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A COMPUTER PROGRAM FOR THE AUTOMATIC INVERSION OF SCHLUMBERGER SOUNDINGS USING MULTI-LAYER INTERPRETATION FOLLOWED BY DAR ZARROUK REDUCTION

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curve, the reduced model is fed back into the least squares inversion procedure.

PROGRAM RESINV

The automatic inversion program for Schlumberger soundings consists of a main program (RESINV) and 11 subroutines. The complete source listing of this interactive program is given in the Appendix. This program was developed using DEC's VAX 11/780 computer, and was successfully transferred to FUJITSU's FACOM M200 and IBM 4341/10 computers. The notation used in the subsequent description is listed in Table 1.

TABLE 1

ND	:number of data points
AB(I)	:half electrode spacing, AB/2
RF(I)	:observed apparent resistivity
RS(I)	:smoothed apparent resistivity
RC(I)	:calculated apparent resistivity
NL	:number of layers
NP	:number of layer parameters = 2•NL-1
P(J)	:layer parameters (NL resistivities followed by NL-1 thicknesses)
INV	:inversion no.
IC	:iteration counter for STEEP
ITE	:iteration counter for RIDGE
Q	:error criterion, eq.(1) or eq.(3)
RE	:relative error, eq.(2)
A(I,J)	:sensitivity matrix, eq.(7) or eq.(14)
G(J)	:negative gradient, eq.(6)
ST	:step length, eq.(11)
FM	:Marquardt factor
T(L+I-1)	:resistivity transform, eq.(21)
D(L+I-1)	:derivative of T, eq.(15)
FC(L)	:convolution filter
NC	:number of coefficients
TT(I)	:transverse resistance
SS(I)	:longitudinal conductance
DZR(I)	:Dar Zarrouk resistivity
DZD(I)	:Dar Zarrouk depth

The calculation of apparent resistivity is done by subroutine APPRES using the linear filter method (Murakami and others, 1984). Depending on the value of MODE supplied by the subroutine FDATA, either of the filters, YM6 (6 points per log-decade) or YM10 (10 points per log-decade), is called.

The array RS stands for the smoothed apparent resistivity curve. Because there are no reliable automatic means of smoothing noisy field curves, RS values can be used to replace the noisy field data (RF) after the first inversion (INV=1) when computing equivalent solutions with a smaller number of layers. This can be applied because we assume that the initial multi-layer interpretation fits the global features of the observed curve, and that any further improvement will end up fitting the noise.

The subroutine AGUESS automatically prepares an initial guess of parameters. Next, two inversion schemes, steepest descent and ridge regression programmed in subroutines STEEP and RIDGE, are called successively. The steepest descent iterative inversion method is relatively insensitive to poor initial guesses, but it converges very slowly after a fit of about 2% is obtained. The program then transfers to the ridge regression inversion because this converges faster as the layer model approaches the least squares fit.

When no parameters are fixed (automatic initial guess mode), the subroutine AUTODZ tries to obtain equivalent solutions composed of a smaller number of layers. This reduction is performed in the Dar Zarrouk domain. The fundamental idea of this reduction, developed by Zohdy (1975), is that two similar Dar Zarrouk curves will give similar apparent resistivity curves. If the Dar Zarrouk curve, calculated from the detailed model, can be approximated using a smaller number of Dar Zarrouk points, this reduced Dar Zarrouk curve is transformed into the corresponding layering model. This reduced layering will be almost equivalent to the original layering in the apparent resistivity domain. Zohdy (1974), however, found models where two almost identical Dar Zarrouk curves with very long steeply descending branches gave appreciably different resistivity curves, pointing out the need for careful smoothing of long steeply descending Dar Zarrouk branches. In order to better fit the field data this reduced model is fed back to the inversion schemes STEEP and RIDGE to obtain the least squares fit in the apparent resistivity domain.

If no reduction is performed by subroutines AUTODZ, the computer asks the user to choose one of four options. By keying in 1, the user can change the initial guess. In this mode the user can input his own initial guess and fix some of the parameters by putting negative signs before them. By keying in 2, MANUDZ, the manual reduction subprogram using the Dar Zarrouk parameters, is called. By keying in 3, printouts of the correlation matrix, resolution matrix, and information density matrix are obtained. By keying in 4, the program stops.

SUBROUTINE AGUESS

The subroutine AGUESS automatically constructs an initial guess, where the number of layers NL is set equal to the number of observed data points ND. The observed apparent resistivities are used as the initial layer resistivities, and AB/2 values are used as the initial layer depths.

SUBROUTINE STEEP

The subroutine STEEP performs the steepest descent inversion. This iterative inversion scheme is chosen because it is insensitive to poor initial guesses. The convergence speed is initially very rapid, but slows as model refinement progresses. When a fit of 2% is obtained the ridge regression inversion scheme is called.

The most common error criterion to be minimized in the least squares inversion is the sum of the squares of either the relative error:

$$Q = \sum_{I=1}^{ND} (RE(I))^2 \qquad (1)$$

where

$$RE(I) = \frac{RC(I)}{RS(I)} - 1 \qquad (2)$$

and RC = calculated resistivities, RS = field resistivities; or the logarithmic differences between the values calculated from the model and the observed data:

$$Q = \sum_{I=1}^{ND} (\ln RC(I) - \ln RS(I))^2 \qquad (3)$$

Due to the small difference between the relative error and the logarithmic difference (see Table 2) they are frequently interchanged.

TABLE 2

Difference between the relative error (RE) and the logarithmic (LOG) difference. As long as the percent error is between ±5, RE and LOG DIFFERENCE are almost the same. LOG DIFFERENCE is a better estimation of difference, because the apparent resistivity is a positive-valued function.

% ERROR	RE	LOG DIFF.
-50	-.50	-.69315
-20	-.20	-.22314
-10	-.10	-.10536
- 5	-.05	-.05129
- 2	-.02	-.02020
- 1	-.01	-.01005
0	.00	.00000
1	.01	.00995
2	.02	.01980
5	.05	.04897
10	.10	.09531
20	.20	.18231
50	.50	.40547
100	1.00	.69315

Vozoff (1958), Wang and Treitel (1973), and Koefoed (1979) have presented algorithms for the steepest descent method. The version by Koefoed is used here. The basic idea of the steepest descent method is to change the value of the layer parameters P(J) (resistivities, thicknesses) into the direction of the steepest descent of the error criterion, that is, in the direction of the negative gradient of the error criterion:

$$\text{Set } \ln P(J) = \ln P(J) + C \cdot G(J) \tag{4}$$

or

$$\begin{aligned}\text{Set } P(J) &= P(J)\exp(C \cdot G(J)) \\ &\cong P(J)(1 + C \cdot G(J))\end{aligned} \tag{5}$$

where the negative gradient G(J) is defined by

$$\begin{aligned}G(J) &= \frac{-\partial Q}{\partial \ln P(J)} \\ &= -\sum_{I=1}^{ND} 2(\ln RC(I) - \ln RS(I)) \frac{\partial \ln RC(I)}{\partial \ln P(J)} \\ &\approx -\sum_{I=1}^{ND} 2 \cdot RE(I) A(I,J)\end{aligned} \tag{6}$$

and the sensitivity matrix A(I,J) is defined by

$$A(I,J) = \frac{\partial \ln RC(I)}{\partial \ln P(J)} = \frac{P(J)}{RC(I)} \frac{\partial RC(I)}{\partial P(J)} \tag{7}$$

The value of C in equations (4) and (5) is chosen following the suggestion of Vozoff (1958) and Koefoed (1979, p. 163), as:

$$C = Q/(GR)^2 \tag{8}$$

where

$$GR = |\nabla Q| = (\sum_{J=1}^{NP} G(J)G(J))^{1/2} \tag{9}$$

Substituting equation (8) into equation (5) we obtain

$$\text{Set } P(J) = P(J)(1 + ST \cdot G(J)/(GR)) \tag{10}$$

where the step length ST is given by

$$ST = Q/GR \tag{11}$$

As discussed by Koefoed (1979, p. 163-164), the step length given by equation (11) may be overestimated when Q approaches very near to its minimum value. When a new P leads to an increase in Q, ST is reduced by a factor of 0.8, and remains unchanged until Q increases. For each increase in Q,ST is multiplied by .8 until it becomes smaller than 0.0001. At that point iteration is stopped. The iteration is also stopped if Q becomes smaller than the fitting tolerances (2%).

To enhance convergence, at every fourth iteration the value of P(J) is changed by setting

$$P(J) = P(J) \cdot P(J)/PI(J) \tag{12}$$

where PI(J) is the value of P(J) four iterations before. This is done because the ratio P(J)/PI(J) is considered to be the trend of convergence.

SUBROUTINE DERIV

The subroutine DERIV calculates the sensitivity matrix A(I,J), defined by equation (7). The analytical derivatives are obtained through the following relations (Koefoed, 1979, p. 164):

$$RC(I) = \sum_{L=1}^{NC} T(L+I-1)FC(L) \tag{13}$$

$$A(I,J) = \frac{P(J)}{RC(I)} \sum_{L=1}^{NC} D(L+I-1,J)FC(L) \tag{14}$$

where

$$D(L+I-1,J) = \frac{\partial T(L+I-1)}{\partial P(J)} \tag{15}$$

with

$$\frac{\partial T}{\partial t_i} = \frac{\partial T_1}{\partial t_i} = \frac{\partial T_1}{\partial T_2}\frac{\partial T_2}{\partial T_3} \cdots\cdots\cdots\cdots \frac{\partial T_i}{\partial t_i} \tag{16}$$

$$\frac{\partial T}{\partial \rho_i} = \frac{\partial T_1}{\partial \rho_i} = \frac{\partial T_1}{\partial T_2}\frac{\partial T_2}{\partial T_3} \cdots\cdots\cdots\cdots \frac{\partial T_i}{\partial \rho_i} \tag{17}$$

$$\frac{\partial T_i}{\partial T_{i+1}} = \frac{1-\tanh^2(\lambda t_i)}{(1+T_{i+1}\tanh(\lambda t_i)/\rho_i)^2} \tag{18}$$

$$\frac{\partial T_i}{\partial t_i} = \frac{\rho_i - T_{i+1}^2/\rho_i}{(1 + T_{i+1}\tanh(\lambda t_i)/\rho_i)^2} \frac{\lambda}{\cosh^2(\lambda t_i)} \tag{19}$$

$$\frac{\partial T_i}{\partial \rho_i} = \frac{\tanh(\lambda t_i)\ (1+T_{i+1}^2/\rho_i^2 + 2\tanh(\lambda t_i)(T_{i+1})/\rho_i)}{(1+T_{i+1}\tanh(\lambda t_i)/\rho_i)^2} \qquad (20)$$

and

$$T_i = \frac{T_{i+1} + \rho_i \tanh(\lambda t_i)}{1+T_{i+1}\tanh(\lambda t_i)/\rho_i} \qquad (21)$$

If P(J) is negative, the derivative with respect to P(J) is set to a very large value:

$$A(I,J) = 10^{20}, \text{ for } I=1,2,\ldots\ldots,ND \qquad (22)$$

so that the increment on P(J) becomes negligibly small.

