

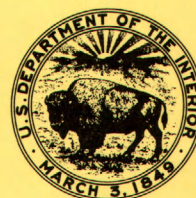
UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

FLOW MODEL OF SAGINAW RIVER
NEAR SAGINAW, MICHIGAN

U.S. GEOLOGICAL SURVEY

Open-File Report 81-1061

Prepared in cooperation with the East Central
Michigan Planning and Development Region



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September 1981



UNITED STATES DEPARTMENT OF THE INTERIOR

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GEOLOGICAL SURVEY

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DEFINITION OF TERMS

Cubic feet per second (ft³/s). A unit expressing rate of discharge.

One cubic foot per second is equal to the discharge of a stream through a square cross-section 1 foot wide and 1 foot deep at an average velocity of 1 foot per second. (Negative discharge values as used in this report indicate upstream flow.)

Gaging station. A particular site on a river or reservoir where systematic time-series observations of gage height (water-surface elevation) or discharge are recorded.

International Great Lakes Datum of 1955 (IGLD 1955). A vertical control datum used on the Great Lakes System. In the Saginaw, Michigan area, to convert elevations based on National Geodetic Vertical Datum of 1929 to IGLD 1955, add about 1.3 ft. Exact conversion factors vary with latitude and elevation.

Manning's roughness coefficient. A measure of the frictional resistance exerted by a channel on flow.

Momentum coefficient. A measure of the nonuniformity in velocity distribution through a channel cross-section.

Lake seiching. In this report, an oscillation of the surface of Lake Huron and Saginaw Bay over a period of a few minutes to a few hours, as a result of atmospheric disturbances.

Slope-rating. A method of computing discharge developed from observations of stage (water-surface elevation) at a base gage, discharge, and the fall of the water surface between the base gage and an auxiliary stage gage.

Water-surface drag coefficient. A measure of the relative efficiency of energy transfer between wind and water.

CONVERSION FACTORS

<u>Inch-pound units</u>	<u>Multiply by</u>	<u>Metric (SI) units</u>
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
square mile (mi ²)	2.590	square kilometer (km ²)
cubic foot per second (ft ³ /s)	0.02832	cubic meter per second (m ³ /s)

FLOW MODEL OF SAGINAW RIVER NEAR SAGINAW, MICHIGAN

By

David J. Holtschlag

ABSTRACT

An unsteady-flow simulation model was applied to a 19.5-mile reach of Saginaw River. The model provides a method of determining instantaneous discharge for flows from -8,000 to 12,000 cubic feet per second. The currently used slope-rating method can be utilized to compute discharge only under steady and high-flow conditions. Unsteady flow frequently occurs in the Saginaw River as a result of lake seiching.

Model computations are based on solution of the continuity and momentum flow equations, on hydraulic characteristics of Saginaw River, and on time-dependent boundary conditions. An implicit, finite-difference technique is used to solve the one-dimensional flow equations. Channel storage and conveyance characteristics were obtained from data collected during a 1979 field survey and through model calibration. Boundary conditions are specified by stage or discharge data at the model extremities. Optionally, wind velocity data are incorporated in the flow simulations.

The model can simulate instantaneous stage and discharge data and summarize or plot the data. Simulations of low-flows are sensitive to small errors in stage data and to gentle breezes. Simulation of high flows for present channel conditions requires additional data and further study.

INTRODUCTION

Saginaw River drains Michigan's largest basin, 6,278 mi² (square miles), in the east-central Lower Peninsula (fig. 1). It flows in a northeasterly direction about 22.2 mi (miles) and discharges into the southern end of Saginaw Bay, an arm of Lake Huron. Four major sub-basins, the Tittabawassee, Shiawassee, Flint, and Cass, comprise 96 percent of Saginaw River basin. The lower 19 mi reach of Saginaw River is navigable by Great Lakes vessels.

The slope-rating method presently used to compute discharge of Saginaw River is applicable when flow exceeds 10,000 ft³/s (cubic feet per second), or about 10 percent of the time. Unsteady flow, caused by lake seiching, prevents the development of a slope-rating for low and medium flows. Moreover, using this technique one can compute discharge at only one point, nearly 18.5 mi upstream from the mouth. Thus, flow information is inadequate for accurately determining loads of suspended and dissolved materials transported to Saginaw Bay and subsequently to Lake Huron.

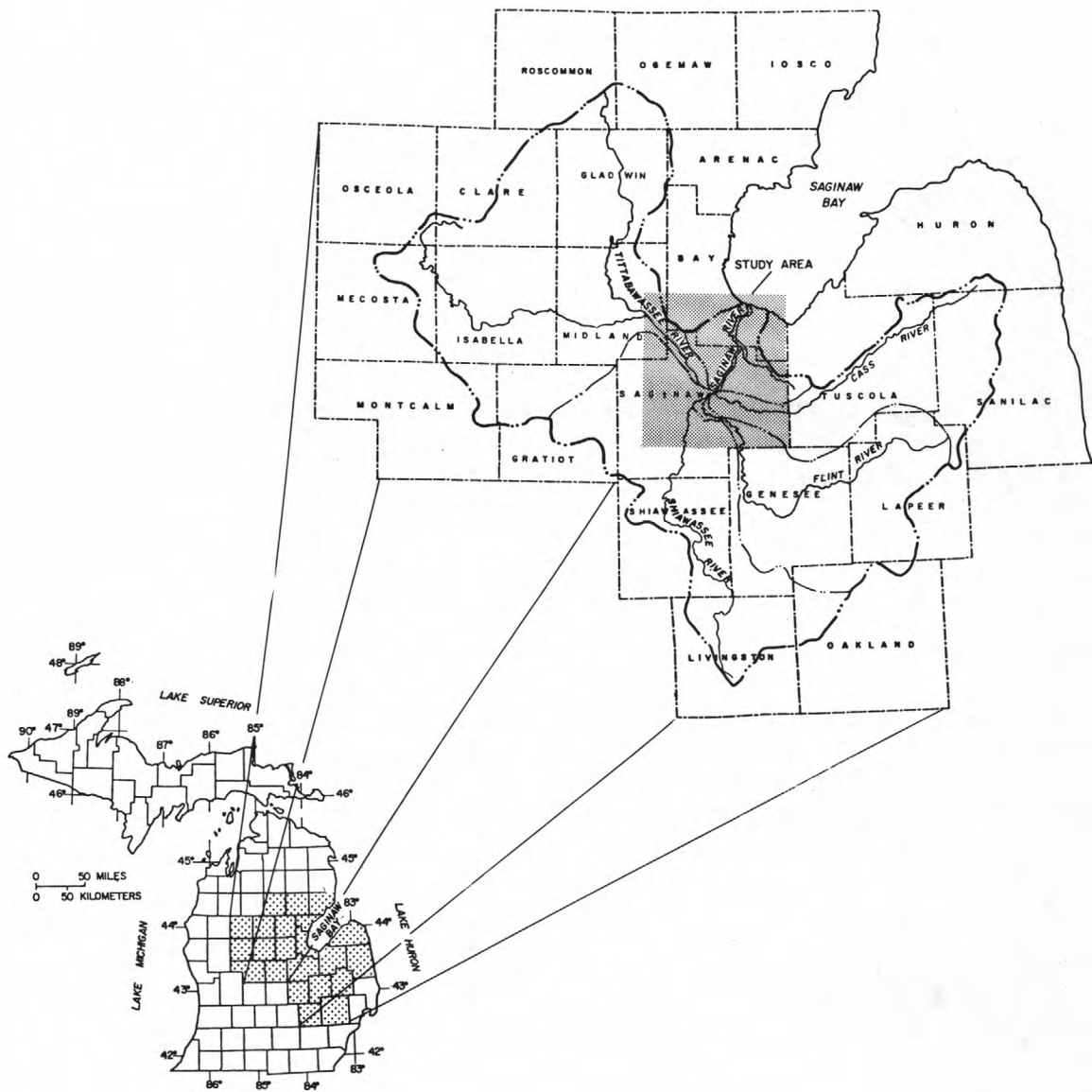


Figure 1.--Saginaw River basin and study area in Michigan's east-central Lower Peninsula.

