SEDIMENT-TRANSPORT CURVES

by G. Douglas Glysson

U.S. GEOLOGICAL SURVEY
Open-File Report 87—218

Reston, Virginia
1987

DEPARTMENT OF THE INTERIOR
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CONVERSION FACTORS

For use of readers who prefer to use metric units, conversion factors for terms used in this report are listed below:

Multiply	By	To Obtain
inch (in)	2.540	centimeter (cm)
square inch (in^2)	6.452	square centimeter (cm^2)
foot (ft)	0.3048	meter (m)
foot per second (ft/s)	0.3048	meter per second (m/s)
foot squared (ft^2)	0.09294	square meter (m^2)
cubic foot (ft^3)	0.02832	cubic meter (m^2)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
mile (mi)	1.609	kilometer (km)
square mile (mi^2)	2.590	square kilometer (km^2)
acre	0.4047	hectare
acre-foot (acre-ft)	1,233.	cubic meter (m^3)
ton (t)	0.9072	megagram (Mg)
ton per square mile (t/mi^2)	0.3503	megagram per square kilometer (Mg/km^2)
gallon (gal)	3.785	liter (L)
ounce (oz)	28.35	gram (g)

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ABSTRACT

This report describes the process of developing sediment-transport curves. It discusses the choice of dependent and independent variables, procedures for developing a transport curve, and the effects that seasons, major sediment transporting events, and timing of peaks can have on the shape of sediment-transport curves. Examples of the visual fit, linear regression, and group average methods are given. Problems associated with computer generated transport curves and potential errors are also discussed.

INTRODUCTION

The relation between water discharge and sediment discharge for a sediment-sampling site is frequently expressed by an average curve. This curve, generally referred to as a sediment-transport curve, is constructed on logarithmic paper. It is widely used to estimate sediment concentration or sediment discharge for periods when water-discharge data are available but sediment data are not. This relation is sometimes referred to as a 'sediment-rating' curve. The term is not descriptive because it infers a cause and effect relation and that a specific value of sediment concentration exists for each discrete value of streamflow.

The transport curve should not be considered as a reliable substitute for detailed observed data when planning the data-collection phase of a project. The reliability of sediment discharges computed from the transport curve depends upon the quantity and reliability of data used to define the curve and whether the data are representative of water and sediment discharge occurring during the period for which sediment discharges are estimated. Colby (1964, p. A2-3) states:

> The relationship of sediment discharge to characteristics of sediment, drainage basin, and streamflow are complex because of the large number of variables involved, the problems of expressing some variables simply, and the complicated relationship among the variables. At a cross section of a stream, the sediment discharge may be considered to depend on depth, width, velocity, energy gradient, temperature, and turbulence of the flowing water; on size, density, shape, and cohesiveness of particles in the banks and beds at the cross section and in upstream channels; and on the geology, meteorology, topography, soils, subsoils, and vegetal cover of the drainage area. Obviously, simple and satisfactory mathematical expression for such factors as turbulence, size, and shape of the sediment particles in the streambed, topography of the drainage basin, and rate, amount, and distribution of precipitation are very difficult, if not impossible, to obtain.

In order to develop meaningful and useful sediment-transport curves, the causes of sediment movement and the source of the sediments must be understood. It is also important to have a good understanding of the surface-water hydrology of the basin being studied.

PURPOSE AND SCOPE

The purpose of this report is to give the reader a general introduction to developing sediment-transport curves and to point out some potential problems associated with developing and using sediment-transport curves. It is not meant to be a rigorous statistical analysis of the development of transport curves or of the errors associated with using them. The report includes discussions of the choice of dependent and independent variables, types of transport curves, how to develop a transport curve, problems associated with computer-fitted curves, and addresses potential sources and magnitudes of errors in estimating sediment discharge from transport equations.

CHOICE OF DEPENDENT AND INDEPENDENT VARIABLES

In most regression studies that relate to quality of water, the statistically independent variables are often interrelated, some of them closely interrelated. The study of sediment transport is no exception. In general, regression relations merely indicate how one variable changes with changes in other variables. In the case of sediment-transport curves, the change in sediment discharge is estimated by changes in water discharge.

Sediment-transport curves should be constructed with sediment discharge as the dependent variable. Convention requires the dependent variable be plotted as the ordinate. Figure 1, based on Colby (1956), shows two curves derived with the same data but with different independent variables. The curve represented by the solid line assumes that water discharge is the independent variable and thus should be used to compute average sediment discharge for a given water discharge. The dashed curve of figure 1 assumes that water discharge is the dependent variable and is based on the average water discharge values for a small range in sediment discharge. If there is a wide scatter of points about this curve, its use may produce incorrect results throughout the upper and/or the lower end of the curve. Upward or downward extension of the dashed curve will also give inaccurate sediment discharges.

The independent variables should not only be chosen correctly, but the variables should be expressed in meaningful terms. Water discharge should not be used directly as an independent variable for a relation to be applied to drainage basins of different sizes, but should be expressed as flow per unit area or as a ratio to average flow. The need for meaningful terms for the independent variables generally is greater if the defined relations are to be applied at more than one site. Meaningful terms also are desirable for relations at one site.

Sediment-transport curves may be constructed with either sediment concentration or sediment discharge as the dependent variable. In graphical analyses, the plot of sediment discharge against water discharge has less

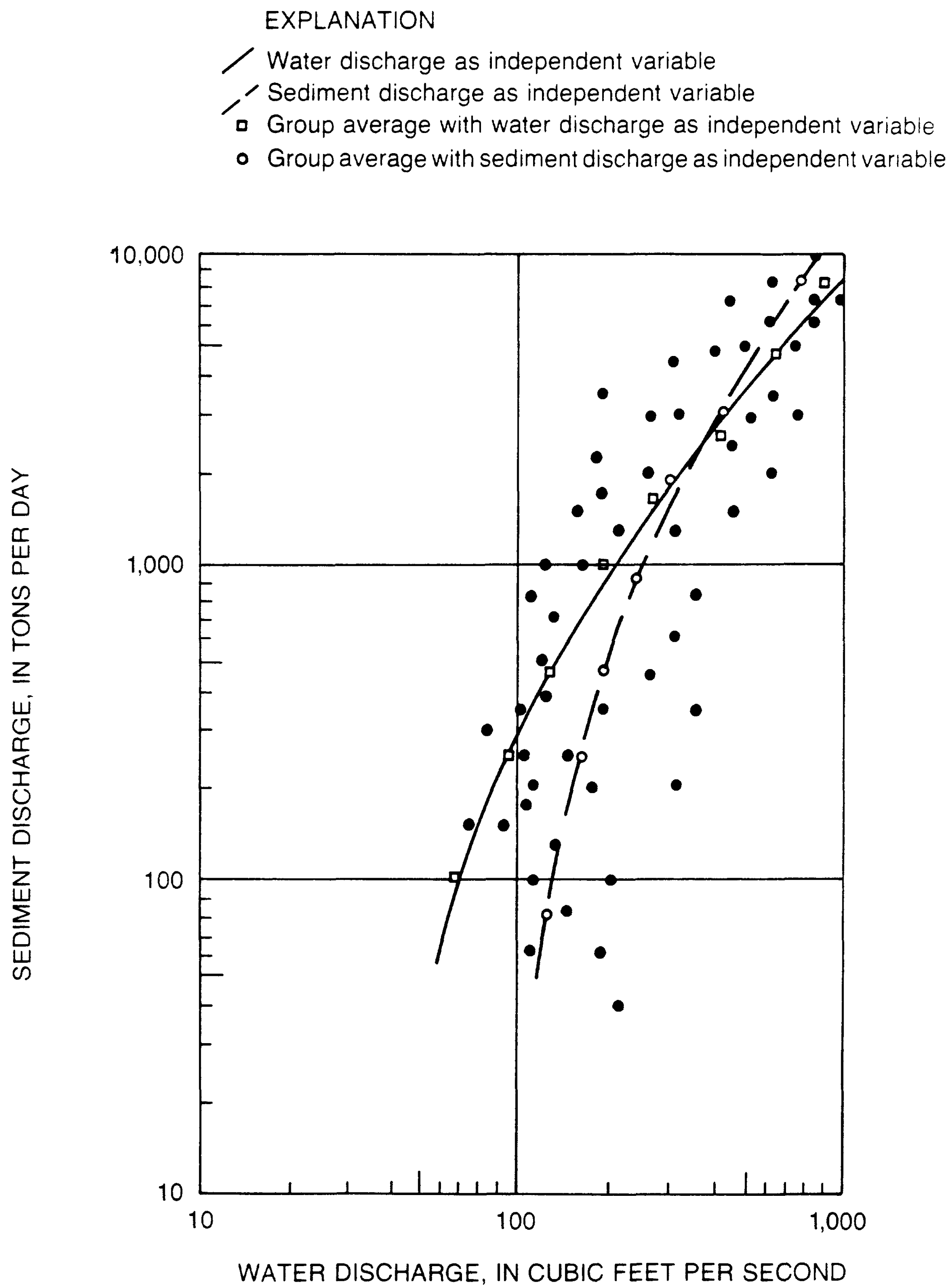

Figure 1.--Comparison of curves based on the same data but with different independent variables (based on Colby, 1956).

scatter than does the plot of sediment concentration and can be better fitted by eye. Mathematically, however, the two relations will produce identical results (Rantz, 1968). It is a common practice to construct the transport curve in the variable that is to be estimated; that is, if concentrations are to be estimated, then the curve is constructed as concentration vs. water discharge and likewise if sediment discharge is to be estimated, then the sediment discharge vs. water discharge form is used.

TYPES OF TRANSPORT CURVES

A sediment-transport curve is the curve that defines the average relation between sediment discharge and water discharge. According to Colby (1956), sediment-transport curves may be classified according to either the period of the basic data that defines a curve or the kind of sediment discharge that a curve represents. Sediment-transport curves based on the period of the basic data may be classified as instantaneous, daily, monthly, seasonal, annual, or flood- or storm-period curves. The instantaneous sediment-transport curves are defined by concurrent measurements of sediment discharge and water discharge for periods too short to be substantially affected by changes in flow or concentration during the measurements. Daily, monthly, seasonal, annual, and flood-period sediment-transport curves usually are defined by and expressed as average sediment (tons per day) and water discharges (cubic feet per second) for periods of days, months, years, or storm periods. They can be defined by and expressed as total quantities of sediment (tons) and water discharges (acre-feet) during the respective lengths of time.

On the basis of the kind of sediment that the data represent, sediment-transport curves may be used to define the suspended-sediment load, unsampled- or unmeasured-sediment load, bedload, bed-material load, and total-sediment load. These transport curves may be further subdivided according to size of particles for which the defining sediment discharges were computed.

DEVELOPING A SEDIMENT-TRANSPORT CURVE

It should be pointed out that just because a straight line may be fitted through a set of points, this does not mean that it will accurately define the relation between the variables. Consider, for example, the curves shown in figure 2. Figure 2a shows an example of how a straight line was incorrectly fit to a much more complex relation. Figure 2b is an example of where three straight line curves would much more accurately define the relation between x and y than would a single straight line curve.

The sediment-transport curve is normally plotted on log paper. Commonly 5 by 3 log cycle paper is used, with the sediment discharge being plotted on the 5-cycle side. If additional cycles are needed, they may be cut and spliced on. Use of a standard log paper facilitates comparison of plots from different years at the same site and between different sites.

Methods commonly used to construct the line that represents the relation between streamflow and sediment discharge include visual fit, group average, and linear regression of log-transformed data.

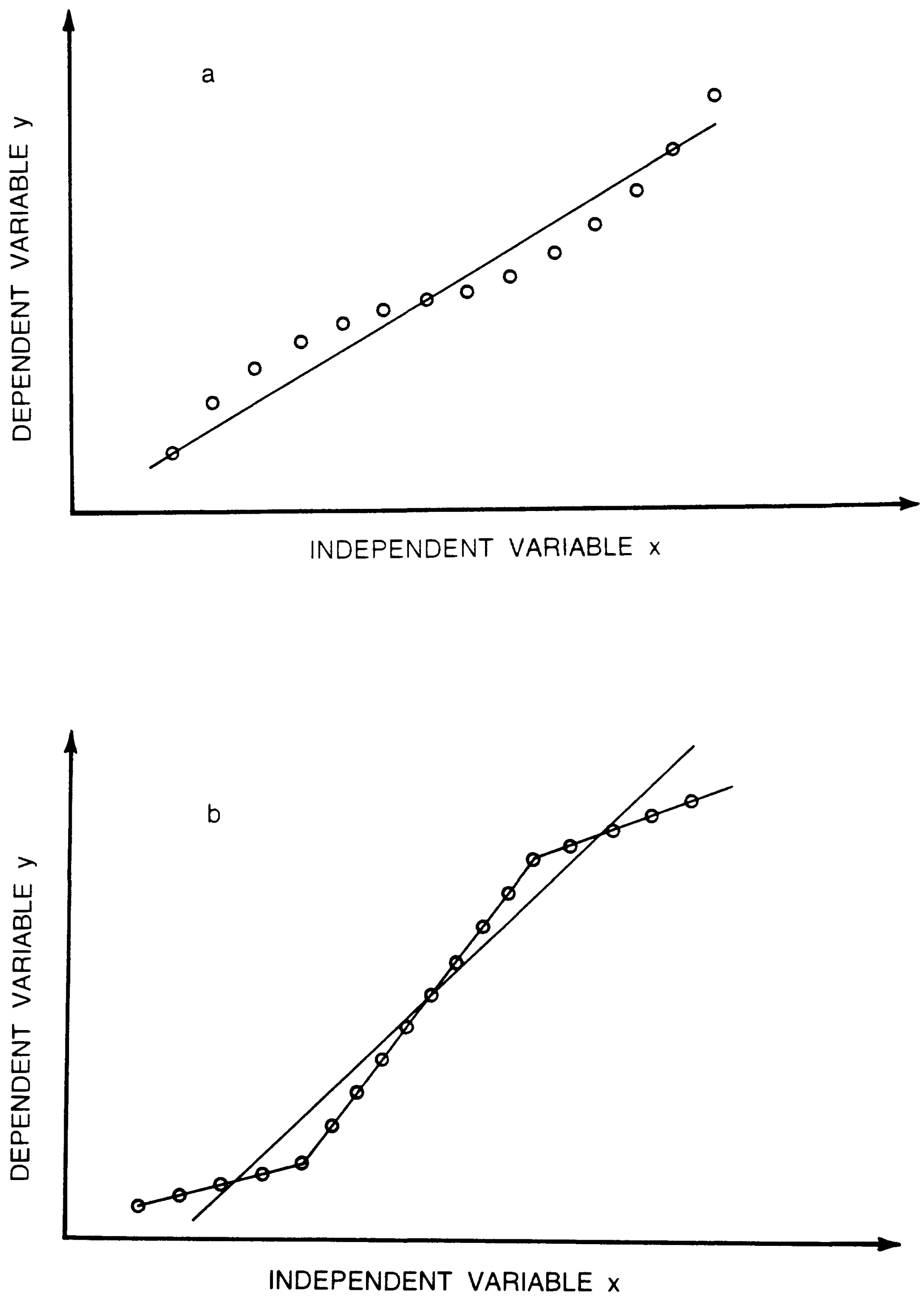

Figure 2.--Examples of single straight lines not adequately defining the relation between dependent and independent variables.

To avoid misinterpretation, a preliminary graphical analysis should always be the first step in developing a sediment-transport curve. This graphical analysis may reveal significant relations that might never be noted or understood if mathematical analysis were applied without a preliminary analysis. A simple plot of sediment discharge or concentration versus water discharge often will indicate whether the relation is simple or complex. If the relation is complex, examination of a plot of the data may indicate a rational method for applying a correct mathematical solution. Some of the questions that should be considered during the preliminary analysis include:

1. Should the relation be one or more straight lines or a curve?

2. Are the data adequate to establish a relation over the entire range of water discharge expected at the site being rated?

3. Do the data cover both dry and wet periods, winter and summer seasons, and all phases of the hydrograph?

4. Are there atypical years or events contained in the data which could incorrectly bias the relation?

5. Especially at the upper end of the curve, are the data representative of a number of events or are they predominantly from a single event?

There are several factors that can have an effect on the shape, slope, and intercept of the sediment-transport curve. Some of the more major ones are: (1) seasons, (2) timing between sediment concentration peak and water discharge peak, and (3) extreme high-water events.

Seasons can have a significant effect on sediment yield, especially in the more humid areas. During winter the ground may be frozen and precipitation may be in the form of snow. As the snow melts, it runs off the frozen ground. The factors of (1) the absence of raindrop impact to loosen the soil and (2) the frozen ground holding together better, combine to produce lower yields. During the summer when high intensity storms are prevalent, raindrop impact is high and thus sediment concentrations are higher. An additional complication may also occur where a large area of the drainage basin is used for agricultural purposes, such as the Midwest. Typically the fields are bare during the winter and spring, but as the crops, such as corn, soybeans, and wheat, grow, the soil becomes protected from erosion by the plants. In these cases, sediment yields for a given discharge may be low in the winter (frozen ground) and summer (high crops) and higher in the spring (before planting and growth of the crops) and fall (after harvest).

