UNITED STATES DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY

Plane wave reflection coefficients for a compressional wave in water incident on a "soft" seafloor with gas-charged sediment

by

Gary Boucher

Open-File Report 80-620

1980

Plane wave reflection coefficients for compressional wave in water incident on a "soft" seafloor with gas-charged sediment

Gary Boucher

Interest in gas-charged sediment at shallow depths beneath the seafloor (Carlson and Molnia, 1978; Kvenvolden and others, 1977; Molnia and others, 1978) raises a requirement for quantitative or semi-quantitative techniques to study the nature and distribution of such sediment. A number of seismic experiments may be envisioned to study the effects of gas in the sediment, some of which involve only a reinterpretation of existing data. Holmes and Thor (1980), for example, interpreted the absence of a direct water-wave arrival on multichannel seismic traces in areas of known gas-charged sediments as being due to refraction of acoustic energy away from the water layer. As an aid to thinking about this particular problem, as well as the more direct problem of designing acoustic experiments for analysis of gas-charged sediments, I have written an interactive computer program to make quick calculations of the plane-wave reflection coefficients for the interface between a liquid and a solid halfspace for arbitrary values of the elastic constants and densities, as a function of angle of incidence of the sound wave (fig. 1). Some illustrative results are presented here, and the computer program is included as an appendix.

Ewing, Jardetzky, and Press (1957), pages 76-81, give expressions for the plane wave reflection and transmission coefficients for the interface between a liquid and a solid halfspace. The formulas for the reflection coefficient for the reflected P wave, and the transmission coefficients for the transmitted P-wave and the converted S-wave are, respectively,

$$P_{refl} = \frac{D - 2\, r\, a'\, c^4/s'^2}{D}$$

$$P_{tran} = \frac{2\ r\ a\ c^2(c^2/s'^2 - 2)}{D}$$

$$S = \frac{4\ r\ a\ a'\ c^2}{D}, \text{ where}$$

$$D = r\ a'\ c^4/s'^2 = r's'^2 a((c^2/s'^2-2)^2 + 4\ a'\ b').$$

Hre r is the density, a and b are the contangents of the angles of incidence for compressional waves and shear waves, resp., c is the phase velocity, and s is the shear-wave velocity. Primes (') refer to the solid medium. The quantities a' and b' may be imaginary, so the reflection and transmission coefficients are complex. Other quantities that may be of interest are the energy partition ratios for the various wave types E_p, E'_p, and E'_s, which are real, given by

$$E_p = P_{refl}^2$$

$$E'_p = (r'a'/ra)P_{tran}^2$$

$$E'_s = (r'b'/ra)S^2, \text{ where}$$

$$E_p + E'_p + E'_s = 1$$

The computer program, named pwrefcoeff is written in PL1 language for the Honeywell 68/80 Multics system, but could easily be converted to Fortran language. It is included as an appendix for the convenience of those interested in making their own calculations.

The particular case of interest in this paper is that of gas-charged sediment on the seafloor. The compressional velocity may be substantially less than that of sea water, typically by as much as 25%. This condition may prevail in nature when gas in the bubble phase is present in a large percentage of the pore space of the sediment. In soft, porous sediments the shear velocity is typically less than 300 m/sec. The effect of free gas displacing the water in the sediment would be an increase in the shear velocity relative to the fluid-saturated case, in contrast to its effect on the compressional velocity.

The Table shows the results of the computer program for two examples. The data for the square root of the energy ratios are more appropriate for actual measurements than are the complex reflection coefficients, since the former are real numbers. For observations in the liquid layer, they are easily interpreted as amplitude reflectivities.

Having in mind the practical problem of "prospecting" for gas-charged sediments by measuring the amplitude of waves reflected from the sea floor, I shall limit the rest of the discussion to consideration of the reflected compressional wave. The compressional wave reflected from the bottom is frequently a strong arrival whose arrival time is known exactly, making it convenient for semiquantitative analysis.

