UNITED STATES

DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY

A STOCHASTIC STREAMFLOW MODEL OF THE PLATTE RIVER AT OVERTON, ODESSA, AND GRAND ISLAND, NEBRASKA

By Aldo V. Vecchia, Jr.

Open File Report 81-1188

Denver, Colorado

1981

UNITED STATES DEPARTMENT OF THE INTERIOR

JAMES G. WATT, Secretary

GEOLOGICAL SURVEY

Dallas L. Peck, Director

For additional information write to:

U.S. Geological Survey
Water Resources Division
Mail Stop 413, Box 25046
Denver Federal Center
Denver, Colorado 80225

For purchase, write to:

Open File Services Section
U.S. Geological Survey
Box 25425
Denver Federal Center
Denver, Colorado 80225
(303) 234-5888; FTS 234-5888

METRIC CONVERSION

Multiply inch-pound unit	By	To obtain metric unit
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)

CONTENTS

ILLUSTRATIONS

TABLES

Page

A STOCHASTIC STREAMFLOW MODEL OF THE PLATTE RIVER AT OVERTON, ODESSA, AND GRAND ISLAND, NEBRASKA

By Aldo V. Vecchia, Jr.

ABSTRACT

A stochastic model is developed to simulate flows for three seasons (September through February, March and April, May through August) at Overton, Odessa, and Grand Island, Nebraska on the Platte River. The model preserves the first and second order moment properties of the historical flow series, including significant autocorrelations within each station and cross-correlations between the stations. Higher order moments are preserved by a transformation technique that allows the residuals from the model to be approximately normal.

An algorithm is given for easy simulation of combined-station flows, and some simulations are carried out to determine the likelihood of specified flow shortages that could be detrimental to the wildlife habitat in the reach of the Platte River between Overton and Grand Island, Nebraska.

INTRODUCTION

Two primary characteristics that make a riverine habitat suitable for use by migratory waterfowl, especially sandhill cranes and whooping cranes, are a wide, open river channel and a wet meadow environment. A wide river channel containing only sparse vegetation affords roosting birds some degree of protection from predators, while wet meadow environment enhances the capability of the birds to locate and feed on invertebrates that are critical to their dietary needs.

To maintain desirable river channel characteristics, sufficient streamflow must be available during the seed-falling period of May through August to prevent germination on the channel bars. Similarly, river stages must be high enough during the months of March and April to raise ground-water levels in the wet meadows sufficiently so that the birds can locate invertebrates within several inches of the land surface. Probabilities of flow events involving these two periods are of great interest to wildlife managers for planning habitat-management alternatives in the event of flow shortages. Therefore, in this report a calendar year is divided into three seasons as they affect these processes. They are: (1) March and April, when river stage influences ground-water levels; (2) May through August, when streamflow may inhibit seedling germination and establishment; and (3) September through February, which is generally the low-flow period.

PURPOSE AND SCOPE OF STUDY

This report: (1) Describes the development of a 3-season flow-series model for Overton, Odessa, and Grand Island, Nebraska (fig. 1); (2) demonstrates how this model can be used for simulation and evaluation of certain probabilities of interest; and (3) gives results of some simulation studies. The 3-season flow series for a particular station is derived by aggregating monthly average-flow series over three predetermined seasons. A precise description of the series will be given in the next section. Seasonal sums instead of averages are used for computational convenience; however, both series (sums or averages) contain the same information.

The model is designed to maintain cross correlations between stations so that simulation of simultaneous three-station series can be made. However, to

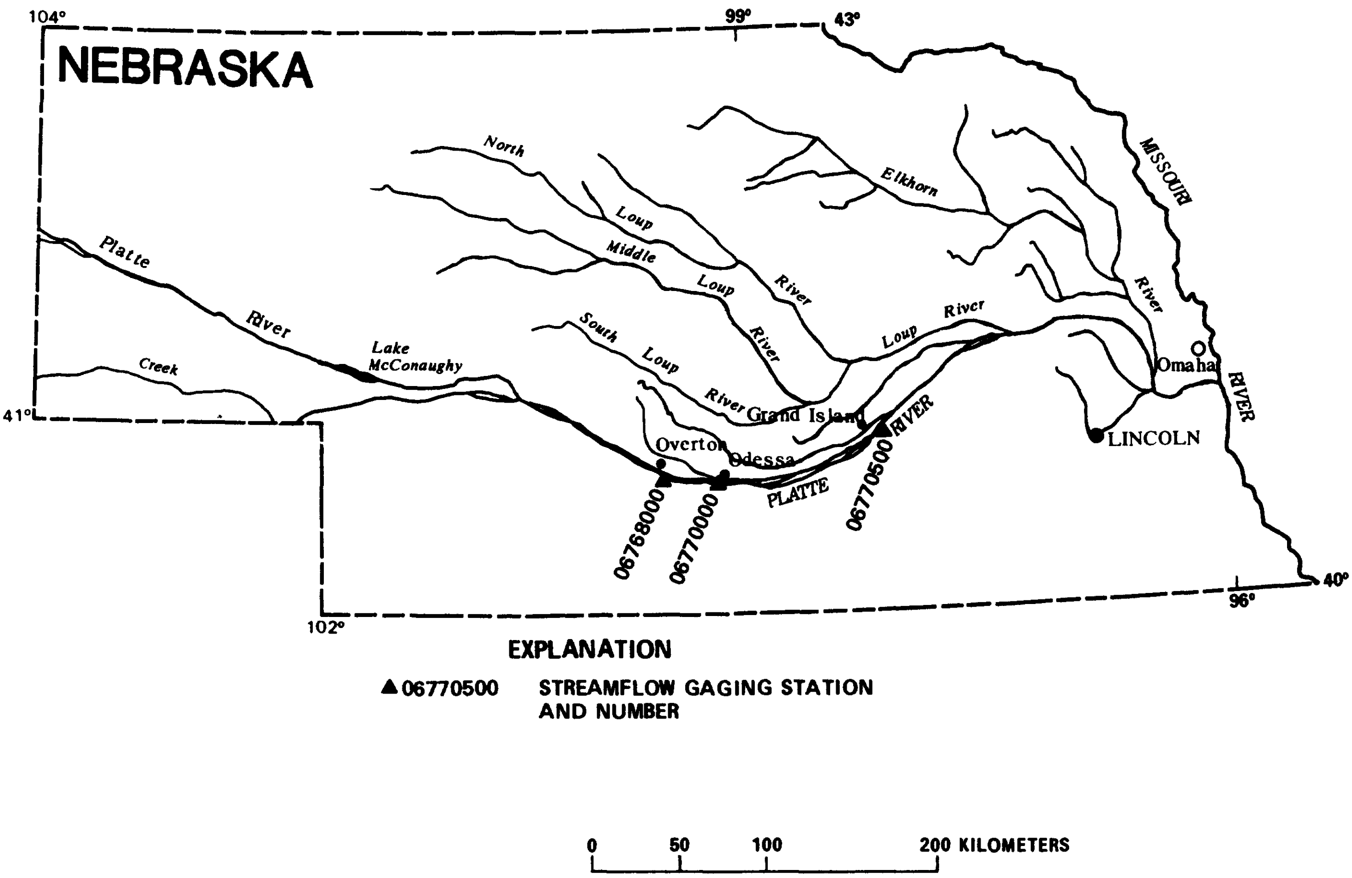

Figure 1.-- Map of Nebraska showing location of streamflow gaging stations.

allow single-station simulations, individual models for each station remain autonomous. For example, the individual flow series for Odessa can be simulated without the Grand Island or Overton series. The correlation structure between stations is maintained by allowing residuals from single-station models to be correlated.

The individual models in this report are periodic autoregression models (Pagano, 1978). A modified Box-Cox transformation technique (Box and Cox, 1964) is used to determine a transformation that results in residuals from the fitted models approximately following a normal distribution. The primary means of checking model adequacy will be to compare the statistical properties of the historical series to the properties of model simulations, as well as to check that the model assumptions are satisfied within a reasonable degree of accuracy.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to the following for their valuable assistance: Duane Boes, M. R. Karlinger, R. F. Hadley, and C. F. Nordin, Jr.

STATISTICAL PROPERTIES OF HISTORICAL-FLOW SERIES

The 3-season flow series for each station are determined by aggregating the monthly flow series. This monthly series consists of average ft^3/s (cubic feet per second) from October, 1942 to August, 1979 (water years 1943-1979) plus September, 1942, and can be obtained from Petsch and others, (1980). Season-one flows consist of the sum of the monthly series from September to February; season-two, the sum of March and April; and season-three the sum of May through August. Sums are used instead of means because the sums are more convenient. The year of season one will be designated by the year in which it ends (that is, season one of 1943 consists of September, 1942 to February, 1943).

The historical record in this report will include 111 seasonal flow totals in the 37-year period from water year 1943 to 1979 for each station. Graphs of these series are presented in figures 2 through 4 for the three stations. Seasonal statistics for Overton, Odessa, and Grand Island are summarized in table 1. The means and standard deviations appear to be significantly different across seasons; (parameters will be included in the models which can account for these differences). Secondly, the positive skewness and kurtosis coefficients indicate that the flows for individual seasons are not normally distributed. One way to solve this distribution problem is to transform the original flows to achieve a model in which residuals are nearly normal; this transformation will be discussed in a later section of this report.

Autocorrelations of the flow series will be important in determining the appropriate model to fit to the data. In ordinary autoregressive-moving average (ARMA) time-series modeling, the autocorrelation structure of the series is assumed to be the same for each season, an assumption which does not appear to be true in this case (fig. 5). An explanation of the term seasonal autocorrelations follows. Let $X_{t(n,k)}$, n = 0,1,2,...N-1, k = 1,2..., s, be a seasonal time series, where t(n,k) = ns+k, n is the year index (these are s seasons per year), and k is the season index. The lag defined for members of the series $X_{t(n,k)}$ is a seasonal lag and not a yearly lag. For example, $X_{t(n,k)-2}$ where k=3 is season number one of year n, and $X_{t(n,k)-4}$ where k=3 is

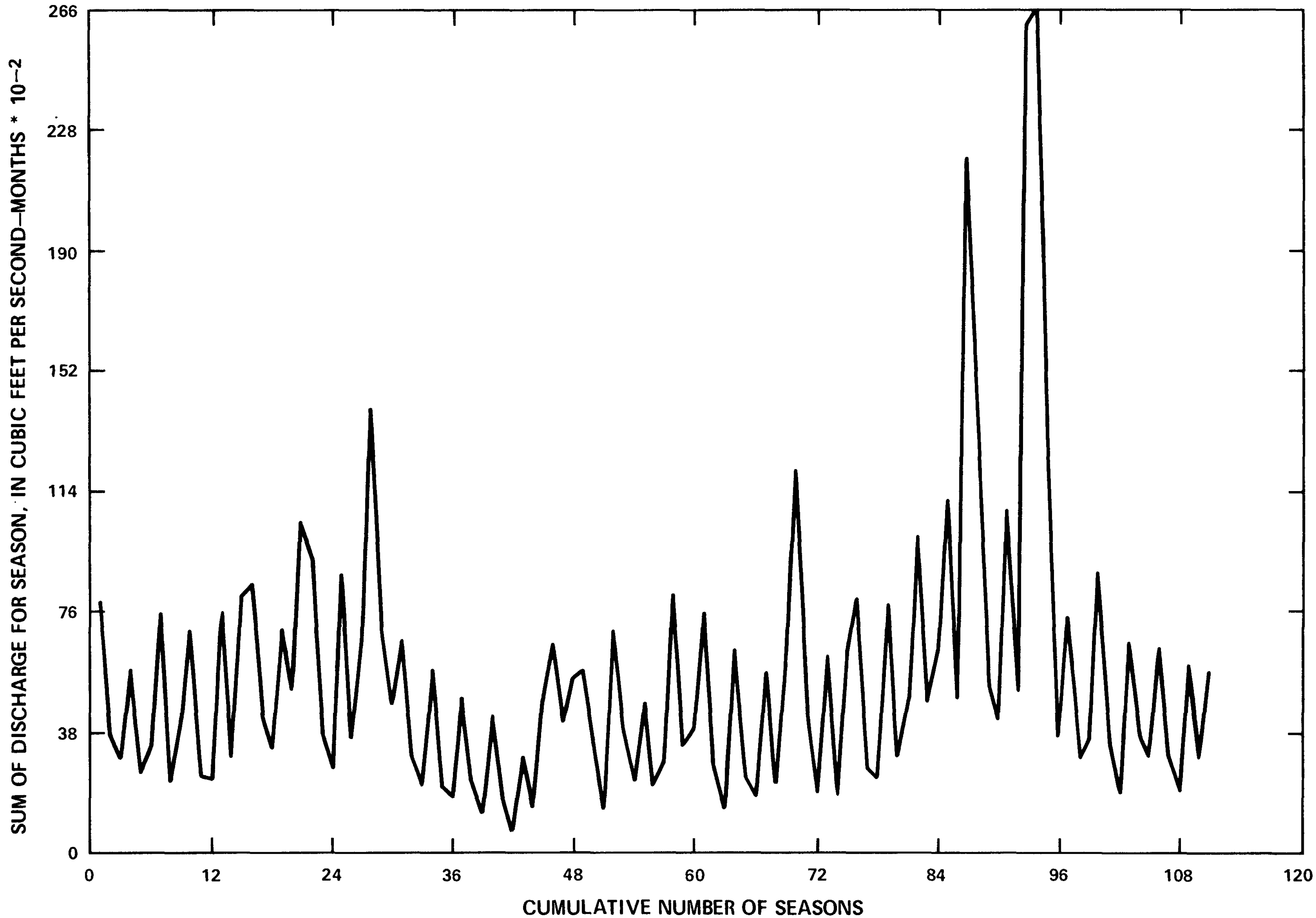

Figure 2.-- Historical sequence of seasonal discharges at Overton, Nebraska gaging station, water years 1943-1979.

