this Fairfax example, the analytical procedures are equally applicable for use at any time, and may provide the technical basis for establishing emergency operating plans.

The first section of this report describes the methods used to reconstruct a suitable hydrologic record for use in the risk analyses. The second part describes a general risk analysis model (GRAM) and gives some results. This model simulates reservoir operation under a seasonally varying rate of water withdrawal and a set of emergency procedures that are to be invoked when reservoir storage falls below certain prescribed levels. The result of the risk analysis is an estimate of the probability of having to invoke any one of these emergency procedures.

The third part of this report describes a special form of risk analysis called position analysis. In position analysis the model takes the current reservoir contents and a projected rate of withdrawal over the next several months and estimates a probability of being unable to achieve these anticipated withdrawals.

Much of the information required to analyze the Occoquan Reservoir example was provided by the Interstate Commission on the Potomac River Basin and the Fairfax County Water Authority. The author wishes to thank specifically Daniel P. Sheer and Floyd Eunpu of these respective agencies for the information and encouragement they have supplied in pursuing this work.

Three questions are addressed in the application of the models to this example. The first question is: As of October 1, 1977, when the reservoir contents were at their historical record low level, what were the risks for the upcoming fall and winter period, and how would they change if the rates of water use were reduced? The second question is: What can be said about the reliability of the existing water-supply systems over the next few years? The third is: How can historical streamflow data be used, on an ongoing basis by water supply agencies, to help them evaluate their systems and the risks associated with operational decisions?

DATA USED

In making an assessment of risk in a water supply system, it is desirable to make use of all available hydrologic information. The ideal information in such a situation is a long continuous record of discharges of all of the streams feeding into the reservoir. Such a data set is often unavailable. In the case of the Occoquan Reservoir, a period of at least 49 years of good hydrologic information can be derived by judicious use of records from six stream gages. The gaging-station names, periods of record, gage numbers, and drainage areas are given in table 1. Figure 1 is a map of the watershed, showing the location of the reservoir, the streams, and the gages. For the 49 water years (1928-1976) used in the risk analysis, some drainage area adjustment or correlation with other gaging stations is necessary to reconstruct an appropriate record of reservoir inflows.

The drainage area at the Occoquan Reservoir dam is 570 square miles. For the period October 1937 through March 1956, the flow record of Occoquan Creek near Occoquan was used with an area adjustment factor of $1.044 = (570 \div 546)$ to account for the flow from the ungaged area. For April 1956 through March 1968, the gages on three tributaries, Bull Run near Manassas, Broad Run at Buckland, and Cedar Run near Catlett, were used. A drainage area adjustment factor of 1.95 was applied to the sum of their flows to account for the contribution of the ungaged area $1.95 = 570 \div (93.4 + 50.5 + 148)$. For April 1968 through September 1976, a new gage on the Occoquan River (near Manassas) was in operation. For this period it and the Bull Run gage were used with an area adjustment factor of 1.16 applied to the sum of their flows to account for the ungaged area $1.16 = 570 \div (343 + 148)$.

The technique of multiplying flow records by an area adjustment factor assumes that the flow from the ungaged area is equal to the flow from the gaged area, on a unit area basis. The

Table 1.--Stream gage information.

Stream and location	Beginning of record Month	Beginning of record Year	End of record Month	End of record Year	U.S. Geological Survey gage No.	Drainage area in square miles
Cedar Run near Catlett	Oct.	1950	Sept.	1976	01656000	93.4
Broad Run at Buckland	Oct.	1950	Sept.	1976	01656500	50.5
Occoquan River near Manassas	Apr.	1968	Sept.	1976	01656700	343
Bull Run near Manassas	Oct.	1950	Sept.	1976	01657000	148
Occoquan Creek near Occoquan	Oct.	1937	Mar.	1956	01657500	546
Rappahannock River at Kellys Ford	Oct.	1927	Sept.	1952	01664500	641

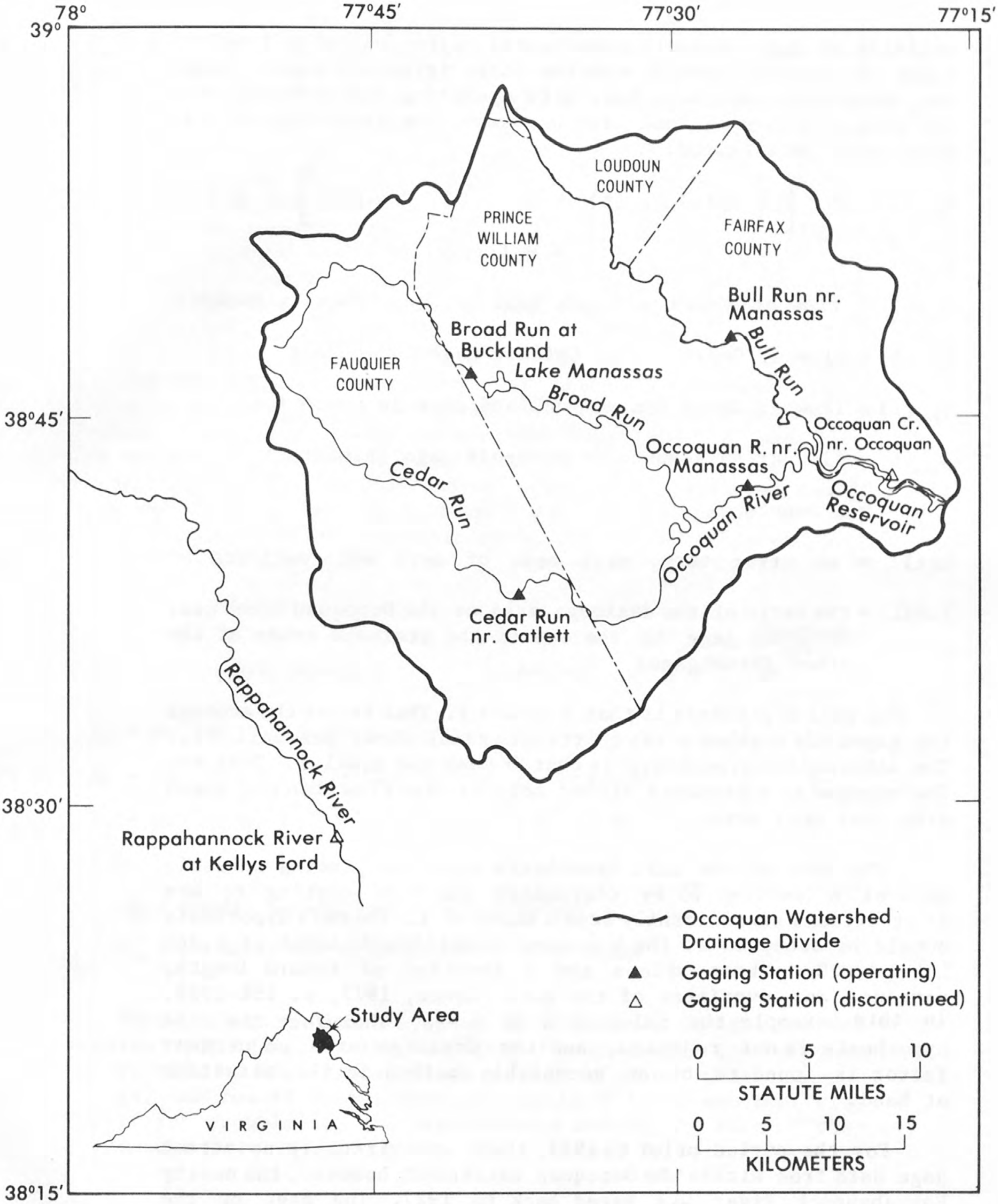

Figure 1.--Map of Occoquan watershed showing location of streams, gages, and reservoirs.

validity of this assumption was tested in the following fashion. Using the 66-month period when the three tributary gages, Cedar Run, Broad Run, and Bull Run, were operating concurrently with the gage on Occoquan Creek near Occoquan, the following statistical model was tested:

$$Q_o(t) = b \cdot \left[\left(Q_o(t)\ Q_{br}(t) + Q_{bu}(t) \right) \cdot 1.871 \right] + E(t)$$

$$t = 1,2,\ldots,66$$

$Q_o(t)$ = flow at Occoquan Creek near Occoquan gage in month t

$Q_c(t)$ = flow at Cedar Run at Catlett gage in month t

$Q_{br}(t)$ = flow at Broad Run at Buckland gage in month t

$Q_{bu}(t)$ = flow at Bull run near Manassas gage in month t

b = a constant

E(t) = an error term, with mean of zero and variance S^2

1.871 = the ratio of the drainage area at the Occoquan Creek near Occoquan gage to the sum of the drainage areas of the other three gages

The null hypothesis is that b equals 1. That is, on the average the gaged and ungaged areas contribute equal flows per unit area. The alternative hypothesis is that b does not equal 1. That is: The ungaged area produces either more or less flow than the gaged area, per unit area.

The test of the null hypothesis involves finding an estimate of b (denoted $\hat{b}$) by regression and then testing to see if it differs significantly from a value of 1. The null hypothesis should be accepted at the 5 percent significance level if $0.980 \leq \hat{b} \leq 1.020$. These values are a function of record length, variance, and covariance of the data (Seber, 1977, p. 191-192). In this example the value of $\hat{b}$ is 0.998. Therefore the null hypothesis is not rejected, and the drainage area adjustment factor is found to be an acceptable method in the situation at hand.

For the period prior to 1937, there are virtually no stream gage data from within the Occoquan watershed; however, the nearby Rappahannock River was gaged back to 1927. The gage on the

Rappahannock River at Kellys Ford covers a drainage area similar in size and physiography to that of the Occoquan River. Both are largely Piedmont watersheds, although the Rappahannock extends slightly into the Blue Ridge. The basins are adjacent, so they would both be subject to many of the same meteorological conditions. An examination of the correlation between the flows at these two gages for the 180-month overlap period shows their correlation to be significant at the 1 percent level. This clearly indicates that the Rappahannock record can provide useful information about Occoquan flows for the 10-year period from October 1927 through September 1937.

The statistical model that was selected to estimate the values of Occoquan flows for those 10 years was a set of three seasonal relationships in which the logarithms of the flows were assumed to be linearly related. For each month of the 1927 through 1937 period, the estimated value of the Occoquan runoff (expressed in inches per month) is calculated from the appropriate regression equations using the value of Rappahannock runoff for that month.

Relationship	Months	R^2	Standard error in percent
$Q_o = \exp\,[-0.300 + 1.239\,(\log Q_R)]$	Oct,Nov,Dec,Jan	.83	58
$Q_o = \exp\,[-0.283 + 1.249\,(\log Q_R)]$	Feb,Mar,Apr,May	.72	38
$Q_o = \exp\,[-0.572 + 0.923\,(\log Q_R)]$	June,July,Aug,Sept	.62	68

where

Q_R = monthly Rappahannock runoff in inches

Q_o = estimated monthly Occoquan runoff in inches

These relationships are shown in figures 2, 3, and 4. One important point illustrated in all of these figures is that when the runoff in the Rappahannock basin is low, then, on the average, the runoff in the Occoquan basin will be substantially lower. For example, if the Rappahannock had 0.100 inch of run-off in October, then the expected value for the Occoquan (using figure 2) would be 0.043 inch. In contrast to the low flow situation, if the October runoff of the Rappahannock were 5 inches, then the expected value for the Occoquan would be 5.44 inches. The virtue of using the logarithms of the flow values rather than the untransformed flow values is that the statistical model has the capability of depicting these differences in base flow characteristics as well as the differences in high flow characteristics.

It should be noted that a record extended by the method of regression estimates will produce records with less variability than actual historical records. If one were interested in the statistical properties (means, variances, covariances) of the flow records themselves these extended records based on regression could not be used without making corrections such as those described by Matalas and Jacobs (1964). Even though these values based on regression estimates do not give a valid statistical representation of the Occoquan streamflows, each value is, in and of itself, the best least square estimate of the streamflow for the month in question.

An additional adjustment was made to the streamflow records to attempt to account for the effects of Lake Manassas, an impoundment located on Broad Run. The drainage area of this reservoir is approximately 60 square miles. It began filling in 1968, and thus its effect is explicitly considered in the period 1968 through 1976 because the gage on the Occoquan River near Manassas is downstream from the reservoir. For the pre-impoundment portion of the record (1927-1968) adjustments were needed to enable the record to simulate conditions with the impoundment.

For the two-year period 1975 and 1976, it is possible to estimate the effect of this impoundment and diversion. For these two years there is not only the upstream gage record of Broad Run at Buckland (drainage area 50.5 square miles), but also the downstream record of Broad Run near Bristow (drainage area 89.6 square miles, U.S. Geological Survey gage number 01656450). For these two years the observed runoff at the upper gage was 18.4 inches per year while at the lower gage it was 15.6 inches per year. Relying, once again, on the assumption of equal flows from equal areas, it appears that approximately 30 percent of the flow at the upper gage is lost by evaporation and diversions at this reservoir. Thus, an amount equal to 30 percent of the flow at the gage on Broad Run at Buckland is subtracted from the total flow as calculated above.

