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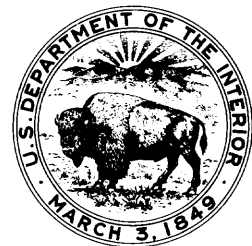
GROUND WATER OF COAL DEPOSITS,  
BAY COUNTY, MICHIGAN

By J. R. Stark and M. G. McDonald

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Technological University



Lansing, Michigan

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UNITED STATES DEPARTMENT OF THE INTERIOR

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## GLOSSARY

Aquifer--A formation, group of formations, or part of a formation that contains sufficient saturated permeable material to yield significant quantities of water to wells and springs.

Confining bed--A body of relatively impermeable material stratigraphically adjacent to one or more aquifers.

Hydraulic conductivity--The volume of water at the prevailing kinematic viscosity that will move in unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow. In general terms, hydraulic conductivity is the ability of a porous medium to transmit water.

Potentiometric surface--A surface which represents the static head in an aquifer. It is defined by the levels to which water will rise in tightly cased wells. Where head varies appreciably with depth, more than one potentiometric surface is required to describe the distribution of head. The water table is a particular potentiometric surface.

Recharge--The process by which water is absorbed and is added to the zone of saturation. Also, the quantity of water added to the zone of saturation.

Specific storage--The volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head.

Transmissivity--The ability of aquifer material to transmit water. It is equal to the product of hydraulic conductivity and thickness.

## CONVERSION FACTORS

| <u>Metric units</u>                      | <u>Multiply by</u> | <u>Inch-pound units</u>                  |
|--|--------------------|--|
| cubic meter (m <sup>3</sup> )            | 35.311             | cubic foot (ft <sup>3</sup> )            |
| hectare (ha)                             | 2.471              | acre                                     |
| kilometer (km)                           | 0.622              | mile (mi)                                |
| meter (m)                                | 3.281              | foot (ft)                                |
| meter per day (m/d)                      | 3.281              | foot per day (ft/d)                      |
| meter per kilometer (m/km)               | 5.280              | foot per mile (ft/mi)                    |
| metric ton                               | 1.102              | ton, short (2,000 lb)                    |
| millimeter (mm)                          | 0.039              | inch (in)                                |
| square kilometer (km <sup>2</sup> )      | 0.386              | square mi (mi <sup>2</sup> )             |
| square meter per day (m <sup>2</sup> /d) | 10.764             | square foot per day (ft <sup>2</sup> /d) |

# GROUND WATER OF COAL DEPOSITS, BAY COUNTY, MICHIGAN

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## ABSTRACT

A coal deposit in Bay County, Michigan, typical of other coal deposits in the State, was studied to determine the degree to which hydrologic factors might affect mining. This coal deposit, which averages about 0.5 meters in thickness, lies 50 meters below land surface. It is part of a multilayered aquifer system. Hydrologic characteristics (hydraulic conductivity and storage coefficient) of each unit were evaluated by aquifer test analyses and a finite-difference ground-water flow model. A flow model simulating ground-water flow to a hypothetical mine in the study area was then developed. Results of the study indicate that seepage will probably not be great enough to preclude mining coal. Also, pumping water to keep the mine dry will have little effect on heads in aquifers outside the mine during the first decade of mining.

Although coal was mined in Michigan during 1860-1950, significant deposits remain. These deposits, part of the Saginaw Formation of Pennsylvanian age, are near the industrialized parts of the State. The pumpage needed to keep mines dry and the effect of this pumpage on aquifers surrounding the mines will be major factors in determining the feasibility of opening new mines.

## INTRODUCTION

Michigan's coal was mined extensively during the early part of the 20th century. Competition from the East gradually made mining uneconomical, and, by 1950, production had ceased even though substantial reserves of bituminous coal remained. The cost of pumping water from the mines was a significant factor in the decline of the industry. Technologic advances in mining and increased demand for coal, however, suggest that mining once again is economically feasible if water problems can be controlled.

### Purpose and Scope of Study

The principal purpose of this study was to develop an understanding of the ground-water system associated with a typical coal deposit in the Lower Peninsula and to determine effects of hydrologic factors on a hypothetical mining operation. By simulation with a digital flow model, these factors were studied and their magnitude evaluated. Because hydrologically similar coal areas may have similar problems, results of the study probably can be applied to other areas in Michigan.

## Location and Extent of Area

The study area, 52 km<sup>2</sup>, is in T. 16 N., R. 3 E., in Bay County, Mich. (fig. 1). The area was selected because the geology and hydrology are similar to that of other areas containing coal deposits and because a substantial amount of data from domestic water wells, coal test holes, and gas exploratory wells were available (fig. 2). The area was also the subject of a detailed geologic report by Matthews (1965).

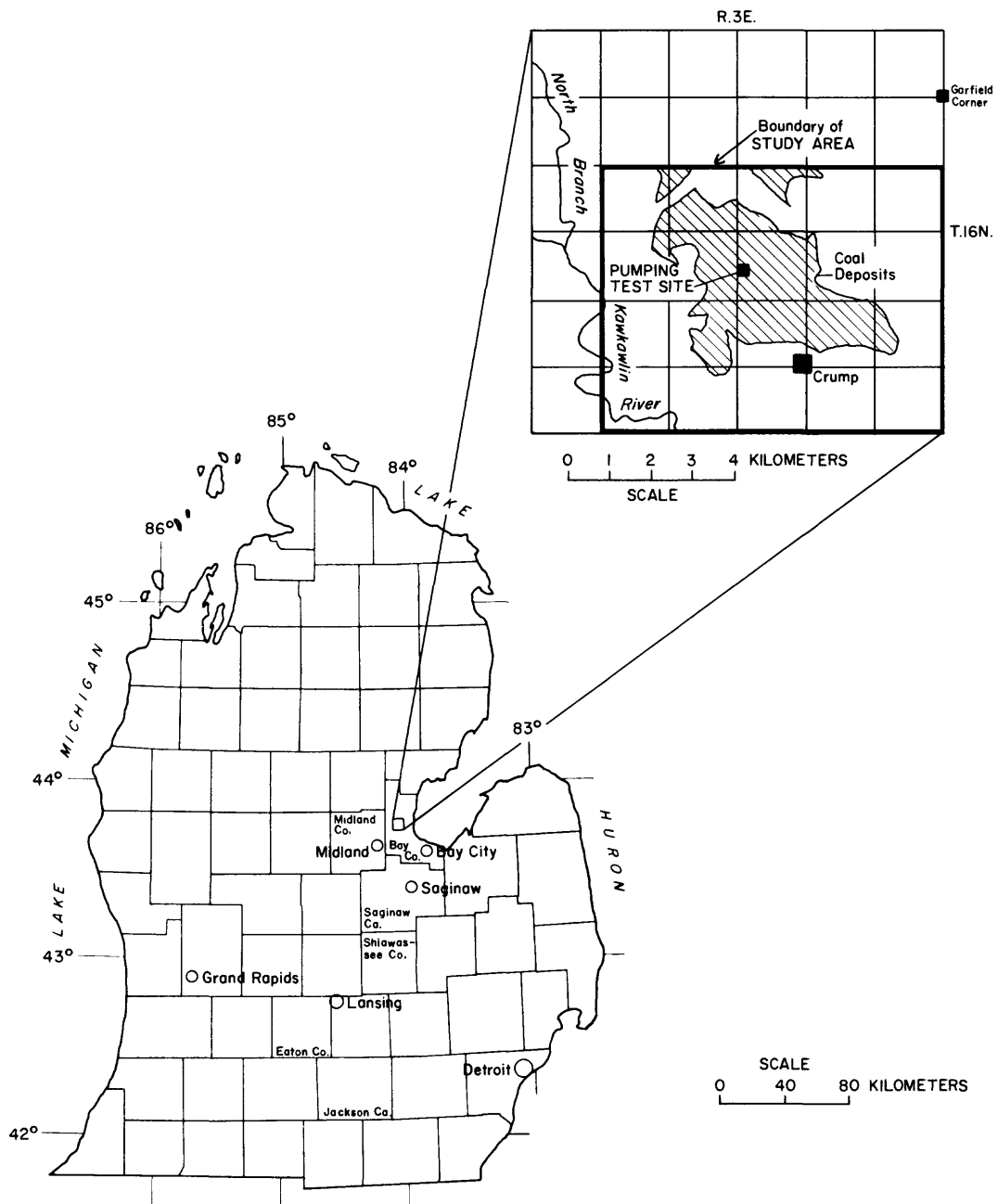


Figure 1.--Location of study area in Bay County, Michigan.

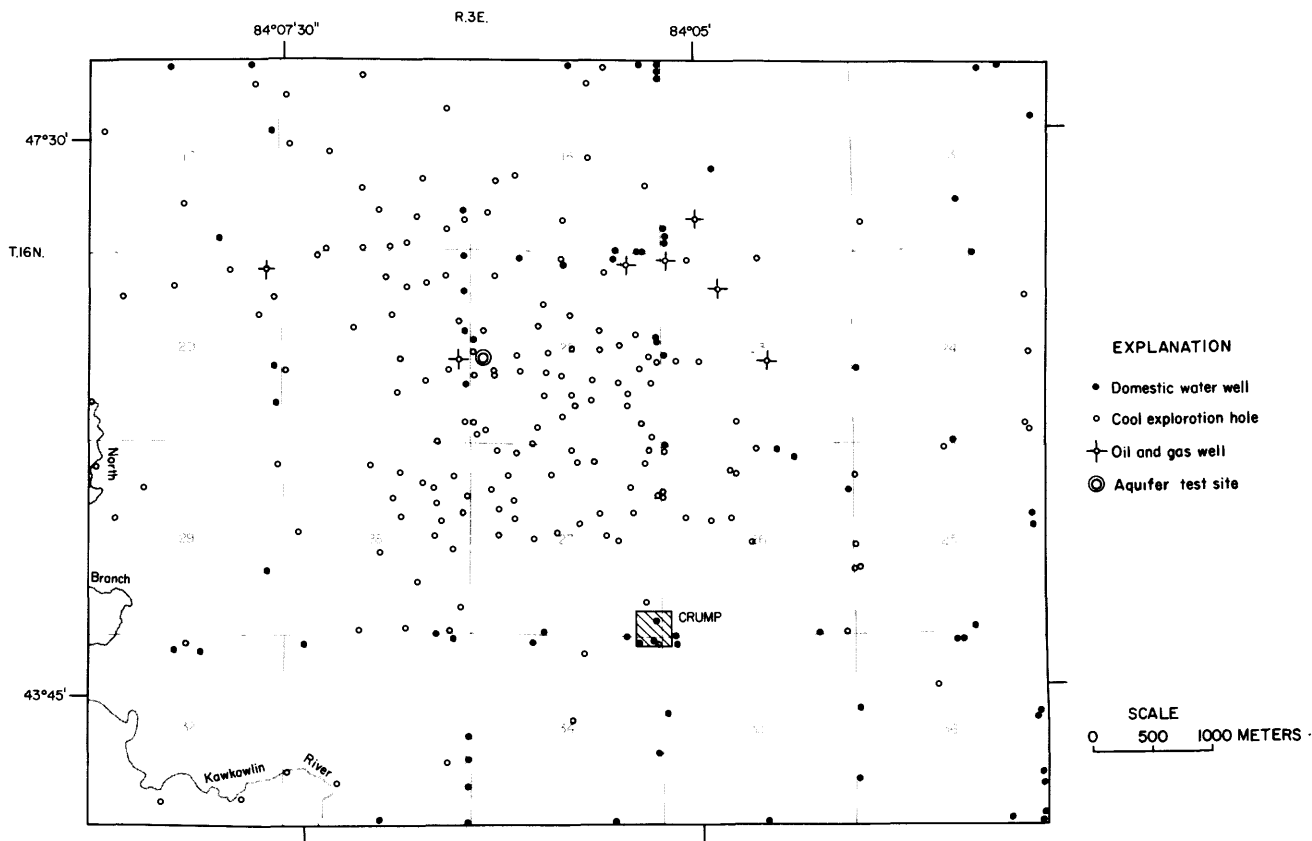


Figure 2.--Location of wells from which geologic and hydrologic data were used for this study.

### Cooperation

This study was made during 1977-79 as part of a national program by the U.S. Geological Survey to investigate the hydrology of coal deposits. The Michigan Department of Natural Resources and Michigan Technological University cooperated with the Geological Survey in the study. The Michigan Department of Natural Resources participated by providing data and technical assistance. Directed by Professor J. Kalliokoski, personnel from Michigan Technological University logged and inventoried wells and prepared geologic sections, maps of potentiometric surface, maps of thickness of lithologic units, and a map of bedrock surface. Much of this information was used in developing the digital model.

## Method of Investigation

Geologic and hydrologic data from wells were collected and analyzed to provide the necessary information to develop a digital model of the hydraulics of a coal deposit and overlying and underlying units. In addition, six wells were installed in section 22 (fig. 2). These wells were used during two aquifer tests. Pumping wells were completed in each of the two principal aquifers. Observation wells were completed in the two principal aquifers and two confining units. Standard analytic techniques and digital model simulation of drawdowns observed during the aquifer tests were used to estimate the hydrologic characteristics (hydraulic conductivity and storage coefficient) for each stratigraphic unit. A digital model was constructed to simulate ground-water flow to a hypothetical mine. Hydrologic characteristics estimated from aquifer test analyses and geologic information were used as model input. By using this model, quantity of seepage to a hypothetical mine was estimated, and the effect of mine-pumpage on water levels in aquifers near the mine was evaluated.

## Coal Deposits in Michigan

### Occurrence and Characteristics

The Michigan coal basin extends over 30,000 km<sup>2</sup> in the central part of Michigan's Lower Peninsula. Within the basin, rock units dip toward the center at about 3 m/km on the south flank and at about 4 m/km on the north flank. The rock unit containing coal, the Saginaw Formation, has a maximum thickness of about 225 m in Midland County (fig. 3). The major known coal deposits are on the eastern flank of the coal basin.

