UNITED STATES
DEPARTMENT OF THE INTERIOR
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# ESTIMATING PROBABILITIES OF RESERVOIR STORAGE

# FOR THE UPPER DELAWARE RIVER BASIN

Open-File Report 81-478

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By Robert M. Hirsch

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Reston, Virginia

June 1981

UNITED STATES DEPARTMENT OF THE INTERIOR

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GEOLOGICAL SURVEY

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# ESTIMATING PROBABILITIES OF RESERVOIR STORAGE

# FOR THE UPPER DELAWARE RIVER BASIN

Robert M. Hirsch

## ABSTRACT

A technique for estimating conditional probabilities of reservoir system storage is described and applied to the upper Delaware River Basin. The results indicate that there is a 73 percent probability that the three major New York City reservoirs (Pepacton, Cannonsville, and Neversink) would be full by June 1, 1981, and only a 9 percent probability that storage would return to the "drought warning" sector of the operations curve sometime in the next year. In contrast, if restrictions are lifted and there is an immediate return to normal operating policies, the probability of the reservoir system being full by June 1 is 37 percent and the probability that storage would return to the "drought warning" sector in the next year is 30 percent.

## NON-MATHEMATICAL DESCRIPTION OF POSITION ANALYSIS

The underlying idea of position analysis is this: A monthly streamflow value can be viewed as the sum of two components. The first is a carry-over effect from the past month, the second is a random effect. We can look over a historical flow record and compute values for this random component for each month. We can check to see if they are, in fact, random by determining if they are correlated with the flows in recent months. If they appear to be random, then we may use these past records of the random component as a sample from the population of random components which may occur in the upcoming month or months.

If, say, we are at the end of February, then we may take the random component values from each historical March and add them to the carry-over effect from this February and consider each of these sums to be an equally likely value of March flow. We can extend this to produce sets of equally likely traces for March, April, May.... We do this by taking each historical trace of March, April, May...random components and add the March random component to the February carry-over component to get a March flow, then add the April random component to the March carry-over to get an April flow, and so on. Then each of these traces may be used in a simulation of the future flows (say as inputs to a reservoir) and the outputs of the simulation (storages under some operating policy) may be viewed as equally likely future traces.

The name "position analysis" refers to the fact that we are evaluating probabilities (of flows or storages) from the present position where the position is defined in terms of the recent history of flows and the present storage.

## DESCRIPTION OF POSITION ANALYSIS

### Flow Trace Generation

We have a historical time series of monthly flow data X(i) i=1,2,...nm, where nm is the index of the present month. For all of the data from month j (j = 1,2,...,12) we can compute a set of 4 parameters which have the effect of normalizing and standardizing the data: [for the method of parameter estimation see Hirsch (1979)].

$$Y(i) = \begin{cases} \ln X(i) & X \leq b_j \\ a_j \ln(X(i)) + (1-a_j)\ln b_j & X > b_j \end{cases} \tag{1}$$

$$Z(i) = \frac{Y(i) - ybar_j}{ystd_j} \tag{2}$$

The relationship between i and j is j = mod(i-1,12)+1

The Z(i) are the normalized and standardized flow values

We can compute the input sequence e(i) recursively

$$e(i) = Z(i) - \Phi_j Z(i-1) \qquad i = 2,3,\ldots,nm \tag{3}$$

where $\Phi_j = r_1(j)$ for j = 1,2,...,12

where $r_1(j)$ is the sample correlation coefficient of the Z data for month j and the month preceding it.

Now: we wish to generate a set of future flow traces for the upcoming $\ell$ months. Find the largest integer value of K such that nm + 1 - 12K $\geq$ 2 and use this K to generate the first sequence:

$$\begin{aligned} Z_f(nm+1) &= \Phi_j Z(nm) + e(nm+1 - 12K) \\ Z_f(nm+2) &= \Phi_j Z_f(nm+1) + e(nm+2 - 12K) \\ &\vdots \\ Z_f(nm+i) &= \Phi_j Z_f(nm+i-1) + e(nm+i - 12K) \\ &\vdots \\ Z_f(nm+\ell) &= \Phi_j Z_f(nm+\ell-1) + e(nm+\ell - 12K) \end{aligned} \tag{4}$$

Note that the index j is always set by the value of the time index on the Z value on the left hand side ($i\ell hs$): j = mod($i\ell hs$-1,12) + 1

To generate the second trace, decrement K by 1 and so on until K reaches a value such that the time index on the e sequence proceeds to times in the future. That is, the time index on the e values is greater than nm.

Let the number of complete traces of $[Z_f(nm+1),\ldots,Z_f(nm+\ell)]$ be n. These n $Z_f$ series can be transformed back to n flow series $[X_f(nm+1),\ldots Xf(nm+\ell)]$ by applying the inverse of equations 2 and 1 to the $Z_f$ data.

## Reservoir Simulation

In the case of the Delaware system (see fig. 1 for a map of the upper Delaware River basin), the X sequence is the "net inflow" to the New York City Reservoir System. Then the n $X_f$ sequences are assumed to be n equally likely traces of net inflows over the upcoming $\ell$ months. We can simulate the storage in the reservoir month by month for each of these n traces by doing a mass balance which requires knowing the initial volume (end of month nm), the net inflow sequence $X_f$, the diversions to New York City, and the release to meet downstream flow requirements. To determine the appropriate value of this release to meet downstream flow requirements for each month for each of the n traces one must know the uncontrolled flow to the Montague, N.J., gage and the required flow at that gage. The uncontrolled flow record at Montague is developed by exactly the same procedure as is used to develop the reservoir inflow record. The inputs for each of the n trials uses e values from the same year for developing each of the two flow records (thus approximately preserving the site-to-site correlation). The rule for simulating the magnitude of the monthly release to achieve the downstream flow requirement is set by a formula developed by the New York State Department of Environmental Conservation. The reason for needing this simulation rule is that the simulation occurs at a monthly time step and the actual flows at Montague will fluctuate widely during the course of the month. The relationship is depicted in figure 2.

