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DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

A FINITE-ELEMENT MODEL STUDY OF THE IMPACT OF THE PROPOSED
I-326 CROSSING ON FLOOD STAGES OF THE CONGAREE RIVER NEAR
COLUMBIA, SOUTH CAROLINA

By Jonathan K. Lee and Curtis S. Bennett, III

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A FINITE-ELEMENT MODEL STUDY OF THE IMPACT OF THE
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ABSTRACT

A two-dimensional finite-element surface-water model developed by Norton and King was used to study the hydraulic impact of the proposed Interstate Route 326 crossing of the Congaree River near Columbia, S.C. A major accomplishment of this application was the assessment of the finite-element model as a potential operational tool for analyzing complex highway crossings and other modifications of river flood plains.

The rapid expansion of the flood plain of the Congaree River upstream from the proposed highway crossing, an extensive dike system, and highly variable roughness combine to cause significant lateral velocities and variations in stage during floods. Thus, use of a two-dimensional model was warranted.

Infrared aerial photography was used to define regions of homogeneous roughness in the flood plain. Finite-element networks approximating flood-plain topography were designed using elements of three roughness types. High-water marks established during an 8-year flood that occurred in October 1976 were used to calibrate the model.

The maximum flood of record, an approximately 100-year flood that occurred in August 1908, was modeled in three cases: dikes on the right bank, dikes on the left bank, and dikes on both banks. In each of the three cases, simulations were performed both without and with the proposed highway embankments in place. Detailed information was obtained about backwater effects upstream from the proposed highway embankments, changes in flow distribution resulting from the embankments, and local velocities in the bridge openings.

On the basis of results from the model study, the South Carolina Department of Highways and Public Transportation changed the design of several bridge openings. A simulation incorporating the new design for the case with dikes on the left bank indicated that both velocities in the bridge openings and backwater were reduced.

A major problem in applying the model was the difficulty in predicting the network detail necessary to avoid local errors caused by roughness discontinuities and large depth gradients.

INTRODUCTION

A two-dimensional finite-element surface-water model developed by Norton and King (Norton and others, 1973; Norton and King, 1973; King and Norton, 1978) was used to study the hydraulic impact of the proposed Interstate Route 326 crossing of the flood plain of the Congaree River near Columbia, S.C. The rapid expansion of the flood plain of the river upstream from the proposed highway crossing, an extensive dike system, and highly variable roughness combine to cause significant lateral velocities and variations in stage during floods. Two sewage disposal

factors for converting inch-pound units to metric units is provided at the front of the report. All data supporting the conclusions of this report are available in the files of the Gulf Coast Hydrosience Center of the U.S. Geological Survey at NSTL Station, Miss., or the South Carolina District office of the Geological Survey at Columbia, S.C.

MODEL DESCRIPTION

The formulation and development of the model have been reported elsewhere (Norton and others, 1973; Norton and King, 1973; Tsenq, 1975; and King and Norton, 1978); therefore, only the equations solved and a brief outline of the technique used to solve them are presented here.

Flow Equations

Under the usual assumptions (for example, hydrostatic pressure and equating to one the momentum correction factors), two-dimensional surface-water flow in the horizontal plane is described by two equations for conservation of momentum and one for conservation of mass:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial h}{\partial x} + g \frac{\partial z_o}{\partial x} - \frac{\epsilon_{xx}}{\rho} \frac{\partial^2 u}{\partial x^2} - \frac{\epsilon_{xy}}{\rho} \frac{\partial^2 u}{\partial y^2} \\ - 2\omega v \sin \phi + \frac{gu}{C^2 h} (u^2 + v^2)^{1/2} - \frac{\zeta}{h} V_a^2 \cos \psi = 0, \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial h}{\partial y} + g \frac{\partial z_o}{\partial y} - \frac{\epsilon_{yx}}{\rho} \frac{\partial^2 v}{\partial x^2} - \frac{\epsilon_{yy}}{\rho} \frac{\partial^2 v}{\partial y^2} \\ + 2\omega u \sin \phi + \frac{gv}{C^2 h} (u^2 + v^2)^{1/2} - \frac{\zeta}{h} V_a^2 \sin \psi = 0, \end{aligned} \quad (2)$$

with the specification of zero normal flow (tangential flow) at the boundaries, has been documented by King and Norton (1978), Gee and MacArthur (1978), and Walters and Cheng (1978, 1980) for the mixed-interpolation formulation of the surface-water flow equations.

The model has the capability of integrating the flow across a line following element sides and beginning and ending at element vertices. Thus, conservation of mass, which is not automatically satisfied, can be checked (King and Norton, 1978).

DESCRIPTION OF THE STUDY AREA

General Site Description

The Congaree River originates at the confluence of the Broad and Saluda Rivers at Columbia, flows southeastward for 51.5 mi, and joins the Wateree River near the head of Lake Marion to form the Santee River. (Both the Broad and Saluda River basins are located in the Piedmont Region of South Carolina, with headwaters in southwestern North Carolina.) The drainage area of the Congaree River at Columbia, 1.7 mi above the study area, is 7,850 mi². Of this drainage area, 5,240 mi² are in the Broad River basin and 2,520 mi² are in the Saluda River basin.

The reach of the Congaree River studied in this report lies between river miles 166.9 and 173.1 (fig. 1). (Zero river mile is defined as the mouth of the Santee River.) The reach is between 42.2 and 48.4 mi upstream from where the Congaree River joins the Santee River. The flood plain expands from a width of approximately 700 ft at the upper end of the study area to a width of approximately 4 mi at the proposed Interstate Route 326 crossing less than 3 mi downstream (fig. 2). The streambed generally consists of alluvial

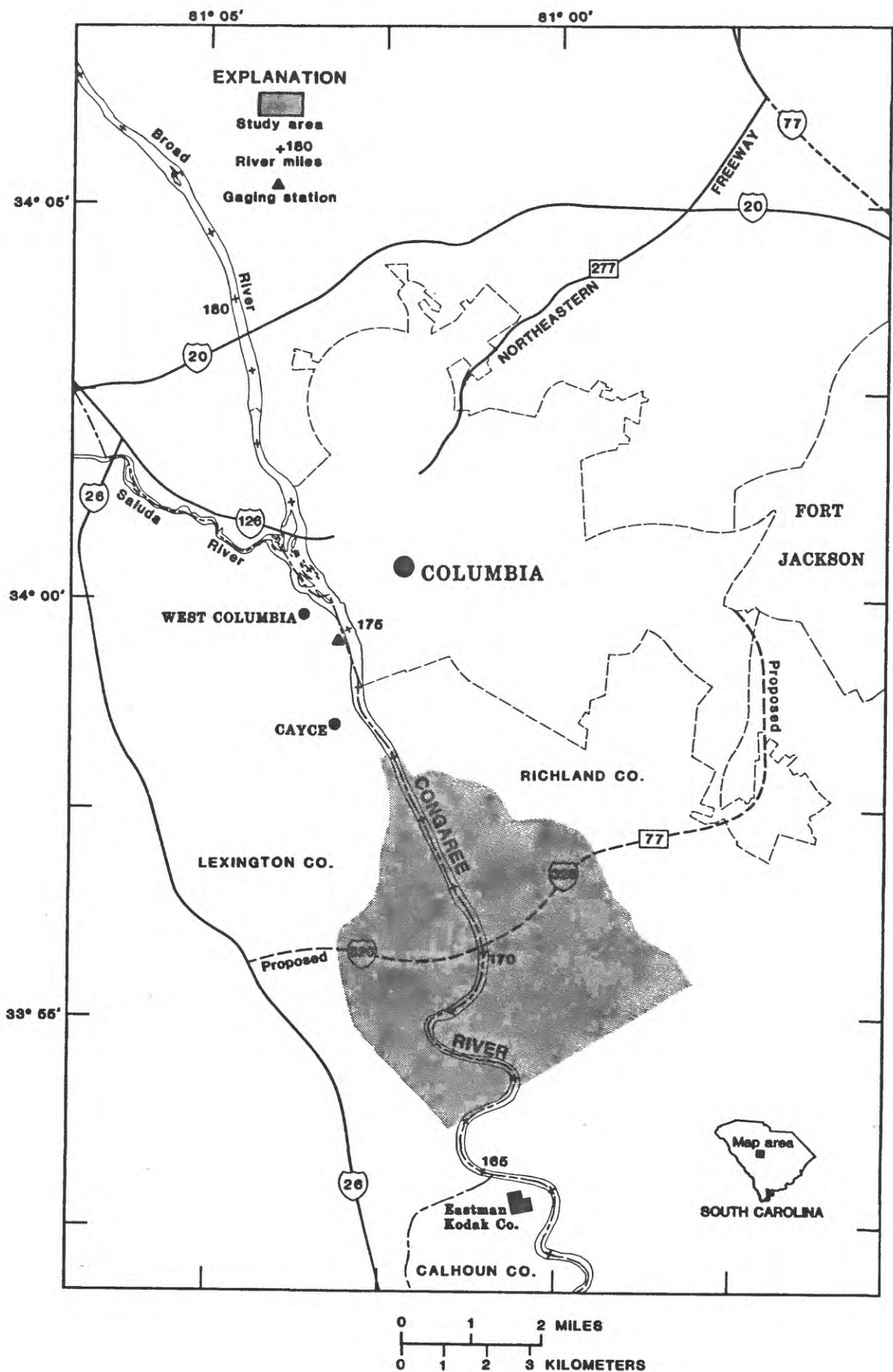


Figure 1.--Congaree River near Columbia, S.C.

sands blanketed with finer soils. About half the flood plain is covered with a combination of dense timber and underbrush. Most of the rest of the flood plain is occupied by cultivated fields, interspersed among the wooded areas.

Although there is limited residential or commercial development on the flood plain, the left side is partially protected by an earth-fill dike (Manning dike). Existing encroachments include a small subdivision (River Bluff Estates) on the right flood plain at the upper end of the study area, a private school (Heathwood Hall Episcopal School) on the left approximately 1.1 mi above the proposed highway crossing, the City of Cayce sewage disposal plant on the right approximately 1.1 mi above the proposed highway crossing, and the City of Columbia sewage disposal plant on the left approximately 0.3 mi above the proposed highway crossing. Earth-fill dike systems are in place to protect the City of Cayce and the City of Columbia disposal plants. A dike (Otarre dike) has been proposed for the right flood plain. The location of the originally proposed Otarre dike and other key features of the flood plain are shown in figure 2.

Interstate Route 326 Roadway System

Interstate Route 326 (I-326) will be a part of the Southeastern Beltway system, which extends around the south and east sides of Columbia. It will connect I-26 on the southwest side of Columbia with I-77 and I-20 on the northeast side (fig. 1). It will also serve the city as an additional crossing over the Congaree River.

The I-326 crossing of the Congaree River flood plain will be a controlled-access, six-lane, divided highway. The crossing is roughly at a right angle to the longitudinal axis of the flood plain, with the

highway running east and west and the river flowing from north to south. An intersection with the Twelfth Street Extension on the west or Lexington County flood plain is in the planning stages of development (fig. 2). Twelfth Street will run parallel to the river. Its effect on flood flow was not considered in this study but was analyzed in the South Carolina Department of Highways and Public Transportation's studies. On the eastern edge of the flood plain, I-326 will intersect South Carolina Route 48, Bluff Road.

There will be seven dual bridges in the flood plain. The number and name of each bridge are given in table 1. During the study, three of the seven bridges were redesigned by the South Carolina Department of Highways and Public Transportation. The beginning and ending stations and length of each bridge, for both the original and revised designs, are also given in table 1. The original bridges are shown in figure 2, and the revised bridges in figure 3. All of these bridges are designed to serve as flood-relief bridges during major floods on the Congaree River. In addition to the bridges, there will be a culvert on the east side of the flood plain within the Route 48 interchange. Its primary purpose is to handle local drainage. Some floodwater will flow through the overpass over Route 48, but this flow will be relatively insignificant and was not considered in this study.

HYDROLOGY OF THE STUDY AREA

Flood Data

Streamflow data have been collected at the Geological Survey gaging station, Congaree River at Columbia, from October 1939 to the current year (1980). Gage-height records were collected at a site 1,000 ft above the present gaging station from October 1891 to December 1933. The maximum flow that has occurred since at least October 1891

Table 1.--Bridges proposed for the Interstate Route 326 crossing of the flood plain of the Congaree River

Bridge number	Bridge description	Original design			Revised design		
		Beginning station	Ending station	Length (ft)	Beginning station	Ending station	Length (ft)
1	Twin overflow bridges 1	448 + 00	455 + 80	780	Bridges not revised		
2	Twin overpasses over Old State Road (Road S-66)	502 + 60	511 + 00	840	495 + 10	510 + 10	1,500
3	Twin bridges over Congaree Creek	516 + 25	530 + 65	1,440	Bridges not revised		
4	Twin bridges over Congaree River	549 + 35	562 + 55	1,320	Bridges not revised		
5	Twin bridges over Metro Lane	578 + 25	579 + 75	150	Bridges not revised		
6	Twin overflow bridges 2	604 + 00	610 + 60	660	601 + 00	614 + 20	1,320
7	Twin overflow bridges 3	634 + 75	641 + 35	660	635 + 65	640 + 45	480
Culvert	Three 8-foot-by-8-foot reinforced-concrete box culverts	656 + 25	-	24	Culverts not revised		