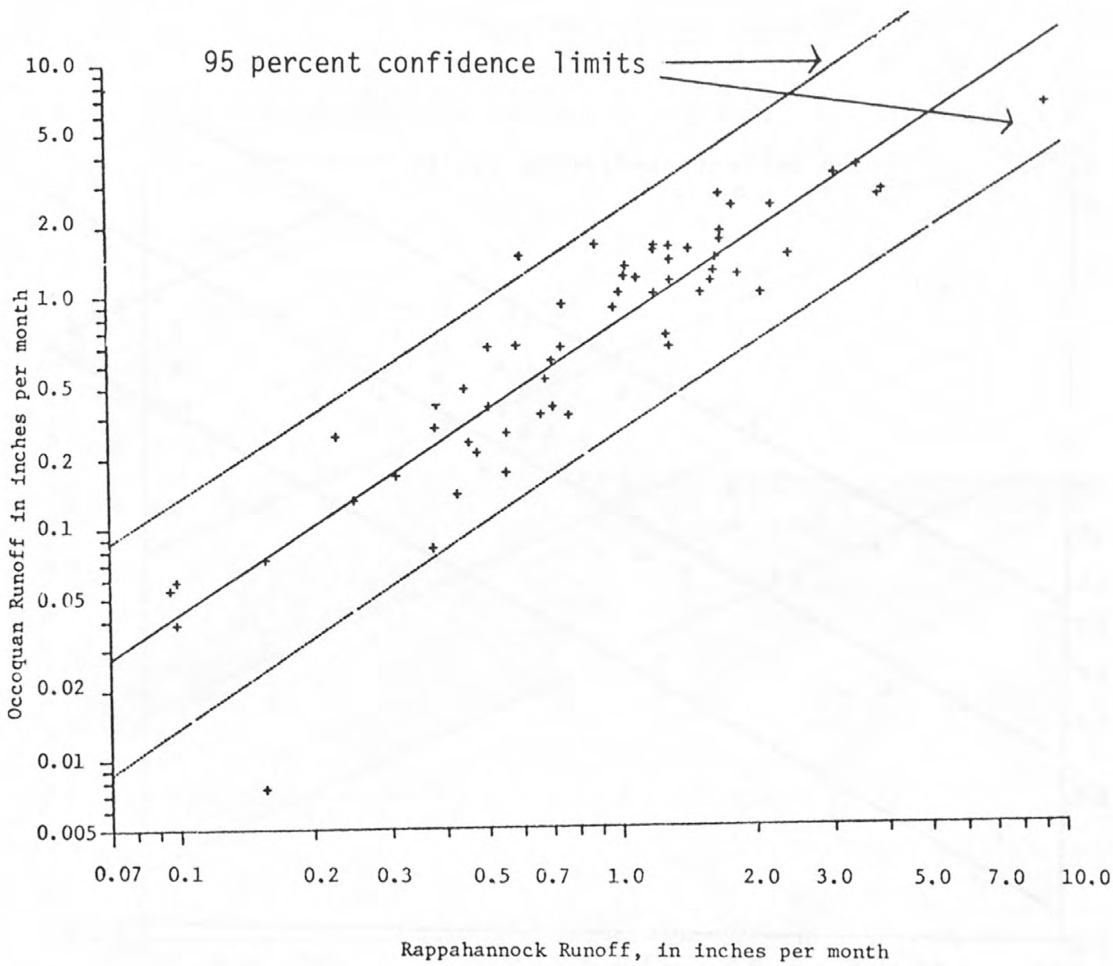

Figure 2.--Regression relationship between the runoff of the Rappahannock River at Kellys Ford and the Occoquan Creek near Occoquan, October through January, with 95 percent confidence limits for predicted values.

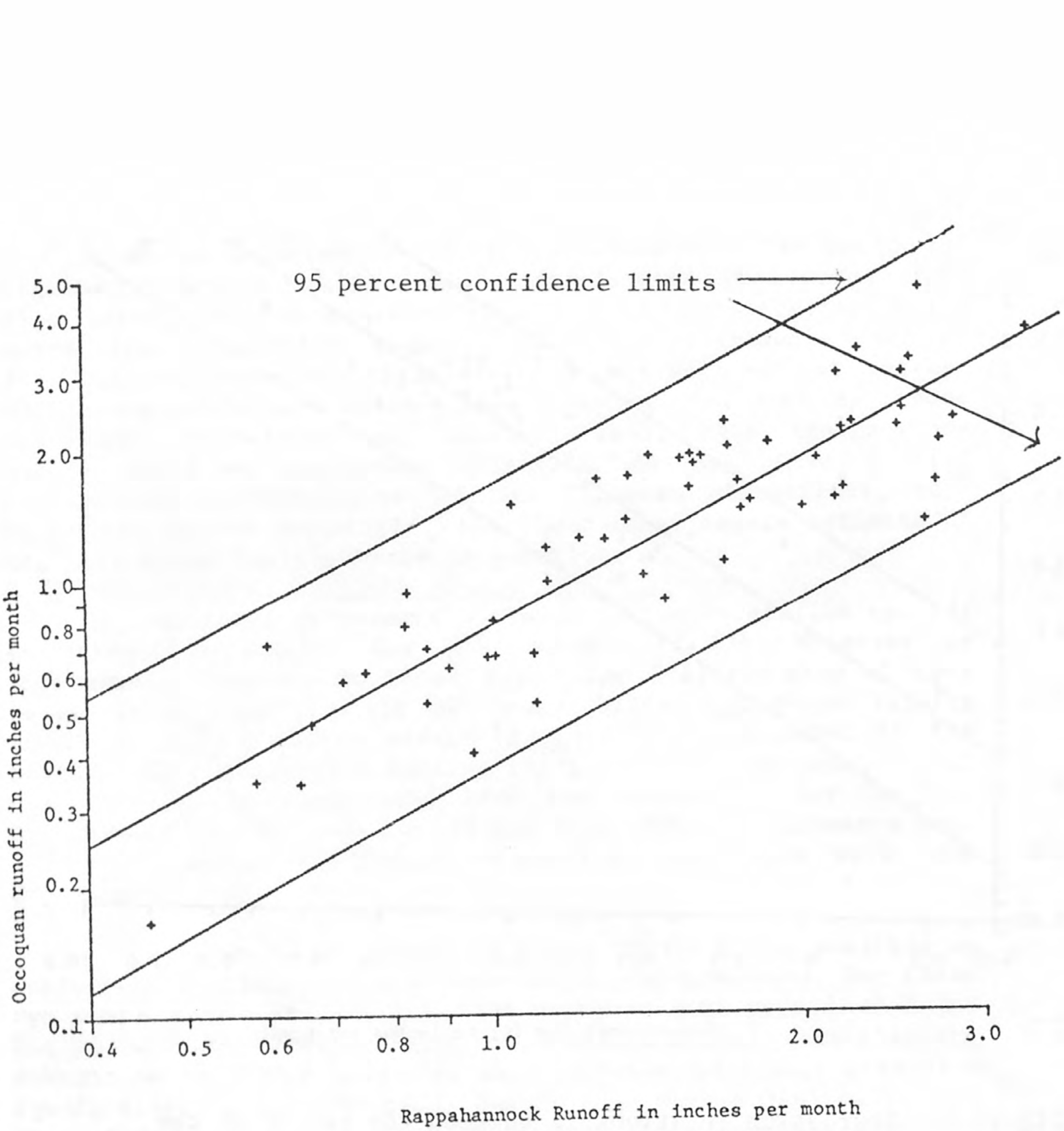

Figure 3.--Regression relationship between the runoff of the Rappahannock River at Kellys Ford and the Occoquan Creek near Occoquan, February through May, with 95 percent confidence limits for predicted values.

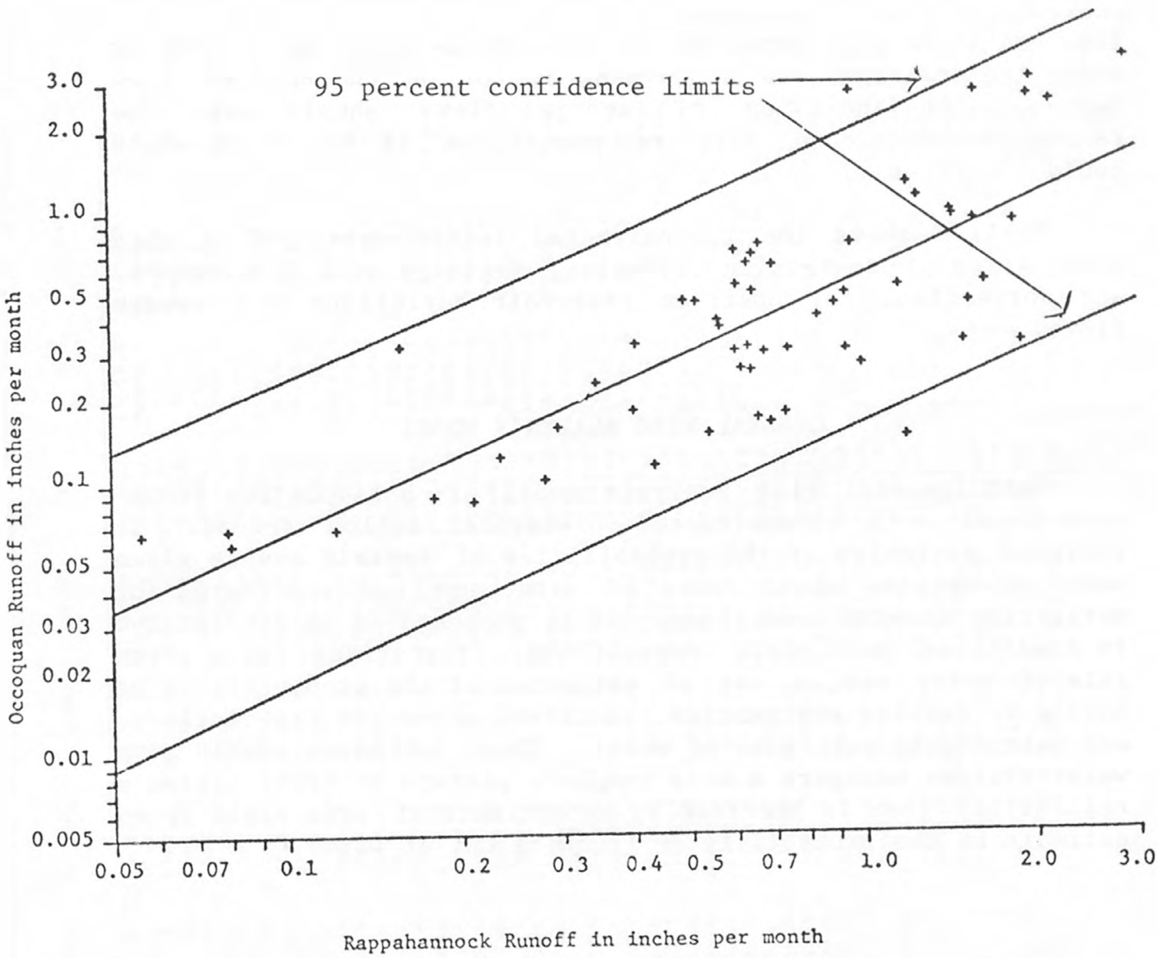

Figure 4.--Regression relationship between the runoff of the Rappahannock River at Kellys Ford and the Occoquan Creek near Occoquan, June through September, with 95 percent confidence limits for predicted values.

The final adjustment of these flow records is the adjustment for sewage inflows. At present there is an inflow of approximately 5 Mgal/d (million gallons per day) of treated sewage to streams of the Occoquan watershed (oral communication, John Thillman, 1977). This amount has grown rapidly in the past 15 years. About 3 Mgal/d of this effluent enters upstream from the Bull Run gage and 2 Mgal/d downstream from it. The reconstructed flow records were adjusted by adding the full 5 Mgal/d to the pre-1963 records and adding an amount decreasing from 5 to 2 Mgal/d from 1963 to 1976. Also, some flow had to be subtracted due to the double counting induced by using the drainage area adjustment factor on the Bull Run flow values. The inclusion of sewage flows should make the reconstructed record more representative of the flows which could occur today.

Table 2 shows the reconstructed record developed by this combination of regression estimates, drainage area adjustments, and corrections for upstream reservoir depletions and sewage flows.

GENERAL RISK ANALYSIS MODEL

GRAM (general risk analysis model) is a simulation procedure based on a reconstructed historical inflow record. It provides estimates of the probabilities of certain events given some assumptions about rates of water use and procedures for mitigating drought conditions. It is proposed as an alternative to traditional safe yield computations. It provides, for a given rate of water use, a set of estimates of the probabilities of having to declare emergencies, institute water-use restrictions, and make costly purchases of water. These estimates should give water-systems managers a more complete picture of their system's reliability than is provided by an estimate of safe yield or an estimate of the probability of "running out of water."

Table 2.--Reconstructed historic stteamflow record for the Occoquan Reservoir, in Mgal.

