PROBABILITY MODELS AND COMPUTER SIMULATION OF LANDSCAPE EVOLUTION

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Open-File Report 81-656

1981

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Abstract

A model of landscape evolution based on a random-walk probability experiment provides a method by which the effects of uplift and-or denudation can be studied, in order to determine the rates of tectonic uplift that leave characteristic topographic imprints. The model, derived from a generalization of the Leopold and Langbein (1962) random-walk experiments, is a discharge-related, rather than elevation-dependent, model.

Implementation of the model on the U. S. Geological Survey Honeywell computer through computer program UDL (Uplift and Denudation of Landscapes) allows visual inspection of the simulated landforms either by three dimensional mesh perspective plots on a cathode ray tube (CRT), contour maps, or contour perspective diagrams. Statistics describing the elevation distribution, slope distribution, and hypsometric integral are printed, with the option of producing frequency-distribution bar graphs at any stage of the simulation. Program testing in a Basin-and-Range type mountain block, plateau escarpment, and mesa setting show results consistent with qualitative understanding of landform evolution. Processes such as pedimentation, scarp retreat, and dissection are interpreted as the real world equivalents of the simulated landscape changes, which operate at realistic rates for the denudation rates used.

Introduction

Landforms evolve within the context of tectonic movements and erosion, which can maintain or modify morphometric characteristics such as relief, channel slope, and convexity of valley side-slopes. Davis (1899) described the topography of recently uplifted areas as youthful. The morphologic attributes of such areas distinguish them from areas that have had no uplift for a longer period of time. Davisian concepts of the geographic cycle are still applied to landforms that experience rapid uplift followed by tectonic quiescence. The Davisian scheme becomes cumbersome in areas of slow, continual uplift, where basin morphometry appears to have adjusted to processes operating today (Hack, 1960), rather than to the decay of an initially youthful landscape. Continuous adjustment of process and form in dynamic equilibrium has led some to conclude that landforms can be independent of time (Chorley, 1962, p. 7).

The present work examines the response of landforms to uplift along nearby faults. Ultimately, landforms may be used in inferring uplift history and uplift rates. We will provide some preliminary theoretical and empirical justification for a probabilistic approach to landform evolution and present a computer program, based on a probabilistic approach, which models landform change through time under various rates of uplift and denudation. Eventually our methodology may show how computer simulation can lead to quantitative procedures for using landforms in discerning the complex interaction between uplift and erosion.

Some mathematical approaches to landform evolution

Several workers have tried to explain landforms purely on theoretical or deductive grounds (Culling, 1960; Schiedegger, 1961; Leopold and Langbein, 1962; Hack, 1965; Ahnert, 1970a; Kirkby, 1971). Culling (1960), for example, uses Fourier theory of heat flow in solids to develop differential equations describing stream profiles and valley side slopes. In a similar study, Devdariani (1967), examined the interaction of uplift and denudation on a two-dimensional anticlinal uplift. Devdariani tried to explain the Davisian cycle of erosion in mathematical terms. In many other similar studies the most common models use equations of continuity or diffusion. Sprunt (1972) developed a computer program to simulate the development of drainage basins, their networks and topography. Sprunt's work has a distinct advantage over prior attempts at network simulation (Leopold and Langbein, 1962; Scheidegger, 1967) in that the new topography associated with drainage development was graphically displayed. The graphic display permitted more meaningful comparison between real and simulated drainage networks.

A random-walk approach to probability modelling

Particularly relevent to the present study is the probability model of Leopold and Langbein (1962), who modelled longitudinal stream profiles by using a random-walk method. In their procedure, a pack containing 5 white cards and 1 black card is prepared. Each white card represents a unit of elevation above base level. A card is selected at random. If a white card is selected, the profile is decreased one unit of elevation, but if a black card is selected, the next point in the profile remains at the same elevation. Regardless of which card is selected, it is replaced by a black card. Results of their experiment are shown in Figure 1 and Table 1. An important property of this experiment is that the outcome of any one card selection depends on the previous outcome. Experiments or processes that have this property are termed simple Markov-chain (Hoel, Port, and Stone, 1972) or first order autoregressive processes (Nelson, 1973).

Leopold and Langbein (1962) assert that Markov-chain generated profiles are comparable to actual stream profiles in overall shape. We can consider this experimental method a model of the process of transporting water and sediment through a system. Because gravity is the driving force, the possible extremes are for material to move down vertically or not to move at all; the most probable material paths lie between these extremes. In actual drainage basins, material is eroded from hillslopes and channels and is eventually transported out of the system. The transport path of a packet of sediment is the topography over which it travels. If the transport paths can be modelled, then topographic form is implicitly modelled as well. Random-walk simulation of stream profiles is useful because in modelling the process the topographic form is predicted.

Leopold and Langbein (1962) interpreted the number of possible downward steps as elevation above base level, which leads to the conclusion that the longitudinal profile of a stream is dependent on elevation above base level. Langbein (1964), perhaps aware of problems in the original random-walk interpretation, tried to show that profiles actually depend on discharge rather than elevation. He also maintained, however, that the number of downward steps is equivalent to elevation above base level, an assumption which precludes a dependence on discharge.

Clearly the number of cards used in the experiment completely determines the probability of a downward step at each point in the sequence of card selection. To illustrate this we can construct an experimental sample space. Figure 2 illustrates the possible choices at each card selection and all possible outcomes. For example, when the first card is selected, choice 1, there are 3 D's (downward steps) and 1 S (same elevation). Thus, the probability of a downward step is the number of D's divided by the total number of possibilities, or 3/4 (0.75). The "elevation" at choice one is therefore 3.0-0.75, or 2.25. When the second card is selected, the probability of a downward step is 9/16 (0.5625), because at choice 2 there are 16 possible outcomes, 9 of which are a downward step (see figure 2). These probabilities can also be found by using binomial distributions. Table 2 is a modified binomial table where n is the choice number. For example, when $n = 1$, the first card is selected and when $n = 8$, the eighth card is selected.

The probability parameter, θ is related to the number of cards in the deck (similar to the transition probability in the Markov sense), because initially there is only one black card. If the initial card deck consists of 10 cards, $\theta = 0.10$, whereas if only two cards are used, $\theta = 0.50$. The entries for θ in table 2 include non-integer deck sizes as well. The fewer the cards the greater the θ, or the smaller the probability of a downward step. For the example shown in figure 2, there are 4 cards, of which one is black, and $\theta = 1/4 = 0.25$. Under $\theta = .25$ and $n = 1$ (see table 2), we find the probability of a downward step is 0.75 and for $n = 2$ the probability of a downward step is 0.5625, as was calculated.

Any geometric meaning of the method relies on assigning a distance quantity to the choice number. If we consider each card selection to represent a fixed length of channel, then larger θ (fewer cards) results in shorter streams and smaller θ (more cards) results in longer streams. The shapes of the resulting profiles are directly related to the number of downward steps, which is a measure of elevation above base level. A great deal of empirical data relating stream length and discharge exist (Horton, 1945; Leopold and Maddock, 1953; Leopold, Wolman and Miller, 1964) indicating that discharge increases exponentially with stream length. Thus long streams have larger discharge and lower slopes than shorter streams. Since large discharge streams have lower slopes than small streams, the probability of a downward step must be smaller for larger streams. When each card selection represents a fixed distance, however, large θ results in shorter streams. This contradiction suggests that each card selection should not represent the same length for all streams.

Instead, the stream length represented by a card selection should be proportional to the total length. Consider any length stream to be divided into n reaches. A stream of 100 km length can be divided in 10 reaches so that each card selection represents 10 km, or a 10-km long stream can be divided in 10 reaches so that each card selection represents 1 km. When a card selection is defined in this way, θ can vary between 0 and 1 and is proportional to discharge. The larger the discharge the greater the θ and the smaller the probability of a downward step, so that profile shape depends on discharge and elevation above base level. This interpretation of the random-walk experiment allows a more general application of the method. For example, the original interpretation results in a single longitudinal profile for a given relief. The present interpretation leads to a family of longitudinal profiles, in which each profile can drop a given elevation, but each has a different length. Such flexibility is needed for modelling because, in real situations, discharge and relief both vary.

Statistical comparison between theoretical and actual profile

Theoretical stream profiles must be compared with actual profiles in order to evaluate the adequacy of the random-walk (Markov chain) model. This can be done by testing the null hypothesis that the actual and the theoretical profile elevations have the same distribution. The two profiles can be statistically compared using a Chi-square, goodness-of-fit test (Gibbons, 1976). The Chi-square test examines how much the theoretical profile deviates from the actual profile. The result of such a test, that is, the equality between the two profiles, can be determined for a desired confidence coefficient, α. The test statistic, Q, measures the sum of the squares of the differences between profiles: when Q is very large, the profiles do not match well. If the null hypothesis is accepted, that is if that Q is not too large, then the actual profile is reasonably approximated by the probability model. Acceptance of the null hypothesis does not exclude other probability distributions which may also be adequate. How large Q becomes before the null hypothesis is rejected depends on how sure one wants to be that the two profiles match. To be conservative, the test is performed at a small α, such as 0.05 or 0.10.

To illustrate this procedure, a theoretical profile of Chalmers Creek, Virginia is compared with a plot of the actual profile of the stream (Figure 3; Table 3). Elevations (H) for the theoretical profile are calculated from

$$H = R \cdot p + H_{min} , \qquad (1)$$

where R is the total drop in channel elevation or relief, p is the probability of a downward step (from Table 2, using $\theta = 0.25$) for each unit distance or card selection, and H_{min} is the lowest elevation of the channel. The Chi-square test was applied to these profiles (Table 4). The test statistic, Q, is the distributed Chi-square with n-1 degrees of freedom. The null hypothesis, that the two profiles are equal, would be accepted where $\alpha = 0.10$. This evaluation shows that theoretical profiles can be reasonable approximations of actual stream profiles.

The probability-model profiles of graded streams can be used to statistically test for anomalous or non-graded segments of actual stream profiles. Anomalous stream profiles are those which are steeper or shallower, as a whole or in segments, than one would expect for a given climate, discharge and channel material. Such profiles may result from a variety of independent variables, including tectonism.

