

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

In Situ Geomechanics of Crystalline and Sedimentary Rocks, Part II:
Two BASIC Computer Programs for the Determination of In Situ
Stresses Using the CSIRO Hollow Inclusion Stress
Cell and the USBM Borehole Deformation Gage

By

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Preface

This report is the second of a series summarizing the results of the U.S. Geological Survey's research program in geomechanics aimed at investigating and assessing the potential of crystalline and sedimentary rock masses as geologic repositories of nuclear waste. The initial report was by W. Z. Savage and H. S. Swolfs, 1980, The long-term deformation and time-temperature correspondence of visco-elastic rocks -- an alternative theoretical approach: U. S. Geological Survey Open-File Report 80-708.

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ABSTRACT

The mathematical method of determining in-situ stresses by overcoring, using either the U.S. Bureau of Mines Borehole Deformation Gage or the Commonwealth Scientific and Industrial Research Organisation Hollow Inclusion Stress Cell, is summarized, and data reduction programs for each type of instrument, written in BASIC, are presented. The BASIC programs offer several advantages over previously available FORTRAN programs. They can be executed on a desk-top microcomputer at or near the field site, allowing the investigator to assess the quality of the data and make decisions on the need for additional testing while the crew is still in the field. Also, data input is much simpler than with currently available FORTRAN programs; either English or SI units can be used; and standard deviations of the principal stresses are computed as well as those of the geographic components.

INTRODUCTION

The determination of the in-situ state of stress in a rock mass is one of the most important problems in geomechanics, both for the design of underground openings and for assessing the effects of external influences, for example, heating by stored radioactive material. A common method of stress determination is the overcoring method: a small-diameter borehole is drilled in a rock mass; an instrument for measuring strain or deformation in several directions is placed in the borehole; and a larger hole is then drilled around the instrument. Because the specimen is now released from the in-situ stress field, the resulting deformations give a measure of the amount of stress relieved.

Two instruments in common use today are U.S. Bureau of Mines (USBM) Borehole Deformation Gage and the Commonwealth Scientific and Industrial Research Organisation (CSIRO, Australia) Hollow Inclusion Stress Cell.

The USBM instrument, developed in the mid-1960's, measures the change in borehole diameter along three directions in a single plane perpendicular to the borehole axis. Measurements in three or more nonparallel boreholes are required for a complete stress determination.

The CSIRO cell utilizes three strain gage rosettes mounted around the circumference of a hollow epoxy cylinder which is cemented to the borehole wall prior to overcoring. Because of the three-dimensional orientation of the nine strain gages, it is possible to obtain a complete stress determination in a single borehole.

The mathematical solution for the stress components is rather complex and a digital computer is required to facilitate the reduction of the field measurements. In the past, the usual procedure has been to record the data in the field, return to the office, and perform the data reduction using a FORTRAN program and a large mainframe computer. Recent advances in computer technology and the availability of desk-top microcomputers have made it possible to perform immediate data reduction onsite in a suitably equipped instrument truck or van, and in addition to record the instrument response in real time during the overcoring. In the field, this capability allows the investigator to analyze the data immediately, to assess its quality, and to make decisions to either abandon a particular test or to conduct additional tests while the drilling crew is still onsite. Most microcomputers in common use are programable in BASIC. This report describes two BASIC data-reduction programs, one for use with the USBM Borehole Deformation Gage and the other for the CSIRO Hollow Inclusion Stress Cell. These programs offer the following advantages over the previously available FORTRAN programs:

1. They can be run on a desk-top microcomputer which can be transported to the field site in a van or trailer, allowing data analysis to be performed onsite.
2. The programs are shorter, although slightly more specialized, because the built-in matrix routines in BASIC eliminate the need for many subroutines.
3. Data input is greatly simplified by assigning the known orientation angles to the individual strain gages.
4. The programs work with any consistent set of units (English or metric).
5. Standard deviations of the principal stresses are computed, as well as those of the geographic components.

Coordinate Systems and Conventions

Two right-handed orthogonal coordinate systems are used in the analysis of stress determination by use of the USBM and CSIRO cells. The first of these is a local, or borehole coordinate system, h_1, h_2, h_3 , as defined by Panek (1966) and illustrated in figure 1. The h_2 axis coincides with the axis of the borehole. The h_1 axis is horizontal and extends to the right as the observer looks into the borehole. The h_3 axis lies in a vertical plane and generally points upward. The h_3 axis is truly vertical only if the borehole is horizontal; it is horizontal for a vertical drill hole. The h_1 axis will coincide with the right-hand handle of the setting tool when either cell is properly oriented in the hole. This is important because constant measurement angles for each of the two types of instrument are referred to this axis.

The second coordinate system is the global, or geographic system, x, y, z , which gives a common base to which the computations are referred. The system used here is the one most commonly used by engineers and surveyors; that is, x is positive east, y is positive north, and z is positive up.

Bearings are given in degrees clockwise from north, and inclinations are in degrees from the horizontal, positive upwards.

For purposes of this analysis it is necessary to define the relationship between the borehole coordinate system and the geographic coordinate system in terms of a direction cosine matrix

$$\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

where l_1 is the cosine of the angle between the $+x$ and $+h_1$ axes, m_1 the cosine of the angle between the $+y$ and h_1 axes, . . . , and n_3 the cosine of the angle between the $+z$ and h_3 axes. For the coordinate systems and sign

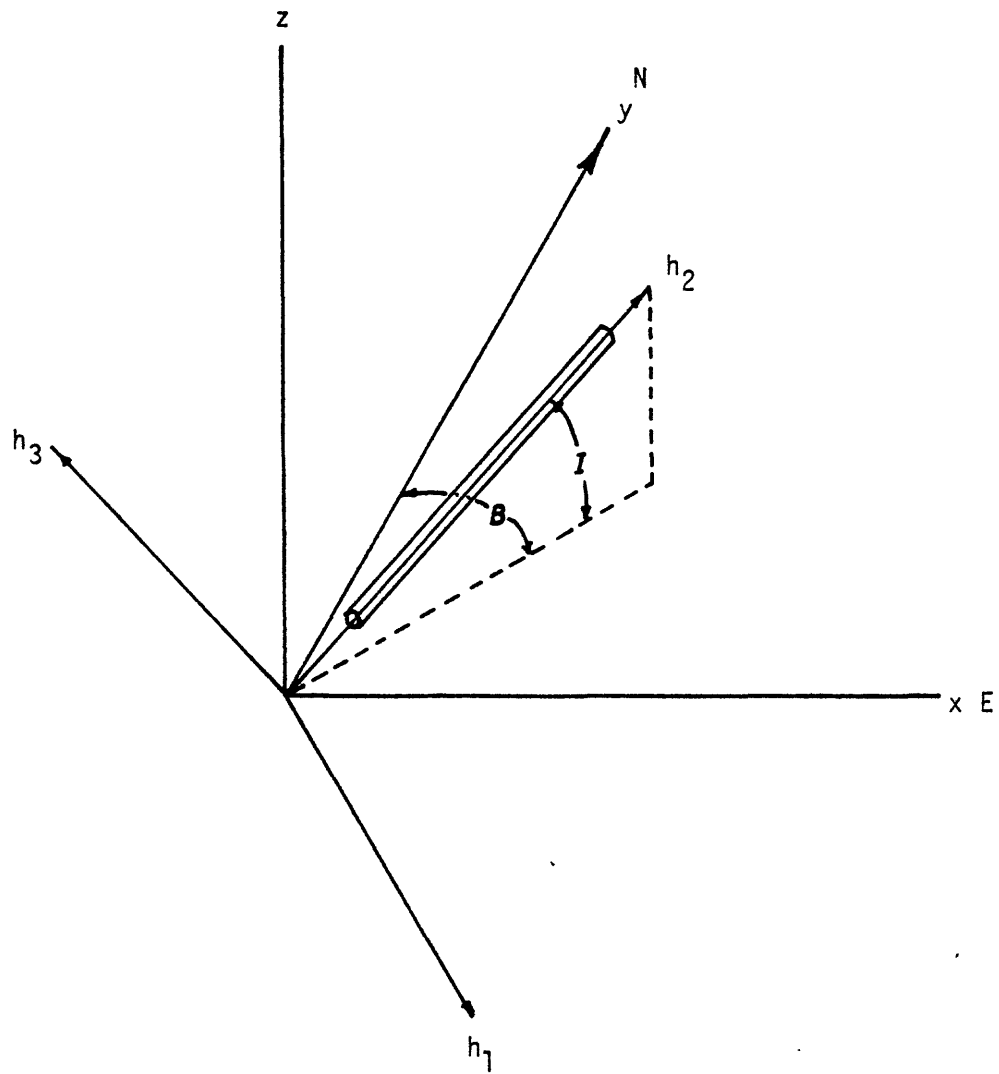


Figure 1. Coordinate systems and sign conventions.

conventions used here, this relationship is

$$\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{matrix} x \\ y \\ z \end{matrix} \begin{matrix} h_1 & h_2 & h_3 \\ \cos B & \sin B \cos I & -\sin B \sin I \\ -\sin B & \cos B \sin I & -\cos B \sin I \\ 0 & \sin I & \cos I \end{matrix} \quad (1)$$

where B is the bearing and I is the inclination of the borehole.

A third, implied, coordinate system is the principal stress system 1-2-3, which is used in order to compute the standard deviations of the principal stresses.

The sign convention for stresses, strains, and displacements (tension positive or negative) is up to the user. It should be noted that the overcoring method measures released stress and that the in-situ stress will be of opposite sign. This can be taken into account either by changing the signs on the printed output or by changing the signs of the input data so that the printed results represent the in-situ stresses.

Standard matrix notation is used throughout this report. An element of a two-dimensional array will be expressed in the form c_{ij} where i is the row number and j is the column number. Tensor notation is not used; that is, c_{ij} represents a diagonal element of a matrix ($i=j$) and does not represent summation from $i = 1$ to 3. Three-dimensional tensor quantities such as stress will be represented in two-dimensional matrix form.

