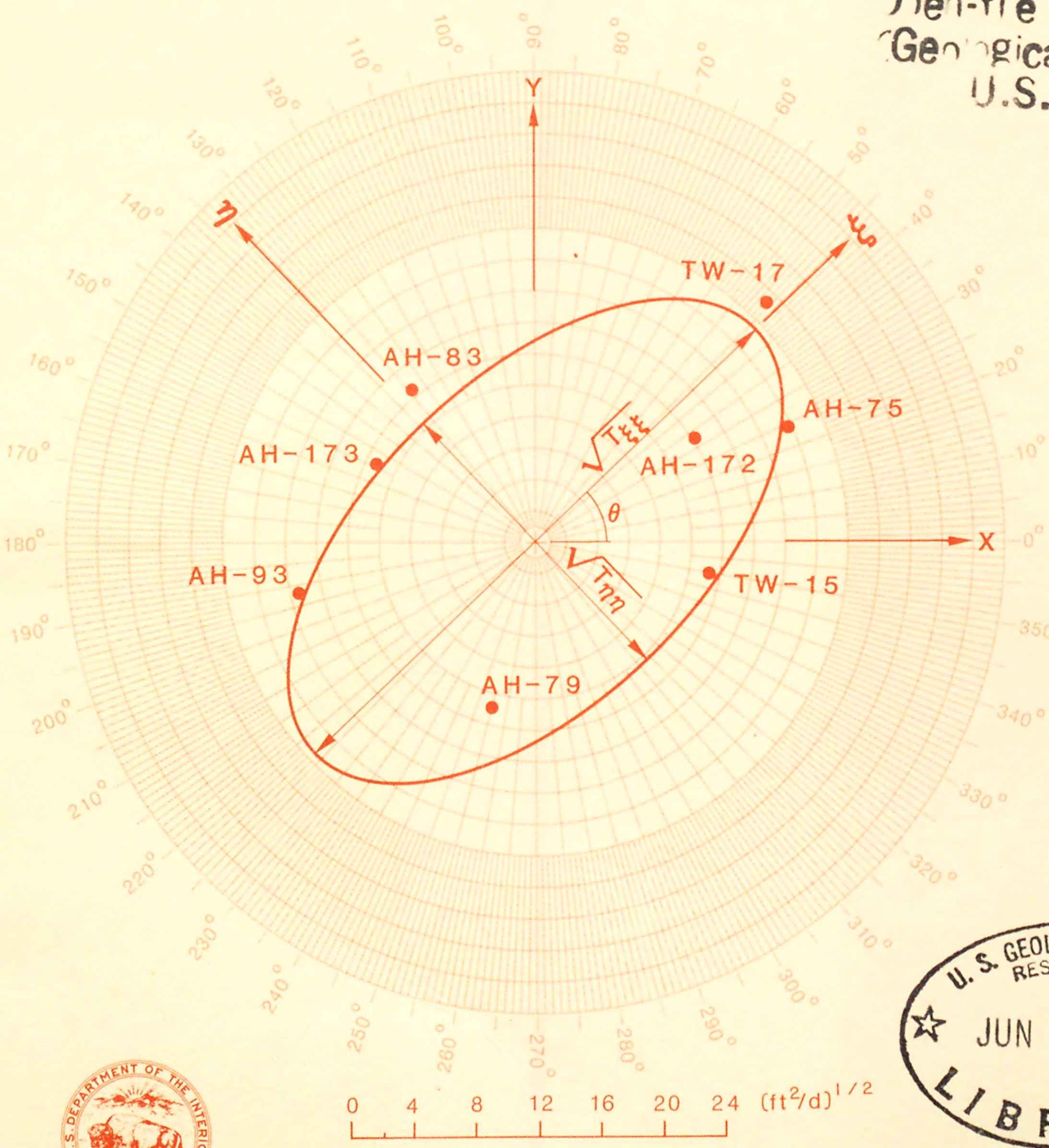
METHODS AND COMPUTER PROGRAM DOCUMENTATION FOR DETERMINING ANISOTROPIC TRANSMISSIVITY TENSOR COMPONENTS OF TWO-DIMENSIONAL GROUND-WATER FLOW



U.S. GEOLOGICAL SURVEY

PREPARED IN COOPERATION WITH
THE CITY OF BRUNSWICK, AND GLYNN COUNTY, GEORGIA

OPEN-FILE REPORT 86-227

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By Morris L. Maslia and Robert B. Randolph

UNITED STATES GEOLOGICAL SURVEY

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CITY of BRUNSWICK, and
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Doraville, Georgia

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CONVERSION FACTORS

For those readers who may prefer to use metric units rather than the inch-pound unit, the conversion factors for the terms used in this report are listed below:

Multiply inch-pound	By	To obtain metric unit
	Length	
inch (in.)	25.40	millimeter (mm)
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Area	
square mile (mi^2)	2.590	square kilometer (km^2)
	Volume	
gallon (gal)	3.785 x 10-3	cubic meter (m^3)
	3.785	liter (L)
	Flow	
gallon per minute (gal/min)	6.309 x 10-3	cubic meter per second (m^3/s)
	0.06309	liter per second (L/s)
	Transmissivity	
foot squared per day (ft^2/d)	0.09290	meter squared per day (m^2/d)

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ABSTRACT

This report describes the theory of anisotropic aquifer hydraulic properties and a computer program, written in Fortran 77, developed to compute the components of the anisotropic transmissivity tensor of two-dimensional ground-water flow. To determine the tensor components using one pumping well and three observation wells, the type-curve and straight-line approximation methods are developed. These methods are based on the equation of drawdown developed for two-dimensional nonsteady flow in an infinite anisotropic aquifer. To determine tensor components using more than three observation wells, a weighted least-squares optimization procedure is described for use with the type-curve and straight-line approximation methods.

The computer program described in this report allows the type-curve, straight-line approximation, and weighted least-squares optimization methods to be used in conjunction with data from observation and pumping wells. Three example applications using the computer program and field data gathered during hydrogeologic investigations at a site near Dawsonville, Ga., are provided to illustrate the use of the computer program. The example applications demonstrate the use of the type-curve method using three observation wells, the weighted least-squares optimization method using eight observation wells and equal weighting, and the weighted least-squares optimization method using eight observation wells and unequal weighting. Results obtained using the computer program indicate major transmissivity ($T_{\xi\xi}$) in the range of 347 to 296 feet squared per day, minor transmissivity ($T_{\eta\eta}$) in the range of 139 to 99 feet squared per day, aquifer anisotropy ($T_{\xi\xi}/T_{\eta\eta}$) in the range of 3.54 to 2.14, principal direction of flow in the range of N. 45.9° E. to N. 58.7° E., and storage coefficient (S) in the range of 6.3×10^{-3} to 3.7×10^{-3}. The numerical results are in good agreement with field data gathered on the weathered crystalline rocks underlying the investigation site.

Supplemental material provides definitions of variables, data requirements and corresponding formats, input data and output results for the example applications, and a listing of the Fortran 77 computer code.

INTRODUCTION

The equations that represent the movement of water in an aquifer when water is being withdrawn from a well form the basis of methods used to analyze aquifer-test data. The equations were derived under the assumption of aquifer isotropy and are not valid for the analysis of anisotropic aquifers that include, for example, flow in some secondary permeability terraines and fractured rocks. Methods for analyzing aquifer-test data for such aquifers must be based on equations which describe the distribution of drawdown around a well of constant discharge in an infinite anisotropic aquifer. In conjunction with aquifer-test data, these equations can be used to determine aquifer anisotropy and the components of the anisotropic transmissivity tensor.

Several methods have been developed to compute drawdown in an anisotropic aquifer and to determine the tensor components. Among the methods described in the literature are those by Papadopulos (1965), Hantush (1966a, b), Hantush and Thomas (1966), Way and Mckee (1982), Neuman and others (1984), and Hsieh and others (1985).

The purpose of this report is to describe the method of Papadopulos (1965) as it is applied to aquifer hydraulic data to determine the components of the anisotropic transmissivity tensor. Additionally, this report describes the use of a computer program, TENSOR2D, which automates the solution of hydraulic parameters and tensor components for an anisotropic aquifer. The rigorous application of the Papadopulos method (1965) requires data for one pumping well and three obsevation wells. To determine tensor components and aquifer hydraulic parameters, analysis of aquifer-test data using the type-curve and straight-line approximation methods are developed. Furthermore, in this report the Papadopulos method of analysis is extended to allow for more than three observation wells by developing a weighted least-squares optimization procedure for use with the type-curve and straight-line approximation methods.

To demostrate the use of the computer program that automates the solution process for the anisotropic aquifer hydraulic parameters and tensor components, three example applications are provided. The data for these example applications, obtained during hydrogeologic investigations at a site near Dawsonville, Ga. (Stewart, 1964; Stewart and others, 1964), demonstrate the use of the type-curve method using three observation wells, the weighted least-squares optimization method using eight observation wells and equal weighting, and the weighted least-squares optimization method using eight observation wells and unequal weighting.

The work and computer simulation presented in this report were done in cooperation with the city of Brunswick and Glynn County, Ga.

THEORY OF ANISOTROPIC AQUIFER HYDRAULIC PROPERTIES

A porous medium is considered to be isotropic if all significant properties of the medium are independent of direction (Lohman and others, 1972, p. 9). If, however, at an arbitrary point in the medium the properties vary with direction, the medium at that point is referred to as anisotropic (Bear, 1972, p. 134). In considering two-dimensional ground-water flow, some aquifers are anisotropic. For example, in carbonate rock aquifers, flowing ground water dissolves the rocks, producing solution channels primarily along the direction of flow. The rocks then become anisotropic making the aquifer more permeable along the solution channels.

In an anisotropic aquifer, $\underline{\underline{T}}$ is defined as a second rank tensor quantity of transmissivity (Bear, 1972, p. 137; Bear, 1979, p. 72). It is a linear transformation relating hydraulic gradient, $\underline{J}$ (in the downstream direction), to the discharge, $\underline{q}^*$, averaged over the thickness of the aquifer per unit width normal to the flow direction (fig. 1). $\underline{\underline{T}}$ can be represented with respect to an arbitrary set of orthogonal axes (x-y) by a 2 x 2 matrix, such that

$$\underline{\underline{T}} = \begin{bmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{bmatrix} . \quad (1)$$

Because the transmissivity tensor is symmetric (Bear, 1979, p. 72), $T_{xy} = T_{yx}$. Additionally, the determinant, D', of $\underline{\underline{T}}$ is defined as

$$D' = T_{xx} T_{yy} - T_{xy}^2 . \quad (2)$$

In an anisotropic aquifer, the hydraulic gradient, $\underline{J}$, and discharge, $\underline{q}^*$, are not necessarily in the same direction (fig. 1a). However, in certain directions, termed the principal directions, $\underline{J}$ and $\underline{q}^*$ are parallel (fig. 1b). These principal directions correspond to greatest and least-preferred flow directions. In these directions the ratio between $\underline{q}^*$ and $\underline{J}$ is known as the principal value of the transmissivity tensor or principal transmissivity. Since the principal values are all distinct, these principal directions are mutually orthogonal and can be used to define the principal coordinate system. For the principal ξ-η coordinate system, $\underline{\underline{T}}$ has the form

$$\underline{\underline{T}} = \begin{bmatrix} T_{\xi\xi} & 0 \\ 0 & T_{\eta\eta} \end{bmatrix} , \quad (3)$$

where $T_{\xi\xi}$ and $T_{\eta\eta}$ are defined as the major and minor or principal components of transmissivity, respectively.

(a) Hyraulic gradient (J̲) and discharge (q*) aligned along different directions in an anisotropic aquifer.

(b) Hydraulic gradient (J̲) and discharge (q*) are parallel and aligned along the principal directions in an anisotropic aquifer.

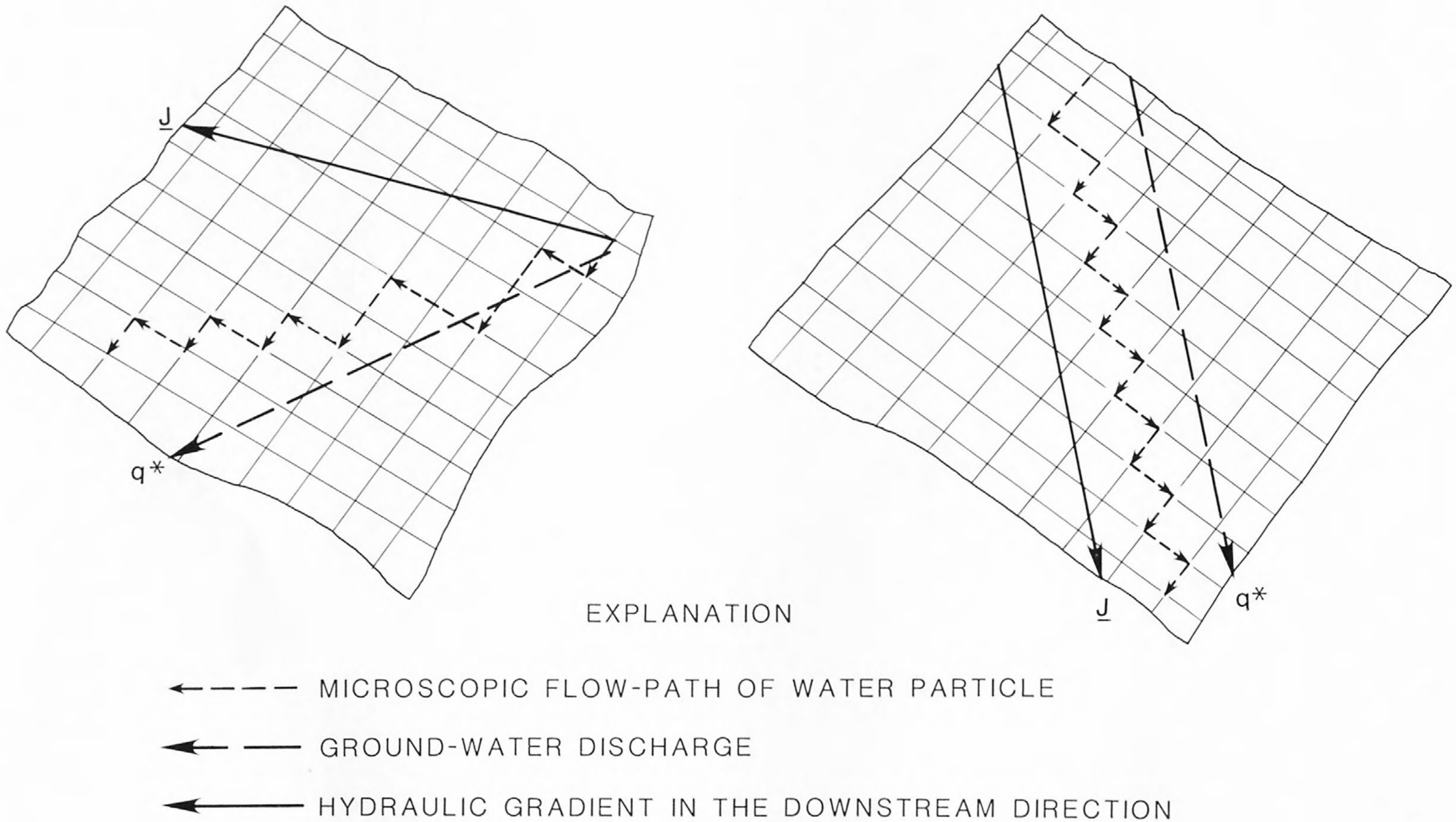


Figure 1.—Relationships between the hydraulic gradient (J̲) and discharge (q*) in an anisotopic aquifer.

The distribution of drawdown around a fully penetrating well of constant discharge in an infinite, anisotropic, confined aquifer is described by the following equation (Papadopulos, 1965, p. 22):

$$T_{xx}\frac{\partial s^2}{\partial x^2} + 2T_{xy}\frac{\partial s^2}{\partial x \partial y} + T_{yy}\frac{\partial s^2}{\partial y^2} + Q\delta(x)\delta(y) = S\frac{\partial s}{\partial t} \tag{4}$$

subject to the following initial and boundary conditions:

$$s(x,y,0) = 0 \tag{5}$$

$$s(\pm\infty,y,t) = 0 \tag{6}$$

$$s(x,\pm\infty,t) = 0\ , \tag{7}$$

where s = the drawdown, (L),

T_{xx}, T_{yy}, T_{xy} = components of the anisotropic transmissivity tensor, (L^2/T),

S = storage coefficient, (L°),

Q = discharge of the well, $(L^3/T)/(L^2$ of aquifer),

δ = Dirac delta function,

x, y = coordinates of an arbitrary set of orthogonal axes with the origin at the discharge well, (L), and

t = time since pumping started, (T).

Under the assumption of aquifer homogeneity, T_{xx}, T_{yy}, and T_{xy} are assumed to be constant over the contributing volume of the aquifer under consideration.

The problem is solved by using and applying initial condition equation 5 and the Laplace transformation with respect to time (t) to solve equation 4. Then the complex Fourier transform with respect to x and y is applied with boundary condition equations 6 and 7. The formal solution to equation 4 given by Papadopulos (1965) is

$$s = \frac{Q}{4\pi\sqrt{D'}} W(u_{xy}) , \tag{8}$$

where $W(u_{xy})$, known as the Theis well function, is defined as:

$$W(u) = \int_u^\infty \frac{e^{-\nu}}{\nu} d\nu \tag{9}$$

in which

$$u_{xy} = \frac{S}{4t} \frac{[T_{xx}(y^2) + T_{yy}(x^2) - 2T_{xy}(xy)]}{D'}, \tag{10}$$

where D' is defined by equation 2.

METHODS FOR DETERMINING ANISOTROPIC TRANSMISSIVITY TENSOR COMPONENTS

In an anisotropic aquifer, the drawdown caused by pumping is directionally dependent--that is, it is not radially symmetric. Therefore, during an aquifer test the drawdown at each observation well must be analyzed and a plot of observed drawdown (s) versus time (t or 1/t) must be made. Either the type-curve (Theis, 1935) or the straight-line method (Cooper and Jacob, 1946; Jacob, 1950) can be used to analyze the observation-well data. In order to compute the tensor components and the anisotropic aquifer parameter values, the four constants in equation 10 (T_{xx}, T_{yy}, T_{xy}, and S) need to be determined. Therefore, one pumping well located at the origin of an arbitrary Cartesian coordinate system and a minimum of three observation wells are required (fig. 2). While the distribution of the wells around the pumping well is arbitrary as long as no two observation wells are radially aligned with the pumping well, the degree of radial distribution of observation wells tends to influence the results of the tensor analysis.