Purpose and Scope

The purpose of this study was to apply an unsteady-flow model to Saginaw River which could provide discharge data throughout a major portion of the river's length. The modeled portion is a 19.5-mi reach extending from a point 0.3 mi downstream from the confluence of the Tittabawassee and Cass Rivers, south of the City of Saginaw (Site 7, fig. 2), to Schearmann Avenue in Essexville, 2.4 mi upstream from Saginaw Bay (Site 2). Three minor tributaries, Cheboyganing Creek, Dutch Creek, and the embayment, Crow Island West adjoin Saginaw River within the model area. Flow between Saginaw River and Crow Island West and the lower portions of Cheboyganing Creek and Dutch Creek, are accounted for in the model.

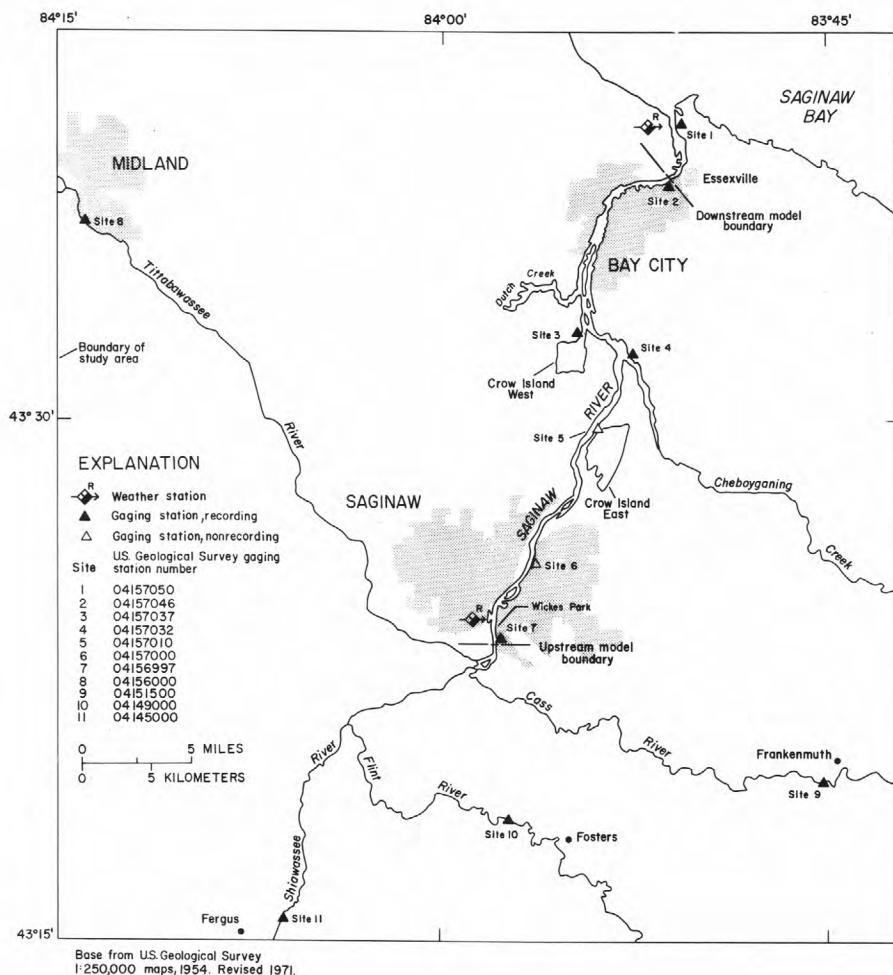


Figure 2.--Saginaw River and tributaries near Saginaw, Michigan.

SAGINAW RIVER UNSTEADY-FLOW MODEL

A branch network unsteady-flow model (Schaffranek and others, 1981) is used to simulate stage and discharge of Saginaw River. This one-dimensional model is based on an implicit, finite-difference formulation of the continuity and momentum flow equations. The model can accommodate local inflow and diversion, account for water-surface drag created by the wind, and compensate for nonuniform velocity distribution in a cross-section through a momentum coefficient. A modeling data base system supports model implementation by providing computer programs for analyzing cross-sectional geometry and for processing time-dependent, boundary-value data.

The model of Saginaw River includes the river and three minor tributaries. Flow between Saginaw River and the three tributaries, Crow Island West, and the downstream reaches of Cheboyganing Creek and Dutch Creek, is controlled by the water level of Saginaw River. Flow between the river and Crow Island East, a managed wetland area, is controlled by operation of a dam.

Schematization

For modeling purposes, a numerical description, or schematization, was made for Saginaw River and its tributaries. In the schematization (fig. 3), the river is depicted as a series of 11 channel reaches or branches. Three additional branches represent Crow Island West, Cheboyganing Creek, and Dutch Creek. Branches are further subdivided into segments to better estimate local hydraulic characteristics.

Junctions delimit and (or) connect branches. Five external junctions delimit upstream and downstream boundaries of Saginaw River and upstream boundaries of the three tributaries. Either stage or discharge data are specified at the external junctions in order to simulate flow. Ten internal junctions connect two or more branches and permit local inflow description. An internal junction is used on Saginaw River at the outlet of Crow Island East.

Implementation

Conveyance and storage characteristics for each segment of Saginaw River were defined by: cross-sectional geometry, segment length, Manning's roughness coefficient, momentum coefficient, channel orientation, and water-surface drag coefficient. Some properties were obtained through direct measurement; others were not directly measurable and required initial approximation and subsequent refinement through model calibration.


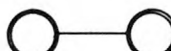

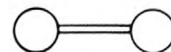









Withdrawals of water for municipal, industrial, commercial, and domestic use and the ultimate return of some water to the Saginaw River were also investigated to determine the affect on flow. Data provided by R. E. Ransom, East Central Michigan Planning and Development Region (written commun., 1979) indicate that, at the present rate of withdrawal and return, local water use does not significantly affect flow in Saginaw River.

Geometry

Cross-sectional geometry was measured in July 1979 by using a sonic depth finder and boat for below-water portions of the channel, and by a profile leveling survey for the overbank portions. The geometry data were prepared in the form of stage-area-width tables for model input. Segment length was determined by measuring along the centerline of the channel as shown on 1:24,000 U.S. Geological Survey quadrangle maps.

Saginaw River geometry, in general, is regular and has a gradually expanding cross-sectional area in the downstream direction. Width increases from 400 ft (feet) at Wickes Park (Site 7) in Saginaw to 800 ft near Essexville (Site 2). Depth varies from about 10 ft near the park to 30 ft at Essexville. An abrupt change in depth occurs downstream of Section 2-3 where dredging operations maintain a deeper navigational channel.

EXPLANATION

-  External junction cross section
-  Internal junction cross section
-  Intermediate cross section
-  Discharge measurement cross section
-  Recording stage gaging station
-  Non-recording stage gaging station
-  Mile marker referenced from Section I-I
-  3-I Branch segment identification
-  Recording weather station
-  Bridge
-  Drawbridge
-  Railroad bridge
-  Railroad drawbridge