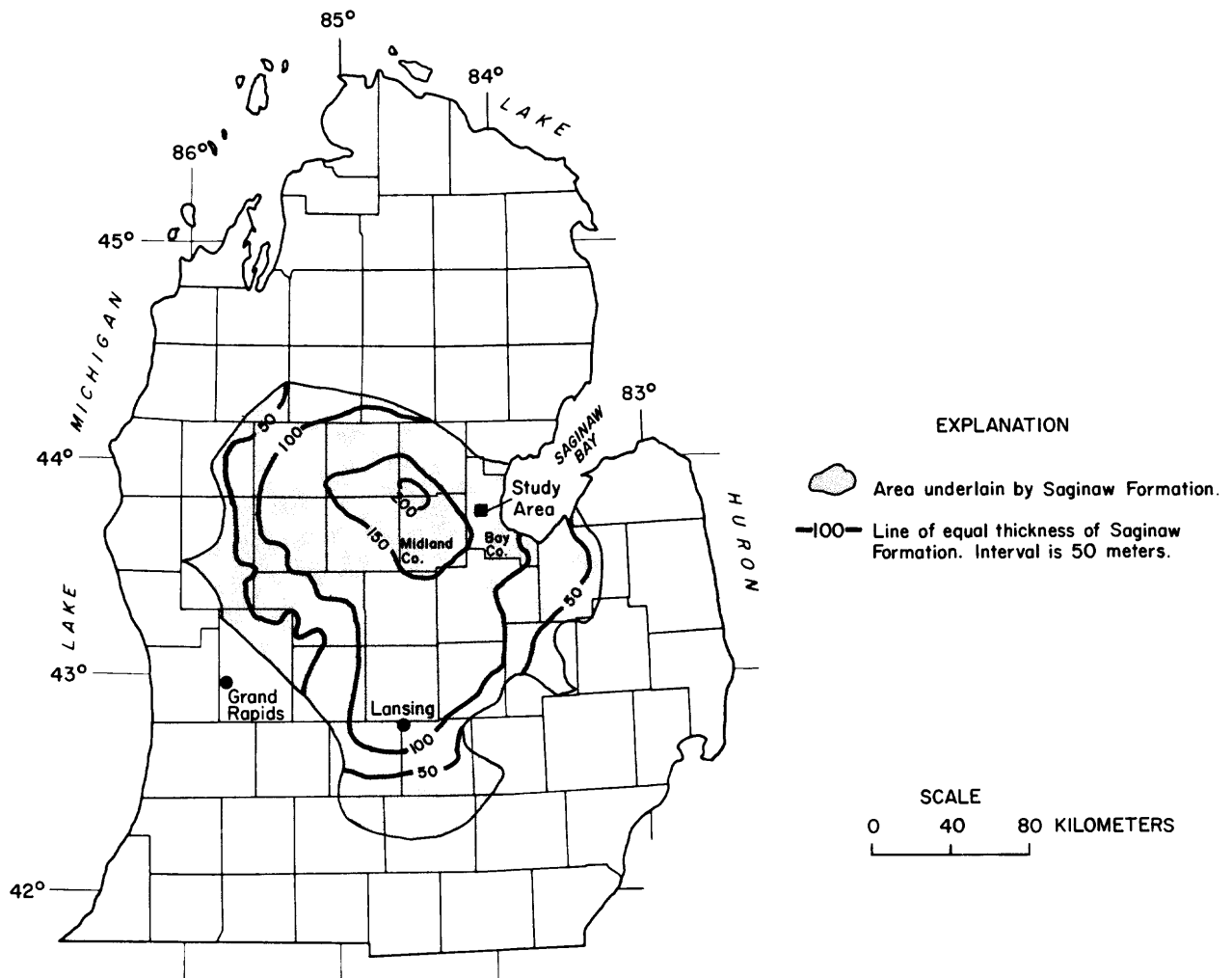


Figure 3.--Thickness and areal extent of Saginaw Formation.

Coal deposits in the Saginaw Formation generally occur as lenticular beds less than 1 m in thickness and less than 60 ha in areal extent. In the Saginaw Bay area, individual beds have been named and correlated (Cohee and others, 1950).

The quality of Michigan's coal, all of which is bituminous, varies greatly. The contents of ash, volatile matter, and fixed carbon range from 3 to 9 percent, 31 to 41 percent, and 39 to 53 percent, respectively. Btu values range from 10,500 to 12,300, and moisture content ranges from 8 to 13 percent. Sulfur content ranges from 1 to 3 percent in the vicinity of Saginaw Bay and is higher elsewhere (Cohee and others, 1950).

## Development and Production

Development of Michigan's coal began about 1835 when small quantities of coal were extracted from exposures in Eaton, Jackson, and Shiawassee Counties. By 1860, about 2,100 metric tons per year were being mined (Martin, 1920). Production increased dramatically after the opening of underground mines in Saginaw and Bay Counties, reaching a maximum annual production rate of about 1.85 million metric tons in 1907 (fig. 4). After 1907, however, production gradually decreased. In 1950 production ceased. The total amount of coal produced in the State was nearly 69 million metric tons (Cohee and others, 1950).

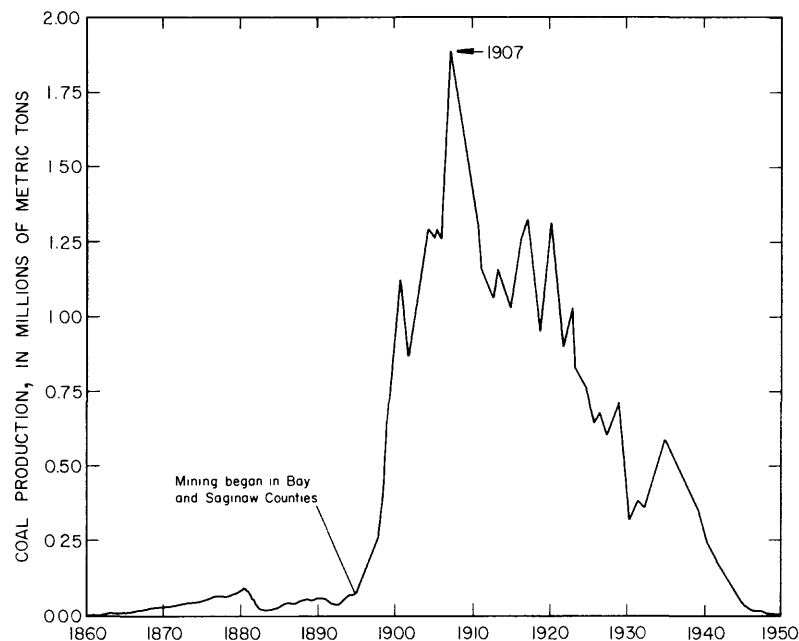


Figure 4.--Coal production in Michigan, 1860-1949.

## Potential Development

Potentially recoverable coal reserves in Michigan were estimated by Cohee and others (1950) to be 100 million metric tons, assuming a mine recovery rate of 50 percent. Kalliokoski and Welch (1977), in a revision of Cohee's work, calculated the State's reserves at 115 million metric tons, about the same as Michigan's current annual coal consumption (Michigan Department of Commerce, 1977). Most reserves are in Bay and Saginaw Counties.

A factor in favor of mining Michigan's coal is the proximity of the deposits to large local markets, particularly the heavily industrialized Bay City-Midland-Saginaw area. Because coal is currently shipped to this area from other states, shipping costs could be greatly reduced by

developing local coals. An increase in demand and higher prices may make mining the State's coal reserves economically feasible if seepage to mines can be controlled and if water pumped from the mines can be disposed of adequately.

## GEOLOGY

Seven major lithologic units have been defined in the study area (fig. 5). Four units--sandstone, lower shale, main coal and upper shale--are shallow water-laid sediments of the Saginaw Formation of Pennsylvanian age. The remaining three units--sand and gravel, clay (includes hardpan), and sand--are glacial deposits of Pleistocene age.

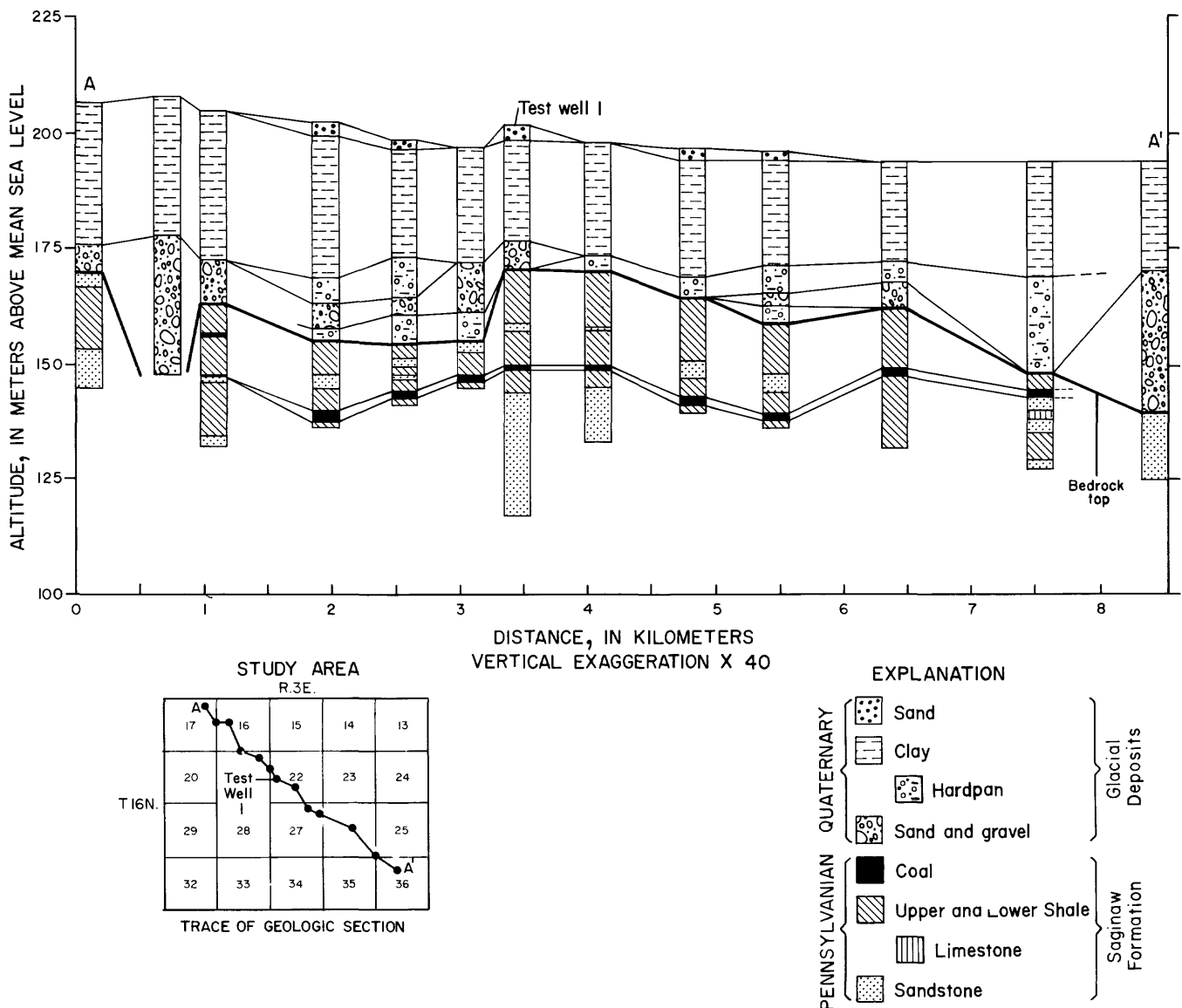


Figure 5.--Northwest-southeast geologic section through study area showing lithologic character of Saginaw Formation and glacial deposits.

The bedrock surface is the eroded Saginaw Formation. The surface is irregular, having altitudes ranging from about 170 m near the center of the study area to about 130 m in the buried valleys (fig. 6).

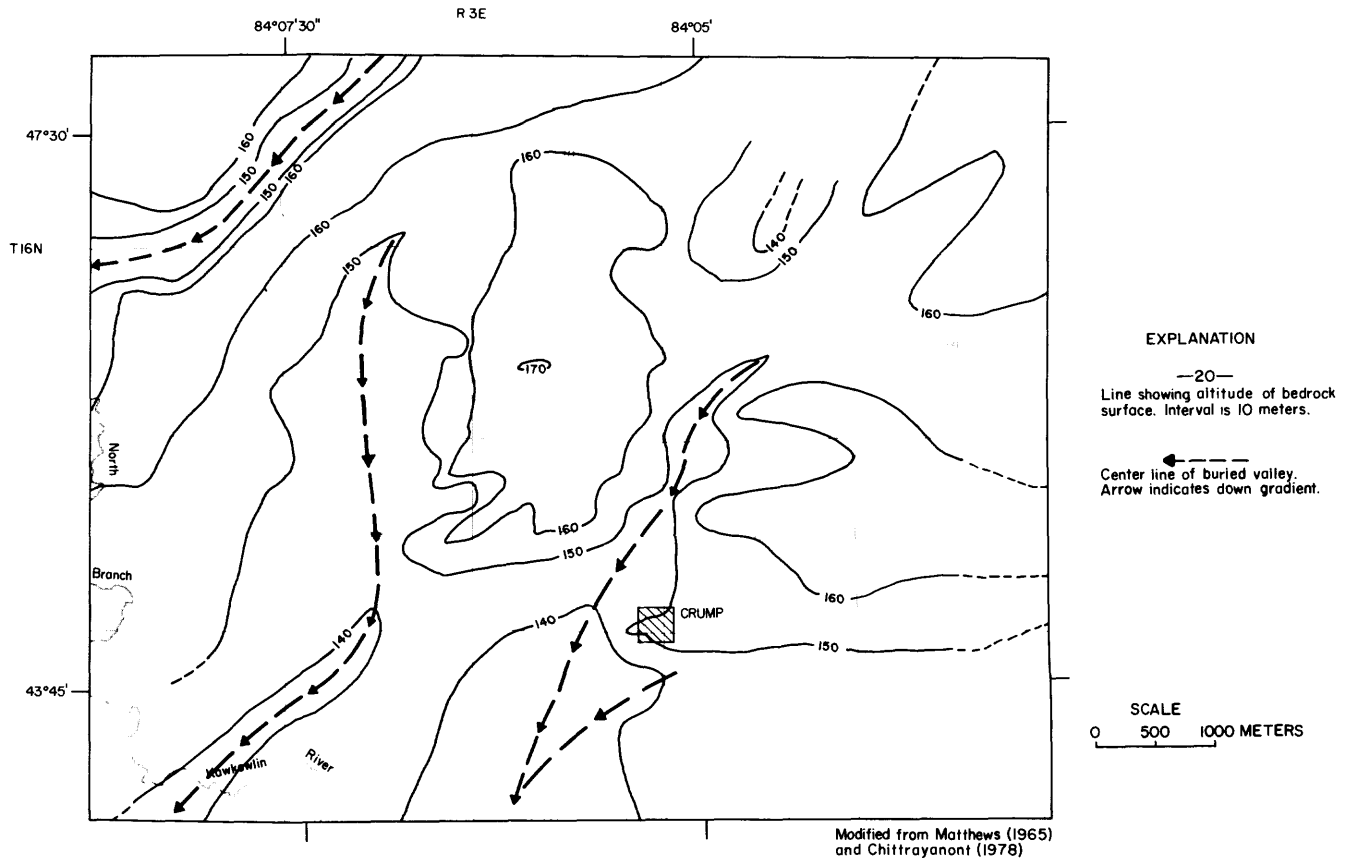


Figure 6.--Elevation of bedrock surface.