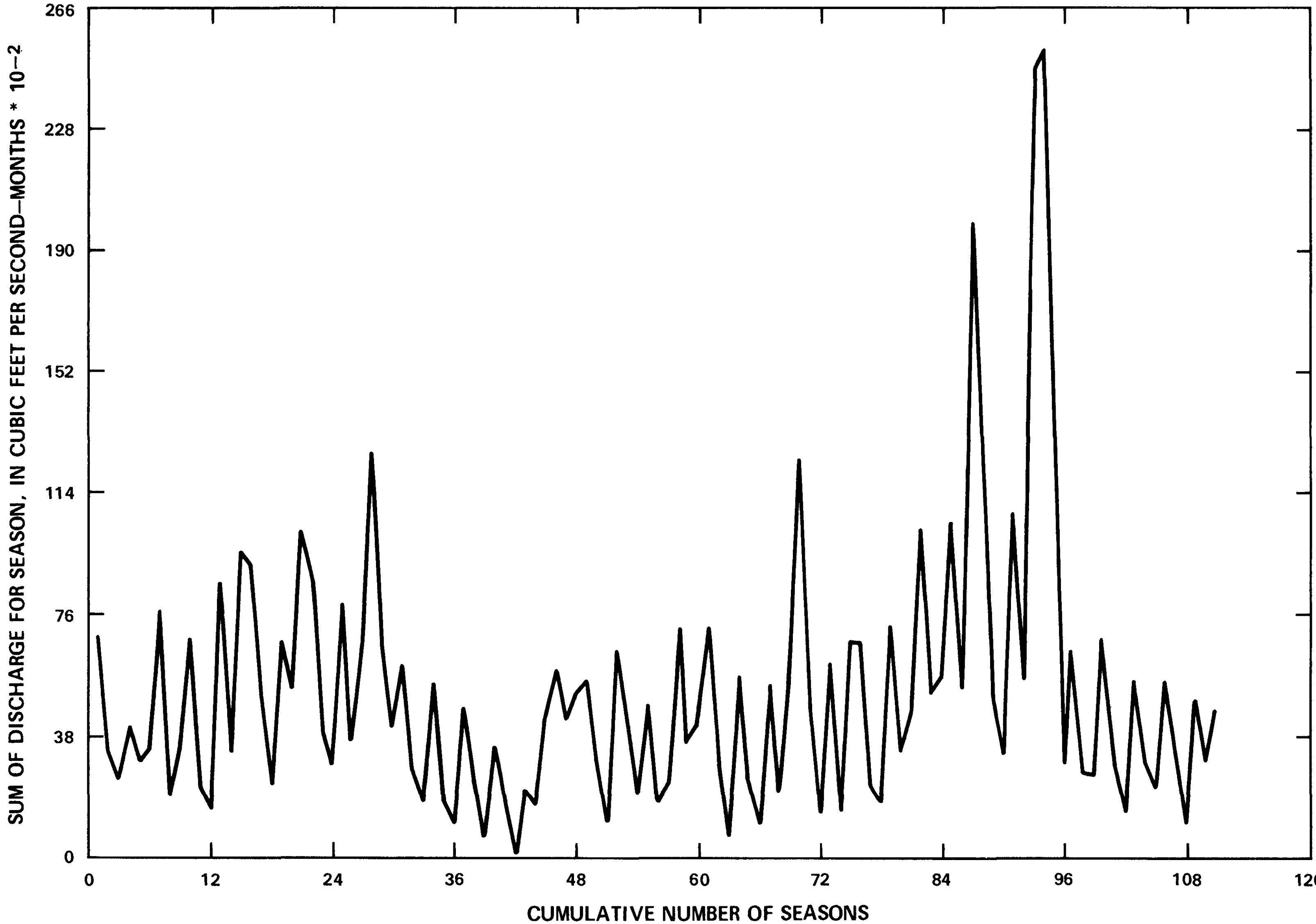

Figure 3.-- Historical sequence of seasonal discharges at Odessa, Nebraska gaging station, water years 1943-1979.

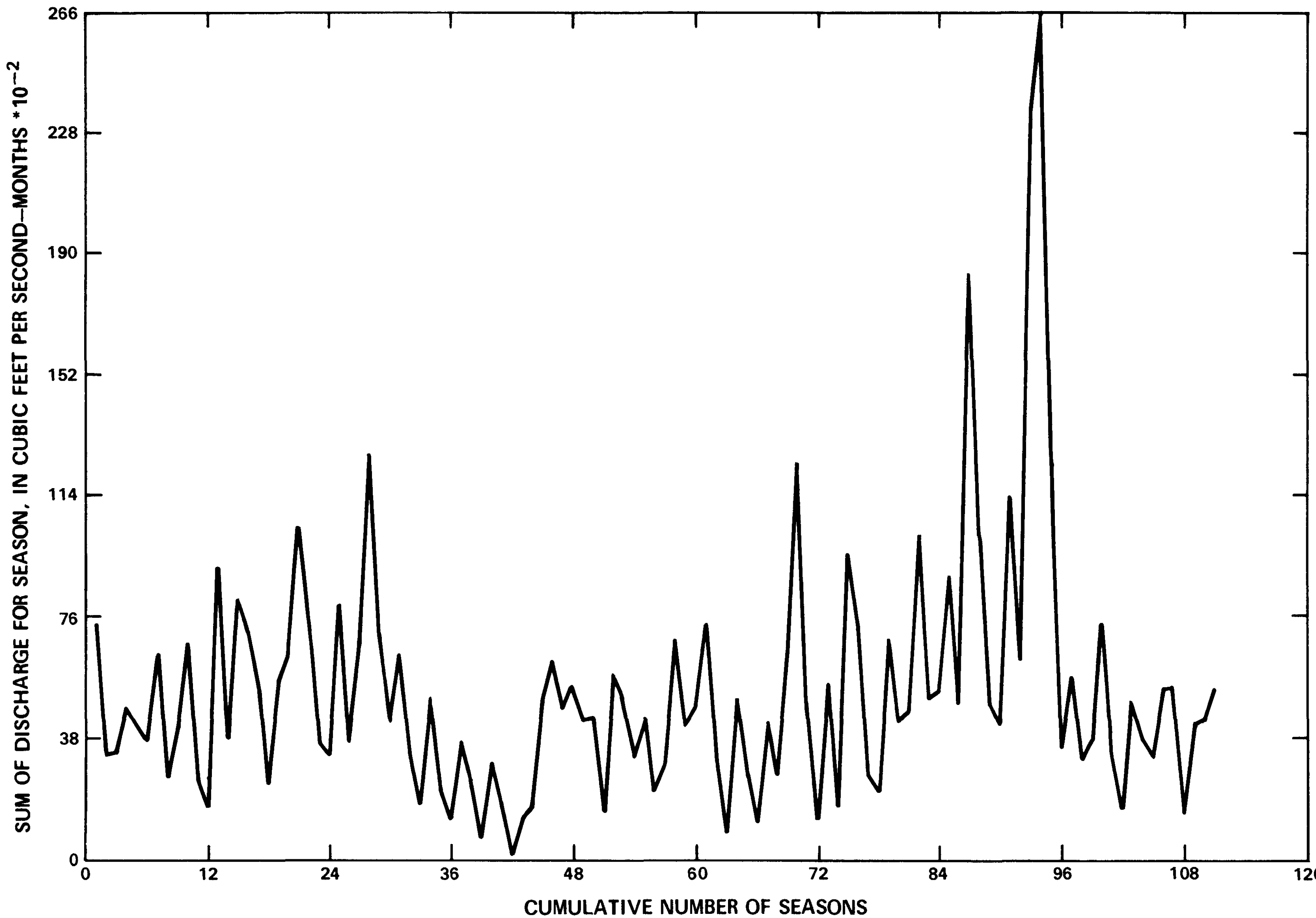

Figure 4.-- Historical sequence of seasonal discharges at Grand Island, Nebraska gaging station, water years 1943-1979.

Table 1.--*Flow statistics for Overton, Odessa, and Grand Island, Nebraska, water years 1943 to 1979*

	Season	Average season total (ft^3/s-months)	Standard deviation (ft^3/s-months)	Skewness	Kurtosis
Overton	1	8093.2	3922.6	2.918	11.068
	2	3641.9	2044.5	3.043	11.927
	3	4809.7	5108.1	2.904	8.554
Odessa	1	7585.5	3864.5	2.676	9.757
	2	3662.9	2062.8	2.618	9.438
	3	4363.8	4960.7	2.669	7.368
Grand Island	1	7219.4	4184.1	2.860	10.962
	2	4068.6	1973.4	2.065	6.564
	3	4653.6	4643.8	2.448	6.456

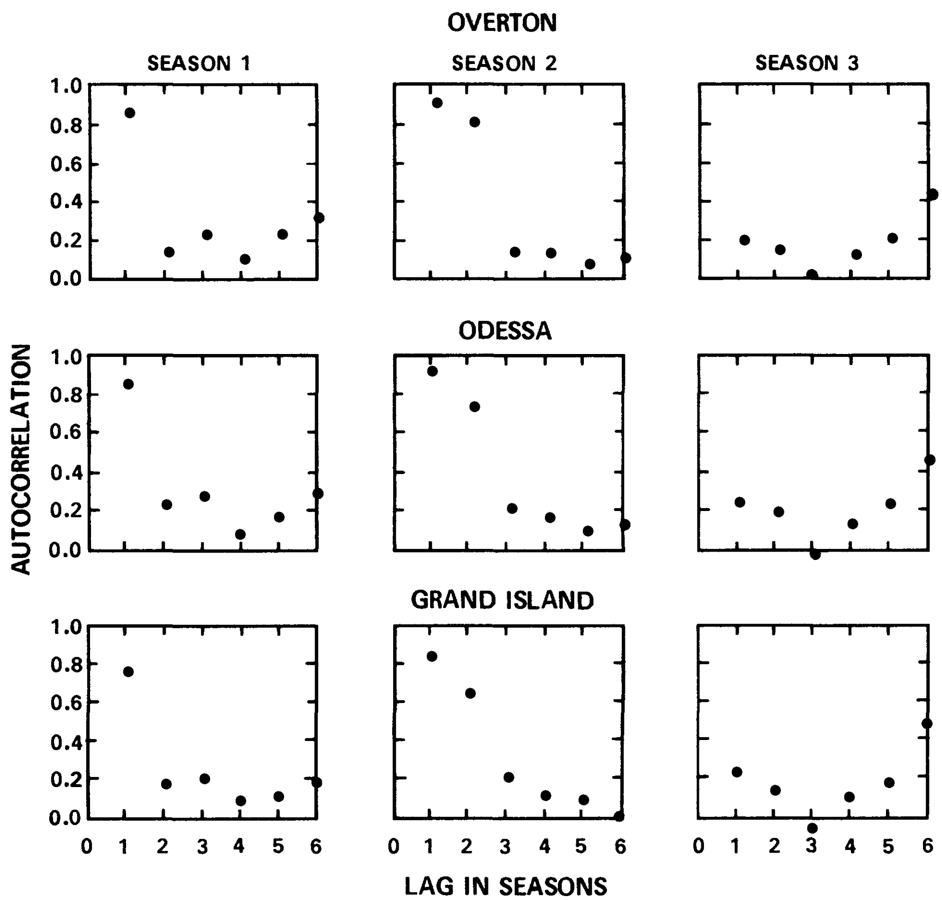

Figure 5.-- Seasonal autocorrelations.