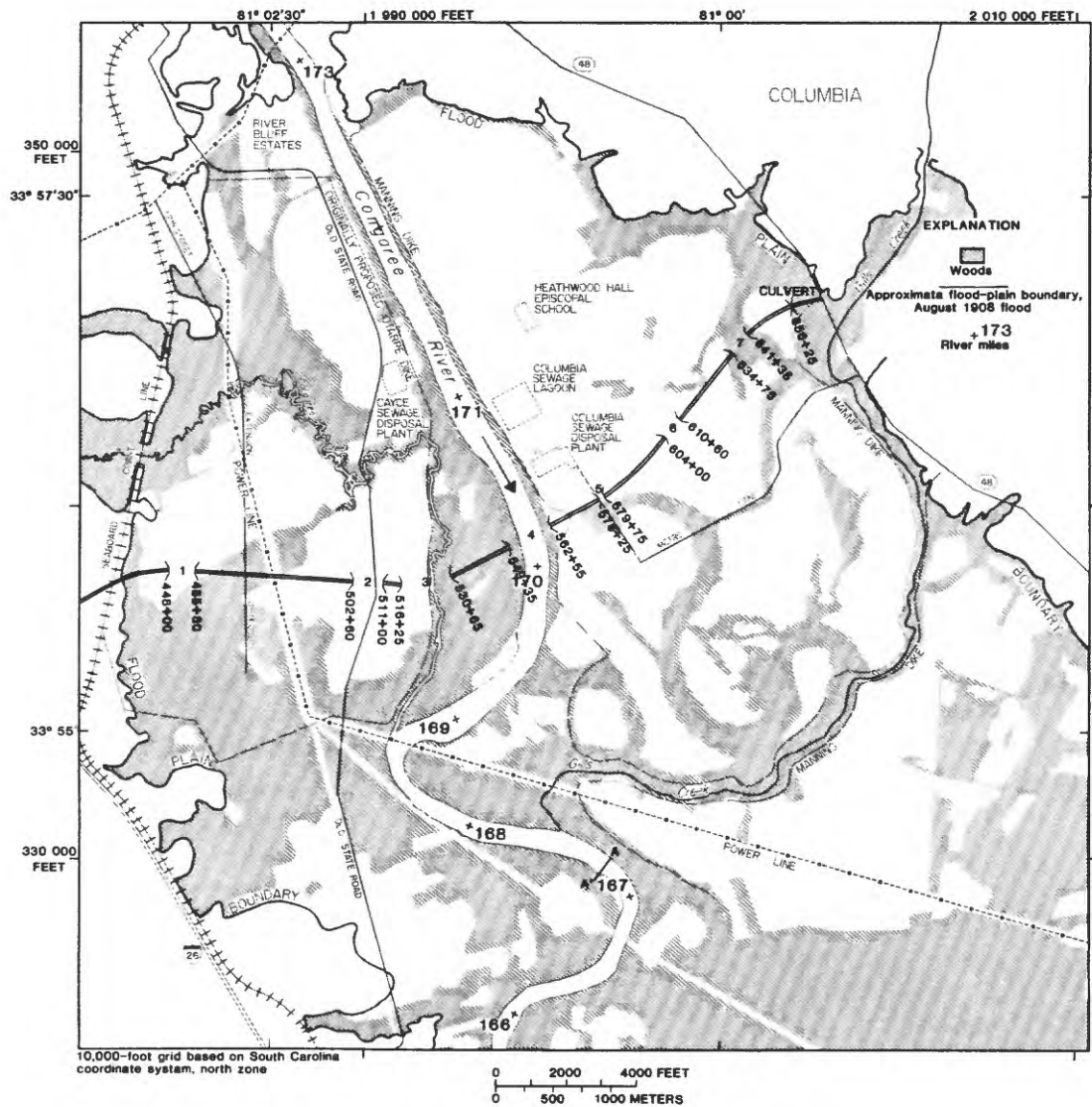


Figure 2.--Congaree River flood plain near Columbia, S.C., showing proposed bridges (original design).
Cross-section A-A' is shown in figure 5.

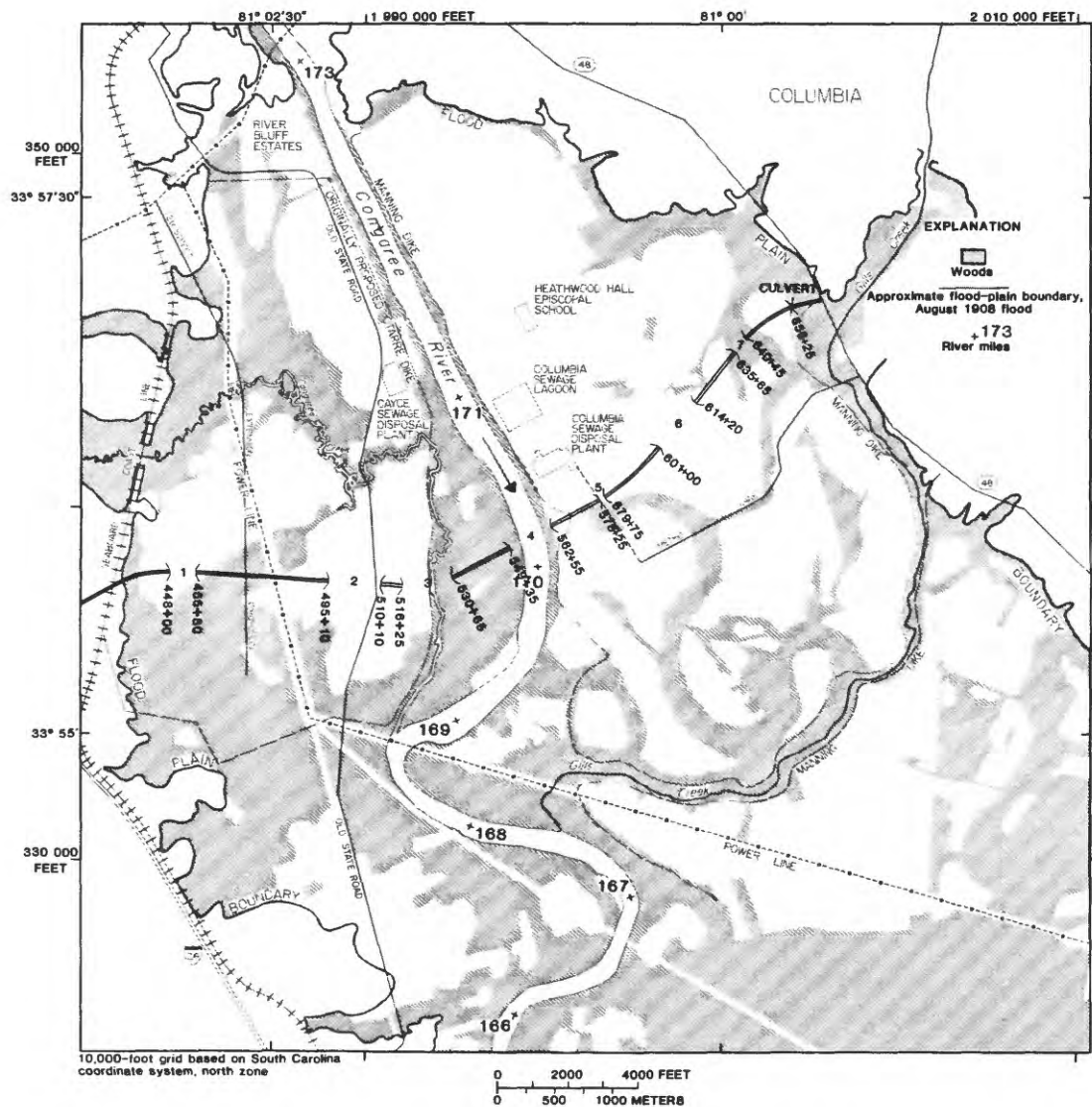


Figure 3.--Congaree River flood plain near Columbia, S.C.,
showing proposed bridges (revised design).

was caused by a tropical storm in August 1908. The storm caused major flooding throughout the basin. The peak flow of the Congaree River at Columbia was 364,000 ft³/s. The water-surface elevation for this flood was 152.8 ft NGVD.

A high-water mark of 136.5 ft NGVD for the 1908 flood was established at the Route 48 crossing of Gills Creek. Other high-water marks have been established at the same location: 133.6 ft in 1916, 135.5 ft in 1928, 135.3 ft in 1929, and 130.2 ft in 1936.

The flood of October 1976, the maximum stage and discharge since April 1936, was used to calibrate the two-dimensional finite-element model. A peak elevation of 142.8 ft NGVD, corresponding to a discharge of 155,000 ft³/s, was recorded at the Columbia Congaree River gage on October 11, 1976. A large portion of the flood plain in the study area was inundated after the dike along the left side of the main channel was breached in the vicinity of the Columbia sewage disposal plant. High-water marks were established by personnel of the South Carolina Department of Highways and Public Transportation at River Bluff Estates, along Old State Road at Congaree Creek, above and below the proposed highway route on the right flood plain, and near the left bank of the main channel at the proposed highway route. High-water marks were also established by Corps of Engineers personnel at River Bluff Estates and later by Geological Survey personnel near River Bluff Estates, near the Cayce sewage disposal plant, and at the downstream end of the study reach. A peak of 127.0 ft NGVD, which was used to define the lower end of the flood profile, was observed on the staff gage at the Eastman Kodak Company plant at river mile 164.5.

Congaree Creek, with a drainage area of 136 mi², empties into the Congaree River from the west between miles 168 and 169. Gills Creek, with an even smaller drainage area, empties into the Congaree River from the east between miles 167 and 168. The record (since 1959) flow of Congaree Creek was 1,840 ft³/s. Because of their insignificance compared to the flow of the river, the flows of these creeks were disregarded in this study.

Flood Frequency

Numerous dams on the Broad and Saluda Rivers have significantly influenced flood magnitude on the Congaree River. The largest of these structures, Saluda Dam, located 12 mi above the mouth of the Saluda River, may have the most pronounced effect on flood flows. Saluda Dam, completed in 1930 by the South Carolina Electric and Gas Company, forms Lake Murray, which has a surface area of about 51,000 acres at maximum power pool. Since this reservoir is operated for hydropower generation only, inflow during major floods creates temporary storage above maximum operating pool levels. During major floods, safety considerations for the earth-fill dam necessitate releases through spillway gates in addition to discharges through power turbines to lower the reservoir to required maximum pool levels as soon as possible. Hence, any flood control that occurs as a result of the operation is coincidental.

Because of the complexity of adjusting the flood frequency for flow regulation by upstream dams, the maximum flood of record (the August 1908 flood) was used for hydraulic design purposes by the South Carolina Department of Highways and Public Transportation and for input to the two-dimensional model. A Log Pearson Type III frequency analysis for the total period of record, with no adjustment for regulation, shows

that the exceedance interval for a flood of this magnitude is approximately 100 years, as illustrated by figure 4. The exceedance interval for the October 1976 flood, used to calibrate the two-dimensional model, is 8 years.

APPLICATION OF THE FINITE-ELEMENT MODEL TO THE PROPOSED INTERSTATE ROUTE 326 CROSSING OF THE CONGAREE RIVER

Outline of the Modeling

High-water marks established during the October 1976 flood (155,000 ft³/s) were used to calibrate the finite-element model. The values of the Chézy coefficients determined during calibration were used to run the model for the August 1908 flood (364,000 ft³/s) for three flood-plain configurations: (1) the existing Manning dike along the left bank of the main channel in place, (2) the Manning dike removed and the originally proposed Otarre dike along the right bank of the main channel in place, and (3) both dikes in place. Each flood-plain configuration was run both without and with the proposed highway embankments in place. Flood-plain configuration (1) was run for both the original and revised designs of bridge 2. The run with the original design did not include spur dikes; the run with the revised design included spur dikes as specified by the South Carolina Department of Highways and Public Transportation. By mistake, bridge 1 was located too far to the right in the original-design run. This mistake was corrected in the revised-design run.

Data Collection

One-dimensional step-backwater profiles for floods of various magnitudes were obtained from the South Carolina Department of Highways and Public Transportation and the Corps of Engineers for use in

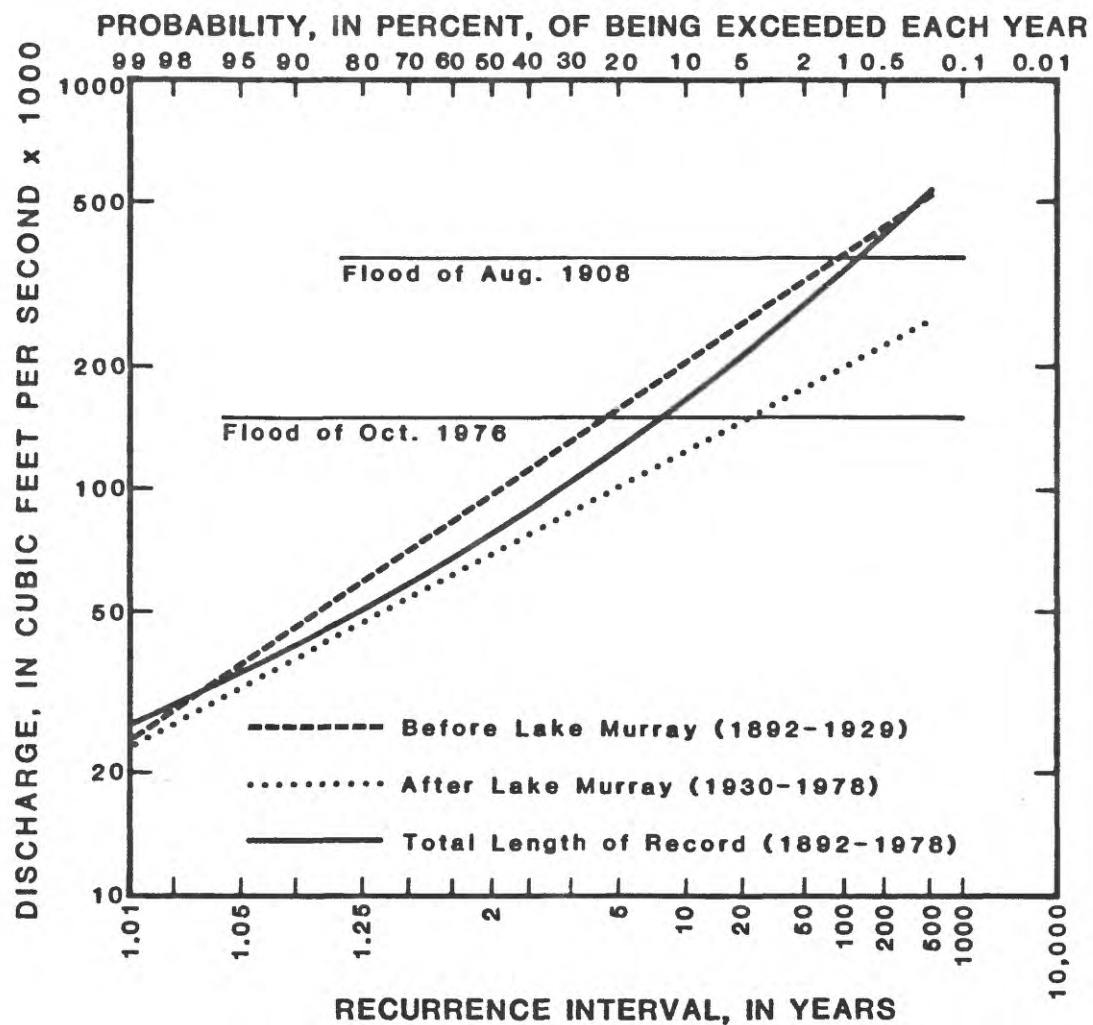


Figure 4.--Frequency curves for the Congaree River at Columbia, S.C.

approximating inundation boundaries and estimating downstream water-surface elevations. High-water marks established for the October 1976 flood were assembled. Information was collected from maps and field surveys on study-area topography, channel cross sections, the location and top elevation of existing and proposed dikes, and proposed highway embankments and spur dikes. Infrared aerial photographs of the study area were made to determine vegetation type and density.

