UNITED STATES DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY

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by

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Abstract

Using modifications of previously developed methods, measurements were made of the resistivity and the dielectric constant of two similar quartz sands of different porosity over the frequency range 10^2-10^8 Hertz for various water contents and water resistivities. Dispersion is pronounced in all the resistivity data above 10^6 Hz. As water content decreases, resistivity dispersion becomes noticeable at lower frequencies. The resistivity data at all frequencies, however, fit an empirical prediction formula relating observed resistivity to water saturation and resistivity of the saturated sample. The data suggest that the dispersion of resistivity of some earth materials with frequency may be predicted on the basis of curve matching of the data obtained in this investigation with the resistivity of the material at a frequency of 100 Hz or lower.

The dispersion of dielectric constant with frequency is pronounced at all frequencies, being more pronounced with increasing water content. The shape of the dispersion curve is dependent on the resistivity of the water in the sample.

Introduction

The phenomena of the dispersion of resistivity (ρ) and dielectric constant (κ) with frequency in rock and soil material are well known (Howell and Licastro, 1961; Keller and Licastro, 1959; Smith-Rose, 1933; Frood and Sharf, 1954). Typically, large values of κ are measured at low frequency with κ decreasing as frequency increases. Resistivity also shows a decrease with increasing frequency, but in a manner different from dielectric constant.

These phenomena are attributed to the presence of dissolved salts in the water within the pore structure of the material. The salt ions are the primary current carriers in geologic materials of interest to the electromagnetic pulse (EMP) program of the Air Force Weapons Laboratory (AFWL). Polarization arises from accumulation of ions at points within the rock structure. Such accumulations can be caused by variations in ion mobility from point to point within the material, by variations in viscosity, pore size, and pore structure, and by the presence of solid conductors and ion exchangeable materials. This polarization manifests itself in an observed frequency dispersion of the resistivity and dielectric constant (Keller and Frischknecht, 1966).

To date, no systematic effort has been made to study these dispersion effects. As a knowledge of these parameters is necessary to determine EMP vulnerability criteria, the USGS (U.S. Geological Survey) at the request of AFWL, has undertaken a systematic investigation of the parameters affecting dispersion. If dispersion of the electrical properties at a site of interest can be predicted on the basis of a few simple *in situ* measurements, for example, water content, coupled with simple resistivity measurements at low frequency, a considerable saving of effort would result. Such a study is also of considerable utility in the understanding of wave propagation in earth materials.

Quartz sands were used as the initial step in this investigation because they are a pure material and could be considered as representative of "clean" earth, that is, material relatively free of polarizable materials such as clay. Predictive criteria relating resistivity to water content, salinity, and porosity are known to exist in such clay-free materials at frequencies less than 1 KHz. The behavior of materials containing clay are considered as a separate problem because of their special electrical properties.

Background

Previous work by the USGS for AFWL had been to develop a method for the laboratory measurement of dielectric constant in the frequency range 10^2-10^6 Hz (Scott and others, written commun., 1964), which described the development of a new type of electrode for use in measuring the dielectric constant of wet rocks. Further work was done (Judy and Eberle, 1969) to extend the frequency range to 10^8 Hz.

Equipment

The electronic equipment used to determine dielectric constant and resistivity from 10^2 to 10^8 Hz consists of commercially available impedance bridges with associated oscillators, detectors, and a frequency meter. The equipment is described in detail in the two references cited above, along with the design of the test cell and electrode system.

Prior to commencing the studies of the quartz sands additional calibrations of the sample cells were made for greater accuracy at the low dielectric constants (<10). A simple Wheatstone bridge was constructed and calibrated to make electrical measurements below 10^2 Hz. However, the results of the initial measurements below 10^2 Hz were inconsistent throughout the work. The behavior of the electrode system used and described herein has not been studied below this frequency and it is possible that the system is not reliable in this region. Because the checks for electrode reliability below 100 Hz require extensive investigation it was felt that time could be more probitably spent in making the measurements in the 10^2 to 10^8 Hz range at this time.

Electrode system

Briefly, the electrode consists of a platinized platinum disc with a specially treated piece of blotter paper between the disc and the sample. The blotter paper is treated by dipping it successively in solutions of 0.1 N silver nitrate, hydroxylamine hydrochloride, and sodium chloride and drying it slightly after each solution by pressing it between layers of towelling. This procedure precipitates metallic silver and silver chloride in the blotter paper. The purpose of this blotter is to provide an electrochemically reversible interface between the metal electrode and the sample. It was demonstrated (Scott and others, written commun., 1964) that this system effectively eliminated polarization at the sample-electrode interface at the lowest frequency of interest--100 Hz. Because the objective of the present work was to vary the water content and salinity in a controlled manner, it became obvious that the method just described for preparing and using the blotters would not be satisfactory. The variety of ions remaining in the blotter after the precipitation process could diffuse into the sample and alter the conductivity of the contained water; also, moisture from the blotter could migrate into the sample and alter the water content.

To overcome these difficulties, a revised method for preparing the blotters was devised, based on the assumption that the precipitated silver and silver chloride are the active elements and that the presence of the residual ions from the precipitation process is extraneous to the functioning of the system. After dipping the blotters in the three reagents to precipitate the silver and silver chloride, the blotters were placed in beakers of distilled water. The water was repeatedly changed to dilute out the residual ions. The blotters were then air dried before use. (The soaking and drying were done in light-proof containers to prevent photoreduction of the silver chloride.)

Sample preparation and treatment

The study was made using two quartz sand samples of mixed grain sizes. A sample of pegmatitic quartz was crushed and sieved into three parts (35 to 75 mesh, 75 to 150 mesh, and -150 mesh). Each part was cleaned by leaching with aqua regia and nitric acid, thoroughly washed with distilled water, and dried. The parts were then remixed in different proportions to obtain two experimental mixtures. Sample A had a fractional porosity (ϕ) of 0.51 and sample B a fractional porosity of 0.32.

The clean dry sample was placed in a test cell with a dry blotter and platinized electrode at each end, and sealed closed. A measured amount of saline water was then injected into the sample, using a hypodermic syringe, through a small-diameter port in the side of the sample holder. The sample was allowed to come to equilibrium (approximately 72 hrs) after which electrical measurements were made to determine the resistivity and dielectric constant. Reduction and evaluation of the data required an additional 2 days. Additional moisture was then added, allowed to equalize, and measured. This cycle was repeated until the relative water content (S_w) equaled 0.8.

$$\left(S_w = \frac{\text{volume of water}}{\text{volume of pores}}\right) \qquad (1)$$

After the measurements at $S_w = 0.8$ the sample cell was opened and the blotters removed, weighed, dried, and reweighed to determine the amount of moisture absorbed from the sample. This amounted to 7 percent of the total volume of water within the system. This value was assumed to be constant at all levels of water content and this correction subtracted to obtain the true water content of the samples.

The sample itself was removed, washed repeatedly with distilled water, and dried in preparation for the next run using water of a different salinity. Washing was done by placing the sample in a large beaker filled with distilled water. The sample was thoroughly agitated then allowed to stand until the fine grains had settled to the bottom (approximately 24 hrs), the water was then decanted off. This process was repeated five times.

On a few occasions the measurements gave irregular results because of problems with the electrodes (imperfect plating and(or) platinizing). This required stopping the run, removing the sample and cleaning it, refinishing the electrodes, and starting the run over again.

Results

Measurements were made on samples A and B for three values of water resistivity (ρ_w = 0.1, 1.0, and 10 ohm-m), at five relative water contents (S_w = 0.047, 0.093, 0.28, 0.47, and 0.74) over the frequency range 10^2-10^8 Hz. Over 200 data points were obtained.

Resistivity

Typical resistivity data are plotted on figures 1, 2, and 3 as a function of frequency. On figure 1, the resistivity is constant at low frequencies but begins to decrease as the frequency increases. The frequency at which the decrease in resistivity begins increases with S_w at given values of ϕ and ρ_w (water resistivity). The dashed lines A and B on figure 1 connect the points (indicated by x's) at which the curves have dropped to values 10 percent and 30 percent, respectively, of the value at 10^2 Hz. On figure 2 it may be seen that the frequency at which the decrease in resistivity begins becomes higher with decreasing ρ_w. Figure 3 illustrates the effect of porosity on the dispersion. The frequency at which the decrease in resistivity begins is lower for the lower porosity, this effect being noticeable only at the lower water contents.

Figures 4 and 5 are typical plots of ρ versus S_w at 10^2 Hz and 10^8 Hz, respectively. Extensive investigations have established a relationship (Keller and Frischknecht, 1966)

$$\frac{\rho}{\rho_{1.00}} = S_w^{-m} \qquad (2)$$

where $m \approx 2$ and $\rho_{1.00}$ is the resistivity of the sample at 100 percent saturation of the pores ($S_w = 1.0$); a ratio known to be generally true for direct current and low-frequency alternating current. The graphs indicate that this same relationship holds true up to 10^8 Hz, though m is somewhat less than 2 at that frequency.