WATER YEAR	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.
1928	15810.4	8891.2	14573.9	5679.9	11893.2	8325.0	19102.9	11991.5	3899.6	3025.7	12955.1	6965.5
1929	3775.7	2685.6	5564.1	8625.7	10252.6	19439.1	22890.7	21725.3	7234.7	4162.1	1481.6	908.6
1930	20961.2	7529.7	6342.1	6112.7	10988.6	10996.0	7759.4	3193.1	1527.8	612.5	212.9	368.4
1931	169.4	299.3	467.5	1290.2	685.0	2050.9	3915.8	5967.3	2137.2	2383.3	3268.9	1351.1
1932	412.0	438.0	699.6	5636.9	6085.8	13169.4	9786.1	24663.3	3163.1	2318.2	880.4	436.7
1933	7076.1	32216.4	10623.8	15358.9	13431.1	13664.3	42834.4	17775.5	4033.3	3999.9	6374.5	4392.6
1934	2369.0	1783.0	1983.8	3099.5	1419.4	16688.2	8696.5	5546.4	4656.5	1915.5	2699.3	10486.4
1935	4507.2	5691.6	21053.6	16927.5	21916.6	16157.4	25680.6	9845.2	3635.0	3536.2	2193.9	8666.5
1936	2904.2	6777.4	8608.8	29443.9	32580.5	51559.9	14716.7	5148.6	2374.2	2392.1	1380.3	733.1
1937	4014.2	1493.3	12250.3	32124.4	28073.8	12895.6	66387.6	16114.1	8338.8	3936.5	10730.3	4794.7
1938	24966.1	14037.1	5766.1	10974.7	11591.4	9728.9	6564.6	3508.7	3136.6	4531.9	4579.8	973.5
1939	861.6	1760.2	13368.3	15441.5	28431.6	16286.7	12098.5	3468.7	3220.7	9418.3	4810.9	1342.6
1940	1421.8	5842.2	3364.0	5918.8	18559.6	16579.5	32121.1	10727.1	3165.9	3012.3	1542.6	1726.1
1941	1491.4	14684.9	11629.4	13676.3	6872.4	12200.1	14718.7	1755.4	3894.5	5247.6	1260.2	1003.7
1942	723.0	222.2	1799.0	2157.9	7728.1	18674.4	6222.4	2874.4	3067.0	3696.5	28185.3	7231.1
1943	60417.7	6490.4	17718.3	9726.3	19851.9	21969.0	14075.2	10021.8	3045.4	1141.4	785.4	725.3
1944	679.2	14096.2	2540.8	15683.2	7072.0	28440.7	13374.5	4029.5	2369.0	805.8	3210.6	3053.5
1945	8464.1	3914.2	14762.5	11456.7	14536.7	7926.3	5820.5	6612.3	6873.9	25706.9	25336.5	24360.3
1946	4263.6	8798.6	25510.6	16267.0	18328.4	8805.6	5181.6	44366.8	8715.9	10643.7	11953.1	1904.0
1947	2588.7	2377.5	3304.5	12012.9	4671.8	15800.1	6096.4	9240.3	6649.3	4903.5	1794.8	1594.7
1948	941.7	9647.1	3268.7	12618.4	16801.6	17689.6	15628.9	30486.3	4429.0	2724.0	33656.8	2600.8
1949	10920.2	22772.4	33543.9	31014.3	21269.1	18265.9	16098.4	13108.3	3237.8	8558.0	4447.8	1858.7
1950	3070.1	3028.3	5803.9	5107.1	18761.8	22259.3	5196.7	19501.2	5111.3	7216.4	1687.1	9069.2
1951	9793.9	11282.3	26277.6	9694.8	23506.4	21527.1	19927.0	6681.9	23146.1	6083.0	2146.7	800.8
1952	527.3	2729.5	15265.5	22768.2	14163.0	22332.2	35371.1	14815.7	4000.8	5185.9	2630.7	6185.7
1953	1256.3	24927.7	14217.4	26574.4	11660.4	29544.0	13738.9	20465.2	7644.8	2275.5	998.2	966.9
1954	686.9	850.7	7162.6	9424.9	3550.9	11866.5	5342.5	4448.5	1016.2	814.3	1158.4	723.3
1955	591.7	819.8	4720.2	2280.1	7909.3	18702.0	6485.8	11412.0	18159.1	2565.7	46782.1	2377.5
1956	3609.0	2673.7	1736.3	3661.1	17503.0	20169.0	9174.4	1841.2	1419.2	41279.3	5317.4	5511.8
1957	11076.7	13643.6	11900.6	13748.2	20337.4	17455.9	16734.6	2141.6	1372.6	466.0	405.1	1872.7
1958	4115.8	4454.6	32409.5	24280.5	17441.3	36290.9	19722.7	18333.4	2313.6	10534.2	3162.2	831.8
1959	945.5	1140.0	1408.4	5653.7	4702.4	9918.5	15713.4	2771.8	2704.2	1121.2	978.8	5614.5
1960	1747.3	3764.8	7134.2	11010.3	26783.6	15675.0	24419.9	19998.5	8921.2	1965.6	6778.6	7735.7
1961	1399.8	1452.5	1733.2	15670.9	46472.7	22835.3	23193.5	9851.4	4666.8	2822.4	2361.0	3063.4
1962	1490.9	2818.6	11097.9	11280.8	17947.5	39552.2	19248.4	7279.3	3821.2	1998.4	618.1	494.6
1963	596.5	6649.5	4757.0	17369.6	7605.2	35209.5	3324.4	1637.7	7336.8	590.1	675.9	382.1
1964	331.5	8729.5	6057.6	35012.7	22708.3	16330.7	19039.1	6557.4	803.5	973.1	405.9	894.5
1965	2078.5	2416.9	8489.6	19626.2	25239.1	32438.3	8258.1	2437.8	748.5	669.3	1795.6	537.2
1966	1265.5	485.6	555.3	840.7	16991.0	12109.8	10347.6	12126.2	1394.3	396.9	302.7	16839.4
1967	13758.2	3078.7	9106.0	15584.5	11873.9	32104.6	3534.6	8213.8	1653.9	1959.7	30325.3	1443.8
1968	1688.2	1620.5	27114.7	25467.6	6459.5	21605.8	1456.3	7585.7	14806.4	4121.0	1061.3	499.9
1969	443.0	4769.0	4998.0	9911.4	11834.1	12117.8	3834.9	1405.8	3777.0	2077.7	6131.5	6027.9
1970	1019.6	2307.2	17989.6	19239.4	25558.2	13583.1	28394.2	5107.4	2457.0	12423.2	1426.0	526.8
1971	687.4	23509.0	16195.4	19231.6	51213.3	18552.5	19993.4	35189.3	12346.8	1189.5	2674.9	1917.4
1972	16902.0	14527.2	11655.1	10876.9	48075.0	18025.2	22016.8	21452.0	127064.4	7567.4	1428.9	611.3
1973	4840.5	41490.0	50509.7	23612.9	26770.6	17480.6	49442.9	15386.9	4568.9	2239.9	8559.8	3683.5
1974	3211.4	1470.1	33873.4	27400.0	7143.9	11214.6	13029.6	8578.8	8675.7	1041.4	1674.4	6064.0
1975	1058.6	700.1	40260.4	17579.7	15439.6	37977.7	7683.0	8071.0	2615.3	13334.6	5546.7	71291.6
1976	10746.0	9837.2	11757.4	45301.7	12763.8	10321.9	19236.2	3386.0	2233.3	2051.8	642.6	1995.2

The GRAM simulation for the Occoquan Reservoir can be characterized by the following statement of the continuity equation:

$$S^*(t+1) = S(t) + I(t) + M(t) - E(t) - W(t)$$

$$t = 1,2,3,\ldots,588 \qquad (1)$$

$$S(t+1) = \begin{cases} 0 & \text{if } S^*(t+1) \le 0 \\ C & \text{if } S^*(t+1) \ge C \\ S^*(t+1) & \text{otherwise} \end{cases} \qquad (2)$$

$$t = 1,2,3,\ldots,588$$

$$S(1) = C \qquad (3)$$

where

$S(t)$ = storage at the beginning of month t in Mgal
$I(t)$ = reconstructed historic inflow in month t in Mgal
$M(t)$ = purchased inflow from Lake Manassas in month t in Mgal
$E(t)$ = evaporation from the reservoir in month t in Mgal
$W(t)$ = withdrawal from the reservoir in month t in Mgal
$S^*(t+1)$ = tentative storage at beginning of month t+1 in Mgal
$S(t+1)$ = storage at begining of month t+1 in Mgal
C = capacity of the reservoir in Mgal

Equation 1 simply accounts for all additions and depletion of storage to calculate the beginning storage for the next month. Equation 2 assures that storages are neither greater than capacity nor less than zero. Equation 3 is the initial storage condition.

The time step of the model is monthly. This biases the results slightly towards higher estimates of reliability because a drought ending in the middle of some month will appear in the simulations to have ended at the end of the previous month, and any deficiency of water in the early part of that month will go uunnoted.

The underlying assumption of GRAM is that the record of past hydrologic events is a representative sample, and therefore provides the best estimate of possible future conditions. That is, the best way to estimate probabilities of certain types of events (such as storage falling below some specified level) occurring in the future is to determine the frequency with which they would have occurred in the past. One must, therefore, be willing to accept the assertion that any event has the same probability of occurring in the future as it did in the past (the process is stationary). Because the past record is finite, it is never possible to determine exactly what the future probability is, but it is possible to make estimates of the probability which incorporate this uncertainty. These probability-estimating techniques will be described following a a more detailed description of the GRAM simulation procedure.

Implementation of the GRAM Simulation for the Occoquan Reservoir

The reservoir has a capacity (C) of 9,800 Mgal (million gallons). The relationship of surface area to volume is shown graphically in figure 5. Evaporation is computed as the product of surface area and the monthly evaporation rate (table 3). The surface area for month t referred to here is calculated as the average of the area corresponding to a storage of $S(t)$ and the area corresponding to a storage of $\hat{S}(t+1)$ where

$$\tilde{S}(t+1) = S(t) + I(t) + M(t) - W(t)$$

and

$$\hat{S}(t+1) = \begin{cases} 0 & \text{if } \tilde{S}(t+1) \leq 0 \\ C & \text{if } \tilde{S}(t+1) \geq 0 \\ \tilde{S}(t+1) & \text{otherwise} \end{cases}$$

The rate of withdrawal $W(t)$ is computed as

$$W(t) = \left[1.05 \cdot P(t) + 1\right] \cdot L(k) \qquad (4)$$

$$P(t) = u(t) - F(t) \qquad (5)$$

$$u(t) = \alpha(t) \cdot \theta(k) \cdot \overline{u} \qquad (6)$$

t = index of time in months $\quad t = 1,2,\ldots,588$

k = index of the month of the water year $\quad k = 1,2,\ldots,12$
(if y is the water year, $y = 1928, 1929,\ldots,1976$, then $12 \cdot (y - 1928) + k = t$)

$P(t)$ = rate of production in month t in Mgal/d (these are the values reported by the Fairfax County Water Authority (FCWA)

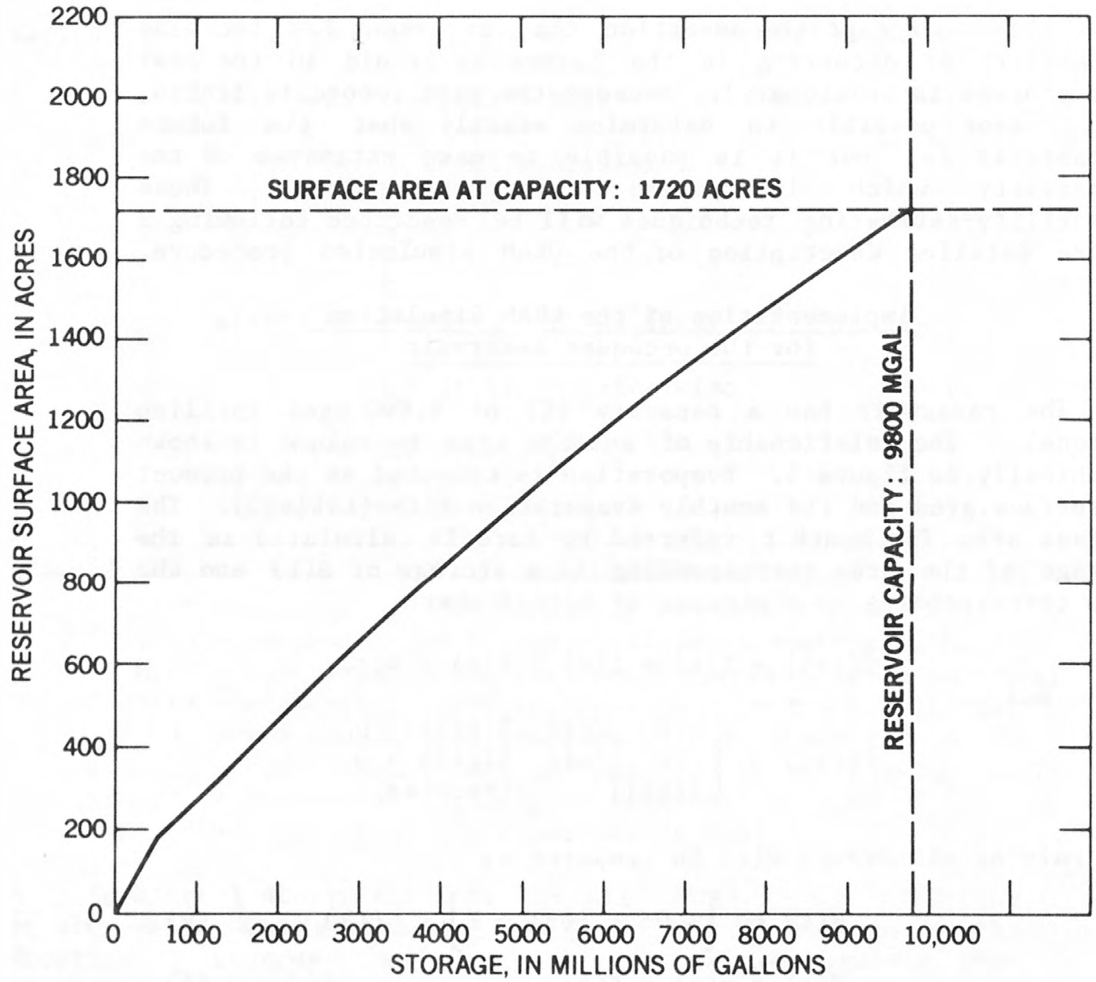

Figure 5.--Relationship of surface area to volume for the Occoquan Reservoir.

Table 3.--Evaporation rates in inches per month, developed from Kohler and others (1959).

Month	Evaporation in inches
Jan	1.3
Feb.	1.1
Mar.	1.5
Apr.	2.4
May	3.3
June	4.1
July	4.6
Aug.	5.0
Sept.	4.5
Oct.	3.7
Nov.	2.7
Dec.	1.8
Annual Total	36.0

$L(k)$ = number of days in month k

$u(t)$ = rate of water use by FCWA customers in Mgal/d in month t

$F(t)$ = rate of finished water purchases by FCWA in Mgal/d

$\alpha(t)$ = conservation factor for month t

$$0 \leq \alpha(t) \leq 1$$

If $\alpha(t) = 1$, customers are unrestricted in their use of water.

If $\alpha(t) < 1$, customers restrict their use of water in response to voluntary or mandatory water use restrictions.

$\alpha(t)$ represents the fraction of ordinary use which occurs in month t.

$\theta(k)$ = seasonality factor for month k

$\bar{u}$ = average (unrestricted) rate of water use by FCWA customers in Mgal/d

The reasons for the difference between the withdrawals ($W(t)$) and the rate of production ($P(t)$) shown in equation 4 are these: The water treatment process uses (for filter backwashing) and then discharges to a point below the dam an amount equal to about 5 percent of the water the FCWA delivers to its customers. In addition, the FCWA delivers about 1 Mgal/d of raw water to the Lorton Correctional Facility, and this amount is not reported in FCWA production figures.

