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Spectral Whitening in the Frequency Domain

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SPECTRAL WHITENING IN THE FREQUENCY DOMAIN

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ABSTRACT

The basic principle of spectral whitening in the frequency domain was investigated and compared to the conventional spiking deconvolution technique. When the source signature is a zero-phase wavelet and earth reflectivity is assumed to be an uncorrelated random sequence, the spectral whitening method can be used as a deconvolution technique (such as spiking deconvolution). This study also shows that spectral whitening can be used as a signal-to-noise enhancement filter or a pulse-compression method irrespective of the phase of the source wavelet.

INTRODUCTION

Increasing vertical resolution (or time resolution) of the seismic trace is one of the main purposes for seismic data processing, and the conventional spiking deconvolution technique is the most widely used method for this purpose. The basic assumptions necessary for spiking deconvolution are: (1) that the earth reflectivity is an uncorrelated random sequence, and (2) that the source signature is a minimum-phase wavelet. When the source is not a minimum phase, spiking deconvolution does not perform as desired and, in certain instances, shows severe phase distortion of the seismic event. Furthermore, when the seismic section has low signal-to-noise ratio, undesirable spiking deconvolution effects are noticeably worsened.

In order to process certain types of seismic data that were difficult to deconvolve by the conventional deconvolution method, a spectral whitening method was introduced by the author as an alternative technique. Hutchinson and Grow (1985) published some examples of seismic data processed by spectral whitening instead of spiking deconvolution. Also, S. T. Harding (1985, personal communication) has extensively incorporated this spectral whitening technique into processing of Mini-Sosie* seismic data.

This paper presents the theoretical aspect of spectral whitening in the frequency domain and provides some synthetic examples. The spectral whitening technique introduced here is defined as $A^{\alpha}(\omega)$ where $A(\omega)$ is the amplitude spectrum of the seismic trace and α is an arbitrary constant. When α approaches zero (under the assumption that the earth reflectivity is an uncorrelated random function and the source signature is a zero-phase wavelet), the effect of spectral whitening becomes similar to the application of the inverse wavelet filter to the seismic trace. Because spectral whitening operates only on the amplitude spectrum of the seismic trace, the phase spectrum of the original seismic trace is retained.

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*Mini-Sosie is a trademark of SNEA(P).

Theoretically, the spectral whitening technique can be applicable as a powerful pulse-compression method without concern for the phase of the wavelet. Therefore, when structural interpretation is the main purpose of seismic data processing, the spectral whitening technique could be a viable alternative to other deconvolution methods. However, in stratigraphic interpretation, one must exercise care in applying spectral whitening, because the original wavelet phase is still present.

THEORY

Seismic traces can be represented by the following convolutional model:

$$s(t) = r(t) * w(t) + n(t), \qquad (1)$$

where

- $s(t)$: seismic trace,
- $r(t)$: reflectivity function,
- $w(t)$: source wavelet, and
- $n(t)$: noise.

The objective of deconvolution is to remove the wavelet effect $w(t)$ from the seismic trace and recover the earth reflectivity function.

Defining the Fourier transform as:

$$S(\omega) = \int_{-\infty}^{\infty} s(t)\, e^{-i\omega t}\, dt$$
$$\hat{=} A_s(\omega)\, e^{i\phi_s(\omega)},$$

where capital letters indicate the Fourier transformed quantity, $A(\omega)$ is the amplitude spectrum, and $\phi(\omega)$ is the phase spectrum. Subscripts of $A(\omega)$ and $\phi(\omega)$ indicate the corresponding quantities shown in equation (1).

If the source waveform is known, the deconvolution operator, $F_1(\omega)$, can be written by the following formula in the frequency domain:

$$F_1(\omega) = \frac{W^*(\omega)}{W(\omega)W^*(\omega) + \lambda} \qquad (2)$$
$$\cong \frac{e^{-i\phi_w(\omega)}}{A_w(\omega)}$$
$$= \frac{1}{W(\omega)}$$

where $W^*(\omega)$ represents the complex conjugate of $W(\omega)$, and λ is the white-noise component, which stabilizes the deconvolution operator.

As shown in equation (2), the deconvolution operator is equal to the inverse of the wavelet when the noise component is not present in the seismogram.

Let $S_1(\omega)$ be the deconvolved output using $F_1(\omega)$ as the deconvolution operator, then:

$$
\begin{aligned}
S_1(\omega) &= F_1(\omega) S(\omega) \qquad (3) \\
&= R(\omega) \frac{W(\omega) W^*(\omega)}{W(\omega) W^*(\omega) + \lambda} + N(\omega) \frac{W^*(\omega)}{W(\omega) W^*(\omega) + \lambda} \\
&\cong R(\omega).
\end{aligned}
$$

Therefore, the filtered output $S_1(\omega)$ approaches the desired reflectivity $R(\omega)$, when the signal-to-noise ratio becomes high. This kind of deconvolution is possible only when the source waveform is known or can be estimated.

The source waveform is rarely known in most seismic data processing, and the above approach cannot be applied. However, under certain assumptions, the deconvolution operator similar to $F_1(\omega)$ can be derived from the seismic trace without knowing the source waveform. Assuming that the earth reflectivity and noise are uncorrelated random functions, then the following formula can be derived:

$$
\begin{aligned}
S(\omega) S^*(\omega) &= R(\omega) R^*(\omega) W(\omega) W^*(\omega) + N(\omega) N^*(\omega) \\
&\triangleq \sigma_R W(\omega) W^*(\omega) + \sigma_N \qquad (4) \\
&\cong \sigma_R W(\omega) W^*(\omega) \qquad \text{when } \sigma_R > \sigma_N
\end{aligned}
$$

where

$$\sigma_R = R(\omega) R^*(\omega)$$

$$\sigma_N = N(\omega) N^*(\omega)$$

Equation (4) implies that the autocorrelation of the seismic trace is close to the scaled autocorrelation function of the source wavelet. Therefore, if we know the phase of the wavelet or we can assume that the source signature is a minimum phase wavelet, the source waveform can be estimated using the autocorrelation function of the seismic trace.

