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Scattering and Attenuation of High-Frequency Seismic Waves: Development of the Theory of Coda Waves

by

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ABSTRACT

Coda waves are one of the most striking features recorded on seismograms, and their study has bloomed over the last 10 years. Scattering and attenuation of high-frequency seismic waves constitute key subjects in both theoretical and applied research in present-day seismology. However, the narrow relations among coda waves, scattering, and attenuation are not taken into account often enough, nor is an overall comprehension of these subjects always offered. Coda waves of local earthquakes can be considered as backscattered S to S waves, and the quality factor which accounts for their decay, Q_c, expresses both absorption and scattering-attenuation effects. How scattering constructs coda waves and contributes to their attenuation is not completely understood; work now in progress will improve the understanding of scattering theory. This review describes coda waves in detail and discusses their relation to attenuation in the context of research on seismic-scattering problems. Care has been taken to explain the statistical models used to deal with the heterogeneities responsible for scattering. The important role played by observational data is also stressed.

GENERAL INTRODUCTION

The assumption of lateral homogeneity in the propagation of seismic waves through the Earth has decisively influenced the development of new concepts and applied procedures in seismology. The classical Earth model, based upon this assumption, is one example of its usefulness.

Seismologists were very soon aware of the large departure from the real Earth that the above assumption brings forth. The effects of scattering and attenuation, not explicable without accepting lateral heterogeneity, became evident during the early 1960's when more accurate data were provided by modern seismic arrays. Since that time, interaction of elastic waves and small obstacles, cracks, wedges, interfaces, etc., has received preferential attention. (See Pilant, 1979, for references up to 1979.) The difficulties of explaining the new observations using methods based on the classical model motivated the development of a new procedure specially designed to deal with seismic waves in a laterally heterogeneous Earth. A wider comprehension of scattering and attenuation processes was required as a basis for the new methodology being proposed.

These requirements become particularly pressing when short-period waves, strongly affected by the shallower crust, are used to study source parameters of local earthquakes and site effects (Espinosa, 1968, 1969, 1971; Espinosa and Algermissen, 1972a, b, 1973; Bard and Gariel, 1986). The high sensitivity of these waves to the details of source and wave path increases their importance. This sensitivity also makes their study more difficult because of the perturbations in amplitude and phase of the primary waves caused by the path heterogeneities. Removing this effect to extract path and source information separately is an extremely important objective.

Some of these problems began to be solved when some seismologists started to use statistical techniques bypassing the usual deterministic approach. Lateral inhomogeneity could no longer be approached in a deterministic way due to the large number of parameters to be considered.

Under the new approach, only a small number of statistical parameters were sufficient to characterize the heterogeneities of the Earth. This work began in the late 1960's and early 1970's using explosion data (Wesley, 1965; Nikolayev, 1968; Nikolayev and Tregub, 1970; Levin and Robinson, 1969; Dunkin, 1969; Greenfield, 1971), teleseismic records (Haddon, 1972; Cleary and Haddon, 1972; Aki, 1973; King and others, 1973; Capon, 1974), and lunar seismograms (Nakamura and others, 1970; Dainty and others, 1974). Its application to high-frequency records was pioneered by Aki (1969). In order to justify the statistical treatment, attention was directed to the coda part of the seismograms, the part after all the direct waves such as P, S, and surface waves have arrived, and where the backscattered waves were more likely to be found. This type of wave is considered as the superposition of many secondary waves generated by the incidence of primary waves upon the heterogeneities. Because they are the result of random processes, they can be statistically analyzed. Moreover, the great variety of paths traveled by these waves provides information concerning the average properties of the medium instead of just the characteristics of a particular path. Thus, the possibility of discriminating source and path effects is firmly established.

The theory required for these early studies just cited and for further investigations was initially borrowed from other fields of physics where scattering had already been studied; for example, astrophysics (Chandrasekhar, 1960), and atmospheric sciences (Tatarskii, 1961). The work by Chernov (1960) on scattering of acoustic waves was a breakthrough. The first adopted steps consisted of a simplified model in which scalar waves were singly scattered by an isotropic heterogeneous medium. The results obtained principally from coda analysis, attenuation estimates, and heterogeneity studies have shown the possibilities and shortcomings of the assumptions just mentioned. The requirement of a more realistic model, including vector wave and anisotropic multiple scattering, has been clearly shown, and most recent studies move in this direction.

The intent of this paper is to summarize the more significant work published to date stressing both observational contributions and progressive improvements in the scattering model. In Section 1, a glossary of the most important terms used in the work, including a detailed definition of two main items studied—coda and attenuation—is given. For perspective, a global description of scattering is also included. Sections 2 and 3 review the effects of inhomogeneities in the lithosphere on short-period waves and their influence in the assumed scattering model. In Section 2, both theoretical aspects (Part I) and applications (Part II) of coda-wave analysis are studied. The attenuation analysis included in the last part describes Q_β estimates and Q_β–Q_c relations. The resulting model supposes that the coda is composed of single backscattered S to S waves coming out from heterogeneities randomly distributed. Also, it is shown how the Q estimates just cited pose the necessity of improving the basic assumptions of the model.

P codas and temporal variations of coda-wave shapes are described in Parts III and IV of this same section.

Section 3 compiles the analysis of wave propagation in heterogeneous media considered as "random media". The results obtained by using Chernov's theory are compared to those arising from the use of the "mean wave" and the "traveltime corrected mean wave" formalisms. Several deterministic results are also described. All these results cast new light on the need to improve scattering theory and on ways to change it. The ongoing attempts to extend the theory to multiple anisotropic scattering of vector waves are presented in Section 4.

SECTION 1: DESCRIPTION OF SEISMIC-WAVE SCATTERING TOPICS

This section has two main goals: to define some frequently used terms and to overview the seismic-scattering problem.

First, a glossary in which "coda" and "attenuation" are defined is presented in detail. Second, a reference frame will be established in which to place observational contributions, the approaches taken by different investigators, and the search for new solutions.

GLOSSARY

Terms are grouped herein according to their necessary relation with the following physical phenomena: waves, media, fluctuations, scattering, and attenuation. This classification is used in table 1, where symbols and mathematical relations are defined. Only the meaning used in this review is presented.

wave Disturbance which is propagated through a medium without involving net movement of material.

primary wave Wave generated at the earthquake source and not due to scattering.

secondary wave Wave generated at the heterogeneities upon the incidence of primary waves or other secondary waves.

scalar wave Wave in which the physical quantity involved is a scalar with three components of ground motion.

vector wave Wave in which the physical quantity involved is a vector. S waves constitute a seismic example.

coda wave Initially the word "coda" was used to refer to the oscillations of the ground continuing long after the slowest theoretical surface arrival time (Jeffreys, 1929; Ewing and others, 1957). A variant of the word, "cauda", was also taken from Latin to name the tail of surface waves following the maximum displacement. It was usually abbreviated in such a way that these waves became "C" waves (Båth, 1973). Another meaning of "coda" has been used to identify the tail part of seismograms of local earthquakes. This part corresponds to the recorded energy after the passage of all primary waves. This latter meaning was used in the first studies related to local earthquake waves (Aki, 1969; Takano, 1971).

If only body waves are considered, a distinction can be made between P and S codas. The former is the train of waves placed between direct P and S phases. The latter means the set following the direct S waves (fig. 1). The S coda is more noticeable on local earthquakes and has received much attention lately. Unless otherwise noted, "coda" refers to S coda.

The beginning of the coda was originally assigned to the point where the amplitude decay starts to be regular. After the Rautian and Khalturin study (1978), the basic procedure is to place the coda-start where the time measured from the earthquake-origin time, usually called lapse-time, is twice the S-wave traveltime. If this rule becomes too demanding, the coda-start can be placed closer to the S-wave time of arrival. In any case, it is necessary to avoid contamination with the contribution made by direct S phases.

The end of the coda is usually placed where the signal-to-noise ratio reaches a chosen value, generally 1 or 2. Care must be taken in following the above procedure since it

TABLE 1.--Main symbols used in this study

[Dim, designates dimensionless]

Symbol(s)	Meaning	Dimension
A	Wave amplitude------------------	L
f, ω	Wave frequency------------------	T^{-1}
T	Wave period---------------------	T
v	Wave velocity-------------------	LT^{-1}
α	P-wave velocity-----------------	
β	S-wave velocity-----------------	
λ	Wavelength ($\lambda = v/f$)-----------	L
κ	Wave number ($\kappa = 2\pi/\lambda$)---------	L^{-1}
L	Medium extent length------------	L
n	Density of discrete-scatterers--	
n_s	Number scatterers/area----------	L^{-2}
n_v	Number scatterers/volume--------	L^{-3}
R, Δ	Source-receiver distance--------	L
r	Scatterer-receiver distance-----	L
r_o	Reference distance--------------	L
$\tilde{R}$	Source-scatterer distance-------	L
$\mu(\vec{r})$	Fluctuation (usually velocity fluctuation).	Dim
$N(\vec{r})$	Correlation function------------	Dim
α	Correlation distance------------	L
D	Wave parameter ($D = 4L/\kappa\alpha^2$)---	Dim
$\langle\mu^2\rangle$	Mean-square of velocity fluctation.	Dim
σ	Scattering cross section--------	L^2, L
ℓ	Mean free path------------------	L
ζ	Mean free time ($\zeta = \ell/v$)------	T
x^*	Average distance that wave travels before its energy is totally attenuated.	L
$g(\theta)$	Differential scattering coefficient.	L^{-1}
g_π	Backscattering coefficient------	L^{-1}
g	Scattering or turbidity coefficient.	L^{-1}
$\Delta I/I$	Scattered energy ratio----------	Dim
Q_c	Quality factor from coda waves--	Dim
Q_β	Quality factor from S waves-----	Dim
Q	Quality factor------------------	Dim
Q_i	Quality factor accounting for absorption (intrinsic attenuation).	Dim
Q_s	Quality factor accounting for scattering (apparent attenuation).	Dim

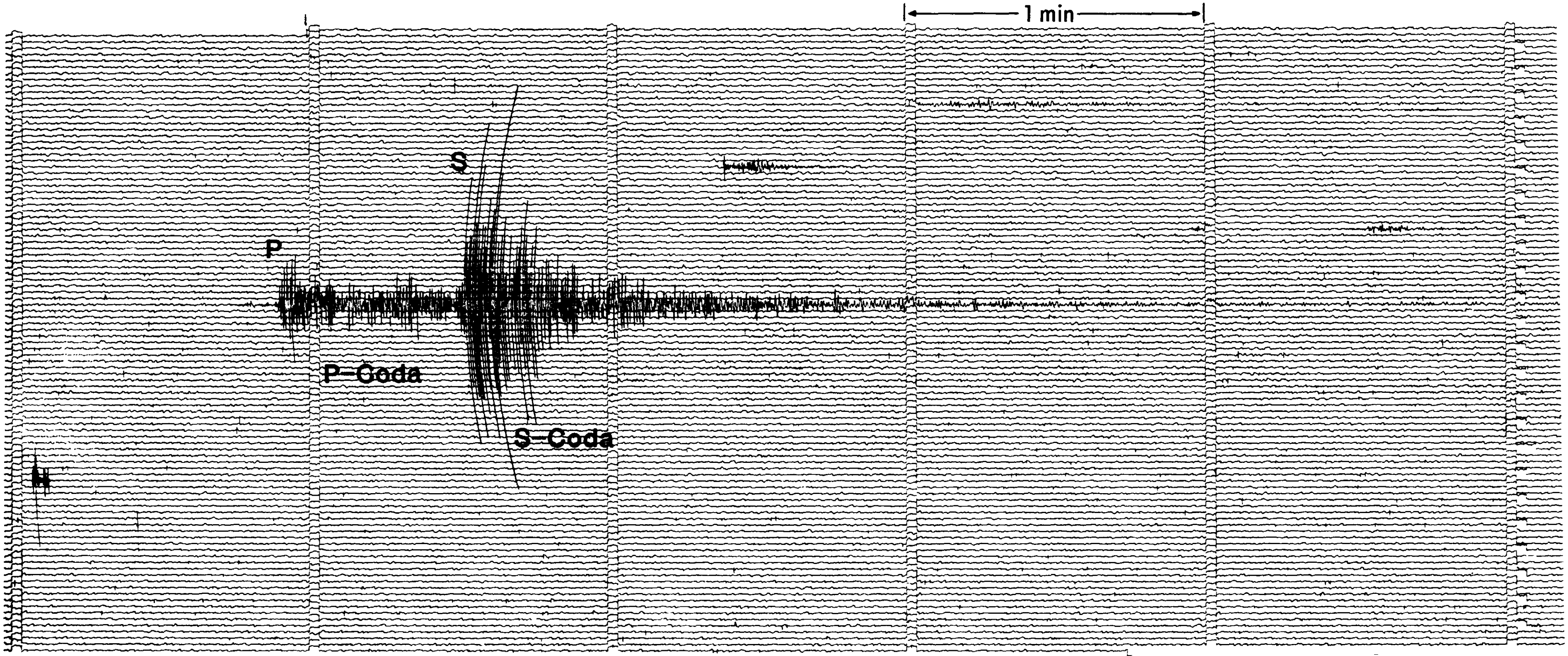

Figure 1.— Sample seismogram, showing P and S codas, recorded at station GUD (Madrid, Spain), with an $M_L = 3.7$ at an epicentral distance of 337 km.

tends to minimize the site amplification effect on the recorded signatures. In addition, coda duration can be affected by the background noise of the station site.

medium Place in which the scattering process occurs.

homogeneous medium with discrete scatterers Unperturbed medium with uniform velocity and density in which fluctuations are introduced by discrete scatterers: cracks, faults, density contrasts, and velocity anomalies. The number of heterogeneities per unit volume inside a homogeneous medium is represented by n_v. When a two-dimensional homogeneous medium is considered, n_v becomes n_s: the number of the heterogeneities per unit surface. Both n_v and n_s are contained in n, which is the density of scatterers.

heterogeneous medium The whole space is taken as inhomogeneous. In this case the scattering field is treated as a continuous medium where an inhomogeneous wave equation should be solved.

random medium Region with average characteristics in which deviations from these mean values produce random heterogeneities. The fluctuating parameters can be velocity, density, or the Lamé parameters.

medium-extent length, L The medium where scattering can take place, also called "travel distance" as is the distance in which waves can travel.

fluctuation

fluctuation, $\mu(\vec{r})$ Variable of a random medium which expresses the difference between the actual and the average value of parameter μ at a point defined by $\vec{r}$. Media with $\langle \mu^2 \rangle \ll 1$ can be considered as weakly inhomogeneous media. Hereafter the triangular brackets represent average values.

correlation function, $N(\vec{r})$ Mathematical expression which gives the spatial correlation of fluctuations. It can be expressed as:

$$N(\vec{r}) = \langle \mu(\vec{r}_1), \mu(\vec{r}_2) \rangle \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ define the points at which a fluctuation is measured and the average is made over an ensemble of the random media. Initially, $N(\vec{r})$ adopted Gaussian or exponential forms. In the latest studies Von Karman functions are preferred.

correlation distance, α It expresses the separation for which fluctuations become uncorrelated in the statistical study. It is also called "inhomogeneity scale length".

uniform random field, $N(\vec{r})$ depends only on the difference $\vec{r}_1 - \vec{r}_2$.

isotropic random field, $N(\vec{r})$ depends only on the modulus of $\vec{r}_1 - \vec{r}_2$.

normal random field, $N(\vec{r})$ has the form of a Gaussian curve

$$N(\vec{r}) = N(\vec{r}_0) e^{-r^2/\alpha^2} \tag{2}$$

wave parameter, D Ratio of the size of the first Fresnel zone to the scale length of inhomogeneities. It can be given by the expression:

$$D = \frac{4L}{\kappa \alpha^2} \tag{3}$$

where κ is the wave number.

scattering

scattering cross section, σ (also called **effective cross section**) Represents the ratio of the time average of the scattered wave energy per unit time to the mean energy flux density of the incident wave (Landau and Lifshitz, 1959).

mean free path, ℓ Parameter which controls the energy transferred from the primary to the scattered waves throughout the traveled path. The scatterers reduce the mean energy flux density of the incident plane wave by $e^{-x/\ell}$, x being the distance along the propagation direction.

mean free time, ς Time necessary for a wave with velocity v to travel the mean free path.

direct path, path connecting source and receiver.

forward scattering Process in which most of the scattered energy is pumped forward in the incident primary wave direction. The opposite result constitutes backward scattering.

differential scattering coefficient, $g(\theta)$ 4π times the fractional loss of energy by scattering per unit travel distance of the primary wave, and per unit solid angle at the radiation direction θ . This angle is measured from the direction of the incident primary wave so that $\theta < \pi/2$ represents the forward direction.

backscattering coefficient, g_π Value of $g(\theta)$ for $\theta = \pi$. Physically $g(\theta)$ expresses 4π times the fractional energy lost by backscattering into a unit solid angle around θ for every kilometer traveled by the primary wave.

scattering coefficient, g Average of $g(\theta)$ over θ. It evaluates the capacity of the medium to originate scattering. For an isotropic scattering $g(\theta) = g$. If the medium contains discrete heterogeneities "g" can be obtained multiplying n (density of scatterers) by σ (scattering cross section). The scattering coefficient is also called "turbidity coefficient". From energy considerations, Chernov (1960) defined the turbidity coefficient as:

$$g = \frac{\Delta I}{IL} \tag{4}$$

where ΔI represents the loss of energy by scattering when a wave of energy I passes through a layer of thickness L.

weak scattering The fluctuations of perturbed parameters are small when compared with their corresponding mean values. The fractional energy loss from primary waves, $\Delta I/I$, is very small and Born's approximation can be applied. Any other situation different from the above definition implies "strong scattering".

single scattering Process in which, if discrete obstacles inside a homogeneous medium are assumed, it may be sufficient to consider only one wave-scatterer encounter. Weak scattering is required. In "strong scattering" it is necessary to consider "multiple scattering". If the scattering is very strong it can be treated as a "diffusion process".

Born's approximation Scattering analysis method in which both the loss of energy from the primary waves and multiple scattering are neglected (Born and Wolf, 1965). This approximation, which violates the energy conservation law, has been accepted in various physical problems (for example, sound turbulence (Lighthill, 1953); sonic boom spikes (Crow, 1969); atomic collision (Mott and Massey, 1965)), but it can break down in many seismic cases.

Rayleigh scattering A particular but important situation when the obstacle size is much smaller than the incident wavelength, that is $\kappa\alpha \ll 1$. In this case, σ is inversely proportional to the fourth power of the wavelength and proportional to the square of the particle size (Rayleigh, 1896).

Mie scattering The particles are comparable to the wavelength (Mie, 1908).

attenuation Expresses the wave amplitude decay that takes place when a wave propagates through real media and cannot be attributed to geometrical spreading. Attenuation is usually expressed by the quality factor Q and is caused by two main sources: scattering due to heterogeneities and intrinsic absorption. Coda wave analysis can be a useful tool for distinguishing both contributions (Gao, 1984). Dainty and Toksöz (1981) summed up this fact by the relation:

$$\frac{1}{Q} = \frac{1}{Q_i} + \frac{1}{Q_s} \tag{5}$$

where Q is the quality factor obtained by experimental measurements based on the decay of seismic waves. Q_i^{-1} considers the intrinsic absorption and Q_s^{-1} represents the attenuation due to scattering. Dainty (1981) writes expression (5) as

$$\frac{1}{Q} = \frac{1}{Q_i} + \frac{g\mathrm{v}}{\omega} \tag{6}$$

where g, v, and ω are the turbidity coefficient, the velocity, and the frequency of the wave under consideration, respectively. The last equation is used by Dainty to explain Q observations and by several authors to estimate absorption and turbidity coefficients (Del Pezzo and Zollo, 1984; Castellano and others, 1986) or the mean free path (Rovelli, 1983b).

It has been noticed that the relative importance of these factors in a given geographical area determines the characteristics of seismograms of local earthquakes. Negligible absorption and strong scattering produce very long time duration seismograms. This is the case for lunar seismograms. On the contrary, an area with high absorption diminishes the scattering process and shortens time duration on the recordings. Hence, we see that the coda is affected by attenuation and therefore the coda recordings can be used to estimate its attenuation value. In fact, since 1975, coda waves have proved to be a useful tool to evaluate regional Q, joining the more conventional methods based on body waves, surface waves, and free oscillations of the Earth. For the sake of clarity, Q measurements based on coda waves will be identified as Q_c. It has been shown for regions of Dodaira and Tsukuba in Japan that for frequencies higher than 1.5 Hz, Q_c estimates agree closely with those obtained for S waves and identified as Q_β (Aki 1980 a, b). This is one of the facts which supports the model of coda waves as S to S backscattered waves. At this point it must be recalled that Q_β values may depend on "how to window" S waves; either from the peaks measured using band-pass data analysis or by using the measurements obtained from Fourier Transforms from digital data. The difference is the amount of forward scattering that is pumped back into primary waves.

Expression (5) indicates that estimates of Q based on the amplitude decay rate of coda waves, namely Q_c, include not only scattering attenuation but also absorptive attenuation as well.

DEFINITION OF SEISMIC-WAVE SCATTERING

The scattering problem demands special attention; it is a rather new concept in seismology and occupies a decisive place in this study.

A simple definition of scattering is the process in which a primary wave interacts with a heterogeneity of the medium and produces new secondary waves. This oversimplified description can be applied to many different physical fields, but every case presents some peculiar characteristics. A general overview of the just cited common background is found in Uscinski (1977) or in Ishimaru (1978). The seismic case can be better understood if the different elements which take part in the phenomenon are considered. The adopted starting point supposes a seismic wave travelling within an elastic medium. P and S waves are particularly interesting because they are considered to make up the major part of coda waves of local events. Although Knopoff and Hudson (1964, 1967) showed that $P \rightarrow S$ and $S \rightarrow P$ conversions can be neglected when high frequencies are considered, the complete treatment must be accomplished studying vector waves and including the possibility of $P \rightarrow S$ and $S \rightarrow P$ conversions. However, because of the difficulty posed by vector waves, most of the early contributions to this field have considered only scalar waves. This approach has been followed to study local coda waves (Section 2) and, in a great measure, velocity fluctuations (Section 3).

The medium where the scattering takes place can be basically considered as homogeneous or heterogeneous. In the first case discrete obstacles such as cracks, faults, low- or high-density inclusions, and low- or high-velocity inclusions are considered inside the homogeneous medium. Aki (1969), Aki and Chouet (1975), Dainty (1981), and Kikuchi (1981), for instance, have utilized this discrete scatterer model with a random uniform distribution.

If the medium is taken as inhomogeneous, the scattering field is treated as a continuous medium where inhomogeneous wave equations should be solved. The problem may become extremely complicated. An approach to this case is considering a "random medium"; that is, a space with average characteristics in which deviations from these mean values produce random heterogeneities. The fluctuating parameters can be velocity, density, or the Lamé parameters. This idea constitutes the basis of Chernov's (1960) and Karal and Keller's (1964) models which will be applied to analyze the Earth's small-scale heterogeneity. These models, which are examples of the use of stochastic theory in seismology (Hudson, 1982), will be explained in detail in Section 3. An updated study of the theoretical background can be found in Sobczyk (1985). They generally assume that the dimension of the chosen medium, L, or, in other words, the wave travel distance, is much greater than the correlation distance, that is $L \ll \alpha$.

In scattering processes, the relation between the wavelength "λ" and the linear dimension of the heterogeneous medium "L" affects the characteristics of the phenomenon. These variables are lumped in the parameter κL; and the greater κL, the more difficult it is to study the problem by using deterministic approaches.

Another important parameter is $\kappa\alpha$. When $\kappa\alpha$ is comparable to unity or greater, the effect of scatterer shape must be considered. Scattering by regular shaped obstacles has been a classical problem in wave propagation (Morse and Feshbach, 1953; Morse, 1968) and the case of a spherical intrusion has received particular attention in seismology (Ying and Truell, 1956; Knopoff, 1959a, b; Einspruch and others, 1960; Yamakawa, 1962; Dainty, 1981; Wu and Aki, 1985a). The obstacle shape is not important for the case of Rayleigh scattering ($\kappa\alpha \ll 1$). If

$\kappa a \lll 1$ or $\kappa a \ggg 1$, the waves are not affected by the obstacles and the medium acts like a homogeneous body (fig. 2). If $1 < \kappa a < 10$, the waves travel in a medium with heterogeneity scale similar to wave-length and scattering can become strong. This case is frequently met in many physical situations. In the seismic case, a discrete obstacle or a random heterogeneous medium can be characterized by perturbations of elastic parameters ($\delta\mu$, $\delta\lambda$) and density ($\delta\rho$) which can modify the amplitude and the velocity of incident waves. If it is assumed that these perturbations are relatively small to their corresponding mean value, this case becomes the "weak scattering" case. Impedance (density times velocity) fluctuations tend to introduce mainly backscattering, while velocity perturbations without impedance fluctuations create more forward scattering. However, in both cases, the results depend strongly on other factors: relative signs of anomalies; κa value; type of incident wave, etc.

The interaction between wave and inhomogeneity; that is, the scattering process core, depends on many parameters, defined in the Glossary. The mean free path, ℓ, evaluates the scatterers' distribution in the Earth and provides useful information on the tectonic characteristics. If ℓ is greater than the travel distance from the source, there will be little need to consider multiple scattering. Single scattering theory is used in most coda models, but multiple-scattering theory is needed in some cases. This problem has been addressed recently by several investigators.

The opposite situation to single scattering takes place when seismic waves are affected by scatterers so strongly that energy is diffused through the medium. This is the diffusion model introduced by Wesley (1965) and applied later to explain the scattering found on lunar seismograms.

When scattering is weak, the Born approximation can be used. However, it has been shown (Cleary and Haddon, 1972) that strong scattering can be present even in the mantle-outer core transitions. A discussion of the Born approximation can be found in Kennett (1972) and Hudson and Heritage (1981); this last work, which is not based on observational data, offers a too pessimistic opinion. A discussion founded on actual data is presented in Aki (1973). In high-frequency seismic-wave analysis, the Born approximation is used frequently to explain coda decay or to deal with attenuation of seismic waves. The discrimination between weak or strong scattering processes can be made considering if the ratio $\Delta I/I$ between the energy carried by the scattered waves, ΔI, and the primary energy, I, is small or not.

Another problem to be considered is in the angular dependence in the scattering process. This matter was studied by Yamakawa (1962) for different shaped inclusions and by Chernov for random fluctuations of the wave propagation velocity. Initial models explaining coda formation consider only backscattering or isotropic scattering, but the angular dependence has received a careful treatment in the latest studies of Sato (1984) and Wu and Aki (1985a). The early arriving coda waves are more sensitive to the angular dependence of scattering than the latter parts of the record, which are more affected by multiple scattering. The early part is also more sensitive to the asymmetry of source radiation pattern, another factor which has been considered in recent studies.

The results presently available show that scattering problems can be classified according to a few parameters whose values condition the mathematical procedures which can be used to solve each case. These parameters are κa, κL, $\Delta I/I$, and D. Figure 3 shows the general distribution of seismic scattering problems according to the above parameters (left), and the area dealing

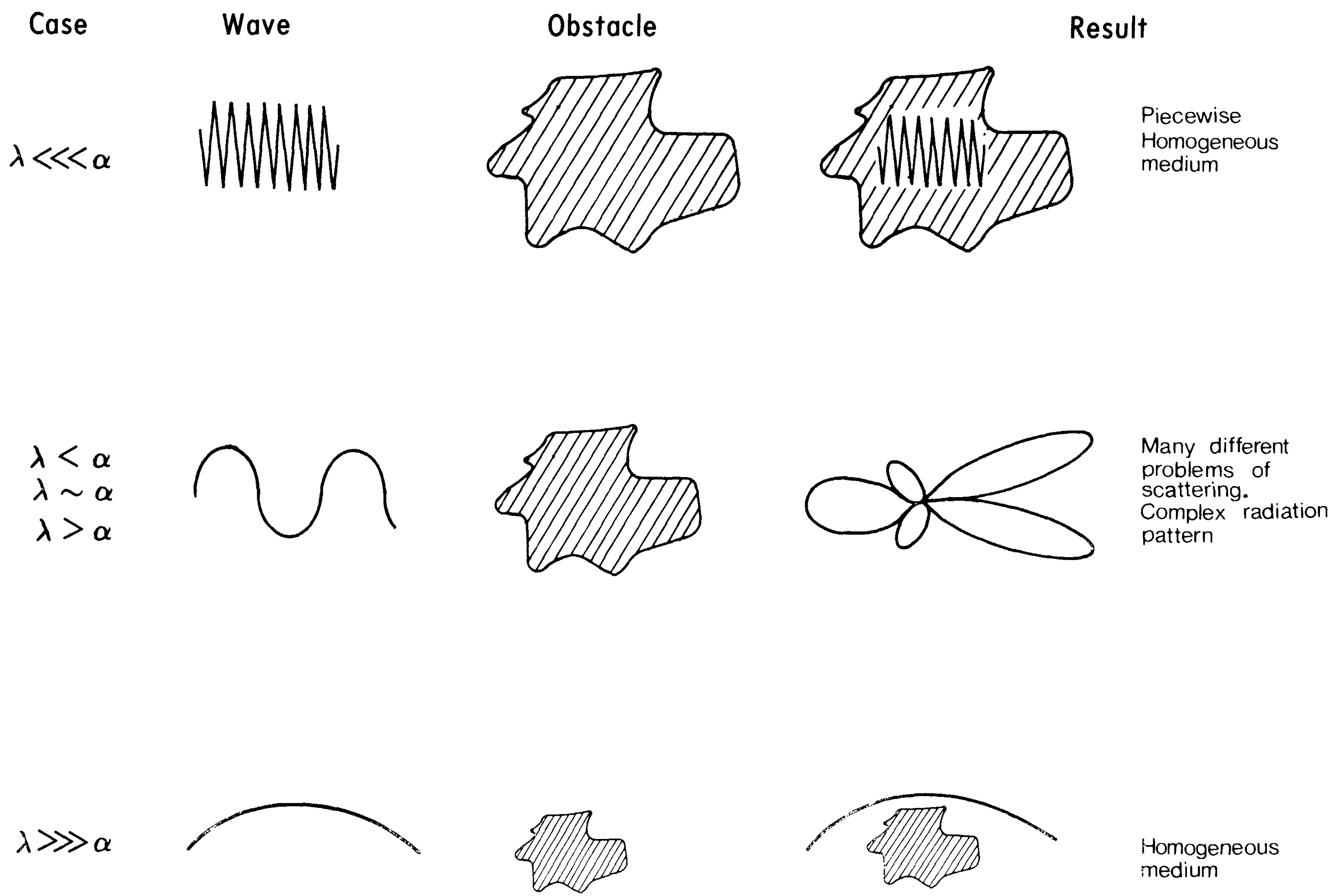

Figure 2.— Simplified schematic of the relation between obstacle and wavelength sizes. λ and α represent wavelength and inhomogeneity scale length respectively.

with the subject matter which is going to be treated in this paper (right). First of all, it can be noticed that the area of interest presented in this paper is characterized by the condition $L \gg \alpha$, namely the travel distance is longer than the inhomogeneity scale length. It seems that a solution cannot be obtained by using numerical methods such as finite differences (F.D.) or finite elements (F.E.). The F.D. case must be reconsidered after the results obtained by Frankel and Clayton (1984, 1986). The enlargement at the right in figure 3 indicates that the seismic problems that will be studied include the possibility of "strong scattering". When $\kappa\alpha \ll 1$, the cases $D \ll 1$ and $D \gg 1$ may be distinguished. The former can be treated by ray theory and the latter (Fraunhofer case) requires diffraction theory. Coda-wave analysis comprehends the situation $\kappa\alpha \simeq 1$, that is, the scattering in which wavelength and obstacle have similar dimensions and scattering can be particularly strong. Figure 3 places the coda wave analysis within the global scattering problem and reveals the importance of the parameters described in the Glossary.

Summarizing Section 1 it is possible to state that a complete solution to the scattering problem will have to deal simultaneously with the following physical phenomena:

vector waves and $P \rightarrow S$ and $S \rightarrow P$ conversions
heterogeneous three dimensional and anisotropic media
scatterers with a size comparable to the wavelength
multiple and anisotropic wave-scatterers interaction

SECTION 2: CODA-WAVE ANALYSIS

PART I: MODELING CODA WAVES

Introduction

The study of coda analysis occupies a substantial part of this review not only for its importance in scattering theory but also because coda waves have become a powerful resource in many fields of seismological research.

The first observation about coda waves, relevant to the backscattering model, was that the total time duration of seismic waves is independent of epicentral distance, at least when the epicentral distance is shorter than about 100 km (Soloviev, 1965). Other investigators had also observed the similarity in the spectral content of coda waves for events with different epicenter-station paths (Aki, 1956). These results received new attention when the need for a practical method of dealing with seismic waves in a laterally heterogeneous medium led to the search for statistical methods. They also increased the interest in backscattering waves formed by the superposition of waves generated by the interaction between scatterers and primary waves. Since these backscattering waves can be considered as the superposition of many independent events, they are more suitable to be treated statistically. If these waves exist, they must be located in time after the passage of all the primary waves, that is, in the coda. The above observations were supported and confirmed in Aki's (1969) paper in which the aftershocks of the June 1966 Parkfield, California, events were studied. Coda waves appeared to be insensitive to the nature of direct path and to have similar amplitude and spectral contents for different stations and for a given source as backscattered waves should. The path independence suggested the separation of source and path effects in the coda power spectrum values. This poses the

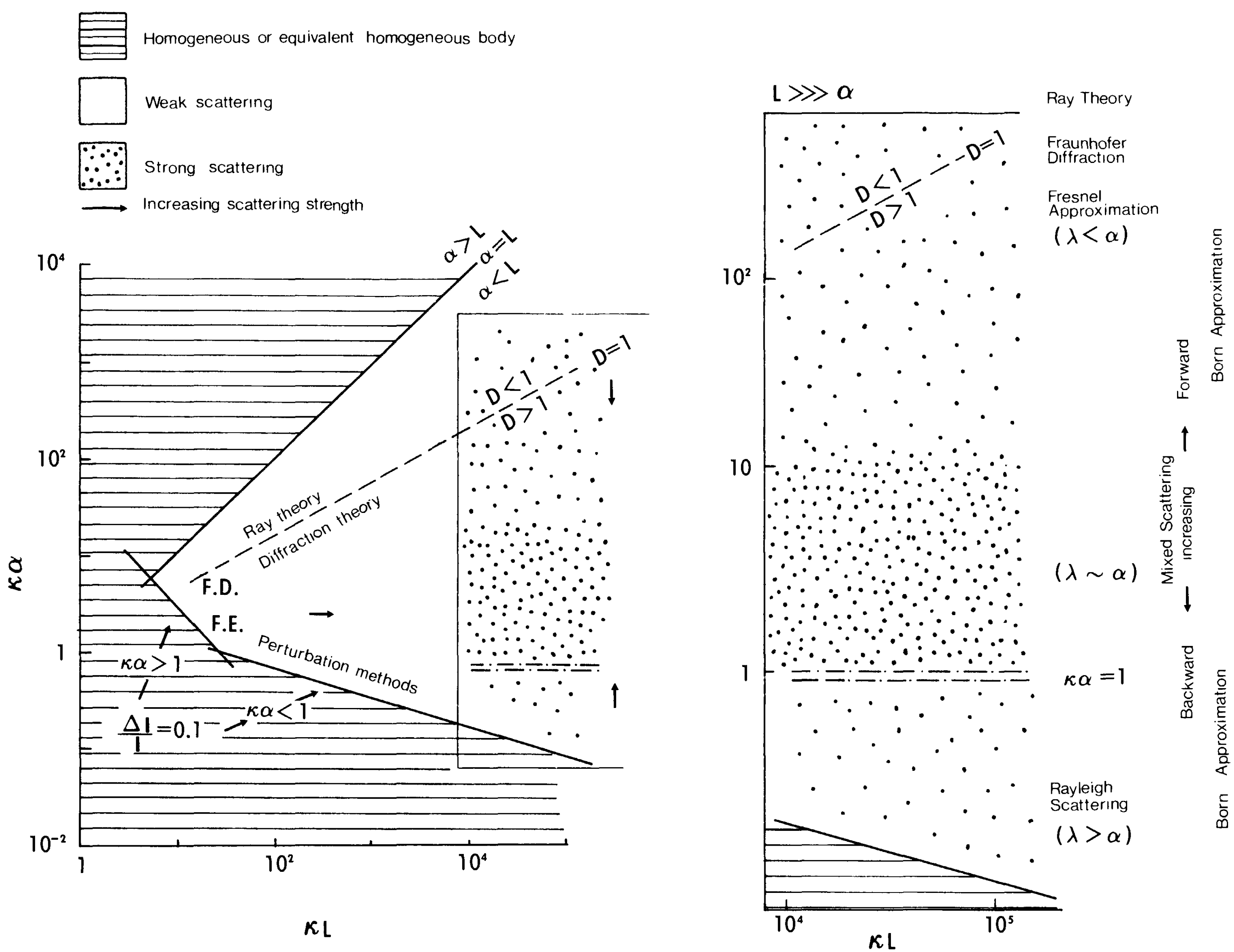

Figure 3.— Diagram of seismic-scattering problems. Left: general overview. Right: region of interest presented in this review. See text and Table 1 for explanation of symbols. [Adapted from **Quantitative Seismology**, vol. II, by K. Aki and P. G. Richards, copyright W. H. Freeman and Company, 1980.]

possibility of finding an expression with the form:

$$P(\omega|t) = S(\omega)C(\omega|t) \tag{7}$$

where $P(\omega|t)$ is the power spectrum of the coda for frequency ω at lapse-time t. $S(\omega)$ includes only the source parameters and $C(\omega|t)$ represents the effect of a large geographical area and is independent of distance and details of path connecting source and station.