season number two of year (n-1). The autocorrelation for season k at lag j is defined to be $\rho_{k,j} = \mathrm{Corr}(X_{t(n,k)}, X_{t(n,k)-j})$ and is assumed throughout to be independent of n. The values graphed in figure 5 are estimates $\hat{\rho}_{k,j}$ of $\rho_{k,j}$, k = 1, 2, 3, j = 1, 2, ... 6 obtained by using the formula

$$\hat{\rho}_{k,j} = \frac{1}{N}\left[\sum_{n=0}^{N-1} (Y_{t(n,k)} \cdot Y_{t(n,k)-j})\right] \quad \text{where } Y_{t(n,k)} = X_{t(n,k)} - \frac{\sum_{n=0}^{N-1} X_{t(n,k)}}{N}$$

and $Y_{t(n,k)-j} = 0$ for $t(n,k)-j \leq 0$. The graph seems to indicate the three stations are so similar that the same model could be used for all three. However, the station models differ somewhat, as will become evident later in this report. Cross-correlation between stations is an important statistical aspect of the historical series; this will be examined in a later section in which individual station models are combined.

PERIODIC AUTOREGRESSIONS

The model used to describe the single-station series falls under the class of periodic autoregressions, for which a substantial base of theory has been developed (Pagano, 1978; Parzen and Pagano, 1979). The relatively small number of years of data available and the irregular nature of the flow series caused by man-induced interventions preclude the use of many asymptotic results. The objective is to adequately describe the 1st and 2nd order moment properties (means, variances, and autocorrelations) of the series with the simplest model, and to use this model for simulation.

Let $\{X_{t(n,k)}, k = 1,2,..,s, n = 0,1,...\}$ be a seasonal time series, where $t(n,k) = ns + k$. Assume that $E(X_{t(n,k)}) = 0$, which can be achieved in practice by standardizing via subtraction of the seasonal means. To simulate from the model, it is necessary to assume that the information available to predict $X_{t(n,k)}$ consists of the past, $X_o, X_1, \ldots, X_{t(n,k)-1}$. A reasonable model to postulate for the series would be $X_{t(n,k)} = \underline{X}'_{n,k}\, \underline{\alpha}_k + \varepsilon_{t(n,k)}$ (' denotes transpose) where $\underline{X}'_{n,k}$ is a 1 x p_k vector of values of the series prior to $X_{t(n,k)}$ useful in predicting $X_{t(n,k)}$, $\underline{\alpha}'_k = \left[\alpha_{k1}, \alpha_{k2}, \ldots, \alpha_{kp_k}\right]$ is a 1 x p_k vector of parameters, and $\varepsilon_{t(n,k)}$ is a random variable with $\left\{\varepsilon_{t(n,k)}\right\}_{n=0}^{\infty}$, an iid (independent identically distributed) sequence, $E(\varepsilon_{t(n,k)}) = 0$, Var $(\varepsilon_{t(n,k)}) = \sigma^2{}_k$. If $\underline{X}'_{n,k} = \left[X_{t(n,k)-\ell_{k1}}, \ldots X_{t(n,k)-\ell_{kp_k}}\right]$, the numbers $\ell_{k1}, \ldots, \ell_{kp_k}$ will be called the lag values for season k.

Since the random errors, $\left\{\varepsilon_{t(n,k)}\right\}$, are assumed to be independent from year to year but may be correlated among seasons within years, it is convenient to write the model in matrix form. Let $\underline{X}'_n = \left[X_{t(n,1)}, \ldots, X_{t(n,s)}\right]$, $\underline{\varepsilon}'_n = \left[\varepsilon_{t(n,1)}, \ldots, \varepsilon_{t(n,s)}\right]$, $\underline{\alpha}' = \left[\underline{\alpha}'_1, \ldots, \underline{\alpha}'_s\right]$, and let

$$\underline{X}_n = \begin{bmatrix} \underline{X}'_{n,1}, & \underline{0}, \ldots, & \underline{0} \\ \underline{0}, & \underline{X}'_{n,2}, \ldots, & \underline{0} \\ \underline{0}, & \underline{0} \ldots, & \underline{X}'_{n,s} \end{bmatrix}$$

be an s x $\sum_{k=1}^{s} p_k$ matrix of predictors. Write the model as $\underline{X}_n = \underline{X}_n\, \underline{\alpha} + \underline{\varepsilon}_n$, n = 0, 1, ..., where $\underline{\varepsilon}_n$ is an iid sequence with $E(\underline{\varepsilon}_n) = \underline{0}$, $\mathrm{Var}(\underline{\varepsilon}_n) = \underline{\Sigma}$.

The parameter vector, $\underline{\alpha}$, is unknown and must be estimated from the observed data (say there are N years observed). If $\underline{\Sigma}$ were known in advance, the weighted least squares (WLS) estimate of $\underline{\alpha}$ would be

$$\hat{\underline{\alpha}} = \left[\sum_{n=0}^{N-1} \underline{X}'_n\, \underline{\Sigma}^{-1}\, \underline{X}_n\right]^{-1} \sum_{n=0}^{N-1} \underline{X}'_n\, \underline{\Sigma}^{-1}\, \underline{X}_n$$ (see Graybill, p. 207).

Compare this to the ordinary least squares (OLS) estimate obtained from

$$\underset{\sim}{\tilde{\alpha}} = \left[\sum_{n=0}^{N-1} \underline{\chi}_n' \underline{\chi}_n\right]^{-1} \left[\sum_{n=0}^{N-1} \underline{\chi}_n' \underline{X}_n\right].$$

$\tilde{\underline{\alpha}}$ minimizes $\sum_{n=0}^{N-1} (\underline{X}_n - \underline{\chi}_n \underline{\alpha})' (\underline{X}_n - \underline{\chi}_n \underline{\alpha})$

while $\hat{\underline{\alpha}}$ minimizes $\sum_{n=0}^{N-1} (\underline{X}_n - \underline{\chi}_n \underline{\alpha})' \underline{\Sigma}^{-1} (\underline{X}_n - \underline{\chi}_n \underline{\alpha})$.

The weighted least squares estimated should be used if possible when $\underline{\Sigma} \neq \sigma^2 \underline{I}$. Since $\underline{\Sigma}$ is generally not known in advance, an iterative technique should be used to get $\hat{\underline{\alpha}}$, as follows:

1. Obtain $\tilde{\underline{\alpha}}_o$, the ordinary least squares estimate of $\underline{\alpha}$.
2. Obtain an estimate of $\underline{\Sigma}$, say $\hat{\underline{\Sigma}}_o$, from (1) by using
   $$\hat{\underline{\Sigma}}_o = \frac{1}{N} \sum_{n=0}^{N-1} (\underline{X}_n - \underline{\chi}_n \tilde{\underline{\alpha}}_o) (\underline{X}_n - \underline{\chi}_n \tilde{\underline{\alpha}}_o)'.$$
3. Plug $\hat{\underline{\Sigma}}_o$ into the weighted least squares equation to get a new estimate, $\hat{\underline{\alpha}}_1$, of $\underline{\alpha}$ from which a new estimate, $\hat{\underline{\Sigma}}_1$, of $\underline{\Sigma}$ is obtained.
4. Repeat until convergence takes place in the sense that $\hat{\underline{\alpha}}_k - \hat{\underline{\alpha}}_{k-1}$ is small. Denote the obtained estimates as $\hat{\underline{\Sigma}}$, $\hat{\underline{\alpha}}$.

Sampling properties of the ordinary least squares and weighted least squares estimates will not be discussed in the report.

The estimation procedure given above presupposes that p_k and $(\ell_{k1}, \ldots, \ell_{kp_k})$ are already known for each season; in fact, these must also be determined from available data. Their determination is called model identification, and techniques of identification have been set forth. In keeping with the general linear model approach in this report, stepwise regression is used (for each season separately) to choose the values that significantly contribute to the prediction of $X_{t(n,k)}$.

TRANSFORMATION TO NORMALITY

In the model developed in the previous section, no mention was made of the distribution of the random component $\underline{\varepsilon}_n$ other than E $(\underline{\varepsilon}_n) = \underline{0}$. However, if the model is to be used for simulation, the distribution of $\underline{\varepsilon}_n$ must be specified. A technique for determining a transformation of the original time series that causes residuals to be approximately normally distributed is described in this section; this technique is applied to the flow series in the next section.

Box and Cox (1964) consider power transformations of the form $Z_t^{(\lambda)} = \{ (X_t^\lambda - 1)/\lambda,\ \lambda \neq 0, \text{ and } \log_e (X_t),\ \lambda = 0 \}$ where X_t is a strictly positive random variable for which a transformation to near normality is desired. These authors give a maximum likelihood technique for estimating λ based on a random sample $X_1, \ldots, X_N$, assuming that $Z_t^{(\lambda)}$ is normally distributed for some λ. Their technique can be generalized to include random vectors. Suppose one has a random sample of strictly positive random vectors, $\underline{X}_1, \ldots, \underline{X}_N$, where $\underline{X}_n' = \left[X_{n1}, X_{n2}, \ldots, X_{ns} \right]$, and let $(\underline{Z}_n^{(\lambda)})' = \left[Z_{n1}^{(\lambda_1)}, Z_{n2}^{(\lambda_2)}, \ldots, Z_{ns}^{(\lambda_s)} \right]$ be normally distributed for some $\underline{\lambda}' = \left[\lambda_1, \ldots, \lambda_s \right]$. This assumption will not be exact in practice unless X_{nk} is log normally distributed for each k (that is, unless $\underline{\lambda} = \underline{0}$), but $\underline{Z}_n^{(\underline{\lambda})}$ can be approximately normal for other values of $\underline{\lambda}$.

Under this assumption and using transformation techniques, it can be shown that the maximized log likelihood in relation to the original observations $\underline{x}_1, \ldots, \underline{x}_N$, is proportional to

$$-\frac{N}{2} \log \left| \hat{\underline{\Sigma}} (\underline{\lambda}) \right| + \sum_{i=1}^{s} (\lambda_i - 1) \sum_{n=1}^{N} \log x_{ni} = M(\underline{\lambda})$$

where $\hat{\underline{\Sigma}}(\underline{\lambda}) = \frac{1}{N} \sum_{n=1}^{N} (\underline{z}_n^{(\underline{\lambda})} - \hat{\underline{\mu}}(\underline{\lambda}))(\underline{z}_n^{(\underline{\lambda})} - \hat{\underline{\mu}}(\underline{\lambda}))'$ and $\hat{\underline{\mu}}(\underline{\lambda}) = \sum_{n=1}^{N} \underline{z}_n^{(\underline{\lambda})}/N$. The value of $\underline{\lambda}$ for which $\underline{z}_n^{(\underline{\lambda})}$ is normally distributed is not known in advance, so use $\hat{\underline{\lambda}}$ as an estimate of $\underline{\lambda}$ where max $M(\underline{\lambda}) = M(\hat{\underline{\lambda}})$. The reader should observe that if $\underline{Z}_n^{(\underline{\lambda})}$ is normally distributed and $\underline{\lambda}^* \neq \underline{\lambda}$, then $M(\lambda^*)$ is not the true value of the maximized log likelihood, a fact that casts some doubt upon the validity of using $\hat{\underline{\lambda}}$ as the maximum likelihood estimate of $\underline{\lambda}$. Hernandez and Johnson (1980) explore transformations of the same form as above and give a method for determining the value $\tilde{\underline{\lambda}}$ that minimizes the Kullback-Liebler information between the density of $\underline{Z}_n^{(\underline{\lambda})}$ and a normal density function, assuming that the density function of $\underline{X}_n$ is known. In most practical problems, one can consider a finite set of values, $\underline{\lambda}_1, \ldots, \underline{\lambda}_\ell$. In this case, $\hat{\underline{\lambda}}$ converges to $\tilde{\underline{\lambda}}$ with probability one, so for large N, $\hat{\underline{\lambda}}$ is seen to be approximately that value of $\underline{\lambda}$ that minimizes the Kullback-Liebler distance between the density of $\underline{Z}_n^{(\underline{\lambda})}$ and a normal density.