SUBROUTINES RIDGE, NEWP, AND MAINE

The subroutine RIDGE performs the Gauss-Newton inversion of the linearized least squares problem. Ridge regression is used for its stable inversion of the system matrix. The formulation based on the solution of normal equations was studied by Inman (1975) and Rijo and others (1977). The computer program used by Rijo and others (1977) was published by Sandberg (1979). Another technique known as the generalized matrix inverse was discussed by Johansen (1977). Here we follow the method of Rijo and others (1977).

The minimization of the error criterion Q, defined by equation (3), is simplified by the linearization approximation of the apparent resistivity function RC. Expanding lnRC in a Taylor series and discarding terms higher than first order, we obtain

$$Q = \sum_{I=1}^{ND} \left(\ln RC(I) + \sum_{J=1}^{NP} \frac{\partial \ln RC(I)}{\partial \ln P(J)}\,\delta \ln P(J) - \ln RS(I)\right)^2$$

$$= \sum_{I=1}^{ND} \left(\sum_{J=1}^{NP} A(I,J)X(J) + \ln\frac{RC(I)}{RS(I)}\right)^2 \qquad (23)$$

where

$$X(J) = \delta \ln P(J) \qquad (24)$$

By taking the partial derivative of equation (23) with respect to X(K) and setting it equal to zero, we obtain the normal equations:

$$\frac{\partial Q}{\partial X(K)} = 2\cdot\sum_{I=1}^{ND} \left(\sum_{J=1}^{NP} A(I,J)X(J) + \ln\frac{RC(I)}{RS(I)}\ A(I,K)\right) = 0$$

or

$$\sum_{J=1}^{NP} \left(\sum_{I=1}^{ND} A(I,J)A(I,K))X(J) \right) = -\sum_{J=1}^{ND} \ln\frac{RC(I)}{RS(I)} A(I,K) \tag{25}$$

The matrix representation of equation (25) is

$$A^T A \cdot \chi = -A^T b \tag{26}$$

where

$$A^T A(K,J) = \sum_{I=1}^{ND} A(I,J)A(I,K) \tag{27}$$

$$\chi = (X(1),(X(2),\ldots\ldots\ldots,X(NP))^T \tag{28}$$

$$A^T(K,I)=A(I,K) \tag{29}$$

and

$$b = \left(\frac{\ln RC(1)}{\ln RS(1)}, \frac{\ln RC(2)}{\ln RS(2)},\ldots\ldots\ldots,\frac{\ln RC(ND)}{\ln RS(ND)}\right)^T \tag{30}$$

The formal solution of equation (26) is given by

$$\chi = -(A^T A)^{-1} A^T b \tag{31}$$

In order to enhance the stability of the matrix inversion in equation (31), the concept of scaling is utilized. In the original linear simultaneous equations:

$$A \cdot \chi = -b \tag{32}$$

the matrix A and the vector χ are scaled in the following way:

$$A'(I,J)=A(I,J)W(J) \tag{33}$$

and

$$X'(J)=\frac{X(J)}{W(J)} \tag{34}$$

with

$$W(J) = \left(\sum_{I=1}^{ND} A(I,J)^2\right)^{-1/2} \tag{35}$$

After scaling, the diagonal components of the matrix $A'^T A'$ become unit.

If the initial guess is so poor that the linearization approximation in equation (23) fails, the step X(J), given by equation (24), may not be adequate. For noisy field data this problem becomes more serious. At the minimum point of the error criterion Q, the negative gradient, given by equation (6) as:

$$-\nabla Q = 2A^T B \qquad (36)$$

must be zero, showing that the matrix A^T and the residual vector b must be orthogonal. The larger the residual vector is, the more complete-orthogonality is required. Thus, for noisy field data where the residual vector never becomes zero, the validity of the linearization approximation is limited.

If the step X(J) given by equation (24) fails to obtain a smaller Q, the increment vector χ is changed into the steepest descent direction. This is accomplished by modifying equation (31) into

$$\chi = -(A^T A + \lambda I)^{-1} A^T b \qquad (37)$$

where λ is called the Marquardt factor. For larger λ, the increment vector becomes more aligned to the steepest descent direction. The technique of modifying the value of λ at each iteration is known as the ridge regression method. In this program, the algorithm by Sandberg (1979) for determining λ is applied.

Two more measures are taken in order to avoid anomalously large values of X(J). First, the residual ln (RC(I)/RS(I)) greater than 2% is set equal to 2% in order to obtain a gradual fit to the abnormal residuals. Second, the increment X(J) if greater than .2, is set to .2, in order to make the change of parameter less than 22% at each iteration.

SUBROUTINE RESULT

The subroutine RESULT prints out the results of the inversion. The Dar Zarrouk parameters of the layering model obtained are calculated through the formula:

$$TT(I) = \sum_{J=1}^{I} h_j \rho_j \qquad (38)$$

$$SS(I) = \sum_{j=1}^{I} h_j / \rho_j \qquad (39)$$

$$DZR(I)=(TT(I)/SS(I))^{1/2} \qquad (40)$$

$$DZD(I)=(TT(I)SS(I))^{1/2} \qquad (41)$$

To help reduce the number of layers, the Dar Zarrouk interpolation is calculated. For the three consecutive points, I-1, I, and I+1, the DZ interpolation connecting the points I-1 and I+1 is calculated through the formula:

$$DZRI(I) = -U+(U^2+R^2)^{1/2} \qquad (42)$$

where

$$R = ((TT(I+1)-TT(I-1))/(SS(I+1)-SS(I-1)))^{1/2} \qquad (43)$$

$$U=DZD(I-1)(R^2-DZR^2(I-1))/(2 \cdot DZD(I)DZR(I-1)) \qquad (44)$$

If the percent difference between DZRI(I) and DZR(I) is relatively small, point I may be eliminated. The value of R, given by equation (43), gives a rough estimate of the resistivity of the combined layer defined by the Dar Zarrouk points I-1 and I+1.

The second and fourth columns of the output show estimates of the percent error of parameters due to the uncertainty in the determination of these parameters (Rijo and others, 1977). This should be interpreted as the relative certainty in the estimated value.

SUBROUTINE AUTODZ

The subroutine AUTODZ performs the Dar Zarrouk reduction automatically. The idea of automatic reduction by means of equivalence in the Dar Zarrouk curves was developed by Zohdy (1975). The computer program based on this idea was published in Zohdy (1973) as subroutine (DZSMTH). Subroutine AUTODZ is a modified version of DZSMTH that includes a modified fitting tolerance (FT) and other reduction criteria.

The leftmost DZ branch is a horizontal line. The first NN points are averaged to give

$$DZRA\ (NN) = \exp\left(\frac{1}{NN}\sum_{I=1}^{NN} \ln DZR(I)\right) \qquad (45)$$

where DZR is the Dar Zarrouk resistivity. If the logarithmic difference ln(DZR(I)/DZRA(NN)) is smaller than FT (fitting tolerance) as defined by Zohdy (1975) for every I from 1 to NN, this average is accepted and the next point is tried. Otherwise, the NN-th point cannot be included in this branch, and the value DZRA(NTAP) is set to the first DZ resistivity, where NTAP=NN-1 is the number of points averaged so far.

Further smoothing is based on fitting straight lines to the linear representation of the DZ curve, which is a plot of T versus S where:

$$T=A0+A1 \cdot S \qquad (46)$$

For the first branch (NBR=1) the two coefficients are given by

$$A0(1)=0 \qquad (47)$$

$$A1(1)=(DZRA(NTAP))^2 \tag{48}$$

For the (NBR+1)-th branch, these are given by

$$A1(NBR+1) = \frac{NN \cdot \Sigma TS - \Sigma T \cdot \Sigma S}{NN \cdot \Sigma S^2 - (\Sigma S)^2} \tag{49}$$

$$AO(NBR+1) = \frac{\Sigma T - A1(NBR+1)\Sigma S}{NN} \tag{50}$$

where the summation is from NTAP to NEXT and NN=NEXT-NTAP+1 is the number of summed points. The point NTAP is the last point on the previous branch, and at the same time the first point on the present branch. The intersection of the (NBR+1)-th branch and the NBR-th branch in the linear DZ domain is obtained by

$$SSR(NBR) = \frac{AO(NBR+1)-AO(NBR)}{A1(NBR)-A1(NBR+1)} \tag{51}$$

$$TTR(NBR) = AO(NBR)+A1(NBR)SSR(NBR) \tag{52}$$

If the intersection is not in the first quadrant (SSR and/or TTR are negative), the point NEXT is rejected from the (NBR+1)-th branch. The Dar Zarrouk point corresponding to this intersection is given by

$$DZDRF(NBR) = TTR(NBR) \cdot SSR(NBR) \tag{53}$$

$$DZRRF(NBR) = TTR(NBR)/SSR(NBR) \tag{54}$$

If the abscissa (DZDRF) of this reduced DZ point is greater than the abscissa ((DZD(NTAP+1)) of the second point on the (NBR+1)-th branch, the point NEXT is rejected. Also if DZDRF is smaller the DZD(NTAP-1) of the second to the last point of the previous branch, the point is also rejected.

