NEAR-STATION TERRAIN CORRECTIONS

FOR GRAVITY DATA

BY A SURFACE-INTEGRAL TECHNIQUE

by

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ABSTRACT

A new method of computing gravity terrain corrections by use of a digitizer and digital computer can result in substantial savings in the time and manual labor required to perform such corrections by conventional manual ring-chart techniques. The method is typically applied to estimate terrain effects for topography near the station, for example within 3 km of the station, although it has been used successfully to a radius of 15 km to estimate corrections in areas where topographic mapping is poor.

Points (about 20) that define topographic maxima, minima, and changes in the slope gradient are picked on the topographic map, within the desired radius of correction about the station. Particular attention must be paid to the area immediately surrounding the station to ensure a good topographic representation. The horizontal and vertical coordinates of these points are entered into the computer, usually by means of a digitizer. The computer then fits a multiquadric surface to the input points to form an analytic representation of the surface. By means of the divergence theorem, the gravity effect of an interior closed solid can be expressed as a surface integral, and the terrain correction is calculated by numerical evaluation of the integral over the surfaces of a cylinder,

The vertical sides of which are at the correction radius about the station, the flat bottom surface at the topographic minimum, and the upper surface given by the multiquadric equation.

The method has been tested with favorable results against models for which an exact result is available and against manually computed field-station locations in areas of rugged topography. By increasing the number of points defining the topographic surface, any desired degree of accuracy can be obtained. The method is more objective than manual ring-chart techniques because no average compartment elevations need be estimated.

INTRODUCTION

Terrain corrections to gravity data collected in areas of moderate to rugged topography are a necessary but time-consuming and expensive step in the reduction of gravity data. Such corrections are commonly made using ring charts on topographic maps (Hayford and Bowie, 1912; Hammer, 1939) or using computer methods that utilize a digital image of the topography (Kane, 1962; Plouff, 1966). Both computer methods require a division of the topography of the region into approximately square cells to which an average elevation is assigned; from this digital image, prisms of height equal to the difference between the cell altitude and station altitude can be defined. The terrain effect is then computed by summing the computed gravity effects of all the prisms within a specified radius of the station. These computer methods have not been sufficiently accurate for the near-station (<2.5 km) part of the terrain correction, which has consequently been done by using ring chart methods that are tedius and error-prone.

In the United States, the recent availability of very detailed digital topographic models has permitted the use of the method of Plouff (1966), with some modifications, to compute terrain corrections to within about 70 m of the station with satisfactory results except in the case of very rugged topography near the station. A second successful approach (Krohn, 1976) has been to use the digitized topography (typically at 1 km intervals) to define an analytic surface about the station that is then used to generate (by computer) the average elevations of ring segments. The gravity effects of these compartments are computed and summed, thus achieving a computer analog of the manual ring-chart method.

The method described in this paper also utilizes the fitting of an analytic surface representation to the topography but calculates the terrain correction from a direct numerical integration of the surface, thus abandoning ring compartments and prisms. Points used to define the topographic surface are manually chosen such that near the station the surface can be specified in more detail if necessary. This technique permits estimates of near-station terrain corrections to be easily made in areas where detailed topographic maps are lacking, and it eliminates the tendency to overestimate the terrain correction because of the use of flat-topped prisms in the ring-chart method. Once some facility is gained in the choosing of points that adequately specify the topography, and if an electronic digitizer is used to input the points, this method yields fast and accurate near-station terrain corrections with modest computer times.

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MATHEMATICAL DEVELOPMENT

The topography within some radius R_D about the station can be represented by a multiquadric equation that represents a series of cones (Hardy, 1971). Real topography is commonly conical in character, and, as shown by Hardy (1971), this surface representation does not introduce spurious hills or valleys, which a Fourier series or polynomial surface is apt to do; thus such a surface is a good choice for topographic representation.

