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PRECIPITATION MODEL FOR THE PLATTE RIVER VALLEY FROM GOTHENBURG TO GRAND ISLAND, NEBRASKA

by Aldo V. Vecchia, Jr.

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METRIC CONVERSION

Multiply inch-pound unit	By	To obtain metric unit
mile	1.609	kilometer
inch	25.40	millimeter

PRECIPITATION MODEL FOR THE PLATTE RIVER VALLEY FROM GOTHENBURG TO GRAND ISLAND, NEBRASKA

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Aldo V. Vecchia, Jr.

ABSTRACT

A trigonometric linear model of the 1934 to 1978 monthly precipitation series for Gothenburg, Kearney, and Grand Island, Nebraska, is presented. Statistical comparisons supporting the validity of the simulated sequences to represent the historical series include quantile comparisons of this latter series to simulated monthly values along with comparisons of actual annual statistics to annual statistics obtained by aggregating the simulated monthly values.

The models are used to determine probabilities of within-year droughts and probabilities of drought durations for a 50-year simulation period. For purposes of this study, a drought occurs when precipitation is less than 2/3, 1/2, or 1/3 of the historical mean value at each station for May-June, June-July, and May-June-July.

INTRODUCTION

The preservation or deterioration of the migratory bird habitat along the Platte River in south-central Nebraska is very dependent on the hydrology of the area. Data indicating the probability of occurrence of certain precipitation and runoff events can be used in making decisions for man-induced changes in the river system to achieve desired features of the habitat. These features include wide, shallow channels characterized by sparse vegetation on the river banks and channel islands. One hydrologic condition that affects the amount of vegetation within the habitat is little or no precipitation during various parts of the year. The impact of these precipitation shortages would be twofold: (1) The lack of surface moisture would naturally reduce the potential for the germination of new seeds and (2) little or no precipitation would cause an increase in ground-water pumping in the surrounding agricultural areas, thereby reducing the subsurface moisture content for existing vegetation along the river, which is hydraulically linked to the aquifer system.

The purpose of this report is to describe the design and development of a stochastic model of the precipitation in a critical habitat reach of the Platte River. The critical reach is defined in this report as the reach of the Platte River from Lexington to Grand Island, Nebraska (fig. 1). The precipitation in this reach is assumed to be adequately described by the monthly series of data collected at the Gothenburg, Kearney, and Grand Island precipitation stations. The model of the joint monthly precipitation for the three stations will be used to obtain the probabilities of within-year droughts of given durations and intensities during a 50-year planning period in order to evaluate the management alternatives affecting the habitat along the critical reach.

Two families of models are considered candidates to describe the monthly precipitation series: (1) The autoregressive integrated moving average (ARIMA) time series model and (2) the trigonometric linear model. A desirable property of either model is an adequate degree of accuracy in the reproduction and forecasting of precipitation while still maintaining parsimony of the parameters.

The ARIMA model fit to the precipitation data will be shown to be nearly identical to a sinusoidal model of period 12. The latter model is selected over the ARIMA model, because of the ease of use in simulating series, and because of the property of having a periodically stationary mean.

The sinusoidal model with a period 12 component was inadequate for use in simulation of some of the individual months, so components of period 3, 4, and 6 were added to the model. These components were not strong enough to be included as nonstationary components in the ARIMA model, so the original ARIMA model was not extended to compare with the final trigonometric model.

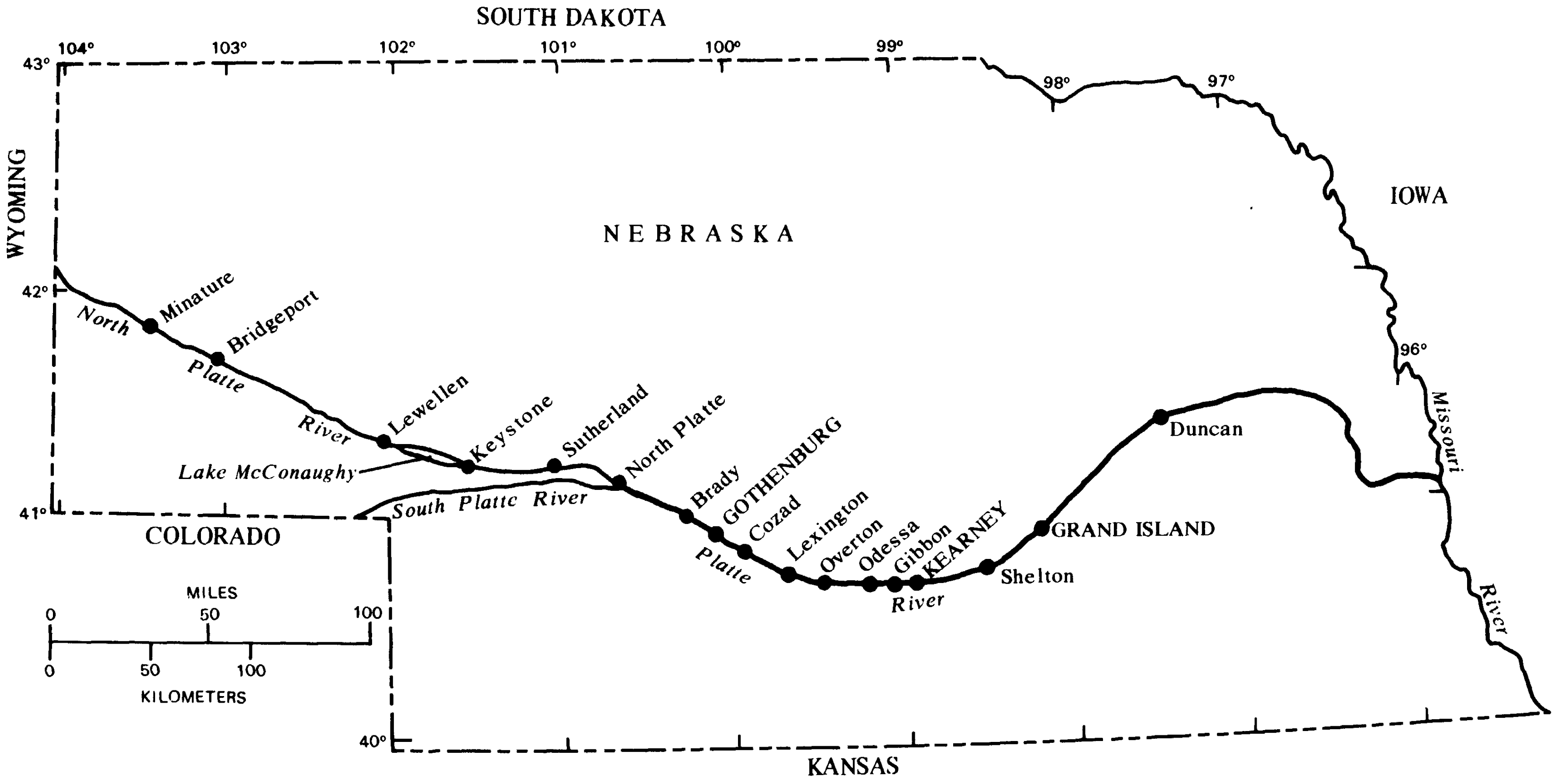

Figure 1.--Map showing location of the Platte River study area from Gothenburg to Grand Island.

A transformation of the data which allows adequate representation of the monthly values is determined, even when using the same transformation for each of the 12 months; this transformation eliminates the need to estimate the mean and standard deviation of each month separately to standardize the data. This latter procedure involves estimating 24 parameters, each having only a few degrees of freedom, while the proposed transformation uses the entire data set for the estimation of each needed parameter.

PRELIMINARY DATA ANALYSIS

The monthly means and standard deviations of the precipitation series from 1934 to 1978 for Gothenburg, Kearney, and Grand Island are listed in table 1. The large difference in standard deviations among the months is important to note. For example, the standard deviation of the June values for Kearney is about 2.8, while the standard deviation for January is about 0.4, a disparity ratio of about 7. Most time-series models or linear models assume a constant error variance for the whole series; thus, a transformation of the data to alleviate this problem is proposed. Graphs of the Gothenburg, Kearney, and Grand Island historical series from January 1, 1939, to December 31, 1978 (480 months) are shown in figures 2-4. In these figures, the months characterized by smaller average precipitation have a smaller variance than wetter months. The data also are truncated at zero and positively skewed, indicating that if a model were fit to the untransformed data, the random component would not be normally distributed. For example, if the deterministic part of the model gives a value of 0.5 inch for January, then the random component for that month cannot be less than -0.5 inch, because negative precipitation values are impossible. An objective of this report is to make the random component of the model approximately normally distributed for convenience in simulations and evaluation of probabilities. A Box-Cox transformation (Box and Cox, 1964), as described in the following section, was used for that purpose.

Table 1.--*Precipitation statistics for Gothenburg, Kearney, and Grand Island, 1934 to 1978*

Month	Gothenburg		Kearney		Grand Island	
	Mean (inches)	Standard deviation (inches)	Mean (inches)	Standard deviation (inches)	Mean (inches)	Standard deviation (inches)
Jan.	0.440	0.381	0.511	0.401	0.525	0.390
Feb.	.500	.440	.688	.539	.786	.641
Mar.	1.220	.881	1.357	1.138	1.238	1.067
Apr.	2.156	1.516	2.420	1.670	2.421	1.493
May	3.431	2.120	3.915	2.227	3.864	2.080
June	4.118	2.279	4.141	2.781	3.871	2.331
July	2.587	1.610	2.962	1.911	2.710	2.040
Aug.	2.493	1.285	2.410	1.479	2.447	1.621
Sept.	1.892	1.453	2.451	1.986	2.443	2.153
Oct.	1.045	1.163	1.348	1.252	1.031	.951
Nov.	.597	.588	.730	.849	.787	.837
Dec.	.473	.344	.576	.473	.603	.550

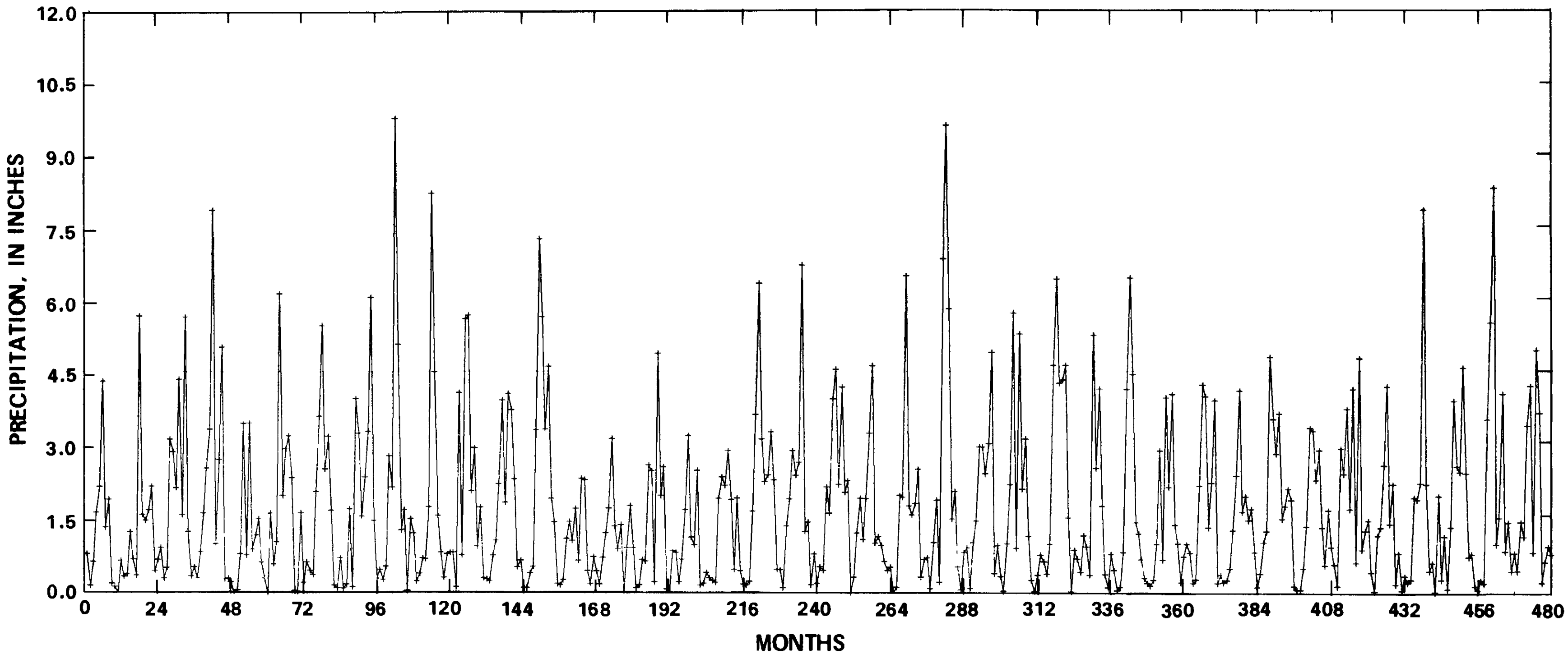

Figure 2.--Monthly precipitation at Gothenburg, January 1, 1939 through December 31, 1978.