An empirical method of profile analysis

A method of stream-profile analysis was developed from a different perspective by Hack (1973), who demonstrated that in erosional environments, the relative magnitudes of stream power can be approximated by a stream gradient index. When a longitudinal stream profile (or a reach along the profile) plots as a straight line on a graph in which the horizontal axis is the natural log of distance from the headwaters and the vertical axis is elevation, the equation of the line is (Hack, 1973)

$$H = C - k \ln L \qquad (2)$$

Here H is elevation, L is stream length measured from the headwaters, and C and k are constants. The slope of the actual stream, S, may be expressed as the derivative with respect to L of equation 2 where

$$S = \frac{k}{L} \qquad (3)$$

The constant, k, is the slope of equation 2 and may be expressed as

$$k = SL = \frac{H_1 - H_2}{\ln L_2 - \ln L_1}, \qquad (4)$$

where SL is the stream-gradient index, H_1, H_2 are map elevations at the end of each reach, and L_1, L_2 are the respective distances from the headwaters measured along the longest channel.

Real streams may be concave up, straight, or convex up in profile, depending on downstream variations in load, discharge, bed material, hydraulic geometry and tectonism. However, the profiles of most natural streams approximate a series of connected segments of various lengths, each logarithmic in form (Hack, 1973). For each segment the stream gradient index (SL) is constant, and each segment is also straightforwardly described by probability modelling. Thus, the profile of Chalmers Creek (figure 3) can be modelled with an SL value of 40.

Applications of the probability model and the stream-gradient index model to landform change

Assuming a constant rate of increase in discharge downstream and homogeneous lithologies along stream length, a theoretical analog to a natural stream profile can be constructed. The theoretical profile is constructed using the probability parameter θ, which is related to discharge, and to channel relief (the total drop in stream channel elevation), which is given. Tectonic uplift increases channel relief whereas erosion can decrease channel relief. A probability profile that models a stream before uplift is modelling a stream adjusted to hydraulic variables. Similarly, following uplift, a probability profile models the stream that now has greater channel relief as it theoretically will appear when it too is adjusted to hydraulic variables. The time needed for the stream to adjust to hydraulic conditions following uplift depends on how fast the channel can erode its floor.

When the rate of increase in discharge does not constantly increase downstream, however, or when resistant and non-resistant rocks crop out alternately along the stream channel, several probabilities are needed to construct a theoretical profile. If discharge actually decreases downstream, the probability of a downward step becomes higher downstream, resulting in less concavity in the profile or even convexity. Coarse load introduced by resistant lithologies along a channel may affect discharge by increasing the channel roughness, which would also result in a steepening of the channel. Under these complicated situations, shorter reaches must be modelled separately along the length of the stream. Meyer and Kramer (1969) have shown that hillslopes also tend towards concavity, a feature analogous to streams. Material can either move down slope or not move. We assume therefore, that hillslopes may be modelled in a similar manner (Meyer and Kramer, 1969), keeping in mind that variations in features such as vegetation or aspect may necessitate a complex probability assignment. For present purposes, a constant rate of increase in discharge is assumed.

Base-level fall

Random-walk stream profiles derived from probability considerations can be used to predict changes in stream profiles caused by uplift and erosion. Figure 4 shows a hypothetical, initial stream channel whose relief is increased 100 meters by block faulting located at base level. Since the stream has not changed in map length, θ remains the same. Change of the faulted profile over time may be modelled by comparing the initial profile with a profile that has adjusted to the new base-level. The major modifications in the initial stream profile (figure 4; table 5) that followed faulting and subsequent stream downcutting occurred where stream power increased the most, that is, in the area of the greatest modification in channel slope. If the initial channel were preserved as a strath terrace, the familiar divergence of strath terrace from the present channel towards the fault would be apparent. Such terrace divergence is a clue that a tectonic perturbation has taken place.

The stream gradient index (SL) can also be used to determine where change is likely to occur. Ideally the SL value is constant if the stream slope is adjusted to discharge. Tectonism (as well as outcrop of resistant rocks and coarse material) can cause a steepening of the slope that is reflected as an increase in the SL value for that reach of the profile. For example, a fault offset across a channel may produce a steeper slope and thus a higher SL value for that reach of the stream. Because work expended on a channel is proportional to the SL value, it will be concentrated at this tectonic nickpoint. With time, the nickpoint will be flattened until SL values for each reach become uniform and work is distributed more evenly along the channel. The even distribution of work has been described as the tendency for least work (Langbein and Leopold, 1966). Thus the trend towards a uniform stream gradient index, a crude indicator of stream power, indicates that the tendency for least work is analogous to a minimization of stream power.

Base-level stability

Probability profiles can also be used to model change under the restriction that base level is fixed but relief is reduced. Figure 5 illustrates a situation where the initial relief of 1100 meters was reduced to 1000 m. The profiles, constructed using $\theta = 0.5$, show that the maximum change in channel elevation occurs at the headwaters and progressively decreases towards base level. This result is consistent with the observations of Bull (1979), who deduced that the tendency for downcutting under conditions of base-level stability is a maximum in the headwater reaches and progressively decreases downstream.

Three-dimensional simulation

Three dimensional simulation of landform change is preferable to two dimensional simulation, because many landform parameters can be measured only when three dimensional data are available. Also, three dimensional displays allow more critical evaluation of the simulation results.

A computer program called UDL, written to simulate the evolution of topography, produces perspective diagrams of three-dimensional surfaces for plotting on pen-plotters or interactive display on CRT-terminals. In addition, the program calculates parameters such as the hypsometric integral, slope and relief, and plots histograms displaying the frequency distributions of slope and elevation.

Input for the program is a two-dimensional array of elevations, or an elevation matrix. Elevation arrays for parts of the U.S. at a scale of 1:250,000 are available on computer tape from the National Cartographic Information Center of the U.S. Geological Survey, or the user can construct his own array by cell digitizing or by line digitizing followed by interpolation to cell format. H_{ij} refers to any specific elevation in the array where i is the row coordinate and j is the column coordinate.

The algorithm used in program UDL

The basis for the three-dimensional simulation is an array of elevations that is modified through time. Alternatively, three-dimensional modelling could be accomplished by modelling a network of profiles and then contouring the predicted elevations. Hillslope profiles would have to be included as well. This requires modelling a large number of separately digitized but spatially coordinated profiles, using a modified stream-gradient index to predict where change would take place. Digitized stream profile information is not available, however, and would have to be generated specifically for use in the model.

Digitized topographic data are available, so the program is structured to use it. The algorithm makes several crude approximations which are needed in order to use elevation arrays. Maximum elevation in the array is considered analogous to the elevation of the headwaters (that is, the drainage divide). The distance between each point in the array and the drainage divide (the denomination is 4) is crudely approximated by dividing the difference in elevation between the highest point and each point in the array by the mean slope.

A logarithmic model can then be applied to the data by

$$K_{ij} = \frac{H_{max} - H_{ij}}{\ln \dfrac{H_{max} - H_{ij}}{S}} , \qquad (4)$$

where K_{ij} is a function analogous to the stream gradient index. H_{max} is the maximum elevation in the data set used to approximate the divide, and S is the mean slope of the area above base level. The denudation rate at each point is assumed to be proportional to K_{ij}. K_{ij} is used as an indication of the relative contribution of a specific cell to the overall denudation rate.

Given an average denudation rate, the total volume of material removed per increment of time is

$$V_d = A_T \, D(t) \triangle t \qquad (5)$$

where V_d is the total volume removed, A_T is the total area denuded, D(t) is the average denudation rate, during time t, and $\triangle t$ is the increment of time over which the region is denuded subject to erosion. V_d is assumed to be proportional to the sum total of K_{ij}, so

$$V_d = C \sum_{i=1}^{r} \sum_{j=1}^{s} K_{ij} \qquad (6)$$

where C is a constant of proportionality. For any location in the elevation array, the denudation rate may be expressed as

$$D_{ij}(t) = \frac{C}{A_{ij} \triangle t} K_{ij} \qquad (7)$$

where $D_{ij}(t)$ is a cell denudation rate, and A_{ij} is the area of the cell.

The program is iterative; the total time of deformation and erosion is divided into segments of equal length ($\triangle t$). During an iteration the mean slope (S) of an eroding landscape decreases and the total area above base level (A_T) also decreases. These decreases affect the denudation rate. After each iteration, the denudation rate is recalculated to reflect the magnitude of these changes. Because the denudation rate is modified after each iteration, it can be considered a function of time.

Uplift

Uplift is incorporated into the algorithm by adding an increment of elevation to each cell in the elevation array, every iteration. In the case of no erosion this is simply

$$\triangle H_{ij} = U \triangle t \qquad (8)$$

where $\triangle H_{ij}$ is the increment of elevation, U is the average rate of uplift and $\triangle t$ is the length of the iteration. Either the entire area or a part of the area can be uplifted as a simple fault-bounded block with no tilting or warping. The faults across which uplift occurs are located by specifying an elevation; uplift occurs at all cells with elevations greater than that specified. If the entire terrain is raised, these faults lie outside and below the area.

Uplift and Erosion

When both uplift and erosion are applied to a landscape, the elevation increment added to each cell is calculated by

$$\triangle H_{ij} = U\triangle t - D_{ij}(t)\triangle t \quad . \qquad (9)$$

As noted above, when the net effect is lowering a landscape, the mean denudation rate is decreased after each iteration. Similarly, if the net effect is uplift, the mean denudation rate increases. The increase in denudation rate is considered to be proportional to mean slope; as uplift increases the mean slope, the denudation rate will increase in direct proportion. When very high relief develops, this proportionality to slope becomes less responsive, so that as some threshold in relief is reached, the mean denudation rate is considered to be proportional to relief (Ahnert, 1970b). The proportion used is

$$D^*(t) = 7.35 \times 10^{-5} \text{ (relief)}.$$

When $D^*(t)$ becomes greater than $D(t)$, $D^*(t)$ is the denudation rate used.

For landscapes with a hypsometric integral below 0.5, the distribution of denudation is weighted towards the higher elevations, so that the denudation function for each cell, K_{ij}, is proportional to relief. The hypsometric integral is calculated by the method of Pike and Wilson (1971).

The material eroded from the uplifted block is not treated by the algorithm. Thus transport and accumulation of sediments is ignored, and basin elevations remain unchanged. Because parameters such as relief and slope are calculated at each iteration, it would be possible to use published statistical relationships between sediment characteristics and source-terrain characteristics to model aggradation in the sedimentary basin as well.

