A COMPUTER PROGRAM FOR THE AUTOMATIC INVERSION OF
SCHLUMBERGER SOUNDINGS USING MULTI-LAYER INTERPRETATION
FOLLOWED BY DAR ZARROUK REDUCTION

By

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Open-File Report 86-288
1986

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curve, the reduced model is fed back into the least squares inversion procedure.

PROGRAM RESINV

The automatic inversion program for Schlumberger soundings consists of a main program (RESINV) and 11 subroutines. The complete source listing of this interactive program is given in the Appendix. This program was developed using DEC's VAX 11/780 computer, and was successfully transferred to FUJITSU's FACOM M200 and IBM 4341/10 computers. The notation used in the subsequent description is listed in Table 1.

| TABLE 1 |
|-----------------|----------------------|
| ND | number of data points |
| AB(I) | half electrode spacing, AB/2 |
| RF(I) | observed apparent resistivity |
| RS(I) | smoothed apparent resistivity |
| RC(I) | calculated apparent resistivity |
| NL | number of layers |
| NP | number of layer parameters = 2*N-L-1 |
| P(J) | layer parameters (N-L resistivities followed by N-L thicknesses) |
| INV | inversion no. |
| IC | iteration counter for STEEP |
| ITE | iteration counter for RIDGE |
| Q | error criterion, eq.(1) or eq.(3) |
| RE | relative error, eq.(2) |
| A(I,J) | sensitivity matrix, eq.(7) or eq.(14) |
| G(J) | negative gradient, eq.(6) |
| ST | step length, eq.(11) |
| FM | Marquardt factor |
| T(L+I-1) | resistivity transform, eq.(21) |
| D(L+I-1) | derivative of T, eq.(15) |
| FC(L) | convolution filter |
| NC | number of coefficients |
| TT(I) | transverse resistance |
| SS(I) | longitudinal conductance |
| DZR(I) | Dar Zarrouk resistivity |
| DZD(I) | Dar Zarrouk depth |
The calculation of apparent resistivity is done by subroutine APPRES using the linear filter method (Murakami and others, 1984). Depending on the value of MODE supplied by the subroutine FDATA, either of the filters, YM6 (6 points per log-decade) or YM10 (10 points per log-decade), is called.

The array RS stands for the smoothed apparent resistivity curve. Because there are no reliable automatic means of smoothing noisy field curves, RS values can be used to replace the noisy field data (RF) after the first inversion (INV=1) when computing equivalent solutions with a smaller number of layers. This can be applied because we assume that the initial multi-layer interpretation fits the global features of the observed curve, and that any further improvement will end up fitting the noise.

The subroutine AGUESS automatically prepares an initial guess of parameters. Next, two inversion schemes, steepest descent and ridge regression programmed in subroutines STEEP and RIDGE, are called successively. The steepest descent iterative inversion method is relatively insensitive to poor initial guesses, but it converges very slowly after a fit of about 2% is obtained. The program then transfers to the ridge regression inversion because this converges faster as the layer model approaches the least squares fit.

When no parameters are fixed (automatic initial guess mode), the subroutine AUTODZ tries to obtain equivalent solutions composed of a smaller number of layers. This reduction is performed in the Dar Zarrouk domain. The fundamental idea of this reduction, developed by Zohdy (1975), is that two similar Dar Zarrouk curves will give similar apparent resistivity curves. If the Dar Zarrouk curve, calculated from the detailed model, can be approximated using a smaller number of Dar Zarrouk points, this reduced Dar Zarrouk curve is transformed into the corresponding layering model. This reduced layering will be almost equivalent to the original layering in the apparent resistivity domain. Zohdy (1974), however, found models where two almost identical Dar Zarrouk curves with very long steeply descending branches gave appreciably different resistivity curves, pointing out the need for careful smoothing of long steeply descending Dar Zarrouk branches. In order to better fit the field data this reduced model is fed back to the inversion schemes STEEP and RIDGE to obtain the least squares fit in the apparent resistivity domain.

If no reduction is performed by subroutines AUTODZ, the computer asks the user to choose one of four options. By keying in 1, the user can change the initial guess. In this mode the user can input his own initial guess and fix some of the parameters by putting negative signs before them. By keying in 2, MANUDZ, the manual reduction subprogram using the Dar Zarrouk parameters, is called. By keying in 3, printouts of the correlation matrix, resolution matrix, and information density matrix are obtained. By keying in 4, the program stops.

SUBROUTINE AGUESS

The subroutine AGUESS automatically constructs an initial guess, where the number of layers NL is set equal to the number of observed data points ND. The observed apparent resistivities are used as the initial layer resistivities, and AB/2 values are used as the initial layer depths.
SUBROUTINE STEEP

The subroutine STEEP performs the steepest descent inversion. This iterative inversion scheme is chosen because it is insensitive to poor initial guesses. The convergence speed is initially very rapid, but slows as model refinement progresses. When a fit of 2% is obtained the ridge regression inversion scheme is called.

The most common error criterion to be minimized in the least squares inversion is the sum of the squares of either the relative error:

\[ Q = \sum_{I=1}^{\text{ND}} (\text{RE}(I))^2 \]  

where

\[ \text{RE}(I) = \frac{\text{RC}(I)}{\text{RS}(I)} - 1 \]

and RC = calculated resistivities, RS = field resistivities; or the logarithmic differences between the values calculated from the model and the observed data:

\[ Q = \sum_{I=1}^{\text{ND}} (\ln \text{RC}(I) - \ln \text{RS}(I))^2 \]  

Due to the small difference between the relative error and the logarithmic difference (see Table 2) they are frequently interchanged.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\% ERROR & RE & LOG DIFF. \\
\hline
-50 & -.50 & -.69315 \\
-20 & -.20 & -.22314 \\
-10 & -.10 & -.10536 \\
-5 & -.05 & -.05129 \\
-2 & -.02 & -.02020 \\
-1 & -.01 & -.01005 \\
0 & .00 & .00000 \\
1 & .01 & .00995 \\
2 & .02 & .01980 \\
5 & .05 & .04897 \\
10 & .10 & .09531 \\
20 & .20 & .18231 \\
50 & .50 & .40547 \\
100 & 1.00 & .69315 \\
\hline
\end{tabular}
\caption{Difference between the relative error (RE) and the logarithmic (LOG) difference. As long as the percent error is between ±5, RE and LOG DIFFERENCE are almost the same. LOG DIFFERENCE is a better estimation of difference, because the apparent resistivity is a positive-valued function.}
\end{table}
Vozoff (1958), Wang and Treitel (1973), and Koefoed (1979) have presented algorithms for the steepest descent method. The version by Koefoed is used here. The basic idea of the steepest descent method is to change the value of the layer parameters $P(J)$ (resistivities, thicknesses) into the direction of the steepest descent of the error criterion, that is, in the direction of the negative gradient of the error criterion:

$$\text{Set } \ln P(J) = \ln P(J) + C \cdot G(J) \quad (4)$$

or

$$\text{Set } P(J) = P(J) \exp(C \cdot G(J))$$

$$= P(J)(1 + C \cdot G(J)) \quad (5)$$

where the negative gradient $G(J)$ is defined by

$$G(J) = \frac{-\partial Q}{\partial \ln P(J)}$$

$$= - \sum_{I=1}^{ND} 2(\ln RC(I) - \ln RS(I)) \frac{\partial \ln RC(I)}{\partial \ln P(J)}$$

$$- \sum_{I=1}^{ND} 2 \cdot \text{RE}(I)A(I,J) \quad (6)$$

and the sensitivity matrix $A(I,J)$ is defined by

$$A(I,J) = \frac{\partial \ln RC(I)}{\partial \ln P(J)} = \frac{P(J)}{RC(I)} \frac{\partial RC(I)}{\partial P(J)} \quad (7)$$

The value of $C$ in equations (4) and (5) is chosen following the suggestion of Vozoff (1958) and Koefoed (1979, p. 163), as:

$$C = Q/(GR)^2 \quad (8)$$

where

$$GR = \left| VQ \right| = (\sum_{J=1}^{NP} G(J)G(J))^{1/2} \quad (9)$$

Substituting equation (8) into equation (5) we obtain

$$\text{Set } P(J) = P(J)(1 + ST \cdot G(J)/(GR)) \quad (10)$$

where the step length $ST$ is given by

$$ST = Q/GR \quad (11)$$

As discussed by Koefoed (1979, p. 163-164), the step length given by equation (11) may be overestimated when $Q$ approaches very near to its minimum.
value. When a new P leads to an increase in Q, ST is reduced by a factor of 0.8, and remains unchanged until Q increases. For each increase in Q, ST is multiplied by 0.8 until it becomes smaller than 0.0001. At that point iteration is stopped. The iteration is also stopped if Q becomes smaller than the fitting tolerances (2%).