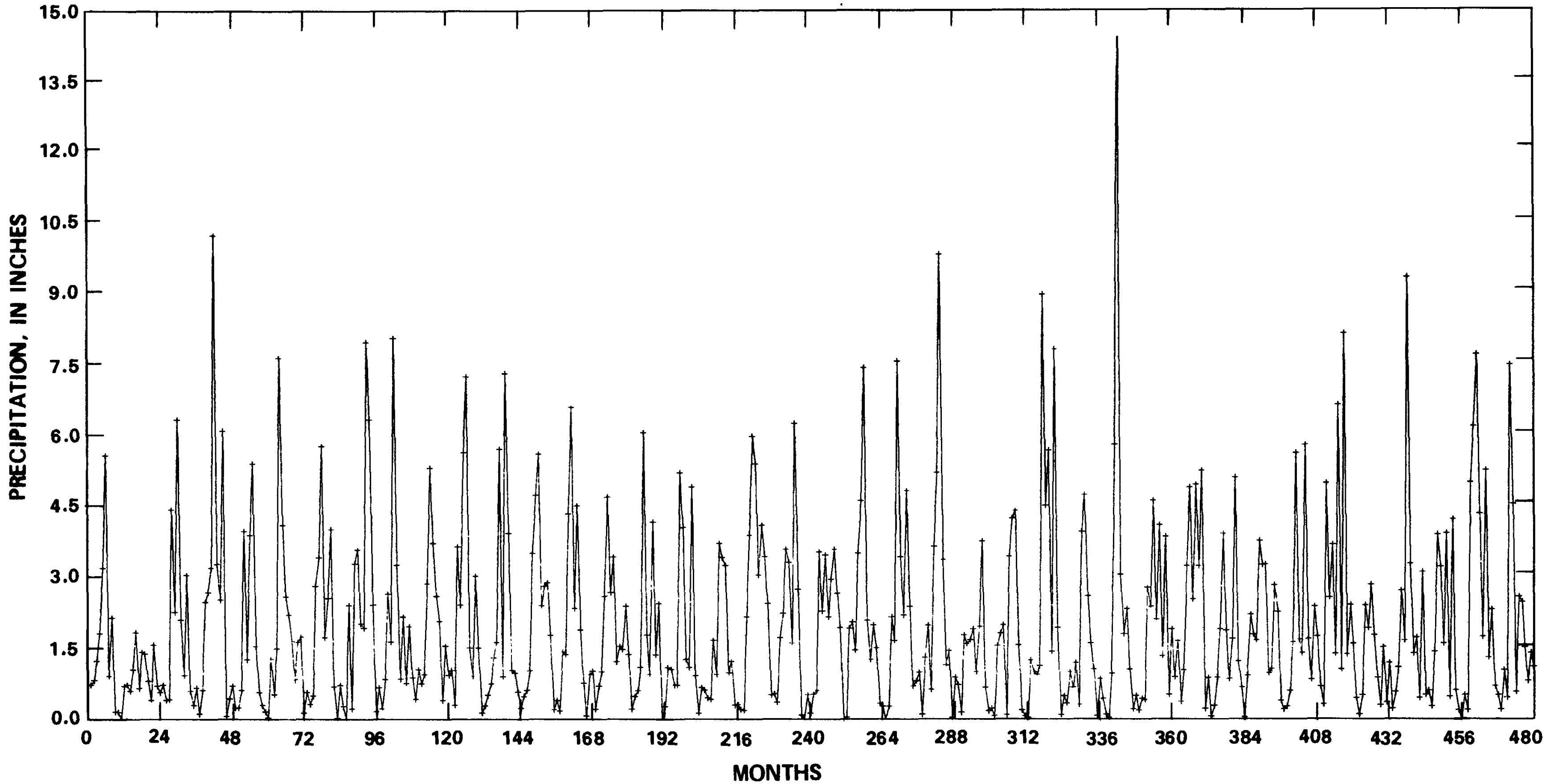

Figure 3.--Monthly precipitation at Kearney, January 1, 1939, through December 31, 1978.

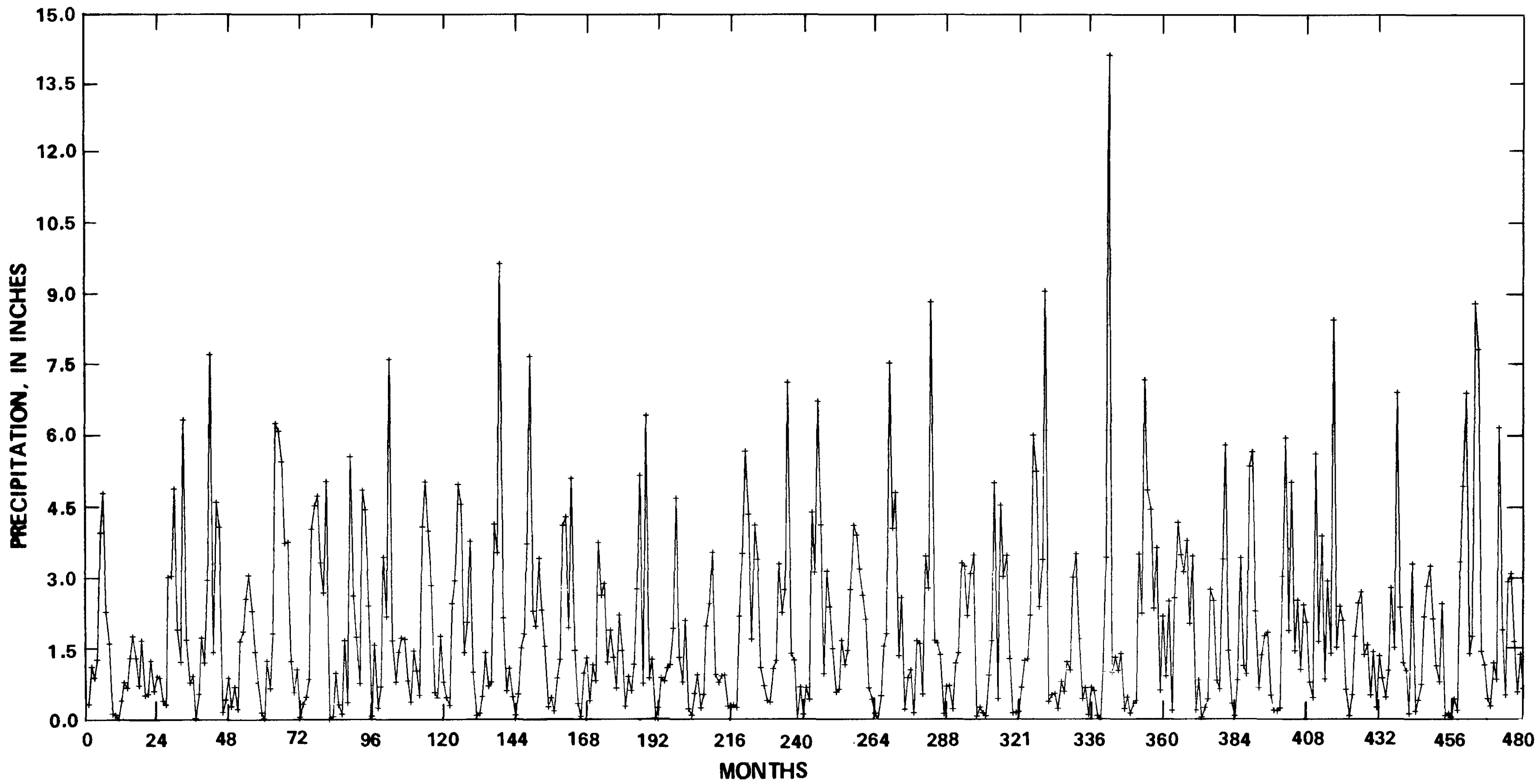

Figure 4.--Monthly precipitation at Grand Island, January 1, 1939, through December 31, 1978.

DETERMINATION OF THE TRANSFORMATION PARAMETER

Consider transformations of the form:

$$Z_t^{(\lambda)} = \frac{(X_t+c)^{\lambda}-1}{\lambda} \text{ , } \lambda\neq 0 \text{ ; and} \quad (1)$$

$$Z_t^{(\lambda)} = \log_e(X_t+c) \text{ , } \lambda=0 \text{ ;} \quad (2)$$

where

$Z_t^{(\lambda)}$ = the transformed data value at time t;

X_t = actual data value at time t; and

c,λ = constants to be determined.

Box and Cox (1964) explain how λ and c can be estimated with the other parameters in the model using the method of maximum likelihood. Their method assigns a larger likelihood to those values of λ which make the residuals approach a normal distribution. This functional form of the transformation was chosen because it is continuous at $\lambda=0$, because:

$$\lim_{\lambda\to 0} \frac{(X_t+c)^{\lambda}-1}{\lambda} = \lim_{\lambda\to 0} \frac{(X_t+c)^{\lambda}\log(X_t+c)}{1} = \log(X_t+c) \quad (3)$$

by applying l'Hopital's rule. All that is required of c is that it be a constant that makes each value X_t+c positive, so it was chosen to be 0.01 and only λ was estimated.

Consider the estimation of λ for the Kearney monthly precipitation series based on the assumption that this series follows an ARIMA model. The autocorrelations and the spectral density of the monthly precipitation totals at Kearney for 1934-78 are shown in figure 5. There is a periodicity of 12 evident in both, indicating that an operation be performed on the series X_t by the operator $1-1.73B+B^2$, where $B^kX_t = X_{t-k}$. In other words, a new series Y_t is formed from the original series by defining:

$$Y_t = (1-1.73B+B^2)X_t = X_t - 1.73X_{t-1} + X_{t-2} \text{ .}$$

This operator was suggested by Gray and others (1978) to remove sinusoidal seasonality of period 12. The usual Box-Jenkins operator, $1-B^{12}$, also removes sinusoidal seasonality of period 12 as well as many other types of seasonality, but, in this instance, $1-1.73B+B^2$ adequately removes the nonstationarity.

No parameters need be estimated to remove the seasonality, because the operator is adaptive in amplitude. This can easily be seen by noting that:

$$(1-1.73B+B^2)(\alpha \sin\frac{2\pi t}{12}) = \alpha \sin\frac{2\pi t}{12} - 1.73\sin\frac{2\pi(t-1)}{12} + \sin\frac{2\pi(t-2)}{12} = 0, \quad (4)$$

regardless of the value of α. The autocorrelations of the series $(1-1.73B+B^2)X_t$ clearly indicated that it followed a moving average of order two, due to nonzero estimated autocorrelations for lags one and two, and nearly zero values for the remaining lags. Therefore, the following parametric family was postulated for the Kearney station:

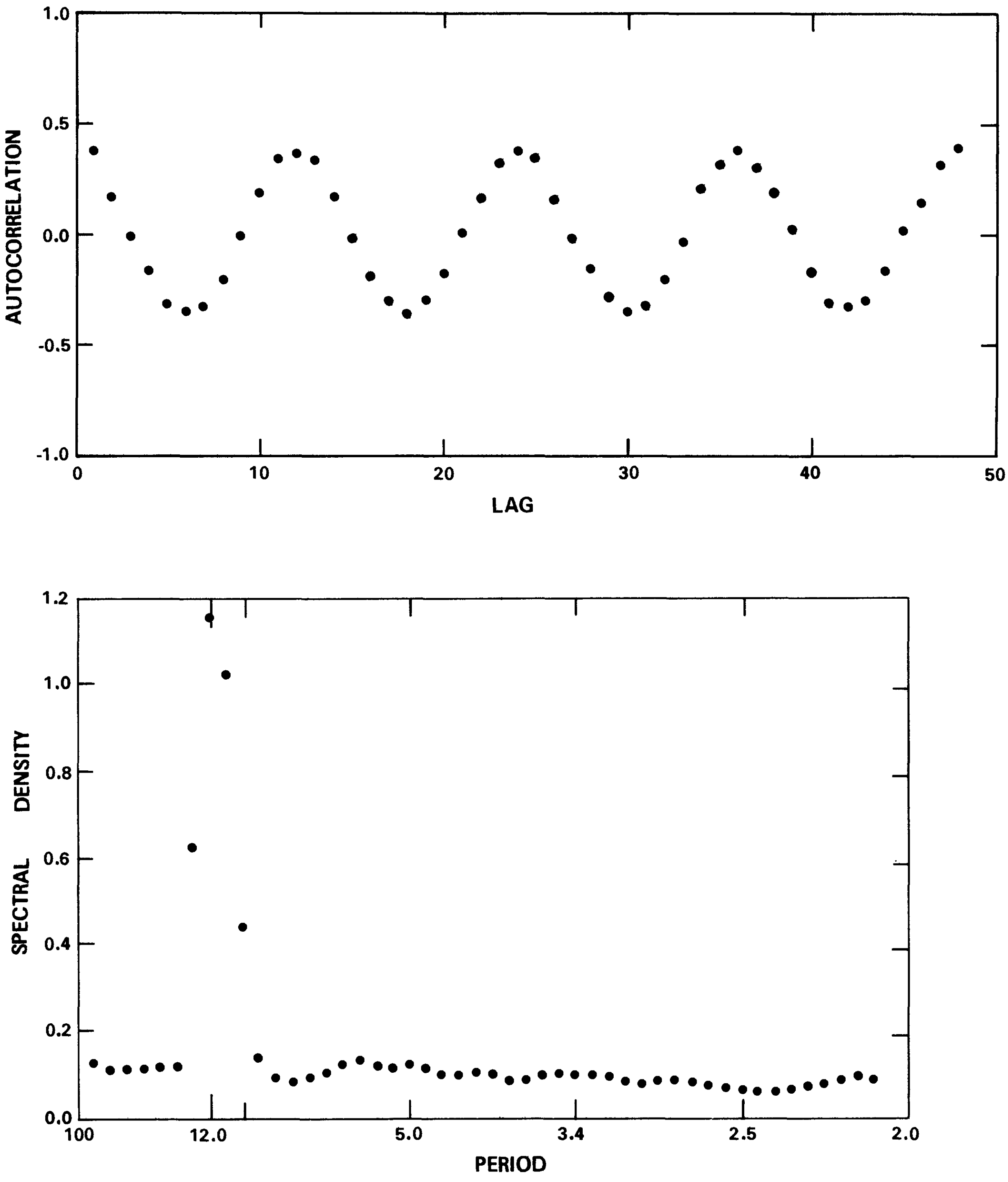

Figure 5.--Autocorrelations and spectral density of the monthly precipitation totals at Kearney, January 1, 1939, through December 31, 1978.

$$(1-1.73B+B^2)Z_t^{(\lambda)} = \mu + (1-\theta_1 B-\theta_2 B^2)A_t \ , \qquad (5)$$

where

$Z_t^{(\lambda)}$ is the transformed data;

$\{A_t\}$ is a sequence of independent, identically distributed (iid) random variables, $A_t \sim N(0, \sigma^2)$; and

$\mu, \theta_1, \theta_2, \sigma^2$ are model parameters.

The natural log of the likelihood function can be shown to be approximately (Hipel and others, 1977):

$$-\frac{n}{2}\log \frac{SS}{n} + (\lambda-1) \sum_{t=1}^{n} \log(X_t+c) \equiv L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) \ , \qquad (6)$$

where

SS = the sum of squares of the residuals from the model, which (given the data) is a function of the parameters; and

n = the adjusted series length (in this instance, 538, because two observations are lost due to the nonstationary operator).

