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# Shape of Roll-type Uranium Deposits

By

C. G. Warren, H. C. Granger, and J. H. Schock

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## Abstract

The Wyoming roll-type uranium deposits occur at the margins of tongues of altered rock that are formed within pyrite-bearing host sandstones. The host sandstones are confined between impermeable mudstone strata which largely prevent the escape of ground water with its load of oxygen and ore bearing components. Oxygen dissolves in the water at the surface and is transported to the end of the altered tongue by two processes (1) velocity transport in which oxygen is carried by moving ground water, and (2) dispersion transport in which oxygen moves from regions of high concentrations to regions of low concentration by diffusion. The altered tongues develop as a result of a chemical reaction between oxygen and pyrite in which both components are consumed. The size of the altered tongue provides a record of how much oxygen was delivered. The shape of the oxidation interface (end of the altered tongue) provides information on the relative effects of velocity and dispersion transport.

Oxygen delivery can be simulated in a computer by mathematically describing the movement of oxygen as the sum of velocity and dispersion transport in a matrix of small cells. In this study a complete description of oxygen movement near the end of the altered tongue is obtained after examining a total of 3100 cells in a cross section which is 100 cells long and 31 cells high. When oxygen is delivered to a cell containing pyrite, the computer program permits them mutually to destroy one another. The entire process is iterated until the oxidation interface takes on a fixed shape although the interface continues slowly to migrate downstream.

The following two models were tested: in the "flow model," no oxygen is permitted to escape across the confining upper and lower boundaries, but in the "leaky model" oxygen is lost from the system by diffusion across these boundaries. The typical roll shape developed only in flow models where water flow was restricted or stagnant in a zone along the upper and lower boundaries of an aquifer. This zone where water movement is restricted is narrow, probably less than 10 cm across. The elongation of the roll is directly proportional to the ground-water velocity. In computer simulation of the leaky model, oxygen was allowed to escape slowly across the mudstone boundaries. Even small rates of oxygen leakage with yearly fractional losses 4 x $10^{-4}$ from the entire aquifer produce the typical roll shape. Although the leaky moder produces typical roll shapes in a computer model, field evidence does not support the idea because oxidation penetrates the mudstone only superficially.

## Introduction

According to generally accepted concepts (fig. 1), the Wyoming roll-type deposits were formed by oxygen-charged solutions which percolated down gradient, reacted with the reduced components in the host rocks, and deposited their load of uranium. During this process an irregular, tongue-shaped, altered zone was formed, and the uranium minerals were concentrated, almost as if they had been filtered out, at the interface between the altered and reduced rock. When the wide diversity of permeabilities among the individual cut-and-filled sandstones, siltstones, and mudstones deposited by both meandering and braided fluvial systems is considered, it seems remarkable that the tongue-like altered zones are as geometrically regular as they are. Although the oxidation-reduction interface or distal surface of the altered tongue, is never a perfect C-shape, it is commonly so nearly so that numerous

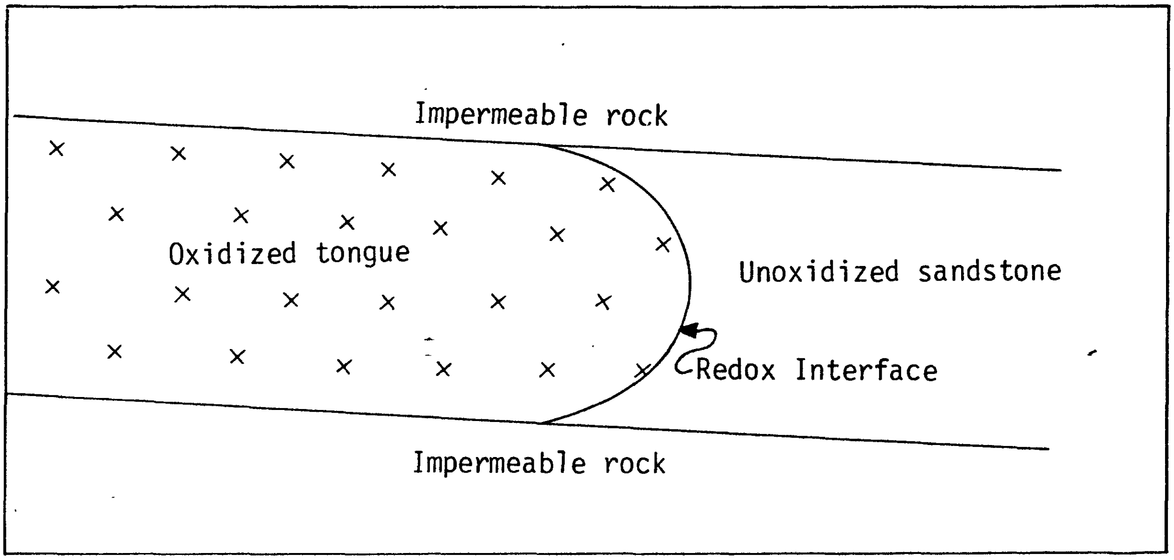

Figure 1. Idealized uranium roll

geologists have idealized it in illustrations that indicate near bilateral symmetry about a subhorizontal plane (Adler and Sharp, 1967; Evseeva, Ivanov, and Kochetkov, 1963; Germanov, 1960; Harshman, 1962; Kashirtseva, 1964; Kholodov and others, 1961; King and Austin, 1965; Lisitsyn and Kuznetsova, 1967; Shawe and Granger, 1965). Either the flow rates of solutions (and their dissolved reagent oxygen), were not in proportion to the permeabilities of the rock units they traversed, or some process tended to even out the delivery of reagents along the roll surface.

This study concerns the mechanisms by which reagents were delivered to the oxidation-reduction interface of roll-type uranium deposits. Until recently (Granger, 1976) we had assumed that dissolved oxygen had simply been carried along the flow paths of ground water by velocity transport to the surfaces of the altered tongue where minor diffusion occurred on a micro scale, allowing the oxygen to contact and react with pyrite and other oxidizable components of the rock and ore. Further investigation suggested, however, that diffusion played a much more important role than first suspected, and this study was undertaken to determine the relative effects that flow and diffusion might have had on the shapes of ore rolls.

The mathematical methods developed in this paper are also applicable without extensive modification to in situ solution mining. The movement of oxidants that are deliberately injected into ore deposits is controlled by the same laws that control the movement of natural oxidants.

## Geologic descriptions of rolls

Uranium ore rolls are ordinarily found in gently dipping aquifer sandstones that are sandwiched between less permeable mudstone beds. In vertical sections (fig. 1) an altered tongue containing iron-oxide minerals and lacking organic material projects into the unaltered rocks containing disseminated authigenic pyrite and scattered coaly fossil-plant remains. Epigenetic accumulations of uranium, selenium, molybdenum and other elements commonly form the ore zone, largely in the reduced rock, adjacent to or near the borders of the oxidized tongue. In plan, (fig. 2) most oxidized tongues associated with economic ores extend over several tens of square kilometers and have a highly irregular almost amoeboid outline on their advancing edge.