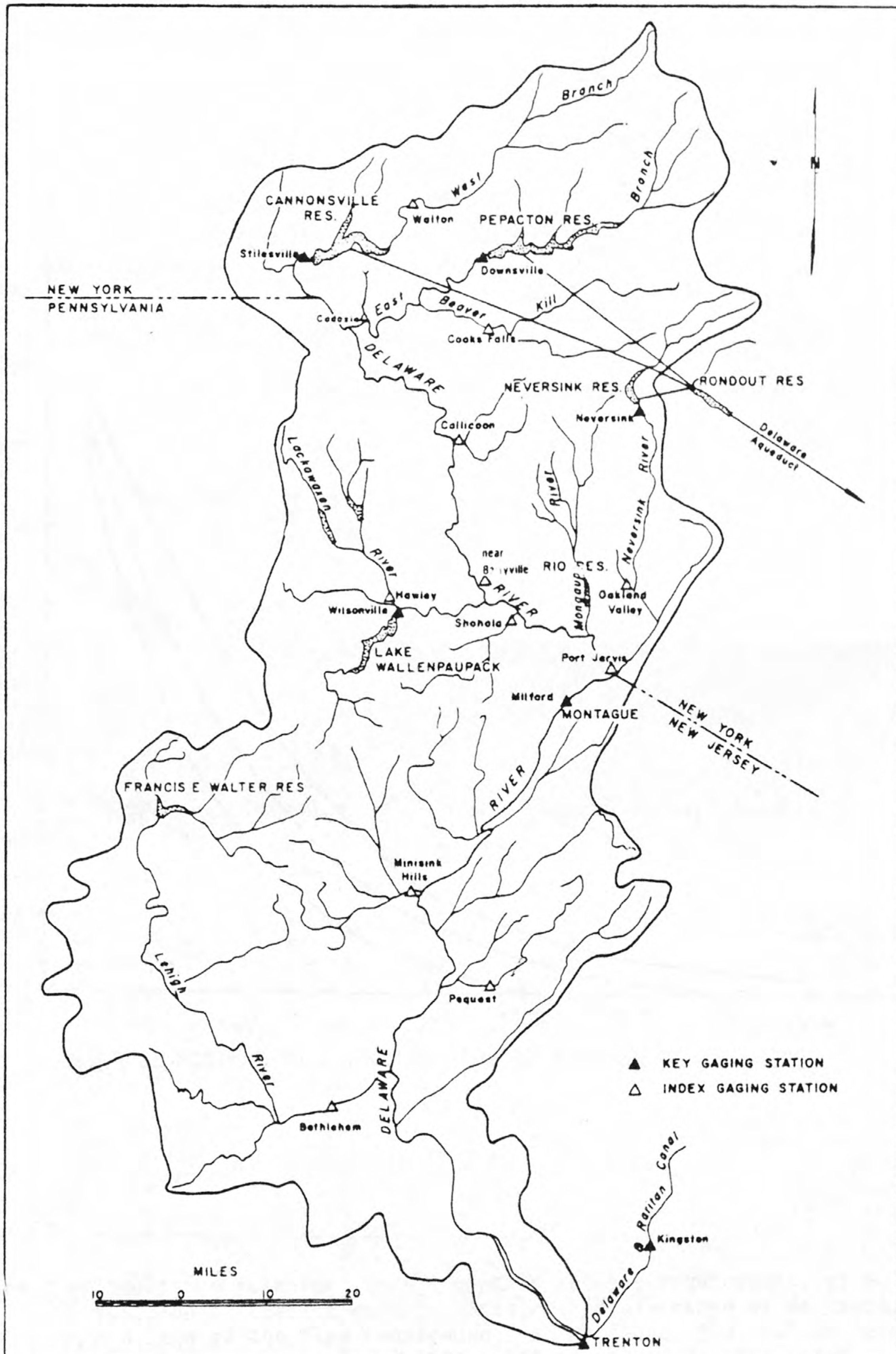


Figure 1.--Delaware River basin above Trenton, N.J., showing Cannonsville, Pepacton, and Neversink reservoirs and the Montague gage.

