

**DEPARTMENT OF THE INTERIOR**

**U.S. GEOLOGICAL SURVEY**

Differential Displacements and Differential Spectra  
for the February 20, 1988 Hollister, California, earthquake

by

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Open-File Report No. 88-679

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# Differential Displacements and Differential Spectra for the February 20, 1988 Hollister Earthquake

G. N. Bycroft and P. N. Mork

## Abstract

Differential ground motion under the base of a structure imposes direct strain on the structure in addition to the strains caused by inertial loading. In order to assess the importance of these differential strains, differential arrays of digital accelerometers with pre-event memory were installed at El Centro and Hollister, California. These arrays produced useful data for the 1979 El Centro, the 1984 Morgan Hill, the 1981 Westmorland, and the 1986 and 1988 Hollister earthquakes in California. This report presents differential displacements and spectra for the ML = 5.3 1988 Hollister earthquake of February 20, 0839 G.M.T. The differential displacements and spectra for the first 4 earthquakes are discussed in Bycroft and Mork, 1987. In certain circumstances the differential strains are significant.

## Introduction

Aseismic design generally has assumed that all points on the ground move in unison with the free-field motion over a region that is larger than the foundation of the structure. This assumption is based on the notion that seismic waves are propagated substantially in high-wave-velocity basement rock and transmitted vertically to the region of interest through lower velocity layers. However, surface waves propagating horizontally through surface layers may have wavelengths along the surface of approximately the dimensions of a large structure (Trifunac, 1972; Wong and Trifunac, 1974; Bycroft, 1980). Further, differential ground motion may be caused by local inhomogeneity. The foundation of the structure would then undergo differential motions that would cause additional strains to be superimposed on those due to inertial loading. Thus, for example, adjacent bridge piers would move relative to each other and cause stresses in the piers and the bridge decking. Structures built on spread footings, dams and pipelines would be similarly affected.

To study such motion, differential ground motions must be measured. Methods of utilizing this information in seismic design should be developed. Arrays of seismometers have been installed at El Centro and Hollister, California. These arrays are discussed in Bycroft (1982, 1983). Figure 1 shows the configuration of the Hollister array.

## Differential Displacements

The processing of accelerograms to give displacements has long been a problem due to the double integrations of base-line error, long-period noise quantification errors, and problems

related to the mechanics of film transport. The original processing was directed towards digitized film records and was developed initially at the California Institute of Technology. Many changes have been made at the U.S.G.S. resulting in a present form known as AGRAM (Converse, 1984). This program has many options available depending on the judgement of the user for his particular application or data. The processing includes instrument corrections, base line corrections and high pass filtering to eliminate long period noise and quantification errors.

In studying the direct strains on a structure displacements themselves are of no interest because only the difference of displacements occur in the structural equations. Consequently, only differential displacements are considered. This difference will eliminate those real long-period seismic signals whose wavelengths are large compared to the spacing of the stations.

### Differential Spectra

In order to examine the significance of a structure subjected to differential strain loading in addition to the inertial loading, a simple model, that is affected by both these loadings was investigated. Figure 2 shows this structure as a simple single-span bridge structure with two vertical piers connected by a deck. Transverse horizontal ground motions are applied to this structure so that shear stresses are developed in the three components. The masses are lumped together where the deck joins the piers. The masses are equal to  $m$ , the shear stiffness of the piers is  $k_1$  and the shear stiffness of the deck is  $k_2$ . Different horizontal displacements  $y_1(t)$  and  $y_2(t)$  are applied to the base of each pier. A response spectrum  $R$  was calculated. This response spectrum is defined as the ratio of the maximum strain in pier 1 when the displacement inputs are  $y_1(t)$  and  $y_2(t)$  to that of the maximum strain in pier 1 when  $y_1(t) = y_2(t)$ . This ratio  $R$  is then a suitable measure of the effect of the addition of differential strain to the inertial loading. It is readily shown that

$$(1) \quad R = \frac{A}{B}$$

where

$$(2) \quad A = \frac{\text{Max}}{2} \int_0^t C \, d\tau$$

where Max means the maximum value of the integral during the length of the input and where

$$(3) \quad C = \frac{e^{-\lambda\omega_1(t-\tau)} \{\ddot{y}_1(\tau) + \ddot{y}_2(\tau)\} \sin \omega_1(t-\tau)}{\omega_1} + \frac{e^{-\lambda\sqrt{\omega_1^2 + 2\omega_2^2}(t-\tau)} [\ddot{y}_1(\tau) - \ddot{y}_2(\tau) + 2\omega_2^2 \{y_1(\tau) - y_2(\tau)\}] \sin [\sqrt{\omega_1^2 + 2\omega_2^2}(t-\tau)]}{\sqrt{\omega_1^2 + 2\omega_2^2}}$$