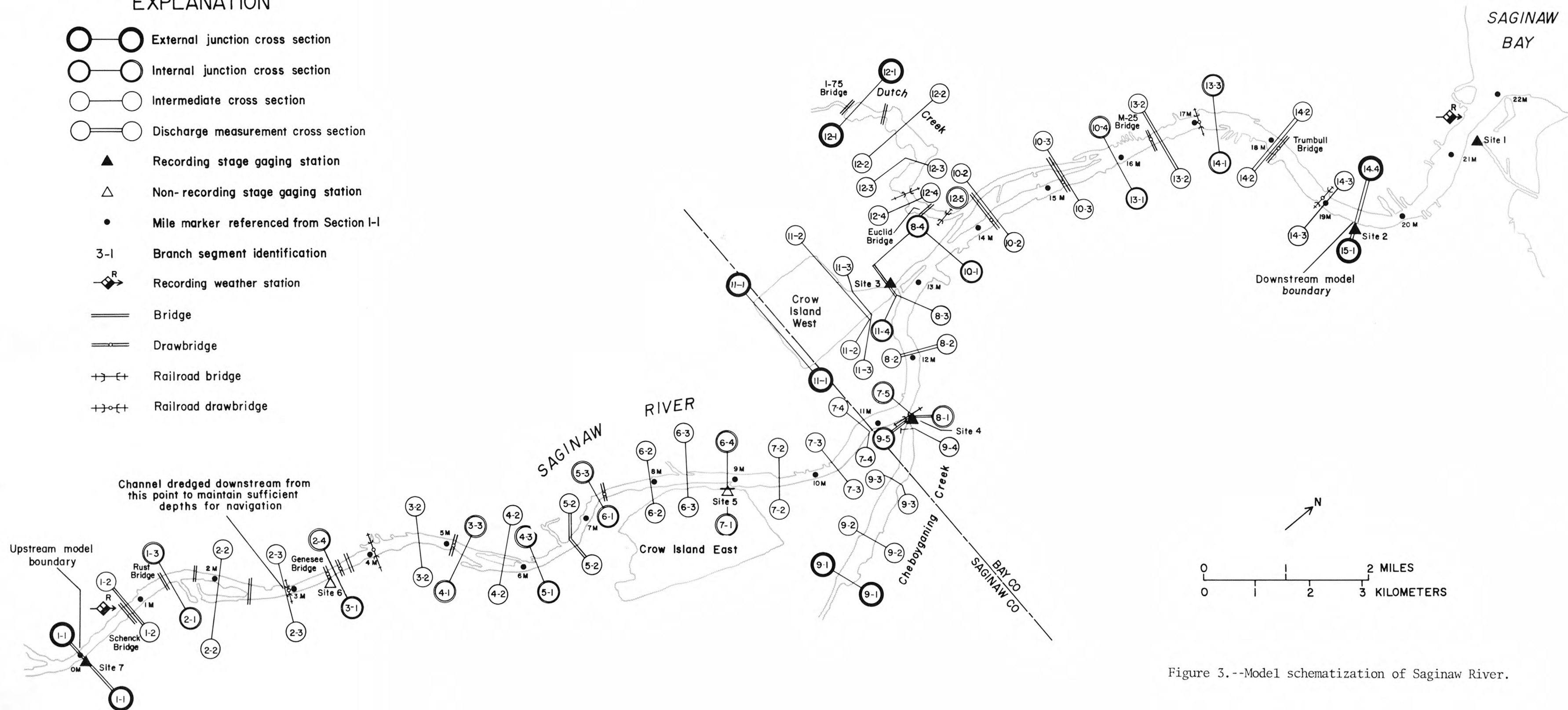


Figure 3.--Model schematization of Saginaw River.

Hydraulic Considerations

Wind can significantly affect flow on Saginaw River when the average water-surface slope decreases below 10^{-5} ft/ft. Such slopes are common on Saginaw River except during spring runoff periods. In order to evaluate this affect, the orientation of each channel segment, relative to true north, was measured. Also a value of 0.0026 was assigned to the water-surface drag coefficient after initial approximation and subsequent calibration of the model.

Channel geometry irregularities and channel roughness cause a nonuniform distribution of velocities over a channel section (fig. 4). As a result of this nonuniform distribution, momentum is always greater than that computed by assuming a uniform velocity distribution to prevail. Correction of the momentum flow equation for this effect is accomplished by use of a momentum coefficient. The value of this coefficient for the Saginaw River flow model was estimated to be 1.06. This determination was made on the basis of vertical velocity profiles (Chow, 1959, p. 29) made at discharge measurement sections.

Initial values for the Manning's roughness coefficients were based on the size of river bottom material. The channel bottom is clay overlain by sand and silt, indicating a Manning's roughness coefficient of about 0.024 (Chow, 1959, p. 118). Because of the change in depth at Section 2-3, a 4.4 percent higher Manning's roughness coefficient was used upstream of this point. Initial estimates of Manning's roughness coefficients were refined during calibration of the model.

Boundary-Condition Data

Water-stage and wind-velocity data describe forces which determine flow in the Saginaw River. To obtain stage data, four new gaging stations, Sites 2, 3, 4, and 7 (fig. 3), and an existing staff gage, Site 5, were used. (Elevations of gaging stations are referenced to IGLD 1955.) Synchronous recording of stages were obtained using precision crystal timers. Two new weather stations were installed adjacent to Saginaw River in Saginaw near Site 7, and in Bay City near Site 1 (fig. 3). Wind speed and direction, precipitation, and temperature are recorded continuously on pressure sensitive analog charts at these locations.

Calibration

Calibration is the process by which values of model parameters are adjusted until results of simulations correspond to measured conditions. For the Saginaw River model, Manning's roughness coefficient and a water-surface drag coefficient were adjusted to simulate mean monthly and instantaneous measured discharge data.

Long-Term Comparison

Mean monthly discharge of Saginaw River was estimated on the basis of discharge from the major subbasins measured at Sites 8, 9, 10, and 11 (fig. 2). The sum of mean daily discharges from the subbasins was multiplied by the ratio of drainage area of Saginaw River at the mouth

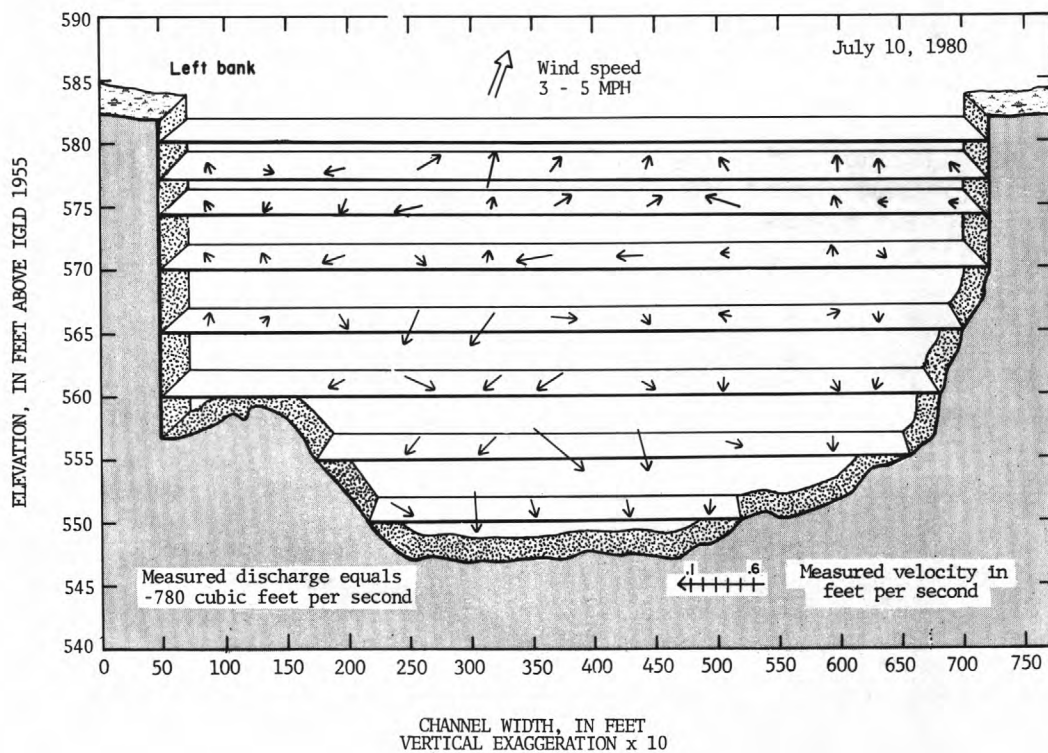
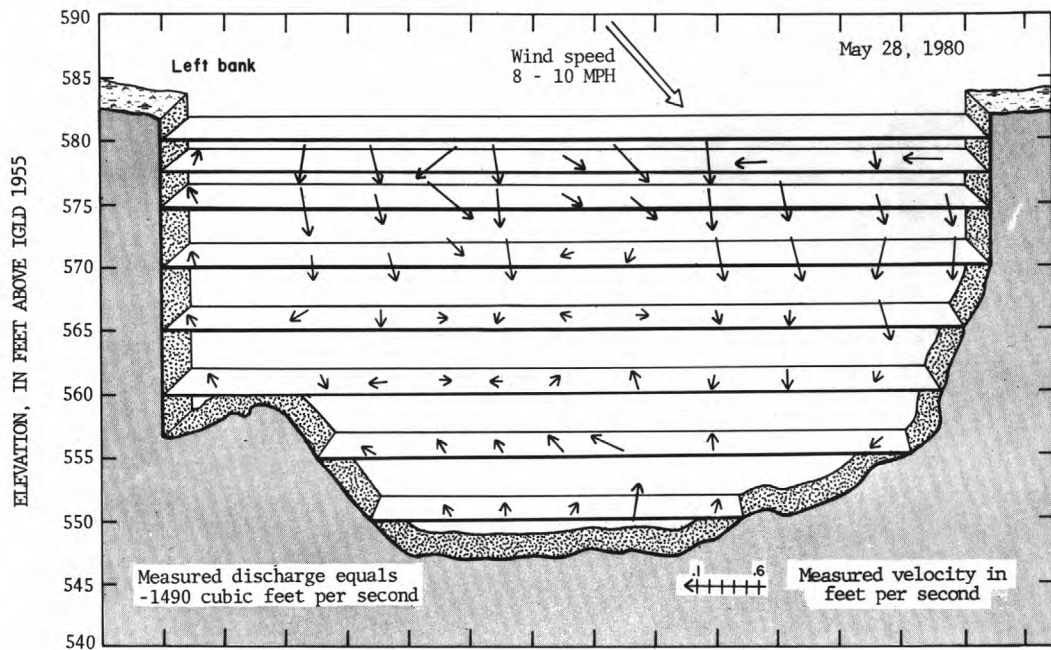