Network Layout

Networks for all model runs were designed to be identical where they overlapped to ensure that results from different runs would be comparable. On the basis of the infrared aerial photography and ground inspection, it was decided to use three roughness types, corresponding to the main channel, wooded ground, and cleared ground. Each element was designed to be of nearly homogeneous roughness and was assigned one of these three types.

Main channel features were ignored in earlier two-dimensional finite-element studies (Franques and Yannitell, 1974; Tseng, 1975; U.S. Army Corps of Engineers, 1976; King and Norton, 1978). In this study, topographic variations of the main channel of the Congaree were modeled because it was anticipated that a large proportion of the flow would remain in the main channel. Four criteria were used in designing the finite-element network for the main channel. First, because numerical experiments showed that a cross-channel change in depth of more than about 250 percent across an element was likely to cause model divergence, the anticipated ratio of the maximum to minimum depth on any one element was kept less than about 2.5. This constraint complicated modeling of

the drop from the flood plain to the channel bottom. In many places, this drop involved an increase in depth from about 5 ft to 30 ft or more. Second, the area and shape of the channel cross section were approximated as accurately as possible. Third, excessively long, narrow elements were avoided. Fourth, as few elements as possible were used in order to minimize computer core requirements and computational time.

Several approximations of the main channel, both triangular and trapezoidal in shape, were investigated numerically. On the basis of these experiments and the criteria given above, a symmetrical, trapezoidal shape was selected to describe the channel, as shown in figure 5.

Ground-surface gradients smaller than those between the flood plain and the main channel, where coupled with roughness discontinuities, were found to cause serious local inconsistencies in the solution. In many cases, it was possible to reduce these errors by moving nodes, adjusting ground-surface elevations, and shifting element sides to lie parallel to streamlines. In problem areas where such adjustments failed to eliminate the errors, more detail was added to the network. Because these changes had to be made on a trial-and-error basis, the process of network adjustment and refinement was tedious, time-consuming, and expensive.

The use of isoparametric elements permitted flexibility in network design. Curved-sided elements were used, not only in defining smooth tangential-flow boundaries, but also in approximating the bends of the main channel (fig. 6) and the embankments and spur dikes of the highway crossing (fig. 15).

Boundary Conditions and Model Coefficients

A starting value for each flow component at each inflow node was estimated and then adjusted after the first iteration until the total

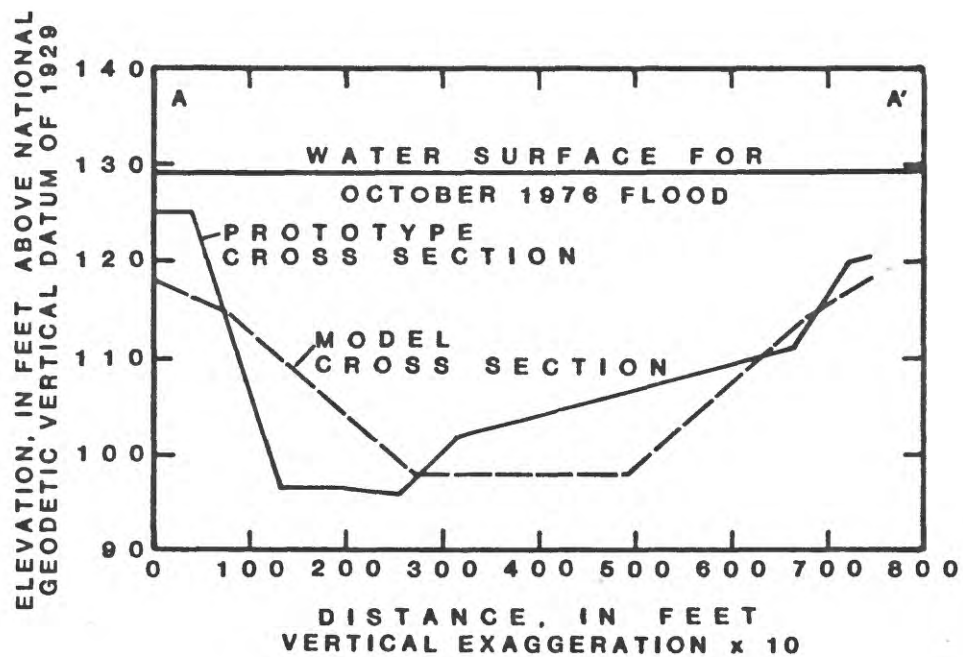


Figure 5.--Prototype and model cross sections of the main
channel at section A-A' (fig. 2).

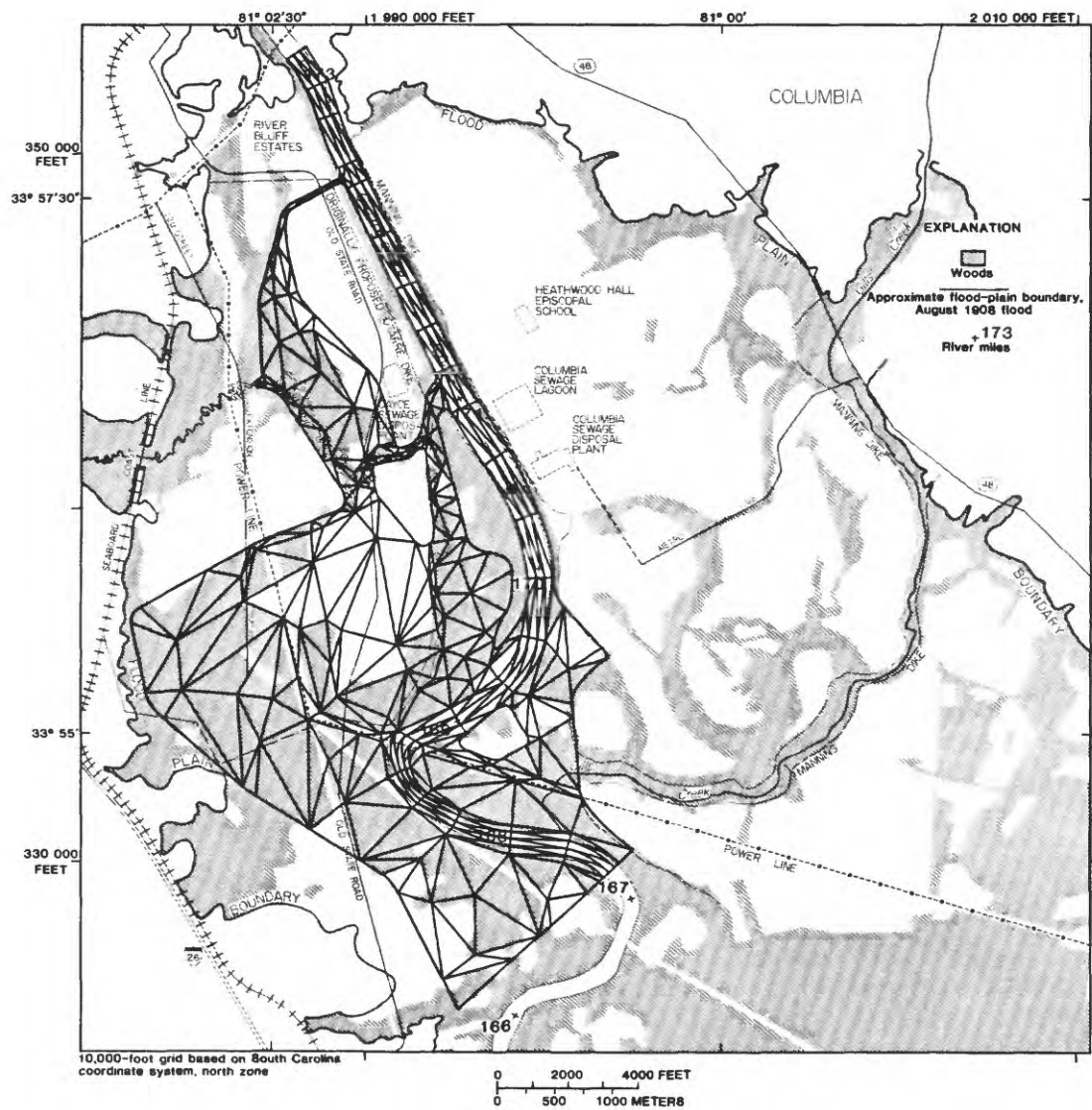


Figure 6.--Finite-element network, Manning dike in place,
October 1976 flood.

computed inflow was correct. Because the inflow in all cases was specified only across the width of the main channel (fig. 6), model results more than two rows of elements downstream from the upstream boundary were not influenced by the inflow distribution. A tributary was defined in several networks (fig. 6), but zero inflow was specified at the tributary boundary nodes in the results reported here.

Water-surface elevations at the downstream outflow boundary were estimated from the one-dimensional step-backwater profiles. In each run, the downstream water surface was assumed to be horizontal.

As stated above, the solution is obtained by an iterative procedure. Initially the model assumes zero velocity and a constant water-surface elevation equal to the downstream water-surface elevation (downstream boundary condition) at each node. Because of the slope of the flood plain, this procedure would result in negative depths in the upstream part of the model if the correct downstream water-surface elevation were used. The initial downstream water-surface elevation was set as high as necessary to avoid this difficulty. It was then decreased 1 or 2 ft per iteration until the specified value was reached. Additional iterations were run until the change in the solution between two successive iterations was less than 1 percent.

The values of the Chezy coefficients determined during the calibration run were used in the seven model runs for the August 1908 flood. For these subsequent runs, the height of the water surface and thus the location of the lateral boundaries of the model could only be estimated on the basis of the step-backwater profiles. In several instances, the water surface computed by the model was lower than initially estimated and resulted in such small depths near

the edges of the flood plain and in other shallow areas that the model diverged. Model ground-surface elevations in these trouble spots were then temporarily lowered to permit convergence. On the basis of the resulting improved estimate of the water-surface elevation, the lateral boundaries of the model were moved into deeper water and network detail was adjusted, after which ground-surface elevations were restored to their correct values and the model was run again.

Nonzero eddy viscosities are necessary for convergence of the Norton-King model. It is difficult to determine what realistic values of these coefficients are and whether values large enough to ensure convergence are not unrealistically large. In general, increasing the values of the eddy viscosities increases the water-surface slope. Model divergence occurred for values of the eddy viscosities as large as 50 lb-s/ft². Hence, to avoid convergence problems and because of a lack of information about their correct values, values of 100 lb-s/ft² were arbitrarily used throughout the study. It was determined by numerical experiment that, once the values of the eddy viscosities were set high enough to ensure model convergence, the solution was considerably less sensitive to changes in their values than to changes in the values of the Chezy coefficients or to changes in network detail.

RESULTS OF THE SIMULATIONS

Interpretation of the Results

Several factors combine to introduce error into the velocities and water-surface elevations reported here: (1) the assumptions that are made in deriving equations 1 through 3 may not be completely valid throughout the study area; (2) model discretization involves approximations

to prototype topography, roughness, velocities, and depths; (3) because of model constraints and the limitations of the calibration data, the values of the model coefficients used may distort the results; and (4) boundary conditions, especially the downstream water-surface elevations, are not known precisely. Common sense and care in model application have reduced the error from these sources to a minimum, and much more information is obtained than can be obtained from one-dimensional models, but error does remain in the computed velocities and water-surface elevations. It should also be kept in mind that the velocities reported in this study are vertically averaged velocities. Thus, a high velocity along a spur dike may indicate a potential for scour, but it is not a point velocity at or near the bed.

October 1976 Flood with the Manning Dike in Place

The high-water marks from the October 1976 flood were used to calibrate the model. Although the Manning dike along the left bank of the main channel was breached in several places during the 1976 flood, it was assumed that the outflows were insignificant and could be ignored for modeling purposes. Thus, the Manning dike was treated as a tangential-flow boundary (fig. 6). An upstream inflow of 155,000 ft³/s and a downstream water-surface elevation of 129.2 ft NGVD were used as boundary conditions. Zero inflow was specified for the tributary (Congaree Creek) on the right side of the flood plain. Zero normal flow was specified at all other boundary nodes.

After model convergence was obtained for nominal values of the Chézy coefficients, the values were adjusted by trial and error until the computed water-surface elevations matched the observed high-water marks (pl. 1) as closely as possible. The location reference numbers, observed

high-water-mark elevations, and computed water-surface elevations at the seven locations where data were available are given in table 2. Fits within 0.5 ft were obtained at locations 1, 2, 3, 4, and 7. No combination of values of the three Chezy coefficients accounted for the large drop in the water surface between locations 3 and 4 and locations 5 and 6. The large difference between the observed and computed water-surface elevations at locations 5 and 6 suggests that the assumption of steady flow is not valid for the 1976 flood. The duration of the peak discharge was probably too short for the water on the right flood plain to rise to the level it would have attained under steady-flow conditions. The values of the Chezy coefficients determined in this calibration process were $63.0 \text{ ft}^{1/2}/\text{s}$ for the main channel, $20.3 \text{ ft}^{1/2}/\text{s}$ for the wooded areas, and $89.7 \text{ ft}^{1/2}/\text{s}$ for the cleared areas. These values were used in all subsequent model runs.