### Saginaw Formation

The Saginaw Formation is a sequence of sandstone, shale, and coal that, in places, contains limestone and siltstone (fig. 5). At the base of the formation is a coarse-grained quartz sandstone. The average thickness of the formation in the study area is slightly greater than 100 m (Chittrayanont, 1978). The Saginaw Formation unconformably overlies the Bayport Limestone of Mississippian age. A brief description of the four lithologic units of the Saginaw Formation follows.

## Sandstone Unit

The sandstone unit, the deepest unit penetrated during test drilling, is gray to greenish gray, fine grained, and rich in quartz. Most grains are less than 2 mm in diameter and are well sorted and rounded. Drill cuttings from the unit contain 15 percent clay, probably due to the presence of thin beds of shale (Chittrayanont 1978).

The thickness of the sandstone unit is not well known. The deepest well drilled during this investigation penetrated about 30 m of sandstone, which contained only a few beds of shale. Data suggest that 30 m is the maximum thickness of the unit. Because the sandstone was deposited in stream channels, its thickness is highly variable. The altitude of the top of the unit ranges from 115 to 150 m; it is highest along a northwest-southwest line that bisects the study area (Kalliokoski, 1979).

## Lower Shale Unit

The lower shale unit overlies the sandstone unit and underlies the coal. The shale consists primarily of gray to greenish-gray soft shale containing some thin beds of gray shaly quartz-rich sandstone. Some wells contain thin beds of limestone. The average thickness of the unit is 10 m; the range in thickness is from 2 to 20 m (fig. 7).

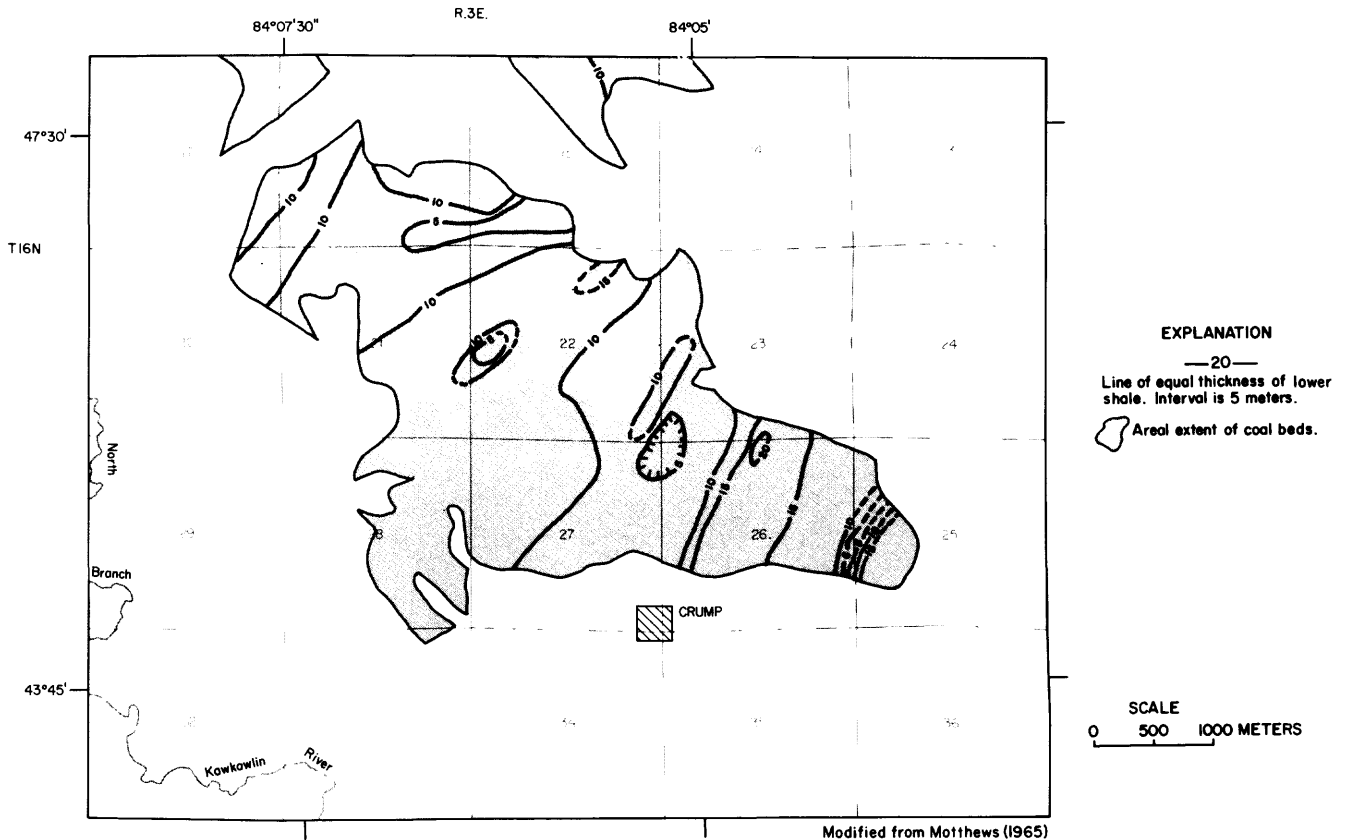


Figure 7.--Thickness of lower shale and areal extent of coal beds of Saginaw Formation.

### Main Coal Unit

One to three beds of coal underlie 13 km<sup>2</sup> of the study area. One bed, called the main coal in this report, is usually thicker than the other beds, having an average thickness of about 0.5 m. It is thickest along a northwest-southeast line that bisects the area (fig. 8). A series of small northwest-southeast trending folds affect the attitude of the main coal bed; the maximum dip of this bed is 80 m/km.

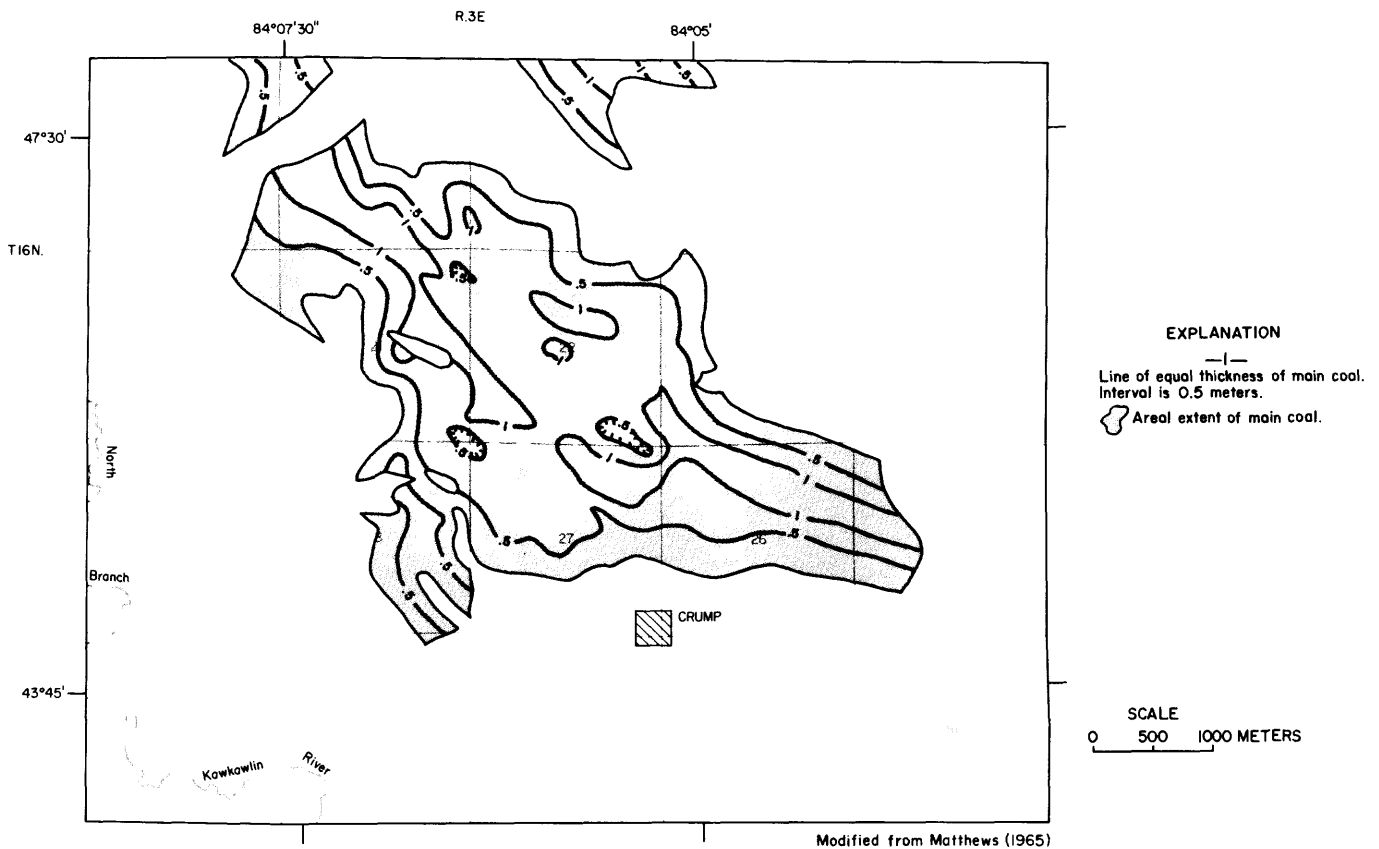


Figure 8.--Thickness of main coal unit of Saginaw Formation.

## Upper Shale Unit

The upper shale unit overlies the main coal unit and is the uppermost bedrock unit. In addition to shale, the unit contains thin beds of coal, quartz-rich sandstone, and hard mudstone and siltstone. The shales are of two types: one is hard and fissile containing carbonaceous material and pyrite; the other is soft, silty, and massive.

The upper shale has an average thickness of about 15 m and generally ranges in thickness from 5 m to 30 m (fig. 9). The thickest section lies along a northwest-southeast trending line that bisects the study area.

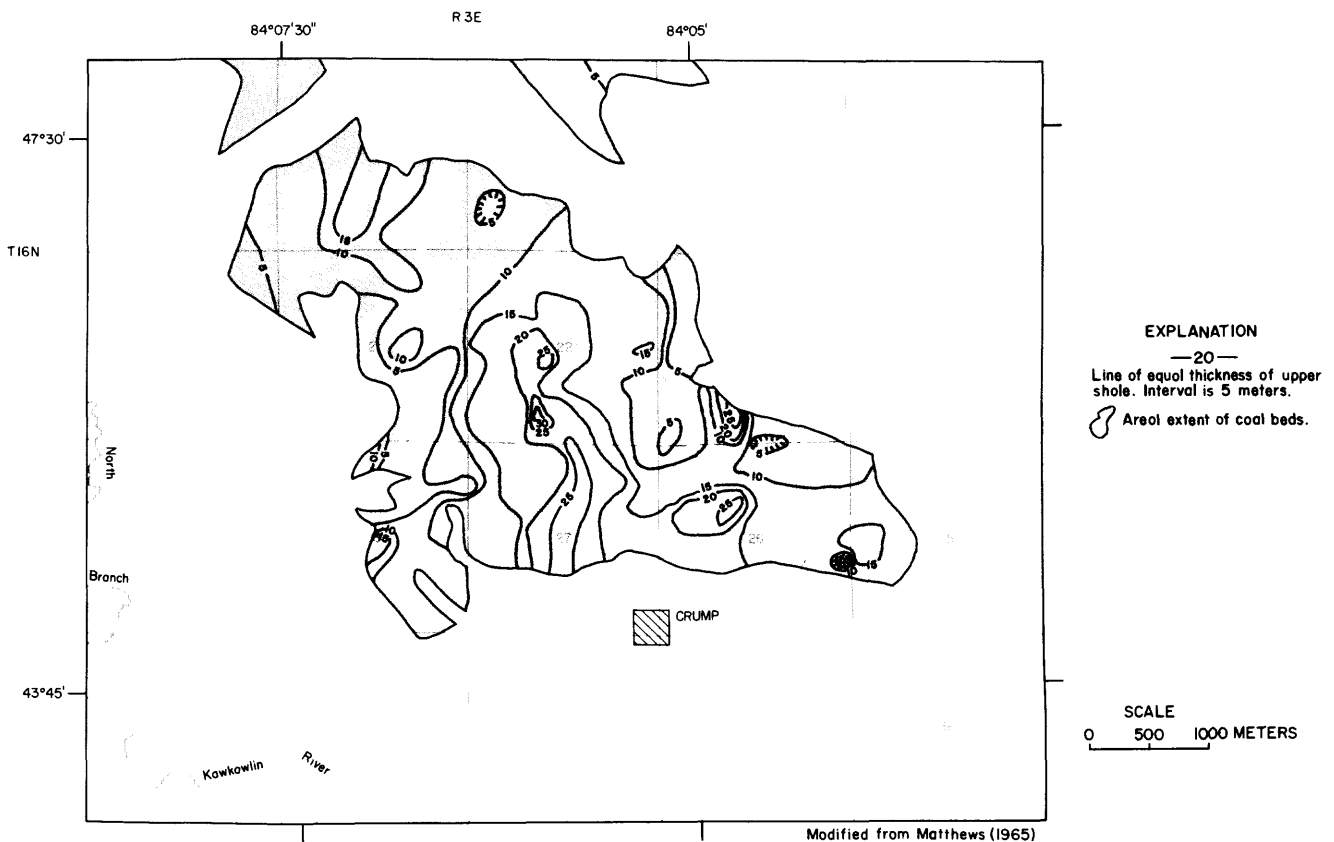


Figure 9.--Thickness of upper shale unit overlying the main coal unit of Saginaw Formation

## Glacial Deposits

Glacial deposits in the study area consist of a discontinuous basal sand and gravel, a thick sequence of clay, and an upper sand which occurs in narrow ridges at land surface. Figure 5 shows the relation of the glacial deposits to the Saginaw Formation.