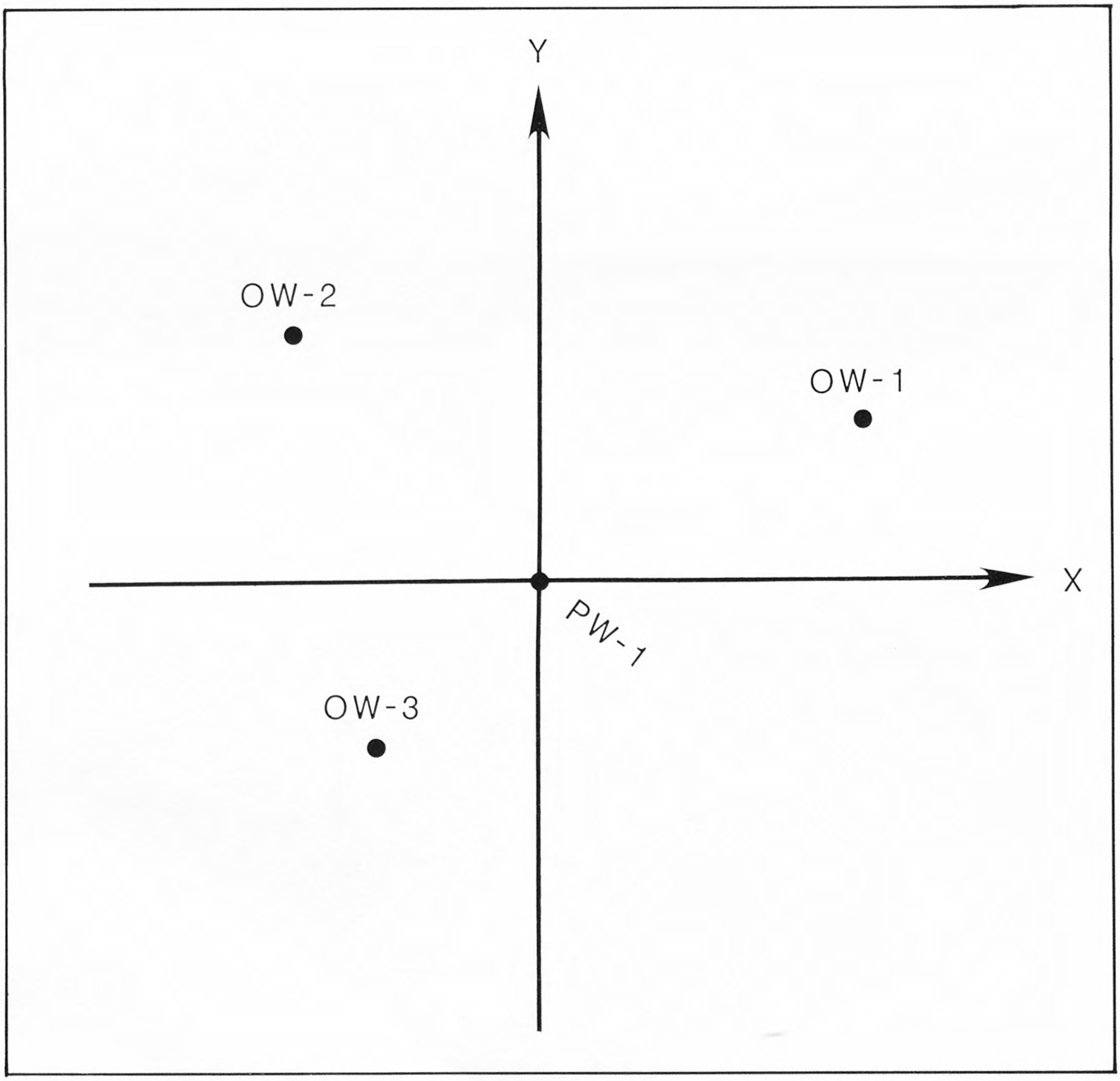


Figure 2.—Arbitary Cartesian coordinate system aligned with reference to the pumping well (PW-1) and observation wells OW-1, OW-2, and OW-3.

Type-Curve

For each observation well a log-log plot of observed drawdown versus time (or inverse time) is graphically (or numerically) matched with the Theis type curve resulting in match point values of s^*, t^*, $W(u)^*$, and u^* for each of the three observation wells. The drawdown (s^*), well function ($W(u)^*$), and the flow rate of the pumping well (Q) are then substituted into equation 8 to solve for the determinant (D') for each set of observation-well data as follows

$$D' = \left\{ \frac{Q}{4\pi s^*} W(u)^* \right\}^2 . \tag{11}$$

D' should have approximately the same value for each observation well. If not, an average value should be selected. Rearranging equation 10 results in

$$ST_{xx}(y^2) + ST_{yy}(x^2) - 2ST_{xy}(xy) = 4tu_{xy}D' . \tag{12}$$

Replacing values of u_{xy}, x, and y for each observation well and D' from equation 11, results in a system of three simultaneous equations of the general form

$$\underline{\underline{A}}\,\underline{X} = \underline{B} , \tag{13}$$

where

$$\underline{\underline{A}} = \begin{bmatrix} y_1^2 & x_1^2 & -2x_1y_1 \\ y_2^2 & x_2^2 & -2x_2y_2 \\ y_3^2 & x_3^2 & -2x_3y_3 \end{bmatrix} , \tag{14}$$

$$\underline{X} = \begin{Bmatrix} ST_{xx} \\ ST_{yy} \\ ST_{xy} \end{Bmatrix} , \text{ and} \tag{15}$$

$$\underline{B} = \begin{Bmatrix} 4t_1^*u_1^*D' \\ 4t_2^*u_2^*D' \\ 4t_3^*u_3^*D' \end{Bmatrix} . \tag{16}$$

In equation 14, x_i and y_i (i = 1, 2, 3) are the coordinate values of the three observation wells with respect to the arbitrary Cartesian coordinate system shown in figure 2. The values of $(u^*)_i$ (i = 1, 2, 3) in equation 16, are determined from the Theis curve match for each observation well and D' is the determinant derived from equation 11.

Equation 13 can be solved by any number of simultaneous equation solvers. In this report, LU decomposition by the Crout method is employed (Stewart, 1973). In the code listing (supplemental data IV) IMSL[1] routines LUDATF and LUELMF are used to solve equation 13. Upon solving equation 13, values are obtained for ST_{xx}, ST_{yy}, and ST_{xy}.

Multiplying both sides of equation 2 by S^2 and rearranging, yields

$$D'S^2 = (ST_{xx})(ST_{yy}) - (ST_{xy})^2 \quad . \tag{17}$$

The storage coefficient for the anisotropic system is then obtained by solving equation 17

$$S = \sqrt{\frac{(ST_{xx})(ST_{yy}) - (ST_{xy})^2}{D'}} \quad , \tag{18}$$

where ST_{xx}, ST_{yy}, ST_{xy} are obtained by solving the system of equations 13 and D' is the determinant derived from equation 11. Using the computed value of S from equation 18 and the three values previously obtained from equation 13, the components of $\underline{\underline{T}}$ can be determined such that

$$T_{xx} = (ST_{xx})/S \tag{19}$$

$$T_{yy} = (ST_{yy})/S \tag{20}$$

$$T_{xy} = (ST_{xy})/S \quad . \tag{21}$$

To determine the principal values of $\underline{\underline{T}}$, the eigenvalue problem

$$\underline{\underline{T}}\,\underline{X} = \lambda \underline{X} \tag{22}$$

is solved by substituting for the components of $\underline{\underline{T}}$ and rearranging

$$\begin{bmatrix} T_{xx} - \lambda & T_{xy} \\ T_{xy} & T_{yy} - \lambda \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad . \tag{23}$$

[1]Use of brand/trade names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

Setting the determinant of the matrix in equation 23 to zero, multiplying, and rearranging results in:

$$\lambda^2 - \lambda(T_{xx} + T_{yy}) + T_{xx}T_{yy} - T_{xy}{}^2 = 0 , \qquad (24)$$

which is a quadratic equation. Because $\underline{\underline{T}}$ is symmetric, there will be two real roots. These roots are the principal values of the transmissivity tensor, which can be expressed as

$$T_{\xi\xi} = 1/2 \cdot \{(T_{xx} + T_{yy}) + \sqrt{(T_{xx} - T_{yy})^2 + 4T_{xy}{}^2}\} \qquad (25)$$

$$T_{\eta\eta} = 1/2 \cdot \{(T_{xx} + T_{yy}) - \sqrt{(T_{xx} - T_{yy})^2 + 4T_{xy}{}^2}\} . \qquad (26)$$

Aquifer anisotropy is now defined as the ratio $T_{\xi\xi}/T_{\eta\eta}$. The angle (Θ) between the x-axis and the maximum principal direction can be found as follows

$$\Theta = \tan^{-1} \frac{T_{\xi\xi} - T_{xx}}{T_{xy}} . \qquad (27)$$

Using the computed principal values, the equation of the theoretical transmissivity ellipse is determined as

$$\xi^2/T_{\xi\xi} + \eta^2/T_{\eta\eta} = 1 , \qquad (28)$$

where ξ, η = the axes of the principal coordinate system rotated by Θ degrees from the arbitrary x-y coordinate system,

$\sqrt{T_{\xi\xi}}$ = the major axis of the transmissivity ellipse, and

$\sqrt{T_{\eta\eta}}$ = the minor axis of the transmissivity ellipse.

The components of the transmissivity tensor can be graphically determined by plotting equation 28 on polar-coordinate paper (fig. 3). Alternatively, using the equation by Hantush and Thomas (1966)

$$1/T_\rho = (1/T_{\xi\xi})\cos^2\beta + (1/T_{\eta\eta})\sin^2\beta , \qquad (29)$$

where T_ρ = the theoretical directional transmissivity, and

β = the direction of T_ρ from the origin with respect to the ξ-η coordinate system,

the transmissivity ellipse can be obtained by plotting $\sqrt{T_\rho}$ in the direction of β on polar-coordinate paper (fig. 3).

The directional transmissivity with respect to flow can be calculated using data from each observation well by (Hantush, 1966b, p. 422)

$$T_d = \frac{Sr^2}{4u^*t^*}, \tag{30}$$

where: T_d = the directional transmissivity at the observation well,

S = the composite storage coefficient as defined by equation 18,

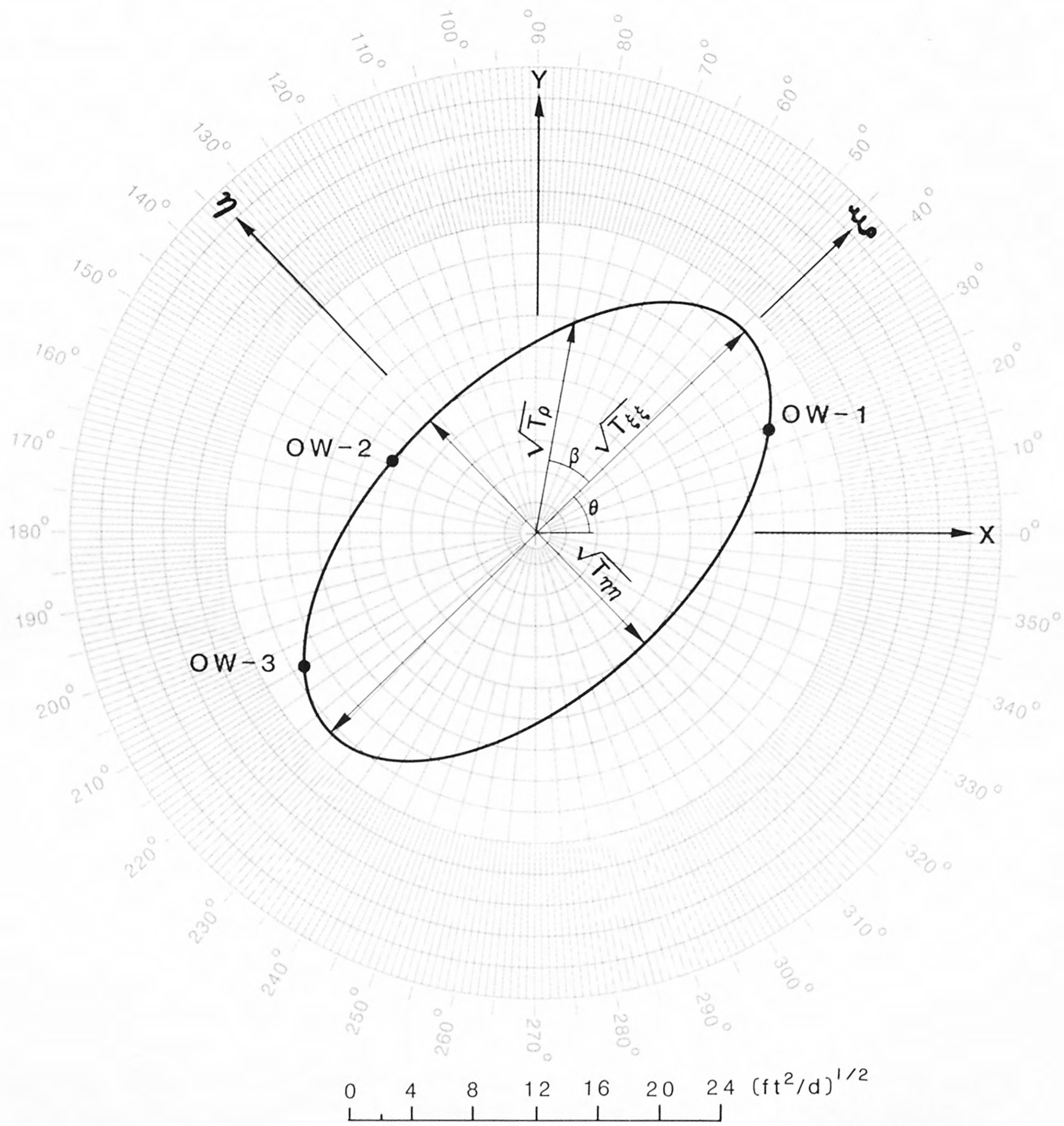
r = the radial distance from the origin of the arbitrary x-y coordinate system to the observation well (fig. 2),

t^* = the time at the match point determined by Theis curve matching at each observation well, and

u^* = the variable of the well function at the match point for the observation well.

A plot of $\sqrt{T_d}$ in the direction of the observation well on polar-coordinate paper (positive is counterclockwise from the +x axis on fig. 2) should coincide with the transmissivity ellipse computed using equation 28 or 29 (fig. 3). The ellipse can therefore be interpreted as the magnitude of transmissivity as a function of angle θ.

Alternatively, by dividing both sides of equation 30 by S (storage coefficient), a plot of directional diffusivity ($\sqrt{T_d/S}$) in the direction of the observation well on polar-coordinate paper should coincide with the aquifer diffusivity ellipse. The diffusivity ellipse can be computed by replacing the principal transmissivities ($T_{\xi\xi}$, $T_{\eta\eta}$) in equation 28 or 29 with the principal diffusivities ($T_{\xi\xi}/S$ and $T_{\eta\eta}/S$, where S is the storage coefficient defined by equation 18). This ellipse will be proportional to the transmissivity ellipse computed as described above by a factor of $1/\sqrt{S}$.



EXPLANATION

● OW-1 DIRECTIONAL TRANSMISSIVITY ($\sqrt{Td}$) COMPUTED USING EQUATION 30 AND WELL IDENTIFICATION NUMBER

——— THEORETICAL TRANSMISSIVITY ELLIPSE COMPUTED USING EQUATION 28 OR 29

Figure 3.—Comparison of theoretical transmissivity ellipse and directional transmissivity.

It is important to note if the term $(ST_{xx})(ST_{yy}) - (ST_{xy})^2$ in equation 18 is negative, then no physically plausible solution exists for the components of T with the observation well data being used. That is, there is no possible way to mathematically fit a transmissivity ellipse to the given observation-well data. A plot of $\sqrt{T_d/S}$ in the direction of the observation wells on polar-coordinate paper should indicate the data are scattered and it is not possible to fit a single ellipse through the three points. This may indicate that the field data are in error, the assumption of aquifer homogeneity is incorrect, the aquifer cannot be conceptualized as an anisotropic porous medium, or the quantity and distribution of observation wells are insufficient to describe the flow regime of the aquifer.

Straight-Line Approximation

For small values of u, $(u < 0.01)$ equation 9 can be approximated (Cooper and Jacob, 1946; Jacob, 1950) such that

$$W(u) = 2.303 \log_{10} \left(\frac{2.25}{4u}\right) . \tag{31}$$

Substituting equations 31 and 10 into equation 8 yields:

$$s = \frac{2.303Q}{4\pi\sqrt{D'}} \log_{10} \left\{ \frac{2.25t}{S} \left[\frac{D'}{T_{xx}(y^2) + T_{yy}(x^2) - 2T_{xy}(xy)}\right] \right\} . \tag{32}$$

For each of the three observation wells, plotting drawdown (s) versus time (t) on semi-log graph paper with t on the logarithmic axis, equation 32 plots as a straight line with:

$$m = \frac{2.303Q}{4\pi\sqrt{D'}} \text{ , and} \tag{33}$$

$$t_0 = \frac{S}{2.25} \left[\frac{T_{xx}(y^2) + T_{yy}(x^2) - 2T_{xy}(xy)}{D'} \right] , \tag{34}$$

where: m = the slope of the line defined by equation 32, which is Δs per log cycle, and

t_0 = the intercept of the straight line with the time axis when $s = 0$.

Rearranging equations 33 and 34 yields

$$D' = \{\frac{2.303Q}{4\pi m}\}^2 \text{ , and} \tag{35}$$

$$ST_{xx}(y^2) + ST_{yy}(x^2) - 2ST_{xy}(xy) = 2.25t_oD' \text{ .} \tag{36}$$

The slope of the drawdown versus time data for each observation well should be approximately the same, thereby giving the same value for D' for each well (as previously discussed). By substituting the computed value of D' from equation 35 into equation 36, a linear system of three simultaneous equations can be written in the same form described by equation 13. $\underline{\underline{A}}$ and $\underline{X}$ are defined by equations 14 and 15, respectively, and $\underline{B}$ has the form

$$\underline{B} = \begin{Bmatrix} 2.25(t_o)_1 \, D' \\ 2.25(t_o)_2 \, D' \\ 2.25(t_o)_3 \, D' \end{Bmatrix} \text{ ,} \tag{37}$$

in which $(t_o)_i$ (i = 1,2,3) is the intercept of the straight line with the t-axis at s = 0 for each observation well and D' is defined by equation 35. The system of three simultaneous equations (equation 13) can now be solved by the methods previously described. Components of $\underline{\underline{T}}$, the principal values of $\underline{\underline{T}}$ and the principal direction of anisotropy can also be computed following the procedures described in equations 17 through 27.

The directional transmissivity (T_d) using the straight-line data for each observation well can be computed by substituting for u^* (equation 10) in equation 30 and simplifying such that

$$T_d = r^2 \{\frac{D'}{T_{xx}(y^2) + T_{yy}(x^2) - 2T_{xy}(xy)}\} \text{ .} \tag{38}$$

Rearranging equation 34 yields

$$\frac{S}{2.25t_o} = \{\frac{D'}{T_{xx}(y^2) + T_{yy}(x^2) - 2T_{xy}(xy)}\} \text{ ,} \tag{39}$$

and substituting equation 39 into equation 38 results in

$$T_d = \frac{Sr^2}{2.25t_o} \quad . \tag{40}$$

As previously discussed, a plot of $\sqrt{T_d}$ in the direction of each observation well on polar-coordinate paper should coincide with the transmissivity ellipse computed using equations 28 or 29 and will be proportional to plot of $\sqrt{T_d/S}$ by a factor of $1/\sqrt{S}$.