Gage Orientation Angles

With the USBM Borehole Deformation Gage, displacements are measured across three diameters in a plane perpendicular to the borehole axis. The orientation of these measurements is specified by the angle θ which is measured to the diametral axis from the h_1 axis toward the h_3 axis in the h_1 - h_3 plane, as shown in figure 2. The orientation of the instrument in the borehole is controlled by the orientation lugs and the setting tool. The program as written assumes the orientation shown in figure 2(a). It is also possible, by orienting the lugs in a vertical plane, to have the orientation shown in figure 2(b), which would require changing three statements in the program.

Because of the three-dimensional nature of the CSIRO cell, two angles, α and β , are required to specify the orientation of each of the nine strain gages. The angle α is the circumferential position of the gage, measured clockwise from the $+h_3$ axis toward the $+h_1$ axis; β is the angle of the gage with respect to the borehole ($+h_2$) axis. The layout of the strain gages for the CSIRO cell is shown in figure 3 and the orientation angles for each of the nine gages for the standard installation (center of #6 gage at bottom of hole) is given in table 1.

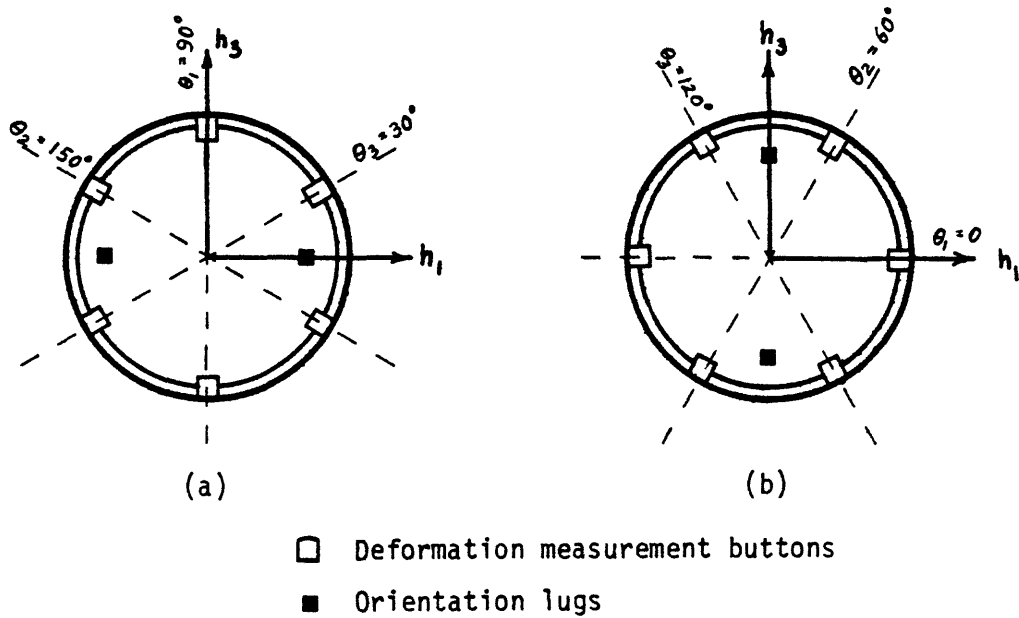


Figure 2. Orientation angles for USBM Borehole Deformation Gage.
 (a) Standard installation assumed in data reduction program
 (b) Alternative installation

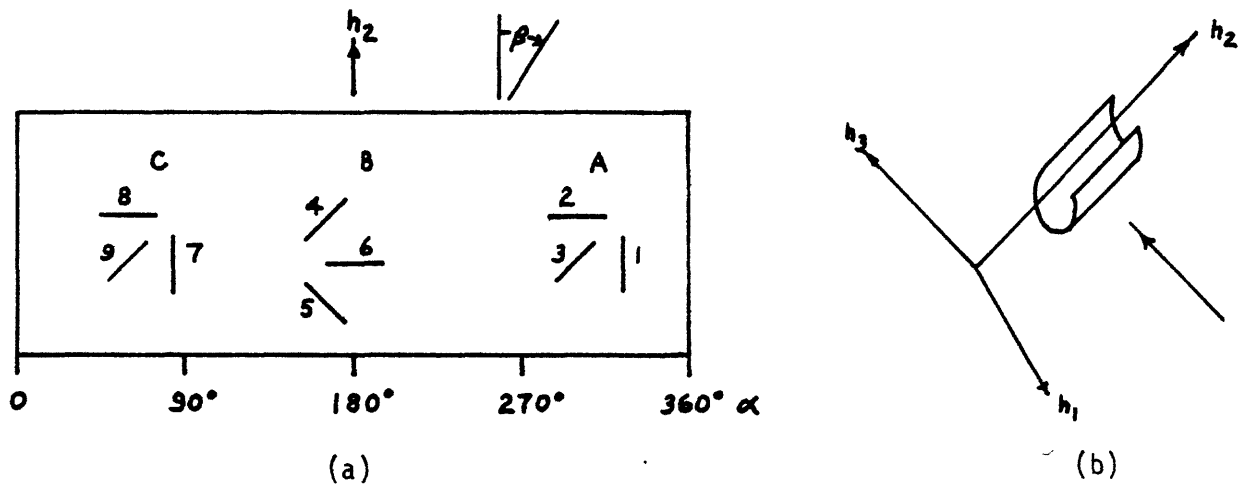


Figure 3. Orientation angles for CSIRO Hollow Inclusion Stress Cell.
 (a) Gage layout on surface of cylinder
 (b) View direction for (a)

Table 1. Orientation angles for strain gages in CSIRO Hollow Inclusion Stress Cell (from Commonwealth Scientific and Industrial Research Organisation, 1979, p. 23)

Rosette	Gage No.	α ($^{\circ}$)	β ($^{\circ}$)
A	1	322.9	0
	2	300.0	90
	3	300.0	45
B	4	163.6	45
	5	163.6	135
	6	180.0	90
C	7	82.9	0
	8	60.0	90
	9	60.0	45

METHOD OF SOLUTION

The solution for the in-situ state of stress determined from measurements of diametral deformations in three or more nonparallel boreholes has been published in some detail by Panek (1966). Only that part of the solution necessary to understand the programs will be summarized here.

Stress Components

The method is essentially a multiple linear regression analysis of the displacement (or strain) measurements (U factors) and another set of factors (J factors), which depend upon the elastic properties of the rock and the orientation of the measurement in space, to arrive at the least-squares solution for the stress components and their variations. The relationship between the two factors for a single measurement is given by

$$U = b_1J_1 + b_2J_2 + b_3J_3 + b_4J_4 + b_5J_5 + b_6J_6 \quad (2)$$

where the b_i are the six independent components of the stress tensor:

$$[B] = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} \quad (3)$$

Thus, for each measurement of strain or deformation there is an associated set of J factors so that we have two arrays U and J:

$$\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ \cdot & \cdot & \cdot & & & \\ J_{n1} & J_{n2} & J_{n3} & J_{n4} & J_{n5} & J_{n6} \end{bmatrix} \cdot$$

To solve for the stress components b_i we first define a 6X6 matrix [A] and a 6X1 matrix [G] where

$$a_{ij} = \sum_{n=1}^m J_{ni} J_{nj} \quad (4)$$

and

$$g_j = \sum_{n=1}^m U_n J_{nj} \quad (5)$$

The least-squares solution for the stress components b_i is the solution of the matrix equation

$$[A] [B] = [G]. \quad (6)$$

Thus,

$$[B] = [A]^{-1} [G] = [C] [G] \quad (7)$$

where the matrix [C] is the inverse of [A].

Definitions of the U and J Factors

The U factors in the above procedure are simply the borehole deformation (in μ -in. or μ m) in the case of the USBM Borehole Deformation Gage, and the strain values in the case of the CSIRO Hollow Inclusion Cell.

To calculate the J factors for the USBM gage, we first define four F factors as follows:

$$f_1 = [d(1 + 2 \cos 2\theta)(1 - \nu^2) + d\nu^2]/E \quad (8a)$$

$$f_2 = -d\nu/E \quad (8b)$$

$$f_3 = [d(1 - 2 \cos 2\theta)(1 - \nu^2) + d\nu^2]/E \quad (8c)$$

$$f_4 = d(4 \sin 2\theta)(1 - \nu^2)/E \quad (8d)$$

where d is the hole diameter, ν is Poisson's ratio, E is Young's modulus, and θ is the angle of measurement in the hole. The J factors can then be expressed by the matrix equation

$$[J_1 \dots J_6] = [f_1 \dots f_4] \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2l_1m_1 & 2m_1n_1 & 2l_1n_1 \\ l_2^2 & m_2^2 & n_2^2 & 2l_2m_2 & 2m_2n_2 & 2l_2n_2 \\ l_3^2 & m_3^2 & n_3^2 & 2l_3m_3 & 2m_3n_3 & 2l_3n_3 \\ l_1l_3 & m_1m_3 & n_1n_3 & l_1m_3+ \\ & & & l_3m_1 & m_1n_3+ \\ & & & & m_3n_1 & l_1n_3+ \\ & & & & & l_3n_1 \end{bmatrix} \quad (9)$$

where the l_i , m_i , n_i are direction cosines as described earlier.