Figure 4.--Velocity distribution measured at Section 14-4.

(6,278 mi²) and the total drainage area measured at the four gaging stations (5,066 mi²), to estimate mean daily discharge of Saginaw River. Estimated mean daily discharges were then averaged over a period, usually one month.

Mean monthly discharge of Saginaw River was simulated using stage data at Sites 2 and 7. No wind data was entered because the net effect of wind on discharge over an extended period, a month or longer, is believed to be small. Local inflow was assumed to be zero. Mean daily discharges were simulated at one-hour time steps for a continuous five-month period between April 1 and August 31, 1980. Mean monthly discharge was computed from mean daily discharge values.

Estimated and simulated discharge near the mouth of Saginaw River correlate closely as indicated in the following table:

Period	Period of simulation (1980) <u>1/</u>	Simulated discharge (ft ³ /s)	Period of estimate (1980) <u>1/</u>	Estimated discharge (ft ³ /s) <u>3/</u>	Percent difference in discharge
1	Apr. 1-30	9,207	Mar. 30-Apr. 28	9,611	-4.20
2	May 1-31	4,629	Apr. 29-May 29	4,954	-6.56
3	June 1-30	2,880	May 30-June 28	3,027	-4.86
4	July 1-25 <u>2/</u>	1,700	June 29-July 23	1,782	-4.60
5	July 26-Aug. 31	1,855	July 24-Aug. 29	1,973	-5.98

1/ The two days difference between simulated periods and estimated periods represent the time required for water to flow from gaged points on tributaries to the mouth of Saginaw River.

2/ Storm discharge from tributaries had not passed through Saginaw River at month's end.

3/ Estimated discharge is based on measured discharge of the major subbasins adjusted for drainage.

The simulated discharge is on the average 5.2 percent less than the estimated discharge for the five periods studied. If local inflow, which represents the flow from about 4 percent of Saginaw River basin, had been included at internal junctions, the resulting simulated discharge at Section 14-4 would have been greater thereby improving the agreement. Calibration using this comparison resulted in Manning's roughness coefficients of 0.0235 for the undredged channel and 0.0225 for the dredged channel.

Simulated and estimated mean daily discharges of Saginaw River from April 1 to August 31, 1980 are shown in figure 5. During April and May, simulated and estimated discharges are similar; however, during June, July, and August mean daily simulated and estimated discharges are quite different. Under these lower flow conditions, simulated discharges are thought to be more accurate than discharge estimated from the major subbasins because the model accounts for changes in flow and channel storage in Saginaw River resulting from frequent lake seiching.

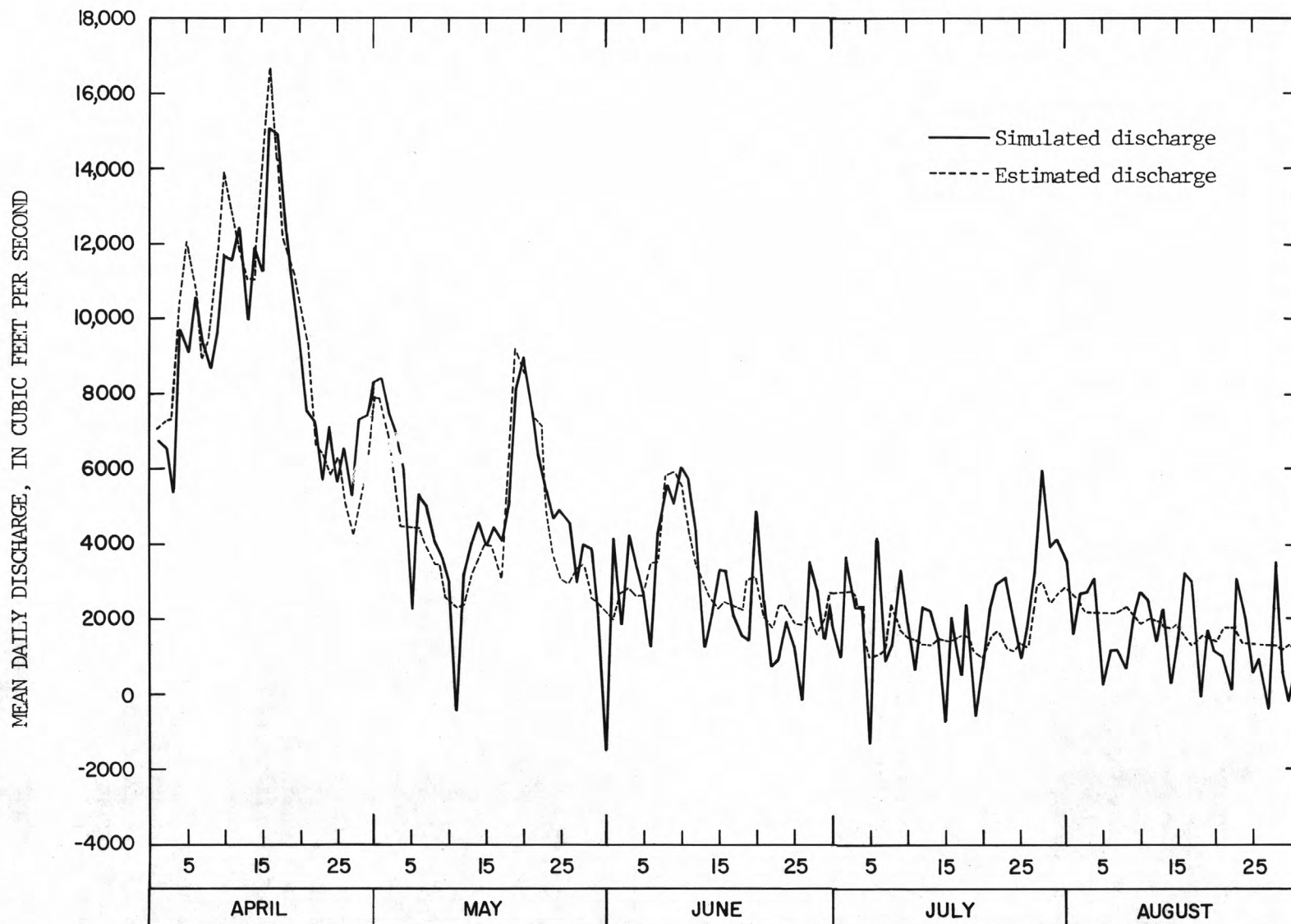


Figure 5.--Simulated and estimated mean daily discharge of Saginaw River, April 1 to August 31, 1980.

