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Simple models of frictional heating by an earthquake

by

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Introduction. When fault surfaces move past one another an amount 2u against a frictional (or other dissipative) resistance τ, work is converted to heat in the amount $2\tau u$ per unit area of fault. This results in a local coseismic temperature rise (θ) that will subsequently dissipate, presumably by conduction. This effect has been discussed in terms of analytical results for an infinitesimally thin fault by Sibson (1975) and McKenzie and Brune (1972). Cardwell et al. (1978) considered the problem in terms of a numerical result for the fault of finite width illustrated in figure 1. A simple form of the solution for the fault of figure 1 has been applied by Lachenbruch (1980) to a related problem and here we adapt it to a convenient general scheme for estimating the magnitude of the coseismic temperature rise and its post-seismic dissipation. For intuitive convenience, graphical results are presented in a general dimensional form.

Coseismic frictional heating. Let t^* be the duration of the slip event and $2v = 2u/t^*$ be the slip velocity (see fig. 1). We assume heat is transferred exclusively by conduction, deformation is homogeneous, and slip velocity and fault width are constant during the event. The simplest case occurs when the fault-width 2a is so large that little heat escapes by conduction from the fault zone during the short time t^* of the event. (This condition is $a > \sqrt{4\alpha t^*}$ where the right side is the effective distance heat can diffuse in time t^* in a medium of thermal diffusivity α which we assume to be 0.01 cm^2/sec). Denoting density by ρ and heat capacity by c and assigning them reasonable values ($\rho = 2.65$ g/cm^3, $c = 0.95$ J/g °C whence $\rho c = 2.5$ J/cm^3 = 0.60 cal/cm^3 °C $\sim$25 bars/°C), we can write the temperature rise θ in terms of stress (τ) and strain (u/a).

$$\theta = \frac{\tau}{\rho c}\frac{vt}{a} \equiv \frac{\tau}{\rho c}\frac{u}{a} \tag{1a}$$

$$\sim 4°\mathrm{C} \times \frac{u}{a} \text{ if } \tau = 100 \text{ bars} \tag{1b}$$

$$\sim 40°\mathrm{C} \times \frac{u}{a} \text{ if } \tau = 1000 \text{ bars} \tag{1c}$$

Thus for fault strains greater than unity ($u/a \gtrsim 1$) coseismic heating can be large.

If the fault is narrower than the distance heat can diffuse during the event (i.e., if $a < \sqrt{4\alpha t^*}$) or equivalently if the event lasts longer than the time constant ($\lambda = a^2/4\alpha$) for the fault, then the temperature rise on the axial plane of the shear zone depends explicitly on the duration of slip t^*, and (1a) is replaced by the more general expression (Lachenbruch, 1980, equation 45a)

$$\theta(0,t) = \frac{\tau}{\rho c}\frac{vt}{a}\left[1 - 4i^2 \operatorname{erfc}\frac{a}{\sqrt{4\alpha t}}\right], \quad \text{all } a, \qquad 0 < t < t^* \tag{2a}$$

$$\cong \frac{\tau}{\rho c}\frac{vt}{a}, \qquad a/\sqrt{4\alpha t} > 1 \tag{2b}$$

$$\rightarrow 2\frac{\tau}{\rho c}\frac{v\sqrt{t}}{\sqrt{\pi\alpha}}, \qquad a/\sqrt{4\alpha t} \rightarrow 0 \tag{2c}$$