Next, each point to be included in this branch is compared with the value calculated from the reduced branch, and if any one of them exceeds the fitting tolerance, this branch is not accepted and the point NEXT is rejected. Because different fitting tolerances should be used depending on the slope of the DZ curve (Zohdy, 1975), the fitting tolerance TOL is defined by

$$\begin{aligned} TOL &= (FT-.02) \cdot SLP+FT \quad \text{for } SLP<0 \\ &= -.02 \cdot SLP+FT \quad \text{for } SLP\geq 0 \end{aligned} \tag{55}$$

which takes a value of .02 at SLP=-1, FT at SLP=0, and FT-.02 at SLP=1. The slope of the reduced DZ curve at the abscissa DZD(I) is calculated by

$$SLP = \frac{\ln DZR2-\ln DZR1}{\ln DZD2-\ln DZD1} \qquad (56)$$

where DZR2 and DZR1 are the DZ resistivities on the reduced DZ curve at the abscissas DZD2=DZD(I)•1.02 and DZD1=DZD(I)/1.02, respectively. The DZ resistivity of the reduced DZ curve is calculated by (Zohdy, 1975, p. E29) as:

$$DZRA(I) = \frac{W + (W^2 + Z)^{1/2}}{2 \cdot DZD(I)DZRF(NN)} \qquad (57)$$

where

$$W = -DZDF(NN)(P(NN)^2-DZRF(NN)^2) \qquad (58)$$

$$Z = 4 \cdot (DZD(I)DZRF(NN)P(NN))^2 \qquad (59)$$

The automatic DZ reduction discussed above has some difficulty in reducing points near sharp deflections. If two consecutive points in the resultant DZ curve are near enough to each other such that,

$$\frac{DZDRF(I+1)}{DZDRF(I)} < 1.5 \qquad (60)$$

then these points are merged by taking their geometric mean.

Initially the fitting tolerance FT is set equal to .08. If no reduction is obtained, FT is increased by .02 until it exceeds .12 where it remains fixed.

SUBROUTINE MANUDZ

In subroutine MANUDZ the user manually reduces the number of layers. There are two modes for manual reduction, in MODE 1 the user specifies the Dar Zarrouk points to be deleted, and in MODE 2 the user specifies the maximum percent difference of the DZ points to be deleted.

SUBROUTINE MATRIX

The subroutine MATRIX prints out the correlation matrix C, the resolution matrix R and the information density matrix S. These matrices are defined as follows:

$$C(I,J) = \frac{ATAINV(I,J)}{(ATAINV(I,I)ATAINV(J,J))^{1/2}} \qquad (61)$$

$$R = (A^TA+\lambda I)^{-1}A^TA \qquad (62)$$

$$S = A(A^TA+\lambda I)^{-1}A^T \qquad (63)$$

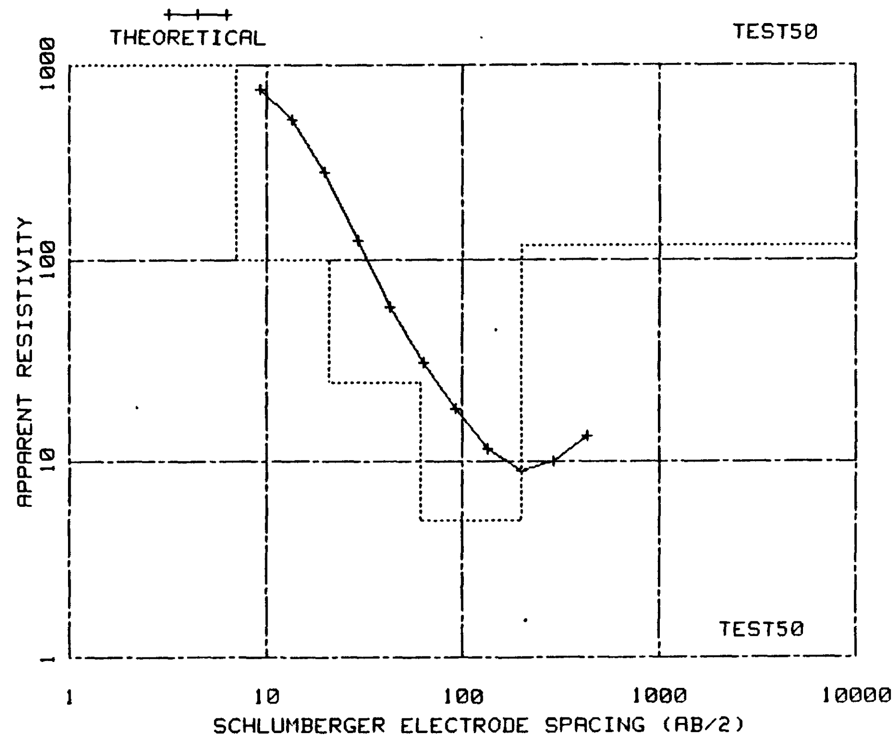


Figure 1. Theoretical Schlumberger sounding curve represented as plus signs, calculated from the model represented by the dashed line.

Rigorously speaking, these matrices must be calculated with λ equal to zero. To avoid extra calculation, this subroutine calculates them using the data after the last inversion, where λ will have a finite but very small value.

EXAMPLES

In this section two examples are given of the application of the automatic interpretation. In the first example a theoretical sounding curve is used as input to the inversion program, and in the second example an actual field curve is used.

EXAMPLE 1

Figure 1 shows the theoretical apparent resistivity curve calculated for the five layer model (TEST50) with resistivities of 1000, 100, 25, 5, and 120 and thicknesses of 7, 14, 40, and 140. For the purpose of illustration the units are unimportant. The units that come out of the inversion routine are the same as those that go in.

The output of the inversion of TEST50, obtained by the program RESINV, is given in Table 3. AUTODZ reduced the number of layers directly to five, and the resultant model is almost identical to the original.

TABLE 3 COMPUTER OUTPUTS FOR TEST50

TITLE= TEST50 ND= 11 MODE= 6

NO	AB/2	RF
NO= 1	AB/2= 9.28	RF= 755.28
NO= 2	AB/2= 13.63	RF= 527.17
NO= 3	AB/2= 20.00	RF= 284.74
NO= 4	AB/2= 29.36	RF= 126.65
NO= 5	AB/2= 43.09	RF= 58.24
NO= 6	AB/2= 63.25	RF= 30.97
NO= 7	AB/2= 92.83	RF= 18.29
NO= 8	AB/2= 136.26	RF= 11.27
NO= 9	AB/2= 200.00	RF= 8.63
NO=10	AB/2= 293.56	RF= 9.69
NO=11	AB/2= 430.89	RF= 13.17

AUTOMATIC INVERSION STARTS
RIDGE Q = 0.00095 AT ITE= 14 FM= 0.006738

INVERSION OF FIELD DATA: TEST50 AT INV= 1

NO	RESIS	PCT	THICK	PCT	DEPTH	DZR	DZD	%DIFF
1	1013.00	0.18	4.97	0.98	4.97	1013.00	4.97	-3.97
2	799.59	0.93	2.66	1.00	7.63	932.55	7.68	-25.42
3	77.38	0.78	3.41	1.03	11.04	376.84	19.70	2.22
4	112.20	0.80	7.58	1.03	18.61	262.82	31.49	-6.62
5	37.82	0.97	8.67	1.06	27.28	157.00	54.80	-1.74
6	23.98	0.71	20.32	1.00	47.60	87.17	104.28	-1.25
7	18.31	1.00	22.31	0.92	69.91	62.72	151.44	-3.60
8	4.31	0.96	45.77	1.07	115.68	27.28	355.38	0.18
9	4.65	1.03	62.81	1.06	178.49	19.40	514.93	0.22
10	5.80	1.06	13.38	1.06	191.87	18.68	538.92	4.70
11	93.87	0.52				19.76	572.97	

NO	AB/2	RHOCAL	RHOOBS	% ERROR
1	9.28	755.632	755.276	0.0471
2	13.63	526.671	527.168	-0.0942
3	20.00	284.897	284.738	0.0560
4	29.36	126.658	126.648	0.0077
5	43.09	58.213	58.244	-0.0526
6	63.25	31.001	30.973	0.0907
7	92.83	18.263	18.285	-0.1215
8	136.26	11.288	11.271	0.1547
9	200.00	8.617	8.625	-0.0924
10	293.56	9.704	9.688	0.1632
11	430.89	13.174	13.170	0.0278

AUTOMATIC DZ REDUCTION REDUCED NL TO 5
RIDGE Q = 0.00094 AT ITE= 3 FM= 0.004087

INVERSION OF FIELD DATA: TEST50 AT INV= 2

NO	RESIS	PCT	THICK	PCT	DEPTH	DZR	DZD	%DIFF
1	998.57	0.21	7.02	0.49	7.02	998.57	7.02	-38.33
2	97.67	0.64	14.48	0.74	21.50	232.98	36.18	-14.17
3	23.86	0.68	42.06	0.46	63.56	70.13	134.49	-9.51
4	4.54	0.93	121.04	1.05	184.60	18.68	534.35	58.89
5	88.10	0.48				26.24	786.72	

TABLE 3 (continued)

NO	AB/2	RHOCAL	RHOOBS	% ERROR
1	9.28	755.107	755.276	-0.0224
2	13.63	527.370	527.168	0.0382
3	20.00	284.773	284.738	0.0123
4	29.36	126.556	126.648	-0.0726
5	43.09	58.299	58.244	0.0947
6	63.25	30.956	30.973	-0.0565
7	92.83	18.276	18.285	-0.0517
8	136.26	11.292	11.271	0.1897
9	200.00	8.608	8.625	-0.1937
10	293.56	9.711	9.688	0.2398
11	430.89	13.170	13.170	0.0015

PARAMETER CORRELATION MATRIX

R(1)	1.00								
R(2)	0.59	1.00							
R(3)	0.18	0.61	1.00						
R(4)	-0.00	0.04	0.32	1.00					
R(5)	-0.01	-0.09	-0.27	0.21	1.00				
T(1)	-0.82	-0.90	-0.40	-0.01	0.05	1.00			
T(2)	-0.32	-0.86	-0.89	-0.16	0.19	0.63	1.00		
T(3)	0.09	-0.02	-0.66	-0.76	0.19	-0.07	0.33	1.00	
T(4)	-0.00	0.01	0.15	0.89	0.62	-0.00	-0.06	-0.52	1.00
	R(1)	R(2)	R(3)	R(4)	R(5)	T(1)	T(2)	T(3)	T(4)

RESOLUTION MATRIX

R(1)	0.98								
R(2)	-0.04	0.81							
R(3)	-0.01	-0.12	0.78						
R(4)	0.00	-0.01	-0.09	0.60					
R(5)	0.00	0.01	0.04	-0.04	0.89				
T(1)	0.04	0.13	0.06	0.00	-0.01	0.89			
T(2)	0.02	0.19	0.21	0.05	-0.03	-0.10	0.75		
T(3)	-0.00	0.00	0.10	0.15	-0.02	0.01	-0.05	0.90	
T(4)	0.00	-0.00	-0.05	-0.40	-0.15	0.00	0.02	0.12	0.49
	R(1)	R(2)	R(3)	R(4)	R(5)	T(1)	T(2)	T(3)	T(4)