The epigenetic or ore stage minerals appear to be incidental to the rolls, which invariably consist of an oxidized tongue adjacent to pyrite-bearing sandstone. This means that an essential process in the genesis of roll deposits is the oxidation of pre-existing pyrite. Dissolved oxygen is carried by flowing ground water and reacts with pyrite very soon after the two reagents come in contact. The oxidized tongue continues to advance as long as oxygen is supplied to the roll. Scattered plant debris, if present, also consumes oxygen and may slow the advance of the roll but organic material does not appear to be an essential component of the ore-forming process.

## Processes that affect roll shape

Meteoric water equilibrates with oxygen in the atmosphere and then percolates downward to the water table. If oxygen is not consumed by reactions within the host rock, the water at the water table contains about 8 ppm dissolved oxygen at sea level, according to Henry's Law. As this recharged water continues to move down dip, below the water table, it can no longer equilibrate with the atmosphere, and its oxygen concentration decreases through reactions with the surrounding environment. The position and shape of a sandstone roll is largely dependent on the reactions between this dissolved oxygen and pyrite in the host rock.

The variables that have a major effect on the shape of the roll are related principally either to the distribution of pyrite or to the relative

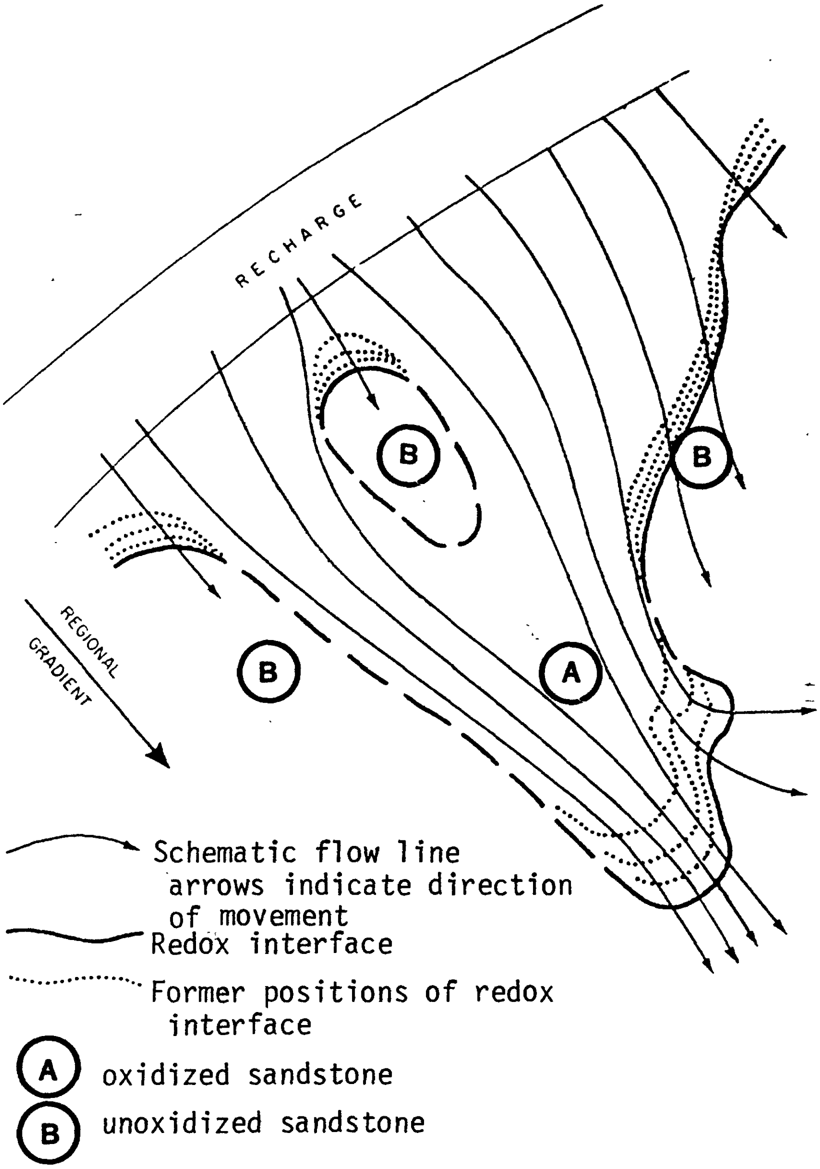

Figure 2. Plan view of an idealized oxidized tongue produced by a variable flux of oxygen charged ground water.

rates of oxygen delivery. If the pyrite distribution is uneven, the advancing oxidation interface tends to be impeded in places where pyrite is most abundant. Conversely, it advances more rapidly where pyrite is sparse. Oxygen is carried to the vicinity of the roll front (fig. 2) principally by the flow of ground water in which it is dissolved. Therefore, the flow of ground water is the dominant control of oxygen delivery at points along the oxidation interface. Because the permeability of fluvial sandstone can vary from depositional unit to depositional unit and even from point to point within depositional units, ground-water velocities are irregular within most aquifers and the delivery of oxygen must vary from point to point. These variations can produce prominent bulges or irregularities in the oxidation interface.

The effects of flow are especially evident when an entire basin is considered (fig. 2). The relatively large distances that are involved emphasize velocity transportation of oxygen and the shapes of the oxidized tongue are predominantly dependent on the movement of water. When shorter distances are considered, as in figure 1 which shows a vertical cross section of an aquifer, oxygen diffusion becomes much more important. The diffusion process has a distinct moderating effect on the delivery of oxygen.

The familiar roll shape of an oxidation interface can be created through either or both of two conditions. (1) If the concentration of pyrite is greater near the mudstone boundaries than at the center of the sandstone unit, then the oxidation interface will penetrate farthest at the center even with a uniform delivery of oxygen. (2) If the rate of delivery of oxygen at the center exceeds oxygen delivery at the boundaries, then the oxidized interface will protrude farthest at the center even with a uniform distribution of pyrite. The second condition can be realized for two situations: (a) The water velocity is impeded near the boundaries or (b) some factor causes the concentration of dissolved oxygen to be depleted at the boundaries.

If the delivery of oxygen was controlled exclusively by "velocity transportation," this would be the only transport mechanism that needs to be considered. However, diffusion tends to equalize concentrations of oxygen whenever some factors such as a chemical reaction cause a concentration imbalance to occur. In solutions which are moving through a porous medium, the effects of diffusion are modified. Although the augmented process is termed dispersion, it still has the same general characteristics as diffusion. Dispersion tends to obscure the effects of velocity transportation and to smooth the smaller irregularities in the oxidation interface.