The finished water purchase ($F(t)$) comes from a number of sources although the great majority of it is treated Potomac River water. This purchase ranges from a minimum of 5 Mgal/d up to 19 Mgal/d. The actual amount is determined according to the emergency procedures described below.

The conservation factor $\alpha(t)$ is also determined by the emergency procedures. It ranges in value from 0.8 to 1.0. The values assigned to $\alpha(t)$ in the GRAM simulation are based on subjective estimates by FCWA officials (oral communication, F. Eunpu and J. Warfield, 1977) of the effect of certain requests for voluntary or mandatory water use reductions.

The seasonality factors $\theta(k)$ represent the ratio of water use rates in month k to the annual average water use rate ($\bar{u}$). These seasonality factors were estimated from several years of FCWA production and sales data. Ten different GRAM simulations were carried out, each with a different value of $\bar{u}$ (from 40 Mgal/d to 85 Mgal/d). Table 4 shows, for each of these $\bar{u}$ values, the 12 monthly production rates and the annual average of these production rates assuming F(t) = 5 Mgal/d and $\alpha(t) = 1$. The variable $\bar{P}$ (average annual production) is defined $\bar{P} = \bar{u} - 5$ Mgal/d.

The values of the variable $\alpha(t)$, F(t), and M(t) used in the GRAM simulations are all determined in accordance with a set of emergency procedures. They are based on a memorandum to the Fairfax County Board of Supervisors from the County Executive (Leonard Whorton September 21, 1977). The procedures used in GRAM approximate those described in the memorandum but, of necessity, are much less flexible. The procedures used in GRAM involve four different emergency stages denoted I, II-A, II-B, and III. The water level at the beginning of the month determines which emergency stage is to be invoked for the coming month. The procedures are set forth in table 5.

To illustrate the way these procedures are incorporated in the simulation, consider the following example: Suppose at the end of September the reservoir contents were 1,800 Mgal, then for October the Stage II-B emergency procedures would be invoked. Suppose further that the water use (u(t)) in October would have been 65 Mgal/d in the absence of an emergency, then the water use would be reduced by conservation to 52 Mgal/d (a 20 percent reduction). The finished water purchases (F(t)) to satisfy this requirement would be 19 Mgal/d so that the Occoquan Reservoir would have to produce (P(t)) 33 Mgal/d. The inflow to the reservoir would be the natural plus sewage inflow plus an additional 38.7 Mgal/d from Lake Manassas [38.7 Mgal/d is 1,200 Mgal divided evenly over 31 days.]. This additional flow would not occur, however, if such a release had already occurred since the preceding spring. The actual withdrawal (W(t)) from the Occoquan Reservoir which would be required in the month would be 1,105 Mgal [33 Mgal/d production, plus 5 percent used for treatment, plus 1 Mgal/d to Lorton, for a daily rate of withdrawal of 35.7 Mgal/d which, when multiplied by 31 days, is 1,105 Mgal].

Table 4.--Average water use ($\overline{u}$), average production (P) and monthly production rates (P(k)), in millions of gallons per day, for the 10 GRAM simulations.

Simulation No.	1	2	3	4	5	6	7	8	9	10
Average annual use, $\overline{u}$	40.0	45.0	50.0	55.0	60.0	65.0	70.0	75.0	80.0	85.0
Average annual production, $\overline{P}$	35.0	40.0	45.0	50.0	55.0	60.0	65.0	70.0	75.0	80.0
Monthly production P(k)										
Oct.	33.4	38.2	43.0	47.8	52.6	57.4	62.2	67.0	71.8	76.6
Nov.	32.6	37.3	42.0	46.7	51.4	56.1	60.8	65.5	70.2	74.9
Dec.	32.2	36.8	41.5	46.1	50.8	55.4	60.1	64.7	69.4	74.0
Jan.	31.0	35.5	40.0	44.5	49.0	53.5	58.0	62.5	67.0	71.5
Feb.	31.4	35.9	40.5	45.0	49.6	54.1	58.7	63.2	67.8	72.3
Mar.	31.4	35.9	40.5	45.0	49.6	54.1	58.7	63.2	67.8	72.3
Apr.	35.4	40.4	45.5	50.5	55.6	60.6	65.7	70.7	75.8	80.8
May	36.2	41.3	46.5	51.6	56.8	61.9	67.1	72.2	77.4	82.5
June	40.2	45.8	51.5	57.1	62.8	68.4	74.1	79.7	85.4	91.0
July	40.6	46.3	52.0	57.7	63.4	69.1	74.8	80.5	86.2	91.9
Aug.	39.4	44.9	50.5	56.0	61.6	67.1	72.7	78.2	83.8	89.3
Sept.	36.0	41.1	46.2	51.4	56.5	61.6	66.7	71.9	77.0	82.1

Table 5.--Emergency procedures for GRAM

Storage at the beginning of the month: Water level in feet above sea level	Storage at the beginning of the month: volume in storage in Mgal	Emergency stage invoked for the month	Emergency measures: Purchase of finished water F(t) Mgal/d	Emergency measures: Purchase of raw water M(t) Mgal	Emergency measures: Anticipated conservation α(t)
120.0	9,800				
		Ø	5	0	1.0
107.0	4,300				
		I	12	0	.95
102.5	3,400				
		II-A	19	0	.90
94.5	1,900				
		II-B	19	1,200*	.80
88.0	1,100				
		III	19	1,200*	.80**
55.0	0				

* A purchase of 1,500 Mgal is made from Lake Manassas; of this 1,200 Mgal is expected to reach the Occoquan Reservoir. This purchase can be made only once per year. That is, once a purchase is made, no further purchase can be made until the following autumn.

** No estimate is made of the amount of conservation in this stage. The economic consequences of this stage are severe. When it occurs the local governments will require the curtailment of all water uses not essential to life, health, and safety. That is, schools and businesses would be closed. At this point the amount of useable water would be less than 400 Mgal, because the lowest major intake pipe at the dam becomes inoperable at a storage of 700 Mgal.

Results of GRAM

Tables 6-9 give the results of four of the ten GRAM simulations at average production levels ($\overline{P}$) of 50, 60, 70, and 80 Mgal/d. These tables show the year and month when the emergency would have commenced (for the period October 1927 through September 1976), the maximum severity both in terms of reservoir contents and emergency stage, and the duration of the emergency. It is clear from these results that with increasing rates of production, the frequency, duration, and severity of emergencies all increase. At the higher rates of production there is a strong tendency for emergencies to occur in groups of two or more consecutive years. This suggests that there is some year-to-year persistence (serial dependence) of low flows in the Occoquan watershed. For the 70 Mgal/d production simulation (table 8) there are 13 years of emergencies, 4 of which occurred alone, 4 in 2 back-to-back pairs, and 5 in a run of 5 consecutive years. The hypothesis that the occurrence of emergencies are independent of each other was tested by the runs test (Siegel, 1956, p. 52-58), and it was rejected at the 0.05 probability level.

In estimating the probability of entering a certain emergency stage, the problem can be posed in two different ways: (1) What is the probability of entering an emergency in any year? (known as the marginal probability) or, (2) What is the probability of entering an emergency next year given what has happened this year? (known as the conditional probability).

The estimation of the marginal probabilities of emergencies relies on binomial sampling theory. Consider, for example, the problem of estimating the probability that some emergency will occur in a year given an average production rate of 70 Mgal/d. Table 8 indicates that emergencies would occur in 13 of the simulated years. This can be viewed as 13 occurrences in 48 trials (there are 49 years in the simulation but only 48 complete summer-autumn-winter sequences). The actual probability (π) of an emergency is unknown but a likelihood function L (π) for this probability π may be approximated

$$L(\pi) = \frac{\Gamma(14)\Gamma(36)}{\Gamma(49)} \pi^{13}(1 - \pi)^{48-13}$$

Table 6.--Results of GRAM simulation #4. ($\bar{u}$ = 55 Mgal/d
$\bar{P}$ = 50 Mgal/d)

Month and year in which emergency commences	Most severe stage declared	Duration of emergency - (months)	Minimum end-of month storage (Mgal)
November 1930	II-A	6	1,970
December 1954	I	1	3,720
November 1963	I	1	4,140
December 1965	II-A	3	3,340

Table 7.--Results of GRAM simulation #6. ($\bar{u}$ = 65 Mgal/d
$\bar{P}$ = 60 Mgal/d)

Month and year in which emergency commences	Most severe stage declared	Duration of emergency - (months)	Minimum end-of month storage (Mgal)
October 1930	II-B	7	1,200
November 1943	I	1	4,120
October 1954	II-A	3	2,760
November 1963	II-A	1	2,820
October 1964	I	2	3,600
October 1965	II-B	5	1,860

Table 8.--Results of GRAM simulation #8 ($\overline{u}$ = 75 Mgal/d
$\overline{P}$ = 70 Mgal/d)

Month and year in which emergency commences		Most severe stage declared	Duration of emergency - (months)	Minimum end-of-month storage (Mgal)
Sept.	1930	III	9	50
Nov.	1931	II-B	3	1,690
Dec.	1941	II-A	3	3,260
Nov.	1943	II-A	1	2,800
Dec.	1953	II-A	1	3,010
Oct.	1954	II-B	3	1,870
Sept.	1957	II-A	2	3,240
Nov.	1962	II-A	1	3,140
Oct.	1963	II-B	2	1,850
Sept.	1964	II-A	4	2,590
Oct	1965	II-B	5	1,380
Sept.	1966	II-A	1	3,630
Nov.	1968	I	1	4,230

Table 9.--Results of GRAM simulation #10. ($\overline{u}$ = 85 Mgal/d
$\overline{P}$ = 80 Mgal/d)

Month and year in which emergency commences		Most severe stage declared	Duration of emergency - (months)	Minimum end-of-month storage (Mgal)
Sept.	1930	III	9	0*
Oct.	1931	III	4	840
Nov.	1941	II-A	4	2,350
Oct.	1943	II-A	2	2,160
Nov.	1953	II-A	2	2,110
Sept.	1954	III	6	1,080
Sept.	1957	II-A	3	1,960
Jan.	1959	I	1	3,690
Oct.	1962	II-A	2	2,180
Oct.	1963	III	2	910
Sept.	1964	II-B	4	1,610
Sept.	1965	III	6	0**
Sept.	1966	II-A	1	2,580
Nov.	1968	II-A	1	3,260
Nov.	1970	I	1	3,880

* Empty at the end of four months. Total deficit during emergency is 1,840 Mgal.

** Empty at the end of one month only. Total deficit during emergency is 4 Mgal.

The likelihood function takes on its maximum value where $\pi = \frac{13}{48} = 0.271$. Thus the maximum likelihood estimate (MLE) of π is 0.271. In general the MLE of π, denoted $\tilde{\pi}$ is

$$\tilde{\pi} = \frac{x}{n}$$

where x is number of occurrences in n trials.

There is a drawback to expressing the results of the GRAM simulations as the MLE of π and that arises where x is very small and particularly where x is zero. If $x = 0$ then $\tilde{\pi} = 0$ which implies that an event which is never observed in the set of trials has a zero probability of occurring in the future.

To avoid giving such misleading estimates a Bayesian estimate of π (Box and Tiao, 1973, p. 34-36) denoted π^* is used. It is the mean of the posterior distribution of π where a non-informative conjugate prior is applied to the likelihood function.

$$\pi^* = \frac{x + \frac{1}{2}}{n+1}$$

thus when x is 0 and n is 48, π^* is 0.01, and when x is 13, π^* is 0.276.

Using π^* thus avoids the unreasonable result of estimating π as either 0 or 1 but is generally very close to the MLE of π. The advantage of the Bayes estimation, π^*, is that it incorporates in the estimation procedure the subjective notion that just because something did not happen in the past does not mean it will not happen in the future. Table 10 gives the Bayesian estimates of the probabilities of the various emergency levels at all 10 rates of production.

Table 10 also gives an estimate of the conditional probability of emergencies. Consider again the 70 Mgal/d results in table 8. Of the 13 emergencies 6 occurred in a year which was preceded by a year which had an emergency. The other 7 emergencies were not preceded by years with emergencies. To estimate the probability of an emergency "next year" given the existence of an emergency "this year," the sample size (n) is 13, and the number of occurrences (x) is 6. So the Bayesian estimate of the probability is 0.464 (6.5/14). Similarly the estimate of the probability of an emergency next year given no emergency this year is 0.208(n = 35, x = 7). These results are given in the last two columns of table 10. They show clearly that the probability of having an emergency in a year following an emergency is higher than the marginal probability of having an emergency.

Table 10.--Bayesian estimates of the probabilities of water emergencies.

GRAM simulation #	Average production in Mgal/d	Estimated probability of a Stage I emergency or worse in percent	Estimated probability of a Stage II-A emergency or worse in percent	Estimated probability of a Stage II-B emergency or worse in percent	Estimated probability of a Stage III emergency in percent	Estimated probability of dry reservoir in percent	Estimated probability of an emergency given the occurrence of an emergency in the prior year	Estimated probability of an emergency given no occurrence of an emergency in the prior year
1	35	3	(*)	(*)	(*)	(*)	(**)	(**)
2	40	3	3	(*)	(*)	(*)	(**)	(**)
3	45	5	3	(*)	(*)	(*)	(**)	(**)
4	50	9	5	(*)	(*)	(*)	(**)	(**)
5	55	11	7	3	(*)	(*)	42	8
6	60	13	9	5	(*)	(*)	36	10
7	65	25	13	7	3	(*)	50	18
8	70	27	25	11	3	(*)	46	21
9	75	27	25	11	5	3	46	21
10	80	32	27	13	11	5	47	25

* No occurrences in the 49-year simulation. The estimate of the probability is 1 percent, but the estimate is unreliable.