To enhance convergence, at every fourth iteration the value of P(J) is changed by setting

\[ P(J) = P(J) \cdot P(J)/P_J \]  

(12)

where \( P_J \) is the value of \( P(J) \) four iterations before. This is done because the ratio \( P(J)/P_J \) is considered to be the trend of convergence.

**SUBROUTINE DERIV**

The subroutine DERIV calculates the sensitivity matrix \( A(I,J) \), defined by equation (7). The analytical derivatives are obtained through the following relations (Koefoed, 1979, p. 164):

\[ R_C(I) = \sum_{L=1}^{NC} T(L+I-1)FC(L) \]  

(13)

\[ A(I,J) = \frac{P(J)}{R_C(I)} \sum_{L=1}^{NC} D(L+I-1,J)FC(L) \]  

(14)

where

\[ D(L+I-1,J) = \frac{\partial T(L+I-1)}{\partial P(J)} \]  

(15)

with

\[ \frac{\partial T_1}{\partial t_1} = \frac{\partial T_1}{\partial t_1} - \frac{\partial T_2}{\partial t_1} \]  

(16)

\[ \frac{\partial T_1}{\partial \rho_1} = \frac{\partial T_1}{\partial \rho_1} - \frac{\partial T_2}{\partial \rho_1} \]  

(17)

\[ \frac{\partial T_1}{\partial t_1+1} = \frac{1 - \tanh^2(\lambda t_1)}{(1 + \tanh^2(\lambda t_1) + \tan^2(\lambda t_1)/\rho_1)^2} \]  

(18)

\[ \frac{\partial T_1}{\partial t_1} = \frac{\rho_1 - T_{i+1}^2/\rho_1}{(1 + T_{i+1}^2/\rho_1)^2} \frac{\lambda}{\cosh^2(\lambda t_1)} \]  

(19)
\[
\begin{align*}
\frac{\partial T_i}{\partial \rho_i} &= \frac{\tanh(\lambda t_i) \left(1+T_i^2/\rho_i^2 + 2\tanh(\lambda t_i)(T_i+1)/\rho_i\right)}{(1+T_i+1\tanh(\lambda t_i)/\rho_i)^2} \\
\text{and} \\
T_i &= \frac{T_i+1 + \rho_i \tanh(\lambda t_i)}{1+T_i+1\tanh(\lambda t_i)/\rho_i}
\end{align*}
\]

If \( P(J) \) is negative, the derivative with respect to \( P(J) \) is set to a very large value:

\[
A(I,J) = 10^{20}, \text{ for } I=1,2,\ldots,\text{ND}
\]

so that the increment on \( P(J) \) becomes negligibly small.

**SUBROUTINES RIDGE, NEMP, AND MAINE**

The subroutine RIDGE performs the Gauss-Newton inversion of the linearized least squares problem. Ridge regression is used for its stable inversion of the system matrix. The formulation based on the solution of normal equations was studied by Inman (1975) and Rijo and others (1977). The computer program used by Rijo and others (1977) was published by Sandberg (1979). Another technique known as the generalized matrix inverse was discussed by Johansen (1977). Here we follow the method of Rijo and others (1977).

The minimization of the error criterion \( Q \), defined by equation (3), is simplified by the linearization approximation of the apparent resistivity function \( R_C \). Expanding \( \ln R_C \) in a Taylor series and discarding terms higher than first order, we obtain

\[
Q = \sum_{I=1}^{\text{ND}} \sum_{J=1}^{\text{NP}} \left( \ln R_C(I) + \sum_{J=1}^{\text{NP}} \frac{\partial \ln R_C(I)}{\partial \ln P(J)} \delta \ln P(J) - \ln R_S(I) \right)^2
\]

or

\[
Q = \sum_{I=1}^{\text{ND}} \sum_{J=1}^{\text{NP}} \left( \sum_{J=1}^{\text{NP}} A(I,J)X(J) + \ln \frac{R_C(I)}{R_S(I)} \right)^2
\]

where

\[
X(J) = \delta \ln P(J)
\]

By taking the partial derivative of equation (23) with respect to \( X(K) \) and setting it equal to zero, we obtain the normal equations:

\[
\frac{\partial Q}{\partial X(K)} = 2 \sum_{I=1}^{\text{ND}} \sum_{J=1}^{\text{NP}} \left( \sum_{J=1}^{\text{NP}} A(I,J)X(J) + \ln \frac{R_C(I)}{R_S(I)} \right) A(I,K) = 0
\]
or

\[ \sum_{I=1}^{NP} \sum_{J=1}^{ND} A(I,J)A(I,K)X(J) = -\sum_{J=1}^{ND} \ln_{RS(I)}^{RC(I)} A(I,K) \]  

The matrix representation of equation (25) is

\[ A^T A \chi = -A^T b \]  

where

\[ A^T A(K,J) = \sum_{I=1}^{ND} A(I,J)A(I,K) \]  

\[ \chi = (X(1), X(2), \ldots, X(NP))^T \]  

\[ A^T(K,I) = A(I,K) \]  

and

\[ b = (\ln_{RS(1)}^{RC(1)}, \ln_{RS(2)}^{RC(2)}, \ldots, \ln_{RS(ND)}^{RC(ND)})^T \]  

The formal solution of equation (26) is given by

\[ \chi = -(A^T A)^{-1} A^T b \]  

In order to enhance the stability of the matrix inversion in equation (31), the concept of scaling is utilized. In the original linear simultaneous equations:

\[ A^T \chi = -b \]  

the matrix \( A \) and the vector \( \chi \) are scaled in the following way:

\[ A^*(I,J) = A(I,J)W(J) \]  

and

\[ X^*(J) = \frac{X(J)}{W(J)} \]  

with

\[ W(J) = \left( \sum_{I=1}^{ND} A(I,J)^2 \right)^{-1/2} \]
After scaling, the diagonal components of the matrix $A^T A$ become unit.

If the initial guess is so poor that the linearization approximation in equation (23) fails, the step $X(J)$, given by equation (24), may not be adequate. For noisy field data this problem becomes more serious. At the minimum point of the error criterion $Q$, the negative gradient, given by equation (6) as:

$$-\nabla Q = 2A^T b$$  \hspace{1cm} (36)

must be zero, showing that the matrix $A^T$ and the residual vector $b$ must be orthogonal. The larger the residual vector is, the more complete-orthogonality is required. Thus, for noisy field data where the residual vector never becomes zero, the validity of the linearization approximation is limited.

If the step $X(J)$ given by equation (24) fails to obtain a smaller $Q$, the increment vector $\chi$ is changed into the steepest descent direction. This is accomplished by modifying equation (31) into

$$\chi = -(A^T A + \lambda I)^{-1} A^T b$$  \hspace{1cm} (37)

where $\lambda$ is called the Marquardt factor. For larger $\lambda$, the increment vector becomes more aligned to the steepest descent direction. The technique of modifying the value of $\lambda$ at each iteration is known as the ridge regression method. In this program, the algorithm by Sandberg (1979) for determining $\lambda$ is applied.

Two more measures are taken in order to avoid anomalously large values of $X(J)$. First, the residual in $(RC(I)/RS(I))$ greater than 2% is set equal to 2% in order to obtain a gradual fit to the abnormal residuals. Second, the increment $X(J)$ if greater than .2, is set to .2, in order to make the change of parameter less than 22% at each iteration.