In a homogeneous sandstone aquifer it is reasonable to assume that the cross sectional roll shape would be symmetrical and a smooth curve. Small random inhomogeneties would not noticeably disrupt the shape because of the smoothing affects of dispersion. However, continental sandstones are never perfectly homogeneous and the cross sections of natural rolls are never symmetrical or regular. Nevertheless, many geologists have preferred to emphasize the underlying regularities and to describe idealized rolls in terms of the symmetrical "C" shape.

## Description of the computer model

The model is based on a permeable sandstone layer containing disseminated pyrite and enclosed between relatively impermeable mudstones. Water containing dissolved oxygen is percolated through the system parallel to the sandstone-mudstone contacts and the oxygen is allowed to react with the disseminated pyrite. Pyrite is spatially fixed within the model until it is destroyed by the oxidation but oxygen is free both to be transported with the water by flow and through the water by dispersion. The rate of flow of water in the model can be arbitrarily adjusted to account for changes in permeability across the aquifer. Two different versions of the basic model described above were tested. In the first, the mudstone boundaries were tightly sealed and neither oxygen nor water was allowed to escape from the aquifer. In the second, just enough oxygen was allowed to leak out of the aquifer across the mudstones to noticeably affect the roll shape.

For purposes of the computer program, a cross section of the aquifer-mudstone system is divided into a matrix of cells, 100 cells long and 31 cells high (fig. 3). At the beginning of a program each of the 3100 cells is assumed to contain the same amount of disseminated pyrite. Water containing a given amount of dissolved oxygen is then allowed to flow through the matrix and the oxygen reacts stoichiometrically with the pyrite in the first column of cells. As soon as the pyrite in a cell is used up, the oxygen is allowed to pass into the next cell, and so on.

To simplify, the permeability and flow rate are kept uniform throughout each horizontal row of cells. The profile of flow velocities across the aquifer was arbitrarily assumed to be symmetrical. Flow in the mudstone is set at zero. Two general types of flow profiles were tested. The first is termed parabolic flow (fig. 4D). It has maximum flow velocity at the center row in the aquifer and the velocity decreases steadily in each row toward the edge of the aquifer. The other type we have termed step flow. Here the velocities are equal and uniform across all the central rows of its aquifer and then drop abruptly to zero in one or more steps in the last few rows along the aquifer boundaries (fig. 4A, 4B, 4C). This decrease in flow is analogous to a geologic setting where increased clay/sand ratios occur near the mudstone.

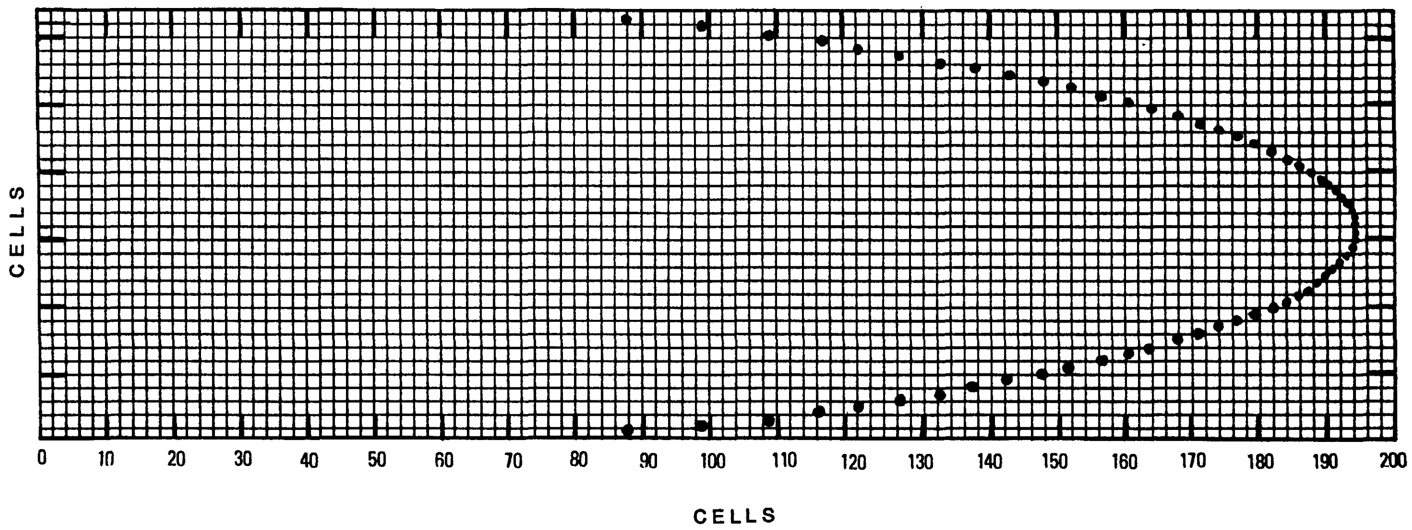

Figure 3.-Matrix representation containing a roll. A 64 by 200 matrix showing the redox interface at high resolution is superimposed on a standard 32 by 100 matrix. See table 1 line 10 for additional information concerning this redox interface.