The computed water-surface profiles in the main channel for this and the subsequent runs are plotted in figure 7. Plate 1 is a plot of the velocity field and water-surface elevations for this run. Water-surface contour lines were obtained from plots generated by the computer program Surface Approximations and Contour Mapping (SACM) developed by Applications Consultants, Inc., and modified by the Geological Survey (U.S. Geological Survey, written commun., 1975). Examination of plate 1 reveals that the flow directions in several small areas are not completely realistic. As stated above, these problems are probably caused by variable topography and roughness and could be resolved by further refining the network shown in figure 6.

Table 2.--Elevations of the observed high-water marks and the computed water surface for the October 1976 flood

Location reference number ¹	Elevation of observed high-water marks above NGVD (ft)	Elevation of computed water surface above NGVD (ft)
1	138.0 - 139.6	139.0
2	135.4	135.9
3	133.2 - 133.4	133.2
4	134.0 - 134.2	134.6
5	130.5 - 130.6	133.1
6	130.2 - 130.8	133.1
7	129.7	130.1

¹Location reference numbers are shown on plate 1.

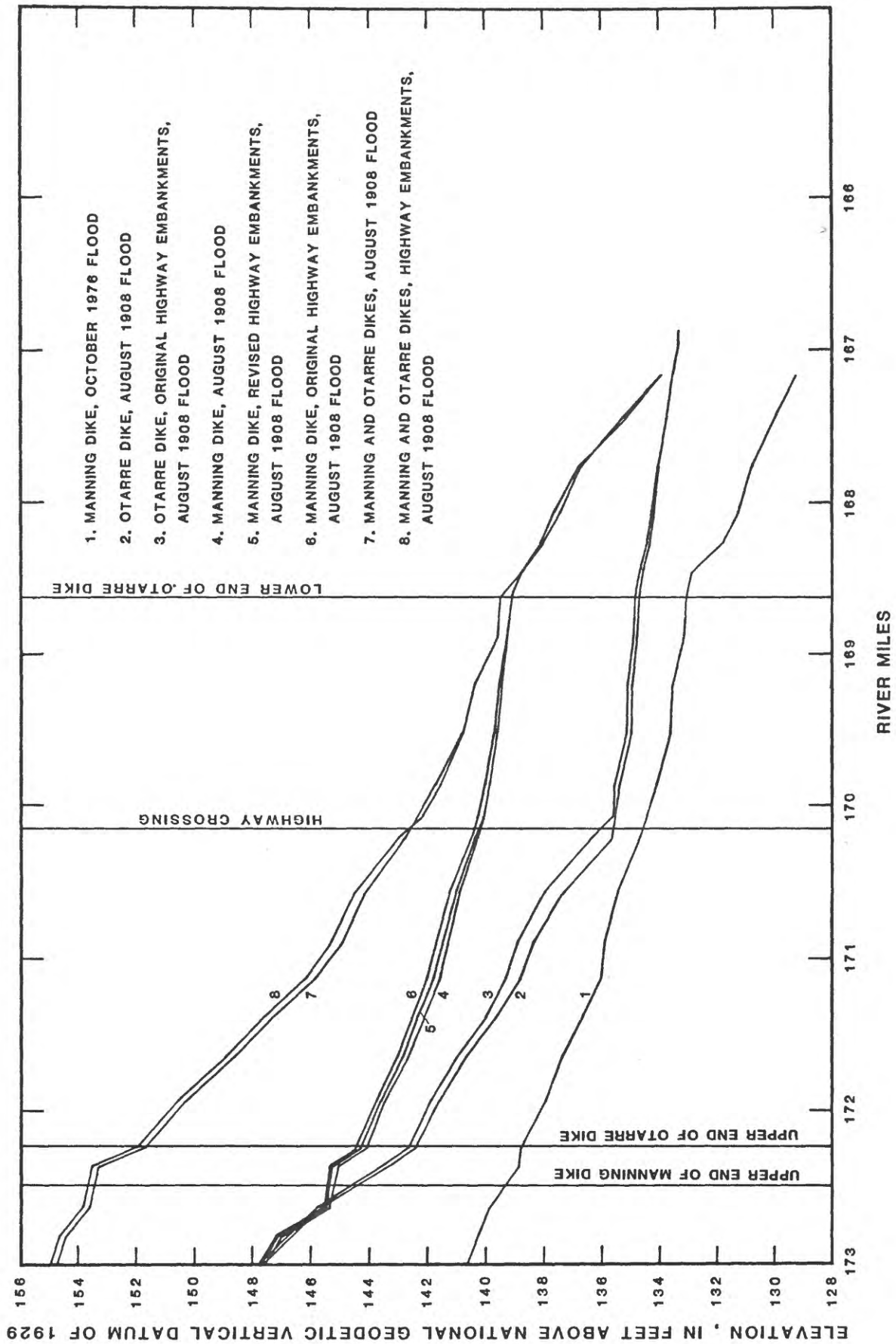


Figure 7.--Water-surface profiles for the main channel.

August 1908 Flood with the Manning Dike in Place

The August 1908 flood was modeled under the assumption that no flow occurred through the Manning dike. This assumption, made throughout the study for both the Manning and Otarre dikes, was based on the South Carolina Department of Highways and Public Transportation's need to quantify the potential effect of the proposed I-326 embankments for dikes raised and strengthened to withstand the 1908 flood. In this case, the assumption forces the maximum possible amount of the flow onto the right flood plain. Because of the increased depth of flow, model boundaries at the right edge of the flood plain (figs. 8, 9, 10) were different from those used for the 1976 flood.

The model was first run without highway embankments (fig. 8). Next, it was run with embankments corresponding to the original bridge design (including the error in the location of bridge 1) but without the proposed spur dikes (fig. 9). To reduce the backwater on the right flood plain and lower high velocities at the right edge of bridge 2, the South Carolina Department of Highways and Public Transportation shifted the twin overpasses over Old State Road toward the right and increased their length from 840 ft to 1,500 ft. Then the model was run again with embankments corresponding to the revised design (fig. 10). This time the proposed spur dikes were included, and bridge 1 was located properly. An upstream inflow of $364,000 \text{ ft}^3/\text{s}$ and a downstream water-surface elevation of 133.85 ft NGVD were used as boundary conditions in all three runs.

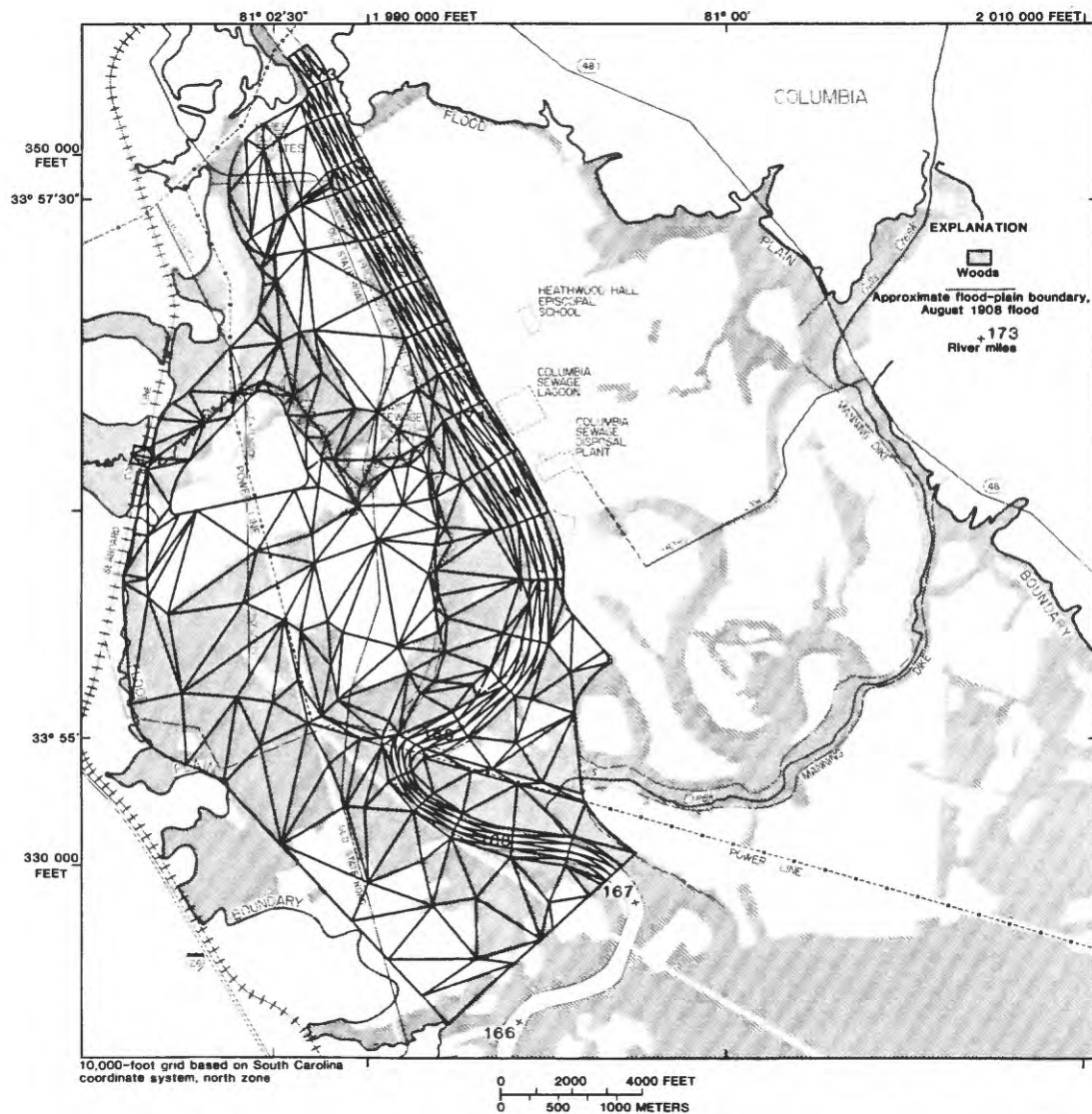


Figure 8.--Finite-element network, Manning dike in place,
August 1908 flood.

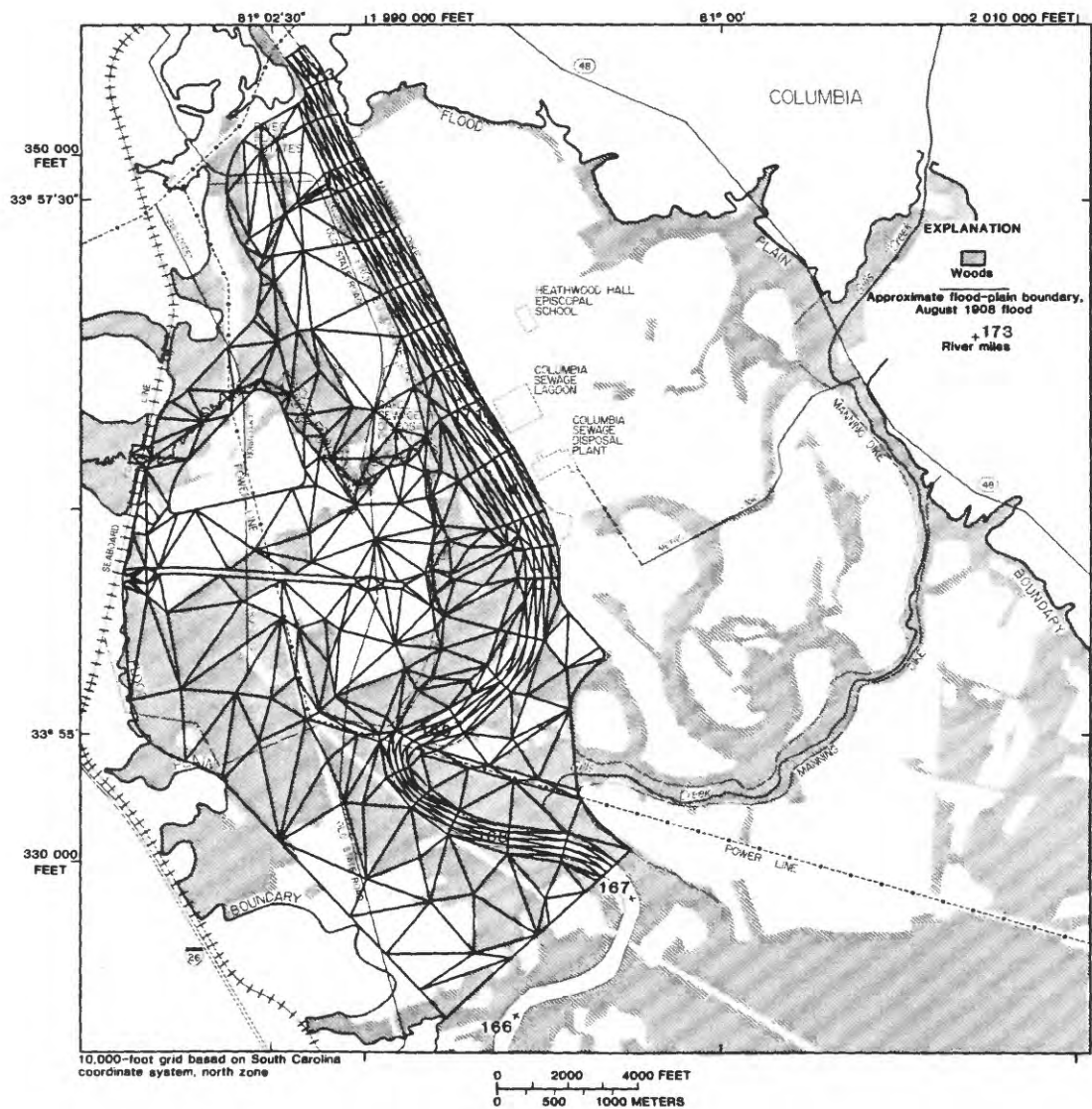


Figure 9.--Finite-element network, Manning dike and
original highway embankments in place,
August 1908 flood.