For the CSIRO cell the computation is similar but slightly more complex due to the fact that measurements are made in three dimensions rather than in a plane. This cell requires the use of six F factors as follows (Commonwealth Scientific and Industrial Research Organisation, 1979, app. 2):

$$f_1 = [(1-\nu^2) \cos 2\alpha (1-\cos 2\beta)K_2 + .5(K_1-\nu) - .5(K_1+\nu) \cos 2\beta]/E \quad (10a)$$

$$f_2 = [.5(1-K_4\nu) + .5(1+K_4\nu) \cos 2\beta]/E \quad (10b)$$

$$f_3 = [(\nu^2-1) \cos 2\alpha (1-\cos 2\beta)K_2 + .5(K_1-\nu) - .5(K_1+\nu) \cos 2\beta]/E \quad (10c)$$

$$f_4 = 2(1+\nu) \cos \alpha \sin 2\beta K_3/E \quad (10d)$$

$$f_5 = -2(1+\nu) \sin \alpha \sin 2\beta K_3/E \quad (10e)$$

$$f_6 = 2(\nu^2-1) \sin 2\alpha (1-\cos 2\beta) K_2/E \quad (10f)$$

where K_1 , ..., K_4 are strain correction factors described by Worotnicki and Walton (1976). The J factors can now be expressed in a manner analogous to that for the USBM gage:

$$[J] = [F] \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2l_1m_1 & 2m_1n_1 & 2l_1n_1 \\ l_2^2 & m_2^2 & n_2^2 & 2l_2m_2 & 2m_2n_2 & 2l_2n_2 \\ l_3^2 & m_3^2 & n_3^2 & 2l_3m_3 & 2m_3n_3 & 2l_3n_3 \\ l_1l_2 & m_1m_2 & n_1n_2 & l_1m_2+ \\ & & & l_2m_1 & m_1n_2+ \\ & & & & m_2n_1 & n_1l_2+ \\ & & & & & n_2l_1 \\ l_2l_3 & m_2m_3 & n_2n_3 & l_2m_3+ \\ & & & l_3m_2 & m_2n_3+ \\ & & & & m_3n_2 & l_2n_3+ \\ & & & & & l_3n_2 \\ l_1l_3 & m_1m_3 & n_1n_3 & l_1m_3+ \\ & & & l_3m_1 & m_1n_3+ \\ & & & & m_3n_1 & l_1n_3+ \\ & & & & & l_3n_1 \end{bmatrix} \quad (11)$$

From a programming standpoint, it is more convenient to define the six J factors for each measurement explicitly rather than to compute them by matrix multiplication.

Statistical Parameters

In addition to the most probable values of the stress components determined by the above process, we are also interested in the variability indicated by the measurements. Whenever there are more than six statistically independent measurements of deformation or strain we have a redundant system of equations, which allows us to assess the accuracy of the most probable values. The parameters of interest in this case are the standard deviations of the stress components and the correlation coefficient.

First we define the following summations:

$$s_1 = \sum_{i=1}^m U_i^2 \quad (12)$$

$$s_2 = b_1g_1 + b_2g_2 + \dots + b_6g_6 = [B] \cdot [G] . \quad (13)$$

The error sum of squares, s_3 , is then

$$s_3 = s_1 - s_2 \quad (14)$$

and the square of the multiple correlation coefficient is

$$R^2 = s_3/s_1 . \quad (15)$$

The standard deviation about the regression line is

$$s_4 = (s_3/(m-6))^{1/2} \quad (16)$$

and the standard deviations, v_i , of the stress components b_i are given by

$$v_i = s_4 c_{ij}^{1/2} . \quad (17)$$

Solution for Principal Stress Values

Once the values of the stress components b_i have been determined using the above procedure, the next step is to determine the values of the principal stresses. This is most easily done by first rewriting the [B] array as the symmetric stress tensor [S] in matrix form:

$$[S] = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} b_1 & b_4 & b_6 \\ b_4 & b_2 & b_5 \\ b_6 & b_5 & b_3 \end{bmatrix} \quad (18)$$

in which the main diagonal elements s_{ij} represent normal stresses along the coordinate axes and the off-diagonal elements are the shear stresses. The principal stresses are the eigenvalues of the matrix and are the three roots of the cubic characteristic equation

$$-\sigma^3 + I_1\sigma^2 - I_2\sigma + I_3 = 0 \quad (19)$$

where σ is the principal stress and I_1, I_2, I_3 are the three invariants of the stress tensor:

$$I_1 = s_{11} + s_{22} + s_{33} \quad (20a)$$

$$I_2 = s_{11}s_{33} + s_{22}s_{33} + s_{11}s_{22} - s_{23}^2 - s_{12}^2 - s_{13}^2 \quad (20b)$$

$$I_3 = |S| = \text{determinant of } [S] \quad (20c)$$

For ease of solution it is convenient to multiply the characteristic equation through by -1:

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \quad (21)$$

Because the stress tensor is real and symmetric the principal stresses are real and the cubic equation (21) can be solved by the method of trigonometric substitution.

Orientation of the Principal Stresses

The direction cosines of the three principal stresses (eigenvalues) are given by the associated set of three normalized eigenvectors, which are the solutions to the matrix equation

$$[[S] - \sigma[I]][n] = 0 \quad (22)$$

where σ is a principal stress value, $[I]$ is a 3X3 identity matrix and $[n]$ is the 3X1 matrix of direction numbers (eigenvector). Because there are an infinite number of non-trivial solutions, it is necessary to assume a value for one of the n_i , say $n_1 = 1$, solve for the other n_i , and then scale the resulting vector according to the condition that $n_1^2 + n_2^2 + n_3^2 = 1$ to obtain the direction cosines. For the 3X3 stress matrix a direct algebraic solution is easier than the more complicated elimination techniques required for large matrices.

First define the matrix $[Y]$

$$[Y] = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} = \begin{bmatrix} s_{11}-\sigma & s_{12} & s_{13} \\ s_{21} & s_{22}-\sigma & s_{23} \\ s_{31} & s_{32} & s_{33}-\sigma \end{bmatrix} = [S] - \sigma[I] \quad (23)$$

Equation (22) then becomes

$$\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (24)$$

giving the three simultaneous equations

$$y_{11}n_1 + y_{12}n_2 + y_{13}n_3 = 0 \quad (25a)$$

$$y_{21}n_1 + y_{22}n_2 + y_{23}n_3 = 0 \quad (25b)$$

$$y_{31}n_1 + y_{32}n_2 + y_{33}n_3 = 0 \quad (25c)$$

Assuming a value of $n_1 = 1$, solving equation (25a) for n_2 and (25b) for n_3 , and noting that $y_{ij} = y_{ji}$, we have

$$n_3 = (y_{11}y_{22} - y_{21}^2)/(y_{12}y_{23} - y_{22}y_{13}) \quad (26a)$$

$$n_2 = -(y_{11} + y_{13}n_3)/y_{12} \quad (26b)$$

$$n_1 = 1 \quad (26c)$$

These values are then normalized to obtain the direction cosines by dividing each n_i by $(1 + n_2^2 + n_3^2)^{1/2}$.

Standard Deviations of the Principal Stresses

Once the orientations of the principal stresses are computed it is possible to compute the standard deviations of the principal stresses. Unfortunately, the standard deviations of stress components do not obey the laws of tensor transformation from one coordinate system to another, and it is necessary to recompute the J factors using direction cosines which relate the borehole axis system to the principal stress directions, and to repeat the regression analysis using these new values. The relationship of the borehole axes to principal axes is given by Panek (1966, p. 22) as:

$$\begin{matrix} h_1 \\ h_2 \\ h_3 \end{matrix} \begin{bmatrix} 1'' & 2'' & 3'' \\ l_1'' & m_1'' & n_1'' \\ l_2'' & m_2'' & n_2'' \\ l_3'' & m_3'' & n_3'' \end{bmatrix} = \begin{matrix} h_1 \\ h_2 \\ h_3 \end{matrix} \begin{bmatrix} x & y & z \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \times \begin{matrix} 1' & 2' & 3' \\ l_1' & l_2' & l_3' \\ m_1' & m_2' & m_3' \\ n_1' & n_2' & n_3' \end{matrix} \quad (27)$$

where the double primes represent the direction cosines between the borehole axes and principal axes, the single primes between the geographic and principal axes, and the unprimed between the geographic and borehole axes. The J factors are computed using these new direction cosines and the regression analysis is repeated. The result is that the stress components b_1 , b_2 , b_3 are the principal stresses computed previously and b_4 , b_5 , and b_6 are zero, or very nearly so.

PROGRAM OPERATION

The two programs presented here, one for the USBM Borehole Deformation Gage and the other for the CSIRO Hollow Inclusion Cell, are interactive programs written in the BASIC language and were developed on Hewlett-Packard 9800 series microcomputers. An effort was made to ensure that the programs conform to ANSI standard BASIC in order to increase program portability. Exceptions to standard BASIC are in the output formatting statements (IMAGE statements) and the first five lines of each program. Specifically, the OPTION BASE 1 statement sets the base subscript for arrays to be 1, rather than the system's default value of 0. Memory requirements to run these programs are 18 K bytes for CSIRO and 28 K bytes for USBM.

Units

Both programs will work with either English or SI (metric) units. Both programs write prompts on the CRT or printer requesting the necessary information. The basic input data for both programs are the material properties (Young's modulus and Poisson's ratio), the borehole orientations, and the gage or channel numbers and the associated microstrain readings.

For the CSIRO cell, the stress values are in the same units as used for Young's modulus (usually lb/in.² or MPa). Data input for the USBM gage is more complicated, because data sets for at least three nonparallel holes are required for a complete stress determination. The USBM instrument must be calibrated prior to use in order to convert the microstrain readings (or other readout units) to corresponding deformations. The calibration curves will be highly linear, but will be different for each of the three channels. The calibration factors will be in units of microlength per microstrain. Table 2 shows the required input and corresponding output units for the USBM gage.

Table 2. Input and output units for USBM data reduction program.

Input			Output
Calibration factor	Diameter	Young's modulus	Stress
$\mu\text{-in.}/\mu\epsilon$ $\mu\text{m}/\mu\epsilon$	in. m	10^6 lb/in.^2 GPa	lb/in.^2 kPa

Sorting of Principal Stresses

As a convenience to the user, the computed values of the principal stresses are sorted in order of decreasing algebraic value. If the particular BASIC compiler or interpreter being used has built-in MAX and MIN functions the 27 statements in the sort routine may be replaced by the following routine:

```

xxxx   Z1 = MAX (Z(1,1), Z(2,1), Z(3,1))
xxxx   Z3 = MIN (Z(1,1), Z(2,1), Z(3,1))
xxxx   Z(1,1) = Z1
xxxx   Z(2,1) = I1 - Z1-Z3
xxxx   Z(3,1) = Z3

```

where the variable I1 is the first invariant of the stress tensor computed prior to solving the cubic equation for the principal stresses and the xxxx's are statement numbers.