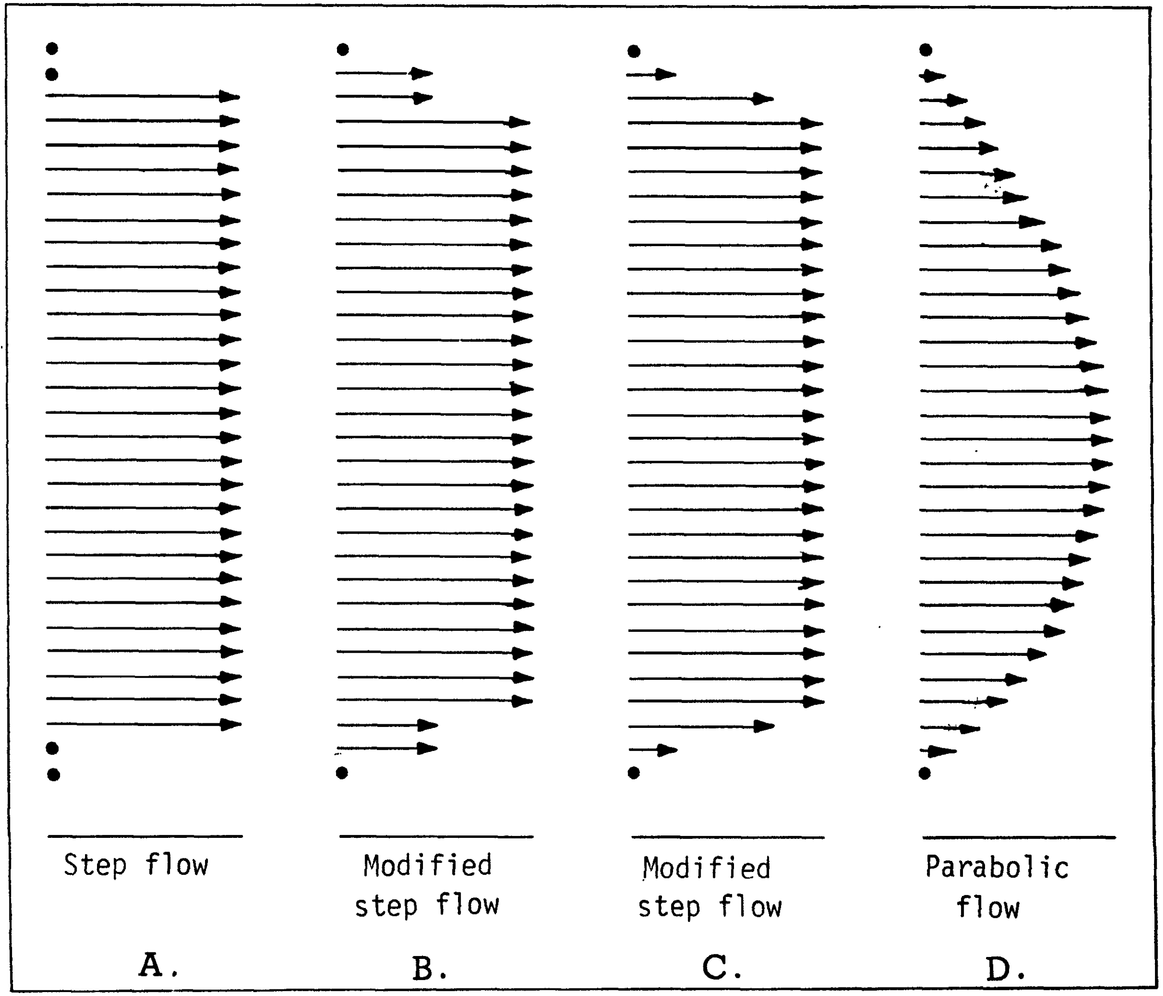

Figure 4. Flow profiles. Vertical cross sections of selected ground water flows are shown.

The movement of oxygen through the matrix of cells is controlled both by flow and dispersion. Flow is assumed to carry the oxygen straight ahead through each row of cells. Dispersion permits oxygen to move into adjacent cells, in any direction by a diffusion-controlled process.

A problem exists in matching the boundaries of the computer model with actual deposits. In the model, three regions can be identified; these are characterized by the importance of diffusion transport (D) relative to velocity transport (V). In the large central region, oxygen is delivered by both processes. In the narrow border regions where water velocity is reduced to near zero, oxygen is only delivered by diffusion processes. Ultimately the aquifer is completely sealed and oxygen cannot move by any process. Here D and V are both zero. No experimental information exists concerning D and V in actual deposits. This means that the mudstone-sandstone contact may mark either the position where the aquifer is completely sealed to both diffusion and water movement or the position where only the ground-water velocity becomes zero while oxygen diffusion remains significant. The difference between the two is the position of the narrow zone of restricted water flow in which diffusion is important. It may reside entirely in the sandstone, the mudstone, or some combination of the two.

If the mudstones are somewhat permeable it is conceivable that oxygen may diffuse across the entire mudstone layer and ultimately leak out of the aquifer. This possibility was investigated in the leaky oxygen model. Studies of this model were simplified by assuming a uniform ground-water flow throughout the sandstone. The shape of the oxidation front is due entirely to small amounts of oxygen leaking from the aquifer.

## Flow and dispersion

Although ground-water movement can be completely characterized by a velocity profile of a cross section of the aquifer, flow is described in this paper in terms of the average flow and a flow index. The flow index is the average flow divided by the maximum velocity that occurs in the velocity profile. It is a measure of the fraction of all the ground water in the aquifer that is found in rapidly moving channels. The symbol "S" is assigned to the term, 1 - flow index (an adaption from Rouse, 1961, p. 198). It is a measure of the fraction of the ground water in the aquifer that moves slowly.

Fick's first law of diffusion states that the flux of a solute in a stagnant solution is proportional to the concentration gradient. For a solution that is moving through a porous medium such as a sandstone, however, the flux of solutes becomes more complex. In such an environment, diffusion is modified by factors related to (1) the flow rate of the solution, (2) the porosity of the rock, and (3) tortuosity, which is a measure of the increased distances solute particles must travel around sand grains in moving through a porous medium. Diffusion modified by these factors is known as dispersion. From a practical standpoint, at the very slow flow rates we are concerned with only the tortuosity and porosity have any significant effects and a fairly simple approximate relationship exists between diffusion and dispersion (Fried and Combarmous, 1971). In a typical, moderately well sorted sandstone, tortuosity reduces the effects of diffusion only by about 1/3 (Fried and Cambarmous, 1971); because only the interstices are occupied by fluid through which diffusion can occur, diffusion transport is directly proportional to porosity. Therefore the dispersion coefficient is about 2/3 PD, or 2/3 times the porosity times the diffusion coefficient.

The ratio of D/V is a measure of the relative importance of dispersion transport and velocity transport. This ratio is especially simple if both D and V are expressed in terms of diffusion and interstitial velocity, as the following derivation demonstrates.

$$\frac{D}{V} = \frac{2/3 \times P \times \text{Diffusion Coefficient}}{P \times \text{Interstitial Velocity}}$$

$$\frac{D}{V} = \frac{2/3 \times \text{Diffusion Coefficient}}{\text{Interstitial Velocity}}$$

where D is the dispersion coefficient

V is the bulk velocity

P is the porosity

All results of this paper that are dependent on D/V are given in terms of the dispersion coefficient for 50 percent porosity and bulk velocity.

## Mathematical Procedure

The rules governing the movement of oxygen in the oxidized tongue as the ground water approaches the orebody are given in equation (1).

$$\frac{\partial Z}{\partial t} = - V \frac{\partial Z}{\partial X} + D \frac{\partial^2 Z}{\partial X^2} + D \frac{\partial^2 Z}{\partial Y^2} - \frac{\partial R}{\partial t} \qquad (1)$$

Where

Z = two dimensional concentration of oxygen--actually a volume concentration over a unit area one unit thick;

R = two dimensional concentration of pyrite, expressed in units that are equivalent to the oxygen on a stoichiometric basis;

X = longitudinal distance in the direction of water flow;

$Y$ = transverse distance;

$D$ = dispersion coefficient;

$V$ = velocity of ground water;

$t$ = time; and

$\partial$ = partial derivative.