### Sand and Gravel Unit

The sand and gravel unit is the basal unit of the glacial deposits. It is poorly sorted, consists of coarse sand, gravel, and pebbles as large as 50 mm in diameter, and contains minor amounts of clay and silt. Sand grains are predominantly quartz and are angular and poorly rounded. Gravel fragments consist of sandstone, carbonate rocks, quartzite, chert and igneous and metamorphic rocks.

The sand and gravel unit ranges in thickness from a featheredge to 40 m (fig. 10). It is usually thin or missing in areas of bedrock highs, but thick in the bedrock valleys.

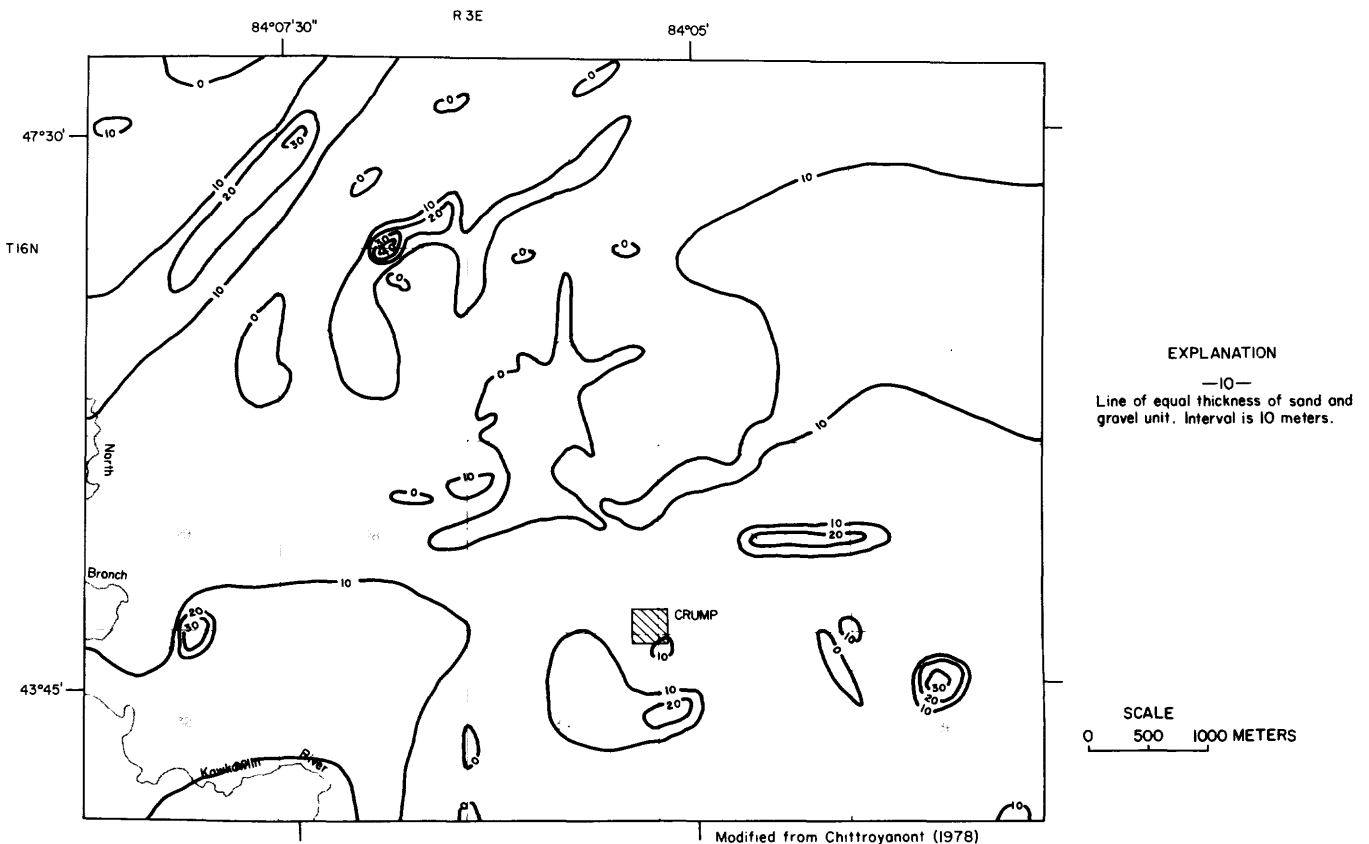


Figure 10.--Thickness of sand and gravel unit of glacial deposits.

## Clay Unit

The clay unit, which overlies the sand and gravel unit, is mostly clay and silt, but contains sand, gravel, and a clay and gravel mix ("hardpan"). The percentage of clay and silt in the unit increases as depth increases. The predominant clay minerals are illite, kaolinite, and chlorite (Chittrayanont, 1978). Pebbles consist primarily of quartzite, sandstone, siltstone, mafic igneous rocks, granite, and carbonates. Color ranges from gray to greenish gray to brown. The thickness of clay unit averages about 30 m and ranges from 10 to 50 m (fig. 11).

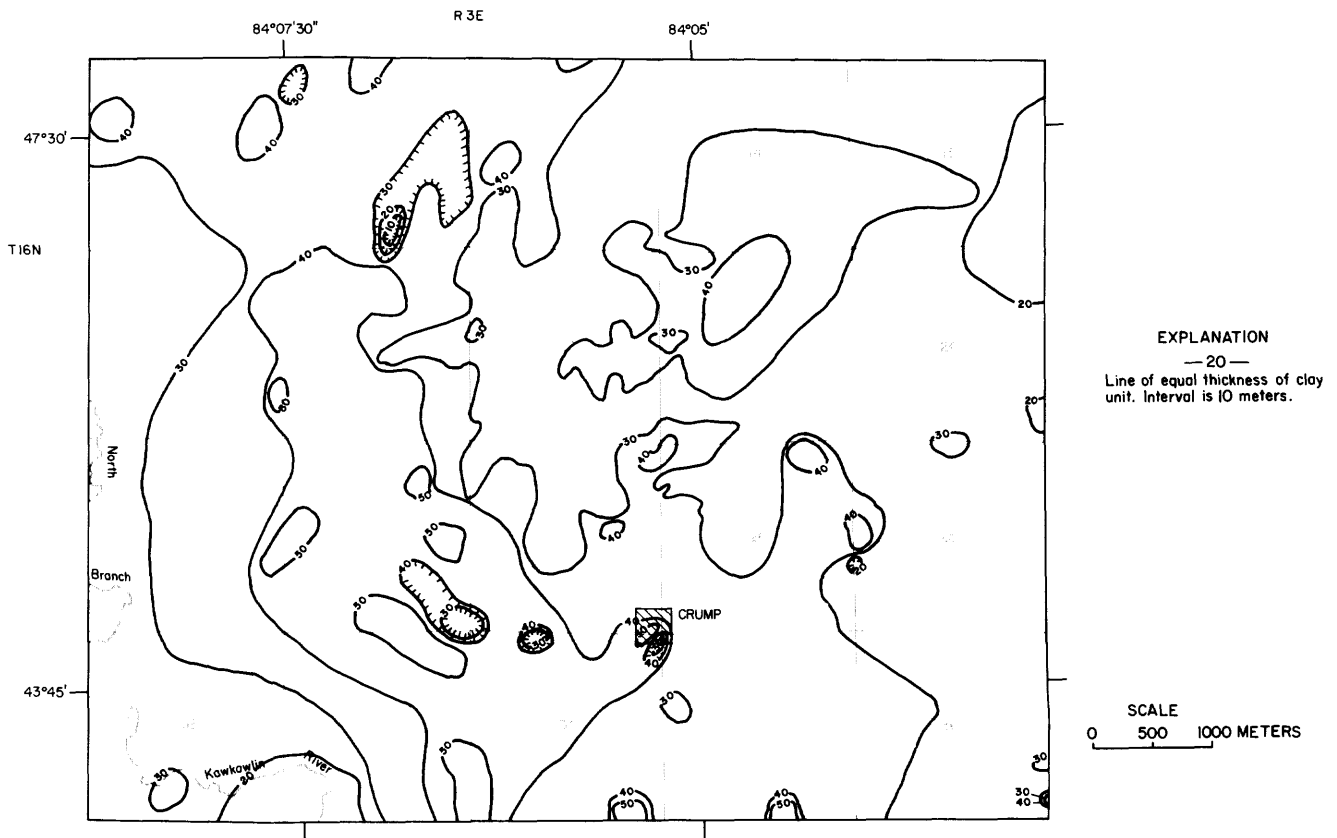


Figure 11.--Thickness of clay unit of glacial deposits.

## Sand Unit

The sand unit forms discontinuous narrow ridges that probably represent nearshore deposition along proglacial lakes at the close of the Pleistocene. The unit is predominantly well-sorted and rounded quartz sand that is brown at the surface but is yellow below the surface. The thickness of the unit ranges from 0 to 6 m.

## HYDROLOGY

The study area is a poorly drained lowland that, in places, has been made suitable for agriculture by extensive ditch and tile systems. The southwest half is drained by the North Branch of the Kawkawlin River (fig. 1) which flows to Saginaw Bay near Bay City, Mich. The northeast half drains directly to Lake Huron through open drains.

Ground water occurs within a sequence of permeable and relatively impermeable lithologic units. Water in the sandstone, lower shale, main coal, upper shale, and sand and gravel units is under confined conditions. Static water levels in wells open to one or more of these units stand about 10 m below land surface.

The direction of lateral flow in the system is to the east or southeast, where the water is discharged to drains, streams and Lake Huron. Analysis of potentiometric head data suggests that water in the sandstone and sand and gravel aquifers flows laterally under a hydraulic gradient of 0.003. In the vicinity of the aquifer test site, water in the system has a downward component of flow produced under the influence of a vertical gradient of 0.03.

The two principal aquifers in the system, and the chief sources of domestic water supply, are the sand and gravel unit in the glacial deposits and the sandstone unit in the Saginaw Formation.

## Water Quality

Water for chemical analysis was collected from wells completed in the sandstone unit (well 1), the sand and gravel unit (well 2), and the upper shale unit (well 5). Results of analyses are given in table 1. The well finished in the main coal (well 6) did not yield sufficient water to obtain a sample. Before sampling, each well was pumped about 1 hour at a rate of 0.001 m<sup>3</sup>/s.

Water in the sandstone unit is the sodium bicarbonate type. In the other two units, the predominant anion is sulfate; there is no predominant cation. Dissolved-solids concentration ranged from 438 to 687 mg/L (milligrams per liter); pH values were 7.8 and 7.9.