INFORMATION DENSITY MATRIX

0.7										
0.3	0.4									
-0.1	0.3	0.6								
-0.1	0.0	0.3	0.6							
0.1	-0.1	-0.1	0.3	0.7						
-0.0	0.0	-0.0	-0.1	0.3	0.7					
-0.0	0.0	0.0	-0.0	-0.1	0.3	0.6				
0.0	-0.0	-0.0	0.0	0.0	-0.1	0.2	0.7			
0.0	0.0	-0.0	-0.0	0.0	-0.0	-0.1	0.3	0.6		
-0.0	-0.0	0.0	0.0	-0.0	0.0	0.0	-0.2	0.3	0.7	
0.0	-0.0	-0.0	0.0	0.0	-0.0	0.0	0.0	-0.1	0.2	0.9

EXAMPLE 2

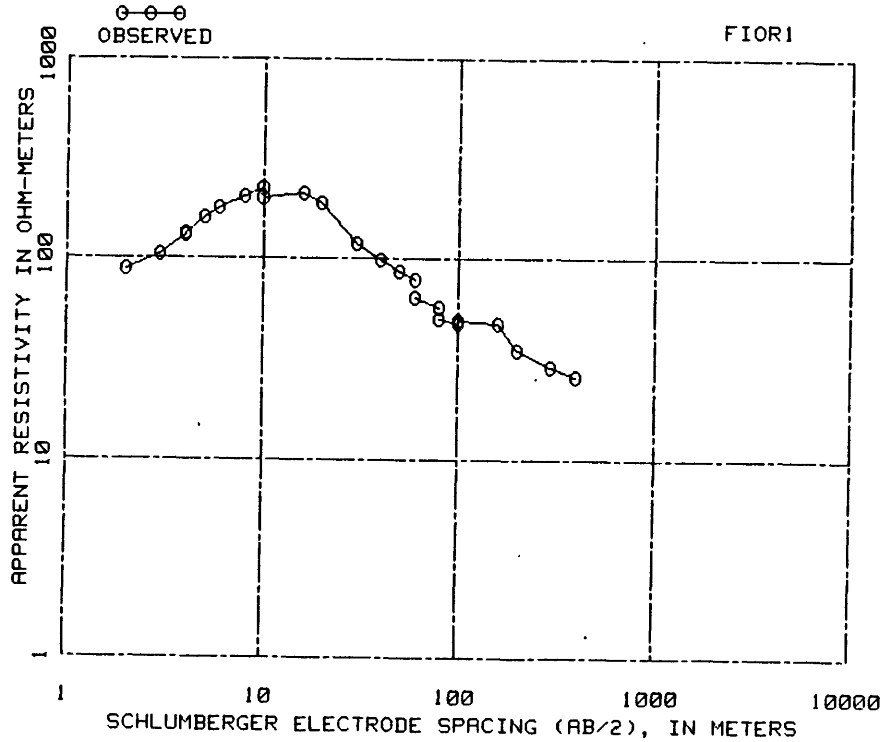
Figure 2 shows a field curve, FIOR1, taken from a set of soundings carried out as a part of an hydrogeological study by the University of Parma (Italy) in the south-west part of the Po plain area (Fiorenzuola d'Arda-Italy). The original field curve (Fig. 2a) was shifted and then digitized at the logarithmically equal interval of six points per log decade (Fig. 2b) using a cubic spline function from Anderson (1971). The digitized apparent resistivity values were fed to the automatic interpretation. Detailed and reduced models obtained by the inversion are presented in Table 4. The initial detailed solution at INV 1 features a very smooth resistivity variation suitable for the multi-layer representation used by Zohdy and Bisdorf (1978). Because of distortions probably due to geologic noise, the initial inversion (INV=1) stopped with a fit of 5.6%. The largest discrepancy between observed and calculated values (15%) occurred at sample point 7. For further calculation of the reduced models, the calculated curve, based on the initial detailed interpretation, was used as the smoothed curve.

The reduced solution, at INV=3, produced five main geoelectric units, as evidenced by the five major breaks of the field curve. This solution agrees well with results from nearby soundings and geological data, especially in the estimates of the depths of layer boundaries. An equivalent six layer model was also constructed. It is worth noting that the construction of equivalent models with different number of layers may give the intepreter a wider possibility of finding geologically convenient solutions.

ACKNOWLEDGMENT

This work was done during our stay at the United States Geological Survey, Denver, Colorado in 1981. We wish to thank Dr. Adel Zohdy for the illuminating discussions, help, and comments.

a.



b.

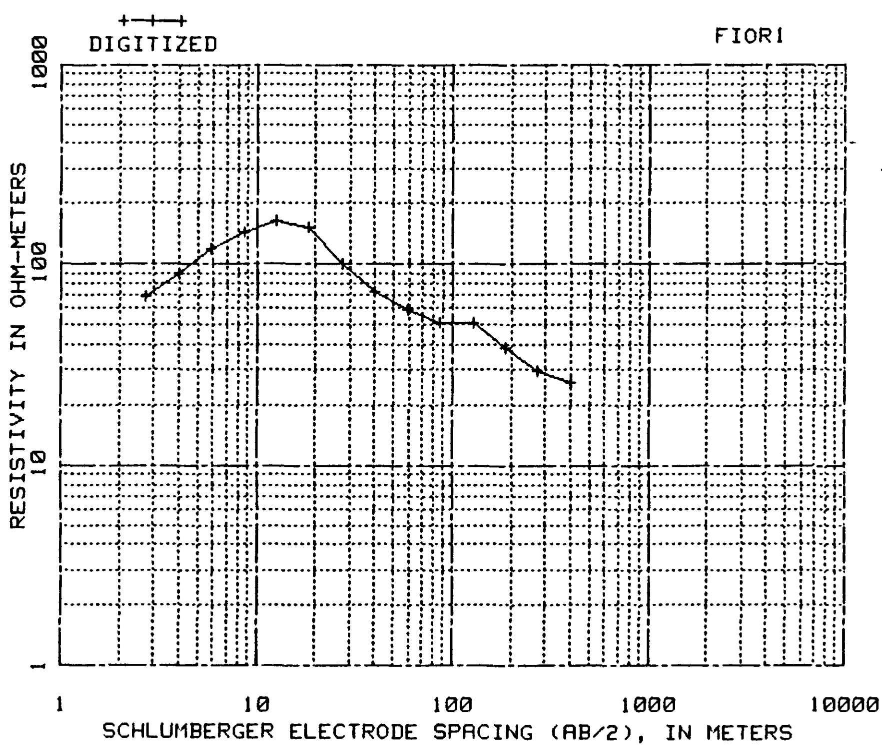


Figure 2. Data for Schlumberger sounding Fior 1. a) Observed field data. b) Shifted and digitized apparent resistivity data.

T A B L E 4 COMPUTER OUTPUTS FOR FIOR1

TITLE= FIOR1 ND= 14 MODE= -6

NO= 1	AB/2= 2.73	RF=	69.00
NO= 2	AB/2= 4.00	RF=	86.40
NO= 3	AB/2= 5.87	RF=	115.00
NO= 4	AB/2= 8.62	RF=	144.00
NO= 5	AB/2= 12.65	RF=	167.00
NO= 6	AB/2= 18.57	RF=	152.00
NO= 7	AB/2= 27.25	RF=	99.90
NO= 8	AB/2= 40.00	RF=	74.10
NO= 9	AB/2= 58.71	RF=	60.30
NO=10	AB/2= 86.17	RF=	50.50
NO=11	AB/2= 126.48	RF=	47.10
NO=12	AB/2= 185.65	RF=	41.10
NO=13	AB/2= 272.50	RF=	30.80
NO=14	AB/2= 399.98	RF=	26.00

AUTOMATIC INVERSION STARTS
RIDGE Q = 0.05598 AT ITE= 5 FM= 0.256505

INVERSION OF FIELD DATA: FIOR1 AT INV= 1

NO	RESIS	PCT	THICK	PCT	DEPTH	DZR	DZD	%DIFF
1	41.35	5.24	1.10	5.28	1.10	41.35	1.10	27.81
2	108.25	5.31	0.93	5.26	2.03	64.35	2.27	20.48
3	364.87	5.50	1.48	5.57	3.50	132.01	5.18	0.88
4	395.83	5.55	2.34	5.57	5.84	188.78	8.52	-8.40
5	189.94	5.57	2.05	5.57	7.89	189.00	10.58	-8.66
6	47.38	5.59	1.03	5.38	8.93	162.30	12.62	-0.78
7	37.39	5.49	9.13	5.41	18.06	86.15	27.74	2.32
8	44.33	5.31	10.31	5.52	28.37	71.65	39.73	2.27
9	51.24	5.22	18.60	5.61	46.97	64.35	59.05	1.62
10	55.96	5.34	26.60	5.59	73.57	61.62	85.83	-3.84
11	44.33	5.54	32.82	5.62	106.39	56.22	119.93	-6.81
12	25.82	5.63	36.75	5.62	143.13	46.51	165.40	-3.91
13	18.91	5.52	122.54	5.34	265.68	31.58	316.95	2.04
14	21.23	5.22				28.24	446.50	

NO	AB/2	RHOCAL	RHOOBS	% ERROR
1	2.73	70.041	69.000	1.5092
2	4.00	91.150	86.400	5.4976
3	5.87	116.524	115.000	1.3253
4	8.62	140.490	144.000	-2.4378
5	12.65	153.319	167.000	-8.1921
6	18.57	144.932	152.000	-4.6501
7	27.25	115.195	99.900	15.3104
8	40.00	80.343	74.100	8.4245
9	58.71	58.810	60.300	-2.4703
10	86.17	51.093	50.500	1.1737
11	126.48	46.739	47.100	-0.7673
12	185.65	40.173	41.100	-2.2563
13	272.50	32.052	30.800	4.0646
14	399.98	25.625	26.000	-1.4440

AUTOMATIC DZ REDUCTION REDUCED NL TO 6
RIDGE Q = 0.00086 AT ITE= 5 FM= 0.001503

T A B L E 4 (continued)