This is similar to equations for heat flow given by Carslaw and Jaeger (1959) in which oxygen concentrations replace expressions for heat content. The equation is applicable to a small cell such as the one shown on figure 5. It provides information on the increase or decrease of oxygen in the cell per unit time. Similar information for all cells of the oxidized tongue provides a complete picture of oxygen movements relative to time.

The term, $V\frac{\partial Z}{\partial X}$, of equation (1), describes the transportation of oxygen by the ground-water movement. The amount of oxygen moving across a vertical boundary of the cell per unit time is proportional to the product $VZ$, the velocity of water times the concentration of oxygen. The concentration of oxygen, $Z$, is a variable and changes from point to point. Thus, if $Z$ is larger as the water enters the cell than it is as water leaves the cell and oxygen is not consumed within the cell, the oxygen concentration in the cell must increase. If the situation is reversed then the oxygen concentration must decrease. The increase or decrease of oxygen in the cell is equal to the product of the velocity times the difference in concentration of oxygen entering and leaving the cell. The term $V\frac{\partial Z}{\partial X}$ determines how fast the oxygen content of the cell increases or decreases because of the movement of ground water.

The expression, $D\frac{\partial^2 Z}{\partial X^2} + D\frac{\partial^2 Z}{\partial Y^2}$ deals with transportation of the oxygen by dispersion. The first and second terms deal respectively with longitudinal and transverse dispersion. The product of the dispersion coefficient and the

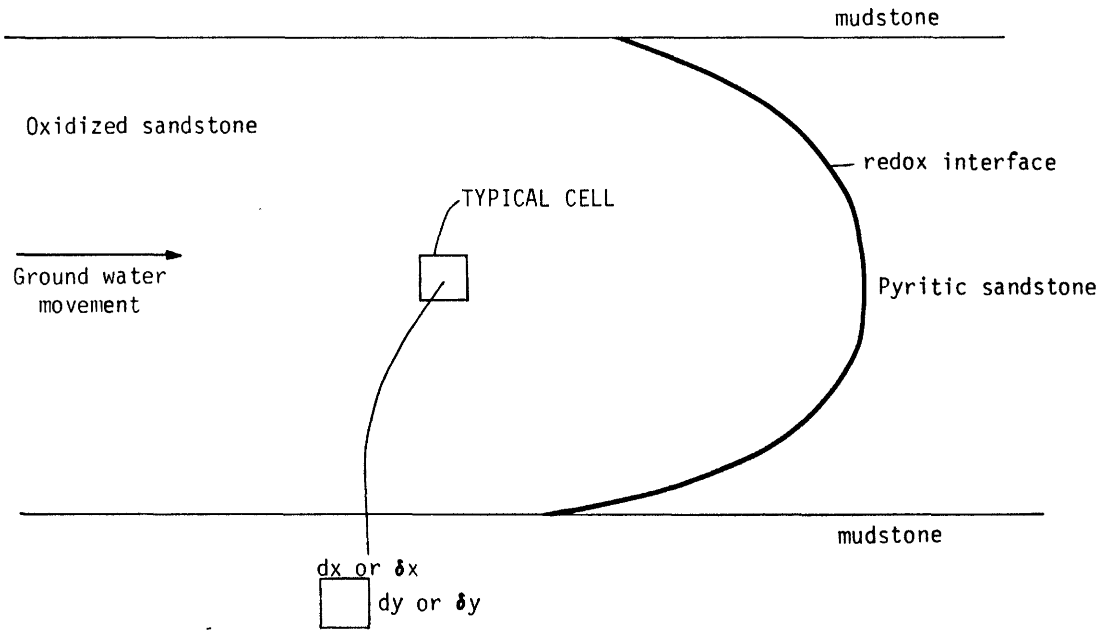

Figure 5. Representative cell. Oxygen in the cell shown is controlled by dispersion and ground water movement.

concentration gradient is equal to the rate at which the oxygen is transferred by the dispersion process. If the concentration gradient remains constant across the cell, the same amount of oxygen is diffusing into the cell as is diffusing out of the cell. However, if the concentration gradient is large at one side of cell and small at the opposite side, then the oxygen content of the cell must increase. If the situation is reversed, the cell loses oxygen. The loss or gain of oxidant is equal to the product of the dispersion coefficient and change in the concentration gradient across the cell,

$$D\frac{\partial^2 Z}{\partial X^2} + D\frac{\partial^2 Z}{\partial Y^2},$$

The term $\partial R/\partial t$, representing the change in the amount of pyrite in a cell is zero except along the redox interface. In the oxidized tongue where pyrite is absent, the amount of pyrite cannot change and, therefore, $\partial R/\partial t = 0$. In the unaltered rock where the oxygen content is zero and the pyrite is neither being oxidized or destroyed, the amount of pyrite cannot change and, therefore, $\partial R/\partial t = 0$. Along the redox interface where both oxygen and pyrite can coexist briefly in the same cell, a chemical reaction occurs which affects the concentration of both components. The computer is programmed so that the rules of equation (1) are applied in turn to each cell in 3100 separate calculations. A typical calculation that is shown in the following example 1 adjusts the oxygen concentration of the central cell of figure 6. An identical computation is then repeated for the next cell. After each cell is examined, the concentrations of all cells are simultaneously adjusted for the calculated changes in concentration. The results provide an equivalent of one frame in a motion picture of the oxygen concentration in the ground water.

Example 1

<u>Given</u>: V = 0.2 cell/unit time (arbitrary)

D = 0.1 $\text{cell}^2$unit time (arbitrary)

cu = concentration units

(from figure 6:)

<u>Oxygen added to the cell by flow</u>

(0.64-0.48) 0.2 = 0.032 cu x cell volume/unit time

<u>Oxygen added to the cell by dispersion</u>

0.1 [(0.64-0.48)-(0.48-0.40)] = 0.008 cu x cell volume/unit time

0.1 [(0.96-0.48)-(0.48-0.16)] = 0.016 cu x cell volume/unit time

Total change in the central cell = 0.032 + 0.008 + 0.016 =

0.056 cu x cell volume/unit time.

<u>The following calculation occurred after all 3100 cells were examined</u>:

New concentration of central cell after one time unit is

0.480 + 0.056 = 0.536 cu.

The calculation shown is applicable to the body of the tongue. An additional step is required for the cells along the redox interface where pyrite and oxygen briefly coexist.