Assuming that the source signature is minimum phase and the earth reflectivity is an uncorrelated random function, the deconvolution operator, $F_2(\omega)$, can be expressed by:

$$F_2(\omega) \cong \frac{e^{-i\tilde{\phi}_W(\omega)}}{\tilde{A}_W(\omega)} \tag{5}$$

In equation (5), $\tilde{A}_W(\omega)$ is the derived amplitude spectrum of the source wavelet from the autocorrelation of the seismic trace and $\tilde{\phi}_W(\omega)$ is the assumed minimum phase spectrum of the source. This is the essence of the spiking deconvolution technique in the frequency domain.

Now, let's define the spectral whitening in the frequency domain as:

$$\begin{aligned} S_3(\omega) &\triangleq A_S^{\alpha}(\omega)\, e^{i\phi_S(\omega)} \\ &= A_S(\omega)\, A_S^{\alpha-1}(\omega)\, e^{i\phi_S(\omega)} \\ &= S(\omega)\, A_S^{\alpha-1}(\omega), \end{aligned} \tag{6}$$

where $S_3(\omega)$ is the output of the spectral whitening process. Equation (6) can be represented by the following filter equation:

$$S_3(\omega) = F_3(\omega)\, S(\omega), \tag{7}$$

where

$$F_3(\omega) = A_S^{\alpha-1}(\omega).$$

From equation (7), the following is obvious:

$$\lim_{\alpha \to 0} F_3(\omega) = \frac{1}{A_S(\omega)}.$$

If the reflectivity is assumed to be a random function, then $A_S(\omega) \doteq \tilde{A}_W(\omega)$.

Therefore, under the random reflectivity assumption, the following can be established:

$$\lim_{\alpha \to 0} \tilde{F}_{3}(\omega) \longrightarrow \frac{1}{\tilde{A}_{W}(\omega)} . \qquad (8)$$

By comparing equations (8) and (5), one can see that spectral whitening is similar to spiking deconvolution except for the fact that it operates only on the amplitude spectrum. Therefore, if the source waveform is a zero-phase wavelet, the spectral whitening method performs in a similar manner to the spiking deconvolution technique.

The amount of spectral whitening is governed by the parameter α. As shown in the above derivation, spectral whitening performs like deconvolution or pulse compression when α becomes small. By choosing an appropriate α, spectral whitening can be used as an alternate processing tool (for example, in signal-to-noise enhancement).

DISCUSSION

In order to compare spectral whitening with spiking deconvolution, the earth reflectivity function (fig. 1) was analyzed. The top part of figure 1 shows the earth reflectivity sequence; the bottom part, the amplitude spectrum. The earth reflectivity series was generated by Mendel and Kormylo (1978), and is a Poisson impulse sequence with Gaussian uncorrelated amplitudes. Because the reflectivity sequence is a random function, its amplitude spectrum is nearly flat.

Figure 2 shows the deconvolution results when the source is a minimum-phase wavelet. Figure 2a shows the earth reflectivity function convolved with 4/8 - 150/200 Hz zero-phase wavelet; figure 2b shows the input seismic trace convolved with a minimum-phase wavelet given by

$$W(t) = e^{-0.15t} \sin\left(\frac{\pi t}{20}\right).$$

Figure 2c shows the result of spiking deconvolution applied to figure 2b with an operator length of 40 ms followed by 4/8 - 150/200 Hz zero-phase bandpass filtering; figure 2d shows the result of spectral whitening of figure 2b with $\alpha = 0.1$ followed by 4/8 - 180/200 Hz zero-phase bandpass filtering. Comparing figure 2c with figure 2a, it is obvious that the spiking deconvolution performed almost perfectly in this instance. This is not surprising because all the assumptions for the spiking deconvolution technique are satisfied in the input trace (shown in fig. 2b).

However, the result of spectral whitening (fig. 2d) is quite different from that of spiking deconvolution. Figure 3 shows the amplitude spectrum of the input trace (top) and the spectral whitened trace (bottom). Figure 3 indicates that the spectral whitening increased the high-frequency content and flattened the amplitude spectrum as desired. In consequence, the spectral whitened trace shows higher resolution of seismic events, but the waveform of the seismic events shows a substantial phase distortion.