INVERSION OF FIELD DATA: FIOR1 AT INV= 2

NO	RESIS	PCT	THICK	PCT	DEPTH	DZR	DZD	%DIFF
1	40.96	1.32	1.13	1.93	1.13	40.96	1.13	15.22
2	90.17	1.91	0.55	1.45	1.68	53.43	1.80	18.84
3	333.81	1.15	5.93	1.42	7.62	200.79	10.34	-62.23
4	34.64	1.40	11.72	1.24	19.34	79.82	31.11	13.78
5	53.57	0.59	83.11	0.98	102.45	59.77	116.03	-28.19
6	20.15	0.29				37.67	228.54	

NO	AB/2	RHOCAL	RHOOBS	% ERROR
1	2.73	69.957	69.000	1.3869
2	4.00	91.285	86.400	5.6533
3	5.87	116.523	115.000	1.3242
4	8.62	140.234	144.000	-2.6150
5	12.65	153.128	167.000	-8.3064
6	18.57	145.002	152.000	-4.6037
7	27.25	115.302	99.900	15.4172
8	40.00	80.311	74.100	8.3821
9	58.71	58.801	60.300	-2.4862
10	86.17	51.125	50.500	1.2385
11	126.48	46.730	47.100	-0.7861
12	185.65	40.163	41.100	-2.2801
13	272.50	32.061	30.800	4.0951
14	399.98	25.623	26.000	-1.4489

AUTOMATIC DZ REDUCTION REDUCED NL TO 5
RIDGE Q = 0.00080 AT ITE= 7 FM= 0.000553

INVERSION OF FIELD DATA: FIOR1 AT INV= 3

NO	RESIS	PCT	THICK	PCT	DEPTH	DZR	DZD	%DIFF
1	43.01	1.42	1.43	1.65	1.43	43.01	1.43	183.39
2	328.28	1.25	6.20	1.66	7.63	200.75	10.45	-61.84
3	32.38	2.18	10.07	1.86	17.70	81.69	29.67	14.45
4	53.32	0.59	84.93	1.04	102.63	59.62	116.61	-28.52
5	20.11	0.28				37.44	231.31	

NO	AB/2	RHOCAL	RHOOBS	% ERROR
1	2.73	69.992	69.000	1.4377
2	4.00	91.286	86.400	5.6547
3	5.87	116.573	115.000	1.3678
4	8.62	140.289	144.000	-2.5770
5	12.65	153.182	167.000	-8.2740
6	18.57	145.043	152.000	-4.5770
7	27.25	115.290	99.900	15.4055
8	40.00	80.281	74.100	8.3410
9	58.71	58.807	60.300	-2.4752
10	86.17	51.134	50.500	1.2561
11	126.48	46.717	47.100	-0.8126
12	185.65	40.158	41.100	-2.2919
13	272.50	32.068	30.800	4.1164
14	399.98	25.620	26.000	-1.4603