**SUBROUTINE RESULT**

The subroutine RESULT prints out the results of the inversion. The Dar Zarrouk parameters of the layering model obtained are calculated through the formula:

$$TT(I) = \sum_{j=1}^{I} h_j \rho_j$$  \hspace{1cm} (38)

$$SS(I) = \sum_{j=1}^{I} h_j / \rho_j$$  \hspace{1cm} (39)

$$DZR(I) = (TT(I)/SS(I))^{1/2}$$  \hspace{1cm} (40)

$$DZD(I) = (TT(I)SS(I))^{1/2}$$  \hspace{1cm} (41)

To help reduce the number of layers, the Dar Zarrouk interpolation is calculated. For the three consecutive points, $I-1$, $I$, and $I+1$, the DZ
interpolation connecting the points I-1 and I+1 is calculated through the formula:

\[ DZRI(I) = -U + (U^2 + R^2)^{1/2} \quad (42) \]

where

\[ R = \left( \frac{(TT(I+1) - TT(I-1))/(SS(I+1) - SS(I-1))}{2} \right)^{1/2} \quad (43) \]

\[ U = DZD(I-1)((R^2 - DZR^2(I-1))/(2 \cdot DZD(I)DZR(I-1))) \quad (44) \]

If the percent difference between DZRI(I) and DZR(I) is relatively small, point I may be eliminated. The value of R, given by equation (43), gives a rough estimate of the resistivity of the combined layer defined by the Dar Zarrouk points I-1 and I+1.

The second and fourth columns of the output show estimates of the percent error of parameters due to the uncertainty in the determination of these parameters (Rijo and others, 1977). This should be interpreted as the relative certainty in the estimated value.

**SUBROUTINE AUTODZ**

The subroutine AUTODZ performs the Dar Zarrouk reduction automatically. The idea of automatic reduction by means of equivalence in the Dar Zarrouk curves was developed by Zohdy (1975). The computer program based on this idea was published in Zohdy (1973) as subroutine (DZSMTH). Subroutine AUTODZ is a modified version of DZSMTH that includes a modified fitting tolerance (FT) and other reduction criteria.

The leftmost DZ branch is a horizontal line. The first NN points are averaged to give

\[ DZRA(NN) = \exp \left( \frac{1}{NN} \sum_{I=1}^{NN} \ln DZR(I) \right) \quad (45) \]

where DZR is the Dar Zarrouk resistivity. If the logarithmic difference \( \ln(DZR(I)/DZRA(NN)) \) is smaller than FT (fitting tolerance) as defined by Zohdy (1975) for every I from 1 to NN, this average is accepted and the next point is tried. Otherwise, the NN-th point cannot be included in this branch, and the value DZRA(NTAP) is set to the first DZ resistivity, where NTAP=NN-1 is the number of points averaged so far.

Further smoothing is based on fitting straight lines to the linear representation of the DZ curve, which is a plot of T versus S where:

\[ T = A_0 + A_1 \cdot S \quad (46) \]

For the first branch (NBR=1) the two coefficients are given by

\[ A_0(1) = 0 \quad (47) \]
\[ A_1(1) = (DZRA(NTAP))^2 \]  

For the \((NBR+1)\)-th branch, these are given by

\[ A_1(NBR+1) = \frac{NN\cdot \Sigma TS - \Sigma \cdot \Sigma S}{NN\cdot \Sigma S^2 - (\Sigma S)^2} \]  

\[ AO(NBR+1) = \frac{\Sigma T - A_1(NBR+1)\Sigma S}{NN} \]  

where the summation is from \(NTAP\) to \(NEXT\) and \(NN = NEXT - NTAP + 1\) is the number of summed points. The point \(NTAP\) is the last point on the previous branch, and at the same time the first point on the present branch. The intersection of the \((NBR+1)\)-th branch and the \(NBR\)-th branch in the linear DZ domain is obtained by

\[ TTR(NBR) = AO(NBR) + A_1(NBR)SSR(NBR) \]  

If the intersection is not in the first quadrant (SSR and/or TTR are negative), the point \(NEXT\) is rejected from the \((NBR+1)\)-th branch. The Dar Zarrouk point corresponding to this intersection is given by

\[ DZDRF(NBR) = TTR(NBR) \cdot SSR(NBR) \]  

\[ DZRRF(NBR) = \frac{TTR(NBR)}{SSR(NBR)} \]  

If the abscissa \((DZDRF)\) of this reduced DZ point is greater than the abscissa \((DZD(NTAP+1))\) of the second point on the \((NBR+1)\)-th branch, the point \(NEXT\) is rejected. Also if \(DZDRF\) is smaller the \(DZD(NTAP-1)\) of the second to the last point of the previous branch, the point is also rejected.

Next, each point to be included in this branch is compared with the value calculated from the reduced branch, and if any one of them exceeds the fitting tolerance, this branch is not accepted and the point \(NEXT\) is rejected. Because different fitting tolerances should be used depending on the slope of the DZ curve (Zohdy, 1975), the fitting tolerance \(TOL\) is defined by

\[ TOL = (FT - .02) \cdot SLP + FT \quad \text{for} \quad SLP < 0 \]  

\[ = -.02 \cdot SLP + FT \quad \text{for} \quad SLP \geq 0 \]  

which takes a value of \(.02\) at \(SLP = -1\), \(FT\) at \(SLP = 0\), and \(FT - .02\) at \(SLP = 1\). The
slope of the reduced DZ curve at the abscissa DZD(I) is calculated by

\[
SLP = \frac{\ln DZR2 - \ln DZR1}{\ln DZD2 - \ln DZD1}
\]

where DZR2 and DZR1 are the DZ resistivities on the reduced DZ curve at the abscissas DZD2=DZD(I)\cdot1.02 and DZD1=DZD(I)/1.02, respectively. The DZ resistivity of the reduced DZ curve is calculated by (Zohdy, 1975, p. E29) as:

\[
DZRA(I) = \frac{W + (w^2 + z)^{1/2}}{2 \cdot DZD(I)DZRF(NN)}
\]

where

\[
W = -DZD(NN)(P(NN)^2 - DZRF(NN)^2)
\]

\[
Z = 4 \cdot (DZD(I)DZRF(NN)P(NN))^2
\]

The automatic DZ reduction discussed above has some difficulty in reducing points near sharp deflections. If two consecutive points in the resultant DZ curve are near enough to each other such that,

\[
\frac{DZDRF(I+1)}{DZDRF(I)} < 1.5
\]

then these points are merged by taking their geometric mean.

Initially the fitting tolerance FT is set equal to 0.08. If no reduction is obtained, FT is increased by 0.02 until it exceeds 0.12 where it remains fixed.

SUBROUTINE MANUDZ

In subroutine MANUDZ the user manually reduces the number of layers. There are two modes for manual reduction, in MODE 1 the user specifies the Dar Zarrouk points to be deleted, and in MODE 2 the user specifies the maximum percent difference of the DZ points to be deleted.

SUBROUTINE MATRIX

The subroutine MATRIX prints out the correlation matrix \(C\), the resolution matrix \(R\) and the information density matrix \(S\). These matrices are defined as follows:

\[
C(I,J) = \frac{ATAINV(I,I)ATAINV(J,J)}{(ATAINV(I,I)ATAINV(J,J))^{1/2}}
\]

\[
R = (A^TA+\lambda I)^{-1}A^TA
\]

\[
S = A(A^TA+\lambda I)^{-1}A^T
\]
Theoretical Schlumberger sounding curve represented as plus signs, calculated from the model represented by the dashed line.

Rigorously speaking, these matrices must be calculated with $\lambda$ equal to zero. To avoid extra calculation, this subroutine calculates them using the data after the last inversion, where $\lambda$ will have a finite but very small value.

**EXAMPLES**

In this section two examples are given of the application of the automatic interpretation. In the first example a theoretical sounding curve is used as input to the inversion program, and in the second example an actual field curve is used.