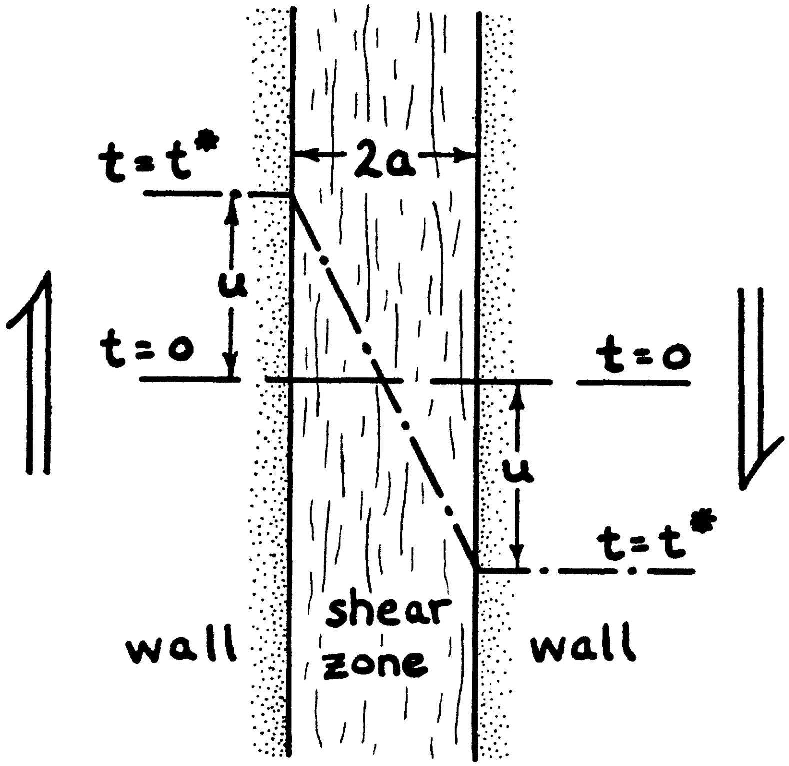

Figure 1. The simple fault model. Dashed line (t=0) is deformed into dash-dot line (t=t*) during slip of amount 2u across shear zone of width 2a in an event of duration t*.

This is illustrated in figure 2 which gives the rise in axial plane temperature as a function of time during faulting for a range of event durations (up to 100 sec) and fault widths (0 to 40 m) that probably brackets most earthquakes. The temperature scale, based on a slip velocity 2v of 25 cm/sec and friction τ of 100b, can be adjusted proportionally for any other choice, e.g., for 2v = 50 cm/sec and τ = 1000 b multiply the temperature scale by 20.

Figure 2 illustrates how wide-fault behavior ($\theta \propto t$, equation 2b) can pass to narrow-fault behavior ($\theta \propto t^{\frac{1}{2}}$, equation 2c) as the event proceeds. With uncertainties of an order of magnitude in friction τ, and at least that much in fault-width 2a, an enormous range of temperatures is possible even if slip velocity 2v and duration t* are known. If friction, fault width, and the slip 2u were known, but duration and velocity were not, a range of temperature is still possible on narrow faults. For example, slip of 50 cm may have resulted from a 1-second event slipping at 50 cm/sec or a 10-second event at 5 cm/sec. For τ = 100b, a 4-mm-wide fault would warm by 500°C in the first case and by 260°C in the second according to figure 2. (In the first case, the ordinate scale is multiplied by 2, and in the second, it is divided by 5 to accommodate the two different velocities.) It is seen that the curve labeled "4 cm" remains linear to t ∿100 sec, probably longer than the duration of local slip in most earthquakes. Hence equation (1) should be useful for predicting coseismic temperature rise on the axis of faults whenever shear zones are at least a few cm wide. In the Parkfield earthquake, if the slip (2u) were 50 cm and fault width (2a) were 5 cm, then according to (1) or (2) θ would be 40°C for 100 bars friction, 400°C for 1000b. Narrower faults attain much higher temperatures, but as we shall see, they decay very rapidly after the event and are soon indistinguishable from effects of wider faults with the same slip.

Postseismic decay of temperature disturbance. The postseismic conductive decay of temperatures in the fault zone depends upon two time constants: the duration of local seismic slip t*, and the time λ required for heat to diffuse across the half width of the heated shear zone ($\lambda = a^2/4\alpha$). The complete expression for temperature decay (t>t*) within a uniform fault zone ($|x|<a$) and beyond it ($|x|>a$) is (after minor modifications of Carslaw and Jaeger, 1959, equations 9 and 10, p. 80)