The series was transformed with λ = 1, 1/2, 1/3, 1/4, and 0, and the resulting series were each examined to insure that the transformation did not alter the type of ARIMA model followed. The time series parameters were then estimated for each value of λ by the method of maximum likelihood with the following results:

1. $Z_t = X_t$ (λ = 1) (Value of the likelihood is the same as that for $Z_t = (X_t+0.01) -1)$;

   Model: $(1-1.73B+B^2)Z_t = 0.53 + (1-1.69B+0.96B^2)\ A_t$;

   Residual SS = 1,426, $L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) = -262$.

2. $Z_t = (\sqrt{X_t+0.01} -1)*2$ (λ=1/2) ;

   Model: $(1-1.73B+B^2)Z_t = 0.13 + (1-1.64B+0.90B^2)A_t$;

   Residual SS = 563.24, $L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) = -36.78$.

3. $Z_t = (\sqrt[3]{X_t+0.01} -1)*3$ (λ=1/3) ;

   Model: $(1-1.73B+B^2)Z_t = 0.094 + (1-1.62B+0.88B^2)A_t$;

   Residual SS = 495.76, $L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) = -10.054$.

4. $Z_t = (\sqrt[4]{X_t+0.01} - 1)*4 \quad (\lambda=1/4)$;

Model: $(1-1.73B+B^2)Z_t = 0.076 + (1-1.61B+0.87B^2)A_t$;

Residual SS = 488.59, $L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) = -10.141$.

5. $Z_t = \log(X_t+0.01) \quad (\lambda=0)$;

Model: $(1-1.73B+B^2)Z_t = 0.023 + (1-1.61B+0.86B^2)A_t$;

Residual SS = 601.35, $L(\lambda,\theta_1,\theta_2,\mu,\sigma^2) = -48.073$.

The log likelihood for different values of λ assuming that the likelihood is a smooth, continuous function in λ is shown in figure 6.

To see why $\lambda=1/3$ is preferred over $\lambda=1$, or $\lambda=0$, graphs of part of the series of residuals from Z_t for each value of λ are presented. When $\lambda=1$, the residuals are truncated negatively and positively skewed (fig. 7). The data in figure 8 ($\lambda=1/3$) show great improvement, while figure 9 ($\lambda=0$) indicates that taking logs of the data overcorrects, by grouping the positive residual values close to zero and making the negative values too large in absolute value.

JUSTIFICATION FOR USING A PERIODIC MEAN MODEL

The rainfall data showed a strong periodicity of 12, justifying the use of the operator $1-1.73B+B^2$. However, assume a time series follows a deterministic mean that is a mixture of sines and cosines of period 12 with white noise added. Then:

$$X_t = \mu' + \alpha_1 \cos\frac{2\pi t}{12} + \alpha_2 \sin\frac{2\pi t}{12} + \varepsilon_t \ ; \qquad (7)$$

where $\{\varepsilon_t\}$ are distributed iid with mean 0 and constant variance; and:

$$(1-1.73B+B^2)X_t = (1-1.73B+B^2)\varepsilon_t + \mu \ ; \qquad (8)$$

where $\mu = (1-1.73B+B^2)\mu' = 0.27\mu'$. In the rainfall model, after the $1-1.73B+B^2$ operation was performed on $Z_t^{(\lambda)}$, the resulting series followed a moving average of order two with parameters very near to $\theta_1=1.73$, and $\theta_2=-1.0$, regardless of the value of λ. That is, the model was very close to $(1-1.73B+B^2)Z_t^\lambda = \mu + (1-1.73B+B^2)\varepsilon_t$; thus, the driving mechanism behind the precipitation series may be a deterministic one with added white noise.

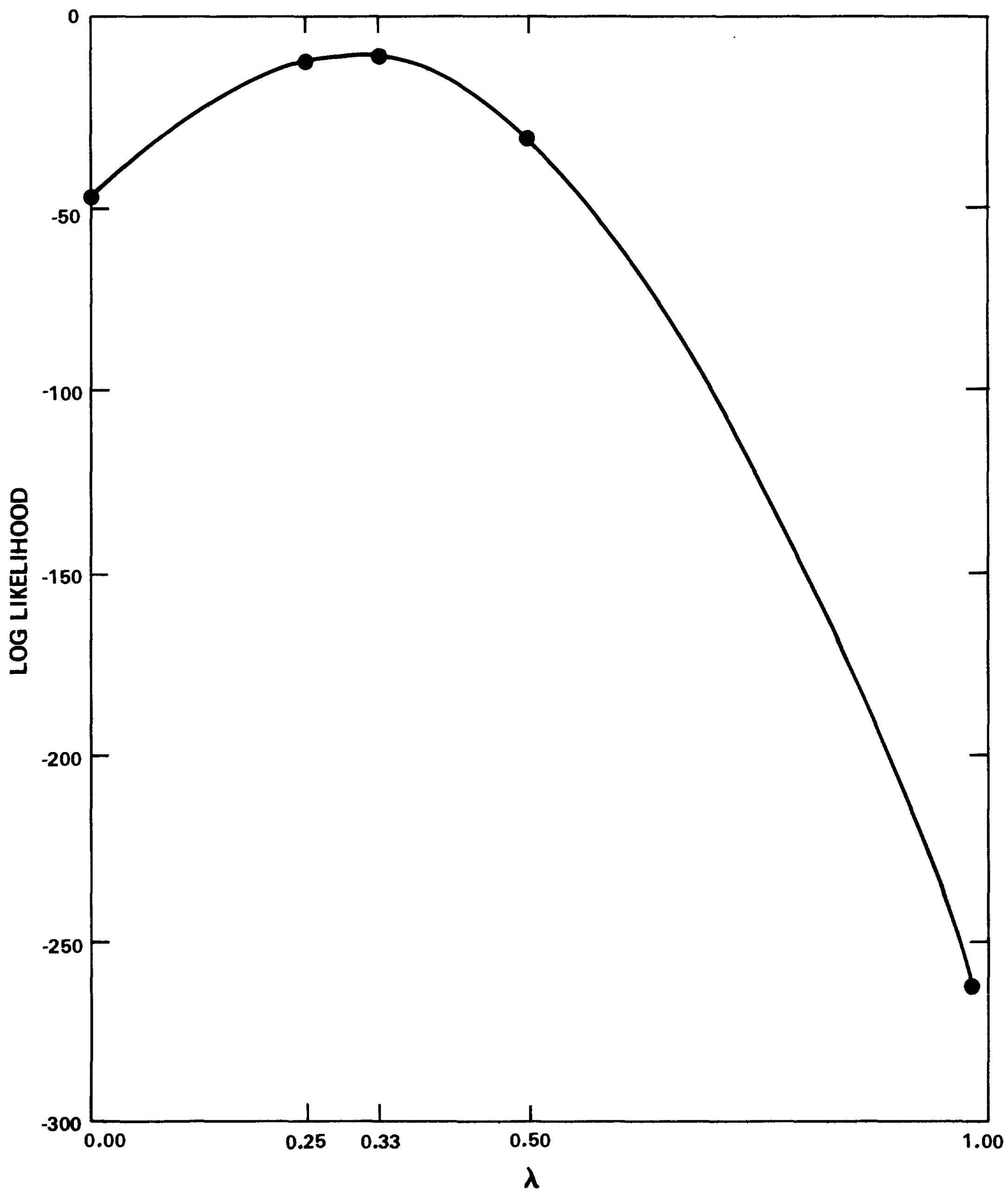

Figure 6.--Log likelihood for different values of the transformation parameter, λ.

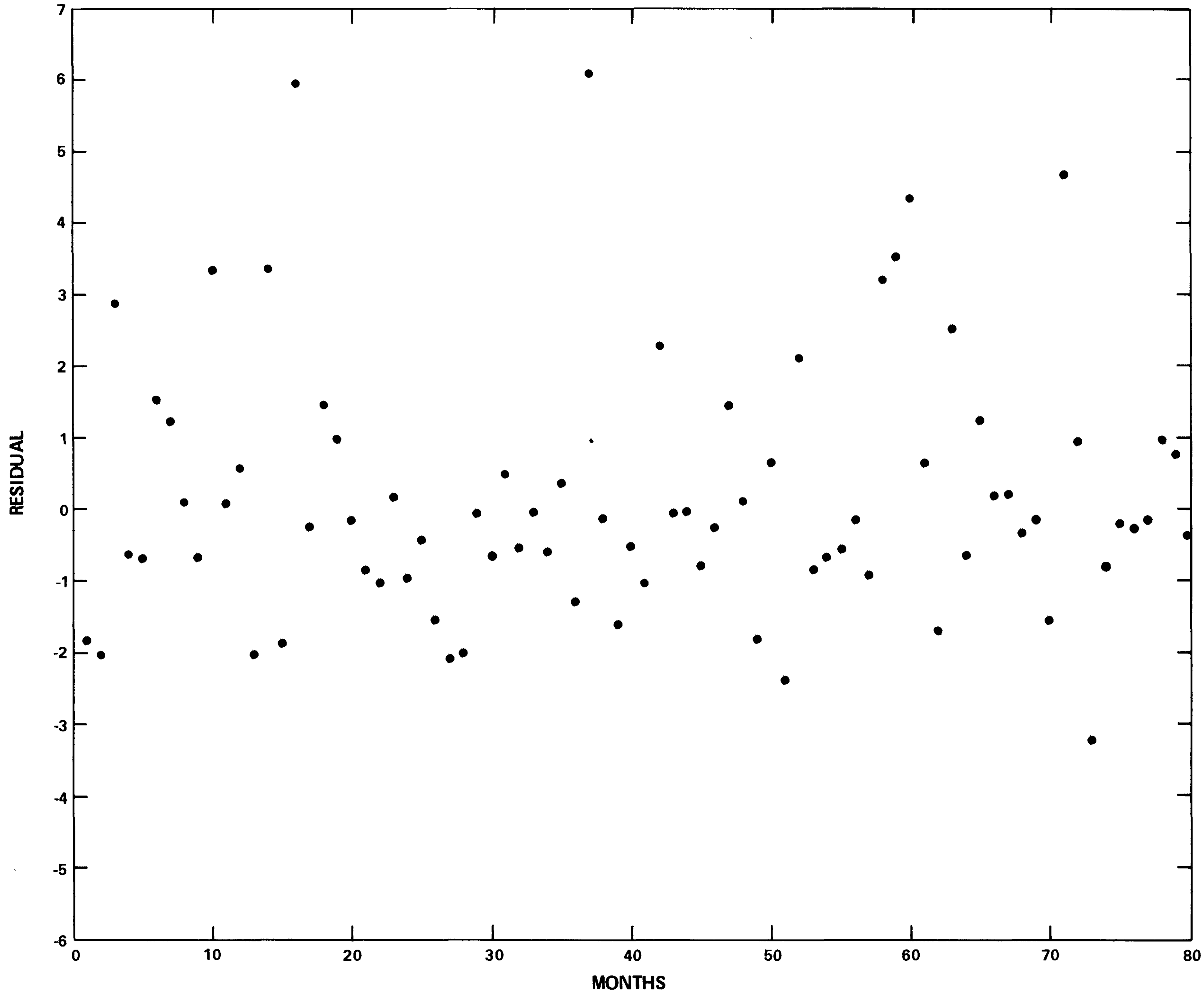

Figure 7.--Residuals for $\lambda = 1$.

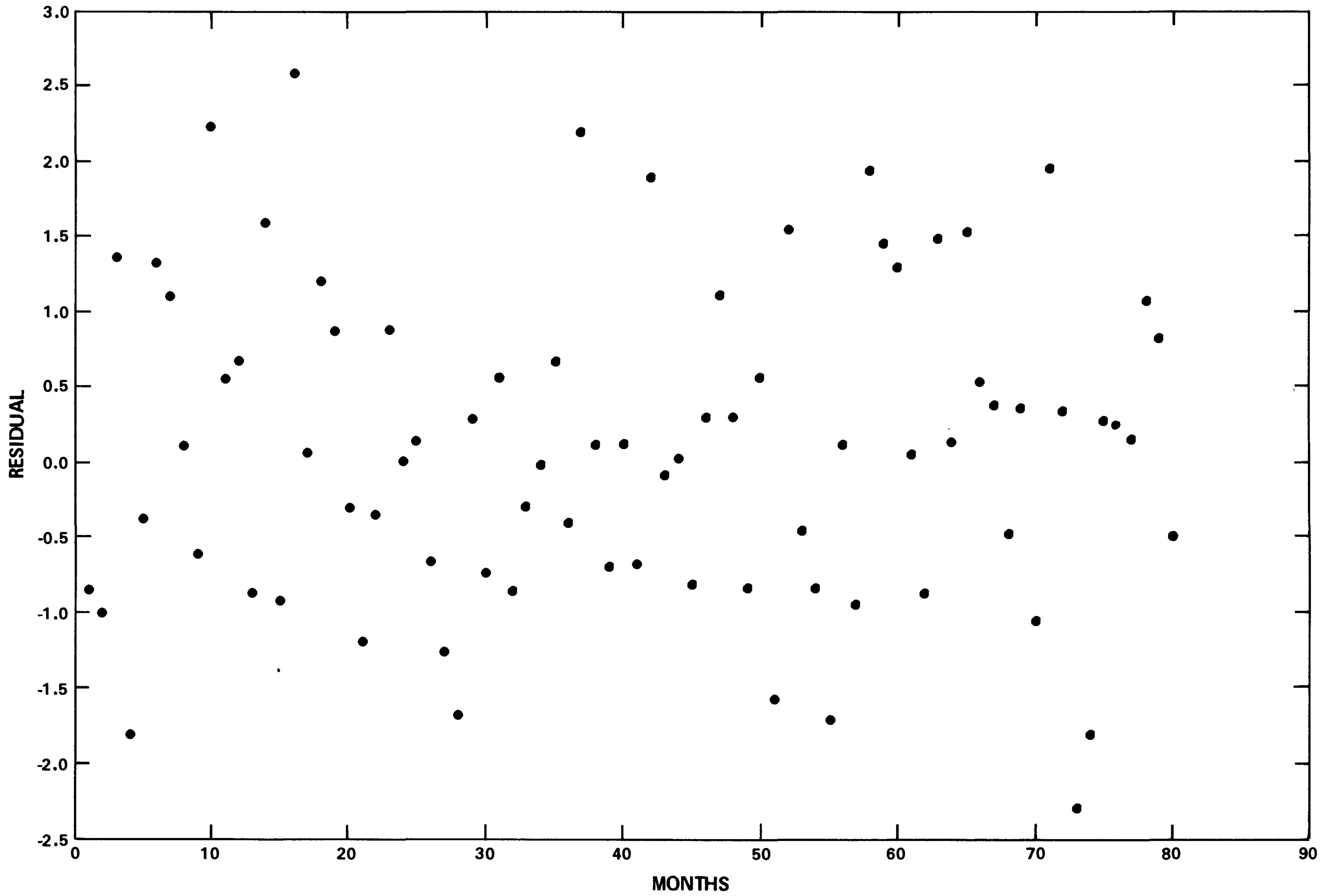

Figure 8.--Residuals for $\lambda = 1/3$.