The algorithm is an approximation of the probability model discussed above. At present it is uncertain how well the algorithm follows the probability model. The simulation of landscape evolution is an initial test. Further work may be aimed at directly incorporating the probability model into the algorithm by transforming the elevation data array into a form more convenient for analysis.

Results of Simulation

Uplifted Mountain Block

We applied program UDL to three specific landscapes -- a Basin-and-Range mountain block, a plateau escarpment, and a basalt-capped mesa. The White Mountains, California, were selected as an example of a fault-bounded mountain range. The topographic data were digitized from a standard 1:250,000-scale topographic map with a contour interval of 200 feet (Mariposa, California). The present topography is shown in figure 6. The 5900-foot contour was arbitrarily chosen to represent both the fault and base level. Figure 7 shows how UDL applies uplift along a fault.

To examine the possible outcome of continual denudation of the White Mountains without any uplift, program UDL was used to simulate 30 million years of erosion with base level at 5900 feet. The initial denudation rate was 0.1 ft per thousand years, a rate suggested by Marchand (1971) for that area. After 30 million years of erosion and tectonic quiescence, landforms associated with tectonic stability were formed (figure 8). The simulation shows well-developed pediments on both fronts of the mountain block, but better developed on the less steep, eastern front than on the steeper western front.

Figure 9 shows the differences in elevation between the initial condition (figure 6) and after 30 million years of erosion over the entire area. The most denudation occurred at the higher elevations. This is what the probability model predicts for a stream profile under conditions of base-level stability (figure 5).

The results of simulation are consistent with Bull and McFadden's (1977) field observations and quantitative characterizations of inactive mountain fronts. For example, the simulation shows pediments, inselberg, pediment pass and a slightly increased mountain-front sinuosity.

Plateau Edge

A small area along the Mogollon Rim in central Arizona was selected to test the UDL algorithm in a plateau-edge setting. The topographic data were digitized from a 1:62,500-scale topographic map with a contour interval of 50 feet (Pine Mountain, Arizona). Figure 10 shows the present Mogollon Rim. Program UDL was used to simulate 10 million years of erosion of the escarpment. Base level was fixed at 5800 feet, approximately the elevation of an erosional surface adjacent to the Rim determined by geologic and altimetric analysis (Mayer, 1979). Estimates of the denudation rate along this section of the Mogollon Rim by Mayer (1979) are a minimum of 0.17 feet (5 cm) per thousand years and a maximum of 0.38 ft (11.5 cm) per thousand years.

To achieve conservative results a denudation rate of 0.13 feet (3 cm) per thousand years was used. Figure 11 illustrates the results of the simulated erosion. The average rate of pedimentation and scarp retreat, determined from the simulation, is consistent with the minimum estimate of scarp retreat proposed by Mayer (1979), 350 m per million years.

Mesa and Canyon

Buckhead Mesa in central Arizona was selected to test UDL on a complex mesa and canyon topography. The topographic data were digitized from the 1:24,000-scale Buckhead Mesa quadrangle, with a contour interval of 40 feet. This sample has the finest grid resolution of all the simulation trials (figure 12). In order to see where material is eroded, the elevations along the perimeter were fixed. Erosion at an initial rate of 0.10 feet (3 cm) per thousand years was used to simulate 10 million years of denudation with no constraint on base level. Figure 13 shows the results. Note that the highest erosion rate occurs at the highest elevation and adjacent to the canyon. The mesa appears to be dissected by downcutting of streams but still has some planar surface.

Discussion

Simulation of prolonged erosion of a Basin-and-Range block uplift shows that pedimentation indicates tectonic stability and only occurs during periods of stability. The occurrence of pediments <u>and</u> uplifted pediments would require episodic rather than continuous uplift. The period of time represented by the pediments can be estimated by simulation of erosion. For the White Mountain example, pediments formed at a maximum rate of 0.25 km/10^6yrs, although this rate is very sensitive to the average denudation rate.

The program could be modified to generate information on morphometric characteristics of drainage basins under specified conditions. For example, drainage basins with a known denudation rate may have a morphometry including stream channel slope, average valley side slope, and a shape that depends on uplift rate. The program may be useful in determining the rate of uplift that best matches the observed morphometry. The estimated rates of uplift could then be plotted on a map for many drainage basins. If a spatial consistency in these rates emerged, the sources of the uplift might be determined.

Simulated erosion of the White Mountains implies that asymmetry in a mountain block caused by tilting persists for millions of years. Asymmetry of ranges that have long been tectonically inactive thus can be used to infer tilting where asymmetry presently exists. This is especially useful in granitic ranges where there are no sedimentary units to document tilting. Simulated erosion of the Mogollon Rim suggests that the program can be used to bracket the age of fault-generated escarpments, a useful tool where stratigraphic control is lacking.

Desirable improvements in the program are the direct incorporation of the probability model and treatment of the aggradational system. Modelling sediment, for example, will yield idealized trends of particle size decrease in the downstream direction under various climatic (denudation rate) and tectonic (uplift rate) regimes. In the southwestern U.S. this would allow more comprehensive interpretation of alluvial fan sedimentology. In the humid and subhumid areas of the U.S., the sedimentology of terrace deposits may provide information on past uplift rates.

Summary

Through Markov-chain random-walk modelling it has been shown that theoretical profiles can be constructed to approximate actual stream profiles. Such theoretical profiles can be described probabilistically. To determine how well the probability profiles approximate the actual profiles, the two can be compared statistically. Such tests have shown that the models work well for streams constantly increasing in discharge downstream. The degree of accuracy can be improved by dividing the stream into segments, each modelled separately.

The stream-gradient index can be used in much the same way. The index allows two stream reaches to be compared, on the same stream or on different streams, without the construction of entire profiles. Both methods of modelling can be used to identify anomalous reaches of stream channels. They can also be used to predict how erosion will be distributed if the channel downcuts in response to a drop in base-level.

The techniques for modelling in two dimensions, modified for simulating erosion and uplift in three dimensions, were incorporated into a computer program called UDL. This program proved useful in evaluating the significance of various landform features. It also shows promise as a tool that could lead to the development of quantitative procedures for using landforms in discerning the complex interaction between uplift and erosion.

Computer Program UDL

Computer program UDL was written in MULTICS Fortran, on the Honeywell Multics system at the U.S. Geological Survey in Menlo Park, California. The program is not portable, but can currently be used by other MULTICS users who can link to or copy the program (table 6). The program is interactive and prompts the user for information it needs (Table 7).

The program can produce tabular and-or graphic output. Frequency distributions of slopes and elevations can be listed or plotted on histograms (Figure 14). The elevation array can be plotted on the screen as a view of a three-dimensional mesh-covered surface, or the data can be stored in file08 and then plotted as a contour map or as a view of a three-dimensional, contoured surface. Two different graphics packages, Surface Display Library (SDL) (Dynamic Graphics Group, 1975) and Display Integrated Software System and Plotting Language (DISSPLA) (ISSCO, 1975) were used to produce the graphs, maps and views of the landform.

DISSPLA can produce graphs and mesh views on a CRT, wheras SDL produces the contour maps and views of the landforms on the Versatec plotter. The user need not restrict himself to uplift along a fault and erosion only above that fault elevation; the entire area can be uplifted and eroded. Alternatively, one can construct a data set that contains a fault and then erode the entire area. This can be done by specifing a rate of uplift, a fault elevation, and no erosion for one iteration and saving the data. These data can then be re-entered into the program and uplifted and eroded with a new set of parameters.

The number of iterations has an effect on the final elevation array even though all other input parameters are identical. The White Mountains elevation array was used to illustrate this. In each of two cases the initial denudation rate was 0.1 feet/1000 years, with no uplift, for a total time of 30 million years. In the first case, there were 30 iterations, each 1 million years long; in the second case there were 3 iterations, each 10 million years long. The statistics generated by the program (table 8, figure 14) show that although the final denudation rates are about equal, the elevation arrays are different. In the second case (3 iterations), the maximum elevation is almost 900 feet higher than in the first case (30 iterations), and the relief is also greater, resulting in steeper slopes. More iterations may produce a landscape closer to reality, however increasing the number of iterations also increases the cost.

Data and Data Format

The maximum size of the elevation array is now 30 rows by 64 columns. This is easily expanded by changing the dimension statements at the beginning of the program and by changing the call to the subroutine "surmat" in the final plotting section of the program. The data must be in file07. The first record contains the number of columns and the number of rows, with the format (3x, i2, 1x, i2). The second record is the grid cell size, the map length of a side of the cell used in digitizing the elevation data, or the map distance between points in the array expressed in feet. All subsequent records contain the values of the elevation array. The format is 8(f8.1, 1x); thus, the number of columns of data must be an integral multiple of eight. The program was written by Larry Mayer in the summer of 1979 and is listed in Table 9.

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Table 1.--Frequencies of random walks having given elevation at various distances from origin in percent, subject to condition that probability of a downward step equals H/6.