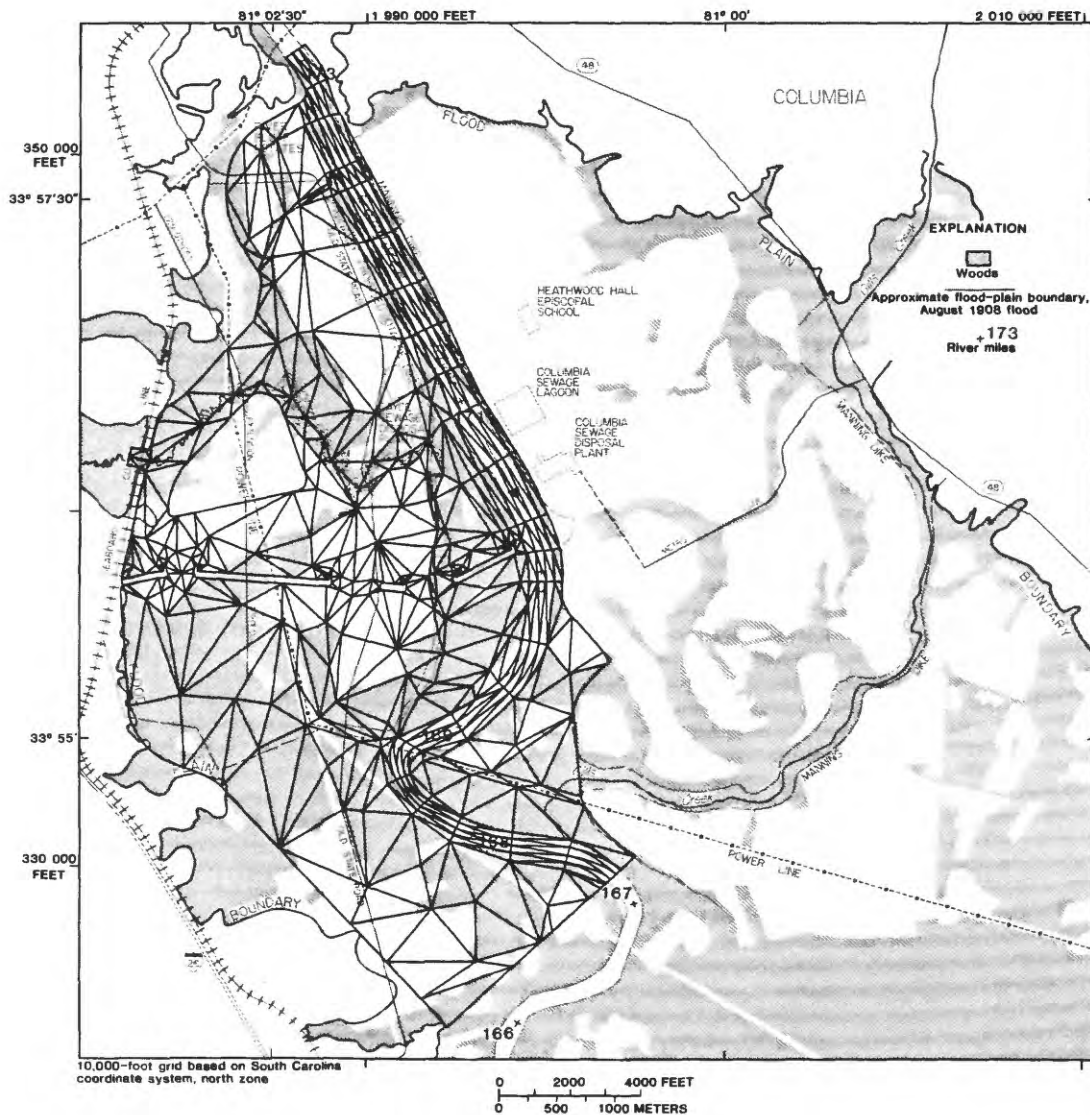


Figure 10.--Finite-element network, Manning dike and revised highway embankments in place, August 1908 flood.

Plate 2 is a plot of the velocity field and water-surface elevations for the run without highway embankments. Plate 3 is a plot of the velocity field and water-surface elevations for the run with the original highway embankments. The continuity-check program option was used to compute the discharge through each of the four bridge openings. Ground- and water-surface elevations across each bridge opening were computed by interpolation from model input data and results. These elevations were then used to compute the cross-sectional area, the average velocity, and the average water-surface elevation at each opening. Together with the maximum velocities at each opening and at the edges of each opening, these are given in table 3.

Plate 4 is a plot of the velocity field and water-surface elevations for the run with the revised highway embankments. The continuity-check option was again used to compute the discharge through each of the four bridge openings. In this case, for each of the openings, the discharge was computed at several cross sections from just upstream of the spur dikes to just downstream of the opening. The computed percentages of the total discharge flowing through bridge 1 are 7.9, 7.7, and 8.4; through bridge 2, 23.8, 25.8, 26.7, and 26.5; through bridge 3, 18.4, 21.1, 18.2, and 20.1; and through bridge 4, 43.2, 43.3, 43.3, and 47.5. The sum of the lowest values is 92.9 percent; the sum of the highest is 103.7 percent. In addition, a continuity check across the entire model gives 99.8 percent one row of elements upstream from the highway crossing and 93.1 percent one row of elements downstream from the crossing. The continuity equation is not satisfied at every node in the Norton-King model, and

Table 3.--Hydraulic properties of the original bridges across the flood plain of the Congaree River with the Manning dike in place for the August 1908 flood

Bridge number	Bridge description	Discharge ¹ (ft ³ /s)	Discharge ¹ (percent of total discharge)	Cross-sectional area (ft ²)	Average velocity (ft/s)	Maximum velocity (ft/s)	Maximum velocity at right edge (ft/s)	Maximum velocity at left edge (ft/s)	Average water-surface elevation above NGVD (ft)
1	Twin overflow bridges ¹	23,240	6.4	6,840	3.4	5.7	2.0	5.7	139.9
2	Twin overpasses over Old State Road (Road S-66)	65,550	18.0	10,400	6.3	9.7	9.7	5.8	139.3
3	Twin bridges over Congaree Creek	72,970	20.0	24,360	3.0	7.5	5.0	2.6	139.9
4	Twin bridges over Congaree River	179,800	49.4	32,750	5.5	7.4	3.0	1.9	140.2

¹The sum of the discharges does not equal the total discharge because the continuity equation is not satisfied at every node.

continuity checks give some indication of the accuracy of the solution (King and Norton, 1978; Gee and MacArthur, 1978; Walters and Cheng, 1978, 1980). Reasonable results at and near the highway crossing were obtained, in spite of the use of a fairly coarse network, by the careful process of network adjustment and refinement discussed above. The discharge obtained from the continuity check following the line closest to the centerline of the roadway at each opening and the other hydraulic properties of each opening are given in table 4.

Water-surface profiles in the main channel for all three runs are shown in figure 7. Figures 11 and 12 are plots of backwater for the original and revised crossing designs. The computer program SACM generated these plots by differencing water-surface elevations obtained from the runs with and without the highway embankments in place. Maximum values of the backwater in the main channel and on the flood plain, together with values of the water-surface elevation and backwater at various man-made encroachments on the flood plain, are given in table 5 for both the original and revised designs.

August 1908 Flood with the Otarre Dike in Place

In the next two runs, it was assumed that the Manning dike was removed and the originally proposed Otarre dike on the right bank of the river was in place (figs. 13, 14). To determine the backwater of the constriction, runs were made both without and with the highway embankments in place. Only the embankments corresponding to the original crossing design were studied. The upstream inflow was again 364,000 ft³/s. Because these two finite-element networks extended slightly farther downstream than did the preceding networks, a downstream water-surface elevation of 133.25 ft NGVD was used. As in all other runs, this downstream boundary

Table 4.--Hydraulic properties of the revised bridges across the flood plain of the Congaree River with the Manning dike in place for the August 1908 flood

Bridge number	Bridge description	Discharge ¹ (ft ³ /s)	Discharge ¹ (percent of total discharge)	Cross-sectional area (ft ²)	Average velocity (ft/s)	Maximum velocity (ft/s)	Maximum velocity at right edge (ft/s)	Maximum velocity at left edge (ft/s)	Average water-surface elevation above NGVD (ft)
1	Twin overflow bridges 1	27,970	7.7	6,710	4.2	5.6	5.1	5.3	139.6
2	Twin overpasses over Old State Road (Road S-66)	93,850	25.8	17,920	5.2	7.4	7.4	6.4	139.6
3	Twin bridges over Congaree Creek	76,920	21.1	24,110	3.2	5.9	5.8	1.7	139.7
4	Twin bridges over Congaree River	172,900	47.5	32,390	5.3	7.2	4.0	1.9	140.1

¹The sum of the discharges does not equal the total discharge because the continuity equation is not satisfied at every node.

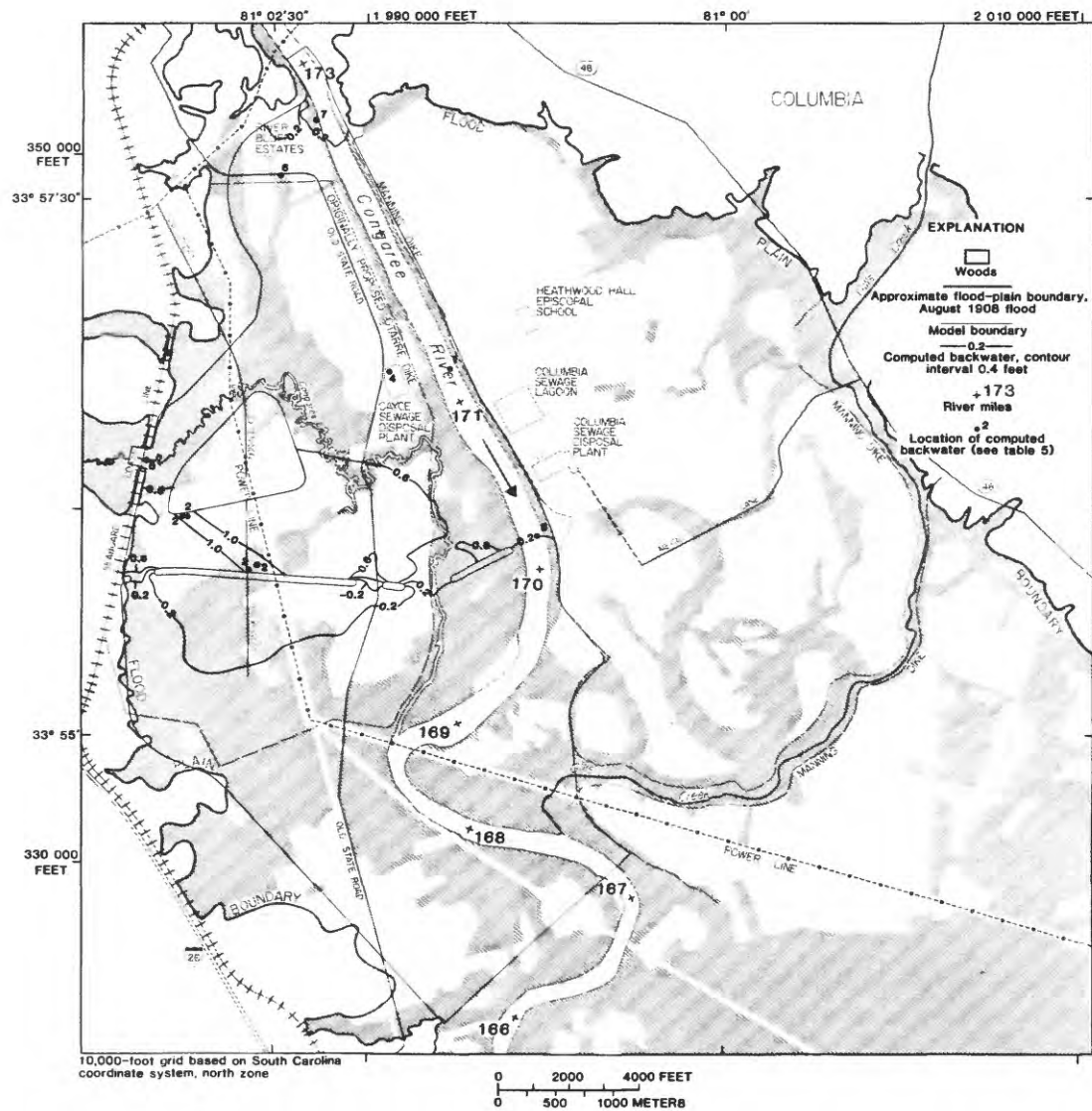


Figure 11.--Backwater, Manning dike and original highway embankments in place, August 1908 flood.