Figure 3 is an example of differences in sediment yields between winter type storms and summer type storms (based on Curtis and others, 1978, p. 51). The two lines fitted to the points are obviously not well defined. There is a considerable amount of scatter and even some overlap between winter and summer type storms. However, it should be noted that it is not uncommon in cases like this to have a constant slope to the sediment-transport curve, with the seasonal effect only being shown in the y intercept. Figure 3 shows how a series of parallel curves might better define the changing sediment-transport relation at the station than would any single curve. Thus by using the constant slope and recent samples, the curve may be adjusted up or down when trying to estimate a sediment discharge for an unsampled storm.

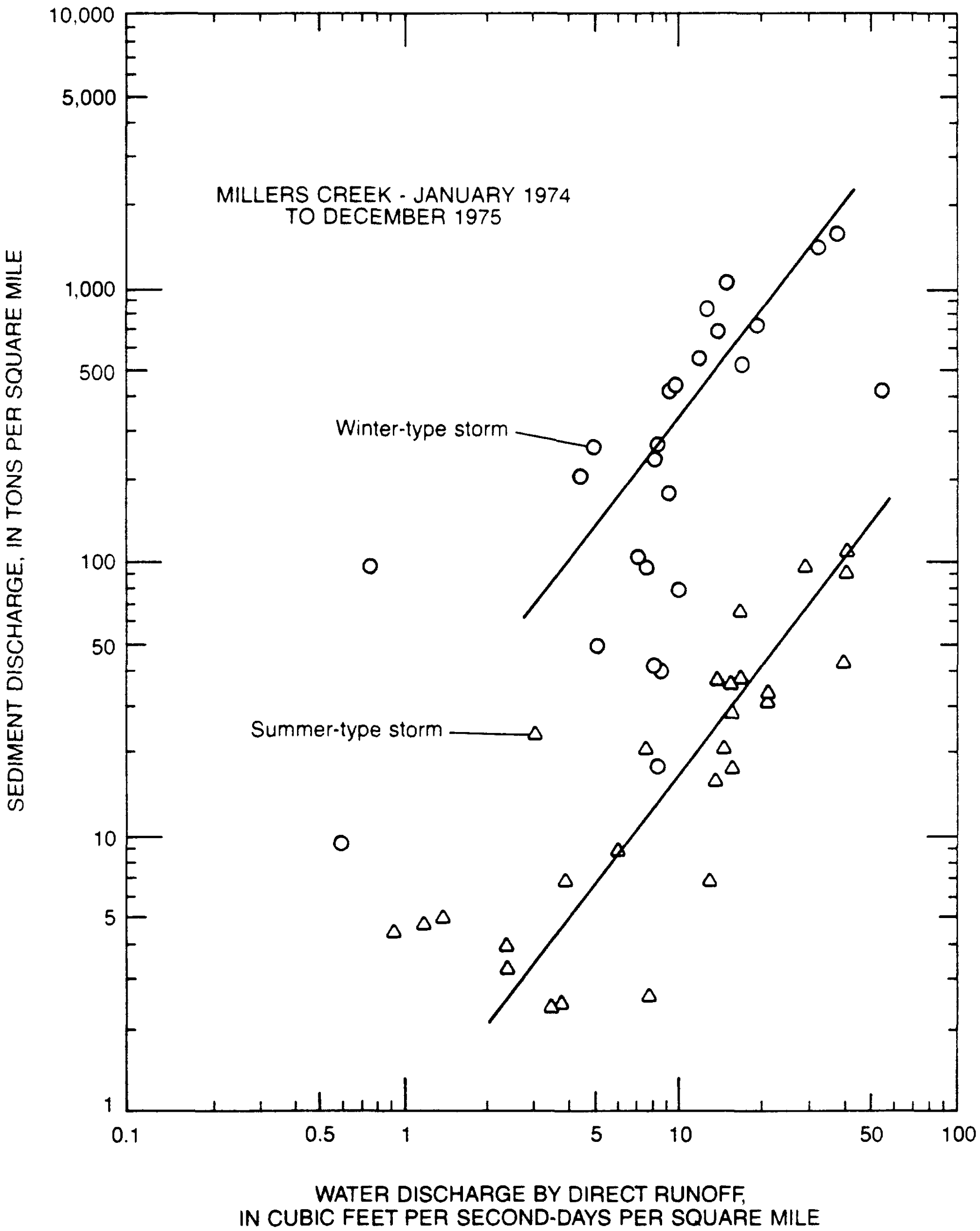

Figure 3.--Relation of sediment discharge to water discharge by storms for Millers Creek near Phyllis, Kentucky (modified from Curtis and others, 1978).

The timing between the sediment concentration peak and the water discharge peak can also drastically affect the shape of the sediment-transport curve. Figures 4, 5, and 6 illustrated this effect. All three figures have the same surface-water hydrograph and sediment hydrograph. The only change is the timing of the peak. In figure 4 the sediment and water peaks coincide, in figure 5 the sediment peak is approximately 5 hours ahead of the water peak, and in figure 6 the sediment peak lags the water peak by 5 hours. These figures not only show how the timing of the peaks can affect the shape of the sediment-transport curve but it also shows how the timing of the samples can affect the results. If samples were only collected on the peak and during recessional periods, no problem would be encountered in figure 4; however, at a station where concentration and water discharge peaks are not coincident, such as figure 5, a completely erroneous curve might be developed. If no samples were collected on the rise at this type station, then only samples such as 5-8 would be plotted. A transport curve such as figure 7 would be drawn, and have very little scatter to it. By not knowing what the rising hydrograph looks like, a peak concentration of about 5,500 milligrams per liter (mg/L) would be estimated for this storm when the true peak concentration was 8,300 mg/L. The same type of problem can arise when the sediment concentration peak lags the water discharge peak.

Quite often a catastrophic event will significantly change the slope and/or shape of the sediment-transport curve. Knott (1971) presented figure 8 which shows sediment-transport curves for the Middle Fork Eel River below Black Butte River near Covelo, California, for the water years 1963-68. On December 22, 1964, a flood having an approximate reoccurrence interval of 75 years (Young and Cruff, 1967) occurred at this site. It is apparent from figure 8 that the 1964 flood caused considerable change to the sediment transport-water discharge relation. Even by 1968, the upper end of the transport curve had not returned to its pre-flood position.

Another major problem one can encounter in constructing sediment-transport curves is when insufficient samples have been collected to define the curve. Consider the example shown in figure 9. These are the same data as presented in figure 6 earlier but with fewer samples shown. A casual look at the plotted data would suggest that the dashed line would not be a bad fit of the data. However, as shown in figure 3, the dashed line would be incorrect. One way to help avoid this kind of error is to plot all the data for a station for the period of record. Subsequent subdivisions by season or rising or falling trends can then be analyzed. At periodically sampled stations, it may take several years of data collection to obtain sufficient samples to adequately define a sediment-transport curve.

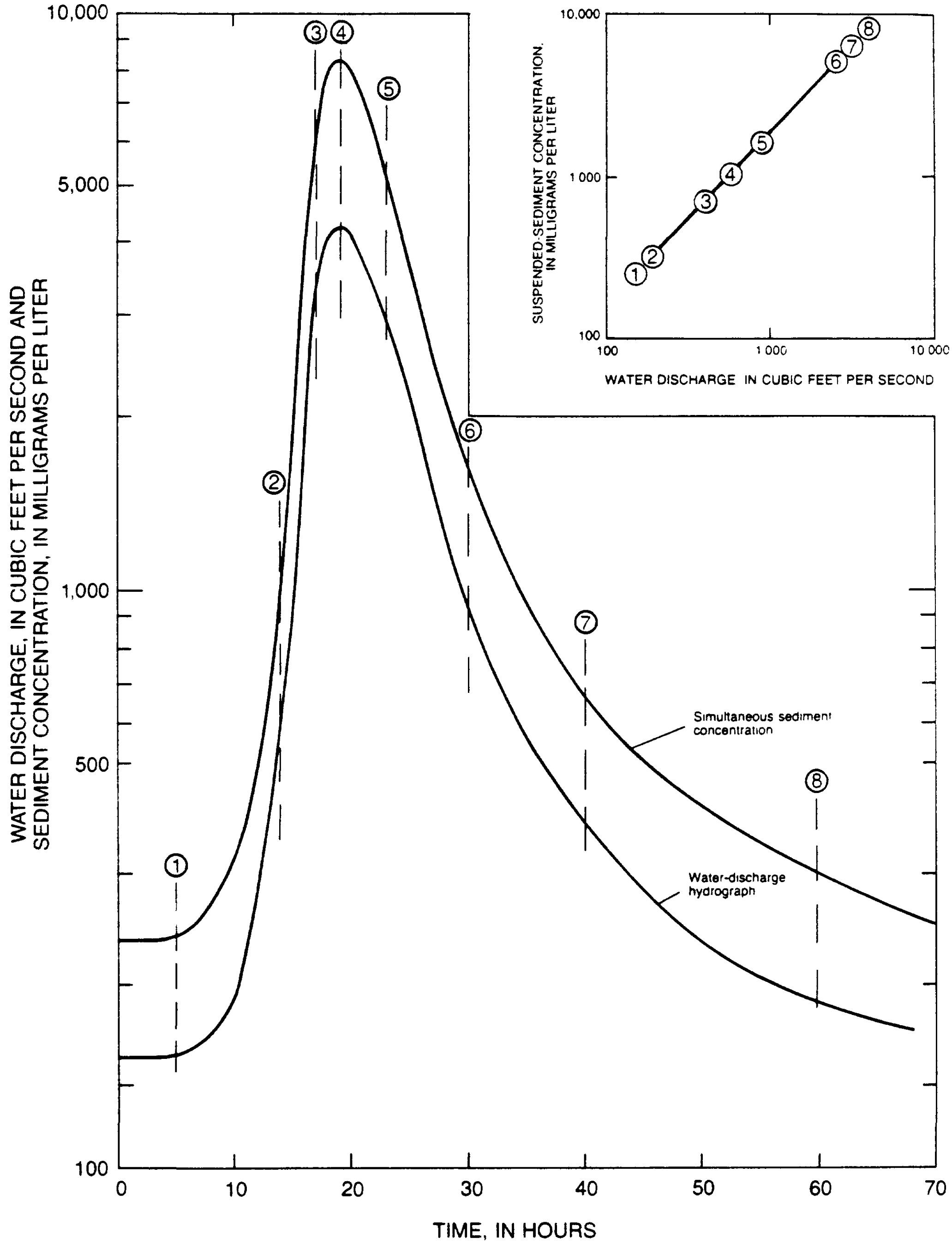

Figure 4.--Simultaneous sediment concentration and water discharge peaks.

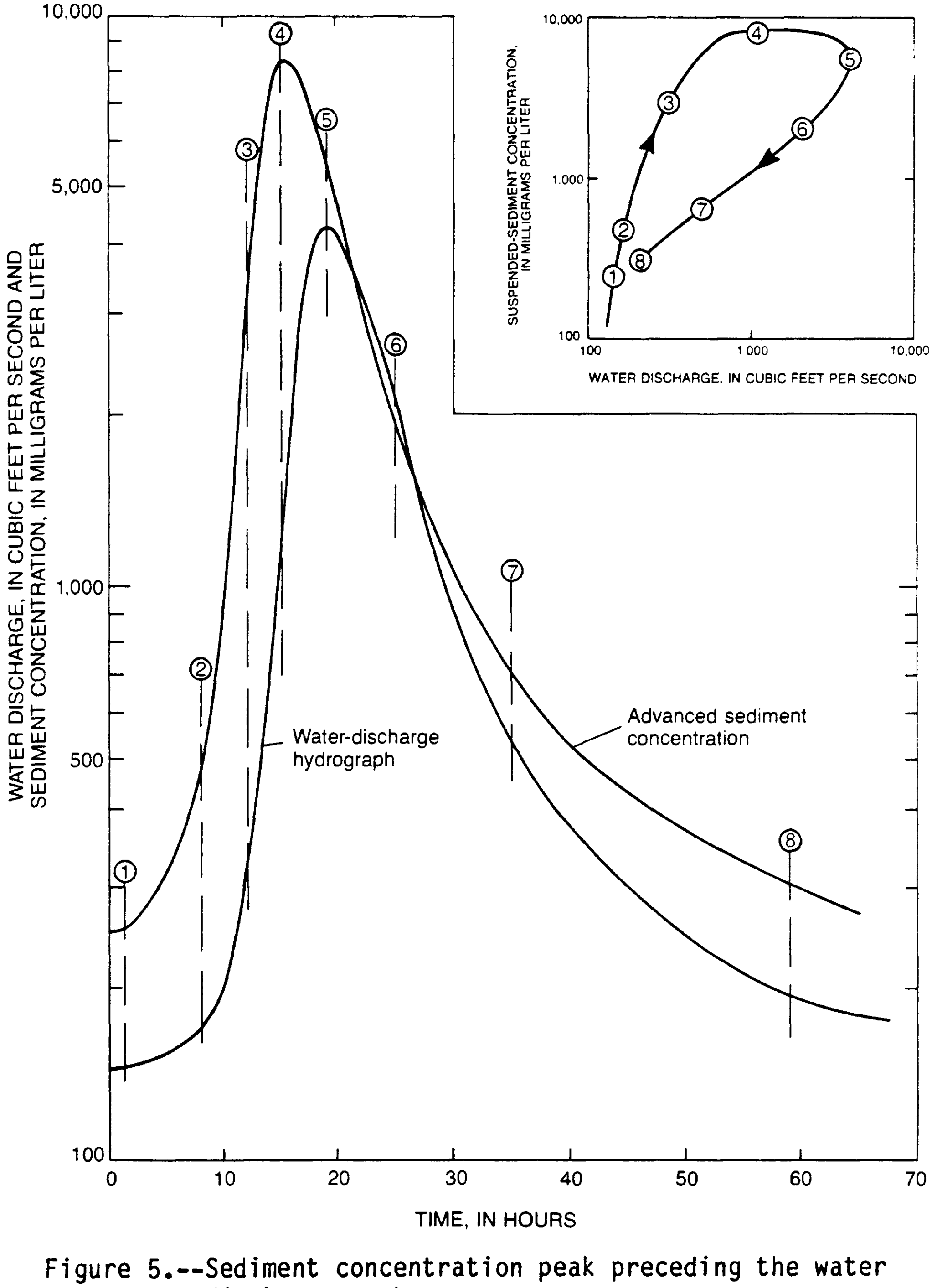

Figure 5.--Sediment concentration peak preceding the water discharge peak.

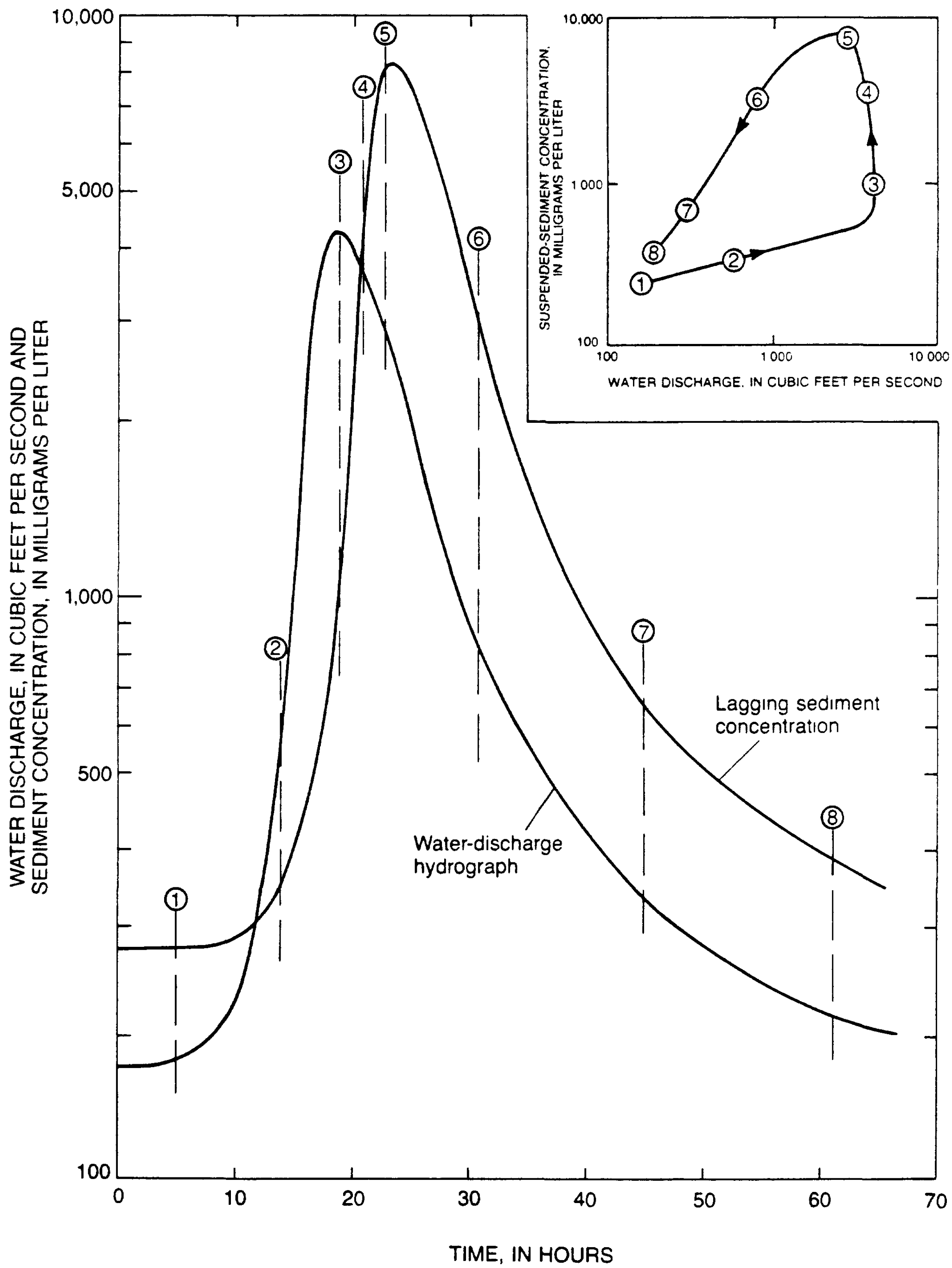

Figure 6.--Sediment concentration peak lagging the water discharge peak.