If $C(\omega|t)$ is common to all the events of the same area and two of these events are considered, the above relation (7) becomes

$$\frac{P_1(\omega|t)}{P_2(\omega|t)} = \frac{S_1(\omega)}{S_2(\omega)}\frac{C(\omega|t)}{C(\omega|t)} = \frac{S_1(\omega)}{S_2(\omega)} \tag{8}$$

which allows the extraction of source information from the power spectra. In this manner, source effects and path effects are distinctly separated.

Relation (7) constitutes a cornerstone in coda-wave analysis and has been confirmed for many different areas (Aki and Chouet, 1975; Rautian and Khalturin, 1978; Rautian and others, 1978; Tsujiura, 1978; Roecker and others, 1982). It also indicates the two steps which must be accomplished in coda wave analysis:

1. Finding a mathematical expression for $S(\omega)C(\omega|t)$. The results will be tightly linked to the model chosen to explain the coda.
2. Relating $P(\omega|t)$ and coda amplitudes. This step will depend on the kind of data, analog or digital, considered.

The following paragraphs describe the successive attempts to accomplish these steps using different models.

Evolution of Model

The evolution of coda wave models comprises four main stages.

1. The elimination of surface waves as a possible cause for coda waves, at least for Japan, and their progressive substitution by S waves. Investigators have focused on this stage from the initial "surface-wave model" (Aki, 1969) to the studies on Q_β–Q_c relation carried out around 1980. The decisive arguments to draw this result were:
   a. Coda waves corresponding to surface and deep borehole ($\sim$ 3.5 km) sites in Japan share the same main features (Sato, 1978).
   b. Coda and S waves have similar site effects for a wide frequency range (1 to 25 Hz) (Tsujiura, 1978)
   c. Q_β and Q_c are very similar for the above frequency range (Aki, 1980a, b).
2. The abandoning of the diffusion process as a possible origin for terrestrial codas. Although diffusion theory had been previously applied by Wesley (1965) to explosion data, it received new attention when the comparison between lunar and terrestrial seismograms suggested a different origin for each phenomenon (Latham and others, 1970, 1971; Nakamura and others, 1970). Lunar seismograms have a very long duration, exceeding 1 hour, and a very slow growth from the first arrival to the maximum amplitude. This

TABLE 2.--Models proposed to explain coda wave

[Δ, Source-receiver distance; r, scatterer-receiver distance; $\tilde{R}$, source-scatterer distance; ℓ, wave-free path]

Model	References	Assumptions				Observations
		Primary wave	Medium	Heterogeneity	Scattering	
Surface waves----------	Aki (1969)-----------	Surface waves--	Homogeneous, unbounded, isotropic, two dimensions.	Random and uniform distribution. $\Delta<\ell$	Single, weak.	$r \approx \tilde{R}>>\Delta$
	Kopnichev (1975)-----	----do---------	----do----------	----do-------------	Single, isotropic.	Extends the previous model to intermediate distances.
Single backscattering--	Aki and Chouet (1975)	Surface and body waves.	Homogeneous, unbounded, isotropic, two and three dimensions.	----do-------------	Single, weak.	Introduces the Born approximation.
Single isotropic scattering.	Sato (1977a)----------	Body waves-----	Homogeneous, unbounded, isotropic, three dimensions.	Random, uniform, and isotropic distribution. $\Delta>\ell$	Single, isotropic.	Deals with energy.
Diffusion model--------	Wesley (1965), Dainty and others (1974), Aki and Chouet (1975), Kopnichev (1977b).	Surface and body waves.	Homogeneous, unbounded, three-dimensions.	----do--------------	Multiple, strong.	Some assumptions vary with the author and the case, terrestrial or lunar considered.
Multiple scattering----	Kopnichev (1977b)-----	----do---------	Random medium, statistically uniform, isotropic and normal. $\Delta>\ell$		Double, triple, isotropic, strong.	Includes diffusion model.
Do-----------------	Gao and others (1983a, b).	Body waves-----	Homogeneous, unbounded, two and three dimensions.	Random and uniform distribution. $\Delta>\ell$	Multiple, isotropic, weak.	Uses the Born approximation $r \approx \tilde{R}>>\Delta$.

increase takes about 5-10 minutes and does not show any predominant phase; body-wave phases are very small and loom up only in the first minute of the seismogram (Toksöz and others, 1974). The coda is very large and occupies almost the whole seismogram. Its maximum amplitude varies with both epicentral distance and focal depth (Latham and others, 1971) and can be used to estimate the magnitude (Nakamura, 1977a). All these features suggested more intense scattering and much lower absorption on the Moon than on the Earth (Dainty and Toksöz, 1977, 1981), demanding a method based upon diffusion models. This analysis has been developed by several authors who assumed isotropic scattering and studied energy instead of amplitude relations (Berckhemer, 1970; Dainty and others, 1974; Nakamura 1976, 1977b). Dainty and Toksöz (1981), by comparing values of ℓ (mean free path) and x^* (average distance seismic energy travels before being attenuated) showed that the diffusion theory is inapplicable to terrestrial codas although it can be used for lunar data. This same result was also established by Kopnichev (1977b) from a different theoretical approach. In addition, the experimental agreement between Q_c and Q_β, firmly stated in the early 1980's, proved that Q_c accounts for both Q_i and Q_s as a nondiffusive process should do. (For a diffusion model, Q_c is equal to Q.) Nevertheless, diffusion theory has recently been reconsidered to model coda waves for large lapse-times, for both isotropic and anisotropic scattering (Gusev and Lemzikov, 1983; Gusev and Abubakirov, 1986).

3. The consideration of multiple scattering instead of only single scattering. This stage originated in a study by Kopnichev (1977b) and has received a new impulse in the studies of Gao and others (1983a, b).
4. The analysis of scattering with more realistic assumptions, such as non-spherical radiation source, anisotropic scattering, consideration of vector waves, and others. This last stage was initiated in 1982 and constitutes the "ongoing research" which will be summarized in Section 4. Its results have clarified many aspects related to coda waves.

The more important features of the main models proposed during this evolution will be reviewed next.

A. Single-Scattering Models

The groundbreaking work (Aki, 1969) supposed that the coda is comprised of backscattered surface waves generated within an unbounded, homogeneous, and isotropic medium in which the heterogeneities are randomly and two-dimensionally distributed on the surface. The size of scatterers is considered to be greater than the wavelengths, and no velocity perturbations or multiple scattering is allowed. Station and source event are placed at the same point. Under these assumptions expression (7) was written as:

$$P(\omega|t) = \mathrm{M}_0^2 \left|\phi_0(\omega|r_0)\right|^2 2\frac{N(r_0)}{t} e^{-\omega t/Q} \tag{9}$$

where

$$S(\omega) = \mathrm{M}_0^2 \tag{10}$$

and

$$C(\omega|t) = \left|\phi_0(\omega|r_0)\right|^2 2\frac{N(r_0)}{t} e^{-\omega t/Q} \tag{11}$$

In this expression, $N(r_0)$ is the number of scatterers within a radius of r_0. $|\phi_0(\omega|r_0)|$ expresses the absolute value of the Fourier transform of displacement due to secondary waves generated at a scatterer at a distance r_0 by a source of unit moment located at the same distance from the scatterer. The term $e^{-\omega t/Q}$ includes the dissipation introduced by the medium. $P(\omega|t)$ was related to the measured amplitudes $y(t)$ by the relation:

$$\langle y^2(t)\rangle = \sqrt{\frac{Q}{2\pi(-dt/dw_p)}}\, P(\omega_p|t) \tag{12}$$

obtained by approximating $P(\omega|t)$ by an error function with a peak at a frequency p. This peak frequency was determined by measuring the wave period directly on the seismogram at time t. With the preceding equations the seismic moment of small and very small earthquakes can be estimated. Also the dependence of the coda excitation on the geology of the station site can be clearly shown.

This model (hereafter "surface-wave model") has been extremely useful in the development of coda analysis despite its wrong assumptions (at least for Japan, for frequencies higher than 1 Hz) and its oversimplifications. Lately, some modified versions have been established with definite purposes: estimating seismic moment and magnitude (Suteau and Whitcomb, 1979), evaluating Q (Herrmann, 1980), or dealing with poor paper-recorded data (Herraiz, 1982; Herraiz and Mezcua, 1982, 1984).

The "surface-wave model" is incorporated as an asymptotic case in the theory developed by Kopnichev (1975) to explain surface-wave coda of seismograms in the range $100\text{km} < \Delta < 3,000\text{km}$. Kopnichev assumed that:

1. Primary and secondary waves are short-period surface waves of the same type.
2. Primary and scattered waves are monochromatic with period T.
3. Scattering is single and isotropic.
4. The inhomogeneities are random and stationarily distributed in the near surface.

Under these assumptions, this method adjusts coda envelopes, that is, the curves drawn through the peak amplitudes, by the expression:

$$A_i(t, \Delta t) \simeq \sqrt{\frac{\Delta t}{t}}\, e^{-\pi t/QT} \tag{13}$$

where A_i is the square root of energy contained within the time Δt after the reference time t. The agreement of this theoretical expression with coda wave envelopes at epicentral distances from 100 to 1800 km was excellent. This same model was used as a starting point to accomplish the analysis of double and triple scattering that will be described later.

The conclusions from the surface-wave model were checked by Takano (1971) and enriched with new observational data. Particularly interesting are the studies which showed the lack of regular plane waves coming from the epicenter in the analysis of codas recorded by a small aperture array of seismographs (Aki and others, 1958; Aki and Tsujiura, 1959; Scheimer and Landers, 1974). All these results were the basis for improved attempts to explain the origin of coda waves (Aki and Chouet, 1975; Chouet, 1976). These authors proposed two extreme models. The first, the single-backscattering model, considers the coda as the superposition of backscattering wavelets from discrete scattering sources. The second, the diffusion model, assumes the seismic energy transfer as a diffusion process. As this last physical phenomenon has been ruled out as a possible explanation for terrestrial seismic codas, attention will be paid here only to the single-backscattering model.

Its initial assumptions are very similar to those of the surface waves model, but it proposes that body waves are responsible for coda waves, which represents a turning point in the explanation of coda waves.

Under these assumptions and taking into account the geometrical spreading and the attenuation due to the Earth's anelasticity for body waves, expression (7) takes the form:

$$P(\omega|t) = \left|\phi(\omega|r_0)\right|^2 \frac{8r_0^4 \pi n_v}{vt^2} e^{-\omega t/Q} \tag{14}$$

$\left|\phi(\omega|r_0)\right|$ represents the amplitude spectra of the backscattering wavelet from a single scatterer located at a reference distance r_0, from the source (receiver), n_v is the density of scatterers per unit of volume, v is the wave velocity, and t represents the lapse-time. Expression (14) has the form

$$P(\omega|t) = S(\omega)\frac{e^{-\omega t/Q}}{t^m} \tag{15}$$

where $S(\omega)$ is the source term, and $t^{-m}e^{-\omega t/Q}$ stands for the medium effect. A closer look at this last term shows that t^{-m} is the geometrical spreading factor. $e^{-\omega t/Q}$ must represent both intrinsic and scattering attenuation effects although initially Aki and Chouet (1975) thought it only included the intrinsic attenuation. This assumption is correct for the diffusion model but not for the single-scattering model. For its part, $S(\omega)$ sums up the effect of both the primary and secondary wave sources. As the scattering is common to all earthquakes, differences in $S(\omega)$ among different earthquakes should be attributed to the variations in the earthquake source. Writing the $S(\omega)$ expression,

$$S(\omega) = \left|\phi(\omega|r_0)\right|^2 \frac{8r_0^4 \pi n_v}{v} \tag{16}$$

it can be noticed that $S(\omega)$ results from two factors $\left|\phi_0(\omega|r_0)\right|^2$ and $8r_0^4 n_v \pi v^{-1}$. The latter term includes the total number of scatterers and, for a given r_0 remains constant, independent of the source. The former term represents the intensity for a single secondary wave leaving a scatterer at a reference distance r_0. Its relation to the primary source is given by

$$\left|\phi(\omega|r_0)\right| = M_0\left|\phi_0(\omega|r_0)\right| \tag{17}$$

where M_0 is the seismic moment, and $\left|\phi_0(\omega|r_0)\right|$ is independent of source effect (Aki, 1969). Expression (17) assumes that ω is lower than the corner frequency of the earthquake. Both source and station are located at a distance r_0 from the scatterer. If the assumption that scatterers are uniformly distributed in space in the statistical sense holds, $\left|\phi_0(\omega|r_0)\right|$ is constant in a given area and the variations in $S(\omega)$ among the different events will be only due to differences in M_0.

The next step is to relate the power spectra and the seismogram's amplitudes. As was mentioned above, the procedure will be very different depending on whether the data are recorded in digital or analog form. In the first case, Fast Fourier Transforms over consecutive time samples, of amplitudes or of coda-waves, are measured on the band-pass filtered digital record.

Analog data require a more laborious treatment and some different procedures have been applied to accomplish it. Some of the methods include digitizing a manually smoothed coda envelope of a band-pass filtered seismogram (Chouet, 1976) or measuring only the peak amplitudes of the coda (Rautian and Khalturin, 1978). Chouet's method is easy to apply and has achieved very good results.

Chouet (1976) and Rautian and Khalturin (1978) used data recorded by a special filtering seismograph system which registered the signatures through several consecutive channels with independent frequency bandwidths. Chouet employed the spectral analyzing seismograph of Tsujiura (1966, 1967, 1969), while the Russians used the ChISS device designed by Zapolskii (1960, 1971). These recordings allowed the above investigators to study the frequency dependence of coda wave parameters.

Mean values, given by the envelope $\langle f^2(t)\rangle$, are related to power spectrum by using the relationship between coda power and the autocorrelation function:

$$\phi(t,\tau) = \langle f(t)f(t+\tau)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(\omega|t)e^{i\omega\tau}\,d\omega \tag{18}$$

For zero lag ($\tau = 0$), we get $\phi(t,0) = \langle f^2(t)\rangle$ and expression (18) becomes

$$\langle f^2(t)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(\omega|t)\,d\omega \tag{19}$$

where the term on the left represents the mean square of the time series. For a band-passed signal it is possible to make

$$\begin{aligned} &P(\omega|t) = \text{P, constant within a certain frequency band} \\ &P(\omega|t) = 0\text{, otherwise} \end{aligned} \tag{20}$$

and this leads to

$$\langle f^2(t)\rangle = 2P(\omega|t)\Delta f \tag{21}$$

where Δf is the signal bandwidth. If the actual seismogram's recorded amplitude, $A(\omega|t)$, is approximated by the root mean square value of the amplitude, then

$$A(\omega|t) = \sqrt{2P(\omega|t)\Delta f} \tag{22}$$

However, if the peak-to-peak amplitude of the manually smoothed envelope is measured, then it may be roughly approximated by twice the root mean square of $f(t)$, therfore

$$A(\omega|t) = 2\sqrt{2P(\omega|t)\Delta f} \tag{23}$$

Combining expressions (15) and (22) leads to

$$A(\omega|t) = C(\omega)\frac{e^{-\omega t/2Q_c}}{t^a} \tag{24}$$

with

$$a = \frac{m}{2} \tag{25}$$

and

$$C(\omega) = \sqrt{2S(\omega)\Delta f} \tag{26}$$

If natural logarithms are taken, expression (24) gives

$$\ln A(\omega|t) = \ln C(\omega) - a\ln(t) - bt \tag{27}$$

where

$$b = \frac{\omega}{2Q_c} \tag{28}$$

Expression (27) will be useful for estimating source factors and Q_c values. It will be considered again when the coda applications are studied, but it must be borne in mind that expression (27) has been obtained using the single-scattering approximation and thus it will not be suitable in strong-scattering cases.

So far, the model built by Aki and Chouet (1975) to explain the observational features of the codas has been considered. Sato (1977a) succeeded in extending the above single-scattering model to the case in which the source and receiver are not coincident. His model, called the "single isotropic-scattering model" or "SIS model", supposes a three-dimensional infinite and perfectly elastic medium in which the scatterers have a homogeneous and random distribution. This distribution is characterized by the mean free path ℓ.

Only S waves generated from a point source in a short time are considered as primary waves. The medium is characterized by the constant wave-velocity v and the mean free path ℓ, and the scattering is assumed to be single and isotropic.

The energy density of the scattered S waves, $E(t)$, is given by

$$E(t) = \frac{W}{4\pi\ell R^2} K\left(\frac{t}{t_s}\right) \qquad \text{for } t > t_s \tag{29}$$

where t and ℓ have their usual meanings, t_s is the traveltime of S wave, and R is the hypocentral distance to the receiver. Letting $t/t_s = x$, the function $K(x)$ is given by

$$K(x) = \frac{1}{x}\ln\left(\frac{x+1}{x-1}\right) \qquad \text{for } x > 1 \tag{30}$$

This last expression, with some corrections in order to include the anelastic effect, has often been used in practical coda analysis (Jin and Aki, 1986).

The single isotropic-scattering model, designed to study the energy balance more than the amplitude attenuation, was soon improved by including $P \to S$ and $S \to P$ conversions (Sato, 1977b). Its first version was applied to 96 local earthquakes recorded in the Kanto district, Japan, to estimate the mean free path (Sato, 1978). These results will be used in the next section, where a study of parameters describing the medium's heterogeneity will be made. In a more recent study, Sato and Matsumura (1980a) applied the SIS model to study the three-dimensional partition of scattered P waves. The theoretical analysis predicts that the radial component of energy is always larger than the transverse one and that this difference diminishes as t increases before the arrival of the S phase. Application of this model to data obtained at the bottom of a well (Sato, 1978), showed a good agreement only for the horizontal component, suggesting the necessity of including $P \rightarrow S$ conversion in the scattering process.

B. Multiple-Scattering Models

Single-scattering theory used so far is strictly valid only if the mean free paths of the waves between scatterers are greater than the travel distances from scatterers to receiver. Very often, actual physical situations do not satisfy this condition, and the consideration of multiple scattering becomes a pressing necessity (Subash and Gir, 1979). The first attempt to solve this problem was made by Kopnichev (1977b) who assumed a random, uniform, isotropic, and normal medium, and studied three cases: double, triple, and diffusion scattering. This last one was discarded for the Earth because the strong scattering required is hard to find. The double and triple scattering are studied both for surface and body waves assuming that the waves which act as primary and those due to scattering have the same nature. Isotropic scattering was also assumed. Without including absorption, the expression of intensity obtained at the source for body waves and double scattering is :

$$J_2(t, \Delta t) = \pi^4 J_0 r_0^2 n_v^2 \alpha^4 \xi^2 \Delta t \frac{e^{-\xi_s v t}}{t} \tag{31}$$

and for triple scattering:

$$J_3(t, \Delta t) = 2\pi^5 (2 \ln 2 - 1) J_0 r_0^2 n_v^3 \alpha^6 \xi^3 v \Delta t \, e^{-\xi_s v t} \tag{32}$$

where

t	Lapse-time of the signal commencement
Δt	Interval considered such that $\Delta t > \psi$, ψ being the duration of the source pulse
J_0	Energy density of a regular body wave at a reference distance r_0
r_0	Reference distance from the epicenter
n_v	Density of heterogeneities in a three-dimensional medium
α	Medium correlation distance
ξ	Scattering efficiency of the heterogeneity
ξ_s	Factor introduced to account for the energy lost by secondary waves due to scattering. (Not estimated in this study. It is introduced in order to maintain the energy conservation law.)
v	Wave velocity

These expressions are obtained assuming that the total distance traveled by scattered waves is much larger than the epicentral distance. They indicate that the energy of the initial part of the coda is mainly determined by the contribution of singly scattered waves while there is a relative increase of the energy of secondary and tertiary scattered waves as t increases. This result poses the necessity of being cautious if long duration codas are studied.

Gao and others (1983a) dealt with multiple scattering following the single-scattering model of Aki and Chouet (1975). First, they considered a two-dimensional infinite elastic medium in which scatterers having a cross section σ are uniformly distributed with a density n_s. Under these assumptions they calculated the expression of power spectrum $P(\omega|t)$ for double, triple, and quadruple scattering, showing it can be decomposed as

$$P(\omega|t) = P_s(\omega|t) + P_m(\omega|t) \tag{33}$$

where $P_s(\omega|t)$ and $P_m(\omega|t)$ represent the contribution for single and multiple scattering respectively. These terms can be expressed by the following approximate form fitted to the exact numerical results for the first 7^{th} order multiple scattering;

$$P_m(\omega|t) = Y(\omega) e^{-\omega t\left(\frac{0.74}{Q_s} + \frac{1}{Q_i}\right)} \tag{34}$$

and

$$P_s(\omega|t) = \frac{Q_s}{\omega t} Y(\omega) e^{-\omega t\left(\frac{1}{Q_s} + \frac{1}{Q_i}\right)} \tag{35}$$

where Q_s is the quality factor due to single scattering, which according to (6) can be written as

$$Q_s = \frac{\omega t}{2 n_s \sigma r} \tag{36}$$

and

$$Y(\omega) = (n_s \sigma)^2 r_0 \mathrm{v} |\phi(\omega|r_0)|^2 \tag{37}$$

Q_s and Q_i (quality factor for intrinsic absorption) are related to Q by expression (5), given by Dainty (1981). As is already known, $|\phi(\omega|r_0)|$ means the amplitude spectrum of the primary waves at the reference distance r_0.

Expressions (34) and (35) show that at short lapse-time, the power spectral density is mainly composed by single scattering contributions, while as t increases, multiple scattering becomes more important. These results, which agree with Kopnichev's conclusions, can be better understood by defining two new parameters

$$\gamma = 2 n_s \sigma r \tag{38}$$

and

$$\overline{Q} = \frac{Q}{Q_s} \tag{39}$$

and plotting the values of $P_m(\omega|t)$ and $P_s(\omega|t)$ against γ for different $\overline{Q}$ (fig. 4). As γ is the product of the turbidity "$n_s\sigma$" and the travelled distance r, it expresses the power loss due to scattering. It is obvious that γ increases with traveltime. In its turn, if Q is small, namely the scattering loss is smaller than the absorption loss, the coda attenuation before the multiple scattering becomes important. Therefore, in such a case, the single scattering assumption works well. The intersection of $P_s(\omega|t)$ and $P_m(\omega|t)$ curves indicates that the γ value for which the multiple scattering starts to be dominant is $\gamma_c = 0.8$. Thus, it can be seen that for $\gamma < 0.8$ or for the traveltime $t < t_c$, being $t_c = 0.8\ (n_s \sigma \mathrm{v})^{-1}$, the single scattering correctly explains coda power. The turbidity coefficient has been estimated as $0.008\,\mathrm{km}^{-1}$ for the Montana LASA (Large Aperture Seismic Array) area (Aki, 1973). Although this result is for P waves and

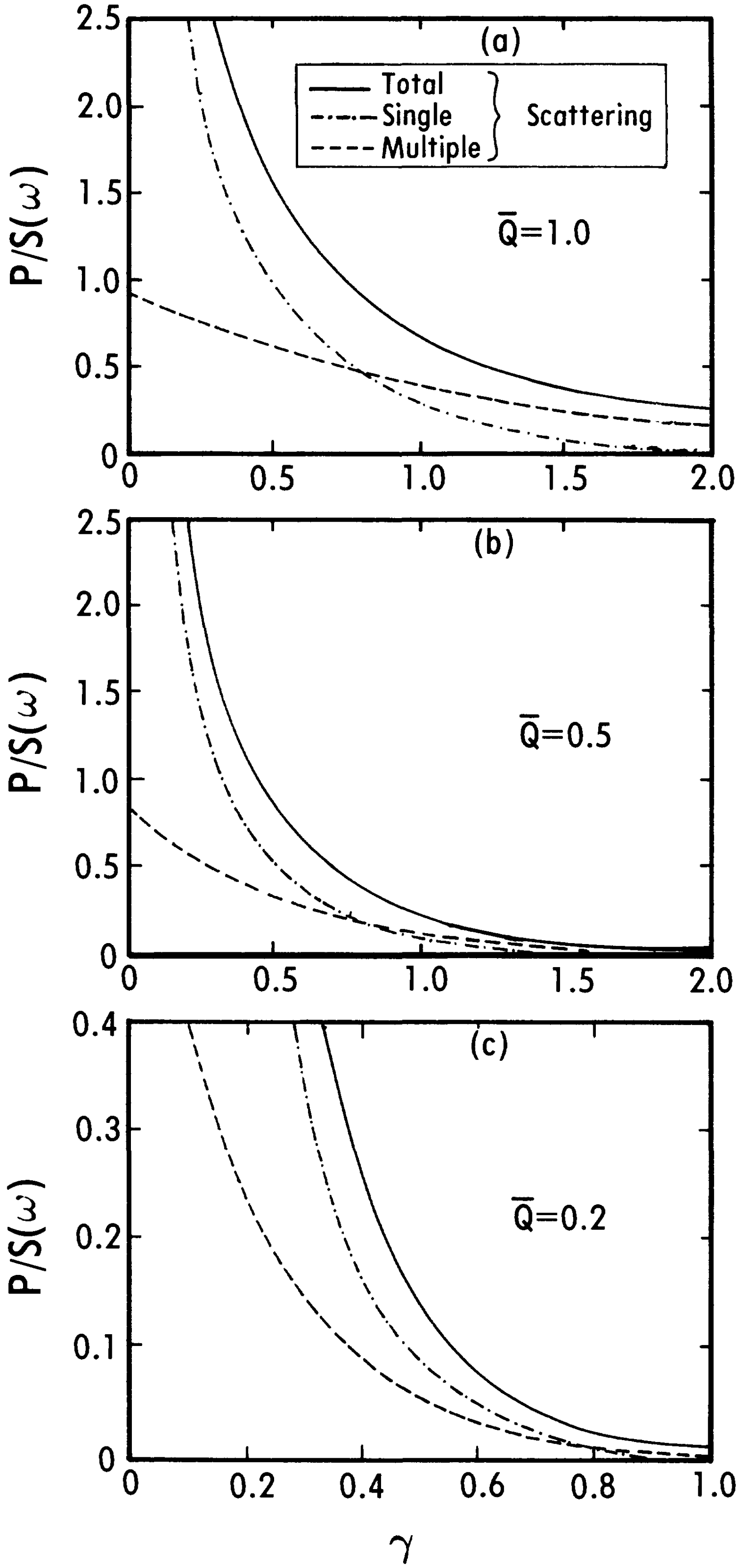

Figure 4.— Relationship of the ratio of coda power P/source factor $S(\omega)$ as a function of γ $(= 2n_s\sigma r)$, for three different $\overline{Q}$ $(= Q/Q_S)$ values for the cases of: total, single, and multiple scattering. [Reproduced with permission from the Seismological Society of America.]

0.5 Hz, and may not apply to coda waves, it implies that, assuming v = 3.5 km/s, t_c will be 29 s, a value which indicates that multiple scattering may be more important than expected. Neglecting this effect implies that for $t \gg t_c$, Q_s values are overestimated by a factor of 0.74. Gao and others (1983b) extended their model to the three-dimensional case showing that γ_c equals 0.65, and the Q_s overestimation is for this situation 0.67.

Summarizing the results of this theoretical development it is possible to conclude that, so far, the coda of local earthquakes is supposed to be the result of S to S single-backscattered waves, although multiple scattering has been introduced in recent models.

This conclusion has been obtained by considering an infinite, homogeneous, and isotropic medium within which the heterogeneities are randomly and uniformly distributed.

These assumptions may seem too simplistic, and the results may seem not to resemble the actual phenomenon. However, these models have provided a basis upon which to develop coda wave analysis and have led to the useful applications reviewed in the next part of this study.

PART II: USING CODA WAVES TO ESTIMATE SEISMIC PARAMETERS

The search for a better explanation of the origin, composition and decay of coda waves yielded many important applications. Since the initial studies, accomplished between 1969 and 1978, it has been clearly proven that coda waves are an effective resource in estimating source parameters and in extracting information about the characteristics of wave paths.

For the sake of clarity, the review of these applications has been divided into three main parts according to whether the results are more related to seismic source, wave path, or station site. However, it must be kept in mind that the tight existing relation linking these effects of the seismic phenomenon makes any separation of the studied parameters artificial.

Source Parameters

As has been previously mentioned, the separation of path and source effects was one of the initial motivations for the statistical option which gave place to the coda wave analysis. In the surface wave model, and assuming that ω is lower than the corner frequency (Aki, 1969), this possibility is given by the expression

$$\mathrm{M_0}\sqrt{2N(r_0)}\,\left|\phi_0(\omega_p|r_0)\right| = \sqrt{t}\,e^{\omega_p t/2Q}\left(-\frac{1}{Q}\frac{dt}{df_p}\right)^{1/4}\sqrt{\langle y^2(t)\rangle} \qquad (40)$$

The left hand side is the product of the seismic moment $\mathrm{M_0}$, which represents the source factor, and the factor $\sqrt{2N(r_0)}\,\left|\phi_0(\omega_p|r_0)\right|$, which describes the mechanism of scattering. These terms have been explained previously.

The right-hand side expresses the path effect. The first factor, $\sqrt{t}$, can be interpreted as a geometrical spreading correction, the second factor as a dissipation correction, and the third as a dispersion correction applied to the observed amplitude. Their product is called the "reduced coda spectrum" and can be evaluated directly from the seismograms. The term which describes the mechanism of scattering is assumed to be the same for every earthquake of a chosen area. It may be determined by calculating the reduced coda spectrum of an earthquake with known $\mathrm{M_0}$. Knowledge of the regional scattering parameter then allows estimation of the seismic moment of all other earthquakes (Aki, 1969; Herraiz and Mezcua, 1982, 1984).

The separation of path and source effects also can be evaluated by using expression (8):

$$\frac{P_1(\omega|t)}{P_2(\omega|t)} = \frac{S_1(\omega)}{S_2(\omega)}\frac{C(\omega|t)}{C(\omega|t)} = \frac{S_1(\omega)}{S_2(\omega)} \tag{8}$$

where $P_1(\omega|t)$ and $S_1(\omega)$ can be given by equations (15) and (16) of the single-backscattering model. The above expression allows calculation of the other source spectra if the estimation of the absolute value of $S(\omega)$ for any one event can be obtained.

In order to calculate the absolute value of a reference source, Aki and Chouet (1975) and Chouet and others (1978) selected the S-wave recordings from a small and nearby event which sustains the least amount of scattering. The Fourier Transform, $F(\omega)$, of S-waves is evaluated from the band-pass filtered trace $A_s(\omega)$, according to

$$A_s(\omega) = 2\Delta f\,|F(\omega)| \tag{41}$$

$|F(\omega)|$ is used in the process of estimating the source spectrum from

$$\langle \mathrm{M}(\omega)\rangle = \frac{1}{c}4\pi\rho\beta^3 R\,|F(\omega)| \tag{42}$$

where $\langle \mathrm{M}(\omega)\rangle$ is the mean Fourier Transform of the seismic moment corresponding to a point dislocation source in an infinite homogeneous medium (Maruyama, 1963), β is the shear wave velocity, ρ is the density, and c is a geometrical factor equal to or less than 1. The source spectrum $\langle \mathrm{M}(\omega)\rangle$ given by expression (42), which is only valid for a homogeneous medium, was taken as an absolute reference value. This method was also independently used by Rautian and Khalturin (1978). Reference earthquakes, together with source factors $S_i(\omega)$, are employed to construct a scaling law, that is, the way in which the spectrum changes with increasing magnitude. Comparison of the scaling law for different regions may offer some useful information about local seismo-tectonics. Applying this method to small earthquakes in Stone Canyon and San Fernando, California; Kilauea, Hawaii; and Oishiyama, Japan, Chouet and others (1978) observed a remarkable departure of the seismic moment-corner frequency relation from the self-similar case in all these regions except Kilauea (fig. 5). This departure corresponds to the variation of stress drop with magnitude over a certain range of magnitude. A possible explanation is the existence of a scale length of heterogeneity unique to each fault zone that affects the rupture process.

A different method to estimate the seismic moment of local earthquakes from coda waves was devised by Vostrikov (1975), who used analog data of the Garm region recorded by a seismograph with a 0.7–10 Hz band-pass filter. Vostrikov assumed that the formation of coda waves was a "black box" process and studied the data without introducing corrections for absorption, divergence and scattering. He calculated the individual envelope $A(t)$ for 30 earthquakes and shifted them in order to fit the overlapping sections of adjacent envelopes. In this way he obtained a set of master curves which give the seismic moment from the intensity of the general shape $C(t)$ of envelopes. This procedure uses the relation between one individual shape $A(t)$ and the general $C(t)$ given by

$$A(t) = h\mathrm{M}_0 C(t) \tag{43}$$

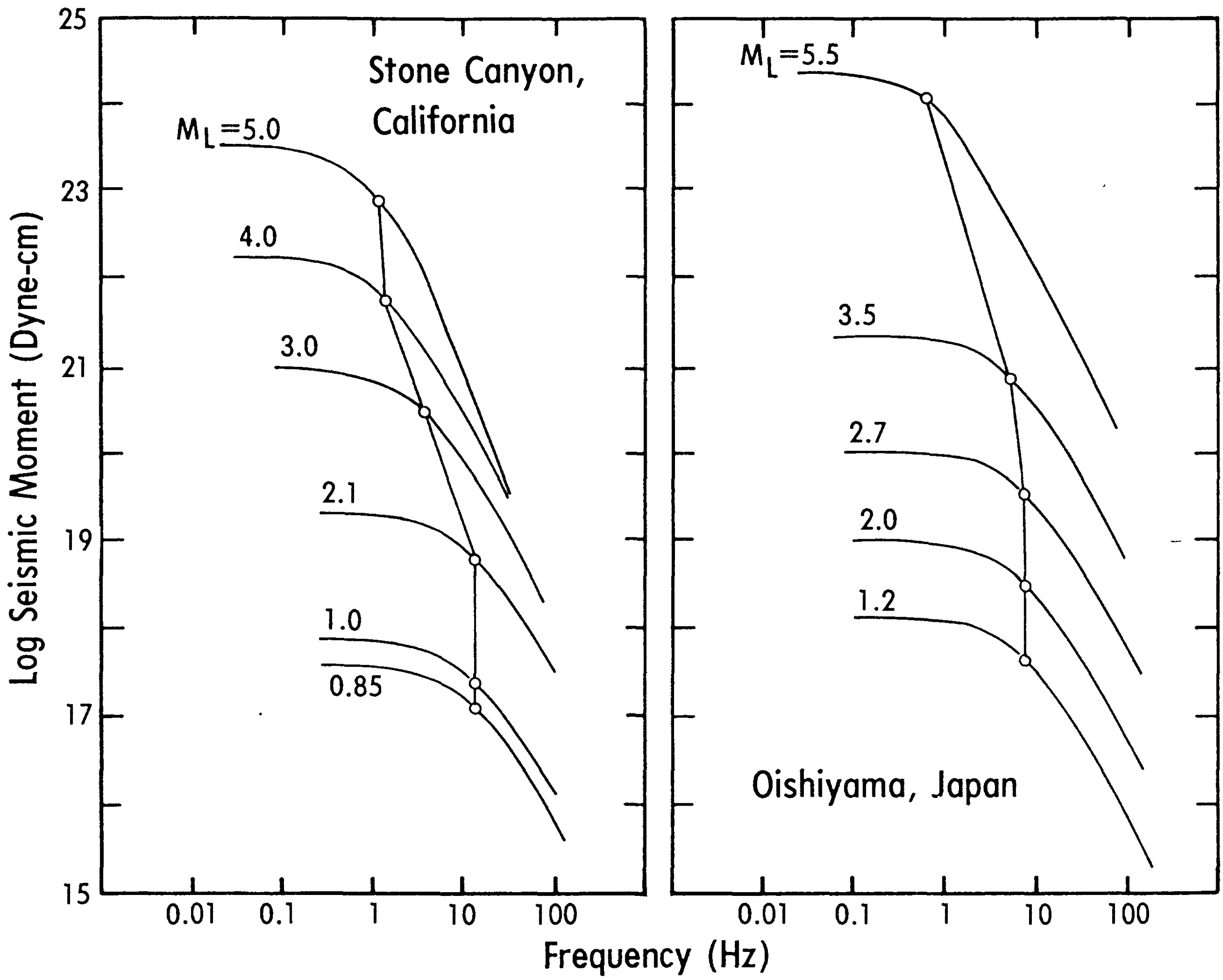

Figure 5.— Comparison of growth of seismic source spectrum, seismic moment, with local magnitude, M_L, for earthquakes within the areas of Stone Canyon, California, and in Oishiyama, Japan. In both cases, the seismic moment corner-frequency relation deviates from the linear trace with a slope −3 expected if the similarity assumption were to hold. [Reproduced with permission from the Seismological Society of America.]

where h represents the source directivity multiplied by a constant and M_0 is the seismic moment. The factor $C(t)$ in Vostrikov's theory is a property of the medium and depends on scattering and absorbing characteristics.

It seems to us that the empirical relations obtained recently by Biswas and Aki (1984) can be more useful. These authors have constructed a new set of relations among seismic moment, coda amplitude, and magnitudes, by using moderate-size Alaskan earthquakes. This procedure can be useful when events with well-known seismic moment are available.

Magnitude is another source parameter which can be easily estimated from coda waves by taking advantage of the independence of total signal duration from epicentral distance. The idea originates from Bisztricsany (1958), who calculated the relationship between magnitude and surface-wave duration at epicentral distances between 4° and 160°. The use of total duration of local earthquake seismograms to obtain the empirical duration-magnitude relationship was started by Soloviev (1965) and Tsumura (1967), and since 1972 has become widespread (see for instance: Lee and others, 1972; Crosson, 1972; Real and Teng, 1973; Herrmann, 1975; Bakun and Lindh, 1977; Saphira, 1980; Chaplin and others, 1980; Havskov and Macías, 1983). Nersesov and others (1975) extended this procedure to distances as great as 3,000 km and introduced a correction for site station effect. In their turn, Suteau and Whitcomb (1979) found a theoretical relationship between the seismic moment of shallow local earthquakes, coda amplitudes, and the total duration of the signal measured from the earthquake origin time. A new coda magnitude, M_c, can also be determined by using this relationship. More recently, Aki (1980c) has reviewed the physical basis for duration-magnitude relationships, recommending the use of coda amplitude, at selected lapse-times, instead of using measurements of coda duration. Bakun (1984) found empirical formulas relating local magnitudes and seismic moments to signal durations and epicentral distances for central California.