**EXAMPLE 1**

Figure 1 shows the theoretical apparent resistivity curve calculated for the five layer model (TEST50) with resistivities of 1000, 100, 25, 5, and 120 and thicknesses of 7, 14, 40, and 140. For the purpose of illustration the units are unimportant. The units that come out of the inversion routine are the same as those that go in.

The output of the inversion of TEST50, obtained by the program RESINV, is given in Table 3. AUTODZ reduced the number of layers directly to five, and the resultant model is almost identical to the original.
### TABLE 3
COMPUTER OUTPUTS FOR TEST50

**TITLE= TEST50**

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<th>AB/2=</th>
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<td>755.276</td>
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**AUTOMATIC INVERSION STARTS**
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EXAMPLE 2

Figure 2 shows a field curve, FIOR1, taken from a set of soundings carried out as a part of an hydrogeological study by the University of Parma (Italy) in the south-west part of the Po plain area (Fiorenzuola d'Arda-Italy). The original field curve (Fig. 2a) was shifted and then digitized at the logarithmically equal interval of six points per log decade (Fig. 2b) using a cubic spline function from Anderson (1971). The digitized apparent resistivity values were fed to the automatic interpretation. Detailed and reduced models obtained by the inversion are presented in Table 4. The initial detailed solution at INV 1 features a very smooth resistivity variation suitable for the multi-layer representation used by Zohdy and Bisdorf (1978). Because of distortions probably due to geologic noise, the initial inversion (INV=1) stopped with a fit of 5.6%. The largest discrepancy between observed and calculated values (15%) occurred at sample point 7. For further calculation of the reduced models, the calculated curve, based on the initial detailed interpretation, was used as the smoothed curve.

The reduced solution, at INV=3, produced five main geoelectric units, as evidenced by the five major breaks of the field curve. This solution agrees well with results from nearby soundings and geological data, especially in the estimates of the depths of layer boundaries. An equivalent six layer model was also constructed. It is worth noting that the construction of equivalent models with different number of layers may give the interpreter a wider possibility of finding geologically convenient solutions.

ACKNOWLEDGMENT

This work was done during our stay at the United States Geological Survey, Denver, Colorado in 1981. We wish to thank Dr. Adel Zohdy for the illuminating discussions, help, and comments.
Figure 2. Data for Schlumberger sounding F101. a) Observed field data. b) Shifted and digitized apparent resistivity data.
### Table 4: Computer Outputs for Fior1

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Automatic Dz Reduction Reduced NL to 6

Ridge q = 0.00086 at Ite = 5 FM = 0.001503
### TABLE 4 (continued)

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**AUTOMATIC DZ REDUCTION REDUCED NL TO 5**

**RIDGE $Q = 0.00080$ AT ITE= 7 FM= 0.000553**

**INVERSION OF FIELD DATA: FIOR1 AT INV= 3**

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**PARAMETER CORRELATION MATRIX**
REFERENCES

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APPENDIX

0010 C PROGRAM RESINV
0020 C DEVELOPED BY YUTAKA MURAKAMI AND ANDREA ZERILLI
0040 C
0050 C TITLE(4) : NAME OF THE VES POINT
0060 C ND : NUMBER OF DATA POINTS
0070 C AB(25) : AB/2 OF EACH DATA POINT
0080 C RF(25) : APPARENT RESISTIVITY
0090 C MODE : POSITIVE FOR EQUI-SPACED AB
0100 C : NEGATIVE FOR ARBITRARY AB
0110 C : +/- 6 FOR USUAL ANALYSIS
0120 C : +/- 10 FOR MORE ACCURATE ANALYSIS
0130 C P(50) : ARRAY OF PARAMETERS OF MODEL
0140 C NL : NUMBER OF LAYER OF MODEL
0150 C NL1 : NL - 1
0160 C NP : NUMBER OF PARAMETERS = NL + NL1
0170 C Q : AVERAGE ERROR
0180 C RC(25) : CALCULATED APPARENT RESISTIVITY
0190 C RS(25) : SMOOTHED APPARENT RESISTIVITY
0200 C A(25,50) : SENSITIVITY MATRIX
0210 C PCT(50) : PERCENT DEVIATION
0220 C NFIX : NUMBER OF FIXED PARAMETERS
0230 C
0240 C COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
0250 1 ,Q,RC(25),RS(25),A(25,50),PCT(50)
0260 C
0270 C *** INPUT OF OBSERVATION DATA
0280 C CALL FDATA(TITLE,ND,AB,RF,MODE)
0290 C
0300 C *** CHOICE OF FILTER
0310 C IF IAMBS(MODE).EQ.6) CALL YM6
0320 C IF IAMBS(MODE).EQ.10) CALL YM10
0330 C
0340 C *** RS IS SET TO RF FOR THE FIRST INVERSION (INV=1)
0350 C DO 5 I=1,ND
0360 5 RS(I)=RF(I)
0370 C
0380 C *** AUTOMATIC GUESS OF INITIAL MODEL
0390 C CALL AGUESS(ND,RF,AB,NL,NL1,NP,P)
0400 NFIX=0
0410 C
0420 WRITE(6,100)
0430 WRITE(8,100)
0440 100 FORMAT(' AUTOMATIC INVERSION STARTS')
0450 C
0460 C INV=INVERSION COUNTER
0470 C INV=0
0480 15 INV=INV+1
0490 C
0500 C *** TWO INVERSION SUBROUTINES ARE CALLED SEQUENTIALLY
0510 C CALL STEEP
0520 C CALL RIDGE
0530 C CALL RESULT(INV)
0540 C
0550 C IF NO PARAMETERS ARE FIXED,
0560 C AUTODZ IS CALLED FOR AUTOMATIC REDUCTION OF LAYER NUMBERS
IF(NFIX.NE.0) GO TO 1000
CALL AUTODZ(JMODE)
IF(JMODE.NE.0) GO TO 1000
C
IF(INV.NE.1) GO TO 10
DO 999 I=1,ND
999 RS(I)=RC(I)
GO TO 10
C
1000 WRITE(6,1001)
1001 FORMAT(/' KEY IN 1 TO CHANGE THE INITIAL GUESS'
* ,/ ' 2 TO REDUCE D-Z POINTS'
* ,/ ' 3 TO PRINT OUT MATRIX INFORMATION'
* ,/ ' 4 TO FINISH')
READ(5,*) IANS
GO TO (2000,3000,4000,5000),IANS
C
C MANUAL INPUT OF INITIAL GUESS
2000 CALL MGUESS(NL,NL1,NP,P)
NFIX=0
DO 20 J=1,NP
20 IF(P(J).LT.0.) NFIX=NFIX+1
GO TO 10
C
C MANUAL DZ REDUCTION
3000 CALL MANUDZ
GO TO 10
C
C PRINT OUT MATRIX INFORMATION
4000 CALL MATRIX
GO TO 1000
C
10 WRITE(6,990) Q
990 FORMAT(/' LAST Q = ',F10.5)
GOTO 15
C
5000 STOP
END
C *******************************************************
C SUBROUTINE FDATA(TITLE,ND,AB,RF,MODE)
C *******************************************************
C READS OBSERVED VES DATA
C
DIMENSION TITLE(4),AB(25),RF(25)
READ(20,1000) TITLE,ND,MODE
WRITE(8,2000) TITLE,ND,MODE
DO 10 I=1,ND
READ(20,3000) AB(I),RF(I)
10 WRITE(8,4000) I,AB(I),RF(I)
RETURN
1000 FORMAT(1X,4A4/2I3)
2000 FORMAT(' TITLE= ',4A4,' ND=' ,I3, ' MODE=' ,I3/)
3000 FORMAT(3X,2F10.3)
SUBROUTINE AGUESS(ND,RF,AB,NL,NL1,NP,P)
C AUTOMATIC GUESS OF INITIAL MODEL
C
DIMENSION RF(25),AB(25),P(50)
NL=ND
NL1=NL-1
NP=NL+NL1
DO 6 I=1,NL
6 P(I)=RF(I)
P(NL+1)=AB(1)
DO 7 I=2,NL1
7 P(NL+I)=AB(I)-AB(I-1)
RETURN
END