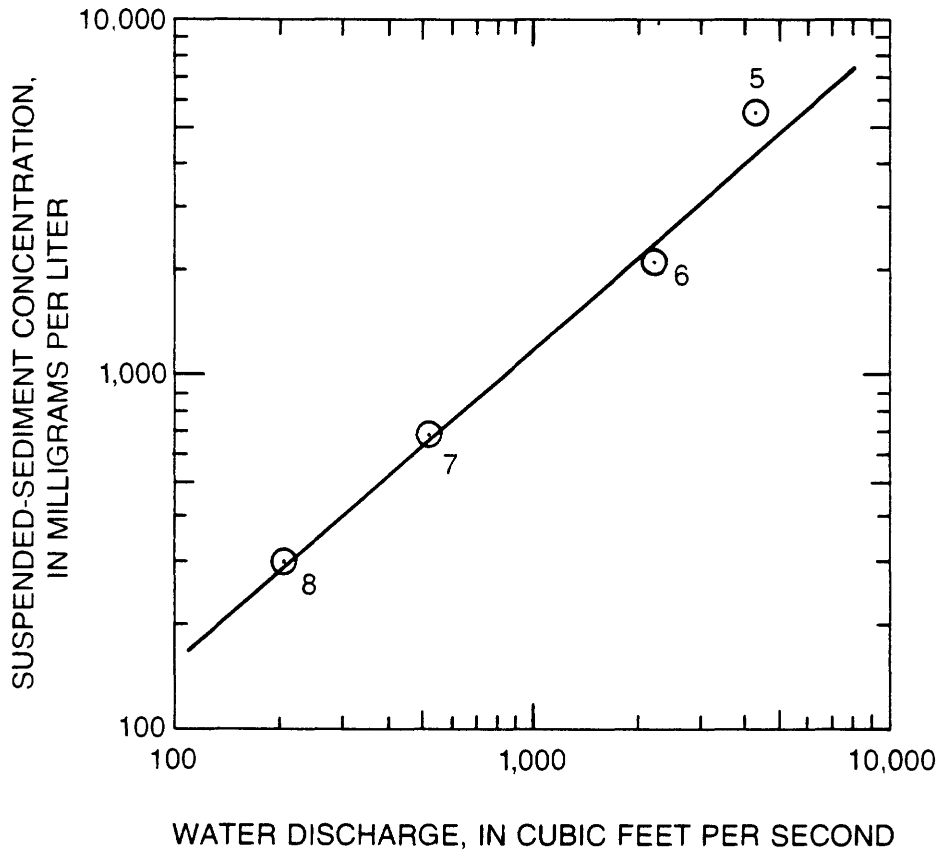

Figure 7.--Sediment-transport curve based on recession samples for a site where the sediment concentration peak precedes the water discharge peak (see fig. 5).

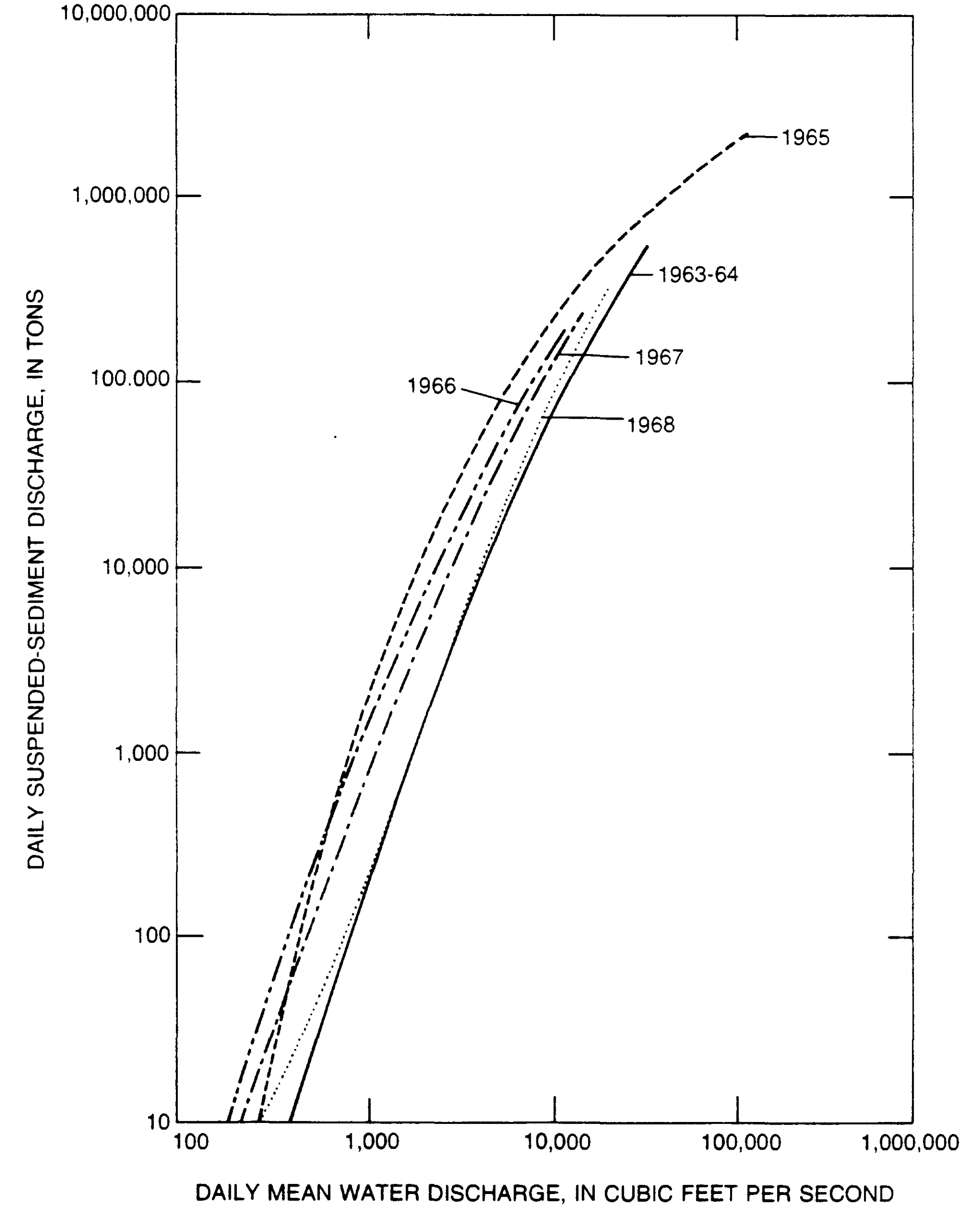

Figure 8.--Relation of suspended-sediment discharge to water discharge at Middle Fork Eel River below Black Butte River near Covelo, California, 1963-68 water years (Knott, 1971).

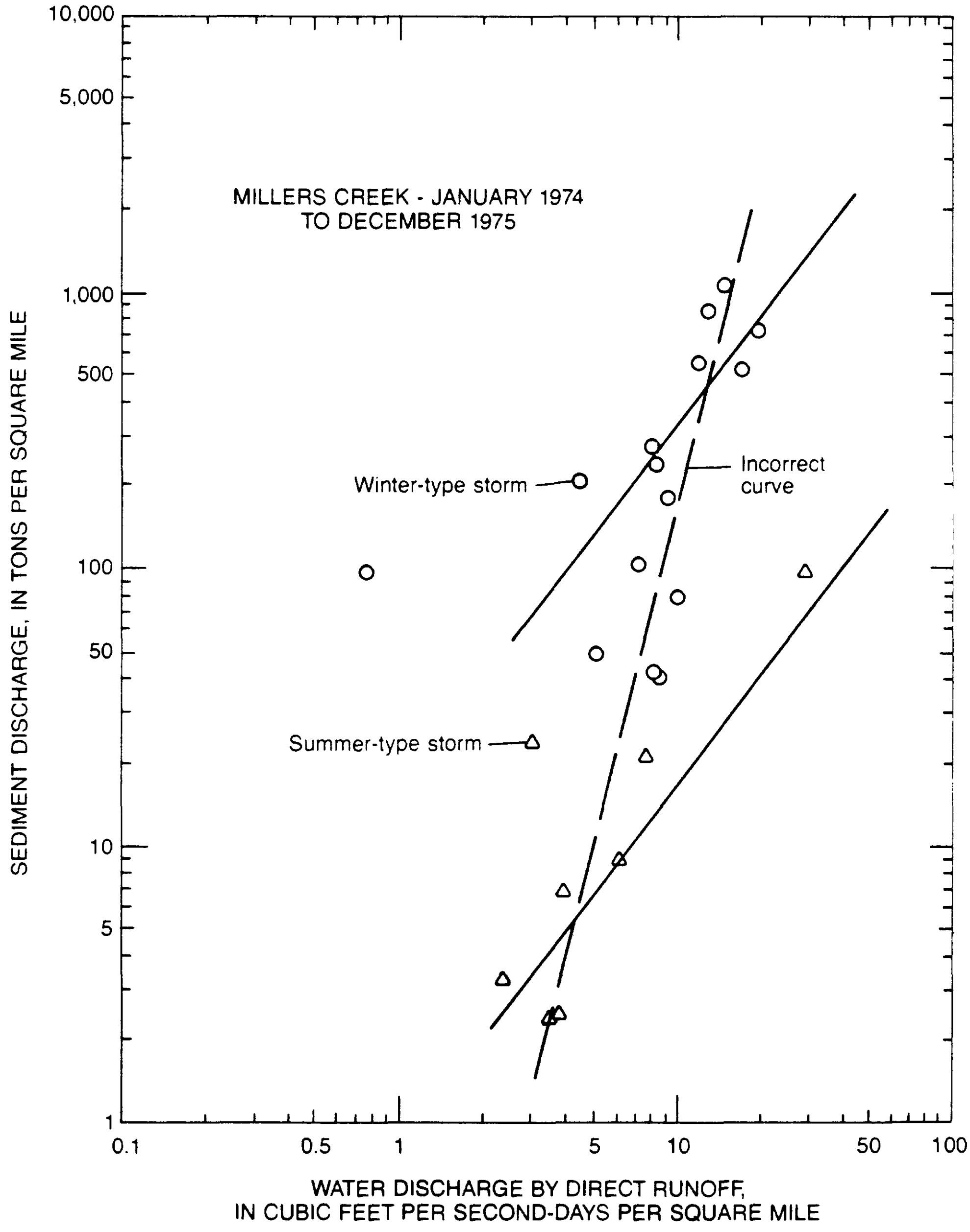

Figure 9.--Incorrect relation of sediment discharge to water discharge by storms for Millers Creek near Phyllis, Kentucky.

VISUAL FIT

A line drawn through the data points, visually, that appears to represent the best average or "fit" is referred to as a visual fit. It is probably the simplest method of constructing a sediment-transport curve and formerly was, and perhaps still is, the most common procedure. Figure 10 is an example of a sediment-transport curve drawn by eye. When the data are tightly grouped as in figure 10, fitting the curve is not much of a problem. However, even if the data do not scatter much, the analyst should be aware of the problems discussed in the previous sections. The data points should be labeled as to date of collection and timing relative to the water peak (that is, rise, peak, or falling) so that a better analysis of the data set can be made.

LINEAR REGRESSION

A linear regression equation (least square) provides a line about which the sum of the squares of the deviations, that is, the difference between the line value and the observed value is a minimum.

If the relation between sediment discharge and water discharge data on logarithmic paper is linear, this relation may be expressed in the form of

$$Q_s = aQ^b \text{ or } \mathrm{Log}\ Q_s = \mathrm{Log}\ a + b\ \mathrm{Log}\ Q \qquad (1)$$

where Q_s is sediment discharge, in tons per day; Q is water discharge in cubic feet per second; and a and b are constants. The constants a and b are solved by applying the equations for a simple linear regression (Riggs, 1968, p. 11) to logarithmic transformed values of water discharge and sediment discharge. The equations are

$$b = \frac{\Sigma \log Q \log Qs - N\ \overline{\log Q}\ \overline{\log Qs}}{\Sigma(\log Q)^2 - N(\overline{\log Q})^2} \qquad (2)$$

$$\log a = \overline{\log Qs} - b\ \overline{\log Q} \qquad (3)$$

$$\text{and } a = 10^{(\overline{\log Qs} - b\ \overline{\log Q})} \qquad (4)$$

where $\overline{\log Q}$ = average of water discharge values (logarithms),

$\overline{\log Qs}$ = average of sediment discharge values (logarithms), and

N = number of paired observations.

Standard statistical computations showing how well the data fit the equation such as the square of the correlation coefficient (r^2), variance of X (S_x^2), variance of y (S_y^2), and standard error of estimate ($S_{y,x}$) can be found in Riggs (1968, p. 11) or most other books on statistics. (Note: The statistical values computed during the regression analysis are based on the logarithmic values and therefore do not minimize the sum of the squared deviations of the actual data from the regression line.)

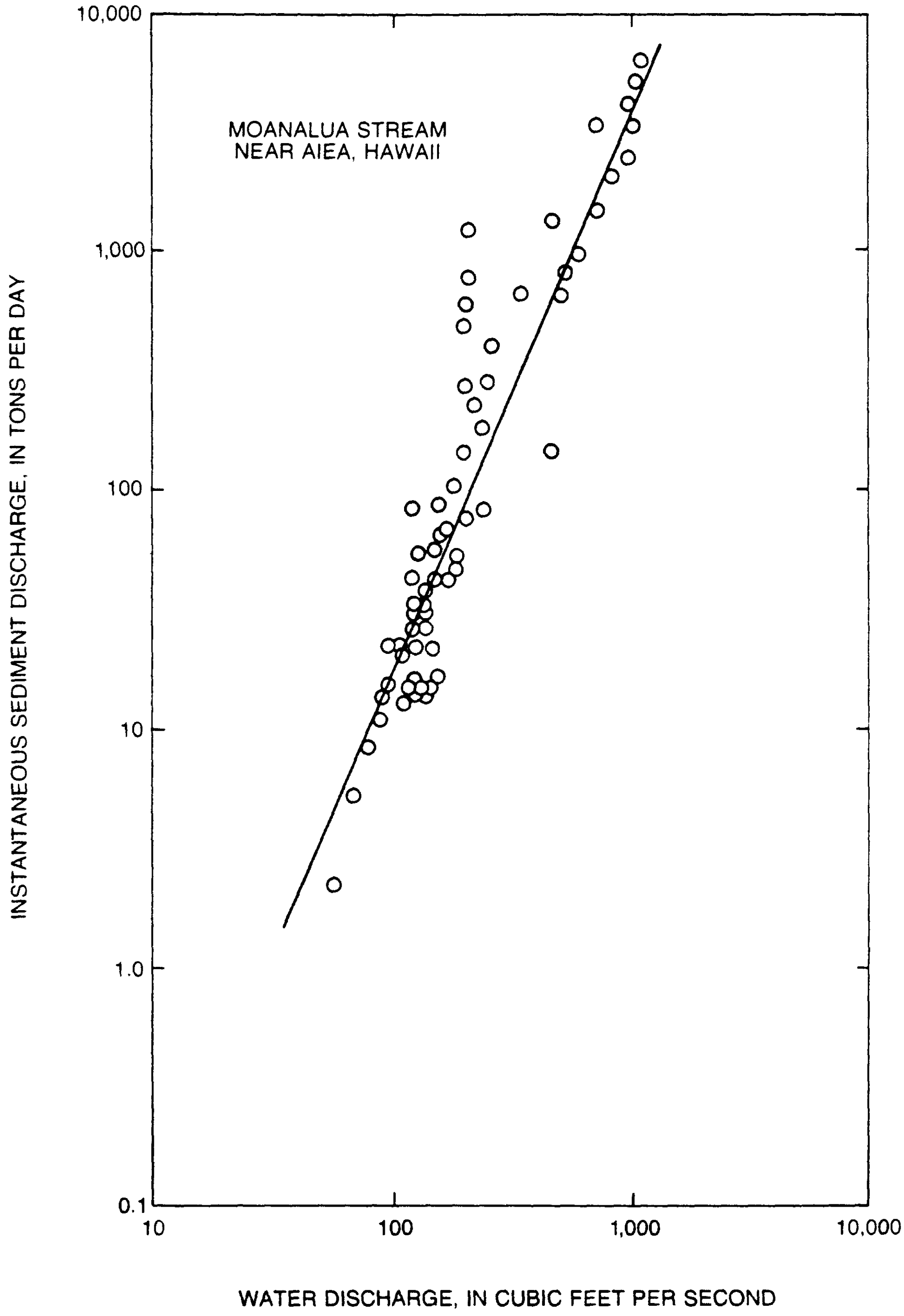

Figure 10.--Instantaneous sediment-transport curve fitted by eye (modified from Jones and Ewart, 1973).