Propagation Medium Parameters

Coda waves can be used to estimate two medium parameters: Q, the attenuation quality factor, and g_π, the backscattering coefficient. This review starts by analyzing Q_c and Q_β estimates, their relation, and frequency dependence. The results will be placed within the general pattern of Q evaluation and used to check the theoretical model. A general review of Q_c versus tectonic and Q_c versus time relations will also be included. Finally, the g_π estimates and their effect on the explanation of coda decay will be considered.

Q_c and Q_β Measurements on Frequency Dependences

The first estimates of Q_c (Aki and Chouet, 1975) were accomplished by applying expression (27) of the single-backscattering model to data from Tsukuba and Oishiyama, Japan, and Stone Canyon, California. Two main features were observed: a clear increase of Q_c with frequency and a remarkable variation in the value at 1 Hz among different regions. The observation was initially attributed to the dependence of Q on depth instead of to some intrinsic Q_c-frequency dependence. Coda waves at 1 Hz were thought to be predominantly composed of surface waves scattered from shallow heterogeneities, while those at 24 Hz would primarily be body waves backscattered from the deep, high-Q lithosphere.

Rautian and Khalturin (1978) and Rautian and others (1978) observed the same increase of Q_c with frequency for the Garm region. By using the single backscattering model and a broader frequency range (from 0.027 to 40 Hz) they obtained the relation

$$Q_c = q\sqrt{f} \tag{44}$$

where f is the frequency of the waves and q varies with the coda interval taken, increasing as longer time intervals are considered. This expression is similar to that obtained by Fedotov and Boldyrev (1969) who used a single-station method to study P- and S-wave attenuation in the Kuril Islands chain. Although no theoretical explanation was given, Rautian and Khalturin's result reinforced the attempts made previously to introduce frequency-dependence in Q (Sato and Espinosa, 1964, 1967; Tsai and Aki, 1969; Nur, 1971; Solomon, 1972; Espinosa, 1977; Lee and Solomon, 1978; Der and others, 1984).

Studying the frequency dependence of Q requires the measurement and interpretation of attenuation for waves in a very broad frequency range. Until the late 1970's the study of low-frequency waves ($f < 1\,\text{Hz}$) had progressed furthest (Anderson and Hart, 1978) because the analysis of primary waves of local events is more complicated: they are strongly affected in amplitude and phase by local heterogeneity (see Aki, 1984, for a review on the main characteristics of short-period seismology). Besides, the amplitude and spectral contents of such waves are very sensitive to the velocity gradient at and near the turning point through which rays emanating from a near-surface source must pass.

The study of the attenuation of high-frequency S waves was made by Aki (1980a), applying a single-station method to minimize site effect and using direct body waves generated by deep sources to eliminate the turning point problem. In addition, the use of coda waves allowed elimination of the source effect without introducing arbitrary assumptions on source spectrum used in earlier single-station methods (Asada and Takano, 1963; Fedotov and Boldyrev, 1969). The method expresses the amplitude-source spectrum for primary S waves as

$$A_s(\omega, R) = S(\omega, \theta)\frac{e^{-\omega R/2\beta Q_\beta}}{R} \tag{45}$$

where R is the distance from the source to receiver, $S(\omega, \theta)$ represents the source factor including the radiation pattern effect, and β is the unperturbed S wave velocity. A more detailed expression for the amplitude of body waves, $A_s(\omega, R, t, \theta, \phi)$, where A_s can be either the amplitude of P or S waves in the frequency or in the time domain, is given by Sato and Espinosa (1967). The observation on coda decay leads to the formula for the coda amplitude spectrum measured at time t_0 as

$$A_c(\omega|t_0) = S(\omega)C(\omega|t_0) \tag{46}$$

where $C(\omega|t_0)$, the path term, is independent of hypocentral distance. The ratio at a given time t is

$$\frac{A_s(\omega, R)}{A_c(\omega|t)} = \frac{S(\omega,\theta)}{S(\omega)R}\,\frac{e^{-\omega R/2\beta Q_\beta}}{C(\omega|t)} \tag{47}$$

Taking logarithms of both sides and averaging the left-hand side over many events which lie in a restricted distance range ($R - \Delta R$, $R + \Delta R$), it is possible to smooth out any systematic variation of focal mechanism with R:

$$\left\langle \ln\left(\frac{R\,A_s(\omega, R)}{A_c(\omega|t)}\right)\right\rangle_{R\pm\Delta R} = a - bR \tag{48}$$

where a is independent of R and

$$b = \frac{\omega}{2\beta Q_\beta} \tag{49}$$

is the slope of the averaged logarithmic ratio versus distance R. It is important to remark that this method for determining Q_β is founded only on observed properties of coda (see expression 46), independent of the models developed to explain them. The coda is used here only to remove the source effect on S waves with the assistance of expression (47) given above.

This method was applied by Aki (1980a) to data from the Kanto region of Japan obtained by Tsujiura (1978) at the Dodaira and Tsukuba stations. Both sources of data were used independently in the application of the single-station method. The resultant values of Q were identical between the two data sets, confirming the validity of the method. The results show a clear frequency-dependence of Q_β of the form f^n, n being 0.5, 0.6 or 0.8 according to the studied area. This dependence is similar to the frequency-dependence of Q_c discussed earlier, which strongly supports the hypothesis that considers the coda composed of S to S backscattered waves. The frequency-dependence of Q_c has been found for many different tectonic regions after Aki and Chouet's (1975), Rautian and Khalturin's (1978), and Rautian and others' (1978) studies. (See for example Console and Rovelli, 1981; Rovelli, 1982; Del Pezzo and others, 1983, 1985; Del Pezzo and Zollo, 1984; Rodriguez and others, 1983.) Herrmann's (1980) results support this theory by finding a similar agreement between Q_c and Q of L_g waves. Espinosa's (1981) results on Q of L_g waves for periods around 1 Hz evaluated in the contiguous United States are similar to those Q_c obtained by Singh and Herrmann (1983). Similar results, on a regional basis, are also shown in studies by Nuttli (1973, 1980), Street (1976), Bollinger (1979), and Roecker and others (1982).

Relation Between Q_c and Q_β

The previous results lead to several conclusions concerning Q_c and Q_β:

1. Q_c and Q_β vary with frequency in the form f^n. Assuming that coda waves are composed of scattered S waves, the frequency dependence of Q_c can be attributed to an intrinsic frequency dependence of Q_β. A previous explanation, which supposed an apparent frequency dependence produced by a combination of depth-dependent Q and a frequency-dependent variable contribution from surface waves to coda waves, can be ruled out (Aki, 1982).
2. Q_c and Q_β agree well for frequencies higher than about 3 Hz at least for the Kanto area of Japan. Q_c is somewhat greater than Q_β for frequencies lower than 3 Hz, suggesting some defect in the S to S single-scattering theory. It is important to recall that Q_c has been calculated using a single backscattering theory, while Q_β does not depend on any coda model we adopt. In an earlier explanation, Aki (1980b) attributed the failure to the presence of multiple scattering. Later, considering that Rautian and Khalturin (1978) and Roecker (1981) obtained a good agreement between Q_c and Q_β at all frequencies in addition to a common increase with depth, Aki (1981b) suggested that Q_c probably represents the value of Q_β below the crust, while Q_β at 1.5 Hz represents the attenuation in the crust.
3. If the agreement between Q_β and Q_c holds, the use of coda waves is the easiest way to estimate Q_β.
4. The values of Q_β and Q_c for different regions tend to converge at high frequencies but are clearly dependent on the degree of current tectonic activity at frequencies around 1 Hz. Higher activity is characterized by lower Q values in such a way that Q can be used as a tectonic activity estimator (Espinosa, 1981, 1984; Pulli and Aki, 1981; Singh and Herrmann, 1983; Rovelli, 1983a). This Q versus activity relation can be observed for frequencies less than 25 Hz and may include differences in Q of more than one order of magnitude between stable and active areas. The hypocentral depth influence in the observed Q_c variations is discussed by Rovelli (1984a, b).

These results confirm a previous study of the lateral variation of Q obtained from P waves generated by explosive sources (Suzuki, 1972). An increase of coda Q with the age of the oceanic crust has been observed by Jin and others (1985). Another effort has been undertaken by Canas and Pujades (oral commun., 1986) in evaluating the spatial Q distribution in the Iberian peninsula.

The importance of these results increases when we consider that the high Q_β values obtained at high frequencies are comparable to those inferred for the lithosphere from the attenuation of long-period surface waves (Kovach, 1978). This agreement reaffirms the possibility of comparing Q_c and Q_β values with Q estimates given by more conventional methods.

Taking into account these results, Aki (1980a, b, 1981b) used only one diagram to plot the estimates of Q^{-1} obtained by different authors employing coda-wave analysis, single-station methods or L_g waves (see fig. 6 and table 3A for references concerning the United States). This figure suggested to Aki that the frequency dependence of Q^{-1} should have a peak around 0.5 Hz. Circles, in figure 6, represent Q^{-1} values obtained from L_g waves, with a period of 1 s, throughout the United States (Espinosa, 1981, 1984). According to Espinosa, the mean Q^{-1} values for different regions in the United States show a definite correlation with crustal thickness and with the general regional physiography. They are characterized as follows:

1. Northeast $Q^{-1} = 0.001$
2. Southeast $Q^{-1} = 0.001$
3. South $Q^{-1} = 0.002$
4. Central United States $Q^{-1} = 0.00089$
5. Basin and Range province $Q^{-1} = 0.0017$
6. The Rocky Mountains $Q^{-1} = 0.0033$
7. Northwest $Q^{-1} = 0.0045$
8. There is good correlation of the Q^{-1} distribution in areas where high heat flow has been reported and characterized with a $Q^{-1} = 0.0056$

The Q^{-1} peak around 0.5 Hz, known as "Aki's conjecture", was seriously taken up by Sato (1982b) in his theoretical work (fig. 7). As will be studied in the next section, the fit of experimental results by the curves expressing the Q-frequency relation for a particular model constitutes a decisive test to check the model's quality for explaining attenuation and scattering mechanisms.

One more aspect of Q versus frequency relations must be considered. It concerns the mechanisms assumed to explain the frequency dependence and how they are affected by the observational results. Aki (1980a), after discarding several possibilities, indicated that only the thermoelastic effect and the scattering process are really viable (Zener, 1948; Savage, 1966). These phenomena require radically different scale lengths of heterogeneities to explain the peak of attenuation around 0.5 Hz: 1 mm and 4 km respectively. Although more recent studies underline the importance of scattering, this discrepancy demands more studies of Q versus frequency relations and points out the importance of the analysis of heterogeneities that will be described later in this paper.

The main features of coda-wave analysis, particularly expression (8), and the regional and frequency dependences of Q have been shown for many different tectonic regions (see table 3B). They can be considered well-established facts. The results obtained under the assumption of constant Q may require future revision if important variations in frequency or different tectonic regions were involved.

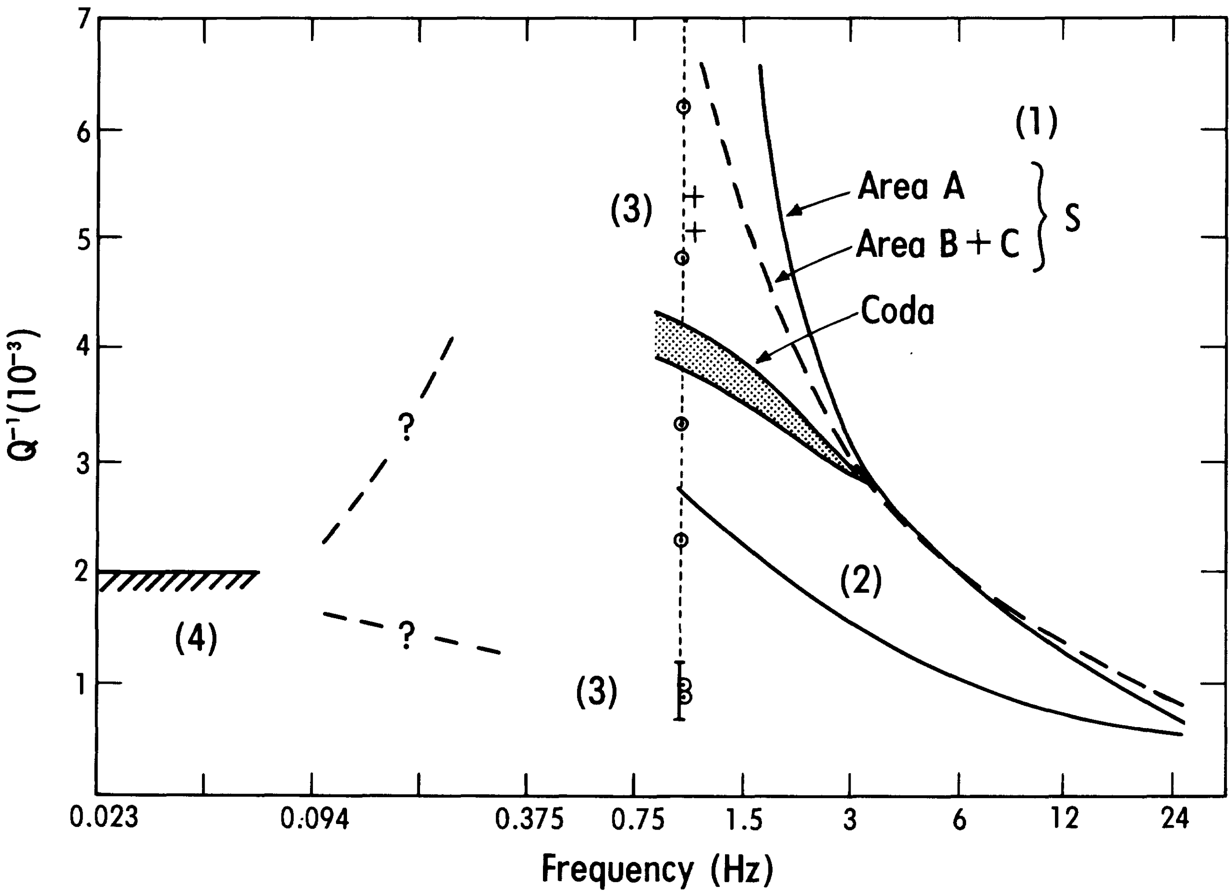

Figure 6.— Q^{-1} vs frequency for the lithosphere in different regions of the world.

(1) Results obtained for two areas of the Kanto region in Japan. S stands for results obtained from S waves (Aki, 1980a). B and C identify the results, also from the Kanto region, which are in a hotter and more complex area than region A. Stippled area identified as Coda implies those values obtained, for the same region, using coda wave analysis (Tsujiura, 1978).

(2) Results for Central Asia obtained from S and coda waves (Rautian and Khalturin, 1978; Rautian and others, 1978).

(3) Estimates obtained from L_g waves for Iran, Midwest, and Eastern U.S. (Nuttli, 1973, 1980; Street, 1976; Bollinger, 1979); open circles, contiguous United States (Espinosa, 1981, 1984).

(4) Results from attenuation of long period surface waves. (Tsai and Aki, 1969; Solomon, 1972). Question marks indicate both lack of data and likely trends of variation.

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Backscattering Coefficient

Information about lithospheric heterogeneity can be obtained from the scattered waves orig-

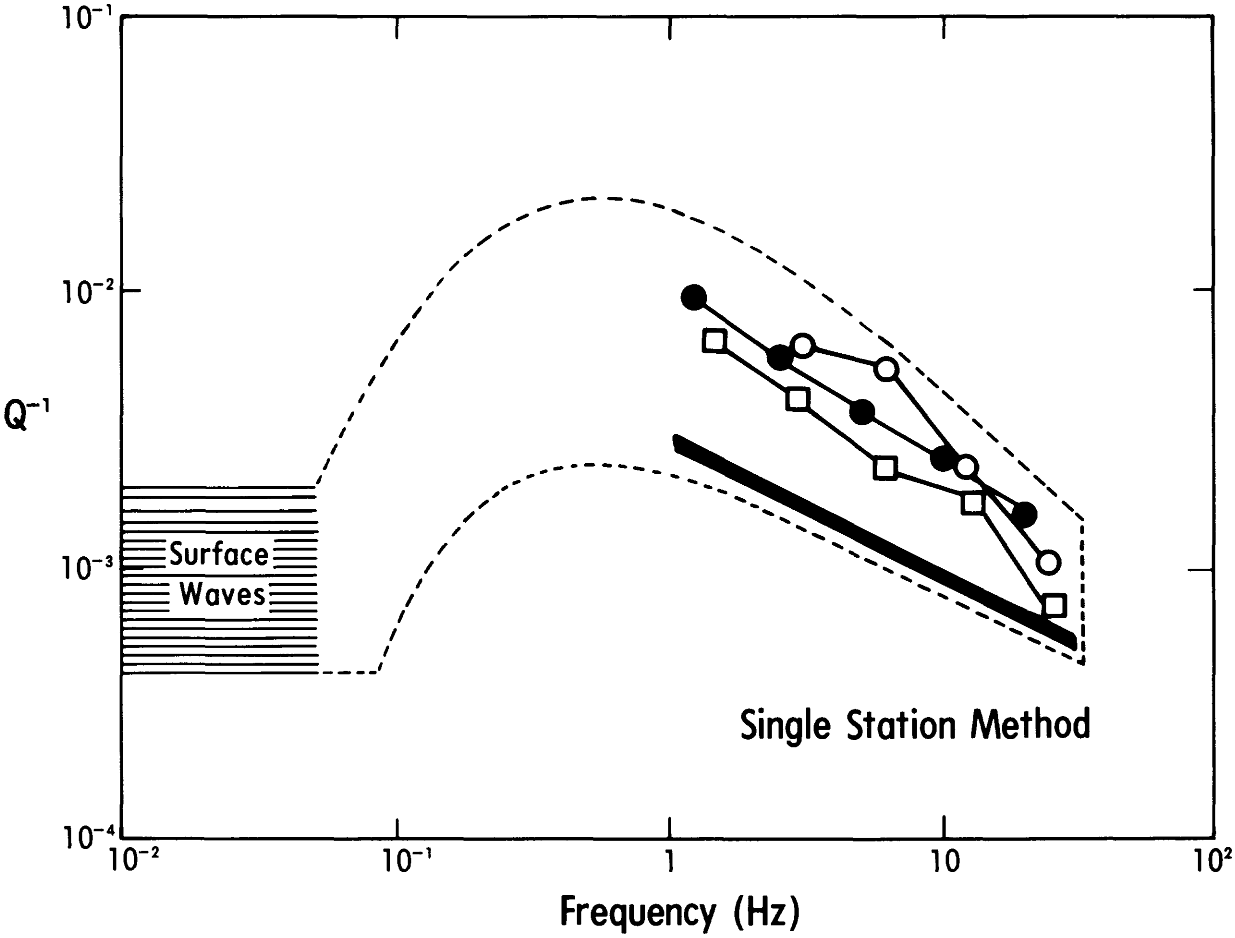


Figure 7.— First display of "Aki's conjecture" made by Sato (1982b) for Q_β^{-1} observational data. On the right are shown several results obtained by the single station method: solid circles, Southern Kuril Island (Fedotov and Boldyrev, 1969); open circles, Iwatsuki, Kanto, Japan (Sato and Matsumura, 1980b); open squares, Dodaira, Kanto, Japan (Aki, 1980a); and bold line, Garm, Central Asia (Rautian and others, 1978). On the left are shown the estimates given by surface wave analysis. The contour, or broken lines, shows the Q^{-1}-frequency trend suggested by "Aki's conjecture" which assumes a maximum for Q^{-1} around 0.5 Hz. [Reproduced with permission from the American Geophysical Union.]

inated by this heterogeneity. Backscattered waves—as coda waves—provide more information about impedance contrasts; forward scattered waves—as primary P and S waves—provide more information about velocity contrasts. However, the large number of variables affecting the scattering process makes any deterministic interpretation difficult. In this discussion, coda waves will be used to estimate the ability of a particular medium to create backscattering. Later, in Section 3, some effects of forward scattering will be employed to improve the knowledge about heterogeneity.

The heterogeneity is usually estimated by the scattering coefficient "g", which measures the intensity of scattering. As has been defined in the glossary, it has a dimension of $(\text{length})^{-1}$ and is the reciprocal of "ℓ", the "mean free path" used by Sato (1978).

TABLE 3A.--Regions of the United States in which Q from Lg waves has been estimated

Region	Reference
Contiguous-------------	Sutton and others (1967), Espinosa (1981, 1984).
Eastern North America--	Nuttli (1973).
Northeastern-----------	Street (1976).
Southeastern-----------	Jones and others (1977), Bollinger (1979).
New Madrid-------------	Nuttli (1978), Dwyer and others (1981).
Central----------------	Dwyer and others (1983).
Basin and Range--------	Mitchell (1981).
Western----------------	Mitchell (1981), Patton (1983).
Colorado Plateau-------	Mitchell (1981).
Southwestern-----------	Pesecki s and Pomeroy (1984).
Great Basin------------	Chávez and Priestley (1986).

A first attempt to estimate the backscattering coefficient, g_π, using coda waves was made by Aki and Chouet (1975) using both single backscattering and diffusion theory. Only the result based on the single backscattering model (Aki, 1981b) will be discussed. Aki related coda power spectrum $P(\omega|t)$ and g_π by the expression:

$$P(\omega|t) = \frac{\beta}{2} g_\pi |S(\omega)|^2 \left(\frac{\beta t}{2}\right)^{-2} e^{-\omega t/Q_c} \tag{50}$$

It is known that $P(\omega|t)$ can be estimated from the seismogram's amplitude $A(\omega|t)$, which is measured on a band-pass filtered record at the center frequency ω in equations (21) and (22). In a similar manner, for coda-wave amplitude $A_c(\omega|t)$, it is possible to obtain the expression

$$A_c(\omega|t) = \sqrt{2P(\omega|t)\Delta f} \tag{51}$$

On the other hand, the Fourier amplitude spectrum $|F(\omega)|$ for S waves may be estimated by equation (41) with an R dependence, which takes the form:

$$A_s(\omega, R) = 2\Delta f |F(\omega)| \tag{52}$$

Note that this last expression is only applicable to an impulsive waveform. This limitation is avoided if the data are digitally recorded because in this case $|F(\omega)|$ can be obtained from Fourier analysis. The relation between the source factor $S(\omega)$ and the amplitude spectrum $|F(\omega)|$ can be written by using expression (44):

$$|F(\omega)| = |S(\omega)| \frac{e^{-\omega R/2\beta Q_\beta}}{R} \tag{53}$$

Expressions (22), (45) and (52) and the calculation of the average of A_s/A_c for different distances, yield:

$$e^a = \frac{\sqrt{\Delta f}\beta t_0}{2\sqrt{\beta g_\pi}} e^{\omega t_0/2Q_c} \tag{54}$$

TABLE 3B.--Regions in which Coda Q has been estimated

Region	References
Oishiyama, Japan	Aki and Chouet (1975).
Tsukuba, Kanto (area T), Japan	Do.
Central California	Aki and Chouet (1975), Phillips (1985).
Kilauea, Hawaii	Chouet (1976).
San Fernando, Calif.	Do.
Crimea	Pustovitenko and Rautian (1977).
Garm, central Asia	Rautian and others (1978).
Stone Canyon, Calif.	Chouet (1979).
Epinal, France	Suhash (1979).
Jura, France	Do.
Bangui, Central African Republic	Do.
Arette, French Pyrenees	Hinderer (1979), Herraiz[1] (1982).
Utah	Herrmann (1980).
Central California	Do.
Southeastern United States	Do.
Kinki, Japan	Akamatsu (1980).
Tangshan, China[2]	Jin (1981).
Songpan, China[2]	Do.
Granada, Spain[1]	Herraiz (1982).
Lorca, southeastern Spain[1]	Do.
Friuli, Italy	Rovelli (1982).
Hindu Kush, Afghanistan	Roecker and others (1982).
Aeolian Islands, Italy	Del Pezzo and others (1983).
Petatlán, Mexico	Rodriguez and others (1983).
Contiguous United States	Singh (1981), Singh and Herrmann (1983).
Sichuan Province, China	Zhang and Yang (1983).
Southern Hawaii	Wilson and others (1983)
Alaska	Biswas and Aki (1984).
New England	Pulli (1984).
Ancona, Italy	Del Pezzo and others (1985).
Beijing, China	Chen and others (1984).
Yu-Nan, China	Do.
South Carolina, United States	Rhea (1984).
Kalapana, Hawaii	Wyss (1985).
Tottori, Japan	Tsukuda (1985).
Northern Baja, Calif.	Rebollar and others (1985).
Petatlán, Mexico	Novelo-Casanova and others (1985).
California	Lee and others (1985, 1986).
Kuril-Kamchatka	Gusev and Lemzikov (1985).
Pacific Ocean: Lithosphere	Jin and others (1985).
Adak, Aleutian Islands	Scherbaum and Kisslinger (1985).
Tangshan, China	Jin and Aki (1986).
Haichen, China	Do.
Central Japan	Sato (1986).
Northwest Pacific	Novelo-Casanova and Butler (1986).
Round Valley, California	Peng and others (1986).
Central California-Sierra Nevada	Phillips and others (1986).
Nevada	Lee and others (1986).

[1]Does not include Q frequency dependence study.
[2]Coda analysis without including Q estimate.

The backscattering coefficient g_π may be calculated directly from this formula because "a" can be found for each frequency band from a linear fit to the ln(S to coda ratio) versus distance relation, and Q_c may be estimated by coda analysis.

This method requires a large number of events to obtain the average value. Therefore, its applicability is very restricted. Aki (1980a, b), by using more than 900 events, obtained $g_\pi = 0.02\,\mathrm{km}^{-1}$ for 1.5 and 3 Hz in the Dodaira and Tsukuba regions of Japan. Physically, this means that S waves with frequencies 1.5 to 3 Hz lose 2 percent of their original energy by scattering for every kilometer traveled if $g(\theta) = g_\pi$ for all θ. This result agrees, within the error of determination, with that deduced from the attenuation of S waves calculated by the single-station method (Aki, 1981a). Thus, the backscattering loss required to explain the observed coda amplitude accounts for the observed attenuation of S waves and it is possible to conclude that the attenuation of S waves for this range of frequencies can be explained by backscattering due to inhomogeneities.

The values of g_π just given are consistent with those obtained by Sato (1978) in the same area but by a different method. His procedure was based on the total energy estimates by the classical Gutenberg-Richter relation between magnitude and energy and the assumption of single isotropic scattering. He obtained g_π values from 0.006 to $0.04\,\mathrm{km}^{-1}$ for the frequency range of 1 to 30 Hz. Note that the use of expression (52) can introduce an overestimation of g_π because of the possible underestimation of S-wave spectra by amplitude measurements on a band-pass filtered record.

There is a final consideration with regard to the method just described: strictly speaking only an average parameter g_π has been obtained. This parameter offers an indirect estimation of the heterogeneity by expressing the energy loss by backscattering; in this way it improves the knowledge about coda synthesis and Q_c–Q_β relations. But it can be more interesting to study the perturbations of wave velocity caused by the heterogeneities and (or) the way they modify density and the Lamé parameters. To evaluate $g(\theta)$ for all values of θ is needed. All these objectives require analysis of more detailed statistical models; the next section will be dedicated to this subject.

A comparison between the observed S-wave displacement spectrum, which is the source spectrum modified by forward scattering and attenuation, and the observed coda-wave spectrum, which is the same source spectrum perturbed by the backscattering effect and attenuation, can shed light on the scattering mechanism. This study was accomplished by Aki (1981b) using the data from the Kanto region with which he had already estimated Q_β and g_π (Aki 1980a, b, respectively). In this method the S spectrum is estimated by expression (52), which after correcting for attenuation yields:

$$F_s(\omega) = \frac{A_s(\omega, R)}{2\Delta f} R\, e^{\omega R/2\beta Q_\beta} \tag{55}$$

where, as usual, R is the distance from the source to the receiver, β is the shear velocity, and Q_β is known from previous studies. On the other hand, the coda spectrum $F_c(\omega)$ is defined as:

$$F_c(\omega) = \frac{A_c}{\sqrt{\Delta f}}\, e^{50\omega/2Q_c} \tag{56}$$

where A_c is the peak to peak ground displacement of coda waves averaged over a fixed time interval around $t = 50\,s$. A_c is measured in the frequency band Δf after correcting for instrument magnification (Aki 1980a, b). As in previous studies, two main tectonic areas were distinguished in the Kanto region.

Comparison of $F_c(\omega)$ and $F_s(\omega)$ for both areas and for crustal and mantle shocks shows that only forward scattering is affected by structural differences. This fact suggests that this scattering is more sensitive to spatial variation of heterogeneity than backscattering. A decrease in the coda spectrum toward the low-frequency end (1.5–3 Hz) is also seen. Since such a decrease is absent in the S spectrum and cannot be attributed to a source effect (Molnar and others, 1973), it must be considered as a result of the weakening of backscattering toward low frequencies. This trend agrees with the frequency dependence of Q_c^{-1} but disagrees with the trend of the Q_β^{-1}-frequency plot shown previously. Aki suggested that this failure can be due to biased observations of Q_β for 1.5 Hz for shocks with focal depth greater than 35 km. Thus, Q_β for 1.5 Hz was mainly attributed to wave paths in the crust while g_π and Q_c were primarily related to mantle paths.

The estimation of scattering coefficients has received careful treatment in some of the most recent studies (Sato, 1982b, 1984; Wu and Aki, 1985b). As will be seen in Section 3, scattering coefficients can also be estimated from phase and amplitude fluctuations of P waves over a large seismic array. The sensitivity of backscattering and forward-scattering coefficients to inhomogeneities with different scales constitutes one of the most interesting points of research.

Station Site Effects: Applications

The influence of station site on short-period seismic waves was identified by Richter (1958) and later, on specific site-amplification studies, by Espinosa (1968, 1969) and Espinosa and Algermissen (1972a, b, 1973). Likewise, the influence of station site effects on coda waves has been identified by Aki's initial work in 1969. He observed that codas corresponding to different stations in the Parkfield-Cholame (California) area had the same behavior in the decay of amplitude but different levels of excitation. Stations on younger sediments showed greater excitation than stations sitting on granitic rock and older sediments. Therefore a correction was necessary before coda amplitudes could be used by seismologists to estimate the seismic moment. This same requirement for station correction was noted by Aki (1981b) while performing the comparison between S-wave and coda-wave spectra in the Dodaira-Tsukuba area.

Tsujiura (1978) compared the site effect for P, S, and coda waves for six stations at Mount Dodaira (Japan). He found a greater stability of amplification factor for coda waves than for P and S waves. The site effect for coda waves seemed to be independent of the direction of wave approach, and its value was about the median of site effects for S waves but different from that for P waves. As was mentioned earlier, this result is a strong argument in favor of the explanation of coda waves as backscattered S waves.

Tsujiura (1978) also noticed that the increase of Q_c with frequency was dependent on the local geology of station site. For frequencies ranging from 0.75 to 24 Hz, he found Q_c variations of 250–800 and 150–2,500 for stations located on sandstone and granitic rocks respectively. The frequency dependence of amplitude also correlates with this local geology. The differences of coda amplitude between stations located on sediment and granite varied from 20 times at 0.75 Hz to less than 1/10 at 24 Hz. Tsujiura attributed these facts, also observed for S waves, to

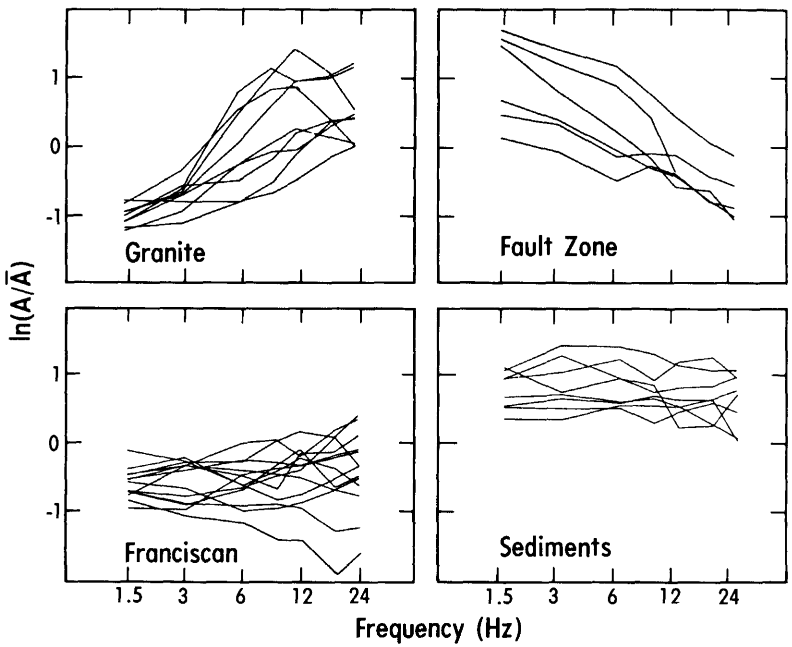


Figure 8a.— Ratio of coda-wave amplitudes showing site effects as a function of frequency for selected CALNET stations grouped by surface geology. $\bar{A}$ is an average coda wave amplitude taken over all sites at which measurement is made for a source and a lapse-time given. Note the different roll-offs with frequency between the four groups of stations, the result of near-site Q variations and the different low-frequency levels. The zero line represents an average station adjustment specially fitted to each group of stations. [Reproduced with permission from Phillips, 1985.]

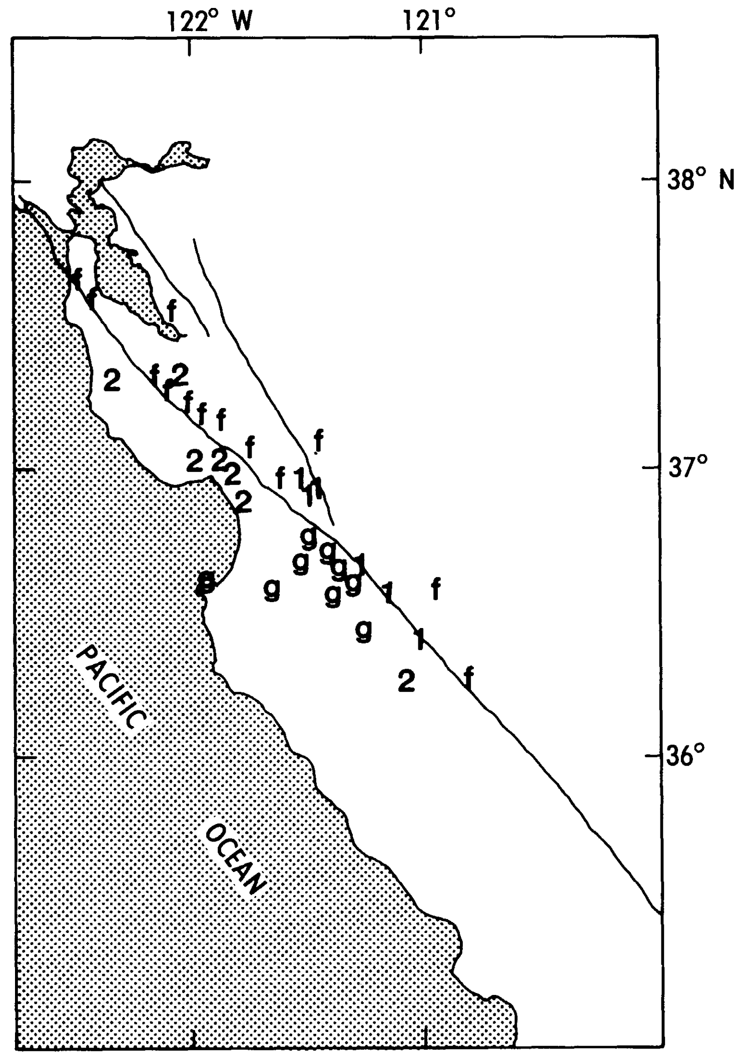


Figure 8b.— CALNET station locations are identified with the following codes: g=Granite, f=Franciscan, 1=fault zone sites, and 2=nonfault zone sediments sites. Solid lines represent the main active faults. [Reproduced with permission from Phillips, 1985.]

differences in intrinsic Q and inhomogeneity changes of the Earth's crust beneath the station. The site dependence of Q_c is inconsistent with the simple S to S backscattering model, because Q_c accounting for path should not be affected by local geologic structure. This subject requires a more careful study.

Site effect has received renewed attention because of its use for earthquake engineering (Espinosa and Algermissen, 1973; Espinosa and Lopez-Arroyo, 1977; Bard and Gariel, 1986). Phillips (1985) and Phillips and Aki (1986) have developed a new method for calculating the coda site effect by using a large amount of network digital data. Its application to over 1,200 codas of 90 California earthquakes recorded at 150 stations shows a different site amplification pattern for low and high frequencies. In agreement with Aki's earlier results, the amplification at 1.5 Hz is very high for young sediments and far lower for granites and old sediments (fig. 8).

It seems that impedance mismatch is the predominant mechanism at low frequencies while absorption becomes more and more important as frequency increases. The possibility of change in site effect must be taken into account in the application of Q temporal fluctuation to earthquake prediction. A recent study of the spatial variations of crustal coda Q across the San Andreas fault in central California and in the Sierra Nevada batholith (Phillips and others, 1986), has introduced a new perspective on this subject. Using only relatively short lapse times in order to avoid upper mantle contributions, these authors obtained a frequency dependent trend opposite to that estimated from longer lapse-times. These results could be explained by assuming a regional and frequency-independent intrinsic attenuation superimposed upon a more slowly varying apparent Q, perhaps due to scattering. In any case, site effect must be considered in the estimation of magnitude based on coda duration (Del Pezzo and others, 1985).