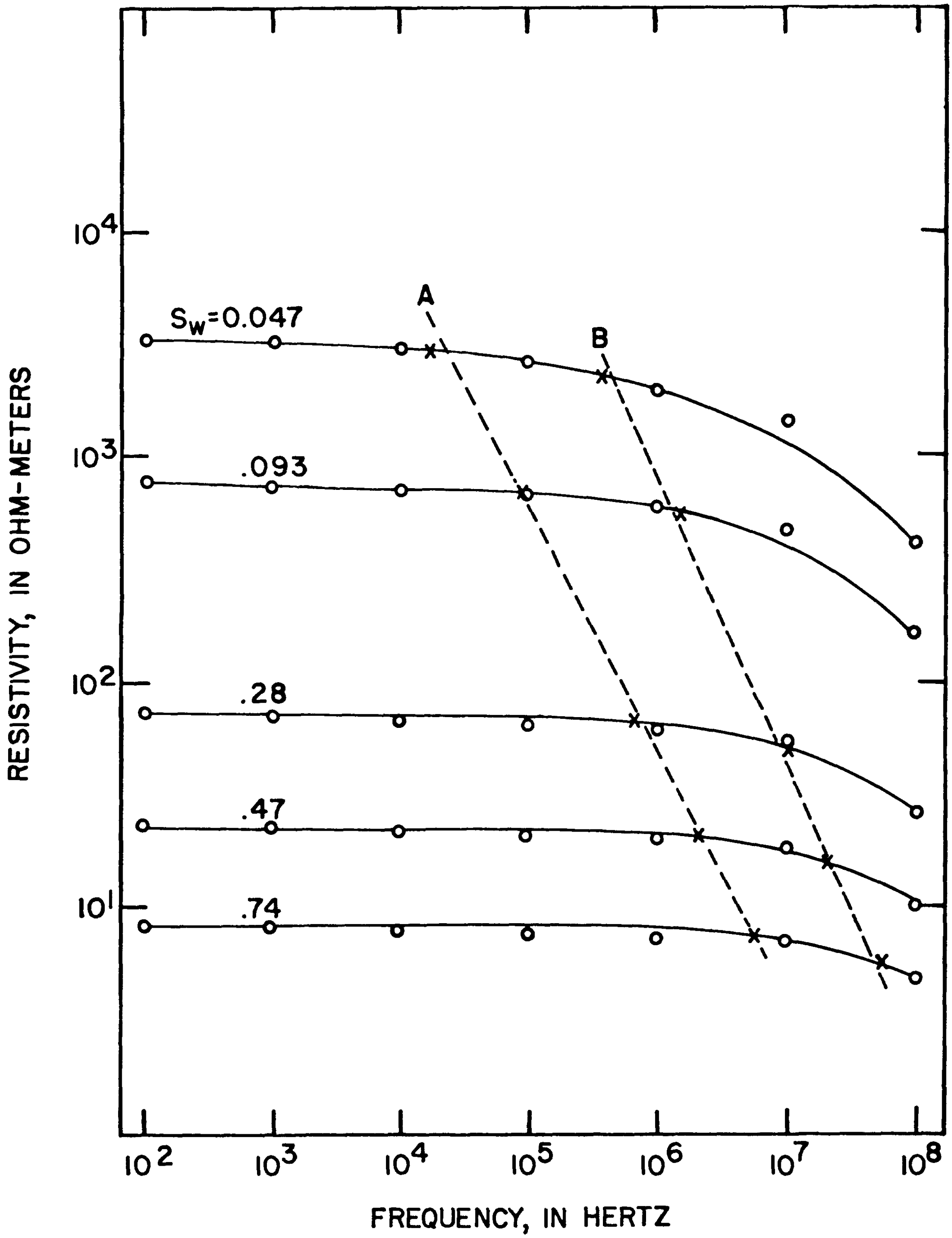

Figure 1.--Resistivity as a function of frequency for various water contents for sample A containing 1.0 ohm-m water. Lines A and B represent the points where the resistivity is down 10 percent and 30 percent from the value at 10^2 Hz.

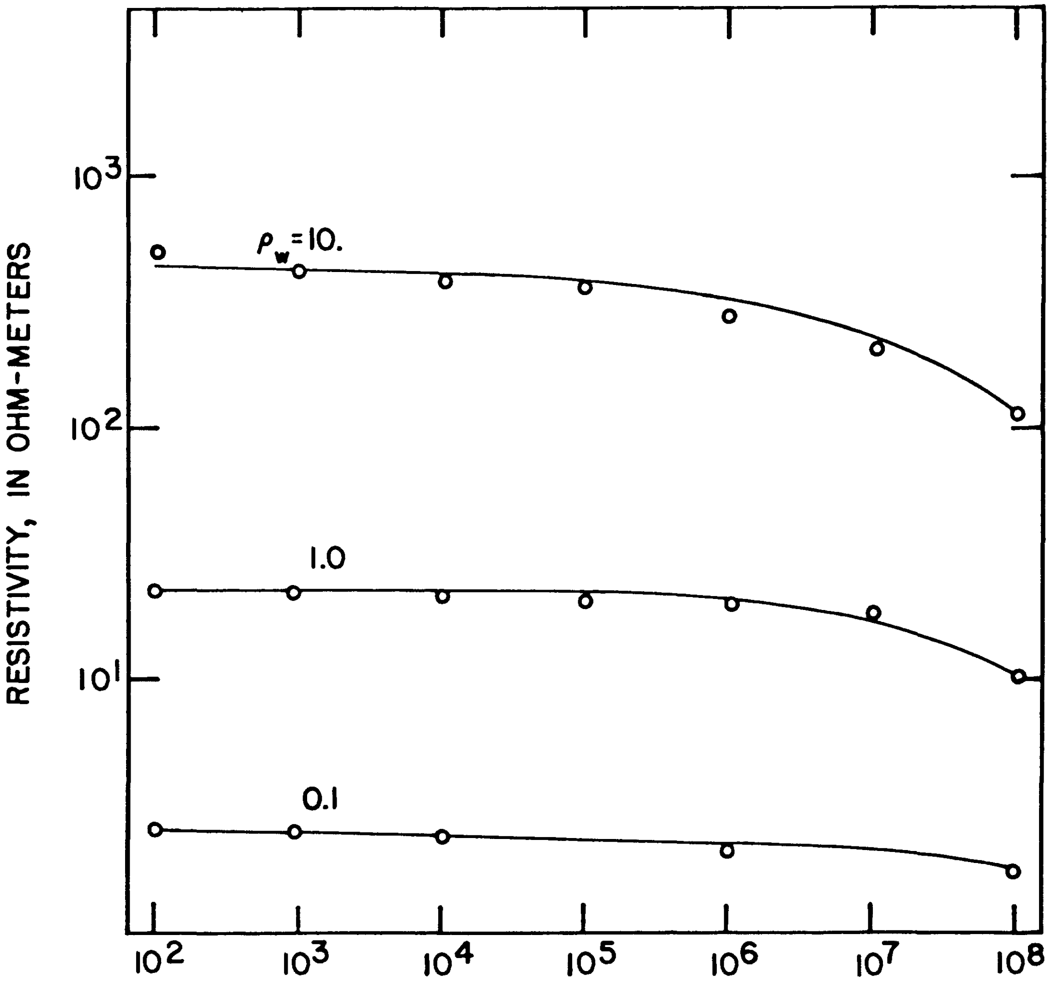

Figure 2.--Resistivity as a function of frequency for various water resistivities for sample A with a fractional water content of 0.47.

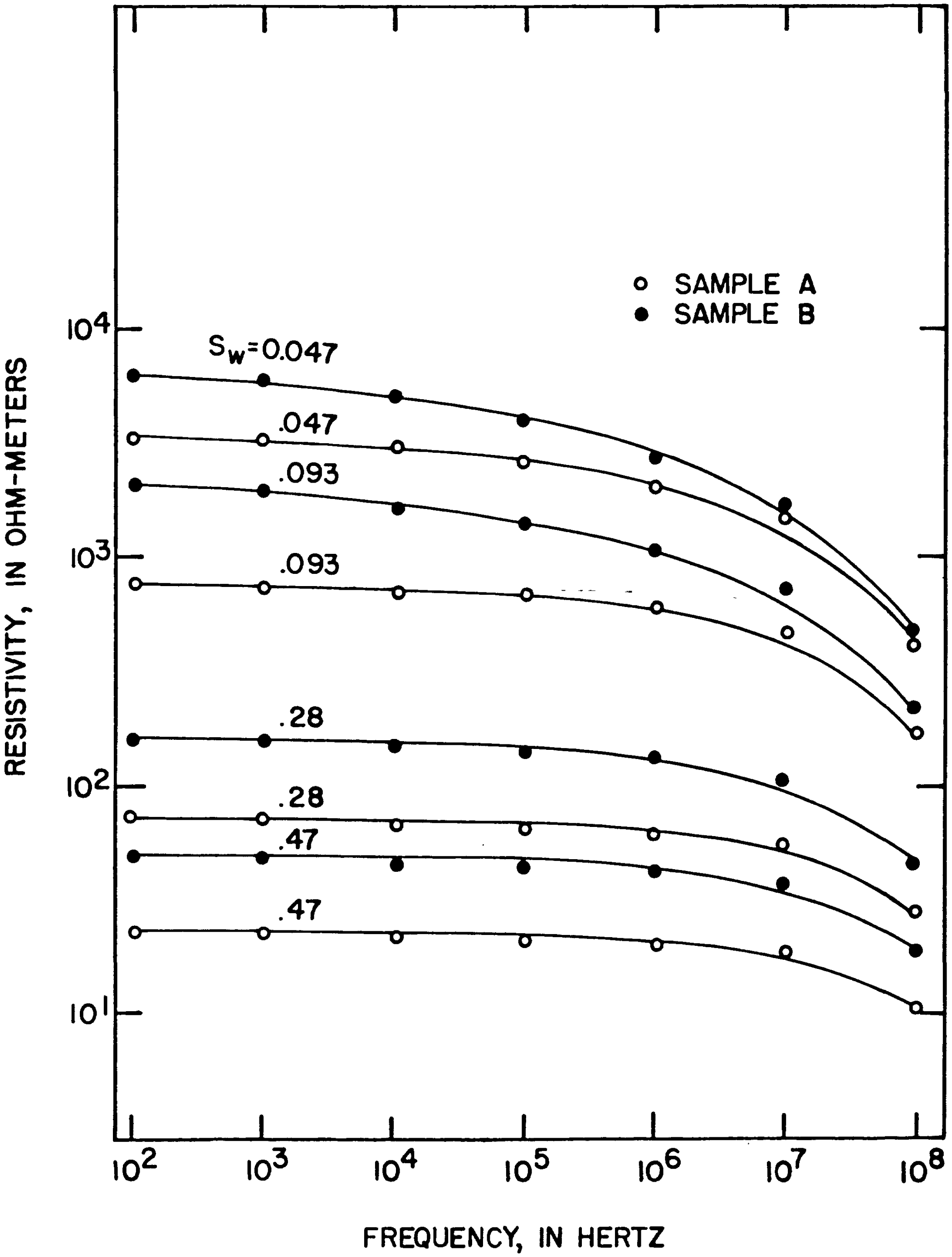

Figure 3.--Resistivity as a function of frequency for various water contents for samples A and B containing 1.0 ohm-m water.

