UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

COST EFFECT STREAM-GAGING STRATEGIES FOR THE LOWER COLORADO RIVER BASIN; THE BLYTHE FIELD OFFICE OPERATIONS

By Marshall E. Moss and Edward J. Gilroy

Open-File Report 80-1048

Reston, Virginia
December 1980

UNITED STATES DEPARTMENT OF THE INTERIOR

CECIL D. ANDRUS, Secretary

GEOLOGICAL SURVEY

H. William Menard, Director

For additional information write to:

U.S. Geological Survey
Water Resources Division
National Center, Mail Stop 430
12201 Sunrise Valley Drive
Reston, Virginia 22092

CONTENTS

FIGURES

TABLES

SYMBOLS

A	Cross-sectional area.
a_i	Coefficients in governing differential equation of discharge-computation problem.
c_i	Coefficient of discharge relation.
F	Matrix of coefficients in the state equation.
F_c	Fixed cost of network operation.
G	Matrix of coefficients in the state equation.
g	Vertical gate opening.
H	Matrix of coefficients in the measurement equation.
$h(t)$	Stage or gage height at time t
$h_u(t)$	Gage height at the CRIR upper main drain.
I	Identity matrix.
I_F	Information content in the forward filter estimate of annual mean discharge.
I_m^b	Information content of the discharge measurements used in a backward-filter estimate of annual mean discharge.
I_m^f	Information content of the discharge measurements used in a forward-filter estimate of annual mean discharge.
I_p	Information content of the a priori knowledge of the statistical parameters of a Kalman filter.
I_r	Redundant information content of the discharge measurements used in forward and backward estimates of annual mean discharge.
I_t	Total information content in a smoothed estimate of annual mean discharge.
$K(t)$	Kalman gain matrix at time t.
M_j	Number of discharge measurements per year at the jth streamgage.
MG	Number of streamgages in the network.
m	Number of discharge measurements available to calibrate the Kalman-filter model.
$\underline{N}$	Vector of numbers of times per year that each route is used.

$\underline{N}^*$ Vector of optimum numbers of times per year that each route is used.

N_i Number of times per year that route i is used.

NR Number of practical routes.

$P(t)$ Matrix of error covariances at time t .

$\dot{P}(t)$ Matrix of first derivatives with respect to time of error covariances at time t.

$p_{ij}(t)$ Covariance of errors of estimation of the ith and jth state variables at time t.

$p^*_{11}(1)$ Variance of the error of the optimal estimate of annual mean discharge.

$\hat{p}_{11}(1)$ Approximation of the variance of the error of the optimal estimate of annual mean discharge.

$p^b_{22}(t)$ Variance of the error of the backward estimate of discharge rate.

p^-_2 Maximum variance of the error of real-time estimates of discharge rate.

Q Vector of spectral densities of the random variables of the state equation.

q Spectral density of a random variable.

Q_C Computed total discharge during a year.

$q_C(t)$ Computed discharge rate at time t.

$q_M(t)$ Mmeasured discharge rate at time$t$.

Q_R Rated total discharge during a year.

$q_R(t)$ Rated discharge at time t.

Q_T True total discharge during a year.

$q_T(t)$ True instantaneous discharge at time t.

$q_u(t)$ Rated discharge at the CRIR Upper Main Drain.

R Covariance matrix of measurement errors.

r Variance of measurement error.

T Indicator of matrix transposition.

T_C Total cost of operating network.

t Time.

t^+ Time just after a discharge measurement.

t^- Time just prior to a discharge measurement.

V Total uncertainty in estimates of annual mean discharge at all streamgages in the network.

$v(t)$ Measurement error at time t.

$\underline{v}(t)$ Vector of measurement errors at time t.

$w(t)$ Random variable at time t.

$\underline{w}(t)$ Vector of random variables at time t.

$\underline{x}(t)$ General vector of state variables.

$\dot{\underline{x}}(t)$ Vector of derivatives with respect to time of the state variables.

$x_1(t)$ Difference between true and rated accumulated discharge since the beginning of a water year at time t.

$\dot{x}_1(t)$ First derivative with respect to time of the difference between true and rated, accumulated discharge since the beginning of the water year.

$\hat{x}_1(t)$ Difference between computed and rated accumulated discharge since the beginning of a water year at time t.

$x_2(t)$ Difference between true and rated discharge rate at time t.

$\dot{x}_2(t)$ First derivative with respect to time of the difference between true and rated discharge rate at time t.

$x_2^*(t)$ Optimal smoothed estimate of the difference between true and rated discharge rate at time t.

$z(t)$ Difference between measured and rated discharge at time t.

$\underline{z}(t)$ Vector of measurements at time t.

α_j Unit cost of a visit to the jth streamgage.

α_t Weight given to forward filter estimate of discharge rate.

β Inverse of the correlation-time constant of a first order Markovian process.

β_i Unit travel cost for route i.

$\Gamma(\Delta)$ Autocovariance of discharge rates separated by a time interval Δ.

γ Error of estimation of total discharge during a year.

Δ Lag time of an autocovariance function.

$\delta(t)$ Difference in two water-surface elevations at time t.

θ Dimensionless parameter in the covariance function of forward and backward estimates of discharge rate.

λ_j Minimum number of visits per year to station j.

π_t	Covariance between forward and backward estimates of discharge rate at time t.
σ_j^2	Variance of discharge at the jth streamgage.
σ_z^2	Variance of the differences between measured and rated discharges.
τ	Time since the last discharge measurement.
ϕ_j	Function relating uncertainty at the jth streamgage to the frequency of discharge measurement.
ψ_i	Time of the ith discharge measurement.
ω_{ij}	Route definition parameter (ω_{ij} = 1 indicates that streamgage j is visited on route i; ω_{ij} = 0, it is not).

FOREWORD

This report describes the theoretical developments and illustrates the applications of techniques that recently have been assembled to analyze the cost-effectiveness of federally funded stream-gaging activities in support of the Colorado River compact and subsequent adjudications. The sample analysis is limited only to the stream gages serviced by the Blythe Field Office of the U.S. Geological Survey that are federally financed under the Geological Survey's authorization for the Collection of Basic Records. Much of this material will be incorporated into the reports that document the analysis of the larger stream gaging network that supports the compact.

SUMMARY

The cost-effectiveness of the current operation of 19 stream gages of the hydrologic network that supports the Colorado River compact and subsequent adjudications is found to be relatively close to the theoretical limits when the total uncertainty in annual-mean-discharge estimates is considered. The current cost (1980) for the 19 stations is $110,900 and the sum of the uncertainties in the annual-mean-discharge estimates for each of the stream gages results in an uncertainty of 113 ft^3/s (cubic feet per second). The major portion of this uncertainty is contributed by the three stream gages that are on the mainstem of the Colorado River. Lesser, but still significant, amounts of uncertainty are derived from the major diversions and return flows. At 11 of the 19 gages at which discharge can be considered as minor return flow, determination of discharge results in less than 1 percent of the total uncertainty.

If a minimum of 12 visits per year to each stream gage is prescribed, the $110,900 budget can be adjusted among the gages so as to reduce the total uncertainty to 94 ft^3/s; with a 6-visit-per-year minimum, uncertainty can be reduced to 87 ft^3/s. In each of these cases funds are diverted from the minor-return-flow gages to the mainstem gages.

On the other hand, a level of uncertainty, 112 ft^3/s, comparable with the current operations can be attained for $95,000 if the 6-visit minimum is maintained and for $101,000 with the 12-visit minimum. Here again the shift of funds from the measurement of minor return flows to more measurements on the mainstem is found to be the most cost effective.

A significantly larger budget of $200,000 will reduce total uncertainty in annual-mean-discharge estimates at the 19 stream gages to 48 ft^3/s.

Subsequent analyses will expand the study to include all stream gages of the Lower Colorado River Basin that are federally financed under the U.S. Geological Survey's authorization for the Collection of Basic Records.

Cost Effective Stream-Gaging Strategies
for the Lower Colorado River Basin

INTRODUCTION

The waters of the Colorado River have been scrutinized as a source of irrigation at least since the time of the Powell Survey (Powell, 1875). As the southwestern United States began to develop, the Colorado River was also seen as a source of municipal and industrial water supply. Because nine states, seven from the United States and two from Mexico, share the drainage basin of the Colorado River, as shown in figure 1, competition for the water became inevitable. In 1922, the drainage basin was partitioned for administrative purposes into the Upper Colorado River Basin and the Lower Colorado River Basin by an interstate compact (Colorado River Compact, 1922). The dividing line is the drainage divide between surface waters that flow into the river above Lee Ferry, Arizona, and those that reach the river below Lee Ferry. This divide is shown in figure 1. In addition to setting up the designation of Upper and Lower Basins, the compact allocated the waters of the river between the Upper and Lower Basins and among the various states represented in the partition. As an aid to the administration of the compact, certain stream-gaging stations were established by the United States Geological Survey where the flow of the main river, its tributaries, and subsequent diversions and return flows were to be measured.

Continued development of the river basin and adjacent areas to which the river's waters were diverted led to still more competition and resulted in a U.S. Supreme Court ruling (Arizona vs. California, 1963) on the relative rights of two of the Lower Basin states, California and Arizona. This court decision and its attendant implementation led to a major increase

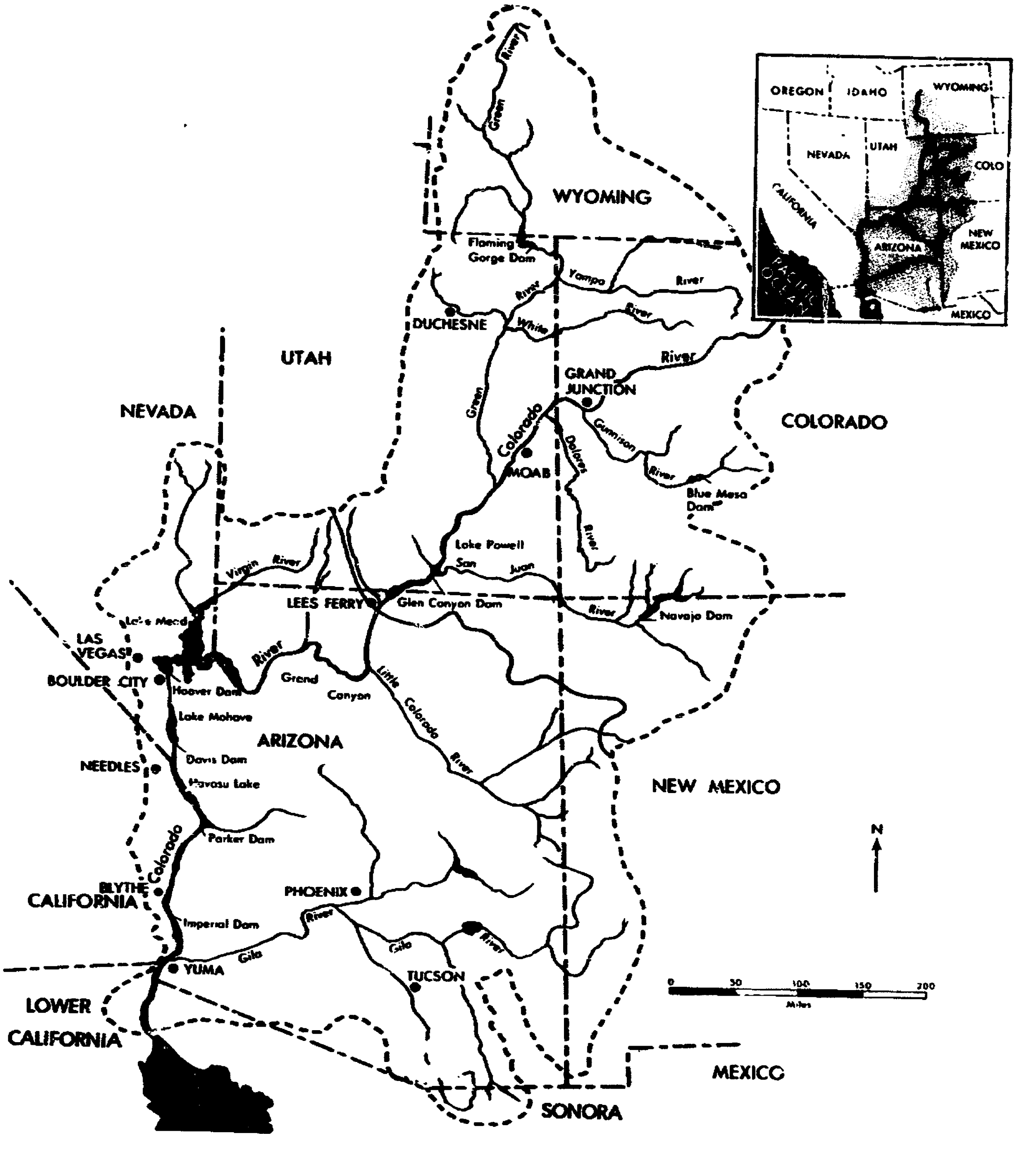


Fig. 1.--The Colorado River drainage basin.

in stream-gaging activity in the Lower Basin by the Geological Survey. Table 1 lists those stream gages in the Lower Basin that are considered by the Geological Survey as being operated primarily in support of the compact and subsequent legal interpretations of the Compact. The financial scope of this network of stream gages is such that it warrants an analysis of the effectiveness of the resulting data in the administration of the compact.

PROBLEM STATEMENT

The multitude of uses that are made of the waters of the Lower Colorado River would result in intractability if a complete economic analysis of the worth of each stream gage were attempted. Therefore the uncertainty or error in the estimation of annual discharges serves in this study as an inverse surrogate for the economic worth of the data. Uncertainty at a particular stream gage is expressed in this study either as the variance of the error of estimate of annual mean flow past the gage or its square root, the standard deviation. A unit of uncertainty is assumed to be as deleterious at any one stream gage as it is at any other in the network. Therefore, the objective of this study is to devise strategies for operating the network of gages that will minimize the total uncertainty in the system for a given operating budget.

APPROACH

The uncertainty in the annual mean discharge at a stream gage is a function of the frequency of visits that are made to the gage to service the recording equipment and to make discharge measurements. Thus, for any particular stream gage its uncertainty is minimized by visiting it as frequently as possible, which entails expending as much money and manpower

Table 1.--Gaging stations of the Lower Colorado River Basin network.

Station No.	Station name
09402500	Colorado River near Grand Canyon, Ariz.
09421500	Colorado River below Hoover Dam, Ariz.-Nev.
09423000	Colorado River below Davis Dam, Ariz.-Nev.
09423550	Topock Marsh Inlet near Needles, Calif.
09423650	Topock Marsh Outlet near Topock, Ariz.
09424150	Colorado River Aqueduct near Parker Dam, Ariz.-Calif.
09427520	Colorado River below Parker Dam, Ariz.-Calif.
09428500	Colorado River Indian Reservation Main Canal near Parker, Ariz.
09428505	Gardner Lateral Spill near Poston, Ariz.
09428510	Poston Wasteway near Poston, Ariz.
09429000	Palo Verde Canal near Blythe, Calif.
09429010	Colorado River at Palo Verde Dam, Ariz.-Calif.
09429030	Colorado River Indian Reservation Palo Verde Drain near Parker, Ariz.
09429060	Colorado River Indian Reservation Lower Main Drain near Parker, Ariz.
09429130	Palo Verde Irrigation District Olive Lake Drain near Blythe, Calif.
09429155	Palo Verde Irrigation District F Canal Spill near Blythe, Calif.
09429160	Palo Verde Irrigation District D-10-11-2 Canal Spill near Blythe, Calif.
09429170	Palo Verde Irrigation District D-10-11-5 Canal Spill near Blythe, Calif.
09429180	Palo Verde Irrigation District D-23 Canal Spill near Blythe, Calif.
09429190	Palo Verde Irrigation District D-23-1 Canal Spill near Blythe, Calif.
09429200	Palo Verde Irrigation District C Canal Spill near Blythe, Calif.
09429210	Palo Verde Irrigation District C-28 Canal Upper Spill near Blythe, Calif.
09429220	Palo Verde Irrigation District Outfall Drain, near Palo Verde, Calif.
09429225	Palo Verde Irrigation District Anderson Drain near Palo Verde, Calif.
09429230	Palo Verde Irrigation District C-28 Canal Lower Spill near Blythe, Calif.

Station No.	Station name
09429280	Cibola Lake Inlet near Cibola, Ariz.
09429290	Cibola Lake Outlet near Cibola, Ariz.
09429490	Colorado River above Imperial Dam, Ariz.-Calif.
09429500	Colorado River below Imperial Dam, Ariz.-Calif.
09520500	Gila River near Dome, Ariz.
09522000	Colorado River at Northerly International Boundary, above Morelos Dam, near Androde, Calif.
09522400	Mittry Lake Diversion at Imperial Dam, Ariz.-Calif.
09522500	Gila Gravity Main Canal at Imperial Dam, Ariz.-Calif.
09522600	North Gila Main Canal near Yuma, Ariz.
09522650	North Gila Canal Number 2 near Yuma, Ariz.
09522700	Wellton-Mohawk Canal near Yuma, Ariz.
09522800	South Gila Canal near Yuma, Ariz.
09522850	Gila Gravity Canal at Pumping Plant near Yuma, Ariz.
09522900	Unit B Main Canal near Yuma, Ariz.
09523000	All American Canal near Imperial Dam, Ariz.-Calif.
09523200	Reservation Main Canal near Yuma, Ariz.
09523400	Titsink Canal near Yuma, Ariz.
09523600	Yaqui Canal near Yuma, Ariz.
09523800	Pontiac Canal near Yuma, Ariz.
09523900	Walapai Canal near Yuma, Ariz.
09524000	Yuma Main Canal at Siphon Drop Power Plant near Yuma, Ariz.
09524500	Diversions from Yuma Main Canal below Siphon Drop Power Plant
09525000	Yuma Main Canal Wasteway at Yuma, Ariz.
09525500	Yuma Main Canal below Colorado River Siphon at Yuma, Ariz.
09526000	Diversions from Yuma Main Canal
09527000	Pilot Knob Power Plant and Wasteway near Pilot Knob, Calif.
09527500	All American Canal below Pilot Knob Wasteway, Calif.
09527900	Mittry Lake Outlet Channel near Yuma, Ariz.
09528600	Laguna Canal Wasteway near Yuma, Ariz.
09528800	Levee Canal Wasteway near Yuma, Aria.
09529000	North Gila Drain Number 1 near Yuma, Ariz.

Station No.	Station Name
09529050	North Gila Drain Number 3 near Yuma, Ariz.
09529100	Fortuna Wasteway near Yuma, Ariz.
09529150	North Gila Main Canal Wasteway near Yuma, Ariz.
09529160	South Gila Pump Outlet Channel Number 3 near Yuma, Ariz.
09529200	Bruce Church Drain near Yuma, Ariz.
09529240	South Gila Pump Outlet Channel Number 2 near Yuma, Ariz.
09529250	Bruce Church Wasteway near Yuma, Ariz.
09529300	Wellton-Mohawk Main Outlet Drain near Yuma, Ariz.
09529360	South Gila Pump Outlet Channel Number 1 near Yuma, Aria.
09529400	South Gila Drain Number 2 near Yuma, Ariz.
09529420	South Gila Terminal Wasteway near Yuma, Ariz.
09529440	South Gila Pump Outlet Channel Number 4 near Yuma, Ariz.
09529600	All American Canal Intercept Number 7 near Bard, Calif.
09529700	All American Canal Intercept Number 6 near Bard, Calif.
09529800	All American Canal Intercept Number 2 near Bard, Calif.
09529900	All American Canal Intercept Number 3 near Yuma, Ariz.
09530000	Reservation Main Drain Number 4 at Yuma, Ariz.
09530200	Yuma Mesa Outlet Drain at Yuma, Ariz.
09530400	All American Canal Intercept Number 11 near Yuma, Ariz.
09530500	Araz Drain 8-B near Yuma, Ariz.
09531800	Wellton-Mohawk M.O.D.E. Number 2 above Morelos Dam, Ariz.
09531850	Cooper Wasteway above Morelos Dam, Ariz.
09531900	Wellton-Mohawk M.O.D.E. Number 3 below Morelos Dam, Ariz.
09532500	Eleven Mile Wasteway below Morelos Dam, Ariz.
09533000	Twenty-One Mile Wasteway near San Luis, Ariz.
09533300	Wellton-Mohawk Drain at Ariz.-Sonora Border near San Luis, Ariz.

Station No.	Station name
09534000	Yuma Valley Main Drain near San Luis, Ariz.
09534300	West Main Canal Wasteway at Arizona-Sonora Boundary near San Luis, Ariz.
09534500	East Main Canal Wasteway at Arizona-Sonora Boundary near San Luis, Ariz.

as possible. With a finite budget for the network, the individual stream gages compete, in the mind of the manager of the network, for the available manpower and money. The manager's decision then becomes the allocation of the funds such that uncertainties at the individual station are reduced as much as possible without doing undue harm to the other stream-gaging records.

Efficient allocation of the stream-gaging budget.

Ideally, the total uncertainty in the network could be minimized if funds were adjusted among gages such that the rate of change of uncertainty with increased funding would be equal at all gages. However, funds are allocated in countable units of varying magnitudes so that derivatives with respect to funding do not exist. For example, a hydrographer does not make two-thirds of a trip to a stream gage or one-sixteenth of a discharge measurement in order to attain a particular cost of operation. Therefore, the budget is expended in integer multiples of unit costs of travel and unit costs of a station visit.

Other aspects of the problem are also integer valued. For example, neither the hydrographer nor the field vehicle are available in less than unit increments. However, for the purpose of this study it is assumed that the time that a hydrographer and his vehicle are used in the network are divisable. Any unused time is assumed to be spent on duties not pertaining to the network.

The problem is further confounded by the fact that the hydrographer does not always take the same route to arrive at the stream gage. The routing depends on the combination of stream gages and perhaps even other activities that the hydrographer must attend on a particular trip. Thus the cost of travel that could be allocated to a gage visit is not always the same.

Because of these complexities, the technique used in this study takes as the network manager's decision variables, the number of times that a particular route of travel is used during the year. A route is defined as a set of one or more gages and the least cost travel that takes the hydrographer from his home base to each of the gages and back to base. A route, therefore, will have associated with it an expected cost for travel and an expected cost of servicing each stream gage visited along the way. In a network of more than a few stream gages there are a great many feasible routes when one considers all possible combinations of gages. However, many of these by the nature of their locations and the connecting road system will be impractical. Therefore, the first step in the analysis is to choose a set of practical routes that might stand a chance of being used in the final solution and to evaluate the unit costs associated with each route. This practical set will contain routes to gages that are in close proximity or that lie along a not too devious route; it will also contain, as individual routes, the path to and from each stream gage with that gage as the lone stop so that the individual needs of a streamgage can be considered in the absence of stops at other gages.

The second step in the procedure is to determine any special requirements for visits to each of the gages. For example, the recording equipment at a gage may not have the capacity to operate longer then 65 days; such a gage would then have to be visited at least six times per year. Another type of constraint can result from auxiliary uses of the streamflow data; water quality samples, for example, might be required monthly entailing 12 visits to the station each year.

The third step is to define the uncertainty in annual mean discharge as a function of the number of visits for each of the stream gages. This step is discussed in detail in a subsequent section of this report.

The final step is to use all of the above to determine the number of times, N_i, that each of the NR routes is used during a year such that (1) the budget for the network is not exceeded, (2) the minimum number of visits to each station is made, and (3) the total uncertainty in the network is minimum. Figure 2 presents this step in the form of a mathematical program.

In its simplest form the function relating uncertainty at a stream gage to the number of visits to the gage is

$$\phi_j(M_j) = \frac{\sigma_j^2}{M_j} \tag{1}$$

where σ_j^2 is the variance of an independent series of streamflows that comprise the annual streamflow record and M_j is the number of visits. In the real world streamflows are not independent in time; the streamflow at one instant gives a good indication of what streamflow will be one minute later and maybe even a day or a week hence. Accounting for the temporal dependence of streamflow results in a form of $\phi_j(M_j)$ that is more complex than equation 1, but uncertainty is still inversely related to the number of visits to the station. This inverse relation, which is nonlinear, precludes the use of classical operations research techniques such as integer programming (Wagner, 1969) to solve for the best set of decisions, $\underline{N}^*$, on how often to use each of the stream gaging routes. Therefore, a direct-search technique was used to specify the values of $\underline{N}^*$ that met all of the criteria described in figure 2. Figure 3 presents a tabular layout of the problem. Each of the

$$\underset{\underline{N}}{\text{Minimize}}\ V = \sum_{j=1}^{MG} \phi_j\ (M_j)$$

$V \equiv$ total uncertainty in the network

$\underline{N} \equiv$ vector of annual number times each route was used

$MG \equiv$ number of gages in the network

$M_j \equiv$ annual number of visits to station j

$\phi_j \equiv$ function relating number of visits to uncertainty at station j

Such that

Budget $\geq T_c \equiv$ total cost of operating the network

$$T_c = F_c + \sum_{j=1}^{MG} \alpha_j M_j + \sum_{i=1}^{NR} \beta_i N_i$$

$F_c \equiv$ fixed cost

$\alpha_j \equiv$ unit cost of visit to station j

$NR \equiv$ number of practical routes chosen

$\beta_i \equiv$ travel cost for route i

$N_i \equiv$ annual number times route i is used (an element of $\underline{N}$)

and such that

$$M_j \geq \lambda_j$$

$\lambda_j \equiv$ minimum number of annual visits to station j

Figure 2.--Mathematical programming form of the optimization of the routing of hydrographers.

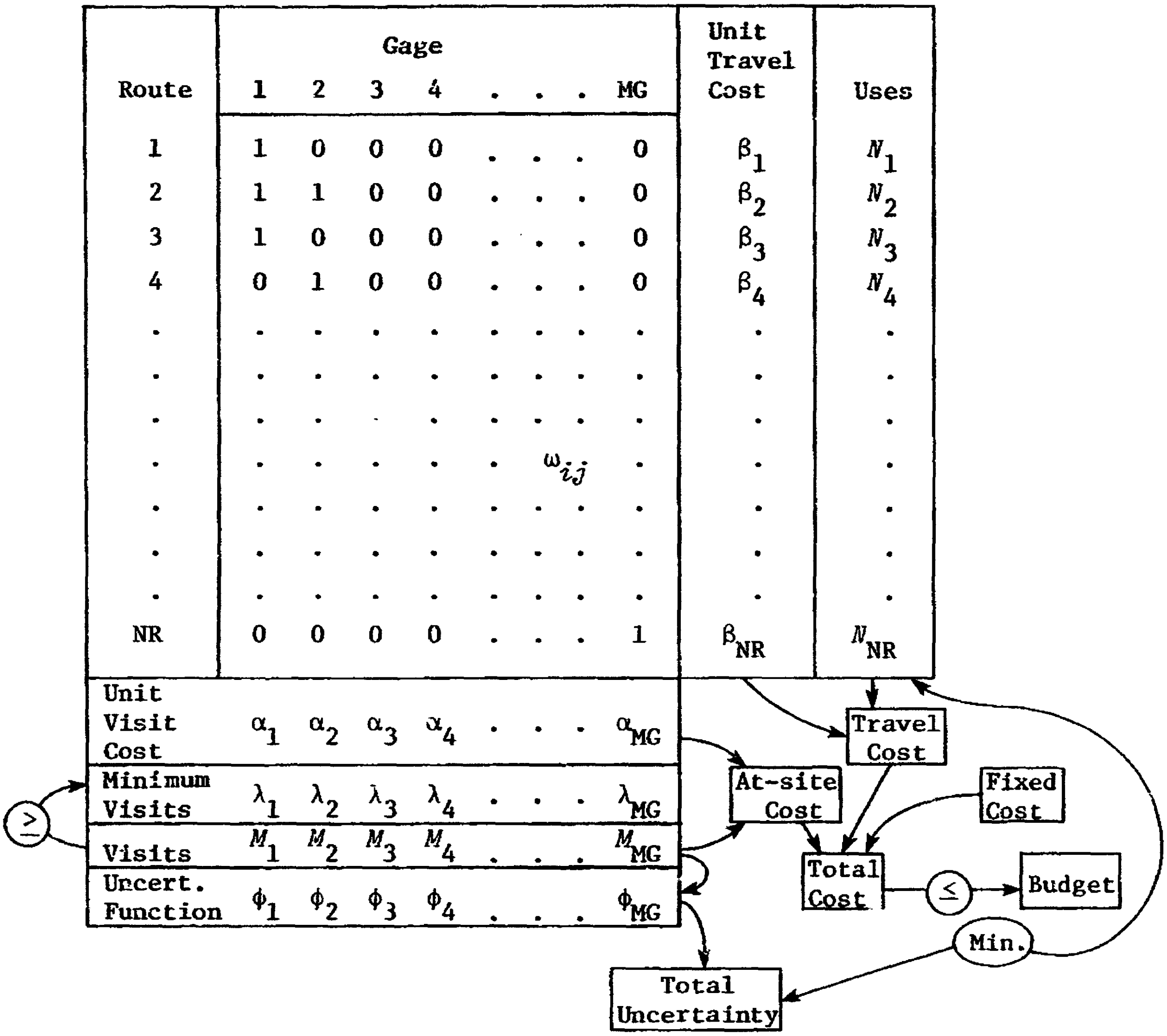


Figure 3.--Tabular form of the optimization of the routing of hydrographers.