Least-Squares Optimization

The assumption of aquifer homogeneity is not always valid in field situations. Where significant heterogeneity occurs, the use of three observation wells in different directions to define the principal transmissivities will not always yield a physically plausible solution ($(ST_{xx})(ST_{yy})-(ST_{xy})^2$ in equation 18 can be negative). For example, one of the wells could be drilled into a local fracture which is not representative of the aquifer penetrated by other wells. Therefore, more than three observation wells may be needed to obtain additional information on the directional characteristics of groundwater flow at the test site. When more than three observation wells are used, the same type-curve and straight-line procedures described previously can be used. However, equation 13 will have the form

$$\begin{bmatrix} y_1^2 & x_1^2 & -2x_1y_1 \\ y_2^2 & x_2^2 & -2x_2y_2 \\ y_3^2 & x_3^2 & -2x_3y_3 \\ \vdots & \vdots & \vdots \\ y_N^2 & x_N^2 & -2x_Ny_N \end{bmatrix} \cdot \begin{Bmatrix} ST_{xx} \\ ST_{yy} \\ ST_{xy} \end{Bmatrix} = \begin{Bmatrix} 4t^*_1u^*_1D' \\ 4t^*_2u^*_2D' \\ 4t^*_3u^*_3D' \\ \vdots \\ 4t^*_Nu^*_ND' \end{Bmatrix} \tag{41}$$

for the type-curve method, and

$$\begin{bmatrix} y_1^2 & x_1^2 & -2x_1y_1 \\ y_2^2 & x_2^2 & -2x_2y_2 \\ y_3^2 & x_3^2 & -2x_3y_3 \\ \vdots & \vdots & \vdots \\ y_N^2 & x_N^2 & -2x_Ny_N \end{bmatrix} \cdot \begin{Bmatrix} ST_{xx} \\ ST_{yy} \\ ST_{xy} \end{Bmatrix} = \begin{Bmatrix} 2.25to_1D' \\ 2.25to_2D' \\ 2.25to_3D' \\ \vdots \\ 2.25to_ND' \end{Bmatrix} \tag{42}$$

for the straight-line method.

Equations 41 and 42 represent a linear system of N simultaneous algebraic equations (N is the total number of observation wells) with three unknowns (ST_{xx}, ST_{yy}, and ST_{xy}). Because the system is over-determined (there are more equations than unknowns), the use of a least-squares optimization procedure is required to solve the system of equations 41 or 42 which are represented by the system of equations 13. Two least-squares procedures may be used to solve the system of equations represented by equation 13--the ordinary least-squares (OLS) method and weighted least-squares (WLS) method.

The solution to equation 13 using the OLS method is computed according to Stewart (1973, p. 221):

$$\underline{X} = (\underline{\underline{A}}^T \underline{\underline{A}})^{-1} \underline{\underline{A}}^T \underline{B} \quad . \tag{43}$$

As long as the deviation of $\sqrt{T_d}$ or $\sqrt{T_d/S}$ from the ellipse computed by the OLS method is only slight, this method works well. (See, for example, Randolph and others, 1985, fig. 7.)

If the test site is characterized by extreme heterogeneity such that the data being analyzed exhibit large deviations, a physically plausible solution may still fail to exist ($(ST_{xx})(ST_{yy}) - (ST_{xy})^2$ in equation 18 is negative). Additionally, if a certain area (or quadrant) is lacking observation-well data (observation wells are clustered about a certain area or quadrant), equation 43 may yield an ellipse that is unrealistically elongated in the direction of the missing data. Another problem that arises in using the OLS method is that elements of $\underline{B}$ in equation 43 are inversely proportioned to directional transmissivity (compare equation 30 and 41). Therefore, the OLS method is more sensitive to smaller values of directional transmissivity. If the data set being considered has significant variations in the values of T_d, the ellipse computed from equation 43 will be biased toward the smaller T_d values. Hsieh and others (1985, p. 1670) have also noted and discussed these difficulties arising from the use of the OLS method in analyzing well data in three dimensions for computing components of the hydraulic conductivity tensor.

To address the problems associated with using the OLS method, an alternative solution methodology, the weighted least-squares method (WLS) can be used. Using the WLS method the solution to equation 13 is computed according to (Draper and Smith, 1979, p. 108; Beck and Arnold, 1977, p. 248):

$$\underline{X} = (\underline{\underline{A}}^T \underline{\underline{\omega}} \underline{\underline{A}})^{-1} \underline{\underline{A}}^T \underline{\underline{\omega}} \underline{B} \quad , \tag{44}$$

where $\underline{\underline{\omega}}$ is an N x N diagonal matrix of selected weights or coefficients. The elements of $\underline{\underline{\omega}}$ are assigned values in such a manner so that large values of T_d are given appropriate weighting in deriving the least-squares transmissivity ellipse and that a physically plausible solution to equation 18 exists ($(ST_{xx})(ST_{yy}) - (ST_{xy})^2$ is positive). It should be obvious that the manner in which the values for elements of $\underline{\underline{\omega}}$ are chosen is subjective. As such, several attempts using different weights may be required to obtain an acceptable solution if the data exhibit a large degree of scatter.

Situations may arise where the scatter of the data is so large that a fit of the field data ($\sqrt{Td}$ or $\sqrt{T_d/S}$) to a computed ellipse is not possible even with the use of the WLS method and a judicious choice of weights. When this occurs, it should be concluded that the aquifer being tested cannot be represented as an anisotropic, homogeneous porous medium on the scale of the aquifer volume being tested. Another indication that the aquifer being tested cannot be considered an anisotropic, homogeneous porous medium will be a lack of fit of s* and t* data to the type curve (or straight line). If the aquifer being tested is sufficiently homogeneous so that the methods described herein can be generally applied (a plot of $\sqrt{T_d}$ or $\sqrt{T_d/S}$ in the direction of the observation wells outlines an ellipse similar to the one derived from equation 43 or 44), then every possible combination of any of the three observation wells in three different directions should yield approximately the same results.

COMPUTER PROGRAM DESCRIPTION

The computer code listing presented in this report (Supplemental Data V) is written in FORTRAN 77 and is intended for use on the PRIME computer system of the U.S. Geological Survey, Water Resources Division. The program, TENSOR2D, is composed of a main program and four subroutine subprograms. A generalized flow chart of TENSOR2D is shown in figure 4. The purpose of the main program and each subroutine are explained below:

MAIN PROGRAM: Dimensions the appropriate arrays and allocates the space in storage vector Y. At the present time enough space is allocated in Y to analyze 25 observation wells. If more space is required, increase the size of Y.

SUBROUTINE TEN3TC: Uses the results of the type-curve method to compute tensor components and aquifer anisotropy for three observation wells. The system of simultaneous equations is solved by LU decomposition using the Crout method.

SUBROUTINE TEN3SL: Uses the results of the straight-line method to compute tensor components and aquifer anisotropy for three observation wells. The system of simultaneous equations is solved by LU decomposition using the Crout method.

SUBROUTINE WLSTC: Uses the results of the type-curve method to compute tensor components and aquifer anisotropy for four or more observation wells. The system of simultaneous equations is solved by a weighted least-squares optimization scheme.

SUBROUTINE WLSL: Uses the results of the straight-line method to compute tensor components and aquifer anisotropy for four or more observation wells. The system of simultaneous equations is solved by a weighted least-squares optimization scheme.

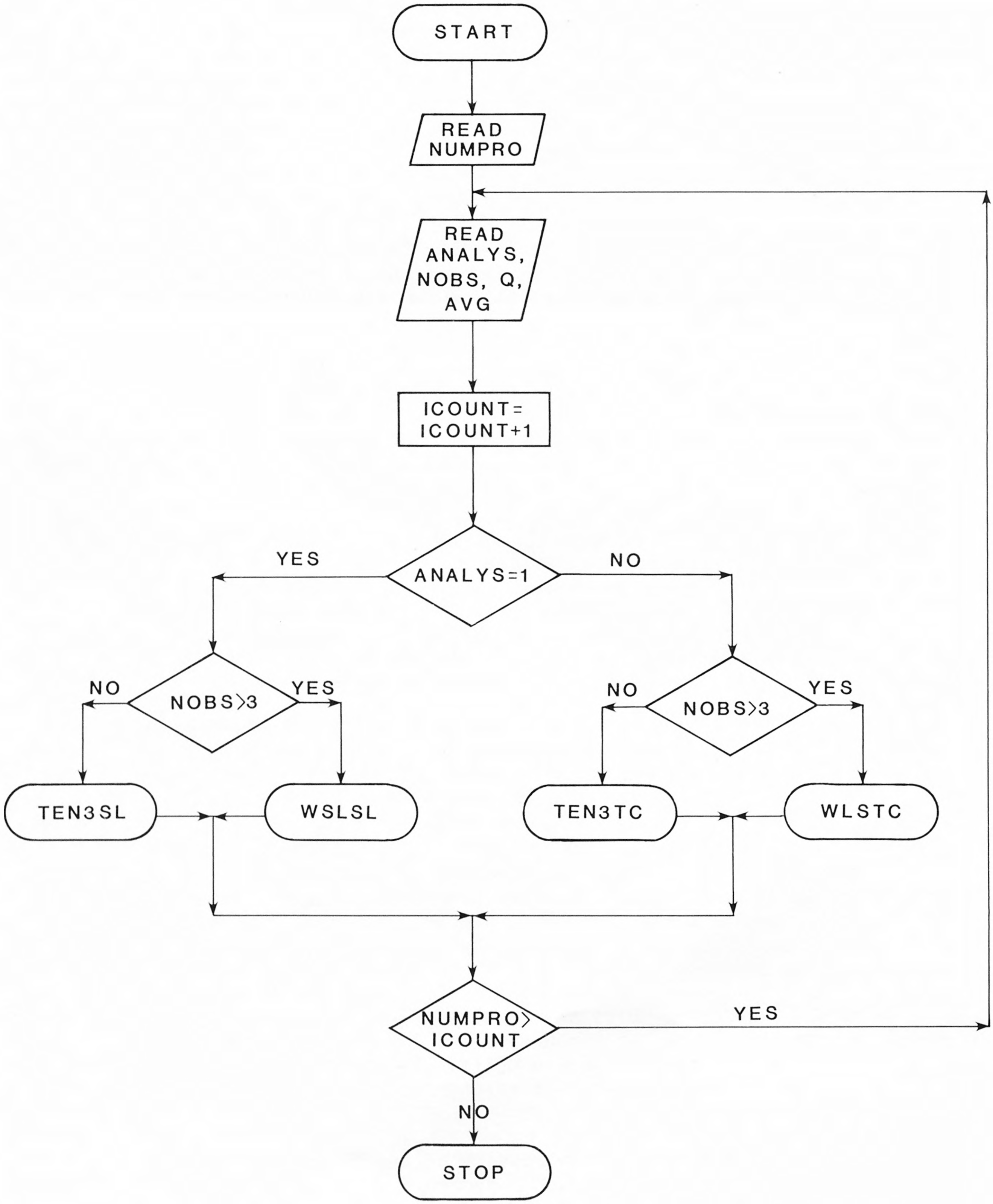


Figure 4.—Generalized flow chart of the computer program.

The definitions of selected variables used in TENSOR2D are listed in Supplemental Data I and formats of required input data are listed in Supplemental Data II. TENSOR2D is written in a modular form to accommodate user modification of input data and output results. Additionally, all input data must be in consistent units.

COMPUTER PROGRAM APPLICATION

Three numerical examples are provided to demonstrate the use of TENSOR2D. Example 1 uses the type-curve method with three observation wells. Examples 2 and 3 use the type-curve method with eight observation wells (weighted least-squares method). In Example 2 the elements of the weight matrix ($\underline{\underline{\omega}}$ in equation 44) are all assigned a value of unity (1.0). This is the same as using the ordinary least-squares method (equation 43). In Example 3, the weights assigned to $\underline{\underline{\omega}}$ are varied in order to demonstrate the effect of weighting on the computed transmissivity ellipse.

The examples use data gathered during hydrogeologic investigations at the site of the Georgia Nuclear Laboratory, located about 4 miles southwest of Dawsonville, Dawson County, Ga., and reported in Stewart (1964) and Stewart and others (1964). Data used in the example problems are listed in tables 1 and 2. Required input data in TENSOR2D format and solutions of the example problems are given in Supplemental Data III and IV, respectively.

Example 1 -- Type-curve method using three observation wells

On March 17-19, 1959, an aquifer test was conducted at the site of the Georgia Nuclear Laboratory to determine the capacity of saprolite, which underlies the test site, to transmit water and to yield water from storage. The estimated saturated thickness of the saprolite at the test site is about 100 feet (Stewart, 1964, p. D51). Discharge from the pumping well (TW-16) was 8.7 gallons per minute for about 30 hours. The location of observation wells AH-75, AH-93, and AH-173 and the arbitrary Cartesian (X-Y) coordinate system used for the tensor analysis are shown in figure 5. All time-drawdown data were matched with the Theis type curve. Coordinate values, radial distances and direction from the pumping well (TW-16), and type-curve match-point values for the three observation wells are listed in table 1. The arbitrary coordinate system was oriented with the Y-axis to the north (fig. 5). As previously discussed, D' (equation 11) should have the same value for each observation well. In this example (and most field situations) D' varies somewhat for each observation well (table 1). Therefore, an arithmetic average of 3.452×10^4 $(ft^2/d)^2$ was used for D' in the tensor analysis. TENSOR2D will calculate an average D' using all the observation wells or the user can specify a D' of his choosing. (See Supplemental Data II and IV.)

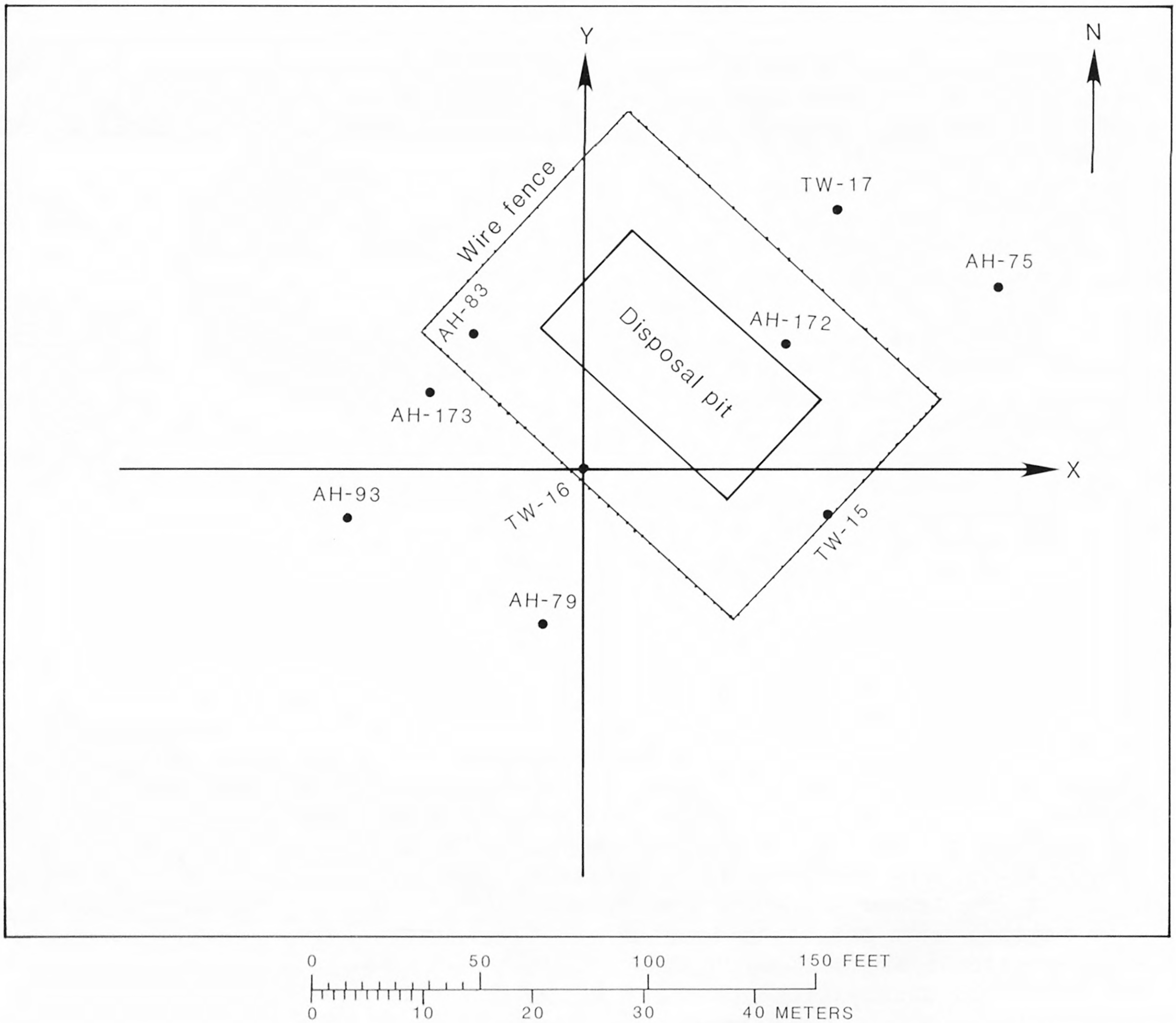


Figure 5.—Location of pumping well (TW-16), observation wells, and arbitary X-Y coordinate system used in the analysis of the March 1959 aquifer test, Georgia Nuclear Laboratory, Dawson County, Georgia. Modified from Stewart (1964).

Table 1.--Cartesian coordinates and curve matching values for observation wells used in example 1

Well identification	x (ft)	y (ft)	r (ft)	Ψ[1] (degrees)	type-curve match points				D'[2] $(ft^2/d)^2$	Td/S[3] (ft^2/d)
					W(u)	u	s (ft)	t (days)		
AH-75[4]	124.24	55.32	136	24°	1.0	1.0	1.23	0.0640	1.174×10^4	7.23×10^4
AH-93	-60.64	-12.89	62	192°	1.0	1.0	.59	.0175	5.103×10^4	5.49×10^4
AH-173	-42.24	20.60	47	154°	1.0	1.0	.66	.0189	4.078×10^4	2.92×10^4

[1]/Direction of observation well; positive is counterclockwise from +x axis.

[2]/See equation 11 for definition of D'.

[3]/See equation 30 for definition of T_d. Computed value of S in Example 1 = 3.71×10^{-3}.

[4]/See figure 5 for well locations.