Along the redox interface the chemical reaction of oxygen and pyrite keeps the oxygen concentration equal to zero. Thus along the interface $\partial Z/\partial t = 0$. Substituting this value into equation (1) the relation of equation (2) is obtained.

$$\frac{\partial R}{\partial t} = V \frac{\partial Z}{\partial X} - D \frac{\partial^2 Z}{\partial X^2} - D \frac{\partial^2 Z}{\partial Y^2} \qquad (2)$$

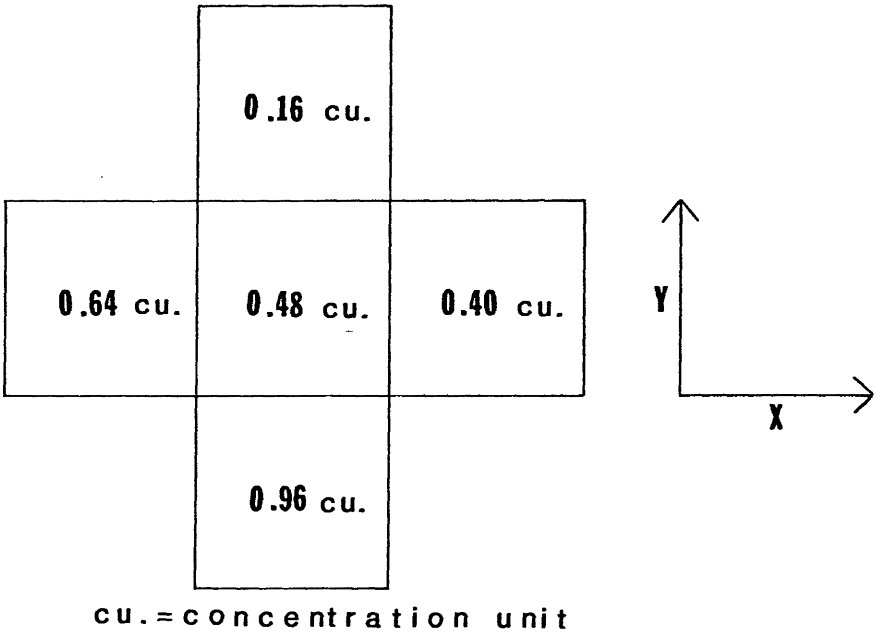

Figure 6. Cell cluster. The five cells shown are used to calculate the new oxygen concentration of the center cell. Actual calculations shown in example 1, p. 18.

The equation indicates that the rate of dissolution of pyrite in chemical equivalences is exactly equal to the rate at which oxygen is delivered to the redox interface. To calculate the dissolution of pyrite in a given time, the amount of oxygen delivered to a cell along the redox interface is determined by the process previously described. The amount of oxygen delivered is then subtracted in chemical equivalents, from the amount of pyrite in the cell. This adjustment is made after the changes in concentration of all 3100 cells are established. Conditions are generally adjusted so that between 7 and 50 iterations are required to completely remove the pyrite from the line of cells along the pyrite interface. This is defined to be one step once the system has reached a steady-state shape.

Flow velocities used in the model can be converted to conventional units through the ratio of D/V. Conversion of V = 0.2 cells/unit time in the model to V in cm/sec is illustrated in example 2.

Example 2

Given:

Cell length = 64.5 cm (arbitrary)

$D = 0.1$ cell length$^2$/unit time (arbitrary)

[1]$D = 6.3 \times 10^{-6} \text{cm}^2/\text{sec}$

The problem is to convert a velocity of 20 cells/unit time to cm/sec.

$D/V = (0.1$ cell lengths$^2$/unit time$)/$

(20 cell length/unit time)

---

[1] The dispersion coefficient of $6.3 \times 10^{-6}$ is 2/3 of the product of the porosity and diffusion coefficient. The diffusion coefficient of $1.9 \times 10^{-5}$ for oxygen was obtained from International Critical Tables. Calculation is for 50 percent porosity.

$D/V = 0.005$ cell lengths

0.005 cell lengths are 0.323 cm

$D/V = 0.323$ cm

$(6.3\times10^{-6}\mathrm{cm}^2/\mathrm{sec})/V = 0.323$ cm

$V = 1.95\times10^{-5}\mathrm{cm/sec} = 6.15$ m/yr.

where $V$ is bulk velocity.

## Results

The shape of the oxidation interface is dependent on the conditions that existed at the time the roll formed. Similarly the shape of a calculated oxidation interface is dependent on the conditions that are used to solve the problem. No computational method exists for starting with a given shape and working backwards to obtain the conditions under which the roll formed. This means that many different conditions must be tested to find which conditions produce the shapes that are similar to the shape of a natural roll. This is done by specifying the conditions in the computer program and then running the model until the shape of the oxidation interface reaches a steady state. The shape usually evolves while it advances the first 150 cell lengths. Once steady state is attained the shape remains fixed regardless of how far the roll advances provided conditions are not changed. The steady-state shape depends only on the conditions specified in computer program.

In the computer models emphasizing flow, all other controls are minimized. The pre-ore pyrite content is held constant. Leakage across the mudstone boundary is prevented by setting the dispersion coefficient in the mudstone to zero. Symmetry and uniformity are used to reduce computer costs and simplify the description of the results.

In computer models where the mudstone boundaries are sealed a crescent-shaped roll (C) is always obtained when ground-water flow is somehow restricted along the mudstone contact. Figure 3 shows the general shape that is obtained. The extent of elongation of a roll is a very sensitive indicator of conditions of formation. Elongation of the roll relative to its height appears to be the most practical measure of the conditions under which the roll formed. Figure 7 shows the measurements involved in deriving the length-to-height ratio. The roll length is the horizontal distance from the tip of the oxidized tongue to the last vestiges of unreacted pyrite along the upper and lower boundaries of the sandstone. The height is the vertical distance across the sandstone. The length-to-height ratio is defined as length/height and is called the length ratio for convenience.

Table 1 shows the length ratios for computer models emphasizing various kinds of ground-water flow. The length ratios are obtained from direct measurements of the computer print out of the pyrite matrix after the roll shape has reached a steady state. Measurements are to the nearest cell length which means a measurement error approaching 3 percent.

The length ratio is proportional to ground-water velocity when other conditions remain constant; for example, only the ground-water velocity varies for lines 12, 13, and 14 of table 1. Examination of this data shows that the length ratio is proportional to ground-water velocity. Comparison of lines 15, 16, and 17 reveals the proportionality of velocity and length ratio for parabolic flow.

For step flows the length ratio is approximately proportional to (1 - flow index) or S. Comparison of lines 8 and 9 (table 1) which have the same ground-water velocity shows the proportionality between S and length ratio. This simple proportionality does not hold very well when the value of S becomes larger than about 5/31. Comparison of lines, 4, 5, 6, and 7 demonstrates the marked deviation from simple proportionality.