Elevation (H)	Distance 0	1	2	3	4	5	6	7	8	9	10
5	100	17	3	--	--	--	--	--	--	--	--
4	--	83	42	17	6	2	--	--	--	--	--
3	--	--	55	56	39	24	14	7	3	2	1
2	--	--	--	27	46	50	45	37	29	20	14
1	--	--	--	--	9	23	36	45	50	52	50
0	--	--	--	--	--	1	5	11	18	26	35
Average elevation	5.0	4.17	3.48	2.90	2.42	2.02	1.68	1.40	1.17	0.98	0.81

Table 2.--Modified table of binomial distributions, where n is the card selection number, θ is the probability parameter. Entries are $(1.0 - \theta)^n$.

n \ θ	.05	.25	.50	.75	.95
1	.9500	.7500	.5000	.2500	.0500
2	.9025	.5625	.2500	.0625	.0025
3	.8574	.4219	.1250	.0156	.0001
4	.8145	.3164	.0625	.0039	.0000
5	.7738	.2373	.0312	.0010	.0000
6	.7351	.1780	.0156	.0002	.0000
7	.6983	.1335	.0078	.0001	.0000
8	.6634	.1001	.0039	.0000	.0000
9	.6302	.0751	.0020	.0000	.0000
10	.5987	.0563	.0010	.0000	.0000
11	.5688	.0422	.0005	.0000	.0000
12	.5404	.0317	.0002	.0000	.0000
13	.5133	.0238	.0001	.0000	.0000
14	.4847	.0178	.0000	.0000	.0000

Table 3.--Elevations along the probability profile for Chalmers Creek, Virginia calculated with $\theta = 0.25$. H is calculated from equation 1 where R = 200 and $H_{min} = 300$.

n	distance(km)	p	H (ft)
1	.7	.7500	450
2	1.4	.5625	413
3	2.1	.4219	384
4	2.8	.3164	363
5	3.5	.2373	348
6	4.2	.1780	336
7	4.9	.1335	327
8	5.6	.1021	320
9	6.3	.0751	315
10	7.0	.0563	311

Table 4--Data for the Chi-square goodness of fit test for real and simulated longitudinal stream profiles at Chalmer Creek, Virginia. Measurement in feet.

nf	F (elevation fall)	e(expected fall)	$\frac{(F-e)^2}{e}$
1	50	50	0
2	40	37	.24
3	30	29	.03
4	20	21	.05
5	10	15	1.67
6	20	12	5.33
7	5	9	1.78
8	3	7	2.29
9	8	5	1.80
10	4	4	0
			Q =13.19

Table 5--Differences in elevation between the two theoretical profiles in Figure 4.

n	distance(km)	initial profile(m)	faulted profile(m)	difference(m)
0	divide	1100	1100	0
1	1	600	550	50
2	2	350	275	75
3	3	225	138	87
4	4	163	69	94
5	5	131	34	97
6	6	116	17	99
7	7	108	9	99

Table 6--Information required to run program UDL

Search rules are needed to use the graphics packages and can be added with the following commands:

```
asr >iml>tcs -after working_dir
asr >iml>disspla -after working_dir
asr >iml>v_plot -after working_dir
asr >iml>SDL -after working_dir
```

Table 7--Explanation of prompts for control arguments for progam UDL.

Prompt	Explanation
Type the initial denudation rate (ft/1000 yrs)	The rate of denudation in feet per 1000 years for the initial landscape. This number is recalculated after each iteration.
Type the rate of uplift (ft/1000 yrs)	The rate of differential uplift in feet per 1000 years for that part of the area above the faults. This rate remains fixed for the entire calculation.
Uplift is to occur above what elevation	For erosion with no uplift, this number is equal to the base level. If a rate of uplift has been specified a fault will occur, the trace of which follows a contour at the given elevation. Erosion will then take place only above the fault.
What is the iteration factor NOTE: factor 1 = 1000 yrs	This number determines how often the denudation rate and the elevation array are recalculated. For example, for a factor of 10, each iteration will be 10,000 years long.
After how many iterations do you want output	This number, the output frequency, determines how often the elevation array is plotted on the screen. To calculate the number of years between plots multiply the iteration factor by 1000 by the output frequency.

How many outputs do you wish to see	In addition to specifying the number of times that the elevation array is plotted on the screen this number determines the total length of time simulated. For example total time = iteration factor 10 x 1000 x output frequency, 5 x the number of outputs, 1 = 50,000 years.

STATISTICS

Do you wish the statistics plotted Type 1 for plot; type 2 for list	For plotting you must have a Tektronix terminal. The statistics can be listed and the elevation array plotted. (see below).
How many intervals for the data grouping	This specifies the number of bars in the histogram. Any integer between 1 and 50 is allowed but 10 to 20 gives the best looking results.

3-D PLOTTING INFORMATION

To suppress 3-D plots type: 1	If you do not have a Tektronics terminal you must type "1", if you do want a plot type another integer. If a plot is not generated on the screen, the elevation array may still be saved and plotted at some future time on a contour map or view of a 3-dimensional, contoured surface can be made with the contour.plt program on the Versatec plotter.
What is the desired view angle	The view distance from the center of the plot and the height above the plot are fixed, but the user can rotate the plot by specifying the view angle in degrees.
Vertical factor for plotting, point 4 works well	This is a scaling factor 0.1 to 1.0 which determines the vertical exaggeration of the mesh-view plot on the screen.
To engage isostatic adjustments Type: 1	This reduces the effective denudation by 1/5 (Schumm, 1963). To ignore this type another integer.
To save the data from the final result in file 08, type a 1	The last elevation array calculated can be saved in a file called file08. The file should be renamed immediately after the run and has the same format as the input data.

Table 8.--The effect of the number of iterations on final elevations, slopes, and other statistics after 30 my of simulated erosion of the White Mountains.

	Original	10 iter	3 iter
Maximum elevation	13185	7923	8806
Minimum Elevation	3945	2716	2974
Range	9240	5207	5832
Mean elevation	7020	4713	5191
Hysometic Integral	0.3828	0.3836	0.3802
Maximum slope	1.1851	1.0869	1.1930
Minimum slope	0.0040	0.0027	0.0030
Range	1.8110	1.0842	1.1900
Mean slope	0.1813	0.1112	0.1248
Original denudation rate		0.1	0.1
Final denudation rate		0.061	0.068
Uplift rate		0	0
Base level		1	1
Number of iterations		30	3
Iteration factor		1000	10000
Total time		30 my	30 my

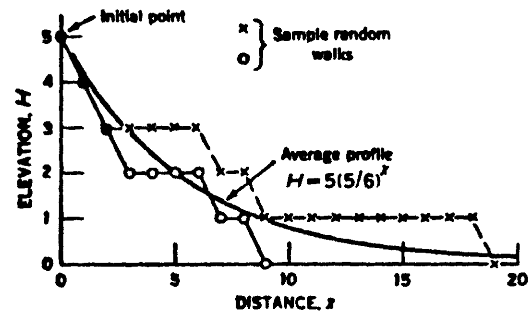

Figure 1. Sample random walks used in the generation of an average stream profile. Deck of six cards initially contains 5 white cards (downward steps) and 1 black card (same elevation) (from Leopold and Langbein, 1962).

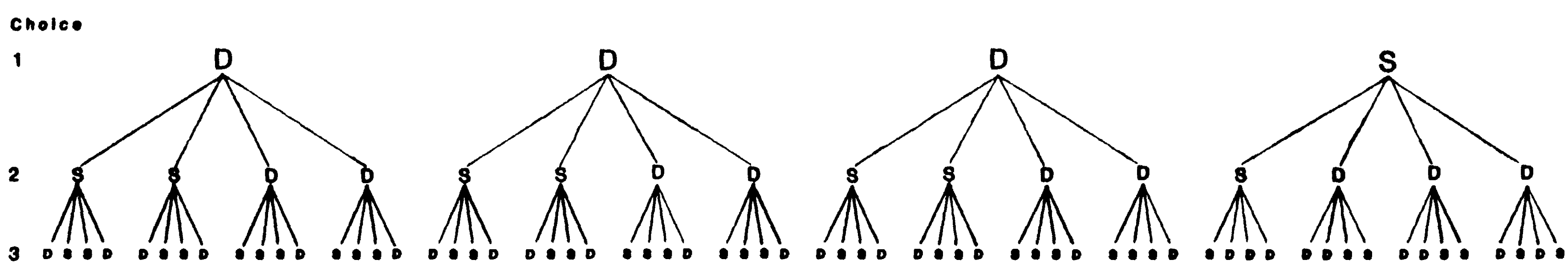

Figure 2. Tree diagram showing the possible choices of each card selection for the first three card selections in a four-card deck. "D" signifies a downward step (white card); "S" signifies staying at the same elevation (black card).

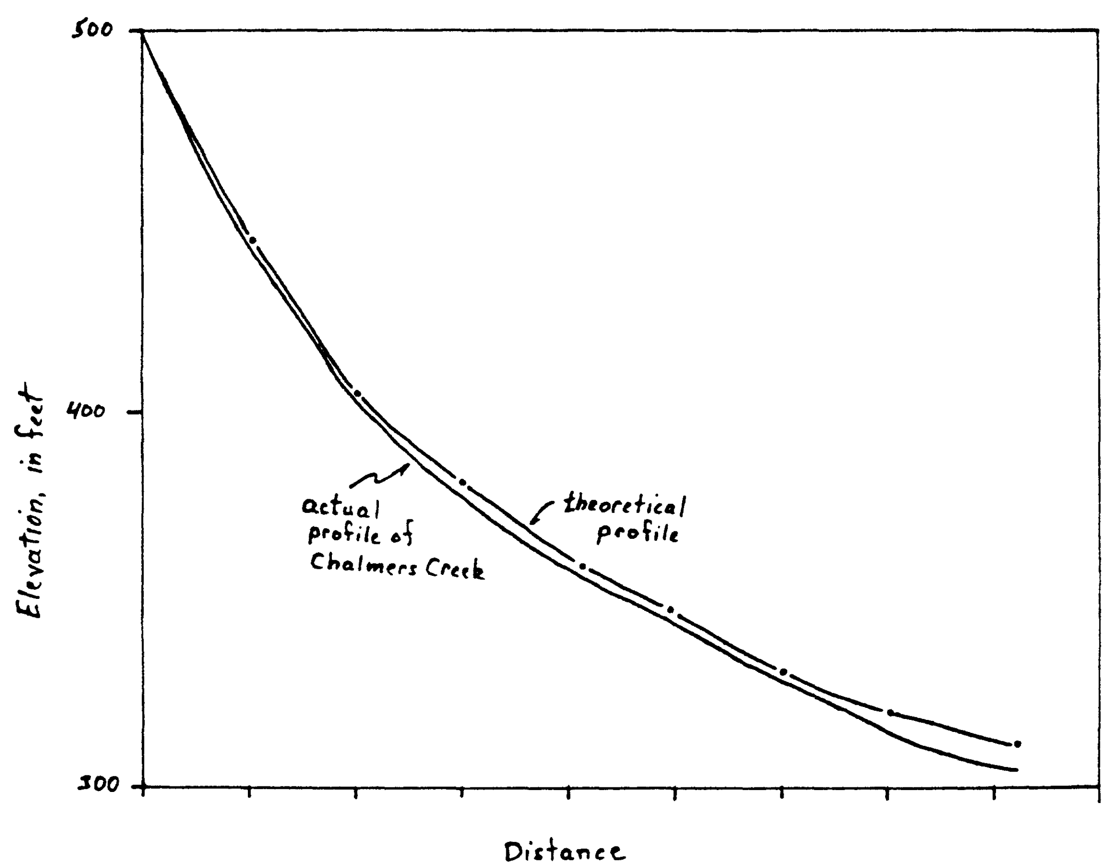

Figure 3. Longitudinal stream profile of Chalmers Creek, Virginia and a probability profile constructed with $\theta = 0.25$.