Fig. 2 illustrates the difference in behavior between the "hard" bottom, typical of gas-free sediment with the speed of sound in the bottom greater than that in the water, and the "soft" bottom, with lower bulk density and sound velocity less than that of water. In the usual seismic situation, the observable data are limited to the reflected compressional wave. Hence the significant difference in the two situations is essentially the strong minimum in reflectivity at moderately large angles of incidence for the "soft" bottom, compared with the very large reflectivity prevailing at all angles greater than 60 degrees for the "hard" bottom.

Judgments to be made in the field are not necessarily so clear-cut as these idealized examples indicate. Reflection coefficients involve products of density and elastic constants, making it necessary to consider variations of both sets of parameters. When this is done, it becomes apparent that variations in density are relatively insignificant compared to variations in compressional wave speed in the bottom, for plausible ranges of both parameters.

For a fixed compressional wave speed less than that of water, the variation in reflectivity at low angles of incidence is more significant than that at high angles, but it is still rather small. The principal effect of decreasing the density is a slow, progressive leftward shift of the location of the minimum reflectivity point. Fig. 3 (a) illustrates this.

In contrast, for a plausible variation of compressional wave speed, with density held constant (fig. 3b), there are dramatic variations in the behavior of the reflectivity of the bottom. In general, for values of compressional wave speed less than that of the overlying water, there is a pronounced minimum of reflectivity at moderately large angles of incidence, the location of the minimum progressing towards smaller angles of incidence as the velocity decreases. For very low values of compressional velocity, the location of this minimum effectively moves far to the left, and for extremely low values (0.8 km/sec) it is absent altogether. A compressional wave speed as low as 0.8 km/sec would be unusual, but Schubel (1974) presents evidence for the existence of gas-charged sediment with a compressional velocity less than 250 m/sec in Chesapeake Bay. Schwartz and others (1973) found velocities near 760 m/sec in Miocene deltaic sands in the Attaka oil field. For wave speeds greater than that of the overlying water, there is no pronounced minimum in the reflectivity function, and reflectivity approaches unity for all moderately large angles of incidence. This range of situations is illustrated in fig. 3 (b), where compressional wave speed is the parameter.

In order to complete the discussion, I should mention two aspects of the problem that I have ignored. First relates to the need for a more elaborate model to represent an actual field situation correctly. Plane wave reflection coefficients are useful, but they are not strictly applicable to spherically propagating waves such as might be generated by an airgun system. Quantitative analysis of the reflection coefficients would benefit from the more elaborate theory. In addition, the ocean bottom is often inhomogeneous, and in particular layered. This may give rise to interesting frequency-dependent effects, which do not appear in the discussion of plane waves propagating in halfspaces.

The second point is that it is tempting to use measurements of the amplitude of the bottom reflection derived from multichannel seismic streamer data to determine behavior as a function of angle of incidence of the reflected waves. In this case it is important to remember that the "sections" of a multichannel streamer are normally made rather long, with the express intent of suppressing low-velocity, obliquely-travelling waves like the bottom reflection. Thus an appropriate correction for the directivity function of the streamer elements would be important. An experiment designed to "prospect" for gas-charged sediments in the way described in this paper would use omnidirectional hydrophones located at moderate depth, and would need to preserve true amplitude data.

Each field situation must be regarded separately, in terms of what is known of bottom geology, density, and the behavior of compressional velocity in the bottom. Some general conclusions may still be derived from this simple series of numerical experiments. The most significant conclusion is that in the range of angles of incidence greater than about 60^0, but less than about 85^0, there is a very large difference in reflectivities to be expected between "soft" and "hard" bottoms, except when the compressional wave speed is quite near to that of sound in sea water. There are differences among the curves at smaller angles of incidence, but they may be too small to be resolved. The most general statement one could make is this: Very high reflectivity in the range of 50^0 to 80^0 angles of incidence is diagnostic of relatively high compressional velocities in the bottom, implying the absence of significant free gas in the sediment.

REFERENCES

Carlson, Paul R., and Molnia, Bruce F., 1978, Submarine Faults and slides on the continental shelf, northern Gulf of Alaska: Marine Geotechnology, v. 2, p. 275-290.

Ewing, M., Jardetzky, W.S., and Press, F.,(1957) Elastic Waves in Layered Media: McGraw-Hill, New York, 380 p.