$$(4) \quad B = \frac{\text{Max} \int_0^t e^{-\lambda\omega_1(t-\tau)} \ddot{y}_1(\tau) [\sin \omega_1(t-\tau)] \, d\tau}{\omega_1}$$

$$(5) \quad \omega_1 = \sqrt{\frac{k_1}{m}}$$

$$(6) \quad \omega_2 = \sqrt{\frac{k_2}{m}}$$

(7)  $\lambda$  = modal damping factor, common to both modes.

Similar equations apply to the longitudinal horizontal motions.

These spectra are for a simple two-degree-of-freedom structure and are relevant to situations where the two stations are close. A multi-degree-of-freedom spectra is being developed that is more relevant to stations farther apart.

### **February 20, 1988 Hollister Records**

The Hollister Differential Array gave useful data from stations 1, 2, 5, and 6 for the February 20, 1988 Hollister earthquake.

#### **Differential Displacements**

The double integration of accelerograms that give satisfactory values of differential displacements is discussed in Bycroft and Mork (1987a). Again, a bidirectional Butterworth filter with a long period cut-off of 5 seconds and order 2 was used. The base line correction was determined by least square fitting a zero line through the first 2 sec. of each record. The displacements at stations 5 and 6 were rotated to give displacements longitudinal and transverse to the joining leg. Figs. 3, 4, 5, and 6 show the corrected accelerograms and the differential displacements between the stations 2 and 1, 6 and 1, 5 and 1, and 6 and 5 for the first 20 seconds. This was a small earthquake and the difference of displacements is correspondingly small.

#### **Differential Spectra**

Differential spectra have been calculated from equation 1 and are shown in figs. 7 through 30 for various parameter values. For low values of  $\omega_1$  the value of R tends to a value close to unity. For high values of  $\omega_1$  the value R is significantly higher than 1 showing that differential strains are now quite significant. In the intermediate range of  $\omega_1$  the value of R is significantly lower than 1 in most but not all cases. This is different from the earthquakes discussed in Bycroft and Mork (1987a and 1987b) and could be a function of the locations of the epicenters relative to that of the array. The effect of damping shown for the cases  $\omega_1$  equals  $\omega_2$  does not follow any recognizable trend.

## Conclusions

Differential displacements contribute significantly to the strains developed in larger structures by an earthquake especially for the higher frequencies. Different earthquakes, recorded at the same array, show significantly different trends in the mid range of frequency presumably because of the different location of the epicenters. In general, no distinct patterns are discernable in the current data and consequently much more data from many different locations and earthquakes is needed.

## References

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# HOLLISTER DIFFERENTIAL DIGITAL ARRAY