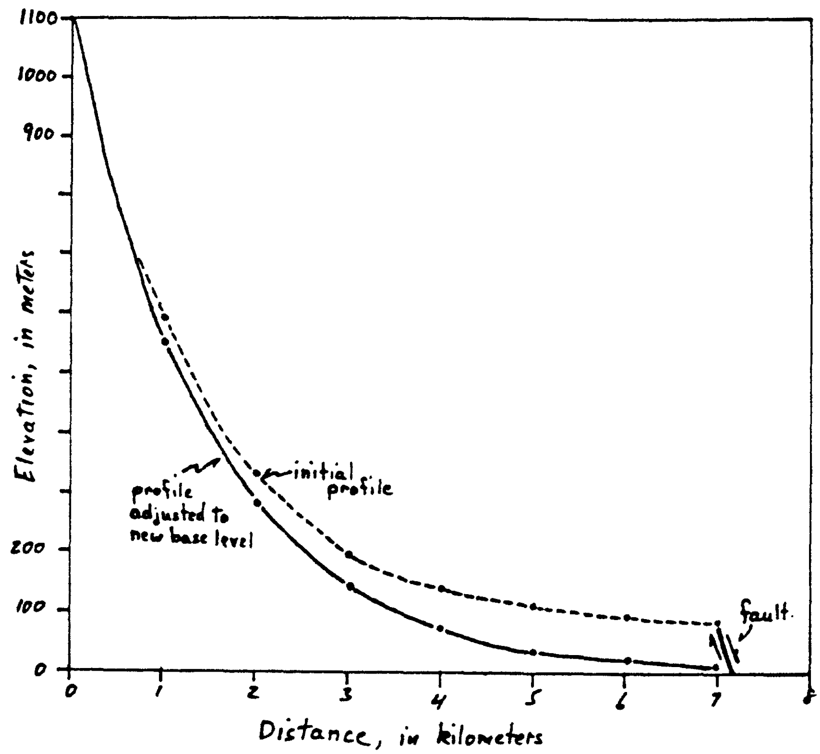

Figure 4. Initial probability stream profile faulted 100 meters at base level and the theoretical profile which is adjusted to the new base level.

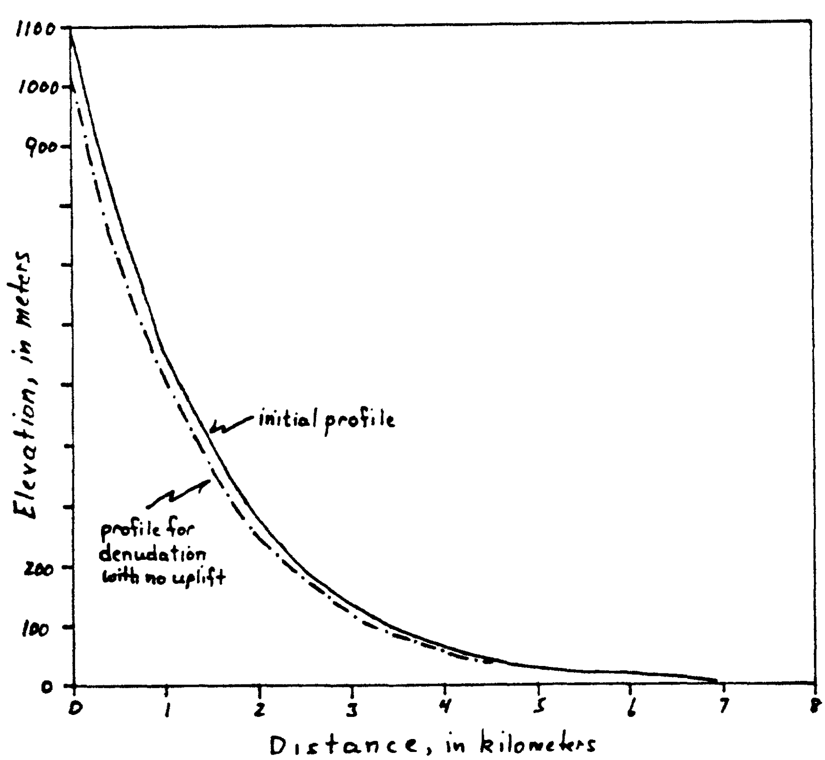

Figure 5. Probability model of a stream profile which is eroded under the condition of base-level stability. See text for analysis.

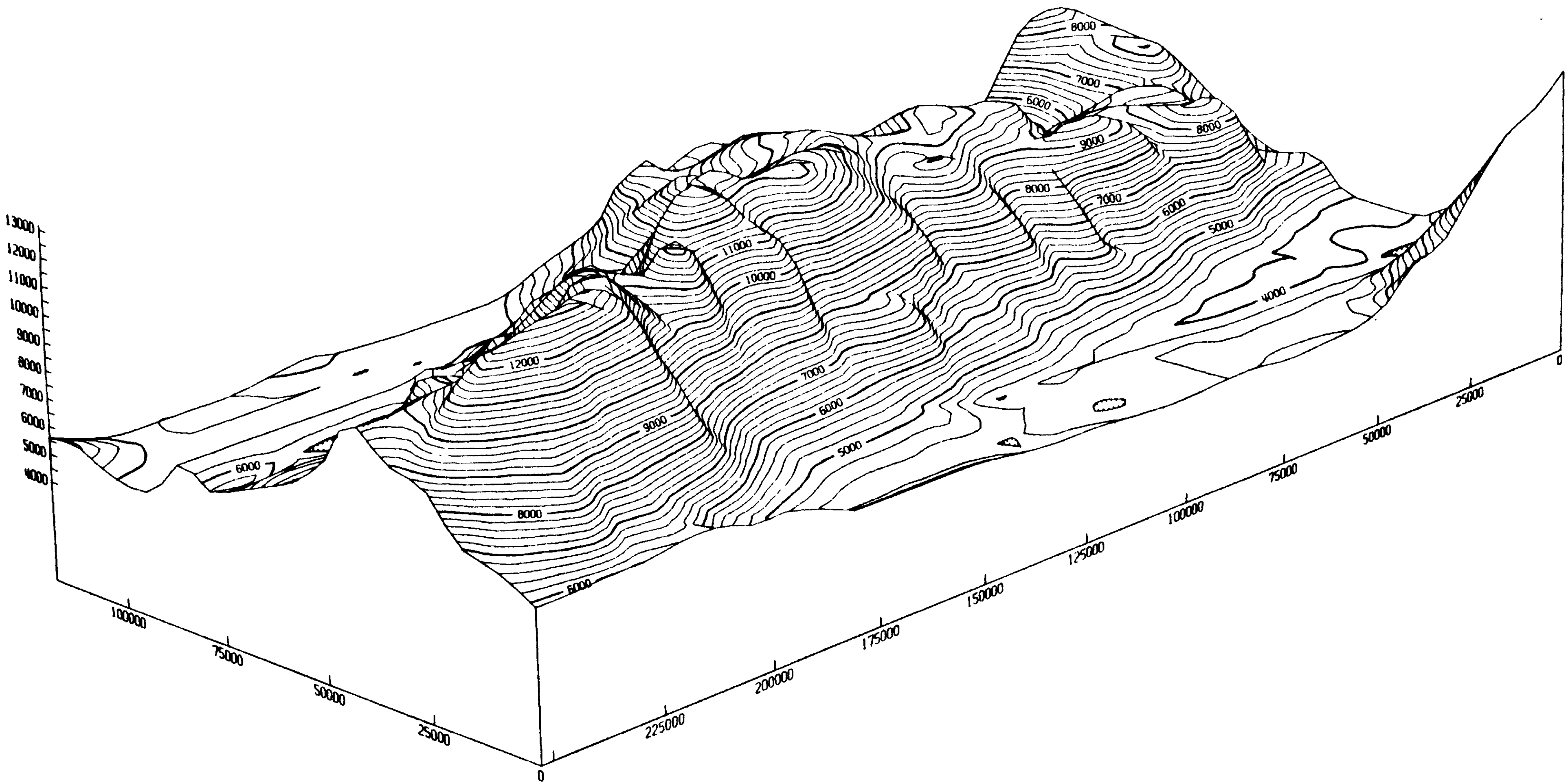

Figure 6A. Computer-generated perspective view of a three-dimensional contour surface of the White Mountains, California. Contour interval is 200 feet. Plots are produced by Surface Display Library (SDL) graphics package.