Discrete Comparison

Thirty-eight discharge measurements of Saginaw River, made between September 1979 and November 1980 at sections shown on figure 3, were used in model calibration. Measured discharges ranged from -8,190 to 11,700 ft³/s. Endeco¹/ current meters were used to measure low flow velocities and directions. Price current meters were used when flow velocities were great enough to ensure accurate measuring. Measurements were conducted from both stationary boats and bridges.

Discharge of Saginaw River was simulated using stage data from Sites 2 and 7, and wind data from the weather stations. Simulated discharges, computed at 15-minute time steps, were averaged for the measurement period, which generally extended from 2 to 4 hours. Discharge simulations often showed unsteady flow during measurement periods as shown in figure 6. Unsteady-flow conditions hampered accurate discharge measurement and subsequent comparison of measured and simulated discharges. However, the analysis of the mean values indicate that the model can accurately simulate unsteady flow on Saginaw River over the range of discharges measured (fig. 7).

A water-surface drag coefficient was determined by comparing the mean simulated and mean measured discharge of Saginaw River through a range of water-surface drag coefficients (fig. 8). On the basis of this analysis and published information (Wilson, 1960), the water-surface drag coefficient value was determined to be 0.0026.

Tributary Influence

The three tributaries which adjoin Saginaw River within the modeled area are Cheboyganing Creek, Dutch Creek, and Crow Island West. Flow in the tributaries is computed on the basis of simulated stage of Saginaw River, near the outlet of the tributary. Tributaries are modeled as storage areas; zero discharge is specified at their upstream model extremities. One measure of the relative significance of each tributary, in comparison to Saginaw River discharge, is indicated by the magnitude of discharge fluctuations during unsteady-flow periods (fig. 9). Discharge measurements on Dutch Creek at Euclid Bridge indicate the accuracy of the simulation.

Calibration Using Historic High-Flow Data

Fifty-eight discharge measurements, ranging from 9,360 to 53,800 ft³/s, were made from Genesee Bridge (Site 6) between 1943 and 1967. These measurements have been used to develop and verify a slope-rating between Sites 6 and 1. The slope-rating is currently used to compute discharge greater than 10,000 ft³/s, a rate at which flow tends to be steady.

¹/ The use of brand names in this report is for identification purposes and does not imply endorsement by the U.S. Geological Survey.

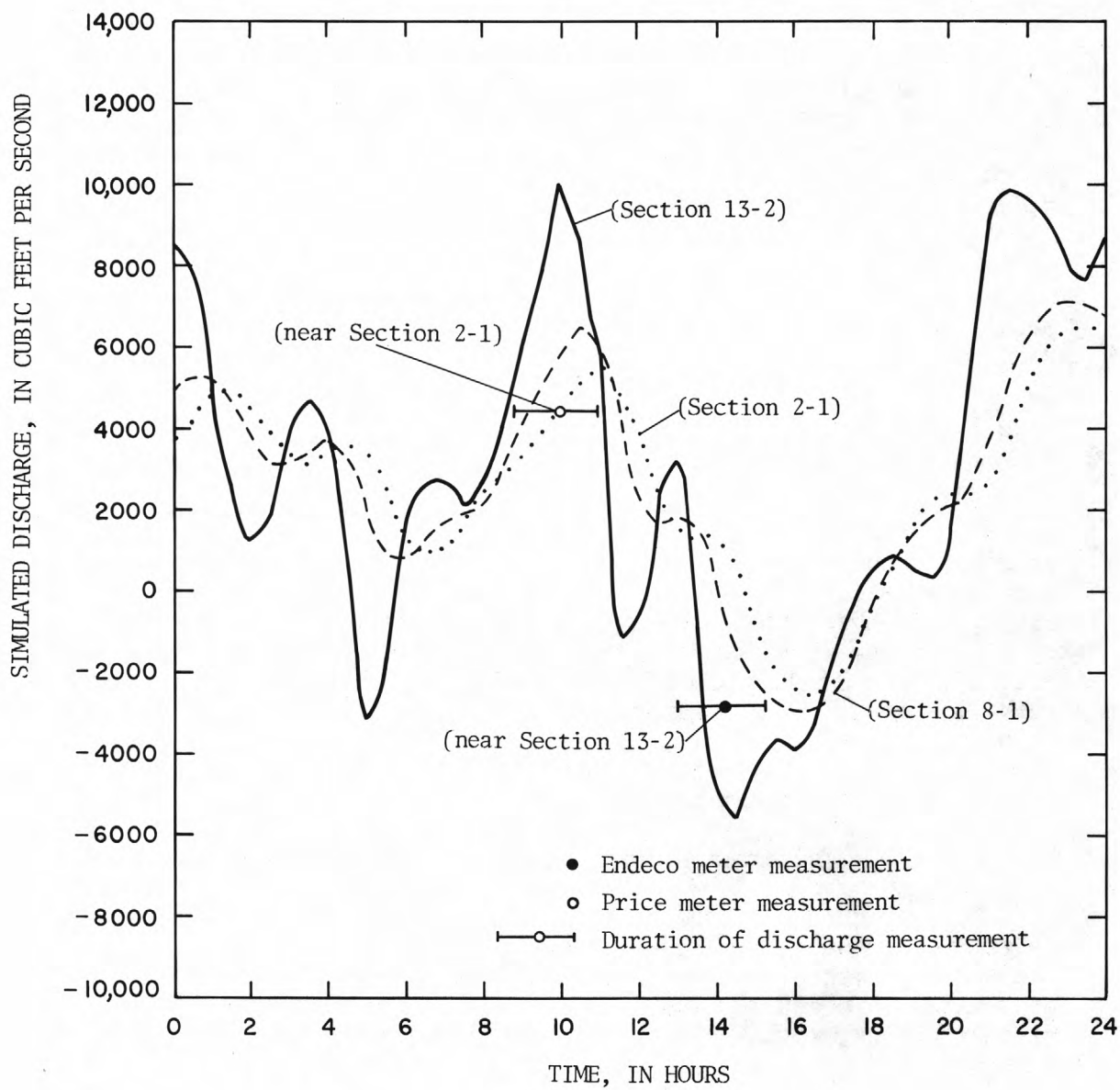


Figure 6.--Simulated and measured discharge of Saginaw River at selected locations, May 28, 1980.

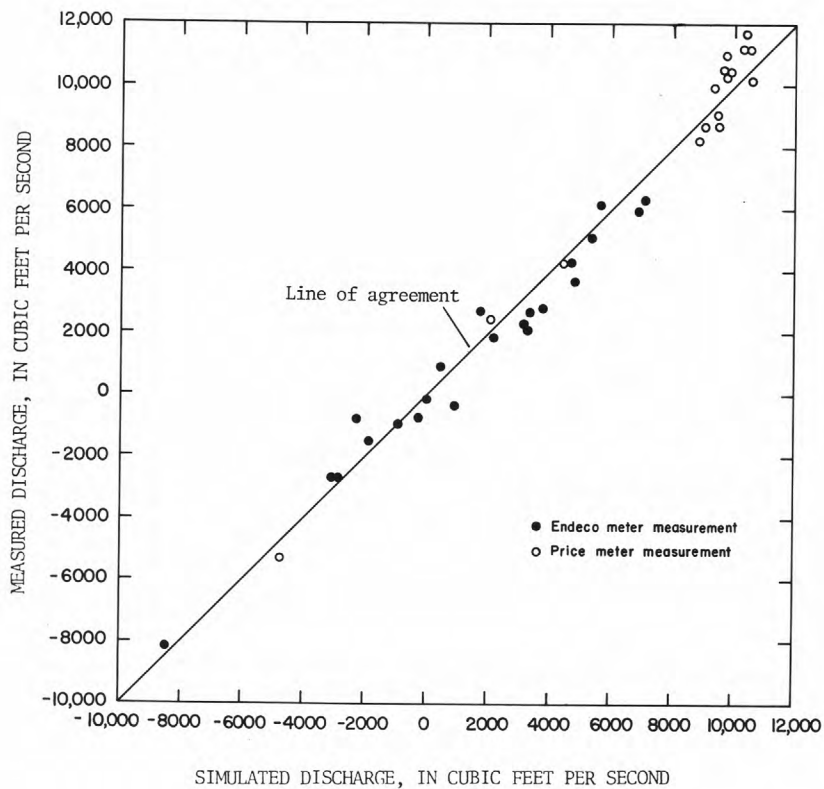


Figure 7.--Relation of simulated to measured discharge of Saginaw River.