INDIVIDUAL-STATION-STREAMFLOW MODELS

A periodic autoregression model was fit to each station by the procedures outlined in the previous sections. An examination of table 1 indicates that the three seasons appear to be similar enough to use the same transformation for each, so $\underline{\lambda}'$ was set equal to $(\lambda, \lambda, \lambda)$ to reduce the number of transformations to be considered. The raw series was transformed with values of λ equal to -1.0, -0.9, ... , 0, 0.1, ... , 1.0, the transformed series mean standardized, and the model parameters estimated by weighted least squares. The identification of p_k and $\ell_k = (\ell_{k1}, \ldots, \ell_{kp_k})$; k = 1,2,3, was carried out for $\lambda = 0$ at each of the stations; this is at, or near, the final values chosen. It became evident that the residuals of the fitted models were essentially uncorrelated from season to season, so $\underline{\Sigma} = \text{cov}\ (\underline{\varepsilon}_n)$ was restricted to be a diagonal matrix: $\underline{\Sigma} = \text{diag}\ (\sigma_i^2)$, where $\sigma_i^2 = \text{Var}\ (\varepsilon_{n,i})$. In this case, the weighted least squares estimates are the same as the ordinary least squares estimates. Results for the three stations are summarized in table 2. The R^2 value for season i is interpreted as the percentage of the variation of the observed data for season i from 1945 to 1979 (allowing for lag considerations) which is accounted for by the nonrandom component of the model. Note that the last season has a low R^2 value. This does not indicate model inadequacy but rather shows that season three variation is nearly random with respect to previous values in the series. However, season 3 is important in predicting season 1 of the following year and hence should be included in the model.

It was assumed that $\left[\underline{\varepsilon}_n,\ n = 0,1...\right]$ are independent, identically distributed normal random variables, with $E\ (\underline{\varepsilon}_n) = \underline{0}$ and $\text{Cov}\ (\underline{\varepsilon}_n) = \underline{D}$ (a diagonal matrix), or that the errors are independent from season to season. To check these assumptions, residuals from the models for 1945 to 1979 were examined. Some residual statistics are in table 3, which reveals no highly significant deviations from normality or from the assumption that $E(\underline{\varepsilon}_n) = \underline{0}$. A graph of the residual series for Odessa from 1945 to 1979 is shown in figure 6; it is observed that no obvious trends are taking place. To check the assumption of no correlation between seasons, lag autocorrelation matrices for the residuals were examined. The lag k autocorrelation matrix of the residual vector $\underline{\varepsilon}_n$ is defined as $\text{Cov}\ (\underline{\varepsilon}_n, \underline{\varepsilon}_{n-k})$ (note that lags are now in years). Estimates of these matrices for k = 0,1,2,3 appear in table 4. Each estimate is based on about 35 observations, and a rough standard error of the estimate

Table 2.--*Individual-station streamflow models*

Overton

$Z_{t,i} = \left[(X_{t,i} + 0.01)^{-.1} - 1\right] / (-0.1) \; - \mu_i$, i = 1,2,3

Seasonal mean vector $\underline{\mu}' = (\mu_1, \mu_2, \mu_3)$ = (5.9000, 5.548, 5.569)

$Z_{t,1} = 0.3632 * Z_{t-1,3} + 0.1449 * Z_{t-1,1} + \varepsilon_{t,1}$

$Z_{t,2} = 1.0756 * Z_{t,1} + \varepsilon_{t,2}$

$Z_{t,3} = 0.4646 * Z_{t,2} + 0.2631 * Z_{t-2,3} + \varepsilon_{t,3}$

Season:	1	2	3
R^2:	67 percent	76 percent	12.4 percent

Overall R^2 = 33 percent

$Cov(\underline{\varepsilon}_t)$ = diag (0.00706, 0.00825, 0.08944)

Odessa

$Z_{t,i} = \log_e (X_{t,i} + 0.01) - \mu_i$, i = 1, 2, 3

$\underline{\mu}'$ = (8.8427, 8.0965, 7.9568)

$Z_{t,1} = 0.3423 * Z_{t-1,3} + 0.1461 * Z_{t-1,1} + \varepsilon_{t,1}$

$Z_{t,2} = 0.9168 * Z_{t,1} - 0.0982 * Z_{t-1,1} + \varepsilon_{t,2}$

$Z_{t,3} = 0.7850 * Z_{t,2} + \varepsilon_{t,3}$

Season:	1	2	3
R^2:	67 percent	66 percent	12 percent

Overall R^2 = 28 percent

$Cov(\underline{\varepsilon}_t)$ = diag (0.05082, 0.06616, 0.77623)

Grand Island

$Z_{t,i} = \log_e (X_{t,i} + 0.01) - \mu_i$, i = 1,2,3

$\underline{\mu}'$ = (8.7668, 8.2185, 8.0729)

$Z_{t,1} = 0.4198 * Z_{t-1,3} - 0.0469 * Z_{t-1,1} + 0.1284 * Z_{t-2,3} + \varepsilon_{t,1}$

$Z_{t,2} = 0.6715 * Z_{t,1} - 0.0949 * Z_{t-1,1} + \varepsilon_{t,2}$

$Z_{t,3} = 0.7757 * Z_{t,2} + \varepsilon_{t,3}$

Season:	1	2	3
R^2:	68 percent	50 percent	8.2 percent

Overall R^2 = 26 percent

$Cov(\varepsilon_t)$ = diag (0.07431, 0.08806, 0.74906)

Table 3.--*Residual statistics from individual-station models, 1945 to 1979 (water years)*

Overton	Sept. to Feb.	March to April	May to August
Mean	0.004	-0.002	0.000
Variance	.007	.008	.089
Skewness	.053	-.055	.253
Kurtosis	-.761	1.216	-.872
Minimum values	-.175	-.261	-.569
Maximum values	.174	.222	.625
Odessa	**Sept. to Feb.**	**March to April**	**May to August**
Mean	0.003	0.000	0.004
Variance	.051	.067	.776
Skewness	.098	-.563	-.045
Kurtosis	-.499	.287	-.107
Minimum values	-.493	-.675	-2.206
Maximum values	.500	.528	1.732
Grand Island	**Sept. to Feb.**	**March to April**	**May to August**
Mean	0.011	-0.002	-0.006
Variance	.074	.088	.749
Skewness	.218	-.110	-.110
Kurtosis	.362	.124	.124
Minimum values	-.658	-.632	-2.259
Maximum values	.610	.654	1.713

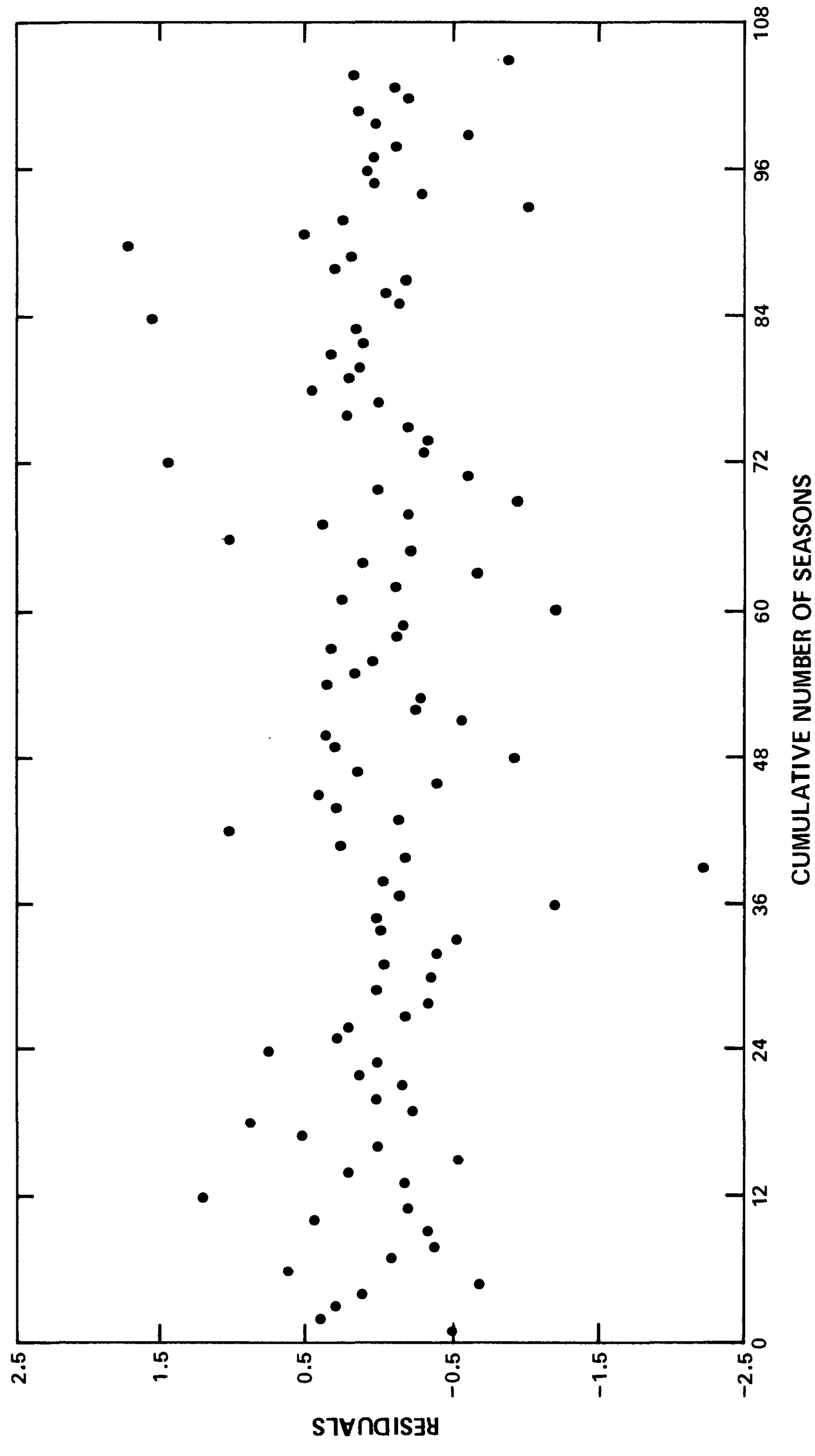

Figure 6.-- Residuals (synthesized minus observed) for Odessa, Nebraska.

Table 4.--*Residual autocorrelation matrices*

Lag (years)		Station								
		Overton			Odessa			Grand Island		
		Season			Season			Season		
	Season	1	2	3	1	2	3	1	2	3
0	1	1.0	-.053	.178	1.0	.064	.207	1.0	-.105	.113
	2	-.053	1.0	-.011	.064	1.0	-.018	-.105	1.0	.016
	3	.178	-.011	1.0	.207	-.018	1.0	.113	.016	1.0
1	1	-.266	-.194	.020	-.214	-.188	-.003	-.117	-.124	.029
	2	-.234	.446	-.150	-.246	.403	-.024	-.243	.396	-.048
	3	-.049	.083	-.070	.047	-.015	-.200	.077	-.046	-.182
2	1	.125	-.247	.032	.024	-.131	.191	-.057	-.074	.099
	2	-.088	.107	-.127	-.178	.110	.004	-.261	-.039	.152
	3	.067	-.058	.045	.119	.059	.231	.012	.160	.214
3	1	-.014	-.197	.006	.024	-.163	.056	.013	-.163	.115
	2	.178	.006	-.204	.120	-.108	-.215	.116	-.097	-.169
	3	.246	-.047	-.042	.200	-.051	-.086	.245	-.104	-.066

is $1/\sqrt{35} = 0.17$. Therefore, any value below about 0.3 in absolute value can be considered insignificant. The only problem is the lag 1 correlation for season 2 ($Corr(\varepsilon_{n,2}\ \varepsilon_{n-1,2})$), which is around 0.4 for each station but drops off to near zero for the remaining lags. This correlation can be preserved in the model by allowing $\varepsilon_{n,2}$ to follow a moving average process of order 1 (see Box and Jenkins, 1975). In other words, $\varepsilon_{n,2} = \Psi_{n,2} + \Theta\ \Psi_{n-1,2}$ with $\{\Psi_{n,2},\ n=0,1...\}$ a white-noise process. Moment estimates of θ for each station were determined to be $\hat{\theta} = 0.63$ for Overton and $\hat{\theta} = 0.5$ for Odessa and Grand Island. A precise statement of the models with moving-average components included will be given in the next section.