Let the set of N topographic points be given by $\{r_j, \theta_j, z_j | j = 1,2, \ldots, N\}$ where the origin of this cylindrical coordinate system is taken so that the z-axis passes through the station. The coordinate z is positive upward. Transform the set $\{r_j, \theta_j, z_j\}$ to an equivalent set of rectangular coordinates x_j, y_j, z_j with the same origin. Then a multiquadric representation of the topographic surface is given by (Hardy, 1971)

$$z(x,y,) = \sum_{j=1}^{N} c_j \left[(x-x_j)^2 + (y-y_j)^2\right]^{\frac{1}{2}} \quad . \tag{1}$$

To find the set of c_j, form the N equations

$$z_i = \sum_{j=1}^{N} c_j \left[(x_j-x_i)^2 + (y_j-y_i)^2\right]^{\frac{1}{2}} \tag{2}$$

where $i = 1,2, \ldots, N$; or in matrix notation,

$$\underline{Z} = X\underline{C} \tag{3}$$

where $\underline{Z}$ and $\underline{C}$ are vectors and X is an NxN matrix. Then

$$\underline{C} = X^{-1}\underline{Z} \tag{4}$$

and the coefficients c_j are determined. The inversion of the well-conditioned, symmetric matrix X can be accomplished by several schemes, for example, Gauss elimination, which was used in this work.

The terrain correction is calculated by computing the gravity effect of a vertical-sided circular cylinder of radius R_o , whose flat (z=constant) bottom is at or below the minimum point of the fitted surface. The top surface of the cylinder is defined by equation (1). Because the terrain correction is defined as the gravity effect of deviations of the topographic surface from the Bouguer plane passing through the station (Grant and West, 1965, p. 239), the terrain correction can be calculated by subtracting the gravity effect of a circular cylinder, of radius R with the same base and a flat top, passing through the station. In the more common case of determining the terrain correction between an inner radius R_i and an outer radius R_o , the above procedure is carried out from R_i to R_o .

The gravity effect, g, of the cylinder is calculated as follows. First, a volume integration is avoided by applying some form of the divergence theorem to convert the volume integral into a surface integral, in this case the formulation of Bodvarsson (1970). Bodvarsson derived for the gravity effect (vertical acceleration) the surface integral

$$\frac{g}{G\rho} \int_S \frac{\hat{\underline{k}} \cdot \hat{\underline{n}}}{r_{ps}} \, d\sigma \equiv I \qquad (5)$$

where G is the gravitational constant, ρ is the uniform density of the body, $d\sigma$ is an infinitesimal element of the surface, $\hat{\underline{k}}$ is the unit vector in the positive (upward) z direction, $\hat{\underline{n}}$ is a unit vector of the outward normal to $d\sigma$ and

$$r_{ps} = (x^2 + y^2 + z(x,y)^2)^{1/2} \qquad (6)$$

is the distance between the station (origin) and the integration element

$d\sigma$ on the surface S of the body. The vertical sides of the cylinder contribute nothing to the integral since $\hat{\underline{k}}\cdot\hat{\underline{n}}$ is everywhere zero on them.

Considering an element of the surface $d\sigma$, the normal $\hat{\underline{n}}$ to the surface is defined by r, θ and $z(r,\theta)$ (or x, y, and z (x,y)) with z related to r and θ(x and y) by equation (1). The projection of $d\sigma$ on the r, θ plane is $r dr d\theta$, as shown in figure 1, and the relation between them is

$$d\sigma \cos\phi = r dr d\theta \tag{7}$$

where ϕ is the angle between the tangent plane to the surface at $d\sigma$ and the r,θ plane. Because $\hat{\underline{k}}$ is the normal to the r,θ plane, $\cos\phi$ is simply $\hat{\underline{k}}\cdot\hat{\underline{n}}$ and we have

$$d\sigma = \frac{r d r d\theta}{\hat{\underline{k}}\cdot\hat{\underline{n}}} \tag{8}$$

and equation (5) becomes

$$I = \int_0^{2\pi}\int_{R_i}^{R_o} \frac{r dr d\theta}{r_{ps}}\Bigg|_{top} + \int_0^{2\pi}\int_{R_i}^{R_o} \frac{r dr d\theta}{r_{ps}}\Bigg|_{bot} \equiv I_{top} + I_{bot} \tag{9}$$

and r_{ps} is calculated from equation (6).