SUBROUTINE MGUESS(NL,NL1,NP,P)
C INPUT OF THE INITIAL GUESS MADE BY THE USER
C
DIMENSION P(50)
1 WRITE(6,100)
100 FORMAT(/' KEY IN NUMBER OF LAYER OF YOUR INITIAL GUESS')
READ(5,*)NL
2 NL1=NL-1
NP=NL+NL1
WRITE(6,150) NL,NL1
150 FORMATC KEY IN',13,' RESISTIVITIES AND',13,' THICKNESSES'
1,' OF YOUR INITIAL GUESS')
READ(5,*)(P(I),I=1,NP)
RETURN
END

SUBROUTINE STEEP
C STEEPEST DESCENT INVERSION
C
COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
1 ,Q,RC(25),RS(25),A(25,50),PCT(50)
COMMON/FLTR/NC,F,Y,FC(141)
DIMENSION PL(50),PI(50),G(50)
IS=0
QV=99.
ITMAX=15
DO 100 IC=1,ITMAX
10 CALL APPRES(RC,1)
IF(QV.GE.Q) GO TO 20
IS = 1
ST=ST*.8
IF(ST.LT..0001) GO TO 110
DO 15 I=1,NP
15 P(I)=PL(I)*.2+P(I)*.8
GO TO 10
20 \text{QV=Q}
1700 \quad \text{IF}(Q.\text{LE.}.02) \text{GO TO 110}
1710 \quad \text{IF}(IC.\text{EQ.}2) \text{GO TO 110}
1720 \quad \text{CALL DERIV}
1730 \quad \text{C}
1750 \quad \text{DO 25 J=1,NP}
1760 \quad 25 \text{G(J)} = 0.
1770 \quad \text{DO 40 I=1,ND}
1780 \quad \text{BA}=\text{RC}(I)/\text{RS}(I)-1.
1790 \quad \text{DO 40 J=1,NP}
1800 \quad 40 \text{IF}(P(J).\text{GT.}0.) \text{G(J)}=\text{G(J)}-2.*A(I,J)*\text{BA}
1810 \quad \text{C}
1820 \quad \text{GR} = 0.
1830 \quad \text{DO 45 J=1,NP}
1840 \quad 45 \text{GR} = \text{GR} + \text{G(J)}*\text{G(J)}
1850 \quad \text{GR} = \sqrt{\text{GR}}
1860 \quad \text{IF}(\text{IS}.\text{EQ.}0) \text{ST} = \text{AMIN1}(.5,\text{Q*Q})*\text{FLOA}(\text{ND})/\text{GR}
1870 \quad \text{DO 50 J=1,NP}
1880 \quad \text{PL(J)} = P(J)
1890 \quad 50 \text{P(J)} = P(J)*((1.+\text{ST})*\text{G(J)}/\text{GR})
1900 \quad \text{IF}(\text{IC}/4.4.\text{NE.}\text{IC}) \text{GO TO 100}
1910 \quad \text{IF}(\text{IC.\text{EQ.}4}) \text{GO TO 60}
1920 \quad \text{DO 55 J=1,NP}
1930 \quad 55 \text{P(J)}=\text{P(J)}*\text{P(J)}/\text{PI(J)}
1940 \quad \text{DO 70 J=1,NP}
1950 \quad 70 \text{PI(J)}=\text{P(J)}
1960 \quad \text{QV=99.}
1970 \quad \text{CONTINUE}
1980 \quad \text{IC}=\text{IC}-1
1990 \quad \text{C}
2000 \quad \text{WRITE(6,1100) Q,IC,ST}
2010 \quad 1100 \text{FORMAT(' STEEP Q=',F10.5,' AT IC=',I3,' ST=',F10.6)}
2020 \quad \text{RETURN}
2030 \quad \text{END}
2040 \quad \text{C} \quad \text{*******************************************************************************}
2050 \quad \text{SUBROUTINE RIDGE}
2060 \quad \text{C} \quad \text{*******************************************************************************}
2070 \quad \text{C} \quad \text{RIDGE REGRESSION INVERSION}
2080 \quad \text{C} \quad \text{MARQUARDT FACTOR IS DETERMINED THROUGH THE ALGORITHM}
2090 \quad \text{C} \quad \text{GIVEN BY SANDBERG (1979)}
2100 \quad \text{C}
2110 \quad \text{COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP}
2120 \quad 1 \quad ,Q,RC(25),RS(25),A(25,50),PCT(50)
2130 \quad \text{COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)}
2140 \quad \text{DOUBLE PRECISION ATA,ATAINV,W,DSUM}
2150 \quad \text{DIMENSION PL(50),XX(4),YY(4),PCT1(50)}
2160 \quad \text{DO 10 J=1,NP}
2170 \quad \text{PL(J)}=P(J)
2180 \quad 10 \text{PCT(J)=0.}
2190 \quad \text{CALL APPRES(RC,1)}
2200 \quad \text{QL=Q}
2210 \quad \text{XCT=-4.}
2220 \quad \text{XCM=2.}
2230 \quad \text{C}
2240 \quad \text{DO 100 ITE=1,15}
CALL DERIV
DO 107 J=1,NP
DSUM=1.D-25
DO 106 I=1,ND
106 DSUM=DSUM+A(I,J)**2
DO 107 W(J)=1.DO/DSQRT(DSUM)
DO 108 I=1,ND
DO 108 J=1,NP
108 A(I,J)=A(I,J)*W(J)
DO 120 K=1,NP-1
DO 120 J=K+1,NP
SUM=0.
DO 111 I=1,ND
SUM=SUM+A(I,K)*A(I,J)
111 ATA(K,J)=SUM
DO 120 J=1,NP
120 ATA(K,J)=SUM
DO 808 J=1,NP
808 P(J)=PL(J)
XX(I)=XCT
YY(I)=Q
IF(I.I.EQ.4) GO TO 280
IF(I.I.EQ.3) GO TO 820
IF(I.I.EQ.1.AND.XCM.LE.5) XCM=XCM*.9
808 P(J)=PL(J)
XX(I)=XCT
YY(I)=Q
IF(I.I.EQ.4) GO TO 280
IF(I.I.EQ.3) GO TO 820
IF(I.I.EQ.1.AND.XCM.LE.5) XCM=XCM*.9
XCT=XCT*XCM
GO TO 810
820 IF(YY(2).*GT.YY(1).OR.YY(2).*GT.YY(3)) GO TO 812
XCT=-(XX(1)**2*(YY(3)-YY(2))+XX(2)**2*(YY(1)-YY(3)))+1
XX(3)**2*(YY(2)-YY(1)))/(YY(1)*XX(3)-XX(2))+2
YY(2)*XX(1)-XX(3)+YY(3)*XX(2)-XX(1))/2.
820 GO TO 810
812 IF(YY(1).LE.YY(3)) XCT=XX(1)-XCM/2.
812 IF(YY(1).GT.YY(3)) XCT=XX(3)+XCM/2.
810 CONTINUE
IF(ITRAP.EQ.4) GO TO 290
XCT=-4.
XCM=2.
GO TO 800
280 ITRAP=ITRAP+1
IF(ITRAP.EQ.3) GO TO 290
XXT=-4.
XCM=2.
GO TO 800
DO 160 J=1,NP
160 PCT1(J)=(EXP(SQRT(SNGL(ATAINV(J,J)))*Q)-1.)*100.
250 DO 160 J=1,NP
2740 PCT1(J)=(EXP(SQRT(SNGL(ATAINV(J,J)))*Q)-1.)*100.
2750 160 PL(J)=P(J)
2760 IF(Q.LE.0.001) GO TO 300
2770 IF(XCM.GT.5) XCM=XCM/2.
2780 XCT=XCT-XCM
2790 100 QL=Q
2800 C
2900 C
C Q = QL
290 Q = QL
291 ITE = ITE - 1
292 WRITE (6, 3000) Q, ITE, FM
293 WRITE (8, 3000) Q, ITE, FM
294 FORMAT ( ' RIDGE Q = ', F10.5, ' AT ITE = ', I3, ' FM = ' , F10.6)
295 DO 625 J = 1, NP
296 IF (ITE .NE. 0) PCT(J) = AMIN1 (99.99, PCT1(J))
297 RETURN
298 END
299 C **********************************************************
300 C SUBROUTINE DERIV
301 C **********************************************************
302 C ANALYTICAL DERIVATIVE OF LOG(RC) WITH RESPECT TO LOG(P)
303 C
304 COMMON TITLE (4), ND, AB(25), RF(25), MODE, P(50), NL, NL1, NP
305 COMMON Q, RC(25), RS(25), A(25, 50), PCT(50)
306 COMMON /FLTR/ NC, F, Y, FC(141)
307 COMMON MAT /ATA(50, 50), ATAINV(50, 50), H(50, 25), W(50)
308 DIMENSION D(100, 50), RD(25)
309 C
310 IF (MODE .LT. 0) GO TO 200
311 C
312 X = EXP(Y) / AB(1)
313 NT = NC + ND - 1
314 DO 23 L = 1, NT
315 T = ABS(P(NL))
316 D(L, NL) = 1.
317 DO 22 K = 1, NL1
318 IZ = NP + 1 - K
319 IR = NL - K
320 Z = ABS(P(IZ))
321 R = ABS(P(IR))
322 TH = TANH(Z*X)
323 CH = 1.E60
324 IF (Z*X .LT. 130.) CH = COSH(Z*X)
325 B2 = (1. + T*TH/R)**2
326 DTDT = (1. - TH**TH) / B2
327 D(L, IZ) = (R - T*TH) / B2**X / CH / CH
328 D(L, IR) = TH*(1. + T*T/(R*R) + 2.*TH*T/R) / B2
329 IF (K .EQ. 1) GO TO 32
330 IS = NP + 2 - K
331 DO 30 IZ = IS, NP
332 D(L, IZ) = DTDT*D(L, IZ)
333 DO 31 IZ = IS, NL
334 D(L, IZ) = DTDT*D(L, IZ)
335 T = (T + TH*R) / (1. + TH*T/R)
336 X = X / F
337 DO 35 I = 1, ND
338 DO 35 J = 1, NP
339 SUM = 0.
340 DO 37 L = 1, NC
341 SUM = SUM + FC(L)*D(L + I - 1, J)
342 A(I, J) = ABS(P(J)) / RC(I)*SUM
343 GO TO 300
344 C
DO 210 I=1,ND
X=EXP(Y)/AB(I)
DO 223 L=1,NC
T=ABS(P(NL))
D(L,NL)=1.
DO 222 K = 1,NL1
IZ = NP+1-K
IR = NL-K
Z=ABS(P(IZ))
R=ABS(P(IR))
TH = TANH(Z*X)
CH = 1.E60
IF(Z*X.LT.130.) CH=COSH(Z*X)
B2 = (1.*T*TH/R)**2
DTDT = (1.-TH*TH)/B2
D(L,IZ) = (R-T*T/R)/B2*X/CH/CH
D(L,IR) = TH*(1.+T*T/(R*R) + 2.*TH*T/R)/B2
IF (K.EQ.1) GO TO 232
IS=NP+2-K
DO 230 IZ = IS,NP
D(L,IZ) = DTDT*D(L,IZ)
DO 231 IZ = IS,NL
D(L,IZ) = DTDT*D(L,IZ)
T = CT+TH*R)/C1.+TH*T/R)
X = X/F
DO 210 J=1,NP
SUM=0.
DO 237 L=1,NC
SUM=SUM+FCCL)*DCL,J)
A(I,J)=ABS(P(J))/RC(I)*SUM
DO 310 J=1,NP
IF(CPCJ).GT.0.) GO TO 310
DO 320 1=1,ND
AU,J)=1.E20
CONTINUE
CONTINUE
RETURN
END