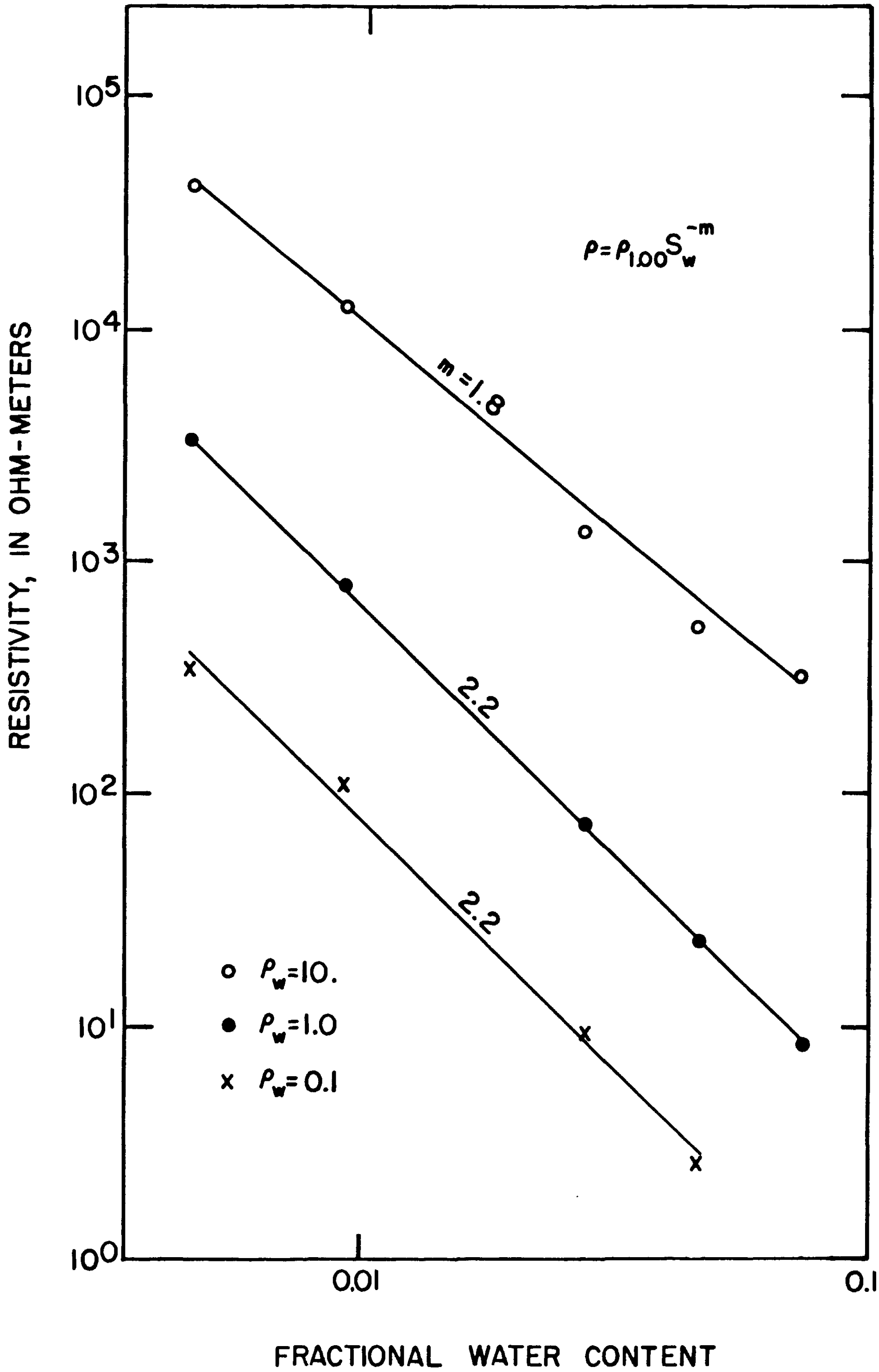

Figure 4.--Resistivity as a function of fractional water content for various water resistivities for sample A at a frequency of 100 Hz.

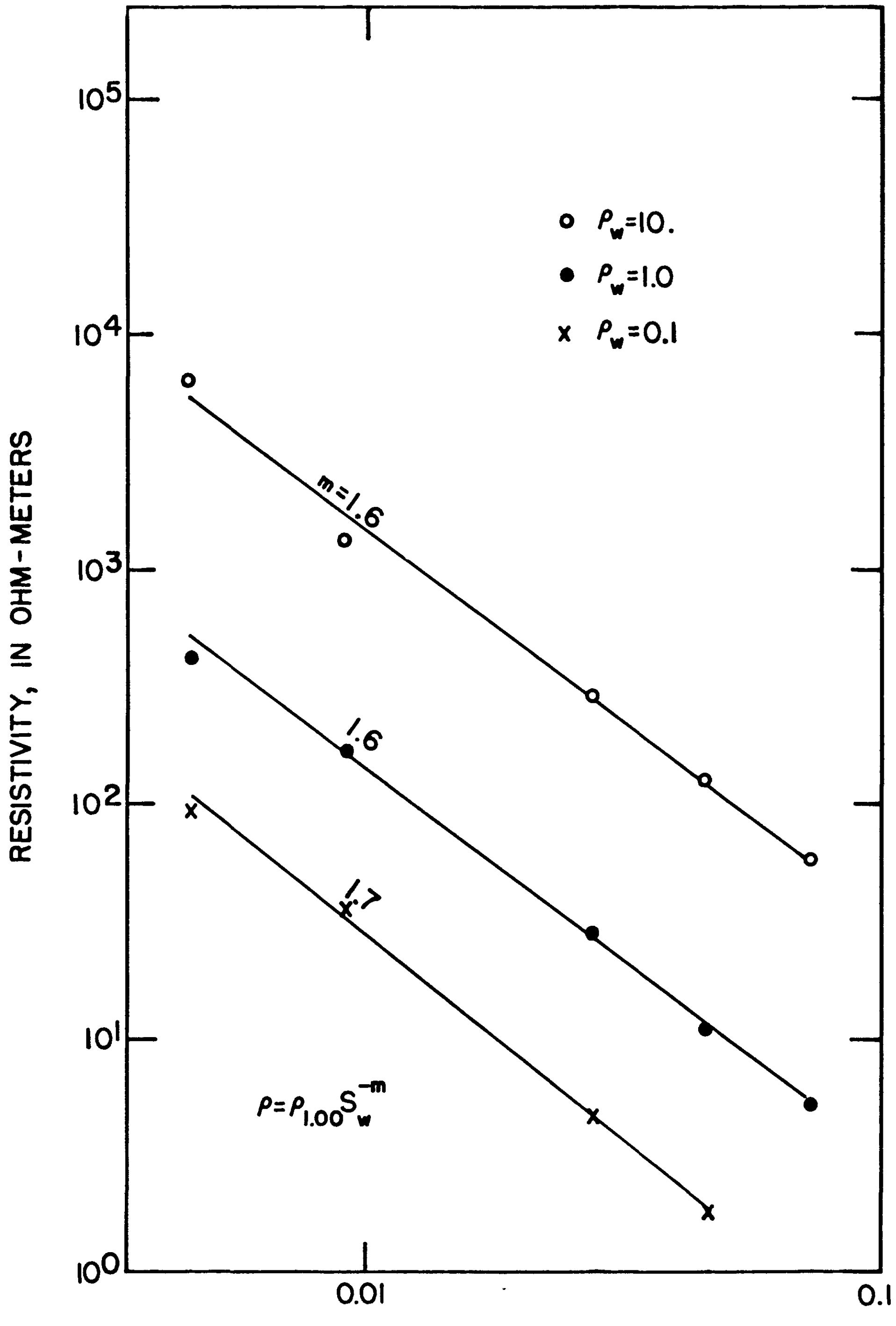

Figure 5.--Resistivity as a function of fractional water content for various water resistivities for sample A at a frequency of 100 MHz.

Grouping all the data at all frequencies and saturations by normalizing to equation (2) results in the plot of figure 6. The value for the resistivity of the saturated sample was determined by extrapolating data shown on figures 4 and 5 to $S_w = 1.00$. The significance of the result is that for a clean earth material one may expect to be able to predict the resistivity at any frequency if the saturated resistivity of the earth at the frequency of interest and the fractional water content are known. It should be borne in mind, of course, that the saturated-earth resistivity differs with frequency. The prediction criteria offered by equation (2) are, however, encouraging.

The above observations require that a number of variables be known to predict dispersion. The measurement of some of them in situ is a formidable proposition. The possibility of using simpler prediction criteria is suggested by figures 1 through 3. The observed dispersion becoming more pronounced at lower frequencies as the resistivity at 100 Hz increases (fig. 1) may enable dispersion to be estimated by a curve-matching technique. This involves extrapolating the resistivity obtained from in situ measurements (≈100 Hz) throughout the frequency range to 100 MHz based on the shape of the curves observed in figures 1 through 3. For example, table 1 is a comparison of the actual resistivity measured for a soil sample taken from the I-6 site at Malmstrom AFB, Montana with the results predicted from a mathematical fit to the curve shown on figure 3 for sample B, $S_w = 0.28$. The predicted results were obtained by fitting a third-order polynomial to the indicated