** Insufficient number of cases to make meaningful estimates.

The cause of this persistence is probably the deficiency of carryover storage in the soil and in the aquifers of the watershed. It is likely that year-to-year persistence in rainfall exists (Potter, 1976) that would also contribute to the repetitive nature of emergencies.

Figures 6-9 are another type of representation of the results of GRAM. Consider again the probability of an emergency at an average production rate of 70 Mgal/d. Figure 6 plots the MLE of π (.271) and also show the corresponding recurrence interval ($1/\pi$) 3.7 years. In addition to this point estimate, it gives a band of values for π. This band, which runs from π = 0.295 to π = 0.346 is a 68 percent confidence band for π. That is, for the result x = 13 and n = 48 to have occurred indicates that there is a 68 percent chance that π lies in the range 0.295 to 0.346, a 16 percent chance that π is less than 0.295 and a 16 percent chance it is greater than 0.346. The method for deriving these confidence bands is given in appendix A.

The figures 6-9 provide information both on the risks of emergency (the MLE of π) and on the uncertainty about that risk. They show that where the risk is large (π large), the uncertainty is relatively small, and where the risk is small, the uncertainty is large.

The implications for the risks of emergency for the FCWA seen in these figures are the following: As annual production rates rise from the 50 to 60 Mgal/d rates observed in the last few years up to 65 Mgal/d and beyond (as is predicted), the recurrence interval for a Stage I emergency decreases rather sharply from about 10 to 20 years down to 3 to 5 years. For Stage II-A, the first stage in which mandatory water use restrictions are imposed. the increase from 60 Mgal/d up to 70 Mgal/d markedly increases the expected frequency of this emergency from a 10- to 20-year event to a 3- to 5-year event. Looking at figure 9, it is clear that a Stage III emergency (an emergency with great economic impact) is a rather remote risk at 60 Mgal/d and even at 65 or 70 Mgal/d probably has a recurrence interval well above 15 years. As production rises to 75 Mgal/d it ceases to be so remote a risk.

It should be stressed that these results are applicable to the Occoquan Reservoir only under the particular type of seasonal demand pattern and emergency procedures described in this study. Changes in the rules for invoking the emergency procedures, or in the rates and total quantities of purchases would all change the results of the study.

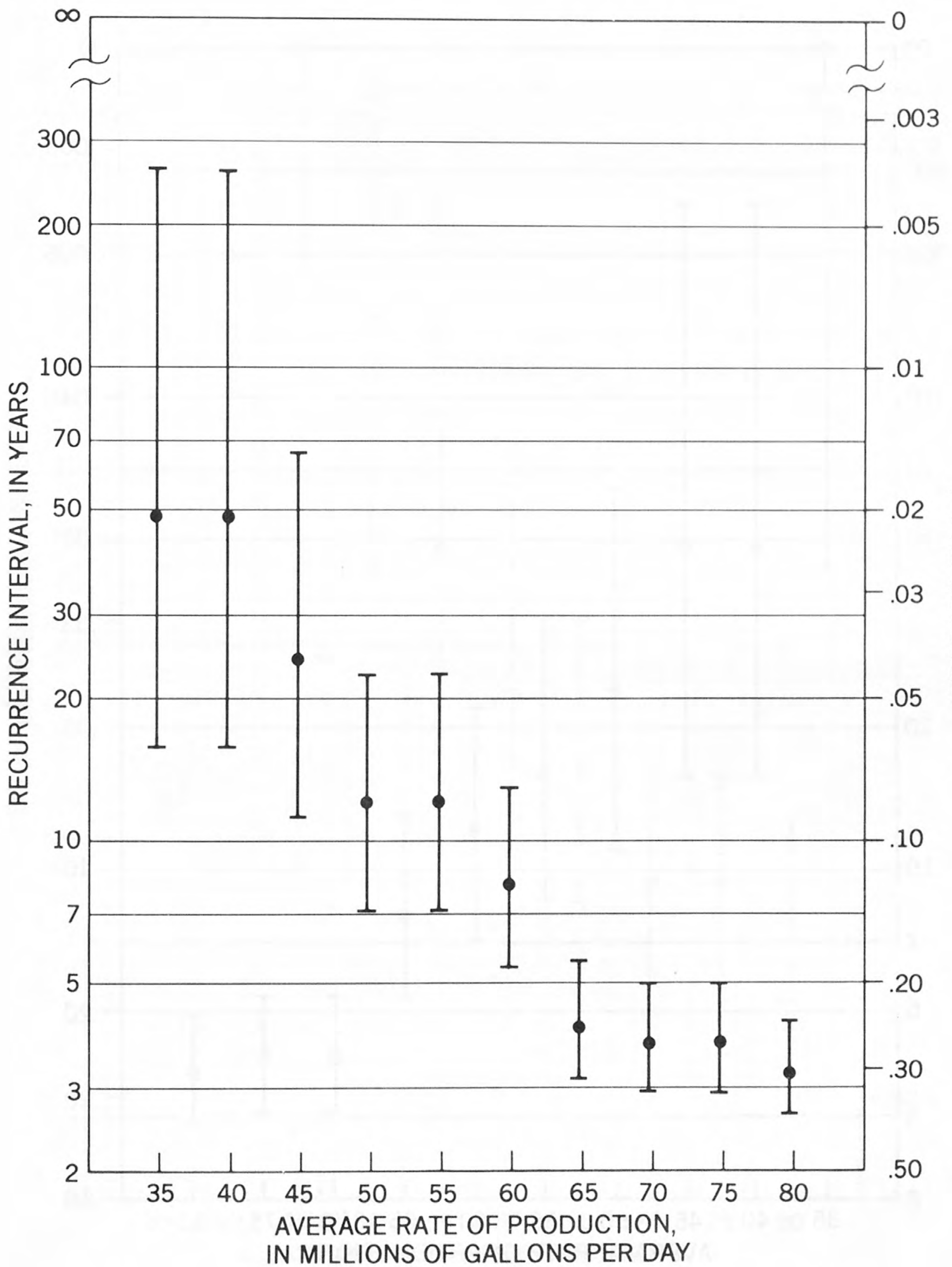

Figure 6.--Probability and recurrence interval of a Stage I emergency (or worse) as a function of the average rate of production. The point indicates the maximum likelihood estimate of π; the bar is the 68 percent confidence band.

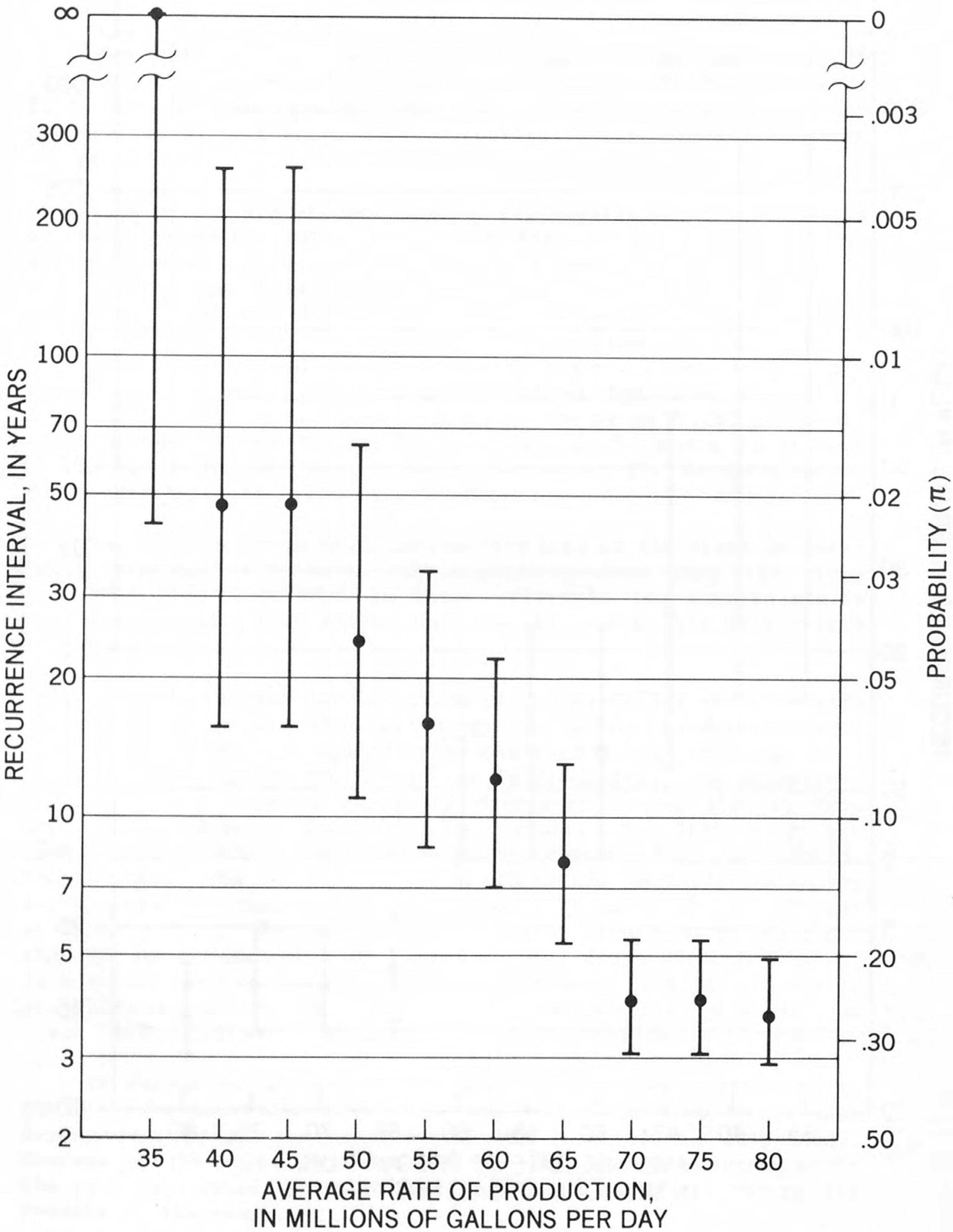

Figure 7.--Probability and recurrence interval of a Stage II-A emergency (or worse) as a function of the average rate of production. The point indicates the maximum likelihood estimate of π; the bar is the 68 percent confidence band.

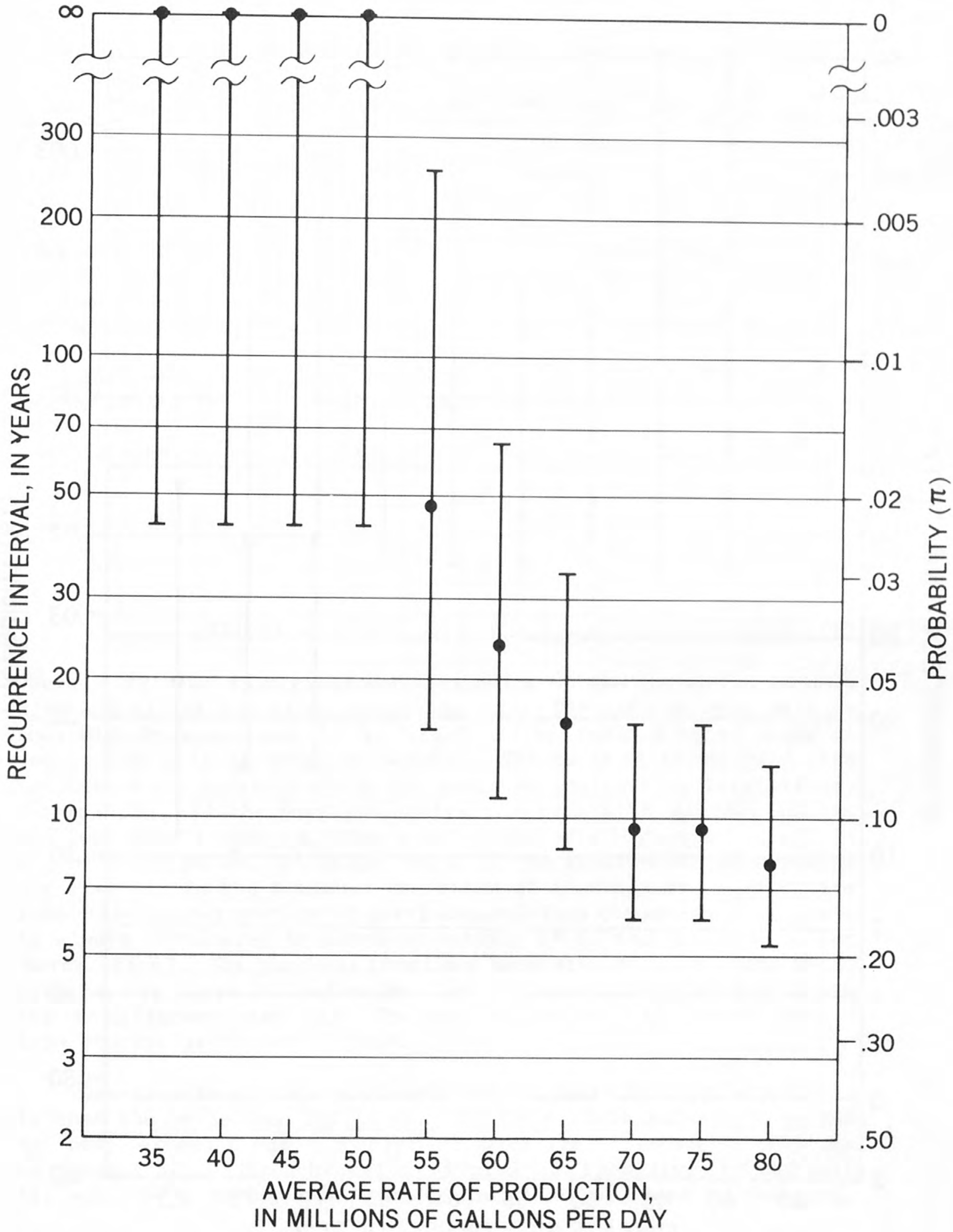

Figure 8.--Probability and recurrence interval of a Stage II-B emergency (or worse) as a function of the average rate of production. The point indicates the maximum likelihood of π; the bar is the 68 percent confidence band.