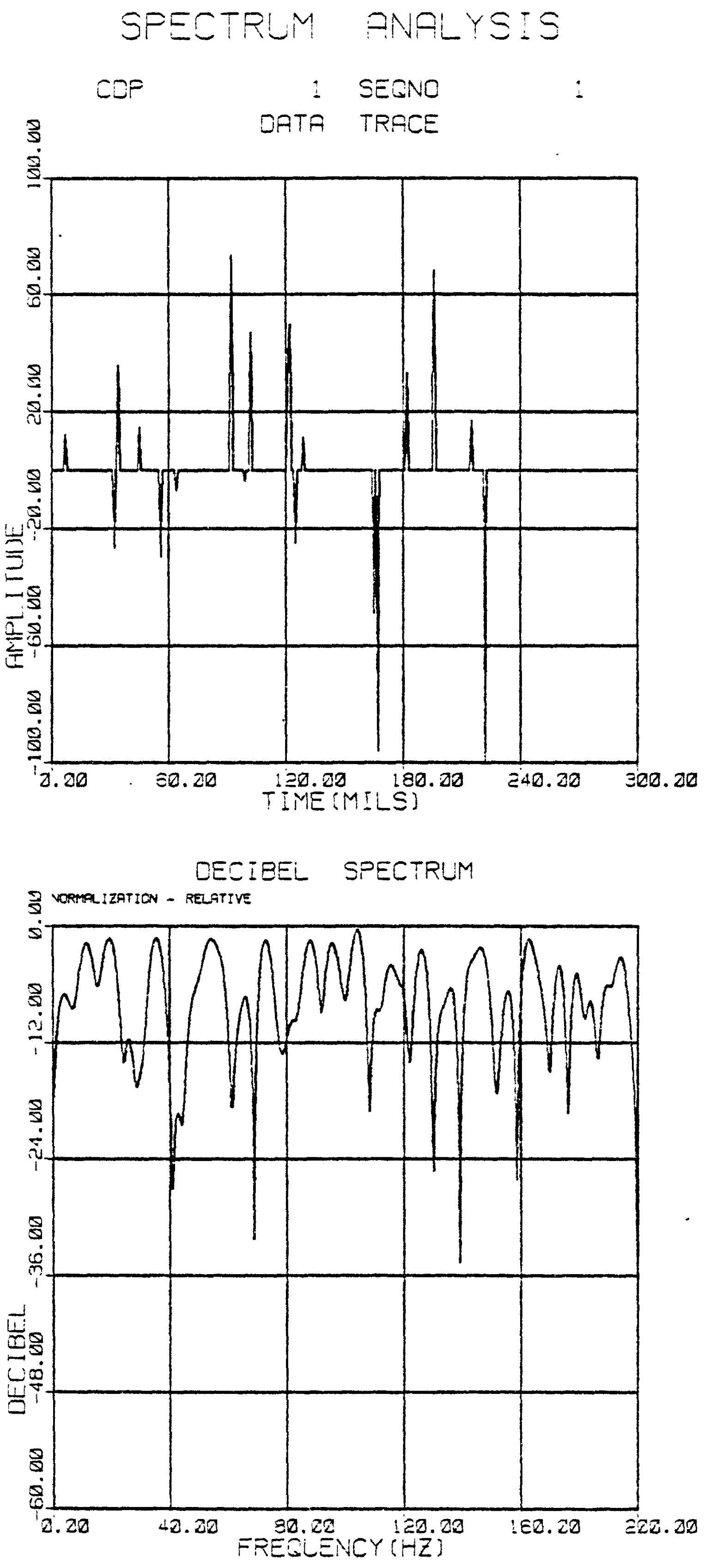


Figure 1.--Earth reflectivity series generated by Mendel and Kormylo (1978). Top: reflectivity series; bottom: amplitude spectrum.

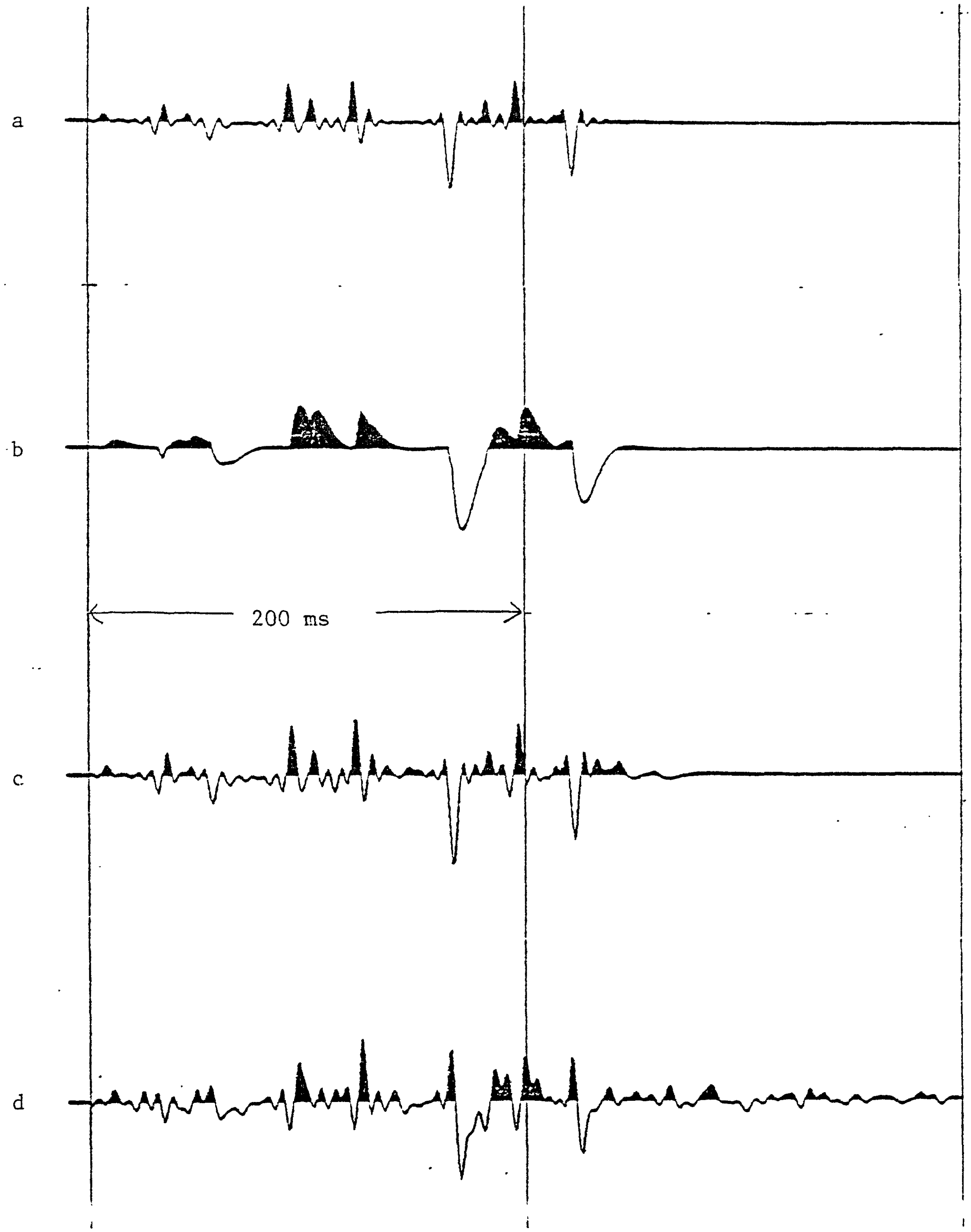
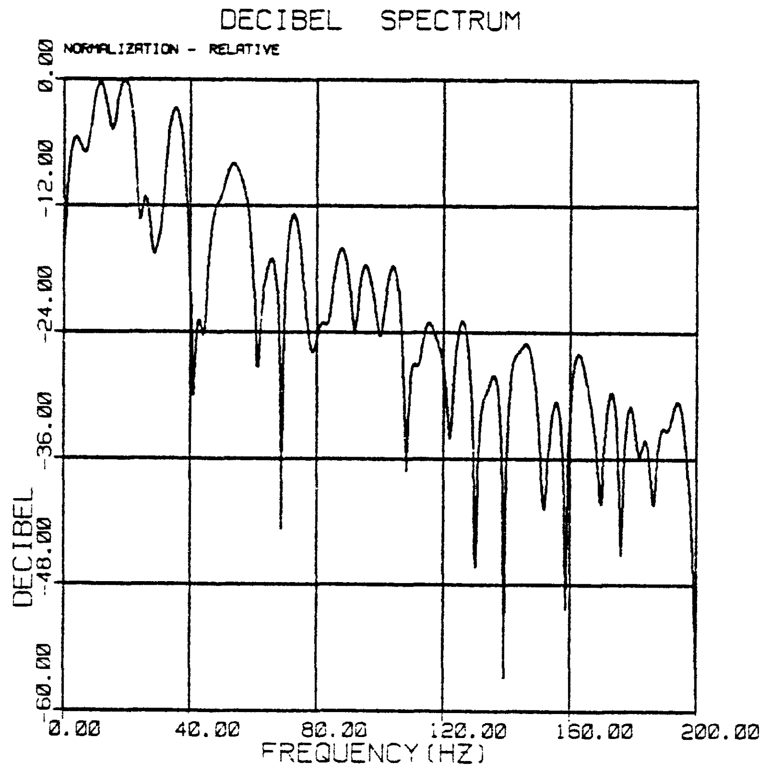