PART III: TEMPORAL VARIATIONS OF CODA-WAVE SHAPES

In addition to the coda characteristics already described, the temporal variations of coda decay, which can be quantified as a temporal change of Q_c, must be considered.

The first observation of this phenomenon was made by Chouet (1979) who, on studying 185 earthquakes recorded in Stone Canyon, California, for 1 year, noticed a marked increase in the energy of the high-frequency coda of earthquakes with similar magnitudes and locations. The change in the spectrum shape was reflected by a remarkable decrease in the corresponding Q_c values. This anomaly was observed over the 1.5 to 24 Hz frequency range. Instrument failure and rainfall were ruled out as possible explanations. Chouet suggested two possible mechanisms based on temporal variations in the source spectra of earthquakes and on changes in the attenuation and (or) scattering properties of the underlying crust. Aki (1980b) recognized that the frequency dependence of spatial variation of Q_c had a shape similar to those due from temporal variation. This similarity suggests an explanation of both phenomena by common origin, namely that heterogeneity is responsible for scattering changes in both time and space. The Q_c fluctuations may be the expression of changes in the heterogeneous structure, also affecting other earthquake precursory phenomena such as anomalous animal behavior, well levels, and geochemical perturbations (Aki, 1985). This hypothesis is supported by several other observations: Q_c changes before pressurizing a hydrofractured reservoir at Los Alamos Hot Dry Rock geothermal site (Fehler, 1979), volcanic eruptions at Mount St. Helens (Fehler and Roberts, oral commun., 1983) and earthquake occurrence (Zhadin, 1979; Jin, 1981; Del Pezzo and others, 1983; Wilson and others, 1983; Rhea, 1984; Gusev and Lemzikov, 1984, 1985; Tsukuda, 1985; Wyss, 1985; Novelo-Casanova and others, 1985; Sato, 1986; Jin and Aki, 1986). Most of the observations show decreases of Q_c of local small earthquakes before major earthquakes and eruptions, as well as fluctuations in the ratio between S-wave and coda-wave amplitudes. It was also observed that b and Q_c variations had the same trend around major earthquakes and opposite around volcanic eruptions. These facts were explained assuming that the b value can increase or decrease when the crustal cracks close according to whether the change of stress condition affects larger cracks more than smaller. As the increase of Q_c always corresponds to the closing of cracks, this parameter seems to be a more reliable indicator of the stress condition than the b value (Aki, 1985). The temporal change in coda Q (Q_c) has been studied by Peng and others (1986) using the recordings from the Round Valley, California, earthquake of November 23, 1984. They found an increase of Q^{-1} for those events which occurred after the main shock in the epicentral area. The opposite effect was observed from those events which were located farther away from the epicentral region. These results are similar to those reported by Tsukuda (1985) using short lapse-time analysis and by other investigators who have used long lapse-time coda-wave studies, but disagree with those expected by Gusev (1986). It seems that the doughnut model proposed by Mogi (1977; see also Kanamori 1981) to explain precursory seismicity patterns is appropriate to fit the Q^{-1} coda changes reported above. All these facts lead toward considering the temporal variations of Q_c^{-1} as an earthquake precursor with these important characteristics:

1. It is a precursor with a known sampling volume; if the station is close to the epicenter, this volume becomes a sphere with radius equal to $\beta t/2$, where β is the velocity of S waves and t is the arrival time of coda waves. This knowledge makes the use of Q_c as a predictor easier.
2. Its measurement may be made without great difficulty, and its variations can be clearly observed.

PART IV: P CODAS; DESCRIPTION AND APPLICATIONS

So far, attention has been focused on the last part of local seismograms, that is, the S coda. Here, a brief review of P-coda waves will be presented. The expression, "P-coda waves" was first used in the late 1960's to refer to the phases following the direct P waves at teleseismic distances (Basham and Ellis, 1969). Greenfield (1971) defined P coda explicitly as "the part of the signal that does not come from the well known arrivals such as PP and PcP". More recently (Hudson, 1977), this term has been applied to any phase placed between P and S direct arrivals. P coda includes several phases fairly well known and some faintest lines considered as their precursors. The main phases have been extensively used to improve knowledge about the Earth's structure and to discriminate between earthquakes and underground explosions.

These precursors demand careful attention, because they are considered to be scattered waves, and because they constitute another thumbprint left by heterogeneities on seismograms.

The first attempt to explain P-coda formation as a scattering process was made by Wesley (1965), who applied the diffusion model to obtain an overall description of near range seismograms. Greenfield (1971) explained the P coda of teleseismic records as a result of conversion of near-source Rayleigh waves to P waves by the rough topography of the surface near the source. In his turn, Kopnichev (1977a) assumed that the scattering occurs singly and isotropically within a medium in which velocity inhomogeneities are random and spatially stationary. Using these assumptions, Kopnichev studied three different situations: scattering in a half-space, scattering in a layer, and scattering in a mixed layer and half-space situation. This analysis constitutes an extension of the single-scattering model. It estimates the Q value from the damping of P coda and the differential scattering coefficient $g(\theta)$ from the ratio of intensities of the P wave to P coda. Using a ChISS station placed at $\Delta = 63°$ and assuming a layer 200 km thick for the last two situations, Kopnichev showed that Q increases somewhat with an increase in frequency for the range of 0.7 to 2.7 Hz and for the three situations. This result is similar to the variation found previously for S coda. In a more recent work, Sato (1984) has outlined the importance of $S \rightarrow P$ scattering near the source region in the formation of P-coda waves.

The scattering mechanism has also been used to explain the precursors to some major phases. For instance, King and others (1974, 1975), King and Cleary (1974), and Cleary and others (1976) so interpret precursors to PP at epicentral distances of around 100°. A similar explanation has been proposed to account for precursors to $PKPPKP$ (King and Cleary, 1974) and to interpret precursors to $PKIKP$ observed in the shadow region of PKP (Haddon and Cleary, 1974; Vinnik, 1974, 1981; Doornbos, 1974, 1976; Husebye and others, 1976). These data suggest the existence of a strong small-scale heterogeneity in the Earth and the necessity of revising several results obtained by methods based on one-dimensional vertical structure (Mereu and Ojo, 1981). According to results obtained by Cleary and Haddon (1972), Doornbos and Husebye (1972), and King and others, (1974), the problem can be particularly serious if discontinuities in the upper-mantle have been considered. Barley and others (1982) have suggested the existence of S to P scattering at the base of the upper-mantle, immediately beneath the source region. Correlation studies between short-period P-coda characteristics and crustal structure were initiated by Hasegawa (1969, 1970), and particle motions of P-coda waves have been analyzed by Sato and Matsumura (1980a).

Most practical analyses previously cited were based on theoretical results for first-order scattering obtained by several authors (see for instance: Miles, 1960; Herrera, 1965; Herrera and Mal, 1965; Haddon, 1972, 1978). Hudson (1977) summarized previous trends of research and pointed out the limitations of results based on acoustic theory.

Finally, let us remark on the stability of the P coda as opposed to the great variability of direct P signals. This last property has been carefully studied by Douglas and others (1973) who compared complexity, m_b, and energy and spectral ratios, for several nuclear tests and earthquakes. Their results confirm the stability of amplitude and spectral contents in P-coda waves, features that we have found earlier in S codas. The ability of P codas to discriminate seismic waves of earthquakes and explosions was also studied by Evernden (1977) and Evernden and Kohler (1979). Other studies of the stability of the P coda and the variability of direct P can be found in Subash and Choudhury (1979), and Aki (1981a). Baumgardt (1985) offers a P and L_g analysis of long term codas recorded at GRAEFENBERG and NORSAR arrays, from possible underground nuclear tests in western Russia.

SECTION 3: STUDY OF INHOMOGENEITY

INTRODUCTION

In the previous section, three important features of seismograms due in great measure to scattering have been considered: coda formation, attenuation, and P-phases precursors. These are, perhaps, the most striking traces left by heterogeneities on seismograms. Let us now consider more subtle effects, namely phase and amplitude fluctuations on direct P waves produced by the interference of scattered waves. This latter effect will be considered carefully because it plays a significant role in our next task: the quantitative study of heterogeneity. This includes the estimation of parameters "α" (scale length of inhomogeneity) and "$g(\theta)$" (differential scattering coefficient) to obtain some quantitative information about the heterogeneities responsible for scattering. A method to evaluate g_π, that is $g(\theta)$ when $\theta = \pi$, by using coda waves has already been described. Their frequency dependence, as well as the Q^{-1}-frequency plot previously described, will play a decisive role in checking scattering models.

There are various ways to estimate the heterogeneity quantitatively, such as the use of well-logging data, the techniques based on inversion of P-time data observed at seismic arrays, and the methods founded on the theory of wave propagation in random media.

As far as we know, the first estimate of small-scale heterogeneities was based on observations of damping of body waves. Jeffreys (1959, p. 114) assumed that the damping factor for waves with periods of 10 s in comparison with those for long periods is ϵ^{-1} for a distance of 5,000 km. He also considered that the wave velocities vary from heterogeneity to heterogeneity by a factor of 0.1. With these main assumptions, Jeffreys attributed a length of 450 cm to the average grain diameter of crustal irregularities. As Aki (1973) pointed out, this value is too low and cannot explain amplitude and phase fluctuations observed in the Montana LASA.

The analyses of precursory phases can be extremely useful to obtain information about the Earth's deep interior, but the subject is out of the scope of this work. Two other methods are more relevant here. The first uses the autocorrelation of seismic velocity measurements taken along a borehole to get information about the fractional velocity fluctuation and the correlation distance. This method has been applied to data from Fenton Hill, New Mexico, by Wu (1982a), and from Shimosha, Japan, by Sato (1979) and Suzuki and others (1981). Its application is very restricted because it can only sample shallow depths ranging between 2,000 and 3,000 m at a very few places.

The other method uses P-time residual data of teleseismic and local earthquakes observed at a seismic array. These data are inverted to obtain a three-dimensional plot of seismic-velocity anomalies under the array (Aki, 1977, 1981c; Aki and others, 1977; Hirn and Nercessian, 1980; Roecker, 1981). The array aperture must be large enough to adequately sample the bottom of the lithosphere with small enough station spacing. In fact, the station spacing limits the scale length of heterogeneity resolvable by this method to about 10 km. The use of damped least-square solution or minimum solution in the method (Aki, 1977) tends to suppress the estimate of mean square fractional velocity fluctuation; therefore, the values given by this method have to be considered as lower bounds.

MODELING RANDOM MEDIA

This research is based on the observation of P-wave phase and amplitude fluctuations recorded by a large aperture seismic array. When a seismic wave travels through a heterogeneous medium, it is modified in amplitude and phase as the wave front encounters random small-scale velocity and density deviations from the mean values. If this medium is situated beneath a seismic array, phase fluctuations are recorded as variations in traveltime of the first P arrival from the value expected for a homogeneous velocity structure (fig. 9). In the same way, amplitudes show large variations across the array from that expected from the homogeneous case. If a mathematical relation between these observed fluctuations and the magnitude and distribution of medium fluctuations can be established, wave fluctuations can be used to deduce characteristics of the medium. This subject constitutes the inverse problem that, generally, does not have a unique solution. The forward problem—to determine wave fluctuations from random deviations of medium parameters—forms part of the theory of wave propagation in random media. In seismology, this forward problem has been mainly treated using one of these statistical approaches: Chernov's theory (1960), the mean wave formalism (Karal and Keller, 1964), and the traveltime corrected mean wave formalism (Sato, 1982a, b). Recently, other numerical methods have been introduced to study the effects of random media on traveltimes and amplitudes of seismic waves (Menke, 1983a, b; Nowack and Aki, 1984; Frankel and Clayton, 1986). Their application to refraction profiles data (Ojo and Mereu, 1986) or to seismic prospecting (Banick and others, 1986a, b) can be highly useful. Concern with the above cited statistical approaches, and because of the similarities between the mathematical model and seismological method, both subjects will be described together.

Chernov's Theory

The theory initiated by Pekeris (1947) and developed by Chernov (1960) on scalar wave propagation in a medium with random heterogeneities has found so large an application in modern seismology (Aki, 1973; Capon, 1974; Berteussen and others, 1975; Wu and Aki, 1985b) that special attention to it is justified. Since the theory is for the scalar wave equation, strictly speaking, an extension to the vector wave scattering analysis is needed for application to seismology. In order to characterize the medium statistically, Chernov introduced the fluctuation of the refractive index, $\mu(r)$, that may be defined as:

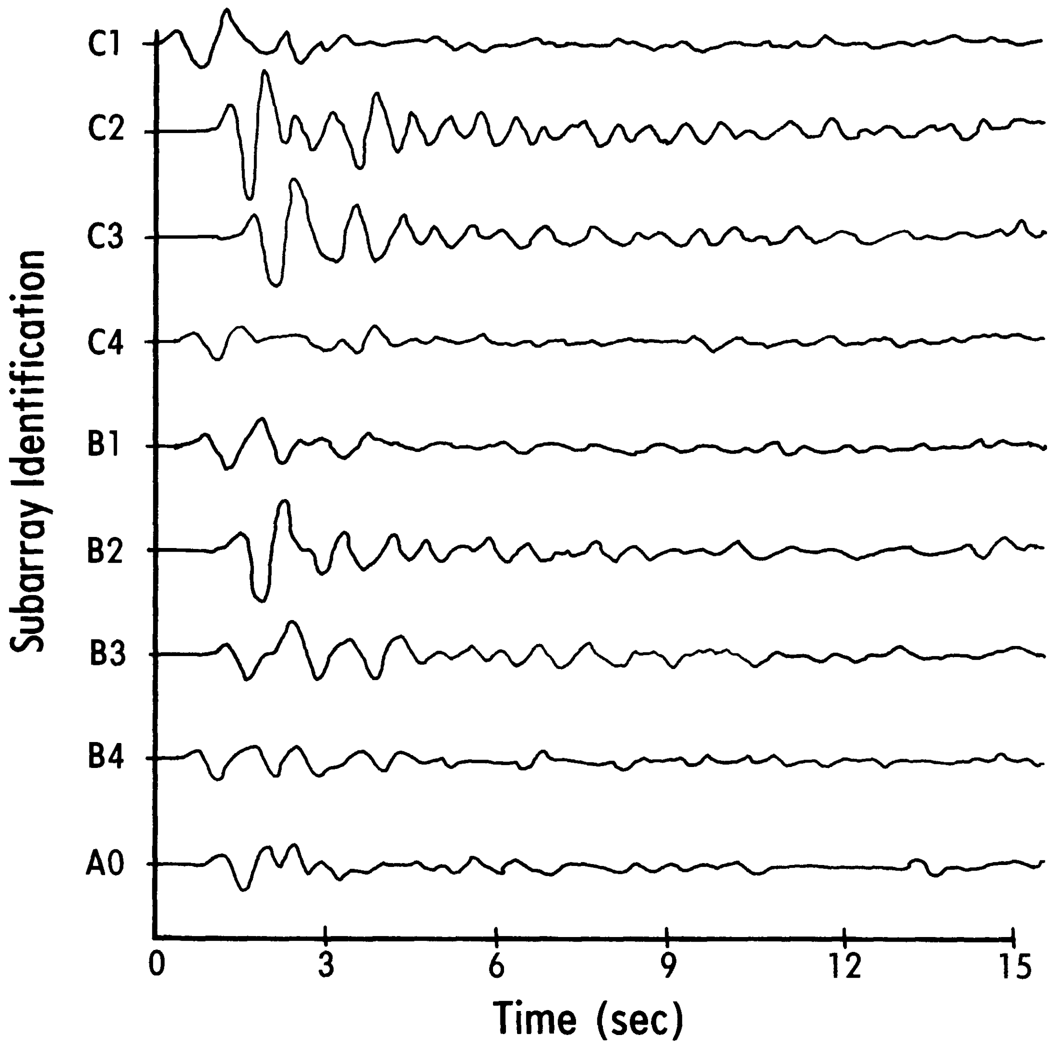
$$\mu(r) = \frac{\langle \mathrm{v} \rangle}{\mathrm{v}(r)} - 1 \tag{57}$$

where $\mathrm{v}(r)$ expresses the velocity and $\langle \mathrm{v} \rangle$ is its average value. The spatial correlation function $N(r)$, which expresses the correlation of fluctuations between two points within the heterogeneous region, is assumed to have Gaussian form. Other assumptions used by Chernov and of special interest in the study are:

1. The medium is weakly inhomogeneous, that is $\langle \mu^2 \rangle \ll 1$
2. The random process is statistically homogeneous and isotropic.
3. The extent of medium is much greater than the correlation distance α: $L \gg \alpha$.

Under these assumptions Chernov proved that if (as happens in hydroacoustic problems) density fluctuations can be neglected, small-scale fluctuations ($\kappa\alpha \ll 1$) generate isotropic scattering. On the contrary, for large-scale fluctuations ($\kappa\alpha \gg 1$) scattering depends strongly on the direction. In this last case, most of the scattered power is concentrated within the small angle θ, $\theta \simeq 1/\kappa\alpha$ in the forward direction.

Scattering due to density fluctuations can be discounted more accurately when only $\kappa\alpha \gg 1$ is considered, because in this case scattering due to velocity fluctuations becomes much more important.



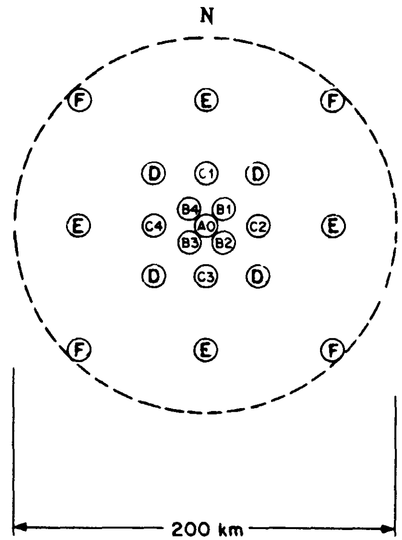


Figure 9.— The P-wave shapes for the December 18, 1966, Eastern Kazakh event, ($\Delta = 83.7°$, $m_b = 5.9$), observed on the center seismometers of subarrays within the C ring at the LASA array. The subarrays used are shown in the adjunct sketch of the LASA geometry. Amplitude and phase fluctuations between subarrays can be clearly observed. [Reproduced with permission from the Seismological Society of America.]

These conclusions, which have been cited before in this work without explaining their origin, are refined in the studies that shall be commented in on the next section.

From considerations of energy flow attenuation, Chernov defined the scattering coefficient g, as given by equation (4)

$$g = \frac{\Delta I}{IL} \tag{4}$$

where ΔI represents the loss of energy by scattering when a wave of energy I passes through the layer of thickness L. Chernov proved that in the case of small-scale inhomogeneities, g is proportional to the fourth power of the frequency (Rayleigh scattering), according to the expression:

$$g = \sqrt{\pi}\ \langle \mu^2 \rangle\, \kappa^4 \alpha^3 \tag{58}$$

For the case of large-scale heterogeneities the formula is:

$$g = \sqrt{\pi}\ \langle \mu^2 \rangle\, \kappa^2 \alpha \tag{59}$$

Hence, the scattering coefficient increases as the square of the frequency. The development of Chernov's theory assumes the Born approximation. It implies that although Chernov defends their applicability (Taylor, 1967; Aki 1973), the formulas which shall be seen later are not applicable beyond the condition for Born's approximation. The ratio $\Delta I/I$ can be evaluated using the expression:

$$\frac{\Delta I}{I} = \sqrt{\pi}\ \langle \mu^2 \rangle\, \kappa^2 \alpha L \left(1 - e^{-\kappa^2 \alpha^2}\right) \tag{60}$$

calculated assuming $\kappa\alpha \gg 1$ This expression will be used by Aki (1973) to check the conditions for meeting the Born approximation.

The study of fluctuations, done in order to get a dependence between the variations of the wave characteristics and those of the medium, assumed the density to be uniform. As in the scattering problem, only a monochromatic plane wave is studied and two ways—small perturbations and Rytov's method— are used. Both these methods give similar results to small-scale inhomogeneities, but Rytov's (1937) method deals better with large scale inhomogeneities. In this case, the Fresnel approximation is introduced. This approximation considers only the effect produced by the inhomogeneities concentrated within a cone with its vertex at the receiving point and with an aperture angle of the order of $1/\kappa\alpha$. In this way, the spherical wave front is approximated by a parabolic one. The Fresnel approximation is only valid for large $\kappa\alpha$ namely when most of the scattered power is concentrated within the solid angle $\theta \simeq 1/\kappa\alpha$. In the case Chernov was interested in, ($\kappa\alpha \gg 1$ for acoustic waves), the scattering produced by density fluctuations is much smaller than that due to velocity anomalies. It must be noticed, however, that real seismological situations can be very different. Taking into account the assumptions described before and introducing the wave parameter D, the mean square phase and logarithmic amplitude fluctuation are respectively given by:

$$\langle S^2 \rangle = \frac{\sqrt{\pi}}{2}\ \langle \mu^2 \rangle\, \kappa^2 \alpha L \left(1 + \frac{1}{D} \arctan D\right) \tag{61}$$

and

$$\langle B^2 \rangle = \frac{\sqrt{\pi}}{2}\ \langle \mu^2 \rangle\, \kappa^2 \alpha L \left(1 - \frac{1}{D} \arctan D\right) \tag{62}$$

The ratio $\langle B^2 \rangle / \langle S^2 \rangle$ is a function of only D and is always less than unity:

$$\frac{\langle B^2 \rangle}{\langle S^2 \rangle} = \frac{1 - \dfrac{1}{D} \arctan D}{1 + \dfrac{1}{D} \arctan D} \tag{63}$$

For $D \gg 1$ (Fraunhofer region), the above expressions become:

$$\langle S^2 \rangle = \langle B^2 \rangle = \frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle \kappa^2 \alpha L \tag{64}$$

So, in the case of a large wave parameter, the mean square amplitude and phase fluctuations are the same and increase proportionally with L. In the opposite case, when D is small, Chernov proved that the amplitude fluctuations decrease faster than phase variations so that they become an order of magnitude smaller for $D \ll 1$. For $D = 0$, equations (61) and (62) reduce to

$$\langle B^2 \rangle = 0 \tag{65}$$

and

$$\langle S^2 \rangle = \sqrt{\pi} \langle \mu^2 \rangle \kappa^2 \alpha L \tag{66}$$

That is, the amplitude fluctuation vanishes and the phase fluctuation becomes dominant.

Chernov further calculated the cross correlation between the phase and log amplitude fluctuations at the same receiver. Also, he evaluated the spatial autocorrelation for fluctuations between different receivers, and the time autocorrelation for amplitude (or phase) variations at the receiver. Only the first two items are treated in this study. For the cross-correlation, $\langle BS \rangle$, between both types of fluctuations the result is

$$\langle BS \rangle = \frac{\sqrt{\pi}}{16} \langle \mu^2 \rangle \kappa^3 \alpha^3 \log \left(1 + D^2\right) \tag{67}$$

So, the correlation coefficient R_{bs} of the amplitude and phase correlations, defined as

$$R_{bs} = \frac{\langle BS \rangle}{\sqrt{\langle B^2 \rangle \langle S^2 \rangle}} \tag{68}$$

takes the form

$$R_{bs} = \frac{\log \left(1 + D^2\right)}{2\sqrt{D^2 - (\arctan D)^2}} \tag{69}$$

At small distances ($D \ll 1$), when the ray approach holds, this expression simplifies to

$$R_{bs} \simeq \frac{\sqrt{3}}{2\sqrt{2}} \sim 0.6 \tag{70}$$

And at large distances

$$R_{bs} = \frac{\log D}{D} \tag{71}$$

The last two expressions show that the correlation between amplitude and phase fluctuation is always positive and vanishes when the distance is large. These results constitute the first known estimate of the correlation between amplitude and phase fluctuation at the receiver.

The spatial autocorrelation evaluates the behavior of fluctuation of one parameter along the space. Chernov examined two cases: when the receivers are colinear with the source (longitudinal autocorrelation), and when they are situated in a plane perpendicular to the direction of wave propagation (transverse autocorrelation).

To cope with the longitudinal autocorrelation Chernov took two points separated by a distance ΔL smaller than their distance to the origin of coordinates. In the region of large values of D and for $\kappa\alpha \gg 1$ and $\Delta L \sim \alpha$ Chernov deduced the expression

$$\langle B_1 B_2 \rangle = \langle S_1 S_2 \rangle = \frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle \kappa^2 \alpha L \tag{72}$$

where $\langle B_1 B_2 \rangle$ is the longitudinal autocorrelation for amplitude fluctuations and $\langle S_1 S_2 \rangle$ the autocorrelation for phase fluctuations. This last result coincides with the expression (64) for the mean square amplitude and phase fluctuations. Mathematically, this result means that at a distance between the receiving points nearly equal to the correlation length, α, both the amplitude and the phase fluctuations are almost perfectly correlated.

If the distance is large compared to α, that is $\Delta L \gg \alpha$, the autocorrelations are

$$\langle B_1 B_2 \rangle = \langle S_1 S_2 \rangle = \frac{\frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle \kappa^2 \alpha L}{1 + \left(\frac{2\Delta L}{\kappa \alpha^2} \right)^2} \tag{73}$$

Defining the correlation coefficients as

$$R_b = \frac{\langle B_1 B_2 \rangle}{\langle B^2 \rangle} \qquad \text{and} \qquad R_s = \frac{\langle S_1 S_2 \rangle}{\langle S^2 \rangle} \tag{74}$$

expressions (64) and (73) lead to:

$$R_b = R_s = \frac{1}{1 + \left(\frac{2\Delta L}{\kappa \alpha^2} \right)^2} \tag{75}$$

For the case $D = \frac{4L}{\kappa\alpha^2} \ll 1$ it is easy to see that there is a complete longitudinal autocorrelation both for amplitude and phase fluctuations.

Finally, the transverse autocorrelation considers two points lying on a plane parallel to the wave-front and separated by a distance d. For $D \ll 1$, the amplitude and phase autocorrelation coefficients are respectively

$$R_b = e^{-d^2/\alpha^2} \left[1 - 2\frac{d^2}{\alpha^2} + \frac{1}{2} \left(\frac{d^2}{\alpha^2} \right)^2 \right] \tag{76}$$

and

$$R_s = e^{-d^2/\alpha^2} \tag{77}$$

So, the phase autocorrelation coefficient R_s has the same shape as the correlation coefficient for $\mu(r)$ fluctuation. Both phase and amplitude fluctuations have a correlation distance similar to that of the refractive index. For the Fraunhofer diffraction and $D \ll 1$, the results are

$$R_b = \frac{e^{-d^2/\alpha^2} - \dfrac{1}{D}\left[\dfrac{\pi}{2} - \mathrm{Si}\left(\dfrac{d^2}{D\alpha^2}\right)\right]}{1 - \dfrac{1}{D}\arctan D} \tag{78}$$

and

$$R_s = \frac{e^{-d^2/\alpha^2} + \dfrac{1}{D}\left[\dfrac{\pi}{2} - \mathrm{Si}\left(\dfrac{d^2}{D\alpha^2}\right)\right]}{1 + \dfrac{1}{D}\arctan D} \tag{79}$$

where $\mathrm{Si}(x)$ is the sine integral function defined as

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\, du \tag{80}$$

Chernov proved that R_b and R_s tend toward unity when $d \ll \alpha$, and decrease to zero when $d \gg \alpha$. These results have been extended to different correlation functions $N(r)$, by Tatarskii (1961). The measurement of R_b and R_s can offer an efficient way to estimate the correlation distance α of the random medium.

So far, the forward problem has been addressed to obtain different correlation coefficients for amplitude and phase fluctuation of the waves from velocity fluctuations of the medium. The inverse problem was investigated by Aki (1973), who studied data from the Montana LASA and succeeded in estimating the correlation length, α, and the mean-square value of the velocity fluctuations, $\langle \mu^2 \rangle$, beneath the array. For this purpose Aki selected 66 seismograms recorded at appropriately spaced stations of the Montana LASA. After applying Fourier Transforms and comparing with the best-fitting plane wave solutions in terms of phase velocity and azimuth of approach, the chosen signals give 44 sets of data; each one with 66 amplitudes and 66 phase-delay measurements. These 44 sets supply the respective values of $\sigma\phi$ (root-mean square phase fluctuation, expressed by $\sqrt{\langle S^2 \rangle}$ in Chernov's theory), σA (root-mean square log amplitude fluctuation, $\sqrt{\langle B^2 \rangle}$ in Chernov's notation), and ρ (Correlation coefficient, R_{bs} in Chernov's theory). Expressions $\sigma\phi$, σA, and ρ correspond to Aki's (1973) notation. It is found that $\sigma A/\sigma\phi$ was less than unity for all but one of the sets. This last result agrees well with Chernov's predictions. Similar results were obtained on the observed correlation coefficients, ρ , between phase and logarithm of amplitudes: 39 out of 44 are positive, as was predicted by Chernov's theory.

The spatial autocorrelations of amplitude and phase were calculated for six data groups separately. These groups comprised events having similar direction of wave approach and considered only data between frequencies of 0.5 and 1.0 Hz. The values obtained for $\sigma A/\sigma\phi$ and ρ from observational data give two independent procedures to estimate the wave parameter D according to expressions (63) and (69). The closer their results agree the better Chernov's theory works. In this way, Aki (1973) showed that Chernov's formula appeared to apply correctly to frequencies around 0.6 Hz but not to those around 1 Hz. The average D value was found to be 5 ± 1 and the transverse autocorrelation function predicted for this value by Chernov agreed well with the observed autocorrelation functions for $f = 0.5$. From the comparison between theoretical and observed curves, α was found to be 10 ± 2 km. On the other hand, for $f = 1$ Hz, observed and predicted curves show departure.

Substituting the values of D and α obtained for $f = 0.5$ Hz in expression (3) which defines D, Aki obtained $L \simeq 60$ km, and from the expression for $\sigma\phi$ deduced $\langle \mu^2 \rangle$. The value of $g(\theta)$ for this case of forward scattering, in which $\theta = 0$, is obtained by approximating to the fractional energy lost by scattering per unit propagation distance

$$g(\theta) = \frac{\Delta I}{IL} \tag{81}$$

Then by substituting expression (60) in (81) and for $\kappa\alpha > 1$ one obtains

$$g(0) = \sqrt{\pi} \, \langle \mu^2 \rangle \, \kappa^2 \alpha \tag{82}$$

formula which has already been seen as Chernov's result for large-scale heterogeneities (59). Expression (82), for $f = 0.5$ Hz and with the estimates of $\langle \mu^2 \rangle$ and α previously obtained, gives a value of $g(0)$ equal to 8×10^{-3} km. The results obtained for α, D, L, $\langle \mu^2 \rangle$, and $g(0)$ characterize the heterogeneity under the Montana LASA. The fractional energy loss from primary waves, $\Delta I / I$, obtained by applying expression (60), is $\simeq 0.5$ for $f = 0.5$ Hz and $\simeq 2$ for $f = 1$ Hz. This result proves that Born's approximation fails for this last case and explains the disagreement between observations and Chernov's theory at this frequency. Finally, the values $\kappa\alpha \simeq 5$ and $\kappa\alpha \simeq 10$ obtained for $f = 0.5$ and 1 Hz respectively show that the Fresnel approximation is met in both cases.

This study strongly supports the Chernov theory and shows the necessity of a better approximation than Born's to describe the amplitude fluctuations for frequencies equal to or greater than 1 Hz. A similar conclusion has been obtained by Powell and Meltzer (1984) from the SCARLET network. The results just described will be compared to values obtained by different methods.

Chernov's theory was also applied to the Montana LASA area by Capon (1974) by using P amplitude and slowness anomalies. Data were recorded within nine subarrays and were not the same as those used by Aki. The frequency chosen was also different: 0.8 Hz. As can be seen in table 4 the results obtained by Aki and Capon agree fairly well. This agreement is remarkable, taking into account that Capon uses slowness measurements instead of phase anomalies as Aki did, and that heterogeneities are confined to different depths in these studies.

A modified version of Chernov's theory was used by Berteussen and others (1975) in modeling the crust and upper mantle beneath the NORSAR and LASA arrays. In the Montana LASA a correlation distance of 15 km was found for two configurations considered. This result agrees closely with those commented on before. However, Berteussen and others show the shortcomings of Chernov's theory to estimate L, that is the thickness of the random medium, and to deal

TABLE 4.--Application of Chernov's theory to the Montana LASA region
[Leaders (---) indicate no data available]

References	Data				Medium parameters obtained		
	Amp/Phase	Frequency range (Hz)	Number of observations	Wave parameter D	Correlation distance α(km)	Extent of medium L (km)	R.M.S.[1] of velocity fluctuation ($\langle u^2 \rangle^{1/2}$(percent))
Aki (1973)------	A and P	0.5-0.7	[2]660 x 2	5	10	60	4
Capon (1974)----	A and P	.7- .9	279 x 2	6	12	136	1.9
Berteussen and others (1975).	A	.6- .8	48	---	15	50-150	3.0-2.0
	P	.6- .8	48	---	15	50-150	.6- .3

[1] Root mean square.
[2] Only the approximated number of observations finally used to estimate D is given.

with processes in which strong scattering can be involved, such as the situations which had been found for frequencies around 1.0 Hz (LASA) and 0.6 Hz (NORSAR) by Aki (1973) and Capon and Berteussen (1973), respectively. As previously stated, the same effect has been observed for the precursors to several P phases, which limits the use of Chernov's theory. The estimate of $\langle \mu^2 \rangle$ by Berteussen and others (1975) was calculated using only phase or amplitude fluctuations and is considerably smaller than that obtained by Aki (1973) and Capon (1974), who employed both amplitude and phase fluctuations (see table 4). Therefore, the estimate obtained by Berteussen and others (1975) will not be used in further comparisons.

Aki and others (1976) applied the three-dimensional inversion method of Aki and others (1977) to 3,206 observations of P time from 178 teleseismic events recorded at the Montana LASA. They obtained a lower limit for $\sqrt{\langle \mu^2 \rangle}$ of 3.2 percent. The corresponding L was estimated between 60 and 120 km. These results allowed them to estimate that 100 km underneath the Montana LASA the rms velocity fluctuation amounts to about 3 percent with a correlation distance of about 10 km.

Mean Wave Formalism

The strong dependence of Chernov's theory on Born's approximation requires a discussion of the drawbacks of this approach. Specifically, the Born approximation may break down because considering the primary wave as an unperturbed wave implies neglecting simultaneously the decay due to the scattering process and the energy supplied back to primary waves by scattered waves. Both phenomena can be important when large regions and (or) strong inhomogeneities are considered. In fact, when Born's criterion ($\Delta I/I \ll 1$) fails, Chernov's theory is not suitable. In addition, as Wu (1982b) pointed out, even when this criterion is satisfied, the frequency dependence of the total field attenuation considering multiple scattering will be different from the single-scattering approximation if $\kappa a \gg 1$. It cannot be forgotten that taking into account multiple scattering may be a more realistic approach in many real problems. These limitations suggest that using another statistical model not based on Born's approximation may better develop the study of seismic wave propagation in random media. And indeed, several authors (Beaudet, 1970; Varadan and others, 1978; Varadan and Varadan, 1979; Sato, 1979; Korvin, 1983) have used the mean vector-wave formalism, which *a priori* has the advantage of being valid for any dimension of the region occupied by the random heterogeneity. Sato's contribution to the subject and the controversy raised by the use of the mean wave model prompt us to offer a short description of its main characteristics.

The mean wave formalism was initiated by Keller (1960, 1964), Bourret (1962a, b), Karal and Keller (1964), and then was renovated in Howe's (1971a, b) studies. The model supposes a family of media in which each member deviates slightly from a homogeneous medium. The deviation is random and is characterized by an autocorrelation function. For each member of the random medium the wave field is determined by using perturbation theory for a given initial condition. The statistical properties of the wave field over all member media are calculated on the basis of the autocorrelation function. All the wave fields share the same statistically average properties. Karal and Keller (1964) obtained an equation satisfied by the average wave and deduced from it the propagation constant of the medium κ. The imaginary part of κ is the attenuation coefficient for the mean wave in the medium. Its expression is

$$Q^{-1} = \operatorname{Im} \kappa = \varepsilon^2 \kappa_0^2 \langle \mu^2 \rangle \int_0^\infty (1 - \cos 2\kappa_0 r) N(r)\, dr \tag{83}$$

where ε^2 measures the departure of the medium from homogeneity, κ_0 is the wave number of the incident wave, $\langle \mu^2 \rangle$ expresses the inhomogeneity and satisfies $\langle \mu(x) \rangle = 0$ and $N(r)$ is the correlation function. Taking $N(r)=e^{-r/\alpha}$ where α is the correlation distance, these authors obtained expressions of κ both for $\kappa_0\alpha \gg 1$ and $\kappa_0 \ll 1$. The study was made initially for scalar waves in a slightly nonuniform infinite medium but was soon extended to vector waves. The application to the elastic case is accomplished assuming perturbation of density and the Lamé parameters, and the results are expressed as a function of their correlation and cross-correlation values. Note that the method establishes a relation between the mean wave characteristics and the fluctuations which define the heterogeneity of the medium. Therefore it is possible to infer medium properties from mean wave characteristics. However, as will be seen later, it is necessary to be aware of the differences between the mean wave and the observed wave when the seismic case is considered.

The Karal and Keller (1964) method was applied to explosion data by Beaudet (1970), who calculated the attenuation factor and the correlation distance for the Nevada Test Site. For the first parameter he obtained a result closely related to Nikolayev's (1968) turbidity coefficient but far from the already-cited result of Aki (1973). The correlation distance was estimated to be about 200 m for waves with frequencies between 2 and 50 Hz, which seems to be too low of a value (Aki, 1973).