Output Description

The output from each program consists of a tabulation of the input data, followed by a summary of the statistical calculations. Next the geographic stress components and their standard deviations are printed with appropriate labels, followed by the sorted principal stresses, standard deviations and orientations. Inclinations of principal stresses are negative, by convention and bearings are adjusted accordingly, if necessary. This facilitates plotting of the principal stresses on lower hemisphere stereographic or equal-area nets if desired. Examples of output from each of the programs are given on pages 20 and 21.

U.S. BUREAU OF MINES BOREHOLE DEFORMATION GAGE DATA REDUCTION

NUMBER OF DEFORMATION MEASUREMENTS = 9

MICRO- STRAIN	DEFORMATION	CH	***** BOREHOLE *****		DIAMETER	E	PR
			BEARING	INCL.			
+736	+.000736	3	225.0	+0.0	1.5000	9.00	.200
+565	+.000565	1	225.0	+0.0	1.5000	9.00	.200
+93	+.000093	2	225.0	+0.0	1.5000	9.00	.200
+546	+.000546	1	315.0	+0.0	1.5000	9.10	.200
+267	+.000267	2	315.0	+0.0	1.5000	9.10	.200
+588	+.000588	3	315.0	+0.0	1.5000	9.10	.200
+322	+.000322	3	0.0	-90.0	1.5000	8.90	.200
+166	+.000166	1	0.0	-90.0	1.5000	8.90	.200
+256	+.000256	2	0.0	-90.0	1.5000	8.90	.200

CHANNEL	ANGLE	CALIB. FACTOR
1	90.0	1.000
2	150.0	1.000
3	30.0	1.000

DEFORMATION SUM OF SQUARES = 1.7815E-06
 [B].[G] = 1.7366E-06
 RESIDUAL SUM OF SQUARES = 4.4847E-08
 STD. DEV. OF FITTED DATA = 1.2227E-04
 GOODNESS OF FIT (R**2) = .97483
 CORRELATION COEFFICIENT = .98733

STRESS COMPONENTS	STD. DEV.
NORMAL N-S = +1031	215
NORMAL E-W = +1285	215
NORMAL VER = +1615	190
SHEAR HOR = +43	113
SHEAR N-V = +617	157
SHEAR E-V = -203	157

PRINCIPAL STRESSES	STRESS	STD. DEV.	BEARING	INCL.
	+2036	214	158.7	-57.0
	+1286	208	253.3	-3.0
	+609	222	345.2	-32.8

SAMPLE OUTPUT FROM USBM PROGRAM
 USING ENGLISH UNITS

*** CSIRO HOLLOW INCLUSION STRESS CELL DATA REDUCTION ***

GAGE NO.	MICRO-STRAIN	ALPHA	BETA
1	+260	322.9	0.0
2	+328	300.0	90.0
3	+350	300.0	45.0
4	-154	163.6	45.0
5	+446	163.6	135.0
6	+397	180.0	90.0
7	+280	82.9	0.0
8	+1255	60.0	90.0
9	+1020	60.0	45.0

BOREHOLE BEARING = 20.4 ; INCLINATION = 37
 MODULUS = 55000 ; POISSON'S RATIO = .25

STRAIN SUM OF SQUARES = 3.3718E-06
 [80.[G] = 3.3689E-06
 RESIDUAL SUM OF SQUARES = 4.8341E-09
 STD. DEV OF FITTED DATA = 4.0390E-05
 GOODNESS OF FIT (R**2) = .99855
 CORRELATION COEFFICIENT = .99927

STRESS COMPONENTS	STD. DEV.
NORMAL N-S = +16.99	1.09
NORMAL E-W = +22.22	.67
NORMAL VER = +20.02	1.08
SHEAR HOR = +10.77	.53
SHEAR N-V = +.38	.75
SHEAR E-V = -1.44	.49

PRINCIPAL STRESSES	STD. DEV.	BEARING	INCL.
+30.76	.89	52.1	-4.8
+20.06	1.20	131.3	-82.4
+8.40	.72	321.6	-5.8

SAMPLE OUTPUT FROM CSIRO PROGRAM
 USING SI UNITS

PROGRAM VARIABLES AND DEFINITIONS

The same names are generally used for comparable variables in each program. Differences are noted as they occur.

<u>Variables</u>	<u>Definition</u>
A1	θ angle in radians for individual measurement (USBM)
B1	Bearing of borehole
B2	Bearing of hole in degrees (CSIRO)
B5	Bearing of principal stress
C1	Borehole diameter (USBM)
D1	Borehole diameter (USBM)
E	Young's modulus (USBM)
E1	Young's modulus $\times 10^{-6}$ (USBM)
F1-F5	Intermediate values used to compute F factors (CSIRO)
I	Index counter
I1	Borehole inclination First invariant of stress tensor
I2	Borehole inclination in degrees (CSIRO) Second invariant of stress tensor
I3	Third invariant of stress tensor
I4	Intermediate value for computing I2
I5	Inclination of principal stress
J	Index counter
K	Index counter
K1-K4	Strain correction factors for CSIRO cell
K9	Counter to test if geographic or principal stress components are being computed

<u>Variables</u>	<u>Definition</u>
L1-L3	Direction cosines with respect to x axis
M	Count of individual measurements
M1-M3	Direction cosines with respect to y axis
N	Index counter
N1-N3	Direction cosines with respect to z axis
N4	Factor for normalizing eigenvector $(n_1^2 + n_2^2 + n_3^2)^{1/2}$
P	Poisson's ratio
Q1-Q3	Coefficients of cubic equation
Q4-Q7	Intermediate values for solution of cubic equation
R	Correlation coefficient
R1	Conversion factor for degrees to radians, 57.29578
R2	Square of correlation coefficient
S1-S4	Statistical parameters
X1-X3	Intermediate values in solution of cubic equation
Z1-Z3	Temporary locations for principal stress values in sorting routine

<u>Arrays</u>	<u>Dimensions</u>	<u>Definition</u>
A	(6,6)	$A_{ij} = \sum_{n=1}^m J_{ni}J_{nj}$
B	(6,1)	Vector of stress components
C	(6,6)	Inverse of [A]
D	(9,2)	Array of α and β angles in degrees ($d_{n1} = \alpha$; $d_{n2} = \beta$) (CSIRO)
E	(9)	Microstrain reading (CSIRO)
F	(9,6) (CSIRO) (50,4) (USBM)	F factors used in computing J factors
G	(6,1)	$g_j = \sum_{n=1}^m U_n J_{nj}$
H	(3)	Calibration factors for respective channels (USBM)
I	(3,3)	Identity matrix
J	(50,6) (USBM) (9,6) (CSIRO)	Array of J factors
K	(50,9) (USBM) (9,9) (CSIRO)	Storage area for direction cosines
O	(50,7)	Storage area for output data tabulation. O_{n1} - microstrain reading; O_{n2} - channel no.; O_{n3} - hole bearing; O_{n4} - hole inclination; O_{n5} - hole diameter; O_{n6} = Young's modulus $\times 10^{-6}$; O_{n7} = Poisson's ratio (USBM only)
P	(3,3)	Direction cosines of the three principal stresses. Also, a dummy matrix for computing I3 by inversion
S	(3,3)	Stress tensor in matrix form

<u>Arrays</u>	<u>Dimensions</u>	<u>Definition</u>
T	(3) (USBM) (9,2) (CSIRO)	In USBM, the θ angle in degrees for each channel. In CSIRO, the α and β angles in radians ($T_{n1} = \alpha$; $T_{n2} = \beta$)
U	(50) (USBM) (9) (CSIRO)	The array of deformation values (USBM) or strain values (CSIRO). Also, microstrain reading for data input in USBM
V	(6)	Standard deviations of stress components
X	(3,3)	Diagonal matrix of a principal stress value
Y	(3,3)	[S] $-\sigma$ [I]
Z	(3,3)	Array of principal stresses, bearings, and inclinations $Z_{n1} = \text{stress}$; $Z_{n2} = \text{bearing}$; $Z_{n3} = \text{inclination}$.