$$\theta(x,t) = \frac{\tau}{\rho c}\frac{v}{a}\{t[1 - 2i^2 \operatorname{erfc}\frac{a-x}{\sqrt{4\alpha t}} - 2i^2 \operatorname{erfc}\frac{a+x}{\sqrt{4\alpha t}}] \tag{3a}$$

$$-(t-t^*)[1 - 2i^2 \operatorname{erfc}\frac{a-x}{\sqrt{4\alpha(t-t^*)}} - 2i^2 \operatorname{erfc}\frac{a+x}{\sqrt{4\alpha(t-t^*)}}]\} \qquad x < a$$

$$= \frac{\tau}{\rho c}\frac{v}{a}\{t[i^2 \operatorname{erfc}\frac{x-a}{\sqrt{4\alpha t}} - i^2 \operatorname{erfc}\frac{x+a}{\sqrt{4\alpha t}}] \tag{3b}$$

$$-(t-t^*)[i^2 \operatorname{erfc}\frac{x-a}{\sqrt{4\alpha(t-t^*)}} - i^2 \operatorname{erfc}\frac{x+a}{\sqrt{4\alpha(t-t^*)}}]\} \qquad x > a$$

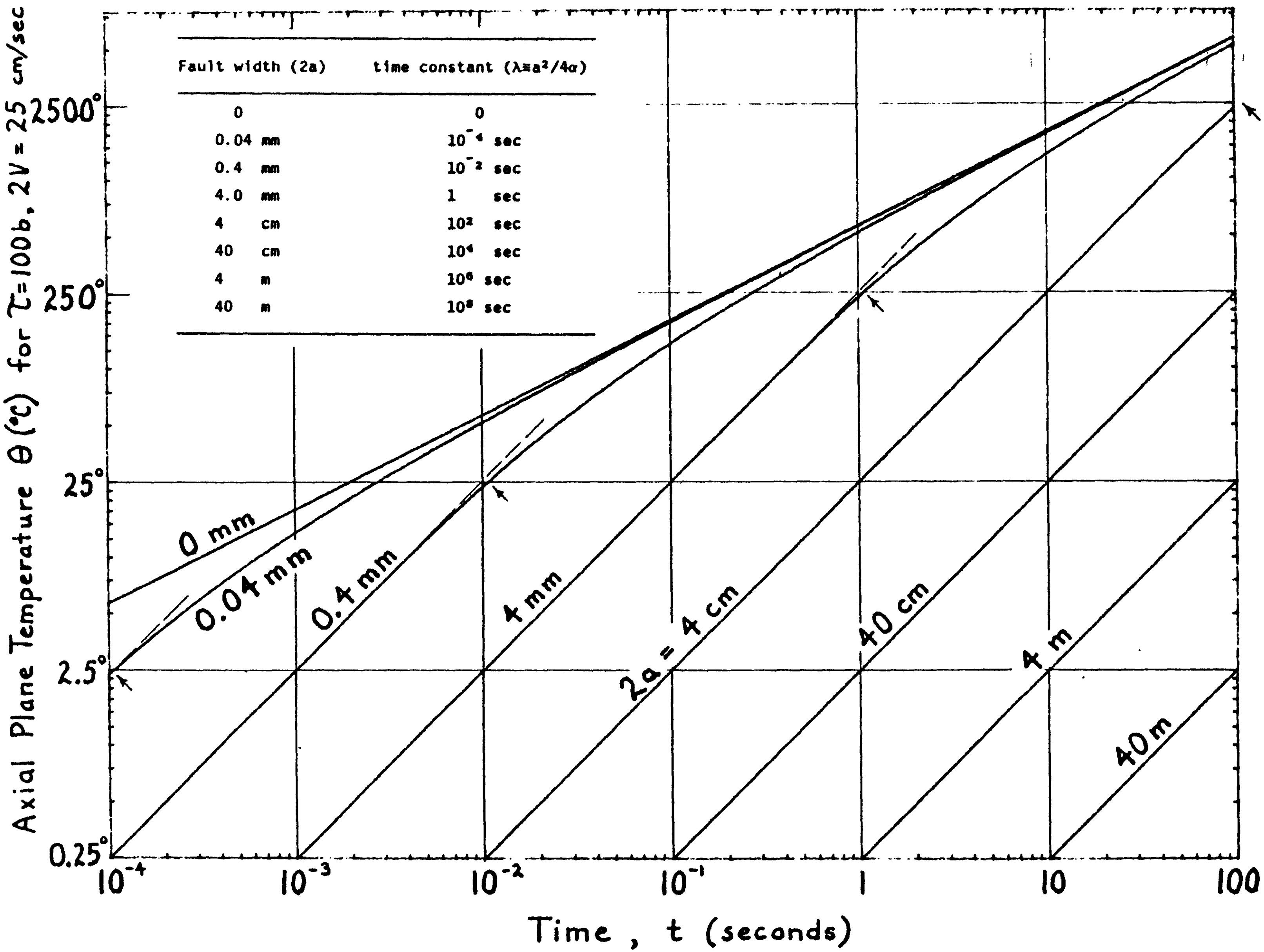

Figure 2. Coseismic temperature rise on axial plane as a function of time since initiation of faulting. Numbers on curves give fault width 2a; inset table gives corresponding time constants. Temperatures are shown for friction τ of 100 bars and slip velocity 2v of 25 cm/sec. They can be scaled to any other values, e.g., for 2v = 50 cm/sec and τ = 1000 bars multiply temperatures by 20. To the left of arrows (at $t=\lambda$) temperature rises linearly with time (equations 1, 2b).

(The general expression for the coseismic ($t<t^*$) temperature rise is obtained by deleting the second lines in equations (3a) and (3b), e.g., by formally setting $t \equiv t^*$.)

The slip duration t^* will generally be only a few seconds and can therefore be neglected for computing the temperature hours, days or weeks after faulting. Neglecting slip duration t^* relative to post-seismic observation time ($t-t^*$), we can treat the faulting as an instantaneous source of strength $\tau u/\rho ca$ distributed throughout the shear zone ($|x| < a$) and (3) is replaced by (Carslaw and Jaeger, equation 3, p. 54)