NR routes is represented by a row of the table and each of the MG stations is represented by a column. The zero-one matrix, $\underline{\omega_{ij}}$, defines the routes in terms of the stations that comprise it. A value of one in row i and column j designates that station j will be visited on route i; a value of zero indicates that it will not. The unit travel costs, $\underline{\beta_i}$, are the per-trip costs of the hydrographer's travel time and any related per diem and the operation, maintenance, and rental costs of vehicles. The sum of the products of β_i and N_i for all i is the total travel cost associated with the set of decisions, $\underline{N}$.

The unit-visit cost, $\underline{\alpha_j}$, is comprised of the average service and maintenance costs incurred on a visit to the station plus the average cost of making a discharge measurement. The set of constraints of minimum visits is denoted by the row, $\underline{\lambda_j}$. The row, $\underline{M_j}$, specifies the number of visits made to each station. M_j is the sum of the products of ω_{ij} and N_i for all i and must equal or exceed λ_j for all j, if $\underline{N}$ is a feasible decision.

The total cost expended at the stations is equal to the sum of the products of α_j and M_j for all j. The cost of record computation, documentation, and publication is assumed to be influenced negligibly by the number of visits to the station and is included along with overhead in the fixed cost of operating the network. The total cost of the network equals the sum of the travel costs, the at-site costs, and the fixed cost and must be less than or equal to the available budget.

The total uncertainty or variance of estimates of annual discharges at the MG stations is determined by summing each of the uncertainty functions, $\underline{\phi_j}$, evaluated at the value of M_j from the row above it.

As stated above, classical techniques for optimization are not adequate for the problem at hand. Therefore, to solve the problem an iterative approach was devised that arrives at an efficient set of decisions, $\underline{N}$, if not at the true optimum. The approach begins with an initial feasible set for $\underline{N}$, which may be supplied externally by the analyst or is developed by the approach itself. If internal specification of the initial set of $\underline{N}$ is desired, the requirements for minimum visits to each of the stations are satisfied in a least cost manner. Any money remaining after these constraints are satisfied is allocated to trips on routes that reduce the total uncertainty in an economically efficient manner. Because these two steps guarantee only a feasible set of decisions, the initial set of decisions is manipulated iteratively, one value of N_i at a time, until no further reduction in uncertainty can be obtained without violating one of the constraints. The locally optimum set of values for $\underline{N}$ obtained in this manner specify an efficient strategy for operating the stream gaging network, which may even be the truly optimum strategy. The true optimum cannot be guaranteed without testing all undominated, feasible strategies, which is a mounumental computational chore for the number of stream gaging stations operated in the Lower Colorado River Basin.

Accuracy of annual mean discharge

In spite of the massive amounts of streamflow data that are collected throughout the world, relatively little has been done to evaluate the accuracy of the resulting numbers. A split-sample approach that is applicable at a gaging station, where sufficient historical discharge measurements exist, was developed by Burkham and Dawdy (1968). In this technique the existing discharge measurements are randomly divided into an analysis group and a control group. Initially the analysis group is small relative to the size of the control group. The analysis group is used by an experienced hydrographer to compute a discharge record whose accuracy can be evaluated by comparison with the measurements in the control group. The process is repeated several times; each time some of the discharge measurements are shifted from the control group to the analysis group. As the amount of data in the analysis group increases, the accuracy of the record tends to increase, that is, the differences between the control discharges and the concurrent computed discharges are less on the average. Such a relation can be expressed graphically as in figure 4, which shows the results of a Burkham-and-Dawdy analysis of the discharge rate at the Colorado River below Davis Dam (station 09423000) for the water years 1956 through 1958. This period of record was chosen because of its high frequency of discharge measurement. The points in figure 4 specify the repetitions of the Burkham-and-Dawdy process with differing amounts of data in the analysis group. It should be noted that the analysis does not seem to point toward an error of zero with continuous measurements as statistical sampling theory dictates. Some possible reasons for this will be discussed later.

Subsequent studies of the accuracy of discharge records are reported by Herschy (1978). However, these later procedures are of little value to the current problem because they ignore the effects of the temporal correlation of errors in the discharge record. Because the correlation in time of discharge errors of estimate is positive, the procedures espoused by Herschy (1978) probably tend to underestimate the error in daily, monthly or annual discharge.

A technique that has been shown to have promise in the determination of the accuracy of water-quality records is Kalman filtering (Moore, 1978).

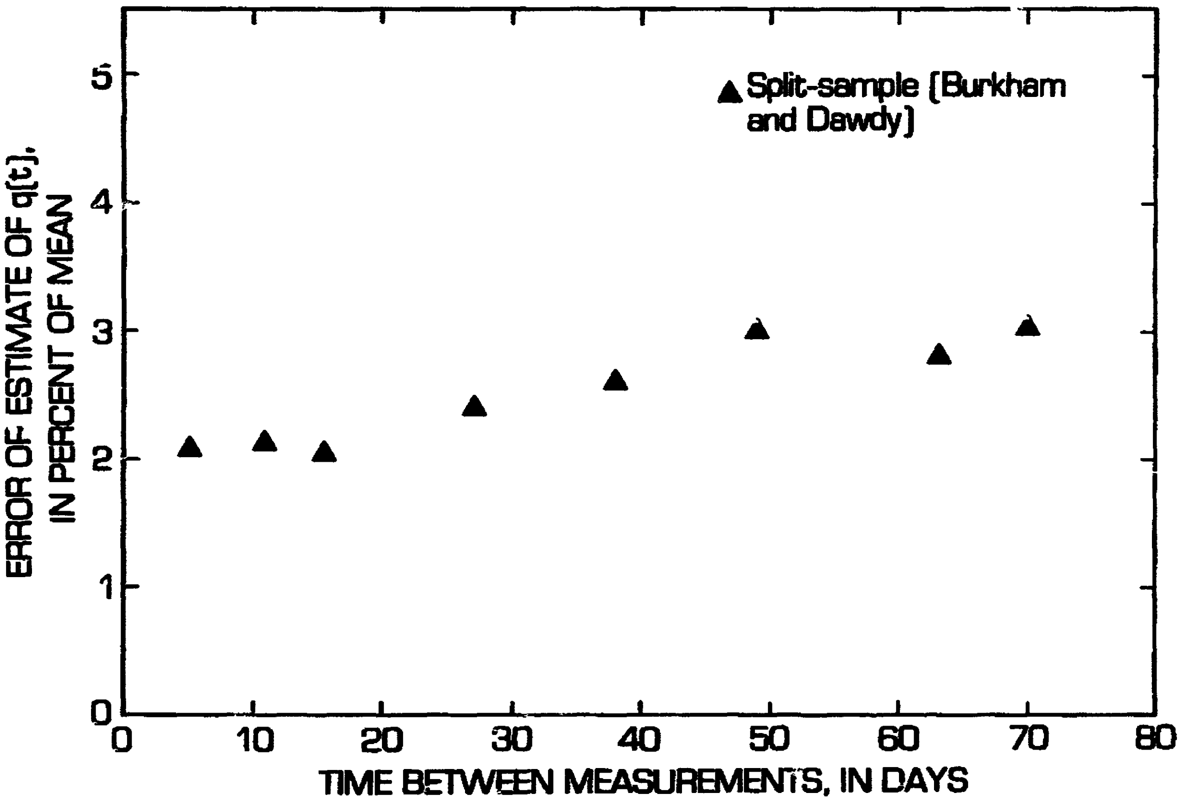


Fig. 4.--Burkham-and-Dawdy accuracy analysis of streamflow data for the Colorado River below Davis Dam.

The Kalman filter provides a framework for optimally estimating values of a random process when only imprecise measurements of the process are available and also yields a measure of the accuracy of these estimates. Temporal correlations are permissible in Kalman filtering.

At most of the stream gages in the Lower Colorado River Basin, there are not enough discharge measurements to perform a series of split-sample analyses to define the error of estimation of discharge as a function of the frequency of discharge measurement at the station. For that reason the approach developed for this study is based on Kalman-filter theory (Gelb, 1974). Because all of the insight of the hydrographer cannot be built into the filter model, certain simplifications were required. These simplifications are enumerated below, and their effects on the estimation of the accuracy of discharge computations are demonstrated by comparison with the split-sample results for the station at the Colorado River below Davis Dam.

In the Kalman-filter analogy of discharge computation, let $q_T(t)$ be the true instantaneous discharge at time t and $q_M(t)$ be a measurement of $q_T(t)$. In actuality the measurement, $q_M(t)$, requires a finite amount of time to accomplish. However, in standard stream gaging procedure, streamflow measurements are made at times when $q_T(t)$ is as constant as possible during the measurement interval. Furthermore, the measurement interval is very brief relative to a year, which is the interval of interest here. Therefore, $q_M(t)$ will be referred to as an instantaneous measurement with little loss of veracity. In addition to the temporal disparity, discharge measurements are subject to several other sources of error (Carter and Anderson, 1963). The total error, $v(t)$, in a measurement is equal to $q_M(t) - q_T(t)$.

Instantaneous discharge can usually be related to correlative data, such as the water-surface elevation, water-surface slope, or head and gate openings in the case of control structures. The relation between instantaneous discharge and the correlative data, known as a discharge rating, is not necessarily constant throughout the life of a stream gaging station. In standard stream gaging procedures, temporary shifts in the discharge rating are handled by adjusting the correlative data, while more persistent shifts are treated by redefinition of the discharge rating. In the long run, shifts of either a temporary or relatively persistent nature may fluctuate about a more or less steady-state discharge relation or the discharge relation may have a definite non-stationarity with respect to time. The latter case is frequently exhibited in alluvial reaches of a stream downstream from a recently constructed reservoir. Impoundment of the sediments carried by the natural stream causes degradation of the stream's channel downstream from the reservoir and a concurrent shift in the relation of discharge to water-surface elevation as the channel erodes. In such a case the most stable discharge relation may be a function of both the correlative data and time.

Even though the correlative data and time can be measured much more precisely than can instantaneous discharge itself, discharge relations are not exact. Let the difference between $q_T(t)$ and rated discharge, $q_R(t)$, based on the correlative data and possibly on time, be denoted $x_2(t)$. Thus measured discharge can be expressed

$$q_M(t) = q_T(t) + v(t) = q_R(t) - x_2(t) + v(t) \tag{2}$$

A typical form of discharge rating is

$$q_R(t) = c_0 + c_1 (h(t) + c_2)^{c_3} \tag{3}$$

where $h(t)$ is the water-surface elevation, also known as stage or gage-height, and c_0, c_1, c_2, and c_3 are coefficients that may or may not be functions of time.

If it is assumed that the correlative data are exact and continuous in time and that sufficient discharge measurements have been made such that the effects of $x_2(t)$ and $v(t)$ are negligible, an exact and continuous trace of $q_R(t)$ can be developed for any period of interest. The unobservable random variable, $x_2(t)$, which is the difference between the trace of $q_R(t)$ and the unobservable trace of $q_T(t)$, is used herein as one of the primary state variables in the Kalman-filter technique; $x_2(t)$ replaces $q_T(t)$ in the formulation because it more nearly satisfies the filter assumptions.

In actuality, the correlative data are neither exact nor continuous. Records are lost when recorders malfunction. Several procedures are available to reconstruct lost record; however, none of these fully replaces the information that was contained in the lost record. For this reason the assumption of exact and continuous correlative data will cause an underestimation of the actual uncertainty in an estimate of annual mean discharge. Because more record will tend to be lost with infrequent visits, uncertainty in the real world will be somewhat more sensitive to visit frequency than the exact-and-continuous assumption will indicate. In this study, the lost-record considerations are considered to be secondary and negligible to the other effects of record computation.

The true total discharge during one year is defined by

$$Q_T = \int_0^1 q_T(t)\, dt \tag{4}$$

The value of Q_T can be approximated from the trace of $q_R(t)$ by

$$Q_T \simeq Q_R = \int_0^1 q_R(t)dt \tag{5}$$

However, because $q_T(t)$ is not perfectly related to $q_R(t)$, Q_R may not be the best estimate of Q_T. Measurements made before, during, and after the year may add additional information by means of the temporary rating shifts mentioned above. By accounting for the temporary shifts, another trace $q_C(t)$, can be developed, and Q_T can be estimated by a discretization of the equation

$$Q_C = \int_0^1 q_C(t)dt \tag{6}$$

The use of Q_C to estimate Q_T is current practice and, the difference between Q_C and Q_T is the error of estimate of annual discharge. This error is denoted γ, and minimization of its expected root-mean square is one of the objectives of stream gaging efforts at many stations.

Although it is not currently used as standard practice in the U.S. Geological Survey, that part of filter theory known as Optimal Estimation, as described by Gelb (1974), could be used to construct the computed trace, $q_C(t)$ and thus the estimate, Q_C, of annual discharge.

The three sources of discharge information, stage, time, and discharge measurements, could be used in a state-space framework (Gelb, 1974) to evaluate the accuracy of Q_C as an estimator of Q_T. However, the nonlinearity of the general formulation of the discharge rating, illustrated in equation 3, would entail unnecessary difficulties. If a "reasonable" number of discharge measurements is available to develop the discharge rating, the information contained in the correlative data can be removed by computing Q_R for the annual time period and subtracting it from both Q_C and Q_T in the definition of the error of annual discharge:

$$\gamma = (Q_C - Q_R) - (Q_T - Q_R) \tag{7}$$

For notational convenience, the following identities will be used for the remainder of this paper:

$$x_1(t) = \int_0^t (q_T(t) - q_R(t))dt \tag{8}$$

and

$$\hat{x}_1(t) = \int_0^t (q_C(t) - q_R(t))dt \tag{9}$$

The error in annual discharge can be redefined

$$\gamma = \hat{x}_1(1) - x_1(1) \tag{10}$$

which is assumed to be a function of the frequency of discharge measurement.

The variable $x_1(1)$ cannot be observed; thus equation 10 cannot be directly evaluated. However the techniques of Kalman filtering (Gelb, 1974) yield an estimate of the variance of γ. If $\hat{x}_1(1)$ meets the assumption of being an unbiased estimator of $x_1(1)$, minimization of the variance of γ is equivalent to the stated objective of minimizing its root mean square.

The general form of the state equation for a system that can be described by a linear differential equation with a random forcing function is

$$\dot{\underline{x}}(t) = F\,\underline{x}(t) + G\,\underline{w}(t) \tag{11}$$

where $\dot{\underline{x}}(t)$ is a vector of length n of the first derivatives with respect to time of the state variables, F and G are matrices of coefficients, $\underline{x}(t)$ is a vector of the state variables, and $\underline{w}(t)$ is a vector of the random noise or forcing variables. The governing differential equation for the discharge-computation problem is assumed to be of the form

$$\frac{d^2x_1(t)}{dt^2} + \frac{a_1dx_1(t)}{dt} + a_0x_1(t) = w(t) \tag{12}$$

which can be translated into the state equation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \tag{13}$$

By comparing equations 11 and 13, it can be seen that n equals 2,

$$F = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and a single, random-forcing function, $w(t)$, drives the system. It should also be noted that $x_2(t)$ equals $q_T(t)$ minus $q_R(t)$.

Identification of the structure of $w(t)$ must be accomplished through analysis of the measurements $q_M(t)$, because $w(t)$ itself cannot be measured directly. For example, if measurements indicate that $x_2(t)$ is a first-order Markovian process, a state equation for $x_2(t)$ is

$$\dot{x}_2(t) = -\beta x_2(t) + w(t) \tag{14}$$

where β is the inverse of the correlation time constant of the Markovian process. By substituting equation 14 into equation 13, the state equation for a Markovian $x_2(t)$ can be written

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\beta \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \tag{15}$$

An error-free discharge measurement at time t would yield a single value of $x_2(t)$ for any discharge rating. Discharge measurements are not error-free, but Kalman filters deal with measurement errors by means of the general measurement equation

$$\underline{z}(t) = H\,\underline{x}(t) + \underline{v}(t) \tag{16}$$

where $\underline{z}(t)$ is the vector of measurements at time t, H is a matrix of coefficients and $\underline{v}(t)$ is a vector of measurement errors. In the discharge-computation problem, measurements are made only at specific times (not continuously), and the measurements result only in estmates of $x_2(t)$ (not of $x_1(t)$). Thus the specific measurement equation is

$$z(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + v(t) \tag{17}$$

where $z(t)$ is the measured discharge, $q_M(t)$, minus the rated discharge, $q_R(t)$, at time t. Equation 17 is applicable only at the specific times of the discharge measurements. Between measurements $v(t)$ can be assumed to have infinite variance, which is the equivalent of no new information being collected.

The proper use of Kalman-filtering techniques requires both $w(t)$ and $v(t)$ be independent, Gaussian random variables. In the case of measurement error, $v(t)$, this requirement probably is not violated grossly. On the other hand, $w(t)$ can be influenced by the choice of the parameters, a_0 and a_1, that are used to describe the time series model of $x_2(t)$. These two parameters must be evaluated on their own merits at each site at which discharge accuracy is to be modeled by a Kalman filter.

As stated earlier, the primary interest of this study is the definition of the variance of the error of estimate of $x_1(1)$ as a function of the frequency of discharge measurement. On the surface this would seem to entail a classical implementation of optimal fixed-point smoothing as described by Gelb (1974, p. 157). However, to perform optimal smoothing, the system must be fully observable. The discharge-computation problem does not meet this criterion because $x_1(t)$ is never measured directly. Its error can be estimated only from that of $x_2(t)$.

The general equation for error propagation of a continuous system between measurements is

$$\dot{P}(t) = FP(t) + P(t)F^T + GQG^T \tag{18}$$

where $P(t)$ is an $n \times n$ matrix of error covariances, and Q is a vector of spectral densities of the random forcing functions, $\underline{w}(t)$. The discharge-computation problem has a single forcing function; therefore Q is a scalar, denoted q. The elements of the matrix, $P(t)$, are as follows: $p_{11}(t)$ is the error variance of $x_1(t)$, $p_{22}(t)$ is the error variance of $x_2(t)$, and $p_{12}(t) = p_{21}(t)$ is the covariance between errors in $x_1(t)$ and $x_2(t)$.

As discussed earlier, discharge measurements are considered to be instantaneous and are obtained at discrete times only. Therefore, the amount of information about $x_1(t)$ and $x_2(t)$ changes abruptly upon completion of a discharge measurement. This abrupt change is measured by a change in the covariance matrix, $P(t)$. Denote the value of $P(t)$ just prior to a measurement at time t by $P(t^-)$ and the value of $P(t)$ just after the measurement as $P(t^+)$. Gelb (1974, p. 109-110) expresses $P(t^+)$ as

$$P(t^+) = [I - K(t)H] \quad P(t^-) \tag{19}$$

where I is an identity matrix and $K(t)$ is known as the Kalman gain matrix and is defined

$$K(t) = P(t^-)H^T[HP(t^-)H^T + R]^{-1} \tag{20}$$

where R is the covariance matrix of measurement errors, the superscript T indicates a matrix transpose, and the superscript -1 indicates a matrix inversion. In the discharge computation problem, the measurements pertain only to $x_2(t)$, not $x_1(t)$, and R is thus a scalar, denoted r, that specifies the variance of the measurement error. The combination of equations 15, 19, and 20 results in

$$P(t^+) = \begin{bmatrix} p_{11}(t^-) - \dfrac{p_{12}^{\,2}(t^-)}{p_{22}(t^-) + r} & p_{12}(t^-) - \dfrac{p_{12}(t^-)p_{22}(t^-)}{p_{22}(t^-) + r} \\ \\ p_{12}(t^-) - \dfrac{p_{12}(t^-)p_{22}(t^-)}{p_{22}(t^-) + r} & p_{22}(t^-) - \dfrac{p_{22}^{\,2}(t^-)}{p_{22}(t^-) + r} \end{bmatrix} \quad (21)$$

An example of the time trace of $p_{11}(t)$ and $p_{22}(t)$ for the case of six equally spaced discharge measurements during a water year is shown in figure 5. The ith measurement during the water year is made at time ψ_i. The variance, $p_{22}(t)$, of the error of estimate of discharge rate can be seen to rise to a peak just before a discharge measurement is made at which time uncertainty is a maximum. Immediately after completion of a discharge measurement, uncertainty is a minimum but begins its increase that ends only when the next measurement is made. If measurements are equally spaced in time, $p_{22}(t)$ is a periodic function.

The variance of total discharge since the beginning of the water year, $p_{11}(t)$, can be seen to increase from a value of zero at the beginning of the year to a maximum at the end of the year.

Figure 5 and equations 18 and 21 pertain to the real-time computation of discharge; that is, the computation of discharge at time t is performed with only discharge measurements and correlative data up to time t. In most cases real-time discharge esimates are not the only requirement at a streamflow station; data from before, during, and after the water year of interest can be used to obtain better estimates than those that may be computed in real time. The process of including all pertinent information in the estimation procedure is known as smoothing in filter-theory parlance. Smoothing is the equivalent of optimally combining two estimates of the unknown states at time t. One estimate is the real-time or forward-filter described above,

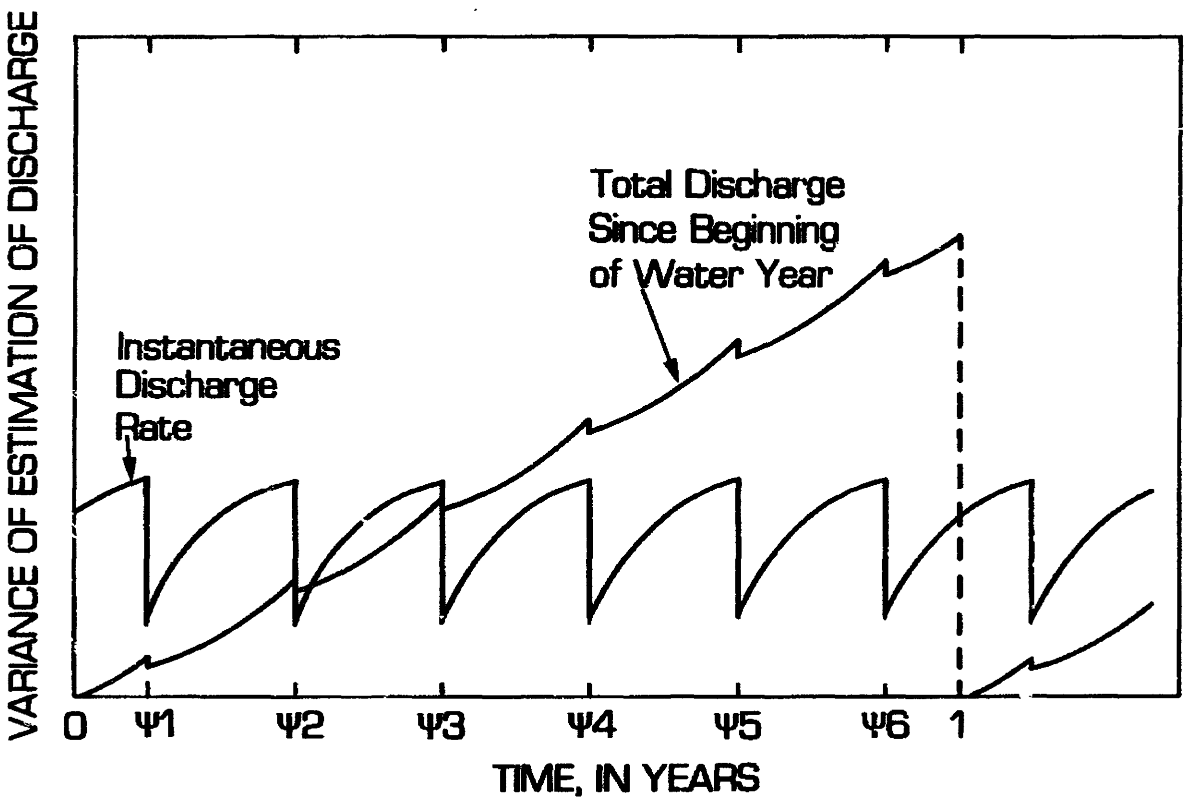


Fig. 5.--Theoretical error variances of real-time estimates of discharge.

the other is the backward-filter estimate that is derived by reversing the filter and using the data collected after time t to project backward to estimate at time t. For a system to be smoothable, it must also be fully observable; a condition that was shown not to be met for the discharge-computation problem. However, if $x_1(t)$ is ignored temporarily, the remainder of the system $x_2(t)$ meets the observability and smoothability requirements.

For a reversible process the equations of the forward and backward filters are identical; only the direction of time is reversed. Under such circumstances the variances of the forward and backward estimates are symmetrical between equally spaced measurements. Figure 6 illustrates the symmetrical nature of the variance of the errors of estimates of $x_2(t)$.

According to Gelb (1974, p. 156-157) the errors of the forward and backward estimates at any time t are uncorrelated; that is, the covariance of the errors is zero. However, it can be shown that the covariance between the estimation errors is non-zero for a first-order Markovian process such as that used to model $x_2(t)$. A paper giving the deviation of this covariance is in the review process now. For uniformly spaced measurements, the covariance is

$$\pi_t = (1 + \theta^2 e^{-2\beta\lambda} - \theta(e^{-2\beta\tau} + e^{-2\beta(\lambda-\tau)}))q/2\beta \qquad (22)$$

where τ is $t - \psi_i$, ψ_i is the time of the last measurement before t, and

$$\theta = \left(\frac{p_\lambda^-}{p_\lambda^- + r}\right) \left[1 - \frac{re^{-2\beta_2}}{p_\lambda^- + r}\right]^{-1} \qquad (23)$$

where p_λ^- is the maximum of $p_{22}(t)$, which occurs just before a discharge measurement. Optimally combining the forward and backward estimates of

$x_2(t)$ results in a new estimate, $x_2^*(t)$, that has a variance of error of estimation,

$$p_{22}^*(t) = \alpha_t^2\, p_{22}(t) + (1 - \alpha_t)^2 p_{22}^b(t) + 2\alpha_t(1 - \alpha_t)\pi_t \qquad (24)$$

where $p_{22}^b(t)$ is the variance of the error of the backward estimate and α_t is the weight given to the forward estimate,

$$\alpha_t = \frac{p_{22}^b(t) - \pi_t}{p_{22}(t) + p_{22}^b(t) - 2\pi_t} \qquad (25).$$

The weight given to the backward estimate is $1-\alpha_t$. Figure 6 shows that the variance of the error of the optimum estimate is less than that of either the forward or backward estimates taken alone.

Use of the Kalman filter to adjust discharges directly, as described herein, is a considerable deviation from the standard computational procedure, which adjusts the correlative data. For a station where discharge is computed from a stage-discharge relation, the record of stage is adjusted on the basis of apparent shifts in the relation at the time of discharge measurement. Because of the form of the stage-discharge relation, the standard procedure would result in larger magnitudes for $\hat{x}_2$ at high stages and smaller magnitudes at low stages. The standard procedure is analagous to the Kalman filter technique with a process noise, q, that is a function of the stage. Equation 24 can be used with the average value of a variable q to approximate the average variance of the estimate of x_2. The effects of these assumptions are examined in the following section.