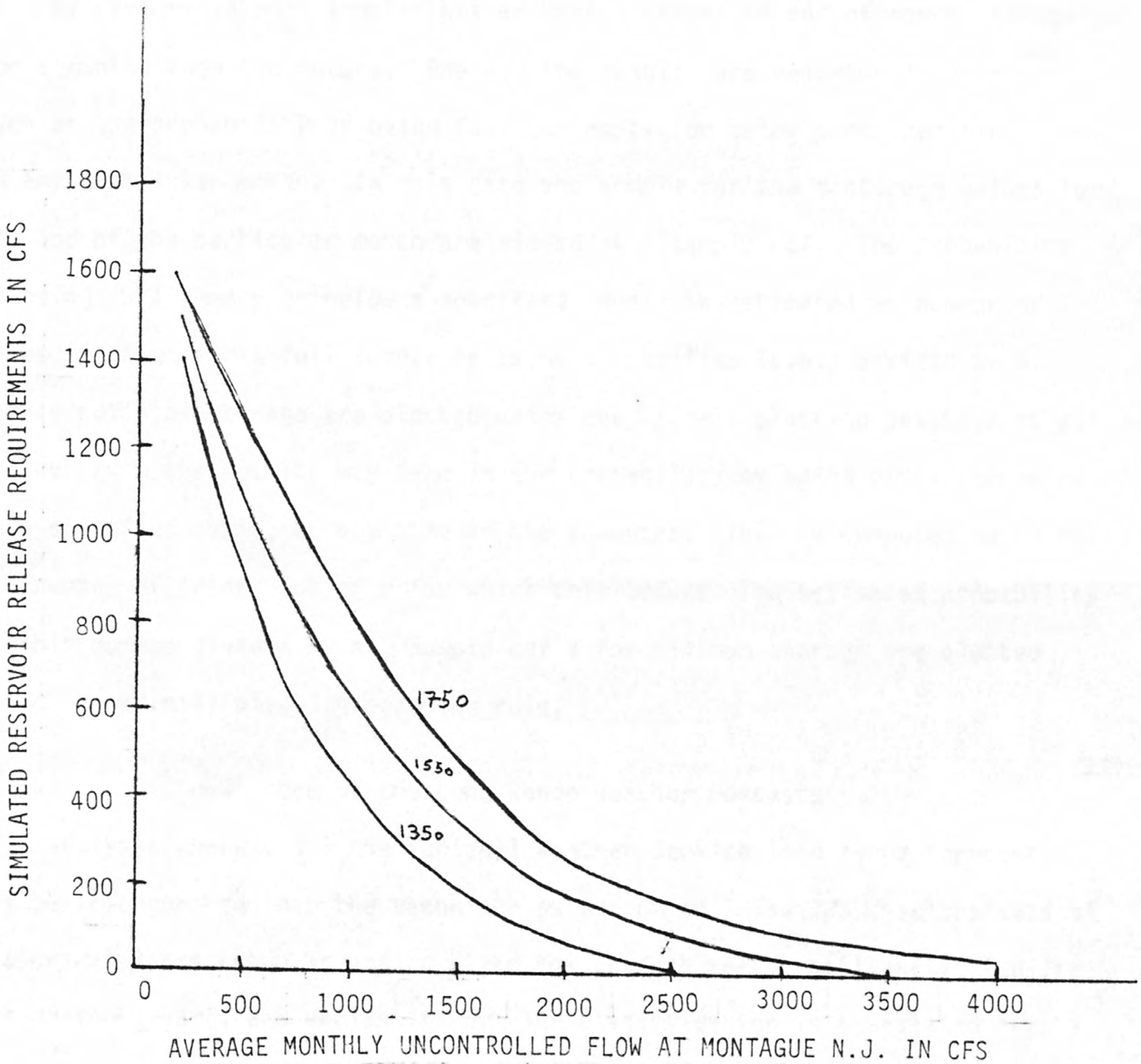


Figure 2.--Simulation rule for average monthly release requirements as a function of average monthly uncontrolled discharge at Montague, N.J. and of the flow requirements at Montague, N.J. Shown here for flow requirements of 1350, 1550 and 1750 cfs, for other values interpolation is used.

## Results Computations

By performing such simulations we have n traces of end of month storage for $\ell$ months into the future. One way the results are reported is in terms such as the probability of being full (or empty, or below some specified level) in any particular month. In this case the sample for the n storage values for the end of the particular month are viewed as a sample cdf. The probability of being full (empty or below a specified level) is estimated as number of cases in which it is full (empty or below a specified level) divided by n. Sample cdf's of storage are plotted using the m/(n+1) plotting position rule. Another form the results may take is the probability of being empty (or below some specified curve) at any time in the $\ell$ months. This is computed by counting the number of traces out of n for which this occurs. The estimated probability is this number divided by n. Sample cdf's for minimum storage are plotted using the m/(n+1) plotting position rule.

## Use of the Long Range Weather Forcasts

Analyses where $\ell \leq 3$ the National Weather Service long range forecast may be incorporated into the technique as an option. The NWS develops sets of historical years known as analog (like the present year), anti-analog (unlike the present year), and unclassified. The classification is translated into a set of probabilities on the n traces. [Note that where no long range weather forecast is used, each of the n traces is assigned a probability 1/n.] In this case each analog year is assigned twice the probability of each non-classified which is in turn assigned twice the probability of each anti-analog year. These probabilities are set so they sum to 1.0. The source of the method is John Schaake (NWS) oral communication 1981.

The two sets of flow data (net inflow and uncontrolled flow to Montague) are 645 months in length (nm = 645). The first 600 values (June 1927-May 1977) were supplied by the New York State Department of Environmental Conservation. The last 45 values (June 1977-February 1981) were supplied by Bob Fish (USGS, Milford, Pennsylvania).

The argument for normalization and fitting parameters to the normalized data is given in Hirsch (1979). Tables 1A and 1B describe the sample moments of the X, Z, and e data for each of the months at each of the sites. The sample size for each month is 52 or 53 values. The X data are not generally normal, but the Z data do not depart significantly from normality (as measured by their skewness and kurtosis). In only 1 out of 24 cases was the coefficient of skewness outside the 90 percent confidence band about zero. In only 1 out of 24 cases was the coefficient of kurtosis outside the 90 percent confidence band about 3.

The adequacy of the auto regressive processes with cyclic parameter $\Phi_j$ is demonstrated by considering the sample correlation coefficients between the e series and lagged values of the Z series. The model would be inadequate if the e series appeared to be dependent on past Z values. Tables 2A and 2B present matrices of these correlation coefficients for the two sites. Of the 288 values in these tables only 8 are outside the 95 percent confidence band about zero.

At both sites these sample correlations appear, in general, not to depart significantly from zero. It is useful to compare the correlations in these tables with the correlations of the Z values (tables 3A and 3B). If one were to employ a no memory model ($\Phi_j = 0 \quad j=1,2,...,12$) then these would be the correlations between the e values and the previous Z values, just like tables 2A and 2B. Comparison of table 3A with 2A or 3B with 2B provides an indication of the extent to which the model has succeeded in depicting the memory, and the room for improvement in the model. For example, it is apparent that e values for March do not depend much on the Z for February or January, but there is a strong suggestion that there is some dependence on Z values from the autumn. The flow generation scheme ignores this possible dependence and looks upon March flows as if they depended only on February flows. In summary, based on tables 2A and 2B, there is no compelling statistical argument against the assertion that each of the n e values for, say, March is equally likely. However, no claim is being made here that this is the "best possible model" in any sense.

Another demonstration of the adequacy of the model is that the cdf of flows several months in the future converges to the historical cdf for that month. It can be shown, based on equations 3 and 4 and provided all $|\Phi_j| \leq 1$, that for a given value of K $\lim_{i\to\infty} Z_f(nm+i) = Z(nm+i-12K)$. The rate of convergence depends on how close the product $\Phi_j \cdot \Phi_{j+1} \cdots \Phi_{j+i}$ is to zero. Generally the convergence is to at least 3 significant figures in the Z values by 12 months.