Regression equations for sediment concentration versus water discharge can be put into a similar form as those for sediment discharge.

If the equation

$$Qs = 0.0027CQ \tag{5}$$

which is used to compute sediment discharge from sediment concentration is substituted into equation 1, the relation between sediment concentration and water discharge becomes

$$0.0027CQ = aQ^b$$

or

$$C = a'Q^{b'} \tag{6}$$

where C = sediment concentration, in milligrams per liter,
$a' = a/.0027$, and
$b' = b - 1$.

The use of the linear regression method generally is preferred over that of the visual method if the preliminary analysis indicates that the transport curve can be represented by four or less straight lines.

Several authors have recently expressed concern about fundamental statistical bias in load estimates from rating curves, particularly when the curves are established by linear regression on log-transformed data (Ferguson, 1986; Koch and Smillie, 1986). The bias can be substantial if the standard error of the log-residuals is large; Ferguson reports systematic underestimation by as much as 50 percent. It is often desirable to eliminate such bias, and the papers (independently) suggest a method for obtaining unbiased estimates. However, the correction they recommend is seriously flawed, and may exacerbate the problem of bias rather than solve it (T. A. Cohn, written commun., 1987). The exact solution to the statistical problem when errors are normally distributed in log space is provided by Bradu and Mundlak (1970). Another possible solution is proposed by Duan (1983), which does not depend on any distributional assumption. However, no careful empirical analysis has been made to date on the performance of any of these solutions to the problem. If care is taken to ensure the transport curves are fitted through the points at the high end of the curve and the variance of the residuals is small, errors in estimating sediment discharges caused by this bias should be small.

Examples of Linear Regression Method

The following are examples of how to compute sediment-transport curves using the linear regression technique. The examples used are for streams typical of the Pacific Slope Region of the western United States (Eel River at Scotia, California) and the Piedmont Region of the eastern United States (Yadkin River near Yadkin College, North Carolina). Sediment discharges estimated by using the regression equation are compared with historical records to illustrate the accuracy of the method.

Eel River at Scotia, California

In this example, 29 samples of suspended-sediment concentration and water discharge obtained at Eel River at Scotia, California, during the 1958-60 water years are used. The procedure for defining the transport curve using the linear regression method is as follows:

Step 1. List instantaneous water discharge and concentration data in chronological order (table 1).

Step 2. Compute sediment discharge using equation 5.
Col. 5 = 0.0027 x col. 3 x col. 4.

Step 3. Plot water discharge (abscissa) and sediment discharge (ordinate) on logarithmic paper (fig. 11).

Step 4. Analyze the plotted data to determine if a straight line could reasonably be fitted to part or the entire range of plotted values. The analysis should begin with the upper part of the graph. These data are most important because they often represent the major fraction of sediment transported in a given year and because they represent the maximum values of measured sediment discharge. Also, often the transport curve must be extrapolated beyond the sampled data to estimate sediment discharge during years of extremely high discharge. In this example, it appears that a straight line could be fitted for flows ranging from about 20,000 to 150,000 cubic foot per second (ft^3/s). Examination of the data for flows less than 20,000 ft^3/s indicates that straight lines could be fitted for the range 2,000 to 20,000 ft^3/s and possibly another straight line for discharges less than 2,000 ft^3/s. In general, 10 or more data points per log cycle of water discharge are needed to define adequately a sediment-transport curve.

After making a preliminary analysis of the available data, it was decided to compute regression equations for three ranges of flow--high flow (20,000 to 151,0000 ft^3/s), medium flow (2,000 to 20,000 ft^3/s), and low flow (98 to 2,000 ft^3/s). It may be desirable to overlap the end points and to use some of the same data points to define two adjacent sections of the separate regression lines.

Step 5. List the data in groupings for the computation of each regression equation (table 2).

Step 6. Transform water discharge and sediment discharge to logarithmic equivalents (columns 4 and 6, table 2).

Step 7. Compute averages, sums, and products of logarithmic equivalents.

Step 8. Compute the exponent b (slope of line) using equation 2 and the intercept by using equation 4; a and b are usually computed to three significant figures. The constants for the three flow ranges are:

Table 1.--Instantaneous suspended-sediment and water discharge, Eel River at Scotia, California, 1958-60 water years

Date (1)	Time (2)	Water discharge (ft^3/s) (3)	Sediment concentration (mg/L) (4)	Sediment discharge (ton/d) (5)
10/04/57	1045	691	14	26
11/18/57	0820	9,880	327	8,720
12/17/57	1600	20,300	1,630	89,300
01/10/58	1700	36,500	1,980	195,000
02/25/58	1800	151,000	4,010	1,630,000
03/12/58	0915	8,110	230	5,040
03/22/58	1200	46,400	1,680	210,000
04/08/58	0900	32,100	663	57,500
05/13/58	0835	5,600	70	1,060
06/07/58	1400	1,970	14	74
09/02/58	0735	147	2	.79
11/05/58	0910	177	1	.48
12/09/58	0950	278	2	1.5
01/12/59	1100	124,000	5,830	1,950,000
02/18/59	0700	79,000	2,040	435,000
03/25/59	0810	6,680	140	2,530
04/29/59	1415	2,390	16	103
06/09/59	0800	546	4	5.9
08/05/59	1030	109	2	.59
09/15/59	1320	98	2	.53
10/20/59	1320	180	1	.49
12/03/59	1200	132	2	.71
01/12/60	1715	4,840	230	3,010
02/11/60	1435	53,300	2,640	380,000
03/02/60	1330	3,750	52	526
04/01/60	1320	4,170	43	484
06/04/60	1915	17,100	539	24,900
07/09/60	0730	580	3	4.7
09/16/60	1615	115	3	.93

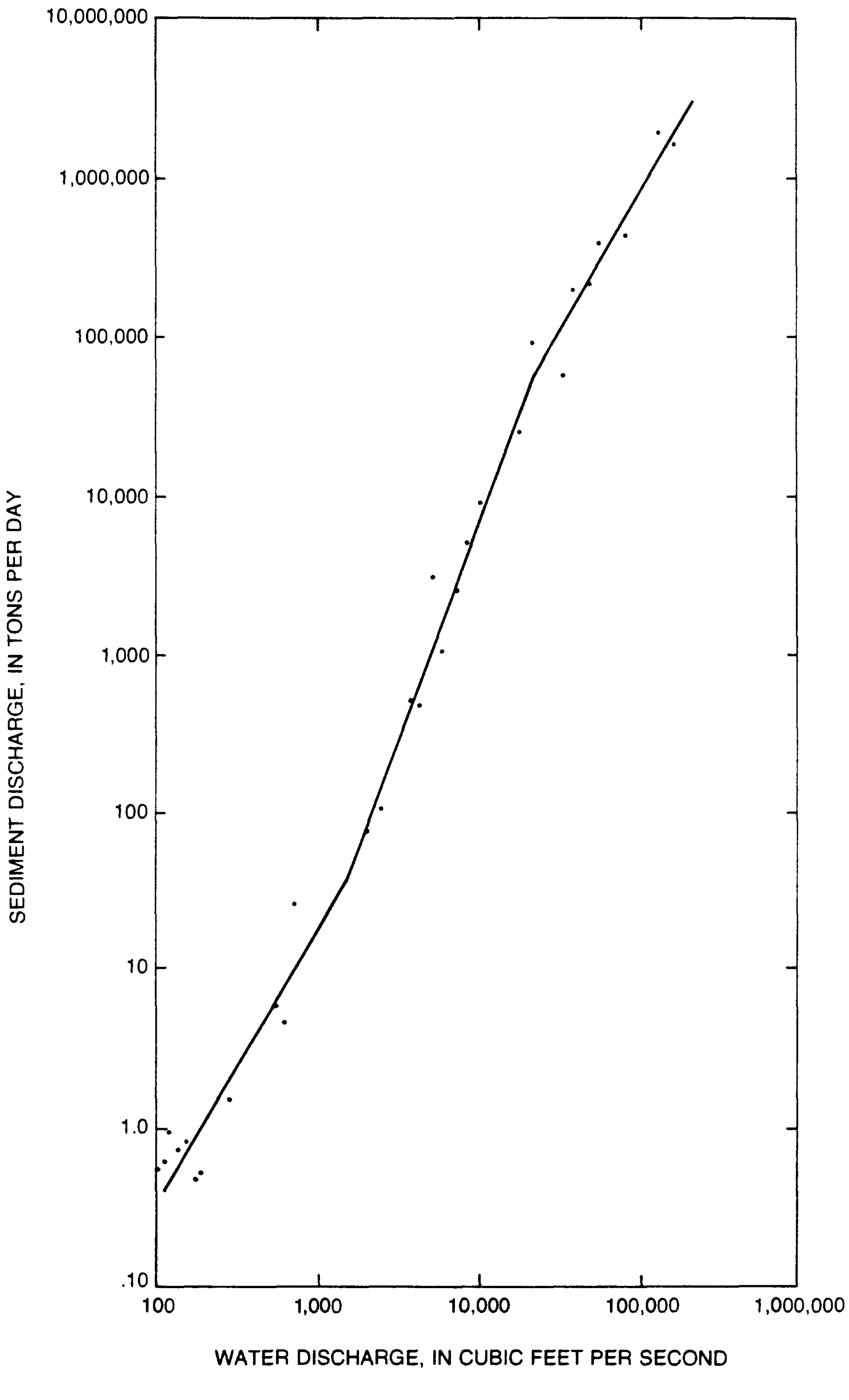

Figure 11.--Sediment-transport curve for Eel River at Scotia, California, 1958-60 water years.

Table 2.--Computation of logarithmic least-squares regression, Eel River at Scotia, California, 1958-60 water years

Flow range (ft^3/s)	Water discharge (ft^3/s)	Sediment discharge (ton/d)	log Q	$(\log Q)^2$	log Qs	log Q log Qs
98 to	98	0.53	1.991226	3.964981	-.275724	-.549029
2,000	109	.59	2.037426	4.151105	-.229148	-.466872
	115	.93	2.060698	4.246476	-.031517	-.064947
	132	.71	2.120574	4.496834	-.148742	-.315418
	147	.79	2.167317	4.697263	-.102373	-.221875
	177	.48	2.247973	5.053383	-.318759	-.716562
	180	.49	2.255273	5.086256	-.309804	-.698693
	278	1.5	2.444045	5.973356	.176091	.430374
	546	5.9	2.737193	7.492226	.770852	2.109971
	580	4.7	2.763428	7.636534	.672098	1.857294
	691	26	2.839478	8.062635	1.414973	4.017785
	1,970	74	3.294466	10.853506	.869232	6.158121
Average			2.413258		.290598	
Sum				71.714555		11.540149
2,000	2,390	103	3.378398	11.413573	2.012837	6.800164
to	3,750	526	3.574031	12.773698	2.720986	9.724888
20,000	4,170	484	3.620136	13.105385	2.684845	9.719504
	4,840	3,010	3.684845	13.578083	3.478566	12.817977
	5,600	1,060	3.748188	14.048913	3.025306	11.339416
	6,680	2,530	3.824776	14.628911	3.403121	13.016176
	8,110	5,040	3.909021	15.280445	3.702431	14.472881
	9,880	8,720	3.994757	15.958083	3.940516	15.741404
	17,100	24,900	4.232996	17.918255	4.396199	18.609093
Average			3.774128		3.262756	
Sum				128.705346		112.241503
20,000	20,300	89,300	4.307496	18.554522	4.950851	21.325771
to	32,100	57,500	4.506505	20.308587	4.759668	21.449468
151,000	36,500	195,000	4.562293	20.814517	5.290035	24.134690
	46,400	210,000	4.666518	21.776390	5.322219	24.836231
	53,300	380,000	4.726727	22.341948	5.579784	26.374116
	79,000	435,000	4.897627	23.986750	5.638489	27.615216
	124,000	1,950,000	5.093422	25.942948	6.290035	32.037803
	151,000	1,630,000	5.178977	26.821803	6.212188	32.172779
Average			4.742446		5.505409	
Sum				180.547465		209.946074

High flow range -

$$b = \frac{209.946074 - (8)(4.742446)(5.505409)}{180.547465 - (8)(4.742446)^2} = 1.728$$

$$\log a = 5.505409 - (1.728)(4.742446) = -2.689538$$

$$a = 10^{(-2.689538)} = 2.044 \times 10^{-3}$$

Therefore:

$$Q_s = 2.04 \times 10^{-3}\ Q^{1.73}$$

Medium flow range -

$b = 2.780$
$a = 5.898 \times 10^{-8}$
$Q_s = 5.90 \times 10^{-8}\ Q^{2.78}$

Low flow range -

$b = 1.709$
$a = 1.457 \times 10^{-4}$
$Q_s = 1.46 \times 10^{-4}\ Q^{1.71}$

Step 9. Determine the exact range of flow that applies to each regression equation. At the point where two adjacent regression lines converge, the sediment discharge and the water discharge values will be equal for both lines, that is

$$Q_{s1} = Q_{s2} \tag{7}$$

and

$$Q_1 = Q_2 \tag{8}$$

where the subscript 1 and 2 indicate line 1 and line 2. Using equations 1 and 7

$$a_1 Q_1^{\ b_1} = a_2 Q_2^{\ b_2} \tag{9}$$

Using logarithmic transformation on equation 9, results in the equation

$$\log a_1 + b_1 \log Q_1 = \log a_2 + b_2 \log Q_2 \tag{10}$$

Using equation 8 and rearranging terms

$$\log Q = \frac{\log a_2 - \log a_1}{b_1 - b_2}$$

or

$$Q = (10)\ \frac{\log a_2 - \log a_1}{b_1 - b_2} \tag{11}$$

The point of convergence between the high and medium flow regression lines occurs at

$$\log Q = \frac{(-7.229320) - (-2.689538)}{(1.728 - 2.780)} = 4.315382$$

$$Q = 10^{(4.315382)} = 20,700 \text{ ft}^3/\text{s}$$

and the point of convergence between medium and low flow occurs at

$$\log Q = \frac{(-3.833660) - (-7.229320)}{(2.780 - 1.709)} = 3.70551$$

$$Q = 10^{(3.170551)} = 1,480 \text{ ft}^3/\text{s}$$

Thus, the range of flow for each regression equation is

$$Q \geq 20,700 \text{ ft}^3/\text{s}, \; Q_s = 2.04 \times 10^{-3} \; Q^{1.73}$$

$$1,480 \leq Q < 20,700 \text{ ft}^3/\text{s}, \; Q_s = 5.90 \times 10^{-8} \; Q^{2.78}$$

$$Q < 1,480 \text{ ft}^3/\text{s}, \; Q_s = 1.46 \times 10^{-4} \; Q^{1.71}$$

Three regression equations generally are needed to completely define a sediment-transport relation at a given site. The relation indicated in figure 11 is typical of many sites, in that the slopes for high and low flow are generally less than that for medium flow.

Yadkin River at Yadkin College, North Carolina

Data used in this example are 39 periodic samples of suspended-sediment concentration and water discharge for Yadkin River at Yadkin College, North Carolina (table 3). Explanatory and computation steps have been omitted because they have been described in the previous example.

An analysis of the data plotted in figure 12 indicates that a regression line could be fitted to the values for discharges less than 15,000 ft^3/s. At higher discharges, however, the points are widely scattered and no clear relation is apparent.