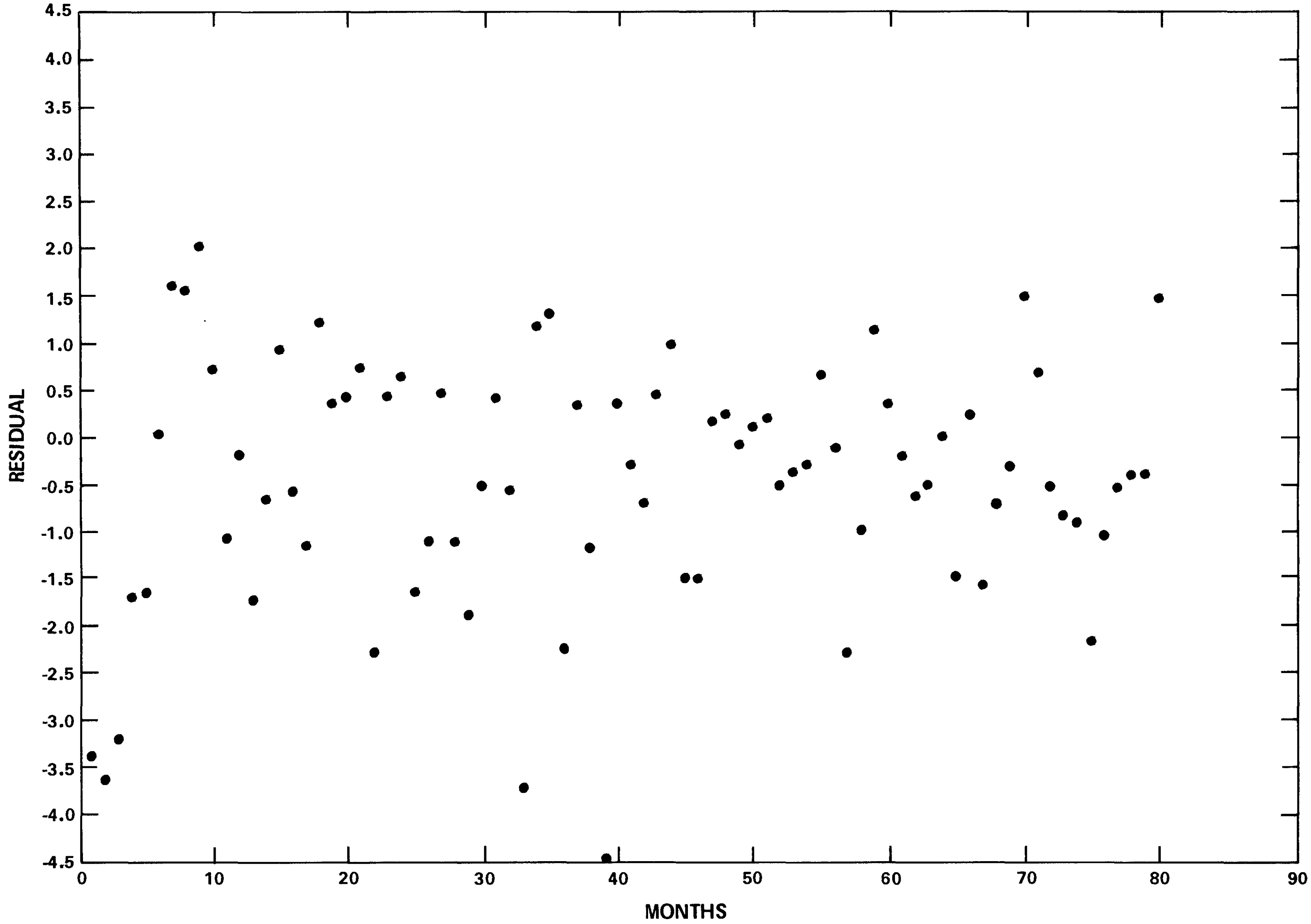

Figure 9.--Residuals for λ = 0.

The preceding paragraph seems to indicate that the ARIMA model is similar in its representation of past precipitation to a trigonometric linear model with period 12 component, the major difference being that the ARIMA model lacks a stationary mean. For example, if the model is assumed to be:

$$(1-1.73B+B^2)Z_t^{(\lambda)} = \mu + (1-\theta_1 B-\theta_2 B^2)A_t,\ A_t \text{ distributed iid } (0,\ \sigma^2), \quad (9)$$

then the mean is not defined at time t unless $Z_{t-1}^{(\lambda)}$ and $Z_{t-2}^{(\lambda)}$ are known. If the model is assumed to be

$$Z_t^{(\lambda)} = \mu + \theta_1 \cos\frac{2\pi t}{12} + \theta_2 \sin\frac{2\pi t}{12} + A_t,\ A_t \text{ distributed iid}(0,\sigma^2), \quad (10)$$

then

$$E(Z_t^{(\lambda)}) = \mu + \theta_1 \cos\frac{2\pi t}{12} + \theta_2 \sin\frac{2\pi t}{12}, \quad (11)$$

a quantity that is easily computed for any month and does not change from year to year. For purposes of this report, the trigonometric linear model gives an adequate representation of the series and allows easier calculation of drought probabilities than the ARIMA model. The estimation of λ for the trigonometric linear model is not presented because the results were nearly identical to those obtained for the ARIMA model.

DETERMINATION OF THE TRIGONOMETRIC LINEAR MODELS

The transformed data were modelled by a deterministic trigonometric component of period 12 with added white noise. However, the precipitation models with harmonics of period 12 only did not fit the data well enough to make inferences about some of the individual months, even though the overall yearly statistics showed a good fit. To improve the fit, harmonics of periods 3, 4, 6, and 12 were considered for inclusion in the model. These periods were chosen partly because of intuitive appeal, and partly because the spectral densities of the residuals from the models with only period 12 showed peaks near periods of 3, 4, and 6. The harmonics to be included in the model for each station were determined by using the forward regression technique (Draper and Smith, 1966) on each station separately. Only those variables that made a significant contribution to the prediction of precipitation were included in the model. It was assumed the Gothenburg and Grand Island stations are similar enough to the Kearney station to warrant using the same transformation for all three stations, because of the proximity of the stations and the similarities of their statistics. The models used are not regression models, but trigonometric linear models, because the independent variables are fixed, not random. This will not change the estimates of the parameters, but does make a difference in their standard errors. The following models were determined to be optimal (t=1 corresponds to January and $Z_t = Z_t^{(1/3)}$). The cube root transformation has been analyzed quantifying an explanation of skewness in precipitation distributions (Studd, 1970):

1. Gothenburg:

$$Z_t = 0.2009 - 1.1956*\cos\frac{2\pi t}{12} - 0.1595*\sin\frac{2\pi t}{12}$$
$$+ 0.1293*\cos\frac{2\pi t}{3} + 0.1089*\sin\frac{2\pi t}{4}$$
$$- 0.1054*\sin\frac{2\pi t}{6} + \varepsilon_t ;$$

where $\{\varepsilon_t\}$ are iid normal $(0, 0.861^2)$; R^2=50.2 percent.

2. Kearney:

$$Z_t = 0.3492 - 1.1637*\cos\frac{2\pi t}{12} - 0.1708*\sin\frac{2\pi t}{12}$$
$$+ 0.1699*\sin\frac{2\pi t}{4} - 0.1367*\sin\frac{2\pi t}{6} + \varepsilon_t ;$$

where $\{\varepsilon_t\}$ are iid normal $(0, 0.898^2)$; R^2=47.0 percent.

3. Grand Island:

$$Z_t = 0.3187 - 1.1174*\cos\frac{2\pi t}{12} + 0.1988*\sin\frac{2\pi t}{4}$$
$$- 0.1216*\sin\frac{2\pi t}{6} - 0.1097*\sin\frac{2\pi t}{12} + \varepsilon_t ;$$

where $\{\varepsilon_t\}$ are iid normal $(0, 0.895^2)$; R^2=45.1 percent.

RESIDUALS CHECK

The residuals from the above models were examined to insure that the following assumptions were not violated: (1) The errors ε_t are mutually independent; and (2) ε_t is distributed normally with mean 0 and variance σ^2 for all t. A visual inspection of the graphs of the residuals versus time for each station indicated no trends or abnormalities. The graphs of the autocorrelations of the residuals for each station (fig. 10) show that the residuals have no significant autocorrelation. A histogram of the residuals for Gothenburg and Grand Island (fig. 11), with the series statistics (table 2), show that the residuals appear to be normally distributed, but that some differences exist in the means and standard deviations among the months. These differences were considered minor enough for the model to be used. The actual data are compared with several simulations from the model in a later section of this report.

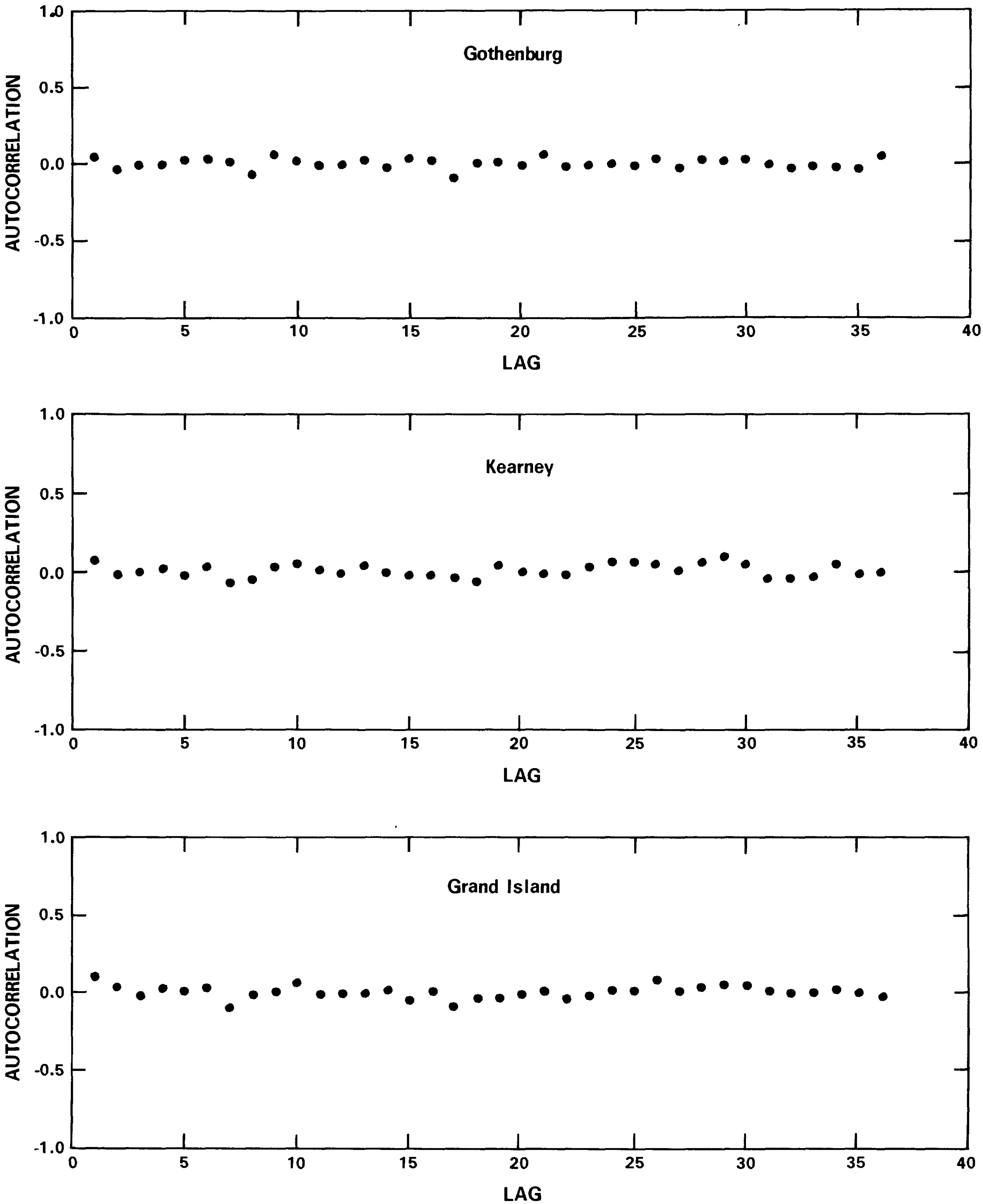

Figure 10.--Autocorrelations for residuals at Gothenburg, Kearney, and Grand Island.