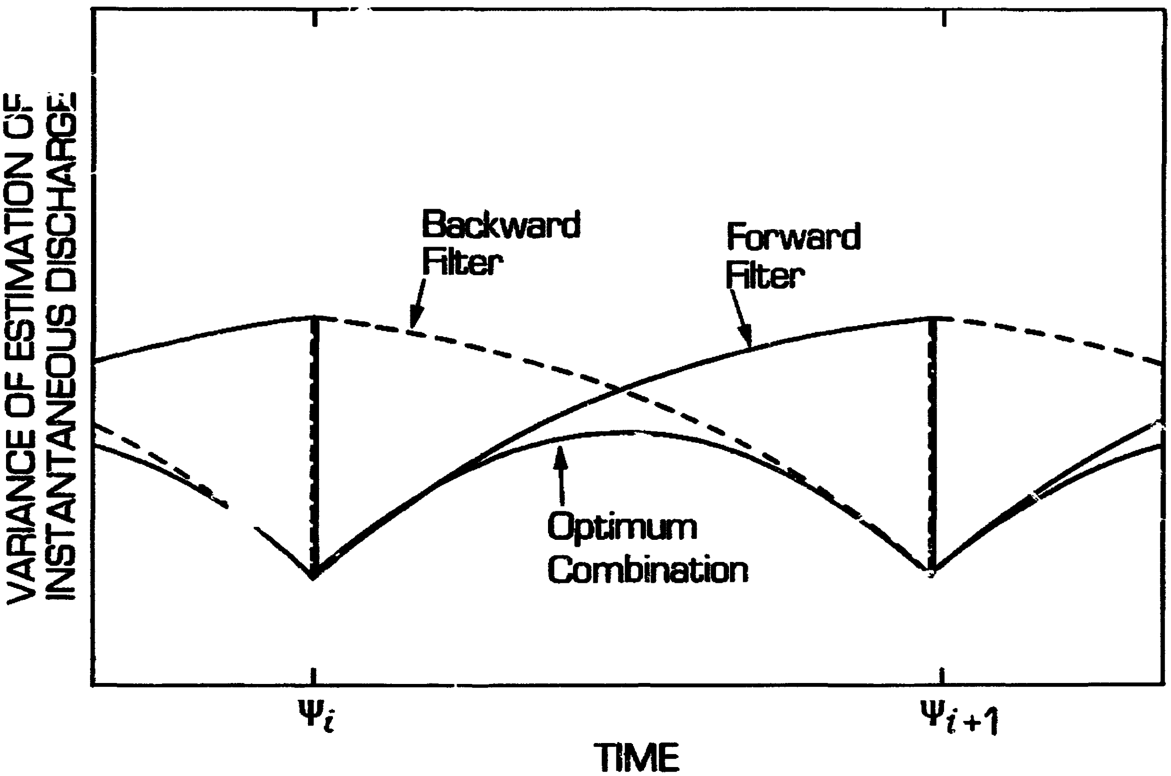


Fig. 6.--Theoretical error variances of smoothed estimates of discharge rate.

Both forward and backward estimates are also available for $x_1(1)$. The forward filter contains discharge-measurement information up to the end of the year; the backward filter contains discharge-measurement information after the end of the year. Both estimates contain information about the statistical nature of the discharge process. Because of the lack of statistical independence in the process, the discharge measurements made during the year of interest and those afterward contain redundant information. Thus, for best results, the two estimates should not be combined as if they were independent. Accounting for the redundancy will yield a better estimate; ignoring the redundancy will result in an overly optimistic estimate of the accuracy of the estimate.

In the case of $x_1(1)$, the information contained in the forward-filter estimate can be expressed in terms of the Fisher (1960) information content

$$I_f = 1/p_{11}(1). \tag{26}$$

The statistical-process information, denoted I_p, that is contained in $\hat{x}_1(1)$ is defined as the reciprocal of $p_{11}(1)$ for the condition where no discharge measurements are available during the year of interest. The discharge measurements prior to the end of the year contain information

$$I_m^f = I_f - I_p \ . \tag{27}$$

Similarly, the backward filter estimate contains discharge-measurement information

$$I_m^b = I_b - I_p \tag{28}$$

where I_b is the reciprocal of $p_{11}^b(1)$. As stated above, there is redundancy, denoted I_r, between I_m^f and I_m^b. Thus, the total information contained in the two estimates is

$$I_t = I_p + I_m^f + I_m^b - I_r \tag{29}$$

and the variance of the optimal estimate of $x_1(1)$ is

$$p_{11}^*(1) = 1/I_t \ . \tag{30}$$

Ideally I_r could be determined in a manner similar to the development of equation 22 for π_t. However, because measurements of $x_1(1)$ are not available and a mathematical derivation based on the nature of $x_2(t)$ is intractable, I_r is assumed to be negligible and the variance of the estimate of annual discharge is estimated by

$$\hat{p}_{11}(1) = (I_p + I_m^f + I_m^b)^{-1}. \tag{31}$$

The amount of redundancy between I_m^f and I_m^b is limited by the minimum of I_m^f and I_m^b. Heuristically, it can be argued that there is more information about annual discharge in a particular year contained in the measurements made during that year than there is in the measurements made subsequent to the end of the year. Therefore, I_r has an upper limit of I_m^b. Figure 7 shows that the redundancy has minimal effect on the estimation of $x_1(1)$ at the Colorado River below Davis Dam. In fact, the backward filter adds little at all in terms of total information.

Equation 31 is the key to obtaining the desired accuracy of annual discharge estimates as functions of the frequency of discharge measurement. The steps to develop the function for a particular stream gage begin with calibration and verification of the Kalman filter model of $x_2(t)$ as described in the next section. Next, a set of frequencies of measurement spanning the possible range of interest is chosen and equation 31 is evaluated for each frequency. This set of paired values of frequency of measurement and variance of estimation of annual mean discharge can be

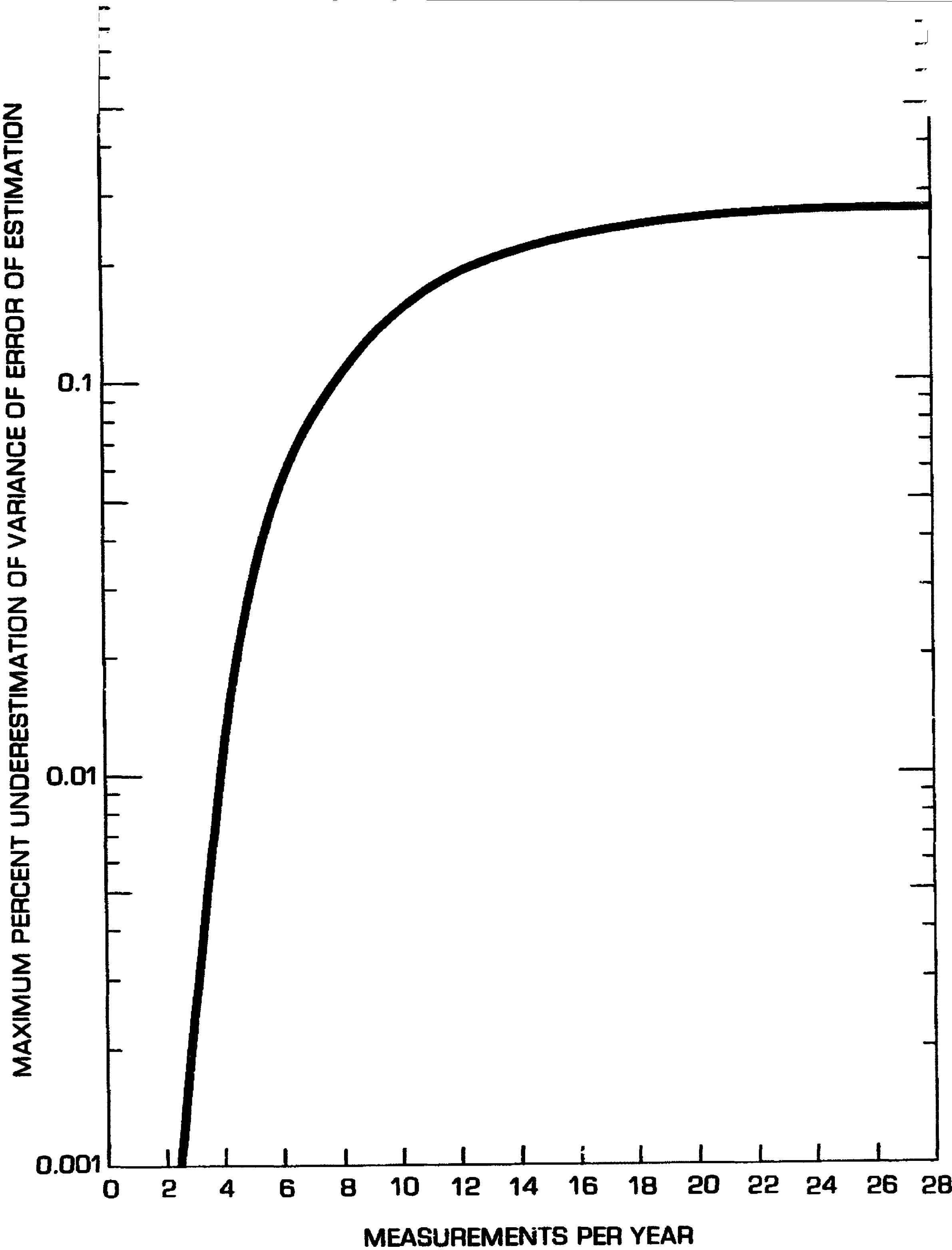


Fig. 7.--Limit of bias introduced by approximating the error variance of estimates of annual mean discharge.

used either in a direct look-up routine or as an empirically fitted function to represent the uncertainty relations, ϕ_j, described in figure 3.

Calibration and verification of the Kalman-filter model

Use of the Kalman-filter analogy of discharge computation, as described above, entails the determination of three parameters at each stream gaging station: (1) r, the variance of the discharge-measurement errors, (2) β, the reciprocal of the correlation time of Markovian structure of the difference between computed and actual instantaneous discharge rate, and (3) q, the spectral density of the white noise that drives the Markovian process. In addition to calibrating the model by finding the best set of values for the three parameters, the assumptions of the model also should be verified. Both the calibration and verification of the model are performed using available discharge measurements and the concurrent correlative data. Ideally, there would be sufficient measurements for a split-sample approach in which the calibration is performed with part of the measurements and the verification is done on the remaining ones. However, for most stations in the Lower Colorado River network the frequency of discharge measurement is too low for split-sampling. In light of this data constraint the approaches described below were used in this study.

The variance of discharge-measurement errors.

Three primary factors were found by Carter and Anderson (1963) to control the scale of the errors in a current-meter measurement of discharge. These are: (1) the number of verticals in the stream cross-section at which velocity and depth are measured, (2) the number and location of the points in each vertical at which velocity is measured, and (3) the duration of each velocity measurement. They found that the standard deviation of the measurement error expressed as a percentage of the true discharge could be estimated as a function of these three factors. Standard practice within the U.S. Geological Survey (Buchanan and Somers, 1969) results in a measurement error of about 2 percent according to the relations of Carter and Anderson (1963). Because the standard deviation of measurement error in percent is relatively constant, the measurement-error variance, r, in discharge units is variable; increasing with increased true discharge. The aim of this study is to estimate the accuracy of future stream gaging strategies and neither the future stage nor the discharge can be prespecified. Therefore, the average estimate of variance of past discharge measurements is used for the value of r. This average is obtained by

$$r = \frac{1}{m} \sum_{k=1}^{m} (0.02 q_M(\psi_k))^2 \tag{32}$$

where ψ_k is the time of the kth discharge measurement and m is the number of measurements available. The effect of the use of equation 32 cannot be studied in isolation; it interacts with the use of an average value for the variable q. Both q and r increase with increasing discharge; therefore, there is positive correlation between these parameters.

Parameters of the Markovian process.

The Kalman filter used in this study assumes that the difference, $x_2(t)$, between the true and the rated instantaneous discharges is a continuous first-order Markovian process that has an underlying Gaussian (Normal) probability distribution with a zero mean value and a variance equal to $q/2\beta$. The zero mean is obtained by developing an unbiased long-term discharge rating on the basis of all available discharge measurements and the associated correlative data.

The gage on the Colorado River below Davis Dam is illustrative of this step. Because of upstream dam construction, the discharge ratings for this site developed by the U.S. Geological Survey in the 1950's showed a steadily degrading channel. In order to obtain a single discharge rating at this station for the 1956 through 1958 water years, the coefficient, c_2, of equation 3 is modeled as an exponential decay with time. The resulting rating curve is

$$q_R(t) = -1.7 + 174(h(t) - 7.21e^{-0.0611t})^{1.98} \tag{33}$$

where $q_R(t)$ is in cubic feet per second, $h(t)$ is in feet above gage datum, and t is in years since February 27, 1956, when the current gage location was established. Figure 8 shows a time series plot of the differences between measured and rated discharges for the 352 measurements made during the period of interest. These residuals, $z(t)$, which are contaminated with measurement errors, contain all of the available information about the structure of $x_2(t)$; that is,

$$z(t) = x_2(t) + v(t) \tag{34}$$

The average residual is -0.04 cubic feet per second; which is not quite equal to the assumed value of zero, but is small enough to be ignored with no ill effect. The variance, σ_z^2, of the residuals, corrected for the lost degrees of freedom of the fitting procedure, is 267,422 feet^6 per second^2. Measurement errors are assumed to be statistically independent from the concurrent values of $x_2(t)$. Therefore,

$$\sigma_z^2 = \frac{q}{2\beta} + r \tag{35}$$

where $q/2\beta$ is the variance of $x_2(t)$ over time. The variance of discharge-measurement errors for this data set was determined to be 103,591 feet^6 per second^2. Therefore, an initial estimate of the variance of $x_2(t)$ can be

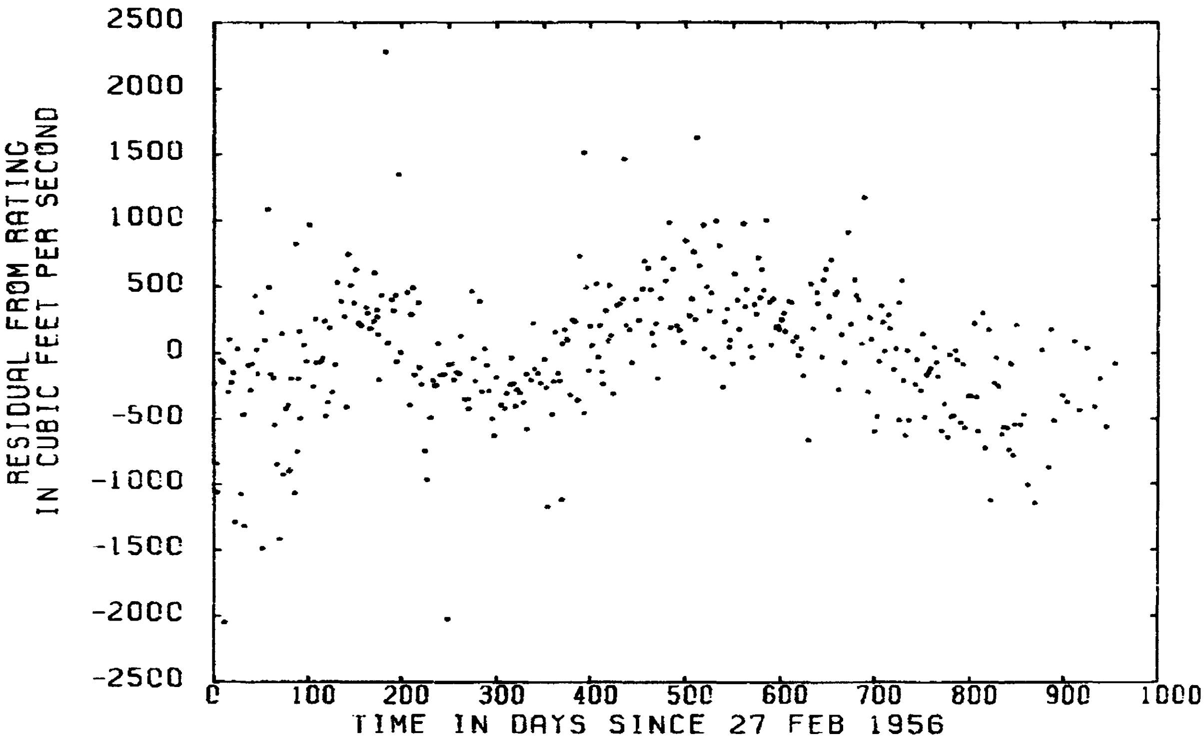


Fig. 8.--Residuals of discharge-rating analysis for the Colorado River below Davis Dam.

obtained by manipulating equation 35,

$$\frac{q}{2\beta} = \sigma_z^2 - r \tag{36}$$

or 163,831 feet^6 per second^2.

Utilization of the Kalman filter requires that the magnitudes of q and β be defined individually. This may be done by estimating the autocovariance function

$$\Gamma(\Delta) = (q/2\beta)e^{-\beta\Delta} \tag{37}$$

where $\Gamma(\Delta$, is the expected value of the product of $x_2(t)$ and $x_2(t+\Delta)$ for any value of t. For Δ equal zero, $\Gamma(0)$ is the variance of $x_2(t)$. Estimates of $\Gamma(\Delta)$ for other values of Δ could be obtained by averaging the products of $z(t)$'s that are Δ apart in time. The $z(t)$'s can be substituted for $x_2(t)$'s for Δ not equal to zero because the measurement errors are assumed to be statistically independent of each other. In the real world of hydrography, there would seldom, if ever, be sufficient discharge measurements displaced by exactly Δ in time to obtain a good estimate of $\Gamma(\Delta)$. To skirt this problem, time intervals of days were used and measurements made 365Δ days apart were averaged to estimate $\Gamma(\Delta)$ for each of several values of Δ. Figure 9 plots estimates of $\Gamma(\Delta)$ for Δ from one to forty days against Δ and shows a non-linear weighted least squares fit of the points where the weights are proportional to the number of pairs that comprise the estimating data for each point. The equation of the line in figure 10 is

$$\Gamma(\Delta) = 116{,}993(0.979)^{\Delta} \tag{38}$$

for Δ in days. This equation translates into a value of β of 7.75 years^{-1} and $q/2\beta$ of 116,993 feet^3 per second^2.

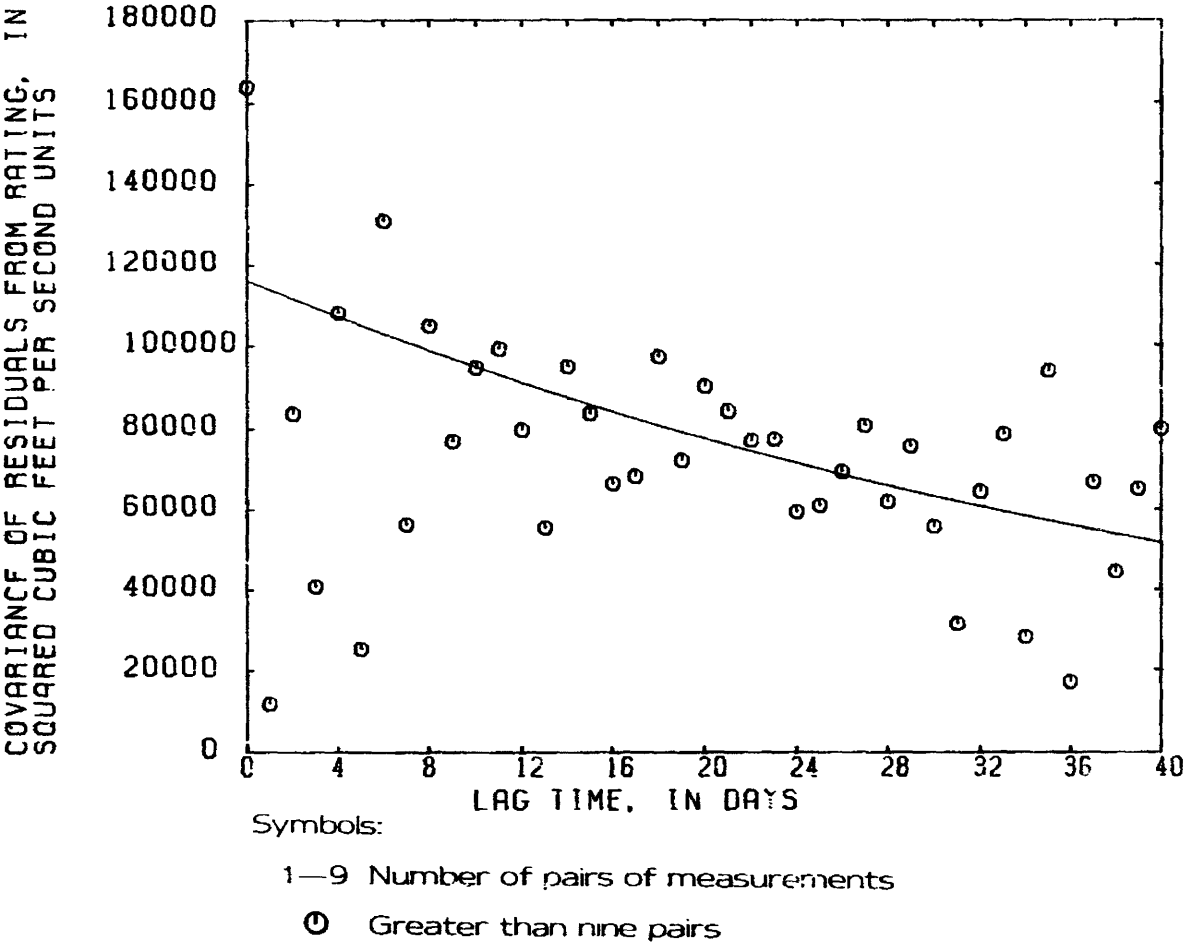


Fig. 9.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River below Davis Dam for the water years 1956-1958.

Verification of the model.

A full verification of the Kalman-filter model is not possible because of the limitations of availabile discharge measurements. Ideally, one would desire a data set that included sufficient measurements that the parameters of the model could be determined with part of the data and the remaining data could be used to draw conclusions about the model's validity. In the Lower Colorado River Basin this luxury is not available; therefore, the following steps were used to explore the credibility of the model.

One check on the validity of equation 24 is provided by comparison with the procedure of Burkham and Dawdy (1968) at a site where sufficient discharge measurements are available. Figure 10 repeats the data, shown earlier as figure 4, with the results of equation 24 superimposed. The Kalman-filter results were transformed into a measure of percent error by dividing the square root of the average value of $p_{22}(t)$ for the indicated frequencies of measurement by the average measured discharge during the 1956-1958 period of analysis.

Equation 24 results in error estimates that are less than those derived by the procedure of Burkham and Dawdy (1968). However, the filter estimates seem to approach the origin of the graph as sampling theory dictates. Because of the disparity between the theoretical filter estimates and the pragmatic Burkham-and-Dawdy estimates, a further test of the filter was conducted by split-sampling. After calibrating the filter with all discharge measurements in the 1956 through 1958 period, the measurements were divided into analysis and control groups in the manner of Burkham and Dawdy (1968).

The analysis group was used to drive the filter, while the control group was used to test the accuracy of the filter estimates at intermediate times between the analysis measurements. The results of this series of

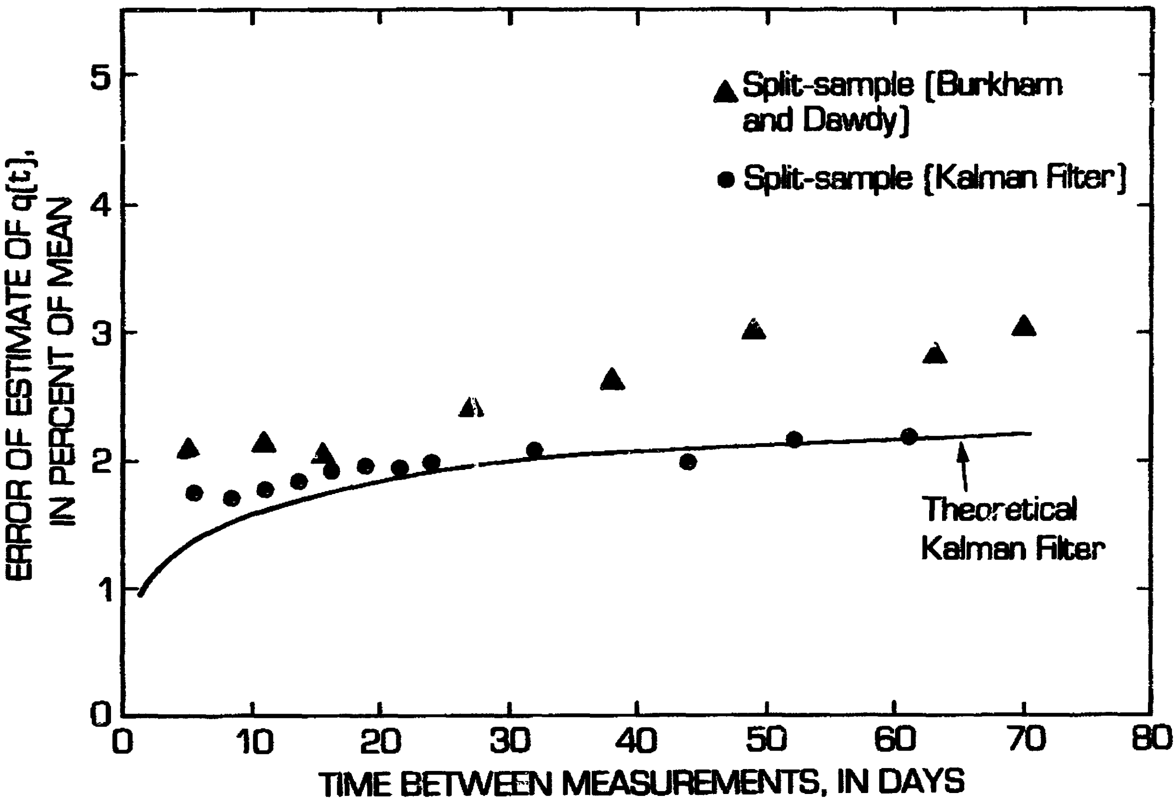


Fig. 10.--A comparison of Kalman-filter results with those of the Burkham-and-Dawdy procedure for error estimation.

tests are plotted as solid dots on figure 10. For infrequent measurements, the dots can be seen to lie close to the theoretical curve. However, as the period between measurements decreases, the dots can be seen to lie above the theoretical curve but still below that of the Burkham-and-Dawdy analysis. The dots also show the characteristic of not seeming to approach the origin. The apparent discrepancy between the theoretical results and both of the split-sample tests at the origin can be explained in part by the non-linearity of the variance relation between measurements as was illustrated in figure 5. As the period between measurements in the analysis group is reduced, the variance relation becomes more non-linear and the number of points in the control group become fewer. Because the variance relation is concave downward, this combination of results causes an overestimation of the average error variance that increases with decreased time between measurements in the analysis group.

Another partial check of the filter model can be obtained by one-step-ahead forecasting; that is, $x_2(\psi_{i+1})$ is forecasted using all measurements prior to ψ_{i+1}. The forecast has an expected error of zero and a variance defined by $p^-_{22}(\psi_{i+1})$ of equation 21. If the difference between the forecast and measured discharge for each measurement is divided by the square root of the sum of $p^-_{22}(\psi_{i+1})$ and the measurement error, r, the resulting series of numbers theoretically should have an expected mean of zero and an expected standard derivation of one. The resulting values for the 1956-1958 data set of 352 discharges measurements are respectively 0.002 and 1.028 which lend credence to the model's use, even at short intervals between measurements, with an average q and an average r as an estimator of the accuracy of streamflow records.