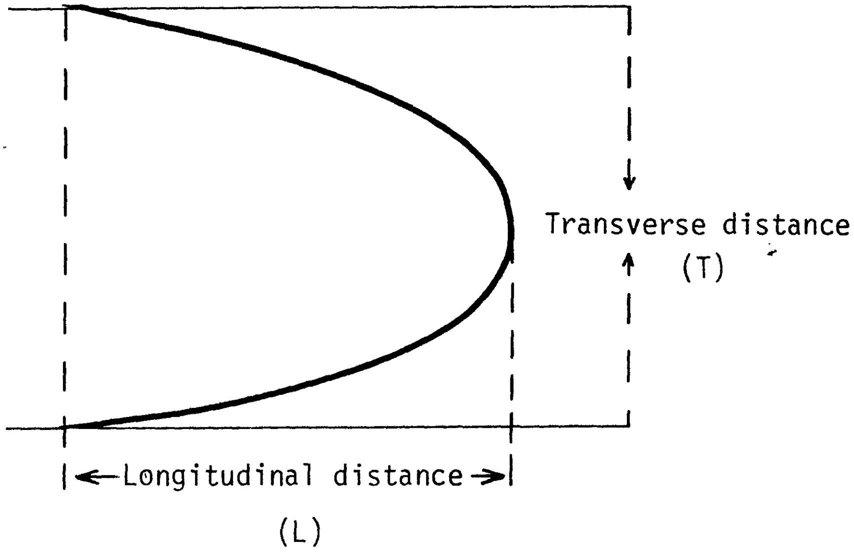

Figure 7. Length ratio. Measurements used in calculating the length ratio are shown. Length ratio = L/T.

Table 1.--Length to height ratios

| Number | S[1] | Maximum[2] bulk velocity in cells/unit time | Average[2] bulk velocity in m/yr | Model length ratio | Calculated[3] length ratio | Type of flow |
|---|---|---|---|---|---|---|
| 1 | 10/31 | 0.30 | 0.0596 | 2.49 | 2.32 | Step. |
| 2 | 10/31 | 0.20 | 0.0397 | 1.65 | 1.55 | do. |
| 3 | 10/31 | 0.15 | 0.0298 | 1.23 | 1.163 | do. |
| 4 | 12/31 | 0.20 | 0.0360 | 1.84 | 1.69 | do. |
| 5 | 10/31 | 0.20 | 0.0397 | 1.65 | 1.55 | do. |
| 6 | 8/31 | 0.20 | 0.0435 | 1.43 | 1.36 | do. |
| 7 | 6/31 | 0.20 | 0.0473 | 1.13 | 1.02 | do. |
| 8 | 4/31 | 0.40 | 0.102 | 1.55 | 1.59 | do. |
| 9 | 2/31 | 0.40 | 0.110 | 0.871 | 0.859 | do. |
| 10 | 2/31 | 0.828 | 0.227 | 1.72 | 1.77 | do.[4] |
| 11 | 1/31 | 1.60 | 0.454 | 1.74 | 1.77 | do.[4] |
| 12 | 8/31 | 0.30 | 0.0653 | 2.16 | 2.04 | do. |
| 13 | 8/31 | 0.20 | 0.0435 | 1.43 | 1.36 | do. |
| 14 | 8/31 | 0.15 | 0.0326 | 1.03 | 1.02 | do. |
| 15 | 1/3 | 0.30 | 0.0587 | 2.13 | N.A.[5] | Parabolic. |
| 16 | 1/3 | 0.20 | 0.0391 | 1.42 | N.A. | Parabolic. |
| 17 | 1/3 | 0.15 | 0.0293 | 1.06 | N.A. | Parabolic. |

[1] S is (1 - flow index).

[2] Conversions according to example 2 and the following units:

D = 0.1 cell$^2$/unit time = 6.3 x $10^{-6}$ cm$^2$sec and a 20 m aquifer.

[3] Calculated by equation 3.

[4] Calculation based on a 64 by 200 matrix. Units are given in terms of 31 by 100 matrix

[5] N.A. not applicable.

Length ratios produced by step flows with S less than 5/31 can be represented by equation 3. This equation relates length ratio to velocity and S for a 20 m thick sand.

$$LR = 121VS \qquad (3)$$

where L is the length ratio
where V is average velocity in m/yr
note V = (flow index) V max = (1-S) V max
S is 1 - flow index or the fraction of groundwater that is relatively stagnant

From the examples used to illustrate equation (3) it might appear that the equation is based on only the four examples of lines 8, 9, 10, and 11. Equation (3) is actually the limit as S approaches zero of a more complex equation (Schock, oral commun., 1979), that can be used to represent all step flows of table 1 including large S values. However, there is no need to consider large S values because these conditions are not significant from a geologic viewpoint. Equation (3) is within the 3 percent measurement error for lines 8, 9, 10, and 11 where S is less than 5/32. The equation gives errors as much as 10 percent for some S values above 5/32 in table 1.

According to geologic evidence presented in a previous paper (Granger and Warren, 1979) water flow during the early stages of ore formation was probably faster than at the present and may have exceeded a bulk flow of 10 m/yr. Many deposits have a length ratio of about five. When these numbers are substituted into equation 3 the indicated S value for 10 m/yr velocity is 0.004 or 8 cm for a 20 m thick sandstone. This means that S values larger than about 0.004 are not geologically significant and that the width of the low velocity zone is probably less than 10 cm for a 20 m thick sandstone. These restrictions exclude all parabolic flows.

As indicated earlier, the manner by which flow is terminated near the mudstone contact is not understood. In an attempt to better visualize the border region the standard model was expanded to a 200 by 64 matrix. Different kinds of terminations are illustrated by the following symetrical flow profiles of 62 entries: (a) 0, 0, 1, 1, ...; (b) 0, 1/2, 1/2, 1, 1,...; (c) 0, 1/3, 2/3, 1, 1,...; and (d) 0, 1/2, 3/4, 7/8, 7/8, 1,... All these flow profiles have S equal to 2/31. When these flow profiles are used as alternates for line 10 table 1 in the computer model the roll shapes were virtually identical to the one shown in figure 4. This confirms that the single most important measure of a flow profile is the S value. Various means by which the ground water might be slowed include undulating contact, infusion of clay a short distance into the aquifer and the sorting of sand grains. If the bulk velocity of the ground water is 10 m/yr the zone of restricted velocity is only about 8 cm wide for a 20 m thick sandstone and a length ratio of 5.

Until now the discussion assumed that the clay-sandstone contact marked the place where the entire system is completely sealed to both flow and diffusion and that ground water flow slowed a few centimeters from this contact. Other possibilities should be recognized. It may be possible that flow continues across the entire sandstone aquifer all the way up to the contact and that diffusion penetrates about 10 centimeters into the mudstone. In this situation, ground-water flow stops at the contact and the narrow stagnant zone resides entirely within the mudstone. The difficulty with this situation is how to justify diffusion for a few centimeters into the mudstone. There are no obvious features in the mudstone which would allow diffusion for only a few centimeters. Field evidence suggests the oxidation does actually penetrate a short distance into the mudstones in some deposits. The few centimeters may represent a practical upper limit on diffusion. If the mudstones are tightly packed, it would take an exceedingly long time for oxygen to penetrate further than this.