As a final check, the models were used to simulate 10 realizations of 37 years each, the same length as the historical series, and the properties of the simulations were compared to the historical series. Some statistics of the simulations are in tables 5, 6, and 7. Care must be taken in interpreting the tables due to high variability of the skewness and kurtosis coefficients for such short data sets. The standard deviations for most of the simulations are lower than the respective observed standard deviations. This could indicate two things: (1) The observed standard deviation is higher than the long term standard deviation; or (2) the standard deviations of model simulations are biased, possibly due to the inverse transformation of model output. However, for purposes of this report, these differences did not warrant further investigation. Seasons 2 and 3 are of primary interest; the only problem is that the second season skewness and kurtosis coefficients for Odessa and Grand Island are consistently low. A graph of a typical simulation for Overton is in figure 7; for Odessa, in figure 8; and for Grand Island, in figure 9. It should be emphasized that these figures are results

Table 5.--*Seasonal statistics for actual streamflow data and 10 model simulations of the same length for Overton, Nebraska*

[Means and standard deviations are in ft^3/s-months]

	September to February				March to April				May to August			
Actual data	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis
	8093	3923	2.918	11.068	3642	2044	3.043	11.927	4810	5108	2.904	8.554
Simulations												
1	8489	2822	.194	-.602	3750	1550	.990	1.411	4952	4399	3.029	11.913
2	6972	2993	1.146	.764	3399	2172	1.811	2.897	3592	2965	1.757	3.838
3	7796	2730	1.132	.558	3705	1676	1.372	1.591	4017	3456	2.489	6.001
4	7990	3471	.899	-.161	3709	1727	.853	.510	4173	4661	3.216	11.305
5	8972	4044	1.795	4.885	4674	2989	2.830	10.202	5367	3749	1.200	1.293
6	8101	3074	1.085	.826	4006	1936	1.789	4.161	5153	4867	3.274	13.135
7	6902	1825	.426	-.178	3145	1040	.629	-.056	3282	1927	.903	-.251
8	7040	2684	.725	.222	3032	1109	.455	-.104	3630	3752	2.838	9.369
9	8159	4082	.933	.222	3705	1809	1.010	.693	5286	5275	2.022	4.708
10	8398	3396	1.994	5.601	3724	1668	3.605	16.009	4218	3806	3.184	12.057

Table 6.--*Seasonal statistics for actual streamflow data and 10 model simulations of the same length for Odessa, Nebraska*

[Means and standard deviations are in ft^3/s-months]

	September to February				March to April				May to August			
Actual data	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis
	7585	3864	2.676	9.757	3662	2063	2.618	9.438	4363	4961	2.669	7.368
Simulations												
1	7325	2612	0.108	-0.785	3423	1469	1.168	2.012	3612	2679	0.984	0.197
2	7645	3642	1.325	1.685	3671	2208	1.438	1.806	4518	4921	2.181	5.366
3	6779	2505	1.189	1.156	3379	1518	.922	.227	3987	6098	4.452	21.502
4	7513	2839	1.638	3.144	3863	1549	1.219	1.180	4645	5465	2.307	4.597
5	7691	2939	1.036	1.075	3526	1705	2.283	7.442	4965	4829	2.079	4.661
6	6533	2666	1.125	.701	3707	1510	1.093	1.422	3349	3755	1.816	2.204
7	7219	3479	1.670	3.358	3487	1839	1.068	.115	3877	4559	3.184	11.803
8	7183	2282	.704	2.169	3311	1340	1.016	1.314	3219	2221	.939	-.153
9	6656	2281	.751	.946	3222	1285	.583	-.084	3483	2695	1.608	2.713
10	6734	1939	.635	-.061	3150	1158	.483	-.616	2833	2103	1.851	3.834

Table 7.--*Seasonal statistics for actual streamflow data and 10 model simulations of the same length for*

Grand Island, Nebraska

[Means and standard deviations are in ft^3/s-months]

	September to February				March to April				May to August			
Actual data	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis	Mean	Standard deviation	Skewness	Kurtosis
	7219	4184	2.860	10.962	4068	1973	2.065	6.564	4653	4644	2.448	6.465
Simulations												
1	7670	4679	2.179	5.272	4228	1874	2.005	6.137	4400	4816	2.877	9.581
2	8815	4068	1.153	1.661	4502	1920	.814	.212	6232	4701	1.233	.856
3	7634	3819	1.654	3.667	3440	1472	.704	-.178	8386	4929	1.785	3.398
4	7784	4386	2.818	10.631	4028	1812	1.002	1.162	5502	8219	4.739	23.758
5	6952	3715	1.536	2.913	3638	1851	1.564	3.146	5903	8254	3.608	15.341
6	6865	3160	1.229	1.382	4279	1671	.707	.186	4414	3996	1.582	1.794
7	6750	3355	1.619	2.105	4203	2028	1.164	.783	4896	6069	2.633	6.810
8	7843	4111	.947	-.210	4727	2956	1.770	4.042	6093	8237	3.968	18.084
9	9154	5016	1.391	1.998	4883	2291	1.331	1.867	7523	8903	2.613	7.681
10	7694	3351	.383	-.942	4173	1968	1.073	.758	4941	4364	.995	-.371

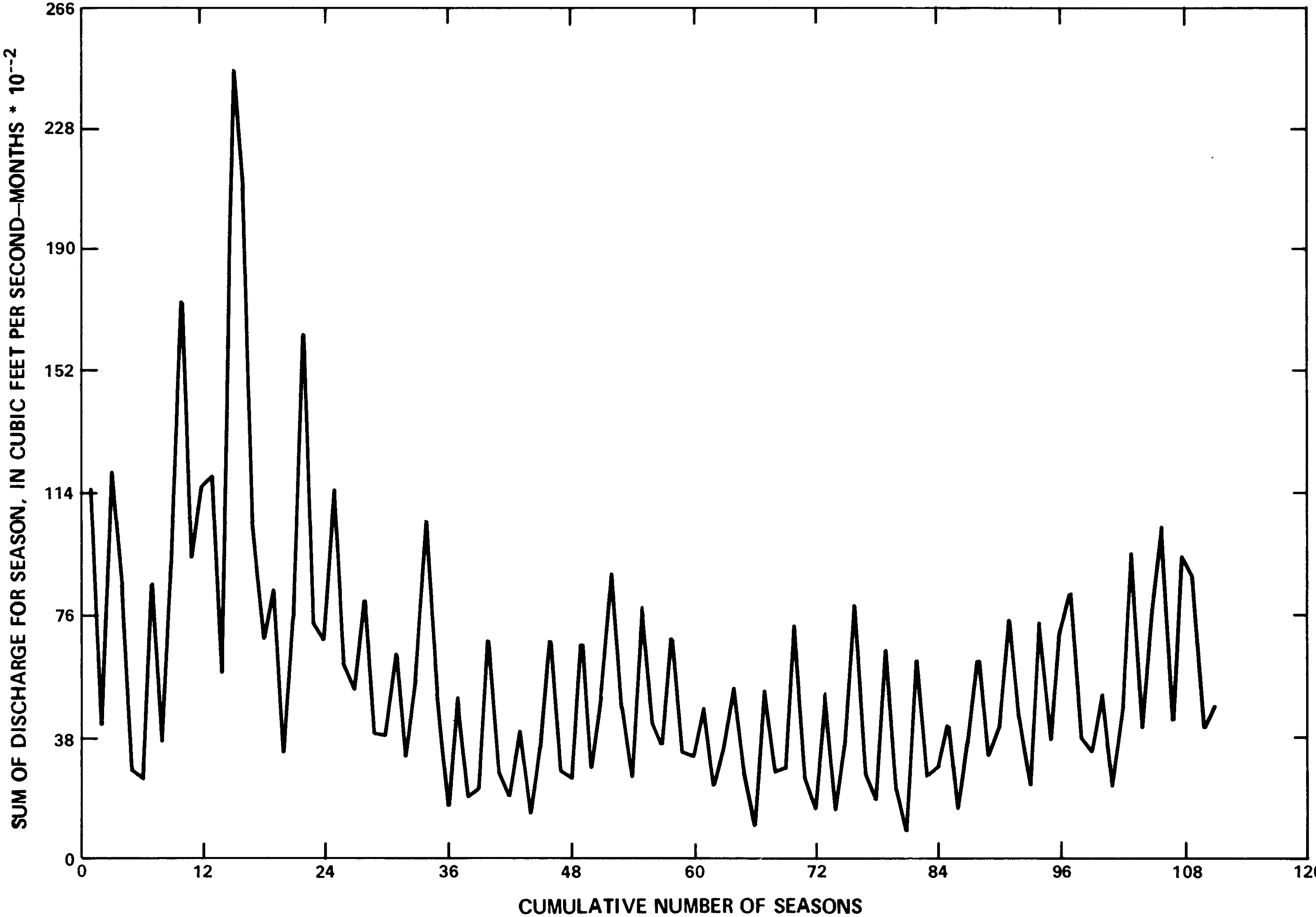

Figure 7.-- Simulated seasonal streamflow for a 37-year sequence at Overton, Nebraska from the individual station model.

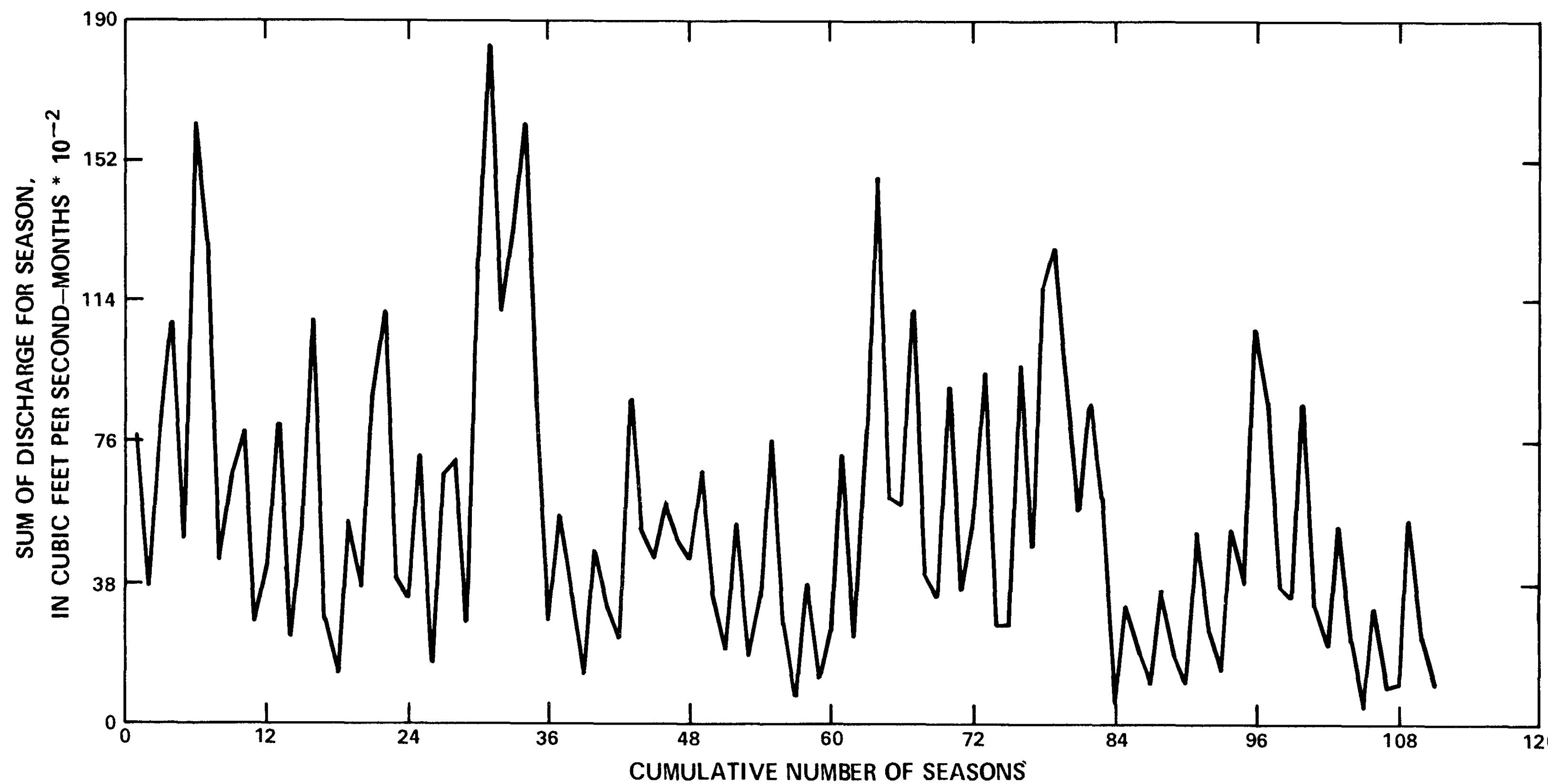

Figure 8.-- Simulated seåsonal streamflow for a 37-year sequence at Odessa, Nebraska from the individual station model.