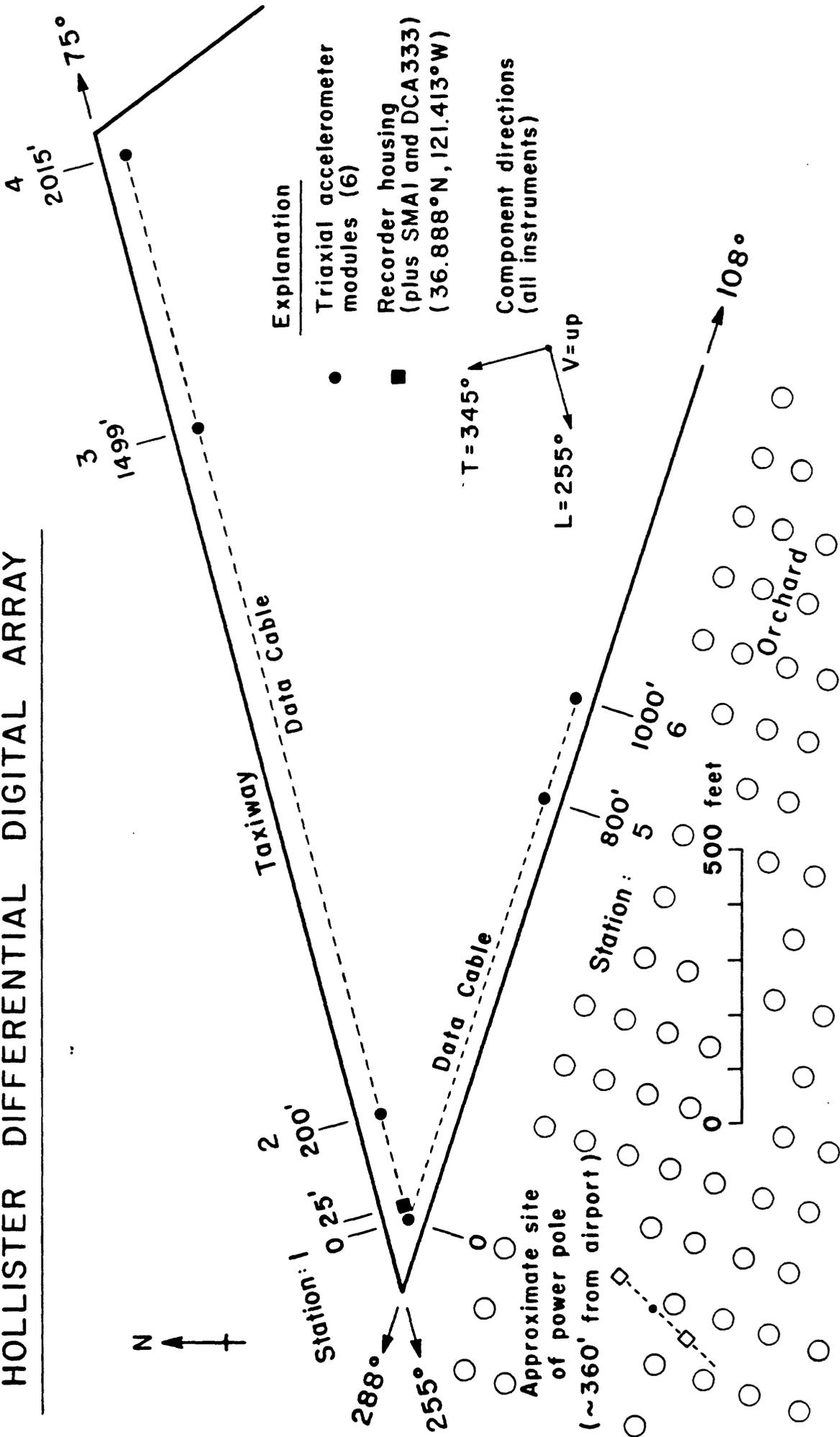


FIGURE 1

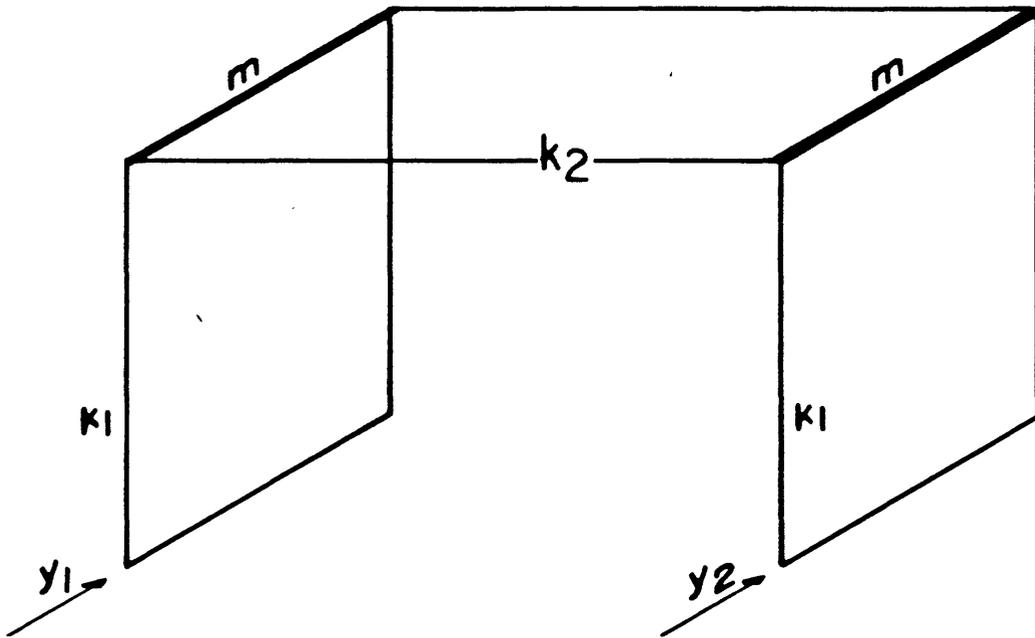


Figure 2. Model for differential spectra.