$$\theta(x,t) = \frac{1}{2}\frac{\tau}{\rho c}\frac{u}{a}\left[\operatorname{erf}\frac{a-x}{\sqrt{4\alpha t}} + \operatorname{erf}\frac{a+x}{\sqrt{4\alpha t}}\right] , \qquad \text{all } x,\ t \gg t^* \qquad (4)$$

Finally, if our observation time is large relative to the time constant λ of the shear zone (for a 4-cm wide shear zone λ is $\sim$100 seconds, but for a 4-m shear zone, it is $\sim$10 days, see inset, fig. 1) the fault can be treated as an instantaneous plane (zero-width) source of strength $2\tau u/\rho c$. This leads to the further simplification (Carslaw and Jaeger, 1959, equation 4, p. 259):

$$\theta(x,t) = \frac{\tau}{\rho c}\frac{u}{\sqrt{\pi\alpha t}}e^{-\frac{x^2}{4\alpha t}} \qquad \text{all } x,\ t \gg t^*,\ \ t \gg \lambda \qquad (5)$$

Temperature rise computed from (5) is shown in figure 3 for a fault slip 2u of 50 cm opposed by frictional resistance τ of 100b. The figure shows that the high axial plane temperatures in the previous numerical examples are short-lived. If the slip were 100 cm, the temperatures of figure 3 would be multiplied by 2, if the friction were 1kb, they would be multiplied by 10. The one-day curve is valid only for faults for which the time constant ($\lambda \equiv a^2/4\alpha$) is much less than a day, e.g., for $2a \lesssim 40$ cm in which case $\lambda \lesssim 3$ hrs (see inset, fig. 1). Temperatures earlier in the postseismic period can be obtained for a fault of any width from the more complete expressions (4) or (3). As the area under curves of the type shown in figure 3 is the source strength $2\tau u/\rho c$, it is conserved. Hence a post-seismic temperature profile across the fault combined with an estimate of slip u provides an opportunity to recover the friction τ.