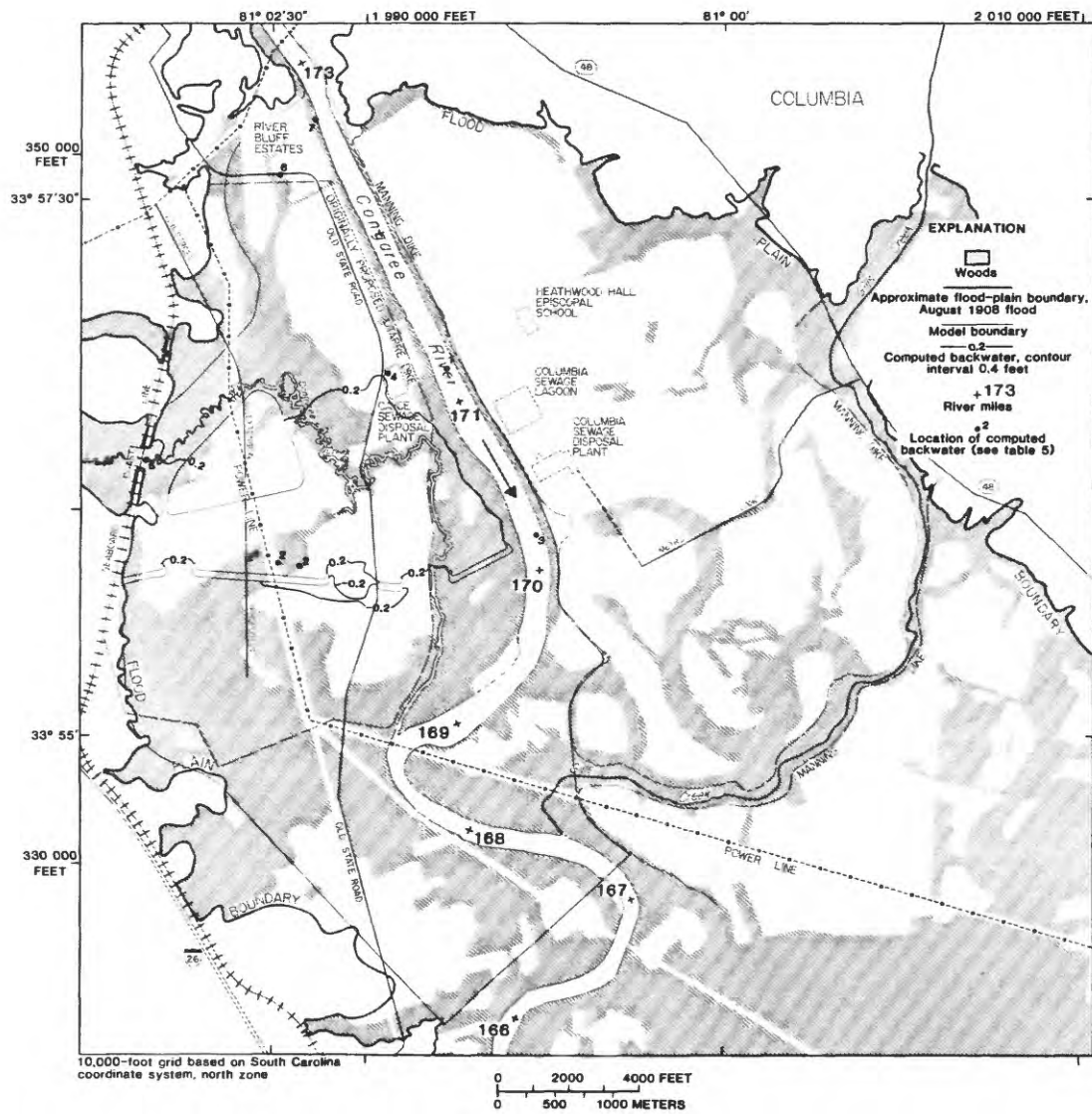


Figure 12.--Backwater, Manning dike and revised highway embankments in place, August 1908 flood.

Table 5.--Water-surface elevations (without and with highway embankments) and backwater at various locations with the Manning dike in place for the August 1908 flood

Location reference number ¹	Location	Water-surface elevation above NGVD without highway embankments (ft)	Water-surface elevation above NGVD with original highway embankments (ft)	Backwater with original highway embankments (ft)	Water-surface elevation above NGVD with revised highway embankments (ft)	Backwater with revised highway embankments (ft)
1	Location of maximum backwater in main channel	141.5	141.9	0.4	141.7	0.2
2	Location of maximum backwater on flood plain	139.8	140.8	1.0	140.3	0.5
3	Main channel at twin bridges over Congaree River ²	140.0	140.2	0.2	140.1	0.1
4	Cayce sewage disposal plant	141.7	142.2	0.5	141.9	0.2
5	Seaboard Coast Line bridge over Congaree Creek	141.7	142.2	0.5	141.9	0.2
6	River Bluff Estates, south side	144.4	144.7	0.3	144.5	0.1
7	River Bluff Estates, east side	145.8	146.0	0.2	145.9	0.1

¹Location reference numbers are shown on plates 3 and 4 and figures 11 and 12.

²The water-surface elevation in the main channel at the twin bridges is an average across the main channel in the case with no highway embankments and an average across the bridge opening in the cases with highway embankments.

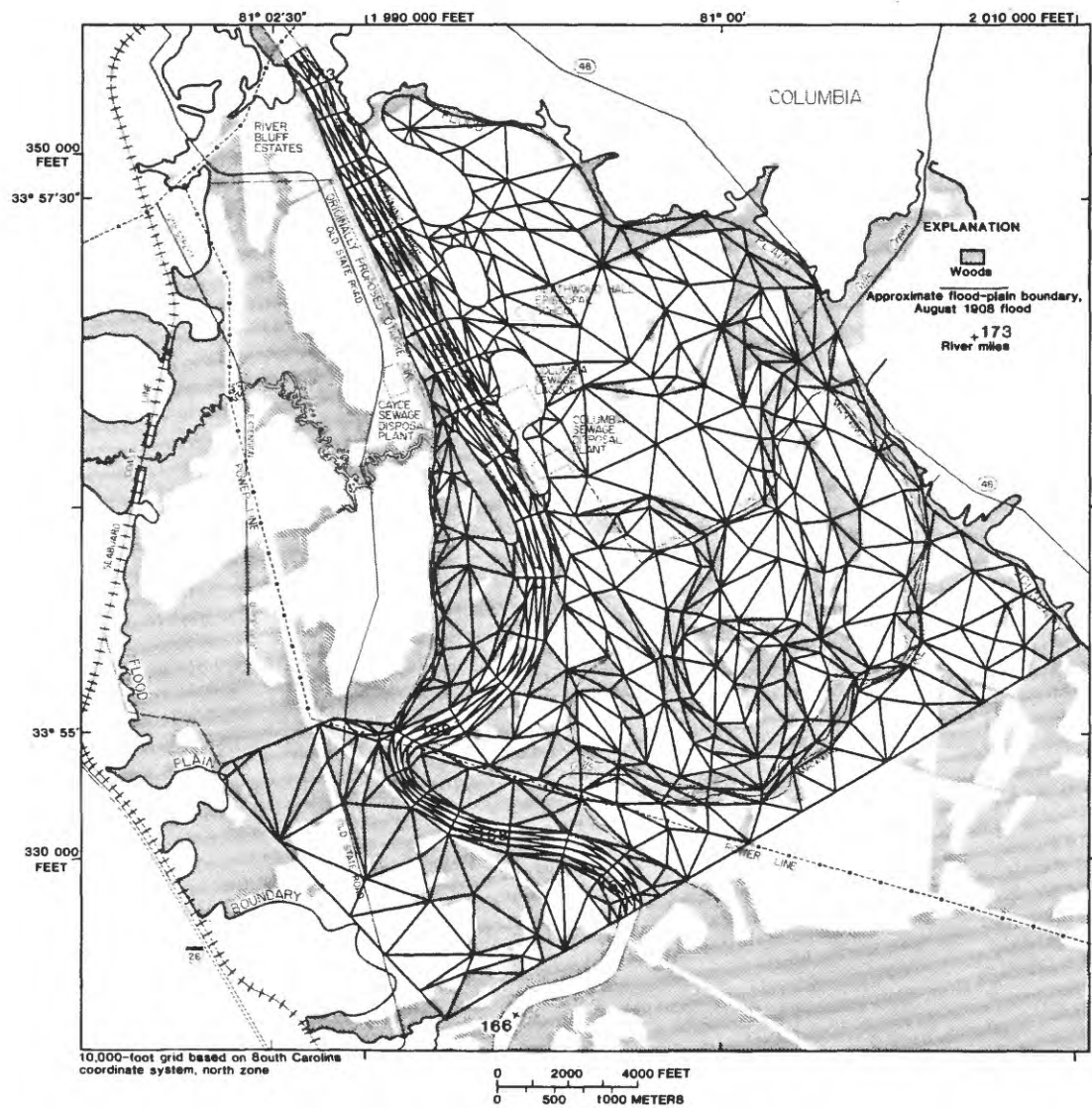


Figure 13.--Finite-element network, Otarre dike in place,
August 1908 flood.

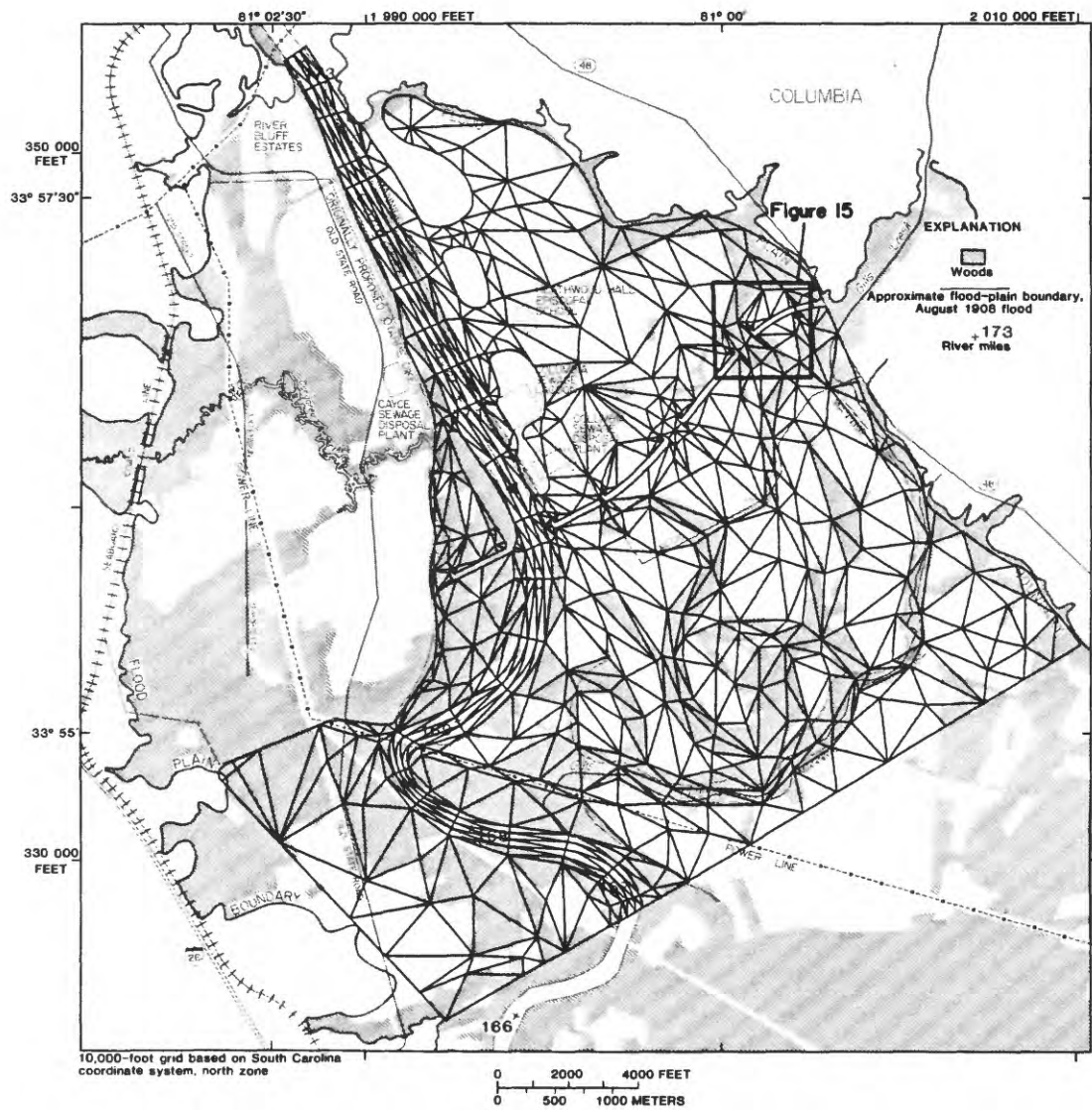


Figure 14.--Finite-element network, Otarre dike and original highway embankments in place, August 1908 flood.

condition was estimated from one-dimensional water-surface profiles calculated by the step-backwater method under the assumption that the Manning dike was in place. Because of the greater width of the flood plain at the downstream end of these models, the correct downstream water-surface elevation would be slightly lower than that used here.

The capability to model different length scales in a single network is one of the advantages of the finite-element method. In this run, for example, open-channel flow through a 24-foot-wide opening corresponding to three 8-foot-by-8-foot reinforced-concrete box culverts at the left edge of the flood plain (fig. 15) was successfully modeled. (Although the culverts were completely submerged, the flow through them was computed as open-channel flow because the finite-element model does not have the capability to model pressure flow through submerged culverts.)

The network with the highway embankments in place was the largest in this study, with 1,000 elements, 2,195 nodes, and 4,611 equations, and required approximately seven megabytes of core on the IBM 3033.

Plate 5 is a plot of the velocity field and water-surface elevations for the run without the highway embankments. Plate 6 is a similar plot for the run with the highway embankments in place. The hydraulic properties of each bridge opening were calculated as for the runs discussed above and are given in table 6.

The main-channel water-surface profiles are shown in figure 7. Figure 16 is a plot of the backwater. Maximum values of the backwater in the main channel and on the flood plain, as well as values of the water-surface elevation and backwater at various man-made encroachments on the flood plain, are given in table 7.