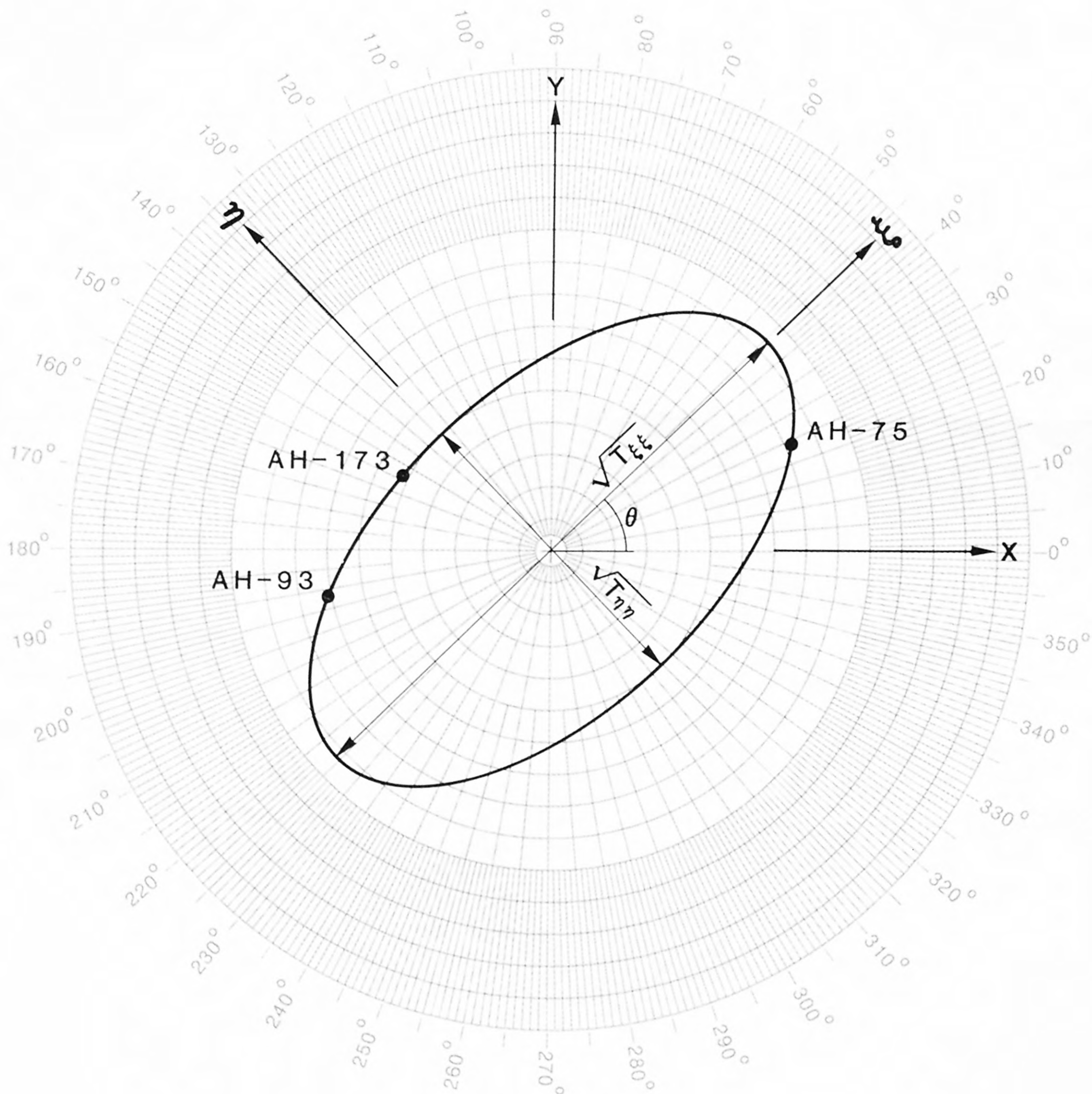
Components of the transmissivity tensor and the storage coefficient computed by TENSOR2D, a plot of the transmissivity ellipse, and the directional transmissivity for each observation well are shown in figure 6. The values computed for the directional diffusivity (T_d/S) are also listed in table 1. The plot of $\sqrt{T_d}$ (equation 30 and table 1) in the direction of the observation well (fig. 6) coincides exactly with the theoretical transmissivity ellipse (computed using equation 28 or 29) since only three observation wells were used. The angle of anisotropy and principal direction of flow computed by TENSOR2D (Θ = 44.1°; N. 45.9° E.) are in good agreement with the alignment of the major axis of the observed cone of depression defined during a June 1958 aquifer test (Stewart and others, 1964, pl. 3). The azimuth of the major axis of this cone is about N. 52° E. and is parallel to the strike of rock foliation in the area of the aquifer test (Stewart and others, 1964, p. F68). The output from Example 1 is provided in Supplemental Data IV.

Example 2 -- Type-curve method and equal weighted least-squares optimization using eight observation wells

In this example, components of the transmissivity tensor and the storage coefficient were computed using the eight observation wells shown on figure 5, and data relative to the same aquifer test described in Example 1. Table 2 lists coordinate values, radial distances and direction of the observation wells from the pumping well (TW-16), type-curve match points, and values of D' (computed using equation 11). As with Example 1, the value of D' varied for each observation well (table 2), so TENSOR2D computed an arithmetic average for use in the tensor analysis. (See output of Example 2 in Supplemental Data IV.) Because there were more than three observation wells the weighted least-squares method was used to solve the over-determined system of equations (subroutine WLSTC of TENSOR2D in fig. 4 and Supplemental Data V). In this example, the weights ($\underline{\underline{\omega}}$ in equation 44) were all assigned a value of 1.0 (Supplemental Data II and III). A justification of these values would be that test data from each observation well are considered to be of equal quality and did not exhibit significant scatter.

Results of the tensor analysis are shown on figure 7. The $\sqrt{T_d}$ (equation 30) for each observation well (T_d/S is listed in table 2) plotted in the direction of the observation well, compares favorably with and outlines the least-squares transmissivity ellipse computed using equation 28 or 29 (fig. 7). Additionally, the ratio of anisotropy (3.5:1) and angle of anistoropy (Θ = 43.4°, N. 46.6° E.) agree well with results from Example 1 and the field observations reported in Stewart and others (1964, pl. 3).

The close agreement between results of Example 1 (three observation wells) and Example 2 (eight observation wells) is one indication that the assumption of aquifer homogeneity is valid for these field data. Another indication that the assumption of a homogeneous porous medium is correct is apparent in the equal weights assigned to the observation well data (ω in equation 44 and WT(I) in Supplemental Data III-B and IV-B). Since all observation wells were equally weighted (assigned a value of 1.0) and the square root of the directional transmissivity ($\sqrt{Td}$) for the wells aligned closely



EXPLANATION

● AH-75 DIRECTIONAL TRANSMISSIVITY ($\sqrt{Td}$) COMPUTED USING EQUATION 30 AND WELL IDENTIFICATION NUMBER

——— THEORETICAL TRANSMISSIVITY ELLIPSE COMPUTED USING EQUATION 28 OR 29

0 4 8 12 16 20 24 $(ft^2/d)^{1/2}$

Tensor analysis results using TENSOR2D		
T_{xx}	=	227.14 ft^2/d
T_{yy}	=	219.31 ft^2/d
T_{xy}	=	123.68 ft^2/d
$T_{\xi\xi}$	=	346.96 ft^2/d
$T_{\eta\eta}$	=	99.49 ft^2/d
S	=	0.00371
θ	=	44.1°
$T_{\xi\xi}/T_{\eta\eta}$	=	3.5

Figure 6.—Comparison of the theoretical transmissivity ellipse and directional transmissivity for example 1, March 1959 aquifer test, Georgia Nuclear Laboratory, Dawson County, Georgia.

Table 2.--Cartesian coordinates and curve matching values for observation wells used in examples 2 and 3

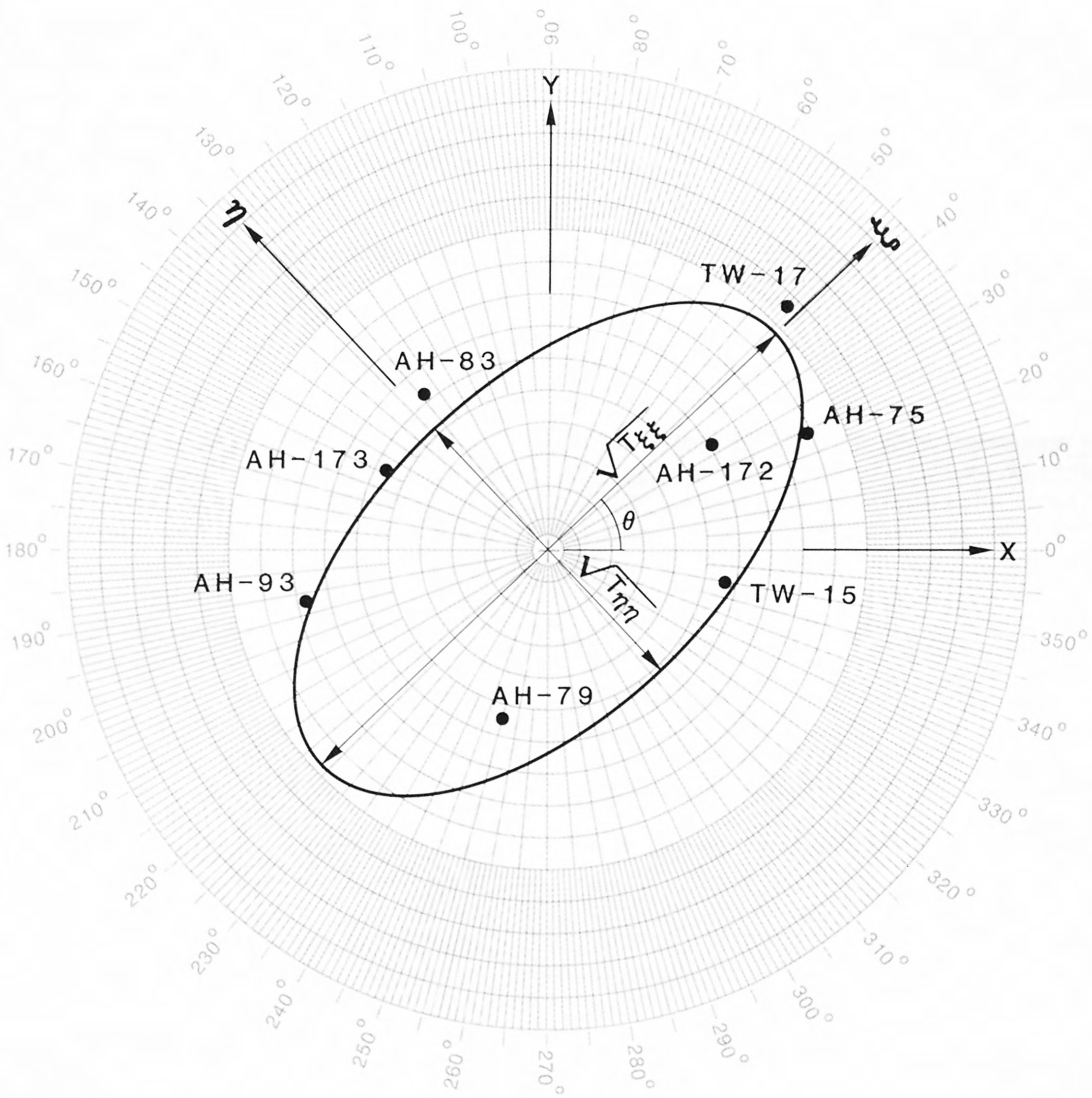
Well identification	x (ft)	y (ft)	r (ft)	Ψ [1] (degrees)	type-curve match points W(u)	u	s (ft)	t (days)	D' [2] $(ft^2/d)^2$	Td/S [3] (ft^2/d)
AH-75 [4]	124.24	55.32	136	24°	1.0	1.0	1.23	0.0640	$1.17x10^4$	$7.22x10^4$
AH-79	-12.68	-47.33	49	255°	1.0	1.0	.80	.0220	$2.75x10^4$	$2.73x10^4$
AH-83	-30.84	38.08	49	129°	1.0	1.0	.51	.0169	$6.86x10^4$	$3.55x10^4$
AH-93	-60.64	-12.89	62	192°	1.0	1.0	59	.0175	$5.10x10^4$	$5.49x10^4$
AH-172	59.88	38.15	71	32.5°	1.0	1.0	.48	.0373	$7.87x10^4$	$3.38x10^4$
AH-173	-42.24	20.60	47	154°	1.0	1.0	.66	.0189	$4.08x10^4$	$2.92x10^4$
TW-15	74.73	-13.85	76	349.5°	1.0	1.0	.66	.0494	$4.08x10^4$	$2.92x10^4$
TW-17	75.70	73.03	108	45.5°	1.0	1.0	1.38	.0284	$9.33x10^3$	$1.03x10^5$

1/Direction of observation well; positive is counterclockwise from +x axis.

2/See equation 11 for definition of D'.

3/See equation 30 for definition of T_d. Computed value of S in Example 2 = $4.38x10^{-3}$. Computed value of S in Example 3 = $6.35x10^{-3}$.

4/See figure 5 for well locations.



EXPLANATION

● AH-75 DIRECTIONAL TRANSMISSIVITY ($\sqrt{Td}$) COMPUTED USING EQUATION 30 AND WELL IDENTIFICATION NUMBER

——— THEORETICAL TRANSMISSIVITY ELLIPSE COMPUTED USING EQUATION 28 OR 29

0 4 8 12 16 20 24 $(ft^2/d)^{1/2}$

Tensor analysis results using TENSOR2D		
T_{xx}	=	251.77 ft^2/d
T_{yy}	=	237.03 ft^2/d
T_{xy}	=	136.46 ft^2/d
$T_{\xi\xi}$	=	381.06 ft^2/d
$T_{\eta\eta}$	=	107.74 ft^2/d
S	=	0.00438
θ	=	43.4°
$T_{\xi\xi}/T_{\eta\eta}$	=	3.5

Figure 7.—Comparison of least-squares transmissivity ellipse and directional transmissivity for example 2, March 1959 aquifer test, Georgia Nuclear Laboratory, Dawson County, Georgia.

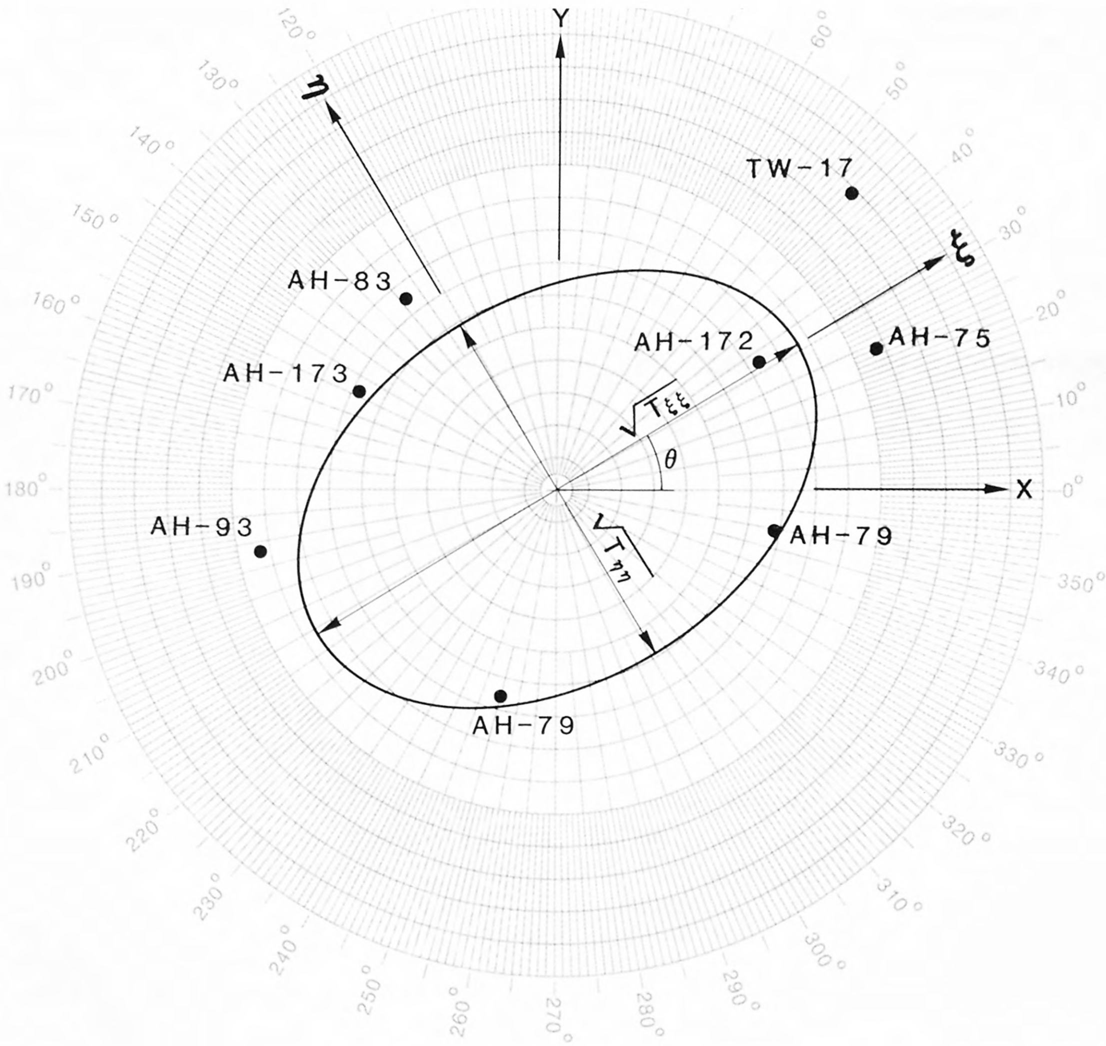
with the computed transmissivity ellipse, the assumption of aquifer homogeneity appears to be valid. If the test data had exhibited significant scatter, indicating possible aquifer heterogeneities, different weighting values may have been needed to be assigned to the observation wells in order to compute the tensor components and anisotropic aquifer parameter values.

Example 3 -- Type-curve method and unequal weighted least-squares optimization using eight observation wells

Example 3 is provided to demonstrate the effect of assigning different values of weight ($\underline{\underline{\omega}}$ in equation 44) to the test data on the computed transmissivity ellipse and components of the transmissivity tensor. All input data are the same as in Example 2 (table 2) with the exception of the weighting values (compare Supplemental Data III-B and III-C). Wells AH-79, AH172, AH-173, and TW-15 (fig. 5) were arbitrarily assigned a weight of 2.0, whereas wells AH-75, AH-83, AH-93, and TW-17 were assigned weights of 0.1, 0.25, 0.75, and 0.1, respectively. This implies that during the solution process of equation 44, wells AH-75 and TW-17 will be given the least amount of weight, whereas wells AH-79, AH-172, AH-173, and TW-15 will be weighted the most. It should be noted again that these weights were assigned arbitrarily to demonstrate the effect of using the weighted least-squares method.

Results of the tensor analysis using the weighting distribution described above are shown on figure 8. A plot of $\sqrt{T_d}$ (equation 30) for each observation well (T_d/S is listed in table 2) in the direction of the well shows that the wells that were weighted the most (AH-79, AH-172, AH-173, and TW-15) align most closely with the computed transmissivity ellipse. Additionally, the ratio of anisotropy has been reduced from 3.5:1 (Example 2) to 2.1:1. Computed values of the tensor components, the angle of anisotropy, and the storage coefficient are also shown in figure 8.

An important point demonstrated by Example 3 is that the weighted least-squares method allows one to use subjective judgement in evaluating the quality of data from the observation wells. Additionally, if some heterogeneities are present at the test site, they can be taken into account by the assignment of different weights ($\underline{\underline{\omega}}$ in equation 44) during the solution procedure.



EXPLANATION

● AH-75 DIRECTIONAL TRANSMISSIVITY ($\sqrt{Td}$) COMPUTED USING EQUATION 30 AND WELL IDENTIFICATION NUMBER

——— THEORETICAL TRANSMISSIVITY ELLIPSE COMPUTED USING EQUATION 28 OR 29

0 4 8 12 16 20 24 $(ft^2/d)^{1/2}$

Tensor analysis results using TENSOR2D		
T_{xx}	=	253.75 ft^2/d
T_{yy}	=	181.11 ft^2/d
T_{xy}	=	70.00 ft^2/d
$T_{\xi\xi}$	=	296.29 ft^2/d
$T_{\eta\eta}$	=	138.56 ft^2/d
S	=	0.00649
θ	=	31.3°
$T_{\xi\xi}/T_{\eta\eta}$	=	2.1

Figure 8.—Comparison of weighted least-squares transmissivity ellipse and directional transmissivity for example 3, March 1959 aquifer test, Georgia Nuclear Laboratory, Dawson County, Georgia.

SUMMARY

The computer program, TENSOR2D, described in this report can be used to compute the anisotropic aquifer hydraulic parameters and components of the transmissivity tensor for two-dimensional ground-water flow. The program is based on the equation of drawdown formulated by Papadopulos (1965) for nonsteady flow in an infinite anisotropic aquifer. Using aquifer-test data for one pumping well and three observation wells, the type-curve and straight-line approximation methods are developed for computing anisotropic aquifer hydraulic properties and components of the transmissivity tensor. Additionally, the method of Papadopulos (1965) as originally developed has been extended to allow for the analysis of more than three observation wells by applying a weighted least-squares optimization procedure to the type-curve and straight-line approximation methods.