Figure 2.--Comparison of spectral whitening with spiking deconvolution when the input wavelet is a minimum phase: (a) earth reflectivity series convolved with a 4/8 - 150/250 Hz zero-phase bandpass filter; (b) earth reflectivity series convolved with a minimum phase wavelet, given by $\exp(-0.15t)\sin(\pi t/20)$ (This is the input trace to the following deconvolution process); (c) result of the spiking deconvolution with a 40-ms operator length; and (d) result of spectral whitening with α = 0.1.



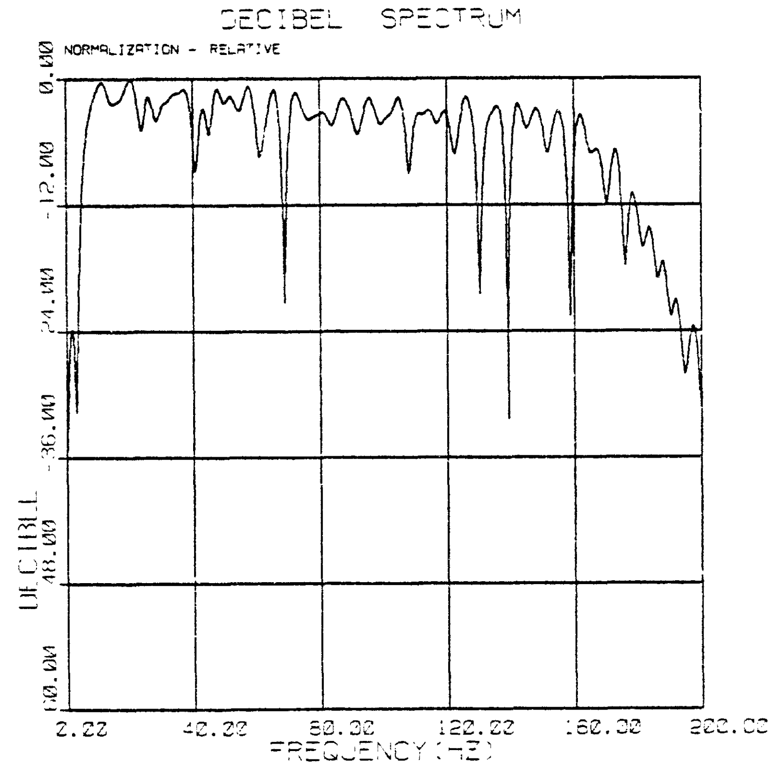


Figure 3.--Amplitude spectrum of the seismic trace. Top: input trace; bottom: spectral whitened trace.

As shown in the theory section, spectral whitening operates only on the amplitude spectrum of the trace; thus the original phase of the input trace is still present on the output trace. This result implies that the spectral whitening technique can be applied in order to increase structural resolution (pulse compression), but caution should be exercised when performing stratigraphic interpretation unless the phase spectrum of the input is known.

Figure 4 shows the result of the analysis when the source signature is a zero-phase wavelet. Figure 4a shows the earth reflectivity function convolved with 4/8 - 80/100 Hz zero-phase bandpass filter; figure 4b shows the input seismic trace convolved with a 40 Hz symmetrical Ricker wavelet. The result of a spiking deconvolution applied to figure 2b with 40 ms operator length is shown in figure 4c, and that of the spectral whitening with $\alpha = 0.1$ is shown in figure 4d. Comparing figure 4a with 4d reveals the success of using spectral whitening for deconvolution, while the spiking deconvolution result was erratic due to the non-minimum phase input wavelet. Therefore, one could conclude that the spectral whitening technique can be used as a deconvolution method when the source signature is a zero-phase wavelet. Figure 5 shows the amplitude spectrum before (top) and after (bottom) spectral whitening. The output amplitude spectrum is flat up to 100 Hz, where the original amplitude spectrum is about 50 db down from the maximum.