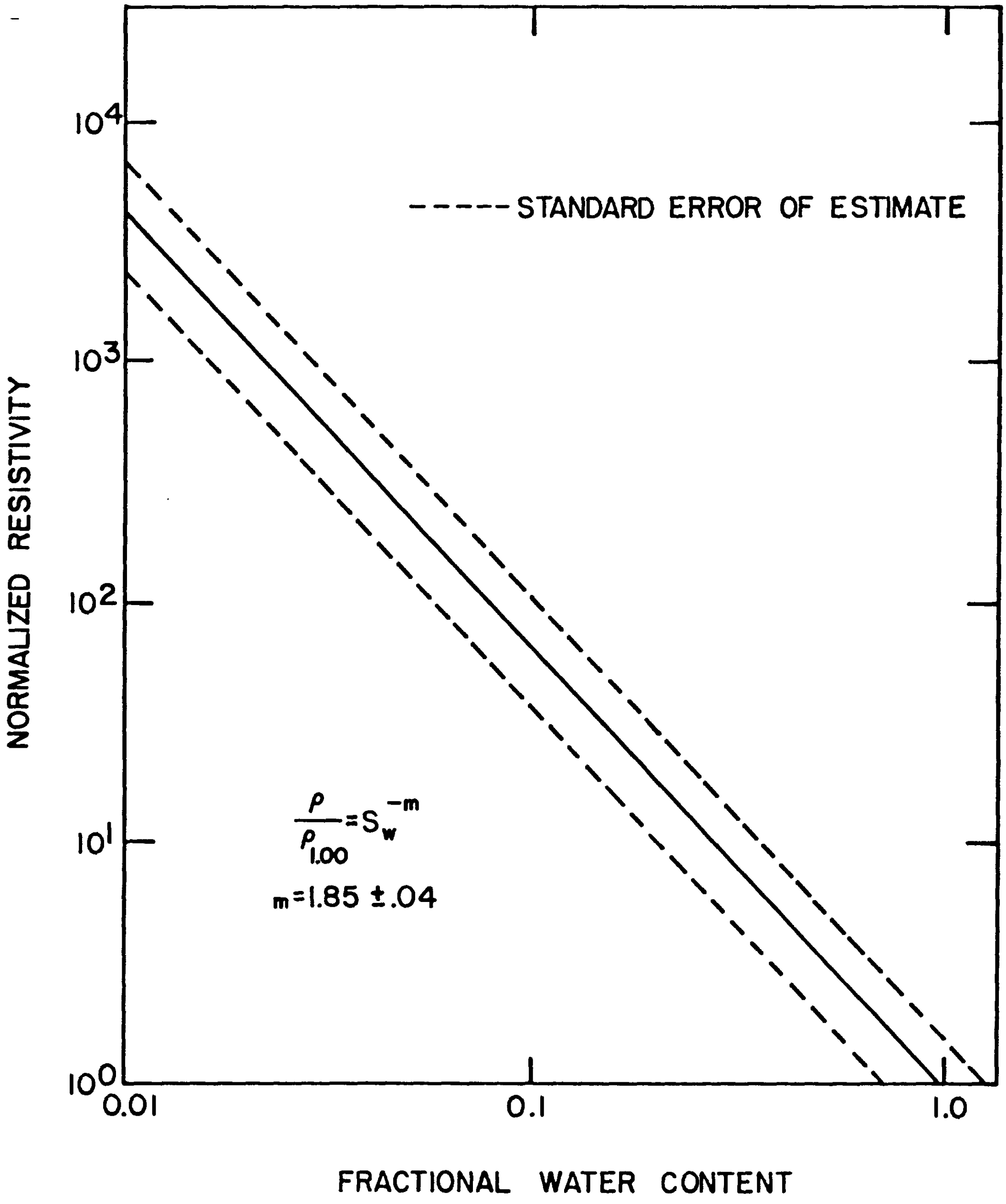

Figure 6.--Normalized resistivity as a function of water content.

curve on figure 3 (this curve was chosen because its value of resistivity at 100 Hz was closest to the I-6 sample), then substituting the value of resistivity for the I-6 sample at 100 Hz into the equation to generate the values given in the table.

Table 1.--Comparison of the measured values of resistivity of a sample of earth material from the I-6 site with the values predicted by the curve-matching technique described in the text

Frequency (Hertz)	Observed resistivity (ohm-m)	Predicted resistivity (ohm-m)
10^2	127	128
10^3	119	115
10^4	112	115
10^5	106	115
10^6	96.1	102
10^7	76.9	70.9
10^8	32.3	34.2

The advantage of this method is that the resistivity at 100 Hz can be measured *in situ* and, consequently, the dispersion can be determined more simply than by equation (2).

There are insufficient data on dispersion in this investigation to enable one to conclude that the method is an unimpeachable one.

Dielectric constant

Figures 7, 8, and 9 present dielectric constant versus frequency data for one sample ($\phi = 0.51$) for 10.0, 1.0, and 0.1 ohm-m water resistivity, respectively. Figures 10, 11, and 12 present the same data for the other sample ($\phi = 0.32$). The curves for $\rho_w = 10.0$ ohm-m (figs. 7 and 10) are relatively smooth concave curves over the whole frequency span. The curves for $\rho_w = 1.0$ ohm-m (figs. 8 and 11) tend to be convex at the lowest frequencies and concave at the higher; in the curves for $\rho_w = 0.1$ ohm-m (figs. 9 and 12) the convex tendency is even more apparent. On the plots for $\rho_w = 0.1$ ohm-m the inflection points tend to fall between 10^4 and 10^5 Hz, while those for $\rho_w = 1.0$ ohm-m tend to fall between 10^3 and 10^4 Hz. The degree of inflection is most pronounced on the curve of dielectric constant versus frequency represented by the parameters $\rho_w = 0.1$ ohm-m, $\phi = 0.51$, and $S_w = 0.47$ and it tends to decrease with water content and porosity.

Figures 13 and 14 are typical plots of κ versus S_w for 10^2 and 10^8 Hz, respectively. For 10^2 Hz the straight lines indicate a power-law fit to the data of the form

$$\frac{\kappa}{\kappa_{1.0}} = S_w^{\,m} \tag{3}$$

where $\kappa_{1.0}$ is the dielectric constant at $S_w = 1.0$. For 10^8 Hz the straight lines indicate a mixing formula of the form

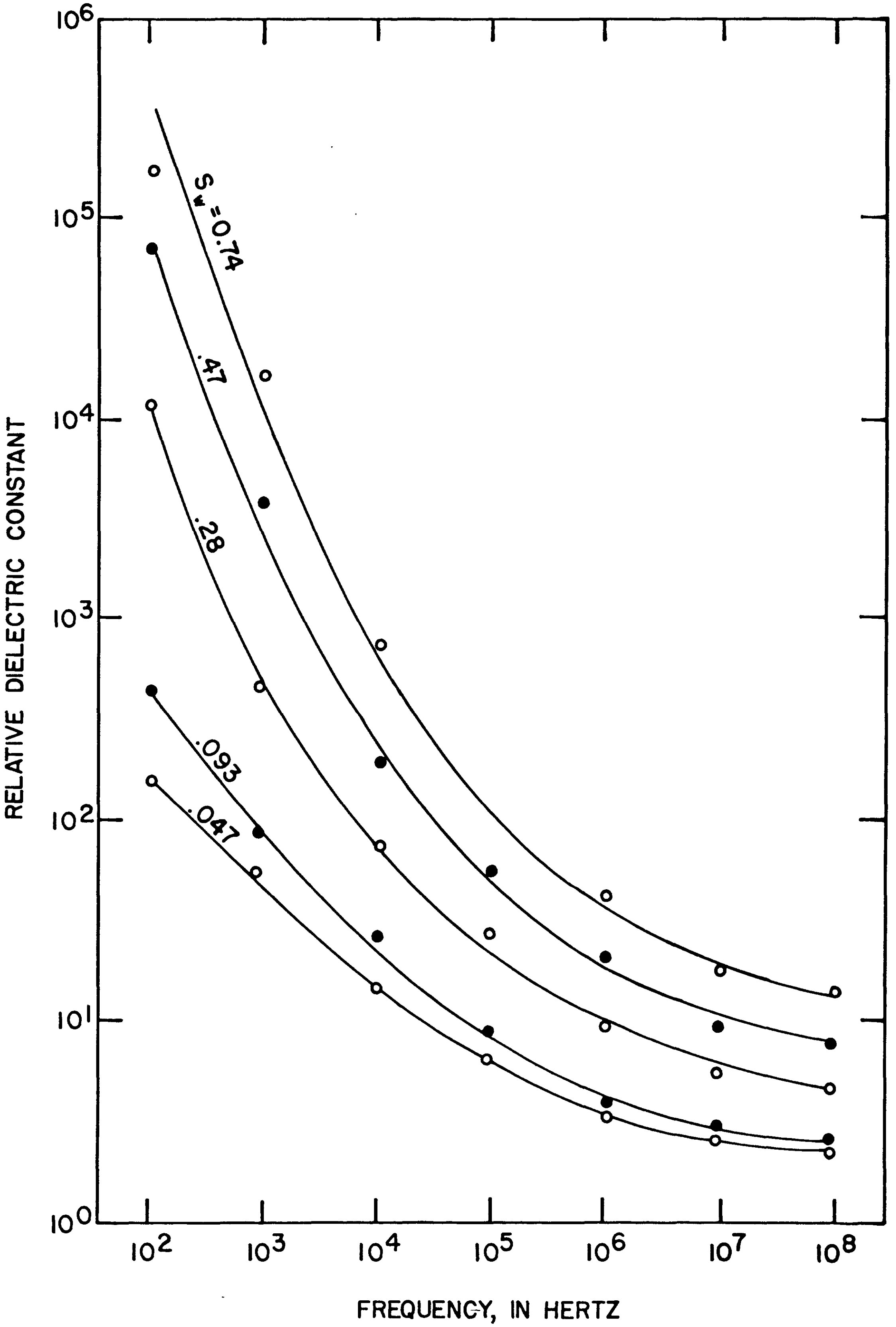

Figure 7.--Dielectric constant as a function of frequency for various water content for sample A containing 10 ohm-m water.