Three example applications using the computer program and field data gathered during hydrogeologic investigations at a site near Dawsonville, Ga. (Stewart, 1964; Stewart and others, 1964) are provided to illustrate the use of the computer program, TENSOR2D. The example applications demonstrate the use of the type-curve method using three observation wells, the weighted least-squares optimization method using eight observation wells and equal weighting, and the weighted-least squares optimization method using eight observation wells and unequal weighting. Results obtained using the computer program indicate major transmissivity ($T_{\xi\xi}$) in the range of 347 to 296 feet squared per day, minor transmissivity ($T_{\eta\eta}$) in the range of 139 to 99 feet squared per day, aquifer anisotropy in the range of 3.54 to 2.14, principal direction of flow in the range of N. 45.9° E. to N. 58.7° E., and computed storage coefficients in the range of 6.3 x 10^{-3} to 3.7 x 10^{-3}. The numerical results are in good agreement with the field data gathered on the weathered crystalline rocks underlying the investigation site.

The names of program variables, data input formats, examples of input data and model output, and the Fortran 77 computer code of TENSOR2D are listed in the "Supplemental Data" sections. The program is written in a modular format to allow user modification of input data and output results.

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SUPPLEMENTAL DATA I

Definition of Selected Variables Used in Computer Program

ANALYS - type of analysis performed on the set of observation wells

AVG - user supplied 'average' value for determinant if type-curve analysis or 'average' value for slope of line if straight-line analysis

D - array of the drawdowns from the Theis curve match points for the set of observation wells (L)

DESCR1 - description to be printed at start of computer output (line 1)

DESCR2 - description to be printed at start of computer output (line 2)

DET - determinant of the matrix of a two-dimensional, symmetric transmissivity tensor based on either the type-curve or straight-line analysis of observation well data $(L^2/T)^2$

DETBAR - arithmetic average of the determinants obtained from the observation wells in an aquifer test $(L^2/T)^2$

NOBS - number of observation wells to be used in an analysis (minimum of three)

NUMPRO - number of problem datasets to be analyzed

Q - pumping rate during an aquifer test (L^3/T)

RATAN - computed ratio of anisotropy (Tss/Tnn)

S - composite storage coefficient resulting from the tensor analysis

SL - array of the slopes resulting from the straight-line fit of the observation well data ($\Delta L/\log T$)

SLBAR - arithmetic average of the slopes resulting from the individual observation wells ($\Delta L/\log T$)

T - array of the times from the Theis curve match points for the set of observation wells (T)

Tnn - principal component (minimum) of the transmissivity tensor (L^2/T)

Tss - principal component (maximum) of the transmissivity tensor (L^2/T)

Txx - anisotropic transmissivity tensor component along the x-direction of the arbitrary axes chosen (L^2/T)

Txy - cross product component of the transmissivity tensor with reference to the arbitrary axes chosen (L^2/T)

Tyy - anisotropic transmissivity tensor component along the y-direction of the arbitrary axes chosen (L^2/T)

To - array of straight-line intercepts of the time axis from Cooper-Jacob plots of observation well data (T)

THETA - angle of anisotropy, in degrees, from the positive x-axis

THETAR - angle of anisotropy, in radians, from the positive x-axis

U - array of the variable of the well function from the Theis curve match points for the set of observation wells

WELLID - array of well identifications for the set of observation wells

WT - array of weighting factors assigned to observation well data for use with weighted least-squares method.

WU - array of the well function from the Theis curve match points for the set of observation wells

XW - array of x-coordinates of the observation wells with respect to the arbitrary axes chosen

YW - array of y-coordinates of the observation wells with respect to the arbitrary axes chosen

NOTE: Additional variable descriptions may be found in the program listing (Supplemental Data V).

SUPPLEMENTAL DATA II

Data Input Formats

Card	Columns	Format	Variable	Definition
1	1-5	I5	NUMPRO	Number of problem datsets to be analyzed

Group 1: Description and input data for individual problems

NUMPRO number of datasets

Card	Columns	Format	Variable	Definition
2	1-5	I5	ANALYS	Type of analysis performed on the individual wells 0: Theis non-leaky type curve 1: Cooper-Jacob straight line
	6-10	I5	NOBS	Number of observation wells in a problem (minimum of three)
	11-20	G10.0	Q	Pumping rate during aquifer test
	21-30	G10.0	AVG	User supplied 'average' value for determinant if type-curve analysis or 'average' value for slope of line if straight-line analysis. If 0.0, program will internally calculate an arithmetic average.
3	1-80	A80	DESCR1	Any description the user wishes to print on one line at start of output
4	1-80	A80	DESCR2	Any description the user wishes to print on second line at start of output

---

NOTE 1: Consistent units should be used for input data throughout.

NOTE 2: Input data are read on Fortran Unit 5. Output data are written on Fortran Unit 6.

IF ANALYS = 1 THEN GO TO GROUP 1.1-B

Group 1.1-A: Type-curve analysis results (ANALYS = 0)

NOBS number of cards

Card	Columns	Format	Variable	Definition
--	1-10	A10	WELLID	Well identification
	11-20	G10.0	XW	X-coordinate of observation well relative to the pumping well
	21-30	G10.0	YW	Y-coordinate of observation well relative to the pumping well
	31-40	G10.0	T	Time at Theis curve match point
	41-50	G10.0	D	Drawdown at Theis curve match point
	51-60	G10.0	WU	Well function at Theis curve match point
	61-70	G10.0	U	Variable of the well function at Theis curve match point
	71-80	G10.0	WT	Weight factor for observation well data to be used with weighted least-squares method. For equal weighting set WT = 1.0 for all data. WT should be omitted if analyzing only three observation wells.

NOTE 1: Consistent units should be used for input data throughout.

NOTE 2: Input data are read on FORTRAN Unit 5. Output data are written on Fortran unit 6.

Group 1.1-B: Straight-line analysis results (ANALYS = 1)

NOBS number of cards

Card	Columns	Format	Variable	Definition
--	1-10	A10	WELLID	Well description
	11-20	G10.0	XW	X-coordinate of observation well relative to the pumping well
	21-30	G10.0	YW	Y-coordinate of observation well relative to the pumping well
	31-40	G10.0	To	Straight-line intercept of time axis
	41-50	G10.0	SL	Slope of straight line, [Δ(drawdown)/Δ(log(time))]
	51-60	G10.0	WT	Weight factor for observation well data to be used with weighted least-squares method. For equal weighting, set WT = 1.0 for all data. WT should be omitted if analyzing only three observation wells.

NOTE 1: Consistent units should be used for input data throughout.

NOTE 2: Input data are read on FORTRAN Unit 5. Output data are written on FORTRAN Unit 6.

SUPPLEMENTAL DATA III

Input Data for Application Examples

A. Example Problem 1

```
    1
    0     3 1674.8663        0.0
EXAMPLE PROBLEM#1:  MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
3 OBSERVATION WELLS USING TYPE-CURVE MATCH POINTS.  UNITS = FT,DAYS
  AH-75        124.24      55.32     0.0640       1.23        1.0        1.0
  AH-93        -60.64     -12.89     0.0175       0.59        1.0        1.0
  AH-173       -42.24      20.60     0.0189       0.66        1.0        1.0
```

B. Example Problem 2

```
    1
    0     8 1674.8663        0.0
EXAMPLE PROBLEM#2:  MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
EIGHT OBSERVATION WELLS USING TYPE-CURVE MATCH POINTS.  UNITS = FT,DAYS
  AH-75        124.24      55.32     0.0640      1.230        1.0        1.0        1.0
  AH-79        -12.68     -47.33     0.0220      0.804        1.0        1.0        1.0
  AH-83        -30.84      38.08     0.0169      0.509        1.0        1.0        1.0
  AH-93        -60.64     -12.89     0.0175      0.590        1.0        1.0        1.0
  AH-172        59.88      38.15     0.0373      0.475        1.0        1.0        1.0
  AH-173       -42.24      20.60     0.0189      0.660        1.0        1.0        1.0
  TW-15         74.73     -13.85     0.0494      0.660        1.0        1.0        1.0
  TW-17         75.70      77.03     0.0284      1.380        1.0        1.0        1.0
```

C. Example Problem 3

```
    1
    0     8 1674.8663        0.0
EXAMPLE PROBLEM#3:  MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
USE OF DIFFERENT WEIGHTS FOR LEAST-SQUARES.                      UNITS = FT,DAYS
  AH-75        124.24      55.32     0.0640      1.230        1.0        1.0       0.10
  AH-79        -12.68     -47.33     0.0220      0.804        1.0        1.0        2.0
  AH-83        -30.84      38.08     0.0169      0.509        1.0        1.0       0.25
  AH-93        -60.64     -12.89     0.0175      0.590        1.0        1.0       0.75
  AH-172        59.88      38.15     0.0373      0.475        1.0        1.0        2.0
  AH-173       -42.24      20.60     0.0189      0.660        1.0        1.0        2.0
  TW-15         74.73     -13.85     0.0494      0.660        1.0        1.0        2.0
  TW-17         75.70      77.03     0.0284      1.380        1.0        1.0       0.10
```

SUPPLEMENTAL DATA IV

Output of Application Examples

A. Example Problem 1

```
                    TRANSMISSIVITY TENSOR ANALYSIS

                   USING THEIS TYPE-CURVE MATCH POINTS

                   AS DESCRIBED IN WATER-SUPPLY PAPER xxxx
             PROGRAM BY: MORRIS L. MASLIA AND ROBERT B. RANDOLPH
                 U.S.G.S - WRD, DORAVILLE, GEORGIA 30360
                          REVISED: 04-12-85

==========================================================================

EXAMPLE PROBLEM#1:  MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
3 OBSERVATION WELLS USING TYPE-CURVE MATCH POINTS.  UNITS = FT,DAYS

==========================================================================

                              INPUT DATA
                             ============

                   (ALL DATA ARE IN "CONSISTENT UNITS")


 WELL ID.   X-COORD.    Y-COORD.     TIME    DRAWDOWN   W(Uxy)      Uxy
---------- ---------- ---------- -------- -------- -------- --------

  AH-75       124.24       55.32  6.40E-02 1.23E+00  1.00E+00  1.00E+00
  AH-93       -60.64      -12.89  1.75E-02 5.90E-01  1.00E+00  1.00E+00
  AH-173      -42.24       20.60  1.89E-02 6.60E-01  1.00E+00  1.00E+00

--------------------------------------------------------------------------
                   AVERAGE PUMPING RATE: Q = 1.6749E+03
--------------------------------------------------------------------------

        Txx*Tyy - 2*Txy*Txy = (Q*W(Uxy)/(4*PI*D(I)))**2 = DET(I)

           1.1742E+04   5.1031E+04   4.0781E+04

      DETBAR = (DET(1)+DET(2)+ ... +DET(NOBS))/NOBS =  3.4518E+04
```

A. Example Problem 1--Continued

LINEAR EQUATION SYSTEM TO BE SOLVED

A(N,N)			X(N)	B(N)
3.0603E+03	1.5436E+04	-1.3746E+04	STxx	8.8366E+03
1.6615E+02	3.6772E+03	-1.5633E+03	STyy	2.4162E+03
4.2436E+02	1.7842E+03	1.7403E+03	STxy	2.6095E+03

LU DECOMPOSITION OF A(N,N)

LU(N,N)			IPVT(N)
4.2436E+02	1.7842E+03	1.7403E+03	3
3.9154E-01	2.9786E+03	-2.2447E+03	2
7.2116E+00	8.6233E-01	-2.4360E+04	3

SOLUTION VECTOR: X(I)

STxx= 8.4322E-01 STyy= 8.1418E-01 STxy= 4.5914E-01

```
                OUTPUT RESULTS
                ==============

              STORAGE COEFFICIENT
             --------------------
               S = 3.7124E-03

       COMPONENTS OF TRANSMISSIBILITY TENSOR
       ------------------------------------
Txx = 2.2714E+02   Tyy = 2.1931E+02   Txy = 1.2368E+02

   PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR
   ----------------------------------------------
        Tss = 3.4696E+02   Tnn = 9.9485E+01

              RATIO OF ANISOTROPY
             --------------------
              Tss:Tnn =  3.49:1

              ANGLE OF ANISOTROPY
             --------------------
             THETA =  44.09 DEGREES
```

B. Example Problem 2

```
                    TRANSMISSIVITY TENSOR ANALYSIS

                  WEIGHTED LEAST-SQUARES OPTIMIZATION
                   USING THEIS TYPE-CURVE MATCH POINTS

                AS DESCRIBED IN WATER-SUPPLY PAPER xxxx
        PROGRAM BY: MORRIS L. MASLIA AND ROBERT B. RANDOLPH
            U.S.G.S - WRD, DORAVILLE, GEORGIA 30360
                        REVISED: 11-12-85

================================================================================

EXAMPLE PROBLEM#2:  MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
EIGHT OBSERVATION WELLS USING TYPE-CURVE MATCH POINTS.  UNITS = FT,DAYS

================================================================================

                              INPUT DATA
                             ============

                     (ALL DATA ARE IN "CONSISTENT UNITS")

 WELL ID.   X-COORD.    Y-COORD.     TIME    DRAWDOWN   W(U)     U     WEIGHT
---------- ---------- ---------- -------- -------- ----- ----- --------

  AH-75       124.24      55.32  6.40E-02 1.23E+00  1.00  1.00  1.00E+00
  AH-79       -12.68     -47.33  2.20E-02 8.04E-01  1.00  1.00  1.00E+00
  AH-83       -30.84      38.08  1.69E-02 5.09E-01  1.00  1.00  1.00E+00
  AH-93       -60.64     -12.89  1.75E-02 5.90E-01  1.00  1.00  1.00E+00
  AH-172       59.88      38.15  3.73E-02 4.75E-01  1.00  1.00  1.00E+00
  AH-173      -42.24      20.60  1.89E-02 6.60E-01  1.00  1.00  1.00E+00
  TW-15        74.73     -13.85  4.94E-02 6.60E-01  1.00  1.00  1.00E+00
  TW-17        75.70      77.03  2.84E-02 1.38E+00  1.00  1.00  1.00E+00

--------------------------------------------------------------------------------
                    AVERAGE PUMPING RATE: Q = 1.6749E+03
--------------------------------------------------------------------------------

        Txx*Tyy - 2*Txy*Txy = (Q*W(Uxy)/(4*PI*D(I)))**2 = DET(I)

      1.1742E+04   2.7481E+04   6.8565E+04   5.1031E+04   7.8732E+04
      4.0781E+04   4.0781E+04   9.3279E+03

       DETBAR = (DET(1)+DET(2)+ ... +DET(NOBS))/NOBS =  4.1055E+04
```

B. Example Problem 2--Continued

LINEAR LEAST SQUARES PROBLEM TO BE SOLVED

A(M,N)			X(N)	B(M)
3.0603E+03	1.5436E+04	-1.3746E+04	STxx	1.0510E+04
2.2401E+03	1.6078E+02	-1.2003E+03	STyy	3.6128E+03
1.4501E+03	9.5111E+02	2.3488E+03	STxy	2.7753E+03
1.6615E+02	3.6772E+03	-1.5633E+03		2.8739E+03
1.4554E+03	3.5856E+03	-4.5688E+03		6.1254E+03
4.2436E+02	1.7842E+03	1.7403E+03		3.1038E+03
1.9182E+02	5.5846E+03	2.0700E+03		8.1125E+03
5.9336E+03	5.7305E+03	-1.1662E+04		4.6639E+03

RESIDUAL VECTOR: R = B - A*X

-6.7863E+02	1.6921E+03	-1.2168E+03	-1.9394E+02	3.5276E+03
-2.5819E+02	8.6272E+02	-8.6071E+02		

MATRIX CONDITION NUMBER: CONNUM = 1/TOL = 1.04915E+01

SOLUTION VECTOR: X(I)

STxx= 1.1033E+00 STyy= 1.0386E+00 STxy= 5.9796E-01

B. Example Problem 2--Continued

```
                 OUTPUT RESULTS
                 ==============

               STORAGE COEFFICIENT
              ---------------------
               S = 4.3820E-03

       COMPONENTS OF TRANSMISSIVITY TENSOR
      -------------------------------------
Txx = 2.5177E+02   Tyy = 2.3703E+02   Txy = 1.3646E+02

   PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR
  -----------------------------------------------
        Tss = 3.8106E+02   Tnn = 1.0774E+02

               RATIO OF ANISOTROPY
              ---------------------
               Tss:Tnn =  3.54:1

               ANGLE OF ANISOTROPY
              ---------------------
              THETA =  43.45 DEGREES
```

C. Example Problem 3

TRANSMISSIVITY TENSOR ANALYSIS

WEIGHTED LEAST-SQUARES OPTIMIZATION
USING THEIS TYPE-CURVE MATCH POINTS

AS DESCRIBED IN WATER-SUPPLY PAPER xxxx
PROGRAM BY: MORRIS L. MASLIA AND ROBERT B. RANDOLPH
U.S.G.S - WRD, DORAVILLE, GEORGIA 30360
REVISED: 11-12-85

==

EXAMPLE PROBLEM#3: MARCH 17-19, 1959 AQUIFER TEST - DAWSON COUNTY, GA.
USE OF DIFFERENT WEIGHTS FOR LEAST-SQUARES. UNITS = FT,DAYS

==

INPUT DATA
============

(ALL DATA ARE IN "CONSISTENT UNITS")

WELL ID.	X-COORD.	Y-COORD.	TIME	DRAWDOWN	W(U)	U	WEIGHT
AH-75	124.24	55.32	6.40E-02	1.23E+00	1.00	1.00	1.00E-01
AH-79	-12.68	-47.33	2.20E-02	8.04E-01	1.00	1.00	2.00E+00
AH-83	-30.84	38.08	1.69E-02	5.09E-01	1.00	1.00	2.50E-01
AH-93	-60.64	-12.89	1.75E-02	5.90E-01	1.00	1.00	7.50E-01
AH-172	59.88	38.15	3.73E-02	4.75E-01	1.00	1.00	2.00E+00
AH-173	-42.24	20.60	1.89E-02	6.60E-01	1.00	1.00	2.00E+00
TW-15	74.73	-13.85	4.94E-02	6.60E-01	1.00	1.00	2.00E+00
TW-17	75.70	77.03	2.84E-02	1.38E+00	1.00	1.00	1.00E-01

AVERAGE PUMPING RATE: Q = 1.6749E+03

Txx*Tyy - 2*Txy*Txy = (Q*W(Uxy)/(4*PI*D(I)))**2 = DET(I)

1.1742E+04 2.7481E+04 6.8565E+04 5.1031E+04 7.8732E+04
4.0781E+04 4.0781E+04 9.3279E+03

DETBAR = (DET(1)+DET(2)+ ... +DET(NOBS))/NOBS = 4.1055E+04

C. Example Problem 3--Continued

LINEAR LEAST SQUARES PROBLEM TO BE SOLVED

A(M,N)			X(N)	B(M)
9.6775E+02	4.8812E+03	-4.3468E+03	STxx	3.3236E+03
3.1680E+03	2.2738E+02	-1.6975E+03	STyy	5.1093E+03
7.2504E+02	4.7555E+02	1.1744E+03	STxy	1.3877E+03
1.4389E+02	3.1846E+03	-1.3539E+03		2.4888E+03
2.0583E+03	5.0708E+03	-6.4613E+03		8.6626E+03
6.0014E+02	2.5233E+03	2.4611E+03		4.3894E+03
2.7128E+02	7.8978E+03	2.9275E+03		1.1473E+04
1.8764E+03	1.8121E+03	-3.6880E+03		1.4748E+03