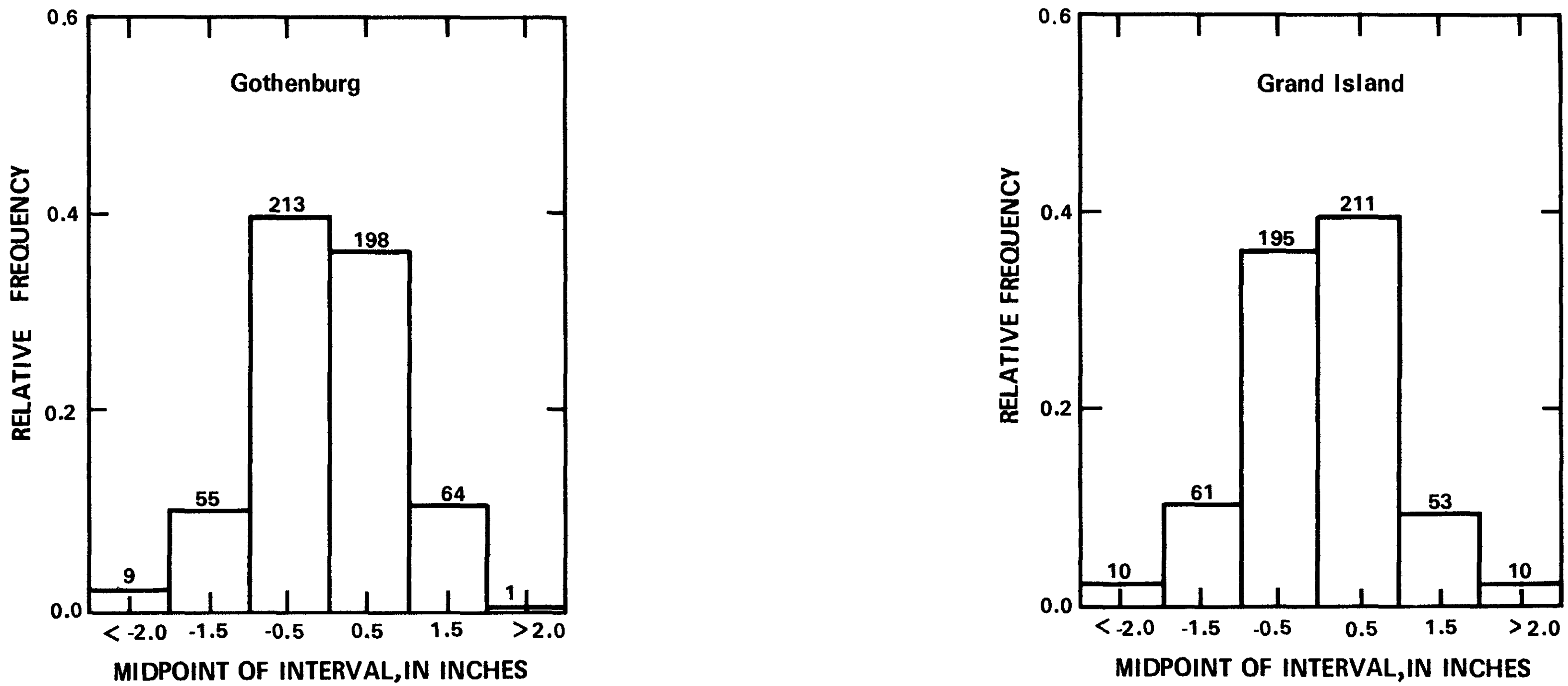

Figure 11.--Histograms of the residuals for Gothenburg and Grand Island.

Table 2.--*Residual statistics for Gothenburg, Kearney, and Grand Island*

Month	Gothenburg		Kearney		Grand Island	
	Mean (inches)	Standard deviation (inches)	Mean (inches)	Standard deviation (inches)	Mean (inches)	Standard deviation (inches)
Jan.	0.059	0.690	-0.054	0.631	-0.120	0.653
Feb.	-.100	.669	-.026	.714	.041	.705
Mar.	-.023	.756	.093	.870	-.003	.848
Apr.	-.030	.947	-.064	.894	-.044	.895
May	.021	1.017	-.046	1.064	.001	.912
June	.117	.911	.099	1.003	.105	.915
July	-.128	.934	-.016	.869	-.099	1.042
Aug.	.169	.718	-.093	.805	.010	.831
Sept.	-.158	1.048	.088	1.056	.125	1.079
Oct.	-.065	1.025	.011	.969	-.234	1.002
Nov.	.074	.887	-.089	1.074	.108	.999
Dec.	.062	.628	.098	.779	.109	.784
Total:	0.0		0.0		0.0	

MULTIVARIATE STRUCTURE

The models were determined for each station individually. However, if joint probability statements are to be made about the three stations, a multivariate structure must be added to the model. Because the stations are geographically near each other, they are assumed to be similar in their precipitation patterns. Therefore, if the error in predicting rainfall at any one station is a large positive number (the rainfall is much greater for that month than the model predicts), then the errors at the other stations should tend to be positive. It is reasonable to expect the error sequences to be correlated. This is indeed the case, as the estimated covariance matrix of the computed residuals is:

$$\hat{\underline{\Sigma}} = \begin{array}{l} \text{Gothenburg} \\ \text{Kearney} \\ \text{Grand Island} \end{array} \overset{\begin{array}{ccc}\text{Gothenburg} & \text{Kearney} & \text{Grand Island}\end{array}}{\begin{bmatrix} 0.741 & 0.546 & 0.484 \\ .546 & .805 & .622 \\ .484 & .622 & .799 \end{bmatrix}} \; ;$$

the corresponding estimate of the correlation matrix is:

$$\hat{\underline{R}} = \begin{array}{l} \text{Gothenburg} \\ \text{Kearney} \\ \text{Grand Island} \end{array} \overset{\begin{array}{ccc}\text{Gothenburg} & \text{Kearney} & \text{Grand Island}\end{array}}{\begin{bmatrix} 1.0 & 0.706 & 0.628 \\ .706 & 1.0 & .773 \\ .628 & .773 & 1.0 \end{bmatrix}} \; .$$

Therefore, the final model for the joint monthly precipitation series at Gothenburg, Kearney, and Grand Island is:

$$Z_t^{(G)} = \text{deterministic component} + \varepsilon_t^{(G)} ;$$
$$Z_t^{(K)} = \text{deterministic component} + \varepsilon_t^{(K)} ;$$
$$Z_t^{(GI)} = \text{deterministic component} + \varepsilon_t^{(GI)} ;$$

where the deterministic components are given in the regression equations above, and the sequences $\left\{\underline{\varepsilon}_t\right\} = \left\{\left(\varepsilon_t^{(G)}, \varepsilon_t^{(K)}, \varepsilon_t^{(GI)}\right)\right\}$ are independent and identically distributed trivariate normal random vectors with mean $\underline{0}$ and covariance matrix $\hat{\underline{\Sigma}}$. Graphs of 40-year model simulations are presented for Gothenburg (fig. 12), Kearney (fig. 13), and Grand Island (fig. 14).

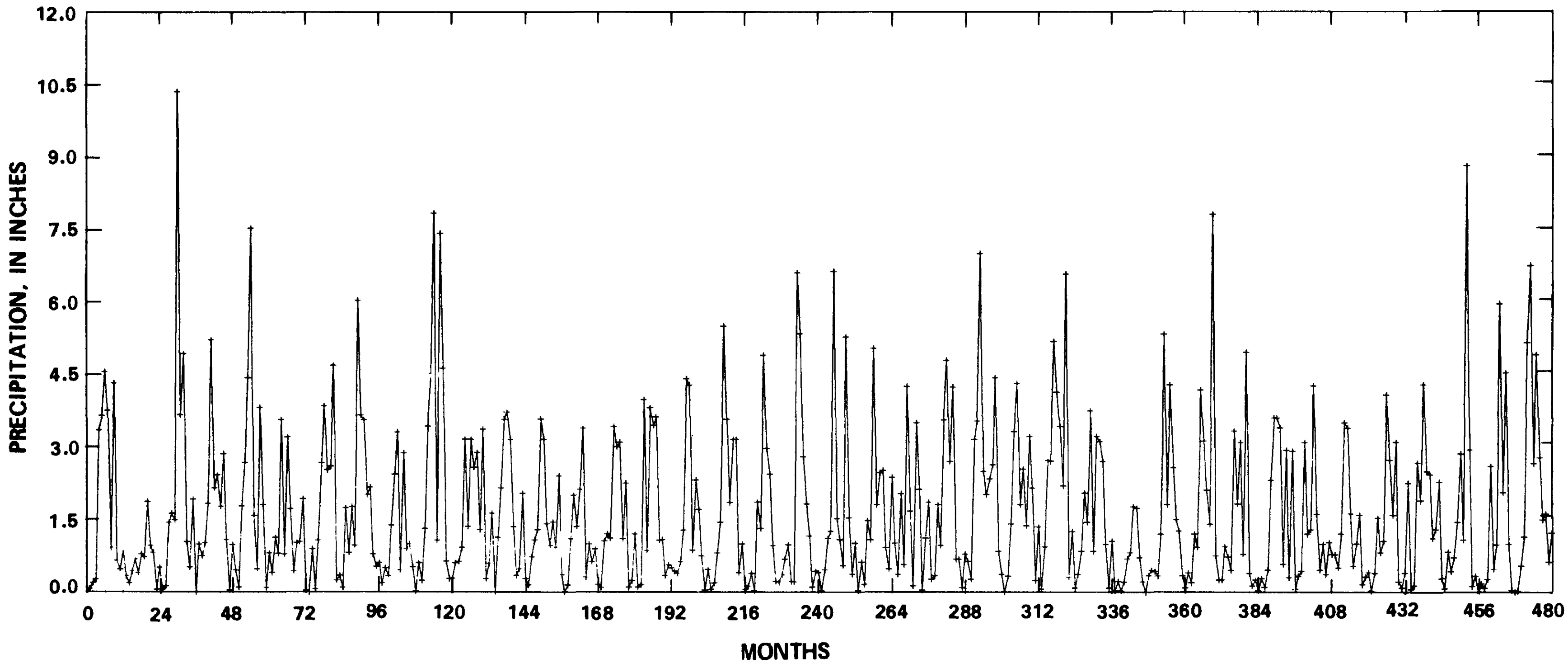

Figure 12.--Monthly precipitation simulation of 40 years for Gothenburg.

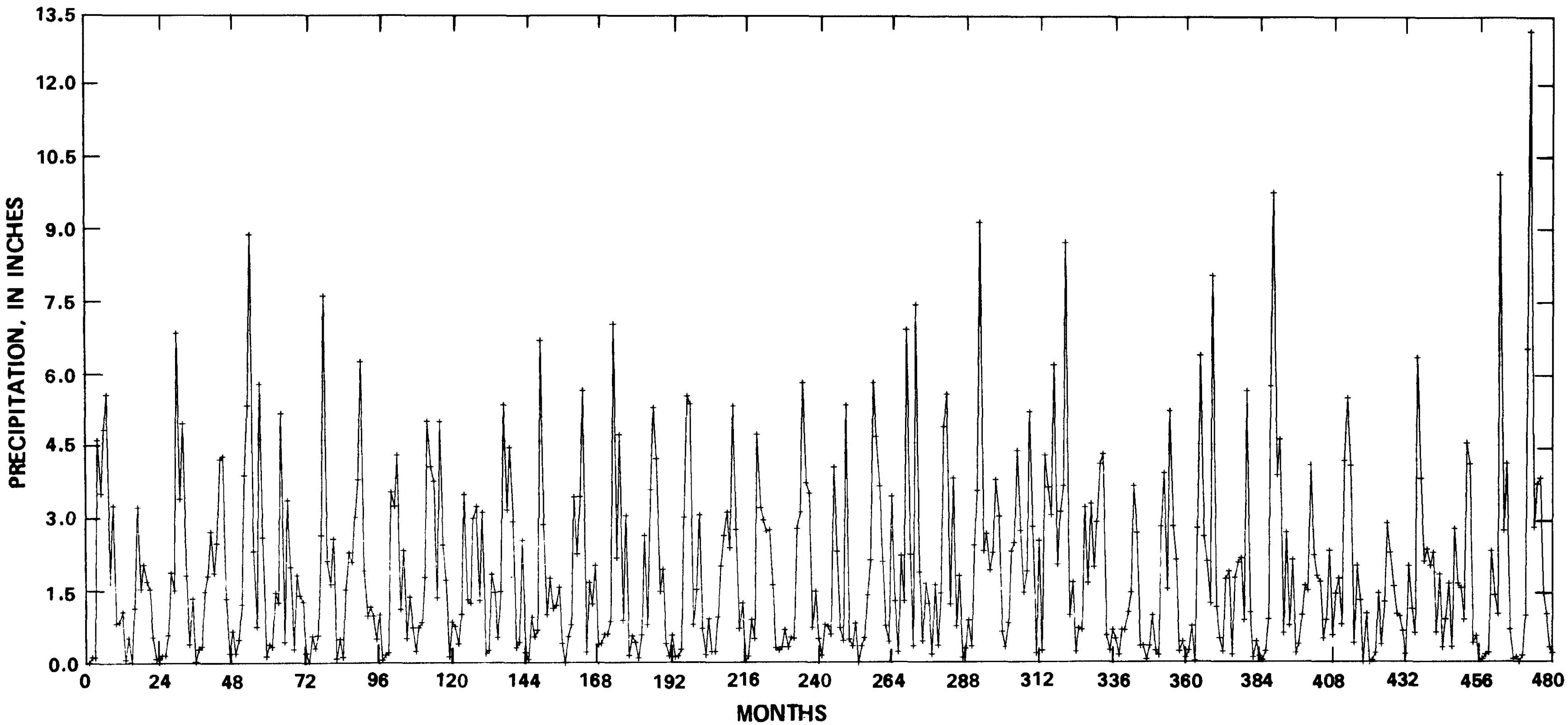

Figure 13.--Monthly precipitation simulation of 40 years for Kearney.

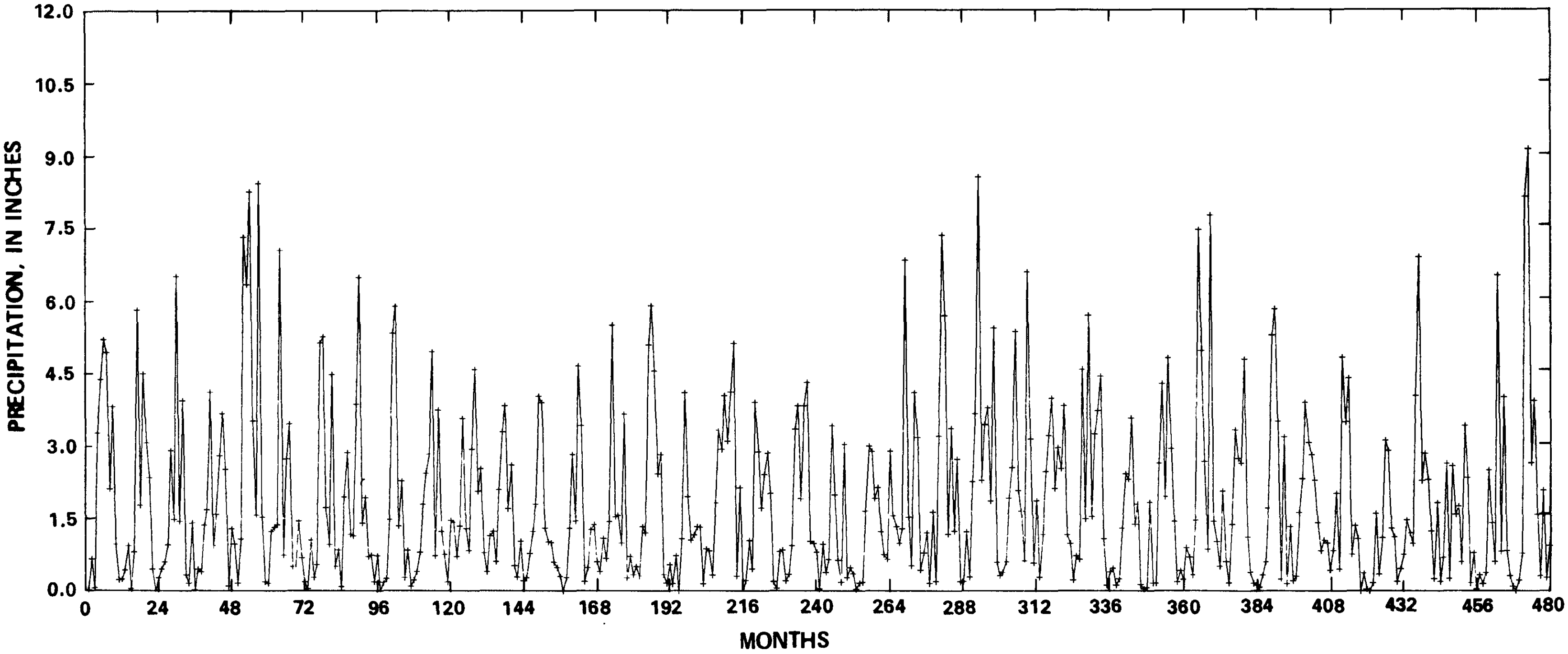

Figure 14.--Monthly precipitation simulation of 40 years for Grand Island.