Holmes, Mark, and Thor, Devin R., 1980, Distribution of gas-charged sediment in Norton Basin, Northern Bering Sea, in International Assoc. of Sedimentologists Symposium: North Sea Analogues, Fall, 1979: (in press).

Kvenvolden, Keith A., Redden, George D., and Carlson, Paul R., 1977, Hydrocarbon gases in sediments of eastern Gulf of Alaska: Am. Assoc. Petrol. Geol. Bull., v. 61, p. 806, (abstract).

Molnia, Bruce F., Carlson, Paul R., and Kvenvolden, Keith, A., 1978, Gas-charged sediment areas in the northern Gulf of Alaska: Geological Society of America Abstracts of Papers, v. 10, no. 7, p. 458.

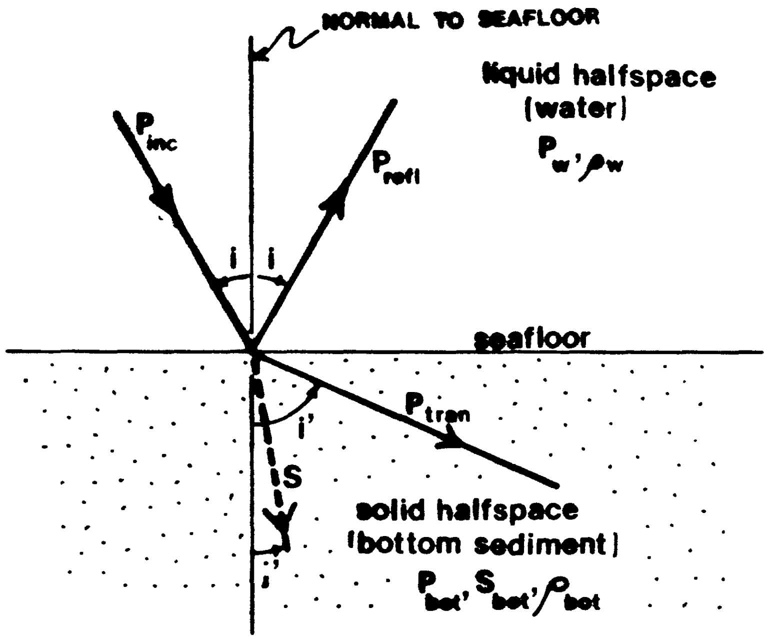
Schubel, J. R., 1974, Gas bubbles and the acoustically impenetrable, or turbid, character of some estuarine sediments in, I. R. Kaplan (e.d.) Natural Gases in Marine Sediments: Plenum Press, New York, pp. 275-297.

Schwartz, C. M., G. H., Laughbaum, Jr., B. S. Samsu, and J. D. Armstrong, 1973, Geology of the Attaka oil field, East Kalimantan, Indonesia, in Proceedings, Second Annual Convention Indonesian Petroleum Association, pp. 195-215.

FIGURE CAPTIONS

1. Schematic diagram of the interface between liquid (sea-water) and a solid (bottom sediment) halfspaces, showing a compressional wave P_{inc} incident on the seafloor at angle of incidence $\underline{i}$, giving rise to a reflected sound wave P_{ref} and, in the solid medium, a transmitted compressional wave P_{tran} and with angles of incidence $\underline{i}$ and $\underline{j}$, respectively, and a converted shear wave S. Heavy lines represent raypaths, perpendicular to plane wavefronts. In this particular situation, the compressional wave speed in the bottom is greater than that in water, whereas the shear wave speed in the bottom is less than either of the other two.
2. Comparison between a "hard" bottom, in which the compressional wave speed is greater than that of water, and a "soft" bottom, in which the compressional wave speed is substantially less than that of water. The latter situation is unlikely to occur unless the sediment pore-space is largely filled with a gas rather than with water. Bulk density is in specific gravity units, such as g/cm^3. The ordinate, the square root of the energy partition ratio, is the same as the amplitude ratio for the liquid medium, except that it is real and positive, and it is related to the amplitude that would be observed on a seismic trace.
3. Plots of the square root of the energy partition ratio for the reflected compressional wave, equivalent to "reflectivity." In (a) bulk density is the parameter for various families of curves. The compressional velocity in the bottom is less than that of water, corresponding to a soft or "gassy" bottom, where other quantities are held fixed. Variations between curves are progressive with increasing bulk density, and are not dramatic.