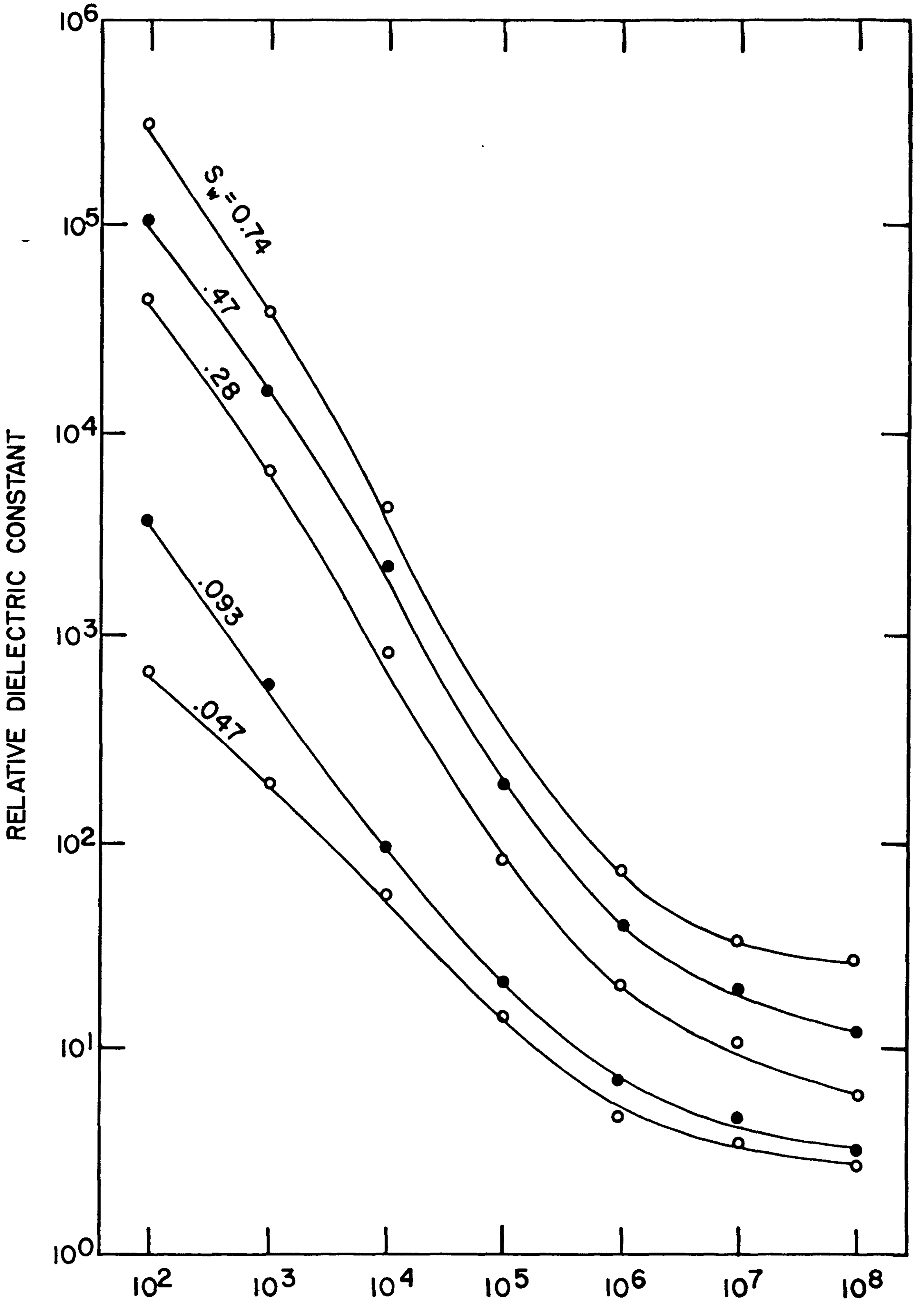

Figure 8.--Dielectric constant as a function of frequency for various water contents for sample A containing 1.0 ohm-m water.

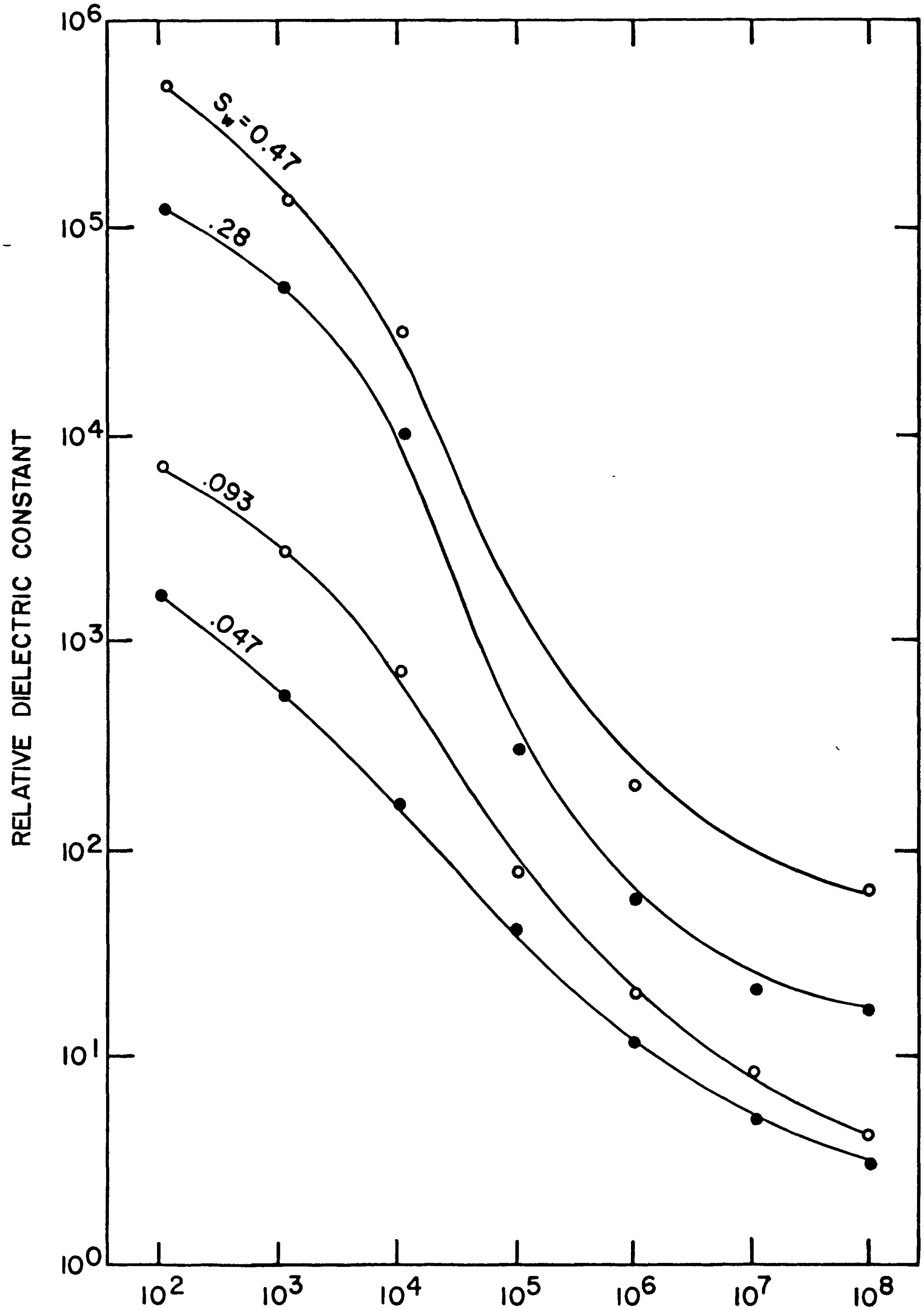

Figure 9.--Dielectric constant as a function of frequency for various water contents for sample A containing 0.1 ohm-m water.

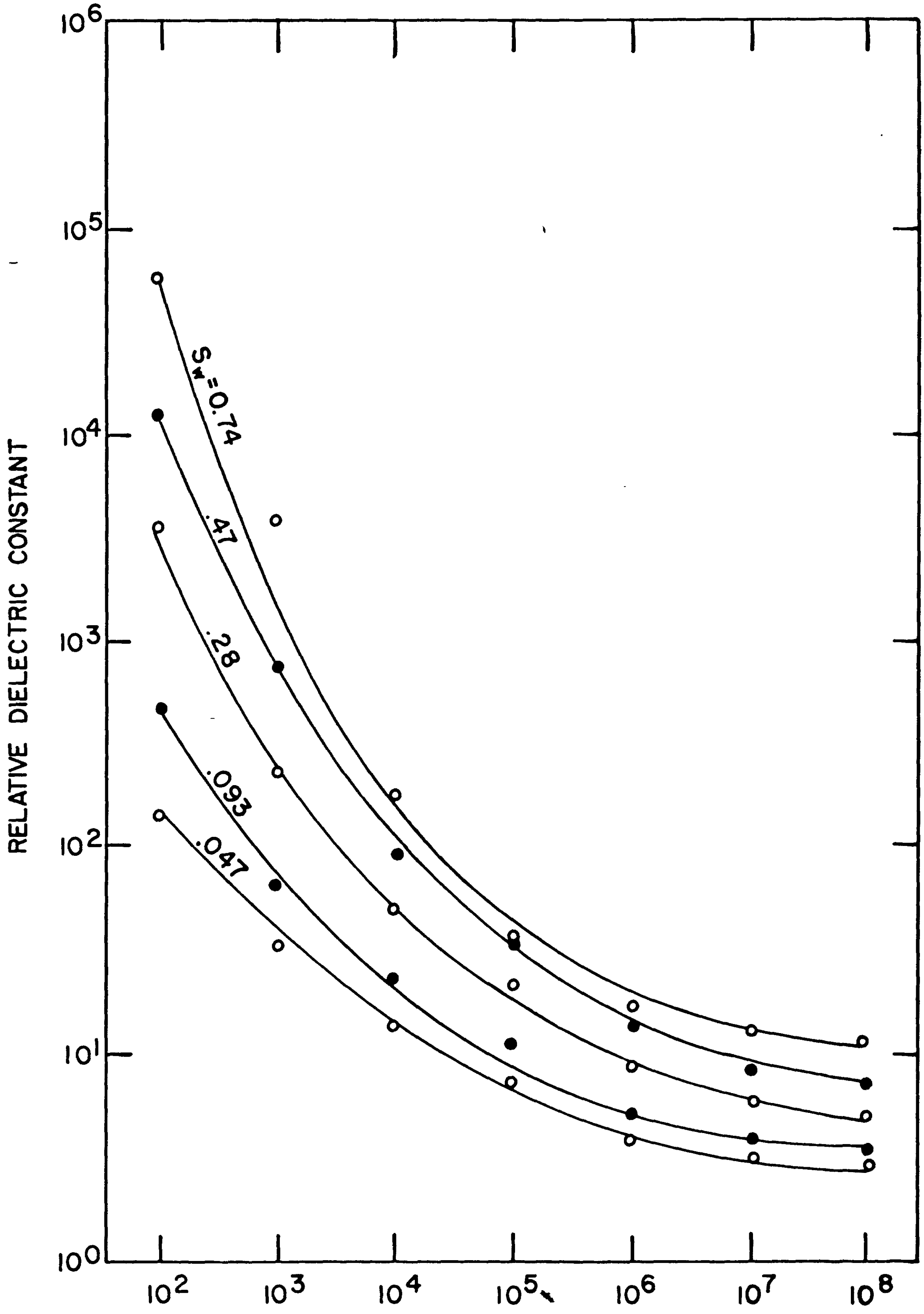

Figure 10.--Dielectric constant as a function of frequency for various water contents for sample B containing 10.0 ohm-m water.