SUBROUTINE NEWPCFM
MARQUARDT INVERSION

COMMON TITLEU),ND,ABC25),RFC25),MODE,PC50),NL,NL1,NP
COMMON/MAT/ATAC50,50),ATAINVC50,50),HC50,25),WC50>
DOUBLE PRECISION ATA,ATAINV,W
DIMENSION XC50)
DO 25 K=1,NP
ATA(K,K)=1.DO+FM
CALL MAINE(NP,ATA,ATAINV)
DO 5 J=1,NP
DO 5 I=1,ND
SUM=0.
DO 4 K=1,NP
SUM=SUM+ATAINV(J,K)*A(I,K)
H(J,I)=SUM
C
DO 35 J=1,NP
SUM=0.
DO 30 I=1,ND
ERROR=ALOG(RC(I)/RS(I))
IF(ERROR.GE. .03) ERROR=.03
IF(ERROR.LT.-.03) ERROR=-.03
30 SUM=SUM+H(J,I)*ERROR
X(J)=SUM*W(J)
IF(X(J).GT. .7) X(J)= .7
IF(X(J).LT.-.7) X(J)=-.7
P(J)=P(J)*EXP(X(J))
35 IF(P(J).GE.99999.) P(J)=99999.
C
CALL APPRESCRC,!
RETURN
END

SUBROUTINE MAINE ( N, A , AINV)
************************************************************
INVERSION OF SYMMETRIC MATRIX AFTER SANDBERG (1979)
REAL* 8 A(50,50),AINV(50,50),R(50),C(50),DEL,CC,RR
DO 10 1=1, N
DO 10 J=1,N
AINV(I,J)=0.DO
DO 40 L=1,N
DEL=A(L,L)
DO 30 I=1,L
CC=0.
RR=0.
DO 20 J=1,L
CC=CC+AINV(I,J)*A(J,L)
20 RR=RR+A(L,J)*AINV(J,I)
40 C(I)=CC
R(I)=RR
45 CONTINUE
DO 40 I=1,L
C(1)=C(1)/DEL
DO 40 J=1,L
40 AINV(I,J)=AINV(I,J)+C(I)*R(J)
RETURN
END

C SUBROUTINE AUTODZ(JMODE)

C AUTOMATIC DAR ZARROUK REDUCTION
C MODIFIED VERSION OF DZSMTH (ZOHDY, 1973)

COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
COMMON/DARZ/DZR(25),DZD(25),TT(25),SS(25),DIFF(25)
DIMENSION DZDR(25),DZRR(25),DZRA(25)
DIMENSION PP(25),SSR(25),TTR(25)
DIMENSION DZDRF(25),DZRRF(25)
FT=.08
FT1=.08
NBR=1
SUMLOG=ALOG(DZR(1))
DZRA(1)=DZR(1)
DO 1 NN=2,NL
SUMLOG=SUMLOG+ALOG(DZR(NN))
DZRA(NN)=EXP(SUMLOG/NN)
DO 1 1 = 1, NN
IF(ABS(ALOG(DZR(I)/DZRA(NN))) .GT.FT1) GO TO 3
CONTINUE
FT1=FT1/1.5
GO TO 2
NTAP=NN-1
PF(1)=DZRA(NTAP)
AOF(1)=0.0
A1F(1)=DZRA(NTAP)**2
IF(CNL-NTAP).EQ.1) GO TO 8
NEXT=NTAP
SUMT=TT(NEXT)
SUMS=SS(NEXT)
SUMSS=SS(NEXT)**2
SUMST=TT(NEXT)*SS(NEXT)
NEXT = NEXT-1
NN=NEXT-NTAP-1
SUMT=TT(NEXT)+SUMT
SUMS=SS(NEXT)+SUMS
SUMSS=SS(NEXT)**2
SUMST = SS(NEXT)*TT(NEXT)+SUMST
A1C(NN)=(NN*SUMST-SUMT*SUMS)/(NN*SUMSS-SUMS*SUMS)
ACO(NN)=(SUMT-A1C(NN)*SUMS)/NN
PP(NN)=SQRT(A1C(NN))
SSR(NBR)=(AO(NN)-AOF(NBR))/(A1F(NBR)-A1(NN))
TTR(NBR)=AOF(NBR)*A1F(NBR)*SSR(NBR)
IF(TTR(NBR).LT.0.OR.SSR(NBR).LT.0.)GO TO 15
DZDR(NN)=SQRT(TTR(NBR)*SSR(NBR))
5050
DZR(NN) = SQRT(TTR(NBR)/SSR(NBR))
5060
IF(NN .EQ. 2) GO TO 21
5070
IF(DZD(NN) .GT. DZD(NTAP+1)) GO TO 15
5080
IF(NTAP .EQ. 1) GO TO 33
5090
IF(DZD(NN) .LT. DZD(NTAP-1)) GO TO 15
5100
IST = NTAP
5110
IF(DZD(NTAP)/DZD(NN) .LE. 1.02) IST = NTAP + 1
5120
C
5130
34 DO 14 I = IST, NEXT
5140
DZD2 = DZD(I) * 1.02
5150
DZD1 = DZD(I) / 1.02
5160
W2 = -DZR(NN) *(PP(NN)**2 - DZR(NN)**2)
5170
W2 = W2**2
5180
ZZ = 4.* (DZD2*DZD(NN) * PP(NN)**2)
5190
DZR2 = (W+SQRT(W2+ZZ))/ (2.*DZD2*DZD(NN))
5200
Z1 = 4.* (DZD1*DZD(NN) * PP(NN)**2)
5210
DZR1 = (W+SQRT(W2+ZZ))/ (2.*DZD1*DZD(NN))
5220
Z = 4.* (DZD(I) * DZD(NN) * PP(NN)**2)
5230
DZRA(I) = (W+SQRT(W2+ZZ))/ (2.*DZD(I)*DZD(NN))
5240
SLP = ALOG10(DZR2/DZR1)/ALOG10(DZD2/DZD1)
5250
IF(SLP .LT. 0.) TOL = (-TOL)**SLP + FT
5260
IF(SLP .GE. 0.) TOL = (-TOL)**SLP + FT
5270
IF(ABS(SLP)/DZRA(I)/DDZRA(I)) .GT. TOL) GO TO 15
5280
CONTINUE
5290
IF(NEXT .EQ. NL) GO TO 16
5300
GO TO 21
5310
C
5320
15 IF(NN .EQ. 2) GO TO 600
5330
NTAP = NN + NTAP - 2
5340
NN = NN - 1
5350
DZDRF(NBR) = DZD(NN)
5360
DZRNF(NBR) = DZD(NN)
5370
NBR = NBR + 1
5380
PF(NBR) = PP(NN)
5390
AOF(NBR) = AO(NN)
5400
A1F(NBR) = A1(NN)
5410
GO TO 22
5420
C
5430
8 NBR = NBR + 1
5440
PF(NBR) = SQRT((TT(NL) - TT(NL-1))/(SS(NL) - SS(NL-1)))
5450
IF(ABS(SLP)/PF(NBR)/PF(NBR-1)) .GT. 1) GO TO 340
5460
NBR = NBR - 1
5470
GO TO 500
5480
C
5490
340 A1F(NBR) = PF(NBR)**2
5500
AOF(NBR) = TT(NL) - PF(NBR)**2*SS(NL)
5510
SSR(NBR-1) = (AOF(NBR) - AOF(NBR-1))/ (A1F(NBR-1) - A1F(NBR))
5520
TTR(NBR-1) = AOF(NBR-1) + A1F(NBR-1) * SSR(NBR-1)
5530
DZRRF(NBR-1) = SQRT(TTR(NBR-1)/SSR(NBR-1))
5540
DZDRF(NBR-1) = SQRT(TTR(NBR-1) * SSR(NBR-1))
5550
GO TO 500
5560
C
5570
16 NBR = NBR + 1
5580
PF(NBR) = PP(NN)
5590
DZRRF(NBR-1) = DZRR(NN)
5600
DZDRF(NBR-1) = DZDR(NN)
5610 500  TTT=DZDRF(NBR'-1)*DZRRF(NBR-1)+999.*PF(NBR)
5620  SSS=DZDRF(NBR'-1)/DZRRF(NBR-1)+999./PF(NBR)
5630  DZDRF(NBR)=SQRT(TTT*SSS)
5640  DZRRF(NBR)=SQRT(TTT/SSS)
5650

C

5660  510  II=1
5670    JJ=1
5680  515  IF(DZDRF(II+1)/DZDRF(II).GT.1.5) GO TO 520
5690  DZDRF(JJ)=SQRT(DZDRF(II)*DZDRF(II+1))
5700  DZRRF(JJ)=SQRT(DZRRF(II)*DZRRF(II+1))
5710  II=II+1
5720  IF(II.EQ.NBR) GO TO 540
5730  GO TO 530
5740  520  DZDRF(JJ)=DZDRF(II)
5750  DZRRF(JJ)=DZRRF(II)
5760  530  II=II+1
5770    JJ=JJ+1
5780  IF(II.NE.NBR) GO TO 515
5790  DZDRF(JJ)=DZDRF(II)
5800  DZRRF(JJ)=DZRRF(II)
5810  540  NBR=JJ
5820  IF(NBR.LT.NL) GO TO 550
5830  C
5840    FT=FT+.02
5850    IF(FT.LE.12) GO TO 2
5860    JMODE=1
5870    IF(NBR.EQ.NL) RETURN
5880  C
5890  550  JMODE=0
5900    WRITE(6,505) NBR
5910    WRITE(8,505) NBR
5920  505  FORMAT(/' AUTOMATIC DZ REDUCTION REDUCED NL TO',I3)
5930    NL=NBR
5940    NL1=NL-1
5950    NP=NL+NL1
5960    DO 555 I=1,NL
5970    DZD(I)=DZDRF(I)
5980  555  DZR(I)=DZRRF(I)
5990    P(I)=DZR(I)
6000    P(NL+1)=DZD(1)
6010    DO 560 I=2,NL
6020    U=DZD(I)*DZR(I)-DZD(I-1)*DZR(I-1)
6030    V=DZD(I)/DZR(I)-DZD(I-1)/DZR(I-1)
6040    IF(I.NE.NL) P(I+NL)=SQRT(U/V)
6050  560  P(I)=SQRT(U/V)
6060  RETURN
6070  C
6080    600  WRITE(6,610)
6090    WRITE(8,610)
6100  610  FORMAT(/' AUTOMATIC DZ REDUCTION IN TROUBLE')
6110    JMODE=1
6120  RETURN
6130  END
6140  C **********************************************************
6150  SUBROUTINE MANUDZ
6160  C **********************************************************
C MANUAL DAR ZARROUK REDUCTION

C COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
C COMMON/DARZ/DZR(25),DZD(25),TT(25),SS(25),DIFF(25)
C DIMENSION IL(25)

C WRITE(8,1000)
1000 FORMAT(' MANUAI DZ REDUCTION')
C WRITE(6,2000)
2000 FORMAT(' KEY IN NUMBER OF D-Z POINTS TO BE DELETED'
1 1/0 TO GIVE MAXIMUM % DIFF VALUE')
C READ(5,*) M
C IF(M.EQ.0) GO TO 300
C WRITE(6;3000) M
C 3000 FORMAT(' KEY IN',I3,' LAYER NUMBERS TO BE DELETED')
C READ(5,*) (IL(I),I=1,M)
C IL(M+1)=0
C II=1
C IJ=1
C DO 360 I=1,NL
C IF(I.EQ.IL(IJ)) GO TO 370
C DZR(I)=DZR(I)
C DZD(I)=DZD(I)
C II=II+1
C GO TO 360
C 370 IJ=IJ+1
C 360 CONTINUE
C NL=II-1
C GO TO 350
C
C 300 WRITE(6,3100)
3100 FORMAT(' KEY IN THE MAXIMUM % DIFF VALUE OF'
1,' DZ POINTS TO BE DELETED')
C READ(5,*) PD
C II=1
C DO 310 I=1,NL1
C IF(ABS(DIFF(I)).LT.PD) GO TO 310
C DZR(I)=DZR(I)
C DZD(I)=DZD(I)
C II=II+1
C 310 CONTINUE
C DZR(NL)=DZR(NL)
C DZD(NL)=DZD(NL)
C NL=II
C 350 NL1=NL-1
C NP=NL+NL1
C P(1)=DZR(1)
C P(NL+1)=DZD(1)
C DO 320 I=2,NL
C U=DZD(I)*DZR(I)-DZD(I-1)*DZR(I-1)
C V=DZR(I)/DZR(I-1)-DZD(I-1)/DZR(I-1)
C IF(I.NE.NL) P(I+NL)=SQRT(U*V)
C 320 P(I)=SQRT(U/V)
C RETURN
C END