Typical roll shapes were also produced in computer models that allowed oxygen to leak from the aquifer. This model gave reasonable results that are compatible with typical length ratios and ground-water velocities. A 20 m thick aquifer leaking a 4 x $10^{-4}$ of its oxygen each year and with ground water moving with a bulk velocity of 1 m/yr produced a roll with a length ratio of 5. Because this is very reasonable, there is nothing about the model itself that would call its validity into question. However, oxygen must pass all the way through the mudstone if it leaks from the aquifer and then all parts of mudstones would be subject to the action of low concentrations of oxygen. The existence of pyrite in the mudstone long after the oxidation interface passed indicates either that oxygen was never present in the mudstone or that the pyrite does not react with low concentrations of oxygen in a mudstone environment. It is difficult to believe that oxygen and pyrite do not ultimately react even under adverse circumstances. The presence of pyrite in the mudstones appears to exclude the possibility that significant amounts of oxygen can leak out of the aquifer by diffusion across the mudstone.

## Conclusions

Computer modeling of geologic processes is not intended to be a substitute for laboratory modeling or field studies. In some instances, however, field studies are inconclusive and laboratory models require more time or equipment than is available. In these instances computer modeling allows a rapid study of dynamic processes. The development of roll type deposits has proved to be ideal for such studies.

The paramount conclusion derived from the present study is that dispersion and diffusion play a much larger part in determining the shapes of roll type deposits than was first suspected. In fact, it was found that the characteristic roll shape is dependent on a combination of fluid flow and diffusion of dissolved oxygen.

In the model presented we have simulated the physical processes of oxygen transport by diffusion and ground water movement, and of chemical reactions between oxygen and pyrite disseminated in the host rock. As oxygen reacts with pyrite, a concentration gradient is established and oxygen moves, by diffusion, to eliminate this gradient. At the upper and lower surfaces of an aquifer, water flow is reduced and oxygen delivery by ground-water movement is restricted. Therefore, ground water near these surfaces cannot react with as much pyrite as ground water near the center of the aquifer. Hence, the redox interface penetrates nearer the center than the margins of the sandstone. Eventually, the process reaches a steady state. This comes when flow and dispersion balance in such a way that equivalent vertical segments of the roll front are all receiving oxygen at an equal rate.

Furthermore, the shape of the roll--its length ratio--can be quantitatively related to the velocity of ground water flow for a given set of initial conditions. The faster the flow the more elongated the roll. Because most natural rolls are not ideal, it is difficult to translate this relation to known roll deposits but preliminary computer studies show that ground water velocities of about 10 m/yr produce rolls in the model that have reasonable ratios and resemble typical shapes observed in the Powder River Basin (Elmer Santos, oral commun., 1978).

Although it was not a part of this study, computer modeling is also applicable to solution mining of uranium ores. Most solution mining depends on oxidation and solution of uranium in much the same way that our model depends on oxidation and solution of pyrite. Hence, with certain modifications in the computer program, the solution and transport of uranium under such artificial conditions can be modeled just as readily as the processes forming natural rolls.

## REFERENCES

Adler, H. H., and Sharp, B. J., 1967, Uranium ore rolls-occurrence, genesis, and physical and chemical characteristics: Utah Geological Society Guidebook No. 21, Uranium District of Southeast Utah, 53-77.

Bear, Jacob, 1972, Dynamics of fluids in porous media: New York, American Elsevier, 764 p.

Carslaw, H. S., and Jaeger, J. C., 1959, Conduction of heat in solids: Oxford, Oxford University Press, 510 p.

Evseeva, L. S., Ivanov, K. E., and Kochetkov, V. I., 1963, Some laws of the formation of epigenetic uranium ores in sandstone derived from expimental and radiochemical data: Atomnaya Energiya, v. 14, no. 5, p. 474-482 [translation New York Consultants Bur., p. 492-498].

Fried, J. J., and Combarmous, M. A., 1971, Dispersion in porous media, in Van Te Chow, ed., Advances in hydroscience: New York, Academic Press, v. 7, p. 170-182.

Germanov, A. N., 1960, Main genetic features of some infiltration-type hydrothermal uranium deposits: Akademiya Nauk SSSR Izvestiya, Seriya Geologicheskaya. no. 8, p. 60-72 [English translation].

Granger, H. C., 1976, Fluid flow and ionic diffusion and their roles in the genesis of sandstone-type uranium ore bodies: U.S. Geological Survey Open-File Report 76-454, 26 p., 11 figs.

Granger, H. C., and Warren, C. G., 1979, Some speculations on the genetic geochemistry and hydrology of roll-type uranium deposits: Wyoming Geological Association Guidebook, Thirtieth Annual Field Conference, 1978, p. 349-361.

Harshman, E. N., 1962, Alteration as a guide to uranium ore, Shirley Basin, Wyoming, in Short papers in geology, hydrology, and topography: U.S. Geological Survey Professional Paper 450-D, p. D8-D10.

Kashirtseva, M. F., 1964, Mineral and geochemical zonation of infiltration uranium ores: Soviet Geology, no. 10, p. 51-65. [English translation].

Kholodov, V. N., Lisitsyn, A. K., Komarova, G. V., and Kondrat'yeva, I. A., 1961, Epigene zoning of uranium mineralization in petroliferous carbonate rocks: Akademiya Nauk SSSR Izvestiya Seriya Geologicheskaya no. 11, p. 43-56.

King, J. W., and Austin, R., 1965, Some characteristics of roll-type uranium deposits at Gas Hills, Wyoming: Distributed by U.S. Atomic Energy Commission, Grand Junction, Colo., 40 p.; abridged publication in Mining Engineering 1966, 18, no. 5, p. 73-80.

Lisitsyn, A. K., and Kuznetsova, E. C., 1967, Role of microorganisms in development of geochemical reduction barriers where limonitization bedded zones wedge-out: Akademiya Nauk SSSR Izvestiya Seriya Geologicheskaya no. 1, p. 31-44. [English translation, International Geology Review v. 9, no. 9, p. 1180-1191, 1967]

Rouse, Hunter, 1961, Fluid mechanics for hydraulic engineers: New York, Dover Publications, Inc., 422 p.

Shawe, D. R., and Granger, H. C., 1965, Uranium ore rolls--an analysis: Economic Geology, v. 60, no. 2, p. 420-450.

Warren, C. G., and Granger, H. C., 1973, The concept of growth and maturity of ore-stage pyrite in roll-type uranium deposits: U.S. Geological Survey Journal of Research, v. 2, no. 2, p. 151-155.

Warren, C. G., 1971, A method of discriminating between biogenic and chemical origins of the ore-stage pyrite in a roll-type uranium deposit. Economic Geology, no. 6, p. 919-128.