Complete sets of model parameters are given in tables 4A and 4B.

Table 1.--Statistics of the X, Z and e series.

## Reservoir Inflow Series

1A

| month | j | X mean | X std dev | X skew | X kurtosis | Z mean | Z std dev | Z skew | Z kurtosis | e mean | e std dev | e skew | e kurtosis |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| O | 1 | 1068.1 | 1130.2 | 1.87 | 6.21 | 0.00 | 1.00 | -0.09 | 2.69 | 0.00 | 0.82 | 0.31 | 2.48 |
| N | 2 | 1836.0 | 1148.8 | 0.71 | 2.77 | 0.00 | 1.00 | -0.40 | 3.02 | -0.00 | 0.75 | 0.43 | 2.63 |
| D | 3 | 2126.0 | 1197.4 | 1.00 | 3.75 | 0.00 | 1.00 | -0.19 | 2.46 | 0.00 | 0.92 | 0.17 | 2.84 |
| J | 4 | 1879.6 | 1084.7 | 0.70 | 3.20 | -0.00 | 1.00 | -0.29 | 2.55 | -0.00 | 0.85 | 0.30 | 2.33 |
| F | 5 | 1798.5 | 1115.2 | 1.53 | 6.30 | 0.00 | 1.00 | 0.16 | 3.28 | 0.00 | 0.99 | 0.40 | 3.58 |
| M | 6 | 3413.4 | 1622.0 | 1.33 | 4.92 | 0.00 | 1.00 | -0.10 | 3.06 | -0.01 | 0.98 | -0.10 | 2.80 |
| A | 7 | 4001.2 | 1537.5 | 1.19 | 4.94 | -0.00 | 1.00 | -0.72 | 4.45 | -0.00 | 0.96 | -0.77 | 4.35 |
| M | 8 | 2156.0 | 928.0 | 0.47 | 2.90 | 0.00 | 1.00 | 0.26 | 2.95 | 0.00 | 1.00 | 0.25 | 2.95 |
| J | 9 | 1155.1 | 761.9 | 1.44 | 5.38 | -0.00 | 1.00 | 0.27 | 2.65 | 0.00 | 0.75 | 0.33 | 2.54 |
| J | 10 | 739.4 | 635.8 | 1.52 | 4.56 | 0.00 | 1.00 | -0.08 | 2.34 | 0.00 | 0.75 | 0.34 | 3.86 |
| A | 11 | 604.5 | 605.7 | 2.54 | 9.35 | -0.00 | 1.00 | -0.23 | 3.36 | -0.00 | 0.86 | 0.68 | 5.33 |
| S | 12 | 753.0 | 841.8 | 2.11 | 6.94 | -0.00 | 1.00 | 0.06 | 2.63 | -0.00 | 0.80 | 0.36 | 2.43 |

## Uncontrolled flow to Montague

1B

| month | j | X mean | X std dev | X skew | X kurtosis | Z mean | Z std dev | Z skew | Z kurtosis | e mean | e std dev | e skew | e kurtosis |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| O | 1 | 2837.6 | 2812.7 | 2.10 | 7.20 | 0.00 | 1.00 | 0.00 | 2.83 | 0.00 | 0.75 | 0.15 | 2.94 |
| N | 2 | 4393.5 | 2868.7 | 1.10 | 4.24 | 0.00 | 1.00 | -0.03 | 2.65 | 0.00 | 0.73 | 0.80 | 3.69 |
| D | 3 | 5221.8 | 2892.9 | 1.04 | 3.71 | 0.00 | 1.00 | 0.09 | 2.56 | 0.00 | 0.87 | 0.52 | 3.35 |
| J | 4 | 4977.3 | 2581.8 | 0.72 | 3.24 | -0.00 | 1.00 | -0.23 | 2.90 | -0.00 | 0.82 | 0.46 | 3.33 |
| F | 5 | 4867.7 | 2506.2 | 0.87 | 3.43 | 0.00 | 1.00 | -0.03 | 2.89 | 0.00 | 0.97 | 0.20 | 4.02 |
| M | 6 | 8703.9 | 3710.7 | 1.92 | 8.85 | -0.00 | 1.00 | -0.08 | 2.85 | -0.01 | 0.99 | -0.11 | 2.60 |
| A | 7 | 9838.4 | 3610.7 | 0.79 | 4.58 | -0.00 | 1.00 | 0.05 | 3.32 | -0.00 | 1.00 | 0.03 | 3.21 |
| M | 8 | 5762.4 | 2299.6 | 0.35 | 2.38 | -0.00 | 1.00 | 0.24 | 2.75 | -0.00 | 1.00 | 0.26 | 2.74 |
| J | 9 | 3544.1 | 2337.1 | 1.89 | 7.37 | -0.00 | 1.00 | -0.08 | 2.44 | -0.00 | 0.75 | 0.13 | 2.20 |
| J | 10 | 2477.1 | 2085.2 | 1.90 | 6.22 | -0.00 | 1.00 | -0.20 | 2.89 | -0.00 | 0.64 | 0.02 | 2.83 |
| A | 11 | 2203.4 | 2071.4 | 2.91 | 14.13 | -0.00 | 1.00 | 0.02 | 3.09 | -0.00 | 0.83 | 1.33 | 6.95 |
| S | 12 | 2245.1 | 1889.4 | 1.64 | 4.85 | -0.00 | 1.00 | -0.02 | 2.26 | -0.00 | 0.77 | 0.07 | 2.38 |

Table 2.--Correlation coefficients between e and lagged Z.