RESIDUAL VECTOR: R = B - A*X

-1.9165E+03	4.9826E+02	-8.4931E+02	-8.0325E+02	2.3873E+03
-5.7298E+02	6.5270E+02	-1.9929E+03		

MATRIX CONDITION NUMBER: CONNUM = 1/TOL = 4.79334E+00

SOLUTION VECTOR: X(I)

STxx= 1.6111E+00   STyy= 1.1499E+00   STxy= 4.4447E-01

```
                 OUTPUT RESULTS
                 ==============


              STORAGE COEFFICIENT
             --------------------
                S = 6.3494E-03


       COMPONENTS OF TRANSMISSIVITY TENSOR
      ------------------------------------
Txx = 2.5375E+02   Tyy = 1.8111E+02   Txy = 7.0002E+01


    PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR
   ----------------------------------------------
        Tss = 2.9629E+02   Tnn = 1.3856E+02


              RATIO OF ANISOTROPY
             --------------------
               Tss:Tnn =  2.14:1


              ANGLE OF ANISOTROPY
             --------------------
             THETA =  31.29 DEGREES
```

SUPPLEMENTAL DATA V

Fortran 77 Computer Code Lisitng

A. Main Program

```
C***********************************************************************
C   PROGRAM NAME:  TENSOR2D                       LAST REVISON:  11-12-85
C***********************************************************************
C
C   THIS PROGRAM USES THE METHOD DEVELOPED BY I. S. PAPADOPULOS TO
C   COMPUTE THE COMPONENTS OF A TWO DIMENSIONAL TRANSMISSIVITY TENSOR
C   AND IS DESCRIBED IN WATER-SUPPLY PAPER xxxx
C-----------------------------------------------------------------------
C   PROGRAM DEVELOPED FOR USE ON THE U. S. GEOLOGICAL SURVEY'S
C   PRIME 750 COMPUTER SYSTEM.  PROGRAM COMPILED IN FORTRAN 77
C   WRITTEN BY MORRIS L. MASLIA AND ROBERT B. RANDOLPH,
C   U.S.G.S - WRD, DORAVILLE, GEORGIA  30360, FTS-242-4858
C-----------------------------------------------------------------------
C   THE PROGRAM CONSISTS OF THE FOLLOWING ROUTINES:
C   TENSOR2D.F77:  MAIN PROGRAM
C   TEN3TC.F77:    SUBROUTINE:  3 OBS. WELLS, TYPE-CURVE ANALYSIS
C   TEN3SL.F77:    SUBROUTINE:  3 OBS. WELLS, STRAIGHT-LINE ANALYSIS
C   WLSTC.F77:     SUBROUTINE:  >3 OBS. WELLS, TYPE-CURVE ANALYSIS
C   WLSSL.F77:     SUBROUTINE:  >3 OBS. WELLS, STRAIGHT-LINE ANALYSIS
C-----------------------------------------------------------------------
C   DEFINITION OF VARIABLES USED IN TENSOR2D
C   NUMPRO:     NUMBER OF PROBLEM DATASETS IN THIS RUN
C   ANALYS:     TYPE OF ANALYSIS PERFORMED ON THE INDIVIDUAL WELLS
C                  0: THEIS NON-LEAKY TYPE CURVE ANALYSIS
C                  1: COOPER-JACOB STRAIGHT LINE ANALYSIS
C   NOBS:       NUMBER OF OBSERVATION WELLS (MINIMUM OF 3)
C   Q:          PUMPING RATE DURING AQUIFER TEST
C   AVG:        USER SUPPLIED 'AVERAGE' VALUE FOR DETERMINANT IF TYPE-
C                  CURVE ANALYSIS OR 'AVERAGE' VALUE FOR SLOPE OF LINE
C                  IF STRAIGHT-LINE ANALYSIS.  IF AVG=0.0 PROGRAM WILL
C                  INTERNALLY CALCULATE  AN ARITHMETIC AVERAGE
C   DESCR1:     80 CHARACTER VARIABLE FOR PROBLEM DESCRIPTION
C   DESCR2:     80 CHARACTER VARIABLE FOR PROBLEM DESCRIPTION
C   WELLID(I):  WELL IDENTIFICATION
C   XW(I):      X-COORDINATE OF WELL
C   YW(I):      Y-COORDINATE OF WELL
C   WT(I):      LEAST-SQUARES WEIGHTING COEFFICIENT
C   ++++++++++ Data From Theis Type-Curve Match ++++++++++
C   T(I):       TIME AT THEIS CURVE MATCH POINT
C   D(I):       DRAWDOWN AT THEIS CURVE MATCH POINT
C   WU(I):      THEIS CURVE MATCH POINT W(U)
C   U(I):       THEIS CURVE MATCH POINT Uxy
C   ++++++++++ Data From Cooper-Jacob Straight-Line Match ++++++++++
C   To(I):      STRAIGHT LINE INTERCEPT OF TIME AXIS
C   SL(I):      SLOPE OF STRAIGHT LINE [ds / dlog(t)]
C-----------------------------------------------------------------------
C   COMPUTED VARIABLES
```

Fortran 77 Computer Code Lisitng--Continued

A. Main Program--Continued

```
C  DET(I):       Txx*Tyy-Txy*Txy = (Q*WU(I)/(4*3.14*D(I)))**2 (THEIS)
C  DETBAR:       (DET(1)+DET(2)+...+DET(NOBS))/NOBS
C  DET:          Txx*Tyy-Txy*Txy = (2.303*Q/4*3.14*SLBAR)**2 (JACOB)
C  SLBAR:        (SL(1)+SL(2)+...+SL(NOBS))/NOBS
C  A(I,1):       YW(I)*YW(I)
C  A(I,2):       XW(I)*XW(I)
C  A(I,3):       -2*XW(I)*YW(I)
C  B(I):         4*T(I)*U(I)*DETBAR (THEIS)
C  B(I):         2.25*To(I)*DET (JACOB)
C  CONNUM:       MATRIX CONDITION NUMBER (RETURNED FROM IMSL ROUTINE)
C  X(I):         SOLUTION TO LINEAR SYSTEM: A(M,N) * X(N) = B(M)
C  DIFF:         X(1)*X(2) - X(3)*X(3)
C  ++++++++++ STORAGE COEFFICIENT ++++++++++
C  S:            SQRT[(X(1)*X(2) - X(3)*X(3))/(DETBAR)] (THEIS)
C  S:            SQRT[(X(1)*X(2) - X(3)*X(3))/(DET)] (JACOB)
C  Txx,Tyy,Txy: COMPONENTS OF THE ANISOTROPIC TRANSMISSIVITY TENSOR
C  Tss,Tnn:      PRINCIPAL COMPONENTS OF THE TRANSMISSIVITY TENSOR
C  RATAN:        RATIO OF ANISOTROPY (Tss:Tnn)
C  THETAR:       ANGLE OF ANISOTROPY IN RADIANS (FROM +X AXIS)
C  THETA:        ANGLE OF ANISOTROPY IN DEGREES (FROM +X AXIS)
C----------------------------------------------------------------------
C        DATA FORMATS (ALL DATA ARE IN 'CONSISTENT UNITS')
C  NUMPRO:  I5
C  ANALYS,NOBS,Q,AVG:  2I5,2G10.0
C  DESCR1:  A80
C  DESCR2:  A80
C  ++++++++++ DATA FROM THEIS TYPE-CURVE MATCH ++++++++++
C  WELLID(1),XW(1),YW(1),T(1),D(1),WU(1),U(1),WT(1): A10,7G10.0
C  WELLID(2),XW(2),YW(2),T(2),D(2),WU(2),U(2),WT(2): A10,7G10.0
C  WELLID(3),XW(3),YW(3),T(3),D(3),WU(3),U(3),WT(3): A10,7G10.0
C     .       .     .      .     .     .     .     . : A10,7G10.0
C     .       .     .      .     .     .     .     . : A10,7G10.0
C  WELLID(M),XW(M),YW(M),T(M),D(M),WU(M),U(M),WT(M): A10,7G10.0
C  ( M = NOBS )
C  ++++++++++ DATA FROM COOPER-JACOB STRAIGHT-LINE MATCH ++++++++++
C  WELLID(1),XW(1),YW(1),To(1),SL(1),WT(1) :A10,5G10.0
C  WELLID(2),XW(2),TW(2),To(2),SL(2),WT(2) :A10,5G10.0
C  WELLID(3),XW(3),YW(3),To(3),SL(3),WT(3) :A10,5G10.0
C     .       .     .     .     .     .     :A10,5G10.0
C     .       .     .     .     .     .     :A10,5G10.0
C  WELLID(M),XW(M),YW(M),To(M),SL(M),WT(M) :A10,5G10.0
C  ( M = NOBS )
C----------------------------------------------------------------------
C**********************************************************************
C*              MAIN PROGRAM:  INITIALIZE CONSTANT PARAMETERS        *
C*                         SET UP STORAGE LOCATIONS                  *
C*                            AND CALL SUBROUTINES                   *
C**********************************************************************
      DIMENSION Y(700), LOC(13)
      INTEGER*2 ANALYS
```

```
      REAL*8 Q,AVG
      CHARACTER*80 DESCR1,DESCR2
      COMMON /PARAM/ M,N,PI,Q,AVG
C------------------- INITIALIZE PARAMETERS -------------------------
      ICOUNT = 0
      READ(5,100) NUMPRO
   10 CONTINUE
      READ(5,110) ANALYS, NOBS, Q, AVG
      READ(5,120) DESCR1
      READ(5,120) DESCR2
      M = NOBS
      N = 3
      MN2 = M * N * 2
      M2 = M * 2
      N2 = N * 2
      PI = 3.141592654
C------------------- INITIALIZE STORAGE LOCATIONS -------------------
      LOC(1)  = 1
      LOC(2)  = LOC(1) + MN2
      LOC(3)  = LOC(2) + M2
      LOC(4)  = LOC(3) + N2
      LOC(5)  = LOC(4) + N2
      LOC(6)  = LOC(5) + M2
      LOC(7)  = LOC(6) + M2
      LOC(8)  = LOC(7) + M2
      LOC(9)  = LOC(8) + M2
      LOC(10) = LOC(9) + M2
          IF(ANALYS .EQ. 1) GO TO 20
      LOC(11) = LOC(10)+ M2
      LOC(12) = LOC(11)+ M2
      LOC(13) = LOC(12)+ M2
   20     ISUM = 1 + MN2 + 2*N2 + 5*M2
          IF(ANALYS .EQ. 0) ISUM = ISUM + 4*M2
      DO 30 I = 1,ISUM
         Y(I) = 0.0
   30 CONTINUE
C------------------- PRINT OUT HEADER INFORMATION -------------------
      ICOUNT = ICOUNT + 1
      WRITE(6,130)
          IF (NOBS.GT.3) WRITE(6,140)
          IF(ANALYS .EQ. 0) WRITE(6,150)
          IF(ANALYS .EQ. 1) WRITE(6,160)
      WRITE(6,170)
      WRITE(6,180)
      WRITE(6,190) DESCR1
      WRITE(6,190) DESCR2
      WRITE(6,180)
          IF(NOBS .LT. 3) THEN
             WRITE(6,200)
             GO TO 1000
```

```
         END IF
C------------------- SUBROUTINES FOR TENSOR ANALYSIS ----------------
      IF(ANALYS .EQ. 1) GO TO 40
C------------------- TYPE - CURVE ANALYSIS -------------------------
      IF (NOBS.GT.3) THEN
        CALL WLSTC( Y(LOC(1)),Y(LOC(2)),Y(LOC(3)),Y(LOC(4)),Y(LOC(5)),
     1             Y(LOC(6)),Y(LOC(7)),Y(LOC(8)),Y(LOC(9)),Y(LOC(10)),
     2             Y(LOC(11)),Y(LOC(12)),Y(LOC(13)) )
      ELSE
        CALL TEN3TC( Y(LOC(1)),Y(LOC(2)),Y(LOC(3)),Y(LOC(4)),Y(LOC(5)),
     1              Y(LOC(6)),Y(LOC(7)),Y(LOC(8)),Y(LOC(9)),Y(LOC(10)),
     2              Y(LOC(11)),Y(LOC(13)) )
      END IF
      GO TO 1000
   40 CONTINUE
C------------------- STRAIGHT - LINE ANALYSIS ----------------------
      IF (NOBS.GT.3) THEN
        CALL WLSSL( Y(LOC(1)),Y(LOC(2)),Y(LOC(3)),Y(LOC(4)),Y(LOC(5)),
     1             Y(LOC(6)),Y(LOC(7)),Y(LOC(8)),Y(LOC(9)),Y(LOC(10)))
      ELSE
        CALL TEN3SL( Y(LOC(1)),Y(LOC(2)),Y(LOC(3)),Y(LOC(4)),Y(LOC(5)),
     1              Y(LOC(6)),Y(LOC(7)),Y(LOC(8)),Y(LOC(10)) )
      END IF
 1000 CONTINUE
C------------------- CHECK FOR ANOTHER DATA SET ---------------------
      IF (NUMPRO .GT. ICOUNT) GO TO 10
C------------------- FORMAT STATEMENTS ----------------------------
  100 FORMAT(I5)
  110 FORMAT(2I5,2G10.0)
  120 FORMAT(A80)
  130 FORMAT(1H1,////,25X,'TRANSMISSIVITY TENSOR ANALYSIS',/)
  140 FORMAT(23X,'WEIGHTED LEAST-SQUARES OPTIMIZATION')
  150 FORMAT(23X,'USING THEIS TYPE-CURVE MATCH POINTS')
  160 FORMAT(20X,'USING COOPER-JACOB STRAIGHT-LINE RESULTS')
  170 FORMAT(/,21X,'AS DESCRIBED IN WATER-SUPPLY PAPER xxxx'/,15X,
     1       'PROGRAM BY: MORRIS L. MASLIA AND ROBERT B. RANDOLPH',/,19X
     2       ,'U.S.G.S - WRD, DORAVILLE, GEORGIA 30360'/,31X,
     3       'REVISED: 11-12-85',/)
  180 FORMAT(/,1X,80(1H=),/)
  190 FORMAT(1X,A80)
  200 FORMAT(//,5X,'***** ERROR: THE MINIMUM NUMBER OF WELLS REQUIRED'
     1      ' FOR THE ANALYSIS IS 3 *****')
C------------------- END MAIN PROGRAM -----------------------------
      STOP
      END
```