EVALUATION OF THE MODEL'S PERFORMANCE

One method of evaluating model adequancy is to determine how well it reproduces monthly means and standard deviations of past data. Note that the historical record is assumed to be one possible realization from an underlying model driving the system. Therefore, the purpose is not to exactly reproduce past statistics, but to determine that the realization that occurred could have come from the developed model within a reasonable margin of accuracy (which depends on the purpose of the model). Monthly means and standard deviations for the actual data and for 10 model simulations of 40 years each are presented in table 3 (Gothenburg), table 4 (Kearney), and table 5 (Grand Island). A graphical display of the means for Kearney is shown in figure 15. Data in the tables indicate that values for January and February seem too large in the simulations for Kearney and Grand Island, and the September simulations may be slightly distorted. Graphs of the last 40 years of actual precipitation and corresponding random simulations for each station, from the model, are compared in figures 2 through 4 and 12 through 14. Some of the simulations appear more like the actual data than others. The purpose of the comparative graphs is to generate many possible future realizations and observe which and how many have a certain event for which a probability is desired.

The distribution of values for all the months combined is another check of the adequacy of the model. Although these values do not represent a random sample from a single distribution, the simulated sequences should closely resemble the past data. Histograms of the actual data (1939-78) are shown in figures 16, 17, and 18, versus a corresponding 40-year simulation for each station (the same simulations as those used for figures 12, 13, and 14); some quantile values of actual versus simulated data are shown in table 6.

As a final check on the monthly model, the generated monthly series were summed to obtain yearly series and compared to the historical record. Statistics of the actual yearly precipitation from 1939 to 1978 compared to model simulations of 40 years are presented in table 7. The simulations used are the same as those shown in figures 12 through 14. The simulations have smaller means than the actual data; however, this is not surprising because of variability of the observations and the relatively small sample size. The autocorrelations up to lag 10 for the historical and simulated sequences are presented in figure 19. No significant autocorrelation is present, indicating that the yearly values are virtually independent.

CALCULATION OF DROUGHT PROBABILITIES

The analysis and discussion in the following paragraphs will illustrate how the precipitation model can be used: (1) To determine periods of rainfall shortages of given severities and durations from the generated sequences, and (2) to obtain the probabilities of these occurrences within the next 50 years. Because no definitive critical period of precipitation for germination is available at this time (U.S. Fish and Wildlife Service, oral communication, 1980), arbitrary choices of May, June, and July were made as the critical months during which the seeds would require precipitation most. With various combinations of these 3 months defining the duration of within-year drought periods, arbitrary choices of severity were taken as 2/3, 1/2, and 1/3 of the

Table 3.--*Monthly means and standard deviations for actual precipitation data and 10 model simulations of 40 years each for Gothenburg statistics*

[Mean values (upper number) in inches]

	Means and standard deviations											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Actual Data	0.440	0.500	1.220	2.156	3.431	4.118	2.587	2.493	1.892	1.045	0.597	0.473
	.381	.440	.881	1.516	2.120	2.279	1.610	1.285	1.453	1.163	.588	.344
Simulations												
1	.502	.553	.968	1.842	3.266	4.146	2.825	1.736	2.169	.918	.447	.423
	.569	.585	.632	1.281	2.070	2.271	1.855	1.160	1.696	.987	.404	.423
2	.475	.506	1.434	2.027	3.198	3.802	2.594	2.182	2.407	1.096	.511	.521
	.361	.659	1.264	1.075	1.633	1.806	1.435	1.496	1.383	1.372	.566	.543
3	.545	.735	1.265	2.557	2.950	3.447	2.694	2.702	2.101	1.146	.517	.432
	.523	.671	1.112	1.503	1.531	1.927	1.414	1.656	1.649	.977	.516	.380
4	.404	.632	1.556	2.300	3.617	3.700	2.606	2.209	2.037	.942	.463	.408
	.548	.520	.940	1.646	2.143	1.981	1.862	1.632	1.596	.773	.436	.374
5	.457	.719	1.306	2.158	3.440	3.774	2.728	2.506	1.883	1.045	.631	.588
	.414	.785	1.127	1.721	2.318	2.015	1.613	2.238	1.744	.727	.619	.622
6	.574	.569	1.133	1.940	3.191	4.446	2.317	2.231	2.260	1.045	.596	.496
	.536	.614	1.112	1.323	1.851	2.384	1.455	1.326	1.495	.936	.679	.478
7	.510	.580	1.292	1.992	2.744	4.097	2.508	2.711	2.524	1.101	.581	.410
	.453	.639	.822	1 065	1.703	1.888	1.686	1.832	1.733	.860	.595	.617
8	.405	.579	1.485	2.405	3.256	3.782	3.134	2.249	1.840	.985	.528	.333
	.602	.572	1.132	1.381	1.767	2.406	2.108	1.518	1.186	.880	.581	.399
9	.450	.491	1.320	2.098	3.290	3.661	2.230	2.110	2.249	.977	.697	.721
	.370	.556	.952	1.441	2.159	2.329	1.182	1.568	1.533	.857	.806	.894
10	.499	.626	1.455	1.940	3.575	4.474	3.439	2.591	2.053	.817	.649	.698
	.597	.562	1.312	1.264	1 929	2.691	2.247	1.673	1.108	.788	.707	.832

Table 4.--*Monthly means and standard deviations for actual precipitation data and 10 model simulations of 40 years each for Kearney statistics*

[Mean values (upper number) in inches]

	Means and standard deviations											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Actual Data	0.511	0.688	1.357	2.420	3.915	4.141	2.962	2.410	2.451	1.348	0.730	0.576
	.401	.539	1.138	1.670	2.227	2.781	1.911	1.479	1.986	1.262	.849	.473
Simulations												
1	.702	.747	1.081	1.950	3.70	3.876	2.868	2.072	2.444	1.122	.591	.372
	.804	.653	.891	1.102	1.863	2.177	1.921	1.482	1.554	.966	.634	.477
2	.730	.635	1.316	2.706	3.878	3.296	3.063	2.829	2.474	1.579	.640	.501
	.570	.780	1.148	1.317	1.776	1.887	1.747	1.663	1.678	1.606	.539	.447
3	.737	.977	1.142	2.960	3.727	3.646	2.915	2.912	2.150	1.506	.713	.457
	.888	1.099	1.142	1.787	2.033	2.122	1.675	1.867	1.432	1.026	.813	.483
4	.537	.663	1.288	2.545	4.208	3.667	3.172	2.387	1.972	1.147	.705	.616
	.827	.589	.799	1.477	2.224	2.157	2.627	1.627	1.402	1.015	.652	.654
5	.629	.927	1.261	2.406	3.936	3.716	2.663	2.710	2.170	1.408	.817	.541
	.663	.992	1.316	1.552	2.163	1.939	1.598	2.237	1.925	1.066	.680	.509
6	.757	.689	1.002	2.422	3.353	4.222	2.951	2.639	2.673	1.283	.904	.530
	.609	.786	1.169	1.530	1.688	2.314	2.100	1.895	2.031	1.453	.739	.509
7	.832	.653	1.176	2.735	3.424	3.835	2.853	2.969	2.636	1.317	.703	.452
	.694	.608	.859	1.605	1.954	1.888	2.056	1.917	1.696	.913	.666	.514
8	.557	.881	1.264	3.065	3.909	3.593	3.274	2.316	1.915	1.038	.644	.396
	.601	.862	.780	1.929	1.801	2.203	2.313	1.417	1.218	1.060	.594	.409
9	.941	.617	1.169	2.483	4.170	4.329	2.589	2.339	2.057	1.233	.659	.728
	1.066	.606	.929	1.531	2.801	2.528	1.494	1.814	1.349	1.171	.655	.948
10	.600	.771	1.379	2.382	4.159	4.568	3.862	2.761	2.330	1.039	.789	.694
	.547	.786	1.179	1.572	1.944	2.870	1.940	1.528	1.580	.840	.794	.733
Combined	.702	.756	1.208	2.567	3.846	3.875	3.021	2.593	2.282	1.267	.717	.529
Simulations	.734	.615	1.025	1.537	2.024	2.202	1.952	1.741	1.586	1.122	.674	.585
1-10	.109	.092	.153	.229	.302	.328	.291	.260	.236	.182	.100	.087

Table 5.--*Monthly means and standard deviations for actual precipitation data and 10 model simulations of 40 years each for Grand Island statistics*

[Mean values (upper number) in inches]

	Means and standard deviations											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Actual Data	0.525	0.786	1.238	2.421	3.864	3.871	2.710	2.447	2.443	1.031	0.787	0.603
	.390	.641	1.067	1.493	2.080	2.331	2.040	1.621	2.153	.951	.837	.550
Simulations												
1	.789	.872	1.241	1.966	3.668	3.559	2.616	1.896	2.249	1.042	.483	.386
	.882	.929	1.051	1.350	2.021	2.217	1.582	1.217	1.541	.904	.506	.457
2	.680	.610	1.435	2.586	3.716	3.148	3.199	2.588	2.432	1.200	.748	.436
	.436	.766	1.239	1.739	2.131	1.761	1.626	1.743	1.491	.937	.687	.350
3	.847	1.086	1.108	2.632	3.323	3.281	3.041	2.770	2.088	1.508	.708	.433
	.712	1.491	.966	1.642	1.943	1.766	1.694	2.023	1.457	1.189	.747	.385
4	.715	.809	1.437	2.287	4.023	3.167	2.682	2.368	1.836	1.130	.704	.650
	1.002	.680	.664	1.364	2.719	1.707	2.128	1.396	1.252	.960	.717	.719
5	.673	1.004	1.276	2.513	3.885	3.124	2.807	2.481	2.034	1.405	.795	.539
	.668	1.108	1.100	1.325	2.292	1.721	1.608	1.880	1.513	1.494	.586	.449
6	.835	.691	1.028	2.349	3.632	3.724	2.555	2.370	2.489	1.132	.888	.607
	.582	.626	1.189	1.529	1.790	2.581	1.797	1.767	1.769	1.009	.723	.565
7	.892	.726	1.298	2.680	3.549	3.721	2.790	2.773	2.319	1.281	.582	.589
	.873	.636	.977	1.413	1.853	1.978	2.221	2.032	1.501	.875	.561	.872
8	.585	.774	1.289	2.724	3.697	3.232	3.084	2.176	1.845	1.188	.711	.421
	.503	.782	.873	1.684	1.481	1.681	1.945	1.171	1.202	1.509	.800	.565
9	.899	.729	1.301	2.716	4.161	4.033	2.383	2.351	2.198	1.118	.817	.742
	.788	.642	1.212	1.944	2.698	2.304	1.186	1.655	1.759	1.023	.819	1.070
10	.609	.738	1.191	2.111	3.849	4.065	3.409	2.876	1.964	.933	.816	.719
	.597	.633	.810	1.392	1.992	2.479	2.005	1.765	1.162	.852	.819	.855

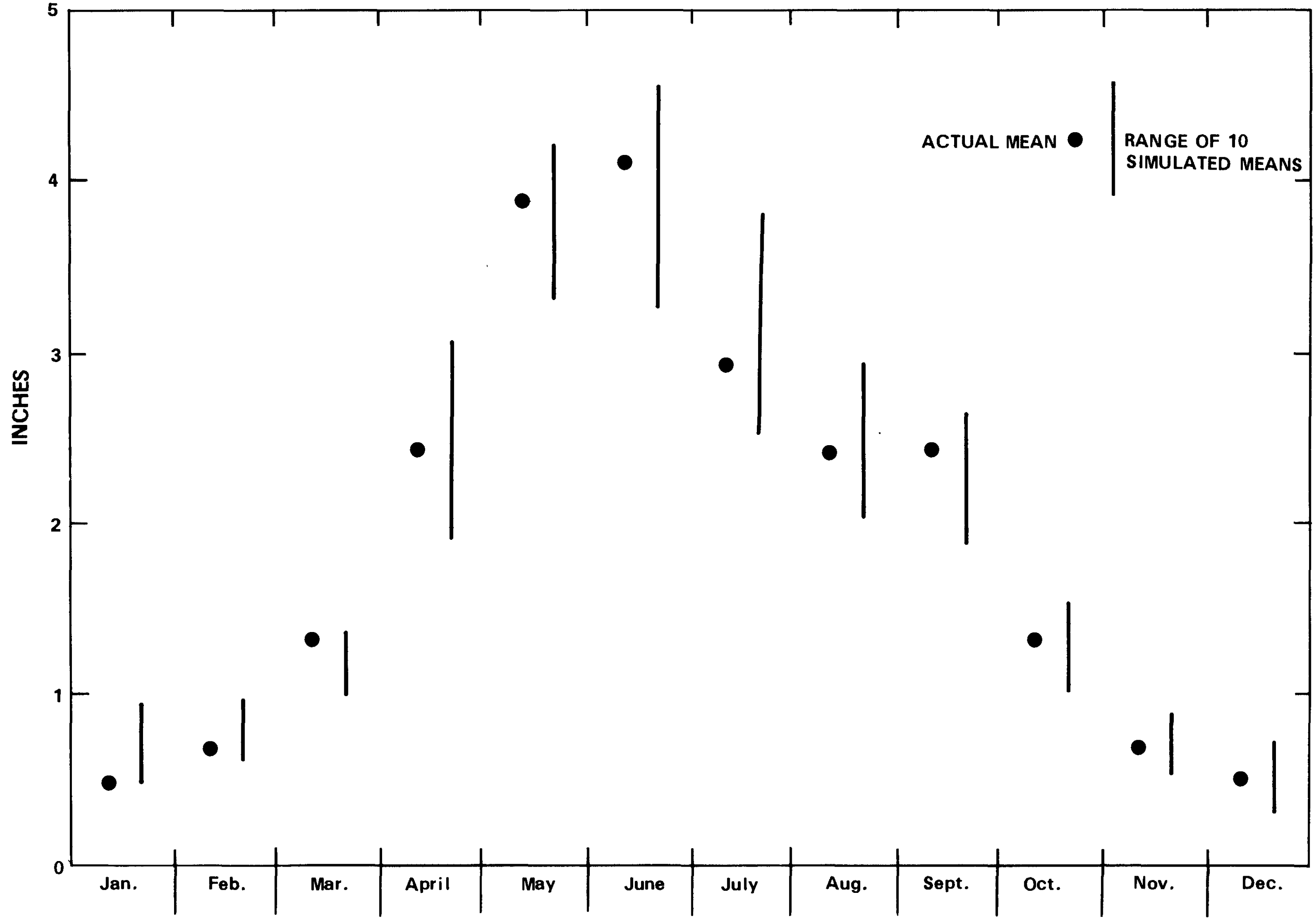

Figure 15.--Actual monthly means versus means from 10 simulations for Kearney.