NETWORK DESCRIPTION

The streamgages in the lower basin that comprise the network in support of the Colorado River Compact were listed in table 1. These gages are serviced primarily from two offices of the U.S. Geological Survey: (1) the Yuma, Arizona, sub-district office, and (2) the Blythe, California, field office whose personnel report to the Yuma office. With one exception, the field operations of the two offices are generally non-overlapping. Figure 11 is a schematic diagram of the river system and the compact stream gages in the reach from Lake Mead to the Cibola Valley. The gages in this reach are serviced from the Blythe office except for the station on the Colorado River below Hoover Dam (09421500), which is serviced once a month from the Yuma office. This station is a part of the U.S. Geological Survey's National Stream Quality Accounting Network (Ficke and Hawkinson, 1975) and requires that monthly water-quality samples be quickly transported to the laboratory in the Yuma office. The water-quality requirements are met most expeditiously by personnel from the Yuma office making the monthly round-trip to the station and back. Should additional discharge measurements be required, they would probably be made by personnel from the Blythe office.

Figure 12 is a schematic of the river reach from Cibola Valley to the boundary between Mexico and Arizona. All stations in the reach are serviced from the Yuma office. Because of the disjoint nature of the field operations, the analysis of the Compact network can be decomposed into sub-analyses of the Blythe operations and the Yuma operations. Cost effective strategies for each component can be developed more efficiently than can a single analysis of the entire network. The individual strategies are functions of the amount of operating capital that is allocated to each component. The overall cost effectiveness is controlled by the amounts allocated to each.

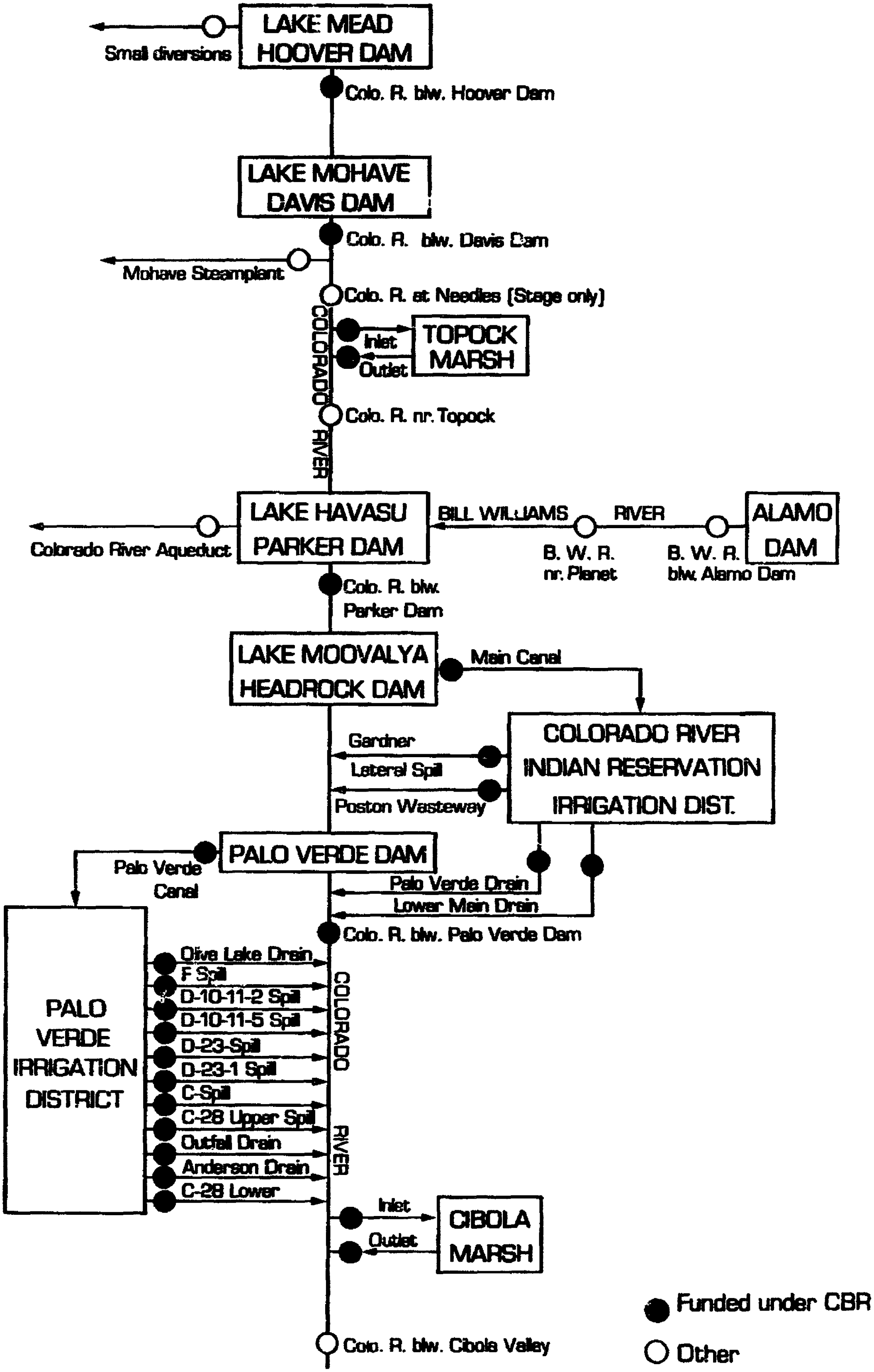


Fig. 11.--Schematic of the network of stream gages serviced from the Blythe Field Office.

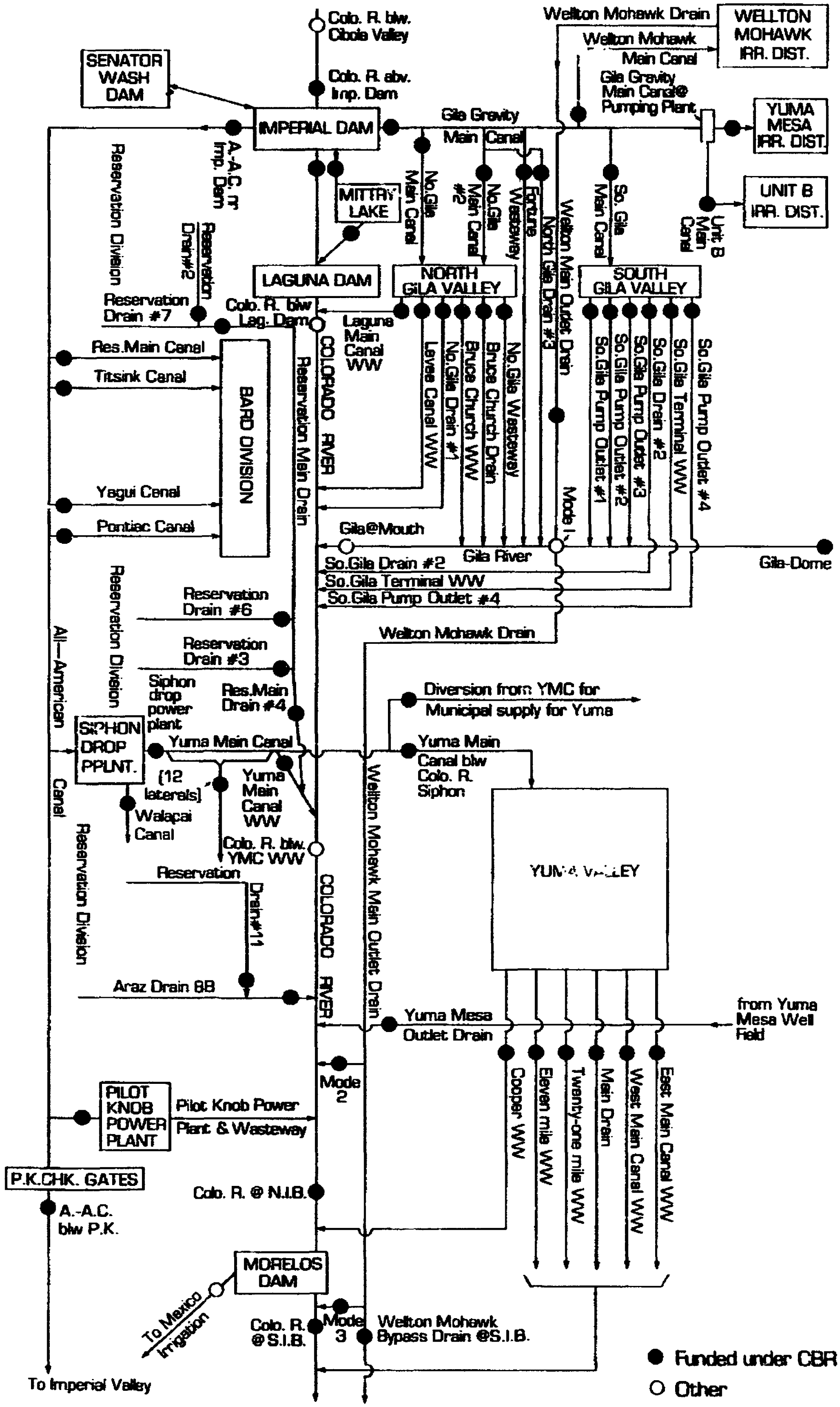


Fig. 12.--Schematic of the network of stream gages serviced from the Yuma Subdistrict Office.

The following describes briefly the hydrographic setting of each of the stations serviced from the Blythe field office and describes the applicability of the Kalman-filter model in defining the accuracy of the streamflow data. With one exception, the relationships that are developed are based only on data collected during the 1976, 1977, and 1978 water years; these were the only data readily available in the Yuma office.

09423000 Colorado River below Davis Dam, Ariz. - Nev.

The stage-discharge relation for this station is controlled by the channel slope, roughness, and geometry at the gage site. However, the closure of Davis Dam in 1950 has caused continual degradation of the channel and consequent changes in the stage-discharge relation. On the basis of 429 stage and discharge measurements made during the 1956-58 and 1976-78 water years, a time dependent rating curve was developed:

$$q_R(t) = 55.0 + 139.82\ (h(t) - 2.5 - 4.5\ e^{-.003t})^{2.05} \tag{39}$$

The estimates of the covariance of the deviations from this rating are shown in figure 13 as a function of the time lag. A weighted least-squares fit of these estimates yields a variance of 134,400 feet^6 per second^2 and a one-day serial correlation coefficient of 0.978. The average variance of measurements errors is estimated to be 153,300 feet^6 per second^2. The relations of the standard error of estimate of annual mean discharge as a function of number of discharge measurements per year is given in figure 14.

The annual mean discharge for the 1978 water year was 11,010 ft^3/s.

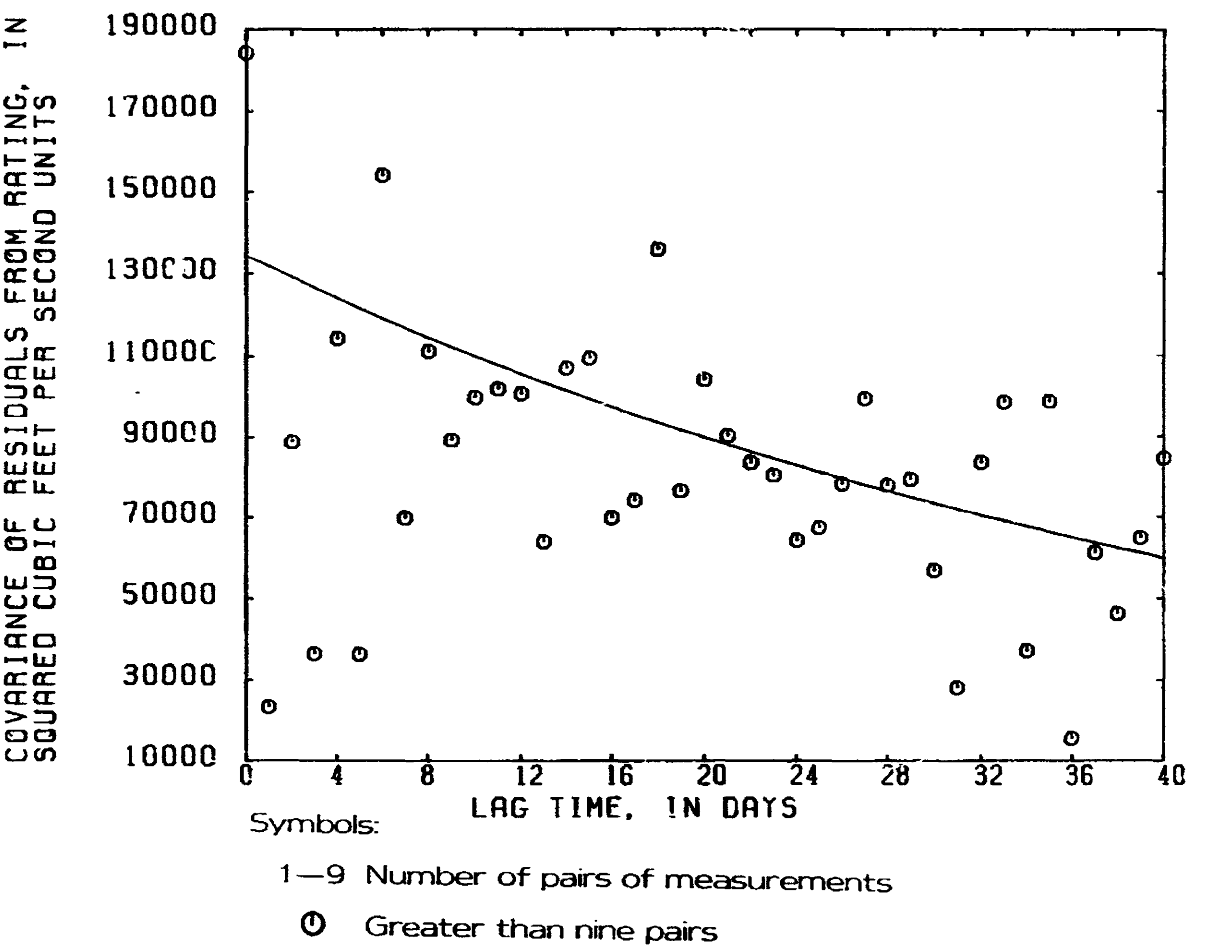


Fig. 13.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River below Davis Dam.

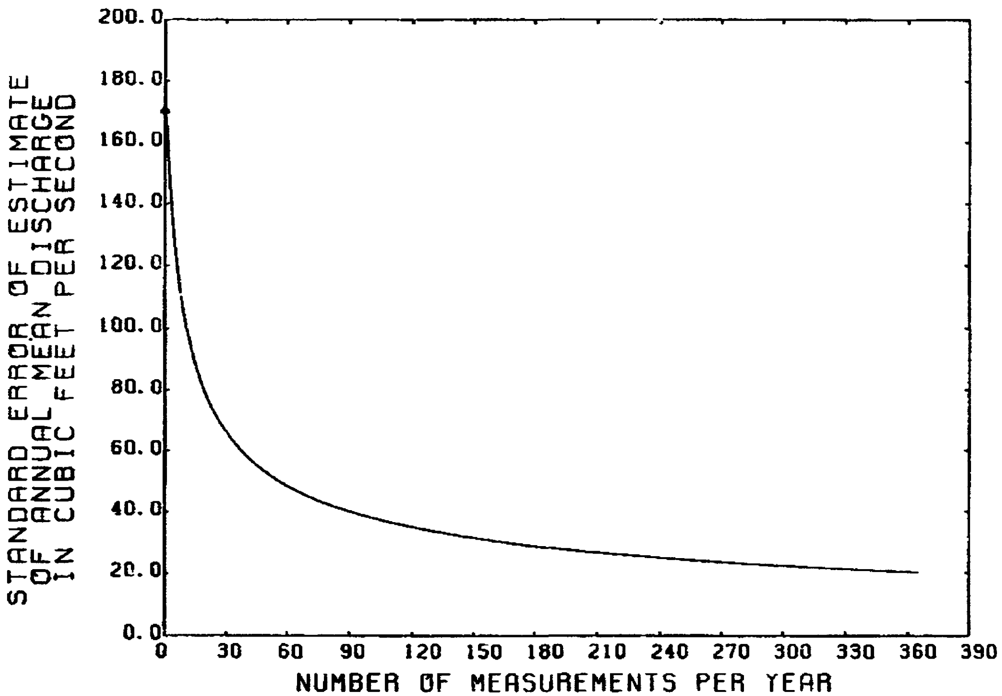


Fig. 14.--Standard error of estimate of annual mean discharge for the Colorado River below Davis Dam.

09423550 Topock Marsh inlet near Needles, Calif.

The amount of water entering the Topock Marsh inlet canal is determined by the openings of three gates located in an inlet structure on the left bank of the Colorado River and the difference in water surface elevations on the upstream and downstream sides of the inlet structure. Let $\delta(t)$ denote the difference in elevations in feet, and g denote the gate opening of the three gates in feet--all three of which are opened the same amount. The discharge is then computed by the formula for a submersible gate

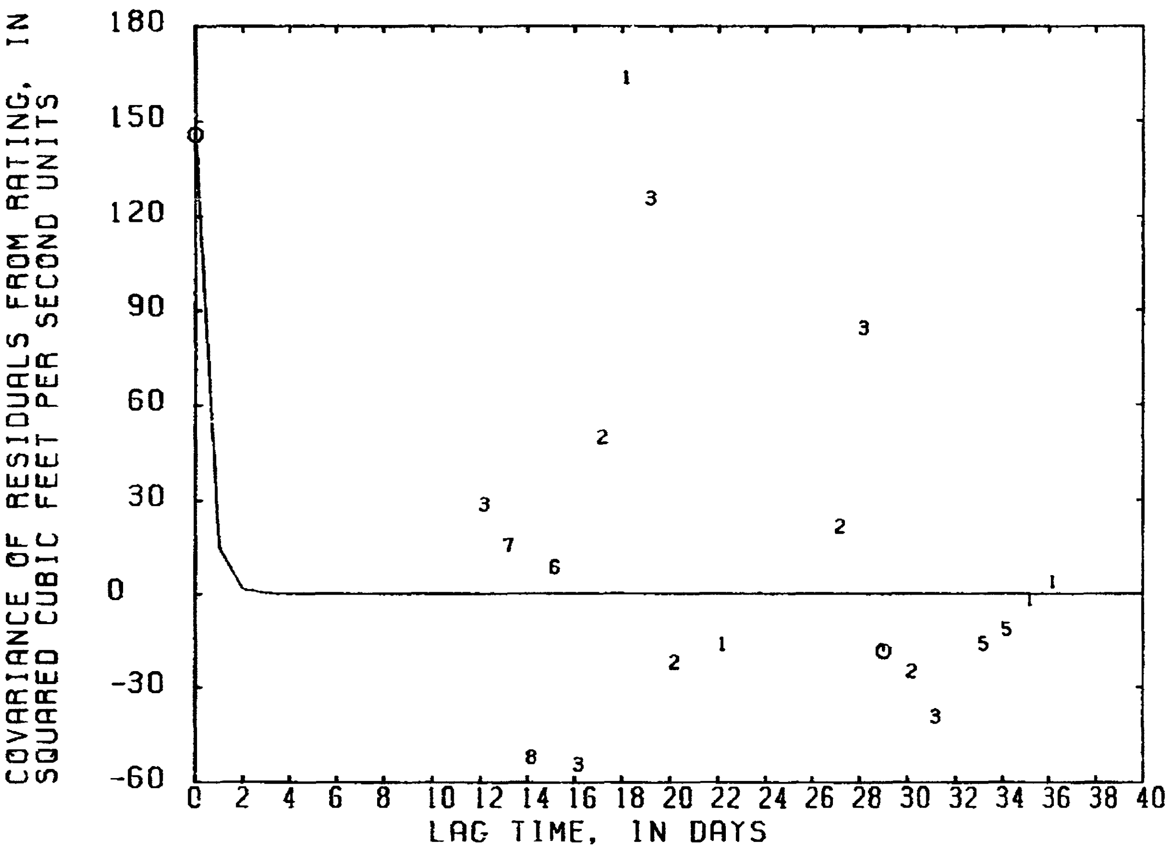
$$q_R(t) = C \cdot A \delta(t)^{0.5} \qquad (40)$$

where A is the area in square feet of the three gate openings and C is a constant to be determined. Each gate is 4 feet by 4 feet and has a sill elevation of 2.08 feet so that the area of the three gates is given by 12 $(g-2.08)$. Based on 48 gate openings, water surface elevations and discharge measurements during the 1976-78 water years the rating curve obtained by least squares is given by

$$q_R(t) = 67.1\ (g-2.08)\delta(t)^{0.5} \qquad (41)$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 15 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 145.69 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.095. The average variance of measurement errors is estimated to be 3.54 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 16.

The annual mean discharge for the 1978 water year was 54.8 ft^3/s.



Symbols:

1—9 Number of pairs of measurements

O Greater than nine pairs

Fig. 15.--Autocovariance function of the differences between true and rated discharge rate for the Topock Marsh Inlet near Needles, Calif.

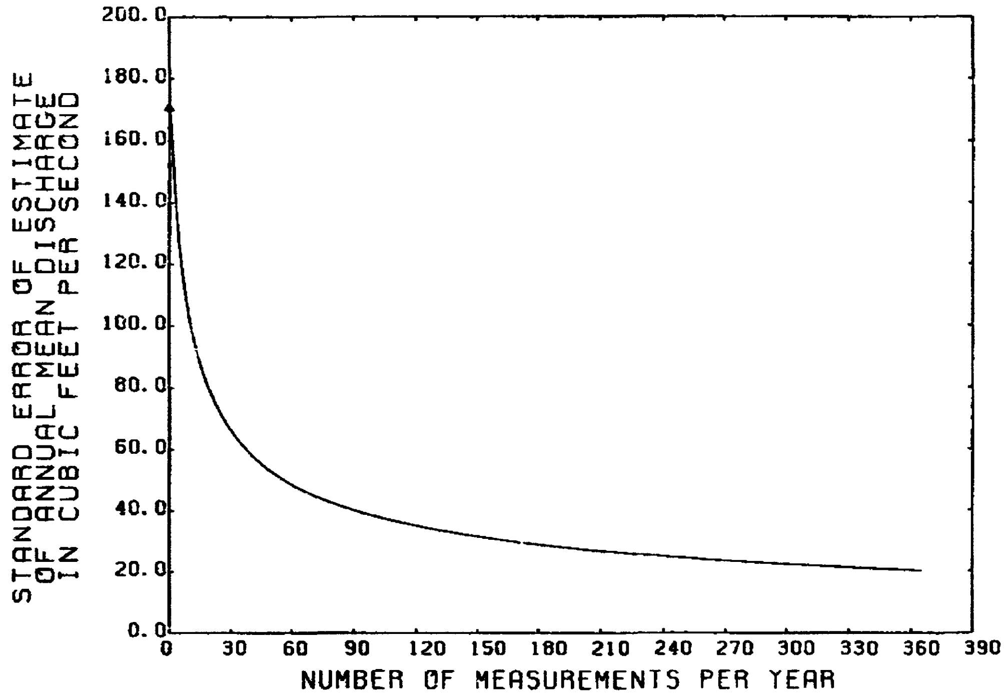


Fig. 16.--Standard error of estimate of annual mean discharge for the Topock Marsh Inlet near Needles, Calif.

09423650 Topock Marsh Outlet near Topock, Ariz.

This station was discontinued as a daily record station on May 31, 1978. Monthly discharges have been estimated since June of 1978; however, estimation procedure is not amenable to the accuracy analysis developed in this study.

09427520 Colorado River below Parker Dam, Ariz.-Calif.

The stage-discharge relationship is determined mainly by channel control except for some backwater conditions. Based on 92 measurements made during the 1975-78 water years the rating for this station was estimated by nonlinear least squares as

$$q_R(t) = 572.9 \cdot (h(t) - 62.38)^{1.55}. \qquad (42)$$

Estimates of the autocovariances of the deviations from this rating as a function of the time lag is shown in figure 17. A weighted nonlinear least squares fit of these estimates yields a variance of 259,707 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.94. The average variance of measurement errors is estimated to be 88,777 feet^6 per second^2. The standard error of estimate of annual mean discharge as a function of the number of discharge measurements per year is given in figure 18.

The annual mean discharge for the water year 1978 is 9,299 ft^3/s.

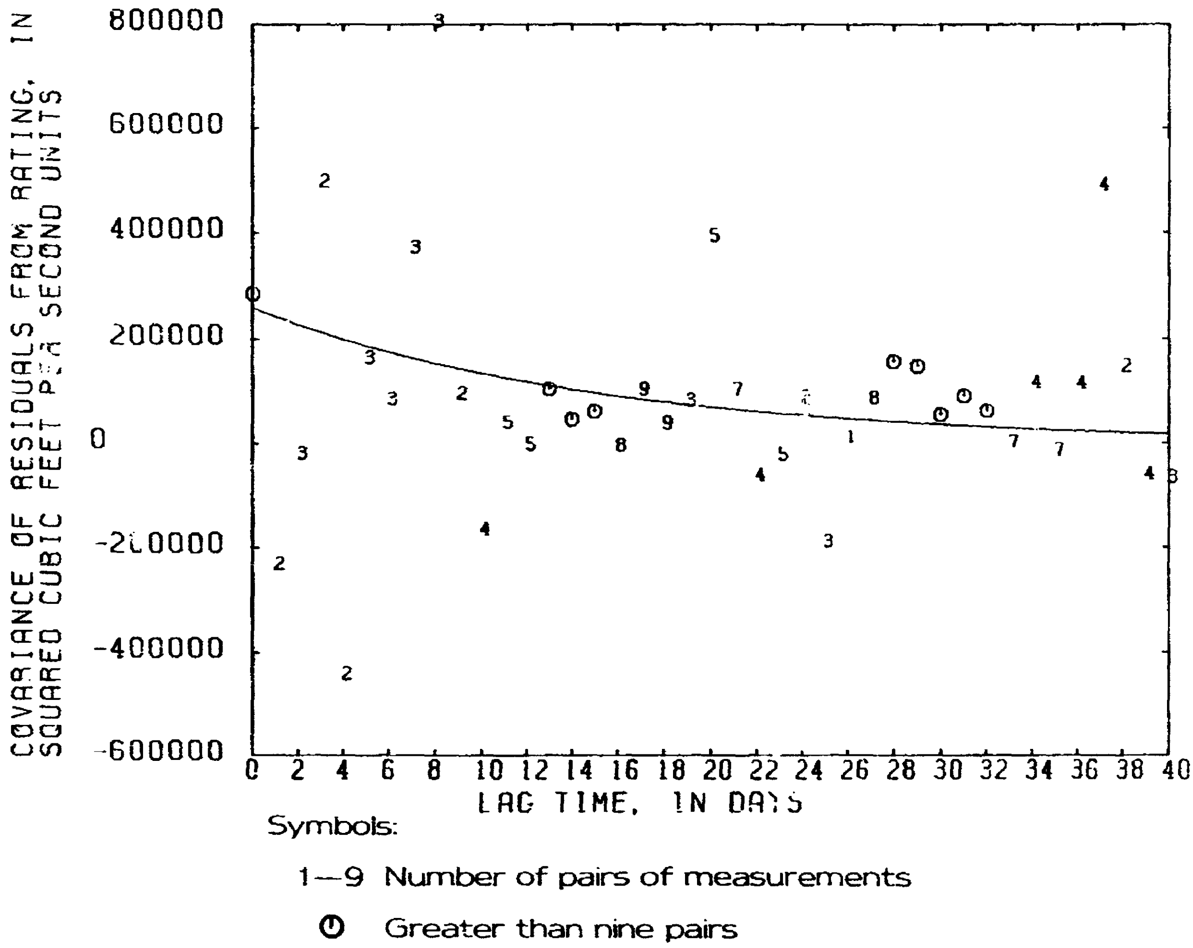


Fig. 17.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River below Parker Dam, Ariz.-Calif.