The mean-wave theory was improved by Howe (1971a, b), who clarified the method and used a more understandable notation. The separation of the wave field $\psi(x,t)$ into two parts introduced by Karal and Keller (1964) is now expressed as

$$\psi(x,t) = \langle \psi(x,t) \rangle + \psi'(x,t) \tag{84}$$

where $\langle \psi(x,t) \rangle$ represents the mean wave and $\psi'(x,t)$ expresses the fluctuations of the field about $\langle \psi(x,t) \rangle$. The wave-velocity perturbation is due to density fluctuations given by $\xi(x)$ which satisfies:

$$\langle \xi(x) \rangle = 0 \tag{85}$$

Howe obtained two equations governing respectively $\langle \psi(x,t) \rangle$ and $\psi'(x,t)$ and considered two kinds of interactions: between the density fluctuation $\xi(x)$ and the mean field, and between $\xi(x)$ and the $\psi'(x,t)$ which is not correlated with $\xi(x)$. The latter type of fluctuation creates the multiple scattering, and neglecting it constitutes the "binary collision approximation". Adopting this approximation, and assuming that $\xi(x)$ is a stationary random function of x, Howe solved the integral differential equation for $\langle \psi(x,t) \rangle$ and obtained the damping of the mean field for both long- and short-wavelength cases. The results show that the damping rate of the meanfield for long waves is twice that for short waves. They are obtained assuming $N(r)=e^{-r^2/\alpha^2}$, that is a Gaussian correlation function for the medium. Later Howe (1971b) extended the study without attributing any particular shape for $N(r)$. The method is applied to very different problems, such as stretched strings and surface gravity waves over shallow water with a rough bed. Because of the use of the binary approximation, this procedure is sometimes called the "first order smoothing method".

Sato (1979, 1981) applied Howe's version of the mean-wave formalism to seismology. He considered seismic waves within a one-dimensional inhomogeneous medium characterized by two independent fluctuations ξ_1 and ξ_2 . For example, in the longitudinal wave case, these fluctuations are given respectively by

$$\xi_1 = \frac{\delta\lambda + 2\delta\mu}{\lambda_0 + 2\mu_0} \tag{86}$$

and

$$\xi_2 = -\frac{\delta\rho}{\rho_0} \tag{87}$$

where $\delta\lambda$, $\delta\mu$, $\delta\rho$ are the fluctuations of the Lamé parameters and density. For any point in the medium the parameters are given by

$$\begin{aligned} \rho(x) &= \langle\rho\rangle + \delta\rho(x) \\ \lambda(x) &= \langle\lambda\rangle + \delta\lambda(x) \\ \mu(x) &= \langle\mu\rangle + \delta\mu(x) \end{aligned} \tag{88}$$

In this case, the cross correlation functions for any couple of points x, y, are

$$R_{ij}(x,y) = \langle\xi_i(x)\xi_j(y)\rangle \qquad i,j = 1,2 \tag{89}$$

The application of this method for the case $N(r){=}e^{-r^2/\alpha^2}$ gives greater values of Q^{-1} for increasing frequencies for both $\kappa\alpha \ll 1$ and $\kappa\alpha \gg 1$ cases. As will be discussed below, this theoretical result disagrees with the observations, and was later revised by considering traveltime corrected mean-wave formalism (Sato, 1982a, b).

Sato's method was also applied to well-logging data from the Simosha deep borehole observatory, and values of Q^{-1} ranging about 1×10^{-3} to 4×10^{-3} were obtained for frequencies between 15 and 130 Hz. At this point it should be stressed that this method, which has the outstanding feature of being able to deal with vector waves, must be applied to seismology bearing in mind the differences between the attenuation of the mean field and the attenuation of the actual observed field (Hudson, 1982). When measurements of seismic-wave amplitude are made, not a mean wave but a peak to peak scaling is measured. In the mean field the values are taken from the ensemble average, whereas in the seismological case attenuation measurements are originated from the real physical phenomena. To obtain a mean value from observed waves, it is necessary to take a spatial average. If this average is not possible, the problem can be considered only under the approach of a "pseudo-random medium" (Frisch, 1968). In addition, if $\kappa\alpha \gg 1$, the spatial average has to be preceded by a phase correction in order to eliminate the phase change caused by the propagation from the reference point to the point in which the measurement is made. Failure to do so leads to mixing waves with very different situations. Hence, it must be pointed out that the attenuation coefficient of the mean field does not evaluate the amplitude attenuation but the loss of coherence among waves considered as a set when they travel through a random medium. Therefore, as Wu (1982b) suggested, it would be more correct to speak about a "randomization coefficient" instead of an "attenuation coefficient" when referring to a mean field. A comparison between the results obtained from the randomization coefficient for the Montana LASA calculated from measured phase fluctuations, and those obtained from the mean-wave formalism is also included in Wu's (1982b) study.

Shortcomings of the Preceding Models

When comparing the method based on Chernov's theory with that founded on mean-wave formalism it must be recalled that the former considers only single scattering under the Born approximation and is thus limited to small distances, whereas the latter includes multiple scattering and supposes an infinite random medium. It has already been seen that the attenuation of the mean wave must be taken more as a measure of randomization than as an estimate of amplitude attenuation. However, if the distance considered is so small that the mean field of this method differs slightly from the primary field deduced from single-scattering theory, the scattered field estimated by the Born approximation will be equal to the fluctuation of the mean field under the binary approximation. Always keeping in mind the meaning of Q^{-1} calculated by the mean field method, notice that both models predict that Q^{-1} increases with frequency even in the high-frequency limit when wavelengths are shorter than the correlation distance. This prediction disagrees with the observational fact that impulsive waves propagate without large attenuation in the high-frequency limit. It also contradicts the observational results alluded to in the last part of Section 2.

The above mentioned difficulties have been recently studied in two different ways by Wu (1982a) and Sato (1982a, b).

Forward Multiple-Scattering Approximation

In order to overcome the difficulties caused by Born's approximation and to account for attenuation, Wu (1982a) introduced a procedure to include the forward multiple scattered energy. In this way, the balance of energy receives a more realistic treatment. This improvement is particularly important for the case $\kappa\alpha \gg 1$ because then the forward scattering becomes predominant.

With this aim, Wu took a scalar plane wave propagating normally into a slab of thickness x_o made of a random medium with correlation distance α and divided the slab into slices with this same thickness (see fig. 10). This medium is considered to be linear, elastic, isotropic, and weakly random. Attention is directed to the case $\kappa\alpha \gg 1$, in which the wave conversion between P and S and the coupling between different polarizations can be neglected (Knopoff and Hudson, 1967). These simplifications constitute the "scalar wave approximation".

Assuming that the back-scattered energy can be considered lost and that the forward scattered energy is preserved by multiple scattering, Wu succeeded in obtaining expressions for amplitude and Q^{-1}. The Q^{-1} result is analyzed for three different forms of correlation functions: exponential, Gaussian and Von Karman. The Von Karman function of order ν is given by

$$N(r) = \frac{1}{2^{\nu-1}\Gamma(\nu)}\left(\frac{r}{\alpha}\right)^{\nu} K_{\nu}\left(\frac{r}{\alpha}\right) \tag{90}$$

where $\Gamma(\nu)$ represents a gamma function and $K_{\nu}\left(\frac{r}{\alpha}\right)$ is a modified Bessel function of order ν (Tatarskii, 1961).

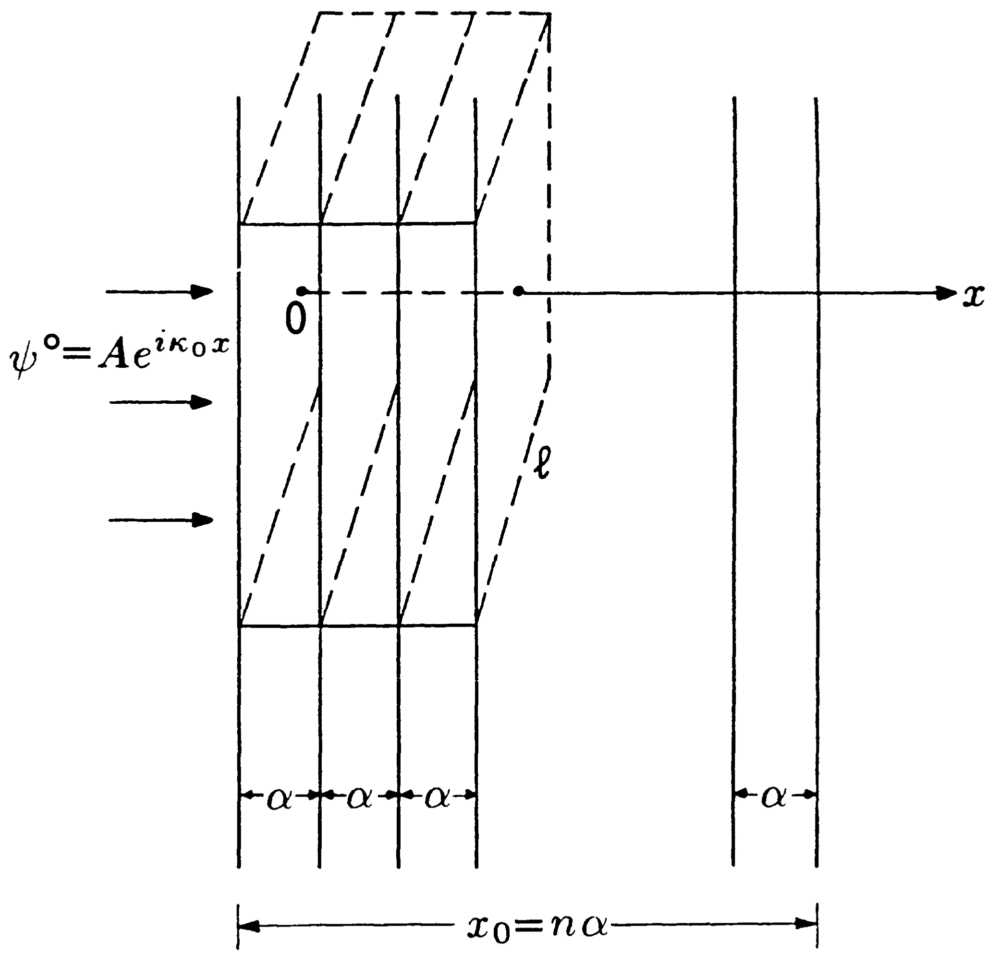


Figure 10.— Schematic of the back halfspace integration approximation: a scalar plane wave, $\psi^0 = Ae^{i\kappa_0 x}$, incident normally on a slab of thickness x_o composed of a random medium with correlation length a. Only the energy backscattered upon the incidence over the slices is considered totally lost. The forward scattered energy is taken into account. [Reproduced with permission from the American Geophysical Union.]

For $\kappa a \ll 1$, low-frequency range, Q^{-1} is proportional to the fourth power of frequency, independent of the form of correlation function. On the other hand, for large scale-inhomogeneity, $\kappa a \gg 1$, the frequency dependence of Q^{-1} varies with the correlation function. For the Von Karman correlation function of order $\nu = 1/3$, Q^{-1} shows a good agreement with results obtained from shear waves by Aki (1980a) for the Kanto region (see fig. 11).

The verification of the expressions used by Wu with observational acoustic-log data from Fenton Hill, New Mexico, also shows better agreement with the exponential and Von Karman correlation functions. However, the unreasonably high value of the velocity fluctuation, $\langle \mu^2 \rangle$, obtained with this method suggests the necessity of introducing some other corrections. The improvement with respect to the preceding methods shows the importance of considering multiple scattering, while the overestimation of $\langle \mu^2 \rangle$ indicates that it is necessary to include higher order forward scattering to evaluate the attenuation more accurately.

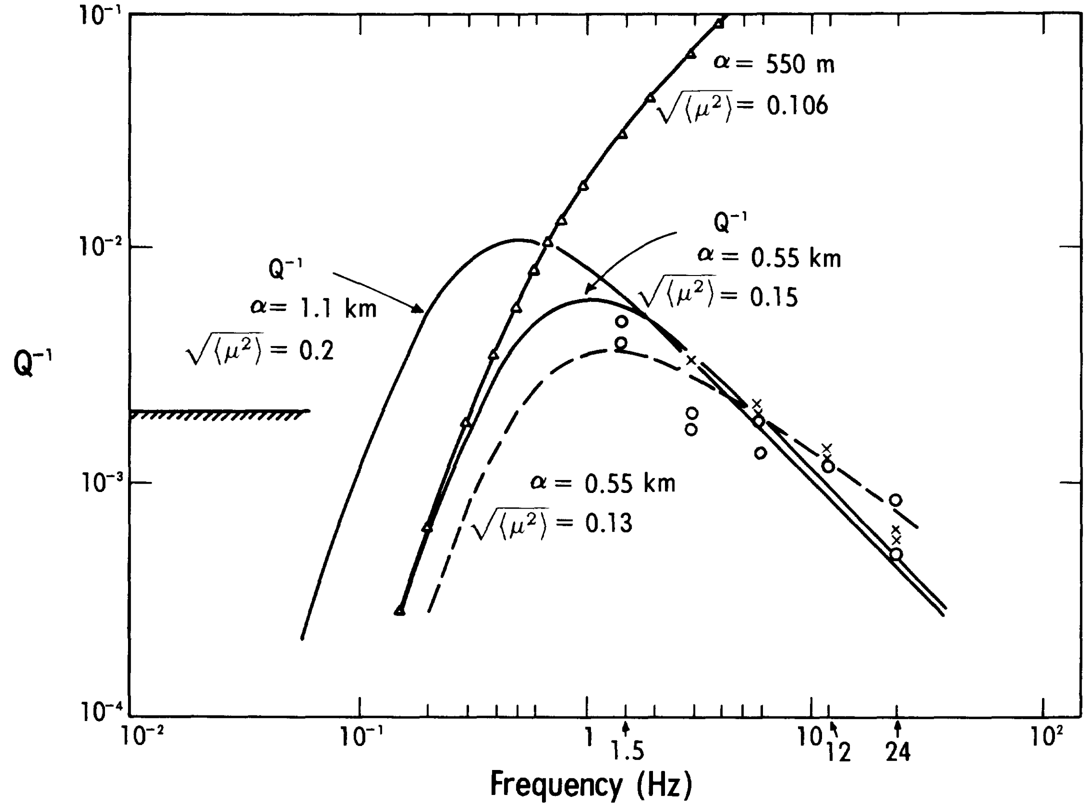


Figure 11.— Comparison of the adjustments for the Q^{-1} vs frequency data (× for area A, ○ for areas B and C) measured in Kanto, Japan (Aki, 1980a) and for the randomization coefficient, shown as triangles, obtained from the "mean field" in a random medium. Three different adjustments for Q^{-1} are shown. The two solid lines correspond to an exponential function $N(r) = e^{-r/\alpha}$, and the broken line is a Von Karman correlation function of order 1/3, $N(r) = \frac{2^{2/3}}{\Gamma(1/3)}(r/\alpha)K_{1/3}(r/\alpha)$. All calculations have been done using the Forward Multiple Scattering Approximation. For the randomization coefficients the adjustment has been given by the "mean field" formalism. The departure of this last adjustment from the measured attenuation of amplitude can be clearly noticed.

In every case chosen values of the correlation length, α, and the r.m.s. of the velocity fluctuation have been used. The region corresponding to results obtained from surface wave analysis (////) is shown on the left. [Reproduced with permission from the American Geophysical Union.]

Traveltime Corrected Mean Wave Formalism

Sato (1982a, b) modified the mean-wave formalism with the introduction of a new statistical averaging method, especially designed to measure the amplitude attenuation of impulsive waves. The goal is to estimate Q^{-1} from the averaged wavelet after correcting the travel time fluctuation due to large-scale heterogeneities. The process is illustrated in figure 12 where it can be easily appreciated that this correction smooths amplitude attenuation.

It is shown (Sato, 1982b) that this correction gives the result on Q^{-1} equivalent to excluding the scattering energy loss inside of a cone around the forward direction. This task is theoretically accomplished in two steps. The first one corrects the traveltime fluctuation caused by the

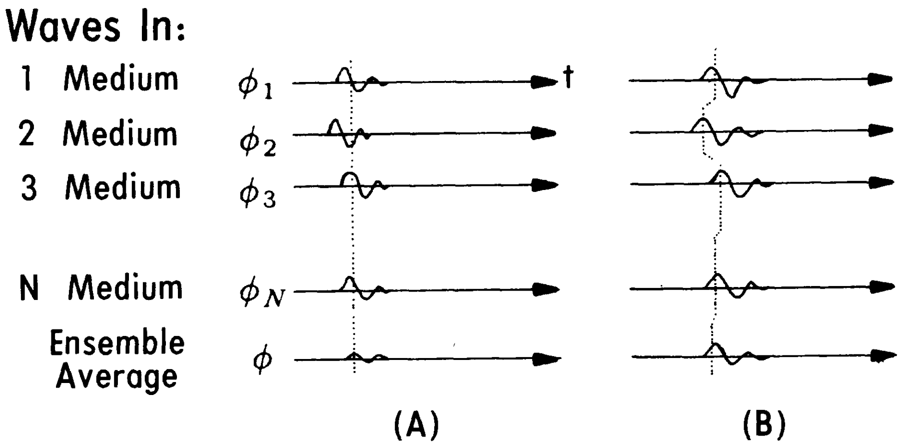


Figure 12.— Schematic illustration of the averaging process for impulsive waves in the high frequency limit ($\kappa a \gg 1$): (A) mean wave formalism, (B) traveltime corrected mean wave formalism. For (A), the mean wave is defined as an ensemble average of wave ϕ_i. If $\kappa a \gg 1$, the traveltime fluctuations become comparable or larger than the λ under consideration. Although the amplitude attenuation would be small, the maximum amplitude of the mean wave will strongly diminish. For (B), before averaging, the waves are shifted in time so their peaks are arranged in line. The attenuation of the ensemble average is more realistic than for (A). [Reproduced with permission from the American Geophysical Union.]

component of the fractional velocity fluctuation longer than twice the wavelength of the wavelet. The second one calculates the mean field from these corrected wavelets. To do the first step, Sato introduced a new wave $\psi(\vec{x},t)$ in which the traveltime fluctuation is corrected:

$$\psi(\vec{x},t) = \psi\left[\vec{x}, t + \delta t^L(\vec{x})\right] \tag{91}$$

$\delta t^L(\vec{x})$ is the traveltime fluctuation caused by the longer wavelength component of the fractional velocity fluctuation ξ^L. It is given by

$$\delta t^L(\vec{x}) = \frac{1}{\mathrm{v}} \int_{\substack{ray\\path}}^{x} \xi^L(\vec{x})\, ds \tag{92}$$

Only the propagation along the x axis is considered. $\psi(\vec{x},t)$ is related to a monochromatic wave $\psi_0(\vec{x})$ of angular frequency $\mathrm{v}\kappa_0$ by the expression:

$$\psi(\vec{x},t) = \psi_0(\vec{x}) e^{-i\mathrm{v}\kappa_0 t} \tag{93}$$

The wave equation for ψ_0 is solved by using the binary interaction approximation on the mean wave formalism and ψ_0 is decomposed as

$$\psi_0 = \langle\psi_0\rangle + \psi_0' \tag{94}$$

where $\langle\psi_0\rangle$ is the mean wave and ψ_0' the fluctuation wave which satisfies

$$\langle\psi_0'\rangle = 0 \tag{95}$$

The wave equation corresponding to $\langle\psi_0\rangle$ is solved for the case

$$\langle\psi_0(\vec{x})\rangle = e^{i\kappa\vec{x}} \tag{96}$$

and Q^{-1} for the mean value $\langle\psi_0\rangle$ is defined as

$$Q^{-1}(\kappa_0) = -\left(\frac{1}{\kappa_0}\right)^2 \mathrm{Im}\,\left(\kappa_0^2 - \kappa^2\right) \tag{97}$$

with $\mathrm{Im}(\kappa_0^2 - \kappa^2)$ being the imaginary part of the dispersion relation for $\langle\psi_0\rangle$. This term is calculated by Sato (1982a) and gives an expression of Q^{-1} as a function of the wave-number vector, the scattering angle θ measured from the x axis and the critical angle $\theta_c \simeq 29°$. Comparison of this result with that obtained from usual statistical theories, shows that this method better accounts for the strong dependence of scattering on the angle measured from the propagation direction. In addition, when the correlation function of the medium is expressed by a Von Karman function such as that defined in expression (90), it is shown that Q^{-1} varies according to the Rayleigh law when $\kappa_0\alpha \ll 1$, and decreases with frequency according to the power law in the high-frequency limit $\kappa_0\alpha \gg 1$. These results remedy the difficulties of the Born single scattering model and classical mean wave formalism, which predicted an increase in Q^{-1} for large frequencies. Figure 13 plots the theoretical Q^{-1} curves for the Von Karman function and the parameters listed in table 5. As can be seen, these curves, especially that marked C, fit fairly well the frequency dependence conjectured by Aki (1980a), on the basis of observed Q^{-1} from various sources, plotted in figure 7. In any case, it should be kept in mind the distinction made earlier between the mean-wave attenuation and the real amplitude attenuation.

TABLE 5.--Parameters chosen by Sato (1982b) to construct the theoretical Q^{-1}-frequency curves for the von Karman autocorrelation function

Set	Order ν	Mean square fractional velocity fluctuation ($\langle\mu^2\rangle \times 10^{-3}$)	Correlation distance α(km)
A	1/2	10	1.6
B	1/2	9.5	3.6
C	1/3	5.0	4.0
D	1/4	2.3	3.5
E	1/10	3.5	6.4

For a general Von Karman autocorrelation function of order ν, the backscattering coefficient g_π^ν is given by Sato (1982b) by the expression:

$$g_\pi^\nu = \frac{8\sqrt{\pi}\,\langle\mu^2\rangle\,\alpha^3\Gamma(\nu+3/2)\kappa_0^4}{\Gamma(\nu)\left(1+4\alpha^2\kappa_0^2\right)^{\nu+3/2}} \tag{98}$$

where $\langle\mu^2\rangle$ represents the mean-square fractional velocity fluctuation. Figure 14 represents the theoretical curves given by equation (98) for the five sets of parameters $(\nu,\langle\mu^2\rangle,\alpha)$ estimated

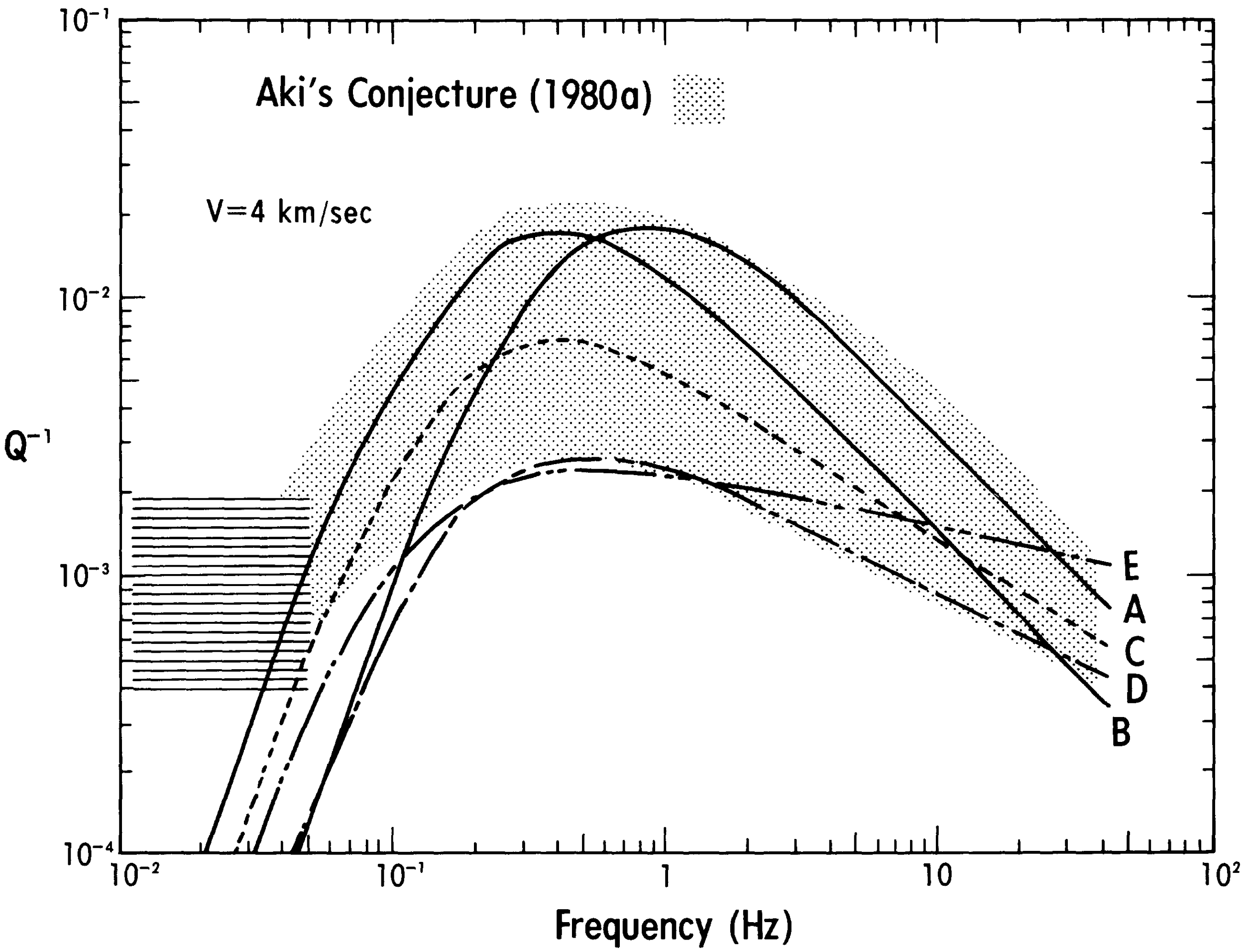


Figure 13.— Sato's (1982a) approach to "Aki's conjecture" which uses the traveltime corrected mean wave formalism. Letters A-E refer to the sets of parameters of the Von Karman auto-correlation function listed in Table 5. [Reproduced with permission from the American Geophysical Union.]

by this method with results obtained from the coda wave analysis by Sato (1978) and Aki (1980b) in the Kanto area. As was seen in Section 2, Aki estimated g_π in the range from 10^{-2} to 4×10^{-2} km^{-1} around 1.5 and 3 Hz, while Sato obtained values of 4×10^{-3} to 4×10^{-2} for frequencies ranging from 1 to 30 Hz.

Figure 14 shows that the value of g_π estimated from coda amplitudes is greater than that calculated from Sato's model. As this model explains the observed Q^{-1} for S waves, this result implies that the observed coda amplitude requires even stronger heterogeneities than those inferred from the attenuation of S waves when all the attenuation is attributed to scattering.

A discrepancy is apparent in figure 15, which shows the results concerning the inhomogeneity of the Earth reported earlier in this section, and those given by this method (see table 6). The broad fluctuation of correlation distance values must be judged bearing in mind that the range of scale length which can be studied depends on the wavelength of seismic waves used, as well as on the size of the region considered (Aki, 1982). Results obtained by the three-dimensional inversion method should be interpreted as a lower bound because this kind of

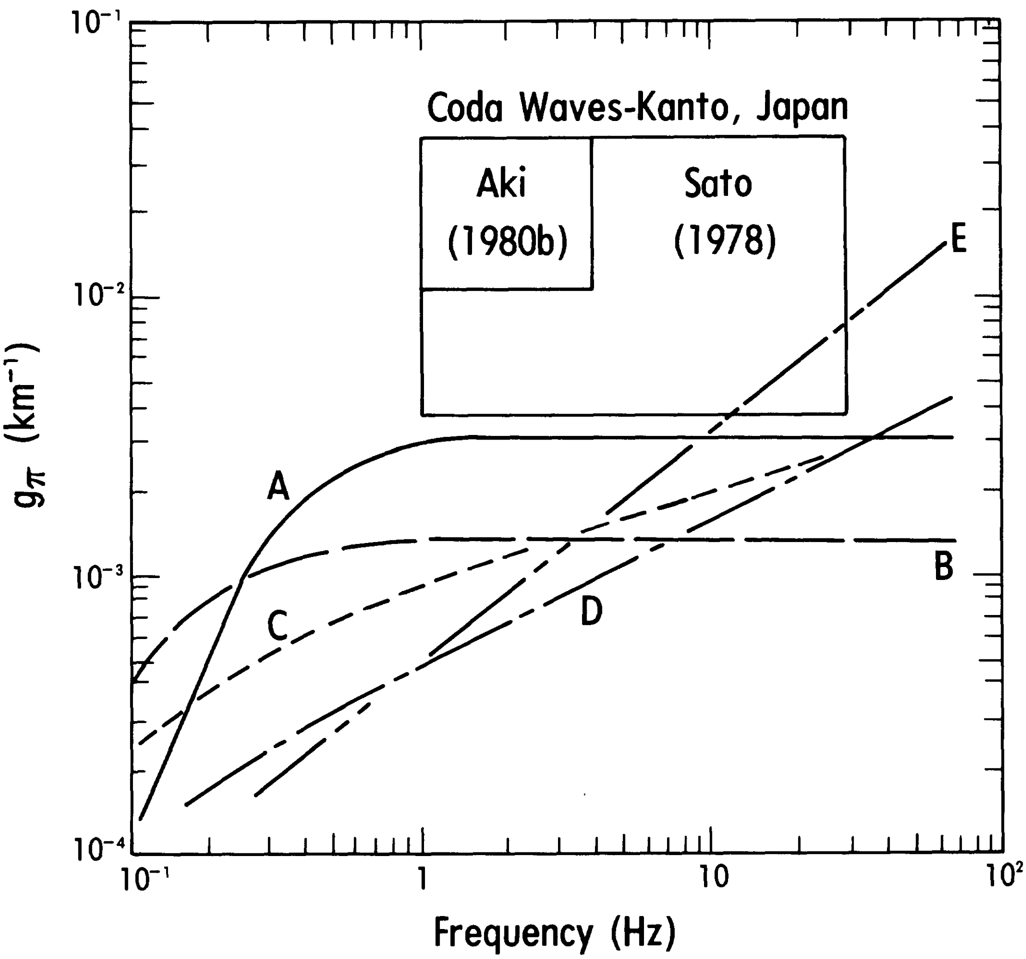


Figure 14.— Curves A-E of the backscattering coefficient, g_π, vs frequency, calculated theoretically by Sato (1982b), which are compared against the results obtained from coda wave analyses in the Kanto district, Japan (Aki, 1980b; Sato, 1978). The observed discrepancy suggested the necessity for improving the models used in order to explain the coda wave excitation. Curves A-E are given by expression (98) in the text and the parameters listed in Table 5. [Reproduced with permission from the American Geophysical Union.]

analysis measures only the longer wavelength component of the velocity fluctuation (longer than the station spacing). The heterogeneity upper bound can be given by estimating the loss by scattering for primary waves propagating in random media, and comparing this result with the observed attenuation. The measurements based on mean wave formalism, such as those made by Beaudet (1970), probably underestimate $\langle \mu^2 \rangle$, while Wu's (1982a) estimate can introduce an overestimation. These differences are due to the kind of waves assumed for calculating the loss of energy of primary waves: the mean wave formalism includes even the waves narrowly focused in the forward direction, which actually modify the phase but not the amplitude, while Wu considered that only backscattered waves are lost by scattering. These arguments suggest that the more accurate estimate of mean-square fractional velocity fluctuations $\langle \mu^2 \rangle$ lies in the region corresponding to values of 10^{-2} to 10^{-3}.

The use of statistical models which consider the Earth's crust as a random medium confirms

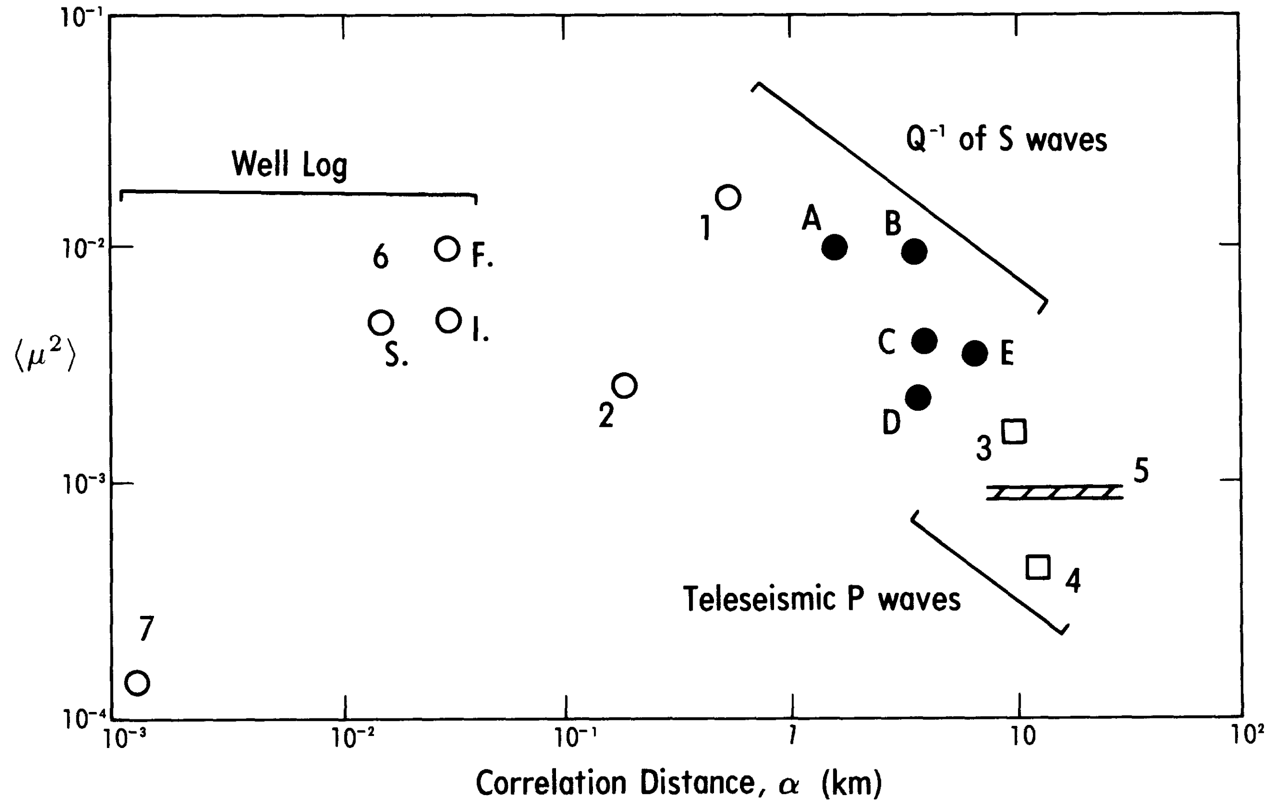


Figure 15.— Plot of mean-square fractional velocity fluctuation $\langle \mu^2 \rangle$, versus correlation distance α. (For symbol identification refer to Table 6.) [Reproduced with permission from the American Geophysical Union.]

the results given by deterministic methods, which prove the existence of strong small-scale heterogeneities. At the same time, the disagreement among the results shows the necessity of improving the models used to obtain them. The next section will be devoted to this subject.

SECTION 4: ONGOING RESEARCH

Figures 13, 14, and 15 of the previous section sum up the main results obtained from the study of the three more striking effects of heterogeneity on seismic waves: formation of codas, attenuation of S and coda waves, and finally, amplitude and phase fluctuations of teleseismic P waves through a large seismic array. The observations play an important role in the development of the theory and reveal the necessity of better models to account for these phenomena. An improved understanding of the scattering process is a particularly pressing need. The theory should include the following more realistic features:

1. Nonuniform distribution of scatterers.
2. Nonspherical sources of radiation.
3. Anisotropic scattering of vector waves due to three independent fluctuations (λ, μ, and ρ).
4. Conversion between P and S waves.
5. Multiple scattering.

TABLE 6.--Mean-square fractional-velocity fluctuations and correlation distances plotted on figure 15

Symbol	Method	Data	Reference
1	Forward multiple-scattering approximation.	S waves in Kanto district Japan.	Wu (1982a).
2	Mean-wave formalism--------	Explosion data in Nevada.	Beaudet (1970).
3	Teleseismic P-wave analysis by the Chernov's theory.	LASA in Montana------	Aki (1973).
4	Do--------------------	----do---------------	Capon (1974).
5	Teleseismic P-wave analysis by the three-dimensional method.	----do---------------	Aki and others (1976).
6	Correlation function for experimental well-log data.	Kanto district, Japan I: Iwatsuki S: Shimosha F: Fuchu	Suzuki and others (1981).
7	Do--------------------	Fenton Hill, N. Mex.---	Wu (1982a).
A-E	Traveltime corrected mean wave formalism.	Single station and surface waves data used to establish Aki's conjecture and plotted on figure 7.	Sato (1982b).

The resulting model must be able to explain the frequency dependence of Q^{-1} and the formation of coda waves, as well as to resolve the disagreements already reviewed between observational results and predictions given by simplified models. This model is far from being achieved, but considerable work is being done to solve partial problems in such a way that a complete solution is coming closer. In this section, recent studies by Sato (1982c, 1984) and Wu and Aki (1985a, b) will be examined.