Data given in table 1 are inadequate to assess the impact of disposing of mine water, particularly if it were to be discharged to streams. Although concentrations of most substances are low, substantially more information needs to be obtained on cadmium, lead, and phenols. Cadmium concentrations of 5 and 6 µg/L (micrograms per liter)

Table 1.--Chemical and physical analyses of ground water (analyses by the U.S. Geological Survey).

| Constituent                                       | Well 1 | Well 2 | Well 5 |
|---|--------|--------|--------|
| Aldrin, total (µg/L) . . . . .                    | 0.00   | 0.00   | 0.00   |
| Alkalinity, total as CaCO <sub>3</sub> (mg/L) . . | 210    | 120    | 160    |
| Aluminum, total (µg/L) . . . . .                  | 60     | 50     | 60     |
| Ammonia, total as N (mg/L) . . . . .              | .67    | .63    | .93    |
| Arsenic, total (µg/L) . . . . .                   | 0      | 1      | 1      |
| Barium, total (µg/L) . . . . .                    | 0      | 0      | 0      |
| Beryllium, total (µg/L) . . . . .                 | 0      | 0      | 0      |
| Bicarbonate (mg/L) . . . . .                      | 260    | 150    | 190    |
| Boron, total (µg/L) . . . . .                     | 610    | 310    | 100    |
| Cadmium, total (µg/L) . . . . .                   | 6      | 6      | 5      |
| Calcium, dissolved (mg/L) . . . . .               | 39     | 85     | 65     |
| Carbon, dissolved organic (mg/L) . . .            | 7.7    | 1.4    | 1.2    |
| Carbonate (mg/L) . . . . .                        | 0      | 0      | 0      |
| Chlordane, total (µg/L) . . . . .                 | .0     | .0     | .0     |
| Chloride, dissolved (mg/L) . . . . .              | 79     | 6.8    | 24     |
| Chromium, total (µg/L) . . . . .                  | <10    | <10    | <10    |
| Cobalt, total (µg/L) . . . . .                    | 1      | 2      | 2      |
| Color (platinum cobalt units) . . . .             | 5      | 10     | 10     |
| Copper, total (µg/L) . . . . .                    | 1      | 2      | 1      |
| Cyanide (mg/L) . . . . .                          | .00    | .00    | .00    |
| DDD, total (µg/L) . . . . .                       | .00    | .00    | .00    |
| DDE, total (µg/L) . . . . .                       | .00    | .00    | .00    |
| DDT, total (µg/L) . . . . .                       | .00    | .00    | .00    |
| Diazinon, total (µg/L) . . . . .                  | .00    | .00    | .00    |
| Dieldrin, total (µg/L) . . . . .                  | .00    | .00    | .00    |
| Endosulfan, total (µg/L) . . . . .                | .00    | .00    | .00    |
| Endrin, total (µg/L) . . . . .                    | .00    | .00    | .00    |
| Ethyl parathion, total (µg/L) . . . .             | .00    | .00    | .00    |
| Ethyl trithion, total (µg/L) . . . . .            | .00    | .00    | .00    |
| Ethion, total (µg/L) . . . . .                    | .00    | .00    | .00    |
| Fluoride, dissolved (mg/L) . . . . .              | .3     | .6     | .5     |
| Hardness, noncarbonate (mg/L) . . . .             | 0      | 200    | 72     |
| Hardness, total (mg/L) . . . . .                  | 150    | 320    | 230    |
| Heptachlor, total (µg/L) . . . . .                | .00    | .00    | .00    |
| Heptachlor epoxide, total (µg/L) . . .            | .00    | .00    | .00    |
| Iron, dissolved (µg/L) . . . . .                  | 500    | 530    | 250    |
| Iron, total (µg/L) . . . . .                      | 890    | 600    | 320    |
| Lead, total (µg/L) . . . . .                      | 150    | 91     | 160    |
| Lindane, total (µg/L) . . . . .                   | .00    | .00    | .00    |
| Lithium, total (µg/L) . . . . .                   | 40     | 30     | 60     |
| Magnesium, dissolved (mg/L) . . . . .             | 13     | 26     | 16     |

Table 1.--Chemical and physical analyses of ground water (analyses by the U.S. Geological Survey).--Continued

| Constituent                              | Well 1 | Well 2 | Well 5 |
|--|--------|--------|--------|
| Malathion, total (µg/L) . . . . .        | 0.00   | 0.00   | 0.00   |
| Manganese, dissolved (µg/L) . . . . .    | 20     | 30     | 50     |
| Manganese, total (µg/L) . . . . .        | 20     | 10     | 50     |
| Mercury, total (µg/L) . . . . .          | <.5    | <.5    | <.5    |
| Methoxychlor, total (µg/L) . . . . .     | .00    | .00    | .00    |
| Methyl parathion, total (µg/L) . . . . . | .00    | .00    | .00    |
| Methyl trithion, total (µg/L) . . . . .  | .00    | .00    | .00    |
| Mirex, total (µg/L) . . . . .            | .00    | .00    | .00    |
| Molybdenum, total (µg/L) . . . . .       | 3      | 8      | 0      |
| Nickle, total (µg/L) . . . . .           | 3      | 6      | 3      |
| Nitrate, total, as N (mg/L) . . . . .    | .00    | .00    | .00    |
| Nitrite, total, as N (mg/L) . . . . .    | .00    | .00    | .01    |
| Nitrogen, total, as N (mg/L) . . . . .   | .83    | .74    | .98    |
| Organic nitrogen, total as N (mg/L) . .  | .16    | .11    | .04    |
| PCB, total (µg/L) . . . . .              | .0     | .0     | .0     |
| PCN, total (µg/L) . . . . .              | .0     | .0     | .0     |
| Perthane, total (µg/L) . . . . .         | .00    | .00    | .00    |
| pH (units) . . . . .                     | 7.9    | 7.8    | 7.9    |
| Phenols (µg/L) . . . . .                 | 0      | 4      | 1      |
| Phosphorous, ortho, total, as P (mg/L).  | .00    | .00    | .00    |
| Phosphorous, total, as P (mg/L) . . . .  | .00    | .01    | .01    |
| Potassium, dissolved (mg/L) . . . . .    | 4.6    | 3.2    | 5.8    |
| Selenium, total (µg/L) . . . . .         | 0      | 0      | 0      |
| Silica, dissolved (mg/L) . . . . .       | 6.1    | 11     | 6.2    |
| Silver, total (µg/L) . . . . .           | 1      | 0      | 0      |
| Silvex, total (µg/L) . . . . .           | .00    | .00    | .00    |
| Sodium, dissolved (mg/L) . . . . .       | 110    | 90     | 110    |
| Sodium (percent) . . . . .               | 60     | 38     | 50     |
| Solids, dissolved, calculated (mg/L) .   | 445    | 677    | 641    |
| Solids, dissolved, (residue at 180°C) .  | 438    | 687    | 614    |
| Specific conductance (µmhos) . . . . .   | 735    | 920    | 880    |
| Strontium, total (µg/L) . . . . .        | 700    | 900    | 1000   |
| Sulfate, dissolved (mg/L) . . . . .      | 64     | 380    | 320    |
| Temperature (°C) . . . . .               | 10.0   | 9.5    | 9.5    |
| Toxaphene, total (µg/L) . . . . .        | .0     | .0     | .0     |
| Turbidity (JTU) . . . . .                | 1      | 1      | 1      |
| Uranium, dissolved (µg/L) . . . . .      | .54    | .01    | .01    |
| Zinc, total (µg/L) . . . . .             | 10     | 10     | 20     |
| 2,4-D, total (µg/L) . . . . .            | .00    | .00    | .00    |
| 2,4-DP, total (µg/L) . . . . .           | .00    | .00    | .00    |
| 2,4,5-T, total (µg/L) . . . . .          | .00    | .00    | .00    |

exceed recommended maximum limits for hard water for aquatic life (U.S. Environmental Protection Agency, 1976). Lead, which ranged from 91 to 160  $\mu\text{g/L}$ , and phenols (4  $\mu\text{g/L}$  in one instance), both exceed recommended maximum permitted limits for domestic use and some aquatic life. Concentrations of lead, cadmium, and phenol are higher than previously collected data indicate are common in most Michigan ground waters. In addition to these trace substances, data indicate that the dissolved-solids concentration of water from the pumping test site is higher than that normally found in waters of streams of the area. Mine waters, discharged to streams, thus could modify the quality of surface waters.

## HYDROLOGIC CHARACTERISTICS OF LITHOLOGIC UNITS

Hydrologic characteristics (hydraulic conductivity and storage coefficient) of lithologic units were estimated by analyses of measured changes in water levels during two aquifer tests. Water levels measured during aquifer tests were analyzed by the nonequilibrium formula. The calculated hydraulic conductivity and storage coefficient values were refined, and the values of the other units were estimated by matching model simulated water levels with those measured in the different lithologic units during the aquifer tests.

The simulation model uses a finite-difference approximation of ground-water flow equations in three dimensions. This type of modeling requires the use of a multilayered grid to represent the ground-water system. The grid is composed of layers of blocks that are assigned values of hydrologic characteristics representative of the lithologic units. The dimensions of individual blocks in the horizontal plane are generally varied to provide necessary precision. Layers typically represent lithologic units with distinct hydrologic properties. The partial differential equation which describes ground-water flow is replaced with a series of linear algebraic equations which are solved simultaneously (Trescott, 1975).

### Aquifer Tests

Six wells were drilled for the aquifer tests (fig. 12). Two were 150 mm diameter pumping wells, the other four were 100 mm diameter wells. The well numbers and the unit tapped by each well are as follows:

| <u>Well<br/>No.</u> | <u>Unit<br/>tapped</u> | <u>Type<br/>well</u> |
|---------------------|------------------------|----------------------|
| 1                   | Sandstone              | Pumping.             |
| 2                   | Sand and gravel        | Do.                  |
| 3                   | Sand and gravel        | Observation.         |
| 4                   | Sandstone              | Do.                  |
| 5                   | Upper shale            | Do.                  |
| 6                   | Main coal              | Do.                  |

Figure 13 shows the materials penetrated by wells and the part of each well open to the formation.

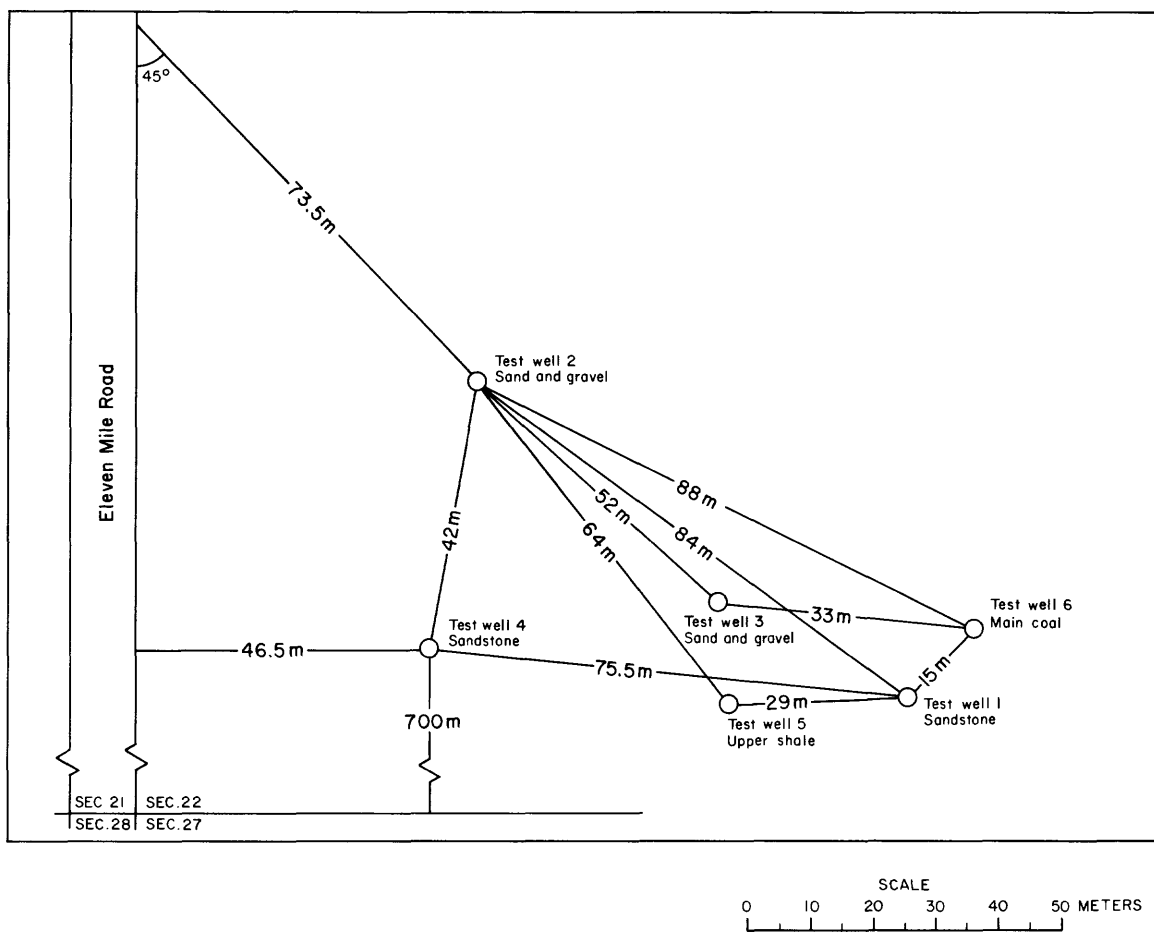


Figure 12.--Location of test wells at aquifer test site.

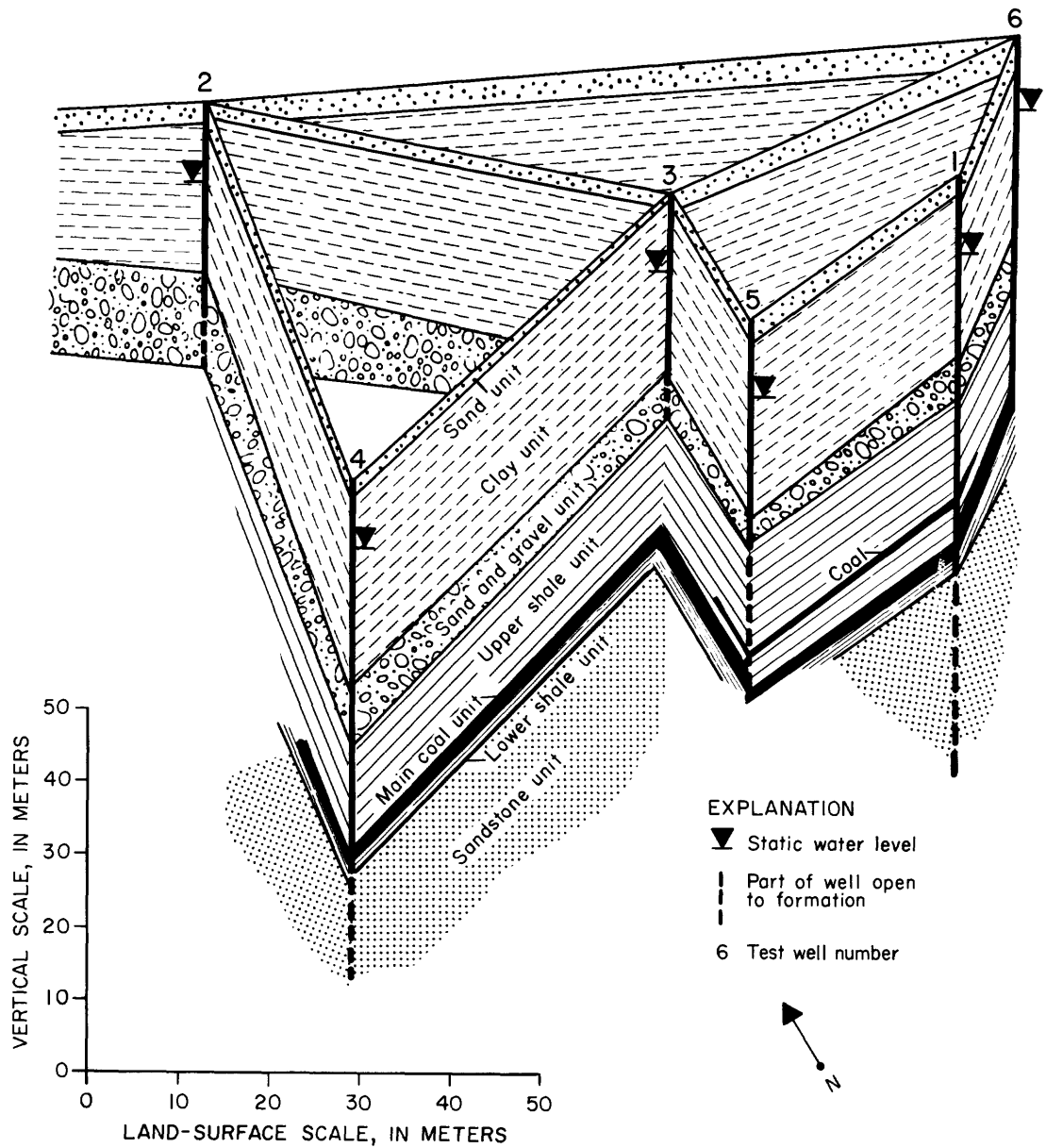


Figure 13.--Units of Saginaw Formation and glacial deposits penetrated by test wells.