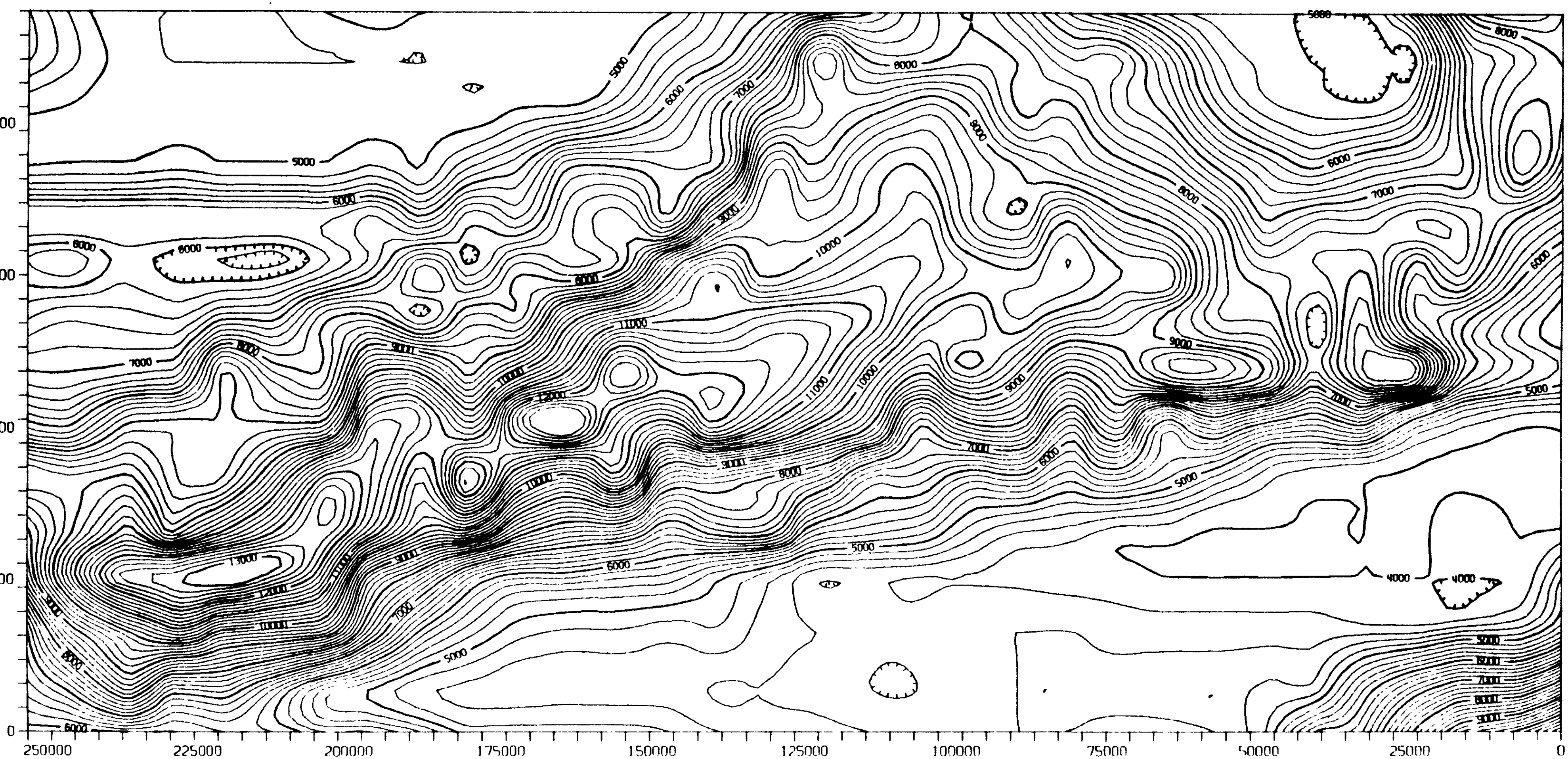

Figure 6B. Computer-generated contour map of the White Mountains, California.

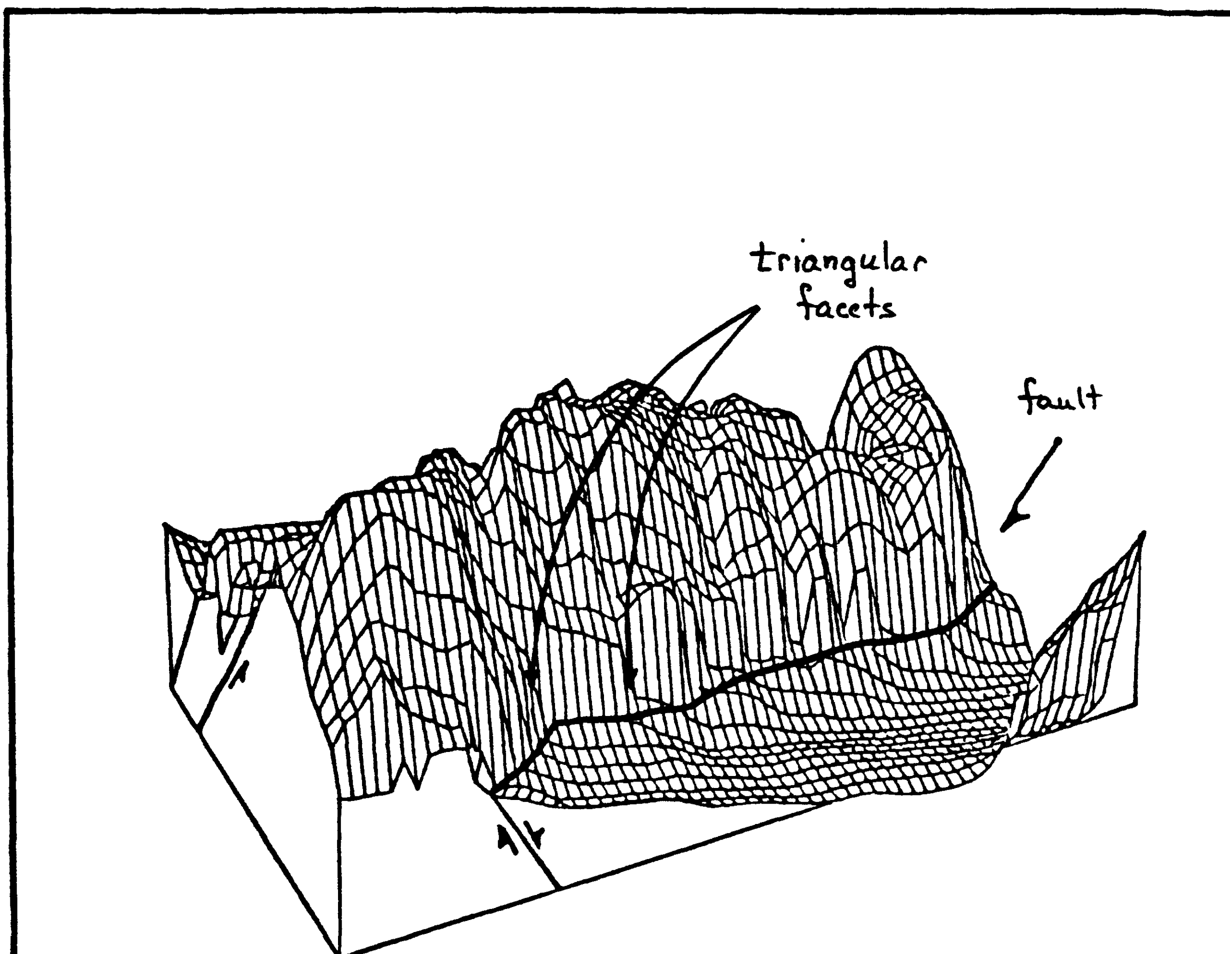

Figure 7. Computer-generated view of a three-dimensional mesh-covered surface of the White Mountains, California showing how uplift is applied along a fault in program UDL. This plot was produced by the DISSPLA graphics package.

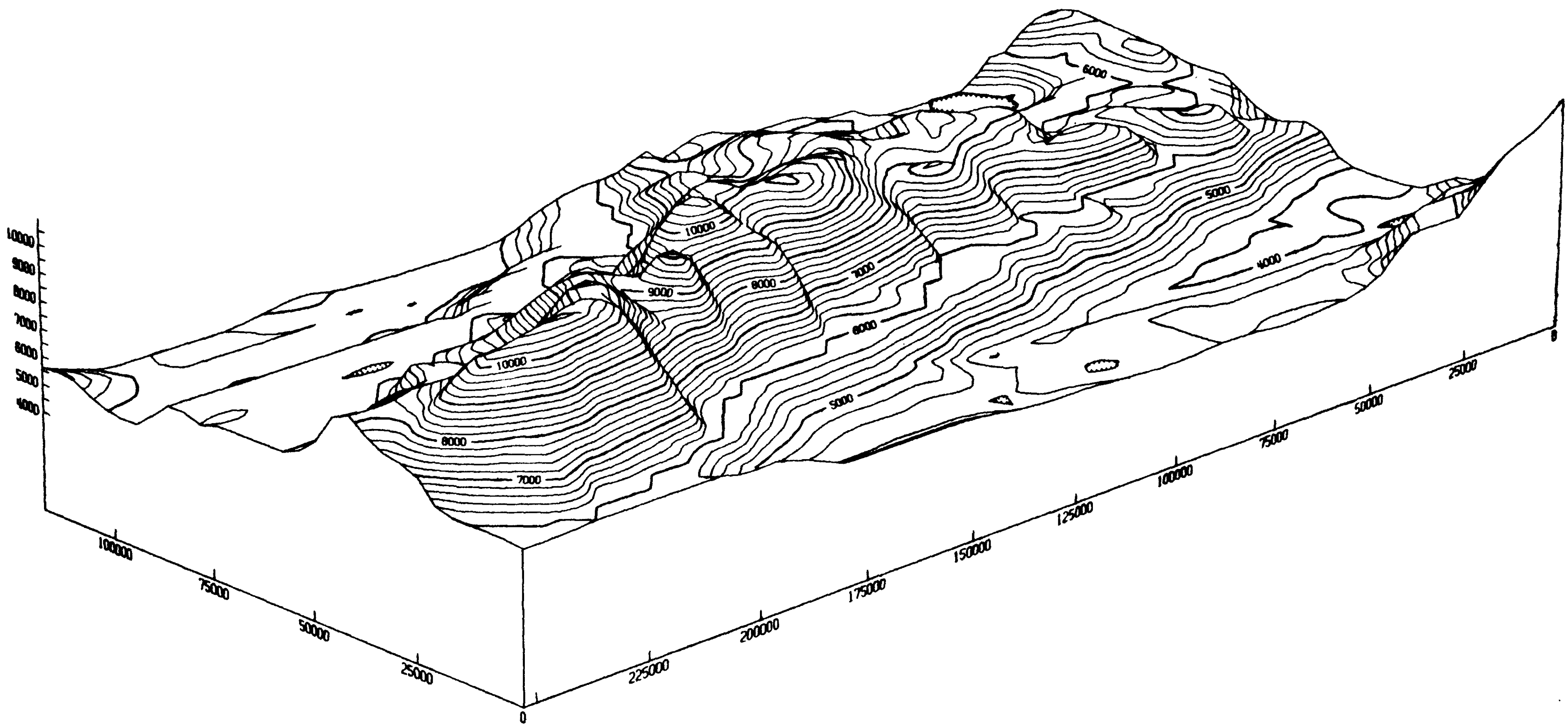

Figure 8A. Computer-generated contour view of the White Mountains, California after erosion for 30 my to a base level at 5900 feet elevation.