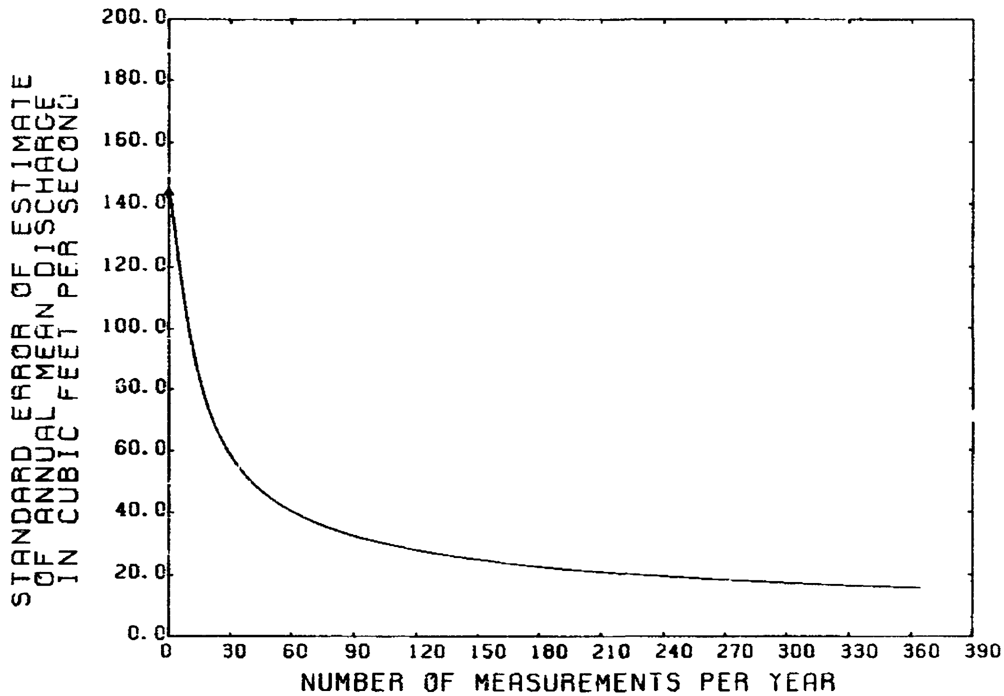


Fig. 18.--Standard error of estimate of annual mean discharge for the Colorado River below Parker Dam, Ariz.-Calif.

09428500 Colorado River Indian Reservation Main Canal near Parker, Ariz.

The control for this station consists of two submersible gates. The rating equation expresses the relation between the discharge and a function of the recorded gate openings and difference in surface water elevations on either side of the gates. Based on 20 discharge measurements for the water years 1976-77 the rating equation obtained by nonlinear least squares is given by

$$q_R(t) = 173.1 \cdot g\delta(t)^{0.5}. \tag{43}$$

The sample estimates of autocovariances of the deviations from this rating are shown in figure 19 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 3651.7 feet^6 per second^2 and a one-day autocorrelation function of 0.97. The average variance of measurement errors is estimated to be 154.8 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 20.

The annual mean discharge for the 1978 water year was 811 ft^3/s.

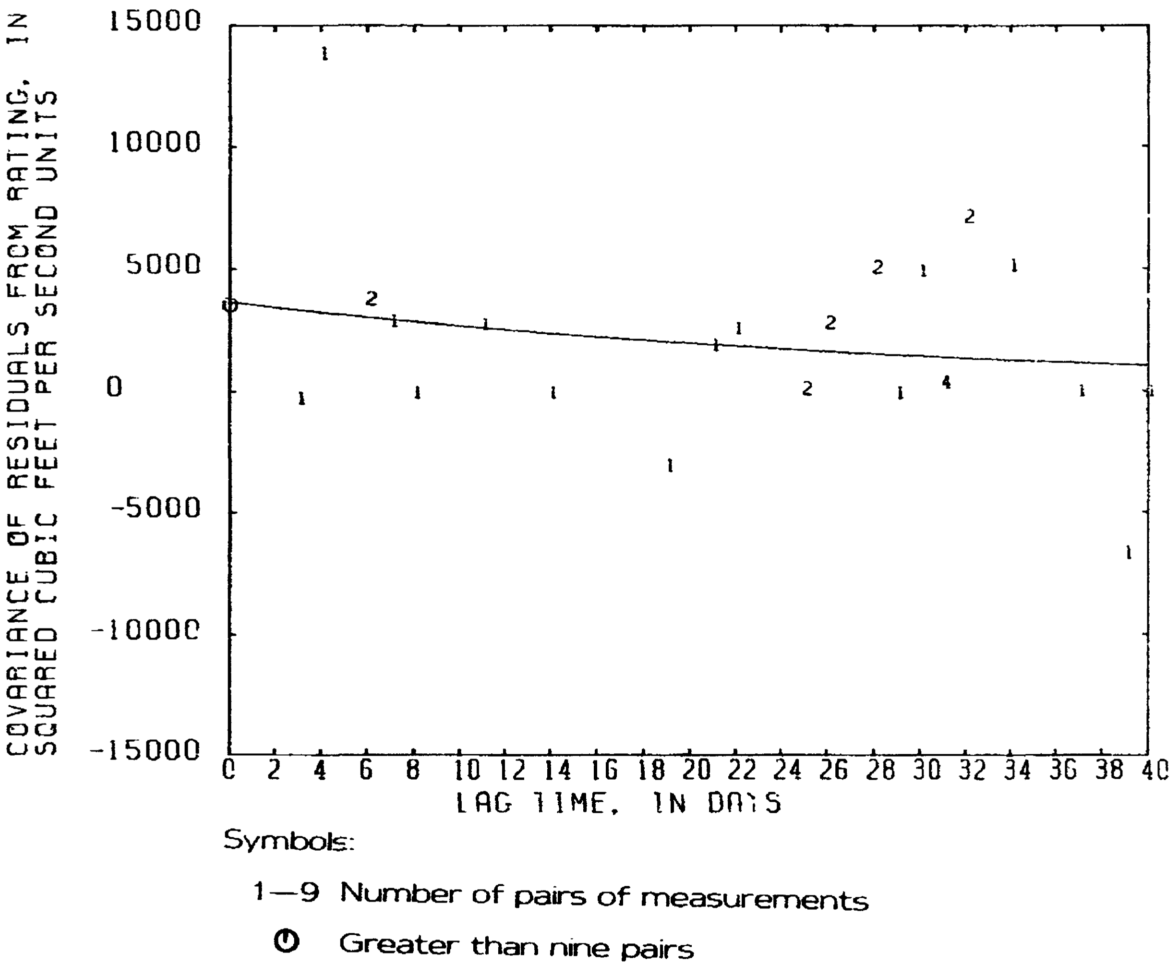


Fig. 19.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River Indian Reservation Main Canal near Parker, Ariz.

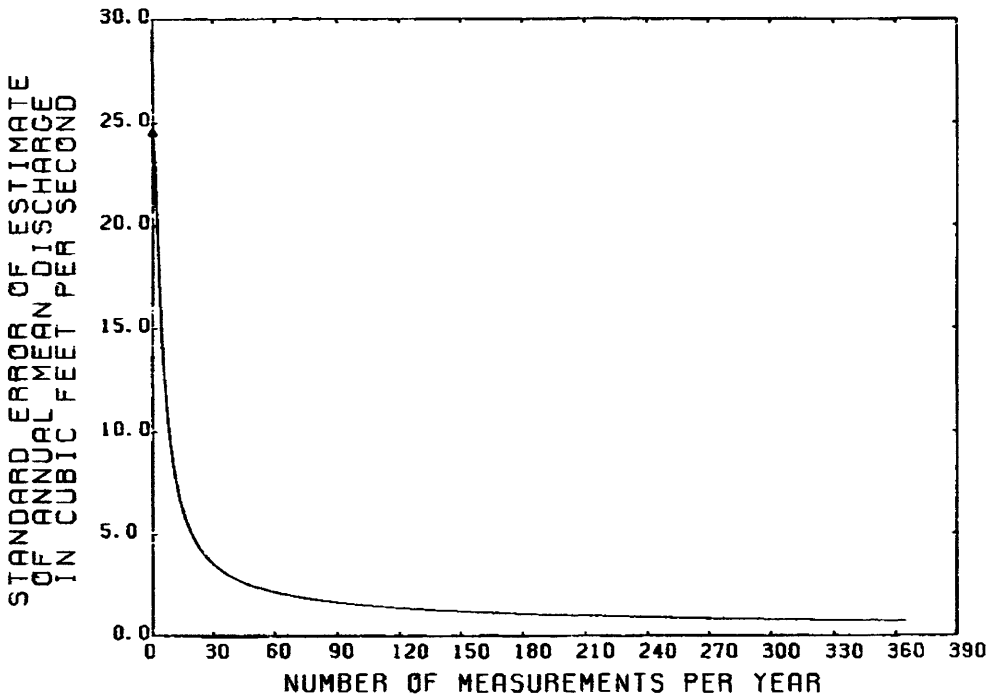


Fig. 20.--Standard error of estimate of annual mean discharge for the Colorado River Indian Reservation Main Canal near Parker, Ariz.

09428505 Gardner Lateral Spill near Parker, Ariz.

A straight channel upstream and downstream of the gage is composed of sand with banks about 12 feet high. The control is a combination of channel control along with section control at a 4 foot diameter culvert approximately 25 feet downstream. Based on 28 discharge measurements during the 1976-78 water years the stage-discharge relationship was determined by nonlinear least squares as

$$q_R(t) = 6.05 \cdot (h(t) - 0.82)^{2.26} \; . \qquad (44)$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 21 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 0.059 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.941. The average variance of measurement errors is estimated to be 0.023 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 22.

This record is 1 of 4 returns to the Colorado River which are published as total return flow on a monthly basis with station 09428500, Colorado River Indian Reservation Main Canal near Parker, Ariz. The mean of the total return flows for the 1978 water year was 365.8 ft^3/s.

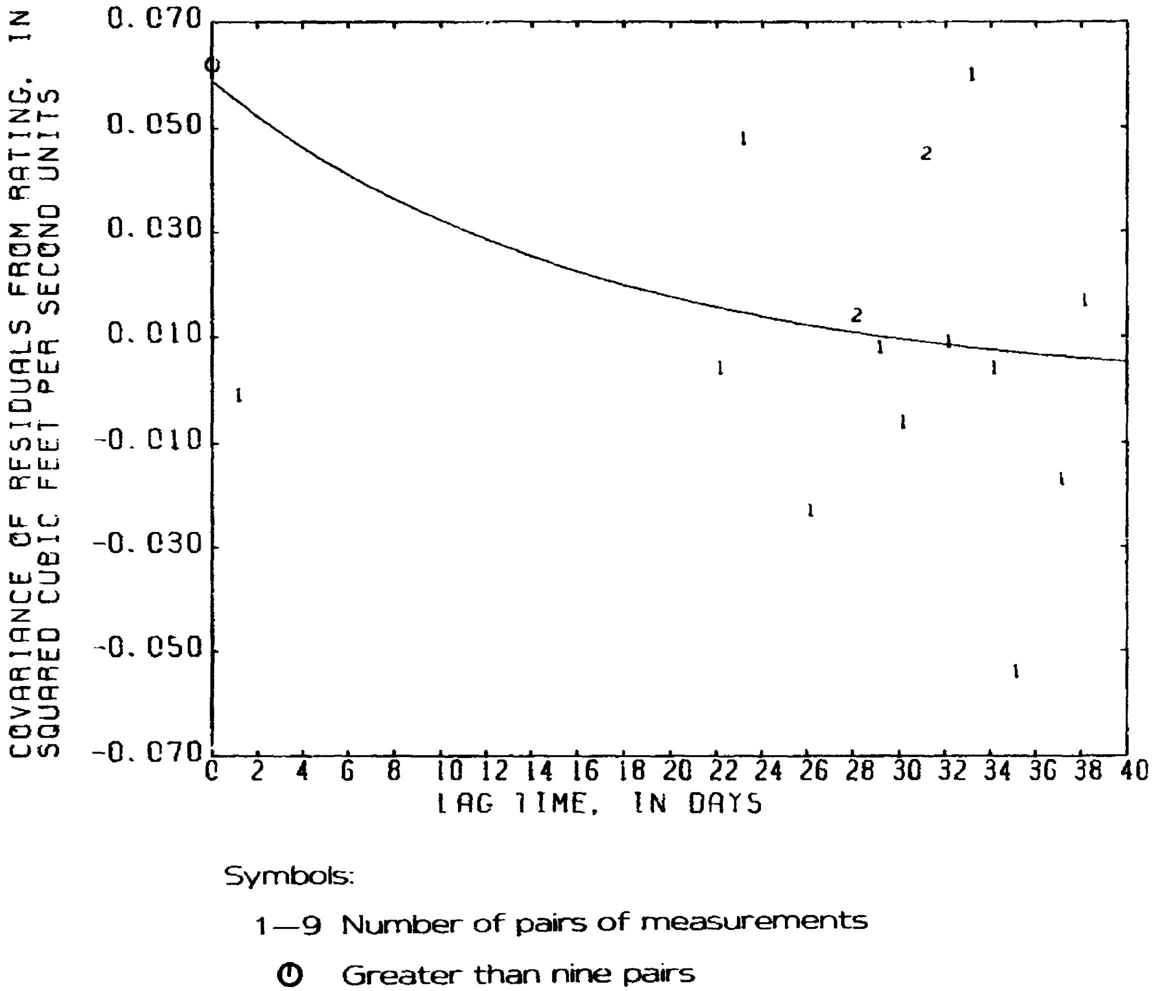


Fig. 21.--Autocovariance function of the differences between true and rated discharge rate for the Gardner Lateral Spill near Parker, Ariz.

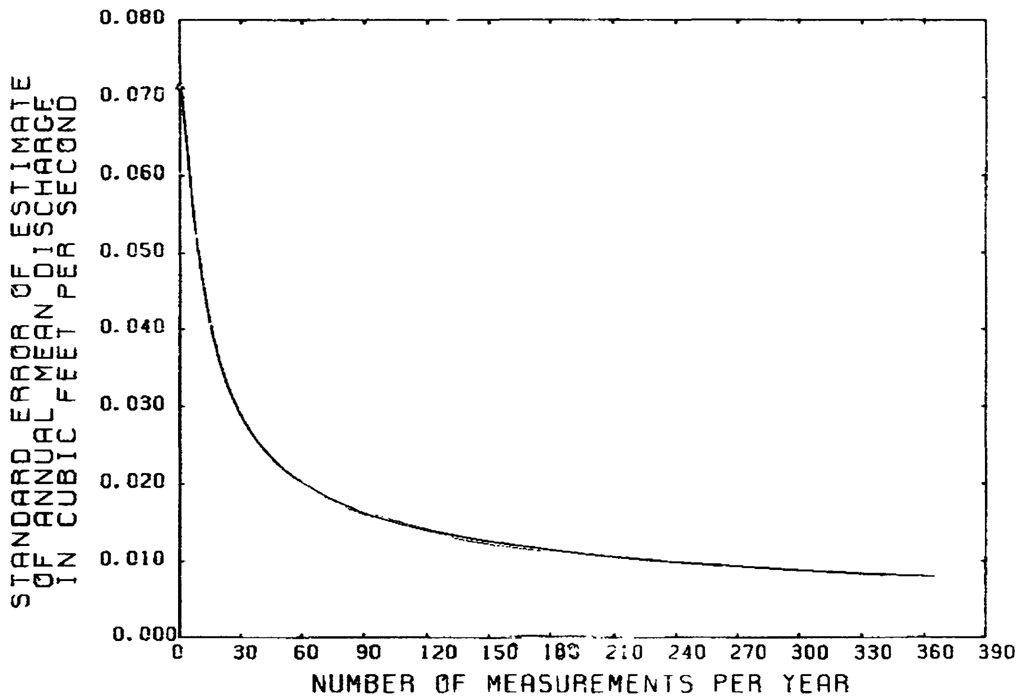


Fig. 22.--Standard error of estimate of annual mean discharge for the Gardner Lateral Spill near Parker, Ariz.

09428510 Colorado River Indian Reservation Poston Wasteway near Parker, Ariz.

The discharge is computed by summing the daily discharge of the Colorado River Indian Reservation Upper Main Drain, station number 09428508, the canal spill, computed as station number 09428511, and an estimated gain or loss. The configuration of these stations is shown in figure 23.

The channel bed at the Upper Main Drain is soft silt and mud with dirt banks. Vegetation growth on the banks and aquatic growth in the channel occurs during the summer months creating negative shifts in the rating. Colorado River Indian Reservation personnel remove this growth at aperiodic intervals creating positive shifts in the rating. Thus, the rating at the Upper Main Drain displays considerable scatter. Based on 88 measurements during the 1976-78 water years a nonlinear least squares estimating procedure yielded the rating

$$q_u = 98.6 \cdot (h_u - 1.93)^{.435} \qquad (45)$$

where q_u is the discharge in ft^3/s at the Upper Main Drain and h_u is the gage height in feet. Deviations from this rating yielded estimates of autocovariances shown in figure 24 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 269.8 feet^6 per second^2 and a one-day autocorrelation of 0.97. The average variance of measurement errors was estimated to be 24.1 feet^6 per second^2.

The rating for the spillway is given by spill rating 1 based on the theoretical curve used by Colorado River Indian Reservation personnel. The form of this rating is not needed for the following development of the uncertainty curve associated with the Poston Wasteway.

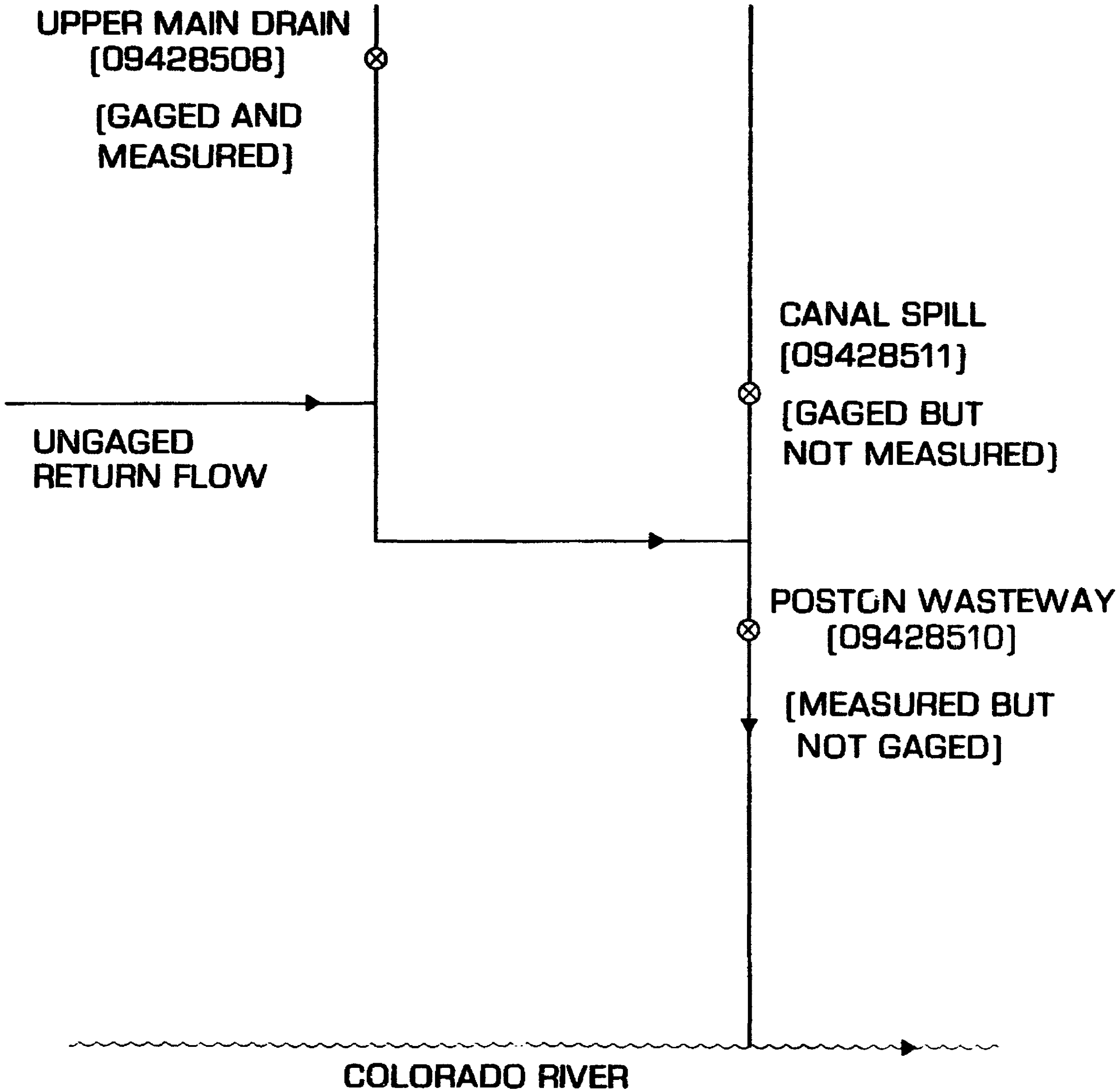


Fig. 23.--Configuration of stations associated with the Poston Wasteway.

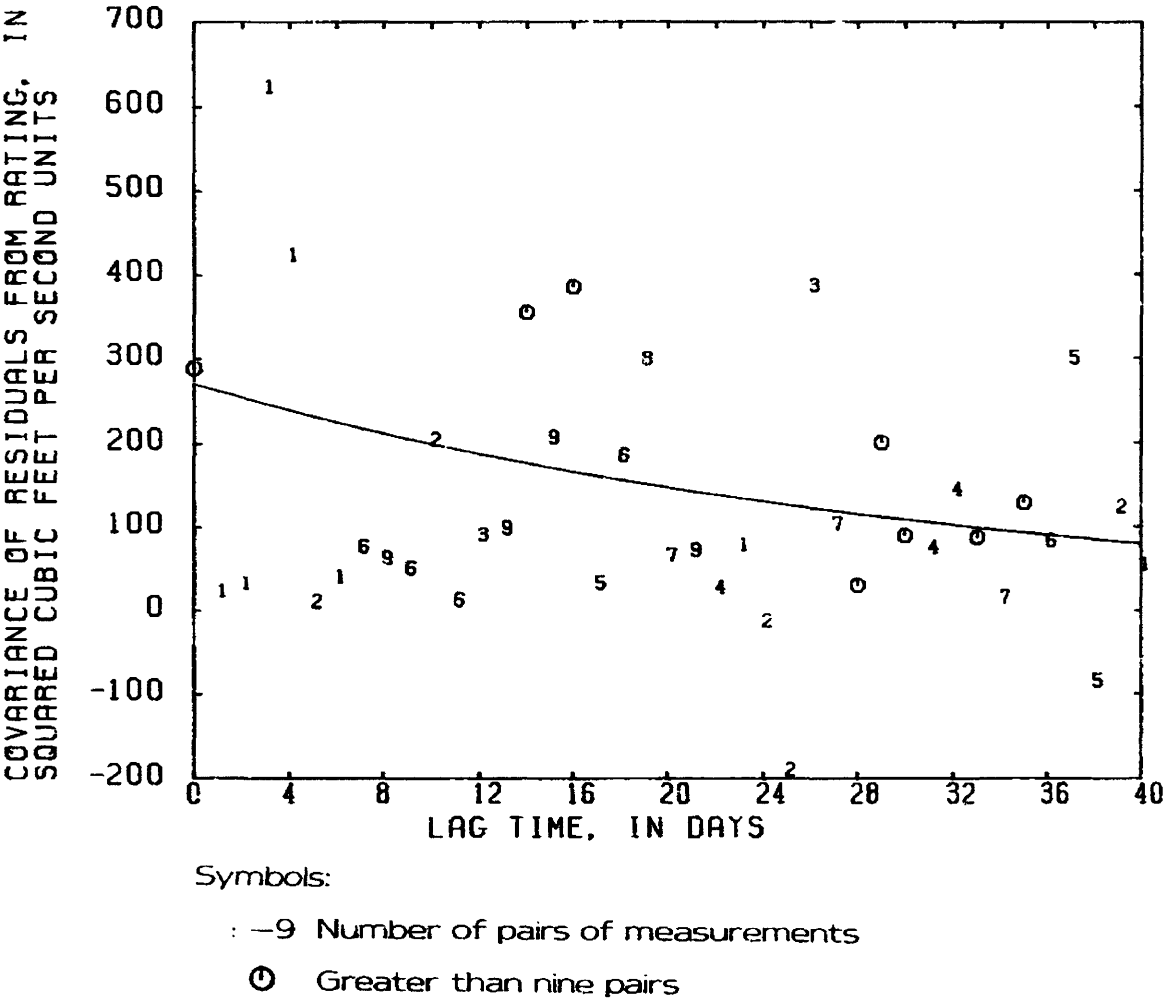


Fig. 24.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River Upper Main Drain.

The gain or loss is estimated as the difference between (1) a discharge measurement at the Poston Wasteway and (2) the sum of a discharge measurement at the Upper Main Drain corrected for an approximate 2½ hour travel time and a rated discharge at the canal spillway. The "rating" for this gain or loss is taken to be the average value of the gains or losses based on 82 measurements and ratings during the 1976–78 water years.

The rating for the Poston Wasteway is then given by

$$q_R = q_u + q_s + 8.9 \tag{46}$$

where q_u is the discharge in ft^3/s at the Upper Main Drain given by equation (45), q_s is the discharge in ft^3/s given by the spillway rating and 8.9 is the average of the gains and losses in ft^3/s. The deviations from the rating in equation (46) yield estimates of autocovariances shown in figure 25 as a function of the time lag. Under the assumption that the deviations from the rating given in equation (45) and the deviations of the spillway flow plus the gain or loss from the rating given by $q_s + 8.9$ are both lag-one autoregressive processes, an estimate of 694.4 feet^6 per second^2 was found for the variance of the deviations from the rating of the spillway plus the gain or loss with a one-day autocorrelation coefficient of 0.98. The average variance of measurement errors was estimated to be 9.1 feet^6 per second^2. The combination of the two lag-one autoregressive processes yields for the Poston Wasteway the relation of standard error of estimate of annual mean discharge to the number of discharge measurements per year shown in figure 26.

This record is 1 of 4 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09428500, Colorado river Indian Reservation Main Canal near Parker, Ariz. The mean of the total return flows for the 1978 water year was 365.8 ft^3/s. The Poston Wasteway contributes approximately 40 percent of the total return flow.