Sato (1982c) extended his single isotropic scattering model (SIS) to the case of nonisotropic-scattering and nonspherical-source radiation. As before, he assumed a random three-dimensional distribution of scatterers and analyzed the energy density of coda waves. Taking the energy density distribution of the SIS model as a reference, Sato showed that the anisotropy of scattering is more effective than the asymmetry of source radiation to modify coda-wave excitation, and that the influence of both anisotropy of scattering and asymmetry of source radiation is more noticeable on the early part of the coda. In addition, the nonspherical radiation of dislocation affects the P coda more strongly than the S coda. In a more recent paper, Sato (1984) extended this study, pursuing two main goals: to explain the frequency dependence of Q^{-1}, and to construct the envelopes of 3-component velocity seismograms produced by nonspherical radiation and nonisotropic scattering. This paper also extends the traveltime correction used previously in the mean-wave formalism (Sato, 1982a) to vector waves singly-scattering under the Born approximation. Sato began by reviewing the scattering process due to weak inhomogeneities contained inside a small volume of finite extent. The density and Lamé parameters are expressed by

$$\begin{aligned} \rho(\vec{x}) &= \rho_0 + \delta\rho(\vec{x}) \\ \lambda(\vec{x}) &= \lambda_0 + \delta\lambda(\vec{x}) \\ \mu(\vec{x}) &= \mu_0 + \delta\mu(\vec{x}) \end{aligned} \tag{99}$$

The subscript "o" in this case stands for the values of the unperturbed medium and $\vec{x}$ is the position vector inside the heterogeneous volume. However, Sato represented the inhomogeneities in terms of ($\delta\rho$, $\delta\alpha$, $\delta\beta$) instead of ($\delta\rho$, $\delta\lambda$, $\delta\mu$); α and β being P and S velocity, respectively. It is assumed that traveltime fluctuations are produced by smoothly varying velocity structure, and scattering amplitudes are due to rapidly varying elastic structure. Spherical coordinates, (r,ψ,ξ), are taken in such a way that ψ expresses the angle between the propagation direction of the scattered wave and the x_3 axis along which the incident wave travels; r represents the travel distance of the scattered wave; and ξ measures the angle between the propagation direction of the scattered wave and the x_1 axis. According to this system, a scattered P wave, at far field, has the r component only, and a scattered S wave has the ψ and ξ components only. Sato succeeded in expressing the scattered displacement $\vec{u}(\vec{x},t)$ into two parts corresponding to the P and S incident waves, respectively. For P waves, $\vec{u}(\vec{x},t)$ depends on the unit amplitude incident P wave and on the scattering amplitude functions F_r^{PP}, F_ψ^{PS}, and F_ξ^{PS} representing the $P \rightarrow P$ and $P \rightarrow S$ conversions. In the case of an incident S wave polarized to the x_1 axis, the result depends on the unit amplitude incident S wave and on the functions F_r^{SP}, F_ψ^{SS} and F_ξ^{SS}, which represent the scattered waves due to $S \rightarrow P$ and $S \rightarrow S$ conversions. By substituting $\vec{u}(\vec{x},t)$ back into the wave equation and using the far-field function, Sato obtained each of the preceding scattering amplitudes F_i^{AB}. They are given as a function of the fractional fluctuations of the medium parameters in the volume L^3, which can be expressed as

$$\frac{\delta\hat{\rho}}{\rho_0} \qquad \frac{\delta\hat{\alpha}}{\alpha_0} \qquad \frac{\delta\hat{\beta}}{\beta_0} \tag{100}$$

where the hat ($\hat{}$) means the Fourier transform in wavenumber space.

F_ξ^{PS}, which represents the P to S conversion in the ξ direction, is equal to zero according to the chosen geometry. Figure 16 plots the radiation patterns of Rayleigh scattering corresponding to the other five components. The following aspects can be outlined again:

1. $P \rightarrow P$ scattering amplitude due to velocity fluctuations is simply isotropic. Fluctuations in P-wave velocity contribute only to $P \rightarrow P$ scattering.
2. The contribution of density fluctuations is zero in the forward direction ($\psi = 0$) and maximum in the backward direction. Results 1 and 2 were predicted in Chernov's theory.
3. As proposed to explain local earthquake coda, $S \rightarrow S$ scattering is generally stronger than $P \rightarrow P$ scattering and shows a more complicated pattern.

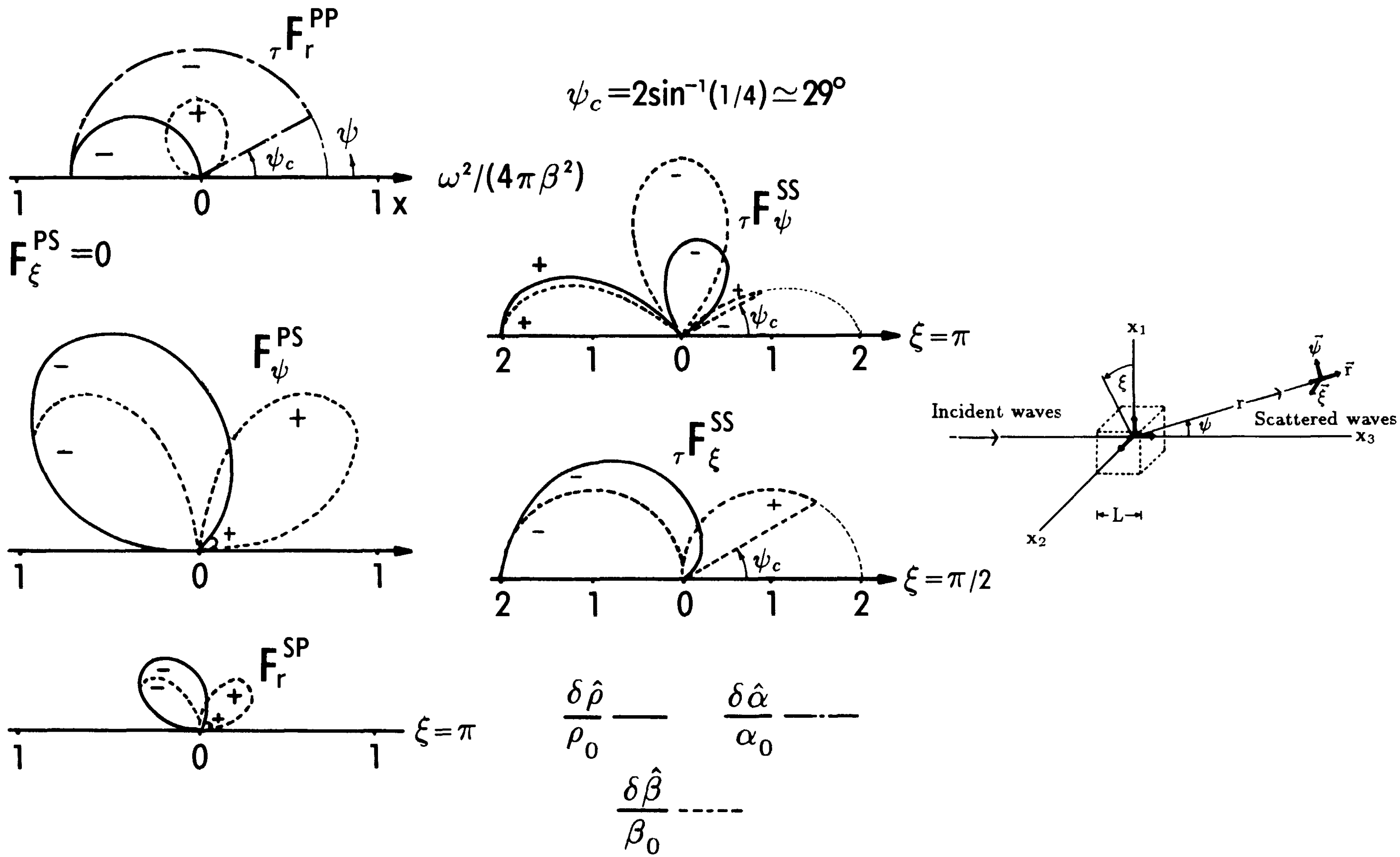


Figure 16.— ψ dependence of coefficients of $\delta\hat{\rho}/\rho_0$, $\delta\hat{\alpha}/\alpha_0$, $\delta\hat{\beta}/\beta_0$ in scattering amplitude for the ordinary and traveltime corrected Born approximation. $\delta\hat{\rho}$, $\delta\hat{\alpha}$, $\delta\hat{\beta}$ are the Fourier transforms in wavenumber space of the perturbation $\delta\rho$ of density and the fluctuations $\delta\alpha$ and $\delta\beta$ of P- and S- wave velocity, respectively. F_i^{AB} represents the i component of he scattered waves with $A \rightarrow B$ conversion. For $P \rightarrow P$ and $S \rightarrow S$ amplitudes, a subindex τ is given to indicate that in these cases the traveltime corrected Born approximation shares the results of the ordinary Born approximation for $\psi > \psi_c$. ψ_c is the critical angle which is used for the traveltime correction. The geometry of the problem is shown in the insert sketch. Signs + and - explain the $\delta\hat{\rho}$, $\delta\hat{\alpha}$, and $\delta\hat{\beta}$ variations. Forward direction is towards the right. (See text for further explanations.) [Reproduced with permission from the American Geophysical Union.]

4. There is no contribution of $P \rightarrow S$ and $S \rightarrow P$ conversion scattering to the forward and the backward directions. This result has also been used in the previous studies.
5. As Sato (1982c) had already indicated, conversion scattering does not excite coda waves at large lapse-time. Results 1–5 allow interpretation of the ψ dependence of figure 16 as that of Rayleigh scattering.

Sato (1984) also applied the correction of traveltime fluctuations to expressions of F_i^{AB} in order that they can be used to calculate Q^{-1} for P and S waves. $S \rightarrow S$ scattering is dominant in Q_β^{-1}, but $P \rightarrow S$ conversion scattering cannot be neglected in Q_α^{-1} estimation. As has been pointed out, this correction extends the method previously applied to scalar waves (Sato, 1982a, b) to vector waves. The dependences of the $\delta\hat{\alpha}$ and $\delta\hat{\beta}$ coefficients on ψ, after being traveltime corrected, are shown by bolder lines in figure 16. For $\psi > \psi_c$ the ordinary and the traveltime corrected Born approximations agree. For $\psi < \psi_c$, the second one is much smaller. ψ_c is the critical angle introduced upon explaining the traveltime correction for mean wave formalism. In this new correction, the scattering due to velocity fluctuations has been excluded within the narrow angle ψ_c in the forward direction, as in the case of scalar waves (Sato, 1982a).

By using an empirical linear correlation between P and S waves and Birch's (1961) law, Sato reduces the number of fluctuations from three, (ρ,α,β), to one, $\xi(\vec{x})$, which is used to characterize the medium. The procedure is similar to that accepted previously (Batchelor, 1953; Sato, 1982a) and the autocorrelation function chosen to describe the fluctuation is the exponential one. This method gives a $Q_\alpha^{-1}/Q_\beta^{-1}$ ratio of about 0.6 for Rayleigh scattering and about 2.41 for the high frequency limit. This last result, which affirms that attenuation is greater for P waves than for S waves, is somewhat surprising. In addition, 2.41 seems to be too high a value because experimental measurements show values for $Q_\alpha^{-1}/Q_\beta^{-1}$ ranging between 0.62 and 1.85 (Bakun and others, 1976; Frankel, 1982; Modiano and Hatzfeld, 1982; Hoan-Trong, 1983). The backward scattering coefficient of S waves, g_π^{SS}, is given by the expression

$$g_\pi^{SS} \simeq 1.62 \left(\frac{\langle \mu^2 \rangle}{\alpha} \right) \tag{101}$$

where $\langle \mu^2 \rangle$ represents the mean square of ξ and α is the correlation distance. Figure 17 represents Q^{-1} versus frequency from observational results obtained in different parts of the world. A theoretical curve using the values of $\langle \mu^2 \rangle = 0.01$ and $\alpha = 2\,\text{km}$ is shown and also the Q^{-1} values for L_g waves and for surface waves are plotted for comparison purposes.

The same procedure has been followed with g_π^{SS} values displayed in figure 18. Both drawings show an improvement over previous adjustments.

The last part of Sato's study is concerned with obtaining the envelopes of short period velocity amplitudes as shown in figure 19. These envelopes are obtained by superposing waves scattered once and by assuming a primary wave source of a point shear dislocation. Results of previous studies (Sato, 1977b, 1982c) are also used. The envelopes correspond to an earthquake of magnitude 3, located 30 km from recorders in the five directions displayed on the same figure. The strong dependence of both P and S codas on the focal mechanism can be clearly observed. P-coda excitation, even in the nodal direction of P wave radiation, can be observed. Something similar occurs for the radial component of S codas. The results of figure 19 suggest that magnitudes of local earthquakes calculated from the maximum amplitudes of vertical components

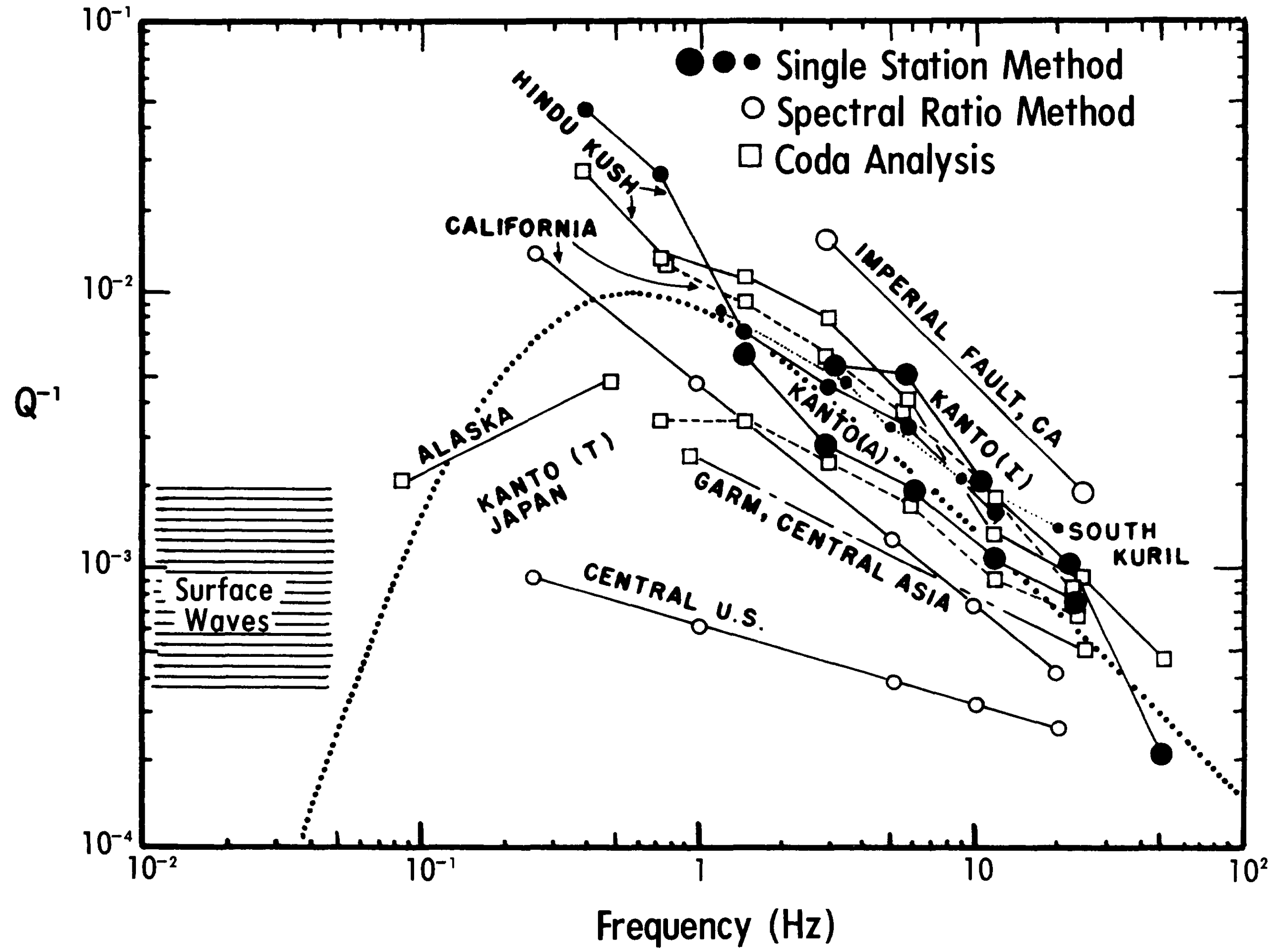


Figure 17.— Observed relation of Q^{-1} of S and coda waves vs frequency and the curve theoretically predicted by Sato (1984), dotted line, by assuming $\langle\mu^2\rangle = 0.01$ and $a = 2\,\mathrm{km}$. This plot can be compared to figures 6, 7, 11 and 13 in order to obtain an overview of the successive attempts which have been made to explain observational results. Kanto (A), (I), and (T) correspond to Aki (1980a), Sato and Matsumura (1980b), and Aki and Chouet (1975) respectively. The other references are found in Table 3A and B and in Sato (1984). [Reproduced with permission from the American Geophysical Union.]

can reflect not only the source effect but also the importance of heterogeneities on the path travelled by the waves.

Sato's model can be modified, for instance, by substituting the traveltime correction for other smoother methods, or by choosing a Von Karman function instead of the exponential one. The method of characterizing the inhomogeneities by only one random fluctuation $\xi(\vec{x})$ seems too restrictive and perhaps could be improved. In addition, velocity envelopes obtained by this method must be compared with observational data. This test, which can be decisive, is presently (1986) being undertaken by Sato.

The study of the scattered field by an elastic inclusion has been performed recently by Wu and Aki (1985a) for the case of weak scattering also. These authors analyzed Rayleigh and high-

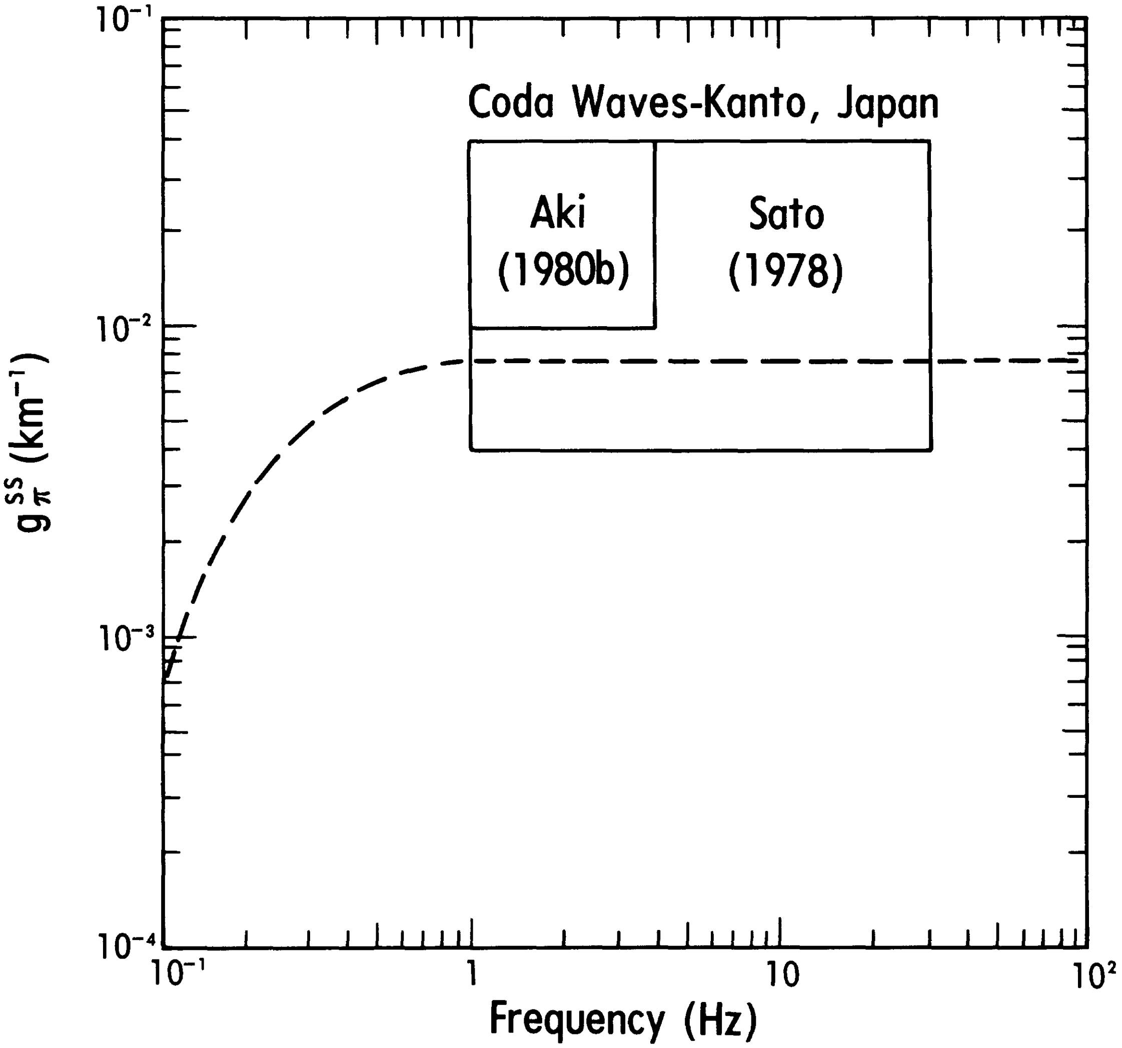


Figure 18.— Application to the estimation of the backscattering coefficient g_π of Sato's theoretical result shown in figure 17. The parameters are: $\langle \mu^2 \rangle = 0.01$, $a = 2\,\text{km}$, and $\beta_0 = 4\,\text{km/sec}$. Comparison to figure 14 shows the improvement obtained. g_π is identified as g_π^{SS} in order to stress the dominance of $S \rightarrow S$ scattering. [Reproduced with permission from the American Geophysical Union.]

frequency scattering of P and S waves by using the Born approximation and the equivalent-source methods. They supposed a general elastic heterogeneity with slightly different density and elastic constants from those values of the surrounding medium. The subscript "0" means the values of the homogeneous medium and the triangular brackets stand for the average excesses. Wu and Aki (1985a) represented the scattered field by a radiation field of the equivalent source due to the presence of the inclusion. In this way, the scattering problem is turned into a radiation problem. For the Rayleigh case, the total equivalent source can be regarded as a point source whose size depends on the total parameter excesses $\delta\rho$, $\delta\lambda$, $\delta\mu$ of the inclusion. For P-wave incidence along $x_1 = x$, $\langle \delta\rho \rangle$ gives a single force in the incident direction while $\langle \delta\lambda \rangle$ and $\langle \delta\mu \rangle$

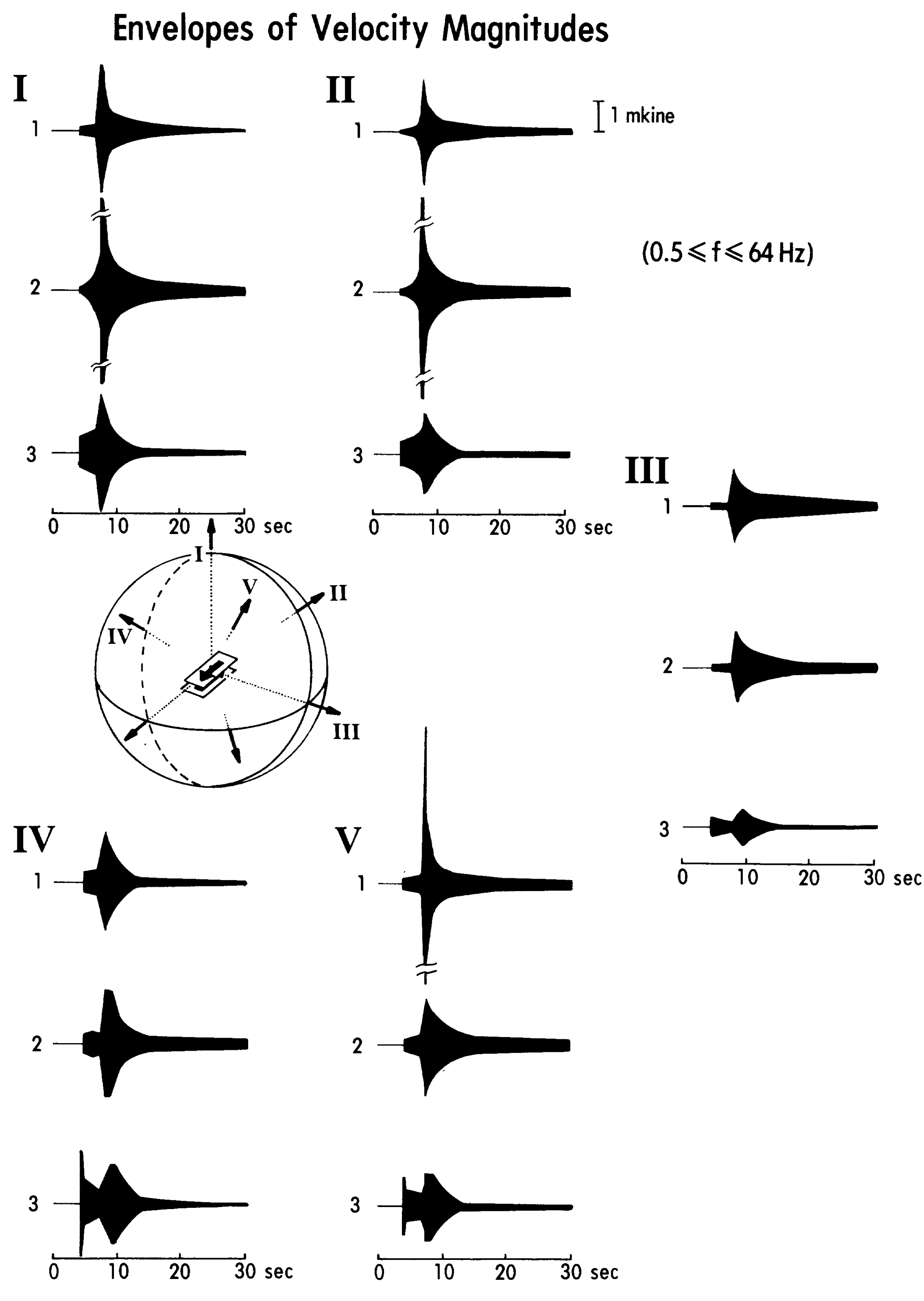


Figure 19.— Theoretical envelopes of three-component velocity amplitudes of an earthquake with an $M_L = 3$ recorded at a distance of 30 km in the displayed directions I to V. 1 and 2 correspond to the horizontal components, and 3 to the vertical component. The vertical and the horizontal scales are linear. These examples show that envelopes of seismograms strongly depend on the focal mechanism and suggest that if the magnitude of local earthquakes is measured from the maximum amplitude of the vertical component, it reflects not only the intensity of source radiation but also the strength of inhomogeneities in the Earth. [Reproduced with permission from the American Geophysical Union.]

produce a moment tensor force as:

$$M = -\frac{i\omega V}{\alpha_0}\begin{pmatrix} \langle\delta\lambda\rangle + 2\langle\delta\mu\rangle & 0 & 0 \\ 0 & \langle\delta\lambda\rangle & 0 \\ 0 & 0 & \langle\delta\lambda\rangle \end{pmatrix} e^{-i\omega t} \tag{102}$$

where V is the volume of the inclusion and α_0 the P-wave velocity.

Expression 102 shows that $V\langle\delta\lambda\rangle$ gives an isotropic explosion-type source, while $V\langle\delta\mu\rangle$ produces an in-line force couple (fig. 20). For an incident S wave with velocity β_0, $\delta\rho$ produces a single force in the direction of particle motion and $\langle\delta\mu\rangle$ introduces a double-couple force in the polarization plane. The moment tensor is now:

$$M = -\frac{i\omega V}{\beta_0}\begin{pmatrix} 0 & \langle\delta\mu\rangle & 0 \\ \langle\delta\mu\rangle & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{-i\omega t} \tag{103}$$

These equivalent forces and their radiation patterns are displayed in figure 21. The scattering patterns vary with the combinations of $\langle\delta\rho\rangle$, $\langle\delta\mu\rangle$, and $\langle\delta\lambda\rangle$. When $\frac{\langle\delta\rho\rangle}{\rho_0} = \frac{\langle\delta\lambda\rangle}{\lambda_0} = \frac{\langle\delta\mu\rangle}{\mu_0}$, there is no velocity contrast between the inclusion and the medium, and only a main lobe on the backward direction is observed. Wu and Aki called this case an "impedance-type" scattering. If $\frac{\langle\delta\lambda\rangle}{\lambda_0} = \frac{\langle\delta\mu\rangle}{\mu_0} = -\frac{\langle\delta\rho\rangle}{\rho_0}$, there is no impedance contrast and there will be only a main lobe in the forward direction. This case will be called a "velocity type" scattering.

These results are valid both for P and S incident waves and confirm the conclusions obtained by Sato (1984). In a similar way, they agree in showing that for both P wave and S wave incidences the converted waves have only side lobes, while the common-mode scattered waves always have the main lobe along the incident direction. The predominance of scattered S waves over P waves for $\langle\delta\lambda\rangle \simeq \langle\delta\mu\rangle$, is also confirmed. (See figs. 22 and 23.)

Wu and Aki obtained these results without assuming any restrictions about the shape and parameter distribution of the inclusion. For a sphere of radius "a" and for low frequencies, the frequency dependence of the amplitude of scattered waves have the known form $\omega^2 a^3$ similar to acoustic scattering.

It can be easily proved that, in general, the scattered field can be decomposed into an "impedance-type" and a "velocity-type".

Beyond Rayleigh scattering, the problem becomes much more complicated because in this case the phase differences of the incident field and of the scattered field along the inclusion cannot be neglected. The Born approximation holds if the total scattered field is still much weaker than the incident one. Assuming this case, Wu and Aki proved that the scattered far field is the product of the elastic-wave Rayleigh scattering and a "volume factor" which expresses the scalar wave scattering pattern for the finite body. This result is obtained by assuming the perturbation parameters $\delta\rho$, $\delta\mu$, and $\delta\lambda$ to have the same spatial distribution within the inclusion.

The "volume factor" is a function of $G(\vec{\xi})$, the normalized parameter variation which has been defined in such a way that:

$$\begin{aligned} \delta\rho(\vec{\xi}) &= \langle\delta\rho\rangle\, G(\vec{\xi}) \\ \delta\lambda(\vec{\xi}) &= \langle\delta\lambda\rangle\, G(\vec{\xi}) \\ \delta\mu(\vec{\xi}) &= \langle\delta\mu\rangle\, G(\vec{\xi}) \end{aligned} \tag{104}$$

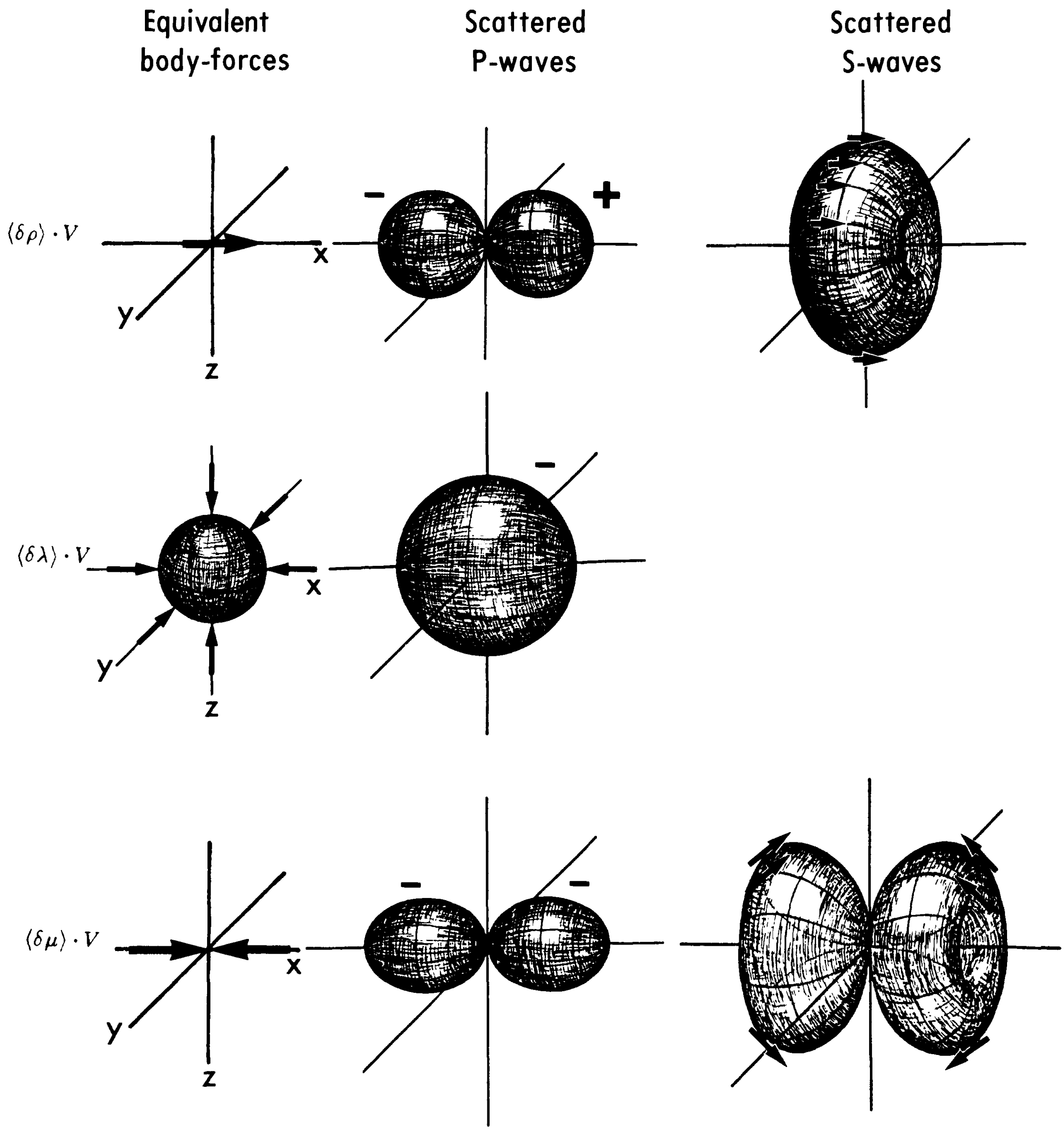


Figure 20.— Rayleigh scattering pattern for different equivalent forces; P-wave case. Wu and Aki (1985a) represented the scattered field by a radiation field of the equivalent source due to the presence of an inclusion. The results for P-wave incidence along the x-axis are plotted in this figure where $\langle\delta\rho\rangle$, $\langle\delta\lambda\rangle$, and $\langle\delta\mu\rangle$ are the average values of density and excesses of the Lamé parameters respectively, due to the inclusion of volume V. Density excess, $\langle\delta\rho\rangle\cdot V$, gives a single force in the incident direction (top). Excesses of the Lamé parameters give an isotropic explosion-type source (middle) and an in-line couple force (bottom). [From Geophysics, vol. 50; reproduced with permission from the Society of Exploration Geophysicists.]

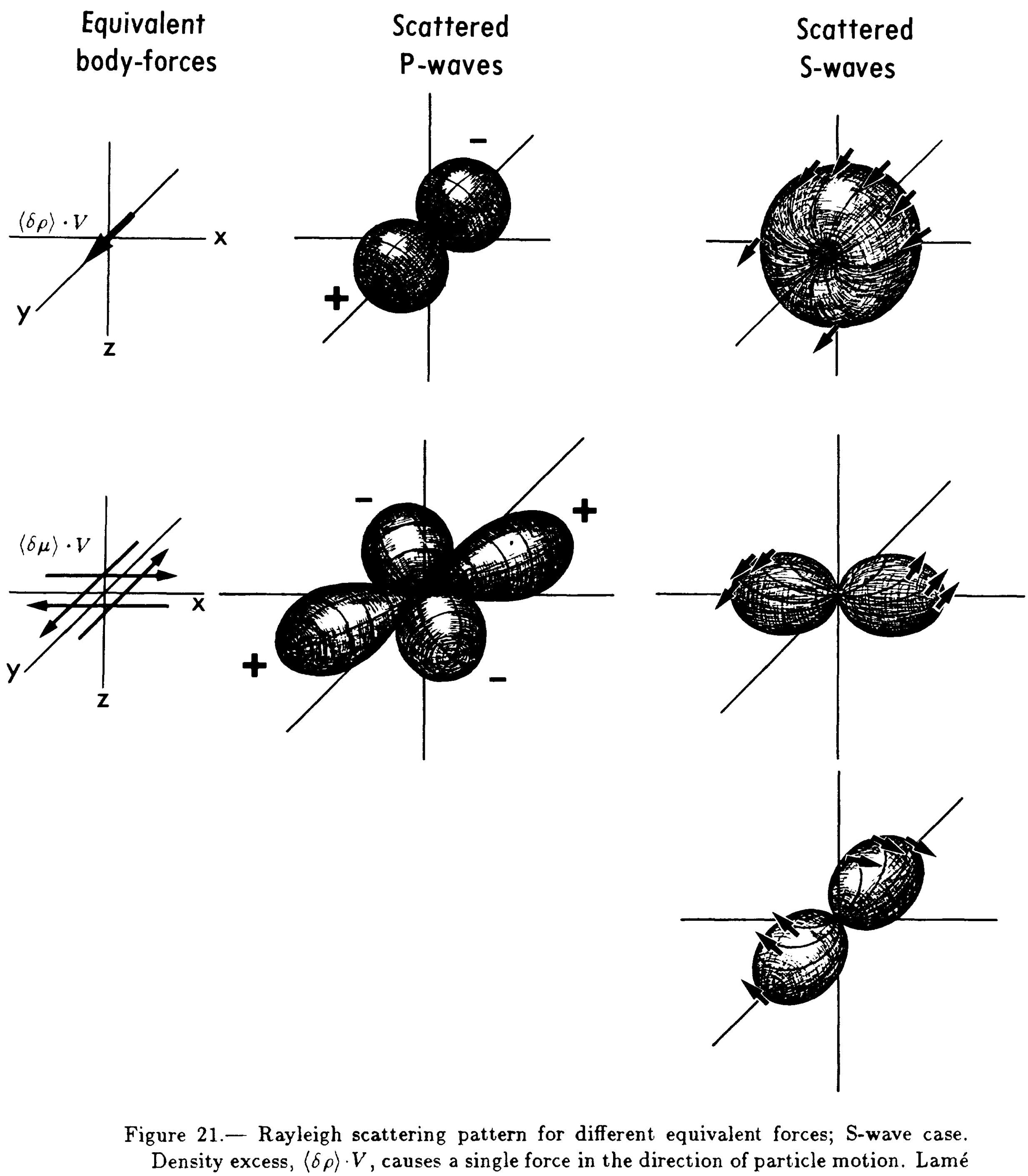


Figure 21.— Rayleigh scattering pattern for different equivalent forces; S-wave case. Density excess, $\langle \delta\rho \rangle \cdot V$, causes a single force in the direction of particle motion. Lamé parameter excess, $\langle \delta\mu \rangle \cdot V$, introduces a double couple force in the polarization plane. Its scattering pattern has been decomposed into two parts, each of which corresponds to a single couple. [From Geophysics, vol. 50; reproduced with permission from the Society of Exploration Geophysicists.]