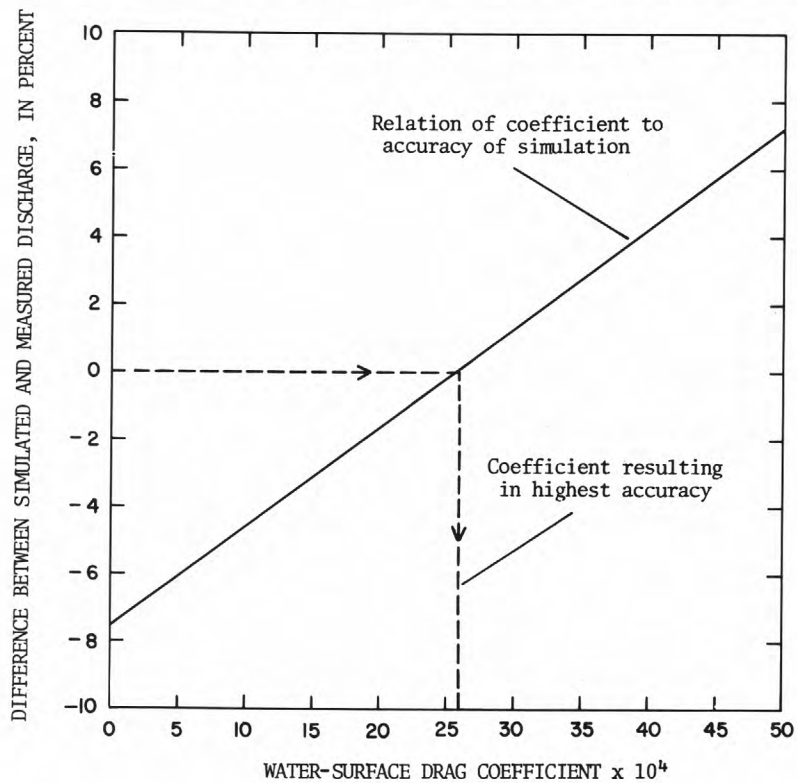


Figure 8.--Relation of water-surface drag coefficient to accuracy of simulated discharge of Saginaw River.

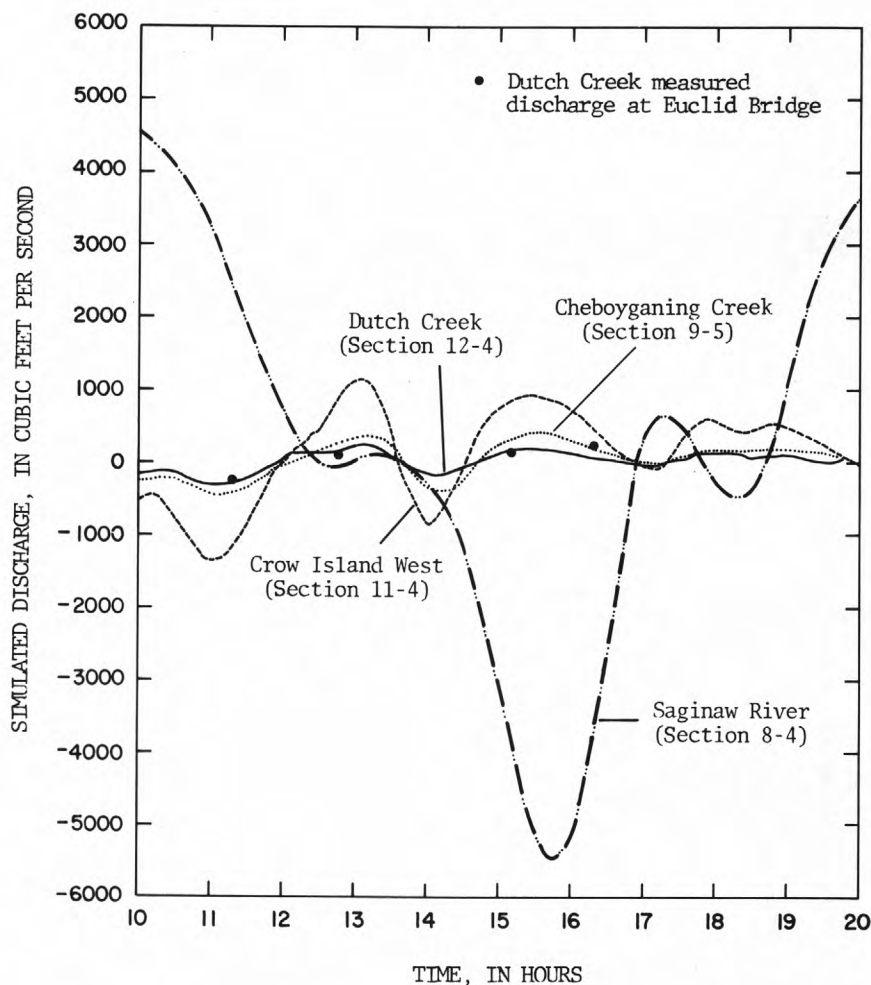


Figure 9.--Simulated discharge of Saginaw River, Crow Island West, Cheboyganing Creek, and Dutch Creek, August 7, 1980.

Historic high-flow data were also used to calibrate the model through comparison of measured and simulated stage data. For the simulation, measured discharge values were specified at Site 7, and stage data, based on measured stage at Site 1, was specified at Site 2. Steady-state solutions were determined. Wind data were not used in the computation.

The initial simulated stage at Section 3-1 averaged about one foot lower than measured stage at Site 6, a short distance upstream. To obtain better accuracy, Manning's roughness coefficients were increased about 17 percent. Using a least squares analysis, roughness coefficients (n), were described as quadratic functions of simulated discharge (Q), as follows:

Undredged channel: $n = 2.6095 \times 10^{-2} + 1.1840 \times 10^{-7} Q - 2.0870 \times 10^{-12} Q^2$
Dredged channel: $n = 2.4985 \times 10^{-2} + 1.1366 \times 10^{-7} Q - 1.9980 \times 10^{-12} Q^2$

The agreement between measured and simulated stage resulting from this modification is shown in figure 10.

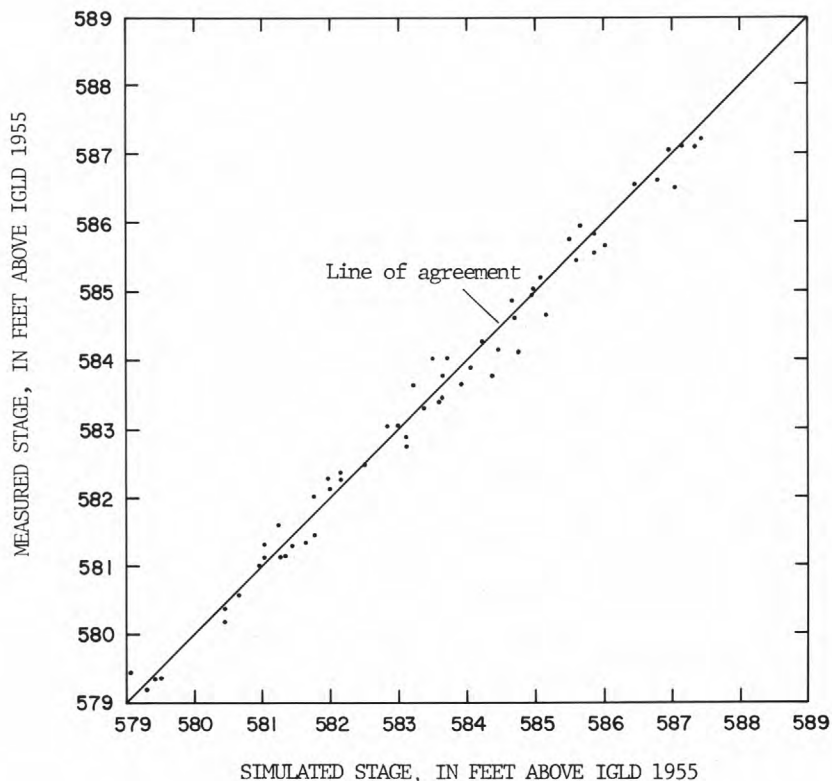


Figure 10.--Relation of simulated stage at Section 3-1 to measured stage at Site 6.