## Sandstone Aquifer Test

Well 1, finished in the sandstone unit, was pumped for 24.75 hours. During the test, pump discharge ranged from 0.0053 to 0.0057 m<sup>3</sup>/s\*. Maximum drawdown was 46 m in the pumped well and 5 m in the sandstone observation well (fig. 14).

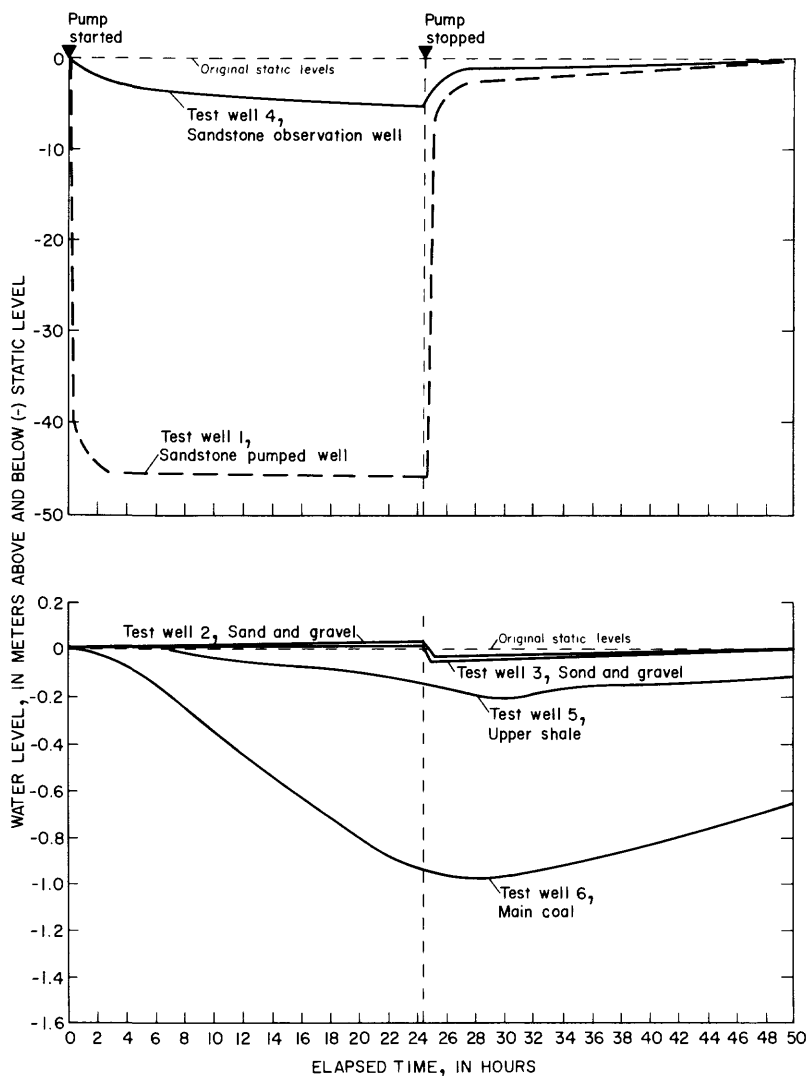


Figure 14.--Water levels in production well and observation wells during sandstone aquifer test.

\*A 5-minute pump failure occurred 13 hours after the test started. Drawdown in wells finished in the sandstone unit, for a 3-hour period following the failure, was visually interpolated by using the plots of drawdown for the first 13 hours and the last 8 hours. Model analysis of the test shows that the difference in drawdown between a test having pump failure and one without it is insignificant. Water-level fluctuations created by the failure, in units other than the sandstone, were negligible.

Maximum drawdown was 0.97 m in well 6, the main coal well, and 0.16 m in well 5, the upper shale well (fig. 14). In both wells, water levels continued to decline for about 3 hours after cessation of pumping.

Water levels in wells 2 and 3, those tapping the sand and gravel unit, rose slightly at the start of pumping. After the initial rise, water levels remained relatively constant during pumping. When pumping stopped, water levels in this unit declined rapidly to about the level which existed before pumping. This effect, observed during other multi-layer aquifer tests (Verruijt, 1969), is probably a result of structural deformation of the aquifer during pumping.

A semilog plot (fig. 15) of drawdown in the sandstone observation well, well 4, indicates that, except for a short period of time at the beginning of the test (less than 0.01 days), the decline is linear until 0.3 days. After 0.3 days, the slope of the line decreases, indicating that an increase in recharge, probably as leakage from overlying layers, is affecting drawdown.

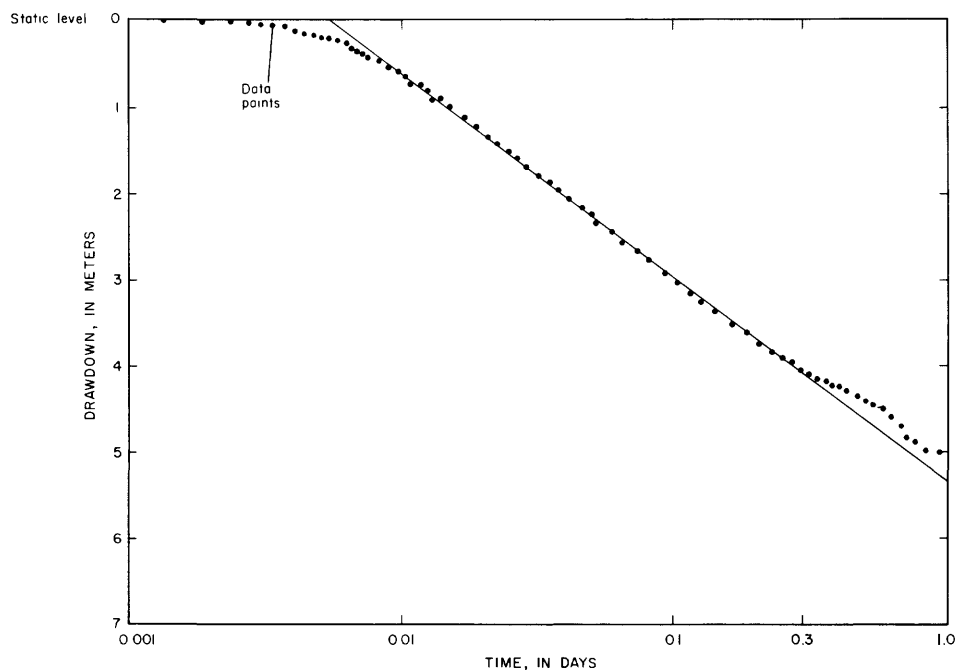


Figure 15.--Semilog plot of data collected from sandstone observation well during sandstone aquifer test.

Time drawdown and recovery solutions to the Theis nonequilibrium equation were used to calculate aquifer transmissivity and storage coefficient. To minimize the effects of leakage on calculated values of transmissivity and storage, only data obtained before 0.3 days were analyzed. Results of these analyses indicate that transmissivity in the sandstone unit is between 28 and 36 m<sup>2</sup>/d and the coefficient of storage is about 0.0001.

### Sand and Gravel Aquifer Test

Well 2, finished in the sand and gravel unit, was pumped for 12.33 hours at 0.0013 m<sup>3</sup>/s. Maximum drawdown was 17.6 m in the pumping well, 1.08 m in the sand and gravel observation well, well 3, and 0.09 m in well 5, finished in the upper shale (fig. 16). No drawdown was observed in other wells.

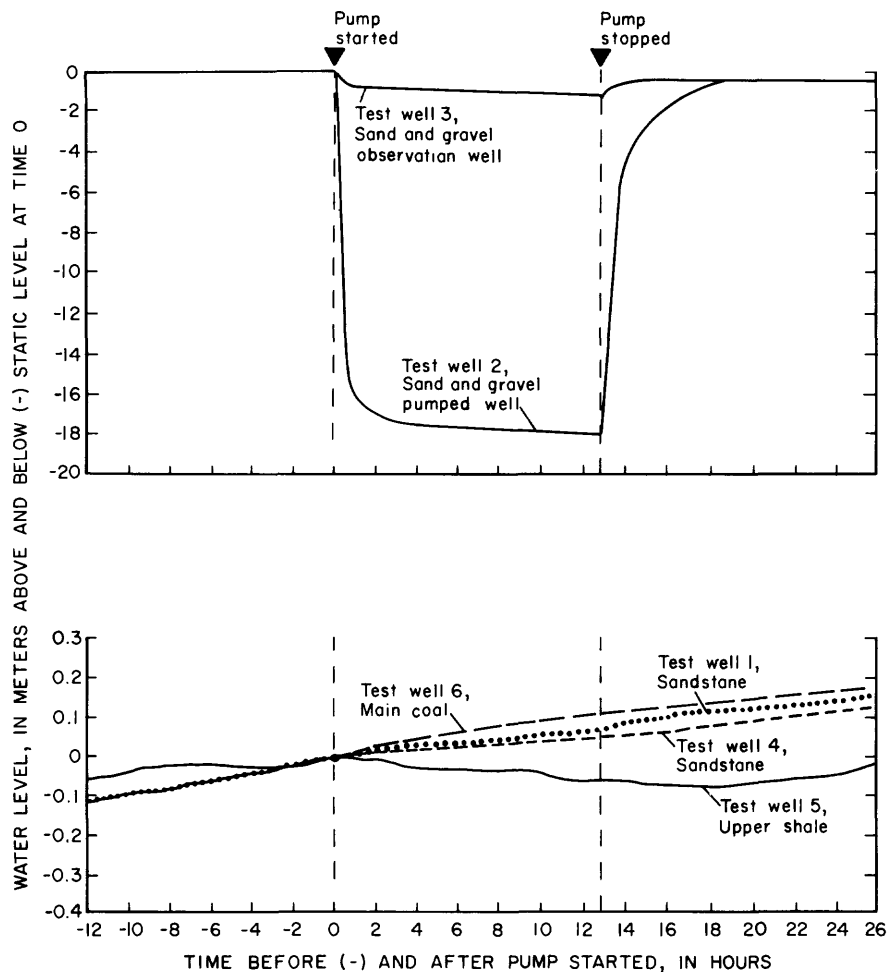


Figure 16.--Water levels in production well and observation wells during sand and gravel aquifer test.

A semilog plot (fig. 17) of drawdown in the sand and gravel observation well indicates that a linear plot applies only for the first 23 minutes (0.02 days) of the test. After that time, the effect of a recharge boundary is noted. Because of this effect, nonequilibrium equation analyses of the aquifer test data measured in the sand and gravel observation well are probably subject to error.

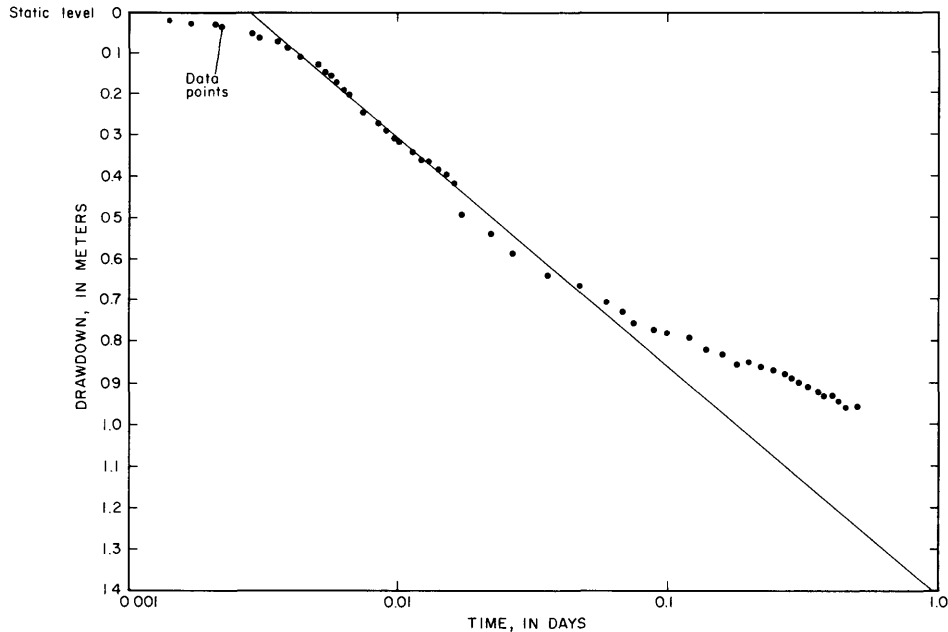


Figure 17.--Semilog plot of data collected from sand and gravel observation well during sand and gravel aquifer test.