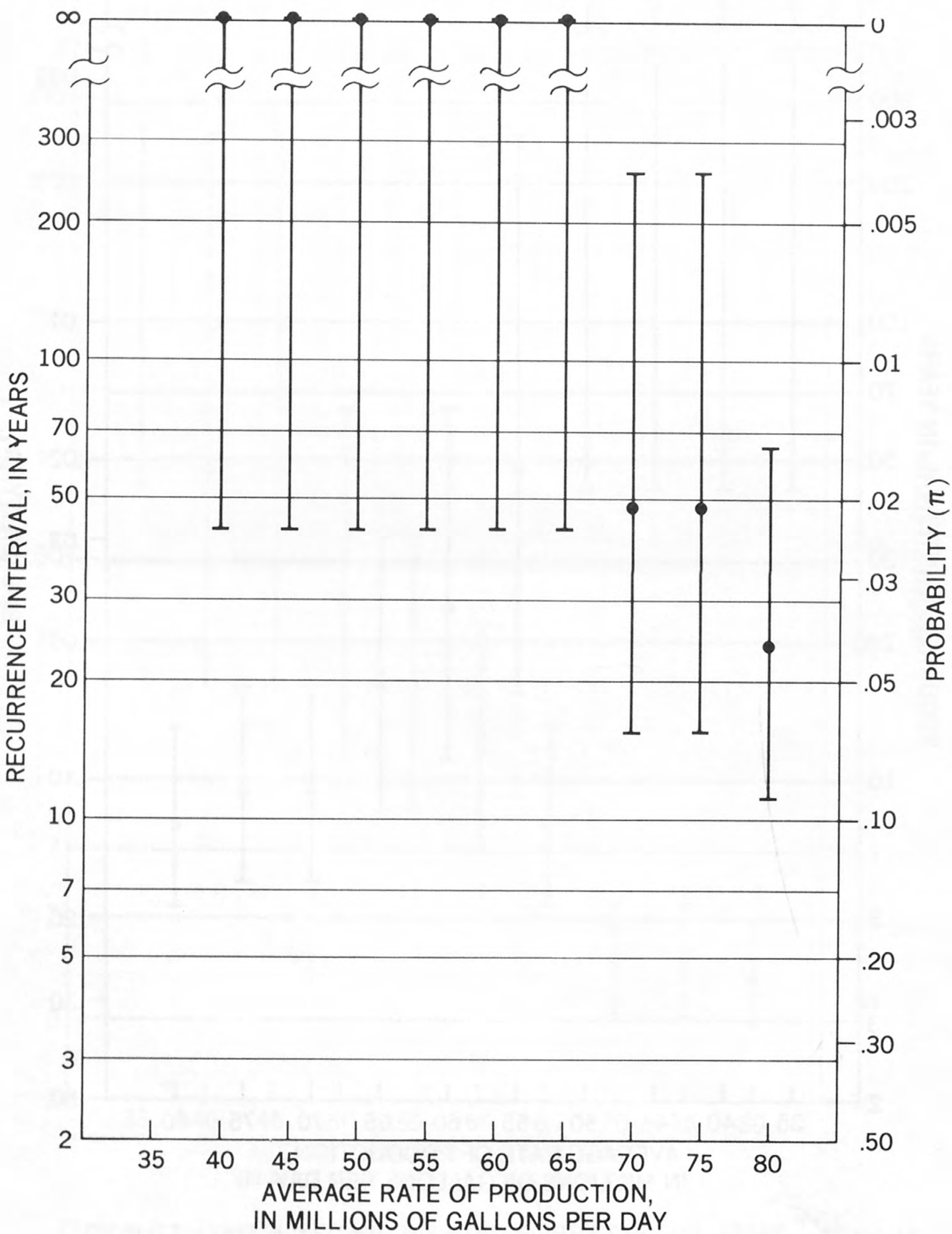

Figure 9.--Probability and recurrence interval of a Stage III emergency as a function of the average rate of production. The point indicates the maximum likelihood estimate of π; the bar is the 68 percent confidence band.

POSITION ANALYSIS

Position analysis is a specialized application of risk analysis. Its purpose is to estimate the risks associated with a given plan of operation over a period of a few months. It relies on a simulation procedure virtually identical to GRAM, using the n year record of reconstructed historical inflow. It differs from GRAM in that it consists of n separate simulations rather than one continuous simulation of length n years. Each of these simulations is initialized with the same reservoir storage value--that storage actually existing in the reservoir at the beginning of the present month. Thus, it is an analysis of risks evaluated from the present "position".

The position analysis simulation for the Occoquan Reservoir is characterized by the following equations:

$$S(i,t+1) = S(i,t) + I(i,t) + M(t) - E(i,t) - W(t) \qquad (7)$$

$$t = 1,2,\ldots,t^*-1$$

$$i = 1,2,\ldots,n$$

$$S(i,1) = V \qquad i = 1,2,\ldots,n \qquad (8)$$

where V is the storage at the beginning of the present month in Mgal and all of the other variables have the same meaning here as they did in equations 1, 2, and 3. The index i is an index of years, and t is an index of months. The index t is defined with respect to the month in which the position analysis is initialized. For example, if the position analysis begins with storage set at the September 1 storage, then $t = 1$ represents September, $t = 2$ is October, and so on. t^* is the index of the final month of concern. For example, in the Occoquan the risks of shortage are nearly zero from the beginning of March until autumn. Thus the position analysis is always terminated in March by setting t^* to the index value for March ($t^*=6$). The purchase from Lake Manassas, M(t), and the withdrawals, W(t), are defined as the plan of operation and do not change for the different years (i). The evaporation, E(i,t), is determined from storage as it was in GRAM.

One important feature of position analysis seen in equation 4 is that the variables S(i,t) are unrestricted in value. They can not only exceed C but can also be negative. As such, they may be thought of as indicators of surplus (if positive) or deficit (if negative), rather than storage in the usual sense of the word.

Having computed the values of $S(i,t)$ for all $i = 1,2,\ldots,n$ and $t = 1,2,\ldots,t^*$ for a given plan of operation defined by $M(t)$ and $W(t)$ for $t = 1,2,\ldots,t^*-1$, the minimum, end of month, storage for each year $S_{min}(i)$ is determined

$$S_{min}(i) = \underset{t = 1,2,\ldots,t^*}{\text{Min}} [S(i,t)]$$

The empirical distribution of this variable is then used to evaluate the risks associated with a given plan of operation.

An Example of Position Analysis

At the beginning of October 1977, the reservoir was at the lowest level in its operating history. Storage (V) was 2,450 Mgal, and average withdrawals were about 44 Mgal/d (production was about 41 Mgal/d, and purchases were about 18 Mgal/d). The Fairfax County Water Authority had arranged to purchase 1,500 Mgal of raw water from Lake Manassas, and this purchase was to commence as soon as the storage in Occoquan Reservoir fell below 1,900 Mgal. Of this 1,500 Mgal about 1,200 Mgal was expected to actually reach the Occoquan Reservoir. The FCWA anticipated that with full use of all possible finished water purchases, and no significant additional conservation effort, it would have to produce about 40 Mgal/d for the remainder of the autumn and winter.

The questions to be addressed by position analysis were these: 1) What is the probability that the storage will fall below 1,100 Mgal (the level at which a Stage III emergency is declared) in the next 6 months, given the purchase from Lake Manassas and the continuing production of 40 Mgal/d? 2) How would this probability change if the production rate for the entire period were changed to 32 Mgal/d or to 48 Mgal/d?

Figures 10, 11, and 12 show the simulated storage records for the three most severe drought episodes in the 1927-1976 record: 1930-1931, 1931-1932, and 1965-1966. Each figure shows the storages at all three rates of production. These figures show that in only 1 year (1930-1931) of the 49 simulated years would there be a Stage III emergency if production were 40 Mgal/d, but in 2 others, the minimum storage would have come quite close to the 1,100 Mgal level at which a Stage III emergency is invoked. An 8 Mgal/d reduction in the rate of production would have made the minimum storages in these three worst droughts 700 to 1,200 Mgal larger than they would have been at 40 Mgal/d, and no Stage III emergencies would occur. On the other hand, an increase in the rate of production to 48 Mgal/d would have resulted in all three of these years having Stage III emergencies.

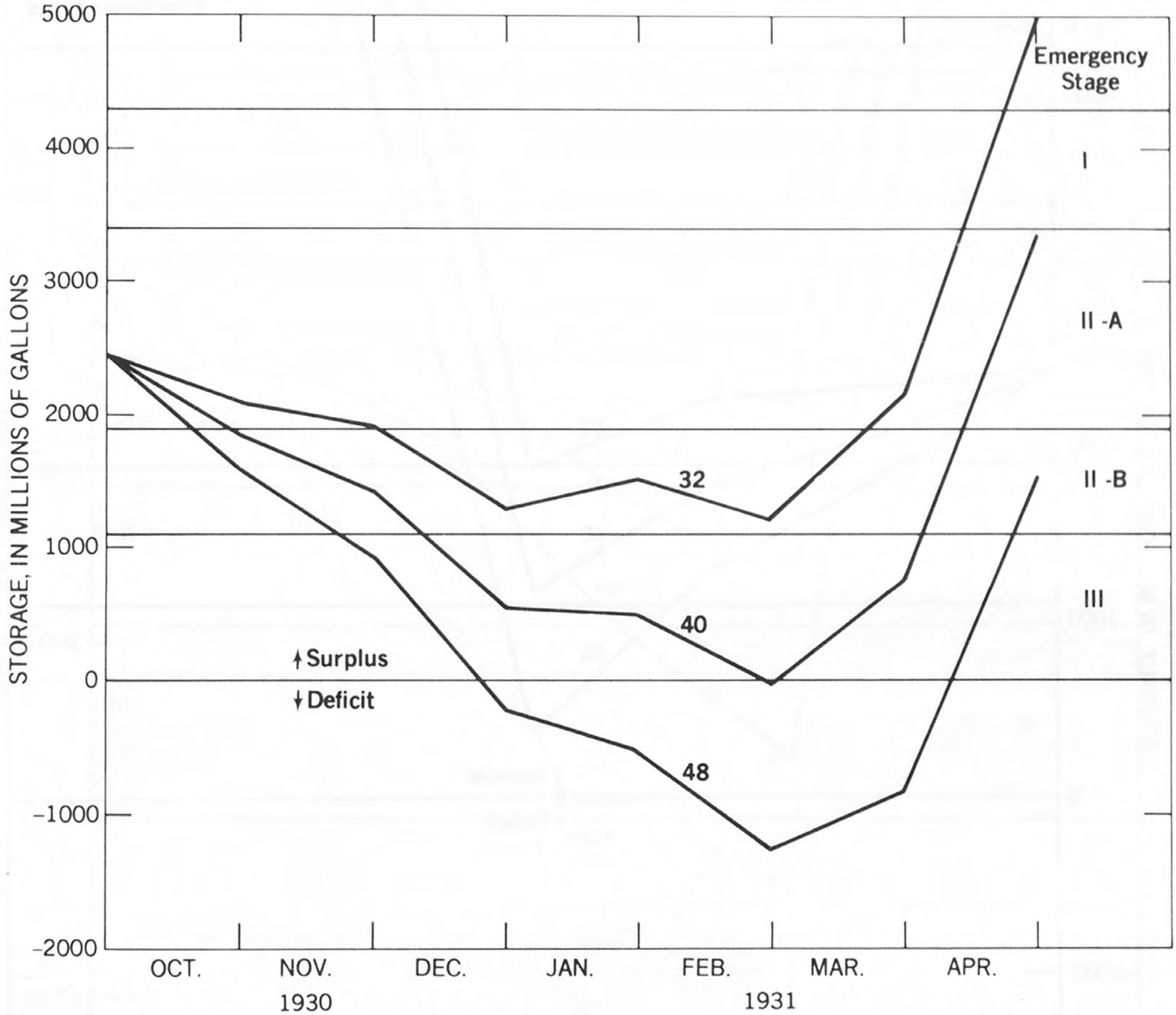

Figure 10.--Position Analysis: Simulated storages for the 1930-1931 drought, at production rates of 32, 40, and 48 Mgal/d.