The flat bottom surface has $\hat{\underline{k}}\cdot\hat{\underline{n}} = -1$ and the contribution to I for the bottom is

$$I_{bot} = 2\pi\left\{(R_o^2+h^2)^{1/2} - |h|\right\} \tag{10}$$

where h is the distance along the z-axis to the bottom from the station. For the case of an annulus,

$$I_{bot} = 2\pi\left\{(R_o^2+h^2)^{1/2} - (R_i^2+h^2)^{1/2}\right\} \tag{11}$$

For the top surface, the integral is

$$I_{top} = \int_0^{2\pi}\int_{R_i}^{R_o} \frac{r dr d\theta}{(r^2+z^2)^{1/2}} \tag{12}$$

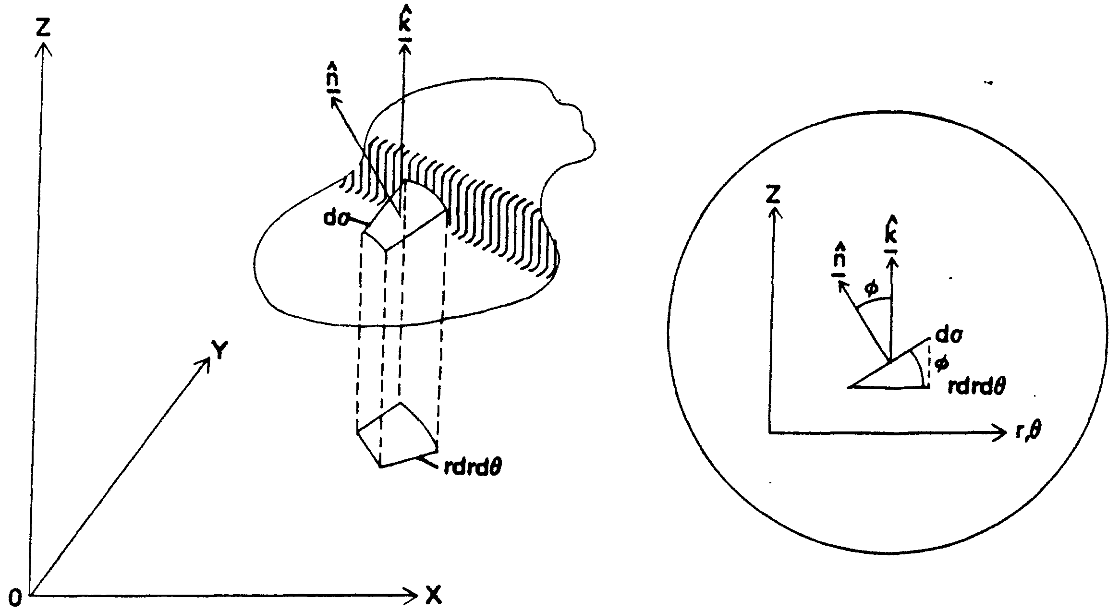

Figure 1.–Geometrical relations between an element of the multiquadric surface dσ and its projections on the r,θ plane. Because the unit vector outward normal to the surface $\hat{n}$ is defined by r,θ, and z(r,θ) and $\hat{k}$ is the unit vector in the +z direction, cos φ is $\hat{k} \cdot \hat{n}$ as shown in the inset. The x-y plane is also the r,θ plane.

where x and y are determined from the transformation equations from r and θ and z_i is given by equation (2). This integral is evaluated numerically, and the gravity effect of the cylinder (or annulus) is

$$g_{cyl} = G\rho \left(I_{top} + I_{bot} \right) \tag{13}$$

Finally, the gravity effect of a flat-topped annular ring from the station to the bottom of the body is given by

$$g_{ar} = 2\pi G\rho \left(R_o - R_i + (h^2 + R_i^2)^{\frac{1}{2}} - (h^2 + R_o^2)^{\frac{1}{2}} \right)^{\frac{1}{2}} \tag{14}$$

and the terrain correction is thus

$$g_{tc} = G\rho \left| \int_{0}^{2\pi} \int_{R_i}^{R_o} \frac{r\,d\,r\,d\theta}{(r^2 + z^2)^{\frac{1}{2}}}\Bigg|_{top} - 2\pi (R_o - R_i) \right| \tag{15}$$

where we take absolute values because the terrain correction for near-station topography is always positive.

METHOD

Although an r, θ template analogous to the ring charts can be drawn up to overlay the topographic map, the most efficient method is to use an electronic digitizer to enter the set of $\{r_j, \theta_j, z_j\}$. In the test cases described here, [~~a circle of radius R was drawn or~~] a transparency with the outer radius R_o drawn to scale on it was overlain centered on the station on the topographic map, which is attached to the digitizer board. Using a grease pencil or similar method, the points defining the surface were marked. For the computer program used in these examples a maximum of 25 points, including the station, was allowed. Then, using the digitizer, the x and y values were recorded and the z values and station name were typed on the digitizer keyboard.