PROGRAM LISTING OF USBM

```

10 WRITE IO 10,5;1
20 WRITE IO 10,4;39
30 WRITE IO 10,6;1
40 WRITE IO 10,5;128
50 OPTION BASE 1
60 REM *** *****
70 REM *** A PROGRAM FOR THE CALCULATION OF STRESS COMPONENTS, PRINCIPAL
80 REM *** STRESSES, AND THEIR STANDARD DEVIATIONS FROM SETS OF READINGS
90 REM *** FROM THE U.S. BUREAU OF MINES BOREHOLE DEFORMATION GAGE IN THREE
100 REM *** OR MORE NON-PARALLEL BOREHOLES.
110 REM *** WRITTEN IN BASIC FOR THE HEWLETT-PACKARD 9800 SERIES MICRO-
120 REM *** COMPUTERS BY WILLIAM K. SMITH, U.S. GEOLOGICAL SURVEY, DEC., 1981.
130 REM *** *****REFERENCES*****
140 REM *** PANEK, LOUIS A., 1966, CALCULATION OF THE AVERAGE GROUND STRESS
150 REM *** COMPONENTS FROM MEASUREMENTS OF THE DIAMETRAL DEFORMATION OF
160 REM *** A DRILL HOLE: U.S. BUREAU OF MINES REPT. OF INV. 6732, 41 P.;
170 REM *** OR ASTM SPEC. TECH. PUB. 402, P. 106-132.
180 REM *** SMITH, WILLIAM K., 1982, TWO BASIC COMPUTER PROGRAMS FOR THE
190 REM *** DETERMINATION OF IN-SITU STRESSES USING THE CSIRO HOLLOW
200 REM *** INCLUSION GAGE AND THE USBM BOREHOLE DEFORMATION GAGE:
210 REM *** U.S. GEOLOGICAL SURVEY OPEN-FILE REPORT 82-489, 40 P.
220 REM *** *****
230 REM *** COORDINATE SYSTEMS:
240 REM *** X-AXIS IS POSITIVE EAST
250 REM *** Y-AXIS IS POSITIVE NORTH
260 REM *** Z-AXIS IS POSITIVE UP
270 REM ***
280 REM *** H1-AXIS IS HORIZONTAL TO THE RIGHT WHEN LOOKING INTO THE HOLE.
290 REM *** H2-AXIS COINCIDES WITH THE BOREHOLE AXIS.
300 REM *** H3-AXIS IS IN A VERTICAL PLANE, POINTS GENERALLY UPWARD.
310 REM *** BEARINGS ARE IN DEGREES CLOCKWISE FROM NORTH.
320 REM *** INCLINATIONS ARE IN DEGREES FROM THE HORIZONTAL, POSITIVE UPWARDS.
330 REM *** BEARING OF A VERTICAL HOLE IS 90 DEGREES COUNTER-CLOCKWISE FROM
340 REM *** DIRECTION OF H1-AXIS, AS DETERMINED FROM THE ORIENTATION OF THE
350 REM *** SETTING TOOL.
360 REM *** UNITS:
370 REM ***
380 REM *** CALIB. FACTOR          DIAMETER      YOUNG'S      STRESS
390 REM *** -----          -----          -----          -----
400 REM *** MICRO-IN/MICROSTRAIN    INCHES      10^6 PSI     PSI
410 REM *** MICRO-M/MICROSTRAIN     METERS      GPA          KPA
420 REM *** *****
430 LET R1=57.29578
440 DIM A(6,6),B(6,1),C(6,6),F(50,4),G(6,1),H(3),I(3,3),J(50,6),K(50,9)
450 DIM O(50,7),P(3,3),S(3,3),T(3),U(50),V(6),X(3,3),Y(3,3),Z(3,3)
460 REM *** DEFINE MEASUREMENT ANGLES FOR EACH CHANNEL
470 LET T(1)=90
480 LET T(2)=150
490 LET T(3)=30
500 REM *** READ CALIBRATION FACTORS FOR EACH CHANNEL
510 PRINT "ENTER CALIBRATION FACTORS FOR CHANNELS 1-3 IN"
520 PRINT "MICRO-LENGTH PER MICROSTRAIN"

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PROGRAM LISTING OF USBM --Continued

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530 INPUT H(1),H(2),H(3)
540 LET K9=1
550 LET M=2
560 LET M=M-1
570 PRINT "ENTER BEARING, INCLINATION, AND DIAMETER OF BOREHOLE,"
580 PRINT "YOUNG'S MODULUS (IN MEGA-STRESS), AND POISSON'S RATIO"
590 INPUT B1,I1,D1,E1,P
600 IF B1=999 THEN 1080
610 LET B1=B1/R1
620 LET I1=I1/R1
630 LET E=E1*1.0E6
640 LET L1=COS(B1)
650 LET M1=-SIN(B1)
660 LET N1=0
670 LET L2=SIN(B1)*COS(I1)
680 LET M2=COS(B1)*COS(I1)
690 LET N2=SIN(I1)
700 LET L3=-SIN(B1)*SIN(I1)
710 LET M3=-COS(B1)*SIN(I1)
720 LET N3=COS(I1)
730 PRINT "ENTER CHANNEL NO. AND MICROSTRAIN READING"
740 PRINT "ENTER '999,999' TO TERMINATE READINGS FOR THIS BOREHOLE"
750 INPUT C1,U(M)
760 LET M=M+1
770 IF C1=999 THEN 560
780 LET A1=T(C1)/R1
790 LET O(M-1,1)=U(M-1)
800 LET U(M-1)=U(M-1)*1.0E-6*H(C1)
810 LET O(M-1,2)=C1
820 LET O(M-1,3)=B1*R1
830 LET O(M-1,4)=I1*R1
840 LET O(M-1,5)=D1
850 LET O(M-1,6)=E1
860 LET O(M-1,7)=P
870 LET K(M-1,1)=L1
880 LET K(M-1,2)=L2
890 LET K(M-1,3)=L3
900 LET K(M-1,4)=M1
910 LET K(M-1,5)=M2
920 LET K(M-1,6)=M3
930 LET K(M-1,7)=N1
940 LET K(M-1,8)=N2
950 LET K(M-1,9)=N3
960 LET F(M-1,1)=(D1*(1+2*COS(2*A1))*(1-P*P)+D1*P*P)/E
970 LET F(M-1,2)=-D1*P/E
980 LET F(M-1,3)=(D1*(1-2*COS(2*A1))*(1-P*P)+D1*P*P)/E
990 LET F(M-1,4)=D1*4*SIN(2*A1)*(1-P*P)/E
1000 LET J(M-1,1)=F(M-1,1)*L1*L1+F(M-1,2)*L2*L2+F(M-1,3)*L3*L3+F(M-1,4)*L1*L3
1010 LET J(M-1,2)=F(M-1,1)*M1*M1+F(M-1,2)*M2*M2+F(M-1,3)*M3*M3+F(M-1,4)*M1*M3
1020 LET J(M-1,3)=F(M-1,1)*N1*N1+F(M-1,2)*N2*N2+F(M-1,3)*N3*N3+F(M-1,4)*N1*N3
1030 LET J(M-1,4)=2*F(M-1,1)*L1*M1+2*F(M-1,2)*L2*M2+2*F(M-1,3)*L3*M3+F(M-1,4)*(L
1*M3+L3*M1)

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PROGRAM LISTING OF USBM -- Continued

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1040 LET J(M-1,5)=2*F(M-1,1)*M1*N1+2*F(M-1,2)*M2*N2+2*F(M-1,3)*M3*N3+F(M-1,4)*(M
1*N3+M3*N1)
1050 LET J(M-1,6)=2*F(M-1,1)*N1*L1+2*F(M-1,2)*N2*L2+2*F(M-1,3)*N3*L3+F(M-1,4)*(N
1*L3+N3*L1)
1060 IF K9=2 THEN 1160
1070 GOTO 730
1080 LET M=M-1
1090 PRINT
1100 PRINT "*U.S. BUREAU OF MINES BOREHOLE DEFORMATION GAGE DATA REDUCTION*"
1110 PRINT
1120 PRINT "NUMBER OF DEFORMATION MEASUREMENTS = ";M
1130 PRINT
1140 PRINT " MICRO-                ***** BOREHOLE *****"
1150 PRINT " STRAIN    DEFORMATION CH BEARING INCL. DIAMETER     E     PR"
1160 FOR K=1 TO M
1170 PRINT USING 1190;O(K,1),U(K),O(K,2),O(K,3),O(K,4),O(K,5),O(K,6),O(K,7)
1180 NEXT K
1190 IMAGE S6D,5X,S.6D,4X,D,3X,3D.D,3X,S2D.D,3X,D.4D,3X,2D.2D,3X,.3D
1200 PRINT
1210 PRINT "                CHANNEL ANGLE CALIB. FACTOR"
1220 FOR K=1 TO 3
1230 PRINT USING 1250;K,T(K),H(K)
1240 NEXT K
1250 IMAGE 21X,D,5X,3D.D,4X,3D.3D
1260 PRINT
1270 REM *** CLEAR A(I,J) AND G(J) ARRAYS
1280 FOR J=1 TO 6
1290 LET G(J,1)=0
1300 FOR I=1 TO 6
1310 LET A(I,J)=0
1320 NEXT I
1330 NEXT J
1340 REM *** COMPUTE [A] AND [G] ARRAYS
1350 FOR K=1 TO M
1360 FOR J=1 TO 6
1370 LET G(J,1)=G(J,1)+U(K)*J(K,J)
1380 FOR I=1 TO 6
1390 LET A(I,J)=A(I,J)+J(K,I)*J(K,J)
1400 NEXT I
1410 NEXT J
1420 NEXT K
1430 MAT C=INV(A)
1440 MAT B=C*G
1450 REM *** COMPUTE STATISTICAL PARAMETERS
1460 IF K9=2 THEN 1680
1470 LET S1=0
1480 FOR K=1 TO M
1490 LET S1=S1+U(K)*U(K)
1500 NEXT K
1510 LET S2=B(1,1)*G(1,1)+B(2,1)*G(2,1)+B(3,1)*G(3,1)+B(4,1)*G(4,1)+B(5,1)*G(5,1
)+B(6,1)*G(6,1)
1520 LET S3=S1-S2

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PROGRAM LISTING OF USBM -- Continued