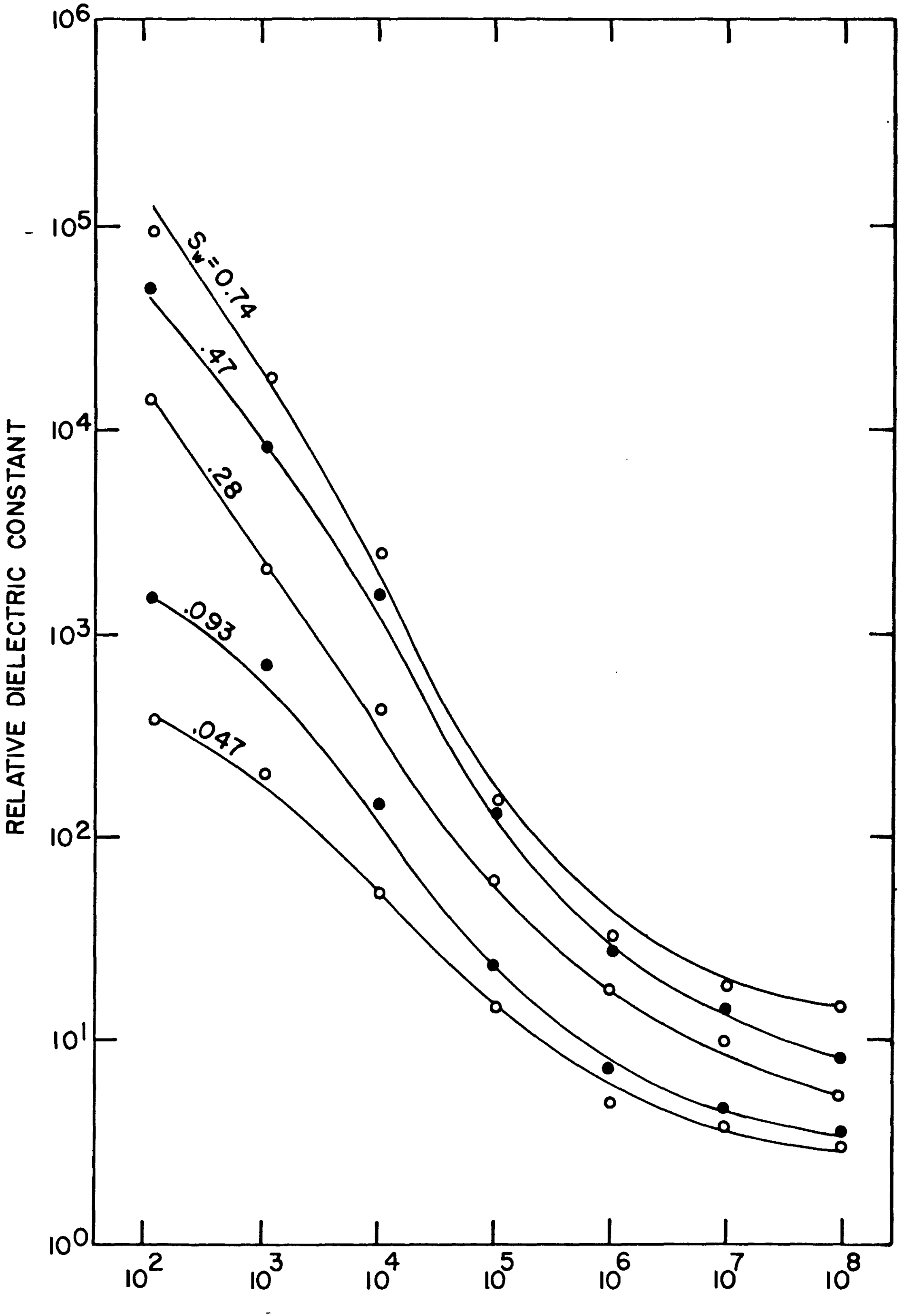

Figure 11.--Dielectric constant as a function of frequency for various water contents for sample B containing 1.0 ohm-m water.

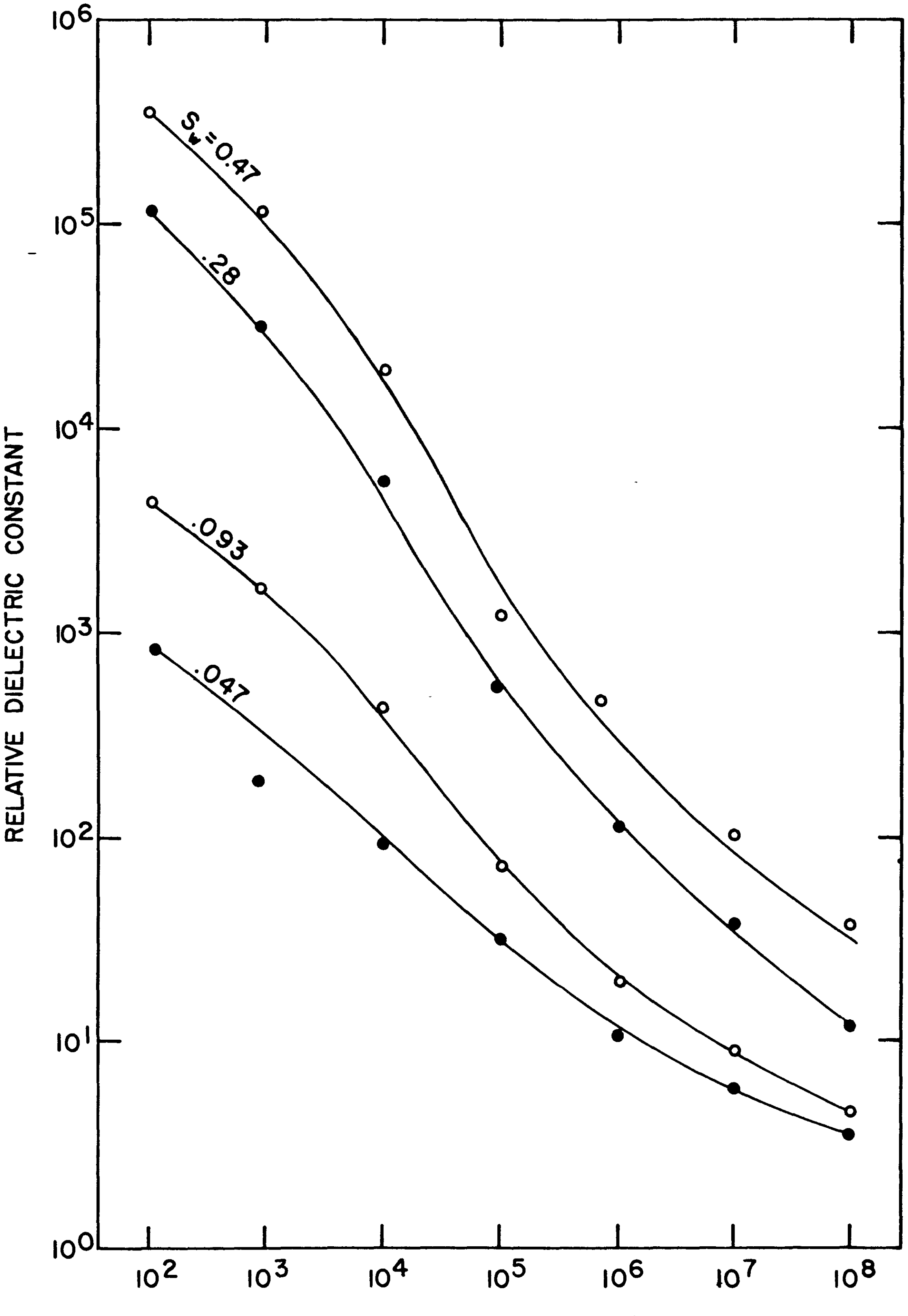

Figure 12.--Dielectric constant as a function of frequency for various water contents for sample B containing 0.1 ohm-m water.

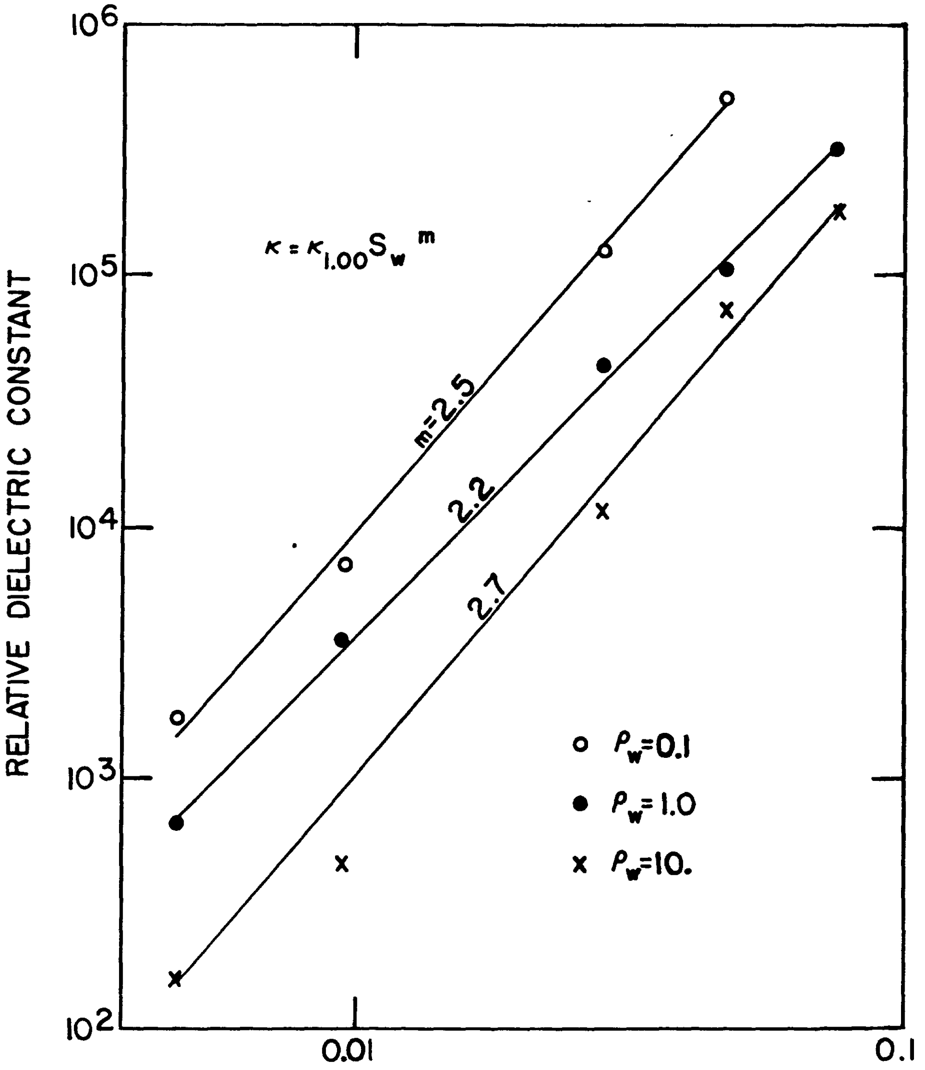

Figure 13.--Dielectric constant as a function of fractional water content for various water resistivities for sample A at a frequency of 100 Hz.