Several approaches can be used to define a relation for the upper part of the graph. One would be to extend the regression line developed for discharges less than 15,000 ft^3/s to include the high flow range. This extension is shown as the dashed line in figure 12. This may or may not be an accurate representation of the sediment transport relation at high flows for this station. Another approach to developing the sediment-transport curve for discharges above 15,000 ft^3/s would be to take a logical look at the data points, keeping in mind some of the things discussed earlier that could affect the shape of the transport curve. To the left of the dashed line lies a group of points between 16,000 and 22,000 ft^3/s. Four of these points were collected during a single storm event. A plot of these points (fig. 13) shows that the sediment concentration peaked about 13 hours prior to the water discharge peak of 22,100 ft^3/s at 2,230. We saw earlier that a stream

Table 3.--Instantaneous suspended-sediment and water discharge, Yadkin River at Yadkin College, North Carolina, 1969-73 water years

Date (1)	Time (2)	Water discharge (ft^3/s) (3)	Sediment concentration (mg/L) (4)	Sediment discharge (ton/d) (5)
10/10/68	0930	995	46	124
10/21/68	1525	9,330	327	8,240
02/19/69	1215	2,280	103	634
03/26/69	1720	8,200	727	16,100
04/16/69	1135	2,980	110	885
06/09/69	1230	1,590	85	365
06/17/69	1335	8,840	431	10,300
08/29/69	1355	1,180	69	220
11/04/69	1315	2,680	116	839
12/02/69	1000	1,460	35	138
12/23/69	1130	2,800	119	900
02/05/70	1230	3,460	216	2,020
03/24/70	----	3,280	124	1,100
05/25/70	1235	1,780	114	548
08/10/70	1200	20,200	2,560	140,000
08/11/70	1100	44,200	1,010	121,000
10/21/70	1420	1,480	45	180
11/18/70	1700	1,940	42	220
02/23/71	0730	16,100	2,600	113,000
02/23/71	1030	18,500	2,760	138,000
02/23/71	1530	21,000	2,210	125,000
02/23/71	2230	22,100	1,620	96,700
03/30/71	1420	3,070	74	613
09/22/71	1600	11,500	742	23,000
11/01/71	1300	3,410	108	994
12/01/71	0945	3,640	202	1,990
03/16/72	1220	2,580	48	334
05/04/72	1305	18,000	1,840	89,400
06/21/72	0730	20,800	1,240	69,600
06/21/72	1900	33,700	1,050	95,500
06/22/72	0700	63,200	2,320	396,000
01/30/73	1500	4,710	221	2,810
02/02/73	2300	24,400	1,790	118,000
02/03/73	1300	34,000	1,100	101,000
04/19/73	1440	4,190	190	2,150
05/28/73	1606	13,200	980	34,900
05/28/73	2145	17,400	1,020	47,900
05/29/73	1255	22,500	931	56,600
09/06/73	1230	3,830	104	1,080

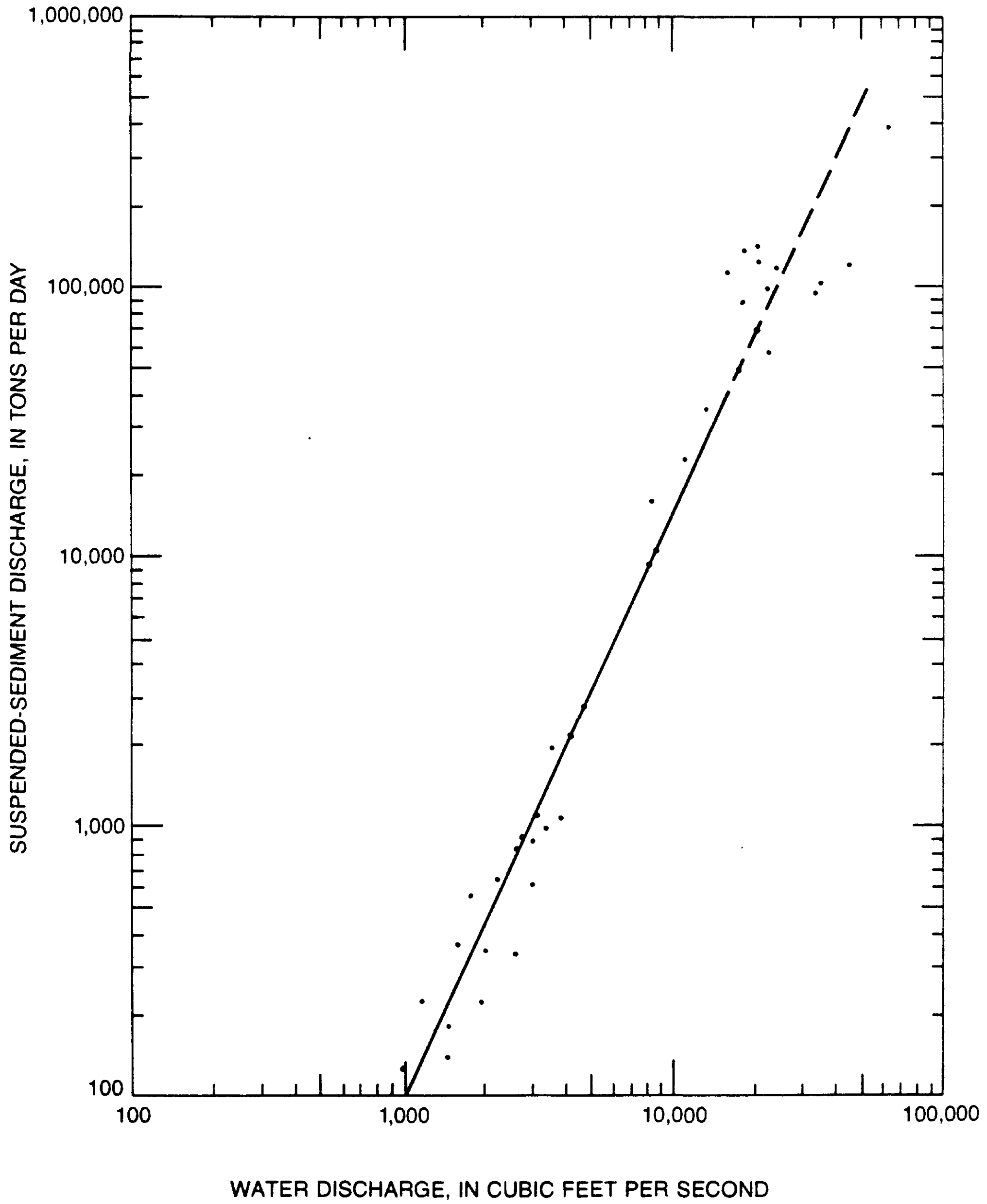

Figure 12.--Relation between sediment discharge and water discharge using linear regression method, Yadkin River at Yadkin College, North Carolina, 1969-73 water years.

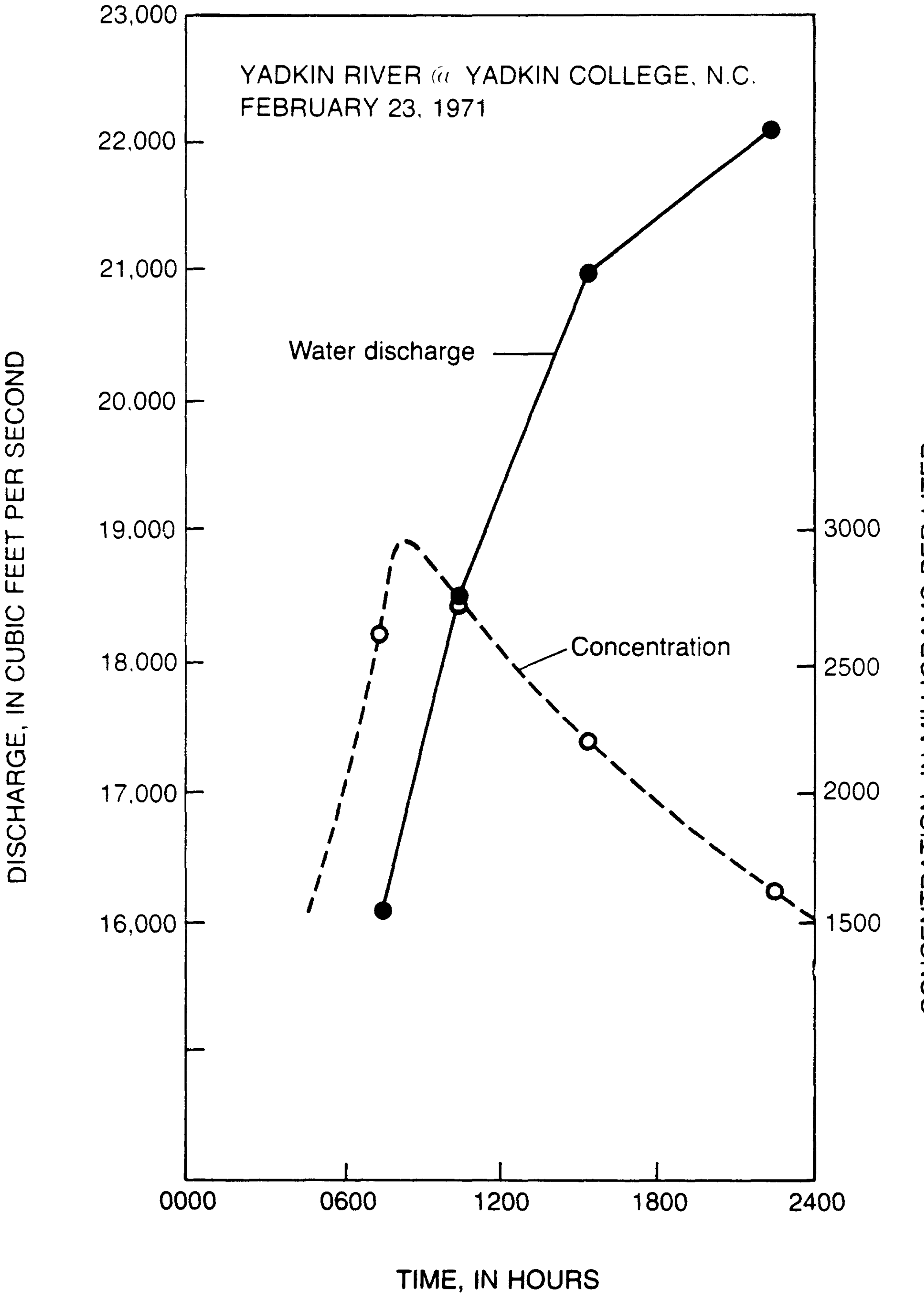

Figure 13.--Water discharge and sediment concentration hydrographs for Yadkin River at Yadkin College, North Carolina, for February 23, 1971.

whose sediment concentration peak preceded the streamflow peak will have a sediment-transport curve that loops to the right (see fig. 5). Figure 14 shows the sediment-transport curve for Yadkin River at Yadkin College, North Carolina, with instantaneous sediment-transport curves for three separate storms drawn on it. It appears from figure 14 that a sediment-transport curve which bends to the right at about 25,000 ft^3/s would better represent the upper end of a mean transport curve than would the straight line extension of the lower curve.

The data points for discharges between 15,000 to 25,000 ft^3/s lie in a transition zone between the upper and lower curves. Their inclusion in the data for the high flow equation is essential to the development of a proper slope for the high flow curve. Using the same linear regression procedure as was used for the Eel River at Scotia, California, example and all data points for discharge greater than 15,000 ft^3/s, the high flow curve shown in figure 15 was developed. The high flow relation for this station would be considered poor because of the large loops in the instantaneous stream transport curves.

The equation for the two regression lines in figure 15 are:

High flow range $\geq$ 23,400 ft^3/s

$$Q_s = 5.78\ Q^{0.962}$$

Medium flow range < 23,400 ft^3/s

$$Q_s = 2.23 \times 10^{-5}\ Q^{2.20}$$

The regression lines converge at a discharge of 23,400 ft^3/s.

GROUP AVERAGE

The equation for the linear regression is convenient for interpolating the data, and the results obtained are definite; subjectivity in interpretation is somewhat eliminated. This is not to say that the linear regression technique will remove all the biases in the data. In fact, it may even create more. One common problem is caused by having more data points at one part of the curve than at another part. Porterfield and others (1978) presented figure 16 which illustrates this problem. A single linear regression analysis on this data will result in the slope of the regression line being influenced by the mass of points at the lower water discharges. The group average method is a simple and effective way of removing this error.

The group averages method is the determination of the average, usually the arithmetic mean, of all values of the dependent variable (sediment discharge) for a small range of the independent variable (water discharge). Average sediment dicharge within each small range of water discharge can then be plotted against the average observed water discharge for that range. A transport curve is fitted to these points. The plotting should be done on logarithmic paper.

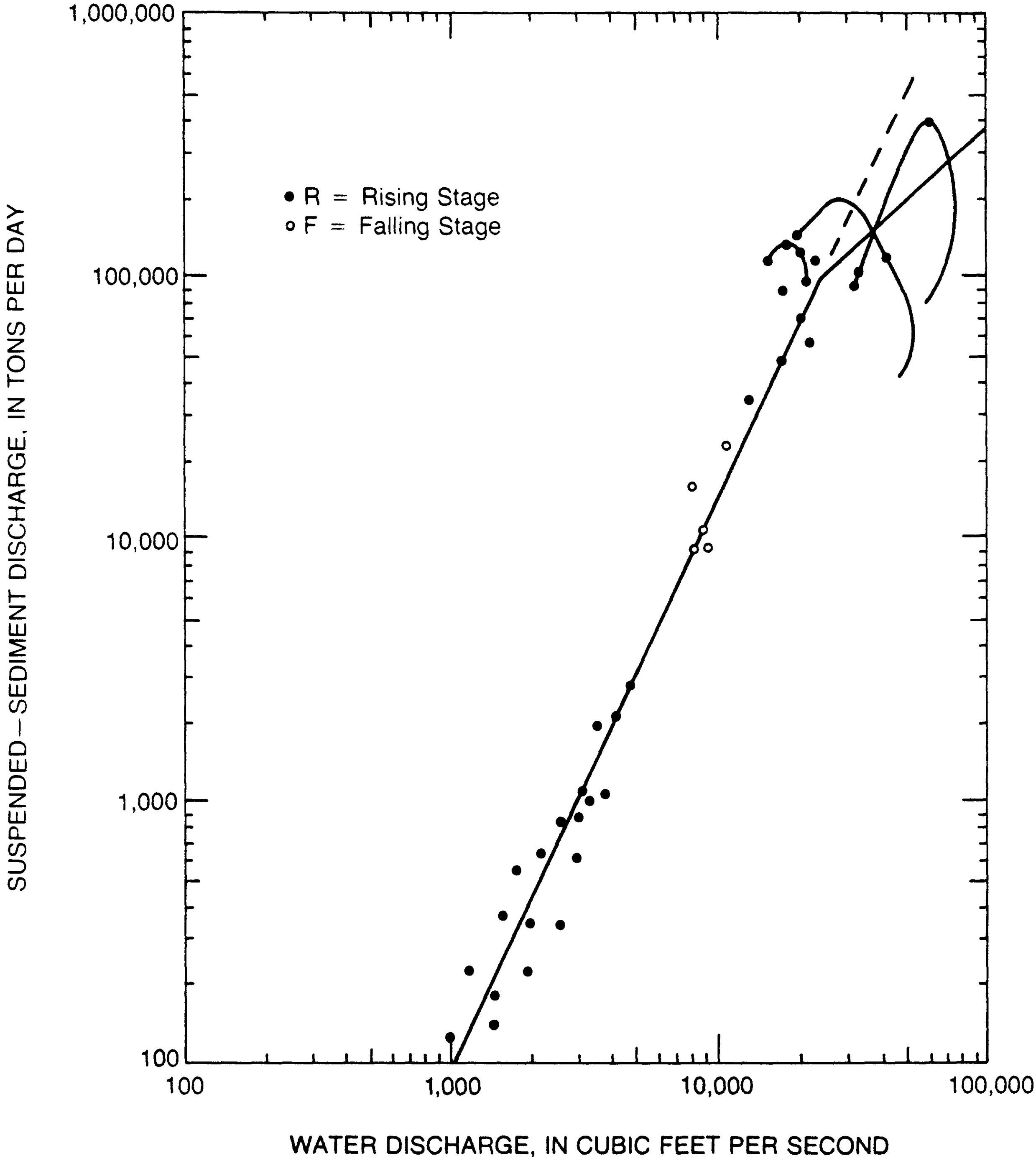

Figure 14.--Relation between sediment discharge and water discharge for three peaks on Yadkin River at Yadkin College, North Carolina.

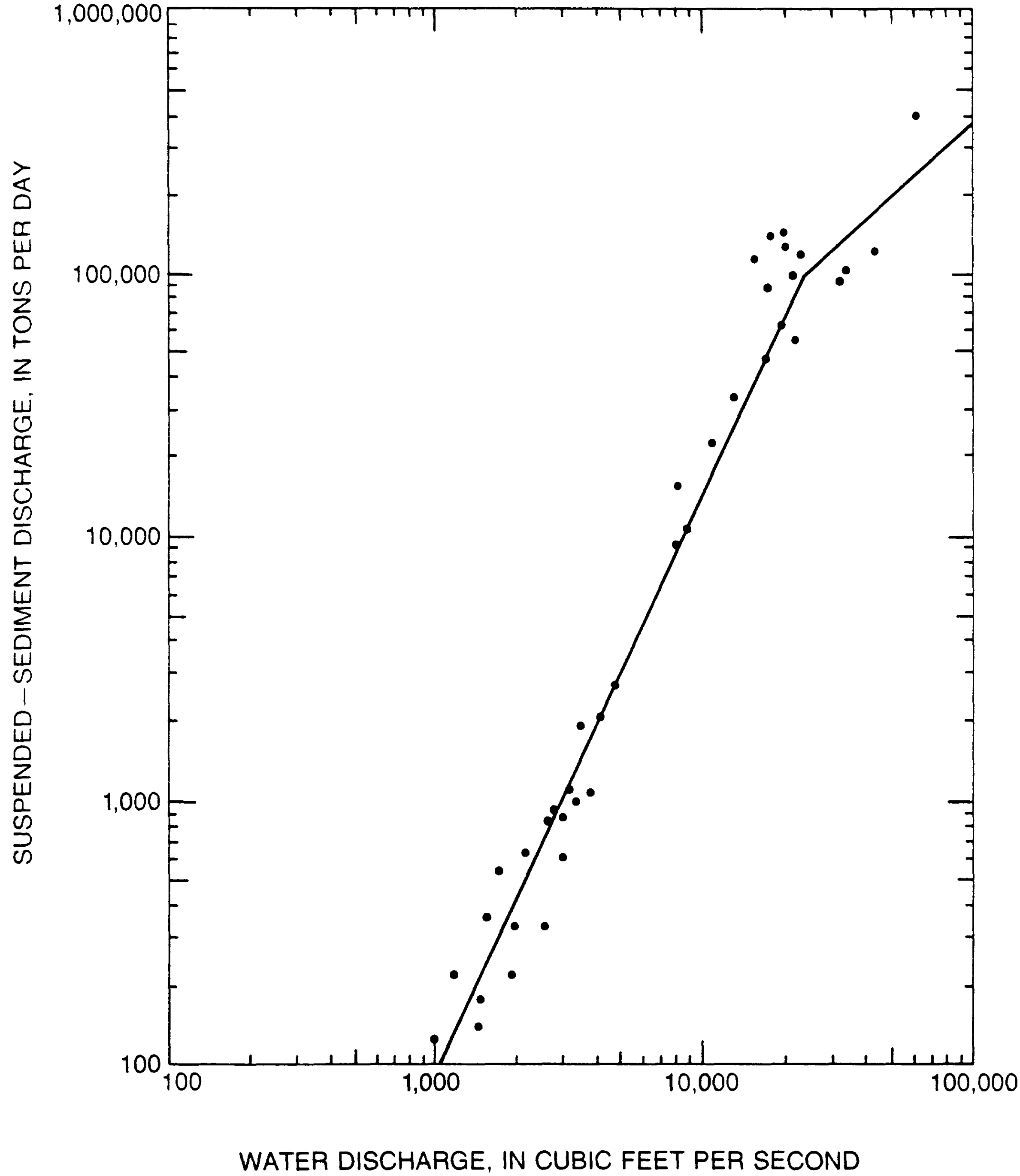

Figure 15.--Best estimate of the relation between sediment discharge and water discharge, Yadkin River near Yadkin College, North Carolina, 1969-73 water years.