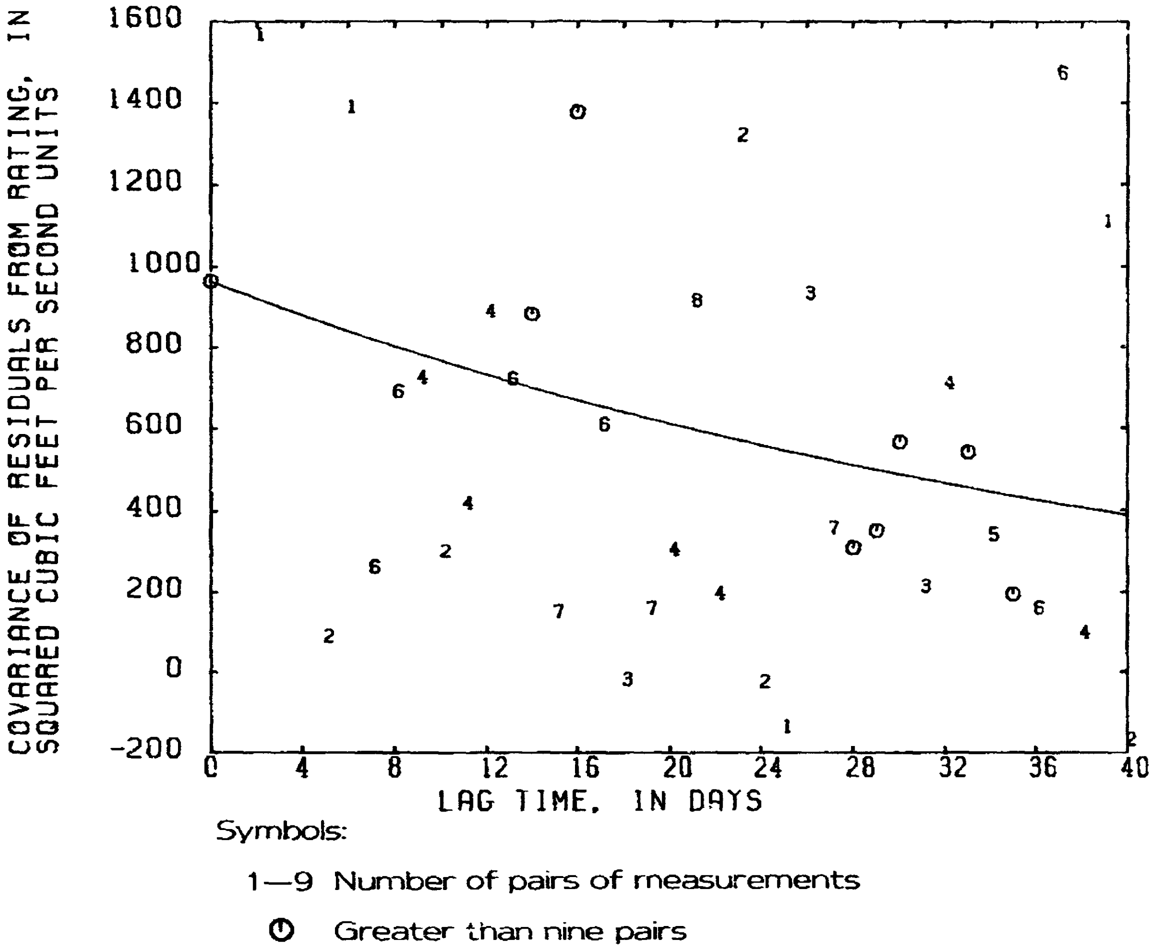


Fig. 25.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River Indian Reservation Poston Wasteway near Parker, Ariz.

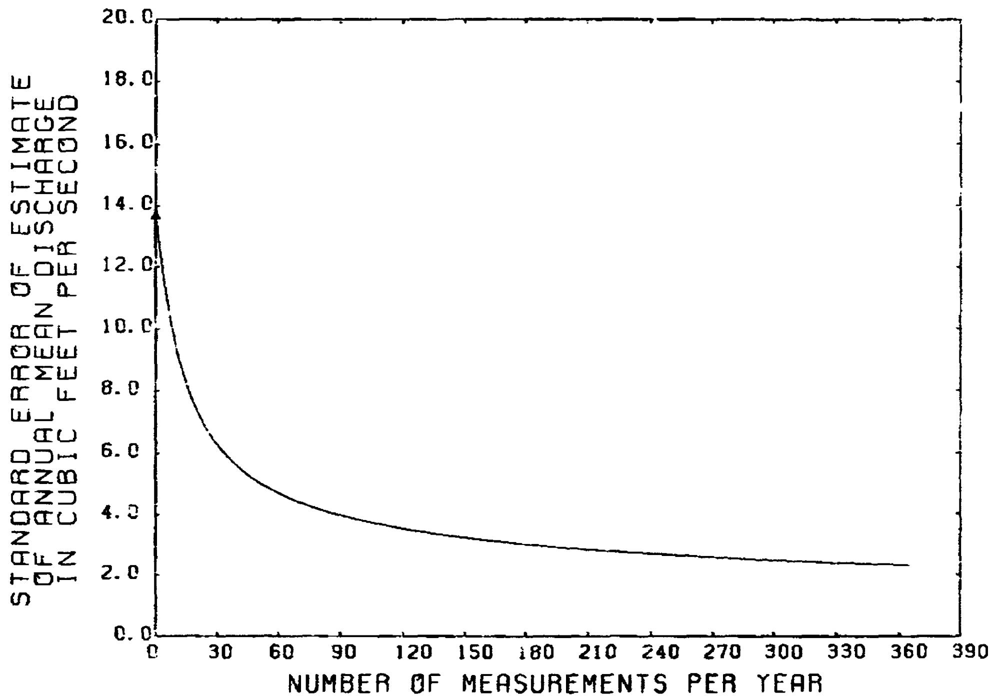


Fig. 26.--Standard error of estimate of annual mean discharge for the Colorado River Indian Reservation Poston Wasteway near Parker, Ariz.

09429000 Palo Verde Canal near Blythe, Calif.

The differences in two water stage records along with a gate opening record are used to compute the discharge at this station. The difference in the stages is the head in feet on the gates denoted by $\delta(t)$. Using 79 measurements from the water years 1976-78 the rating for this station was estimated by nonlinear least squares to be

$$q_R(t) = 75.25g^{1.71}\delta(t)^{0.5} \tag{47}$$

The time series of deviations from this rating yielded estimates of the autocovariance function shown in figure 27 as a function of the time lag. A weighted nonlinear least squares fit of a lag-one autoregressive process covariance function to these estimated covariances gave a variance of 3486.5 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.86. The average variance of the measurement error is estimated to be 462.2 feet^6 per second^2. Figure 28 shows the relation of standard error of estimate of the annual mean discharge to the number of measurements per year.

The annual mean discharge for the 1978 water year was 1244 ft^3/s.

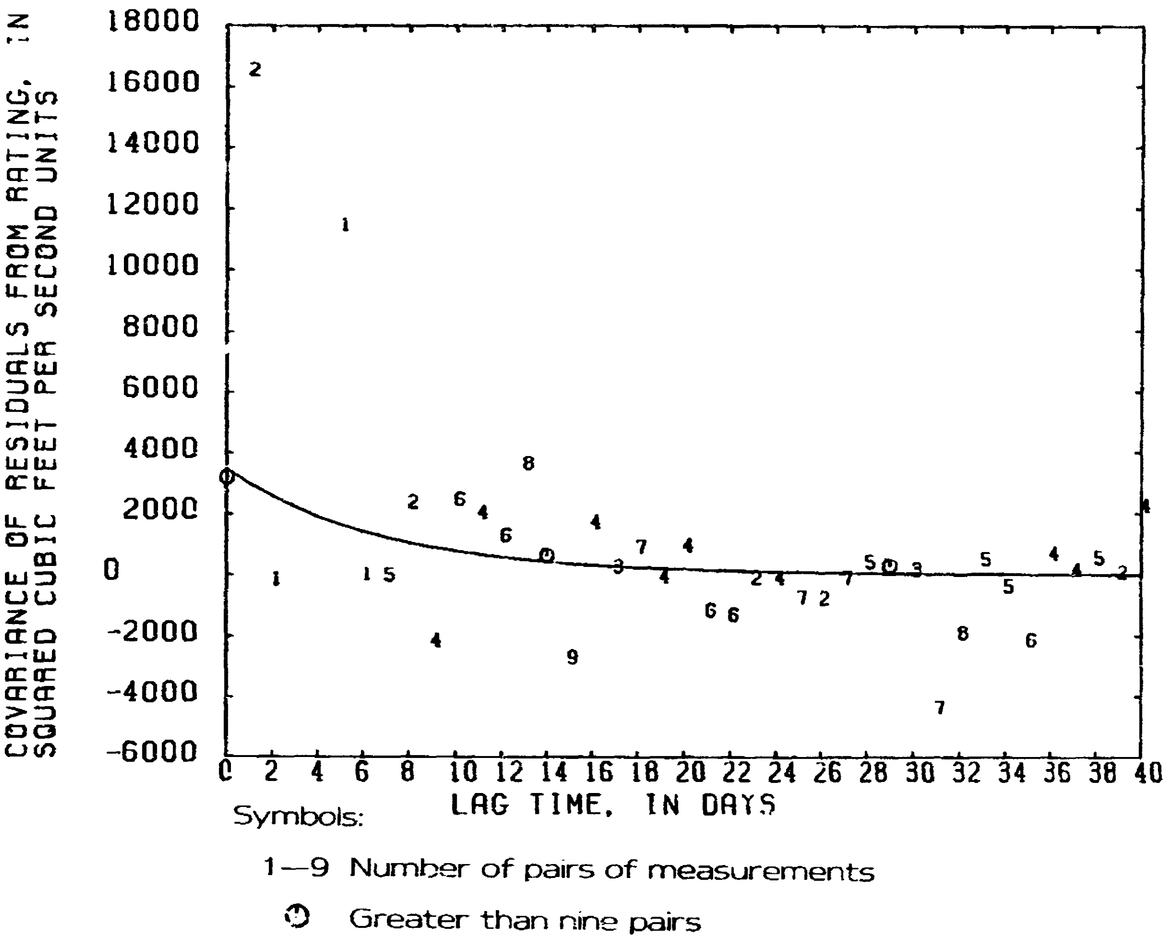


Fig. 27.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Canal near Blythe, Calif.

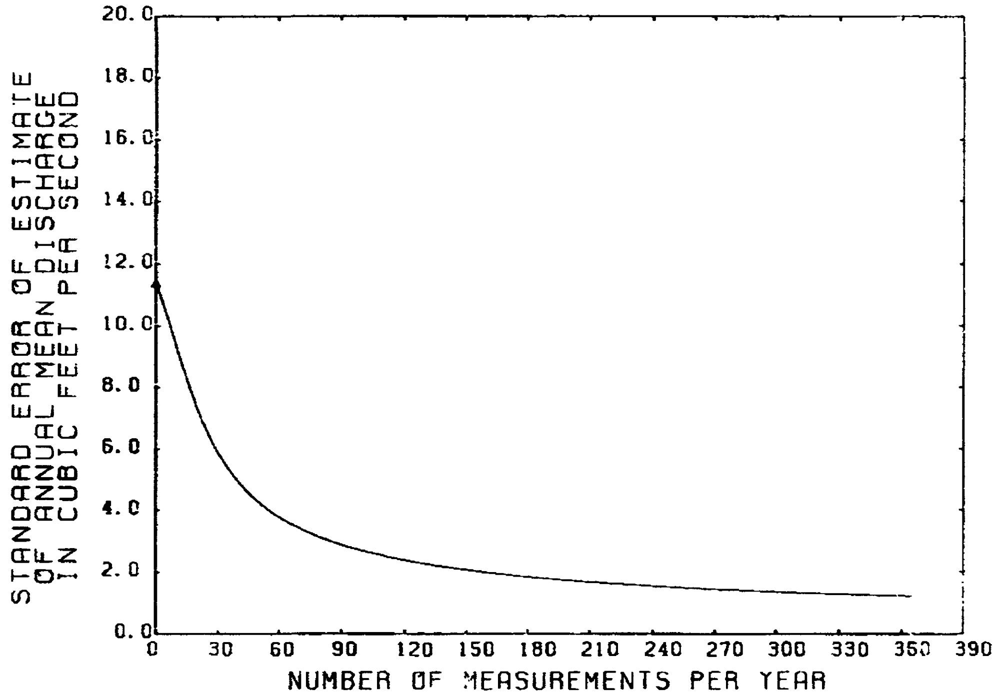


Fig. 28.--Standard error of estimate of annual mean discharge for the Palo Verde Canal near Blythe, Calif.

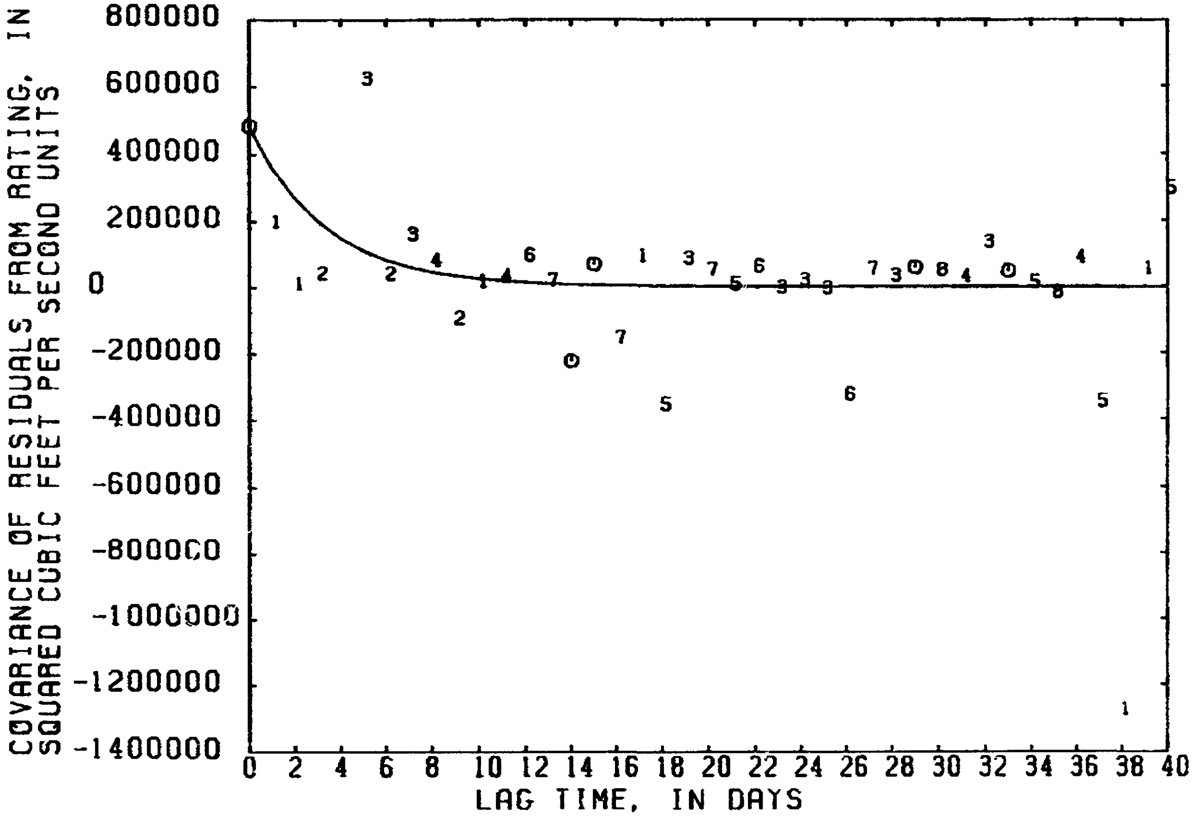
09429010 Colorado River at Palo Verde Dam, Ariz.-Calif.

The control at this station consists of three 50-foot radial gates. Equivalent vertical gate openings along with forebay- and afterbay-water-surface elevations are used to compute discharge. On the basis of 79 concurrent measurements of discharge, during the 1976-78 water years, the rating curve obtained by least squares is

$$q_R(t) = 892.08 \cdot g\delta(t)^{0.5} \tag{48}$$

The deviations from this rating curve yielded estimates of an autocovariance function shown in figure 29 as a function of the time lag. A weighted nonlinear least squares fit of a lag-one autoregressive model covariance function gave a variance of 481,762 feet^6 per second^2 and a one-day autocorrelation coeffficient of 0.74. The estimated average variance of measurement errors is 35,057 feet^6 per second^2. The standard error of estimate of the mean annual discharge as a function of the number of measurements per year is shown in figure 30.

The annual mean discharge for the 1978 water year was 7111 ft^3/s.



Symbols:

1—9 Number of pairs of measurements

Ⓞ Greater than nine pairs

Fig. 29.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River at Palo Verde Dam, Ariz.-Calif.

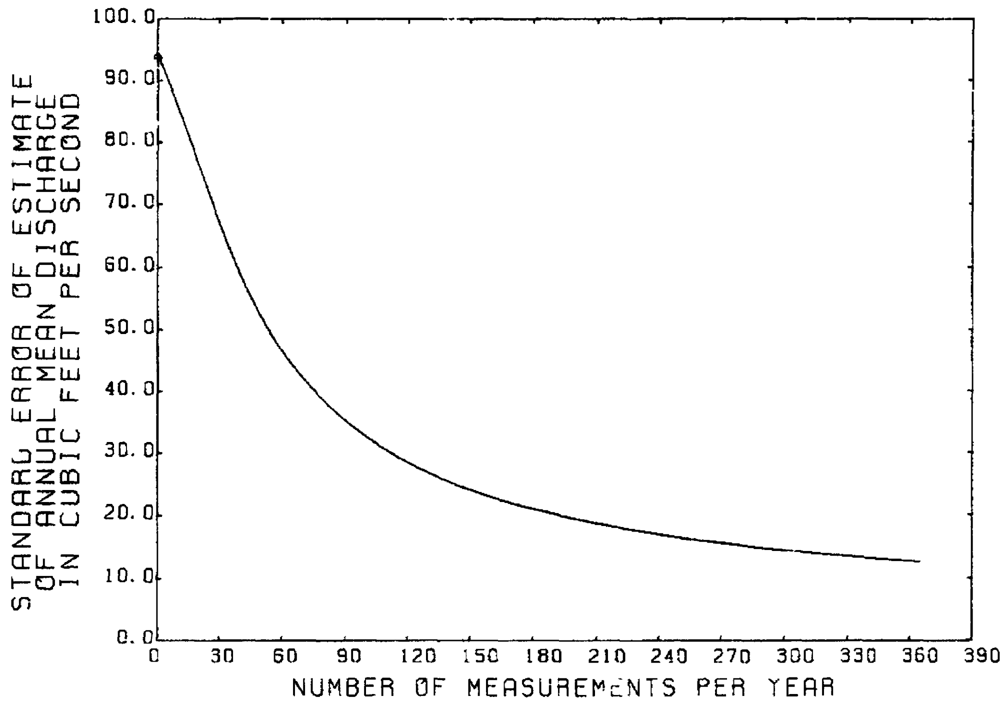


Fig. 30.--Standard error of estimate of annual mean discharge for the Colorado River at Palo Verde Dam, Ariz.-Calif.

09429030 Colorado River Indian Reservation Palo Verde Drain near Parker, Ariz.

No correlative data is collected except when discharge measurements are being made. The rating is taken to be the average of 36 discharge measurements made during the 1976-78 water years along with one additional measurement on September 17, 1976, and one on October 27, 1978. This average is 40.7 ft^3/s. The 38 measurements have a standard deviation of 10.0 ft^3/s with a maximum of 58.7 ft^3/s and a minimum of 23.2 ft^3/s. The estimates of the autocovariances of the deviations from this average value are shown in figure 31. A weighted nonlinear least squares fit of these estimates yields a variance of 97.99 $feet^6$ per $second^2$ and a one-day autocorrelation coefficient of 0.99. The average variance of measurement errors is estimated to be 0.702 $feet^6$ per $second^2$. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 32.

This record is 1 of 4 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09428500, Colorado River Indian Reservation Main Canal near Parker, Ariz. The contribution from this station is about 10 percent of the total return flow. The mean of the total return flows for the 1978 water year was 365.8 ft^3/s.

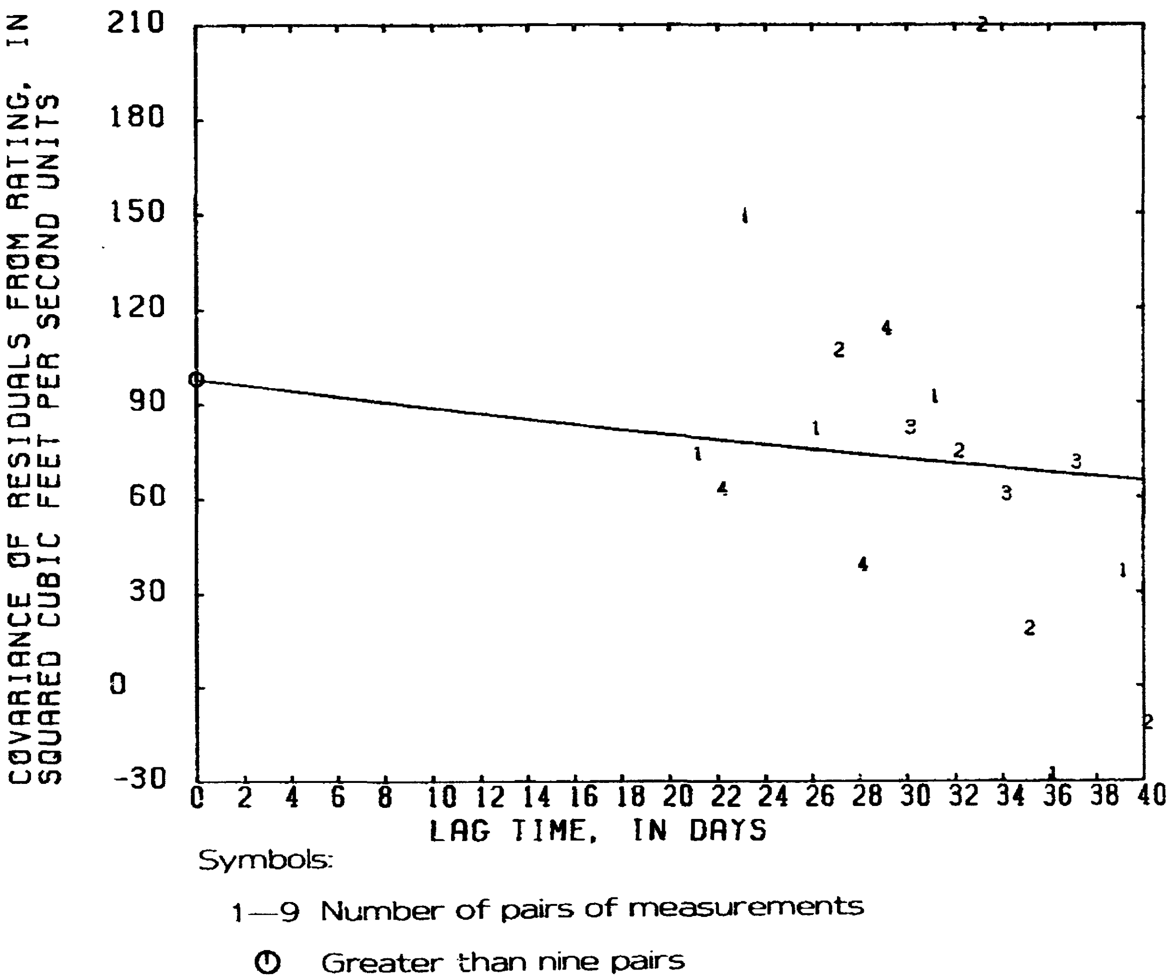


Fig. 31.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River Indian Reservation Palo Verde Drain near Parker, Ariz.

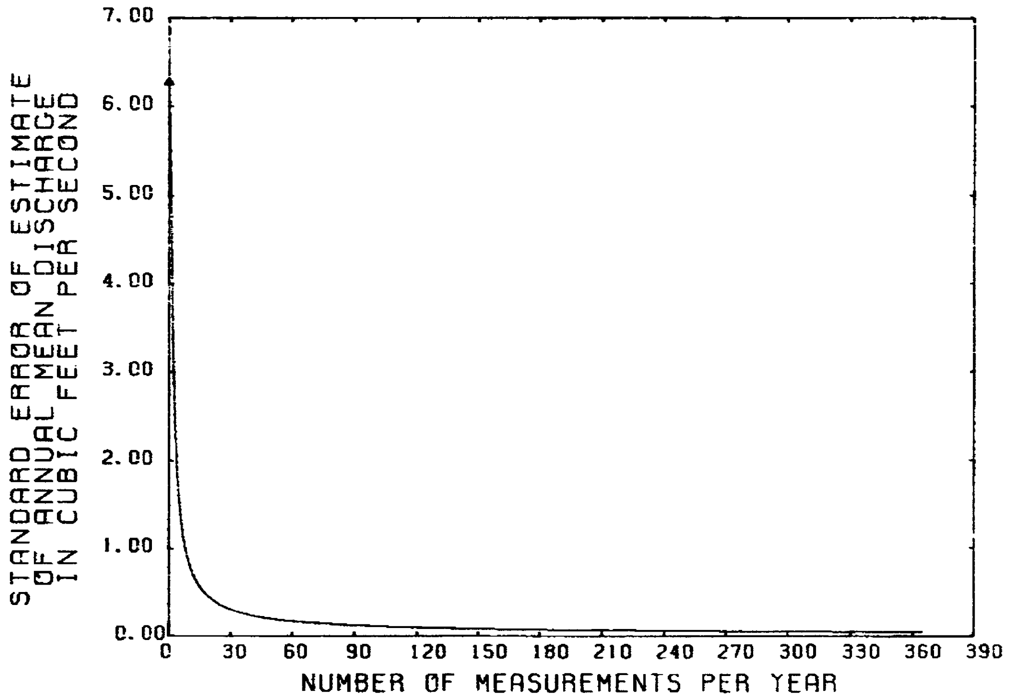


Fig. 32.--Standard error of estimate of annual mean discharge for the Colorado River Indian Reservation Palo Verde Drain near Parker, Ariz.

09429060 Colorado River Indian Reservation Lower Main Drain near Parker, Ariz.

The stage discharge relation is determined by the natural bed and banks of the drain. Considerable seasonal shifting occurs due to growth and removal of water vegetation. Based on 43 discharge measurements made during the 1976-78 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 153.1 \cdot (h(t) - 1.2)^{0.47}. \tag{49}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 33 as a function of the time lag. A weighted nonlinear least squares fit of these covariance estimates yields a variance of 303.7 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.98. The average variance of measurement errors is estimated to be 19.2 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 34.

The record is 1 of 4 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09428500, Colorado River Indian Reservation Main Canal near Parker, Ariz. The annual mean return flow for the 1978 water year was 365.8 ft^3/s.

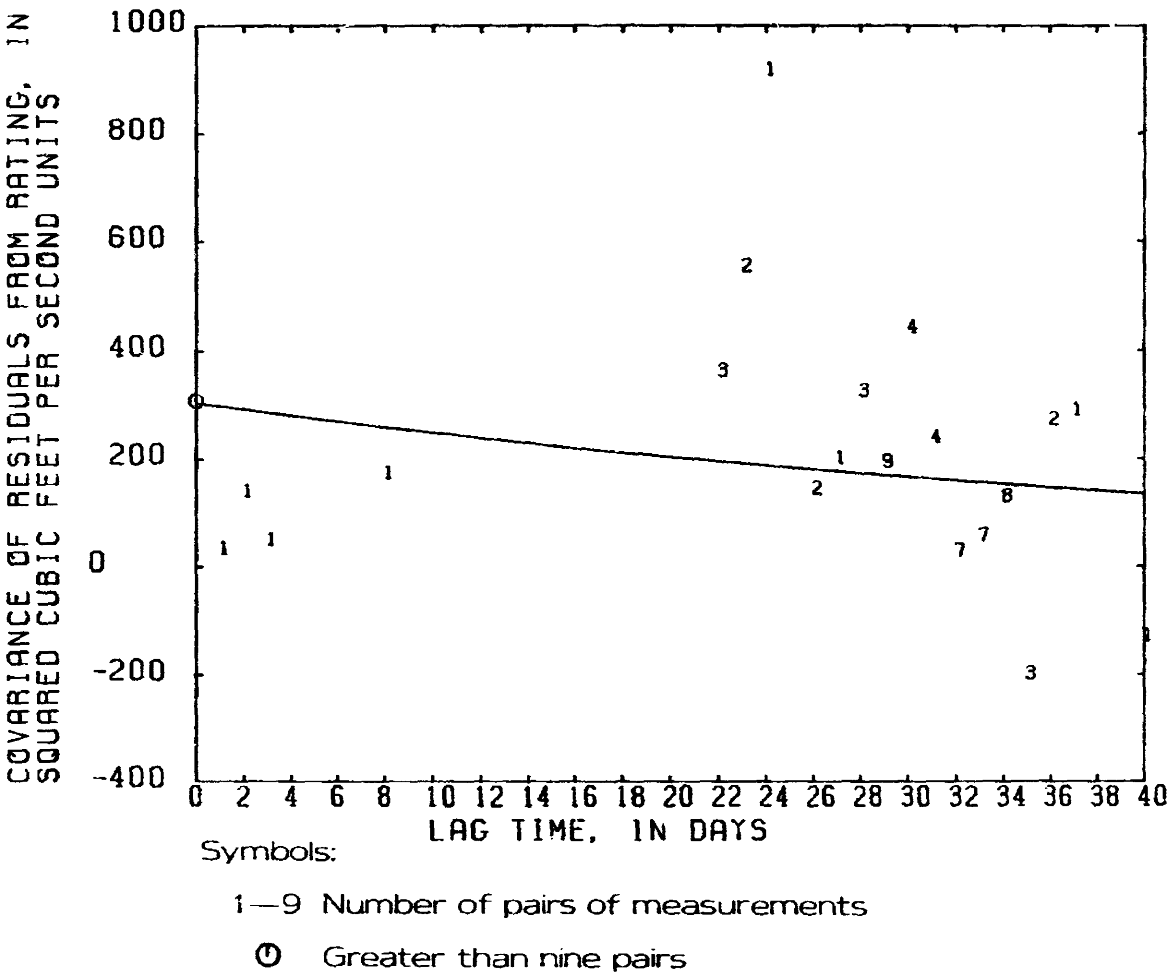


Fig. 33.--Autocovariance function of the differences between true and rated discharge rate for the Colorado River Indian Reservation Lower Main Drain near Parker, Ariz.