Another useful application of spectral whitening is in enhancement of signal-to-noise in certain instances. When the noise has a narrow spectral range, such as in ground roll or 60 Hz electrical noise, spectral whitening enhances the signal-to-noise ratio significantly. Thus spectral whitening with an appropriate α could be used as an alternative to band-rejection or notch-filtering in seismic data processing: the spectral whitening method attempts to balance overall amplitude spectrum instead of rejecting or suppressing the undesired frequency components. This kind of application is shown in figure 6. Figure 6a shows the seismic trace with varying signal-to-noise ratio with an 80 Hz symmetrical Ricker wavelet for the input wavelet and 50 Hz sine wave for noise. When the signal-to-noise ratio is equal or less than 1 (bottom 2 traces), the resolution of the seismic trace is poor. Figure 7 shows the amplitude spectrum of the noise-free seismogram (top) and the noise seismic trace (bottom) with a signal-to-noise ratio of 0.5. As shown in figure 7, the 50 Hz noise is about 12 db higher than the dominant signal component. Figure 6b shows the results of spectral whitening with $\alpha = 0.17$. These results indicate not only the substantical pulse compression but also signal-to-noise ratio enhancement.

SUMMARY

Based on the theory and results shown in this paper and the author's experience with spectral whitening techniques, the following conclusions can be made.

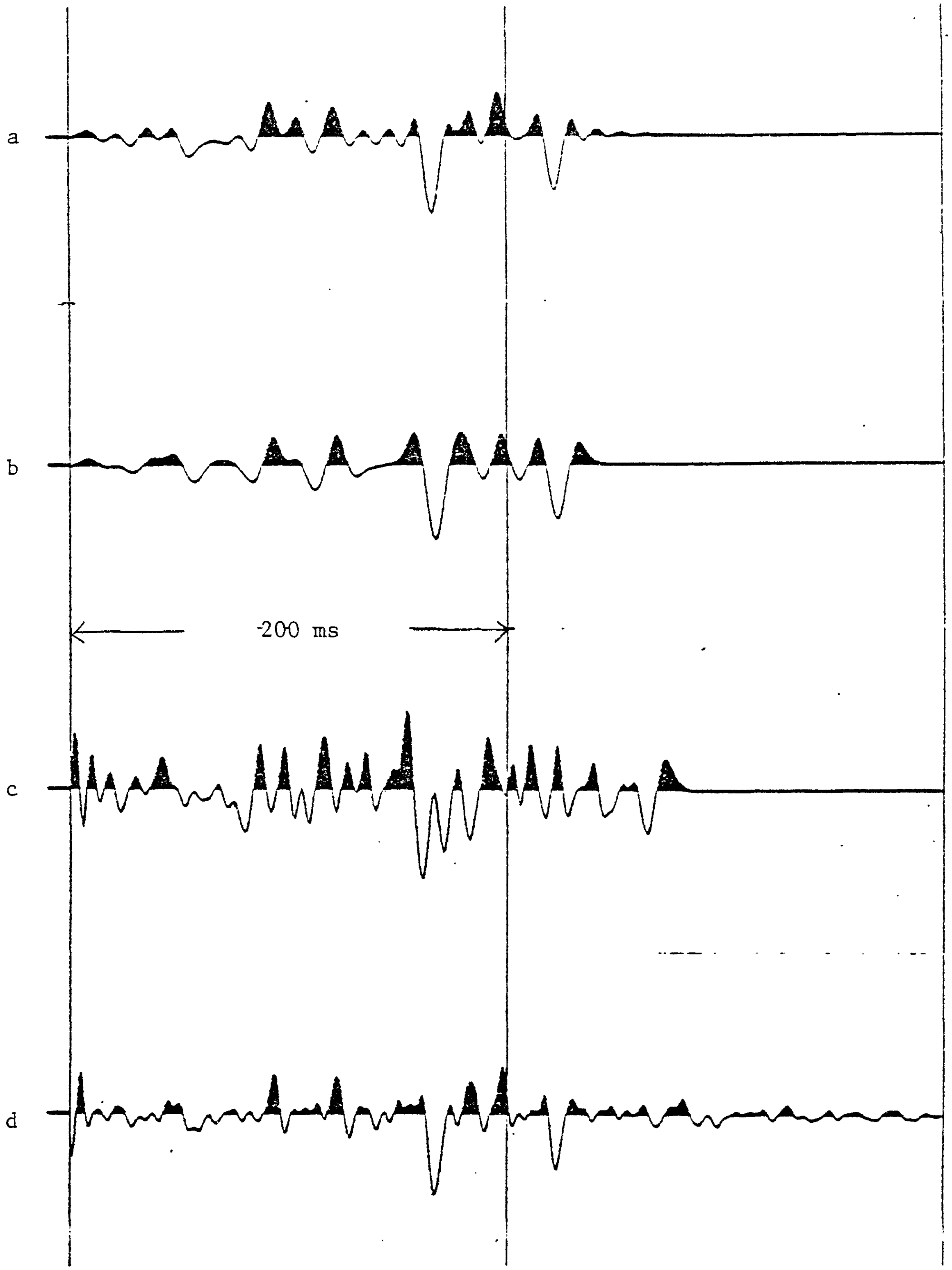
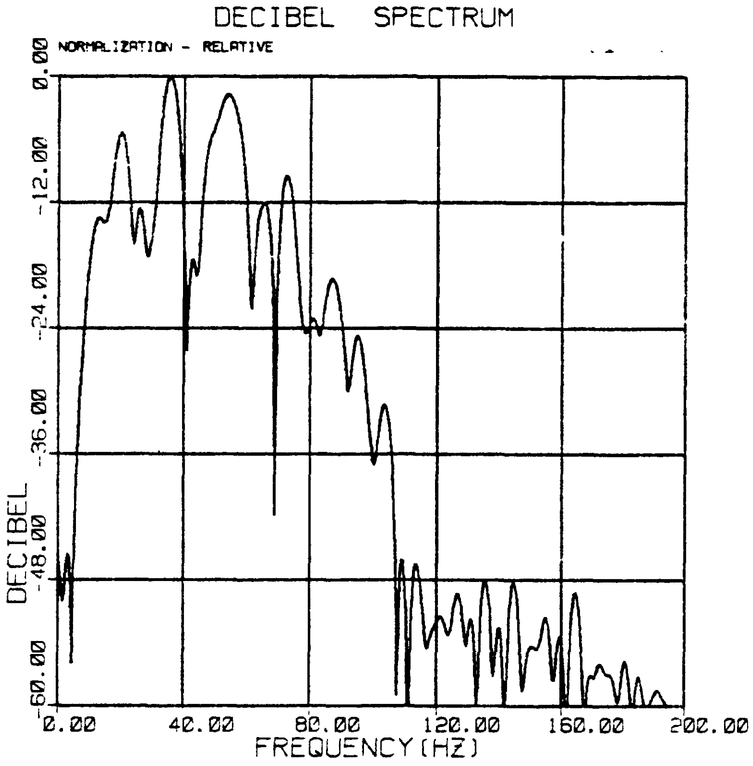