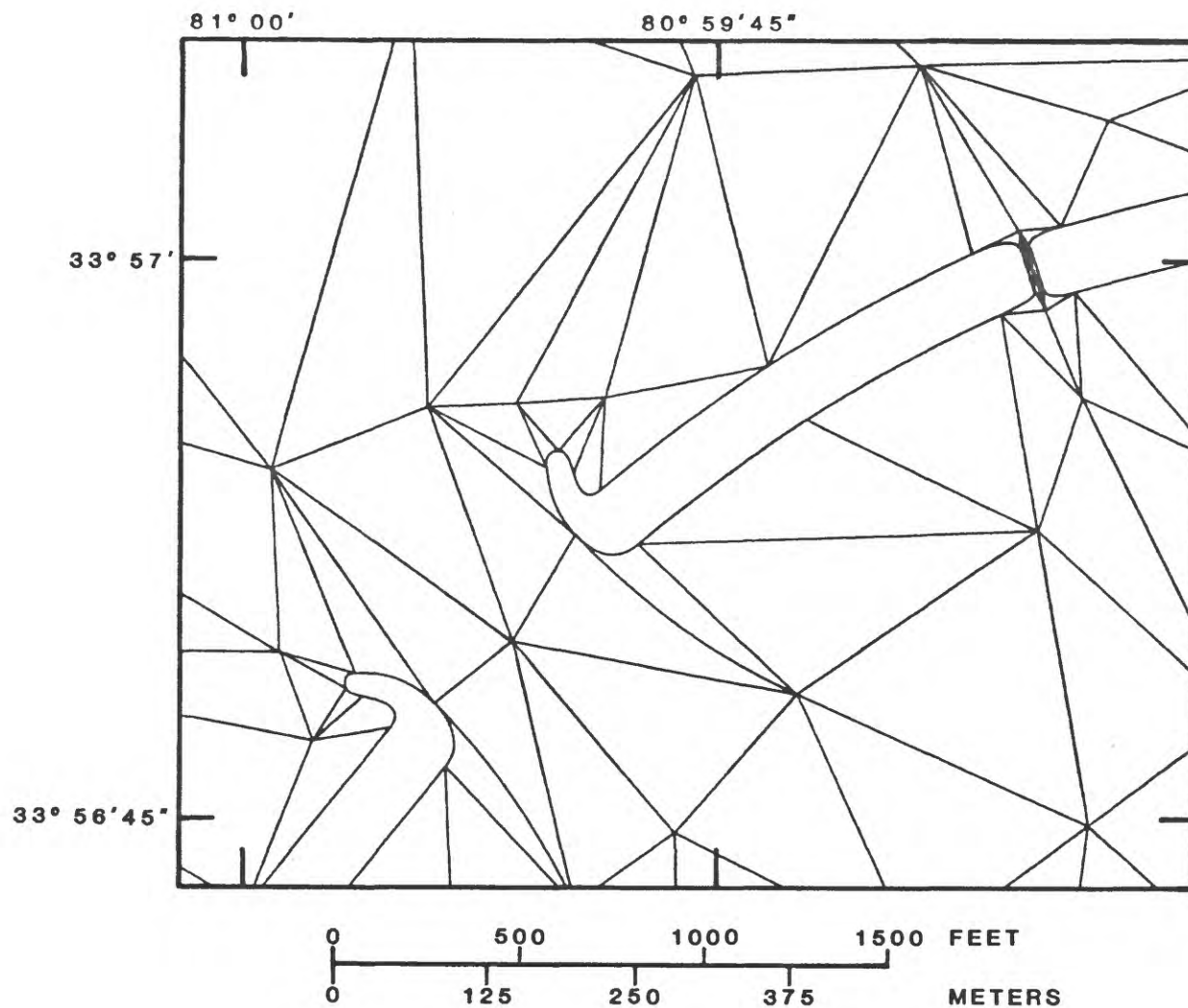


Figure 15.--Section of the finite-element network illustrating the definition of highway embankments, spur dikes, and a culvert using quadratic isoparametric elements, Otarre dike and original highway embankments in place, August 1908 flood. The location of this section is shown in figure 14.

Table 6.--Hydraulic properties of the original bridges across the flood plain of the Congaree River with the

Otarre dike in place and the Manning dike removed for the August 1908 flood

Bridge number	Bridge description	Discharge ¹ (ft ³ /s)	Discharge ¹ (percent of total discharge)	Cross-sectional area (ft ²)	Average velocity (ft/s)	Maximum velocity (ft/s)	Maximum velocity at right edge (ft/s)	Maximum velocity at left edge (ft/s)	Average water-surface elevation above NGVD (ft)
3	Twin bridges over Congaree Creek	11,980	3.3	10,310	1.2	1.4	1.0	1.1	135.1
4	Twin bridges over Congaree River	237,600	65.3	27,580	8.6	10.9	3.9	3.6	135.6
5	Twin bridges over Metro Lane	3,780	1.0	360	10.5	10.9	9.9	10.8	133.8
6	Twin overflow bridges 2	81,990	22.5	6,660	12.3	13.9	12.7	12.1	134.2
7	Twin overflow bridges 3	29,420	8.1	9,030	3.3	4.0	4.0	3.7	135.9
Culvert	Three 8-foot-by-8-foot reinforced-concrete box culverts ²	1,190	0.3	210	5.6	5.6	5.5	5.5	135.9

¹The sum of the discharges does not equal the total discharge because the continuity equation is not satisfied at every node.

²Although the culverts were completely submerged, the flow through them was computed as open-channel flow because the finite-element model does not have the capability to model pressure flow through submerged culverts.

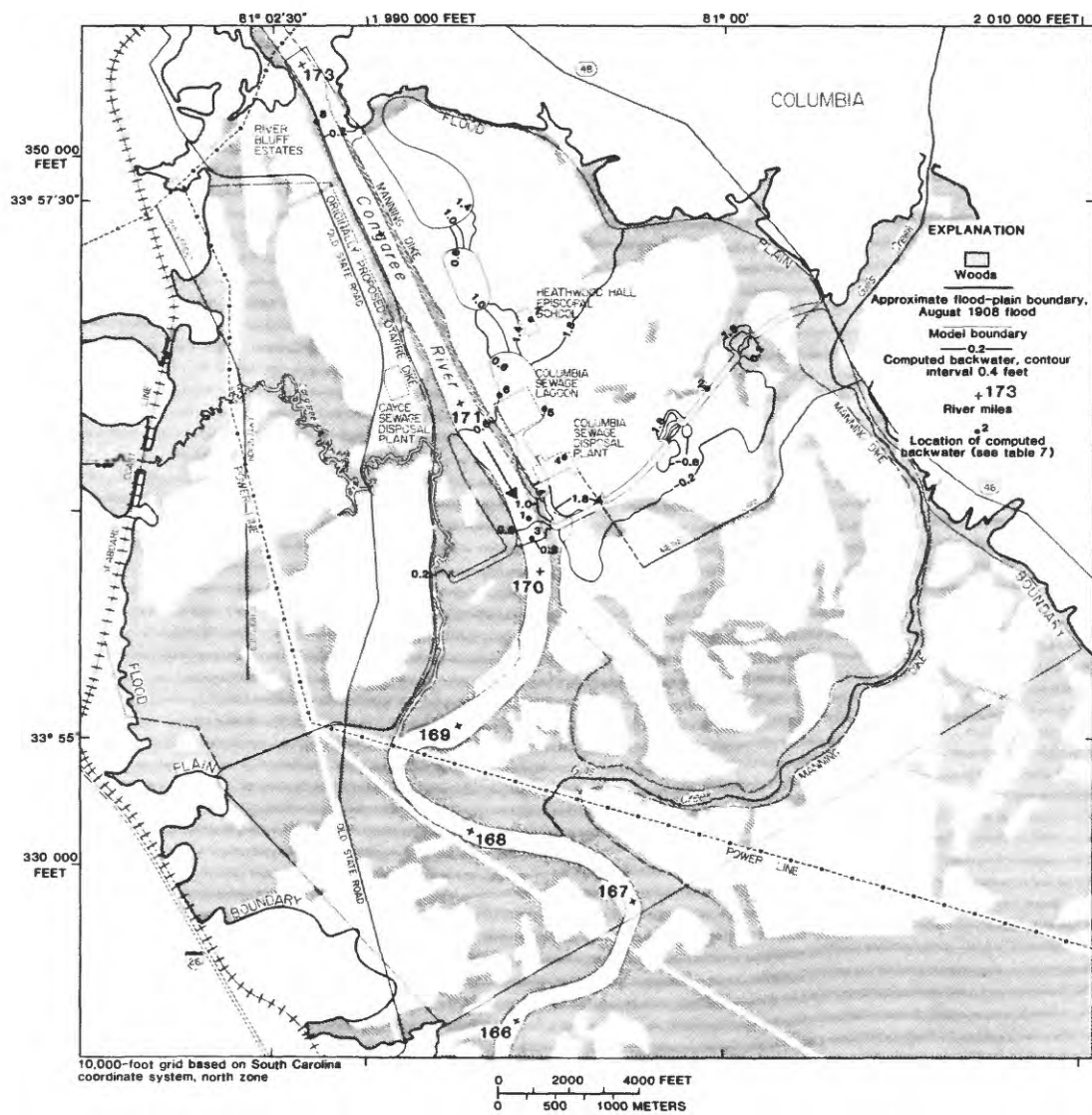


Figure 16.--Backwater, Otarre dike and original highway embankments in place, August 1908 flood.

Table 7.--Water-surface elevations (without and with highway embankments)

and backwater at various locations with the Otarre dike in place and the Manning dike removed for the August 1908 flood

Location reference number ¹	Location	Water-surface elevation above NGVD without highway embankments (ft)	Water-surface elevation above NGVD with highway embankments (ft)	Backwater (ft)
1	Location of maximum backwater in main channel	135.6	136.3	0.7
2	Location of maximum backwater on flood plain	135.1	137.2	2.1
3	Main channel at twin bridges over Congaree River ²	135.5	135.6	0.1
4	Columbia sewage disposal plant, building	135.1	137.1	2.0
5	Columbia sewage disposal plant, lagoon, southeast corner	135.2	137.1	1.9
6	Columbia sewage disposal plant, lagoon, northwest corner	138.3	138.9	0.6
7	Heathwood Hall Episcopal School	136.0	137.4	1.4
8	River Bluff Estates, east side	145.5	145.6	0.2

¹Location reference numbers are shown on plate 6 and figure 16.

²The water-surface elevation in the main channel at the twin bridges is an average across the main channel in the case with no highway embankments and an average across the bridge opening in the case with highway embankments.

The main-channel water-surface profiles are shown in figure 7. In the main channel above the bridge, the maximum increase in the water surface caused by the highway embankments is 0.7 ft. However, water-surface elevations across the entire left flood plain are increased by approximately 2.0 ft by the highway embankments. This lateral variation in the backwater of the constriction could not be detected by a one-dimensional model.

A high-water elevation of 136.5 ft NGVD was established during the 1908 flood at the location shown on plate 5. The computed water-surface elevation without the highway embankments in place is 135.0 ft. Lowering the downstream water-surface elevation and removing the Otarre dike in the model would increase the difference of 1.5 ft even more. However, comparison of the computed and observed values is difficult because it is not known how much of the flood plain was wooded in 1908. It is likely that much more was wooded in 1908 than today. Thus, the values of the Chezy coefficients would be lower for 1908 conditions, and water-surface elevations would be higher.

In response to the backwater of about 2.0 ft on the left flood plain and the high velocities at both edges of bridges 5 and 6, the South Carolina Department of Highways and Public Transportation revised the designs of bridges 6 and 7 (table 1, fig. 3). The model was not run for the revised bridges with the Otarre dike in place and the Manning dike removed.

August 1908 Flood with the Manning and the Otarre Dikes in Place

In the final set of runs, the dikes on both sides of the channel were in place (figs. 17, 18). Runs were made without and with the highway

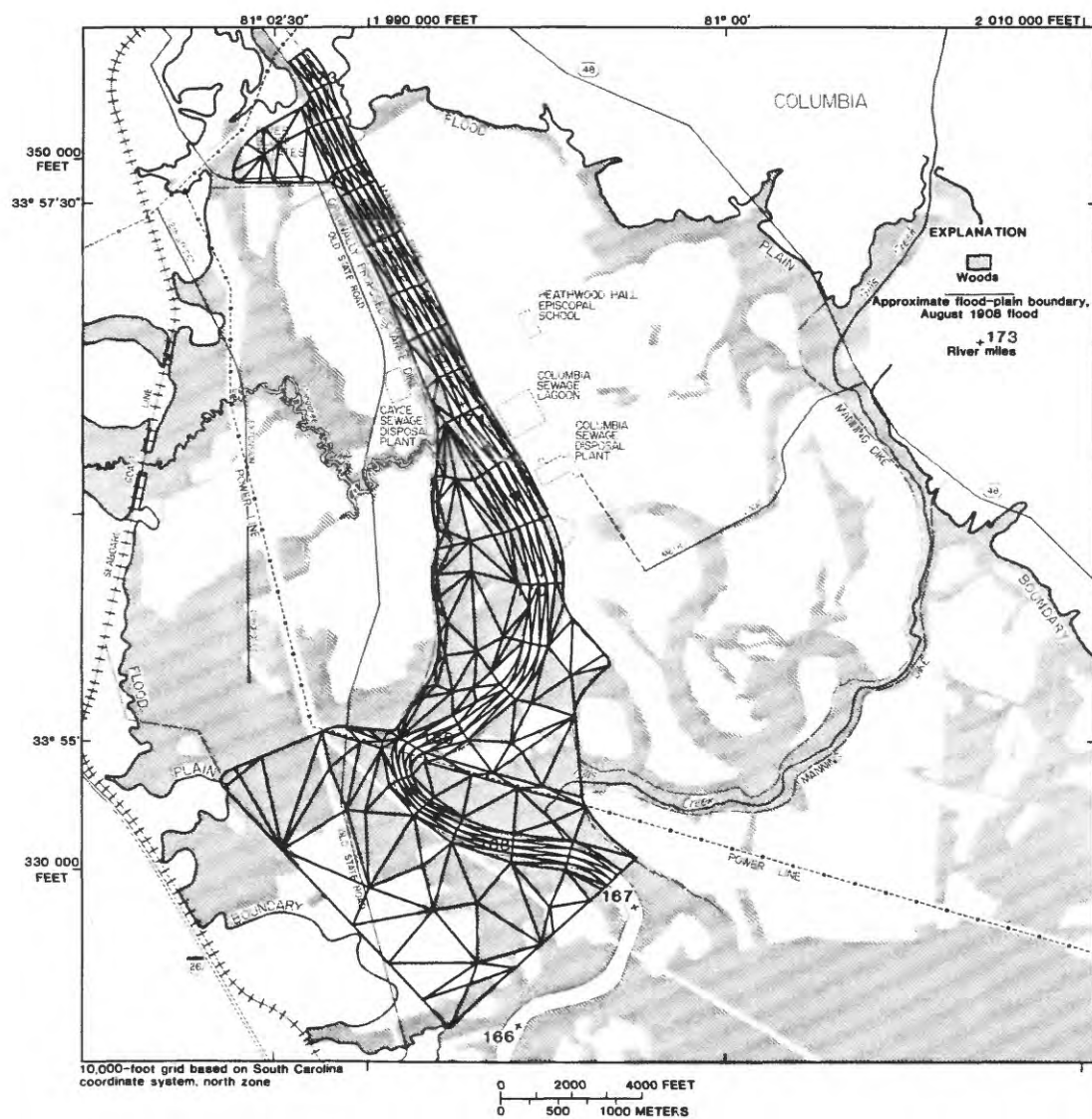


Figure 17.--Finite-element network, Manning and Otterre
dikes in place, August 1908 flood.