2A

Reservoir inflow

| lag | O<br>1 | N<br>2 | D<br>3 | J<br>4 | F<br>5 | M<br>6 | A<br>7 | M<br>8 | J<br>9 | J<br>10 | A<br>11 | S<br>12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | -0.00 | -0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.01 | -0.00 | -0.00 | -0.00 |
| 2 | 0.06 | -0.07 | -0.09 | -0.08 | 0.21 | -0.05 | -0.21 | 0.12 | -0.14 | 0.06 | 0.09 | -0.05 |
| 3 | -0.20 | -0.03 | 0.16 | 0.02 | 0.04 | -0.02 | -0.20 | 0.10 | -0.04 | 0.23 | -0.12 | -0.02 |
| 4 | -0.16 | 0.14 | -0.01 | 0.06 | -0.02 | 0.15 | -0.14 | 0.08 | 0.11 | -0.15 | -0.03 | -0.00 |
| 5 | -0.03 | 0.07 | 0.01 | 0.16 | 0.13 | 0.23 | -0.03 | 0.07 | 0.07 | 0.00 | -0.01 | 0.08 |
| 6 | -0.01 | 0.13 | -0.05 | 0.04 | 0.06 | 0.25 | -0.05 | -0.09 | 0.28 | 0.09 | 0.07 | -0.05 |
| 7 | 0.00 | 0.17 | 0.11 | 0.05 | 0.22 | 0.34 | -0.07 | 0.12 | 0.35 | 0.02 | 0.06 | 0.10 |
| 8 | 0.15 | 0.14 | 0.15 | 0.01 | 0.11 | 0.27 | 0.11 | 0.13 | 0.24 | -0.01 | 0.04 | -0.01 |
| 9 | 0.19 | -0.06 | 0.04 | -0.08 | 0.02 | 0.12 | -0.02 | 0.13 | 0.10 | 0.01 | 0.29 | 0.06 |
| 10 | 0.06 | 0.07 | -0.08 | 0.22 | 0.08 | 0.18 | 0.03 | 0.05 | 0.04 | 0.26 | 0.29 | 0.00 |
| 11 | -0.05 | 0.18 | -0.05 | -0.16 | -0.10 | -0.18 | -0.15 | 0.23 | -0.16 | 0.24 | 0.04 | 0.23 |
| 12 | 0.01 | -0.13 | 0.10 | -0.11 | 0.02 | -0.06 | -0.06 | 0.14 | -0.11 | 0.15 | -0.08 | -0.13 |

2B

Uncontrolled flow to Montague

| lag | O<br>1 | N<br>2 | D<br>3 | J<br>4 | F<br>5 | M<br>6 | A<br>7 | M<br>8 | J<br>9 | J<br>10 | A<br>11 | S<br>12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 | -0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.00 | -0.00 |
| 2 | 0.02 | -0.12 | -0.02 | -0.08 | 0.21 | 0.03 | -0.03 | 0.22 | -0.09 | 0.01 | -0.05 | 0.01 |
| 3 | -0.21 | 0.03 | 0.15 | 0.02 | 0.07 | 0.09 | -0.13 | 0.10 | -0.10 | 0.25 | -0.19 | 0.09 |
| 4 | -0.17 | 0.20 | -0.05 | 0.03 | 0.01 | 0.11 | 0.03 | 0.17 | 0.17 | -0.09 | -0.09 | 0.08 |
| 5 | -0.08 | 0.19 | 0.00 | 0.03 | 0.12 | 0.18 | 0.12 | 0.14 | 0.16 | -0.04 | -0.00 | 0.14 |
| 6 | -0.07 | 0.14 | 0.04 | 0.10 | 0.15 | 0.25 | 0.07 | 0.06 | 0.32 | -0.02 | 0.13 | -0.05 |
| 7 | 0.09 | 0.12 | 0.06 | 0.16 | 0.17 | 0.20 | 0.06 | 0.21 | 0.24 | 0.01 | 0.05 | 0.13 |
| 8 | 0.13 | 0.18 | 0.17 | 0.08 | 0.07 | 0.13 | 0.14 | 0.17 | 0.11 | 0.09 | 0.01 | -0.05 |
| 9 | 0.20 | -0.01 | 0.03 | 0.06 | -0.02 | 0.01 | 0.04 | 0.23 | -0.00 | 0.11 | 0.22 | 0.01 |
| 10 | -0.04 | 0.04 | -0.02 | 0.28 | 0.15 | 0.08 | -0.00 | 0.21 | 0.12 | 0.18 | 0.04 | 0.05 |
| 11 | -0.17 | 0.12 | 0.04 | -0.12 | -0.14 | -0.20 | -0.11 | 0.28 | -0.03 | 0.06 | -0.05 | 0.19 |
| 12 | -0.15 | -0.17 | 0.11 | -0.01 | 0.02 | -0.10 | 0.09 | 0.09 | -0.03 | 0.15 | -0.16 | -0.05 |

Table 3.--Correlation coefficients of the Z value.

Inflows to reservoirs

3A

| lag | O<br>1 | N<br>2 | D<br>3 | J<br>4 | F<br>5 | M<br>6 | A<br>7 | M<br>8 | J<br>9 | J<br>10 | A<br>11 | S<br>12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.58 | 0.66 | 0.40 | 0.52 | 0.16 | -0.20 | -0.27 | -0.02 | 0.66 | 0.66 | 0.50 | 0.60 |
| 2 | 0.40 | 0.33 | 0.18 | 0.14 | 0.29 | -0.10 | -0.15 | 0.13 | -0.12 | 0.48 | 0.41 | 0.26 |
| 3 | -0.01 | 0.24 | 0.28 | 0.11 | 0.06 | -0.08 | -0.17 | 0.10 | 0.05 | 0.09 | 0.14 | 0.23 |
| 4 | -0.00 | 0.09 | 0.09 | 0.19 | -0.00 | 0.13 | -0.11 | 0.08 | 0.15 | -0.08 | 0.03 | 0.08 |
| 5 | 0.02 | 0.05 | 0.04 | 0.18 | 0.16 | 0.22 | -0.06 | 0.07 | 0.10 | 0.10 | -0.05 | 0.08 |
| 6 | 0.04 | 0.12 | -0.03 | 0.06 | 0.09 | 0.20 | -0.11 | -0.09 | 0.26 | 0.14 | 0.11 | -0.07 |
| 7 | -0.04 | 0.16 | 0.15 | 0.03 | 0.23 | 0.31 | -0.12 | 0.12 | 0.21 | 0.18 | 0.12 | 0.15 |
| 8 | 0.21 | 0.08 | 0.20 | 0.08 | 0.11 | 0.21 | 0.02 | 0.13 | 0.26 | 0.13 | 0.13 | 0.07 |
| 9 | 0.20 | 0.09 | 0.07 | 0.04 | 0.04 | 0.09 | -0.08 | 0.13 | 0.16 | 0.17 | 0.32 | 0.12 |
| 10 | 0.12 | 0.18 | -0.03 | 0.22 | 0.09 | 0.16 | 0.01 | 0.05 | 0.11 | 0.31 | 0.33 | 0.19 |
| 11 | 0.07 | 0.21 | 0.02 | -0.16 | -0.06 | -0.20 | -0.18 | 0.22 | -0.09 | 0.26 | 0.19 | 0.38 |
| 12 | 0.23 | -0.06 | 0.17 | -0.08 | -0.00 | -0.04 | -0.00 | 0.14 | 0.06 | 0.06 | 0.06 | 0.01 |