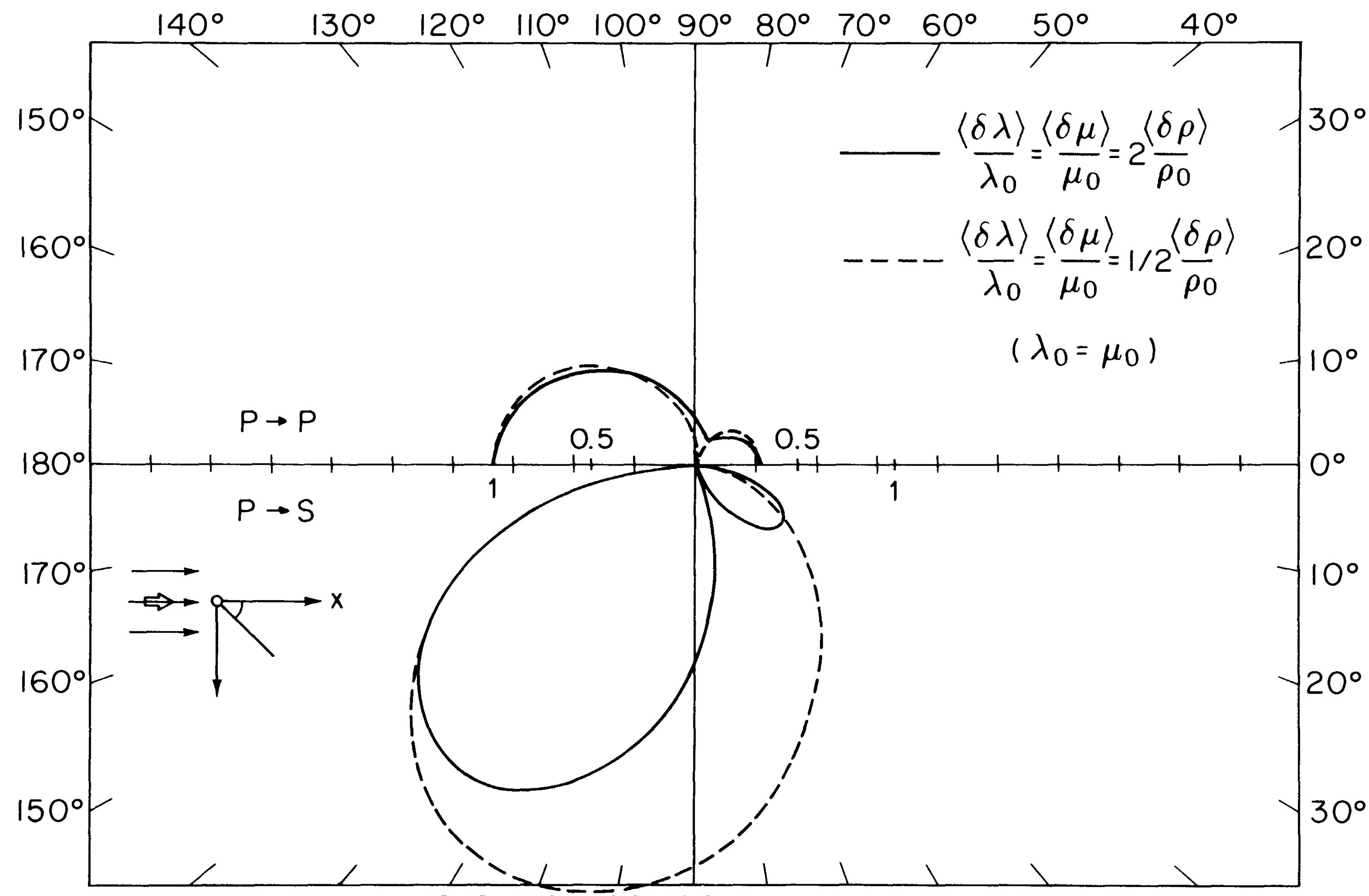


Figure 22.— Rayleigh-scattering patterns for plane P-wave incidence. The upper half corresponds to $P \to P$ scattering and the lower half corresponds to $P \to S$ scattering. All the patterns are symmetric about the x-axis. The common-mode scattered wave $P \to P$ has its main lobe along the incident direction and the converted mode $P \to S$ has only a side lobe. Because $\langle\delta\lambda\rangle$, $\langle\delta\mu\rangle$, and $\langle\delta\rho\rangle$ have the same sign, that is the inclusion is harder and heavier (or softer and lighter) the $P \to P$ scattering has its maximum in the backscattering direction. [From Geophysics, vol. 50; reproduced with permission from the Society of Exploration Geophysicists.]

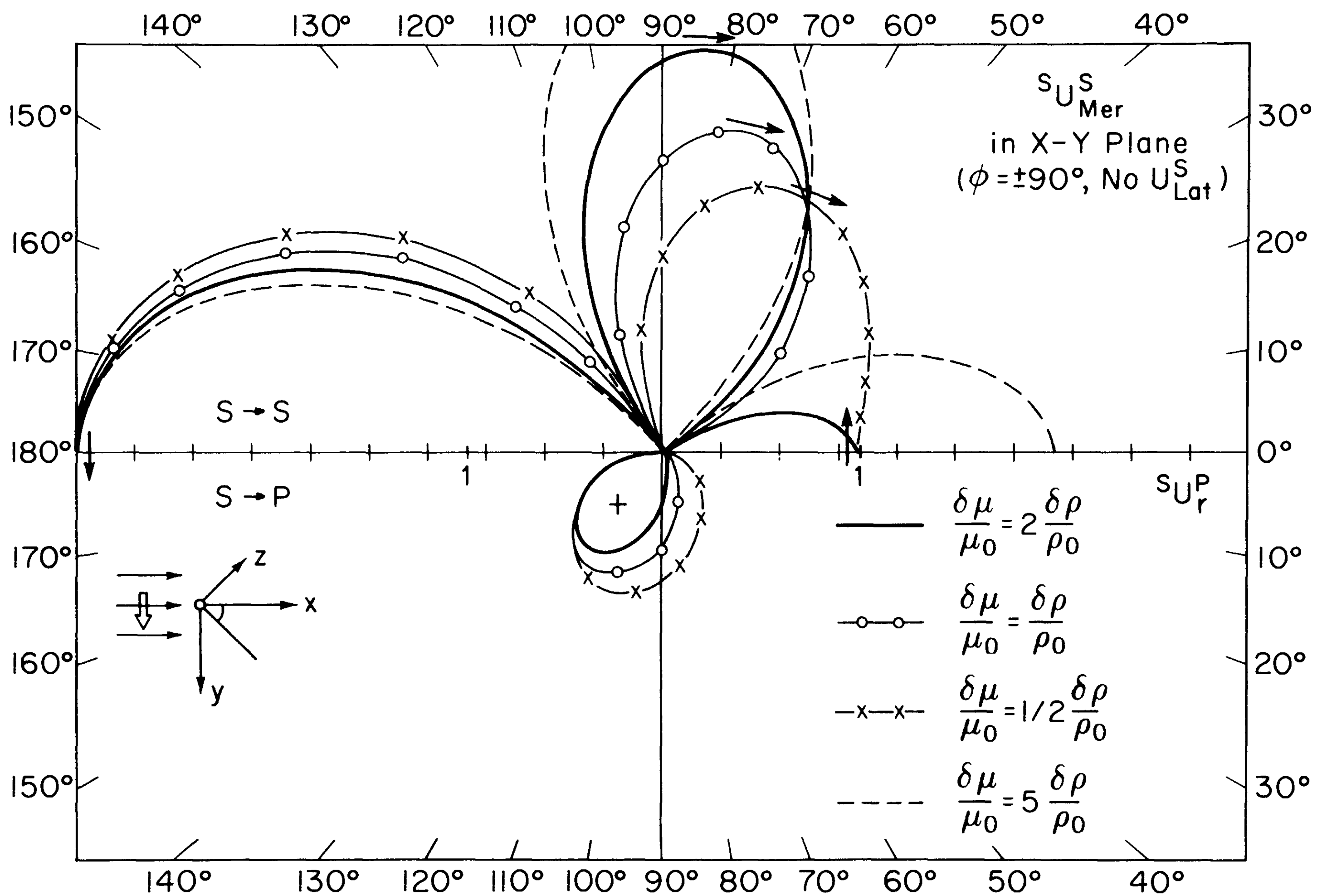


Figure 23.— Same as figure 22 for plane S-wave incidence and x-y plane scattering. The upper and lower halves represent the $S \rightarrow S$ scattering and the $S \rightarrow P$ scattering, respectively. In the x-y plane only the meridian component $^{S}U^{S}_{mer}$ can exist, since in this representation the direction of the particle motion of incident waves (y-axis) is taken as the polar axis, so $\phi = \pm 90°$.

As in the previous case, the common mode of $S \rightarrow S$ has the main lobes in the incidence direction. These lobes are mainly backscattered because the density $\delta\rho$, and the shear modulus $\delta\mu$ variations always have the same sign. When $\delta\mu/\mu_0 = \delta\rho/\rho_0$ there is no scattering in the forward direction. [From Geophysics, vol. 50; reproduced with permission from the Society of Exploration Geophysicists.]

and

$$\frac{1}{V}\int_V G(\vec{\xi})\,dV(\vec{\xi}) = 1 \tag{105}$$

If $G(\vec{\xi})$ is known, the "volume factor" can be calculated either numerically or analytically. Wu and Aki solved this problem for a homogeneous spherical inclusion in which $G(\vec{\xi})$ is assumed to be spherically symmetrical. They found that for $P \rightarrow P$ or $S \rightarrow S$ scattering the "volume factor" always has the main lobe in the incidence direction. On the other hand, for converted waves, the scattering pattern becomes more complex and there is a multi-directional radiation of converted energy. In any case, for very high frequencies, forward scattering and scattering without wave conversion become dominant. As an example, figure 24 sums up the main results for the $P \rightarrow P$ scattering case.

These results generalize the solution to this problem given previously by other authors (Miles, 1960; Yamakawa, 1962).

In a more recent paper, Wu and Aki (1985b) considered the problem of the full elastic wave theory of scattering in random media. Wu and Aki followed Haddon's (1978) approach and applied body-force equivalents and the Born approximation. They assumed a random medium embedded in a homogeneous elastic medium with density ρ_0 and elastic constants λ_0 and μ_0. The random-medium parameters were given as in expression (99), but in this case the vector position within the volume is represented by $\vec{\xi}$ and the average values $\langle \rho(\vec{\xi}) \rangle$, $\langle \lambda(\vec{\xi}) \rangle$, and $\langle \mu(\vec{\xi}) \rangle$ satisfy the following relations:

$$\begin{aligned} \rho_0 &= \langle \rho(\vec{\xi}) \rangle \\ \lambda_0 &= \langle \lambda(\vec{\xi}) \rangle \\ \mu_0 &= \langle \mu(\vec{\xi}) \rangle \end{aligned} \tag{106}$$

The inclusion of a random medium is assumed to be weakly inhomogeneous, that is,

$$\delta\rho \ll \rho_0, \quad \delta\lambda \ll \lambda_0, \quad \delta\mu \ll \mu_0 \tag{107}$$

and statistically homogeneous and isotropic. In addition, $\delta\rho(\vec{\xi})$, $\delta\lambda(\vec{\xi})$, and $\delta\mu(\vec{\xi})$ have autocorrelation functions of exponential form. Considering one sample from an ensemble of random media, Wu and Aki applied the previous method to obtain the scattered field corresponding to P and S incident waves within this particular inhomogeneous medium. The result is also a random wave field which must be considered as a part of the general scattered field. After laborious mathematics, they obtained the mean square amplitudes of the scattered field and the directional scattering coefficients $g(\theta)$ for $P \rightarrow P$, $P \rightarrow S$, $S \rightarrow P$, and $S \rightarrow S$ scattering. Paralleling their previous study, they found that the perturbations can be decomposed into an impedance term and a velocity term with radiation patterns similar to those described before. They proved that when $\kappa a \gg 1$ the scalar wave theory of velocity perturbations is valid for forward scattering of elastic waves. This result explains the applicability of Chernov's theory to the analyses of phase and amplitude fluctuations across large seismic arrays. On the other hand the study of backscattering requires the use of the theory of elastic-wave scattering. Therefore, the coda wave analyses must be improved by using vector wave theory.

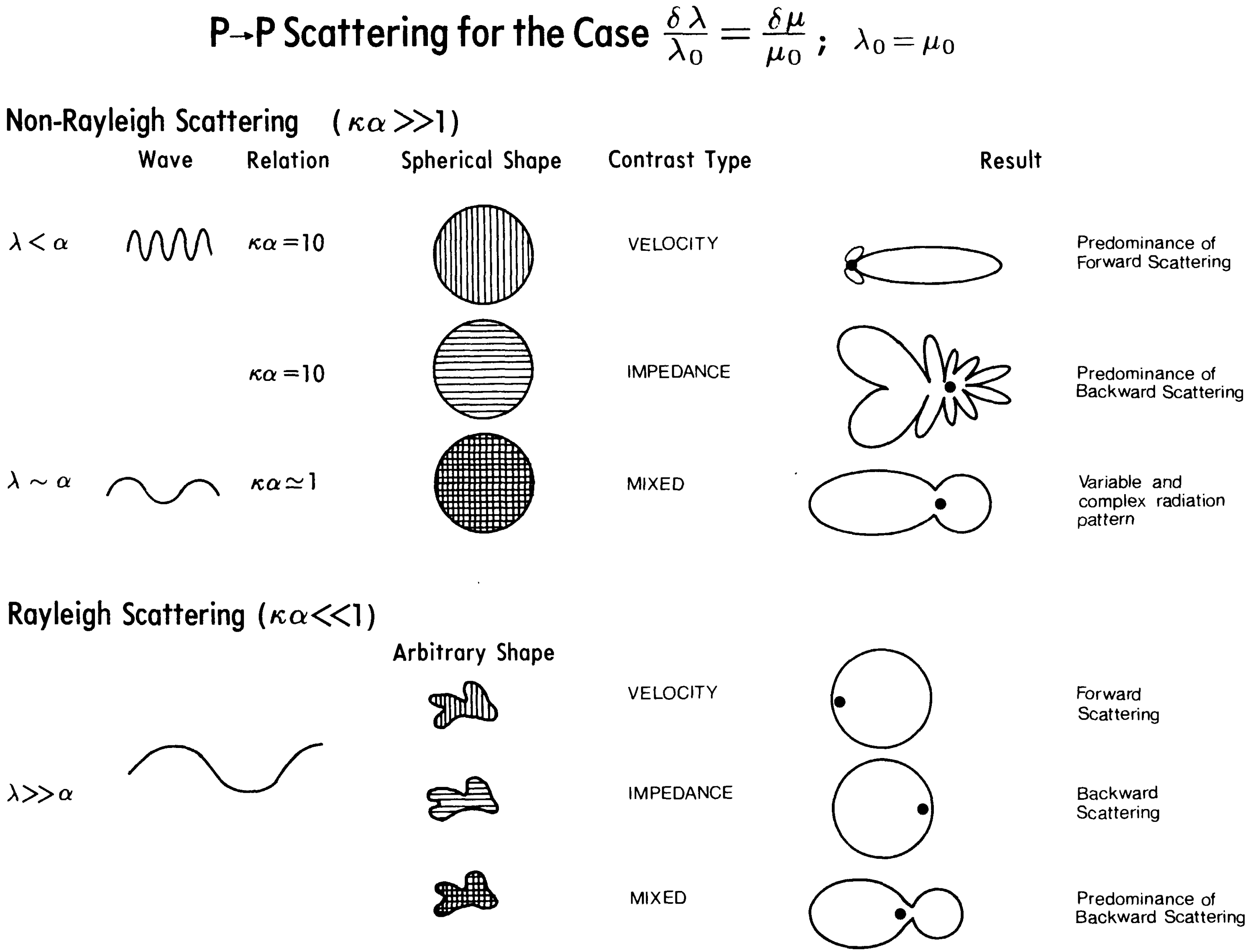


Figure 24.— Schematic of $P \to P$ scattering for different conditions. This figure describes, in full, the problem of a scalar seismic wave scattering from two different obstacle shapes and it complements figures 2 and 3. The wave incidence direction is toward the right. The solid circle represents the point at which the source of scattering takes place.

For the elastic-wave problem, the directional scattering coefficient $g(\theta)$ for low frequencies has the typical characteristics of Rayleigh scattering but its spatial pattern is much more complicated than for the scalar case. For high frequencies, the frequency dependence is more complex, except for the forward and backward scattering cases. For a random continuum, the scattering coefficient, g, is defined as the total power scattered by a unit volume of the random medium for a unit incident field. Its expression is

$$g = \frac{1}{4\pi} \iint_{4\pi} g(\theta)\, d\Omega \tag{108}$$

where $d\Omega$ is the differential solid angle over which $g(\theta)$ is defined. $g(\theta)$ is related to the total scattered power for a unit incident field by

$$W = \frac{1}{4\pi} \iint_{4\pi} V g(\theta)\, d\Omega \tag{109}$$

Wu and Aki obtained the expression for g for P-wave incidence considering $P \rightarrow P$ and $P \rightarrow S$ scattering. At high frequencies, the result coincides with that obtained for scalar waves in a medium with only velocity perturbations. In the low and intermediate frequency range, the elastic-wave coefficient and the scalar-wave one can be quite different from each other. This study also pointed out the importance of $P \rightarrow S$ conversion and the shortcomings of single-scattering theory to deal, even qualitatively, with the energy loss due to high-frequency scattering.

Finally, the application of this theory to previous observations of small-scale inhomogeneities (Aki, 1973, 1977, 1981a, c; Capon 1974; Berteussen and others, 1975), sheds light on some of the disagreements which have been mentioned previously.

First of all, the result just cited, which outlines the necessity of using vector wave theory to account for coda generation, must be recalled. This requirement explains the disagreement shown by Sato (1982b) between the backscattering coefficients needed to explain the coda strength and the results obtained from the calculation of scattering attenuation. In addition, the radical difference between forward and backward scattering can be treated by this theory. The forward scattering is mainly sensitive to large scale velocity inhomogeneities. The backscattering is produced by impedance inhomogeneities with scale-length comparable to wavelength (see fig. 25). This difference suggested the existence of at least two kinds of inhomogeneities within the lithosphere. This hypothesis is strengthened by considering that the theoretical predictions developed in this study cannot match the observed backscattering coefficient if the correlation length has the observed value $\alpha \simeq 10\,\mathrm{km}$. From considerations about the backscattering coefficient, Wu and Aki suggested that phase and amplitude fluctuations at large seismic arrays are caused by velocity inhomogeneities with $\alpha \simeq 10 - 20\,\mathrm{km}$, while local codas are generated by impedance inhomogeneities with $\alpha < 1\,\mathrm{km}$.

The above discussed studies by Sato, and Wu and Aki, constitute important breakthroughs in this field of research. They strongly improve the understanding of scattering phenomenon and, at the same time, suggest new approaches for future research. However, their importance does not lessen that of some other current studies. For instance, multiple scattering, in addition to

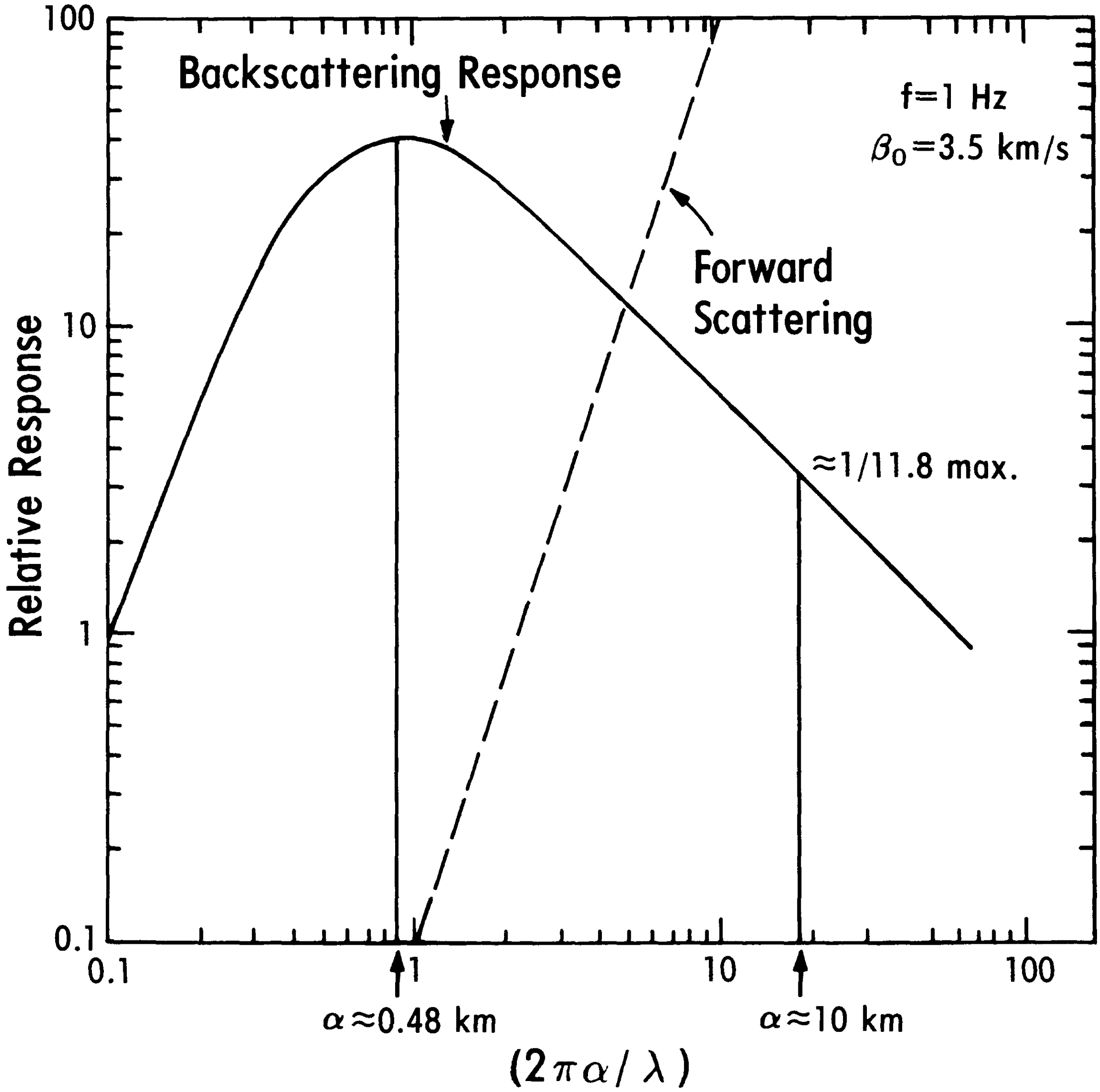


Figure 25.— Responses of inhomogeneities with different correlation lengths to S-waves with a period of 1 Hz and $\beta_0 = 3.5$ km/sec. Backscattering response has its maximum for $\alpha = 0.48$ km. For $\alpha \approx 10$ km, which is the correlation length estimated from LASA data, the backscattering is only 1/11.8 of its maximum value and forward scattering is clearly predominant. [Reproduced with permission from the American Geophysical Union.]

having been incorporated in the new coda models (described in Section 2), is being considered in recent attenuation studies (Wu 1984a, 1985; Menke, 1982, 1983a; Menke and Chen, 1984) and in the analysis of effects introduced by finite obstacles (Varadan and others, 1978; Wu, 1984b). The use of synthetic seismograms to construct coda waves (Herrmann, 1977; Frankel and Clayton, 1984) makes new information available about the effects of scattering in the real Earth and about coda formation. Some recent experimental (Frankel, 1982; Richards and Menke, 1983), and theoretical studies (Malin, 1980; Dainty, 1984; Menke, 1983b, 1984; Frankel and Clayton, 1986) are clarifying the way attenuation effectively acts and its behavior in extreme frequency limits. In the same way, the attention accorded to different concrete problems like attenuation and turbidity in volcanic areas (Castellano and others, 1986), or to the interpretation of the L_g phase from the scattering point of view (Kopnichev, 1980; Der and others, 1984), is supplementing both our observational and theoretical information. Many aspects of the scattering phenomenon as the interaction of a wave with an arbitrary shaped body (Gelchinsky, 1982), or the scattering experienced by specific types of waves (Matsunami, 1981; Malin and Phinney, 1984; Bache and others, 1985; Taylor and Bonner, 1986), are receiving increasing interest. In this field, the work which uses the Born approximation in order to describe surface-wave scattering by three-dimensional heterogeneities (Snieder, 1986; Snieder and Nolet, in prep.) deserves special attention. Much work remains to be done, and some of the most interesting subjects to be dealt with are listed below as part of the conclusion reached in this review paper:

1. The use of random media to analyze scattered waves must be improved by including the features indicated at the beginning of this section, and by using more complete autocorrelation functions.
2. Reaching a way to separate the effects of different scale lengths is an extremely attractive goal with great practical interest.
3. A better comprehension of attenuation mechanisms is another pressing subject. It may facilitate the discrimination between scattering and absorption contributions.
4. Coda-site effect must be studied further to resolve the contradiction between some of its observations and the S to S backscattering theory of coda composition.
5. It is necessary to obtain more observational data in order to test the theories described in this review.
6. Finally, a common notation to name scattering and coda concepts should expressly be adopted.

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REFERENCES

Akamatsu, J., 1980, Attenuation property of coda parts of seismic waves from local earthquakes, Bull. Disas. Prev. Res. Inst., Kyoto Univ., 30, 1–16.

Aki, K., 1956, Correlogram analyses of seismograms by means of a simple automatic computer, J. Phys. Earth, 4, 71–79.

Aki, K., 1969, Analysis of the seismic coda of local earthquakes as scattered waves, J. Geophys. Res., 74, 615–631.

Aki, K., 1973, Scattering of *P* waves under the Montana LASA, J. Geophys. Res., 78, 1334–1346.

Aki, K., 1977, Three-dimensional seismic velocity anomalies in the lithosphere, method and summary of results, J. Geophys., 43, 235–242.

Aki, K., 1980a, Attenuation of shear-waves in the lithosphere for frequencies from 0.05 to 25 Hz, Phys. Earth Planet. Interiors, 21, 50–60.

Aki, K., 1980b, Scattering and attenuation of shear waves in the lithosphere, J. Geophys. Res., 85, 6496–6504.

Aki, K., 1980c, Physical basis for the duration magnitude and recommended practice for coda magnitude determination, Proc. of the 17^{th} Assembly of the ESC, Budapest.

Aki, K., 1981a, Attenuation and scattering of short-period seismic waves in the lithosphere, in Identification of Seismic Sources-Earthquakes or Underground Explosions, E. S. Husebye and S. Mykkelveit, editors, D. Reidel Publishing Co., Dordrecht, The Netherlands, 515–541.

Aki, K., 1981b, Source and scattering effects on the spectra of small local earthquakes, Bull. Seism. Soc. Am., 71, 1687–1700.

Aki, K., 1981c, 3-D inhomogeneities in the upper mantle, Tectonophysics, 75, 31–40.

Aki, K., 1982, Scattering and attenuation, Bull. Seism. Soc. Am., 72, S319–S330.

Aki, K., 1984, Short-period seismology, J. Comput. Phys., 54, 3–17.

Aki, K., 1985, Theory of earthquake prediction with special reference to monitoring of the quality factor of the lithosphere by the coda method, Earthq. Predict. Res., 3, 219–230.

Aki, K., M. Tsujiura, M. Hori, and K. Goto, 1958, Spectral study of near earthquake waves, Bull. Earthquake Res. Inst., Tokyo Univ., 36, 71–98.

Aki, K., and M. Tsujiura, 1959, Correlation study of near earthquake waves, Bull. Earthquake Res. Inst., Tokyo Univ., 37, 207–232.

Aki, K., and B. Chouet, 1975, Origin of coda waves: source, attenuation and scattering effects, J. Geophys. Res., 80, 3322–3342.

Aki, K., A. Christoffersson, and E. S. Husebye, 1976, Three dimensional seismic structure of the lithosphere under Montana LASA, Bull. Seism. Soc. Am., 66, 501–526.

Aki, K., A. Christoffersson, and E. S. Husebye, 1977, Determination of the three-dimensional seismic structure of the lithosphere, J. Geophys. Res., 82, 277–296.

Aki, K., and P. Richards, 1980, Quantitative Seismology, I and II, W. H. Freeman and Co., San Francisco.

Anderson, D. L., and R. S. Hart, 1978, Attenuation models of the earth, Phys. Earth Planet. Interiors, 16, 289–296.

Asada, T., and K. Takano, 1963, Attenuation of short-period *P* waves in the mantle, J. Phys. Earth, 11, 25–34.

Bache, T. C., P. D. Marshall, and L. B. Bache, 1985, Q for teleseismic P waves from central Asia, J. Geophys. Res., 90, 3575–3587.

Bakun, W. H., 1984, Magnitudes and moments of duration, Bull. Seism. Soc. Am., 74, 2335–2356.

Bakun, W. H., C. G. Bufe, and R. M. Stewart, 1976, Body wave spectra of central California earthquakes, Bull. Seism. Soc. Am., 66, 363–384.

Bakun, W. H., and A. G. Lindh, 1977, Local magnitude, seismic moments and coda durations for earthquakes near Oroville, California, Bull. Seism. Soc. Am., 69, 343–355.

Banick, N. C., I. Lerche, and R. T. Shuey, 1986a, Stratigraphic filtering, 1, Derivation of the O' Doherty-Anstey formula, Geophysics, in press.

Banick, N. C., I. Lerche, J. Resnick, and R. T. Shuey, 1986b, Stratigraphic filtering, 2, Model Spectra, Geophysics, in press.

Bard, P. Y., and J. C. Gariel, 1986, The seismic response of two-dimensional sedimentary deposits with large vertical velocity gradients, Bull. Seism. Soc. Am., 76, 343–366.

Barley, B. J., J. A. Hudson, and A. Douglas, 1982, S to P scattering at the 650 km discontinuity, Geophys. J. R. astr. Soc., 69, 159–172.

Basham, P. W., and R. M. Ellis, 1969, The composition of P codas using magnetic tape seismograms, Bull. Seism. Soc. Am., 59, 473–486.

Batchelor, G. K., 1953, The Theory of Homogeneous Turbulence, Cambridge University Press.

Båth, M., 1973, Introduction to Seismology, Birkhauser Verlag, Basel.

Baumgardt, D. R., 1985, Comparative analysis of teleseismic P coda and L_g waves from underground nuclear explosions in Eurasia, Bull. Seism. Soc. Am., 75, 1413–1433.

Beaudet, P. R., 1970, Elastic wave propagation in heterogeneous media, Bull. Seism. Soc. Am., 60, 769–784.

Berckhemer, H., 1970, A possible scattering mechanism for lunar seismic waves, J. Geophys. Res., 36, 523–529.

Berteussen, K. A., A. Christoffersson, E. S. Husebye, and A. Dahle, 1975, Wave scattering theory in analyses of P wave anomalies at NORSAR and LASA, Geophys. J. R. astr. Soc., 42, 403–417.

Birch, F., 1961, The velocity of compressional waves in rocks to 10 kb, part 2, J. Geophys. Res., 66, 2199–2226.

Biswas, N. N., and K. Aki, 1984, Characteristics of coda waves: central and southcentral Alaska, Bull. Seism. Soc. Am., 74, 493–507.

Bisztricsany, E. A., 1958, A new method for the determination of the magnitude of earthquakes, Geofiz. Koszl, 7, 2.

Bollinger, G. A., 1979, Attenuation of the L_g phase and the determination of m_b in the southeastern United States, Bull. Seism. Soc. Am., 69, 45–63.

Born, M., and E. Wolf, 1965, Principles of Optics, 3rd edn., Pergamon Press, Oxford.

Bourret, R. C., 1962a, Stochastically perturbated fields with application to wave propagation in random media, Nuovo Cimento, 26, 1–31.

Bourret, R. C., 1962b, Propagation of randomly perturbated fields, Canad. J. Phys., 40, 782–790.

Capon, J., 1974, Characterization of crust and upper mantle structure under LASA as a random medium, Bull. Seism. Soc. Am., 64, 235–266.

Capon, J., and K. A. Berteussen, 1973, A random medium analysis of crust and upper mantle structure under NORSAR, Semiannual Technical Summary, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Castellano, M., E. Del Pezzo, G. de Natala, and A. Zollo, 1986, Seismic coda and turbidity coefficient at Campi Flegrei Volcanic area: preliminary results, submitted to Bull. Volc.

Chandrasekhar, S., 1960, Radiative Transfer, Dover Publications, New York, (revised version of the work first published in 1950).

Chaplin, M. P., S. R. Taylor, and M. N. Toksöz, 1980, A coda-length magnitude scale for New England, Earthquake Notes, 51, 15-22.

Chávez, D. E., and K. K. Priestley, 1986, Measurement of frequency dependent L_g attenuation in the Great Basin, Geophys. Res. Lett., 13, 551–554.

Chen, P., O. Nuttli, W. Ye, and J. Qin, 1984, Estimates of short-period Q values and seismic moments from coda waves for earthquakes of the Beijing and Yu-Nan regions of China, Bull. Seism. Soc. Am., 74, 1189–1207.

Chernov, L. A., 1960, Wave Propagation in a Random Medium, McGraw-Hill, New York.

Chouet, B., 1976, Source, scattering, and attenuation effects on high frequency seismic waves, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Chouet, B., 1979, Temporal variation in the attenuation of earthquake coda near Stone Canyon, California, Geophys. Res. Lett., 6, 143–146.

Chouet, B., K. Aki, and M. Tsujiura, 1978, Regional variations of the scaling law of earthquake source spectra, Bull. Seism. Soc. Am., 68, 49–79.

Cleary, J. R., and R. A. W. Haddon, 1972, Seismic wave scattering near the core-mantle boundary: A new interpretation of precursors to $PKIKP$, Nature, 240, 549–550.

Cleary, J. R., D. W. King, and R. A. W. Haddon, 1976, Seismic wave scattering in the earth's crust and upper mantle, Geophys. J., 43, 861–872.

Console, R., and A. Rovelli, 1981, Attenuation parameters for Friuli region from strong motion accelerograms, Bull. Seism. Soc. Am., 71, 1981–1991.

Crosson, R. S., 1972, Small earthquakes, structure and tectonics of the Puget Sound region, Bull. Seism. Soc. Am., 62, 1133–1177.

Crow, S. C., 1969, Distortion of sonic bangs by atmospheric turbulence, J. Fluid Mech., 37, 529–563.

Dainty, A. M., 1981, A scattering model to explain seismic Q observations in the lithosphere between 1 and 30 Hz, Geophys. Res. Lett., 8, 1126–1128.

Dainty, A. M., 1984, High-frequency acoustic backscattering and seismic attenuation, J. Geophys. Res., 89, 3172–3176.

Dainty, A. M., M. N. Toksöz, K. R. Anderson, P. J. Pines, Y. Nakamura, and G. Latham, 1974, Seismic scattering and shallow structure of the Moon in Oceanus Procellarum, Moon, 9, 11–29.

Dainty, A. M., and M. N. Toksöz, 1977, Elastic wave scattering in a highly scattering medium, a diffusion approach, J. Geophys., 43, 375–388.

Dainty, A. M., and M. N. Toksöz, 1981, Seismic codas on the Earth and the Moon: a comparison, Phys. Earth Planet. Interiors, 26, 256–266.

Del Pezzo, E., F. Ferulano, A. Giarrusso, and M. Martini, 1983, Seismic coda, Q and scaling law of the source spectra at the Aeolian Islands, southern Italy, Bull. Seism. Soc. Am., 73, 97–108.

Del Pezzo, E., and A. Zollo, 1984, Attenuation of coda waves and turbidity coefficient in central Italy, Bull. Seism. Soc. Am., 74, 2655–2659.

Del Pezzo, E., A. Rovelli, and G. Zonno, 1985, Seismic Q and site effects on seismograms of local earthquakes in the Ancona region (central Italy), Annales Geophysicae, 3, 5, 629–636.

Der, Z., M. E. Marshall, A. O'Donnell, and T. W. McElfresh, 1984, Spatial coherence structure and attenuation of the L_g phase, site effects, and the interpretation of the L_g coda, Bull. Seism. Soc. Am., 74, 1125–1147.

Doornbos, D. J., 1974, Seismic wave scattering near caustics: observations of $PKKP$ precursors, Nature, 247, 352–353.

Doornbos, D. J., 1976, Characteristics of lower mantle inhomogeneities from scattered waves, Geophys. J. R. astr. Soc., 44, 447–470.

Doornbos, D. J., and E. S. Husebye, 1972, Array analyses of PKP phases and their precursors, Phys. Earth Planet. Interiors, 5, 387–399.

Douglas, A., P. D. Marshall, P. G. Gibbs, J. B. Young, and C. Blamey, 1973, Signal complexity reexamined, Geophys. J., 33, 195–221.

Dunkin, J. W., 1969, Scattering of transient, spherical P waves by a randomly inhomogeneous elastic half-space, Geophysics, 34, 357–382.

Dwyer, J. J., R. B. Herrmann, and O. W. Nuttli, 1981, Numerical study of attenuation of high frequency L_g-waves in the New Madrid seismic region, U.S. Geol. Surv., Open-File Rept. 81–112, 131 pp.