In (b), the compressional velocity in the bottom is the variable parameter, and the variations between curves are rather dramatic, particularly at the change from a compressional velocity greater than to less than that of sound in water. The dashed curve for 0.8 km/sec represents an unusual situation, but one that should be interpreted carefully, should it be encountered. The arrow is intended to clarify the progression in the family of curves from higher to lower velocity.



NORMAL TO SEAFLOOR
liquid halfspace
(water)
P_w, ρ_w
P_{inc}
P_{refl}
i
i
seafloor
P_{tran}
i'
S
i'
solid halfspace
(bottom sediment)
P_{bot}, S_{bot}, ρ_{bot}

fig. 1

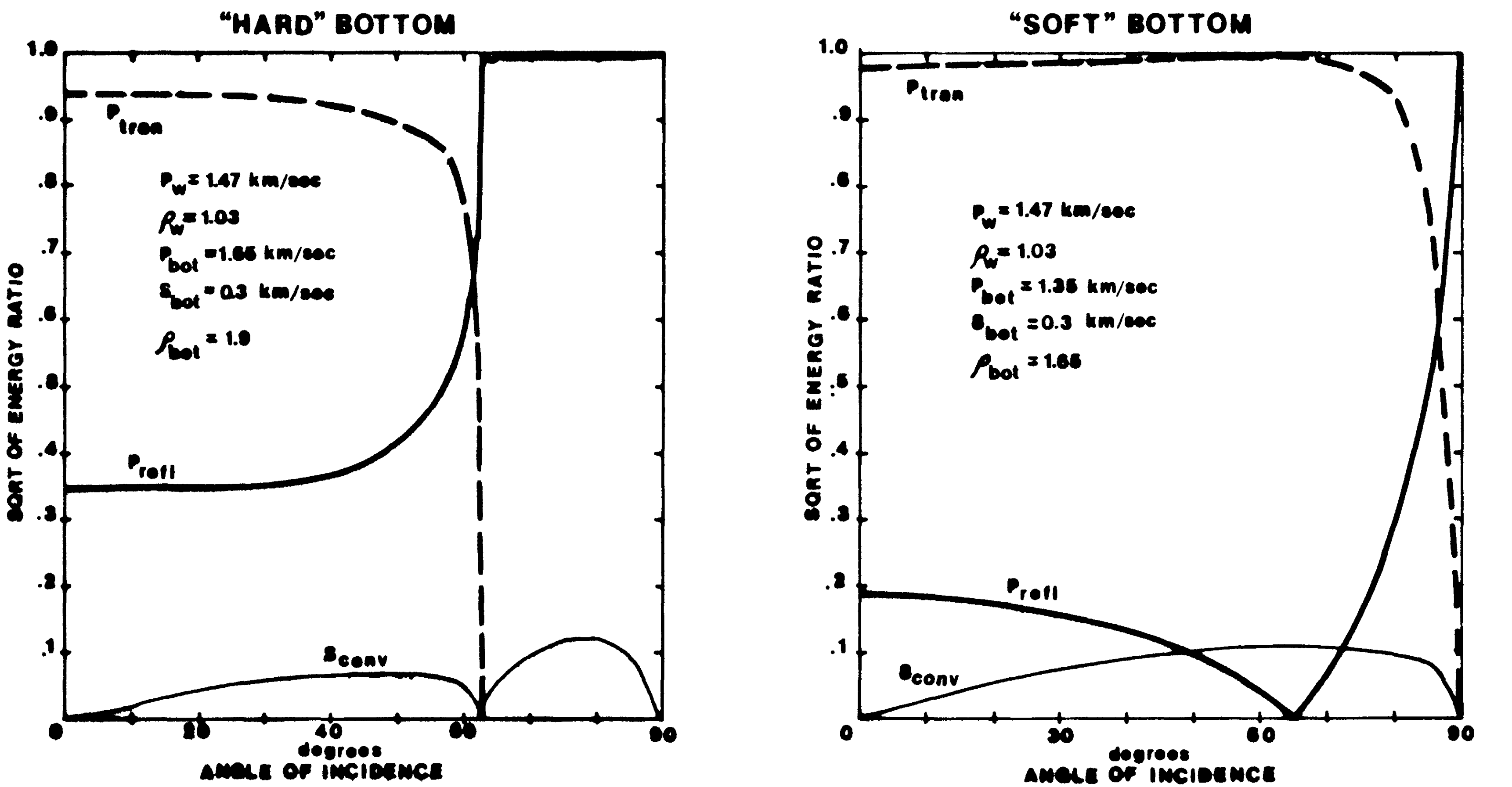
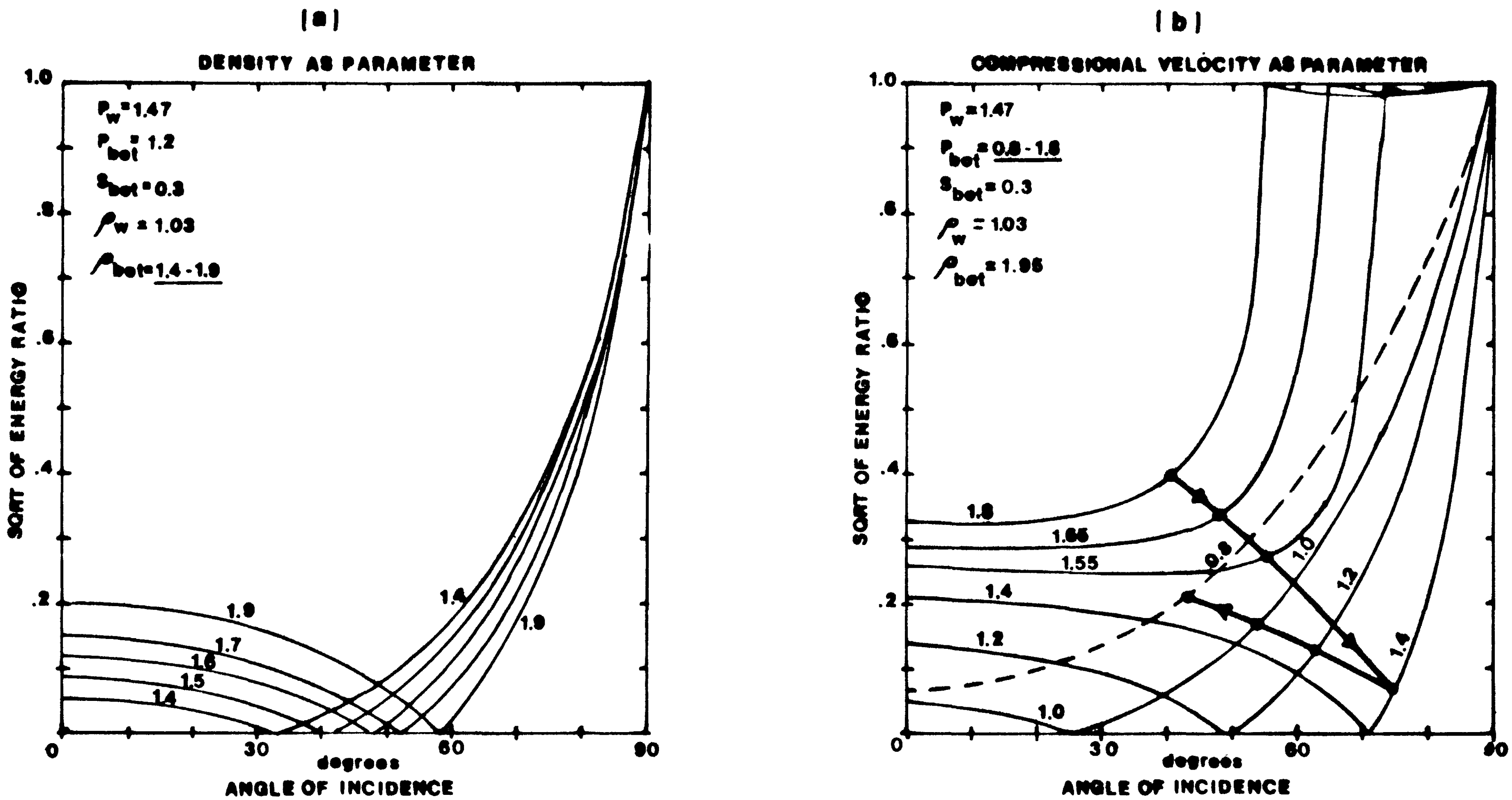