Figure 4.--Comparison of spectral whitening with spiking deconvolution when the input wavelet is a zero-phase: (a) earth reflectivity series convolved with a 4/8 - 80/150 Hz zero-phase bandpass filter; (b) earth reflectivity series convolved with a 40 Hz symmetrical Ricker wavelet (This is the input trace to the following deconvolution process); (c) result of spiking deconvolution with a 40-ms operator length; and (d) result of spectral whitening with $\alpha = 0.1$.



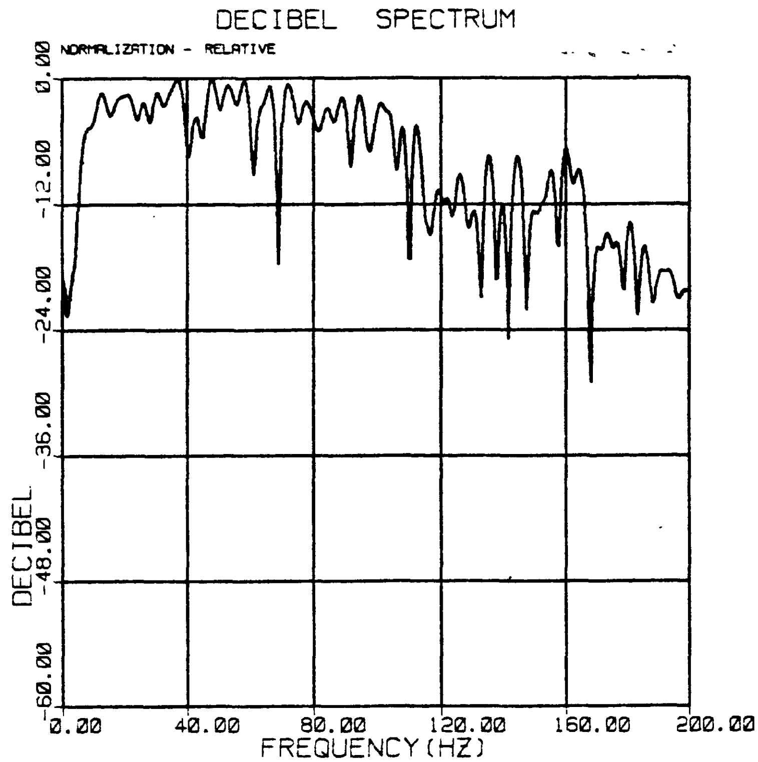


Figure 5.--Amplitude spectrum of the seismic trace. Top: input trace; bottom: spectral whitened traces.