```

1530 LET R2=1-S3/S1
1540 LET R=SQR(R2)
1550 LET S4=SQR(S3/(M-6))
1560 PRINT USING 1620;S1
1570 PRINT USING 1630;S2
1580 PRINT USING 1640;S3
1590 PRINT USING 1650;S4
1600 PRINT USING 1660;R2
1610 PRINT USING 1670;R
1620 IMAGE " DEFORMATION SUM OF SQUARES = ", D.4DE
1630 IMAGE " [B].[G] = ", D.4DE
1640 IMAGE " RESIDUAL SUM OF SQUARES = ", D.4DE
1650 IMAGE " STD. DEV. OF FITTED DATA = ", D.4DE
1660 IMAGE " GOODNESS OF FIT (R**2) = ", D.5D
1670 IMAGE " CORRELATION COEFFICIENT = ", D.5D
1680 FOR I=1 TO 6
1690 LET V(I)=S4*SQR(C(I,I))
1700 NEXT I
1710 IF K9=2 THEN 2830
1720 PRINT
1730 PRINT " STRESS COMPONENTS          STD. DEV."
1740 PRINT USING 1750;B(2,1),V(2)
1750 IMAGE "   NORMAL N-S = ",S6D, 4X, 5D
1760 PRINT USING 1770;B(1,1),V(1)
1770 IMAGE "   NORMAL E-W = ",S6D, 4X, 5D
1780 PRINT USING 1790;B(3,1),V(3)
1790 IMAGE "   NORMAL VER = ",S6D, 4X, 5D
1800 PRINT USING 1810;B(4,1),V(4)
1810 IMAGE "   SHEAR HOR = ",S6D, 4X, 5D
1820 PRINT USING 1830;B(5,1),V(5)
1830 IMAGE "   SHEAR N-V = ",S6D, 4X, 5D
1840 PRINT USING 1850;B(6,1),V(6)
1850 IMAGE "   SHEAR E-V = ",S6D, 4X, 5D
1860 REM *** COMPUTE PRINCIPAL STRESSES AND ORIENTATIONS
1870 LET S(1,1)=B(1,1)
1880 LET S(2,1)=B(4,1)
1890 LET S(3,1)=B(6,1)
1900 LET S(1,2)=B(4,1)
1910 LET S(2,2)=B(2,1)
1920 LET S(3,2)=B(5,1)
1930 LET S(1,3)=B(6,1)
1940 LET S(2,3)=B(5,1)
1950 LET S(3,3)=B(3,1)
1960 REM *** COMPUTE INVARIANTS OF STRESS TENSOR
1970 LET I1=S(1,1)+S(2,2)+S(3,3)
1980 LET I2=S(1,1)*S(3,3)+S(2,2)*S(3,3)+S(1,1)*S(2,2)
1990 LET I4=S(2,3) 2+S(1,2) 2+S(1,3) 2
2000 LET I2=I2-I4
2010 MAT P=INV(S)
2020 LET I3=DET
2030 REM *** SOLVE CUBIC EQUATION BY TRIGONOMETRIC SUBSTITUTION
2040 LET Q1=- I1

```

PROGRAM LISTING OF USBM -- Continued

```

2050 LET Q2=I2
2060 LET Q3=-I3
2070 LET Q4=(3*Q2-Q1*Q1)/3
2080 LET Q5=(2*Q13-9*Q1*Q2+27*Q3)/27
2090 LET Q6=-Q5/2/SQR(-Q43/27)
2100 LET Q7=ACS(Q6)/3
2110 LET X1=2*SQR(-Q4/3)*COS(Q7)
2120 LET X2=2*SQR(-Q4/3)*COS(Q7+120/R1)
2130 LET X3=2*SQR(-Q4/3)*COS(Q7+240/R1)
2140 LET Z(1,1)=X1-Q1/3
2150 LET Z(2,1)=X2-Q1/3
2160 LET Z(3,1)=X3-Q1/3
2170 REM *** END OF CUBIC SOLUTION
2180 REM *** SORT PRINCIPAL STRESSES
2190 IF Z(1,1)>Z(2,1) THEN 2260
2200 IF Z(1,1)>Z(3,1) THEN 2360
2210 IF Z(2,1)>Z(3,1) THEN 2400
2220 LET Z1=Z(3,1)
2230 LET Z2=Z(2,1)
2240 LET Z3=Z(1,1)
2250 GOTO 2430
2260 IF Z(1,1)>Z(3,1) THEN 2310
2270 LET Z1=Z(3,1)
2280 LET Z2=Z(1,1)
2290 LET Z3=Z(2,1)
2300 GOTO 2430
2310 IF Z(2,1)>Z(3,1) THEN 2460
2320 LET Z1=Z(1,1)
2330 LET Z2=Z(3,1)
2340 LET Z3=Z(2,1)
2350 GOTO 2430
2360 LET Z1=Z(2,1)
2370 LET Z2=Z(1,1)
2380 LET Z3=Z(3,1)
2390 GOTO 2430
2400 LET Z1=Z(2,1)
2410 LET Z2=Z(3,1)
2420 LET Z3=Z(1,1)
2430 LET Z(1,1)=Z1
2440 LET Z(2,1)=Z2
2450 LET Z(3,1)=Z3
2460 REM *** END OF SORT
2470 REM *** COMPUTE DIRECTION COSINES OF PRINCIPAL STRESSES
2480 FOR N=1 TO 3
2490 MAT I=IDN
2500 MAT X=(Z(N,1))*I
2510 MAT Y=S-X
2520 LET N3=(Y(1,1)*Y(2,2)-Y(2,1) 2)/(Y(1,2)*Y(2,3)-Y(2,2)*Y(1,3))
2530 LET N2=-((Y(1,1)+Y(1,3)*N3)/Y(1,2))
2540 LET N4=SQR(1+N2*N2+N3*N3)
2550 LET P(1,N)=1/N4
2560 LET P(2,N)=N2/N4

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PROGRAM LISTING OF USBM -- Continued

```

2570 LET P(3,N)=N3/N4
2580 GOSUB 2910
2590 LET Z(N,2)=B5
2600 LET Z(N,3)=I5
2610 NEXT N
2620 REM *** RE-COMPUTE DIRECTION COSINES WITH RESPECT TO PRINCIPAL AXES
2630 REM *** TO COMPUTE STANDARD DEVIATIONS OF PRINCIPAL STRESSES
2640 FOR N=1 TO M
2650 LET L1=K(N,1)*P(1,1)+K(N,4)*P(2,1)+K(N,7)*P(3,1)
2660 LET M1=K(N,1)*P(1,2)+K(N,4)*P(2,2)+K(N,7)*P(3,2)
2670 LET N1=K(N,1)*P(1,3)+K(N,4)*P(2,3)+K(N,7)*P(3,3)
2680 LET L2=K(N,2)*P(1,1)+K(N,5)*P(2,1)+K(N,8)*P(3,1)
2690 LET M2=K(N,2)*P(1,2)+K(N,5)*P(2,2)+K(N,8)*P(3,2)
2700 LET N2=K(N,2)*P(1,3)+K(N,5)*P(2,3)+K(N,8)*P(3,3)
2710 LET L3=K(N,3)*P(1,1)+K(N,6)*P(2,1)+K(N,9)*P(3,1)
2720 LET M3=K(N,3)*P(1,2)+K(N,6)*P(2,2)+K(N,9)*P(3,2)
2730 LET N3=K(N,3)*P(1,3)+K(N,6)*P(2,3)+K(N,9)*P(3,3)
2740 LET J(N,1)=F(N,1)*L1*L1+F(N,2)*L2*L2+F(N,3)*L3*L3+F(N,4)*L1*L3
2750 LET J(N,2)=F(N,1)*M1*M1+F(N,2)*M2*M2+F(N,3)*M3*M3+F(N,4)*M1*M3
2760 LET J(N,3)=F(N,1)*N1*N1+F(N,2)*N2*N2+F(N,3)*N3*N3+F(N,4)*N1*N3
2770 LET J(N,4)=2*F(N,1)*L1*M1+2*F(N,2)*L2*M2+2*F(N,3)*L3*M3+F(N,4)*(L1*M3+L3*M1
)
2780 LET J(N,5)=2*F(N,1)*M1*N1+2*F(N,2)*M2*N2+2*F(N,3)*M3*N3+F(N,4)*(M1*N3+M3*N1
)
2790 LET J(N,6)=2*F(N,1)*N1*L1+2*F(N,2)*N2*L2+2*F(N,3)*N3*L3+F(N,4)*(N1*L3+N3*L1
)
2800 NEXT N
2810 K9=2
2820 GOTO 1270
2830 PRINT
2840 PRINT " PRINCIPAL STRESSES"
2850 PRINT " STRESS STD. DEV. BEARING INCL."
2860 FOR N=1 TO 3
2870 PRINT USING 2890;B(N,1),V(N),Z(N,2),Z(N,3)
2880 NEXT N
2890 IMAGE 3X,S6D,4X,5D,5X,3D.D,3X,S2D.D
2900 END
2910 REM *** COMPUTE BEARINGS AND INCLINATIONS
2920 REM *** OF PRINCIPAL STRESSES
2930 LET I5=90-ACS(P(3,N))*R1
2940 IF P(1,N)<>0 THEN 3030
2950 IF P(2,N)>0 THEN 2990
2960 IF P(2,N)<0 THEN 3010
2970 LET B5=999
2980 GOTO 3140
2990 LET B5=0
3000 GOTO 3140
3010 LET B5=180
3020 GOTO 3140
3030 LET B5=ATN(P(1,N)/P(2,N))*R1
3040 IF P(2,N)<0 THEN 3080
3050 IF P(1,N)>0 THEN 3090

```

PROGRAM LISTING OF USBM -- Continued

```
3060 LET B5=360+B5
3070 GOTO 3090
3080 LET B5=180+B5
3090 IF I5<0 THEN 3120
3100 LET I5=-I5
3110 LET B5=180+B5
3120 IF B5<360 THEN 3140
3130 LET B5=B5-360
3140 RETURN
```