Results from the model, as modified to simulate historic high flow, were compared with those computed by the slope-rating method. Stage at Site 6 was computed using the slope-rating, given historic measurements of discharge at Site 6 and stage at Site 1. The relation of simulated stage to stage computed using the slope-rating method shows that the two methods of computing high-flow produce similar results (fig. 11).

Sensitivity of the Model

An analysis was made to determine the sensitivity of simulated discharge to water-surface fall and wind velocity--the two primary driving forces. Steady-state solutions were determined using average water level conditions on Saginaw River.

Water-surface fall between Sites 2 and 7 is the primary force affecting flow within the modeled portion of Saginaw River. In the sensitivity analysis, stage at Site 2 was set to an elevation of 579.0 ft; stage at Site 7 was varied to produce a water-surface fall which ranged from 0.10 to 1.00 ft through the modeled reach. Results show that simulated discharge is proportional to the square root of water-surface fall (fig. 12).

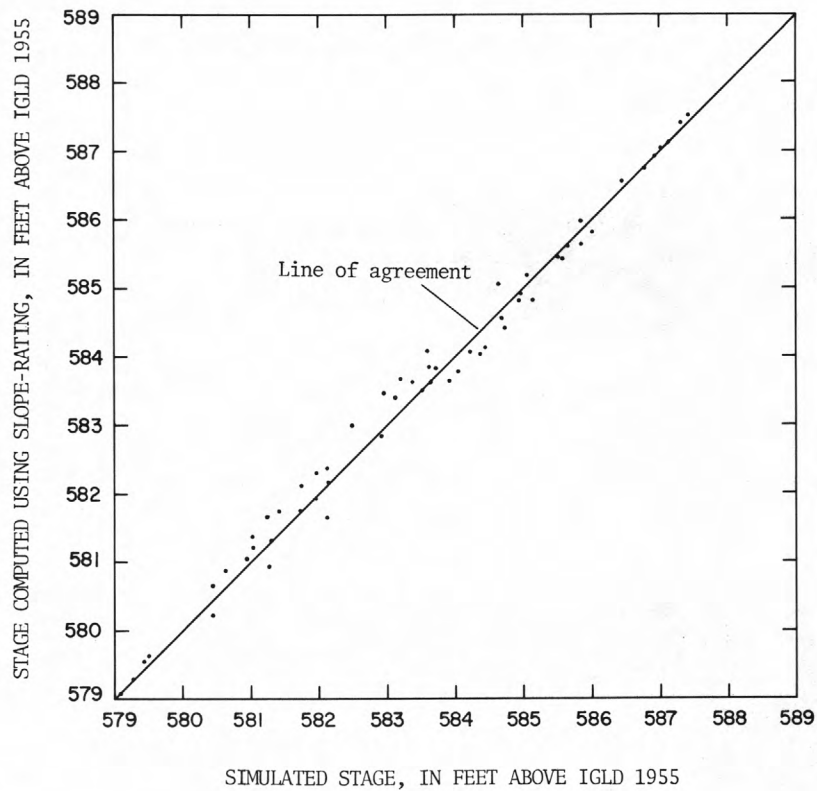


Figure 11.--Relation of simulated stage at Section 3-1 to stage at Site 6 computed using slope rating.

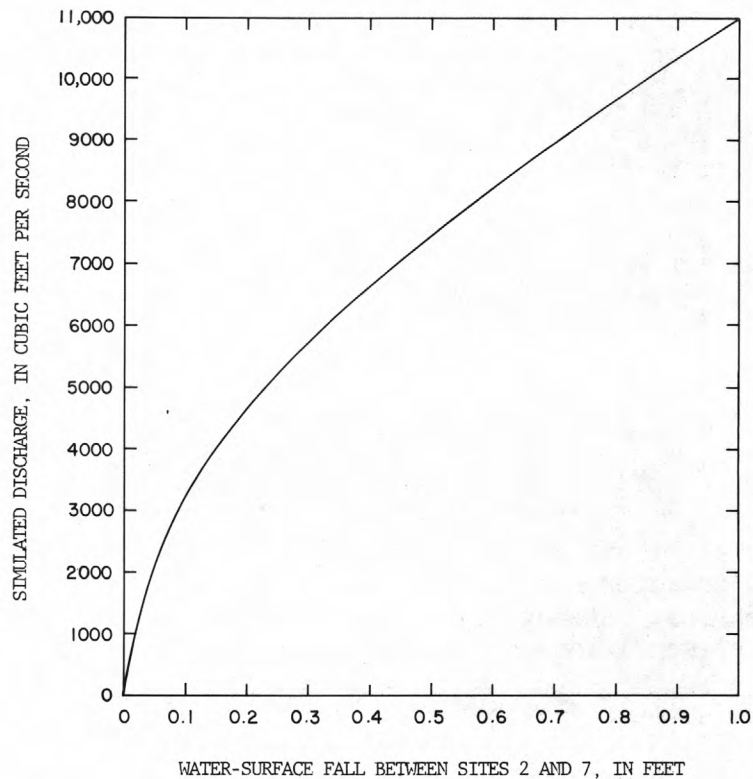


Figure 12.--Relation of water-surface fall to simulated discharge of Saginaw River. (Steady-flow solutions shown for a water-surface elevation of 579 ft above IGLD 1955 at Site 2, zero wind velocity specified.)

The mean discharge of Saginaw River, based on major tributary flow computed previously, is about 4,050 ft³/s. This discharge corresponds to water-surface fall of 0.15 ft or an average water-surface slope of 1.5×10^{-6} ft/ft. At this fall, a 0.02 ft error in stage data between Sites 2 and 7, would result in an error of 280 ft³/s or 7 percent. If the fall were 0.075 ft, a 0.02 ft error would result in a 14 percent error in simulated discharge. Thus, simulations of flow are sensitive to small errors in stage data, particularly at low water-surface fall conditions.

Flow of Saginaw River is also significantly affected by wind conditions. The affect of wind direction on simulated discharge for selected wind speeds is shown in figure 13. For this simulation, water surface fall was set to 0.0 ft. The maximum decrease in simulated discharge occurs when the wind is from N. 20° E.; the maximum increase occurs when wind is from S. 20° W. Wind has the minimum affect on discharge when it is from S. 70° E. or N. 70° W. For example, wind from the S. 35° E. at 5 mi/hr (miles per hour) increased simulated discharge 1,450 ft³/s (figure 13).

The affect of wind speed and water-surface fall on simulated discharge is shown in figure 14. Discharge is directly proportional to wind speed and proportional to the square root of water-surface fall. The effect of wind speed decreases with increasing water-surface fall.