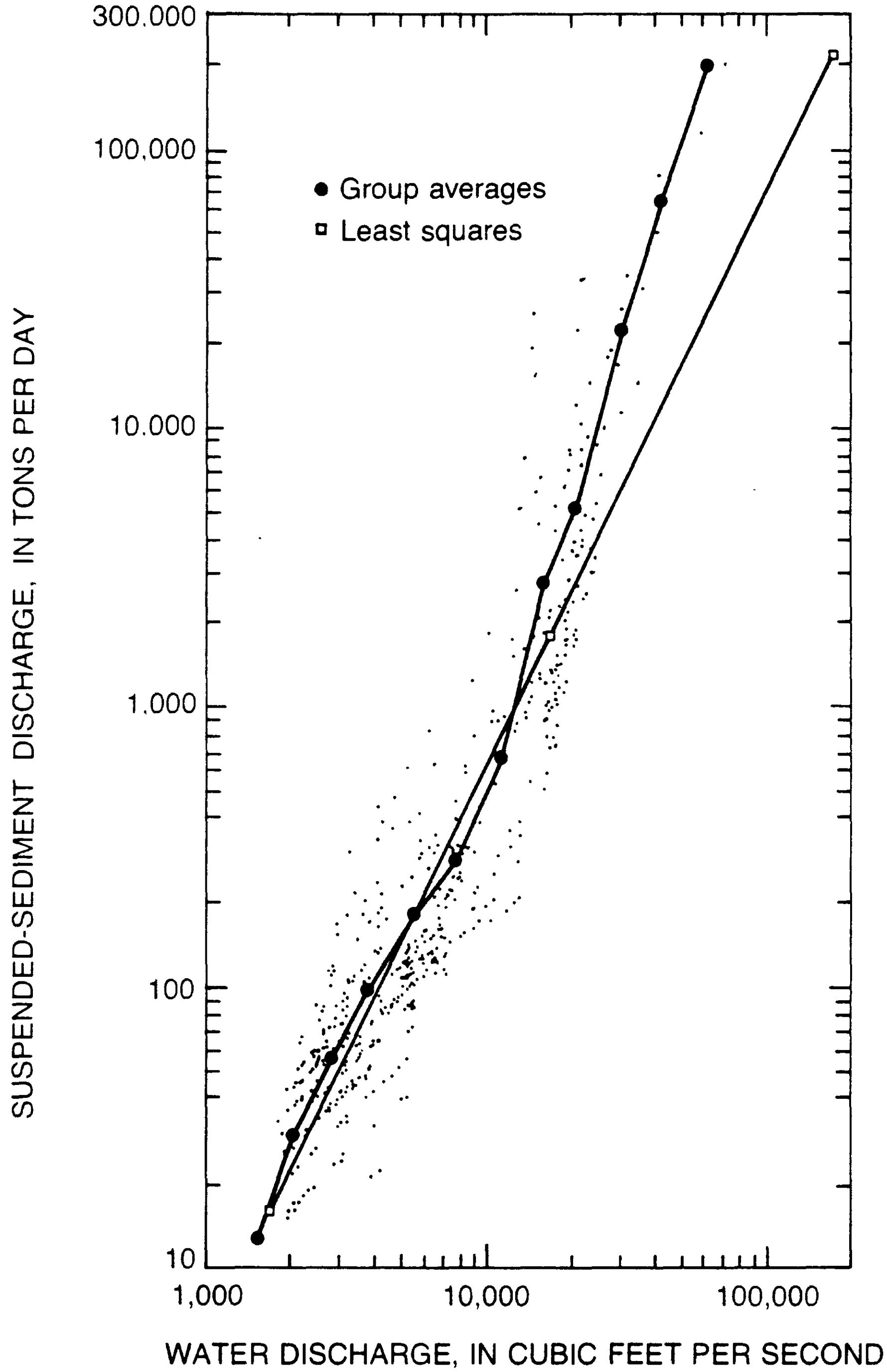

Figure 16.--Relation between streamflow and suspended-sediment discharge, Feather River at Oroville, California, 1958 water year (Porterfield and others, 1978).

Example of Group Average Method

This example uses the Eel River at Scotia data used for the linear regression method. The procedure is as follows:

Step 1. List water discharge and concentration data in chronological order (table 1).

Step 2. Compute sediment discharge using equation 5. Col. 5 = 0.0027 x Col. 3 x Col. 4.

Step 3. Plot water discharge (abscissa) and sediment discharge (ordinate) on logarithmic paper (fig. 11).

Step 4. Analyze the plotted data for unusual or anomalous clusters of data points and obvious outliers. If unusual or outlier data exist, they should be rechecked for possible errors (sampling, laboratory, or computation) and for cause. A large number of samples collected during a short period may be representative of conditions (construction, seasonal effects, wildfires) that would bias the use of the transport curve for estimating long-term conditions. The preliminary analysis for the group averages method usually is minor because the relation is averaged for many small increments of streamflow.

Step 5. Arrange water-discharge values into uniform groups or classes according to magnitude (table 4). Extremes for each group (class limits) can usually be obtained from available flow-duration tables where daily discharge is distributed into about 30 groups. Fewer groups are justified if the data set is small.

For example, to obtain 15 groups for the 1958-60 water years, the procedures would be:

1. Select the upper and lower class limits. Select the lower class limit just smaller than the minimum observed daily discharges for the period and the maximum class limit just larger than the maximum observed daily discharge for the period. For this example, use 70 and 170,000 ft^3/s.

2. Determine the difference of the logarithms of the minimum and maximum class limits.

   Log 170,000 - log 70 = 5.230449 - 1.845098 = 3.385351

3. Divide the difference by the number of groups (that is, 15) minus one.

   3.385351 ÷ (15 - 1) = 0.241811

4. Increment the logarithm of each successive class limit by 0.241811 and determine the antilogarithm (minimum discharge of the group).

Group	Log	Antilog (rounded)
1	1.845098	70
2	1.845098 + 0.241811 = 2.086909	120
3	2.086909 + 0.241811 = 2.328720	210
4	2.328720 + 0.241811 = 2.570531	370
5	2.570531 + 0.241811 = 2.812342	650
6	2.812342 + 0.241811 = 3.054153	1,100
7	3.054153 + 0.241811 = 3.295964	2,000
8	3.295964 + 0.241811 = 3.537775	3,400
9	3.537775 + 0.241811 = 3.779586	6,000
10	3.779586 + 0.241811 = 4.021397	11,000
11	4.021397 + 0.241811 = 4.263208	18,000
12	4.263208 + 0.241811 = 4.505019	32,000
13	4.505019 + 0.241811 = 4.746830	56,000
14	4.746830 + 0.241811 = 4.988641	97,000
15	5.230449	170,000

Step 6. List individual water discharges and associated sediment discharges in groups ranked in ascending or descending order of water discharge. For streams where a significant fraction of the annual sediment load is transported in a few days, list several of the largest discharge values individually to more precisely define the upper end of the transport curve.

Step 7. Compute the arithmetic mean of the water discharges and sediment discharges for each group.

Step 8. Plot the group averages on logarithmic paper (fig. 17).

Step 9. The sediment-transport curve can be obtained from the group average points by two methods: (1) by joining the points with a straight line between points or (2) by developing a curve(s), using the points as a guide.

The dashed line in figure 17 shows a sediment-transport curve drawn using the first method. Several problems arise from this method. The sediment discharge apparently decreases with increasing water discharge between points 1 and 2 and the single points, such as 5 and 11, may cause drastic changes in slope. For these reasons, it is generally perferred to use the second method to define the sediment-transport curve from the group average points. The solid line in figure 17 was developed using the group average points and a linear regression analysis, and dividing the curve into three straight line segments. The linear regression analysis method used was the same as that discussed in the previous section, with the exception that the group average points were used instead of the individual data points.

Table 4.--Computation of sediment-transport relation using group averages, Eel River at Scotia, California, 1958-60 water years

Group	Class limit (ft^3/s)	Water discharge (ft^3/s)		Sediment discharge (ton/d)	
		Data	Mean	Data	Mean
1	70	98	107	0.53	0.68
		109		.59	
		115		.93	
2	120	132	159	.71	.62
		147		.79	
		177		.48	
		180		.49	
3	210	278	278	1.5	1.5
4	370	546	563	5.9	5.3
		580		4.7	
5	650	691	691	26	26
6	1,100	1,970	1,970	74	74
7	2,000	2,390	2,390	103	103
8	3,400	3,750	4,590	526	1,270
		4,170		484	
		4,840		3,010	
		5,600		1,060	
9	6,000	6,680	8,220	2,530	5,430
		8,110		5,040	
		9,880		8,720	
10	11,000	17,100	17,100	24,900	24,900
11	18,000	20,300	20,300	89,300	89,300
12	32,000	32,100	42,100	57,500	211,000
		36,500		195,000	
		46,400		210,000	
		53,300		380,000	
13	56,000	79,000	79,000	435,000	435,000
14	97,000	124,000	138,000	1,950,000	1,790,000
		151,000		1,630,000	
15	170,000	-------		---------	

The equation and range of flow for the regression equations based on the group average data are:

$Q < 1980\ ft^3/s,\ Q_s = 1.02 \times 10^{-4}\ Q^{1.78}$

$1980 \leq Q \leq 16{,}400\ ft^3/s,\ Q_s = 1.85 \times 10^{-8}\ Q^{2.92}$

$Q > 16{,}400\ ft^3/s,\ Q_s = 1.30 \times 10^{-3}\ Q^{1.77}$

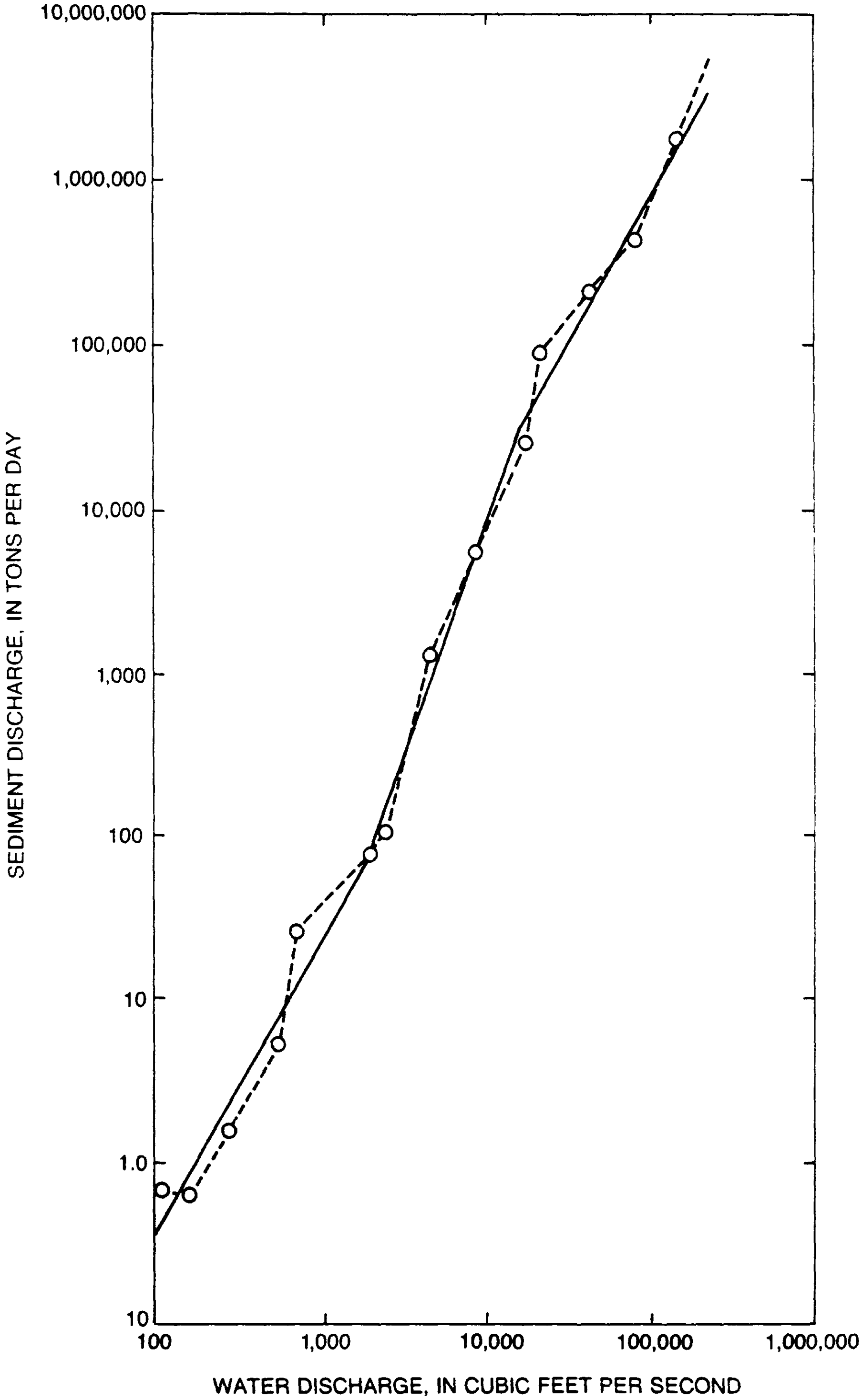

Figure 17.--Sediment-transport curves based on group averages method for Eel River at Scotia, California, 1958-60 water years.

Another major advantage of using a linear regression analysis on the group average points is that the slope of the upper end of the transport curves is defined better. A slight error in the slope at the upper end of the curve can mean a significant difference in the estimated sediment discharge. For example, if the sediment discharge for a water discharge of 200,000 ft^3/s was to be estimated from the curves in figure 17, the first method (dashed line) would produce an estimated sediment discharge of 4.5 million tons per day. Using the solid line, the estimated sediment discharge would be 3.0 million tons per day. This difference may be very significant when we consider that for most streams the majority of the sediment is transported in a few days. The 1.5 million tons per day difference in this example may be equivalent to 40 or 50 percent of the total sediment discharge for the year.

OTHER PROBLEMS ASSOCIATED WITH COMPUTER-FITTED CURVES

Several other problems can arise when computer programs are used to compute sediment-transport curves. Just because a curve "fits" the data points, it does not necessarily make it hydraulicly correct. The following is a discussion of several of the more common types of problems encountered when using computers to generate sediment-transport curves.