C *****************************************************************
SUBROUTINE RESULT(INV)

C ********************************************

COMMON TITLE(4),ND,AB(25),RF(25),MODEL,P(50),NL,NL1,NP

1 Q,RC(25),RS(25),A(25,50),PCT(50)

COMMON/DARRZ,DZR(25),DZD(25),TT(25),SS(25),DIFF(25)

DIMENSION DEP(25),DZRI(25)

WRITE(6,93) TITLE,INV

WRITE(8,93) TITLE,INV

93 FORMAT(/' INVERSION OF FIELD DATA: ',4A4,' AT INV=',I2)

DEP(I)=ABS(P(NL+1))

IF(NL1.EQ.1) GO TO 81

DO 80 I=2,NL1

80 DEP(I)=DEP(I-1)+ABS(P(NL+1))

C

81 TTT=0.

SSS=0.

DO 200 I=1,NL

RI=ABS(P(I))

IF(I.NE.NL) HI=ABS(P(NL+1))

TTT=TTT+HI*RI

SSS=SSS*HI/RI

TT(I)=TTT

SS(I)=SSS

DZR(I)=SQRT(TTT/SSS)

DZD(I)=SQRT(TTT*SSS)

DZRI(I)=(DZR(I)+DZR(2))/2.

DIFF(I)=(DZRI(I)/DZR(I)-1.)*100.

DO 700 I=2,NL1

R2=(TT(I-1)-TT(I-1))/(SS(I+1)-SS(I-1))

U=DZD(I-1)*(R2-DZR(I-1)**2)/(2.*DZD(I)*DZR(I-1))

DZRI(I)=-U+SQRT(U*U+R2)

200 DIFF(I)=(DZRI(I)/DZR(I)-1.)*100.

C

WRITE(6,2500)

WRITE(8,2500)

WRITE(6,2550)(I,P(I),PCT(I),P(I+NL),PCT(I+NL),DEP(I)

1,DZR(I),DZD(I),DIFF(I),I=1,NL1)

WRITE(8,2550)(I,P(I),P(I+NL),PCT(I+NL),DEP(I)

1,DZR(I),DZD(I),DIFF(I),I=1,NL1)

WRITE(6,2560)NL,P(NL),PCT(NL),DZR(NL),DZD(NL)

WRITE(8,2560)NL,P(NL),PCT(NL),DZR(NL),DZD(NL)

2500 FORMAT(/' NO',4X,'RESIS PCT',4X,'THICK PCT',4X

1,'DEPTH',6X,'DZR',6X,'DZD',3X,'DIFF')


2560 FORMAT(I3,F9.2,F8.2,26X,2F9.2)

C

WRITE(8,210)

210 FORMAT(/' NO',11X,'AB/2',9X,'RHOCAL',9X

1,'RHOBS',8X,'% ERROR')

DO 224 I=1,ND

ERROR=(RC(I)/RF(I)-1.)*100.

224 WRITE(8,230)I,AB(I),RC(I),RF(I),ERROR

230 FORMAT(I3,F15.2,F15.3,F15.3,F15.4)

260 RETURN

270 END

C ********************************************
SUBROUTINE MATRIX

C **********************************************************************
C COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
C
1 COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)
C DOUBLE PRECISION ATA,ATAINV,W,DSUM
C DIMENSION B(50),WORK(25)
C
C WRITE(8,1000)
C 1000 FORMAT(/' PARAMETER CORRELATION MATRIX')
C NR=(NP+1)/2
C DO 10 J=1,NP
C DO 10 K=1,J
C 10 ATAINV(J,K)=ATAINV(J,K)*W(J)*W(K)
C NT=NP-NR
C DO 40 J=1,I
C WORK(J)=ATAINV(I,J)/DSQRT(ATAINV(I,I)*ATAINV(J,J))
C IF(I.GT.NR) GO TO 30
C B(I)='R'
C WRITE(8,1100)I,(WORK(J),J=1,1)
C 30 CONTINUE
C WRITE(8,1300)(B(J),J,J=1,NR),(B(K+NR),K,K=1,NT)
C 40 FORMAT(/' RESOLUTION MATRIX')
C DO 60 J=1,I
C SUM=0.
C DO 50 K=1,ND
C SUM=SUM+H(I,K)*A(K,J)
C WORK(J)=SUM
C IF(I.GT.NR) GO TO 80
C B(I)='R'
C WRITE(8,1100)I,(WORK(J),J=1,I)
C 60 CONTINUE
C WRITE(8,1300)(B(J),J,J=1,NR),(B(K+NR),K,K=1,NT)
C 70 FORMAT(/' INFORMATION DENSITY MATRIX')
C DO 110 K=1,ND
C DO 100 J=1,NP
C SUM=SUM+A(I,J)*H(J,K)
C 100 SUM=SUM+A(I,J)*H(J,K)
C
C ************************************************************
SUBROUTINE APPRES(RA,IQ)
C ************************************************************
COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
COMMON/FLTR/NC,F,Y,FC(141)
DIMENSION RA(25),T(100)
C
IF(MODE.LE.0) GO TO 100
X=AB(1)*EXP(-Y)
LE = NC+ND-1
DO 10 L = 1,LE
  B = ABS(P(NL))
DO 9 K = 1,NL1
  RW=ABS(P(NL-K))
  TH = EXP(-2.*ABS(P(NP+1-K))/X)
  TP = 1.+TH
  TM = 1.-TH
  B = RW*(B*TP+RW*TM)/(RW*TP+B*TM)
T(L) = B
X = X*F
10 DO 12 J = 1,ND
  RM = 0.
  DO 11 L = 1,NC
    RM = RM+FC(L)*T(L+J-1)
  RA(J) = RM
12 GO TO 200
110 110 DO 110 I = 1,ND
  RM = 0.
  X=AB(I)*EXP(-Y)
  DO 120 J=1,NC
    B = ABS(P(NL))
    RW=ABS(P(NL-K))
    TH=EXP(-2.*ABS(P(NP+1-K))/X)
    TP=1.+TH
    TM=1.-TH
    B=RW*(B*TP+RW*TM)/(RW*TP+B*TM)
    X=X*F
    RM=RM+FC(J)*B
110 110 RA(I) = RM
200 IF(IQ.EQ.O) RETURN
210 210 DO 210 1=1,ND
  Q=0.
  DO 220 I=1,ND
    Q=Q+(RA(I)/RS(I)-1.)*2
220 220 Q=SQRT(Q/FLOATCND))
RETURN
END
C **********************************************************************
SUBROUTINE YM6

COMMON/FLTR/NC,F,Y1,FCC(141)

NC : NUMBER OF COEFFICIENTS
FCCNC) : FILTER COEFFICIENTS
Yl : ABSCISSA OF FC(l)

F = EXP(D), WHERE SAMPLING DISTANCE D = (LN10)/6

ABSCISSA OF FC(J) = Y1-(J-1)*D

DIMENSION FD(28)

DATA FD/

NC = 28
Yl=7.4222404
DO 100 1=1,NC
100 FC(I)=FD(I)
RETURN
END

SUBROUTINE YM10

COMMON/FLTR/NC,F,Y1,FCC(141)

NC = 70
F=10.**1.1
Yl=12.664218
DO 100 1=1,NC
100 FC(I)=FD(I)
RETURN
END