Uncontrolled flows to Montague

3B

| lag | O<br>1 | N<br>2 | D<br>3 | J<br>4 | F<br>5 | M<br>6 | A<br>7 | M<br>8 | J<br>9 | J<br>10 | A<br>11 | S<br>12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.66 | 0.69 | 0.50 | 0.57 | 0.26 | -0.12 | -0.10 | 0.04 | 0.66 | 0.77 | 0.56 | 0.64 |
| 2 | 0.44 | 0.37 | 0.32 | 0.22 | 0.35 | -0.01 | -0.01 | 0.21 | -0.04 | 0.51 | 0.39 | 0.36 |
| 3 | 0.09 | 0.32 | 0.31 | 0.20 | 0.12 | 0.04 | -0.13 | 0.10 | 0.07 | 0.13 | 0.13 | 0.33 |
| 4 | 0.09 | 0.20 | 0.12 | 0.20 | 0.07 | 0.09 | 0.03 | 0.16 | 0.19 | -0.00 | -0.01 | 0.15 |
| 5 | 0.04 | 0.20 | 0.10 | 0.09 | 0.17 | 0.16 | 0.11 | 0.15 | 0.22 | 0.12 | -0.00 | 0.10 |
| 6 | 0.01 | 0.13 | 0.14 | 0.14 | 0.17 | 0.22 | 0.06 | 0.06 | 0.34 | 0.16 | 0.17 | -0.04 |
| 7 | 0.04 | 0.10 | 0.12 | 0.20 | 0.20 | 0.18 | 0.04 | 0.21 | 0.22 | 0.27 | 0.12 | 0.22 |
| 8 | 0.24 | 0.17 | 0.20 | 0.13 | 0.12 | 0.11 | 0.12 | 0.17 | 0.22 | 0.23 | 0.16 | 0.04 |
| 9 | 0.18 | 0.16 | 0.11 | 0.16 | 0.01 | -0.01 | 0.03 | 0.24 | 0.11 | 0.24 | 0.31 | 0.11 |
| 10 | 0.04 | 0.16 | 0.06 | 0.29 | 0.19 | 0.08 | -0.00 | 0.21 | 0.25 | 0.20 | 0.17 | 0.23 |
| 11 | 0.03 | 0.12 | 0.12 | -0.07 | -0.06 | -0.22 | -0.12 | 0.27 | 0.12 | 0.23 | 0.07 | 0.25 |
| 12 | 0.05 | -0.11 | 0.16 | 0.05 | -0.00 | -0.09 | 0.11 | 0.08 | 0.16 | 0.19 | -0.01 | 0.01 |

Table 4.--Model parameters

4A Reservoir inflow parameters

| | j | $a_j$ | $b_j$ | $ybar_j$ | $ystd_j$ | — | $\beta_j$ | $\phi_j$ |
|---|---|---|---|---|---|---|---|---|
| O | 1 | 0.444 | 180.0 | 5.761 | 0.460 | 0.000 | 0.817 | 0.577 |
| N | 2 | 1.500 | 1589.0 | 7.405 | 0.898 | 0.000 | 0.748 | 0.664 |
| D | 3 | 0.667 | 783.0 | 7.220 | 0.390 | 0.000 | 0.916 | 0.401 |
| J | 4 | 1.500 | 1746.0 | 7.443 | 0.798 | 0.000 | 0.855 | 0.519 |
| F | 5 | 1.500 | 2325.0 | 7.354 | 0.681 | 0.000 | 0.987 | 0.163 |
| M | 6 | 0.667 | 1483.0 | 7.786 | 0.309 | 0.000 | 0.980 | -0.197 |
| A | 7 | 0.667 | 5102.0 | 8.210 | 0.357 | 0.000 | 0.964 | -0.266 |
| M | 8 | 2.250 | 2866.0 | 7.621 | 0.543 | 0.000 | 1.000 | -0.022 |
| J | 9 | 1.500 | 1080.0 | 6.946 | 0.788 | 0.000 | 0.751 | 0.660 |
| J | 10 | 0.667 | 246.0 | 5.995 | 0.583 | 0.000 | 0.754 | 0.657 |
| A | 11 | 0.296 | 210.0 | 5.535 | 0.273 | 0.000 | 0.864 | 0.504 |
| S | 12 | 0.667 | 394.0 | 6.016 | 0.715 | 0.000 | 0.799 | 0.601 |