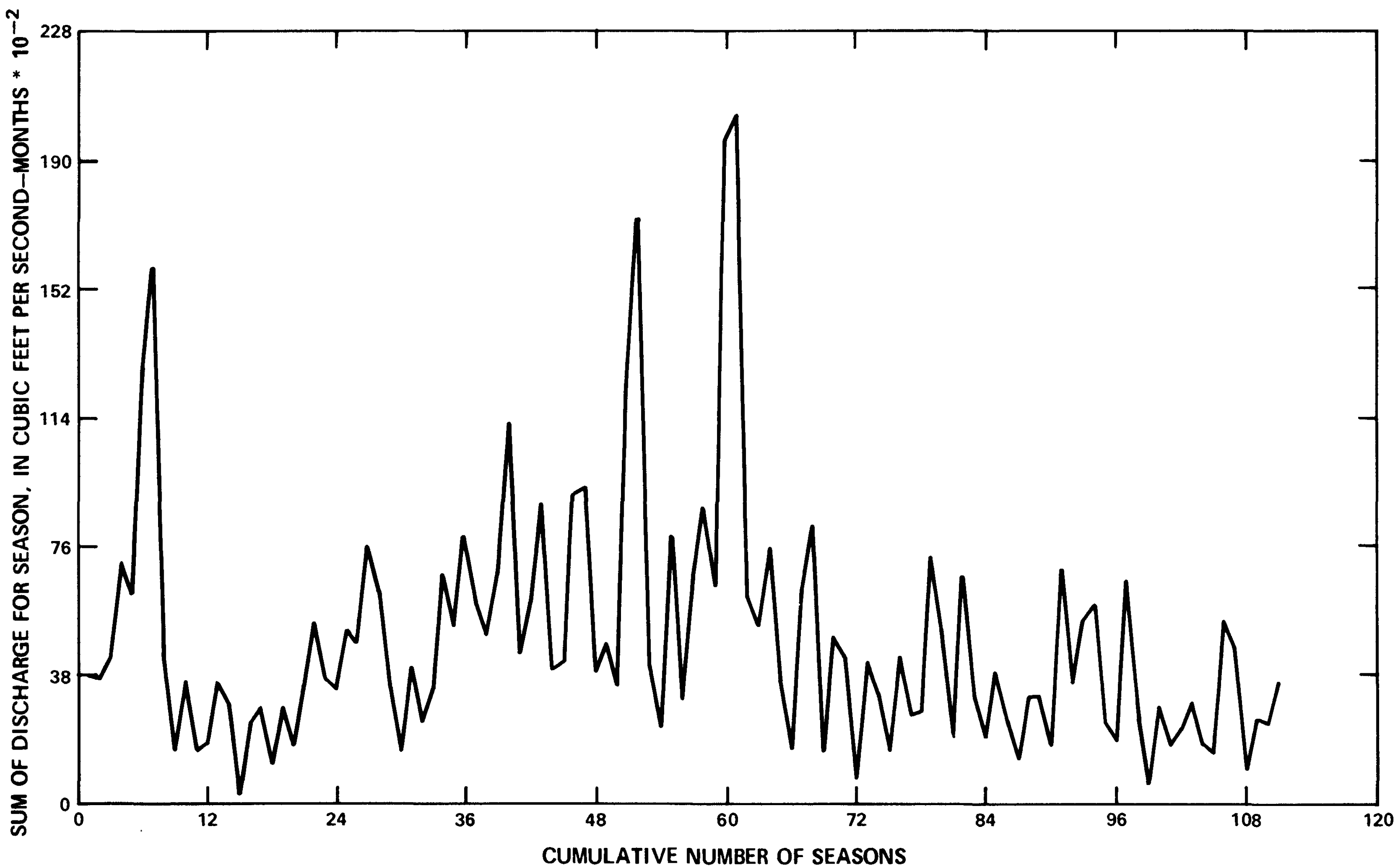

Figure 9.-- Simulated seasonal streamflow for a 37-year sequence at Grand Island, Nebraska from the individual-station model.

of the univariate models and hence generated values will not be correlated across stations. The next section will address the correlation between stations.

COMBINED-STATION MODEL

Thus far, cross correlations between the stations have not been considered. However, if simulations of more than one station are generated, these simulations should maintain continuity between the stations. This will allow simulation of average flow series for two or three of the stations by simulating from the combined-station model and averaging the appropriate values. One method of maintaining station continuity is to allow the residual vectors, $\underline{\varepsilon}_n$, to be correlated from station to station. Let $\underline{\xi}_n$ be the residuals for Overton; $\underline{\gamma}_n$, the residuals for Odessa; and $\underline{\delta}_n$ the residuals for Grand Island. The results are easier to interpret if the residuals are grouped together by season rather than by station, so define $\left[\underline{\varepsilon}_n(i)\right]' = (\xi_{ni}, \gamma_{ni}, \delta_{ni})$ for i = 1,2,3 and $\underline{\varepsilon}_n' = (\underline{\varepsilon}_n(1)', \underline{\varepsilon}_n(2)', \underline{\varepsilon}_n(3)')$. The autocorrelation matrices Corr $(\underline{\varepsilon}_n, \underline{\varepsilon}_{n-k})$ (note that lag values are now in years) were estimated for k = 0,1,2,3; no significant values were found among the submatrices Corr $(\underline{\varepsilon}_n(i), \underline{\varepsilon}_{n-k}(j))$, k=0,..,3 and $i \neq j$. All correlations between the residuals occur within the same season. Among the matrices Corr $(\underline{\varepsilon}_n(i), \underline{\varepsilon}_{n-k}(i))$ the only significant values occur for k = 0, i = 1,2,3; and k = 1, i = 2. These matrices are given in table 8.

Recall from the individual models that a moving average component was added to the residuals for the second season. In particular, we have $\xi_{n2} = \xi_n^* + 0.63\ \xi_{n-1}^*$, $\gamma_{n2} = \gamma_n^* + 0.5\ \gamma_{n-1}^*$, $\delta_{n2} = \delta_n^* + 0.5\delta_{n-1}^*$ where $(\xi_n^*, \gamma_n^*, \delta_n^*)$ and $(\xi_t^*, \gamma_t^*, \delta_t^*)$ are independent for $n \neq t$.

Table 8.--*Significant cross correlations between residuals from individual-station models*

[Lag, in years]

			Overton	Odessa	Grand Island
Season 1					
(September to February)	Lag 0	Overton	1.0	0.941	0.871
		Odessa	.941	1.0	.913
		Grand Island	.871	.913	1.0
Season 2					
(March and April)	Lag 0	Overton	1.0	.893	.864
		Odessa	.893	1.0	.910
		Grand Island	.864	.910	1.0
	Lag 1	Overton	.446	.436	.430
		Odessa	.387	.403	.395
		Grand Island	.420	.410	.395
Season 3					
(May to August)	Lag 0	Overton	1.0	.945	.918
		Odessa	.945	1.0	.988
		Grand Island	.918	.988	1.0

Let $(\underline{\varepsilon}_n^*)' = (\xi_n^*, \gamma_n^*, \delta_n^*)$.

It is straightforward to show that if $\text{Cov}(\underline{\varepsilon}_n^*) = \begin{bmatrix} 0.00591 & 0.01586 & 0.01772 \\ .01586 & .05293 & .05558 \\ .01772 & .05558 & .07045 \end{bmatrix}$

then $\text{Corr}\,(\underline{\varepsilon}_n\,(2), \underline{\varepsilon}_n\,(2)) = \begin{bmatrix} 1.0 & 0.893 & 0.864 \\ .893 & 1.0 & .910 \\ .864 & .910 & 1.0 \end{bmatrix}$

and $\text{Corr}\,(\underline{\varepsilon}_n\,(2), \underline{\varepsilon}_{n-1}\,(2)) = \begin{bmatrix} 0.450 & 0.428 & 0.414 \\ .339 & .400 & .364 \\ .328 & .364 & .400 \end{bmatrix}$

It is evident that the lag 1 matrix for season 2 in table 8 can be explained by the moving average components.

It should be noted that the same parameter estimates obtained for the single-station models were used for the combined model rather than estimating the parameters multivariately. These estimates can easily be shown to correspond to the ordinary least squares estimates. This should not greatly affect the performance of the model for simulation. The main differences between the multivariate and ordinary least squares estimates lie in their standard errors.

The final statement of the model is: Let $\{Z_{t,i}\,(Ov)\}$ be the transformed and mean standardized values for Overton; let $\{Z_{t,i}\,(Od)\}$ be the transformed and mean standardized values for Odessa; and let $\{Z_{t,i}\,(GI)\}$ be the transformed and mean standardized values for Grand Island (see table 2). The final model becomes:

Overton

$$Z_{t,1}(Ov) = 0.3632 * Z_{t-1,3}(Ov) + 0.1449 * Z_{t-1,1}(Ov) + \xi_{t,1}$$

$$Z_{t,2}(Ov) = 1.0756 * Z_{t,1}(Ov) + \xi_t^* + 0.63\ \xi_{t-1}^*$$

$$Z_{t,3}(Ov) = 0.4646 * Z_{t,2}(Ov) + 0.2631 * Z_{t-2,3}(Ov) + \xi_{t,3}$$

Odessa

$$Z_{t,1}(Od) = 0.3423 * Z_{t-1,3}(Od) + 0.1461 * Z_{t-1,1}(Od) + \gamma_{t,1}$$

$$Z_{t,2}(Od) = 0.9168 * Z_{t,1}(Od) - 0.0982 * Z_{t-1,1}(Od) + \gamma_t^* + .5\ \gamma_{t-1}^*$$

$$Z_{t,3}(Od) = 0.7850 * Z_{t,2}(Od) + \gamma_{t,3}$$

Grand Island

$$Z_{t,1}(GI) = 0.4198 * Z_{t-1,3}(GI) - 0.0469 * Z_{t-1,1}(GI) + 0.1284 * Z_{t-2,3}(GI) + \delta_{t,1}$$

$$Z_{t,2}(GI) = 0.6715 * Z_{t,1}(GI) - 0.0949 * Z_{t-1,1}(GI) + \delta_t^* + .5\ \delta_{t-1}^*$$

$$Z_{t,3}(GI) = 0.7757 * Z_{t,2}(GI) + \delta_{t,3}$$

Let $(\underline{\varepsilon}_t\ (1))' = (\xi_{t,1}, \gamma_{t,1}, \delta_{t,1})$, $(\underline{\varepsilon}_t\ (2))' = (\xi_t^*, \gamma_t^*, \delta_t^*)$, $(\underline{\varepsilon}_t\ (3))' = (\xi_{t,3}, \gamma_{t,3}, \delta_{t,3})$. Then: $\underline{\varepsilon}_t\ (j)$, $j = 1,2,3$, $t = 0,1,\ldots$ are independent normal random vectors with

$$\text{Cov}(\underline{\varepsilon}_t\ (1)) = \begin{bmatrix} 0.00706 & 0.01782 & 0.01993 \\ .01782 & .05082 & .05608 \\ .01993 & .05608 & .07431 \end{bmatrix}$$

$$\text{Cov}(\underline{\varepsilon}_t\ (2)) = \begin{bmatrix} 0.00591 & 0.01586 & 0.01772 \\ .01586 & .05293 & .05558 \\ .01772 & .05558 & .07045 \end{bmatrix}$$

$$\text{Cov}(\underline{\varepsilon}_t\ (3)) = \begin{bmatrix} 0.08944 & 0.24913 & 0.23767 \\ .24913 & .77623 & .75345 \\ .23767 & .75345 & .74906 \end{bmatrix}$$

$E\ \underline{\varepsilon}_t\ (j) = \underline{0}$ for $j = 1,2,3$.