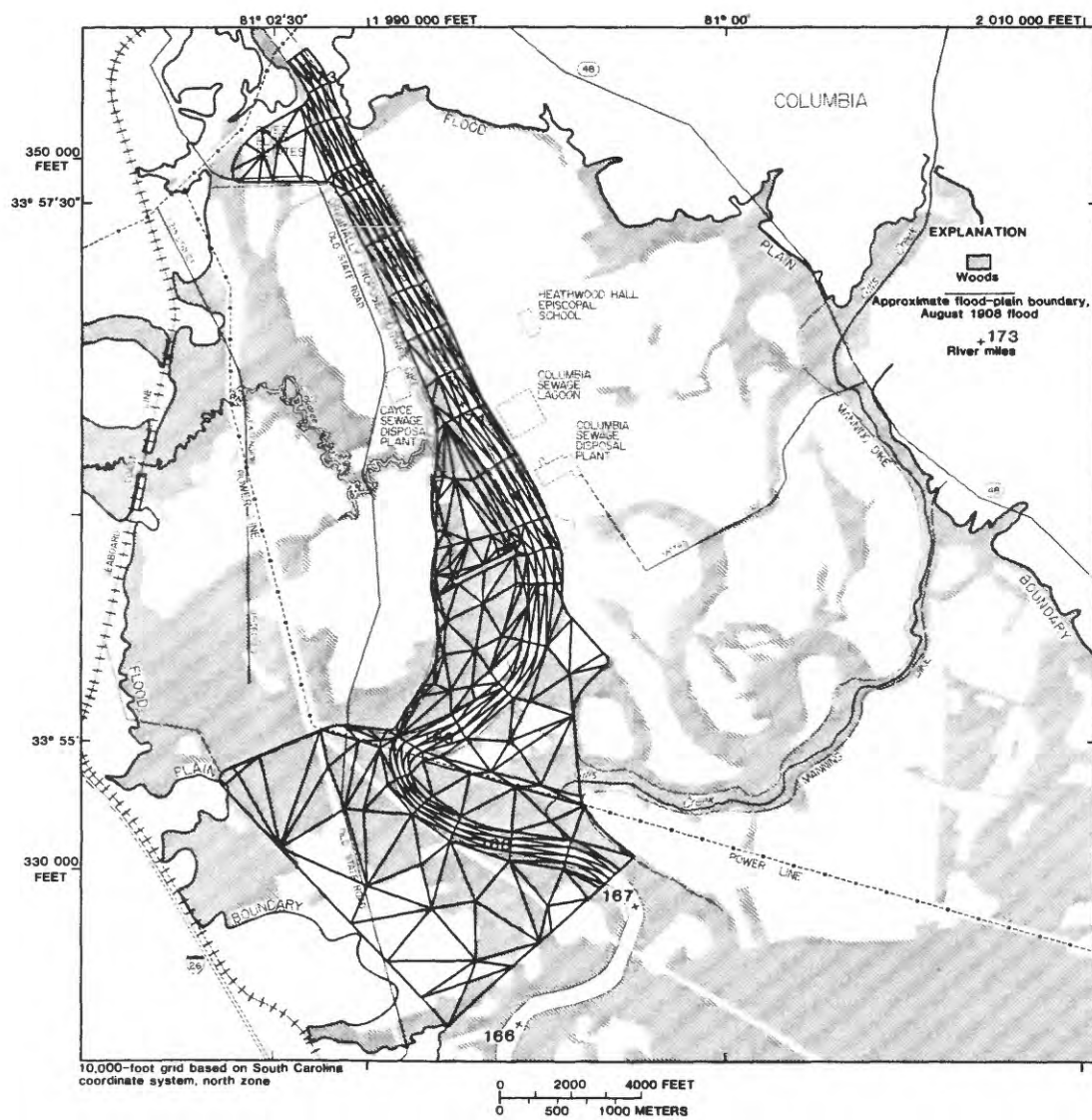


Figure 18.--Finite-element network, Manning and Otarre dikes
and highway embankments in place, August 1908 flood.

embankments in place. Neither of the bridges involved for this dike configuration was modified by the South Carolina Department of Highways and Public Transportation. The upstream inflow was 364,000 ft³/s, and the downstream water-surface elevation was 133.85 ft NGVD.

Plates 7 and 8 are plots of the velocity fields and water-surface elevations for the runs without and with the highway embankments in place, respectively. The hydraulic properties of each bridge opening were calculated as before and are given in table 8.

The water-surface profiles in the main channel are shown in figure 7. Figure 19 is a plot of the backwater. The maximum values of the backwater in the main channel and on the flood plain, as well as values of the water-surface elevation and backwater at various man-made encroachments, are given in table 9.

Relative Effects of the Dikes and Highway Embankments

As throughout this report, the Otarre dike discussed below is the originally proposed Otarre dike. The water-surface elevations are those computed with the highway embankments in place for the revised design of bridge 2 and the original designs of bridges 6 and 7.

As shown in figure 7, the Manning dike alone causes a greater increase in the water-surface elevation at and near the proposed main-channel crossing of Interstate Route 326 than does the Otarre dike alone. At the main-channel crossing of I-326, the water-surface elevation with the Otarre dike in place is 135.6 ft NGVD; with the Manning dike in place, 140.1 ft NGVD; and with both dikes in place, 142.2 ft NGVD. Thus, the water-surface elevation at the crossing is 4.5 ft higher with the Manning dike alone in place than with the Otarre dike alone in place, and the presence of both dikes causes the water-surface elevation there to

Table 8.--Hydraulic properties of the bridges across the flood plain of the Congaree River with both the Manning
and Otterre dikes in place for the August 1908 flood

Bridge number	Bridge description	Discharge ¹ (ft ³ /s)	Discharge ¹ (percent of total discharge)	Cross- sectional area (ft ²)	Average velocity (ft/s)	Maximum velocity (ft/s)	Maximum velocity at right edge (ft/s)	Maximum velocity at left edge (ft/s)	Average water- surface elevation above NGVD (ft)
3	Twin bridges over Congaree Creek	53,800	14.8	14,190	3.8	4.6	2.8	4.5	141.4
4	Twin bridges over Congaree River	318,100	87.4	35,900	8.9	12.3	6.9	3.3	142.2

¹The sum of the discharge does not equal the total discharge because the continuity equation is not satisfied at every node.

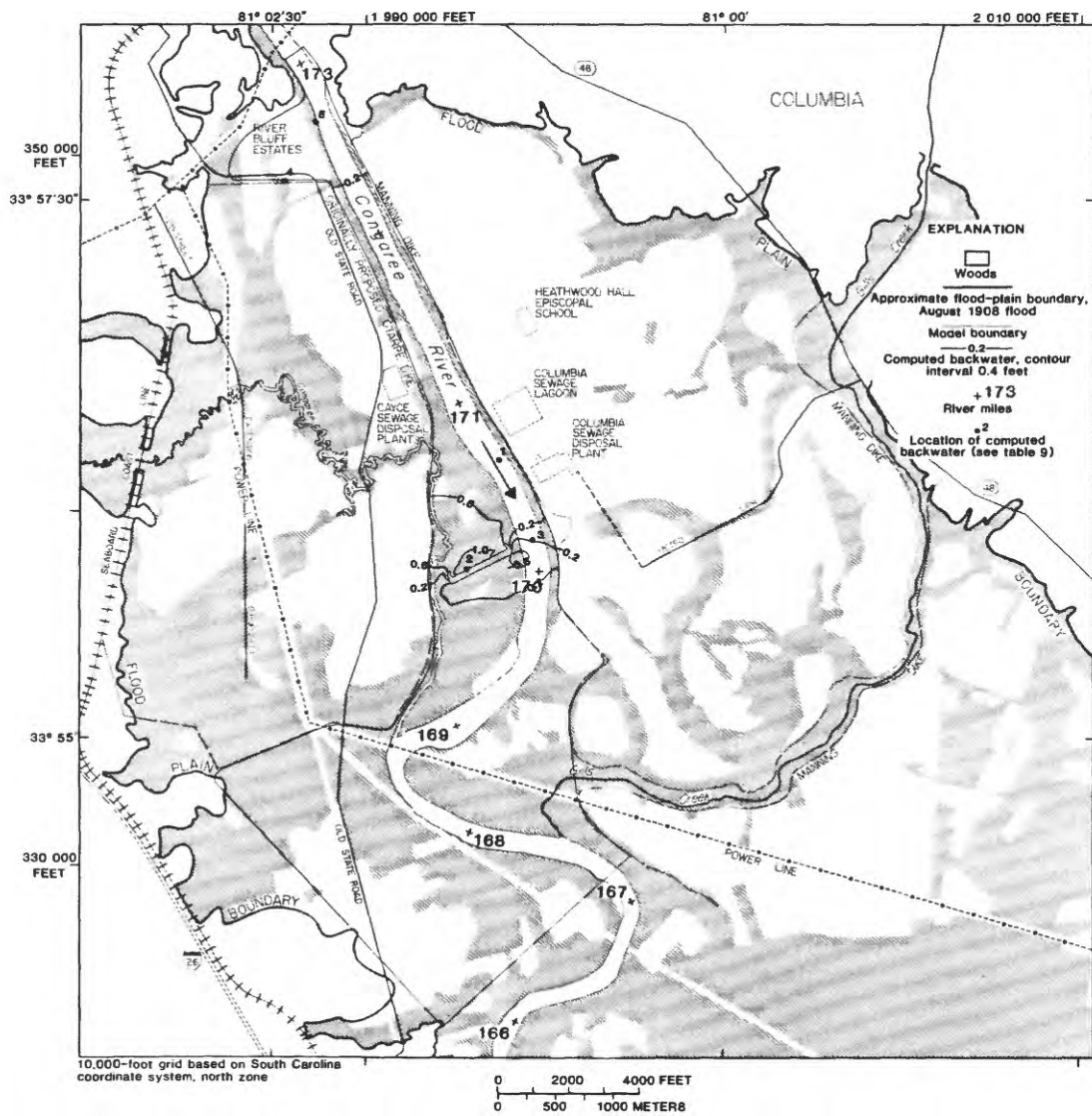


Figure 19.--Backwater, Manning and Otarre dikes and highway embankments in place, August 1908 flood.

Table 9.--Water-surface elevations (without and with highway embankments)

and backwater at various locations with the Manning and Otarre dikes
in place for the August 1908 flood

Location reference number ¹	Location	Water-surface elevation above NGVD without highway embankments (ft)	Water-surface elevation above NGVD with highway embankments (ft)	Backwater (ft)
1	Location of maximum backwater in main channel	144.1	144.5	0.4
2	Location of maximum backwater on flood plain	141.5	142.7	1.2
3	Main channel at twin bridges over Congaree River ²	142.4	142.2	-0.2
4	River Bluff Estates, south side	154.0	154.2	0.2
5	River Bluff Estates, east side	154.1	154.3	0.2

¹Location reference numbers are shown on plate 8 and figure 19.

²The water-surface elevation in the main channel at the twin bridges is an average across the main channel in the case with no highway embankments and an average across the bridge opening in the case with highway embankments.

increase an additional 2.1 ft above the water-surface elevation computed with the Manning dike alone in place. The situation at the east side of River Bluff Estates is different. There the presence of either the Manning dike alone or the Otarre dike alone makes little difference in the water-surface elevation: 145.6 ft NGVD for the Otarre dike and 145.9 ft NGVD for the Manning dike. However, the presence of both dikes causes the water-surface elevation to increase to 154.3 ft NGVD, 8.4 ft higher than the water-surface elevation caused by the presence of the Manning dike alone. At the south side of River Bluff Estates, the presence of both dikes causes the water-surface elevation to increase 9.7 ft above the water-surface elevation caused by the presence of the Manning dike alone.

The models indicate that the effect of the highway embankments is everywhere less than 1.2 ft except on the left flood plain for the case with the Otarre dike alone in place. In that case, with embankments corresponding to the original designs of bridges 6 and 7, the computed backwater on the left flood plain is as much as 2.1 ft. On the basis of these results, bridges 6 and 7 were revised, but the model was not run for these revisions.

SUMMARY AND CONCLUSIONS

The use of a two-dimensional finite-element model to study the hydraulic effect of a highway crossing yields a more detailed evaluation of flow distribution and water-surface elevations than can be obtained by one-dimensional step-backwater and conveyance techniques.

The capability to describe topography, model boundaries, highway embankments, and spur dikes using quadratic isoparametric elements is an important advantage of the Norton-King finite-element model. One of the

most troublesome problems in model application is the difficulty in determining the amount of network detail necessary to avoid local errors caused by large ground-surface gradients, large roughness variations, or both. In general, the model gives reasonable results if a time-consuming and expensive trial-and-error process of network refinement is possible. The need to use arbitrary values of the eddy viscosities must also be considered a disadvantage of the model.

The results of this study suggest that correct and effective application of this model requires considerable experience in its use. For modeling flow on flood plains of highly variable roughness, the testing or development, or both, of other finite-element approaches and comparison with the Norton-King model are suggested, with particular emphasis on the capability of the models to respond accurately to large topographic gradients and highly variable roughness.

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