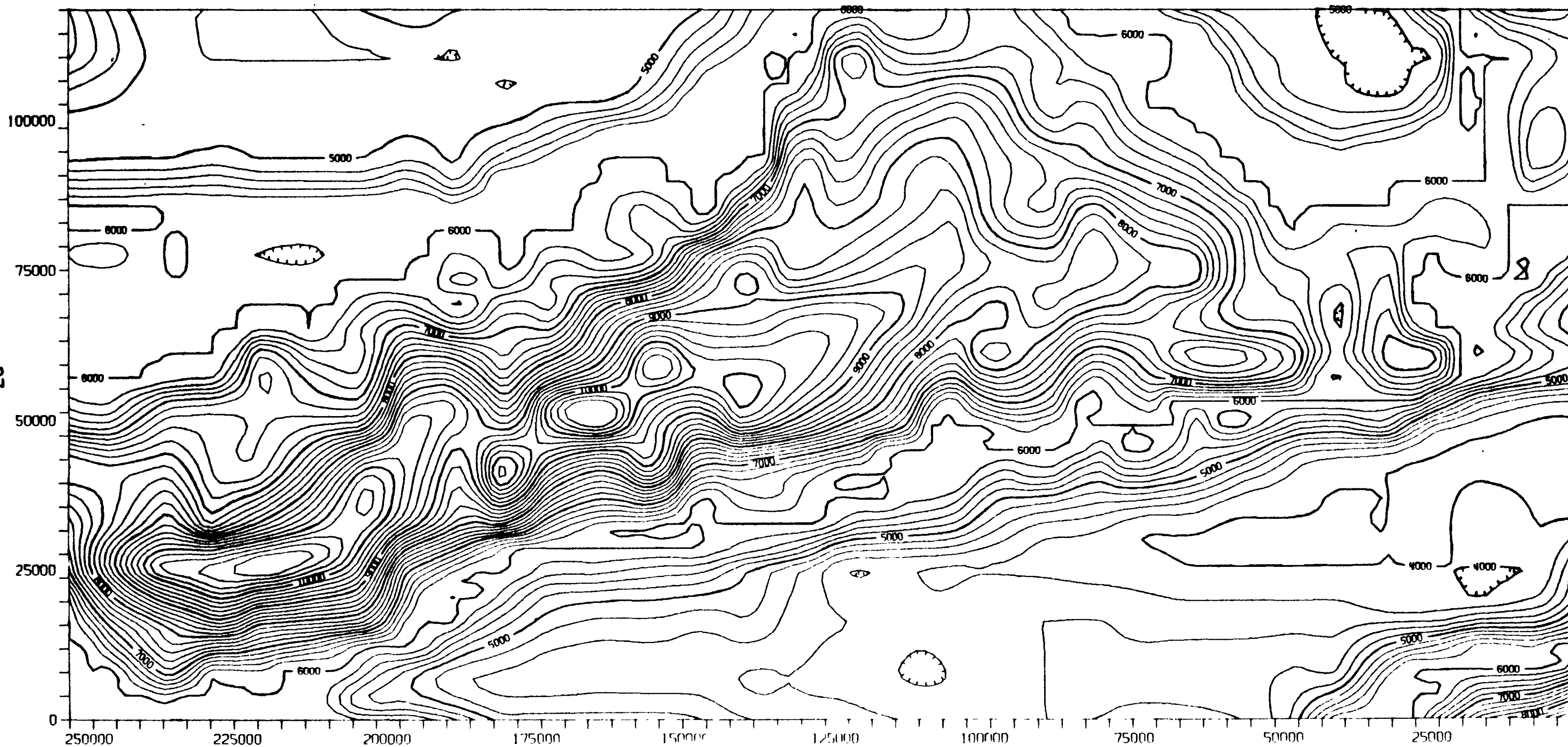

Figure 8B. Computer-generated contour map of the White Mountains, California after erosion for 30 my to a base level at 5900 feet elevation.

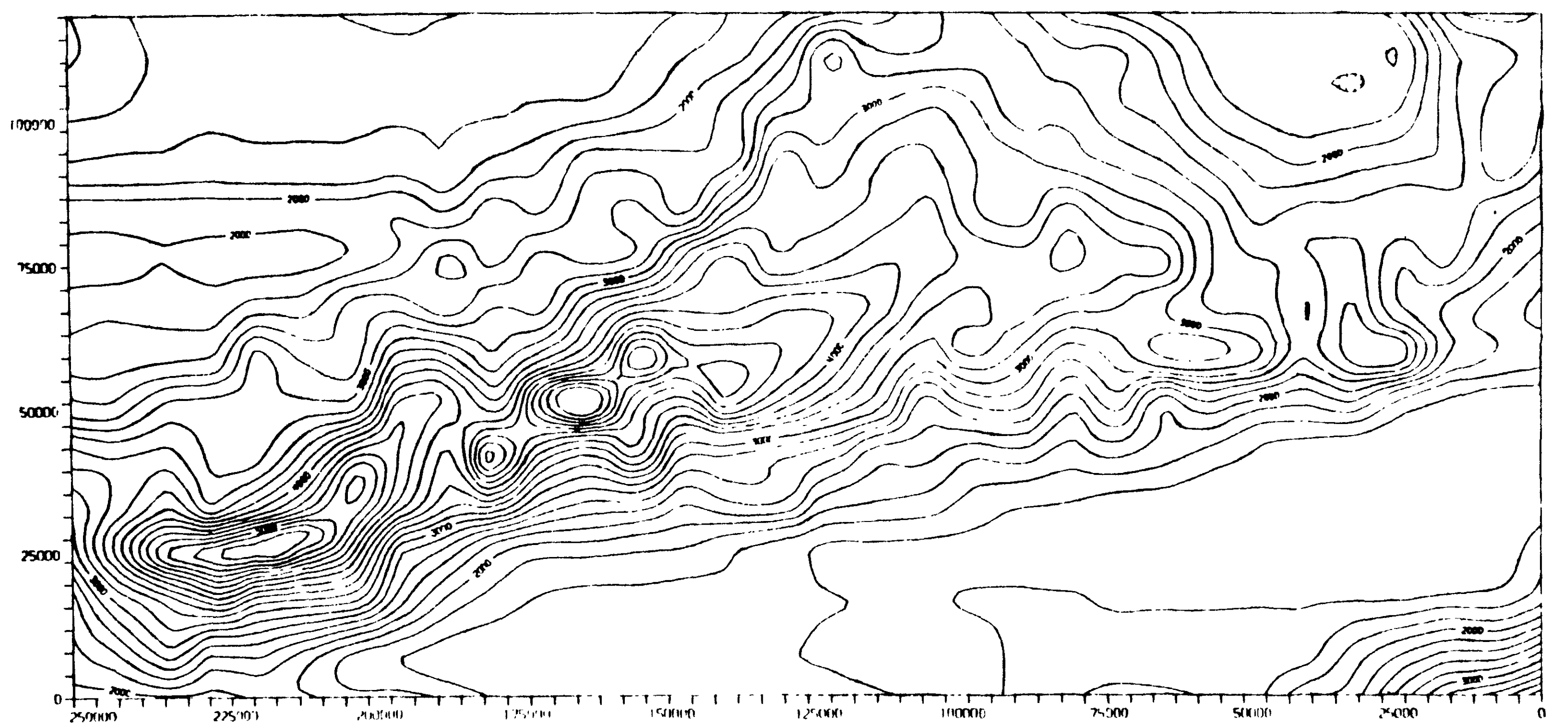

Figure 9. Computer-generated map of the amount of material eroded from the White Mountains over 30 my. Contour lines connect points of equal thickness of material eroded, or also represent the difference between the original topography and the topography after 30 my of simulated erosion.

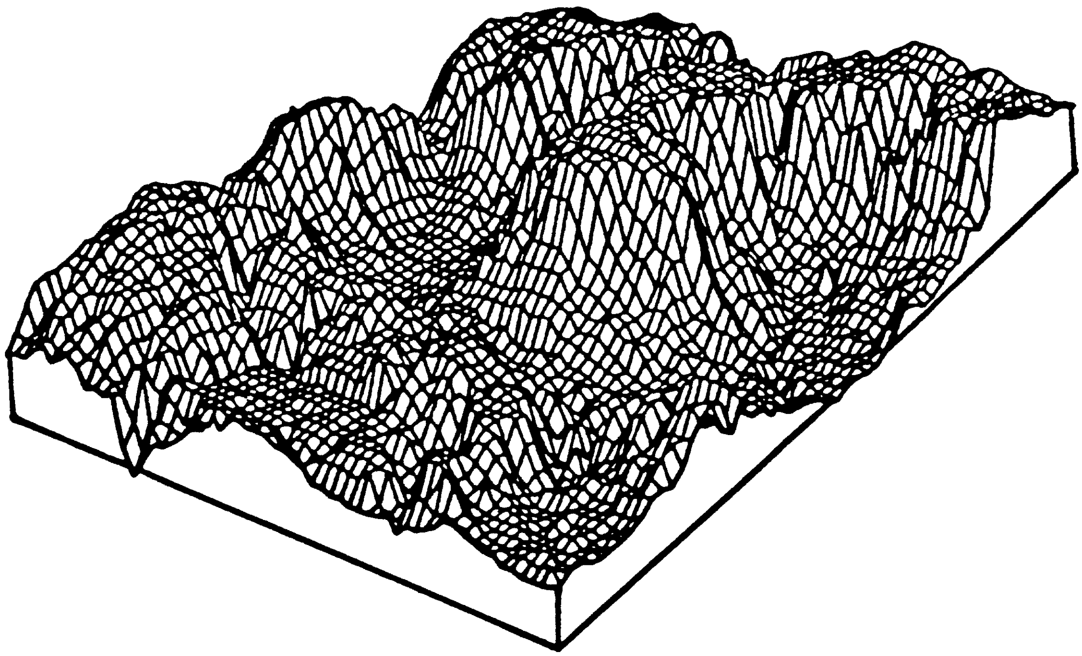

Figure 10. Computer-generated mesh view of the present topography of the Mogollon Rim.