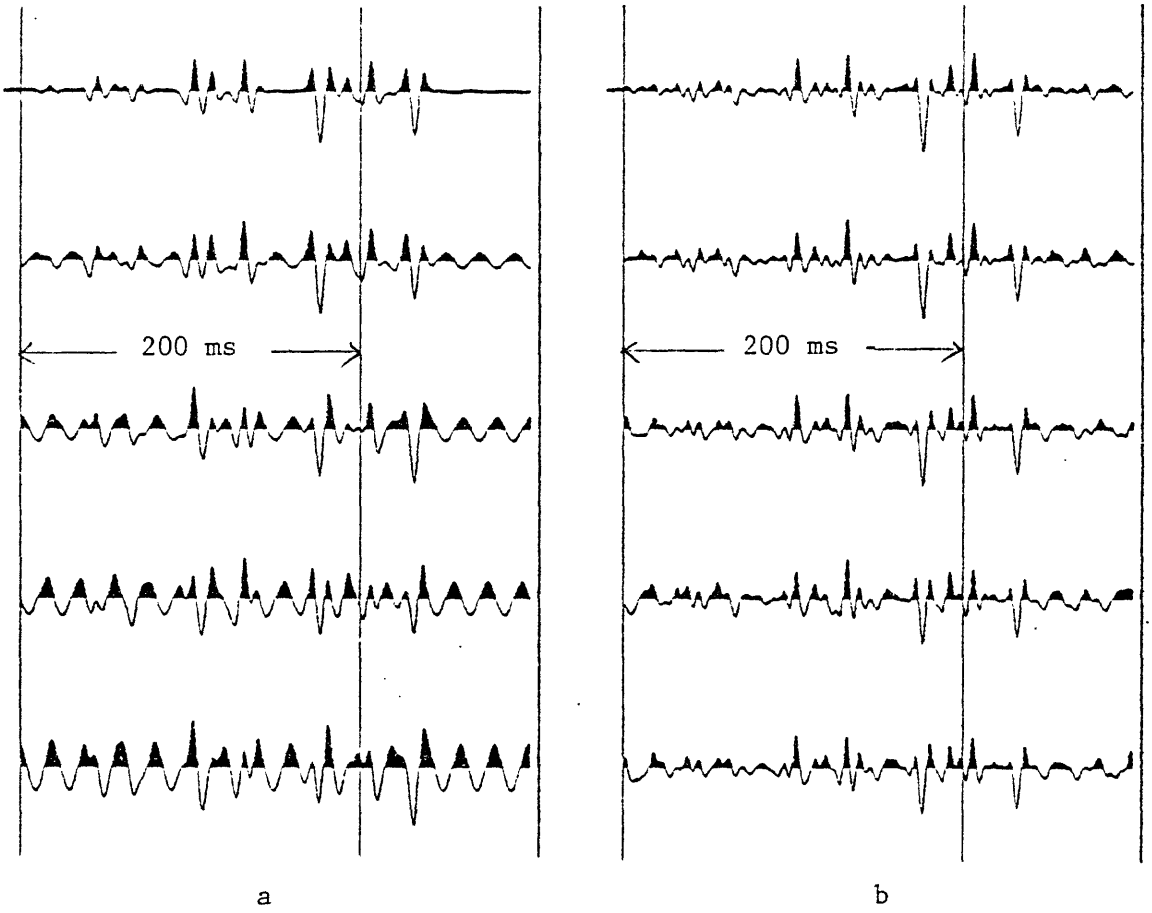
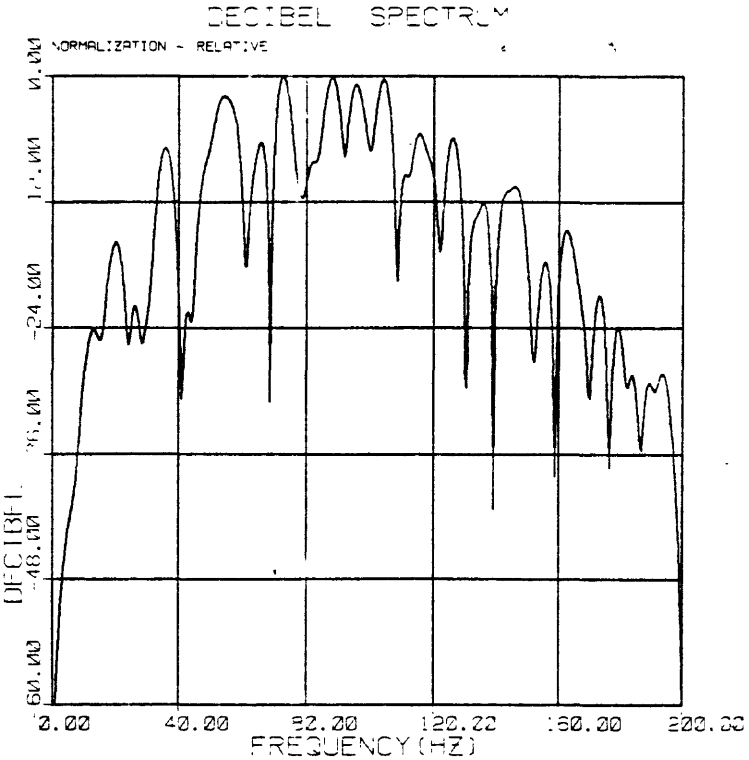


Figure 6.--Application of spectral whitening method as an alternative to a notch filter: (a) earth reflectivity convolved with an 80 Hz symmetrical Ricker wavelet contaminated by 50 Hz sine wave; (b) result of spectral whitening with α = 0.17. Signal-to-noise ratios from the top trace to the bottom traces are 1,000, 5, 2, 1, and 0.5.



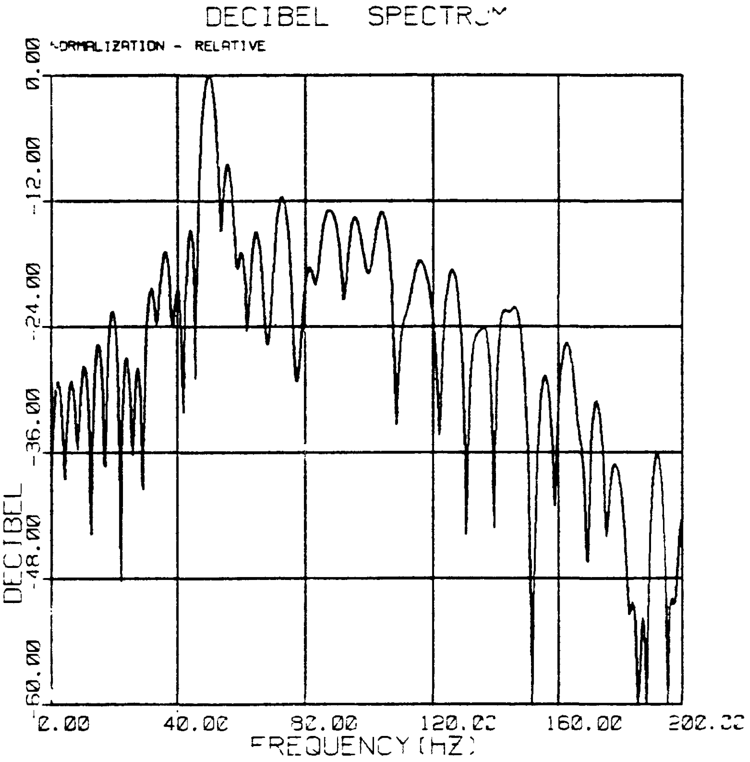


Figure 7.--Amplitude spectrum of the noise-free and noisy traces. Top: noise-free trace; bottom: noisy trace (bottom of figure 6a).