4B Uncontrolled flow to Montague parameters

| | j | $a_j$ | $b_j$ | $ybar_j$ | $ystd_j$ | — | $\beta_j$ | $\phi_j$ |
|---|---|---|---|---|---|---|---|---|
| O | 1 | 0.444 | 1000.0 | 7.180 | 0.410 | 0.000 | 0.748 | 0.664 |
| N | 2 | 1.500 | 4204.0 | 8.257 | 0.838 | 0.000 | 0.726 | 0.688 |
| D | 3 | 1.500 | 9265.0 | 8.420 | 0.572 | 0.000 | 0.868 | 0.496 |
| J | 4 | 1.500 | 4598.0 | 8.458 | 0.698 | 0.000 | 0.823 | 0.569 |
| F | 5 | 1.500 | 6518.0 | 8.381 | 0.587 | 0.000 | 0.965 | 0.261 |
| M | 6 | 0.667 | 11970.0 | 8.985 | 0.359 | 0.000 | 0.993 | -0.117 |
| A | 7 | 1.500 | 10000.0 | 9.176 | 0.451 | 0.000 | 0.995 | -0.096 |
| M | 8 | 2.250 | 7364.0 | 8.624 | 0.511 | 0.000 | 0.999 | 0.043 |
| J | 9 | 0.667 | 5687.0 | 7.979 | 0.566 | 0.000 | 0.749 | 0.663 |
| J | 10 | 0.667 | 1605.0 | 7.418 | 0.576 | 0.000 | 0.642 | 0.767 |
| A | 11 | 0.198 | 532.0 | 6.493 | 0.158 | 0.000 | 0.831 | 0.557 |
| S | 12 | 0.444 | 1398.0 | 7.233 | 0.425 | 0.000 | 0.769 | 0.639 |

There were 4 basic cases considered:

(1) 3-month projection with restricted use

(2) 3-month projection with non-restricted use

(3) 12-month projection with restricted use

(4) 12-month projection with non-restricted use

For cases 1 and 2 the long range weather forecast was used.

The analog years were 1951, 1952, 1955, 1956, 1958, 1962, and 1969.

The anti-analog years were 1949, 1954, 1968, 1978, and 1979.

The draft rate in case 1 was 520 MGD

The draft rate in case 2 was 800 MGD

The required flows at Montegue were:

Case (1) 1350 cfs in March and April and 1600 cfs in May

Case (2) 1750 cfs all three months

The value of n is 53 (53 historical e series of length 3 were used)

Result: probability of being full:

Case (1) 0.722 (if the long range weather forecast is not used it is 0.717)

Case (2) 0.356 (if the long range weather forecast is not used it is 0.358)

In cases 3 and 4, the long range weather forecast was not used

The draft rate in case 3 was 520 MGD

The draft rate in case 4 was 800 MGD

The required flow at Montegue in case 3 is

1350 cfs March-April, 1600 cfs (May-Aug.), 1500 cfs (Sept.-Nov.) and 1300 cfs (Dec.-Feb.)

The required flow at Montegue in case 4 is 1750 cfs in all months.

The value of n is 52 (52 historical e series of length 12 used).

In case (3) the number of trials out of 52 which fall below the warning curve is 5, so the probability estimate is 0.096 (5/52).

In case (4) the number of trials out of 52 which fall below the warning curve is 16, so the probability estimate is 0.308 (16/52).

Figures 3 and 4 display the results of cases 3 and 4 and also show the drought warning curve. In each of these figures the solid line is the historical record of storage up to the end of February 1981 and the 2, 10, 50, 90, and 98 percent probability curves for the end of March 1981 through the end of February 1982. The estimated probability of storage falling below the P percent probability curve (where P = 2, 10, 50, 90 or 98) at any given time is P percent. In general there would be a greater than P percent chance that storage would fall below this curve at some time in the entire 12 month period.