PARAMETER CORRELATION MATRIX

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APPENDIX

```
C       PROGRAM RESINV
C
C       DEVELOPED BY YUTAKA MURAKAMI AND ANDREA ZERILLI
C
C       TITLE(4)   : NAME OF THE VES POINT
C       ND         : NUMBER OF DATA POINTS
C       AB(25)     : AB/2 OF EACH DATA POINT
C       RF(25)     : APPARENT RESISTIVITY
C       MODE       : POSITIVE FOR EQUI-SPACED AB
C                  : NEGATIVE FOR ARBITRARY AB
C                  : +/- 6 FOR USUAL ANALYSIS
C                  : +/- 10 FOR MORE ACCURATE ANALYSIS
C       P(50)      : ARRAY OF PARAMETERS OF MODEL
C       NL         : NUMBER OF LAYER OF MODEL
C       NL1        : NL - 1
C       NP         : NUMBER OF PARAMETERS = NL + NL1
C       Q          : AVERAGE ERROR
C       RC(25)     : CALCULATED APPARENT RESISTIVITY
C       RS(25)     : SMOOTHED APPARENT RESISTIVITY
C       A(25,50)   : SENSITIVITY MATRIX
C       PCT(50)    : PERCENT DEVIATION
C       NFIX       : NUMBER OF FIXED PARAMETERS
C
        COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1        ,Q,RC(25),RS(25),A(25,50),PCT(50)
C
C *** INPUT OF OBSERVATION DATA
        CALL FDATA(TITLE,ND,AB,RF,MODE)
C
C *** CHOICE OF FILTER
        IF(IABS(MODE).EQ. 6) CALL YM6
        IF(IABS(MODE).EQ.10) CALL YM10
C
C *** RS IS SET TO RF FOR THE FIRST INVERSION (INV=1)
        DO 5 I=1,ND
      5 RS(I)=RF(I)
C
C *** AUTOMATIC GUESS OF INITIAL MODEL
        CALL AGUESS(ND,RF,AB,NL,NL1,NP,P)
        NFIX=0
C
        WRITE(6,100)
        WRITE(8,100)
    100 FORMAT(/' AUTOMATIC INVERSION STARTS')
C
C       INV=INVERSION COUNTER
        INV=0
     15 INV=INV+1
C
C *** TWO INVERSION SUBROUTINES ARE CALLED SEQUENTIALLY
        CALL STEEP
        CALL RIDGE
        CALL RESULT(INV)
C
C       IF NO PARAMETERS ARE FIXED,
C       AUTODZ IS CALLED FOR AUTOMATIC REDUCTION OF LAYER NUMBERS
```

```
      IF(NFIX.NE.0) GO TO 1000
      CALL AUTODZ(JMODE)
      IF(JMODE.NE.0) GO TO 1000
C
      IF(INV.NE.1) GO TO 10
      DO 999 I=1,ND
  999 RS(I)=RC(I)
      GO TO 10
C
 1000 WRITE(6,1001)
 1001 FORMAT(/' KEY IN 1 TO CHANGE THE INITIAL GUESS'
     *      ,/'        2 TO REDUCE D-Z POINTS'
     *      ,/'        3 TO PRINT OUT MATRIX INFORMATION'
     *      ,/'        4 TO FINISH')
      READ(5,*) IANS
      GO TO (2000,3000,4000,5000),IANS
C
C     MANUAL INPUT OF INITIAL GUESS
 2000 CALL MGUESS(NL,NL1,NP,P)
      NFIX=0
      DO 20 J=1,NP
   20 IF(P(J).LT.0.) NFIX=NFIX+1
      GO TO 10
C
C     MANUAL DZ REDUCTION
 3000 CALL MANUDZ
      GO TO 10
C
C     PRINT OUT MATRIX INFORMATION
 4000 CALL MATRIX
      GO TO 1000
C
   10 WRITE(6,990) Q
  990 FORMAT(' LAST   Q =',F10.5)
      GOTO 15
C
 5000 STOP
      END
C ****************************************************************
      SUBROUTINE FDATA(TITLE,ND,AB,RF,MODE)
C ****************************************************************
C     READS OBSERVED VES DATA
C
      DIMENSION TITLE(4),AB(25),RF(25)
C
      READ(20,1000) TITLE,ND,MODE
      WRITE(8,2000) TITLE,ND,MODE
C
      DO 10 I=1,ND
      READ(20,3000)   AB(I),RF(I)
   10 WRITE(8,4000) I,AB(I),RF(I)
C
      RETURN
 1000 FORMAT(1X,4A4/2I3)
 2000 FORMAT(' TITLE= ',4A4,'    ND=',I3,'   MODE=',I3/)
 3000 FORMAT(3X,2F10.3)
```

```
 4000 FORMAT(' NO=',I2,6X,'AB/2=',F10.2,6X,'RF=',F10.2)
      END
C ***************************************************************
      SUBROUTINE AGUESS(ND,RF,AB,NL,NL1,NP,P)
C ***************************************************************
C     AUTOMATIC GUESS OF INITIAL MODEL
C
      DIMENSION RF(25),AB(25),P(50)
      NL=ND
      NL1=NL-1
      NP=NL+NL1
      DO 6 I=1,NL
    6 P(I)=RF(I)
      P(NL+1)=AB(1)
      DO 7 I=2,NL1
    7 P(NL+I)=AB(I)-AB(I-1)
      RETURN
      END
C ***************************************************************
      SUBROUTINE MGUESS(NL,NL1,NP,P)
C ***************************************************************
C     INPUT OF THE INITIAL GUESS MADE BY THE USER
C
      DIMENSION P(50)
    1 WRITE(6,100)
  100 FORMAT(/' KEY IN NUMBER OF LAYER OF YOUR INITIAL GUESS')
      READ(5,*)NL
    2 NL1=NL-1
      NP=NL+NL1
      WRITE(6,150) NL,NL1
  150 FORMAT(' KEY IN',I3,' RESISTIVITIES AND',I3,' THICKNESSES'
     1,' OF YOUR INITIAL GUESS')
      READ(5,*)(P(I),I=1,NP)
      RETURN
      END
C ***************************************************************
      SUBROUTINE STEEP
C ***************************************************************
C     STEEPEST DESCENT INVERSION
C
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1      ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/FLTR/NC,F,Y,FC(141)
      DIMENSION PL(50),PI(50),G(50)
      IS=0
      QV=99.
      ITMAX=15
      DO 100 IC=1,ITMAX
   10 CALL APPRES(RC,1)
      IF(QV.GE.Q) GO TO 20
      IS = 1
      ST=ST*.8
      IF(ST.LT..0001) GO TO 110
      DO 15 I=1,NP
   15 P(I)=PL(I)*.2+P(I)*.8
      GO TO 10
```

```
   20 QV=Q
      IF(Q.LE..02) GO TO 110
      IF(IC.EQ.ITMAX) GO TO 110
C
      CALL DERIV
C
      DO 25 J=1,NP
   25 G(J) = 0.
      DO 40 I=1,ND
      BA=RC(I)/RS(I)-1.
      DO 40 J=1,NP
   40 IF(P(J).GT.0.) G(J)=G(J)-2.*A(I,J)*BA
C
      GR = 0.
      DO 45 J=1,NP
   45 GR = GR + G(J)*G(J)
      GR = SQRT(GR)
      IF(IS.EQ.0) ST = AMIN1(.5,Q*Q*FLOAT(ND)/GR)
      DO 50 J=1,NP
      PL(J) = P(J)
   50 P(J) = P(J)*(1.+ST*G(J)/GR)
      IF(IC/4*4.NE.IC) GO TO 100
      IF(IC.EQ.4) GO TO 60
      DO 55 J=1,NP
   55 P(J)=P(J)*P(J)/PI(J)
   60 DO 70 J=1,NP
   70 PI(J)=P(J)
      QV=99.
  100 CONTINUE
      IC=IC-1
C
  110 WRITE(6,1100) Q,IC,ST
 1100 FORMAT(' STEEP  Q =',F10.5,'   AT  IC=',I3,'   ST=',F10.6)
      RETURN
      END
C ****************************************************************
      SUBROUTINE RIDGE
C ****************************************************************
C     RIDGE REGRESSION INVERSION
C     MARQUARDT FACTOR IS DETERMINED THROUGH THE ALGORITHM
C     GIVEN BY SANDBERG (1979)
C
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1      ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)
      DOUBLE PRECISION ATA,ATAINV,W,DSUM
      DIMENSION PL(50),XX(4),YY(4),PCT1(50)
      DO 10 J=1,NP
      PL(J)=P(J)
   10 PCT(J)=0.
      CALL APPRES(RC,1)
      QL=Q
      XCT=-4.
      XCM=2.
C
      DO 100 ITE=1,15
```

```
      CALL DERIV
      DO 107 J=1,NP
      DSUM=1.D-25
      DO 106 I=1,ND
  106 DSUM=DSUM+A(I,J)**2
  107 W(J)=1.DO/DSQRT(DSUM)
      DO 108 I=1,ND
      DO 108 J=1,NP
  108 A(I,J)=A(I,J)*W(J)
      DO 120 K=1,NP-1
      DO 120 J=K+1,NP
      SUM=0.
      DO 111 I=1,ND
  111 SUM=SUM+A(I,K)*A(I,J)
      ATA(J,K)=SUM
  120 ATA(K,J)=SUM
C
      ITRAP=0
  800 DO 810 II=1,4
C
C     MARQUARDT INVERSION
      FM=EXP(XCT)
      CALL NEWP(FM)
      IF(Q.LT.QL) GO TO 250
      DO 808 J=1,NP
  808 P(J)=PL(J)
      XX(II)=XCT
      YY(II)=Q
      IF(II.EQ.4) GO TO 280
      IF(II.EQ.3) GO TO 820
      IF(II.EQ.1.AND.XCM.LE..5)XCM=XCM*.9
      XCT=XCT+XCM
      GO TO 810
  820 IF(YY(2).GT.YY(1).OR.YY(2).GT.YY(3)) GO TO 812
      XCT=-(XX(1)**2*(YY(3)-YY(2))+XX(2)**2*(YY(1)-YY(3))+
     1 XX(3)**2*(YY(2)-YY(1)))/(YY(1)*(XX(3)-XX(2))+
     2 YY(2)*(XX(1)-XX(3))+YY(3)*(XX(2)-XX(1)))/2.
      GO TO 810
  812 IF(YY(1).LE.YY(3)) XCT=XX(1)-XCM/2.
      IF(YY(1).GT.YY(3)) XCT=XX(3)+XCM/2.
  810 CONTINUE
C
  280 ITRAP=ITRAP+1
      IF(ITRAP.EQ.3) GO TO 290
      XCT=-4.
      XCM=2.
      GO TO 800
C
  250 DO 160 J=1,NP
      PCT1(J)=(EXP(SQRT(SNGL(ATAINV(J,J)))*Q)-1.)*100.
  160 PL(J)=P(J)
      IF(Q.LE..001) GO TO 300
      IF(XCM.GT..5) XCM=XCM/2.
      XCT=XCT-XCM
  100 QL=Q
C
```

```
  290 Q=QL
      ITE=ITE-1
  300 WRITE(6,3000) Q,ITE,FM
      WRITE(8,3000) Q,ITE,FM
 3000 FORMAT(' RIDGE  Q =',F10.5,'   AT ITE=',I3,'   FM=',F10.6)
      DO 625 J=1,NP
  625 IF(ITE.NE.0)PCT(J)=AMIN1(99.99,PCT1(J))
      RETURN
      END
C *************************************************************
      SUBROUTINE DERIV
C *************************************************************
C     ANALYTICAL DERIVATIVE OF LOG(RC) WITH RESPECT TO LOG(P)
C
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1      ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/FLTR/NC,F,Y,FC(141)
      COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)
      DIMENSION D(100,50),RD(25)
C
      IF(MODE.LT.0) GO TO 200
C
      X=EXP(Y)/AB(1)