Under the assumption that earth reflectivity is an uncorrelated random sequence, the spectral whitening method presented here:

(1) can replace the conventional spiking deconvolution when the source signature is a zero-phase wavelet, such as Vibroseis(R) data;

(2) is a good pulse-compression method irrespective of the phase of the wavelet (thus, could be used to increase the structural resolution);

(3) can be used as a signal-to-noise enhancement filter, particularly when the noise has a narrow spectral bandwidth; and

(4) could be misleading when performing stratigraphic interpretation unless the input wavelet is zero-phase wavelet.

A final observation in regard to spectral whitening as presented in this paper is that this method cannot handle multiple suppression adequately. Whenever reverberatory energy is significant in the seismic section, some kind of multiple suppression method (such as predictive deconvolution) should be applied before or after spectral whitening in order to enhance the primary seismic events.

REFERENCES CITED

Hutchinson, D. R., and Grow, J. A., 1985, New York Bight Fault: Geological Society of America Bulletin, v. 96, p. 975-989.

Mendel, J. M., and Kormylo, John, 1978, Single-channel white-noise estimates for deconvolution: Geophysics, v. 43, p. 102-124.

APPENDIX

Source waveforms and their amplitude spectra used for this study are shown in figure A-1.

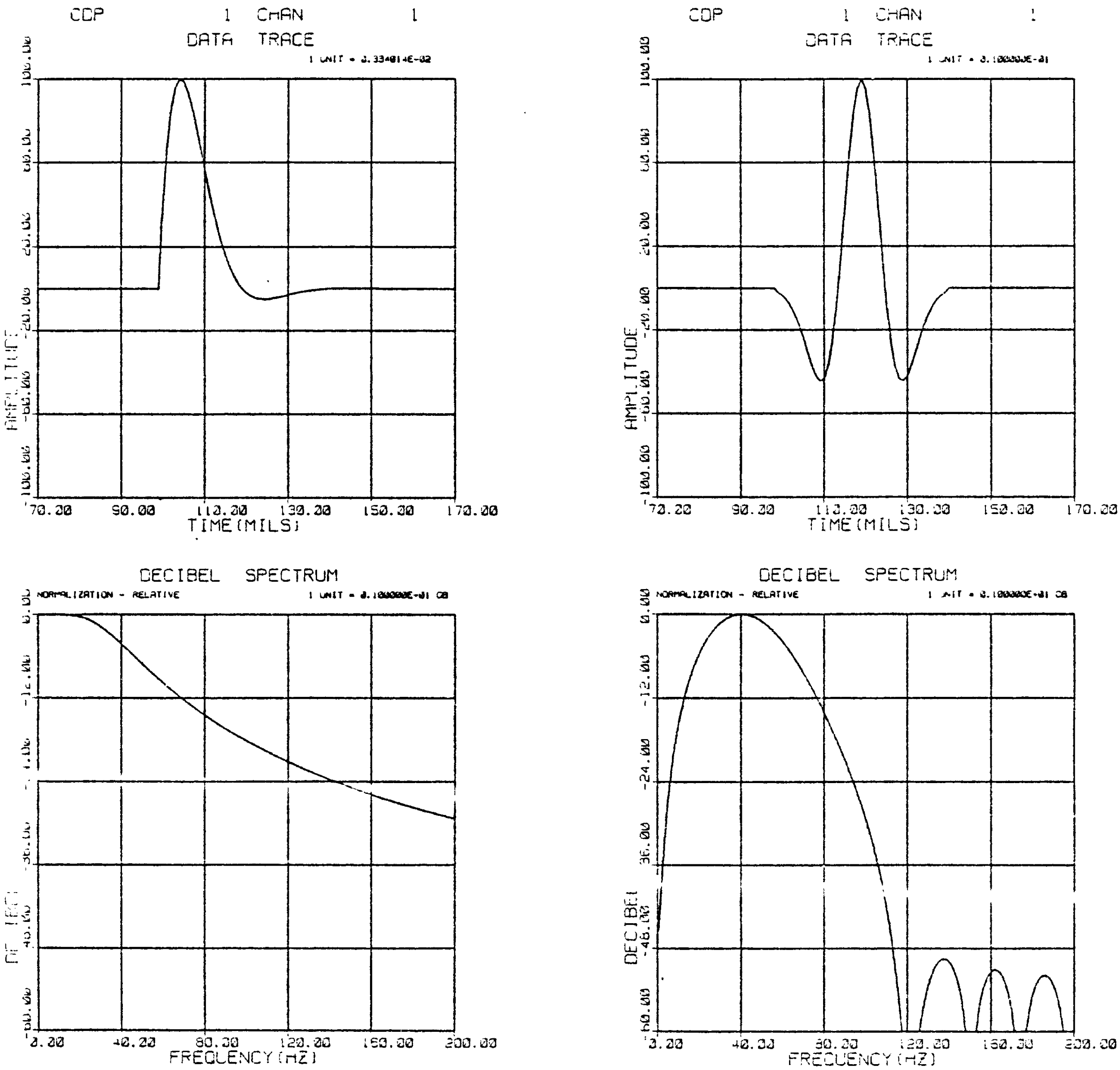


Figure A-1.--Source waveform and its amplitude spectrum. Left: minimum phase wavelet given by $e^{-0.15t}\sin\pi t/20$; right: symmetrical 40 Hz Ricker wavelet.