PROGRAM LISTING OF CSIRO

```

10  WRITE IO 10,5;1
20  WRITE IO 10,4;39
30  WRITE IO 10,6;1
40  WRITE IO 10,5;128
50  OPTION BASE 1
60  REM *** *****
70  REM *** A PROGRAM FOR THE CALCULATION OF STRESS COMPONENTS, PRINCIPAL
80  REM *** STRESSES, AND THEIR STANDARD DEVIATIONS FROM STRAIN READINGS FROM
90  REM *** THE CSIRO HOLLOW INCLUSION STRESS CELL IN A SINGLE BOREHOLE.
100 REM ***
110 REM *** WRITTEN IN BASIC FOR HEWLETT-PACKARD 9800-SERIES MICRO-COMPUTERS
120 REM *** BY WILLIAM K. SMITH, U.S. GEOLOGICAL SURVEY, JAN., 1982.
130 REM *** *****REFERENCES*****
140 REM *** PANEK, LOUIS A., 1966, CALCULATION OF THE AVERAGE GROUND STRESS
150 REM *** COMPONENTS FROM MEASUREMENTS OF THE DIAMETRAL DEFORMATION OF
160 REM *** A DRILL HOLE: U.S. BUREAU OF MINES REPT. OF INV. 6732, 41 P.;
170 REM *** OR ASTM SPEC. TECH. PUB. 402, P. 106-132
180 REM *** WOROTNICKI, G., AND WALTON, R., 1976, TRIAXIAL "HOLLOW INCLUSION"
190 REM *** GAUGES FOR DETERMINATION OF ROCK STRESSES IN SITU:
200 REM *** COMMONWEALTH SCI. AND IND. RESEARCH ORG. (AUSTRALIA) DIV.
210 REM *** APPLIED GEOMECHANICS RESEARCH PAPER 275; OR PROC. SYMP. ON
220 REM *** INVESTIGATION OF STRESS IN ROCK - ADVANCES IN STRESS
230 REM *** MEASUREMENT, SYDNEY, 1976, P. 1-8.
240 REM *** FIELD MANUAL FOR CSIRO HOLLOW INCLUSION GAUGE
250 REM *** (INCLUDES FORTRAN DATA REDUCTION PROGRAM)
260 REM *** SMITH, WILLIAM K., 1982, TWO BASIC COMPUTER PROGRAMS FOR THE
270 REM *** DETERMINATION OF IN-SITU STRESSES USING THE CSIRO HOLLOW
280 REM *** INCLUSION GAGE AND THE USBM BOREHOLE DEFORMATION GAGE:
290 REM *** U.S. GEOLOGICAL SURVEY OPEN-FILE REPT. 82-489, 40 P.
300 REM *** *****
310 REM *** COORDINATE SYSTEMS:
320 REM *** X-AXIS IS POSITIVE EAST
330 REM *** Y-AXIS IS POSITIVE NORTH
340 REM *** Z-AXIS IS POSITIVE UP
350 REM ***
360 REM *** H1-AXIS IS HORIZONTAL TO THE RIGHT WHEN LOOKING INTO THE HOLE.
370 REM *** H2-AXIS IS COINCIDES WITH THE BOREHOLE AXIS.
380 REM *** H3-AXIS IS IN A VERTICAL PLANE, POINTS GENERALLY UPWARD.
390 REM *** BEARINGS ARE IN DEGREES CLOCKWISE FROM NORTH.
400 REM *** INCLINATIONS ARE IN DEGREES FROM THE HORIZONTAL, POSITIVE UPWARDS.
410 REM *** BEARING OF A VERTICAL HOLE IS 90 DEGREES COUNTER-CLOCKWISE FROM THE
420 REM *** DIRECTION OF THE H1-AXIS, AS DETERMINED FROM THE ORIENTATION OF THE
430 REM *** SETTING TOOL.
440 REM *** UNITS FOR THE STRESS VALUES IN THE OUTPUT ARE THE SAME AS THOSE FOR
450 REM *** USED FOR YOUNG'S MODULUS IN THE INPUT.
460 REM *** ORIENTATION ANGLES ALPHA AND BETA AND CORRECTION FACTORS K1-K4 ARE
470 REM *** INITIALIZED IN DATA STATEMENTS.
480 REM *** ALPHA AND BETA ANGLES WILL NOT CHANGE AS LONG AS STANDARD
490 REM *** INSTALLATION PROCEDURES ARE USED. CORRECTION FACTORS MAY CHANGE
500 REM *** DEPENDING ON THE ROCK/EPOXY MODULUS RATIO.
510 DIM A(6,6),B(6,1),C(6,6),D(9,2),E(9),F(9,6),G(6,1),I(3,3),J(9,6),K(9,9)
520 DIM P(3,3),S(3,3),T(9,2),U(9),V(6),X(3,3),Y(3,3),Z(3,3)

```

PROGRAM LISTING OF CSIRO -- Continued

```

530 LET R1=57.29578
540 LET K9=1
550 PRINT "INPUT NUMBER OF STRAIN READINGS TO BE AVERAGED"
560 INPUT M
570 PRINT "INPUT ELASTIC MODULUS AND POISSON'S RATIO"
580 INPUT E1,P
590 PRINT "INPUT BEARING AND INCLINATION OF BOREHOLE"
600 INPUT B2,I2
610 LET B1=B2/R1
620 LET I1=I2/R1
630 REM *** READ ORIENTATION ANGLES ALPHA AND BETA
640 REM *** AND CORRECTION FACTORS K1 THROUGH K4
650 FOR N=1 TO 9
660 READ D(N,1)
670 NEXT N
680 FOR N=1 TO 9
690 READ D(N,2)
700 NEXT N
710 READ K1,K2,K3,K4
720 DATA 322.9,300,300,163.6,163.6,180,82.9,60,60
730 DATA 0,90,45,45,135,90,0,90,45
740 DATA 1.12,1.13,1.08,0.91
750 REM *** CONVERT ALPHA AND BETA TO RADIANS
760 MAT T=(1/R1)*D
770 PRINT "INPUT";M;"GAGE NUMBERS AND CORRESPONDING MICROSTRAINS"
780 FOR N=1 TO M
790 INPUT H(N),E(N)
800 LET U(N)=E(N)*1.0E-6
810 NEXT N
820 PRINT PAGE
830 PRINT " *** CSIRO HOLLOW INCLUSION STRESS CELL DATA REDUCTION ***"
840 PRINT
850 PRINT "          MICRO-"
860 PRINT " GAGE NO.   STRAIN   ALPHA   BETA"
870 FOR N=1 TO M
880 PRINT USING 890;H(N),E(N),D(H(N),1),D(H(N),2)
890 IMAGE (4X,D,6X,S6D,5X,3D.D,4X,3D.D)
900 NEXT N
910 PRINT
920 PRINT " BOREHOLE BEARING = ";B2;"   INCLINATION = ";I2
930 PRINT " MODULUS = ";E1;"   POISSON'S RATIO = ";P
940 PRINT
950 LET L1=COS(B1)
960 LET M1=-SIN(B1)
970 LET N1=0
980 LET L2=SIN(B1)*COS(I1)
990 LET M2=COS(B1)*COS(I1)
1000 LET N2=SIN(I1)
1010 LET L3=-SIN(B1)*SIN(I1)
1020 LET M3=-COS(B1)*SIN(I1)
1030 LET N3=COS(I1)
1040 FOR N=1 TO M

```

PROGRAM LISTING OF CSIRO -- Continued

```

1050 LET K(N,1)=L1
1060 LET K(N,2)=L2
1070 LET K(N,3)=L3
1080 LET K(N,4)=M1
1090 LET K(N,5)=M2
1100 LET K(N,6)=M3
1110 LET K(N,7)=N1
1120 LET K(N,8)=N2
1130 LET K(N,9)=N3
1140 LET F1=COS(2*T(H(N),1))
1150 LET F2=COS(2*T(H(N),2))
1160 LET F3=SIN(2*T(H(N),1))
1170 LET F4=SIN(2*T(H(N),2))
1180 LET F5=P*P
1190 LET F(N,1)=((1-F5)*F1*(1-F2)*K2+.5*(K1-P)-.5*(K1+P)*F2)/E1
1200 LET F(N,2)=(.5*(1-K4*P)+.5*(1+K4*P)*F2)/E1
1210 LET F(N,3)=((F5-1)*F1*(1-F2)*K2+.5*(K1-P)-.5*(K1+P)*F2)/E1
1220 LET F(N,4)=2*(1+P)*COS(T(H(N),1))*F4*K3/E1
1230 LET F(N,5)=-2*(1+P)*SIN(T(H(N),1))*F4*K3/E1
1240 LET F(N,6)=2*(F5-1)*F3*(1-F2)*K2/E1
1250 NEXT N
1260 FOR N=1 TO 9
1270 LET J(N,1)=F(N,1)*L1*L1+F(N,2)*L2*L2+F(N,3)*L3*L3+F(N,4)*L1*L2+F(N,5)*L2*L3
+F(N,6)*L1*L3
1280 LET J(N,2)=F(N,1)*M1*M1+F(N,2)*M2*M2+F(N,3)*M3*M3+F(N,4)*M1*M2+F(N,5)*M2*M3
+F(N,6)*M1*M3
1290 LET J(N,3)=F(N,1)*N1*N1+F(N,2)*N2*N2+F(N,3)*N3*N3+F(N,4)*N1*N2+F(N,5)*N2*N3
+F(N,6)*N1*N3
1300 LET J(N,4)=2*F(N,1)*L1*M1+2*F(N,2)*L2*M2+2*F(N,3)*L3*M3+F(N,4)*(L1*M2+L2*M1
)+F(N,5)*(L2*M3+L3*M2)+F(N,6)*(L1*M3+L3*M1)
1310 LET J(N,5)=2*F(N,1)*M1*N1+2*F(N,2)*M2*N2+2*F(N,3)*M3*N3+F(N,4)*(M1*N2+M2*N1
)+F(N,5)*(M2*N3+M3*N2)+F(N,6)*(M1*N3+M3*N1)
1320 LET J(N,6)=2*F(N,1)*L1*N1+2*F(N,2)*L2*N2+2*F(N,3)*L3*N3+F(N,4)*(L1*N2+L2*N1
)+F(N,5)*(L2*N3+L3*N2)+F(N,6)*(L1*N3+L3*N1)
1330 NEXT N
1340 REM *** CLEAR A(I,J) AND G(J) ARRAYS
1350 FOR J=1 TO 6
1360 LET G(J,1)=0
1370 FOR I=1 TO 6
1380 LET A(I,J)=0
1390 NEXT I
1400 NEXT J
1410 REM *** COMPUTE [A] AND [G] ARRAYS
1420 FOR K=1 TO M
1430 FOR J=1 TO 6
1440 LET G(J,1)=G(J,1)+U(K)*J(K,J)
1450 FOR I=1 TO 6
1460 LET A(I,J)=A(I,J)+J(K,I)*J(K,J)
1470 NEXT I
1480 NEXT J
1490 NEXT K
1500 MAT C=INV(A)