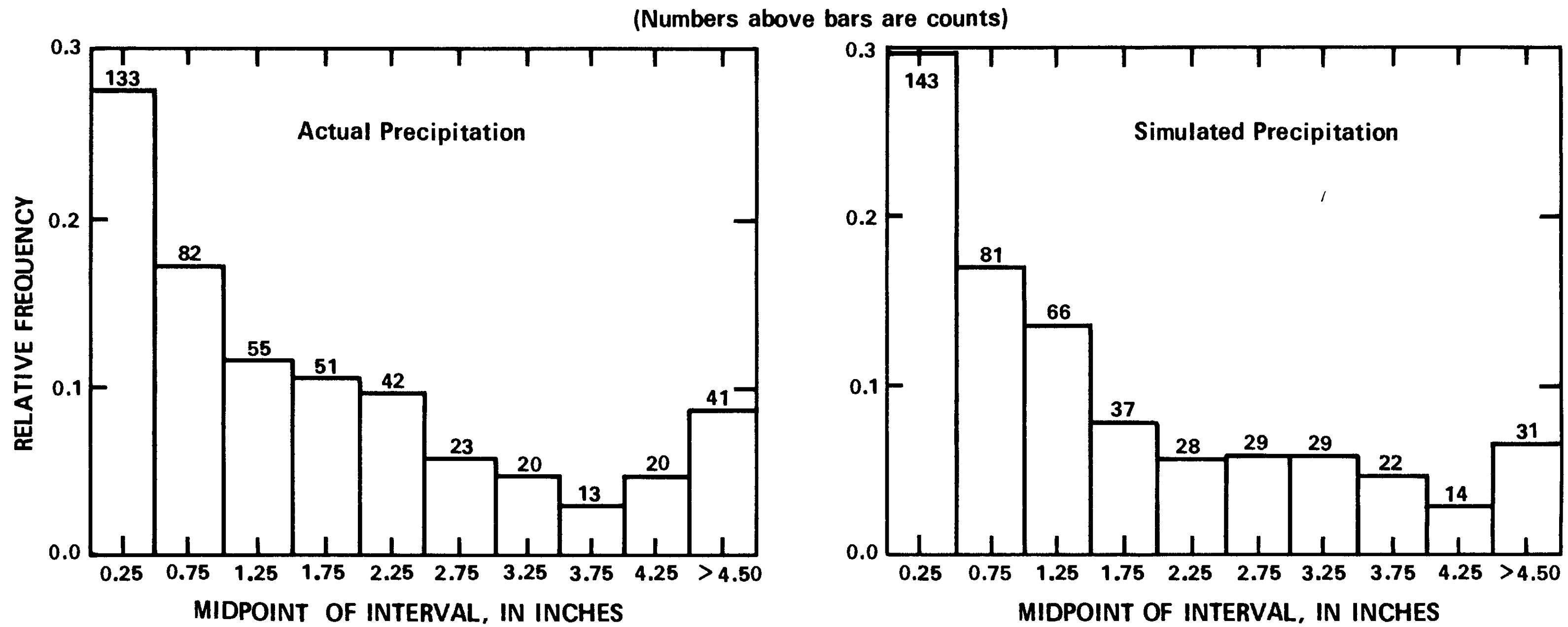

Figure 16.--Histograms of actual data from January 1, 1939, through December 31, 1978, with corresponding simulation for Gothenburg station.

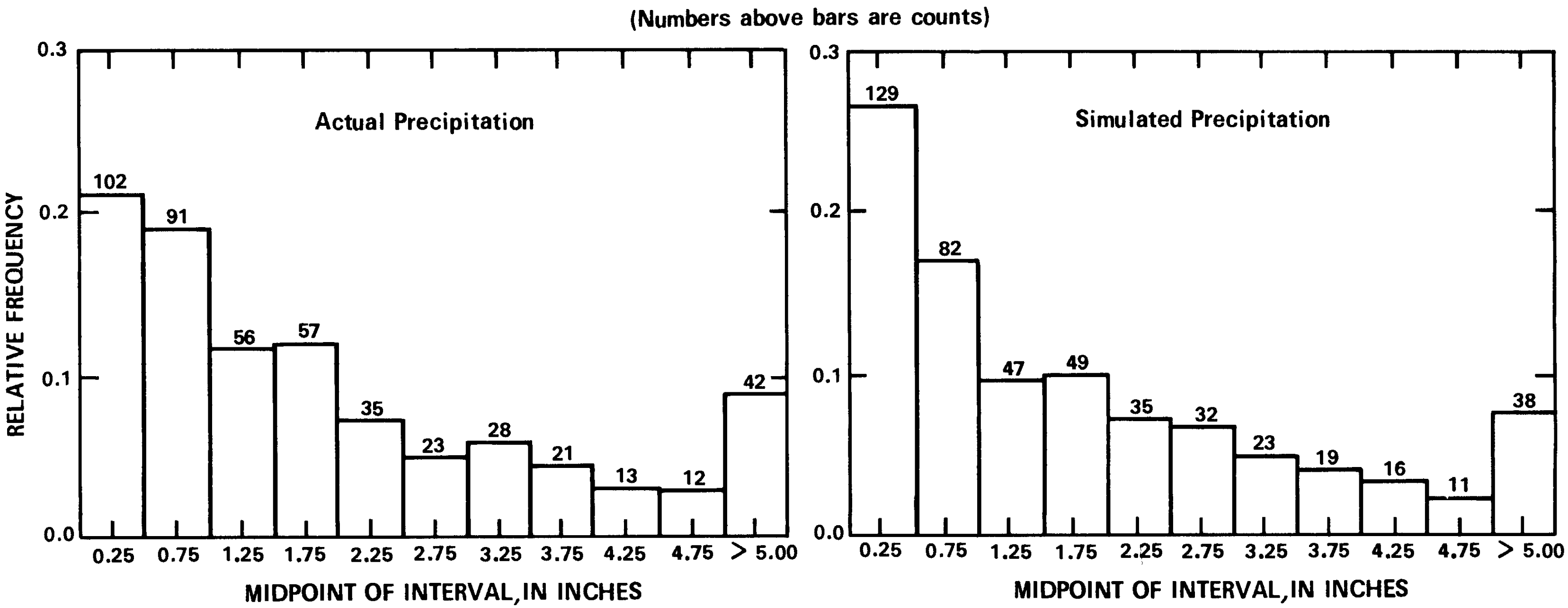

Figure 17.--Histograms of actual data from January 1, 1939, through December 31, 1978, with corresponding simulation for Kearney station.

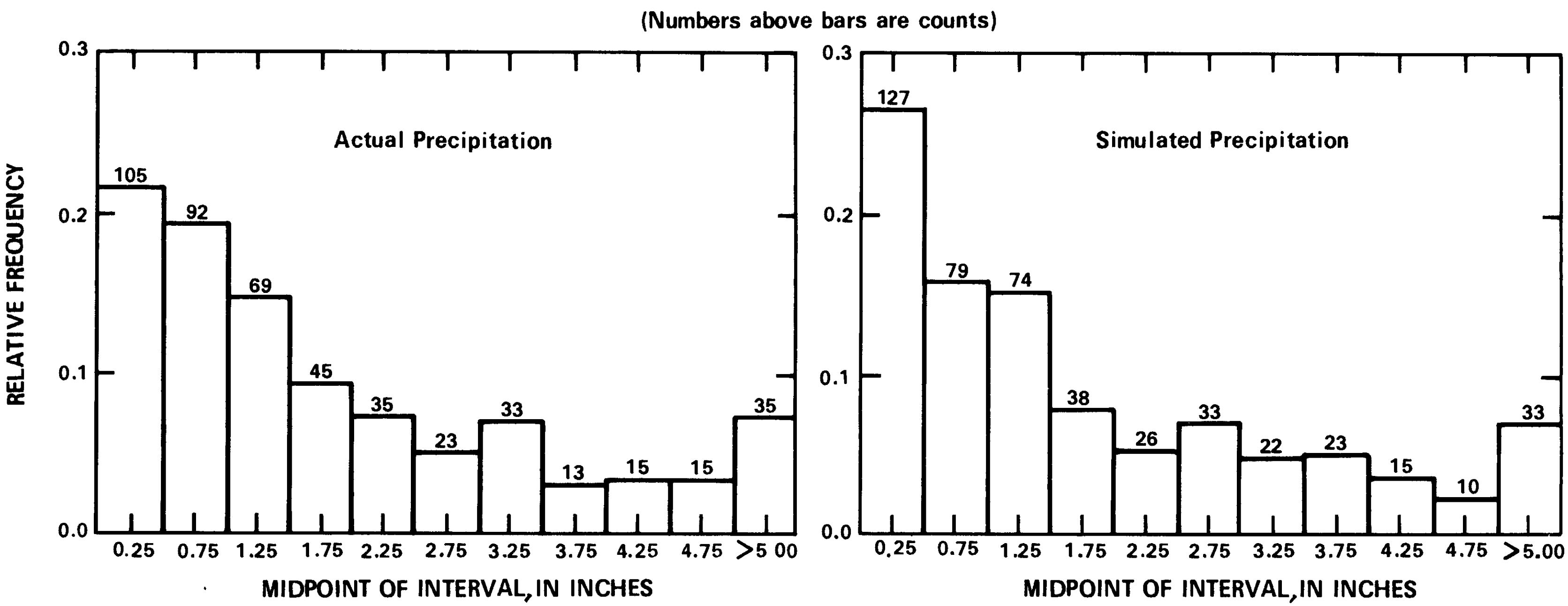

Figure 18.--Histograms of actual data from January 1, 1939, through December 31, 1978, with corresponding simulation for Grand Island station.

Table 6.--*Quantiles (all months combined) of actual data versus simulation*

Rainfall (percent less than tabular values)	Gothenburg		Kearney		Grand Island	
	Actual (inches)	Simulated (inches)	Actual (inches)	Simulated (inches)	Actual (inches)	Simulated (inches)
10	0.16	0.12	0.21	0.15	0.19	0.17
20	.31	.32	.45	.33	.45	.32
30	.58	.50	.70	.56	.68	.60
40	.83	.82	.99	.87	.95	.93
50	1.22	1.07	1.42	1.33	1.30	1.24
60	1.66	1.45	1.79	1.78	1.68	1.58
70	2.19	2.12	2.4	2.32	2.36	2.31
80	2.97	2.97	3.4	3.19	3.26	3.17
90	4.26	3.82	4.71	4.44	4.51	4.31

Table 7.--*Yearly statistics of actual precipitation (1939-78) versus 40-year simulation*

	Gothenburg		Kearney		Grand Island	
	Actual (inches)	Simulated (inches)	Actual (inches)	Simulated (inches)	Actual (inches)	Simulated (inches)
Mean	21.257	19.912	24.192	22.676	23.120	21.951
Variance	23.695	27.853	32.782	31.385	34.568	31.807
Standard deviation	4.868	5.278	5.726	5.602	5.879	5.64
Skewness	.015	.021	.076	.453	.144	.838
Minimum	12.84	6.72	11.65	11.77	11.91	12.39
Maximum	30.81	32.23	35.52	37.45	38.22	39.48