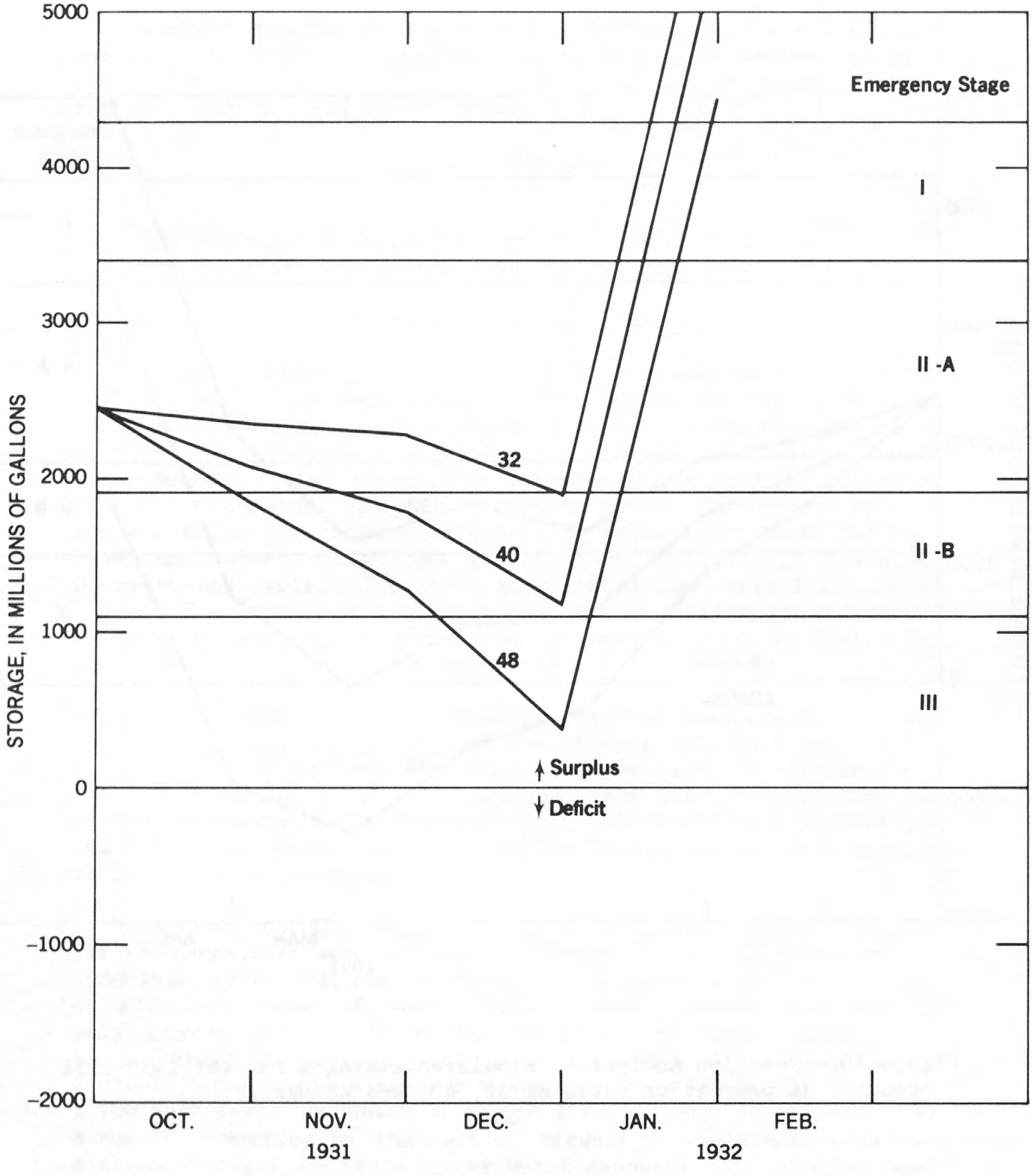

Figure 11.--Position Analysis: Simulated storages for the 1931-1932 drought, at production rates of 32, 40, and 48 Mgal/d.

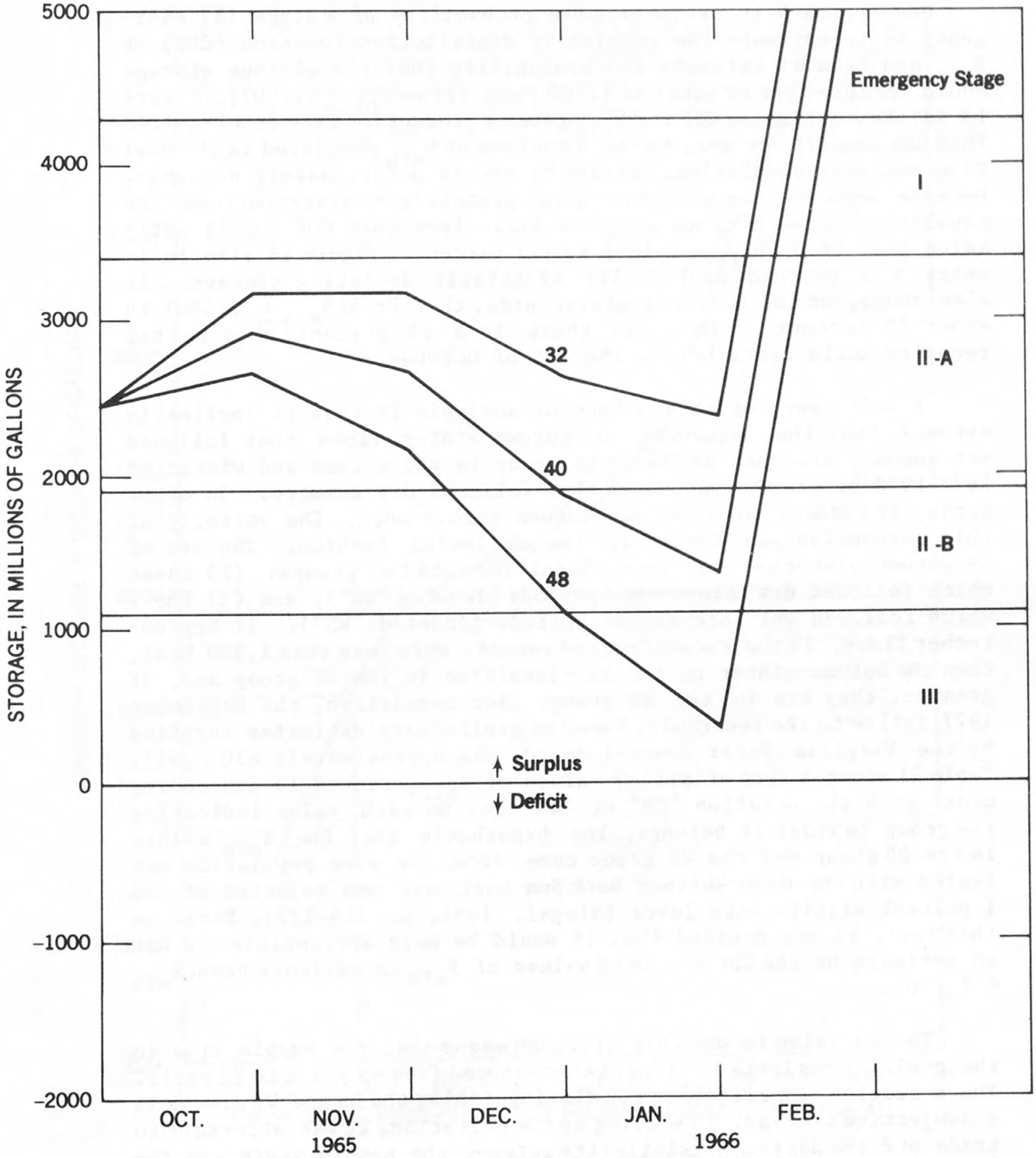

Figure 12.--Position Analysis: Simulated storages for the 1965-1966 drought, at production rates of 32, 40, and 48 Mgal/d.

One approach to estimating the probability of a Stage III emergency is to estimate the cumulative distribution function (CDF) of S_{min} and from it estimate the probability that the minimum storage would be less than or equal to 1,100 Mgal [Prob($S_{min} \leq 1,100$)]. Figure 13 is the estimated CDF for S_{min} given a production rate of 40 Mgal/d. This CDF was fit, by eye, to the 49 values of S_{min} developed in the position analysis simulation. Fitting by eye is unfortunately necessary because none of the commonly used probability distributions are capable of providing an adequate fit. From this CDF it is estimated that Prob ($S_{min} \leq 1,100$) is 4.5 percent. Figure 13 also indicates a 2 percent probability of totally depleting storage. It also shows, on the more optimistic side, that Prob($S_{min} \geq 2,450$) is about 75 percent. That is, there is a 75 percent chance that recovery would begin before the end of October.

A deficiency of this method of analysis is that it implicitly assumes that the sequences of autumn-winter flows that followed wet summers are just as likely to occur in the autumn and winter of 1977-1978 as are the sequences that followed dry summers. In other words, it assumes no summer-to-autumn persistence. The validity of this assumption was tested in the following fashion. The set of 49 autumn-winter periods were classified into two groups: (1) those which followed dry late-summer periods (denoted "DS"), and (2) those which followed wet late-summer periods (denoted "WS"). If September flows, in the reconstructed record, were less than 1,500 Mgal, then the autumn-winter period is classified in the DS group and, if greater, they are in the WS group. [For comparison, the September 1977 inflow to the reservoir, based on preliminary estimates supplied by the Virginia Water Control Board, was approximately 630 Mgal]. Table 11 shows a list of all 49 values of S_{min}, ranked in descending order with the notation "DS" or "WS" beside each value indicating the group to which it belongs. The hypothesis that the S_{min} values in the DS group and the WS group come from the same population was tested with the Mann-Whitney Rank Sum test and was rejected at the 1 percent significance level (Siegel, 1956, p. 116-127). Based on this test, it was decided that it would be more appropriate to use an estimate of the CDF of the DS values of S_{min} to estimate Prob($S_{min} \leq 1,100$).

The decision to use this approach means that the sample size in the position analysis is effectively reduced from 49 years to 22 years. The selection of a criterion for distinguishing the DS and WS groups is a subjective process. In setting up the criterion, it was necessary to trade off the degree of similarity between the sample years and the present year against the reduction in sample size. Many different criteria were tried, and this one was selected because it resulted in a highly significant Mann Whitney test statistic (indicating that the DS S_{min} values come from a different population than the WS S_{min} values), it allowed the three worst droughts to remain in the DS sample, and allowed the sample to be large enough to show much of the inherent variability of the streamflows.

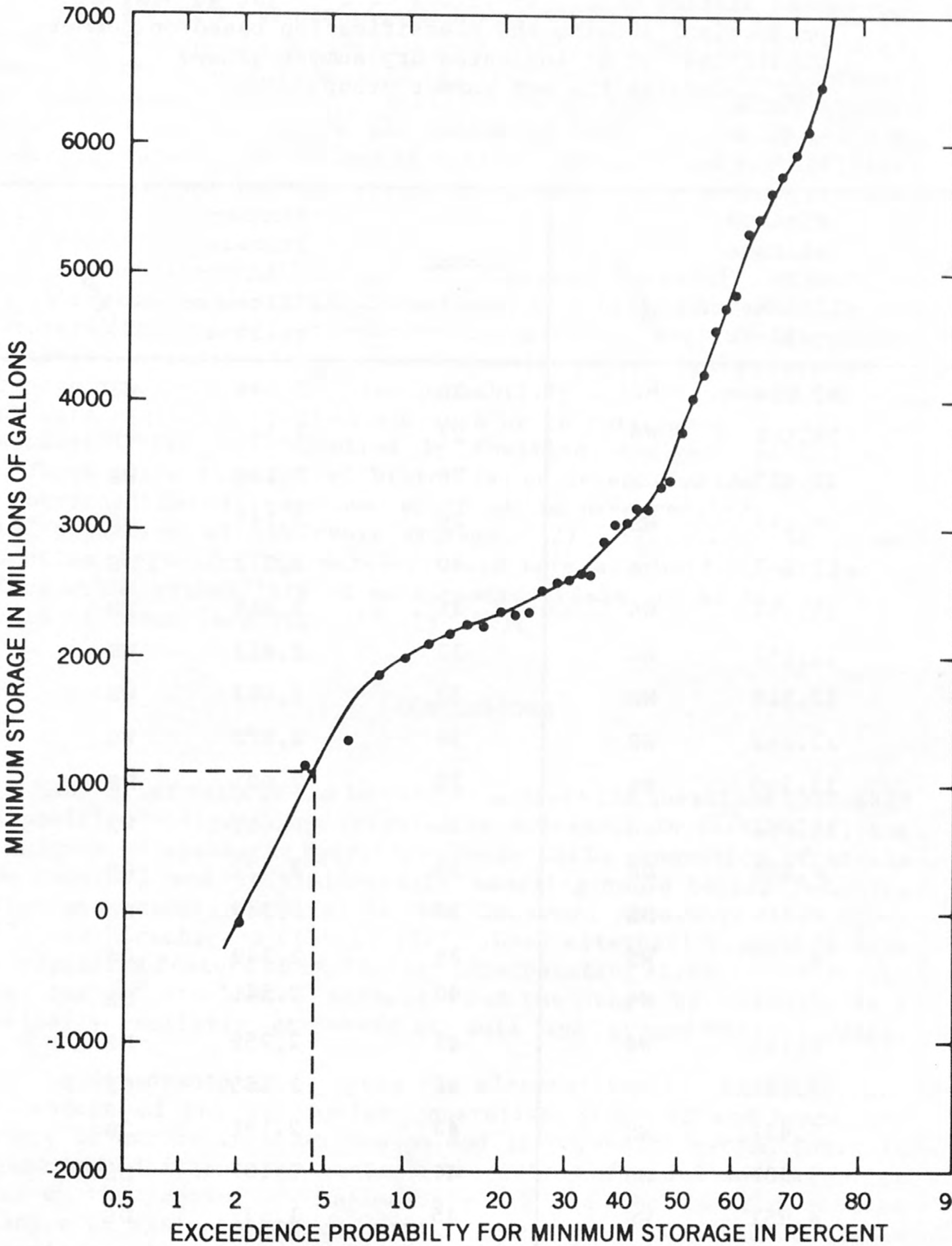

Figure 13.--Cumulative distribution function for S_{min}, 40 Mgal/d production rate, from Position Analysis based on 49 years of record.

Table 11.--Ranked listing of all 49 values of S_{min} (at 40 Mgal/d production) showing the classification based on summer conditions: "DS" indicates dry summer group; "WS" indicates the wet summer group.

Rank	Minimum storage in Millions of gallons	Group	Rank	Minimum storage in Millions of gallons	Group
1	62,014	WS	26	3,399	WS
2	26,564	WS	27	3,341	DS
3	22,557	DS	28	3,146	WS
4	18,498	WS	29	3,145	WS
5	17,407	DS	30	3,077	DS
6	15,354	WS	31	3,055	WS
7	12,667	WS	32	2,913	WS
8	12,516	WS	33	2,683	WS
9	12,342	WS	34	2,678	WS
10	11,390	WS	35	2,605	DS
11	10,064	WS	36	2,601	WS
12	8,686	DS	37	2,522	DS
13	6,466	DS	38	2,349	DS
14	6,136	WS	39	2,349	DS
15	5,914	WS	40	2,341	DS
16	5,748	WS	41	2,259	DS
17	5,647	DS	42	2,255	DS
18	5,411	WS	43	2,191	DS
19	5,305	WS	44	2,107	DS
20	4,851	WS	45	1,997	DS
21	4,711	WS	46	1,881	DS
22	4,546	WS	47	1,363	DS
23	4,234	WS	48	1,140	DS
24	4,036	WS	49	- 32	DS
25	3,728	DS			

Figure 14 shows the estimated CDF of the DS value of S_{min} and the 22 values of S_{min} to which this CDF was fit (once again by eye). The estimated Prob($S_{min} \leq 1,100$) is 10 percent rather than 4.5 percent, and the estimated Prob($S_{min} \geq 2,450$) is 45 percent rather than 75 percent. Figure 15 shows the estimated CDFs for the DS group at production rates of 32, 40, and 48 Mgal/d. The estimated probabilities of various minimum storages are given in table 12 for all three rates of production.