```

PROGRAM LISTING OF CSIRO -- Continued

```

1510 MAT B=C*G
1520 REM *** COMPUTE STATISTICAL PARAMETERS
1530 IF K9=2 THEN 1750
1540 LET S1=0
1550 FOR K=1 TO M
1560 LET S1=S1+U(K)*U(K)
1570 NEXT K
1580 LET
S2=B(1,1)*G(1,1)+B(2,1)*G(2,1)+B(3,1)*G(3,1)+B(4,1)*G(4,1)+B(5,1)*G(5,1)+B(6,1)*
G(6,1)
1590 LET S3=S1-S2
1600 LET R2=1-S3/S1
1610 LET R=SQR(R2)
1620 LET S4=SQR(S3/(M-6))
1630 PRINT USING 1690;S1
1640 PRINT USING 1700;S2
1650 PRINT USING 1710;S3
1660 PRINT USING 1720;S4
1670 PRINT USING 1730;R2
1680 PRINT USING 1740;R
1690 IMAGE " STRAIN SUM OF SQUARES      = ",D.4DE
1700 IMAGE " [B].[G]                    = ",D.4DE
1710 IMAGE " RESIDUAL SUM OF SQUARES    = ",D.4DE
1720 IMAGE " STD. DEV OF FITTED DATA   = ",D.4DE
1730 IMAGE " GOODNESS OF FIT (R**2)    = ",D.5D
1740 IMAGE " CORRELATION COEFFICIENT   = ",D.5D
1750 FOR I=1 TO 6
1760 LET V(I)=S4*SQR(C(I,I))
1770 NEXT I
1780 IF K9=2 THEN 2840
1790 PRINT
1800 PRINT " STRESS COMPONENTS          STD. DEV."
1810 PRINT USING 1820;B(2,1),V(2)
1820 IMAGE "     NORMAL N-S = ",S3D.2D,4X,3D.2D
1830 PRINT USING 1840;B(1,1),V(1)
1840 IMAGE "     NORMAL E-W = ",S3D.2D,4X,3D.2D
1850 PRINT USING 1860;B(3,1),V(3)
1860 IMAGE "     NORMAL VER = ",S3D.2D,4X,3D.2D
1870 PRINT USING 1880;B(4,1),V(4)
1880 IMAGE "     SHEAR  HOR = ",S3D.2D,4X,3D.2D
1890 PRINT USING 1900;B(5,1),V(5)
1900 IMAGE "     SHEAR  N-V = ",S3D.2D,4X,3D.2D
1910 PRINT USING 1920;B(6,1),V(6)
1920 IMAGE "     SHEAR  E-V = ",S3D.2D,4X,3D.2D
1930 REM *** COMPUTE PRINCIPAL STRESSES AND ORIENTATIONS
1940 LET S(1,1)=B(1,1)
1950 LET S(2,1)=B(4,1)
1960 LET S(3,1)=B(6,1)
1970 LET S(1,2)=B(4,1)
1980 LET S(2,2)=B(2,1)
1990 LET S(3,2)=B(5,1)
2000 LET S(1,3)=B(6,1)

```


PROGRAM LISTING OF CSIRO -- Continued

```

2010 LET S(2,3)=B(5,1)
2020 LET S(3,3)=B(3,1)
2030 REM *** COMPUTE INVARIANTS OF STRESS TENSOR
2040 LET I1=S(1,1)+S(2,2)+S(3,3)
2050 LET I2=S(1,1)*S(3,3)+S(2,2)*S(3,3)+S(1,1)*S(2,2)
2060 LET I4=S(2,3)2+S(1,2)2+S(1,3)2
2070 LET I2=I2-I4
2080 MAT P=INV(S)
2090 LET I3=DET
2100 REM *** SOLVE CUBIC EQUATION BY TRIGONOMETRIC SUBSTITUTION
2110 LET Q1=-I1
2120 LET Q2=I2
2130 LET Q3=-I3
2140 LET Q4=(3*Q2-Q1*Q1)/3
2150 LET Q5=(2*Q13-9*Q1*Q2+27*Q3)/27
2160 LET Q6=-Q5/2/SQR(-Q43/27)
2170 LET Q7=ACS(Q6)/3
2180 LET X1=2*SQR(-Q4/3)*COS(Q7)
2190 LET X2=2*SQR(-Q4/3)*COS(Q7+120/R1)
2200 LET X3=2*SQR(-Q4/3)*COS(Q7+240/R1)
2210 LET Z(1,1)=X1-Q1/3
2220 LET Z(2,1)=X2-Q1/3
2230 LET Z(3,1)=X3-Q1/3
2240 REM *** END OF CUBIC SOLUTION
2250 REM *** SORT PRINCIPAL STRESSES
2260 IF Z(1,1)>Z(2,1) THEN 2330
2270 IF Z(1,1)>Z(3,1) THEN 2430
2280 IF Z(2,1)>Z(3,1) THEN 2470
2290 LET Z1=Z(3,1)
2300 LET Z2=Z(2,1)
2310 LET Z3=Z(1,1)
2320 GOTO 2500
2330 IF Z(1,1)>Z(3,1) THEN 2380
2340 LET Z1=Z(3,1)
2350 LET Z2=Z(1,1)
2360 LET Z3=Z(2,1)
2370 GOTO 2500
2380 IF Z(2,1)>Z(3,1) THEN 2530
2390 LET Z1=Z(1,1)
2400 LET Z2=Z(3,1)
2410 LET Z3=Z(2,1)
2420 GOTO 2500
2430 LET Z1=Z(2,1)
2440 LET Z2=Z(1,1)
2450 LET Z3=Z(3,1)
2460 GOTO 2500
2470 LET Z1=Z(2,1)
2480 LET Z2=Z(3,1)
2490 LET Z3=Z(1,1)
2500 LET Z(1,1)=Z1
2510 LET Z(2,1)=Z2
2520 LET Z(3,1)=Z3

```

PROGRAM LISTING OF CSIRO -- Continued

```

2530 REM *** END OF SORT
2540 REM *** COMPUTE DIRECTION COSINES OF PRINCIPAL STRESSES
2550 FOR N=1 TO 3
2560 MAT I=IDN
2570 MAT X=(Z(N,1))*I
2580 MAT Y=S-X
2590 LET N3=(Y(1,1)*Y(2,2)-Y(2,1) 2)/(Y(1,2)*Y(2,3)-Y(2,2)*Y(1,3))
2600 LET N2=- (Y(1,1)+Y(1,3)*N3)/Y(1,2)
2610 LET N4=SQR(1+N2*N2+N3*N3)
2620 LET P(1,N)=1/N4
2630 LET P(2,N)=N2/N4
2640 LET P(3,N)=N3/N4
2650 GOSUB 2920
2660 LET Z(N,2)=B5
2670 LET Z(N,3)=I5
2680 NEXT N
2690 REM *** RE-COMPUTE DIRECTION COSINES WITH RESPECT TO PRINCIPAL AXES
2700 REM *** TO COMPUTE STANDARD DEVIATIONS OF PRINCIPAL STRESSES
2710 FOR N=1 TO M
2720 LET L1=K(N,1)*P(1,1)+K(N,4)*P(2,1)+K(N,7)*P(3,1)
2730 LET M1=K(N,1)*P(1,2)+K(N,4)*P(2,2)+K(N,7)*P(3,2)
2740 LET N1=K(N,1)*P(1,3)+K(N,4)*P(2,3)+K(N,7)*P(3,3)
2750 LET L2=K(N,2)*P(1,1)+K(N,5)*P(2,1)+K(N,8)*P(3,1)
2760 LET M2=K(N,2)*P(1,2)+K(N,5)*P(2,2)+K(N,8)*P(3,2)
2770 LET N2=K(N,2)*P(1,3)+K(N,5)*P(2,3)+K(N,8)*P(3,3)
2780 LET L3=K(N,3)*P(1,1)+K(N,6)*P(2,1)+K(N,9)*P(3,1)
2790 LET M3=K(N,3)*P(1,2)+K(N,6)*P(2,2)+K(N,9)*P(3,2)
2800 LET N3=K(N,3)*P(1,3)+K(N,6)*P(2,3)+K(N,9)*P(3,3)
2810 NEXT N
2820 K9=2
2830 GOTO 1260
2840 PRINT
2850 PRINT " PRINCIPAL STRESSES"
2860 PRINT "      STRESS  STD. DEV.  BEARING  INCL."
2870 FOR N=1 TO 3
2880 PRINT USING 2900;B(N,1),V(N),Z(N,2),Z(N,3)
2890 NEXT N
2900 IMAGE 3X,S3D.2D,4X,2D.2D,5X,3D.D,3X,S2D.D
2910 END
2920 REM *** COMPUTE BEARINGS AND INCLINATIONS
2930 REM *** OF PRINCIPAL STRESSES
2940 LET I5=90-ACS(P(3,N))*R1
2950 IF P(1,N)<>0 THEN 3040
2960 IF P(2,N)>0 THEN 3000
2970 IF P(2,N)<0 THEN 3020
2980 LET B5=999
2990 GOTO 3150
3000 LET B5=0
3010 GOTO 3150
3020 LET B5=180
3030 GOTO 3150
3040 LET B5=ATN(P(1,N)/P(2,N))*R1

```

PROGRAM LISTING OF CSIRO -- Continued

```
3050 IF P(2,N)<0 THEN 3090
3060 IF P(1,N)>0 THEN 3100
3070 LET B5=360+B5
3080 GOTO 3100
3090 LET B5=180+B5
3100 IF I5<0 THEN 3130
3110 LET I5=-I5
3120 LET B5=180+B5
3130 IF B5<360 THEN 3150
3140 LET B5=B5-360
3150 RETURN
```

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