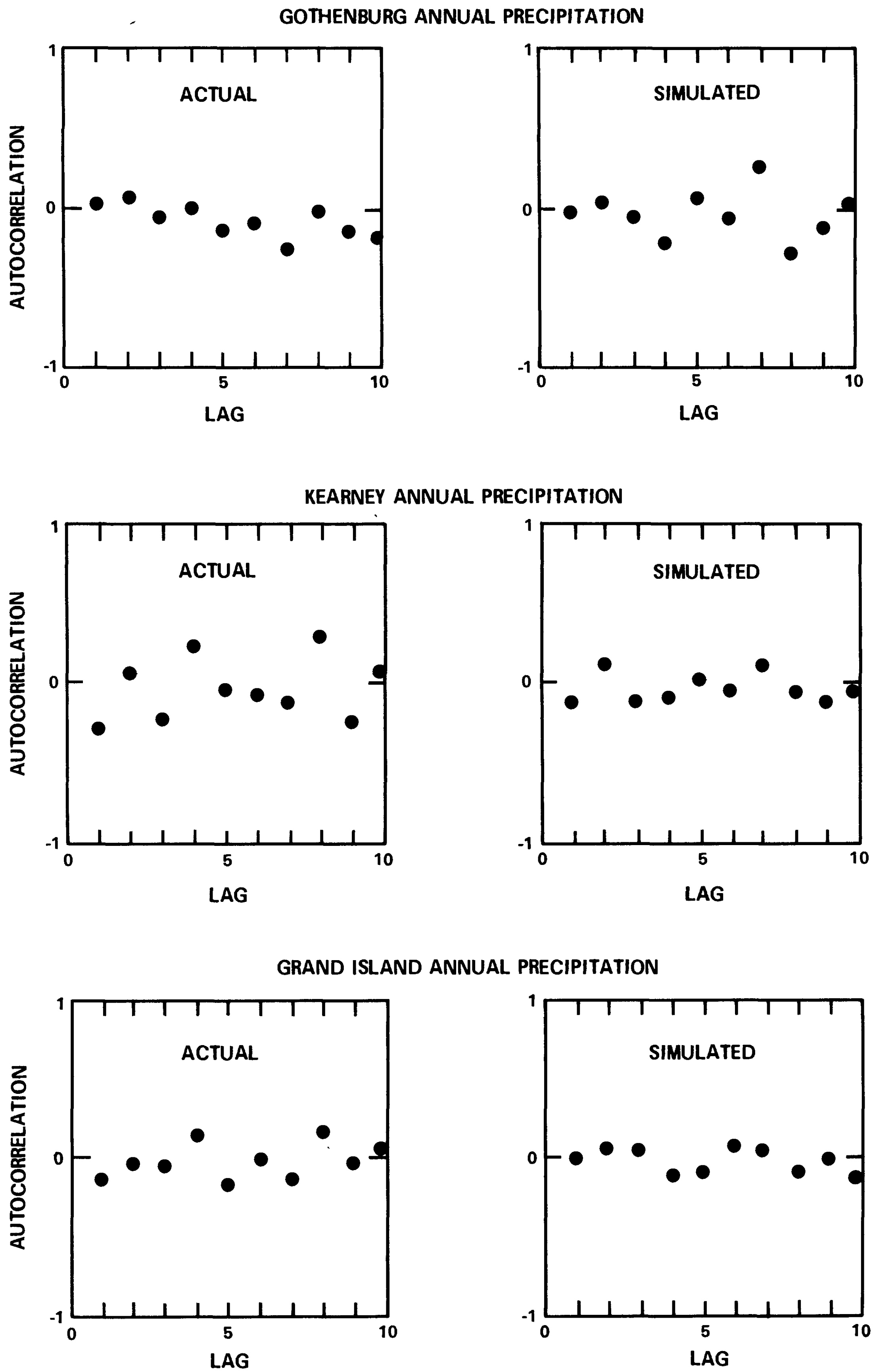

Figure 19.--Autocorrelations for actual and simulated annual precipitation sequences at Gothenburg, Kearney, and Grand Island.

historical mean values of rainfall for each of the respective periods for the three sites. Because no critical drought severity could be defined at this time, the three values were chosen to attempt to envelope the true-critical severity and offer some relationship between drought severity and probability of occurrence, to aid in future decision processes.

In the model discussed, it is assumed that the monthly precipitation values are independent. Hence, any combination of monthly values (say Y) within a year is independent of any combination of monthly values from any other year. This allows easy calculation of the probability of many events involving Y during the next 50 years knowing the cumulative distribution function (cdf) of Y for a single year, because a 50-year series is a random sample from the cdf of Y. Denote this cdf as:

$$F_Y(x) = \text{Probability}[Y \leq x] \; ;$$

and define the sample cdf of Y as

$$\left[\frac{1}{n} \sum_{i=1}^{n} I_{(-\infty,x]}(y_i)\right] \equiv F_n(x) \; ; \tag{12}$$

where $\{y_1, y_2, \ldots, y_n\}$ is a random sample of size n from F_Y; and

$$I_{(-\infty,x]}(y_i) = \begin{cases} 1, & \text{if } y_i \varepsilon (-\infty,x] \\ 0, & \text{otherwise} \end{cases} \; . \tag{13}$$

By the Glivenko-Cantelli theorem (Gibbons, 1971) $F_n(x)$ converges uniformly with probability one to $F_Y(x)$. Therefore, to approximate $F_Y(x)$, a sample of size n can be generated and the corresponding sample cdf calculated. The sample size must be large to achieve accurate results for rare events. As a general rule in this report, the sample cdf will be assumed to adequately estimate the true cdf for all x for which:

$$\sum_{i=1}^{n} I_{(-\infty,x]}(y_i) \geqslant 10 \; .$$

This rule can be used to pick n by noting that:

$$E\left[\sum_{i=1}^{n} I_{(-\infty,x]}(y_i)\right] = nF_Y(x) \; . \tag{14}$$

For example, if a value x_m for which the precipitation exceeds with probability 0.99 is the smallest x value of interest, then $F_Y(x_m) = 0.01$, and n would be 1,000, for one to expect, on the average, that:

$$\sum_{i=1}^{n} I_{(-\infty,x_m]}(y_i) = 10 \; .$$

For the drought analyses in this report, the following random variables were computed for each of 2,000 years of jointly simulated precipitation for Gothenburg, Kearney, and Grand Island:

$$Y_{1t} = X_t^M + X_t^{Ju} \; ; \; F_{Y_1}(x) = P[Y_1 \leq x] \; ;$$

$$Y_{2t} = X_t^{Ju} + X_t^{Jl} \; ; \; F_{Y_2}(x) = P[Y_2 \leq x] \; ;$$

$$Y_{3t} = X_t^M + X_t^{Ju} + X_t^{Jl} \; ; \; F_{Y_3}(x) = P[Y_3 \leq x] \; ;$$

where

X_t^M = total precipitation for May of year t;

X_t^{Ju} = total precipitation for June for year t; and

X_t^{Jl} = total precipitation for July for year t.

Because n was chosen to be 2,000, it is expected that droughts with probability as small as 0.005 will occur about 10 times. For each station, $\{Y_{it}; i=1, 2,3; t=1,2,...,2,000\}$ were computed, and the sample cdf's were used to estimate $F_{Y_i}(x)$, i=1, 2, 3, for the values of x given in table 8. Another, perhaps more important, set of random variables were computed using the three-station average precipitation series. Define $W_t = (X_t^G + X_t^K + X_t^{GI}) \div 3$ = average precipitation series and let

$$U_{1t} = W_t^M + W_t^{Ju} \; ;$$

$$U_{2t} = W_t^{Ju} + W_t^{Jl} \; ; \text{ and}$$

$$U_{3t} = W_t^M + W_t^{Ju} + W_t^{Jl} \; ;$$

where

W_t^M = average precipitation value for May of year t;

W_t^{Ju} = average precipitation value for June of year t; and

W_t^{Jl} = average precipitation value for July of year t.

The W_t series is an areal average of the three stations and is important in evaluating drought probabilities over the entire critical reach. The values at which the cumulative cdf's for U_1, U_2, and U_3 were estimated are given in column 4 of table 8. Forty years of past precipitation records (1939-78) were used to estimate the mean rainfall μ in defining drought severity levels 1/3 μ, 1/2 μ, and 2/3 μ. The rationale for using historical data rather than the model to estimate μ is that the state of the environment in the critical reach is the result of occurrences in the past. Therefore, any events that take place to change the environment from its present state need to be in reference to the past.

Table 8.--*Values at which the cumulative distribution function is estimated*

[μ = Mean of drought period, in inches]

	Fraction of mean	Gothenburg	Kearney	Grand Island	Three-station average
May and June	μ	7.549	8.056	7.735	7.780
	2/3 μ	5.033	5.371	5.157	5.187
	1/2 μ	3.774	4.028	3.867	3.890
	1/3 μ	2.516	2.685	2.578	2.593
June and July	μ	6.705	7.103	6.581	6.796
	2/3 μ	4.470	4.735	4.387	4.531
	1/2 μ	3.352	3.551	3.291	3.398
	1/3 μ	2.235	2.368	2.194	2.265
May, June, and July	μ	10.136	11.018	10.445	10.533
	2/3 μ	6.757	7.345	6.963	7.022
	1/2 μ	5.068	5.509	5.222	5.266
	1/3 μ	3.379	3.673	3.482	3.511

Column one of table 8 is interpreted as follows: The mean of Y_{1t} was computed for 1939-78 at the Gothenburg station and was found to be 7.549, the first entry under Gothenburg. The following three entries under Gothenburg are 2/3, 1/2, and 1/3 of this mean. Similar calculations were performed for Y_{2t} and Y_{3t} and appear below Y_{1t} in the table. The same computations were performed for Kearney, Grand Island, and the three-station average and appear in columns 3, 4, and 5.

The counts:

$$\left(\sum_{t=1}^{2000} I_{(-\infty,x]}(y_{it}) \quad \text{or} \quad \sum_{t=1}^{2000} I_{(-\infty,x]}(U_{it}) \right)$$

and the sample cdf for the corresponding x values in table 8 are given in table 9. The data in table 9 can be used to determine the probabilities of many droughts during the next 50 years. For example, let N be the number of times during the next 50 years for which Y_{1t} is less than or equal to 4.028 (1/2 the mean of Y_{1t} for the past 40 years) at Kearney. The corresponding probability in table 9 is 0.093, so:

$$P[N=k] = \binom{50}{k}(0.093)^k(0.907)^{50-k}$$

because N is distributed as a binomial with parameters 50 and 0.093.

An important question that can be answered by the data in tables 8 and 9 is: What is the probability of a drought of duration m years or longer during the 50-year planning period? Suppose a drought is said to occur whenever Y_{it} is less than a certain amount x, and that the duration of the drought is the number of years in a row that Y_{it} is less than x. The question above would then reduce to finding the distribution of the longest number of successes in a row in a sequence of Bernoulli trials, with success probability equal to probability $[Y_{it} \leq x]$. David and Barton (1962) show that if a sample of size r is drawn from an urn consisting of black and white balls, then, given that r_1 white and $r_2 = r-r_1$ black balls were selected, the distribution of the longest run K of white balls is:

$$\text{Prob}[K \geq m+1] \equiv P_K(m,r,r_1) = 1 - \sum_{t=0}^{\infty} \frac{(-1)^t}{t!}(r_2+1)^{(t)} \frac{r_1^{(tm+t)}}{r^{(tm+t)}} ; \qquad (15)$$

where $z^{(t)} = t^{\underline{th}}$ factorial power of $z = z(z-1)\cdots(z-t+1)$. Let M be the largest number of successes in a row in a sequence of 50 Bernoulli trials with success probability, p; then:

$$P[M \geq m+1] = \sum_{n=0}^{50} P[M \geq m+1 | N = n]\cdot P[N = n] ;$$

where N is the total number of successes in the 50 trials; (hence N has a binomial distribution). Therefore:

Table 9.--*Counts and sample cumulative distribution function (cdf) (corresponding to table 8)*

[μ = Mean of drought period, in inches]

	Fraction of mean	Gothenburg		Kearney		Grand Island		Three-station average	
		count	cfd	count	cfd	count	cfd	count	cfd
May and June	μ	1,213	0.606	1,163	0.581	1,142	0.571	1,175	0.587
	2/3 μ	511	.255	470	.235	441	.220	385	.192
	1/2 μ	192	.096	185	.093	174	.087	133	.066
	1/3 μ	34	.017	26	.013	33	.016	15	.007
June and July	μ	1,150	.575	1,201	.600	1,145	.572	1,186	.593
	2/3 μ	454	.227	518	.259	510	.255	427	.213
	1/2 μ	184	.092	200	.100	199	.099	130	.065
	1/3 μ	39	.019	48	.024	54	.027	32	.016
May,	μ	1,136	.568	1,127	.563	1,093	.546	1,105	.552
June,	2/3 μ	353	.176	336	.168	345	.172	266	.133
and	1/2 μ	100	.050	94	.047	96	.048	58	.029
July	1/3 μ	9	.004	9	.004	10	.005	3	.001

$$P[M \geq m+1] = \sum_{n=0}^{50} P_K(m,50,n) \cdot P[N = n] = \sum_{n=0}^{50} P_K(m,50,n) \cdot \binom{50}{n} p^n (1-p)^{50-n} .$$

where $P_k(m,50,n)$ can be evaluated on a computer as accurately as desired. Note that the summation in $P_K(m,r,r_1)$ is not really an infinite sum for fixed m, r, and r_1, because the terms become zero, whenever $t > r_2 + 1$ or $tm+t > r_1$. In this context, the probability of one or more droughts of m+1 years or longer during the next 50 years would be

$$P[M \geq m+1]$$

where

$$p = \text{Prob}[Y_{it} \leq x]$$

and a drought occurs whenever $Y_{it} \leq x$. The probabilities of some droughts involving Y_{2t} for Grand Island are presented in table 10.

CONCLUSIONS

The precipitation models presented in this report are offered primarily as decision tools for use in the management of the waterfowl habitat along a critical reach of the Platte River. Data from the precipitation stations at Gothenburg, Kearney, and Grand Island, Nebraska, were assumed to be adequate indicators of the precipitation patterns throughout the critical reach. Both ARIMA and trigonometric linear models were considered in representing the precipitation patterns at the three stations, but the trigonometric models were chosen for use in the analysis because they were superior for long-term simulations and drought analyses.

The precipitation models were used to determine the probability of occurrence of periods of less than average rainfall within the reach. The primary problem which the models addressed was delineating the frequency of insufficient rainfall which would inhibit vegetation regrowth along the banks and channel bars of the Platte River. Knowing the critical period and the amount of rainfall necessary to support revegetation along the reach, habitat managers could use tables, such as 8 and 9, to estimate either the number of times this amount will not be attained during the next 50 years, or the number of successive years for which this amount will not be attained during the 50-years. The critical within-year periods used were various month combinations from the set, May-June-July, and the drought limits were defined as 1/3, 1/2, and 2/3 of the respective historical mean precipitation. With the calculated frequency values, managers would have an idea of how often alternative methods might become necessary to remove the vegetation to insure proper habitat characteristics and subsequently, an estimate of the projected costs of this particular aspect of habitat management could be made.

Table 10.--*Probabilities of some droughts of duration m years or longer for Grand Island*

[Total for June and July; μ = mean of drought period, in inches]

m years	Critical level (x)[1/]			
	6.581 = μ	4.387 = 2/3μ	3.291 = 1/2μ	2.194 = 1/3μ
1	1.000	1.000	0.9945	0.7455
2	1.000	.9353	.3581	.0342
3	.9975	.4641	.0413	.0009
4	.9429	.1403	.0041	.0000
5	.7671	.0367	.0004	.0000
6	.5362	.0093	.0000	.0000
7	.3391	.0023	.0000	.0000
8	.2024	.0006	.0000	.0000
9	.1171	.0001	.0000	.0000
10	.0666	.0000	.0000	.0000
11	.0376	.0000	.0000	.0000
12	.0211	.0000	.0000	.0000

[1/] x is the level which defines the occurrence of a drought.

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