      NT = NC+ND-1
      DO 23 L = 1,NT
      T=ABS(P(NL))
      D(L,NL)=1.
      DO 22 K = 1,NL1
      IZ = NP+1-K
      IR = NL-K
      Z=ABS(P(IZ))
      R=ABS(P(IR))
      TH = TANH(Z*X)
      CH = 1.E60
      IF(Z*X.LT.130.) CH=COSH(Z*X)
      B2 = (1. + T*TH/R)**2
      DTDT = (1.-TH*TH)/B2
      D(L,IZ) = (R-T*T/R)/B2*X/CH/CH
      D(L,IR) = TH*(1.+T*T/(R*R)+2.*TH*T/R)/B2
      IF (K.EQ.1) GO TO 32
      IS=NP+2-K
      DO 30 IZ = IS,NP
   30 D(L,IZ) = DTDT*D(L,IZ)
   32 IS=NL+1-K
      DO 31 IZ = IS,NL
   31 D(L,IZ) = DTDT*D(L,IZ)
   22 T = (T+TH*R)/(1.+TH*T/R)
   23 X=X/F
      DO 35 I=1,ND
      DO 35 J=1,NP
      SUM=0.
      DO 37 L=1,NC
   37 SUM=SUM+FC(L)*D(L+I-1,J)
   35 A(I,J)=ABS(P(J))/RC(I)*SUM
      GO TO 300
C
```

```
  200 DO 210 I=1,ND
      X=EXP(Y)/AB(I)
      DO 223 L=1,NC
      T=ABS(P(NL))
      D(L,NL)=1.
      DO 222 K = 1,NL1
      IZ = NP+1-K
      IR = NL-K
      Z=ABS(P(IZ))
      R=ABS(P(IR))
      TH = TANH(Z*X)
      CH = 1.E60
      IF(Z*X.LT.130.) CH=COSH(Z*X)
      B2 = (1. + T*TH/R)**2
      DTDT = (1.-TH*TH)/B2
      D(L,IZ) = (R-T*T/R)/B2*X/CH/CH
      D(L,IR) = TH*(1.+T*T/(R*R) + 2.*TH*T/R)/B2
      IF (K.EQ.1) GO TO 232
      IS=NP+2-K
      DO 230 IZ = IS,NP
  230 D(L,IZ) = DTDT*D(L,IZ)
  232 IS=NL+1-K
      DO 231 IZ = IS,NL
  231 D(L,IZ) = DTDT*D(L,IZ)
  222 T = (T+TH*R)/(1.+TH*T/R)
  223 X=X/F
      DO 210 J=1,NP
      SUM=0.
      DO 237 L=1,NC
  237 SUM=SUM+FC(L)*D(L,J)
  210 A(I,J)=ABS(P(J))/RC(I)*SUM
C
  300 DO 310 J=1,NP
      IF(P(J).GT.0.) GO TO 310
      DO 320 I=1,ND
      A(I,J)=1.E20
  320 CONTINUE
  310 CONTINUE
      RETURN
      END
C ***************************************************************
      SUBROUTINE NEWP(FM)
C ***************************************************************
C     MARQUARDT INVERSION
C
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1       ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)
      DOUBLE PRECISION ATA,ATAINV,W
      DIMENSION X(50)
C
      DO 25 K=1,NP
   25 ATA(K,K)=1.D0+FM
C
      CALL MAINE(NP,ATA,ATAINV)
C
```

```
      DO 5 J=1,NP
      DO 5 I=1,ND
      SUM=0.
      DO 4 K=1,NP
    4 SUM=SUM+ATAINV(J,K)*A(I,K)
    5 H(J,I)=SUM
C
      DO 35 J=1,NP
      SUM=0.
      DO 30 I=1,ND
      ERROR=ALOG(RC(I)/RS(I))
      IF(ERROR.GE. .03) ERROR= .03
      IF(ERROR.LT.-.03) ERROR=-.03
   30 SUM=SUM+H(J,I)*ERROR
      X(J)=SUM*W(J)
      IF(X(J).GT. .7) X(J)= .7
      IF(X(J).LT.-.7) X(J)=-.7
      P(J)=P(J)*EXP(X(J))
   35 IF(P(J).GE.99999.) P(J)=99999.
C
      X(1)=P(NL+1)
      DO 40 I=2,NL1
   40 X(I)=X(I-1)+P(I+NL)
      DO 45 I=NL1,2,-1
      IF(P(I+NL)/X(I).GT.0.1) GO TO 45
      TT=(P(NL+I)+P(NL+I+1))/2.
      P(NL+I  )=TT
      P(NL+I+1)=TT
   45 CONTINUE
C
      CALL APPRES(RC,1)
      RETURN
      END
C *************************************************************
      SUBROUTINE MAINE(N,A,AINV)
C *************************************************************
C     INVERSION OF SYMMETRIC MATRIX AFTER SANDBERG (1979)
C
      REAL*8 A(50,50),AINV(50,50),R(50),C(50),DEL,CC,RR
      DO 10 I=1,N
      DO 10 J=1,N
   10 AINV(I,J)=0.D0
      DO 40 L=1,N
      DEL=A(L,L)
      DO 30 I=1,L
      CC=0.
      RR=0.
      DO 20 J=1,L
      CC=CC+AINV(I,J)*A(J,L)
   20 RR=RR+A(L,J)*AINV(J,I)
      C(I)=CC
      R(I)=RR
   30 DEL=DEL-A(L,I)*CC
      C(L)=-1.
      R(L)=-1.
      DO 40 I=1,L
```

```
      C(I)=C(I)/DEL
      DO 40 J=1,L
   40 AINV(I,J)=AINV(I,J)+C(I)*R(J)
      RETURN
      END
C ***************************************************************
      SUBROUTINE AUTODZ(JMODE)
C ***************************************************************
C     AUTOMATIC DAR ZARROUK REDUCTION
C     MODIFIED VERSION OF DZSMTH (ZOHDY,1973)
C
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
      COMMON/DARZ/DZR(25),DZD(25),TT(25),SS(25),DIFF(25)
      DIMENSION DZDR(25),DZRR(25),DZRA(25)
      DIMENSION PP(25),SSR(25),TTR(25)
      DIMENSION AOF(25),A1F(25),PF(25),AO(25),A1(25)
      DIMENSION DZDRF(25),DZRRF(25)
C
      FT=.08
      FT1=.08
    2 NBR=1
      SUMLOG=ALOG(DZR(1))
      DZRA(1)=DZR(1)
      DO 1 NN=2,NL
      SUMLOG=SUMLOG+ALOG(DZR(NN))
      DZRA(NN)=EXP(SUMLOG/NN)
      DO 1 I=1,NN
      IF(ABS(ALOG(DZR(I)/DZRA(NN))).GT.FT1) GO TO 3
    1 CONTINUE
      FT1=FT1/1.5
      GO TO 2
C
    3 NTAP=NN-1
      PF(1)=DZRA(NTAP)
      AOF(1)=0.0
      A1F(1)=DZRA(NTAP)**2
C
   22 IF((NL-NTAP).EQ.1) GO TO 8
      NEXT=NTAP
      SUMT=TT(NEXT)
      SUMS=SS(NEXT)
      SUMSS=SS(NEXT)**2
      SUMST=TT(NEXT)*SS(NEXT)
   21 NEXT=NEXT+1
      NN=NEXT-NTAP+1
      SUMT=TT(NEXT)+SUMT
      SUMS=SS(NEXT)+SUMS
      SUMSS=SS(NEXT)*SS(NEXT)+SUMSS
      SUMST=SS(NEXT)*TT(NEXT)+SUMST
      A1(NN)=(NN*SUMST-SUMT*SUMS)/(NN*SUMSS-SUMS*SUMS)
      AO(NN)=(SUMT-A1(NN)*SUMS)/NN
      PP(NN)=SQRT(A1(NN))
      SSR(NBR)=(AO(NN)-AOF(NBR))/(A1F(NBR)-A1(NN))
      TTR(NBR)=AOF(NBR)+A1F(NBR)*SSR(NBR)
      IF(TTR(NBR).LT.O..OR.SSR(NBR).LT.O.)GO TO 15
      DZDR(NN)=SQRT(TTR(NBR)*SSR(NBR))
```

```
      DZRR(NN)=SQRT(TTR(NBR)/SSR(NBR))
      IF(NN.EQ.2) GO TO 21
      IF(DZDR(NN).GT.DZD(NTAP+1))GO TO 15
      IF(NTAP.EQ.1)GO TO 33
      IF(DZDR(NN).LT.DZD(NTAP-1))GO TO 15
   33 IST=NTAP
      IF(DZD(NTAP)/DZDR(NN).LE.1.02)IST=NTAP+1
C
   34 DO 14 I=IST,NEXT
      DZD2=DZD(I)*1.02
      DZD1=DZD(I)/1.02
      W=-DZDR(NN)*(PP(NN)**2-DZRR(NN)**2)
      W2=W**2
      Z2=4.*(DZD2*DZRR(NN)*PP(NN))**2
      DZR2=(W+SQRT(W2+Z2))/(2.*DZD2*DZRR(NN))
      Z1=4.*(DZD1*DZRR(NN)*PP(NN))**2
      DZR1=(W+SQRT(W2+Z1))/(2.*DZD1*DZRR(NN))
      Z=4.*(DZD(I)*DZRR(NN)*PP(NN))**2
      DZRA(I)=(W+SQRT(W2+Z))/(2.*DZD(I)*DZRR(NN))
      SLP=ALOG10(DZR2/DZR1)/ALOG10(DZD2/DZD1)
      IF(SLP.LT.0.) TOL=(FT-.02)*SLP+FT
      IF(SLP.GE.0.) TOL=-.02*SLP+FT
      IF(ABS(ALOG(DZR(I)/DZRA(I))).GT.TOL) GO TO 15
   14 CONTINUE
      IF(NEXT.EQ.NL) GO TO 16
      GO TO 21
C
   15 IF(NN.EQ.2) GO TO 600
      NTAP=NN+NTAP-2
      NN=NN-1
      DZDRF(NBR)=DZDR(NN)
      DZRRF(NBR)=DZRR(NN)
      NBR=NBR+1
      PF(NBR)=PP(NN)
      AOF(NBR)=AO(NN)
      A1F(NBR)=A1(NN)
      GO TO 22
C
    8 NBR=NBR+1
      PF(NBR)=SQRT((TT(NL)-TT(NL-1))/(SS(NL)-SS(NL-1)))
      IF(ABS(ALOG(PF(NBR)/PF(NBR-1))).GT..1)GO TO 340
      NBR=NBR-1
      GO TO 500
C
  340 A1F(NBR)=PF(NBR)**2
      AOF(NBR)=TT(NL)-PF(NBR)**2*SS(NL)
      SSR(NBR-1)=(AOF(NBR)-AOF(NBR-1))/(A1F(NBR-1)-A1F(NBR))
      TTR(NBR-1)=AOF(NBR-1)+A1F(NBR-1)*SSR(NBR-1)
      DZRRF(NBR-1)=SQRT(TTR(NBR-1)/SSR(NBR-1))
      DZDRF(NBR-1)=SQRT(TTR(NBR-1)*SSR(NBR-1))
      GO TO 500
C
   16 NBR=NBR+1
      PF(NBR)=PP(NN)
      DZRRF(NBR-1)=DZRR(NN)
      DZDRF(NBR-1)=DZDR(NN)
```

```
  500 TTT=DZDRF(NBR-1)*DZRRF(NBR-1)+999.*PF(NBR)
      SSS=DZDRF(NBR-1)/DZRRF(NBR-1)+999./PF(NBR)
      DZDRF(NBR)=SQRT(TTT*SSS)
      DZRRF(NBR)=SQRT(TTT/SSS)
C
  510 II=1
      JJ=1
  515 IF(DZDRF(II+1)/DZDRF(II).GT.1.5) GO TO 520
      DZDRF(JJ)=SQRT(DZDRF(II)*DZDRF(II+1))
      DZRRF(JJ)=SQRT(DZRRF(II)*DZRRF(II+1))
      II=II+1
      IF(II.EQ.NBR) GO TO 540
      GO TO 530
  520 DZDRF(JJ)=DZDRF(II)
      DZRRF(JJ)=DZRRF(II)
  530 II=II+1
      JJ=JJ+1
      IF(II.NE.NBR) GO TO 515
      DZDRF(JJ)=DZDRF(II)
      DZRRF(JJ)=DZRRF(II)
  540 NBR=JJ
      IF(NBR.LT.NL) GO TO 550
C
      FT=FT+.02
      IF(FT.LE..12) GO TO 2
      JMODE=1
      IF(NBR.EQ.NL) RETURN
C
  550 JMODE=0
      WRITE(6,505) NBR
      WRITE(8,505) NBR
  505 FORMAT(/' AUTOMATIC DZ REDUCTION REDUCED NL TO',I3)
      NL=NBR
      NL1=NL-1
      NP=NL+NL1
      DO 555 I=1,NL
      DZD(I)=DZDRF(I)
  555 DZR(I)=DZRRF(I)
      P(1)=DZR(1)
      P(NL+1)=DZD(1)
      DO 560 I=2,NL
      U=DZD(I)*DZR(I)-DZD(I-1)*DZR(I-1)
      V=DZD(I)/DZR(I)-DZD(I-1)/DZR(I-1)
      IF(I.NE.NL) P(I+NL)=SQRT(U*V)
  560 P(I)=SQRT(U/V)
      RETURN
C
  600 WRITE(6,610)
      WRITE(8,610)
  610 FORMAT(/' AUTOMATIC DZ REDUCTION IN TROUBLE')
      JMODE=1
      RETURN
      END
C ***************************************************************
      SUBROUTINE MANUDZ
C ***************************************************************
```

```
C      MANUAL DAR ZARROUK REDUCTION
C
       COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
       COMMON/DARZ/DZR(25),DZD(25),TT(25),SS(25),DIFF(25)
       DIMENSION IL(25)
C
       WRITE(8,1000)
  1000 FORMAT(/' MANUAL DZ REDUCTION')
       WRITE(6,2000)