Table 5 and figure 18 show daily sediment discharges for a summer peak on a small stream in Pennsylvania, arrows show the progression of the storm days. Figures 19 and 20 are plots of the same storm with the standard linear regression and log-quadratic equations fit to the points respectively. Both

Table 5.--Original data figures 18-21 and 24 are based on

Flow ft^3/s	Load ton
33	0.53
34	0.54
950	2,620.0
528	156.0
291	30.0
174	9.9
131	7.1
88	1.4
72	1.2
62	1.3
57	1.5
54	1.3
46	1.0

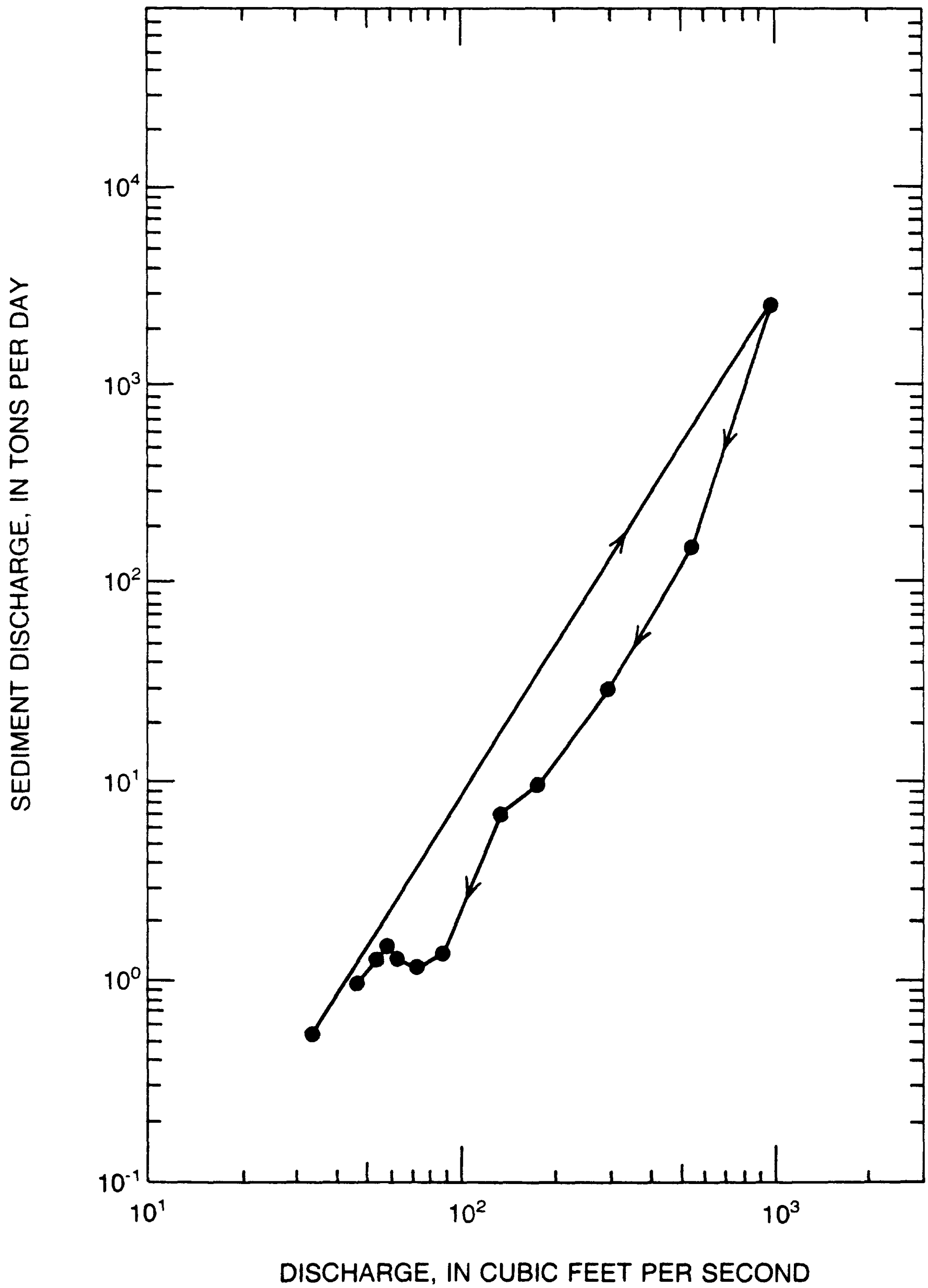

Figure 18.--Daily sediment discharge for a summer peak on a small stream in Pennsylvania.

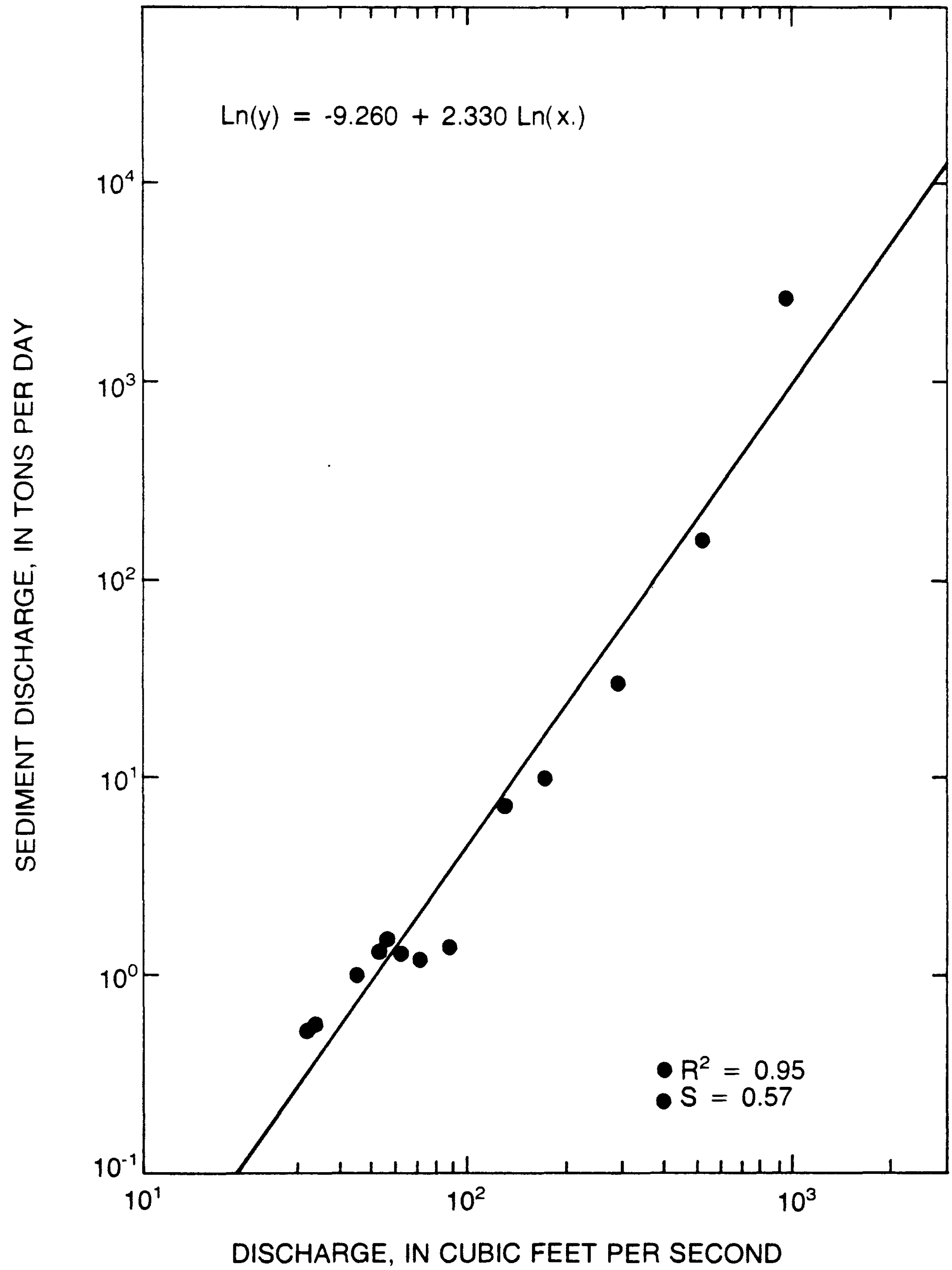

Figure 19.--Sediment-transport curve based on log-linear regression analysis.

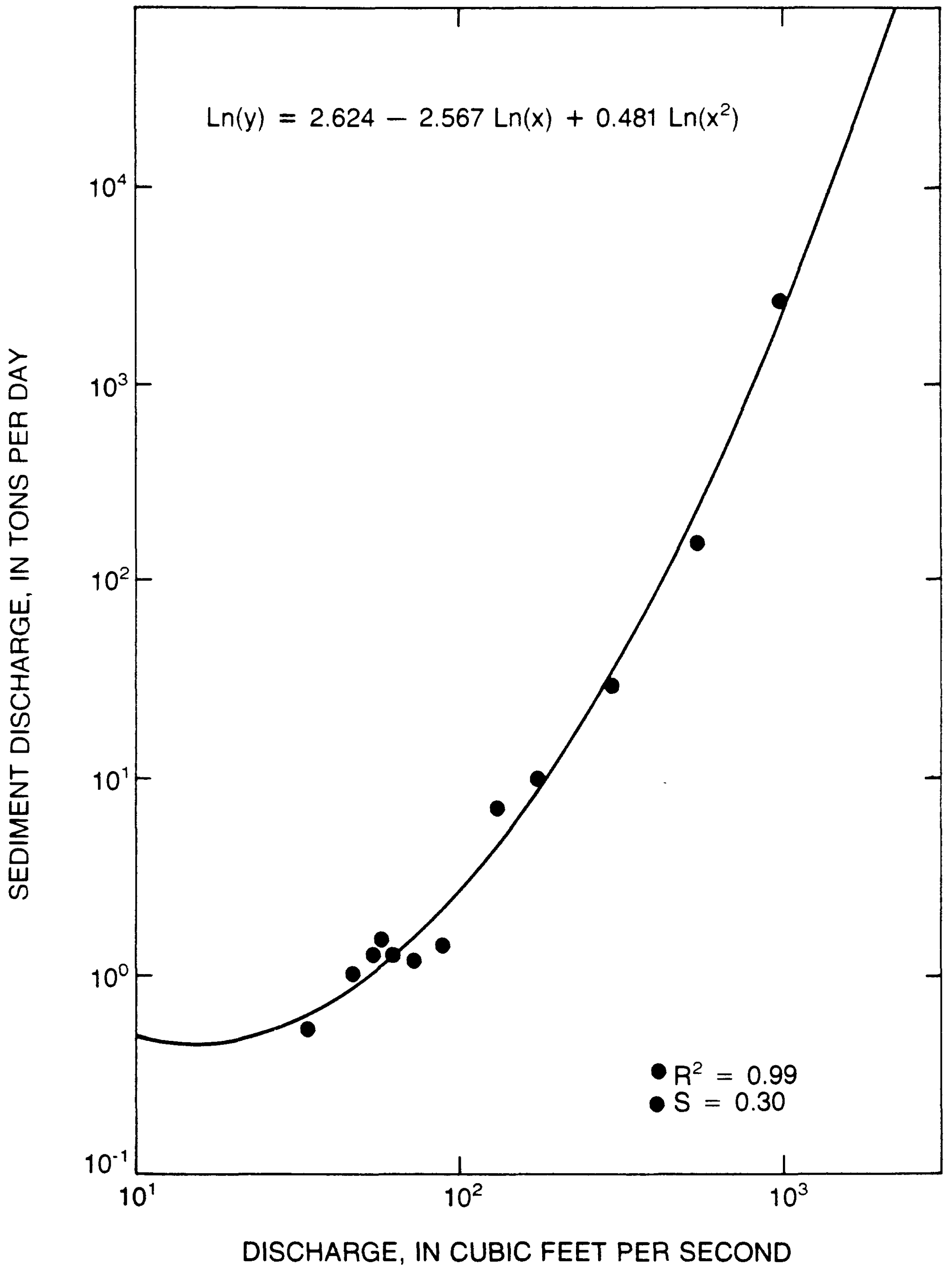

Figure 20.--Sediment-transport curve based on log-quadratic regression analysis.

show good correlation (high R^2 values) and low standard errors (s). The linear-regression curve (fig. 19) underestimates the peak whereas the curve in figure 20 comes very close to correctly estimating the peak sediment discharge. However, if we examine the curve in figure 20 more closely, we see two major problems with it: (1) at the lower end it shows an increase in sediment discharge with decreasing water discharge; and (2) the upper end of the curve is curving upward, thus showing ever-increasing sediment discharge with increasing water discharge. The ultimate result of the second problem would be that with very little increase in water discharge, we would have a very large increase in sediment discharge. The curve in figure 20 does have limited uses, however, and can be used if the range of discharge is equivalent to the range defined by the data points. This does not help when trying to estimate sediment discharges outside this range, especially at the upper end. In this particular case, two curves might be better than one for estimating sediment discharges. Figure 21 shows two curves that might be used, the upper curve to be used on rising and peak days, the lower one for recessional days. Table 6 gives the peak sediment discharges estimated using the curves shown in figures 19, 20, and 21.

Table 6.--Estimated sediment discharge in tons per day for indicated water discharges

Curve type	Water discharge in ft^3/s: 950	Water discharge in ft^3/s: 2,000
Log-log	822	5,000
Log-quad	2,050	65,000
Double straight line	2,620	18,000
	2,620*	

*Actual peak sediment discharge recorded.

Figure 20 also illustrates the problem with computer-generated sediment-transport curves which do not cover the entire range of flows. The problem is that they may have shapes which do not represent the sediment-water relation outside the flows sampled. The data in figure 22 are from a coastal stream in northern California. The third order polynomial fits the data points quite well in the flow ranges sampled. But when the curve is extended to include measured flows at this site (fig. 23), it does not represent the sediment-water relation. Obviously this is an extreme case, but it shows that one must plot out the whole range of flows to be estimated to detect abnormalities.

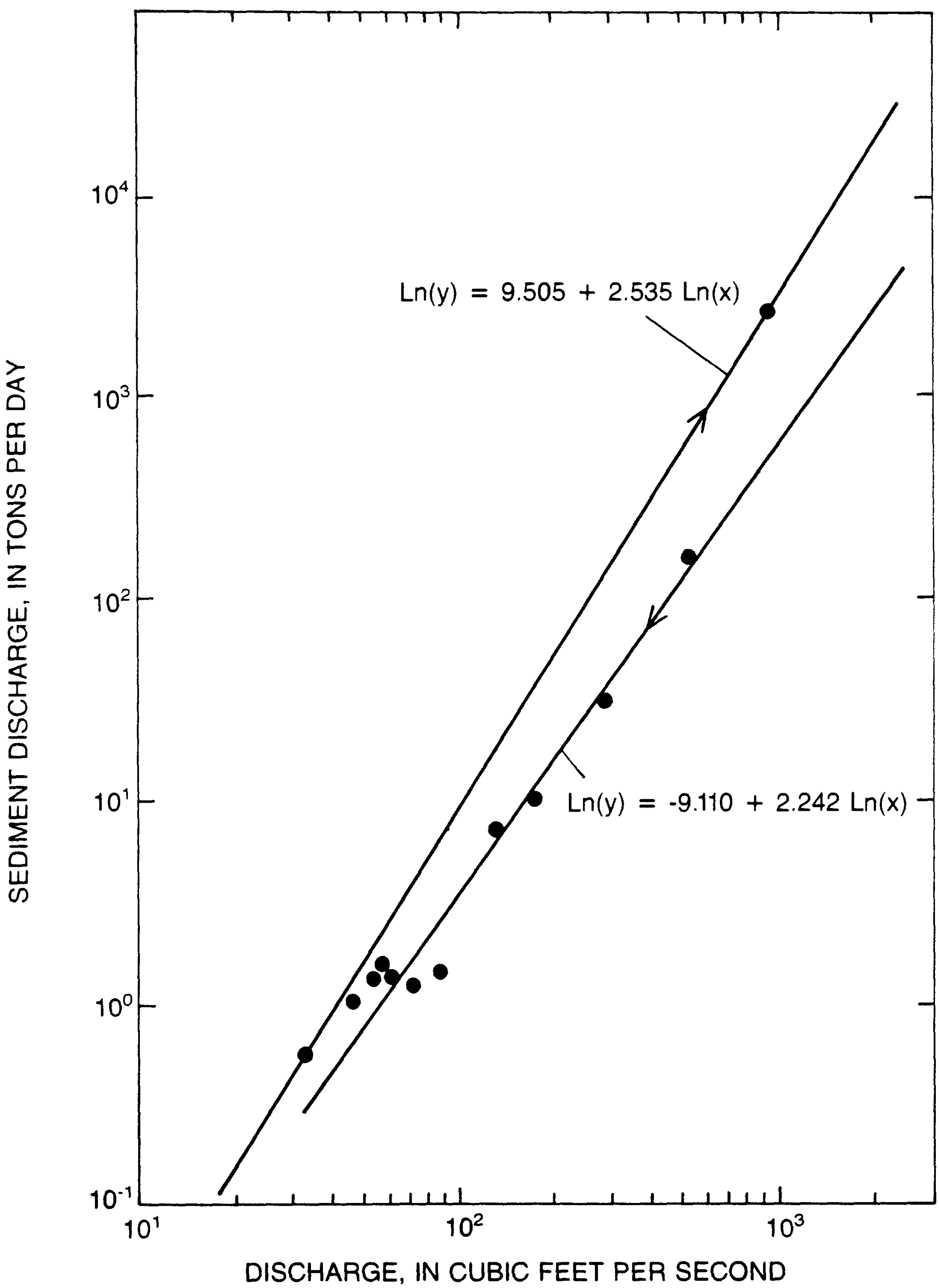

Figure 21.--Example of two log-linear sediment-transport curves, one for rising and peak periods and one for recession periods.