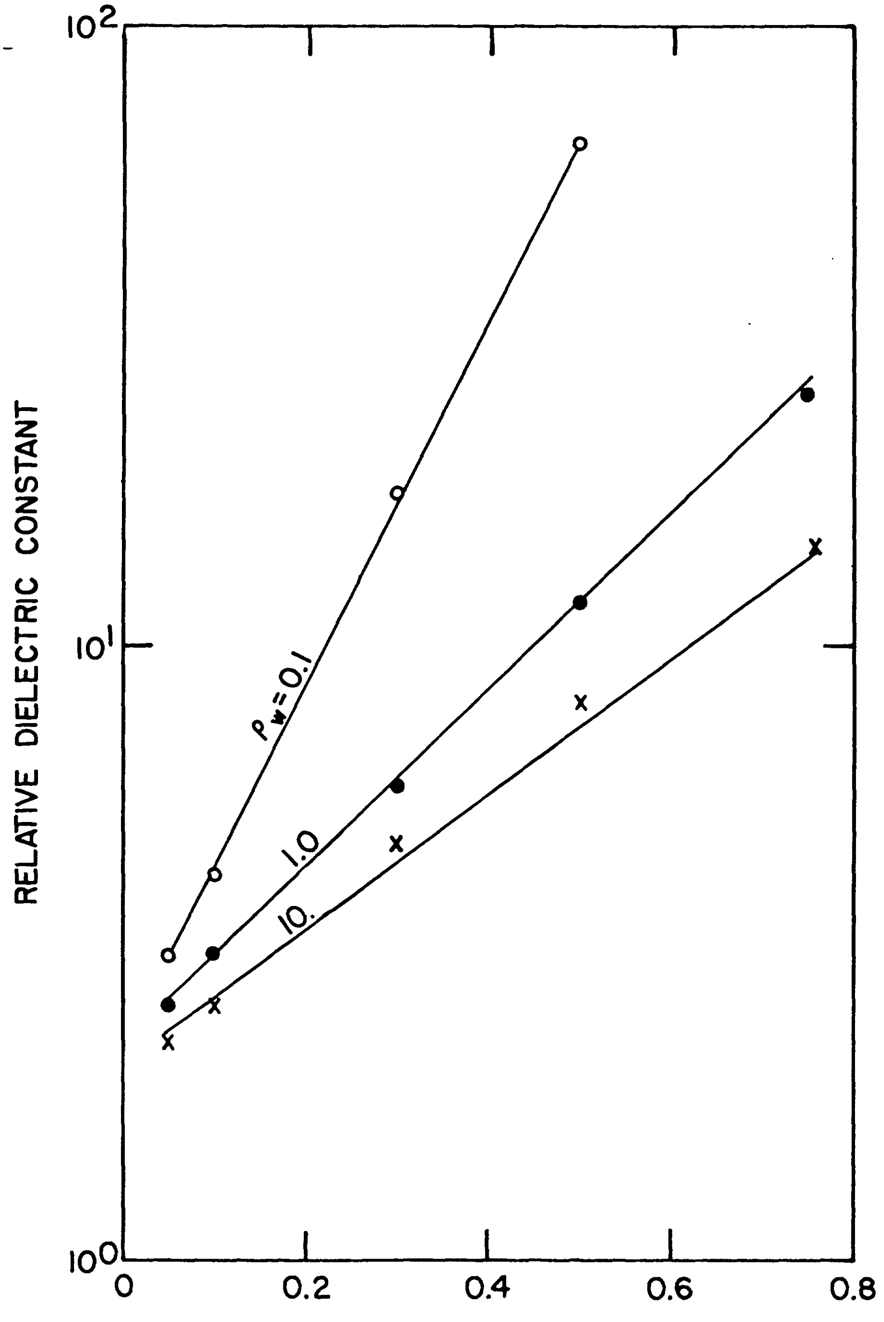

Figure 14.--Dielectric constant as a function of fraction water content for various water resistivities for sample A at a frequency of 100 MHz.

$$\ln \kappa = V_w \ln \kappa_w + V_s \ln \kappa_s \tag{4}$$

where κ_w is the dielectric constant of the water; κ_s, the dielectric constant of the sand; V_w, the volume fraction of water (= ϕS_w); and V_s, the volume fraction of sand (= $1 - \phi$) (Von Hipple, 1954).

In the frequency range 10^5 to 10^6 Hz, neither formulation fits the data. This region appears to be a transition zone between two different effects.

In applying equation (4) to the data at 10^8 Hz, values of 4.0-5.0 were obtained for the dielectric constant of the quartz; these values are in agreement with an accepted value of 4.4. However, the values obtained for the dielectric constant of the saline water in the sample were anomalously large, being 6×10^5 for $\rho_w = 0.1$ ohm-m, 400 for $\rho_w = 1.0$ ohm-m, and 130 for $\rho_w = 10.0$ ohm-m.

Approaches to determine prediction criteria for the dispersion of dielectric constant are not as readily apparent as in the case of resistivity. This is supported by the changes in curve shape discussed above. A further difficulty is that no easily measured in situ electrical parameter is available as in the resistivity case.

In the case of dielectric constant it is felt that the best approach is from the observed values at 100 MHz. Some investigators have observed (Lundien, 1966) that the dielectric constant at radar frequencies is directly dependent on water content. Extrapolating the present data to radar frequencies (10^9 to 10^{10} Hz) indicates this to be the case. Consequently, an in situ measurement of water content (a not too difficult procedure) may yield a first approximation to the dielectric constant at 10^8 Hz. An extrapolation back to the lower frequencies will be more difficult as the curve shape apparently varies depending upon the water resistivity--a parameter not easily determined *in situ*.

Sources of error

Primary sources of error in this work lie in sample and electrode reproducibility. Care was taken in loading, removing, and washing the samples to minimize loss of material. Weighing of the samples at the end of the experiments showed some loss of material through the repeated handling, which increased the porosity of the samples by approximately 2 percent. The correction for the moisture absorbed by the blotters, especially at the low water contents, is the greatest point of uncertainty; however, the fact that very good fits of the resistivity data to equation (2) seems to indicate that the assumption of uniform absorption at all water contents was valid.

Conclusions

Because dispersion curves similar to those presented here for the quartz sands have been observed in real geologic materials (Keller and Licastro, 1959; Smith-Rose, 1933) it is felt that the general results can be applied to real, clay-free geologic materials.

The well-behaved nature of the dispersion of the resistivity while systematically varying the parameters found in nature--porosity, water content, and water resistivity--indicates that it is feasible to predict the dispersion of a "clean" material based on a knowledge of its resistivity at 100 Hz using the laboratory-derived curves.

The behavior of the dielectric constant indicates that it may be feasible to predict the dielectric dispersion at high frequencies (10^6 to 10^8 Hz) based on water content measurements. The variation in curve shape as a function of water resistivity makes extrapolation back to low frequencies a difficult task; however, displacement currents are usually negligible below 10^6 Hz in the types of materials under consideration.

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Appendix

Data tables

RESISTIVITY
(ohm-m)

Sample A (ϕ = .51)

Frequency (Hz)

Fractional water content S_w	10^2	10^3	10^4	10^5	10^6	10^7	10^8
	ρ_w = 0.10 ohm-m						
0.047	0.345 x 10^3	0.344 x 10^3	0.335 x 10^3	0.319 x 10^3	0.278 x 10^3	0.216 x 10^3	0.918 x 10^2
.093	.110 x 10^3	.109 x 10^3	.104 x 10^3	.986 x 10^2	.915 x 10^2	.758 x 10^2	.348 x 10^2
.28	.937 x 10^1	.927 x 10^1	.861 x 10^1	.812 x 10^1	.789 x 10^1	.751 x 10^1	.448 x 10^1
.47	.252 x 10^1	.250 x 10^1	.236 x 10^1	------	.206 x 10^1	------	.175 x 10^1
.74	------	------	------	------	------	------	------
	ρ_w = 1.0 ohm-m						
0.047	0.330 x 10^4	0.323 x 10^4	0.297 x 10^4	0.259 x 10^4	0.198 x 10^4	0.147 x 10^4	0.413 x 10^3
.093	.759 x 10^3	.742 x 10^3	.706 x 10^3	.675 x 10^3	.597 x 10^3	.468 x 10^3	.165 x 10^3
.28	.713 x 10^2	.694 x 10^2	.663 x 10^2	.644 x 10^2	.609 x 10^2	.545 x 10^2	.271 x 10^2
.47	.230 x 10^2	.225 x 10^2	.217 x 10^2	.209 x 10^2	.201 x 10^2	.187 x 10^2	.103 x 10^2
.74	.831 x 10^1	.813 x 10^1	.790 x 10^1	.765 x 10^1	.742 x 10^1	.702 x 10^1	.490 x 10^1
	ρ_w = 10.0 ohm-m						
0.047	0.391 x 10^5	0.364 x 10^5	0.299 x 10^5	0.207 x 10^5	------	0.452 x 10^4	0.605 x 10^4
.093	.117 x 10^5	.113 x 10^5	.100 x 10^5	.805 x 10^4	.527 x 10^4	.296 x 10^4	.123 x 10^4
.28	.130 x 10^4	.116 x 10^4	.112 x 10^4	.103 x 10^4	.780 x 10^3	.541 x 10^3	.271 x 10^3
.47	.506 x 10^3	.418 x 10^3	.393 x 10^3	.375 x 10^3	.292 x 10^3	.207 x 10^3	.118 x 10^3
.74	.315 x 10^3	.195 x 10^3	.170 x 10^3	.162 x 10^3	.131 x 10^3	.884 x 10^2	.534 x 10^2