"HARD" BOTTOM
P_{tran}
$P_w = 1.47$ km/sec
$\rho_w = 1.03$
$P_{bot} = 1.65$ km/sec
$S_{bot} = 0.3$ km/sec
$\rho_{bot} = 1.9$
P_{refl}
S_{conv}
SQRT OF ENERGY RATIO
degrees
ANGLE OF INCIDENCE
"SOFT" BOTTOM
P_{tran}
$P_w = 1.47$ km/sec
$\rho_w = 1.03$
$P_{bot} = 1.35$ km/sec
$S_{bot} = 0.3$ km/sec
$\rho_{bot} = 1.65$
P_{refl}
S_{conv}
SQRT OF ENERGY RATIO
degrees
ANGLE OF INCIDENCE

fig. 2



(a)
DENSITY AS PARAMETER
$P_w = 1.47$
$P_{bot} = 1.2$
$S_{bot} = 0.3$
$\rho_w = 1.03$
$\rho_{bot} = 1.4 - 1.9$
1.9
1.7
1.6
1.5
1.4
1.4
1.9
SQRT OF ENERGY RATIO
degrees
ANGLE OF INCIDENCE
(b)
COMPRESSIONAL VELOCITY AS PARAMETER
$P_w = 1.47$
$P_{bot} = 0.8 - 1.8$
$S_{bot} = 0.3$
$\rho_w = 1.03$
$\rho_{bot} = 1.95$
1.8
1.65
1.55
1.4
1.2
1.0
0.8
1.0
1.2
1.4
SQRT OF ENERGY RATIO
degrees
ANGLE OF INCIDENCE

fig. 3

TABLE

"Density of water =" 1.03000000e+000 "Density of bottom =" 1.65000001e+000
"Vel. of sound in water = " 1.47000000e+000 "km/sec"
"Compressional and shear velocities in bottom, respectively:" 1.30000000e+000 3.00000001e-001

Angle of incidence Degrees	Reflected P real	Reflected P imag	Refracted P real	Refracted P imag	Refracted S real	Refracted S imag	enRfl sqrt	enRfr energy	enS ratio
10.000	0.1669	0.0000	0.7307	0.0000	0.0118	0.0000	0.1669	0.9851	0.0264
15.000	0.1644	0.0000	0.7291	0.0000	0.0174	0.0000	0.1644	0.9851	0.0392
20.000	0.1581	0.0000	0.7269	0.0000	0.0225	0.0000	0.1581	0.9853	0.0514
25.000	0.1497	0.0000	0.7236	0.0000	0.0271	0.0000	0.1497	0.9855	0.0630
30.000	0.1391	0.0000	0.7198	0.0000	0.0311	0.0000	0.1391	0.9859	0.0737
35.000	0.1260	0.0000	0.7145	0.0000	0.0342	0.0000	0.1260	0.9864	0.0834
40.000	0.1099	0.0000	0.7077	0.0000	0.0365	0.0000	0.1099	0.9871	0.0919
45.000	0.0901	0.0000	0.6988	0.0000	0.0379	0.0000	0.0901	0.9880	0.0992
50.000	0.0655	0.0000	0.6869	0.0000	0.0383	0.0000	0.0655	0.9889	0.1052
55.000	0.0344	0.0000	0.6708	0.0000	0.0378	0.0000	0.0344	0.9897	0.1097
60.000	-0.0058	0.0000	0.6485	0.0000	0.0363	0.0000	0.0058	0.9898	0.1127
65.000	-0.0596	0.0000	0.6169	0.0000	0.0336	0.0000	0.0596	0.9877	0.1141
70.000	-0.1339	0.0000	0.5712	0.0000	0.0304	0.0000	0.1339	0.9805	0.1138
75.000	-0.2401	0.0000	0.5035	0.0000	0.0258	0.0000	0.2401	0.9605	0.1113
80.000	-0.3967	0.0000	0.4012	0.0000	0.0199	0.0000	0.3967	0.9084	0.1046
85.000	-0.6337	0.0000	0.2441	0.0000	0.0118	0.0000	0.6337	0.7656	0.0877
90.000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
95.000	-1.5780	0.0000	-0.3853	0.0000	-0.0186	0.0000	1.5780	1.2081	0.1383