Fortran 77 Computer Code Lisitng--Continued

B. Subroutine TEN3TC

```
C***********************************************************************
C*  SUBROUTINE:  TEN3TC                     LAST REVISION:  11-12-85   *
C*              TENSOR ANALYSIS USING 3 OBSERVATION WELLS              *
C*                      THEIS TYPE - CURVE METHOD                      *
C***********************************************************************
      SUBROUTINE TEN3TC(A,B,X,H,XW,YW,T,D,WU,U,DET,WELLID)
C-----------------------------------------------------------------------
      COMMON /PARAM/ M,N,PI,Q,AVG
      PARAMETER (IA=3, IDGT=3)
      INTEGER IPVT(3),IER
      REAL*8 A(N,N),LU(3,3),EQUIL(3),B(N),X(N),XW(N),YW(N),T(N),D(N),WU
     1(N),U(N),DET(N),D1,D2,DETBAR,S,TXX,TYY,TXY,TSS,TNN,THETA,THETAD,
     2RATAN,Q,AVG,DIFF
      CHARACTER WELLID(3)*10, TII(3)*4
      DATA (TII(J),J=1,3) /'STxx','STyy','STxy'/
C-------------------- READ OBSERVATION WELL DATA ----------------------
      DO 10 I=1,M
          READ(5,110) WELLID(I), XW(I), YW(I), T(I), D(I), WU(I), U(I)
   10 CONTINUE
C-------------------- PRINT OBSERVATION WELL DATA ---------------------
      WRITE(6,140)
      WRITE(6,150)
      DO 20 I = 1,N
          WRITE(6,160) WELLID(I),XW(I),YW(I),T(I),D(I),WU(I),U(I)
   20 CONTINUE
      WRITE(6,170) Q
C-------------------- COMPUTE AVERAGE VALUE FOR DETERMINANT -----------
C-------------------- OR USE A USER SUPPLIED AVERAGE VALUE ------------
      DETBAR = 0.00
      DO 30 I = 1,N
          DET(I) = (Q * WU(I) / (4.0 * PI * D(I))) ** 2
   30 CONTINUE
      DETBAR = (DET(1) + DET(2) + DET(3)) / FLOAT(N)
      IF(AVG .GT. 0.00) DETBAR = AVG
C-------------------- FORM LINEAR SYSTEM:  [A](X) = (B) ---------------
      DO 40 I = 1,N
          A(I,1) = YW(I) * YW(I)
          A(I,2) = XW(I) * XW(I)
          A(I,3) = -2.0 * XW(I) * YW(I)
          B(I)   = 4.0 * T(I) * U(I) * DETBAR
 40   CONTINUE
C-------------------- PRINT OUT DETERMINANT AND COMPONENTS OF ---------
C-------------------- [A], (X), AND (B) -------------------------------
      WRITE(6,230)
      WRITE(6,240) (DET(I), I=1,N)
      IF (AVG .GT. 0.00) THEN
         WRITE(6,220) DETBAR
```

```
      ELSE
         WRITE(6,250) DETBAR
      END IF
      WRITE(6,260)
      DO 50 I = 1,N
          WRITE(6,270) (A(I,J), J=1,N), TII(I), B(I)
   50 CONTINUE
      WRITE(6,280)
C------------------- LU DECOMPOSITION OF [A] BY THE CROUT METHOD -----
C------------------- USE IMSL LIBRARY SUBROUTINE 'LUDATF' -----------
C------------------- PRINT DECOMPOSITION AND PIVOT VECTOR -----------
      CALL LUDATF(A,LU,N,IA,IDGT,D1,D2,IPVT,EQUIL,WA,IER)
          WRITE(6,290)
          DO 60 I =1,N
              WRITE(6,300) (LU(I,J), J=1,N), IPVT(I)
   60     CONTINUE
          IF(IER .EQ. 34) WRITE(6,310)
          IF(IER .EQ. 129) WRITE(6,320)
          IF(IER .EQ. 129)      RETURN
C------------------- ELIMINATION AND SOLUTION FOR (X) ---------------
C------------------- USE IMSL LIBRARY SUBROUTINE 'LUELMF' -----------
C------------------- PRINT SOLUTION VECTOR (X) ----------------------
      CALL LUELMF(LU,B,IPVT,N,IA,X)
          WRITE(6,330) X(1), X(2), X(3)
C------------------- SOLVE FOR STORAGE COEFFICIENT ------------------
      DIFF = X(1)*X(2) - X(3)*X(3)
      IF(DIFF .LT. 0.00) THEN
          WRITE(6,335)
          RETURN
      END IF
C
      S = DSQRT(DIFF / DETBAR)
C
      IF(S .LT. E-10) THEN
          WRITE(6,336)
          RETURN
      END IF
C
      WRITE(6,340)
      WRITE(6,350) S
C------------------- SOLVE FOR COMPONENTS OF TRANSMISSIVITY ---------
      TXX = X(1) / S
      TYY = X(2) / S
      TXY = X(3) / S
      WRITE(6,360) TXX,TYY,TXY
C------------------- SOLVE FOR PRINCIPAL COMPONENTS AND -------------
C------------------- ANGLE OF ANISOTROPY ----------------------------
      THETA = 0.D0
      TSS = 0.5 * (TXX + TYY + SQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
      TNN = 0.5 * (TXX + TYY - SQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
```

```
      RATAN = TSS / TNN                                                 TNTC 980
      IF(DABS(TXX - TYY) .LT. 1.E-5 .OR. DABS(TXX-TSS) .LT. 1.E-5)      TNTC 990
     1GO TO 70                                                          TNTC1000
      THETAR = ATAN2((TSS-TXX),TXY)                                     TNTC1010
      THETA = THETAR * 180.00 / PI                                      TNTC1020
      IF(THETA .LT. 0.00) THETA = THETA + 360.00                        TNTC1030
   70 CONTINUE                                                          TNTC1040
      WRITE(6,370) TSS,TNN                                              TNTC1050
      WRITE(6,375) RATAN                                                TNTC1060
      WRITE(6,380) THETA                                                TNTC1070
C-------------------- FORMAT STATEMENTS ------------------------------ TNTC1080
  110 FORMAT(A10,6G10.0)                                                TNTC1090
  140 FORMAT(/,35X,'INPUT DATA',/,34X,12(1H=),//,                       TNTC1100
     1       22X,'(ALL DATA ARE IN "CONSISTENT UNITS")',//)             TNTC1110
  150 FORMAT(4X,'WELL ID.',3X,'X-COORD.',4X,'Y-COORD.',5X,'TIME',4X,    TNTC1120
     1       'DRAWDOWN',3X,'W(Uxy)',6X,'Uxy',/,3X,10(1H-),1X,10(1H-),   TNTC1130
     2       2X,10(1H-),2X,8(1H-),2X,8(1H-),2X,8(1H-),2X,8(1H-),/)      TNTC1140
  160 FORMAT(3X,A10,1X,2(F10.2,2X),1PE8.2,2X,E8.2,1X,2(E9.2,1X))        TNTC1150
  170 FORMAT(//,1X,80(1H-),/,22X,'AVERAGE PUMPING RATE: Q = ',1PE10.4,/,TNTC1160
     1       1X,80(1H-),/)                                              TNTC1170
  180 FORMAT(1H1)                                                       TNTC1180
  220 FORMAT(/,16X,'THE DETERMINANT INPUT BY THE USER IS: ',1PE11.4,//) TNTC1190
  230 FORMAT(11X,'Txx*Tyy - 2*Txy*Txy = (Q*W(Uxy)/(4*PI*D(I)))**2 = DET(TNTC1200
     1I)',/)                                                            TNTC1210
  240 FORMAT((13X,1PE11.4,4(2X,E11.4)))                                 TNTC1220
  250 FORMAT(/,9X,'DETBAR = (DET(1)+DET(2)+ ... +DET(NOBS))/NOBS = ',   TNTC1230
     1       1PE11.4,//)                                                TNTC1240
  260 FORMAT(22X,'LINEAR EQUATION SYSTEM TO BE SOLVED',/,21X,           TNTC1250
     1       37(1H-),//,26X,'A(N,N)',19X,'X(N)',8X,'B(N)',/)            TNTC1260
  270 FORMAT(10X,1PE11.4,2(2X,E11.4),4X,A4,5X,E11.4)                    TNTC1270
  280 FORMAT(/,11X,61(1H=),//)                                          TNTC1280
  290 FORMAT(1H1,///,27X,'LU DECOMPOSITION OF A(N,N)',/,26X,28(1H-),//, TNTC1290
     1       32X,'LU(N,N)',17X,'IPVT(N)',//)                            TNTC1300
  300 FORMAT(17X,1PE11.4,2(2X,E11.4),4X,I2)                             TNTC1310
  310 FORMAT(///,10X,'WARNING: IMSL ERROR. IER=34. ACCURACY TEST',      TNTC1320
     1       ' FAILED. COMPUTED',/,19X,'SOLUTION MAY BE IN ERROR BY',   TNTC1330
     2       ' MORE THAN CAN BE ACCOUNT-',19X,'ED FOR BY THE',          TNTC1340
     3       ' UNCERTAINTY OF THE DATA. SEE IMSL',/,19X,'CHAPTER L',    TNTC1350
     4       ' PRELUDE FOR MORE DETAILS.')                              TNTC1360
  320 FORMAT(///,10X,'WARNING: IMSL ERROR. IER=129. MATRIX A IS',       TNTC1370
     1       ' ALGORITHMICALLY SINGULAR.',/,19X,'SEE IMSL CHAPTER L',   TNTC1380
     2       ' PRELUDE FOR MORE DETAILS.')                              TNTC1390
  330 FORMAT(//,29X,'SOLUTION VECTOR:  X(I)',/,28X,24(1H-),/,           TNTC1400
     1       13X,'STxx=',1PE11.4,2X,'STyy=',E11.4,2X,'STxy=',E11.4)     TNTC1410
  335 FORMAT(//,12X,'**** ERROR:  SQUARE ROOT OF NEGATIVE NUMBER ****', TNTC1420
     1        /,12X,'*    CANNOT COMPUTE STOR. COEF. OR TRANSM.     *', TNTC1430
     2        /,12X,'*       WITH GIVEN OBSERVATION WELL DATA       *', TNTC1440
     3        /,12X,'***********************************************') TNTC1450
  336 FORMAT(//,16X,'**** ERROR:  STORAGE COEFFICIENT = 0.00 ****',     TNTC1460
     1       /,16X,'* CANNOT COMPUTE TRANSMISIVITY COMPONENTS  *',      TNTC1470
```

```
     2       /,16X,'*     WITH GIVEN OBSERVATION WELL DATA     *',
     3       /,16X,'*********************************************')
 340 FORMAT(6(/),33X,'OUTPUT RESULTS',/,33X,14(1H=),/)
 350 FORMAT(30X,'STORAGE COEFFICIENT',/,29X,21(1H-),/,32X,'S =',
     1       1PE11.4)
 360 FORMAT(//,22X,'COMPONENTS OF TRANSMISSIVITY TENSOR',/,
     1       21X,37(1H-),/,13X,'Txx =',1PE11.4,3X,'Tyy =',
     2       E11.4,3X,'Txy =',E11.4,//)
 370 FORMAT(17X,'PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR',/,16X,
     1       47(1H-),/,22X,'Tss =',1PE11.4,3X,'Tnn =',E11.4,//)
 375 FORMAT(30X,'RATIO OF ANISOTROPY',/,29X,21(1H-),/,31X,'Tss:Tnn =',
     1       F6.2,':1',/)
 380 FORMAT(30X,'ANGLE OF ANISOTROPY',/,29X,21(1H-),/,29X,
     1       'THETA = ',F6.2,' DEGREES')
C-------------------- END SUBROUTINE TEN3TC --------------------------
      RETURN
      END
```

Fortran 77 Computer Code Lisitng--Continued

C. Subroutine TEN3SL

```
C***********************************************************************
C*   SUBROUTINE:  TEN3SL                     LAST REVISION:  11-12-85   *
C*                 TENSOR ANALYSIS USING 3 OBSERVATION WELLS            *
C*                     COOPER-JACOB STRAIGHT-LINE METHOD                *
C***********************************************************************
      SUBROUTINE TEN3SL(A,B,X,H,XW,YW,To,SL,WELLID)
C-----------------------------------------------------------------------
      COMMON /PARAM/ M,N,PI,Q,AVG
      PARAMETER (IA=3, IDGT=3)
      INTEGER IPVT(3),IER
      REAL*8 AVG,DIFF
      REAL*8 A(N,N),LU(3,3),EQUIL(3),B(N),X(N),XW(N),YW(N),To(N),SL(N),
     1D1,D2,DET,Q,S,TXX,TYY,TXY,TSS,TNN,THETA,THETAD,SLBAR,RATAN
      CHARACTER WELLID(3)*10, TII(3)*4
      DATA (TII(J),J=1,3) /'STxx','STyy','STxy'/
C------------------- READ OBSERVATION WELL DATA ---------------------
      DO 10 I=1,M
          READ(5,110) WELLID(I), XW(I), YW(I), To(I), SL(I)
   10 CONTINUE
C------------------- PRINT OBSERVATION WELL DATA --------------------
      WRITE(6,140)
      WRITE(6,150)
      DO 20 I = 1,N
   20 WRITE(6,160) WELLID(I), XW(I), YW(I), To(I), SL(I)
      WRITE(6,170) Q
C------------------- COMPUTE AVERAGE VALUE FOR SLOPE OF LINE --------
C------------------- OR USE A USER SUPPLIED AVERAGE VALUE -----------
      SLBAR = 0.00
      DO 30 I = 1,M
            SLBAR = SLBAR + SL(I)
   30 CONTINUE
      SLBAR = SLBAR / FLOAT(M)
      IF(DABS(AVG) .GT. 0.00) SLBAR = AVG
C------------------- COMPUTE DETERMINANT AND FORM -------------------
C------------------- LINEAR SYSTEM:  [A](X) = (B) -------------------
      DET = (2.3025851*Q/(4.0*PI*SLBAR))**2
      DO 40 I = 1,N
          A(I,1) = YW(I) * YW(I)
          A(I,2) = XW(I) * XW(I)
          A(I,3) = -2.0 * XW(I) * YW(I)
          B(I)   = 2.25 * To(I) * DET
   40 CONTINUE
C------------------- PRINT AVERAGE SLOPE, DETERMINANT, AND ----------
C------------------- COMPONENTS OF [A], (X), AND (B) ----------------
      IF(DABS(AVG) .GT. 0.00) THEN
          WRITE(6,220) SLBAR
      ELSE
```

```
          WRITE(6,230) SLBAR
      END IF
      WRITE(6,240) DET
      WRITE(6,260)
      DO 50 I = 1,N
          WRITE(6,270) (A(I,J), J=1,3), TII(I), B(I)
   50 CONTINUE
      WRITE(6,280)
C------------------- LU DECOMPOSITION OF [A] BY THE CROUT METHOD -----
C------------------- USE IMSL LIBRARY SUBROUTINE 'LUDATF' -----------
C------------------- PRINT DECOMPOSITION AND PIVOT VECTOR -----------
      CALL LUDATF(A,LU,N,IA,IDGT,D1,D2,IPVT,EQUIL,WA,IER)
          WRITE(6,290)
          DO 60 I =1,N
              WRITE(6,300) (LU(I,J), J=1,N), IPVT(I)
   60     CONTINUE
          IF(IER .EQ. 34) WRITE(6,310)
          IF(IER .EQ. 129) WRITE(6,320)
          IF(IER .EQ. 129)      RETURN
C------------------- ELIMINATION AND SOLUTION FOR (X) ---------------
C------------------- USE IMSL LIBRARY SUBROUTINE 'LUELMF' -----------
C------------------- PRINT SOLUTION VECTOR (X) ----------------------
      CALL LUELMF(LU,B,IPVT,N,IA,X)
          WRITE(6,330) X(1), X(2), X(3)
C------------------- SOLVE FOR STORAGE COEFFICIENT ------------------
      DIFF = X(1)*X(2) - X(3)*X(3)
      IF(DIFF .LT. 0.00) THEN
          WRITE(6,335)
          RETURN
      END IF
C
      S = DSQRT(DIFF / DET)
C
      IF(S .LT. E-10) THEN
          WRITE(6,336)
          RETURN
      END IF
C
      WRITE(6,340)
      WRITE(6,350) S
C------------------- SOLVE FOR COMPONENTS OF TRANSMISSIVITY ---------
      TXX = X(1) / S
      TYY = X(2) / S
      TXY = X(3) / S
      WRITE(6,360) TXX,TYY,TXY
C------------------- SOLVE FOR PRINCIPAL COMPONENTS AND -------------
C------------------- ANGLE OF ANISOTROPY ----------------------------
      THETA = 0.D0
      TSS = 0.5 * (TXX + TYY + DSQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
      TNN = 0.5 * (TXX + TYY - DSQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
```

```
      RATAN = TSS / TNN
      IF(DABS(TXX - TYY) .LT. 1.E-5 .OR. DABS(TXX-TSS) .LT. 1.E-5)
     1GO TO 70
      THETAR = ATAN2((TSS-TXX),TXY)
      THETA = THETAR * 180.00 / PI
      IF(THETA .LT. 0.00) THETA = THETA + 360.00
   70 CONTINUE
      WRITE(6,370) TSS,TNN
      WRITE(6,375) RATAN
      WRITE(6,380) THETA
C-------------------- FORMAT STATEMENTS -----------------------------
  110 FORMAT(A10,4G10.0)
  140 FORMAT(/,35X,'INPUT DATA',/,34X,12(1H=),//,
     1       22X,'(ALL DATA ARE IN "CONSISTENT UNITS")',//)
  150 FORMAT(8X,'WELL ID.',7X,'X-COORD.',8X,'Y-COORD.',10X,'To',10X,
     1       ' SLOPE  ',/,7X,10(1H-),5X,10(1H-),
     2       6X,10(1H-),6X,8(1H-),6X,8(1H-),/)
  160 FORMAT(7X,A10,5X,2(F10.2,6X),1PE8.2,6X,E8.2)
  170 FORMAT(//,1X,80(1H-),/,22X,'AVERAGE PUMPING RATE: Q = ',1PE10.4,/
     1       ,1X,80(1H-))
  180 FORMAT(1H1)
  220 FORMAT(/,11X,'THE AVERAGE SLOPE (SLBAR) INPUT BY THE USER IS: ',
     1       1PE11.4,//)
  230 FORMAT(/,11X,'SLBAR = [SL(1)+SL(2)+ ... +SL(NOBS)]/NOBS = ',
     1       1PE11.4,//)
  240 FORMAT(11X,'Txx*Tyy - 2*Txy*Txy = [2.30 * Q / (4*PI*SLBAR)]**2 = D
     1ET',//,31X,'DET = ',1PE11.4,/)
  260 FORMAT(22X,'LINEAR EQUATION SYSTEM TO BE SOLVED',/,21X,
     1      37(1H-),//,26X,'A(N,N)',19X,'X(N)',8X,'B(N)',/)
  270 FORMAT(10X,1PE11.4,2(2X,E11.4),4X,A4,5X,E11.4)
  280 FORMAT(/,11X,61(1H=),//)
  290 FORMAT(1H1,////,27X,'LU DECOMPOSITION OF A(N,N)',/,26X,28(1H-),//,
     1       32X,'LU(N,N)',17X,'IPVT(N)',//)
  300 FORMAT(17X,1PE11.4,2(2X,E11.4),4X,I2)
  310 FORMAT(///,10X,'WARNING: IMSL ERROR. IER=34. ACCURACY TEST',
     1       ' FAILED. COMPUTED',/,19X,'SOLUTION MAY BE IN ERROR BY',
     2       ' MORE THAN CAN BE ACCOUNT-',19X,'ED FOR BY THE',
     3       ' UNCERTAINTY OF THE DATA. SEE IMSL',/,19X,'CHAPTER L',
     4       ' PRELUDE FOR MORE DETAILS.')
  320 FORMAT(///,10X,'WARNING: IMSL ERROR. IER=129. MATRIX A IS',
     1       ' ALGORITHMICALLY SINGULAR.',/,19X,'SEE IMSL CHAPTER L',
     2       ' PRELUDE FOR MORE DETAILS.')
  330 FORMAT(//,29X,'SOLUTION VECTOR:  X(I)',/,28X,24(1H-),/,
     1       13X,'STxx=',1PE11.4,2X,'STyy=',E11.4,2X,
     2       'STxy=',E11.4)
  335 FORMAT(//,12X,'**** ERROR:  SQUARE ROOT OF NEGATIVE NUMBER ****',
     1        /,12X,'*    CANNOT COMPUTE STOR. COEFF. OR TRANSM.    *',
     2        /,12X,'*       WITH GIVEN OBSERVATION WELL DATA       *',
     3        /,12X,'************************************************')
  336 FORMAT(//16X,'**** ERROR:  STORAGE COEFFICIENT = 0.00 ****',
```

C. Subroutine TEN3SL--Continued

```
     1        /,16X,'* CANNOT COMPUTE TRANSMISSIVITY COMPONENTS *',
     2        /,16X,'*     WITH GIVEN OBSERVATION WELL DATA     *',
     3        /,16X,'********************************************')
   340 FORMAT(6(/),33X,'OUTPUT RESULTS',/,33X,14(1H=),/)
   350 FORMAT(30X,'STORAGE COEFFICIENT',/,29X,21(1H-),/,32X,'S =',
     1        1PE11.4)
   360 FORMAT(//,22X,'COMPONENTS OF TRANSMISSIVITY TENSOR',/,
     1        21X,37(1H-),/,13X,'Txx =',1PE11.4,3X,'Tyy =',
     2        E11.4,3X,'Txy =',E11.4,//)
   370 FORMAT(17X,'PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR',/,16X,
     1        47(1H-),/,22X,'Tss =',1PE11.4,3X,'Tnn =',E11.4,//)
   375 FORMAT(30X,'RATIO OF ANISOTROPY',/,29X,21(1H-),/,31X,'Tss:Tnn =',
     1        F6.2,':1',/)
   380 FORMAT(30X,'ANGLE OF ANISOTROPY',/,29X,21(1H-),/,29X,
     1        'THETA = ',F6.2,' DEGREES')
C-------------------- END SUBROUTINE TEN3SL --------------------------
       RETURN
       END
```