Using a computer, the information so recorded was then converted from digitizer coordinates to actual distances, the multiquadric surface fit was performed, and the numerical integration was carried out. In these examples integration was done in the r-direction by repeatedly using Simpson's rule (Abramowitz and Stegun, 1965, p. 886) for three equally spaced values of r modified such that the integral contribution between two successive points is the average of the "leading" and "lagging" three points that bracket the interval in question. For the test cases described here, corrections were calculated from Hayford-Bowie (1912) zone "C" (inner radius 0.068 km) through zone "F" (outer radius 2.29 km). Fifty-one points were used from 0.068 to 0.50 km radius, and fifty-one points were used for the interval from 0.50 to 2.29 km. The increment in for integration was 10°, therefore 3,672 points were used in the integration for each station. Note that the integration automatically expands to a larger integration interval in the outer zones. About 5 minutes at the digitizer and 40 seconds of computer time on an IBM 370 were required per station. The computer time per station can be reduced substantially without undue loss of accuracy by reducing the number of points used in the r-integration. Documentation and program listings of the Fortran programs used for these computations are available from the author.

In order to render a faithful representation of the topography, the points that define the maxima and minima and changes in slope of the topography must be carefully chosen, especially near the station. A certain amount of skill must be acquired analogous to estimating average elevations in template compartments in the manual method. Poor choices of points in this method generally result in the generation of too smooth a multiquadric surface such that the terrain correction is usually less than that given by the manual use of ring charts. In the ring-chart method, the tendency is often to overestimate the average compartment elevation, which, if combined with the flat-top prism approximation inherent in this method, will lead to an overestimation of the terrain effect. This overestimation effect was noted by Krohn (1976).

DISCUSSION AND APPLICATION TO TEST CASES

The method was tested against two simple models for which an analytical result was available. In the first test, a cone of 2.29 km radius base and 225 m height was used. The terrain correction from 68 to 2,290 m about the station calculated by the numerical integration method (using the scheme with 3,672 points described above) was smaller than the exact result by 0.06 mgal (0.37 percent). For this test, 21 points were used to define the conical surface. The terrain correction of the cone was also manually calculated using the Hayford-Bowie ring chart of 28 compartments for zone C through F (68 to 2,290 m) and using the average elevation for each compartment given by the equation of the cone in question. The result was a correction that exceeded the exact result by 0.24 mgal (20 percent), which represents the error in the flat-topped prism approximation of the ring-chart method. Because of this error in the ring-chart method, corrections for stations on peaks are generally overestimated and those for valleys underestimated. The amount of error is a function of cone angle and is documented by Krohn (1976); it is of the order of 15 percent.

In the second test, the terrain correction for from 68 to 2,290 m radius for an inverted cone was calculated. The inverted cone had a basal radius of 2,290 m and a height of 500 m. When 17 points were used to define the surface, the result was too large by 0.04 mgal (0.70 percent).

For a field test, an area of dissected flood-basalt topography with about 700 m of relief was selected near Spray, Oregon, U.S.A. The topography of this area is characterized by deep valleys, steep slopes, and abrupt gradient changes and thus provides a good test of any terrain-correction method. Five station locations were chosen: 1) a high, flat-topped peak; 2) the bottom of a deep, narrow stream valley; 3) a bench partway up a hillside; 4) near the end of a more or less flat-topped ridge with very steep sides; and 5) the summit of a sharp ridge. The stations were chosen to simulate typical gravity station locations in fairly rugged topography. Corrections for these locations were made using the Hayford-Bowie ring chart for zones C through F (68 to 2,290 m) consisting of 28 compartments. Corrections for stations 2, 4, and 5 were also made using the Hayford-Bowie subzone chart of 112 compartments.