"Density of water =" 1.03000000e+000 "Density of bottom =" 1.65000001e+000
"Vel. of sound in water = " 1.47000000e+000 "km/sec"
"Compressional and shear velocities in bottom, respectively:" 1.09999999e+000 3.00000001e-001

Angle of incidence Degrees	Reflected P real	Reflected P imag	Refracted P real	Refracted P imag	Refracted S real	Refracted S imag	enRfl sqrt	enRfr energy	enS ratio
5.000	0.0891	0.0000	0.6801	0.0000	0.0066	0.0000	0.0891	0.9959	0.0146
10.000	0.0852	0.0000	0.6782	0.0000	0.0130	0.0000	0.0852	0.9957	0.0291
15.000	0.0786	0.0000	0.6751	0.0000	0.0192	0.0000	0.0786	0.9954	0.0432
20.000	0.0692	0.0000	0.6705	0.0000	0.0249	0.0000	0.0692	0.9950	0.0568
25.000	0.0567	0.0000	0.6643	0.0000	0.0301	0.0000	0.0567	0.9945	0.0699
30.000	0.0408	0.0000	0.6563	0.0000	0.0346	0.0000	0.0408	0.9938	0.0821
35.000	0.0210	0.0000	0.6462	0.0000	0.0383	0.0000	0.0210	0.9928	0.0934
40.000	-0.0032	0.0000	0.6333	0.0000	0.0411	0.0000	0.0032	0.9914	0.1036
45.000	-0.0328	0.0000	0.6172	0.0000	0.0430	0.0000	0.0328	0.9892	0.1126
50.000	-0.0690	0.0000	0.5968	0.0000	0.0438	0.0000	0.0690	0.9859	0.1203
55.000	-0.1134	0.0000	0.5710	0.0000	0.0436	0.0000	0.1134	0.9806	0.1265
60.000	-0.1683	0.0000	0.5361	0.0000	0.0421	0.0000	0.1683	0.9717	0.1309
65.000	-0.2370	0.0000	0.4959	0.0000	0.0395	0.0000	0.2370	0.9568	0.1332
70.000	-0.3236	0.0000	0.4414	0.0000	0.0354	0.0000	0.3236	0.9311	0.1329
75.000	-0.4339	0.0000	0.3706	0.0000	0.0299	0.0000	0.4339	0.8861	0.1286
80.000	-0.5758	0.0000	0.2764	0.0000	0.0224	0.0000	0.5758	0.8038	0.1180
85.000	-0.7598	0.0000	0.1579	0.0000	0.0127	0.0000	0.7598	0.6391	0.0944
90.000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
95.000	-1.3162	0.0000	-0.2079	0.0000	-0.0168	0.0000	1.3162	0.8412	0.1243