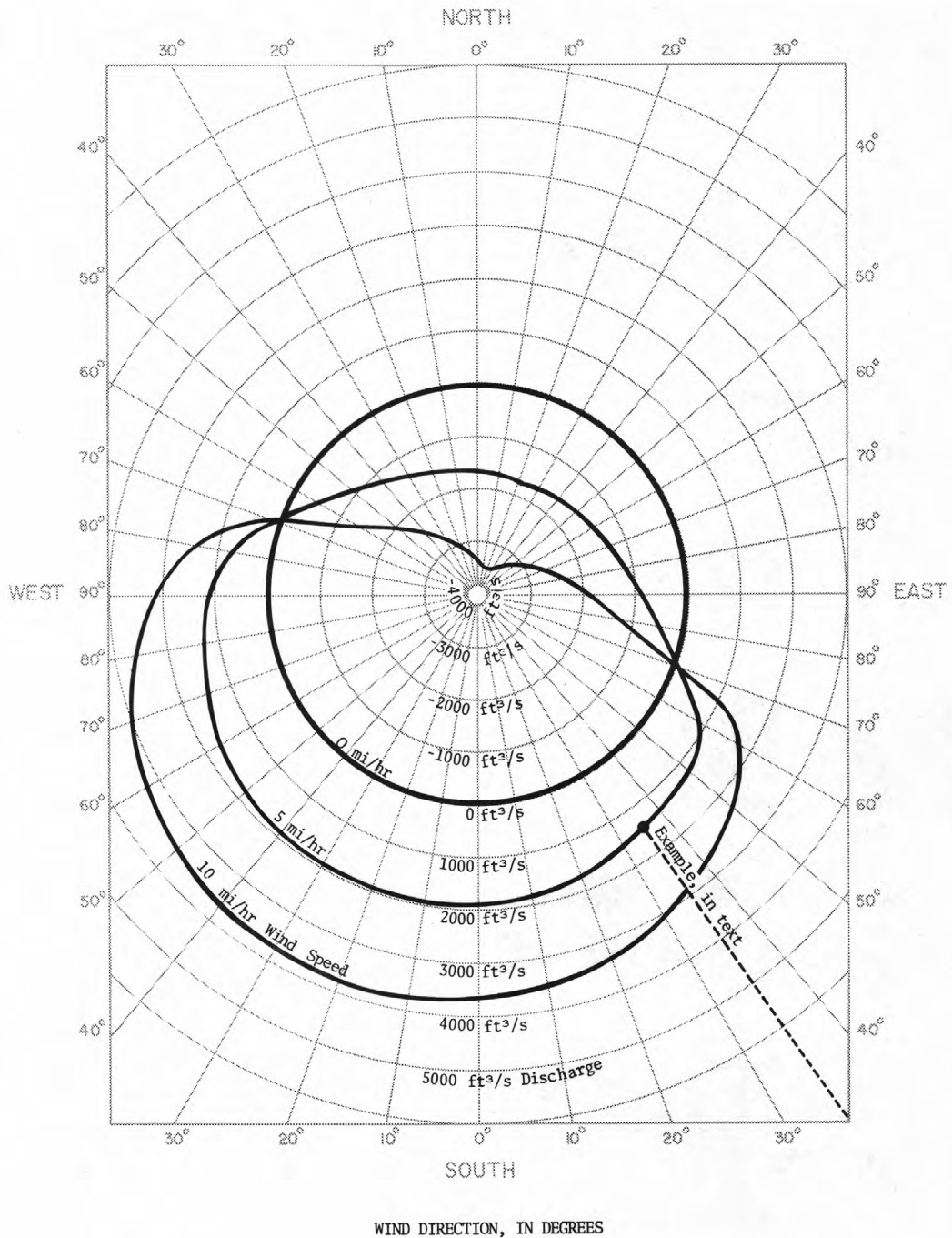


Figure 13.--Relation of wind direction to simulated discharge of Saginaw River for selected wind speeds. (Steady-flow solutions shown for a water-surface elevation of 579 ft above IGLD 1955.)

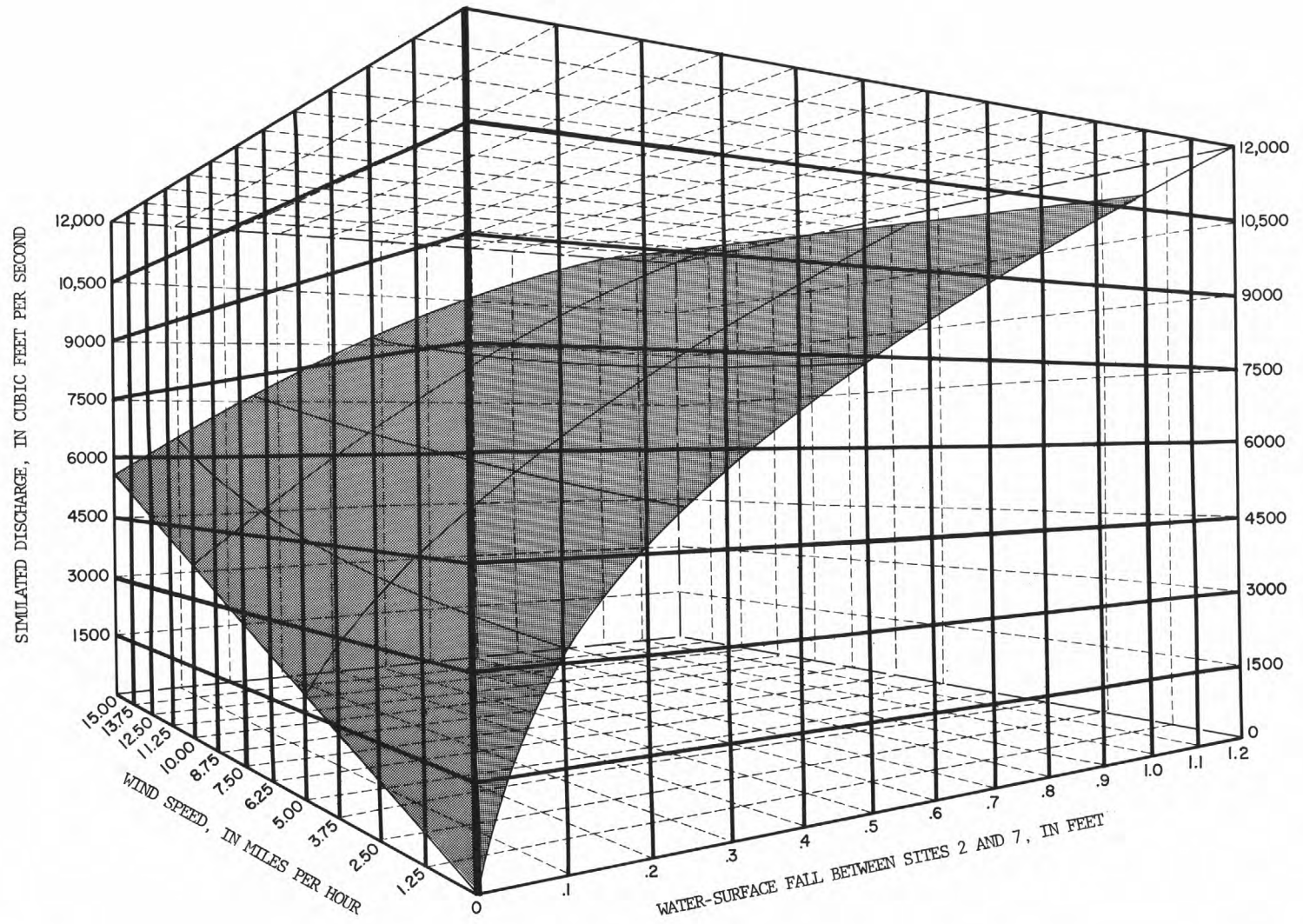


Figure 14.--Relation of wind speed and water-surface fall to simulated discharge of Saginaw River.
(Steady-flow solutions shown for a water-surface elevation of 579 ft above IGLD 1955 at Site 2 and a wind direction of S. 20° W.)

SUMMARY

A one-dimensional finite-difference unsteady-flow model was applied to a 19.5-mi reach of Saginaw River. The model provides a method by which to compute instantaneous discharge and stage data at any cross section within the modeled area. Model output options may be specified which plot or summarize the instantaneous data according to the user's interests.

Analyses of channel geometry, stage, wind and discharge data collected on Saginaw River during 1979 and 1980, lead to the development of a model for flow between -8,000 and 12,000 ft³/s. Flow between Saginaw River and the tributaries, Crow Island West, and the lower portions of Cheboyganing Creek and Dutch Creek, were accounted for in the model. Calibration of the model, through adjustments of Manning's roughness coefficients and a water-surface drag coefficient, was based on comparisons of simulated discharge with measured instantaneous discharge and estimated mean monthly discharge data.

Occasionally, Saginaw River discharge has far exceeded the highest discharge that occurred during the 1979 and 1980 model study period. Data on these historic high flow discharges, which ranged between 9,360 to 53,800 ft³/s, were used to calibrate the model. Manning's roughness coefficients were adjusted in response to measured versus simulated stage comparisons. The model, as modified, computed high-flow data with an accuracy that was similar to the slope-rating method.

A model sensitivity analysis showed that simulations of flow are sensitive to small errors in stage data and gentle breezes during low water-surface fall conditions common to Saginaw River. Simulation of high flows under present channel conditions requires additional discharge data and further calibration.

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