Terrain corrections for these stations were computed as outlined above, with most points chosen at topographic maxima and minima, and the results are shown in table 1 and plotted for comparison in figure 2. Stations 1 and 5 showed the largest disparity and were rerun with a few more points in an attempt to define the topography near the station, in particular the change in gradient to steep slopes. At station 5, an increase in the number of defining points and the use of more compartments in the ring-chart method lead to a convergence of results at approximately 7.2 mgal for this station. The actual topographic surface at station 5 is shown in figure 3, and the 23-point multiquadric surface is shown in figure 4 for comparison. As a result of the station 1 and 5 improvements, it is clear that good agreement between the two methods could be

Table 1.--*Comparison of near-station terrain corrections (68-2,290 m about station) calculated using the surface-integral method with manually computed corrections, Hayford-Bowie method, for five stations in the Spray, Oregon, area*

[Terrain corrections in mgals. "Number of points" is the number of points including the station used to define the multiquadric equation of the topography. Leaders indicate correction not made using that method]

Station	Terrain correction, Hayford-Bowie method (28 compartments)	Terrain correction, Hayford-Bowie method (112 compartments)	Number of points	Terrain correction, surface-integral method (3,672 points)	Terrain description
1	4.10	-	14	1.85	Station on flat-topped mountain top
1	4.10	-	21	3.60	Station on flat-topped mountain top
2	2.34	2.57	14	1.53	Station at bottom of deep, narrow stream valley
3	1.94	-	16	1.63	Station on bench partway up hillside
4	3.48	2.95	18	2.11	Station near end of flat-topped ridge
5	8.79	7.89	17	4.73	Station on summit of sharp ridge
5	8.79	7.89	23	6.67	Station on summit of sharp ridge