APPENDIX

```
Compiled on: 12/20/79  1145.2 pst Thu
     Options: map table

pwrefcoeff : proc;
/*  8888888888888888888888888888888888888888888888888888888888
    Reference is made to page 79 of Ewing, Jardetzky, and Press,
     ELASTIC WAVES IN LAYERED MEDIA.
    Correspondence of program variables to EJP variables:

            Program variable            EJP variable
              -----------------          -----------------
             Plec                        A2/A1 (reflection coefficient for reflectedP)
             Prac                        A'/A1 (amplitude coefficient for refracted P)
             Srac                        B'/A1 (amplitude coefficient for refracted S)
              r                          rho ( density of liquid)
               r1                        rho' (density of solid)
             p                           alpha (speed of sound in liquid)
              p1                         alpha' (speed of sound in solid)
              s                          beta' (shear velocity in solid)
               i                         angle of incidence for incident P in liquid
              c                          c (phase velocity)
               d                          a'
                e                         b'
               D                         denominator of expressions for R.C.
                h                         angle of incidence for refracted s
                 Epl                      square root of energy ratio for reflected P
                   Epr                       "   "   "    "    "   "  refracted P
                 Es                          "   "   "    "    "   "    "      s
*888888888888888888888888888888888888888888888888888888888888888888888888888888888
*                                        */


 dcl (r init(1.03), r1 init(1.9), p init(1.47),p1 init(1.65),
       s1 init (0.3), s,a,c,g,h,e,i) float bin ;
  dcl (d,D,Srac,Prac,Plec) complex float bin ;
 dcl (sysin,sysprint,output_segment) file ;
dcl (firsti init(1.0),lasti init(85.),incr init(5.),Epl,Epr,Es) float bin ;
dcl (sind,cosd) builtin ;
 dcl asind_ entry (float bin) returns (float bin) external ;


 put list ("reflection coefficients for a liquid over a half-space, taken from Ewing,
Jardetzky and Press. Parameters may be changed, or left the
 same by putting a comma.") ;
     open file (output_segment) output stream ;

start: put skip list("Density of water=",r,"New value (-1 to quit):") ;
  get list (r);
  if r < 0.0 then go to finish ;
   put skip file (output_segment) list ("Density of water =",r) ;
  put skip list ("Bottom density =",r1,"New value:") ;
  get list (r1) ;
   put file (output_segment) list ("Density of bottom =",r1) ;
  put skip list ("Vel. of sound in water=",p,"New value:");
  get list (p);
   put skip file (output_segment) list ("Vel. of sound in water = ",p,"km/sec") ;
  put skip list("Vel. of sound in bottom=",p1,"New value:");
```

Continued next page

```
  get list (p1);
  put skip list ("Shear vel. in bottom=",s1,"New value:");
  get list (s1);
   put skip file (output_segment) list ("Compressional and shear velocities in bottom, respectively:",p1,s1);
  put skip list ("Range of angles in degrees- START,END,INCREMENT:");
  get list (firsti,lasti,incr);
  put edit ("Angle of incidence    Reflected P      Refracted P       Refracted S  enRfl   enRfr   enS")(skip,a);
put edit ("   Degrees           real   imag       real   imag       real   imag       sqrt energy ratio")(skip,a);
  put  file (output_segment) edit ("Angle of incidence    Reflected P      Refracted P       Refracted S  enRfl   enRfr   enS")
     (skip,a);
put  file (output_segment) edit ("   Degrees           real   imag       real   imag       real   imag       sqrt energy ratio")
     (skip,a);
  do i = firsti to lasti by incr;
    s= sind(i);
     a= cosd(i)/s ;
    h= asind_ (s1/p*s);
    c = p/s ;
     if c<p1 then d= cplx (0,-sqrt(1-c**2/p1**2)) ;
        else d= sqrt (c**2/p1**2-1) ;
    e= 1./tand(h);
    D= r*d*c**4/s1**2+r1*s1**2*a*((c**2/s1**2-2)**2+4*d*e) ;
    Plec= (D-2*r*d*c**4/s1**2)/D ;
         if c<p1 then Prac = 0; else
    Prac= 2*r*a*c**2*(c**2/s1**2-2)/D ;
    Srac= 4*r*a*d*c**2/D ;
        Epl = abs (Plec) ;
            if c<p1 then Epr = 0.0 ;
                else Epr = sqrt (r1*abs(d)/(r*a))*abs (Prac) ;
         Es= sqrt (r*e/(r*a))*abs (Srac) ;

    put edit (i,Plec,Prac,Srac,Epl,Epr,Es)(skip,x(3),f(8,3),x(8),c(f(7,4)),x(3),c(f(7,4)),x(3), c(f(7,4)),3(f(8,4))) ;
    put file(output_segment) edit (i,Plec,Prac,Srac,Epl,Epr,Es)(skip,x(3),f(8,3),x(8),c(f(7,4)),x(3),c(f(7,4)),x(3), c(f(7,4))
    ,3(f(8,4))) ;
end ;
 go to start ;
 finish: close file (output_segment) ;
    put skip list ("remember to delete 'output_segment' if you don't want to save it");
   end pwrefcoeff ;
```