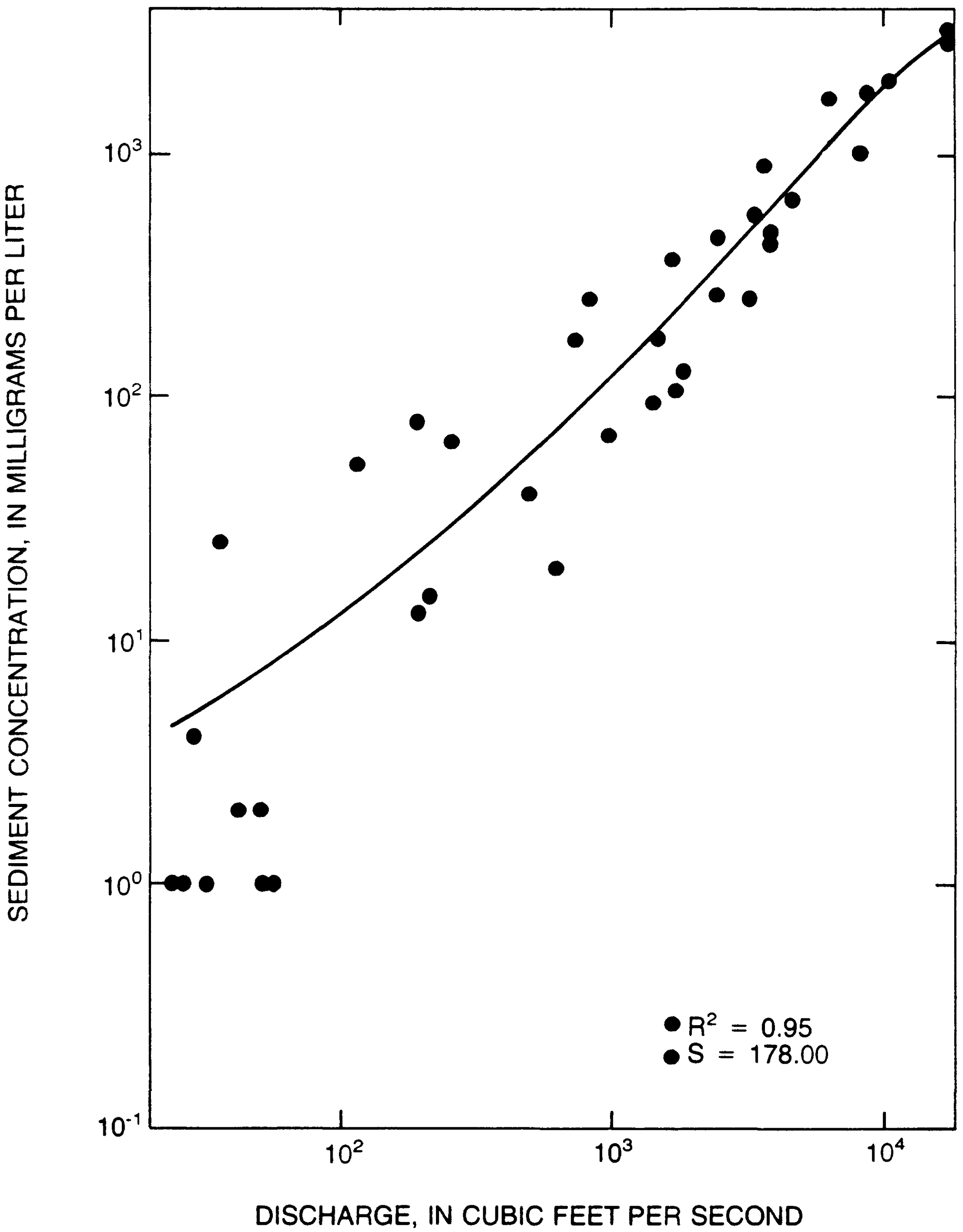

Figure 22.--Example of sediment-transport curve based on the flow range sampled.

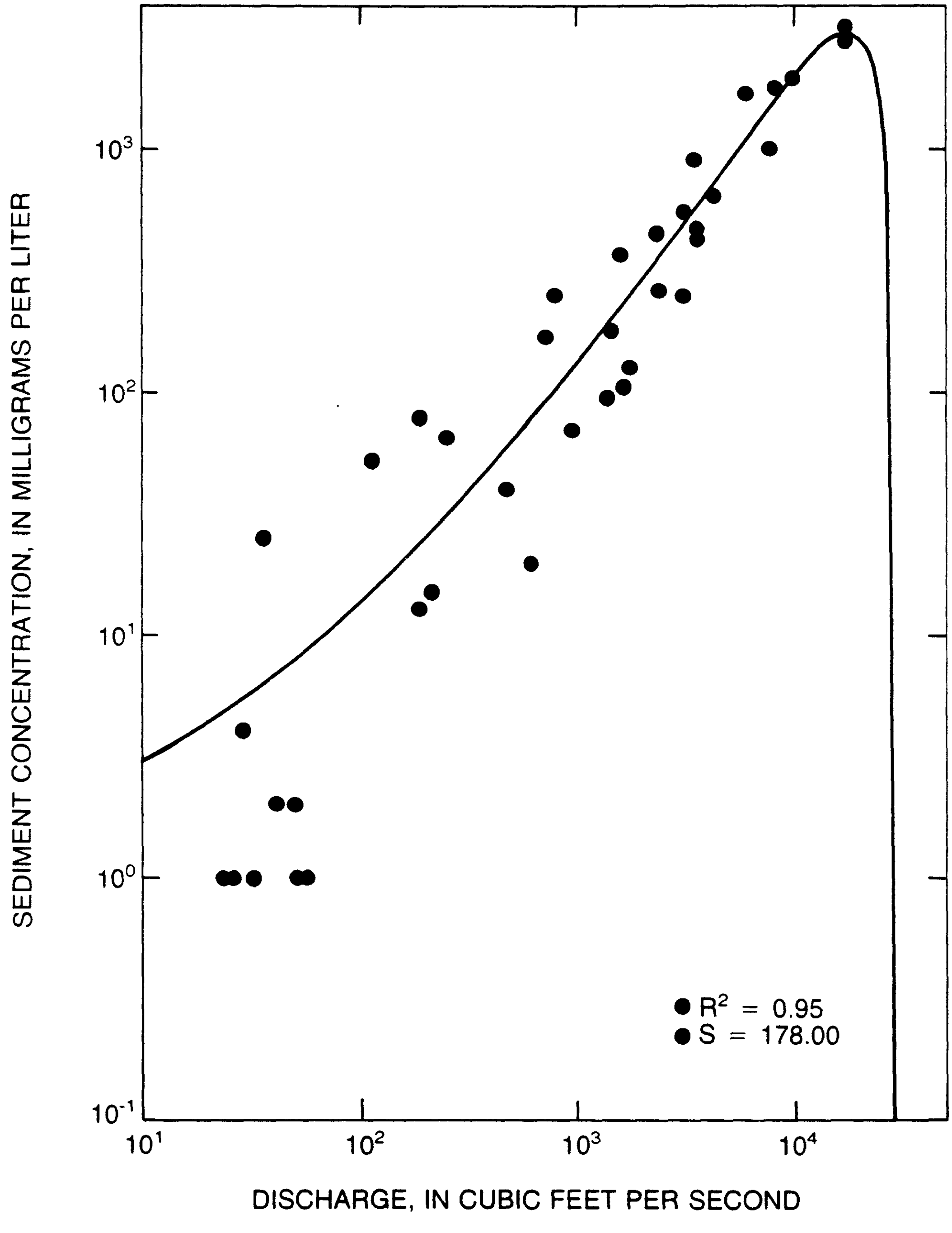

Figure 23.--Example of problem that may be encountered when sediment-transport curve is extended beyond the range of flows sampled.

The last problem to be discussed deals with the tendency to use the statistics produced by the computer fitting programs to judge which curve would be the best to use. Figures 24a, b, and c show the same data as in figure 18. The curves show progressively higher R^2 values with all having relatively low standard errors (s). Figure 24c has a R^2 of 1.00, which would indicate a perfect correlation between water and sediment, which is obviously not true in this case. R^2 should not be used when log transformation has been performed on the data because it is not a measure of the fit of the data but of the fit of the logs of the data. Usually the best curve to use is the simplest one which adequately represents the data and the hydrologic conditions at the site.

POTENTIAL ERRORS

Estimates of sediment discharge based on transport curves are obviously subject to error. The potential magnitude of those errors can be demonstrated using the data shown in figure 18. The curve of a log-quadratic equation is fitted to the data of the original storm (fig. 25, same as fig. 20). In addition, three other storms occurring in April, June, and July of the same year are plotted. Table 7 shows the actual loads for these storms plus the estimated loads based on: (1) log-quadratic equation, (2) log-log equation, and (3) two log-log equations. The percent of error for these three sets of transport curves for these four storms ranged from 0 to 1,760 percent.

Table 7.--Comparison of errors in estimating storm sediment loads based on three sediment-transport curves

Storm	Actual measured load T/D	Log-quadratic[1] equation		Log-log[2] equation		Two log-log[3] equations	
		T/D	percent difference	T/D	percent difference	T/D	percent difference
Original	2,830	2,330	-18	1,120	-60	2,830	0
April	2,410	1,500	-38	1,330	-45	2,100	-13
June	8,380	156,000	+1,760	13,000	+55	33,000	+290
July	65	13	-80	18	-72	26	-60

[1]Ln(y) = 2.624 - 2.567 Ln(x) + 0.481 Ln (x)**2 (fig. 20)
[2]Ln(y) = -9.260 + 2.330 Ln(x) (fig. 19)
[3]Ln(y) = -9.505 + 2.535 Ln(x) (rising stage and peak) (fig. 21)
Ln(y) = -9.110 + 2.242 Ln(x) (recession) (fig. 21)

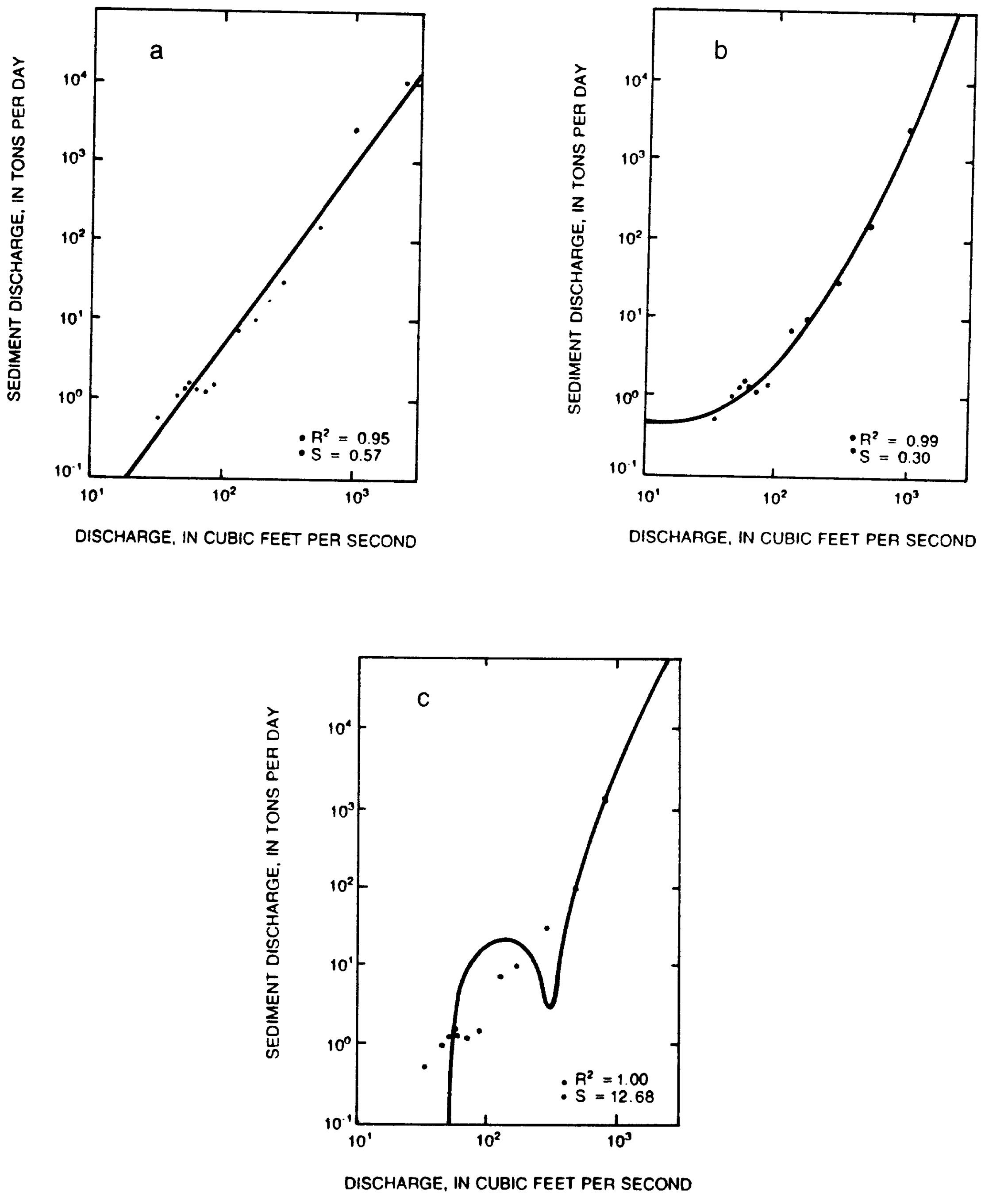

Figure 24.--Example of different types of transport curves fitted to the same data and variations in R^2 and standard error (s).

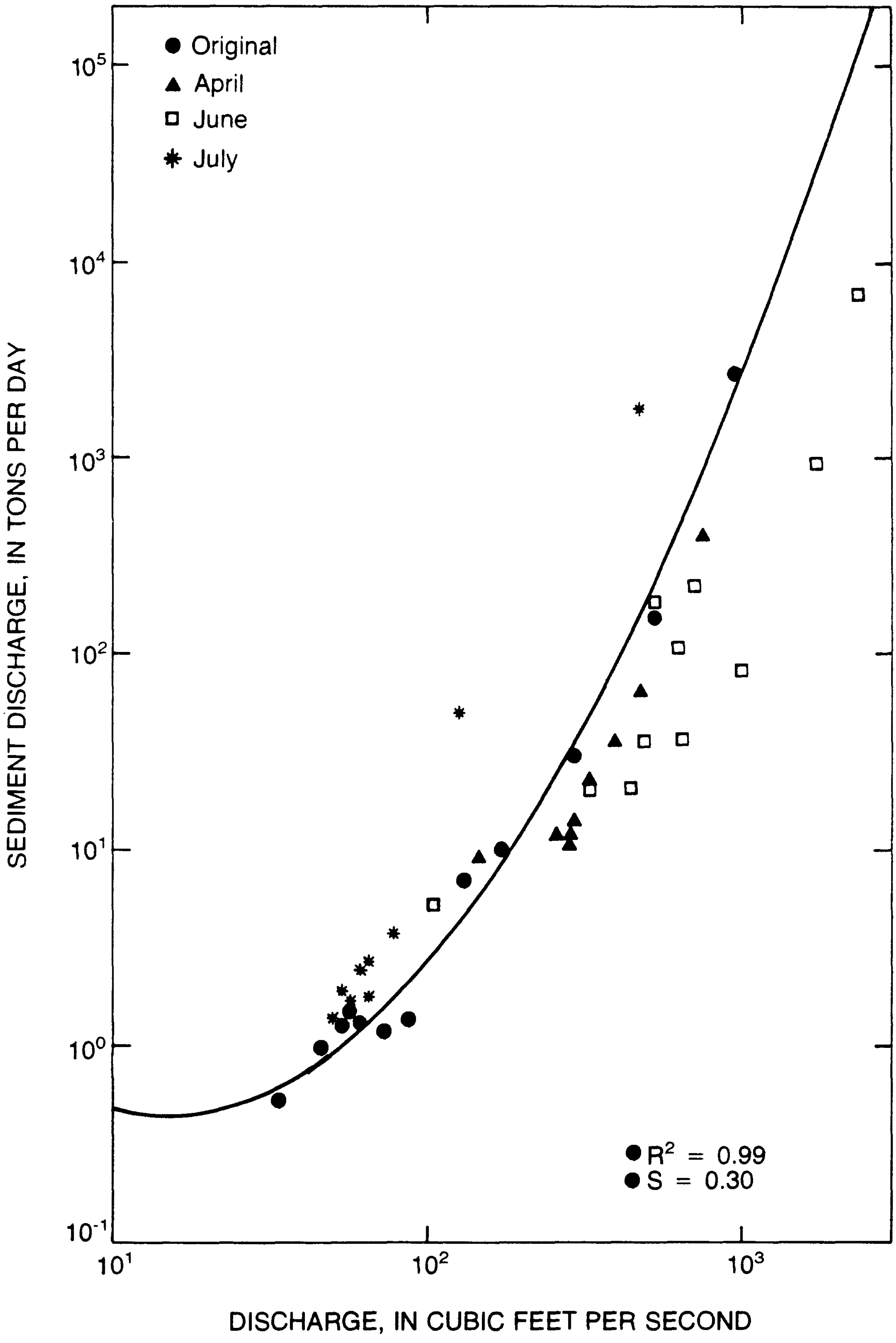

Figure 25.--Example of sediment-transport curve (figure 20) with data for additional storms added.

In this case, the log-log transport curve had the highest percent error when fitted to the original data (-60 percent) but gave the best estimate (+55 percent) of the major peak which occurred in June. This may not always be true, but it has been the author's experience that a log-log or series of log-log transport curves, such as in figures 10, 11, and 15, will normally give the best overall estimate of sediment discharge or concentration.

None of these curves provide what would be considered a "good" estimate of sediment load. In a case such as this example, where large loops appear in the transport curve, no single line transport curve will give an accurate estimate of sediment transport. This example again illustrates the danger of extending log-quadratic type transport curves beyond the data used to define them. In this case, the log-quadratic equation overestimated the June storm by 1,760 percent.

SUMMARY

Sediment-transport curves are very useful when trying to estimate sediment discharge or concentration. Care must be taken in developing these curves so that errors in the estimates are minimized. Sediment-transport curves can be drawn based on either sediment concentration or sediment discharge vs. water discharge. The units in which the curve is drawn should be consistent with the units to be estimated.

Care should be taken to ensure the best fit to the data while not violating any hydrologic or hydraulic principles of sediment transport. Seasons, timing of sediment peaks vs. water discharge peaks, and extreme high sediment events are just some of the things that can affect the slope and shape of sediment-transport curves. If computer programs are used to fit sediment-transport curves to the data points, these curves should be checked for their reasonableness and consistency over the full range of water discharges for which they will be used. Statistical parameters should not be the sole criteria in determining which curve fits the data best. In any case, thorough analysis of all factors affecting the transport curve must be considered when developing and using these curves. Usually the best curve to use is the simplest one that still defines the relation between sediment and water discharge.

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