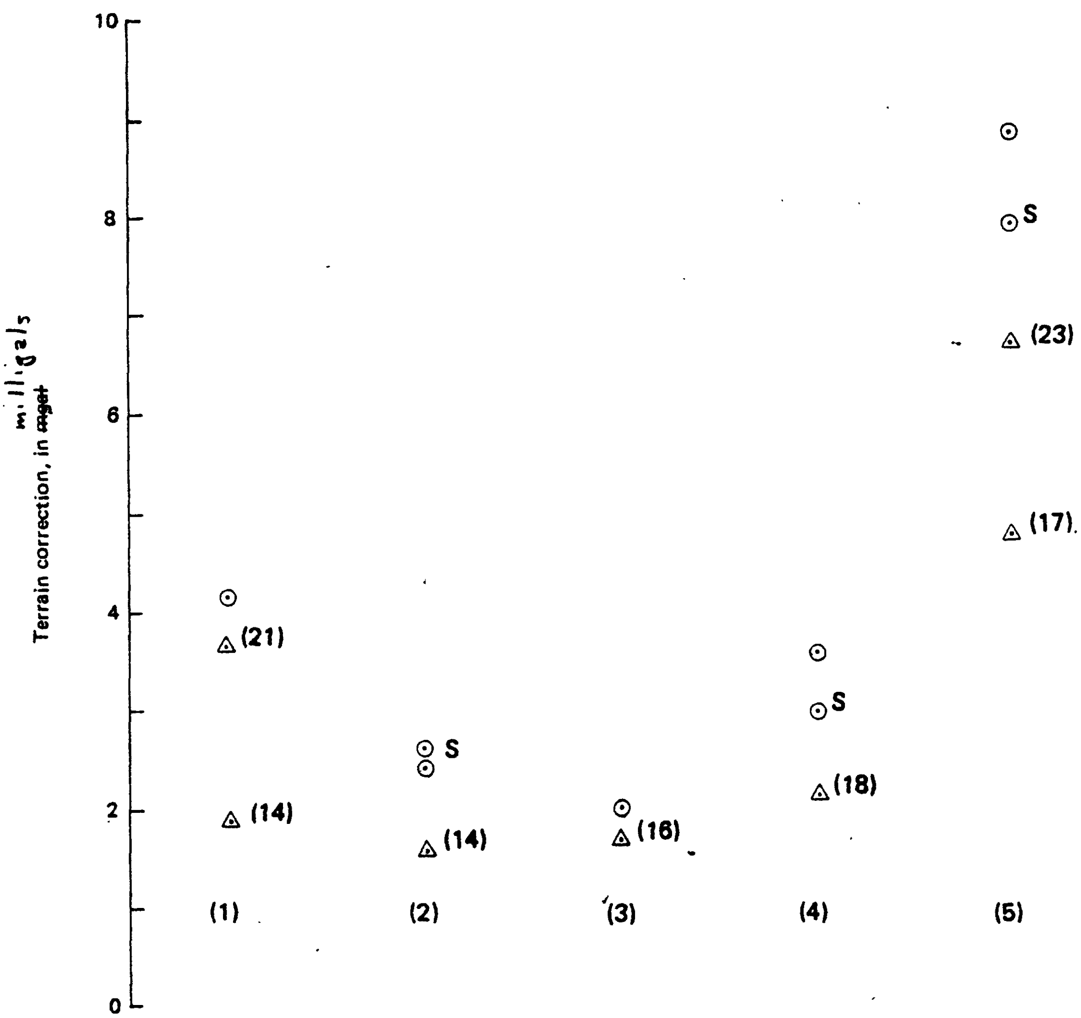

Figure 2.–Scatterplots of terrain-correction estimates for five test station locations computed by manual ring-chart and surface-integral methods. Circles are results for each station using Hayford-Bowie zones C-F (68-2,290 m radius) with 28 compartments. Circles with "S" beside them are corrections computed for same radii using a subzone chart (112 compartments). Triangles are terrain corrections computed by the surface-integral method; the number beside triangle is the number of topographic points used to define the surface; station number is in parentheses below the corrections. The agreement between values at stations 2 and 4 could be improved by including more points near the station.