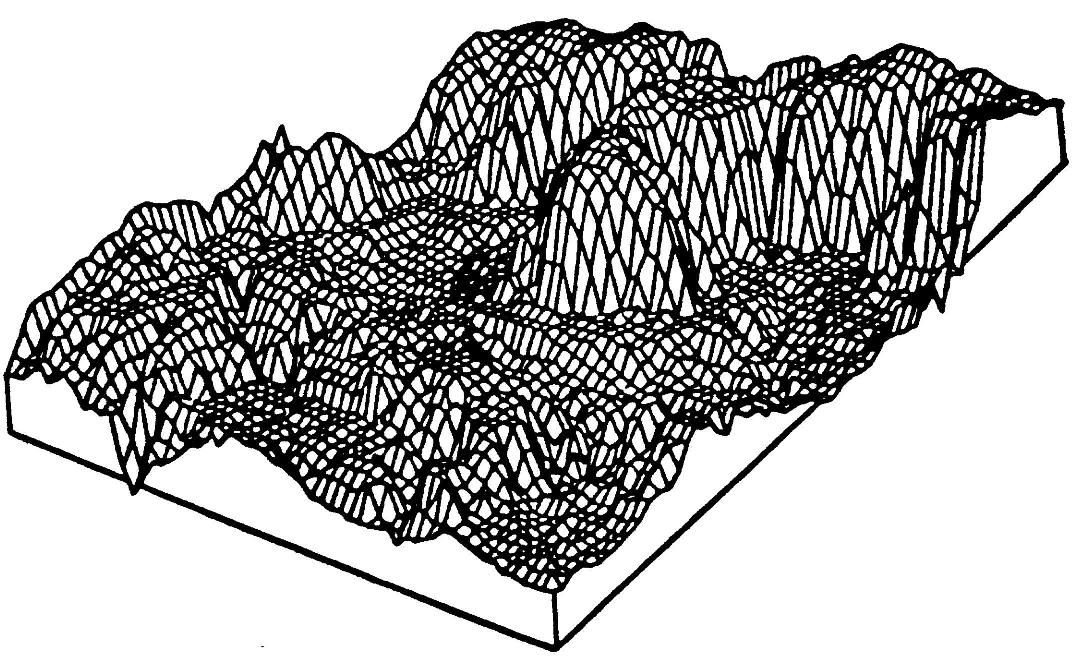

Figure 11. Computer-generated view of the predicted topography of the Mogollon Rim after 10 my of erosion. See text for analysis.

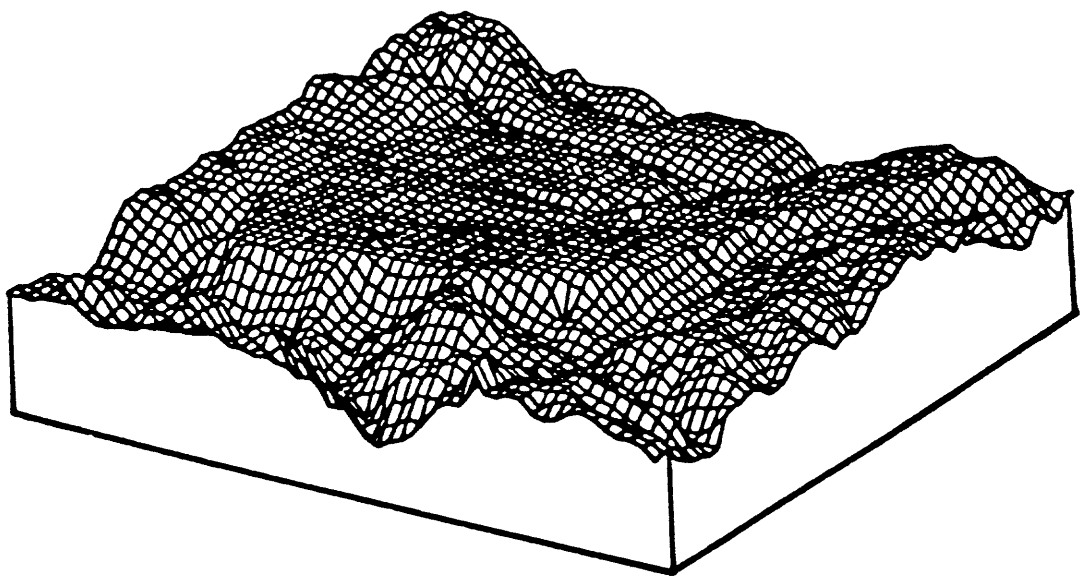

Figure 12. Computer-generated view of the present topography of Buckhead Mesa, Arizona.

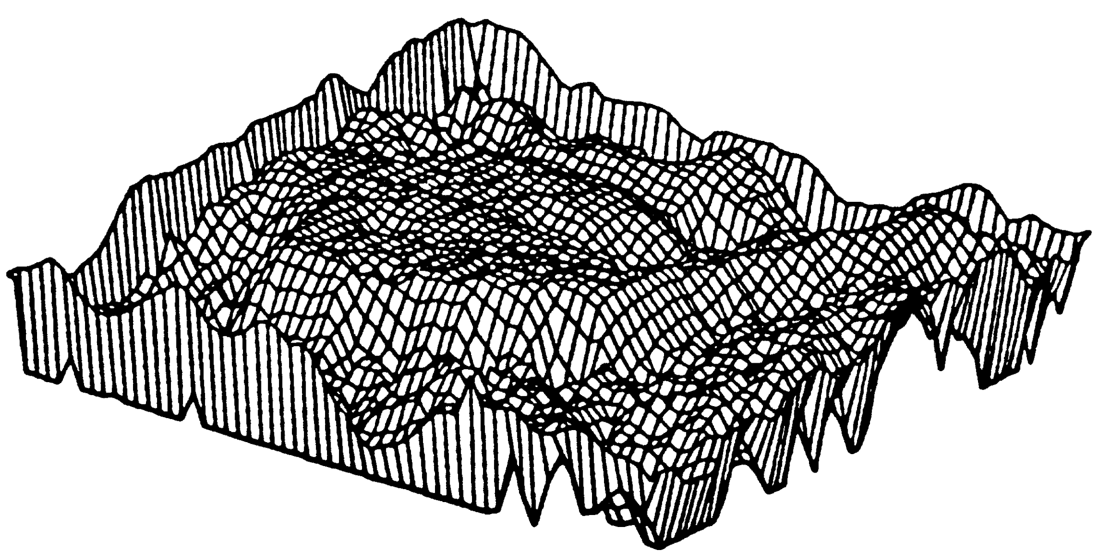

Figure 13. Computer-generated view of the predicted topography of Buckhead Mesa after 10 my of erosion. See text for explanation.

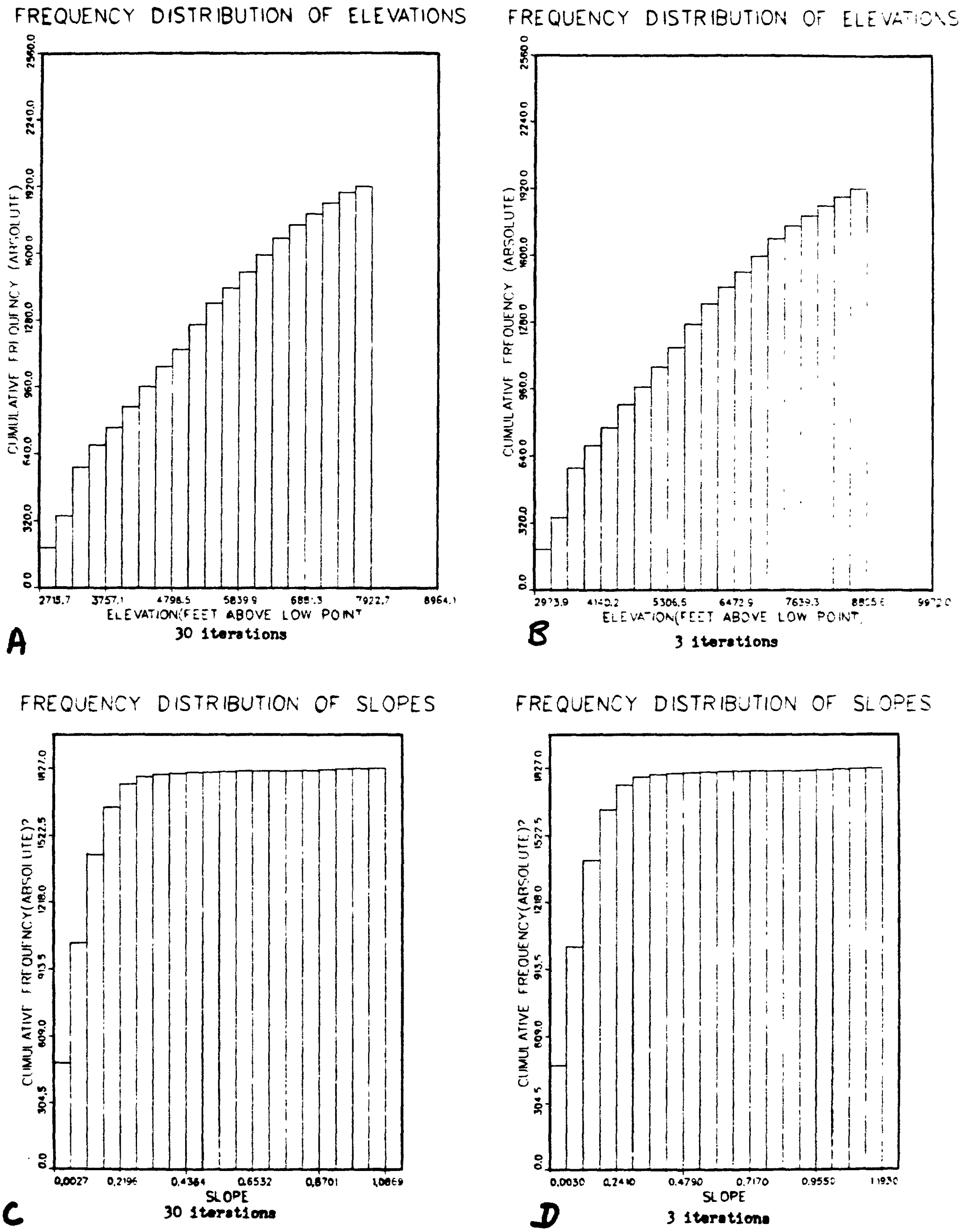

Figure 14. Histograms showing the frequency-distributions of elevatons (A and B) and slopes (C and D) for 30 my divided among 30 iterations (A and C) and 3 iterations (B and D) for the White Mountains. The number of iterations chiefly affects the values of the elevations and slopes rather than their distribution.