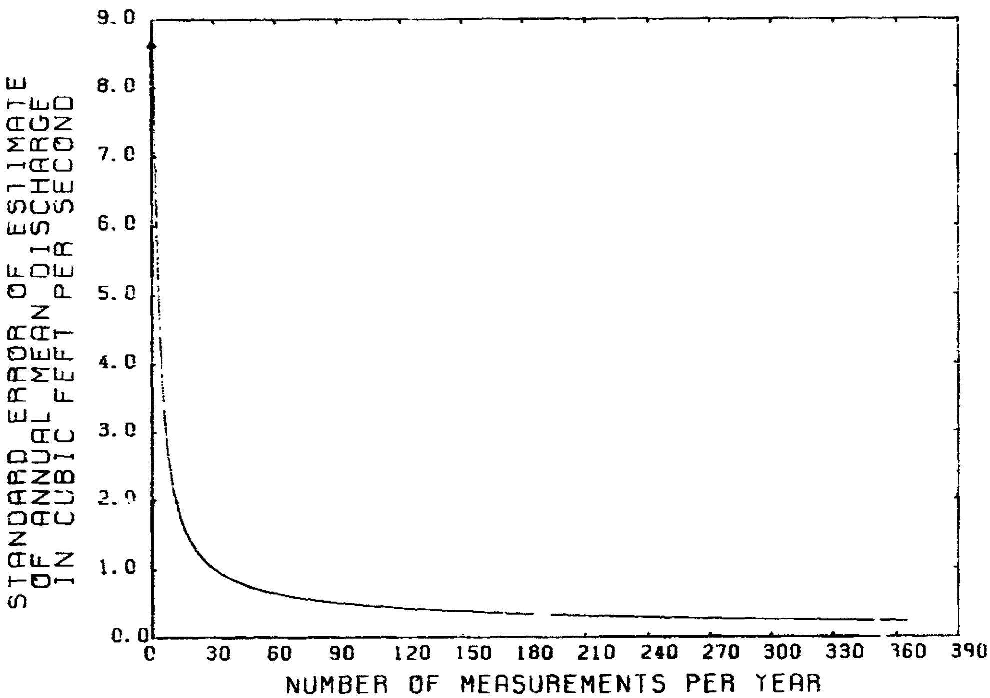


Fig. 34.--Standard error of estimate of annual mean discharge for the Colorado River Indian Reservation Lower Main Drain near Parker, Ariz.

09429130 Palo Verde Irrigation District Olive Lake Drain near Blythe, Calif.

No correlative data is collected except when discharge measurements are being made. The rating is taken to be the average of 24 discharge measurements made during the 1977-78 water years. This average is 9.08 ft^3/s. The 24 measurements have a standard deviation of 4.84 ft^3/s with a minimum value of 0.4 ft^3/s and a maximum of 17.7 ft^3/s. The estimates of the autocovariances of the deviations from this average value are shown in figure 35. A weighted nonlinear least squares fit of these estimates yields a variance of 22.3 $feet^6$ per $second^2$ and a one-day autocorrelation coefficient of 0.98. The average variance of measurement errors is estimated to be 0.15 $feet^6$ per $second^2$. The standard error of estimate of the annual mean discharge as a function of the number of discharge measurements per year is shown in figure 36.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978.

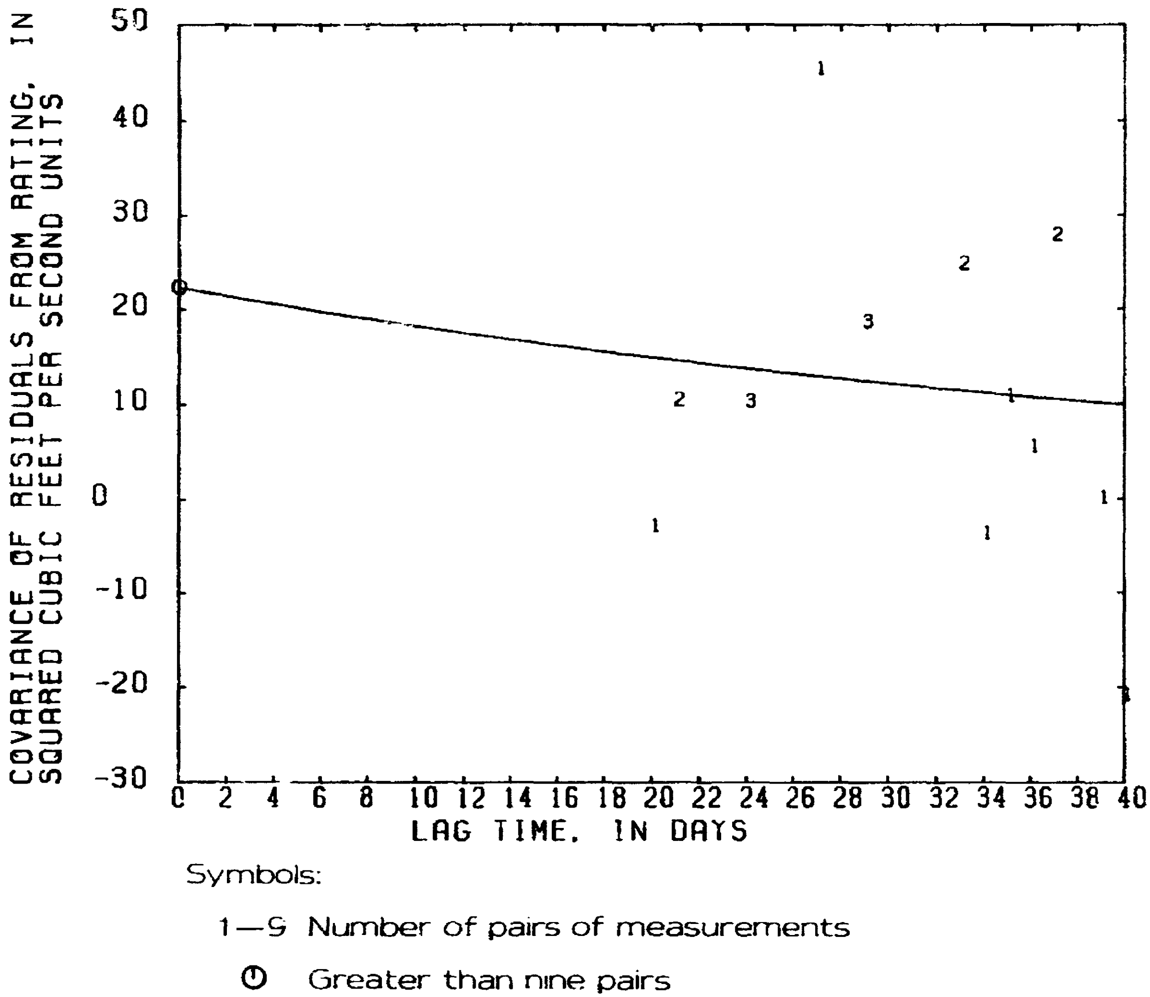


Fig. 35.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District Olive Lake Drain near Blythe, Calif.

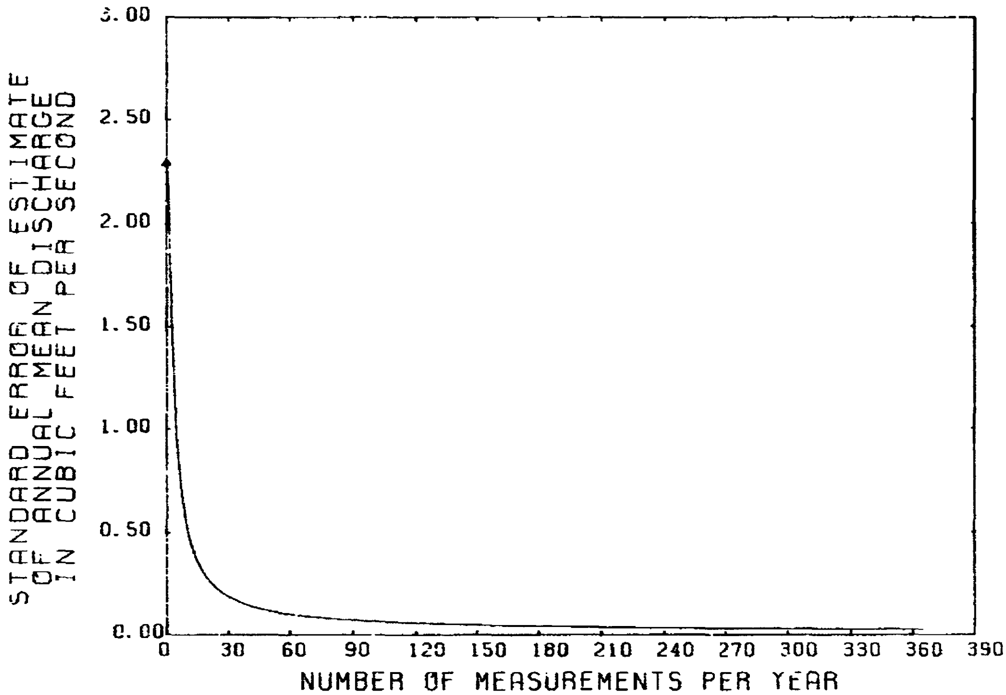


Fig. 36.--Standard error of estimate of annual mean discharge for the Palo Verde Irrigation District Olive Lake Drain near Blythe, Calif.

09429155 Palo Verde Irrigation District F Canal Spill near Blythe, Calif.

The control is a standard five foot concrete Parshall flume. Based on 23 discharge measurements made during the 1976-78 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 22.8 \cdot (h(t) - 15.1)^{1.45}. \tag{50}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 37 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a negligible variance and negligible one-day autocorrelation function. The average measurement variance is estimated to be 0.534 feet^6 per second^2. Because of the negligibleness of the process variance and autocorrelation the standard error of estimate of the annual mean discharge is given as zero for any number of discharge measurements per year.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill represents about 1½ percent of the total return flow.

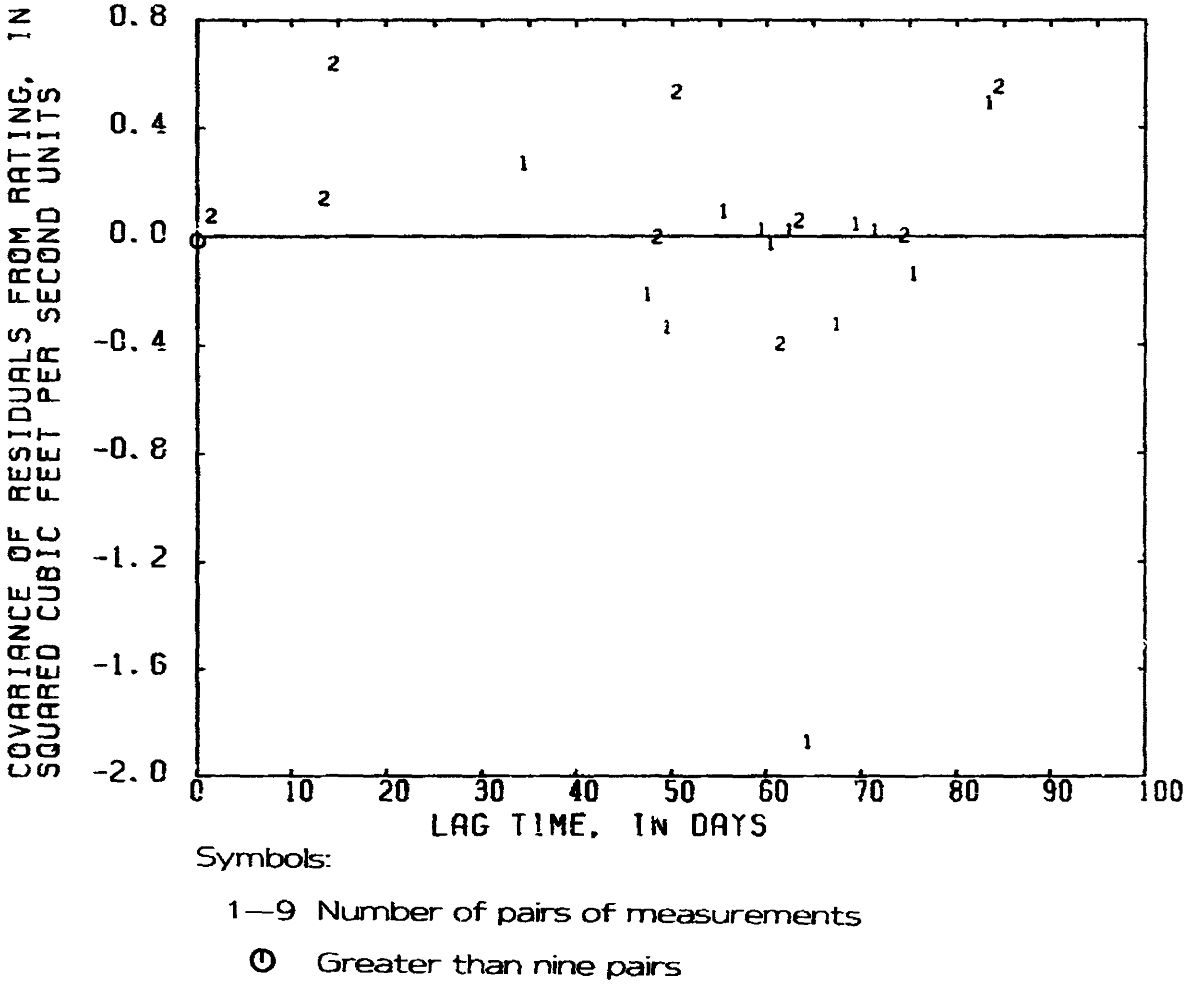


Fig. 37.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District F Canal Spill near Blythe, Calif.

09429170 Palo Verde Irrigation District D-10-11-5 Spill near Blythe, Calif.

Discharge is recorded on a totalizing meter and no discharge measurements are made at this site. The accuracy of the record is fixed by the accuracy of the meter and is, therefore, independent of the frequency of visit to the station.

The record is of 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill amounts to less than 1 percent of the total return flow.

09429180 Palo Verde Irrigation District D-23 Spill near Blythe, Calif.

The control is a 5-foot Parshall flume. Based on 14 discharge measurements made during the 1976-78 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 26.4 \cdot (h(t) - 6.2)^{1.3}. \tag{51}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 38 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 0.557 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.90. The average variance of measurement errors is estimated to be 0.359 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of measurements per year is given in figure 39.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill represents about 2 percent of the total return flow.

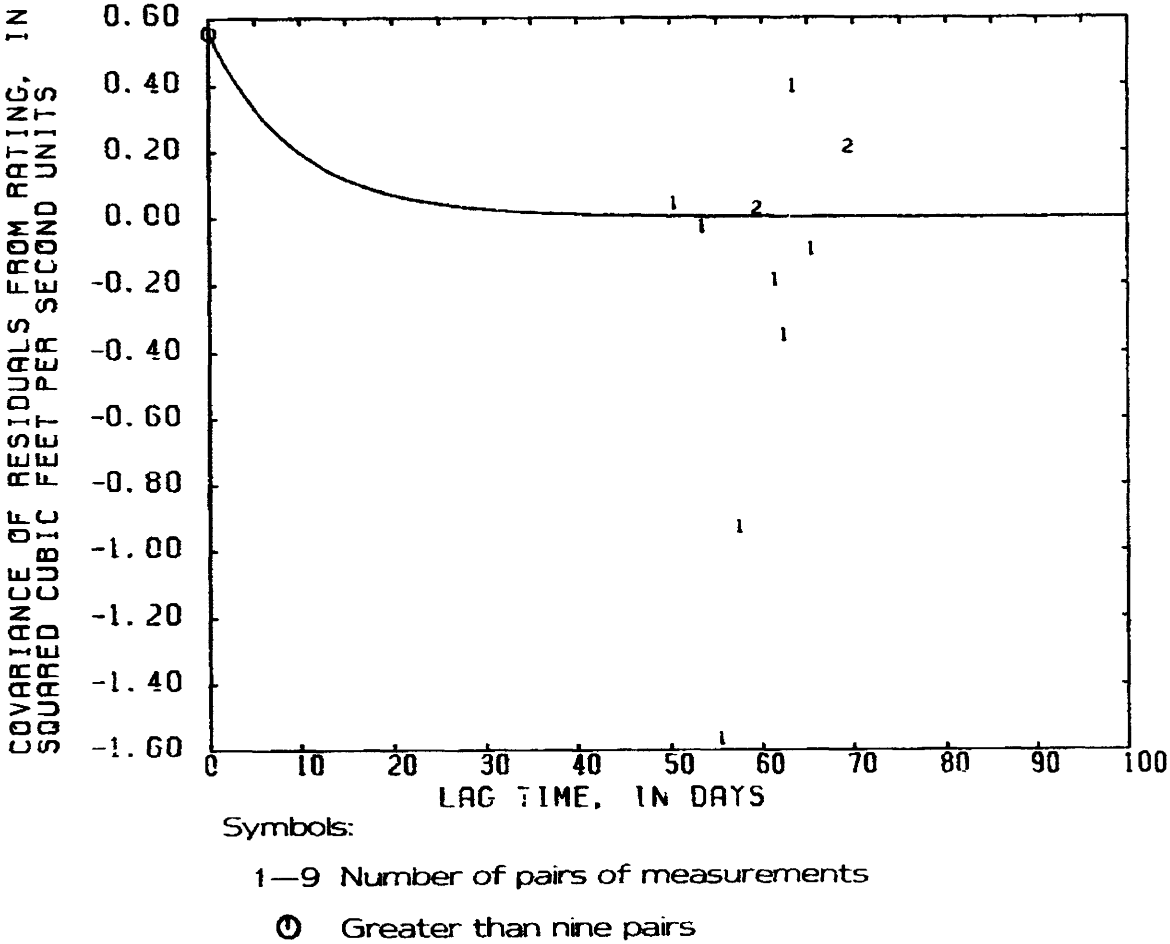


Fig. 38.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District D-23 Spill near Blythe, Calif.

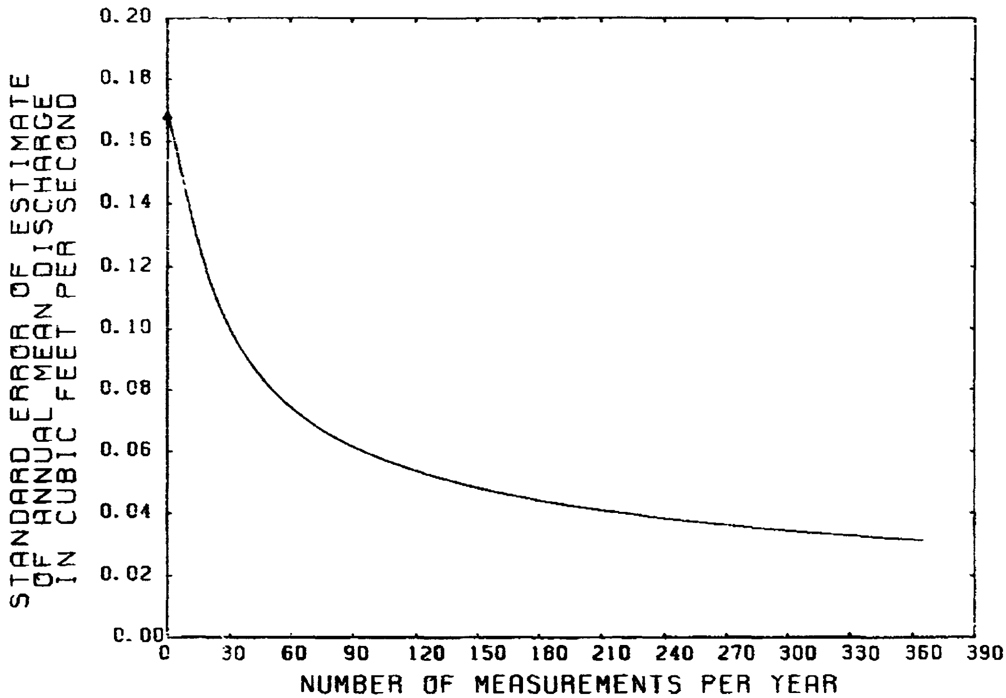


Fig. 39.--Standard error of estimate of annual mean discharge for the Palo Verde Irrigation District D-23 Spill near Blythe, Calif.

09429190 Palo Verde Irrigation District D-23-1 Spill near Blythe, Calif.

Discharge is recorded on a totalizing meter and no discharge measurements are made at this site. The accuracy of the record is fixed by the accuracy of the meter and is, therefore, independent of the frequency of visit to the station.

The record is of 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill amounts to less than 1 percent of the total return flow.

09429200 Palo Verde Irrigation District C Canal Spill near Blythe, Calif.

The control is a 10-foot Parshall flume subject to some moss growth. Based on 33 discharge measurements made during the 1973-78 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 24.1\cdot(h(t) - 0.81)^{2.12} \quad . \tag{52}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 40 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 3.14 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.88. The average variance of measurement errors is estimated to be 0.51 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of measurements per year is given in figure 41.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill represents about 5 percent of this total return flow.

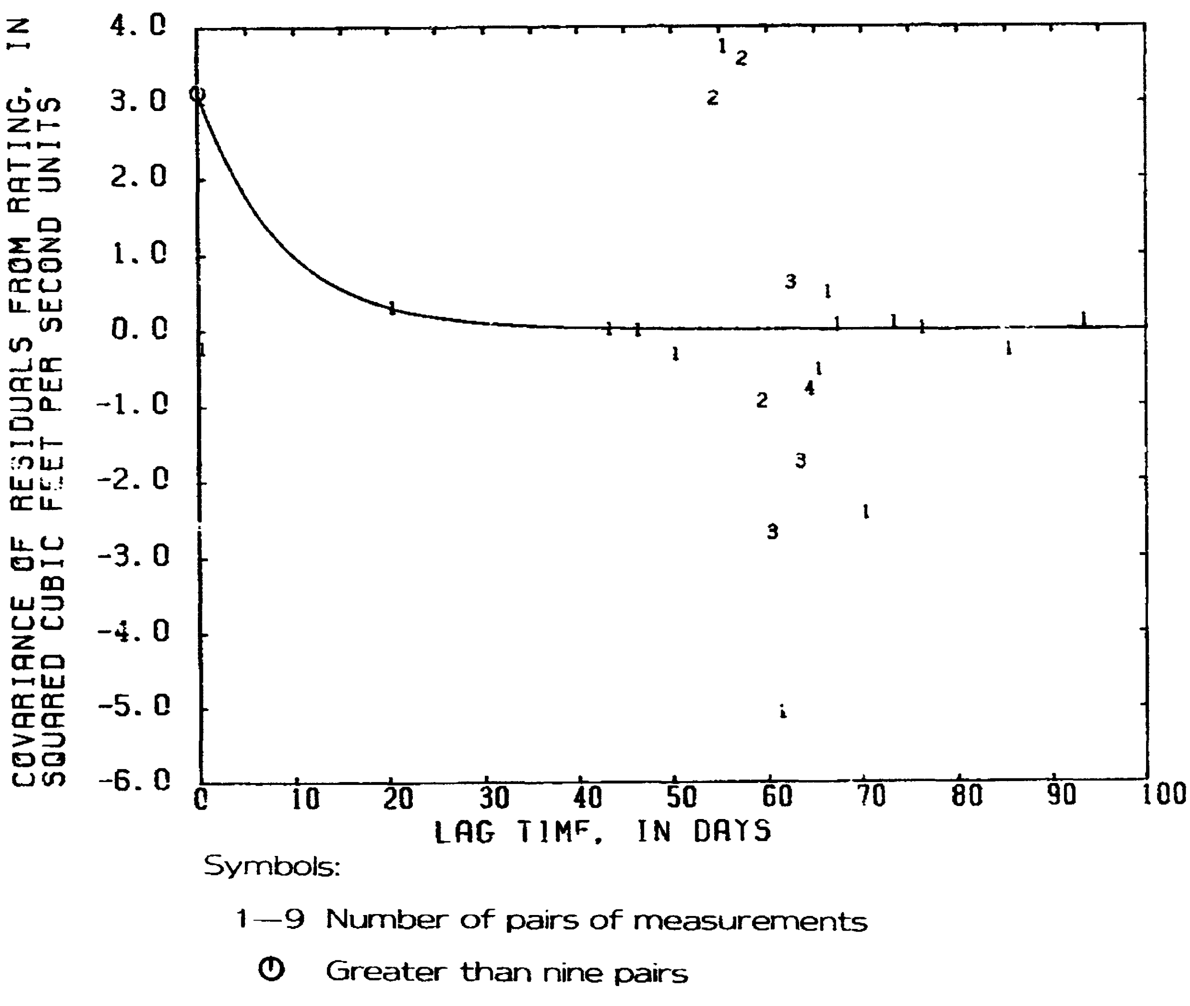


Fig. 40.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District C Canal Spill near Blythe, Calif.

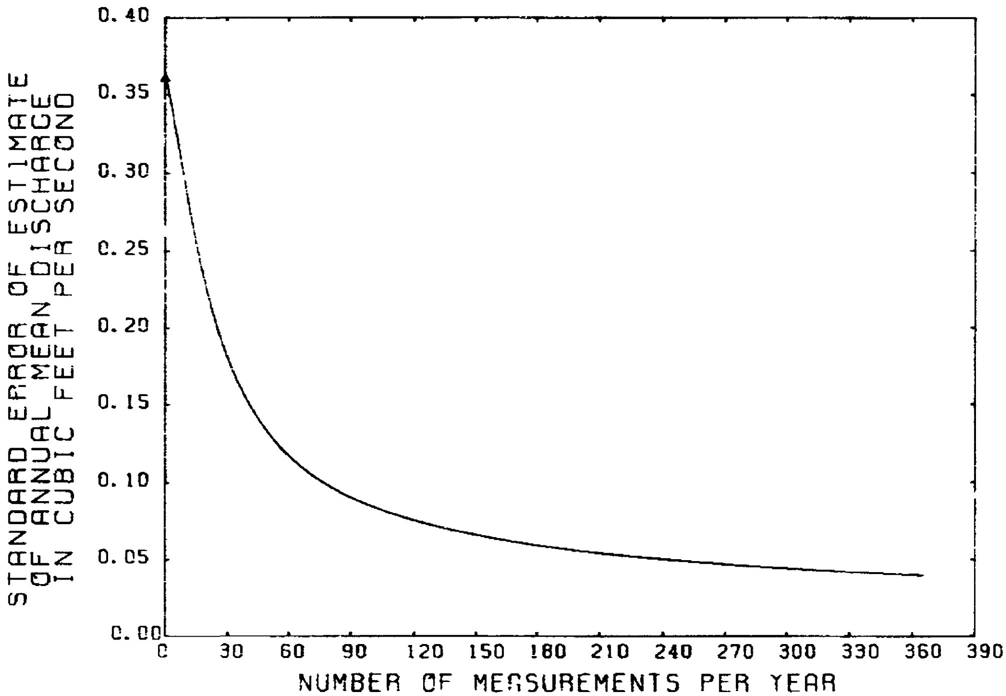


Fig. 41.--Standard error of estimate of annual mean discharge for the Palo Verde Irrigation District C Canal Spill near Blythe, Calif.