In the October 1977 drought the FCWA used the results of position analysis along with other information as the basis for informing its customers that an 8 Mgal/d reduction in water use would bring about a substantial reduction in the risk of a severe water crisis (Stage III). Ultimately it is not the precise probability values shown in table 12 that were useful to the FCWA managers or to customers. Rather, the fundamental findings supplied by Position Analysis were these: 1) There is a substantial historical precedent for the occurrence of continued low streamflows which would have resulted in nearly total depletion of reservoir storage. 2) Efforts at water use reduction beginning in October could have a dramatic, mitigating effect on the probability of an upcoming crisis and on its severity should it occur (see figs. 10, 11, 12).

CONCLUSIONS

Many other methods can be used to address the questions that GRAM and position analysis are intended to address. In particular, the techniques of synthetic hydrology (Monte Carlo generation of stream flow records) and rainfall-runoff modeling could be applied. The latter was actually applied to the Occoquan Reservoir in October 1977: see Hirsch, and others, 1977. These alternative methods have the appealing feature of explicitly incorporating streamflow persistence, one by stochastic methods, and the other by building in a physically realistic structure of soil and ground-water storage.

The disadvantages of these two alternatives are that they are both models of the streamflow generating process, and hence are subject to errors in model design and in parameter estimation. It is generally difficult to estimate the effect of model and parameter error on the probability estimates which are the desired results of any risk analysis. In contrast, the errors in GRAM and position analysis are largely restricted to those resulting from the finite length of the record. These can be dealt with explicitly and analytically (see figures 6, 7, 8, and 9). There are, of course, errors associated with the reconstruction of the historical flow records, but these errors are present in any of the methods.

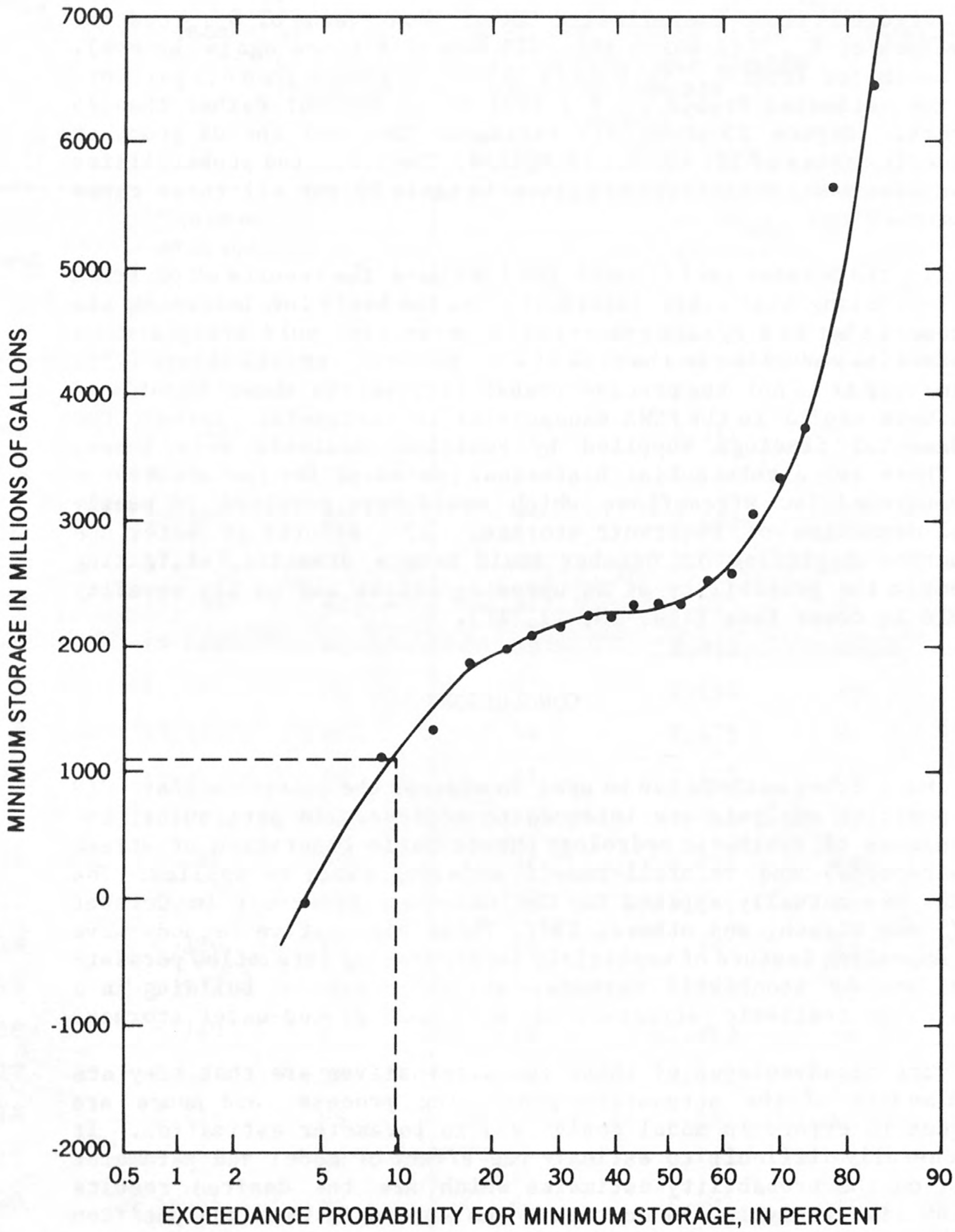

Figure 14.--Cumulative distribution function for S_{min}, 40 Mgal/d production rate, from Position Analysis based on 22 years with dry late-summer periods.

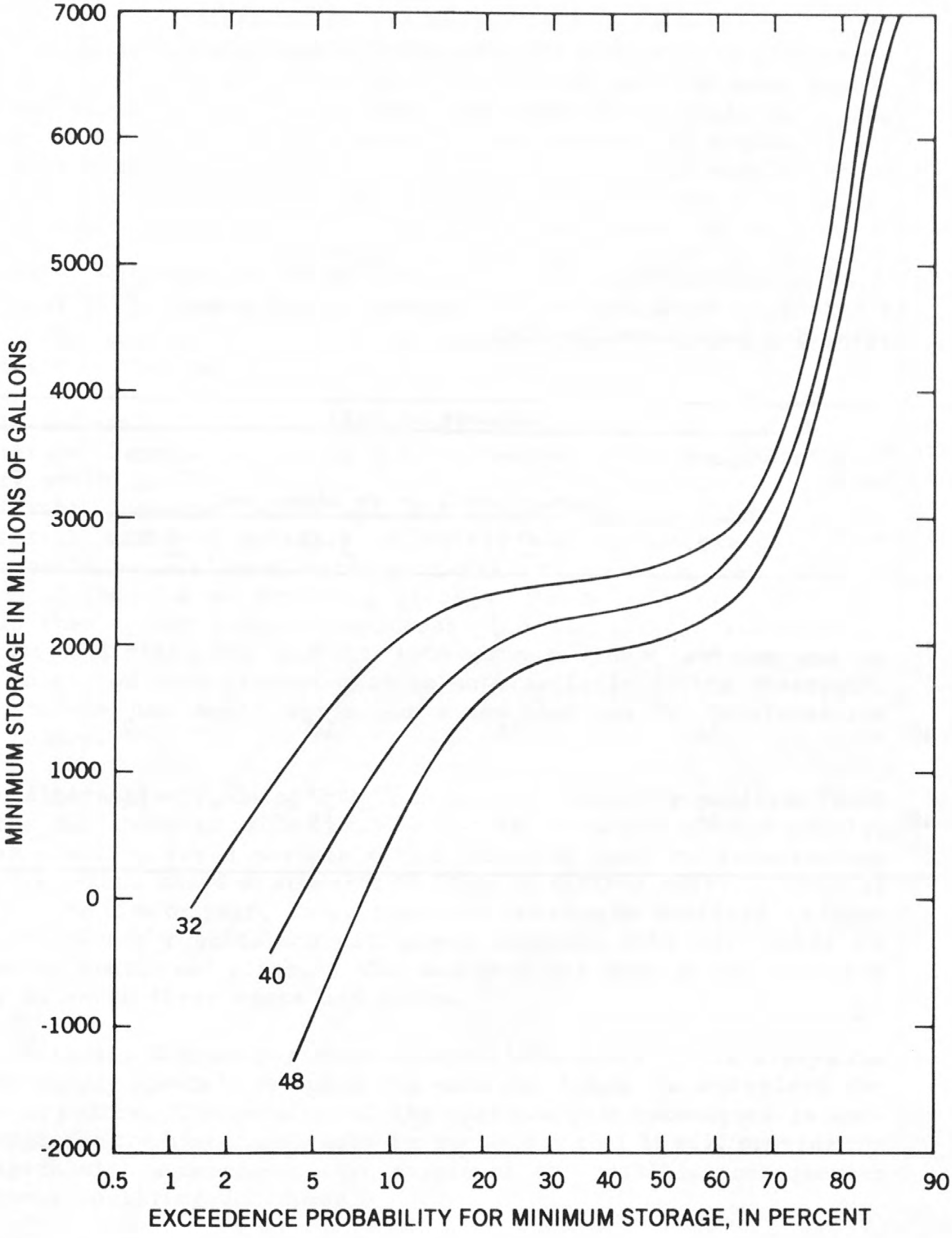

Figure 15.--Cumulative distribution functions for S_{min} at 32, 40, and 48 Mgal/d production rates, from Position Analysis based on 22 years with dry-late summer periods.

Table 12.--Probabilities of storage falling below specified volume, in percent: Based on Position Analysis using sample of 22 years having dry late-summer periods.

Rate of production in Mgal/d	Storage in Mgal				
	≤ 2450	≤ 1700	≤ 1100	≤ 700	≤ 0
	Water level in ft above msl				
	≤ 99	≤ 93	≤ 88	≤ 83	≤ 55
32	26	6	4	3	< 2
40	56	16	10	7	4
48	64	22	14	12	8

Clearly no categorical statements can be made about which approach is better in general. In any case, the comparative advantages would be a function of the method's reliability and the data, money, and manpower available. The GRAM and position analysis have the advantage of simplicity and speed of development. If a good reconstructed historical record is available to a water supply agency when it enters a crisis or when it wishes to evaluate a design or operational change in its system, the analysis could be carried out in a few days by a professional with an understanding of the water supply system and computer programing ability. Position analysis can really be carried out effectively even without a computer. All that is necessary is to solve the storage equation (4) for several months for the few worst years of record.

Position analysis could easily be incorporated into the ongoing operation of a water-supply agency. For example, at the beginning of every month it could test a variety of operating plans based on different projected water use rates for the upcoming months and different mixes of delivery of water from various sources. Each plan could be evaluated both in terms of operating costs and in terms of the risk of depleting storage. The managers of the system could then select a best-compromise plan (an appropriate tradeoff of cost and risk) and put it into effect. Then, at the end of the month, or when storage changes substantially in the reservoir, the process can begin again and a new plan can be developed for the future.

Alternatively, using GRAM, a variety of operating policies (such as the one shown in table 5) may be tested. This can involve complex rules (calling for a certain mix of supplies used and conservation measure taken) based on storages or flows at various water sources as well as the time of year. These policies can then be modified in light of preliminary results and ultimately compared with each other in terms of costs and risks. The managers may then select a policy that balances these costs and risks.

With the GRAM and position analysis techniques, it is always the water-supply agency's managers who make the judgments and select the plan or policy. The purpose of the risk analysis techniques is only to organize the historical data in such a way that it will provide the managers with a representative sample of the possible consequences of their operating decisions.

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APPENDIX A

These confidence bands arise directly from the likelihood function: For the example $x = 13$, $n = 48$ the 68 percent confidence band is bounded by $\pi = 0.295$ and $\pi = 0.346$.

$$\int_{0.295}^{0.346} \frac{\Gamma(14)\,\Gamma(36)}{\Gamma(49)} \pi^{13} (1-\pi)^{35}\, d\pi = 0.68$$

In general where $\pi_{UB}(\alpha)$ is the upper confidence limit on π and $\pi_{LB}(\alpha)$ is the lower confidence limit on π such that

$$\text{Prob}\left(\pi < \pi_{LB}(\alpha)\right) + \text{Prob}\left(\pi > \pi_{UB}(\alpha)\right) = \alpha$$

and

$$\text{Prob}\left(\pi_{LB}(\alpha) \leq \pi \leq \pi_{UB}(\alpha)\right) = 1 - \alpha \quad \text{then}$$

$$\int_{0}^{\pi_{LB}(\alpha)} \frac{\Gamma(x+1)\,\Gamma(n-x+1)}{\Gamma(n+1)} \pi^{x} (1-\pi)^{-x}\, d\pi = \alpha/2 \tag{1}$$

$$\int_{\pi_{UB}(\alpha)}^{1.0} \frac{\Gamma(x+1)\,\Gamma(n-x+1)}{\Gamma(n+1)} \pi^{x} (1-\pi)^{n-x}\, d\pi = \alpha/2 \tag{2}$$

However, if $n = 0$, $\pi_{LB}(\alpha) = 0$ and $\pi_{UB}(\alpha)$ is evaluated with the right hand side of of (2) set to α. Similarly if $n = X$ then $\pi_{UB}(\alpha) = 1.0$

and $\pi_{LB}(\alpha)$ is evaluated with the right hand side of (1) set to α.

The integrals in (1) and (2) are beta functions and have been tabulated by Pearson and others (1956).