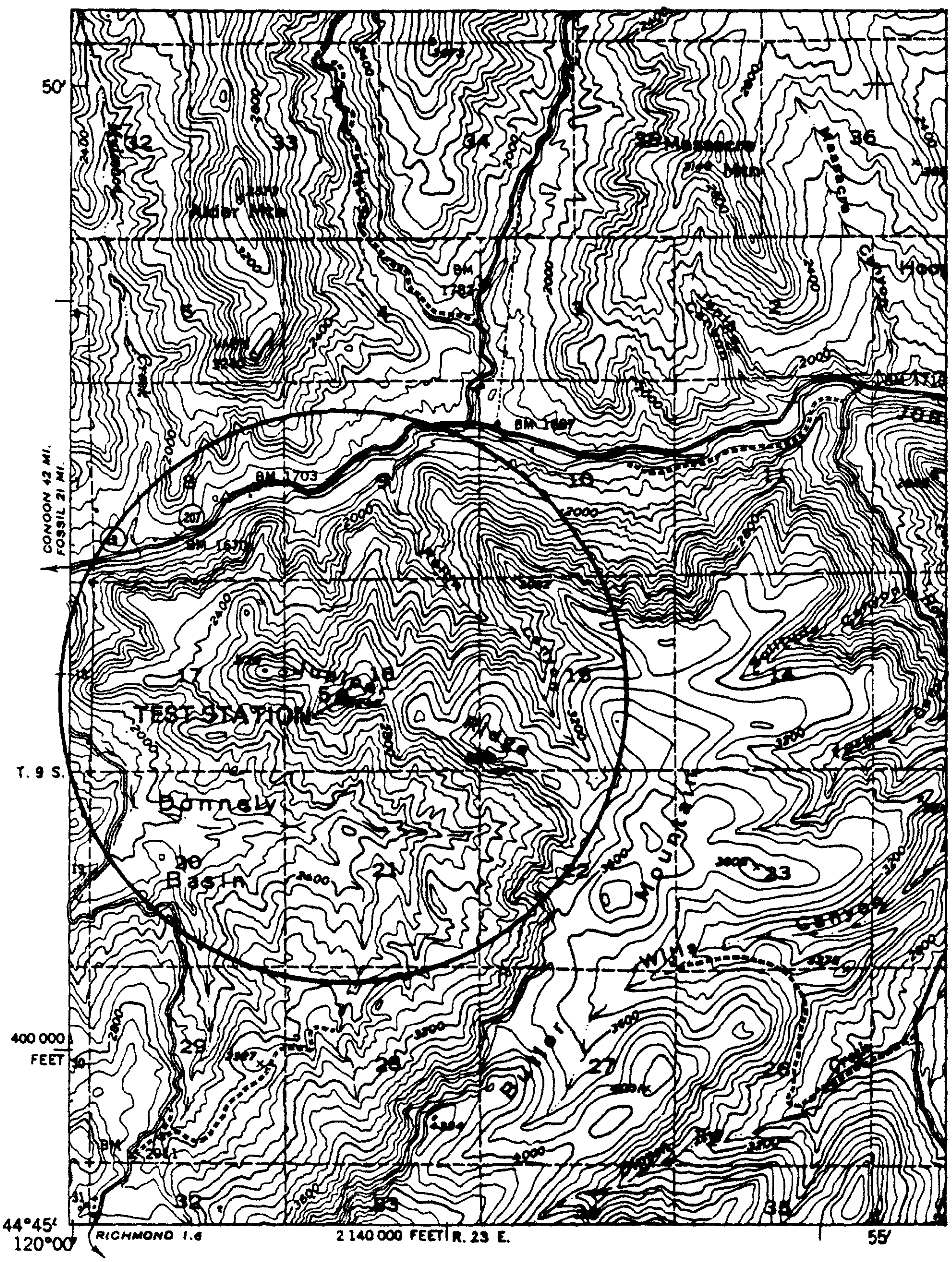

Figure 3.–Topographic map of the area of test station 5. Map is a section of the U. S. Geological Survey (1953) 15-minute series topographic map of the Spray quadrangle, Oregon; contour interval 80 feet, scale 1:62,500. Encircled area is area of terrain correction.

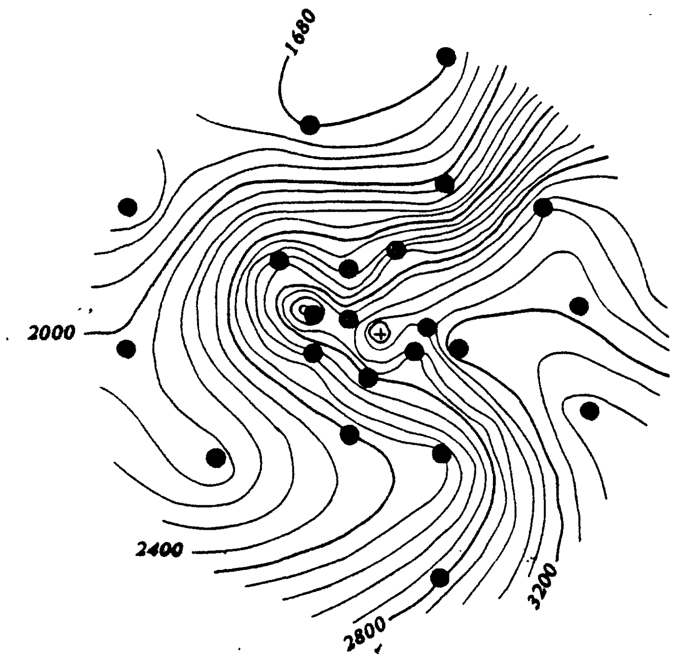

Figure 4.–Fitted multiquadric surface map of location of test station 5. Dots show locations of the 22 control points that were used, along with the station location at the center, to define the surface. Compare with encircled area of figure 3. Contour interval is 80 feet.