Graphs of 50-year simulations from the combined-station model are given in figures 10, 11, and 12. The unusually high discharge values for the 72nd simulated values at Odessa and Grand Island (73,396 ft^3/s-months for Odessa and 96,266 ft^3/s-months for Grand Island) may seem at first to point to a model inadequacy. However, out of five hundred 50-year combined-station simulations from the next section, only eight contained values greater than 90,000, which indicates that this particular simulation happened to be an extreme case.

SIMULATION AND EVALUATION OF PROBABILITIES

The model of the previous section can easily be used for simulation of 3-station, 3-season flow series over the next 50 years. The complicated dependence structure between seasons and between stations causes direct evaluation of probabilities of most events of interest to be difficult. However, if one obtains N 50-year simulations from the model, n of which exhibit a certain event, then an estimate of the probability of that event occurring over the next 50 years would be n/N, assuming that all simulations are equally likely.

Suppose one has data for years 0, 1, ..., t (t corresponds to 1979 here) and wants to simulate from the model for years t+1, ..., t+50. Referring back to table 2 and to the equations of the last section, one would proceed as follows: (call the final model of the last section equation (M))

1. Obtain the values of the series from years t and t-1 that are needed in equation (M) for year t+1; these values are transformed as in table 2.

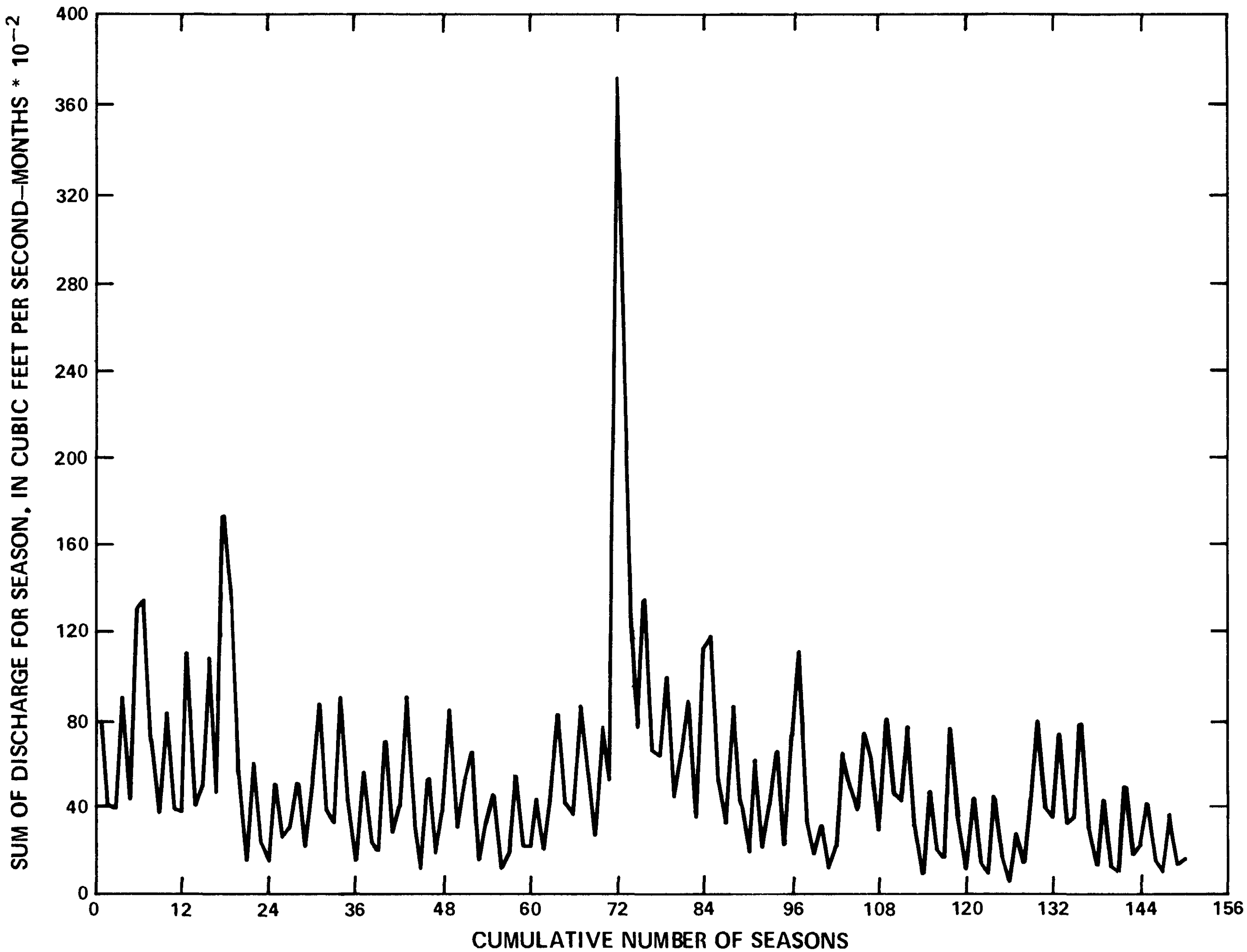

Figure 10.-- Fifty-year simulation of streamflow at Overton, Nebraska from the combined-station model.

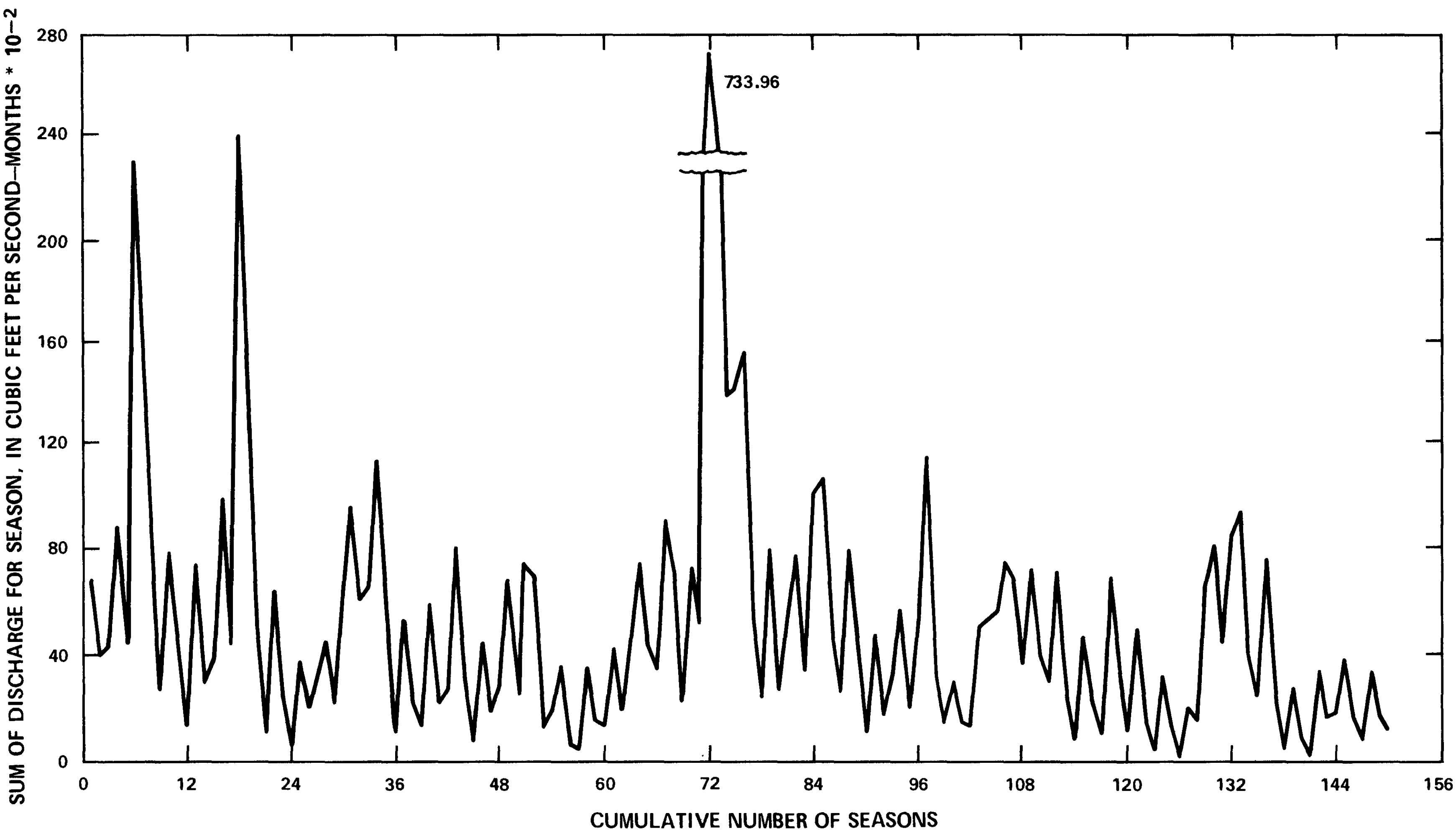

Figure 11.-- Fifty-year simulation of streamflow at Odessa, Nebraska from the combined-statiom model.