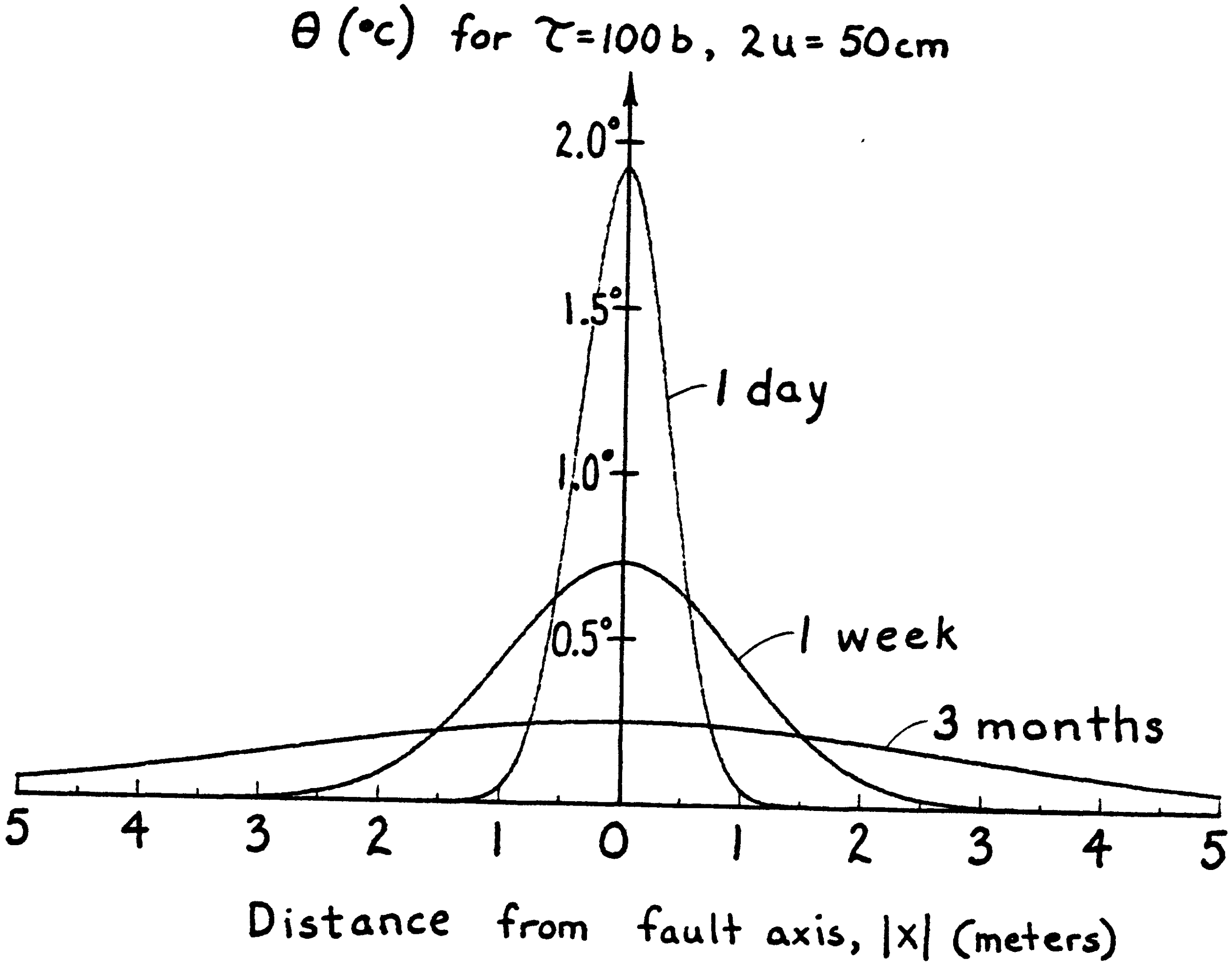

Figure 3. Post-seismic temperature distribution. Temperatures are shown for friction τ of 100 bars and slip 2u of 50 cm; they can be scaled proportionally for any other values. Curves are good approximations as long as the post-seismic time they represent is large relative to the fault's time constant $\lambda(a)$; see inset, figure 2.

References

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