Dwyer, J. J., R. B. Herrmann, and O. W. Nuttli, 1983, Spatial attenuation of the L_g-wave in the central United States, Bull. Seism. Soc. Am., 73, 781–796.

Einspruch, N. G., E. J. Witterholt, and R. Trull, 1960, Scattering of a plane transverse wave by a spherical obstacle in an elastic medium, J. Appl. Phys., 31, 5, 806–818.

Espinosa, A. F., 1968, Ground amplification studies of short-period seismic waves, in Studies in Seismicity and Earthquake Damage Statistics, Part I; Report for Housing and Urban Development agency (HUD), ESSA Technical Report, Coast and Geodetic Survey, 1–12.

Espinosa, A. F., 1969, Ground amplification of short-period seismic waves at two sites near Bakersfield, California, Earthquake Notes, 40, 3–20.

Espinosa, A. F., 1971, Ground surface motion synthesis and theoretical amplification effects due to shear waves [abs.], General Assembly IASPEI, Symposium on Forerunners of Strong Earthquakes, Moscow, SSSR, 3, 34 (I–5), and Acad. Sci. Comptes Rendus, 17, 128.

Espinosa, A. F., 1977, Particle velocity attenuation relations, San Fernando earthquake of February 9, 1971, Bull. Seism. Soc. Am., 67, 1195–1214.

Espinosa, A. F., 1981, Seismic wave attenuation studies in the contiguous United States, in Summaries of Technical Reports, vol. XII, U.S. Geol. Surv., Open-File Rept. 81-833.

Espinosa, A. F., 1984, L_g-wave attenuation in contiguous United States [abs.], EOS Transactions AGU, Spring meeting, American Geophysical Union, Cincinnati, Ohio, 65, 233.

Espinosa, A. F., and S. T. Algermissen, 1972a, Soil amplification studies in areas damaged by the Caracas earthquake of July 29, 1967, Proceedings of the International Conference on Microzonation, 11, 455–464.

Espinosa, A. F., and S. T. Algermissen, 1972b, Study of soil amplification factors in earthquake damaged areas: Caracas Venezuela, National Oceanic and Atmospheric Administration, Technical Report ERL–280–ESL, 31, 201 pp.

Espinosa, A. F., and S. T. Algermissen, 1973, Ground amplification studies in the Caracas Valley and the northern coastal area of Venezuela, Proceedings of the 5th World Conference on Earthquake Engineering, Rome, Italy, 2, 1–6.

Espinosa, A. F., and A. Lopez-Arroyo, 1977, Earthquake instrumental intensity from strong ground motion records: San Fernando earthquake, Proceedings of the 6th World Conference on Earthquake Engineering, New Delhi, India, 2, 1–7.

Evernden, J. F., 1977, Spectral characteristics of the *P* coda of Eurasian earthquakes and explosions, Bull. Seism. Soc. Am., 67, 1153–1171.

Evernden, J. F., and W. M. Kohler, 1979, Further study of spectral composition of *P* codas of earthquakes and explosions, Bull. Seism. Soc. Am., 69, 483–511.

Ewing, W. M., W. S. Jardentzky, and F. Press, 1957, Elastic Waves in Layered Media, McGraw-Hill, New York.

Fedotov, S. A., and S. A. Boldyrev, 1969, Frequency dependence of the body-wave absorption in the crust and the upper mantle of the Kuril Island chain (English Trans.), Izv. Akad. Nauk SSSR, Fiz. Zemli, 9, 553–562

Fehler, M., 1979, Seismological investigation of the mechanical properties of a hot dry rock geothermal system, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Frankel, A., 1982, The effects of attenuation and site response on the spectra of microearthquakes in the northeastern Caribbean, Bull. Seism. Soc. Am., 72, 1379–1402.

Frankel, A., and R. W. Clayton, 1984, A finite difference simulation of wave propagation in two-dimensional random media, Bull. Seism. Soc. Am., 74, 2167–2186.

Frankel, A., and R. W. Clayton, 1986, Finite difference simulations of seismic scattering: implications for the propogation of short-period seismic waves in the crust and models of crustal heterogeneity, J. Geophys. Res., 91, 6465–6489.

Frisch, U., 1968, Wave propagation in random media in Probabilistic Methods in Applied Mathematics, vol. 1, edited by A. T. Bharucha-Reid, pp. 76–198, Academic Press, New York.

Gao, L. S., 1984, Coda waves analysis for distinguishing attenuation due to isotropic scattering from attenuation due to absorption, Pure Appl. Geophys., 122, 1–9.

Gao, L. S., L. C. Lee, N. N. Biswas, and K. Aki, 1983a, Comparison of the effects between single and multiple scattering on coda waves for local earthquakes, Bull. Seism. Soc. Am., 73, 377–389.

Gao, L. S., N. N. Biswas, L. C. Lee, and K. Aki, 1983b, Effects of multiple scattering on coda waves in three-dimensional medium, Pure Appl. Geophys., 121, 3–15.

Gelchinsky, B. Ya., 1982, Scattering of waves by a quasi-thin body of arbitrary shape, Geophys. J. R. astr. Soc., 71, 425–453.

Greenfield, R. J., 1971, Short-period P wave generation by Rayleigh wave scattering at Novaya Zemlya, J. Geophys. Res., 76, 7988–8002.

Gusev, A. A., 1986, Two dilatancy-based models to explain coda-wave precursors and P/S spectral ratio, paper presented at the European Seismological Commission Assembly at Kiel, West Germany.

Gusev, A. A., and V. K. Lemzikov, 1983, Estimation of shear wave scattering in crust and upper mantle of Kamchatka from observations of Shipunsky station (in Russian), Vulk. Seismol., 1, 96–107.

Gusev, A. A., and V. K. Lemzikov, 1984, The anomalies of small earthquake coda wave characteristics before the three large earthquakes in the Kuril-Kamchatka zone (in Russian), Vulk. Seismol., 4, 76-90.

Gusev, A. A., and V. K. Lemzikov, 1985, Properties of scattered elastic waves in the lithosphere of Kamchatka: Parameters and temporal variations, Tectonophysics, 112, 137–153.

Gusev, A. A., and I. R. Abubakirov, 1986, Monte-Carlo simulation of record envelope of a near earthquake, paper presented at the European Seismological Commission Assembly at Kiel, West Germany.

Haddon, R. A. W., 1972, Corrugations on the mantle-core boundary or transitions layers between inner and outer cores? [abs.], EOS Transactions AGU, 53, 600.

Haddon, R. A. W., 1978, Scattering of seismic body waves by small random inhomogeneities in the earth, NORSAR Scientific Report No. 3–77/78.

Haddon, R. A. W., and J. R. Cleary, 1974, Evidence for scattering of PKP waves near the mantle-core boundary, Phys. Earth Planet. Interiors, 8, 211–234.

Hasegawa, H. S., 1969, A study of the effect of the Yellowknife crustal structure upon the P coda of teleseismic events, Geophys. J. R. astr. Soc., 28, 159–175.

Hasegawa, H. S., 1970, Short-period P-coda characteristics in the eastern Canadian Shield, Bull. Seism. Soc. Am., 60, 839–858.

Havskov, J. J., and M. Macías, 1983, A coda-length magnitude scale for some Mexican stations (Spanish abs.), Geofis. Int., 22, 205–213.

Herraiz, M., 1982, Microsismicidad en el campo próximo. Análisis de generación de ondas de coda y parámetros físicos asociados, Tesis Doct., U. Complutense, Madrid, Spain.

Herraiz, M., and J. Mezcua, 1982, Recent improvement of the study of microseismic data based on coda analysis (in Spanish, English abs.), Rvta. Geofís., 38, 167–178.

Herraiz, M., and J. Mezcua, 1984, Application of coda wave analysis to analog recorded data, Annales Geophysicae, 2, 5, 545–552.

Herrera, I., 1965, A perturbation method for elastic wave propagation, III — thin inhomogeneities, Geofis. Int., 5, 1–14.

Herrera, I., and A. K. Mal, 1965, A perturbation method for elastic wave propagation II — small inhomogeneities, J. Geophys. Res., 70, 871–883.

Herrmann, R. B., 1975, The use of duration as a measure of seismic moment and magnitude, Bull. Seism. Soc. Am., 65, 899–913.

Herrmann, R. B., 1977, A method for synthesis of the seismic coda of local earthquakes, J. Geophys., 43, 341–350.

Herrmann, R. B., 1980, Q estimates using the coda of local earthquakes, Bull. Seism. Soc. Am., 70, 447–468.

Hinderer, S., 1979, Etude du facteur de Qualité Q de Région Arette-Larrau (Pyrenées-Atlantiques) a partir de la coda de seismes locaux. Memoire presentée a l'Institut de Physique du Globe de l'Université Louis Pasteur de Strasbourg pour l'obtention de Diplome d'Ingenieur Geophysicien.

Hirn, A., and A. Nercessian, 1980, Mont Doré: étude structurale tri-dimensionelle par des méthodes de sismique transmission, Rep. on contract 578 78 EGF, R and D Program Geothermal Energy, Commission of the European Communities, 118 pp.

Hoang-Trong, P., 1983, Some medium properties of the Hohenzollerngraben (Swabian Jura, W. Germany) inferred from Q_P/Q_S ratio analysis, Phys. Earth Planet. Int., 31, 119–131.

Howe, M. S., 1971a, Wave propagation in random media, J. Fluid Mech., 45, 769–783.

Howe, M. S., 1971b, On wave scattering by random inhomogeneities, with application to the theory of weak bores, J. Fluid Mech., 45, 785–804.

Hudson, J. A., 1977, Scattered waves in the coda of P, J. Geophys., 43, 359– 374.

Hudson, J. A., 1982, Use of stochastic models in seismology, Geophys. J. R. astr. Soc., 69, 649–657.

Hudson, J. A., and J. R. Heritage, 1981, The use of the Born approximation in seismic scattering problems, Geophys. J. R. astr. Soc., 66, 221–240.

Husebye, E. S., D. W. King, and R. A. W. Haddon, 1976, Precursors to $PKIKP$ and seismic wave scattering near the mantle-core boundary, J. Geophys. Res., 81, 1870–1882.

Ishimaru, A., 1978, Wave Propagation and Scattering in Random Media, I and II, Academic Press, New York.

Jeffreys, H., 1929, 1959, The Earth, its origin, history and physical constitution, Cambridge University Press, 2nd edition, 1929, 4th edition, 1959.

Jin, A., 1981, Duration of coda waves and the back-scattering coefficient, paper presented at the Symposium on Seismology in China, State Seismol. Bur., Shanghai, China.

Jin, A., T. Cao, and K. Aki, 1985, Regional change of coda Q in the ocean lithosphere, J. Geophys. Res., 90, 8651–8659.

Jin, A., and K. Aki, 1986, Temporal change in coda Q before the Tangshan earthquake of 1976 and the Haicheng earthquake of 1975, J. Geophys. Res, 91, 665–673.

Jones, F. B., L. T. Long, and J. H. McKee, 1977, Study of the attenuation and azimuthal dependence of seismic-wave propagation in the southeastern United States, Bull. Seism. Soc Am., 67, 1503–1513.

Kanamori, H., 1981, The nature of seismicity patterns before large earthquakes, Earthquake Prediction, An International Review, Maurice Ewing Series, 4, 1–20.

Karal, F. C., and J. B. Keller, 1964, Elastic, electromagnetic and other waves in a random medium, J. Math. Phys., 5, 537–547.

Keller, J. B., 1960, Wave propagation in random media, Proc. 13th Symp. Applied Mathematics, Am. Mathematical Society, New York, 227–246.

Keller, J. B., 1964, Stochastic equations and wave propagation in random media, Proc. 16th Symp. Applied Mathematics, Am. Mathematical Soc., New York, 145–170.

Kennett, B. L. N., 1972, Seismic waves in laterally inhomogeneous media, Geophys. J. R. astr. Soc., 27, 301–336.

Kikuchi, M., 1981, Dispersion and attenuation of elastic waves due to multiple scattering from cracks, Phys. Earth Planet. Interiors, 27, 100–105.

King, D. W., R. A. W. Haddon, and J. R. Cleary, 1973, Evidence for seismic wave scattering in the "D" layer, Earth Planet. Sci. Lett., 20, 353–356.

King, D. W., and J. R. Cleary, 1974, A note on the interpretation of precursors to $PKPPKP$, Bull. Seism. Soc. Am., 64, 721–723.

King, D. W., R. A. W. Haddon, and J. R. Cleary, 1974, Array analysis of precursors to $PKIKP$ in the distance range 128° to 142°, Geophys. J., 37, 157–173.

King, D. W., R. A. W. Haddon, and E. S. Husebye, 1975, Precursors to PP, Phys. Earth Planet. Interiors, 10, 103–127.

Knopoff, L., 1959a, Scattering of compressional waves by spherical obstacles, Geophysics, 24, 30–39.

Knopoff, L., 1959b, Scattering of shear waves by spherical obstacles, Geophysics, 24, 209–219.

Knopoff, L., 1964, "Q", Rev. Geophys., 2, 625–660.

Knopoff, L., and J. A. Hudson, 1964, Scattering of elastic waves by small inhomogeneities, J. Acoust. Soc. Am., 36, 338–343.

Knopoff, L., and J. A. Hudson, 1967, Frequency dependence of amplitude of scattered elastic waves, J. Acoust. Soc. Am., 42, 18–20

Kopnichev, Y. F., 1975, A model of generation of the tail of the seismogram (English Trans.), Dokl. Akad. Nauk SSSR, 222, 333–335.

Kopnichev, Y. F., 1977a, Models for the formation of the coda of the longitudinal wave (English Trans.), Dokl. Akad. Nauk SSSR, 234, 13–15.

Kopnichev, Y. F., 1977b, The role of multiple scattering in the formation of a seismogram's tail (English Trans.), Izv. Akad. Nauk SSSR, Fiz. Zemli, 13, 394–398.

Kopnichev, Y. F., 1980, Statistical models for the generation of coda and short-period L_g-phases and the results of their joint investigation (in Russian), Izv. Akad. Nauk SSSR, Fiz. Zemli, 2, 22–35.

Korvin G., 1983, General theorem on mean wave attenuation, Geophys. Trans., 29, 2, 191–202.

Kovach, R. L., 1978, Seismic surface waves and crustal and upper mantle structure, Rev. Geophys. Space Phys., 16, 1–13.

Landau, L. P., and E. M. Lifshitz, 1959, Fluid Mechanics, Pergamon Press, London.

Latham, G. V., M. Ewing, F. Press, G. Sutton, J. Dorman, Y. Nakamura, N. Toksöz, R. Wiggins, J. Derr, and F. Duennebier, 1970, Passive seismic experiment, Science, 167, 455–457.

Latham, G. V., M. Ewing, F. Press, G. Sutton, J. Dorman, Y. Nakamura, N. Toksöz, F. Duennebier, and D. Lammlein, 1971, Passive seismic experiment. NASA Spec. Publ. SP-272, 133–161.

Lee, W. B., and S. C. Solomon, 1978, Simultaneous inversion of surface wave phase velocity and attenuation: Love waves in western North America, J. Geophys. Res., 83, 3389–3400.

Lee, W. H. K., R. E. Bennett, and K. L. Meagher, 1972, A method of estimating magnitude of local earthquakes from signal duration, Open-file Report, Nat. Center for Earthquake Res., U.S. Geol. Surv., Menlo Park, California, 28 pp.

Lee, W. H. K., K. Aki, B. Chouet, J. T. Newberry, and D. M. Tottingham, 1985, Mapping coda $Q(\omega)$ in California using the U.S.G.S. earthquake data archiving and processing system [abs.], Seism. Soc. Am. 1985 annual meeting, Earthquake Notes, 55, 15.

Lee, W. H. K., K. Aki, B. Chouet, P. Johnson, S. Marks, J. T. Newberry, A. S. Ryall, S. W. Stewart, and D. M. Tottingham, 1986, A preliminary study of coda Q in California and Nevada, Bull. Seism. Soc. Am., 76, 1143–1150.

Levin, F. K., and D. J. Robinson, 1969, Scattering by a random field of surface scatterers, Geophysics, 34, 170–179

Lighthill, M. J., 1953, Interaction of turbulence with sound or shock waves, Proc. Camb. Phil. Soc., 49, 531.

Malin, P. E., 1980, A first-order scattering solution for modeling elastic wave codas, I. The acoustic case, Geophys. J., 63, 361–380.

Malin, P. E., and R. A. Phinney, 1984, On the relative scattering of P and S waves, Geophys. J. R. astr. Soc., 80, 603–618.

Maruyama, T., 1963, On the force equivalents of dynamic elastic dislocations with reference to earthquake mechanism, Bull. Earthquake Res. Inst., Tokyo Univ., 41, 467–486.

Matsunami, K., 1981, Scattering of P waves by random velocity heterogeneities, Bull. Disas. Prev. Res. Inst., Kyoto Univ., 31, 59–78.

Menke, W., 1982, On extending Biot's theory of multiple scattering at low frequencies from acoustic to elastic media, Geophys. J. R. astr. Soc., 69, 819–830.

Menke, W., 1983a, A formula for the apparent attenuation of acoustic waves in randomly layered media, Geophys. J. R. astr. Soc., 75, 541–544.

Menke, W., 1983b, On the effect of P-S coupling on the apparent attenuation of elastic waves in randomly layered media, Geophys. Res. Lett., 10, 1145–1147.

Menke, W., 1984, Asymptotic formulas for the apparent Q of weakly scattering three-dimension al media, Bull. Seism. Soc. Am., 74, 1079–1081.

Menke, W., and R. L. Chen, 1984, Numerical studies of the coda fall-off rate of multiply scattered waves in randomly layered media, Bull. Seism. Soc. Am., 74, 1605–1614.

Mereu, R. F., and S. B. Ojo, 1981, The scattering of seismic waves through a crust and upper mantle with random lateral and vertical inhomogeneities, Phys. Earth Planet. Interiors, 26, 233–240.

Mie, G., 1908, Beitrage zur Optik trüber Medien, speziell holloidaler Metallösungen, Annalen der Physik, 25, 377.

Miles, J. W., 1960, Scattering of elastic waves by small inhomogeneities, Geophysics, 15, 642–648.

Mitchell, B., 1981, Regional variation and frequency dependence of Q_β in the crust of the United States, Bull. Seism. Soc. Am., 71, 1531–1538.

Modiano, T. and D. Hatzfeld, 1982, Experimental study of the spectral content for shallow earthquakes, Bull. Seism. Soc. Am., 72, 1739–1758.

Mogi, K., 1977, Seismic activity and earthquake prediction (in Japanese), Proceedings of the Symposium on Earthquake Prediction Research, 203-214.

Molnar, P., K. H. Jacob, and K. McCany, 1973, Implications of Archambeau's earthquake source theory for step faults, Bull. Seism. Soc. Am., 63, 101–104.

Morse, P. M., 1968, Theoretical Acoustics, McGraw-Hill, New York.

Morse, P. M., and H. Feshbach, 1953, Methods of Theoretical Physics, McGraw-Hill, New York.

Mott, N. F., and H. S. W. Massey, 1965, The Theory of Atomic Collisions, 3rd edition, Oxford University Press.

Nakamura, Y., 1976, Seismic energy transmission in the lunar surface zone determined from signals generated by movement of lunar rovers, Bull. Seism. Soc. Am., 66, 593–606.

Nakamura, Y., 1977a, HFF events: shallow earthquakes, Phys. Earth Planet. Interiors, 16, 271–273.

Nakamura, Y., 1977b, Seismic energy transmissions in an intensively scattering environment, J. Geophys., 43, 389–399.

Nakamura, Y., G. V. Latham, M. Ewing, and J. Dorman, 1970, Lunar seismic energy transmissions [abs.], EOS Transactions AGU, 51, 776.

Nersesov, I. L., Y. F. Kopnichev, and G. A. Vostrikov, 1975, Calibration of earthquakes in terms of magnitude, based on coda waves, at distances as great as 3000 km (English Trans.), Dokl. Akad. Nauk SSSR, 222, 7–9.

Nikolayev, A. V., 1968, Seismic properties of weakly heterogeneous media (English Trans.), Izv. Akad. Nauk SSSR, Fiz. Zemli, 2, 83–87.

Nikolayev, A. V., and F. S. Tregub, 1970, A statistical model of the earth crust: Method and results, Tectonophysics, 10, 573–578.

Novelo-Casanova, D. A., E. Berg, V. Hsu, and C. E. Helsley, 1985, Time-space variation seismic S-wave coda attenuation (Q^{-1}) and magnitude distribution (b-values) for the Petatlán earthquake, Geophys. Res. Lett., 12, 789–792.

Novelo-Casanova, D. A., and R. Butler, 1986, High frequency seismic coda and scattering in the northwest Pacific, Bull. Seism. Soc. Am., 76, 617–626.

Nowack, R., and K. Aki, 1984, The two-dimensional Gaussian beam synthetic method: Testing and application, J. Geophys. Res., 89, 7797–7819.

Nur, A., 1971, Viscous phases in rocks and the low velocity zone, J. Geophys. Res., 76, 1270–1278.

Nuttli, O. W., 1973, Seismic wave attenuation and magnitude relations for eastern North America, J. Geophys. Res., 78, 876–885.

Nuttli, O., 1978, A time-domain study of the attenuation of 10-Hz waves in the New Madrid seismic zone, Bull. Seism. Soc. Am., 68, 343–355.

Nuttli, O. W., 1980, The excitation and attenuation of seismic crustal phases in Iran., Bull. Seism. Soc. Am., 70, 469–486.

Ojo, S. B., and R. F. Mereu, 1986, The effect of random velocity functions on the travel times and amplitude of seismic waves, Geophys. J. R. astr. Soc., 84, 607–618.

Patton, H. J., 1983, L_g excitation and propagation in the western United States, preprint for fifth annual DARPA/AFOSR Symposium, May, 1983.

Pekeris, C. L., 1947, Note on the scattering of radiation in an inhomogeneous medium, Phys. Rev., 71, 268–269.

Peng, J. Y., K. Aki, W. H. K. Lee, B. Chouet, P. Johnson, S. Marks, J. T. Newberry, A. S. Ryall, S. Stewart, and D. M. Tottingham, 1986, Temporal change in coda Q associated with the Round Valley, California, earthquake of November 23, 1984, submitted to J. Geophys.

Res.

Peseckis, L. L., and P. W. Pomeroy, 1984, Determination of Q using L_g waves and its implications for nuclear yield estimation [abs.], EOS Transactions AGU, 65, 995.

Phillips, W. S., 1985, The separation of source, path and site effects on high frequency seismic waves: an analysis using coda wave techniques, Ph. D. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Phillips, W. S., and K. Aki, 1986, Site amplification of coda waves from local earthquakes in central California, Bull. Seism. Soc. Am., 76, 627–648.

Phillips, W. S., W. H. K. Lee, and J. T. Newberry, 1986, Spacial variation of crustal coda Q in California, submitted to Bull. Seism. Soc. Am.

Pilant, W., 1979, Elastic Waves in the Earth, Elsevier, Amsterdam.

Powel, C. A., and A. S. Meltzer, 1984, Scattering of P-waves beneath SCARLET in southern California, Geophys. Res. Lett., 4, 481–484.

Pulli, J. J., 1984, Attenuation of coda waves in New England, Bull. Seism. Soc. Am., 74, 1149–1166.

Pulli, J. J., and K. Aki, 1981, Attenuation of seismic waves in the lithosphere: comparison of active and stable areas, in Earthquakes and Earthquake Engineering – Eastern United States, Edited by J. Beavers, Ann Arbor Science Publishers, Inc., Ann Arbor, Michigan, 129–141.

Pustovitenko, B. G., and T. G. Rautian, 1977, Use of the seismic coda for the study of the attenuation of seismic waves in the Crimean region (in Russian, English abs.), Geofiz. Sb., 78, 3–16.

Rautian, T. G., and V. I. Khalturin, 1978, The use of the coda for determination of the earthquake source spectrum, Bull. Seism. Soc. Am., 68, 923–943.

Rautian, T. G., V. I. Khalturin, V. G. Martinov, and P. Molnar, 1978, Preliminary analysis of the spectral content of P and S waves from local earthquakes in the Garm, Tadjikistan region, Bull. Seism. Soc. Am., 68, 949–971.

Rayleigh, J. W. S., 1896, The Theory of Sound, vol. II, Dover Publications Inc., New York, (1945 edition).

Real, C. R., and T. L. Teng, 1973, Local Richter magnitude and total signal duration in southern California, Bull. Seism. Soc. Am., 63, 1809–1827.

Rebollar, C. J., C. Traslosheros, and R. Alvarez, 1985, Estimates of seismic wave attenuation in northern Baja California, Bull. Seism. Soc. Am., 75, 1371–1382.

Rhea, S., 1984, Q determined from local earthquakes in the South Carolina coastal plain, Bull, Seism. Soc. Am., 74, 2257–2268.

Richards, P. G., and W. Menke, 1983, The apparent attenuation of a scattering medium, Bull. Seism. Soc. Am., 73, 1005–1022.

Richter, C. F., 1958, Elementary Seismology, W. H. Freeman and Co., Inc., San Francisco, California.

Rodriguez, M., J. Havskov, and S. K. Singh, 1983, Q from coda waves near Petatlan, Guerrero, Mexico, Bull. Seism. Soc. Am., 73, 321–326.

Roecker, W. W., 1981, Seismicity and tectonics of the Pamir-Hindu Kush region of central Asia, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge Massachusetts.

Roecker, S. W., B. Tucker, J. King, and D. Hatzfeld, 1982, Estimates of Q in central Asia as a function of frequency and depth using the coda of locally recorded earthquakes, Bull. Seism. Soc. Am., 72, 129–149.

Rovelli, A., 1982, On the frequency dependence of Q in Friuli from short-period digital records, Bull. Seism. Soc. Am., 72, 2369–2372.

Rovelli, A., 1983a, Frequency relationship for seismic Q_β of central southern Italy from accelerograms for the Irpinia earthquake (1980), Phys. Earth Planet. Interiors, 32, 209–217.

Rovelli, A., 1983b, Time-frequency analysis of seismic excitation and estimates of attenuation parameters for the Friuli (Italy) local earthquakes, Phys. Earth Planet. Interiors, 33, 94–110.

Rovelli, A., 1984a, Seismic Q of the lithosphere for Montenegro region (Yugoslavia): frequency, depth and time windowing effects, Phys. Earth Planet. Interiors, 34, 159–172.

Rovelli, A., 1984b, Seismic wave attenuation in southern Italy crest; scattering from random heterogeneties mechanism or intrinsic Q variations with the depth? Problems and methods for lithosphere exploration, (R. Cassinis, editor), in the collection Eltore Majorana International Science Series, Plenum Press, New York, 19, 207–212.

Rytov, S. M., 1937, Diffraction of Light by Ultrasound Waves (in Russian), Izv. Akad. Nauk SSSR, Ser. Fiz., 2, 223.

Saphira, A., 1980, Magnitude determination from coda duration of underground nuclear explosions, Seismological Section, Uppsala, Sweden, Report 2-80, 25 pp.

Sato, H., 1977a, Energy propagation including scattering effects. Single isotropic scattering approximation, J. Phys. Earth, 25, 27–41.

Sato, H., 1977b, Single isotropic scattering model including wave conversions. Simple theoretical model of the short-period body wave propagation, J. Phys. Earth, 25, 163–176.

Sato, H., 1978, Mean free path of S-waves under the Kanto District of Japan, J. Phys. Earth, 26, 185–198.

Sato, H., 1979, Wave propagation in one-dimensional inhomogeneous elastic media, J. Phys. Earth, 27, 455–466.

Sato, H., 1981, Attenuation of elastic waves in one-dimensional inhomogeneous elastic media, Phys. Earth Planet. Interiors, 26, 244–245.

Sato, H., 1982a, Amplitude attenuation of impulsive waves in random media based on travel time corrected mean wave formalism, J. Acoust. Soc. Am., 71, 559–564.

Sato, H., 1982b, Attenuation of S waves in the lithosphere due to scattering by its random velocity structure, J. Geophys. Res., 87, 7779–7785.

Sato, H., 1982c, Coda wave excitation due to nonisotropic scattering and nonspherical source radiation, J. Geophys. Res., 87, 8665–8676.

Sato, H., 1984, Attenuation and envelope formation of three-component seismograms of small local earthquakes in randomly inhomogeneous lithosphere, J. Geophys. Res., 89, 1221–1241.

Sato, H., 1986, Temporal change in attenuation intensity before and after the eastern Yamanashi earthquake of 1983, in central Japan, J. Geophys. Res., 91, 2049–2061.

Sato, H., and S. Matsumura, 1980a, Three-dimensional analysis of scattered P waves on the basis of the PP single isotropic scattering model, J. Phys. Earth, 28, 521–530.

Sato, H., and S. Matsumura, 1980b, Q^{-1} values for S waves (2–32 Hz) under the Kanto district (in Japanese), Zisin, 33, 541–543.

Sato, R., and A. F. Espinosa, 1964, Dissipation in the earth's mantle from waves multiply reflected at the core-mantle boundary [abs.], EOS Transactions AGU, American Geophysical Union 45th Ann. Meeting, Washington, D.C.

Sato, R., and A. F. Espinosa, 1967, Dissipation in the earth's mantle and rigidity and viscosity in the earth's core determined from waves multiply reflected from the core-mantle boundary, Bull. Seism. Soc Am., 57, 829–856.

Savage, J., 1966, Thermoelastic attenuation of elastic waves by cracks, J. Geophys. Res., 71, 3929–3938.

Scheimer, J., and T. E. Landers, 1974, Short period coda of a local event at LASA, Seismic Discrimination, Semiannual Tech. Sum. 42, Lincoln Lab., Massachusetts Institute of Technology, Cambridge, Massachusetts.

Scherbaum, F., and C. Kisslinger, 1985, Coda Q in the Adak seismic zone, Bull. Seism. Soc. Am., 75, 615–620.

Singh, S., 1981, Regionalization of crustal Q in the continental United States, Ph.D. Thesis, St. Louis University, St. Louis, Missouri.

Singh, S. K., and R. B. Herrmann, 1983, Regionalization of crustal coda Q in the continental United States, J. Geophys. Res., 88, 527–538.

Snieder, R., 1986, 3-D linearized scattering of surface waves and a formalism for surface wave holography, Geophys. J. R. astr. Soc., 84, 581–605.

Sobczyk, K., 1985, Stochastic Wave Propagation, Elsevier, Amsterdam.

Solomon, S. C., 1972, Seismic wave attenuation and partial melting in the upper mantle of North America, J. Geophys. Res., 77, 1483–1502.

Soloviev, S. L., 1965, Seismicity of Sakhalin, Bull. Earthquake Res. Inst., Tokyo Univ., 43, 95–102.

Street, R. L., 1976, Scaling northeastern United States/southeastern Canadian earthquakes by their L_g waves, Bull. Seism. Soc. Am., 66, 1525–1537.

Subash, S. M. G., 1979, Interpretation de l'origine de la coda de seismes locaux déduite de leur propietés sur 3 composants, Thése soutenue á L'Université L. Pasteur de Strasbourg.

Subash, S. M. G., and M. A. Choudhury, 1979, Coda power and modulation characteristics of a complex P signal from underground nuclear explosions, Tectonophysics, 53, T33–T39.

Subash, S. M. G., and R. Gir, 1979, Test of surface-body wave hypotheses for the Q-frequency dependence of coda, Tectonophysics, 57, T27–T33.

Suteau, A. M., and J. H. Whitcomb, 1979, A local earthquake coda magnitude and its relation to duration, moment M_o, and local Richter magnitude M_L, Bull. Seism. Soc. Am., 69, 353–368.

Sutton, G. M., W. Mitronovas, and P. W. Pomeroy, 1967, Short-period seismic energy radiation patterns from underground nuclear explosions and small magnitude earthquakes, Bull. Seism. Soc. Am., 57, 249–267.

Suzuki, S., 1972, Anomalous attenuation of P-waves in the Matsushiro earthquake swarm area, J. Phys. Earth, 20, 1–21.

Suzuki, H., R. Ikeda, T. Mikoshiba, S. Kinoshita, H. Sato, and H. Takahashi, 1981, Deep well logs in the Kanto-Tokai area (in Japanese), Rev. Res. Disast. Prev., 65, 1–162.

Takano, K., 1971, Analysis of seismic coda of ultra-microearthquakes in the Matsushiro area, A comparison with Parkfield, California, J. Phys. Earth, 19, 209–216.

Tatarskii, V. I., 1961, Wave Propagation in a Turbulent Medium, McGraw-Hill, New York.

Taylor, L. S., 1967, On Rytov's method, Radio Sci., 2, 437–411.

Taylor, S. R., and B. P. Bonner, 1986, Attenuation and scattering of broadband P and S waves across North America, J. Geophys. Res., 91, 7309–7325.

Toksöz, N., A. M. Dainty, S. C. Solomon, and K. R. Anderson, 1974, Structure of the Moon, Rev. Geophys. Space Phys., 12, 539–567.

Tsai, Y. B., and K. Aki, 1969, Simultaneous determination of the seismic moment and attenuation of seismic surface waves, Bull. Seism. Soc. Am., 59, 275–287.

Tsujiura, M., 1966, Frequency analysis of seismic waves, 1, Bull. Earthquake Res. Inst., Tokyo Univ., 44, 873–891.

Tsujiura, M., 1967, Frequency analysis of seismic waves, 2, Bull. Earthquake Res. Inst., Tokyo Univ., 45, 973–995.

Tsujiura, M., 1969, Regional variation of P-wave spectra, 1, Bull. Earthquake Res. Inst., Tokyo Univ., 47, 613–633.

Tsujiura, M., 1978, Spectral analysis of the coda waves from local earthquakes, Bull. Earthquake Res. Inst., Tokyo Univ., 53, 1–48.

Tsukuda, T., 1985, Coda Q before and after a medium-scale earthquake, paper presented at the 23rd General Assembly of IASPEI, Tokyo.

Tsumura, K., 1967, Determination of earthquake magnitude from total duration of oscillation, Bull. Earthquake Res. Inst., Tokyo Univ., 45, 7–18.

Uscinski, B. DJ., 1977, The Elements of Wave Propagation in Random Media, McGraw-Hill, New York.

Varadan, V. K., V. V. Varadan, and Y. H. Pao, 1978, Multiple scattering of elastic waves by cylinders of arbitrary cross section I, SH waves, J. Acoust. Soc. Am., 63, 1310–1319.

Varadan, V. K., and V. V. Varadan, 1979, Frequency dependence of elastic (SH-) wave velocity and attenuation in anisotropic two phase media, Wave Motion, 1, 53–63.

Vinnik, L. P., 1974, Scattering of longitudinal waves in the earth (English Trans.), Izv. Akad. Nauk SSSR, Fiz. Zemli, 11, 718–728.

Vinnik, L. P., 1981, Evaluation of the effective cross-section of scattering in the lithosphere, Phys. Earth Planet. Interiors., 26, 268–284.

Vostrikov, G. A., 1975, Estimate of the seismic moment of local earthquakes from the tail portion of the seismogram (English Trans.), Dokl. Akad. Nauk SSSR, 220, 9–12.

Wesley, M. P., 1965, Diffusion of seismic energy in the near range, J. Geophys. Res., 70, 5099–5106.

Wilson, M., M. Wyss, and R. Koyanagi, 1983, Temporal attenuation change in the Koae fault system, southern Hawaii [abs.], EOS Transactions AGU, 64, 761.

Wu, R. S., 1982a, Attenuation of short period seismic waves due to scattering, Geophys. Res. Lett., 9, 9–12.

Wu, R. S., 1982b, Mean field attenuation and amplitude attenuation due to wave scattering, Wave Motion, 4, 305-316.

Wu, R. S., 1984a, Seismic wave scattering and the small scale inhomogeneities in the lithosphere. Ph. D. Thesis Massachusetts Institute of Technology, Cambridge, Massachusetts.

Wu, R. S., 1984b, The scattering pattern of a single scatterer in an elastic medium and coda wave, DARPA report.

Wu, R. S., 1985, Multiple scattering and energy transfer of seismic waves, separation of scattering effect from intrinsic attenuation, I., theoretical modeling, Geophys. J. R. astr. Soc., 82, 57–80.

Wu, R. S., and K. Aki, 1985a, Scattering characteristics of elastic waves by an elastic heterogeneity, Geophysics, 50, 582–595.

Wu, R. S., and K. Aki, 1985b, Scattering of elastic waves by a random medium and the small scale inhomogeneities in the lithosphere, J. Geophys. Res., 90, 10261–10276.

Wyss, M., 1985, Precursory phenomena before large earthquakes, Earthq. Predict. Res., 3, 519–543.

Yamakawa, N., 1962, Scattering and attenuation of elastic waves, Geophys. Mag., 31, 63–103.

Ying, C. F., and R. Truell, 1956, Scattering of a plane longitudinal wave by a spherical obstacle in an isotropically elastic solid, J. Appl. Phys., 27, 1087–1097.

Zapolskii, K. K., 1960, Measurements of the level and spectral composition of short period microseisms (in Russian), Trudy Inst. Geol. Geofiz., 10, 153–164.

Zapolskii, K. K., 1971, The frequency selective seismograph station ChISS, Experimental Seismology (in Russian), Nauka Publishing House, Moscow, 20–36.

Zener, C. M., 1948, Elasticity and Anelasticity of Metals, University of Chicago Press, Chicago.

Zhadin, V. V., 1979, Temporal variations in the effective Q factor of the upper layers in the lithosphere that are associated with the Ozernoy earthquake of November 22, 1969 and the Ust Kamchatsk earthquake of December 15, 1971 (in Russian), Dokl. Akad. Nauk SSSR, 248, 580–583.

Zhang, Y. G., and G. Z. Yang, 1983, Determination of Q-value by the coda wave for a portion of Sichuan Province (in Chinese, English abs.), Acta Seism. Sinica, 5, 304–312.

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