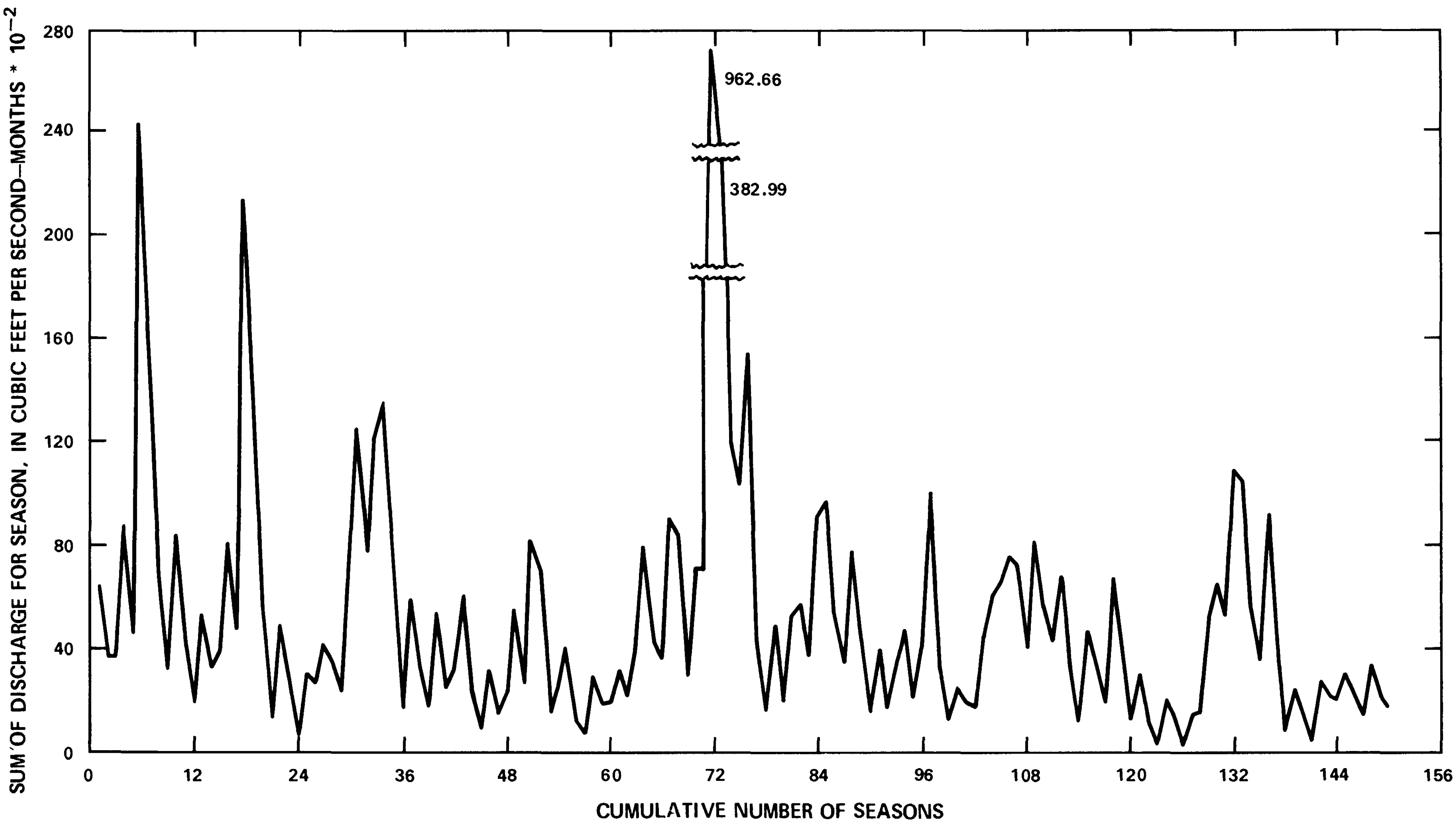

Figure 12.-- Fifty-year simulation of streamflow at Grand Island, Nebraska from the combined-station model.

2. Generate random vectors $\underline{\varepsilon}_{t+j}^{(1)}$, $\underline{\varepsilon}_{t}^{(2)}$, $\underline{\varepsilon}_{t+j}^{(2)}$, $\underline{\varepsilon}_{t+j}^{(3)}$, $j = 1,2, \ldots, 50$. The easiest way to do this is to generate a vector of three independent standard normal random variables for each season and each year and then transform them to obtain the desired covariance matrices. For instance, if Cov $(\underline{\varepsilon}_t(j)) = \underline{\Sigma}_j$, then $\underline{\Sigma}_j$ can be written as $\underline{\Gamma}_j' \underline{\Gamma}_j$, where $\underline{\Gamma}_j$ is 3x3 of rank 3. Hence, if $\underline{e}_{t+1}^{(j)}$ is a 3x1 standard normal random vector (Cov $(\underline{e}_{t+1}^{(j)}) = \underline{I}$), then $\underline{\Gamma}_j' \underline{e}_{t+1}^{(j)} = \underline{\varepsilon}_{t+1}^{(j)}$ is a normal random vector, with covariance matrix $\underline{\Gamma}_j' \underline{\Gamma}_j = \underline{\Sigma}_j$. $\underline{\Gamma}_j'$ for j = 1, 2, 3 is shown in table 9.

   Note that for season 2, $\underline{\varepsilon}_t$ (2) is not observable from the past series. The following is observable:

$$\xi_t^* + 0.63\ \xi_{t-1}^*,\ \gamma_t^* + 0.5\ \gamma_{t-1}^*,\ \delta_t^* + 0.5\ \xi_{t-1}^*.$$

   There are ways to estimate $\underline{\varepsilon}_t(2)$ from the available data, but since the initial effect of $\underline{\varepsilon}_t(2)$ on the simulations will die off quickly and the simulations are so long, the effect of using $\underline{\varepsilon}_t(2)$ generated as above will be negligible.

3. Now all the quantities necessary for the generation of $\underline{Z}_{t+1}$ for each station are secured. Using equation (M), generate $Z_{t+1,1}$ followed by $Z_{t+1,2}$ and $Z_{t+1,3}$ for each station. Once $\underline{Z}_{t+j}$ is generated, $\underline{Z}_{t+j+1}$ can be generated by using $\underline{Z}_{t+j}$ and $\underline{\varepsilon}_{t+j+1}^{(k)}$, k = 1, 2, 3.

4. Untransform the simulated values to get the series in terms of the original units.

Results of a simulation study involving some flow events for season 2 (March through April) and season 3 (May through August) are presented in

Table 9.--*Full-rank decompositions of seasonal-covariance matrices*

$[(\underline{\Sigma}_j = \underline{\Gamma}'_j\,\underline{\Gamma}_j)]$

Season 1:	$\underline{\Gamma}'_1$	=	0.08402	0	0
			.21208	.07643	0
			.23719	.07557	.11108
Season 2:	$\underline{\Gamma}'_2$	=	.07688	0	0
			.20631	.10182	0
			.23050	.07883	.10538
Season 3:	$\underline{\Gamma}'_3$	=	.29906	0	0
			.83303	.28686	0
			.79471	.31873	.12612

table 10. The flow events represent a range of discharges that can be related to certain habitat characteristics.

Based on a study by R. T. Hurr (1981) it was found that ground-water levels at Mormon Island would rise to within 8 inches of the land surface, if the discharge in the channels was 3,000 ft^3/s. For season two, this is equivalent to a seasonal sum of 6,000 ft^3/s-months. A seasonal value of 8,000 ft^3/s-months would raise the levels to within about 4 inches of land surface, while a value of 4,000 ft^3/s-months would drop the levels to about one foot below land surface. These values are assumed to encompass the critical ground-water levels necessary for an acceptable wet-meadow habitat in this area. Probability values based on these flows are presented also for Overton and Odessa, although the groundwater-streamflow relationships regarding possible wet-meadow complexes in these areas have not been determined.

Season 3, May through August, is the seed-germination period within the critical habitat reach. If the process of channel maintenance occurs during this period, the flows should be sufficient to prevent seedling establishment, as well as to transport the sediment necessary for erosional processes of channel formation. Channel-geometry plots from Eschner (1981) show that a width of 500 feet near Odessa is associated with an instantaneous discharge of approximately 3,000 ft^3/s. This width is estimated as the minimum unobstructed width necessary for suitable crane habitat. Therefore, as an approximation for each reach, seasonal streamflow means of 4,000, 12,000, and 20,000 ft^3/s-months were chosen as channel-maintenance discharge reference points for calculation of exceedence probabilities. These streamflow means are presented as seasonal values because of limited model resolution. They do not imply

Table 10.--*Estimates of the probability of level χ being surpassed at least m times over the next 50 years*

Season	χ ft^3/s-months	Station m:	1	2	3	4	5	6	7	8	16	20	24	30
2	4,000	Overton	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.96	0.49	0.25	0.07	0.01
		Odessa	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.56	.25	.06	.00
		Grand Island	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.94	.71	.34	.04
2	6,000	Overton	0.93	0.83	0.67	0.52	0.39	0.27	0.17	.09	.00	.00	.00	.00
		Odessa	.98	.91	.79	.66	.47	.34	.23	.14	.00	.00	.00	.00
		Grand Island	1.0	.99	.95	.90	.80	.67	.54	.43	.00	.00	.00	.00
2	8,000	Overton	.60	.31	.13	.06	.02	.01	.00	.00	.00	.00	.00	.00
		Odessa	.68	.38	.17	.06	.02	.01	.00	.00	.00	.00	.00	.00
		Grand Island	.83	.59	.37	.20	.09	.04	.02	.01	.00	.00	.00	.00
3	4,000	Overton	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.80	.56	.31	.05
		Odessa	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.71	.35	.09	.00
		Grand Island	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.89	.54	.21	.00
3	12,000	Overton	.89	.69	.49	.28	.16	.12	.06	.03	.00	.00	.00	.00
		Odessa	.96	.83	.62	.40	.22	.12	.06	.02	.00	.00	.00	.00
		Grand Island	.97	.90	.71	.53	.33	.18	.11	.06	.00	.00	.00	.00
3	20,000	Overton	.43	.18	.06	.02	.01	.00	.00	.00	.00	.00	.00	.00
		Odessa	.61	.27	.09	.04	.01	.00	.00	.00	.00	.00	.00	.00
		Grand Island	.68	.36	.12	.05	.02	.00	.00	.00	.00	.00	.00	.00

that these discharges are necessary throughout season 3 to maintain the channel; in fact, flows listed in table 10 are probably necessary for no more than 15 percent of season 3 to maintain the channel.

Critical discharge levels for maintenance of both wet meadows and channel cross sections can be compared to the probabilities of achieving these levels over the next 50 years using table 10. This comparison guide should assist habitat managers and water users in the area to plan effective utilization of available flows to satisfy projected water demands. Five hundred 50-year simulations from (M) were generated and the probabilities of the events of table 9 evaluated by counting the number, n, of the 500 simulations that satisfied the event, and taking n/500 as an estimate of the probability. Estimates below about 0.02 are not very reliable.

CONCLUSIONS

The primary problems confronted in modeling seasonal streamflow were: (1) the trade-off between overfitting the historical data versus adequately describing the driving mechanism behind the series; (2) the heterogeneity of the autocorrelation structure among seasons for each station; (3) nonnormality of the marginal distributions; and (4) the need to maintain correlation among stations. The first problem is present in any time-series analysis but was intensified here by the short length of record. The main statistics used to determine the model were seasonal autocorrelations and between-station correlations. Care was taken not to overfit by including nonsignificant values; but, all significant values up to a lag of three years (nine seasons) were included to insure adequate description of the correlation structure. Periodic autoregressions, with moving-average components in the residuals, afforded the flexibility to model the heterogeneous seasonal autocorrelation structure. From a set of power transformations including the natural log, the transformation for which residuals most closely followed a normal distribution was determined. The reader should exercise caution in applying this technique, because it does not guarantee normality; the residuals should be examined to determine if the transformation is adequate. Correlation among stations was maintained by building a correlation structure into the residuals from individual-station models. The reasoning for this approach follows: (1) Any subset of the stations can be generated independently of the others; and (2) the procedures for model identification and parameter estimation are more straightforward than those with all stations included as possible predictors for any one station although both approaches accomplish the same objectives.

The final model is intended to be used as a tool for simulation and evaluation of probabilities of interest, assuming present conditions are maintained. It can be used in conjunction with discharge-versus-channel width models, and discharge-versus-depth to water-table models to determine the need for future wildlife-management alternatives.

REFERENCES

Box, G. E. P., and Cox, D. R., 1964, An analysis of transformations: Journal of the Royal Statistical Society, Ser. B, 26, p. 211-252.

Box, G. E. P., and Jenkins, G. W., 1975, Time series analysis, forcasting and control: San Francisco, Calif., Holden-Day, p. 67-70.

Eschner, T. R., 1981, Morphologic and hydrologic changes of the Platte River, south-central Nebraska: MS Thesis, Fort Collins, Colorado, Colorado State University, 269 p.

Graybill, F. A., 1976, Theory and application of the linear model: North Scituate, Mass., Duxbury Press, 207 p.

Hernandez, F., and Johnson, R. A., 1980, The large sample behavior of transformations to normality: Journal of the American Statistical Assoc. v. 75, no. 372, p. 855-861.

Hurr, R. T., 1981, Ground-water hydrology of the Crane Meadows wildlife area, near Grand Island, Hall County, Nebraska: U.S. Geological Survey Open-File Report 81-1109, 43 p.

Pagano, Marcello, 1978, On periodic and multiple autoregressions: Annals of Statistics, v. 6, no. 6, p. 1310-1317.

Parzen, E., and Pagano, M., 1979, An approach to modeling seasonally stationary time series: Journal of Econometrics, v. 9, p. 137-153.

Petsch, H. E., Jr., Rennick, K. B., and Nordin, C. F., Jr., 1980, Statistical summaries of selected streamflow data, South Platte River in Colorado and Nebraska: North Platte and Platte Rivers in Nebraska, U.S. Geological Survey Open-File Report 80-679, 278 p.