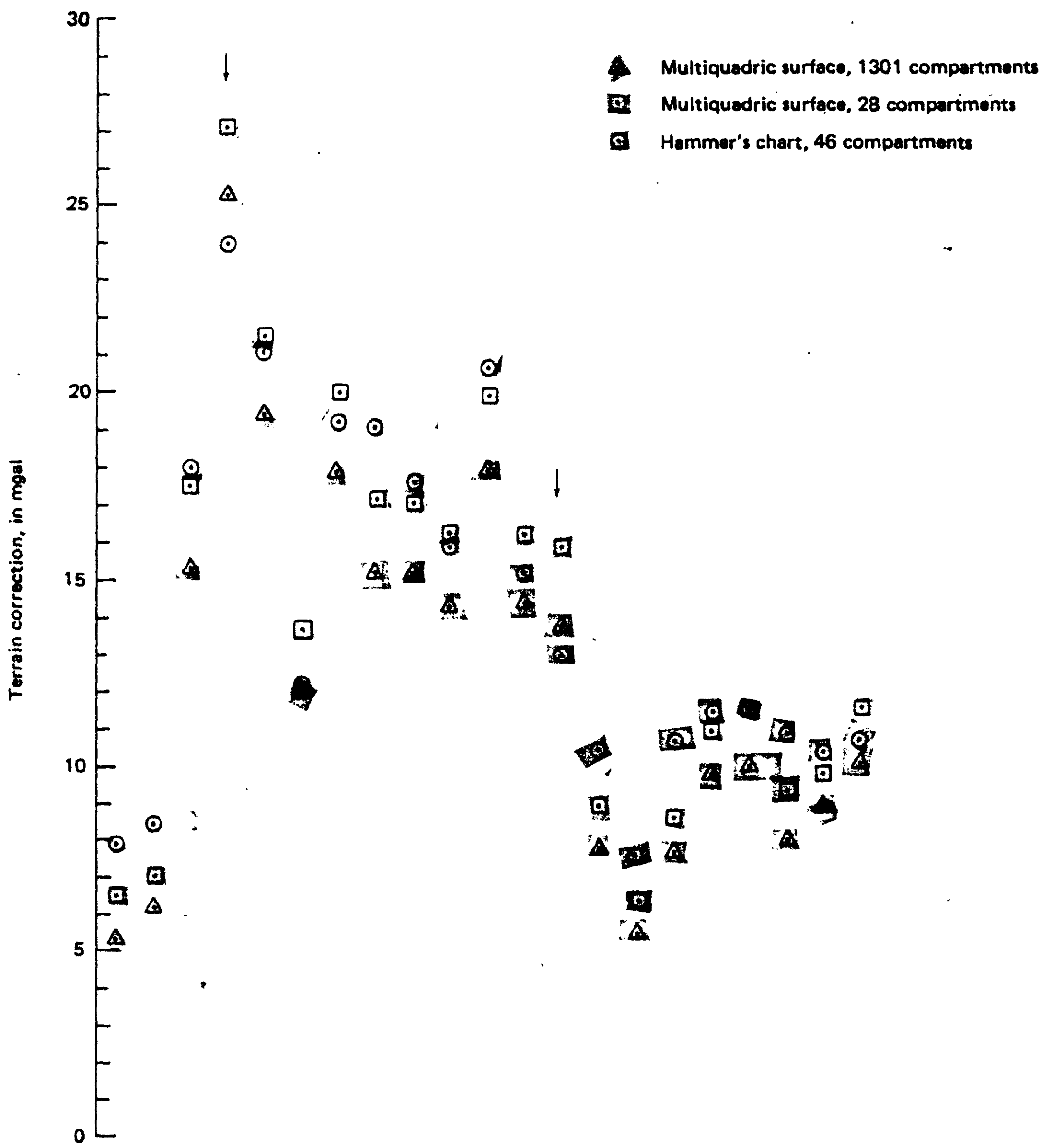

Figure 5.–Comparison scatterplots of terrain-correction estimates using multiquadric surface and manual ring-chart techniques from Krohn (p. 272, 1976). The arrows indicate the two gravity stations for which the highest precision multiquadric-surface terrain correction is greater than the ring-chart result. See text for details.

obtained at stations 2, 3, and 4 by a more careful choice of controlling points for the multiquadric-surface fit.

A similar plot of the data of Krohn (1976, table 1) is shown in figure 5 for 21 gravity stations. With only two exceptions, the highest precision multiquadric-surface terrain correction is systematically less than the ring-chart result. The two exceptions may be cases in which the station is in a valley bottom, where, because of the flat-topped prism approximation, the ring-chart method tends to give too small a correction. It appears therefore that the method is valid and if carefully applied will yield terrain-correction estimates that are equal to or greater than in precision compared to corrections obtained by the manual ring-chart method. The advantage of the method is its objectivity and the large reduction in labor required to complete the corrections. Also less skill is required to choose the points defining the surface than to estimate accurately the average elevations of compartments in rough topography. The method can also be applied to the innermost zones (0-70 m about the station) to compute corrections using estimated elevations obtained during the gravity survey work. Finally, the use of a smooth analytic surface to accomplish the computation rather than flat-topped prisms seems desirable on theoretical grounds.

This method was used to estimate terrain corrections in southwestern Saudi Arabia in the Jizan area (approximately lat 17° N., long 43° E.). Corrections were carried out for approximately 150 stations from 68 m to 15 km about the stations using 3.0 km radius as the point beyond which the integration spacing was increased. Computer time for the calculations averaged 36 seconds per station. In this area and at the time (1975), the only topographic map available was at 1:4,000,000 scale, and thus the large radius of 15 km was used. Beyond a radius of 15 km the method of Plouff (1966) was used. The estimated terrain corrections within a radius of 15 km are probably optimal for the topographic information available and required about 20 hours of time at the digitizer to select and enter the defining points. It is important to use the map elevation of the station rather than that determined in the gravity survey because there can be considerable discrepancies between the two resulting from the generality of the map and datum inconsistencies.

It should be noted that the method as presented should not be used for terrain corrections beyond a radius of 15 km about the station because no allowance for of the Earth's curvature has been included in the theory, and this effect becomes appreciable beyond a radius of 15 km.

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