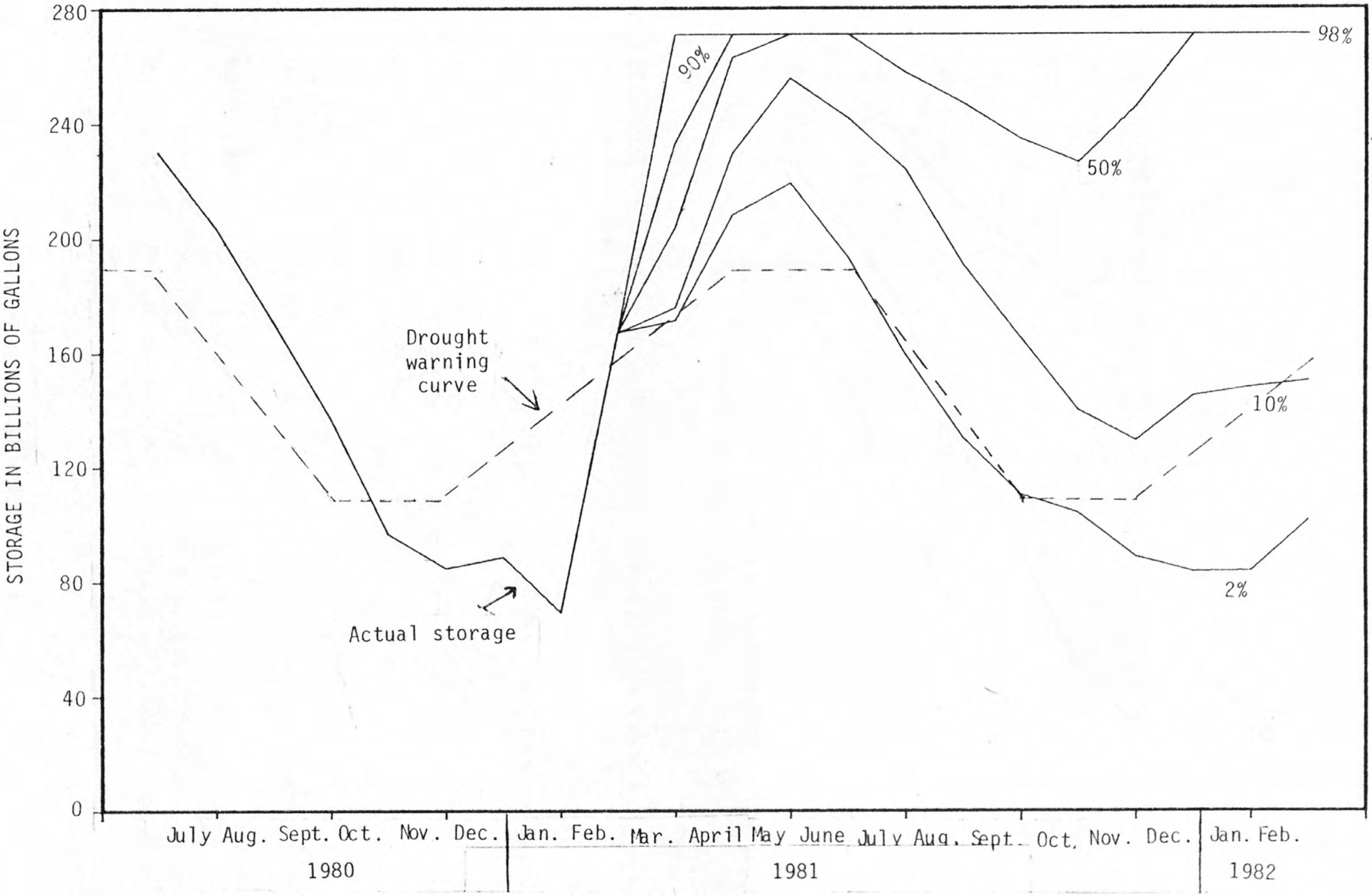


Figure 3.--Results of case 3, showing history of storage from the end of June 1980 to 2, 10, 50, 90 and 98 percent probability values for storage at the ends of March 1981 through February 1982. Dashed line is the Drought Warning Curve. Case 3 is a continuation of the restrictions imposed by the Delaware River Basin Commission on January 15, 1981. (520 million gallons per day to New York City.)

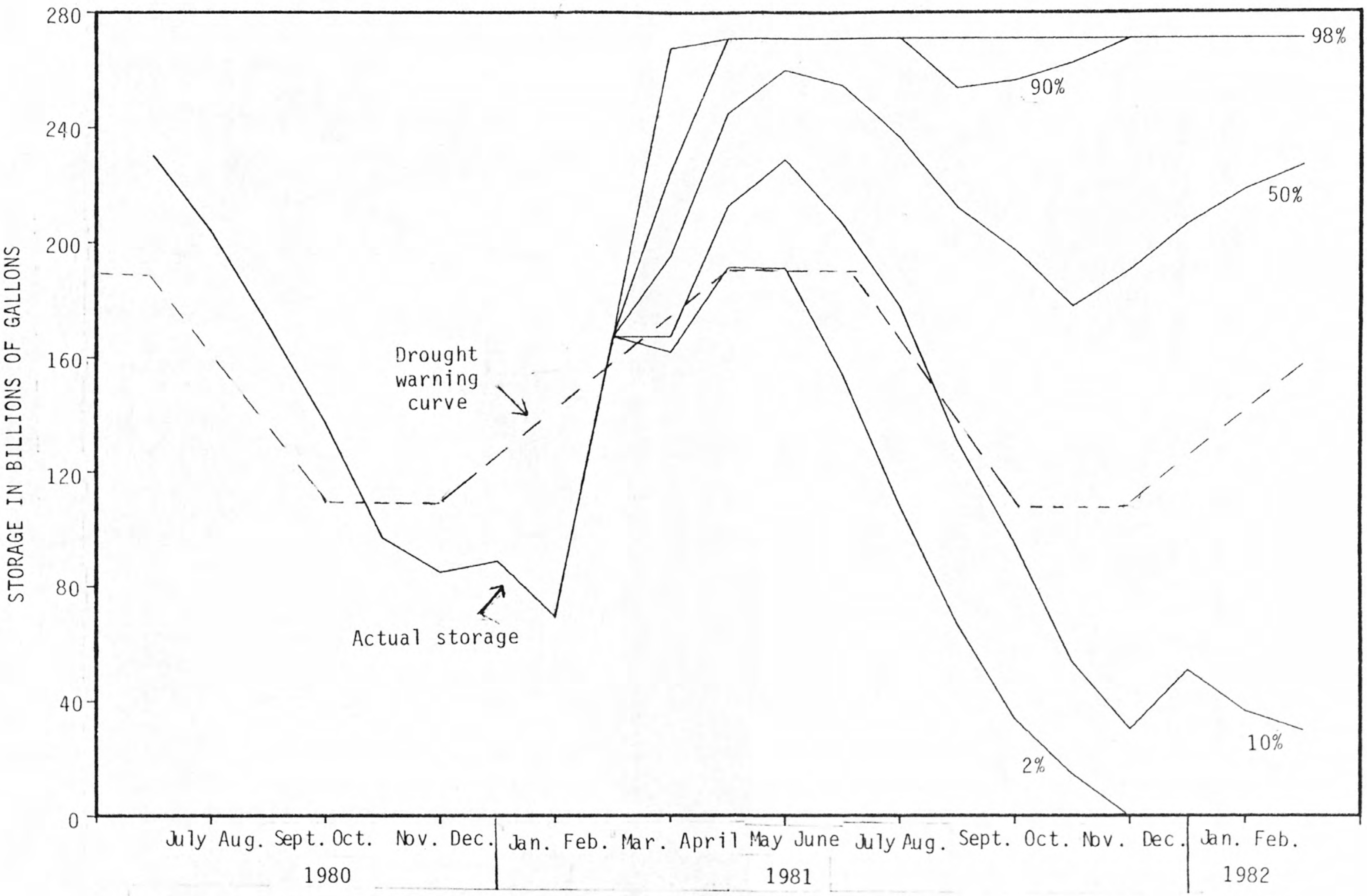


Figure 4.--Results of case 4, showing history of storage from the end of June 1980 to 2, 10, 50, 90 and 98 percent probability values for storage at the ends of March 1981 through February 1982. Dashed line is the Drought Warning Curve. Case 4 assumes the return to customary operation. (800 million gallons per day to New York City.)

# REFERENCE

Hirsch, R.M., 1979, Synthetic hydrology and water supply reliability, Water Resources Research, vol. 15, no. 6., p. 1603-1615.