RESISTIVITY
(ohm-m)
Sample B (Ø = .51)

Frequency (Hz)

Fractional water content S_w	10^2	10^3	10^4	10^5	10^6	10^7	10^8
	ρ_w = 0.10 ohm-m						
0.047	0.121×10^4	0.120×10^4	0.117×10^4	0.104×10^4	0.785×10^3	0.428×10^3	0.122×10^3
.093	$.315 \times 10^3$	$.311 \times 10^3$	$.289 \times 10^3$	$.261 \times 10^3$	$.222 \times 10^3$	$.151 \times 10^3$	$.468 \times 10^2$
.28	$.654 \times 10^2$	$.602 \times 10^2$	$.490 \times 10^2$	$.403 \times 10^2$	$.338 \times 10^2$	$.225 \times 10^2$	$.886 \times 10^1$
.47	$.241 \times 10^2$	$.218 \times 10^2$	$.163 \times 10^2$	$.136 \times 10^2$	------	$.828 \times 10^1$	$.247 \times 10^1$
.74	------	------	------	------	------	------	------
	ρ_w = 1.0 ohm-m						
0.047	0.624×10^4	0.605×10^4	0.516×10^4	0.408×10^4	0.279×10^4	0.172×10^4	0.475×10^3
.093	$.205 \times 10^4$	$.196 \times 10^4$	$.162 \times 10^4$	$.138 \times 10^4$	$.106 \times 10^4$	$.708 \times 10^3$	$.214 \times 10^3$
.28	$.157 \times 10^3$	$.155 \times 10^3$	$.149 \times 10^3$	$.141 \times 10^3$	$.132 \times 10^3$	$.106 \times 10^3$	$.462 \times 10^2$
.47	$.489 \times 10^2$	$.479 \times 10^2$	$.457 \times 10^2$	$.438 \times 10^2$	$.418 \times 10^2$	$.369 \times 10^2$	$.191 \times 10^2$
.74	$.143 \times 10^2$	$.141 \times 10^2$	$.137 \times 10^2$	$.134 \times 10^2$	$.131 \times 10^2$	$.126 \times 10^2$	$.778 \times 10^1$
	ρ_w = 10.0 ohm-m						
0.047	0.247×10^5	0.240×10^5	0.222×10^5	0.172×10^5	0.871×10^4	0.501×10^4	0.164×10^4
.093	$.771 \times 10^4$	$.752 \times 10^4$	$.719 \times 10^4$	$.607 \times 10^4$	$.374 \times 10^4$	$.210 \times 10^4$	$.835 \times 10^3$
.28	$.138 \times 10^4$	$.134 \times 10^4$	$.131 \times 10^4$	$.123 \times 10^4$	$.947 \times 10^3$	$.649 \times 10^3$	$.312 \times 10^3$
.47	$.476 \times 10^3$	$.460 \times 10^3$	$.451 \times 10^3$	$.444 \times 10^3$	$.376 \times 10^3$	$.287 \times 10^3$	$.154 \times 10^3$
.74	$.151 \times 10^3$	$.141 \times 10^3$	$.138 \times 10^3$	$.137 \times 10^3$	$.131 \times 10^3$	$.119 \times 10^3$	$.718 \times 10^2$

DIELECTRIC CONSTANT

Sample A (ϕ = .51)

Frequency (Hz)

Fractional water content S_w	10^2	10^3	10^4	10^5	10^6	10^7	10^8
	ρ_w = 0.10 ohm-m						
0.047	0.169×10^4	0.546×10^3	0.162×10^3	0.424×10^2	0.120×10^2	0.504×10^1	0.312×10^1
.093	$.700 \times 10^4$	$.268 \times 10^4$	$.727 \times 10^3$	$.742 \times 10^2$	$.199 \times 10^2$	$.857 \times 10^1$	$.425 \times 10^1$
.28	$.124 \times 10^6$	$.502 \times 10^5$	$.102 \times 10^5$	$.306 \times 10^3$	$.598 \times 10^2$	$.217 \times 10^2$	$.177 \times 10^2$
.47	$.497 \times 10^6$	$.139 \times 10^6$	$.328 \times 10^5$	-------	$.209 \times 10^3$	-------	$.645 \times 10^2$
.74	-------	-------	-------	-------	-------	-------	-------
	ρ_w = 1.0 ohm-m						
0.047	0.649×10^3	0.194×10^3	0.540×10^2	0.140×10^2	0.453×10^1	0.337×10^1	0.255×10^1
.093	$.350 \times 10^4$	$.582 \times 10^3$	$.945 \times 10^2$	$.204 \times 10^2$	$.692 \times 10^1$	$.459 \times 10^1$	$.315 \times 10^1$
.28	$.429 \times 10^5$	$.617 \times 10^4$	$.830 \times 10^3$	$.813 \times 10^2$	$.203 \times 10^2$	$.106 \times 10^2$	$.589 \times 10^1$
.47	$.103 \times 10^6$	$.155 \times 10^5$	$.216 \times 10^4$	$.189 \times 10^3$	$.389 \times 10^2$	$.188 \times 10^2$	$.117 \times 10^2$
.74	$.316 \times 10^6$	$.372 \times 10^5$	$.416 \times 10^4$	$.135 \times 10^3$	$.725 \times 10^2$	$.334 \times 10^2$	$.260 \times 10^2$
	ρ_w = 10.0 ohm-m						
0.047	0.157×10^3	0.547×10^2	0.150×10^2	0.661×10^1	0.333×10^1	0.254×10^1	0.227×10^1
.093	$.450 \times 10^3$	$.899 \times 10^2$	$.267 \times 10^2$	$.908 \times 10^1$	$.391 \times 10^1$	$.303 \times 10^1$	$.258 \times 10^1$
.28	$.116 \times 10^5$	$.475 \times 10^3$	$.744 \times 10^2$	$.286 \times 10^2$	$.958 \times 10^1$	$.571 \times 10^1$	$.478 \times 10^1$
.47	$.717 \times 10^5$	$.389 \times 10^4$	$.195 \times 10^3$	$.569 \times 10^2$	$.212 \times 10^2$	$.953 \times 10^1$	$.800 \times 10^1$
.74	$.177 \times 10^6$	$.161 \times 10^5$	$.761 \times 10^3$	$.105 \times 10^3$	$.450 \times 10^2$	$.178 \times 10^2$	$.142 \times 10^2$

DIELECTRIC CONSTANT

Sample B ($\phi = .32$)

Frequency (Hz)

Fractional water content S_w	10^2	10^3	10^4	10^5	10^6	10^7	10^8
	$\rho_w = 0.10$ ohm-m						
0.047	0.823×10^3	0.190×10^3	0.923×10^2	0.313×10^2	0.108×10^2	0.591×10^1	0.347×10^1
.093	$.440 \times 10^4$	$.164 \times 10^4$	$.430 \times 10^3$	$.707 \times 10^2$	$.195 \times 10^2$	$.918 \times 10^1$	$.458 \times 10^1$
.28	$.116 \times 10^6$	$.313 \times 10^5$	$.548 \times 10^4$	$.540 \times 10^3$	$.114 \times 10^3$	$.375 \times 10^2$	$.117 \times 10^2$
.47	$.353 \times 10^6$	$.113 \times 10^6$	$.188 \times 10^5$	$.119 \times 10^4$	------	$.102 \times 10^3$	$.378 \times 10^2$
.74	------	------	------	------	------	------	-----
	$\rho_w = 1.0$ ohm-m						
0.047	0.394×10^3	0.202×10^3	0.526×10^2	0.143×10^2	0.495×10^1	0.377×10^1	0.300×10^1
.093	$.152 \times 10^4$	$.746 \times 10^3$	$.145 \times 10^3$	$.240 \times 10^2$	$.725 \times 10^1$	$.481 \times 10^1$	$.353 \times 10^1$
.28	$.141 \times 10^5$	$.211 \times 10^4$	$.450 \times 10^3$	$.605 \times 10^2$	$.181 \times 10^2$	$.959 \times 10^1$	$.573 \times 10^1$
.47	$.486 \times 10^5$	$.835 \times 10^4$	$.160 \times 10^4$	$.127 \times 10^3$	$.278 \times 10^2$	$.147 \times 10^2$	$.835 \times 10^1$
.74	$.926 \times 10^5$	$.180 \times 10^5$	$.263 \times 10^4$	$.143 \times 10^3$	$.318 \times 10^2$	$.180 \times 10^2$	$.152 \times 10^2$
	$\rho_w = 10.0$ ohm-m						
0.047	0.144×10^3	0.326×10^2	0.136×10^2	0.746×10^1	0.387×10^1	0.314×10^1	0.278×10^1
.093	$.473 \times 10^3$	$.652 \times 10^2$	$.231 \times 10^2$	$.116 \times 10^2$	$.503 \times 10^1$	$.385 \times 10^1$	$.333 \times 10^1$
.28	$.360 \times 10^4$	$.231 \times 10^3$	$.495 \times 10^2$	$.234 \times 10^2$	$.870 \times 10^1$	$.589 \times 10^1$	$.504 \times 10^1$
.47	$.127 \times 10^5$	$.736 \times 10^3$	$.875 \times 10^2$	$.358 \times 10^2$	$.136 \times 10^2$	$.827 \times 10^1$	$.727 \times 10^1$
.74	$.565 \times 10^5$	$.391 \times 10^4$	$.178 \times 10^3$	$.354 \times 10^2$	$.168 \times 10^2$	$.127 \times 10^2$	$.116 \times 10^2$