### Model Analyses of Aquifer Tests

Digital model analyses of the aquifer tests were used to establish values of the hydrologic characteristics that would produce drawdowns matching those measured in the field.

#### Model Analysis of the Sandstone Aquifer Test

The model used to simulate ground-water flow during the sandstone aquifer test consisted of six layers. Each layer represented a lithologic unit (fig. 18). The sand unit, which is too thin and sparsely distributed to significantly impact ground-water flow in the area, was not included in the model. Flow in each layer was treated as being radially symmetric about the pumping well. Radial symmetry was justified because of the uniformity in thickness of all units, except the sand and gravel, around the wells. The sand and gravel unit, whose thickness is variable, was assigned a thickness representative of the unit in the immediate vicinity of the aquifer test site.

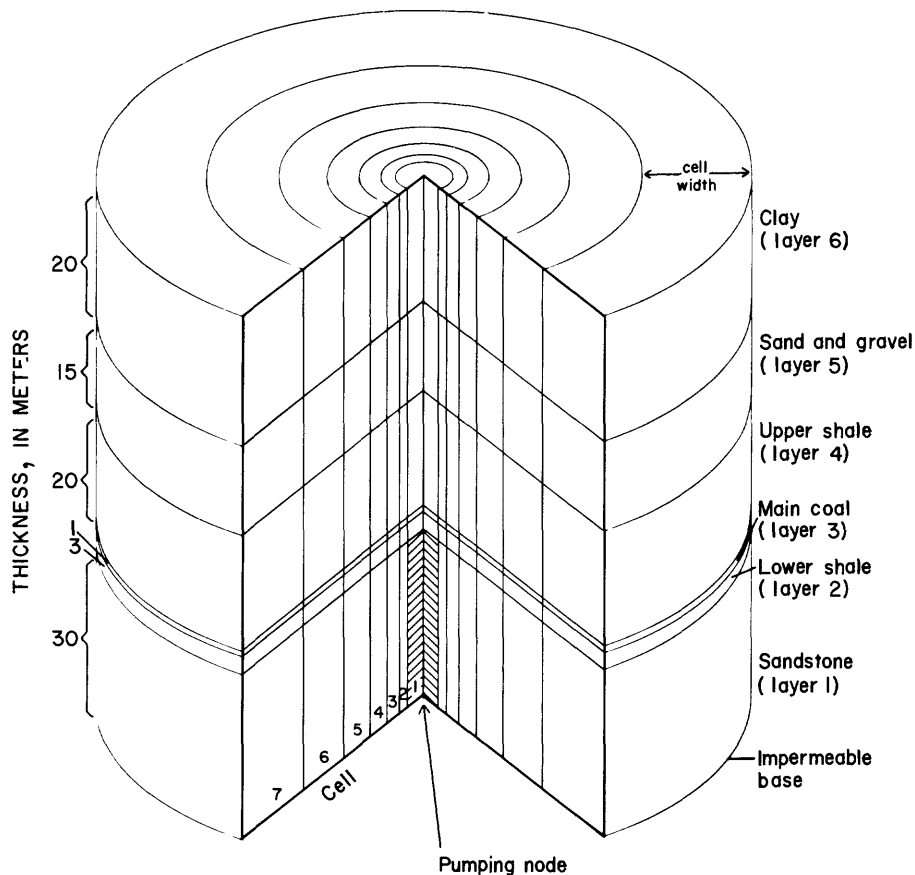


Figure 18.--Inner 7 cells of the 29 cells in the radial model.

A radial flow model, using a three-dimensional computer program developed by Trescott (1975), was used for the simulation. Discussion of the radial flow model is given in the appendix. Use of the radial flow model required substantially less computer time and storage than that required for a comparable cartesian coordinated flow model.

The flow field in each layer was represented as being divided by concentric circles into 29 annular volumes. Each of these volumes was simulated by a cell in the radial flow model. The innermost cell has an outer radius of 0.076 m. The width of successive cells increased by a factor of 1.5 to the outermost cell which has a width of 3834 m and an outer radius of 11,504 m. The innermost cell of layer 1 represented the pumping well. The outermost cells in all layers were held at constant potentiometric head and represented an area beyond the influence of the pumping well.

Water levels before the start of simulated pumping were assumed to be areally constant for each layer. For layers 1, 3, 4, and 5 the water levels were set to the elevation of water surfaces measured in wells at

the pumping test site in the corresponding lithologic units. Water levels in layers 2 and 6 were assumed to be the same as those in layers 1 and 5, respectively.

Calculated values of transmissivity and storage in the sandstone layer (layer 1), based on Theis nonequilibrium equation analysis, were used as initial model input to that layer. Published values of hydraulic conductivity for similar lithologic materials (U.S. Bureau of Reclamation, 1977) and measured thicknesses were used as the model's initial transmissivity input for layers 2, 3, 4, 5, and 6. The ratio of horizontal to vertical hydraulic conductivity was assumed to be 10:1 for sandstone, sand and gravel, and clay layers (layer 1, 5, and 6), and 100:1 for lower shale, main coal, and upper shale layers (layers 2, 3, and 4). Initial model input data for coefficients of storage in layers 2, 3, 4, 5, and 6 were assumed to be  $3.3 \times 10^{-6}$ /m of thickness (Lohman, 1972).

Hydrologic characteristics were varied over a range of two orders of magnitude during simulation until model-calculated drawdowns best matched those measured in the field (fig. 19). Varying hydrologic characteristics in the upper two layers (layers 5 and 6) produced negligible changes in calculated drawdown in the observation wells. Therefore, simulation of the sandstone aquifer test could only be used to refine estimated values of hydrologic characteristics in the lower four units (sandstone, lower shale, main coal, and upper shale).

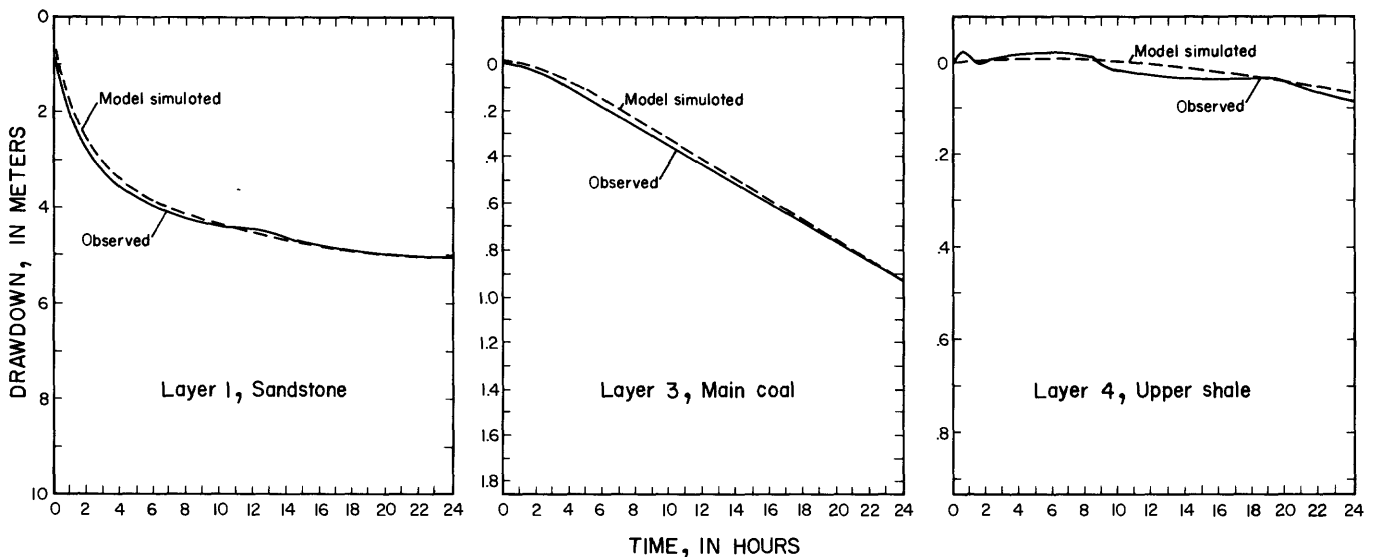


Figure 19.--Comparison of observed drawdown and model-simulated drawdown in layers 1, 3, and 4, during sandstone aquifer test.

## Model Analysis of the Sand and Gravel Aquifer Test

The sand and gravel aquifer test was simulated with a three-dimensional cartesian coordinated flow model. The model consisted of 3 layers representing the upper shale, sand and gravel, and clay. The three-dimensional flow model was used because the thickness of the sand and gravel unit could not be accurately represented in the radial flow model. Each unit was treated as being confined. Vertical differences in potentiometric head within individual units were not simulated.

Each layer consisted of a square area of 148.8 km<sup>2</sup> divided into a rectilinear grid with 31 columns and 31 rows. Grid spacing was 5 m at the center of the grid, which represented the location of the pumping well, and increased by a factor of 1.5 at each successive row and column out to the perimeter of the square. The outer rows and columns were held at constant heads to represent the boundary of the area beyond the influence of pumping. The initial heads in the sand and gravel layer, layer 5, and clay layer, layer 6, were set equal to the water level in the sand and gravel observation well (well 3) before the aquifer test. In the upper shale layer, layer 4, hydraulic heads were assigned values equal to the water level in the well finished in the upper shale (well 5). Initial values used for transmissivity and storage coefficients of layers 5 and 6 corresponded to those used during model analysis of the sandstone aquifer test. Hydrologic characteristics of layer 4 were set equal to the values calculated from model analysis of the sandstone aquifer test.

Hydrologic characteristics in each of the two upper layers, layers 5 and 6, were varied over reasonable ranges until model-calculated drawdown best matched the measured drawdowns (fig. 20). The simulated results are sensitive to changes in hydraulic characteristics of all units, but most sensitive to changes in the sand and gravel unit.

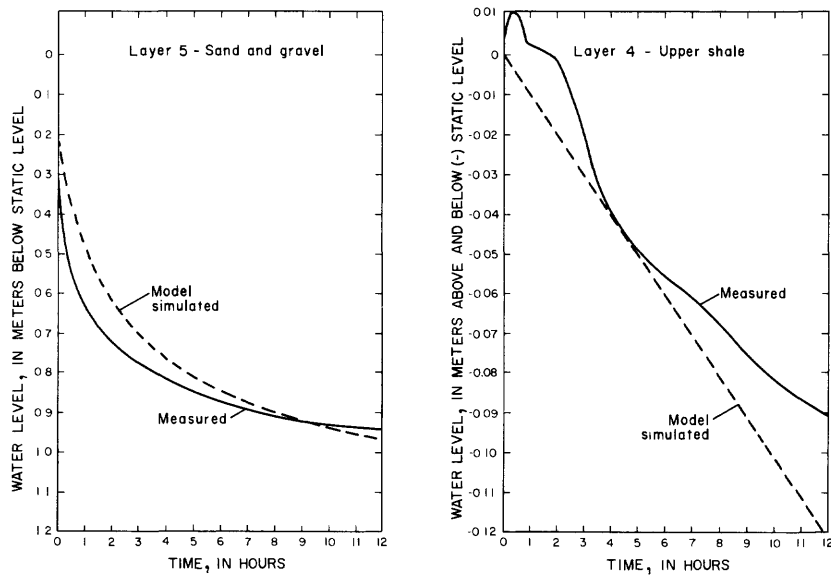


Figure 20.--Comparison of observed and model-simulated drawdown in layers 4 and 5, during sand and gravel aquifer test.

#### Results of Model Analysis of Aquifer Tests

Model analyses of the tests were used to obtain the estimated values of hydrologic characteristics shown in the following table. These values were used to calculate seepage rates into a hypothetical coal mine and to evaluate the effect of mine-dewatering on heads in aquifers near the mine.

|                       | Thickness<br>(m)   | Specific<br>storage<br>( $m^{-1}$ ) | Vertical<br>conductivity<br>anisotropy<br>$K_z/K_x$<br>(assumed) | Transmissivity<br>( $m^2/d$ ) |
|-----------------------|--------------------|-------------------------------------|--|-------------------------------|
| Clay layer            | 13.2               | $3.3 \times 10^{-6}$                | 1/10   | 4.1                           |
| Sand and gravel layer | Variable<br>(0-14) | $1.0 \times 10^{-5}$                | 1/10   | Variable<br>(0-29.4)          |
| Upper shale layer     | 20                 | $2.5 \times 10^{-6}$                | 1/100  | $7.0 \times 10^{-1}$          |
| Main coal layer       | 1                  | $1.4 \times 10^{-4}$                | 1/100  | $1.0 \times 10^{-2}$          |
| Lower shale layer     | 3                  | $8.0 \times 10^{-6}$                | 1/100  | $3.1 \times 10^{-2}$          |
| Sandstone layer       | 30                 | $3.3 \times 10^{-6}$                | 1/10   | 34.5                          |

Model-calculated values of hydrologic characteristics in the sandstone layer and the sand and gravel layer, layers 1 and 5, do not differ from those calculated analytically by more than a factor of two. Values in the other layers, layers 2, 3, 4, and 6, are reasonable and probably have a maximum error of no more than 10.

#### GROUND-WATER MODEL OF HYPOTHETICAL COAL MINE

Water seepage into a hypothetical mine and the effects of mine dewatering on water levels in aquifers surrounding the mine were evaluated by simulation with a transient radial-flow model. This simulation used the estimated hydrologic characteristics, shown in the previous table, as model input. The center of the mine was located at the center of the modeled coal deposit. All lithologic units, except the upper shale and sand and gravel units, were defined as being horizontally bedded, having uniform thickness and being continuous to the edges of the modeled area. The thickness of the upper shale was varied to account for the configuration of its eroded surface. The thickness of the sand and gravel layer was increased in a symmetrical manner outward from the mine.

The model grid consisted of 102 active cells. Each of the six lithologic units described previously (fig. 18) was represented by a layer of 17 cylindrical cells. Cylinders, representing annular volumes, were located about the center of the mine in a manner similar to that for the model simulation of the sandstone aquifer test. The width of each successive cylinder, moving outward from the mine, was increased by a factor of 1.5. The radius of the inner cylinder was 10 m; the outer cylinder had a width of 9,853 m. The boundary of the model was a circle centered about the center of the mine with a radius 29,529 m.