Fortran 77 Computer Code Lisitng--Continued

D. Subroutine WLSTC

```
C***********************************************************************
C*   SUBROUTINE:  WLSTC                      LAST REVISION:  11-12-85   *
C*        TENSOR ANALYSIS USING MORE THAN 3 OBSERVATION WELLS           *
C*               WEIGHTED LEAST - SQUARES OPTIMIZATION                  *
C*                      THEIS TYPE - CURVE METHOD                       *
C***********************************************************************
      SUBROUTINE WLSTC(A,B,X,H,XW,YW,T,D,WU,U,DET,WT,WELLID)
C-----------------------------------------------------------------------
      COMMON /PARAM/ M,N,PI,Q,AVG
      DIMENSION IP(3)
      REAL*8 A(M,N),B(N),X(N),H(N),XW(M),YW(M),T(M),D(M),WU(M),U(M),DET(
     1M),WT(M),DETBAR,S,TXX,TYY,TXY,TSS,TNN,RATAN,THETA,THETAR,Q,TOL,CON
     2NUM,AVG,DIFF
      CHARACTER WELLID(M)*10, TII(3)*4
      DATA (TII(J),J=1,3)/'STxx','STyy','STxy'/
C-------------------- LEAST-SQUARES PARAMETERS FOR 'LLSQF' ------------
      M1 = M
      N1 = N
      IA = M
      KBASIS = N
      TOL = 0.D0
C-------------------- READ OBSERVATION WELL DATA ----------------------
      DO 10 I = 1,M
          READ(5,110) WELLID(I),XW(I),YW(I),T(I),D(I),WU(I),U(I),WT(I)
   10 CONTINUE
C-------------------- PRINT OBSERVATION WELL DATA ---------------------
      WRITE(6,140)
      WRITE(6,150)
      DO 20 I = 1,M
          WRITE(6,160) WELLID(I),XW(I),YW(I),T(I),D(I),WU(I),U(I),WT(I)
   20 CONTINUE
      WRITE(6,170) Q
      IF (M.GT.4) WRITE(6,180)
C-------------------- COMPUTE AVERAGE VALUE FOR DETERMINANT -----------
C-------------------- OR USE A USER SUPPLIED AVERAGE VALUE ------------
      DETBAR = 0.D0
      DO 30 I = 1,M
          WT(I) = DSQRT (WT(I))
          DET(I) = (Q * WU(I) / (4.0 * PI * D(I))) ** 2
          DETBAR = DETBAR + DET(I)
   30 CONTINUE
      DETBAR = DETBAR / FLOAT(M)
      IF (AVG .GT. 0.0) DETBAR = AVG
C-------------------- FOR LINEAR SYSTEM:  [A](X) = (B) ----------------
      DO 40 I = 1,M
          A(I,1) = YW(I) * YW(I) * WT(I)
          A(I,2) = XW(I) * XW(I) * WT(I)
```

```
         A(I,3) = -2.0 * XW(I) * YW(I) * WT(I)
         B(I)   = 4.0 * T(I) * U(I) * DETBAR * WT(I)
   40 CONTINUE
C------------------- PRINT DETERMINANT AND COMPONENTS OF ------------
C------------------- [A], (X), AND (B) ------------------------------
      WRITE(6,230)
      WRITE(6,240) (DET(I), I=1,M)
      IF (AVG .GT. 0.00) THEN
         WRITE(6,220) DETBAR
      ELSE
         WRITE(6,250) DETBAR
      END IF
      WRITE(6,260)
      DO 50 I = 1,M
         IF(I .LE. 3) WRITE(6,270) (A(I,J),J=1,N),TII(I),B(I)
         IF(I. GT. 3) WRITE(6,275) (A(I,J),J=1,N),B(I)
   50 CONTINUE
      WRITE(6,280)
C------------------- SOLUTION OF LINEAR LEAST-SQUARES PROBLEM --------
C------------------- A[M x N] * X(N)  =  B(M) -----------------------
C------------------- USE IMSL LIBRARY SUBROUTINE 'LLSQF' ------------
      CALL LLSQF(A,IA,M1,N1,B,TOL,KBASIS,X,H,IP,IER)
      IF(IER .GT. 0)                        RETURN
      CONNUM = 1.0 / TOL
C------------------- PRINT MATRIX CONDITION NUMBER (CONNUM) ----------
C------------------- RESIDUAL VECTOR (B), AND SOLUTION VECTOR (X) ----
      WRITE(6,310)
      WRITE(6,320) (B(I), I=1,M)
      WRITE(6,325) CONNUM
      WRITE(6,330) X(1), X(2), X(3)
C------------------- SOLVE FOR STORAGE COEFFICIENT ------------------
      DIFF = X(1)*X(2) - X(3)*X(3)
      IF(DIFF .LT. 0.00) THEN
            WRITE(6,335)
            RETURN
      END IF
C
      S = DSQRT(DIFF / DETBAR)
C
      IF(S .LT. 1.E-10) THEN
          WRITE(6,336)
          RETURN
      END IF
C
      WRITE(6,340)
      WRITE(6,350) S
C------------------- SOLVE FOR COMPONENTS OF TRANSMISSIVITY ----------
      TXX = X(1) / S
      TYY = X(2) / S
      TXY = X(3) / S
```

```
      WRITE(6,360) TXX,TYY,TXY
C-------------------- SOLVE FOR PRINCIPAL COMPONENTS AND --------------
C-------------------- ANGLE OF ANISOTROPY ----------------------------
      THETA = 0.D0
      TSS = 0.5 * (TXX + TYY + SQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
      TNN = 0.5 * (TXX + TYY - SQRT((TXX-TYY)**2 + 4.0*TXY*TXY))
      RATAN = TSS / TNN
      IF(DABS(TXX - TYY) .LT. 1.E-5 .OR. DABS(TXX-TSS) .LT. 1.E-5)
     1GO TO 60
      THETAR = ATAN2((TSS-TXX),TXY)
      THETA = THETAR * 180.00 / PI
      IF(THETA .LT. 0.00) THETA = THETA + 360.00
   60 CONTINUE
      WRITE(6,370) TSS,TNN
      WRITE(6,375) RATAN
      WRITE(6,380) THETA
C-------------------- FORMAT STATEMENTS ------------------------------
  110 FORMAT(A10,7G10.0)
  140 FORMAT(/,35X,'INPUT DATA',/,34X,12(1H=),//,
     1       22X,'(ALL DATA ARE IN "CONSISTENT UNITS")',//)
  150 FORMAT(2X,'WELL ID.',3X,'X-COORD.',4X,'Y-COORD.',5X,'TIME',4X,
     1       'DRAWDOWN',3X,'W(U)',4X,'U',5X,'WEIGHT',/,1X,10(1H-),1X,
     2       10(1H-),2X,10(1H-),2X,8(1H-),2X,8(1H-),2X,5(1H-),2X,5(1H-),
     3       2X,8(1H-),/)
  160 FORMAT(1X,A10,1X,2(F10.2,2X),1PE8.2,2X,E8.2,2X,0PF5.2,2X,F5.2,
     1       2X,1PE8.2)
  170 FORMAT(//,1X,80(1H-),/,22X,'AVERAGE PUMPING RATE: Q = ',1PE10.4,/
     1       ,1X,80(1H-),/)
  180 FORMAT(1H1)
  220 FORMAT(/,16X,'THE DETERMINANT INPUT BY THE USER IS: ',1PE11.4,//)
  230 FORMAT(///,11X,'Txx*Tyy - 2*Txy*Txy = (Q*W(Uxy)/(4*PI*D(I)))**2 =
     1DET(I)',/)
  240 FORMAT((7X,1PE11.4,4(2X,E11.4)))
  250 FORMAT(/,9X,'DETBAR = (DET(1)+DET(2)+ ... +DET(NOBS))/NOBS = ',
     1       1PE11.4,//)
  260 FORMAT(17X,'LINEAR LEAST SQUARES PROBLEM TO BE SOLVED',/,16X,
     1       43(1H-),//,26X,'A(M,N)',19X,'X(N)',8X,'B(M)',/)
  270 FORMAT(10X,1PE11.4,2(2X,E11.4),4X,A4,4X,E11.4)
  275 FORMAT(10X,1PE11.4,2(2X,E11.4),12X,E11.4)
  280 FORMAT(11X,59(1H=),//)
  310 FORMAT(1H1,///,25X,'RESIDUAL VECTOR:  R = B - A*X',/,24X,32(1H-))
  320 FORMAT((10X,1PE11.4,4(2X,E11.4)))
  325 FORMAT(/,12X,'MATRIX CONDITION NUMBER: CONNUM = 1/TOL =',1PE15.5)
  330 FORMAT(/,29X,'SOLUTION VECTOR:  X(I)',/,28X,24(1H-),/,
     1       10X,'STxx=',1PE11.4,4X,'STyy=',E11.4,4X,'STxy=',E11.4)
  335 FORMAT(//,12X,'**** ERROR:  SQUARE ROOT OF NEGATIVE NUMBER ****',
     1        /,12X,'*     CANNOT COMPUTE STOR. COEF. OR TRANSM.    *',
     2        /,12X,'*       WITH GIVEN OBSERVATION WELL DATA       *',
     3        /,12X,'**********************************************')
  336 FORMAT(//,16X,'***** ERROR:  STORAGE COEFFICIENT = 0.00 ****'/,
```

D. Subroutine WLSTC--Continued

```
     1       /,16X,'* CANNOT COMPUTE TRANSMISSIVITY COMPONENTS  *',
     2       /,16X,'*     WITH GIVEN OBSERVATION WELL DATA      *',
     3       /,16X,'*********************************************')
 340 FORMAT(/////,33X,'OUTPUT RESULTS',/,33X,14(1H=),/)
 350 FORMAT(30X,'STORAGE COEFFICIENT',/,29X,21(1H-),/,32X,'S =',
     1       1PE11.4)
 360 FORMAT(/,22X,'COMPONENTS OF TRANSMISSIVITY TENSOR',/,
     1       21X,37(1H-),/,13X,'Txx =',1PE11.4,3X,'Tyy =',
     2       E11.4,3X,'Txy =',E11.4,/)
 370 FORMAT(17X,'PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR',/,16X,
     1       47(1H-),/,22X,'Tss =',1PE11.4,3X,'Tnn =',E11.4,/)
 375 FORMAT(30X,'RATIO OF ANISOTROPY',/,29X,21(1H-),/,31X,'Tss:Tnn =',
     1       F6.2,':1',/)
 380 FORMAT(30X,'ANGLE OF ANISOTROPY',/,29X,21(1H-),/,29X,
     1       'THETA = ',F6.2,' DEGREES')
C------------------- END SUBROUTINE WLSTC ---------------------------
      RETURN
      END
```

E. Subroutine WLSSL

```
C***********************************************************************
C*   SUBROUTINE:  WLSSL                     LAST REVISION:  11-12-85   *
C*        TENSOR ANALYSIS USING MORE THAN 3 OBSERVATION WELLS          *
C*               WEIGHTED LEAST - SQUARES OPTIMIZATION                 *
C*                 COOPER-JACOB STRAIGHT-LINE METHOD                   *
C***********************************************************************
      SUBROUTINE WLSSL(A,B,X,H,XW,YW,To,SL,WT,WELLID)
C-----------------------------------------------------------------------
      COMMON /PARAM/ M,N,PI,Q,AVG
      DIMENSION IP(3)
      REAL*8 A(M,N),B(M),X(N),H(N),XW(M),YW(M),To(M),SL(M),WT(M),SLBAR,
     1S,TXX,TYY,TXY,TSS,TNN,RATAN,THETA,THETAR,Q,TOL,CONNUM,DET,
     2DIFF,AVG
      CHARACTER WELLID(M)*10, TII(3)*4
      DATA (TII(J),J=1,3)/'STxx','STyy','STxy'/
C-------------------- LEAST-SQUARES PARAMETERS FOR 'LLSQF' ------------
      M1 = M
      N1 = N
      IA = M
      KBASIS = N
      TOL = 0.D0
C-------------------- READ OBSERVATION WELL DATA ----------------------
      DO 10 I = 1,M
          READ(5,110) WELLID(I),XW(I),YW(I),To(I),SL(I),WT(I)
   10 CONTINUE
C-------------------- PRINT OBSERVATION WELL DATA ---------------------
      WRITE(6,140)
      WRITE(6,150)
      DO 20 I = 1,M
          WRITE(6,160) WELLID(I),XW(I),YW(I),To(I),SL(I),WT(I)
   20 CONTINUE
      WRITE(6,170) Q
C-------------------- COMPUTE AVERAGE VALUE FOR SLOPE OF LINE ---------
C-------------------- OR USE A USER SUPPLIED AVERAGE VALUE ------------
      SLBAR = 0.00
      DO 30 I = 1,M
          SLBAR = SLBAR + SL(I)
          WT(I) = DSQRT (WT(I))
   30 CONTINUE
      SLBAR = SLBAR / FLOAT(M)
      IF(DABS(AVG) .GT. 0.00) SLBAR = AVG
C-------------------- COMPUTE DETERMINANT AND FORM --------------------
C-------------------- LINEAR SYSTEM:  [A](X) = (B) --------------------
      DET = (2.3025851 * Q / (4.0 * PI * SLBAR)) ** 2
      DO 40 I = 1,M
          A(I,1) = YW(I) * YW(I) * WT(I)
          A(I,2) = XW(I) * XW(I) * WT(I)
```

```
         A(I,3) = -2.0 * XW(I) * YW(I) * WT(I)
         B(I)   = 2.25 * To(I) * DET * WT(I)
   40 CONTINUE
C------------------- PRINT AVERAGE SLOPE, DETERMINANT, AND ----------
C------------------- COMPONENTS OF [A], (X), AND (B) ----------------
      IF(DABS(AVG) .GT. 0.00) THEN
         WRITE(6,220) SLBAR
      ELSE
         WRITE(6,230) SLBAR
      END IF
      WRITE(6,240) DET
      WRITE(6,260)
      DO 50 I = 1,M
        IF(I .LE. 3) WRITE(6,270) (A(I,J),J=1,N),TII(I),B(I)
        IF(I .GT. 3) WRITE(6,275) (A(I,J),J=1,N),B(I)
   50 CONTINUE
      WRITE(6,280)
C------------------- SOLUTION OF LINEAR LEAST-SQUARES PROBLEM --------
C------------------- A[M x N] * X(N)  =  B(M) -----------------------
C------------------- USE IMSL LIBRARY SUBROUTINE 'LLSQF' ------------
      CALL LLSQF(A,IA,M1,N1,B,TOL,KBASIS,X,H,IP,IER)
      IF(IER .GT. 0)                     RETURN
      CONNUM = 1.0 / TOL
C------------------- PRINT MATRIX CONDITION NUMBER (CONNUM) ---------
C------------------- RESIDUAL VECTOR (B), AND SOLUTION VECTOR (X) ----
      WRITE(6,310)
      WRITE(6,320) (B(I), I=1,M)
      WRITE(6,325) CONNUM
      WRITE(6,330) X(1), X(2), X(3)
C------------------- SOLVE FOR STORAGE COEFFICIENT ------------------
      DIFF = X(1)*X(2) - X(3)*X(3)
      IF(DIFF .LT. 0.00) THEN
           WRITE(6,335)
           RETURN
      END IF
C
      S = DSQRT(DIFF / DET)
C
      IF(S .LT. 1.E-10) THEN
         WRITE(6,336)
         RETURN
      END IF
C
      WRITE(6,340)
      WRITE(6,350) S
C------------------- SOLVE FOR COMPONENTS OF TRANSMISSIVITY ---------
      TXX = X(1) / S
      TYY = X(2) / S
      TXY = X(3) / S
      WRITE(6,360) TXX,TYY,TXY
```

```
C------------------- SOLVE FOR PRINCIPAL COMPONENTS AND ------------- WLSS 980
C------------------- ANGLE OF ANISOTROPY --------------------------- WLSS 990
      THETA = 0.D0                                                    WLSS1000
      TSS = 0.5 * (TXX + TYY + DSQRT((TXX-TYY)**2 + 4.0*TXY*TXY))     WLSS1010
      TNN = 0.5 * (TXX + TYY - DSQRT((TXX-TYY)**2 + 4.0*TXY*TXY))     WLSS1020
      RATAN = TSS / TNN                                               WLSS1030
      IF(DABS(TXX - TYY) .LT. 1.E-5 .OR. DABS(TXX-TSS) .LT. 1.E-5)    WLSS1040
     1GO TO 60                                                        WLSS1050
      THETAR = DATAN2((TSS-TXX),TXY)                                  WLSS1060
      THETA = THETAR * 180.00 / PI                                    WLSS1070
      IF(THETA .LT. 0.D0) THETA = THETA + 360.00                      WLSS1080
   60 CONTINUE                                                        WLSS1090
      WRITE(6,370) TSS,TNN                                            WLSS1100
      WRITE(6,375) RATAN                                              WLSS1110
      WRITE(6,380) THETA                                              WLSS1120
C------------------- FORMAT STATEMENTS ----------------------------- WLSS1130
  110 FORMAT(A10,5G10.0)                                              WLSS1140
  140 FORMAT(/,35X,'INPUT DATA',/,34X,12(1H=),//,                     WLSS1150
     1       22X,'(ALL DATA ARE IN "CONSISTENT UNITS")',//)           WLSS1160
  150 FORMAT(6X,'WELL ID.',5X,'X-COORD.',6X,'Y-COORD.',8X,'To',8X,    WLSS1170
     1       ' SLOPE  ',3X,'WEIGHT',/,5X,10(1H-),3X,10(1H-),          WLSS1180
     2       4X,10(1H-),4X,8(1H-),4X,8(1H-),3X,8(1H-),/)              WLSS1190
  160 FORMAT(5X,A10,3X,2(F10.2,4X),1PE8.2,4X,E8.2,3X,E8.2)            WLSS1200
  170 FORMAT(//,1X,80(1H-),/,22X,'AVERAGE PUMPING RATE: Q = ',1PE10.4,/ WLSS1210
     1       ,1X,80(1H-))                                             WLSS1220
  220 FORMAT(1H1,///,11X,                                             WLSS1230
     1       'THE AVERAGE SLOPE (SLBAR) INPUT BY THE USER IS: ',      WLSS1240
     2       1PE11.4,//)                                              WLSS1250
  230 FORMAT(1H1,///,11X,'SLBAR = [SL(1)+SL(2)+ ... +SL(NOBS)]/NOBS = ',WLSS1260
     1       1PE11.4,//)                                              WLSS1270
  240 FORMAT(11X,'Txx*Tyy - 2*Txy*Txy = [2.30 * Q / (4*PI*SLBAR)]**2 = DWLSS1280
     1ET',//,31X,'DET = ',1PE11.4,/)                                  WLSS1290
  260 FORMAT(///,17X,'LINEAR LEAST SQUARES PROBLEM TO BE SOLVED',/,   WLSS1300
     1       16X,43(1H-),//,26X,'A(M,N)',19X,'X(N)',8X,'B(M)',/)      WLSS1310
  270 FORMAT(10X,1PE11.4,2(2X,E11.4),4X,A4,4X,E11.4)                  WLSS1320
  275 FORMAT(10X,1PE11.4,2(2X,E11.4),12X,E11.4)                       WLSS1330
  280 FORMAT(11X,59(1H=),//)                                          WLSS1340
  310 FORMAT(1H1,///,25X,'RESIDUAL VECTOR:  R = B - A*X',/,24X,32(1H-)) WLSS1350
  320 FORMAT((10X,1PE11.4,4(2X,E11.4)))                               WLSS1360
  325 FORMAT(//,12X,'MATRIX CONDITION NUMBER: CONNUM = 1/TOL =',1PE15.5)WLSS1370
  330 FORMAT(///,29X,'SOLUTION VECTOR:  X(I)',/,28X,24(1H-),/,        WLSS1380
     1       10X,'STxx=',1PE11.4,4X,'STyy=',E11.4,4X,'STxy= ',E11.4)  WLSS1390
  335 FORMAT(//,12X,'**** ERROR:  SQUARE ROOT OF NEGATIVE NUMBER ****', WLSS1400
     1        /,12X,'*     CANNOT COMPUTE STOR. COEF. OR TRANSM.    *', WLSS1410
     2        /,12X,'*       WITH GIVEN OBSERVATION WELL DATA       *', WLSS1420
     3        /,12X,'**********************************************') WLSS1430
  336 FORMAT(//,16X,'**** ERROR:  STORAGE COEFFICIENT = 0.00 ****',   WLSS1440
     1        /,16X,'* CANNOT COMPUTE TRANSMISSIVITY COMPONENTS *',   WLSS1450
     2        /,16X,'*     WITH GIVEN OBSERVATION WELL DATA     *',   WLSS1460
     3        /,16X,'********************************************')   WLSS1470
```

E. Subroutine WLSSL--Continued

```
 340 FORMAT(////,33X,'OUTPUT RESULTS',/,33X,14(1H=),/)
 350 FORMAT(30X,'STORAGE COEFFICIENT',/,29X,21(1H-),/,32X,'S =',
    1       1PE11.4)
 360 FORMAT(/,22X,'COMPONENTS OF TRANSMISSIVITY TENSOR',/,
    1       21X,37(1H-),/,13X,'Txx =',1PE11.4,3X,'Tyy =',
    2       E11.4,3X,'Txy =',E11.4,/)
 370 FORMAT(17X,'PRINCIPAL COMPONENTS OF TRANSMISSIVITY TENSOR',/,16X,
    1       47(1H-),/,22X,'Tss =',1PE11.4,3X,'Tnn =',E11.4,/)
 375 FORMAT(30X,'RATIO OF ANISOTROPY',/,29X,21(1H-),/,31X,'Tss:Tnn =',
    1       F6.2,':1',/)
 380 FORMAT(30X,'ANGLE OF ANISOTROPY',/,29X,21(1H-),/,29X,
    1       'THETA = ',F6.2,' DEGREES')
C-------------------- END SUBROUTINE WLSSL ---------------------------
      RETURN
      END
```