09429210 Palo Verde Irrigation District C-28 Upper Spill near Blythe, Calif.

Discharge is recorded on a totalizing meter and no discharge measurements are made at this site. The accuracy of the record is fixed by the accuracy of the meter and is, therefore, independent of the frequency of visit to the station.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill amounts to less than 1 percent of the total return flow.

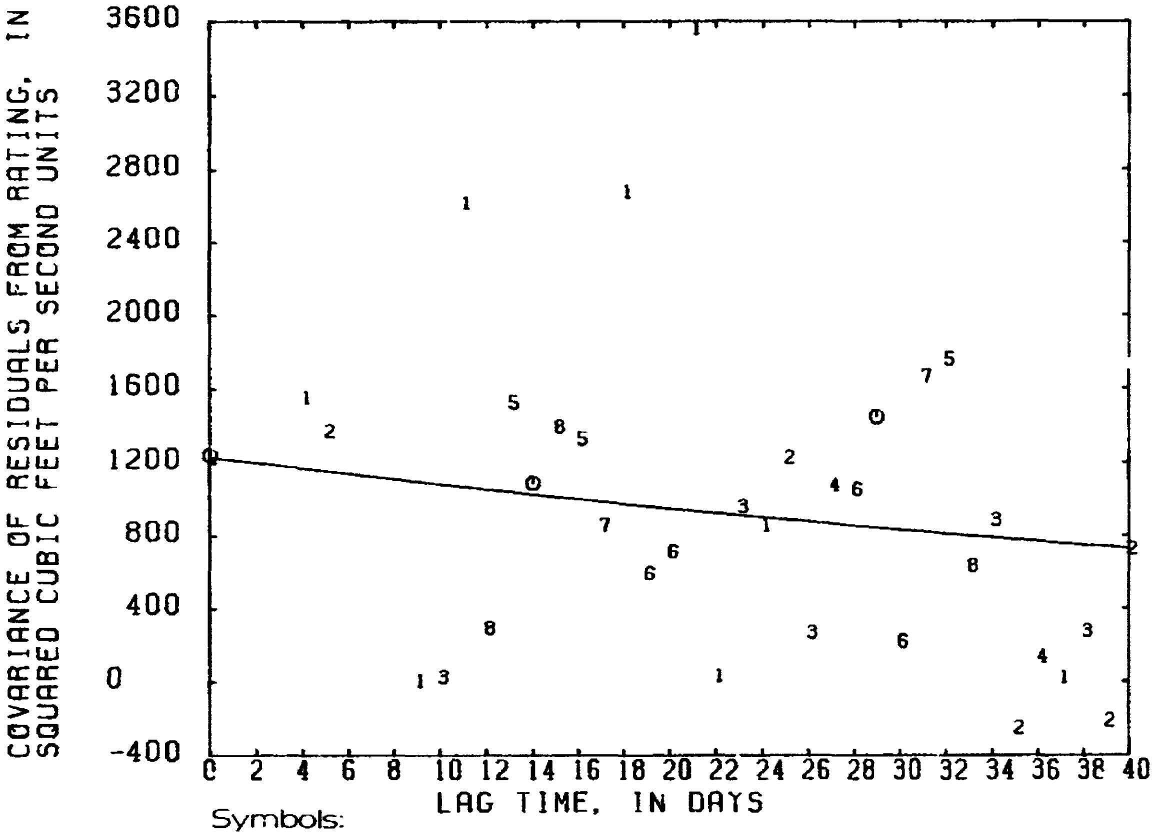
09429220 Palo Verde Irrigation District Outfall Drain near Palo Verde, Calif.

The control is the channel at all stages. Considerable aquatic growth occurs causing seasonal shifting. Based on 70 discharge measurements made during the 1976-78 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 491.1 \cdot (h(t) - 3.6)^{0.43} \tag{53}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 42. A weighted nonlinear least squares fit of these estimates yields a variance of 1234 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.99. The average variance of measurement errors is estimated to be 144 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of measurements per year is given in figure 43.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this station represents nearly 90 percent of the total return flow.



Symbols:

1—9 Number of pairs of measurements

⓪ Greater than nine pairs

Fig. 42.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District Outfall Drain near Palo Verde, Calif.

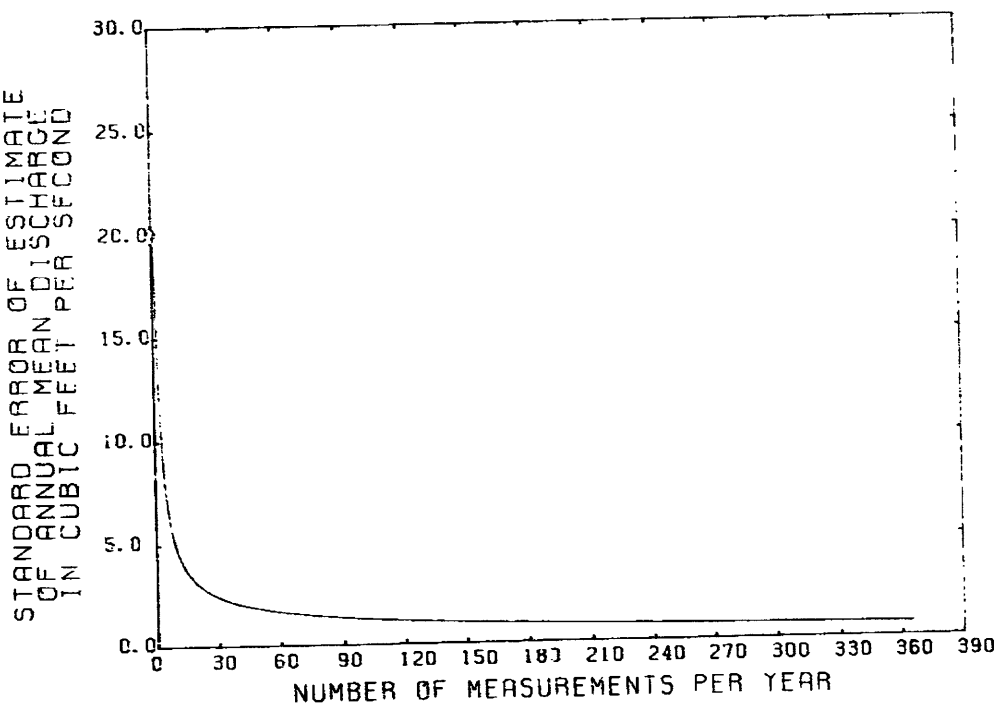


Fig. 43.--Standard error of estimate of annual mean discharge for the Palo Verde Irrigation District Outfall Drain near Palo Verde, Calif.

09429225 Palo Verde Irrigation District Anderson Drain near Blythe, Calif.

Discharge is furnished by Palo Verde Irrigation District. The record is not amenable to the present method of accuracy analysis.

The record is 1 of 11 return flows to the Colorado River which are published as total return flow on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this drain amounts to less than 1 percent of the total return flow.

09429230 Palo Verde Irrigation District C-28 Lower Spill near Blythe, Calif.

The control is a 5-foot Parshall flume subject to some moss growth. Based on 15 discharge measurements made during the 1974-77 water years the rating curve obtained by nonlinear least squares is given by

$$q_R(t) = 14.2 \cdot (h(t) - 0.88)^{1.83}. \tag{54}$$

The estimates of the autocovariances of the deviations from this rating are shown in figure 44 as a function of the time lag. A weighted nonlinear least squares fit of these estimates yields a variance of 0.155 feet^6 per second^2 and a one-day autocorrelation coefficient of 0.96. The average variance of measurement errors is estimated to be 0.11 feet^6 per second^2. The standard error of estimate of the annual mean discharge as a function of the number of measurements per year is given in figure 45.

The record is 1 of 11 return flows to the Colorado River which are published on a monthly basis with station 09429000, Palo Verde Canal near Blythe, Calif. The annual mean return flow for the 11 stations is 644.5 ft^3/s for the water year 1978. The return flow from this spill represents about 2 percent of the total return flow.

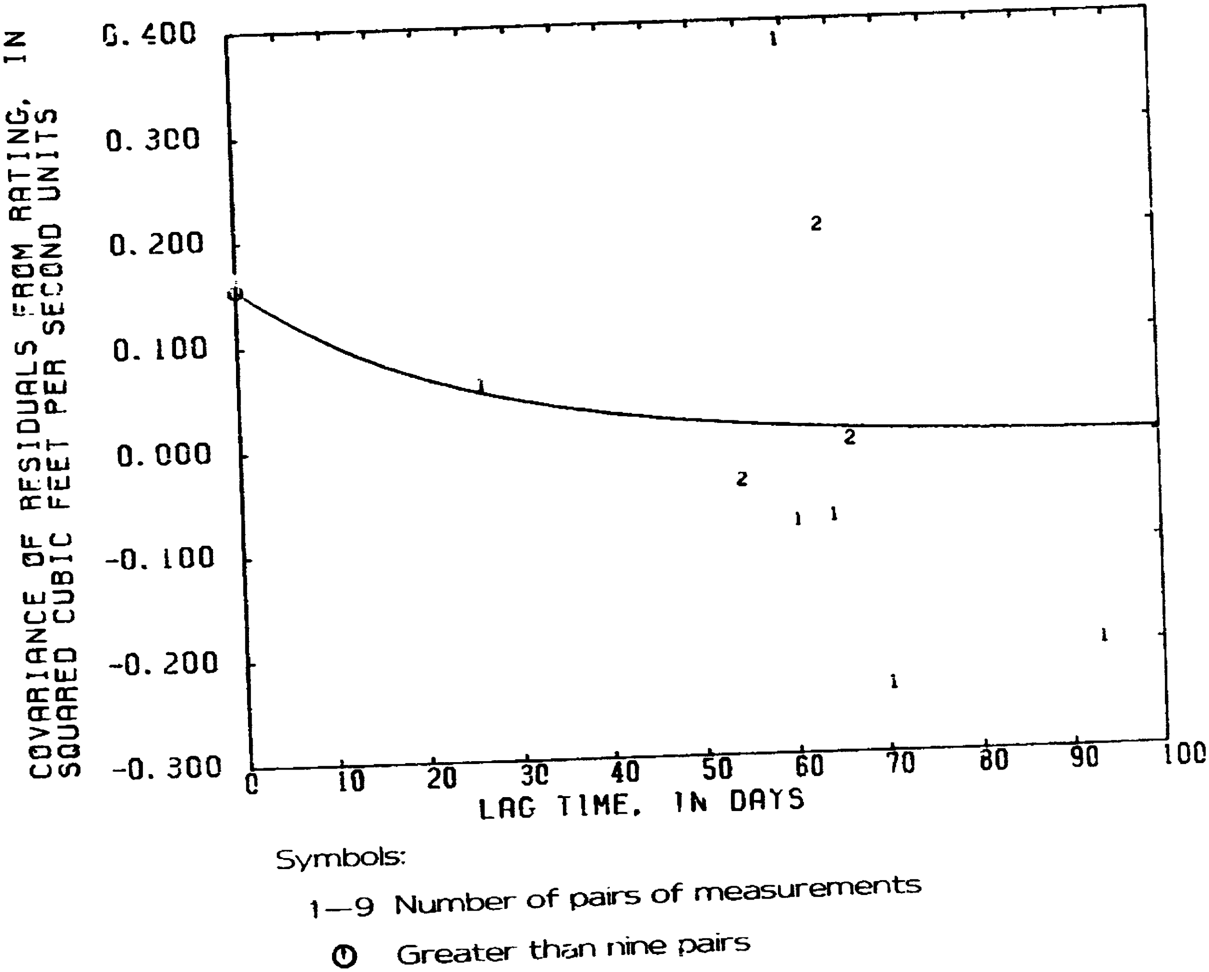


Fig. 44.--Autocovariance function of the differences between true and rated discharge rate for the Palo Verde Irrigation District C-28 Lower Spill near Blythe, Calif.

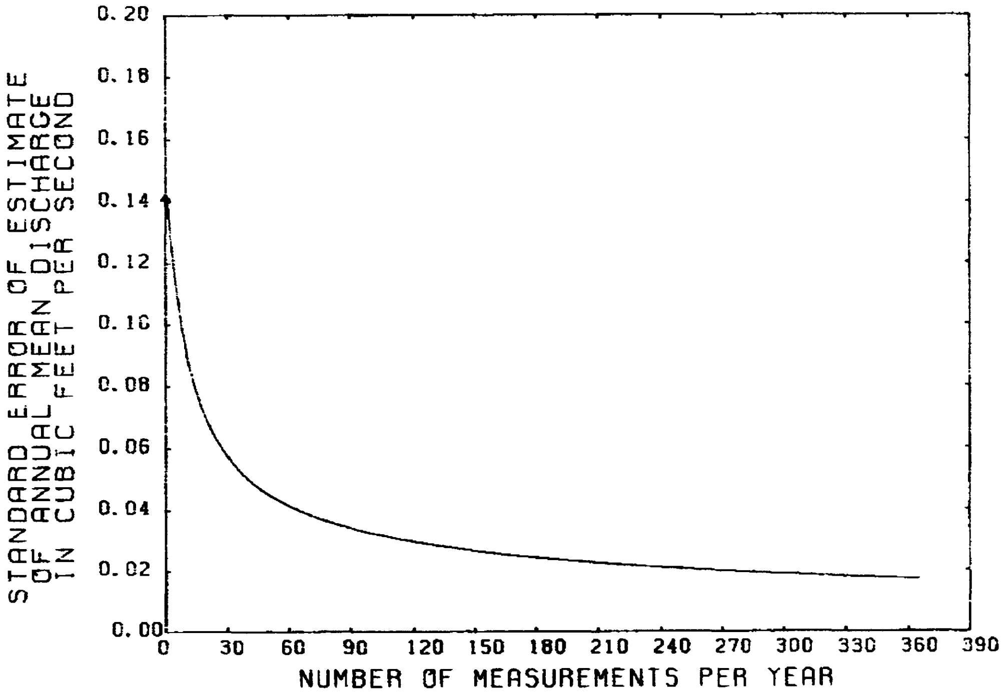


Fig. 45.--Standard error of estimate of annual mean discharge for the Palo Verde Irrigation District C-28 Lower Spill near Blythe, Calif.

09429280 Cibola Lake Inlet near Cibola, Ariz.

The rating is based on area and velocity obtained from a deflection meter. The deflection vane is located in a submerged concrete flume. Only 10 discharge measurements were made during the 1975-78 water years and only 8 of these are usable in obtaining a rating. Not enough data were available to apply the proposed method of accuracy analysis.

Discharge is published on a monthly basis only. The annual mean discharge for water year 1978 was 6.79 ft^3/s.

09429290 Cibola Lake Outlet near Cibola, Ariz.

The control is an adjustable Cipolletti weir. Present setting for the weir does not allow measurement of the low flows (under 20 cfs) normally available. Record is not amenable to the proposed method of accuracy analysis.

Discharge is published on a monthly basis only. The annual mean discharge for water year 1978 was 1.84 ft^3/s.

UNIT COSTS OF STREAM GAGING

As was discussed earlier, the definition of cost-effective stream-gaging strategies requires the specification of four types of costs: (1) visit costs, α_j, for each stream gage, consisting of the average service, maintenance, and measurement costs incurred in a visit to a station, but excluding travel costs to the gage; (2) route costs, β_i, for each route, consisting of the costs of a hydrographer's time to travel the route and any associated per diem and the operation, maintainance, and rental costs of a vehicle; (3) fixed costs, which include the cost of computing, publishing and storing the data; and (4) overhead, which includes salaries of managers and supervisors, technical support, and office rental.

Personnel of the Arizona District Office, the Yuma Subdistrict Office, and the Blythe Field Office developed the required cost data that are shown in table 2 (J. D. Camp, written communication, April 1980). The set of practical routes, also defined in table 2, were jointly developed by the authors and Arizona District personnel.

Overhead is charged as 42 percent of the gross budget.

Table 2.--Unit costs and route definitions for the Blythe Field Office.

Route	Unit cost, in dollars	09423000	09423550	09427520	09428500	09428505	09428510	09429000	09429010	09429030	09429060	09429130	09429155	09429170	09429180	09429190	09429200	09429210	09429220	09429230
		Station																		
1	145	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	240	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	115	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	115	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	115	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	90	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	120	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	40	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	35	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10	35	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
11	55	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
12	55	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
13	55	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
14	50	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
15	35	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
16	20	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
17	25	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
18	35	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
19	55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
20	65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
21	65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
22	65	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
23	70	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
24	35	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0
25	55	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
Visit cost, in dollars		45	40	65	65	15	20	58	60	60	25	15	15	0	15	0	15	0	25	15
Fixed cost, in dollars		2480	3510	2600	2860	1560	5200	2990	4490	780	1430	780	1560	390	1170	390	1430	390	1890	1300

RESULTS

Cost figures of the last section were used in conjunction with the foregoing uncertainty relations to specify cost-effective strategies for several possible budgets. Three sets of minimum-visit constraints, one, six, and twelve visits per year to each of the 19 gaging stations in the analysis, were considered. The one-visit minimum, as a lower limit on the accuracy that can be obtained for annual mean discharge, is not feasible with the equipment that is currently in use to record the correlative data of gage heights and gate openings. This equipment should be serviced bimonthly, six visits per year, in order that reasonably continuous records of the correlative data be available. On the other hand, if monthly mean discharges must also be computed at the end of each month, twelve visits to each station are required every year.

Currently (1980) a minimum of 12 and a maximum of 29 visits are made to any of the gaging stations. However, discharge measurements are not made each time a station is visited. Table 3 provides the visit-frequencies currently used and the resulting total uncertainty, which are integrated with the cost data and presented as a point on figure 46. Figure 46 reveals that a similar level of uncertainty in annual mean discharge can be obtained for a budget of about $95,000 with a six-visit minimum or for about $10[illegible],000 for twelve-visit minimum. The changes in visit frequency entailed by these two latter strategies also are presented in table 3.

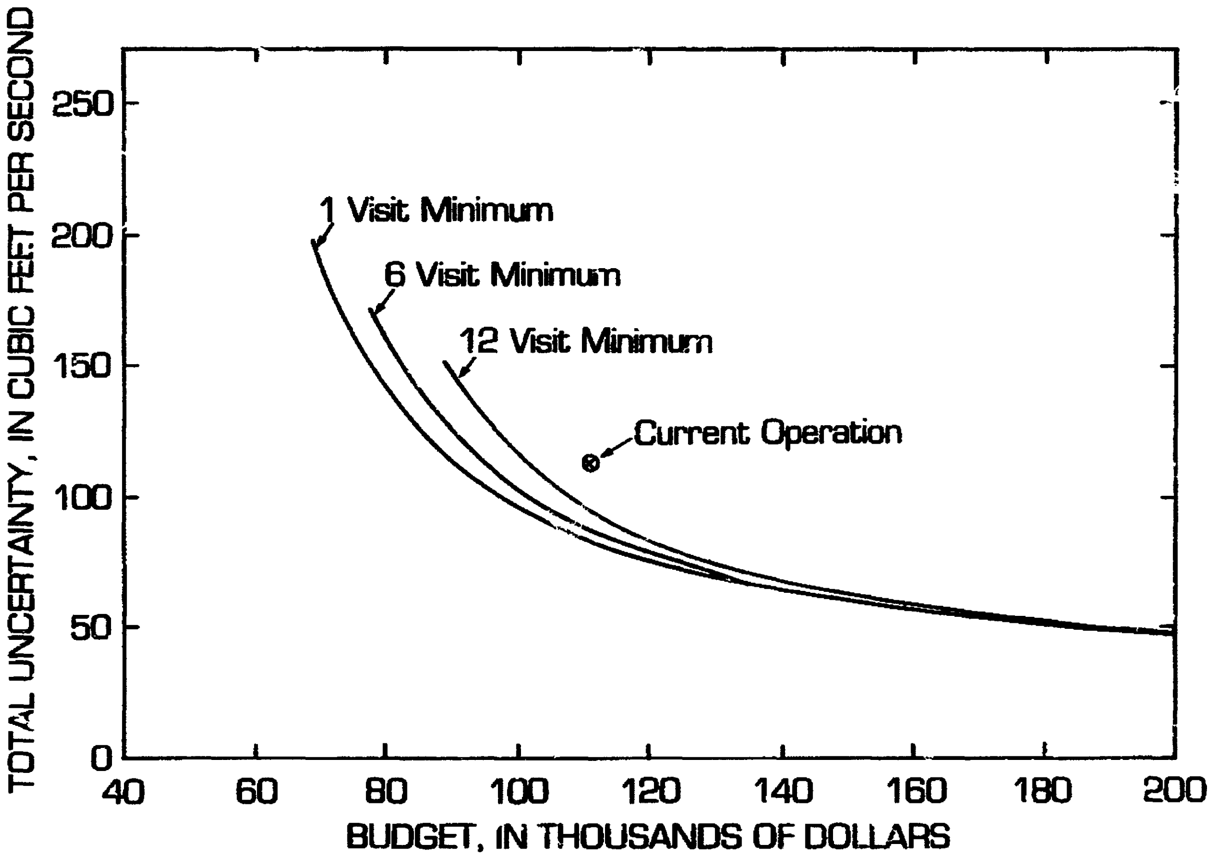


Fig. 46.--The total uncertainty of cost-effective schemes and of the current operation of the Blythe Field Office.

Table 3.--Gaging strategies for the Blythe Field Office

Station	Visits Per Year: Six-visit Minimum*	Twelve-visit Minimum*	Current Operation	Six-visit Minimum+	Twelve-visit Minimum+
09423000	44	37	29	26	25
09423550	6	12	27	6	12
09427520	42	35	29	27	26
09428500	8	12	29	6	12
09428505	6	12	27	6	12
09428510	7	13	27	6	12
09429000	6	12	27	6	12
09429010	74	61	29	43	42
09429030	6	12	12	6	12
09429060	6	12	27	6	12
09429130	6	12	12	7	12
09429155	6	12	18 1/	6	12
09429170	6	12	12 2/	6	12
09429180	6	12	18 1/	6	12
09429190	6	12	12 2/	6	12
09429200	6	12	18 1/	6	12
09429210	6	12	12 2/	6	12
09429220	6	12	24 3/	6	12
09429230	6	12	18 1/	6	12
Budget, in Thousands of 1980 Dollars	110.9	110.9	110.9	95.0	101.0
Uncertainty, in ft^3/s	87	94	113	112	112

*Constant cost network

+Constant uncertainty network

1/ Six discharge measurements.

2/ No discharge measurements (totalizing meter).

3/ Twelve discharge measurements.

The budget for current operations, $110,900, can be expended so as to reduce the total uncertainty in annual-mean-discharge estimates below that derived under the current scheme. If monthly discharge must be computed currently, the total uncertainty can be reduced from 113 ft^3/s to 94 ft^3/s by increasing the frequency of discharge measurement at the three gaging stations on the mainstem of the Colorado River (09423000, 09427520, and 09429010) at the expense of reduced measurement frequency at several off-stream stations. Increases and decreases in measurement frequency can be determined by comparing visit-frequencies in table 3.

An additional reduction of 7 ft^3/s of uncertainty from 94 to 87 can be obtained by relaxing the constraint of a twelve-visit minimum at each site to a six-visit minimum. This difference of 7 ft^3/s of uncertainty in the annual-mean-discharge estimates can be considered a cost of supplying timely monthly-discharge estimates.

Cost-effective gaging strategies for budgets smaller than those shown in table 3 result in reduced measurement frequencies at those stations that are above the minimum-visit constraint--primarily the three mainstem stations. On the other hand, larger budgets will begin to add visits to stations that are at the minimum once the uncertainty is reduced sufficiently at the mainstem stations. However, even with a budget of $200,000 for the 19 stations, only four stations off of the mainstem of the Colorado River are measured more frequently than the twelve-visit minimum. These stations are: 09428500, Colorado River Indian Reservation Main Canal near Parker, Ariz.; 09428510, Colorado River Indian Reservation Poston Wasteway near Poston, Ariz.; 09429000, Palo Verde Canal near Blythe, Calif.; and 09429130, Palo Verde Irrigation

District Olive Lake Drain near Blythe, Calif. The first three of these are the major sources of uncertainty among the non-mainstem stations; the fourth is measured more frequently because of significant uncertainty and a fortuitous location that teams it with two other stations of more significant uncertainty to form a single route. The contributions of uncertainty of the remaining eleven stations that are measured only at the minimum frequency of once per month is 0.2 ft^3/s out of a total of 48 ft^3/s.

CONCLUSIONS

Of the 25 stream gaging stations operated out of the Blythe Field Office, 19 are amenable to a procedure to estimate the accuracy of the estimates of annual mean discharge as a function of the number of discharge measurements made during a year. Comparison of the measurement frequencies currently used at the 19 stations with those determined to be cost-effective in estimating the annual mean discharge shows that a savings of less than $10,000 or 10 percent can be garnered by moving to a cost-effective scheme that provides current monthly discharges within the year. Elimination of the provision of current monthly records can save another $6,000 at the same level of uncertainty in the annual-mean-discharge estimates. On the other hand, expending the current budget in a cost-effective manner can reduce the total uncertainty in annual-discharge estimates by about 20 percent.

The vast majority of the uncertainty is attributable to the stations on the mainstem of the Colorado River. Secondary uncertainties are derived from the main diversions and drains, and insignificant amounts are contributed by the minor drains. The cost-effectiveness of discontinuiance of these minor drains will be explored in a subsequent phase of this study.

The proper allocation of resources to the Blythe operations can not be accomplished in isolation from other demands for the same funds and personnel. This is particularly true of the remainder of the Lower Colorado River Basin network that is operated out of the Yuma Subdistrict Office. The Yuma operations will be subjected to an analysis of the sort contained herein, and the two studies will be merged to define the proper allocation of stream gaging resources between these two subunits of the network.

REFERENCES

Arizona v. California, 1963, 373, U.S. 546; Decree 376 U.S. 340 (1964).

Buchanan, T. J., and Somers, W. P., 1969, Discharge measurements at gaging stations: U. S. Geological Survey Techniques of Water Resources Investigations, Book 3, Chapter A8, 65 p.

Burkham, D. E., and Dawdy, D. R., 1968, Error analysis of streamflow data for an alluvial stream: U.S. Geological Survey Professional Paper 655-C, 13 p.

Carter, R. W., and Anderson, I.E., 1963, Accuracy of current meter measurements: American Society of Civil Engineers Proceedings Journal of the Hydraulics Division, v. 89, no. HY4, p. 105-115.

Colorado River Compact, 1922.70 Congressional Record 324, 325, 1928; Nov. 24.

Ficke, J. F., and Hawkinson, R. O., 1975, The National Stream Quality Accounting Network (NASQAN) - Some questions and answers: U.S. Geological Survey Circular 719, 23 p.

Fisher, R. A., 1960, The design of experiments (8th ed.): New York Hafner Publishing Company, 248 p.

Gelb, A. (editor), 1974, Applied optimal estimation: Cambridge, The Massachusetts Institute of Technology Press, 374 p.

Herschy, R. W., 1978, Accuracy: Chapter 10 *in* Herschy, R.W., ed., Hydrometry, John Wiley and Sons, Ltd., Chichester, England, p. 353-397.

Moore, S. F., 1978, Applications of Kalman filter to water quality studies, *in* Chiu, C. L., ed., Applications of Kalman filter to hydrology, hydraulics, and water resources: University of Pittsburg, Pittsburg, Pa., p. 485-507.

Powell, J. W., 1875, Exploration of the Colorado River of the west and its tributaries: Explored in 1870, 1871, and 1872: Smithsonian Institution, Washington, D.C.

Wagner, H. M., 1969, Principles of operations research: Englewood Cliffs, N. J., Prentice-Hall, Inc., 939 p. plus appendices.