  2000 FORMAT(' KEY IN NUMBER OF D-Z POINTS TO BE DELETED'
      1      /' OR      O TO GIVE MAXIMUM % DIFF VALUE')
       READ(5,*) M
       IF(M.EQ.0) GO TO 300
       WRITE(6;3000) M
  3000 FORMAT(' KEY IN',I3,' LAYER NUMBERS TO BE DELETED')
       READ(5,*) (IL(I),I=1,M)
       IL(M+1)=0
       II=1
       IJ=1
       DO 360 I=1,NL
       IF(I.EQ.IL(IJ)) GO TO 370
       DZR(II)=DZR(I)
       DZD(II)=DZD(I)
       II=II+1
       GO TO 360
   370 IJ=IJ+1
   360 CONTINUE
       NL=II-1
       GO TO 350
C
   300 WRITE(6,3100)
  3100 FORMAT(' KEY IN THE MAXIMUM % DIFF VALUE OF'
      1,' DZ POINTS TO BE DELETED')
       READ(5,*) PD
       II=1
       DO 310 I=1,NL1
       IF(ABS(DIFF(I)).LT.PD) GO TO 310
       DZR(II)=DZR(I)
       DZD(II)=DZD(I)
       II=II+1
   310 CONTINUE
   315 DZR(II)=DZR(NL)
       DZD(II)=DZD(NL)
       NL=II
   350 NL1=NL-1
       NP=NL+NL1
       P(1)=DZR(1)
       P(NL+1)=DZD(1)
       DO 320 I=2,NL
       U=DZD(I)*DZR(I)-DZD(I-1)*DZR(I-1)
       V=DZD(I)/DZR(I)-DZD(I-1)/DZR(I-1)
       IF(I.NE.NL) P(I+NL)=SQRT(U*V)
   320 P(I)=SQRT(U/V)
       RETURN
       END
C ***************************************************************
```

```
      SUBROUTINE RESULT(INV)
C ***************************************************************
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1       ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/DARZ/DZR(25),DZD(25),TT(25),SS(25),DIFF(25)
      DIMENSION DEP(25),DZRI(25)
      WRITE(6,93) TITLE,INV
      WRITE(8,93) TITLE,INV
   93 FORMAT(/' INVERSION OF FIELD DATA: ',4A4,' AT INV=',I2)
      DEP(1)=ABS(P(NL+1))
      IF(NL1.EQ.1) GO TO 81
      DO 80 I=2,NL1
   80 DEP(I)=DEP(I-1)+ABS(P(NL+I))
C
   81 TTT=0.
      SSS=0.
      DO 200 I=1,NL
      RI=ABS(P(I))
      IF(I.NE.NL) HI=ABS(P(NL+I))
      TTT=TTT+HI*RI
      SSS=SSS+HI/RI
      TT(I)=TTT
      SS(I)=SSS
      DZR(I)=SQRT(TTT/SSS)
  200 DZD(I)=SQRT(TTT*SSS)
      DZRI(1)=(DZR(1)+DZR(2))/2.
      DIFF(1)=(DZRI(1)/DZR(1)-1.)*100.
      DO 220 I=2,NL1
      R2=(TT(I+1)-TT(I-1))/(SS(I+1)-SS(I-1))
      U=DZD(I-1)*(R2-DZR(I-1)**2)/(2.*DZD(I)*DZR(I-1))
      DZRI(I)=-U+SQRT(U*U+R2)
  220 DIFF(I)=(DZRI(I)/DZR(I)-1.)*100.
C
      WRITE(6,2500)
      WRITE(8,2500)
      WRITE(6,2550)(I,P(I),PCT(I),P(I+NL),PCT(I+NL),DEP(I)
     1,DZR(I),DZD(I),DIFF(I),I=1,NL1)
      WRITE(8,2550)(I,P(I),PCT(I),P(I+NL),PCT(I+NL),DEP(I)
     1,DZR(I),DZD(I),DIFF(I),I=1,NL1)
      WRITE(6,2560)NL,P(NL),PCT(NL),DZR(NL),DZD(NL)
      WRITE(8,2560)NL,P(NL),PCT(NL),DZR(NL),DZD(NL)
 2500 FORMAT(/' NO',4X,'RESIS     PCT',4X,'THICK     PCT',4X
     1,'DEPTH',6X,'DZR',6X,'DZD',3X,'%DIFF')
 2550 FORMAT(I3,F9.2,F8.2,F9.2,F8.2,3F9.2,F8.2)
 2560 FORMAT(I3,F9.2,F8.2,26X,2F9.2)
C
      WRITE(8,210)
  210 FORMAT(/' NO',11X,'AB/2',9X,'RHOCAL',9X
     1      ,'RHOOBS',8X,'% ERROR')
      DO 224 I=1,ND
      ERROR=(RC(I)/RF(I)-1.)*100.
  224 WRITE(8,230)I,AB(I),RC(I),RF(I),ERROR
  230 FORMAT(I3,F15.2,F15.3,F15.3,F15.4)
      RETURN
      END
C ***************************************************************
```

```
      SUBROUTINE MATRIX
C ***************************************************************
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1      ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/MAT/ATA(50,50),ATAINV(50,50),H(50,25),W(50)
      DOUBLE PRECISION ATA,ATAINV,W,DSUM
      DIMENSION B(50),WORK(25)
C
      WRITE(8,1000)
 1000 FORMAT(/' PARAMETER CORRELATION MATRIX')
      NR=(NP+1)/2
      DO 10 J=1,NP
      DO 10 K=1,J
   10 ATAINV(J,K)=ATAINV(J,K)*W(J)*W(K)
      NT=NP-NR
      DO 40 I=1,NP
      DO 20 J=1,I
   20 WORK(J)=ATAINV(I,J)/DSQRT(ATAINV(I,I)*ATAINV(J,J))
      IF(I.GT.NR) GO TO 30
      B(I)='R'
      WRITE(8,1100)I,(WORK(J),J=1,I)
 1100 FORMAT(' R(',I1,')',15F6.2)
      GO TO 40
   30 IT=I-NR
      B(I)='T'
      WRITE(8,1200) IT,(WORK(J),J=1,I)
 1200 FORMAT(' T(',I1,')',15F6.2)
   40 CONTINUE
      WRITE(8,1300)(B(J),J,J=1,NR),(B(K+NR),K,K=1,NT)
 1300 FORMAT(5X,15(2X,A1,'(',I1,')'))
C
      WRITE(8,2000)
 2000 FORMAT(/' RESOLUTION MATRIX')
      DO 90 I=1,NP
      DO 60 J=1,I
      SUM=0.
      DO 50 K=1,ND
   50 SUM=SUM+H(I,K)*A(K,J)
   60 WORK(J)=SUM
      IF(I.GT.NR) GO TO 80
      B(I)='R'
      WRITE(8,1100)I,(WORK(J),J=1,I)
      GO TO 90
   80 IT=I-NR
      B(I)='T'
      WRITE(8,1200) IT,(WORK(J),J=1,I)
   90 CONTINUE
      WRITE(8,1300)(B(J),J,J=1,NR),(B(K+NR),K,K=1,NT)
C
      WRITE(8,3000)
 3000 FORMAT(/' INFORMATION DENSITY MATRIX')
      DO 120 I=1,ND
      DO 110 K=1,I
      SUM=0.0
      DO 100 J=1,NP
  100 SUM=SUM+A(I,J)*H(J,K)
```

```
  110 WORK(K)=SUM
  120 WRITE(8,3100) (WORK(K),K=1,I)
 3100 FORMAT(1X,26F4.1)
      RETURN
      END
C ****************************************************************
      SUBROUTINE APPRES(RA,IQ)
C ****************************************************************
      COMMON TITLE(4),ND,AB(25),RF(25),MODE,P(50),NL,NL1,NP
     1      ,Q,RC(25),RS(25),A(25,50),PCT(50)
      COMMON/FLTR/NC,F,Y,FC(141)
      DIMENSION RA(25),T(100)
C
      IF(MODE.LE.0) GO TO 100
      X=AB(1)*EXP(-Y)
      LE = NC+ND-1
      DO 10 L = 1,LE
      B=ABS(P(NL))
      DO 9 K = 1,NL1
      RW=ABS(P(NL-K))
      TH = EXP(-2.*ABS(P(NP+1-K))/X)
      TP = 1.+TH
      TM = 1.-TH
    9 B = RW*(B*TP+RW*TM)/(RW*TP+B*TM)
      T(L) = B
   10 X = X*F
      DO 12 J = 1,ND
      RM=0.
      DO 11 L = 1,NC
   11 RM = RM+FC(L)*T(L+J-1)
   12 RA(J) = RM
      GO TO 200
C
  100 DO 110 I=1,ND
      RM=0.
      X=AB(I)*EXP(-Y)
      DO 120 J=1,NC
      B=ABS(P(NL))
      DO 130 K=1,NL1
      RW=ABS(P(NL-K))
      TH=EXP(-2.*ABS(P(NP+1-K))/X)
      TP=1.+TH
      TM=1.-TH
  130 B=RW*(B*TP+RW*TM)/(RW*TP+B*TM)
      X=X*F
  120 RM=RM+FC(J)*B
  110 RA(I)=RM
C
  200 IF(IQ.EQ.0) RETURN
      Q=0.
      DO 210 I=1,ND
  210 Q=Q+(RA(I)/RS(I)-1.)**2
      Q=SQRT(Q/FLOAT(ND))
      RETURN
      END
C ****************************************************************
```

```
      SUBROUTINE YM6
C *************************************************************
      COMMON/FLTR/NC,F,Y1,FC(141)
C     NC     : NUMBER OF COEFFICIENTS
C     FC(NC) : FILTER COEFFICIENTS
C     Y1     : ABSCISSA OF FC(1)
C     F = EXP(D), WHERE SAMPLING DISTANCE D = (LN10)/6
C     ABSCISSA OF FC(J) = Y1-(J-1)*D
C
      DIMENSION FD(28)
      DATA FD/
     1 .86463368E-4,-.36875438E-3, .92111524E-3,-.18772686E-2,
     2 .35343506E-2,-.64612278E-2, .11691267E-1,-.21088677E-1,
     3 .38045092E-1,-.68913866E-1, .12666755E-0,-.24355470E-0,
     4 .52117305E-0,-.12644217E+1, .27992503E+1,-.34853734E+1,
     5 .41912647E+0, .11950174E+1, .61073260E-0, .24298434E-0,
     6 .82207576E-1, .27770876E-1, .87075202E-2, .28615354E-2,
     7 .88399812E-3, .28020133E-3, .10060054E-3, .34147278E-4/
      F=10.**(1./6.)
      NC = 28
      Y1=7.4222404
      DO 100 I=1,NC
  100 FC(I)=FD(I)
      RETURN
      END
C *************************************************************
      SUBROUTINE YM10
C *************************************************************
      COMMON/FLTR/NC,F,Y1,FC(141)
      DIMENSION FD(70)
      DATA FD/
     *-.22247786E-4, .51184989E-4,-.66575186E-4, .86592875E-4,
     *-.11262944E-3, .14649463E-3,-.19054233E-3, .24783420E-3,
     *-.32235248E-3, .41927675E-3,-.54534402E-3, .70931693E-3,
     *-.92259288E-3, .11999962E-2,-.15608086E-2, .20301093E-2,
     *-.26405183E-2, .34344639E-2,-.44671314E-2, .58102992E-2,
     *-.75573279E-2, .98296496E-2,-.12785208E-1, .16629439E-1,
     *-.21629544E-1, .28133073E-1,-.36592072E-1, .47594515E-1,
     *-.61905179E-1, .80518827E-1,-.10472943E+0, .13622036E+0,
     *-.17718202E+0, .23046613E+0,-.29978969E+0, .39001009E+0,
     *-.50751079E+0, .66077997E+0,-.86135847E+0, .11254667E+1,
     *-.14762271E+1, .19413057E+1,-.25117825E+1, .29397638E+1,
     *-.22862253E+1,-.71362115E+0, .41491251E+1,-.23169602E+1,
     *-.16867419E+1,-.32170199E+0, .68963453E+0, .69150854E+0,
     * .54204064E+0, .32222510E+0, .19033795E+0, .99724447E-1,
     * .54063095E-1, .27109364E-1, .14239291E-1, .70240599E-2,
     * .36435998E-2, .17863940E-2, .92183691E-3, .45100602E-3,
     * .23218367E-3, .11353351E-3, .58376418E-4, .28547132E-4,
     * .14666868E-4, .14501929E-4/
      NC=70
      F=10.**.1
      Y1=12.664218
      DO 100 I=1,NC
  100 FC(I)=FD(I)
      RETURN
      END
```