The mining operation was simulated as starting from the center and expanding with radial symmetry at a rate of 12 ha per year. Dewatering was simulated by defining heads in the dewatered cells as constant and equal to the altitude of the bottom of main coal. As the size of the simulated mine increased with time, additional constant head nodes were assigned.

Heads in all layers, except the clay layer, were allowed to respond to the hydraulic stress induced by mine dewatering. Model simulations demonstrated that mine dewatering had little effect on heads in the clay layer; therefore, they were assigned constant values. Other initial conditions were identical to those used in the model simulation of the sandstone aquifer test. Constant head boundary conditions were assigned for edges of the modeled area. These boundaries were located sufficiently far from the mine so that flow toward the mine would not be affected by the boundaries.

Seepage rates into the mine are shown in figure 21. Curve A represents the rates that were calculated by the values of hydrologic characteristics estimated by model simulations of two aquifer tests. Curve B represents the seepage rates that were calculated by increasing the vertical hydraulic conductivity in the lower and upper shales, which underlie and overlie the coal, by an order of magnitude from that used

in the calculation of curve A. Curves A and B depict the expected and anticipated maximum seepage with time, into the hypothetical mine. Although the expected seepage rates are significant, they are probably not great enough to preclude either the economic or engineering feasibility of mining coal at the site.

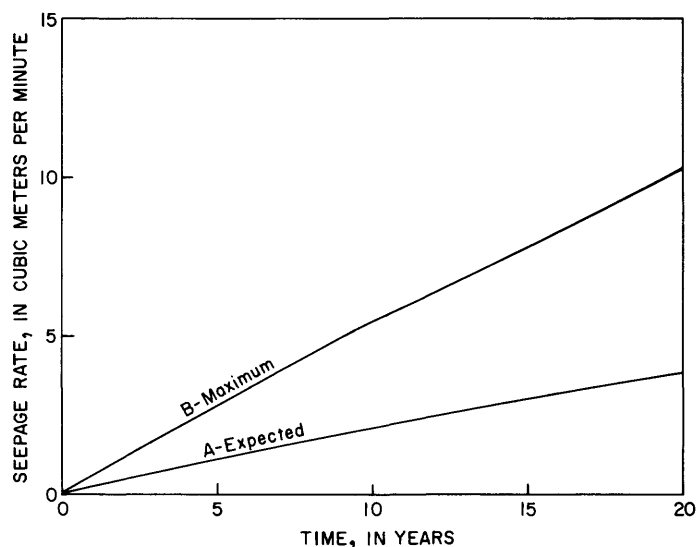


Figure 21.--Seepage rates into simulated mine where mining is expanding at an annual rate of 12 ha.

The simulated effects of mine dewatering on hydraulic heads in aquifers surrounding the coal bed increase with time (fig. 22). During the first decade of mining, at an annual rate of 12 ha, drawdown in the sandstone will be insignificant outside of the mined area. If the mine continues in operation for more than 20 years, some concern about head loss in domestic wells completed in the sandstone may be warranted. Because the sand and gravel was simulated as having zero thickness in the vicinity of the center of the mine, drawdown in the unit was negligible during the first decade of mining.

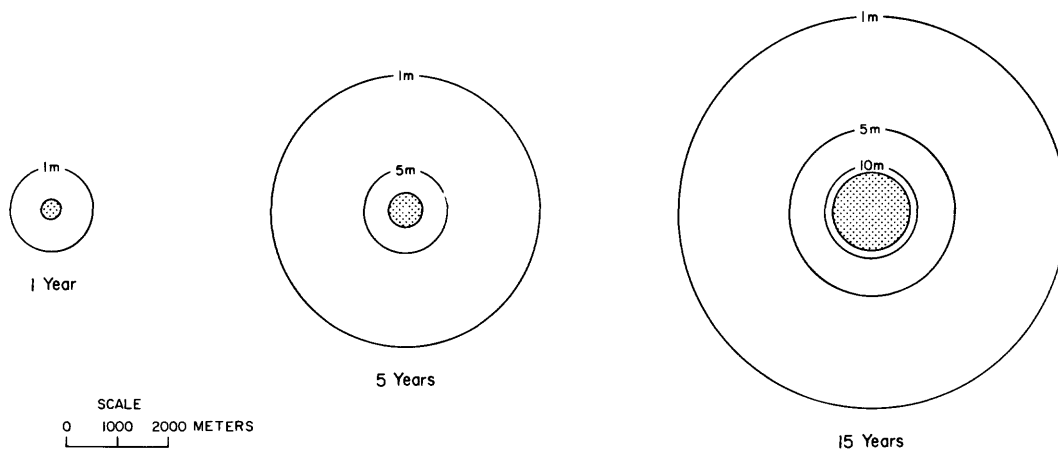


Figure 22.--Extent of cone of depression and amount of drawdown in sandstone around simulated mine with time.

#### Application of Model to Other Sites in the Michigan Coal Basin

Most coal deposits in the Michigan coal basin occur in geologic and hydrologic settings similar to those of the study area. The major differences between mine areas will be in the physical characteristics of the rocks overlying and underlying the coal, the static water levels, and the rate at which the coal can be mined.

Factors that will determine the quantity of water seeping into a mine at any site in the Michigan coal basin are: (1) the rate at which the mine area is expanded, (2) the position of potentiometric surfaces relative to the bottom of the mine, and (3) the thickness and hydrologic characteristics of rock and glacial materials overlying and underlying the coal. Several model simulations of the study area, in which these factors were varied, were used to examine the relative importance of hydrologic, geologic, and mining-rate factors different from those used previously in this report. Results of the model simulations indicate that doubling or halving the heights of the static head above the mine bottom produced a 30 percent increase or decrease in seepage. Halving the thickness of confining units overlying and underlying the coal produce a 50 percent increase in seepage. Doubling or halving the mining rate was found to roughly double or reduce by half the rate of seepage to the mine.

Stratigraphic pinch-outs and lenticular bedding in the Saginaw Formation may result in coal beds terminating against rocks with high hydraulic conductivities, a situation which would increase seepage into a mine. Because many of Michigan's coal deposits occur in isolated

bedrock highs, some coal beds may be truncated by sand and gravel valley fill. Such an intersection would also result in increased seepage into a mine.

#### SUMMARY

Coal was mined from Michigan's Saginaw Formation from 1860 to 1950. Excessive seepage to the mines contributed to the inability of the industry to compete with eastern coal.

Significant reserves of coal remain unmined in Michigan. As energy prices rise, the opening of new mines may become feasible. Hydrologic factors, such as the quantity and quality of water pumped to keep mines dry, will be important to the feasibility of opening new mines.

The ground-water hydrology of coal deposits in Bay County is considered representative of the ground-water hydrology of Michigan's coal deposits. The coal bed at the site averages about 0.5 m thick and is 50 m below land surface. It is part of a multilayered aquifer system that contains rocks of Pennsylvanian age and glacial deposits of Pleistocene age. The rock units, part of the Saginaw Formation, consist of a thick basal sandstone, shale units, which overly and underly the coal, and the main coal unit. The major glacial deposits consist of a sand and gravel unit, which overlies the sedimentary rocks, and a clay unit, which overlies the sand and gravel.

Aquifer test analyses and digital modeling helped determine, within limits, values of hydrologic characteristics of each unit in the system. These values were used as input to a flow model which simulated ground-water flow to a hypothetical mine in the study area. Results of the simulation indicate that seepage will probably not be great enough to preclude either the economic or engineering feasibility of mining the deposit. The pumpage will have little effect on heads in aquifers outside the mined area during the first decade of mining. Because hydrologically similar coal areas will probably have similar problems, results of this study should be applicable to other areas in Michigan.

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## APPENDIX

Modification of three-dimensional ground-water flow program  
for use with radial model

Ground-water flow in three-dimensions can be expressed by the equation (Trescott, 1975):

$$\frac{\partial}{\partial x} (K_{xx} \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (K_{yy} \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (K_{zz} \frac{\partial h}{\partial z}) = S_s \frac{\partial h}{\partial t} + W(x, y, z, t) \quad (1)$$

where  $K_{xx}$ ,  $K_{yy}$ ,  $K_{zz}$  are the principal components of the hydraulic conductivity tensor along the principal axes of  $x$ ,  $y$ , and  $z$ .

$h$  is potentiometric head (L)

$S_s$  is specific storage (1/L)

$W$  is the source term (1/L)

To describe flow in a vertical cross section ( $x$ - $z$  plane) the flow in the  $y$  direction is assumed to be zero; then equation 1, multiplied by aquifer thickness and written in finite difference form is:

$$\begin{aligned} & \frac{1}{\Delta x_j} \left\{ [T_{xx}(j+\frac{1}{2}, k) \frac{(h_{j+1, k} - \hat{h}_{j, k})}{\Delta x_{j+\frac{1}{2}}}] - [T_{xx}(j-\frac{1}{2}, k) \frac{(h_{j, k} - h_{j-1, k})}{\Delta x_{j-\frac{1}{2}}}] \right\} \\ & + \frac{1}{\Delta z_k} \left\{ [(bK_{zz})_{j, k+\frac{1}{2}} \frac{(h_{j, k+1} - \hat{h}_{j, k})}{\Delta z_{k+\frac{1}{2}}}] - [(bK_{zz})_{j, k-\frac{1}{2}} \frac{(h_{j, k} - h_{j, k-1})}{\Delta z_{k-\frac{1}{2}}}] \right\} \\ & = \frac{S'_{j, k}}{\Delta t} (h_{j, k} - \hat{h}_{j, k}) + bW_{j, k} \end{aligned} \quad (2)$$

where  $\Delta x_j$  is the space increment in the  $x$  direction (L)

$\Delta z_k$  is the space increment in the  $z$  direction (L)

$\Delta t$  is the time increment (T)

$T_{xx}(j+\frac{1}{2}, k)$  is the transmissivity between cells  $j$  and  $j+1$  ( $L^2/T$ )

$S'_{j, k}$  is the storage coefficient of cell  $j$  in layer  $k$  (dimensionless)

$W_{j, k}$  is the source term in cell  $j$  of layer  $k$  (1/T)

$\hat{h}_{j, k}$  is potentiometric head (L) at previous time step

When hydraulic head and hydraulic conductivity are radially symmetric equation 1 can be written as:

$$K \frac{\partial^2 h}{\partial r^2} + \frac{\partial K}{\partial r} \frac{\partial h}{\partial r} + \frac{K}{r} \frac{\partial h}{\partial r} + \frac{\partial}{\partial z} (K_{zz} \frac{\partial h}{\partial z}) = S_s \frac{\partial h}{\partial t} + w(r, z, t) \quad (3)$$

Equation 3 (multiplied by  $2\pi r b$ , where  $b$  is the thickness of a hydraulic unit) can be rewritten as:

$$2\pi \frac{\partial}{\partial r} (r T \frac{\partial h}{\partial r}) + 2\pi \frac{\partial}{\partial z} (r b K_{zz} \frac{\partial h}{\partial z}) = 2\pi r S' \frac{\partial h}{\partial t} + 2\pi r b w(r, z, t) \quad (4)$$

where  $S'$  is the storage coefficient (dimensionless)

$T$  is the transmissivity of a hydraulic unit ( $L^2/T$ )

Equation 4 in finite difference form is:

$$\begin{aligned} & \frac{1}{\Delta r_j} \left\{ \left[ (2\pi r_j T)_{j+1/2, k} \frac{(h_{j+1, k} - h_{j, k})}{\Delta r_{j+1/2}} \right] - \left[ (2\pi r_j T)_{j-1/2, k} \frac{(h_{j, k} - h_{j-1, k})}{\Delta r_{j-1/2}} \right] \right\} \\ & + \frac{1}{\Delta z_k} \left\{ \left[ (2\pi r_j b K_{zz}(j, k+1/2)) \frac{(h_{j, k+1} - h_{j, k})}{\Delta z_{k+1/2}} \right] - \left[ (2\pi r_j b K_{zz}(j, k-1/2)) \frac{(h_{j, k} - h_{j, k-1})}{\Delta z_{k-1/2}} \right] \right\} \\ & = \frac{2\pi r_j S'_{j, k}}{\Delta t} (h_{j, k} - \hat{h}_{j, k}) + 2\pi r_j b w_{j, k} \end{aligned} \quad (5)$$

where  $\Delta r_j$  is the radial space increment (the cell width) in annulus  $j$  (L)

$r_{j+1/2}$  is the outer radius of annulus  $j$  (L)

$r_{j-1/2}$  is the inner radius of annulus  $j$  (L)

$r_j$  is the mean of the inner and outer radii of cell  $j$  (L)

$\Delta r_{j-1/2}$  is  $r_j - r_{j-1}$

$\Delta r_{j+1/2}$  is  $r_{j+1} - r_j$

$\Delta z_k$  is the space increment in the  $z$  direction for layer  $k$  (L)

$w_{j-k}$  is the source term for cell  $j$  in layer  $k$  (1/T)

Equations 2 and 5 have the same form. Therefore, the program designed by Trescott (1975) to solve equation 2 can also be used to solve equation 5. The program will treat the radial flow field as a vertical cross section. However, as indicated by equation 5, each hydraulic parameter must be multiplied by  $2\pi r_j$ .

Use of Trescott's (1975) model to simulate radial flow, as described above, causes an incorrect value of transmissivity to be used. The error in transmissivity between mode  $j$  and  $j+1$  is a function of  $r_j$ ,  $r_{j+1}$ ,  $\Delta r_j$  and  $\Delta r_{j+1}$ . If cell width  $\Delta r_j$  increases by a constant factor,  $\phi$ , so that  $\Delta r_{j+1} = \phi * \Delta r_j$ , then the error is a function of  $\phi$  and  $j$ . For this project the factor  $\phi$  was 1.5. The error in transmissivity between modes 1 and 2 was 12 percent. The error in transmissivity between all other modes was less than 1 percent. The accuracy was deemed to be adequate for this project.