HOLLISTER, DIFFERENTIAL ARRAY NO 2 - NO 1  
345 DEGREES  
EARTHQUAKE OF FEBRUARY 20, 1988, 0839:57.5 GMT  
BUTTERWORTH AT .2 HZ, ORDER 2, BASELINE CORR: 0 TO 2 SEC  
PEAK VALUES (CM/SEC/SEC): -68.52, 0.22

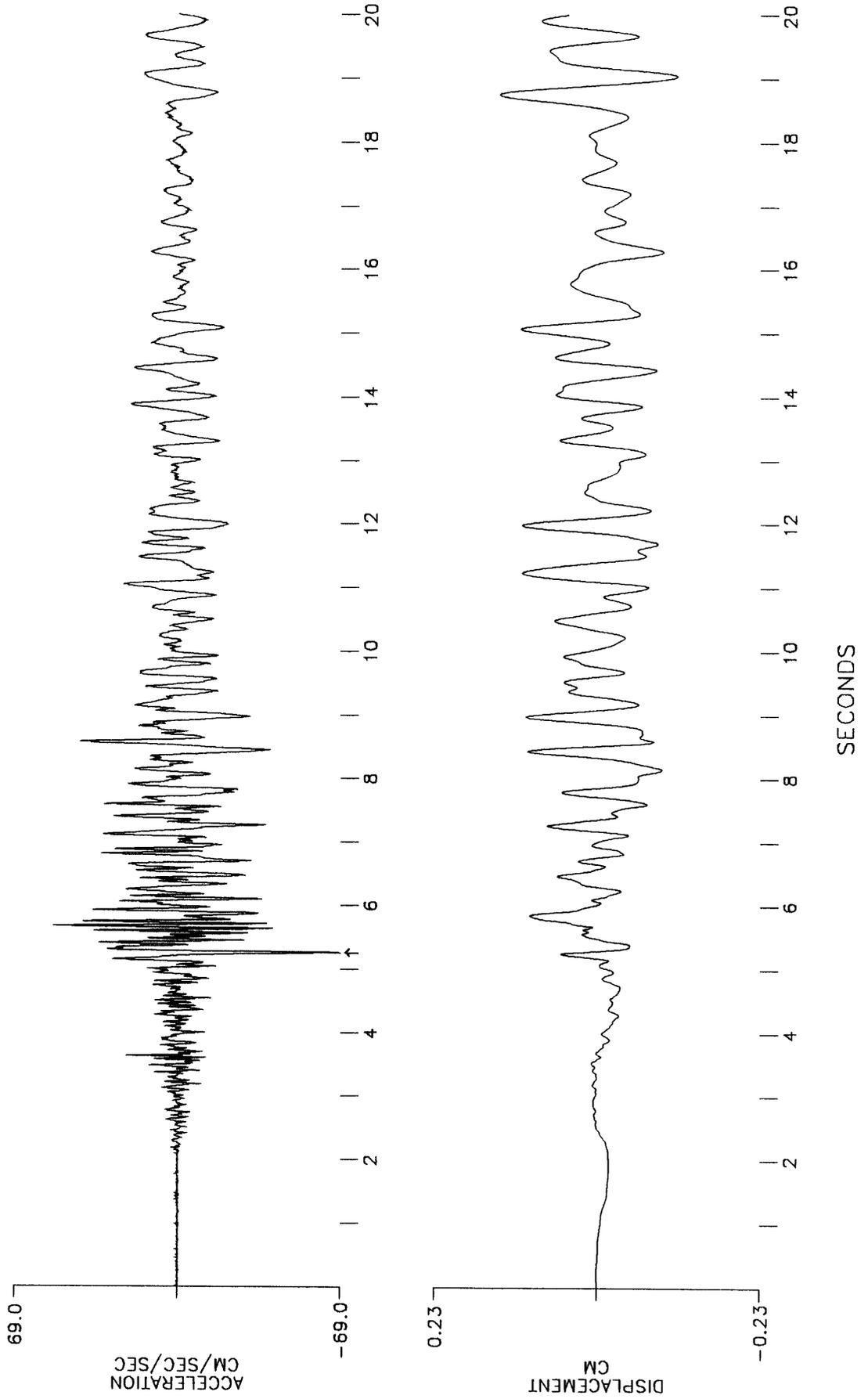
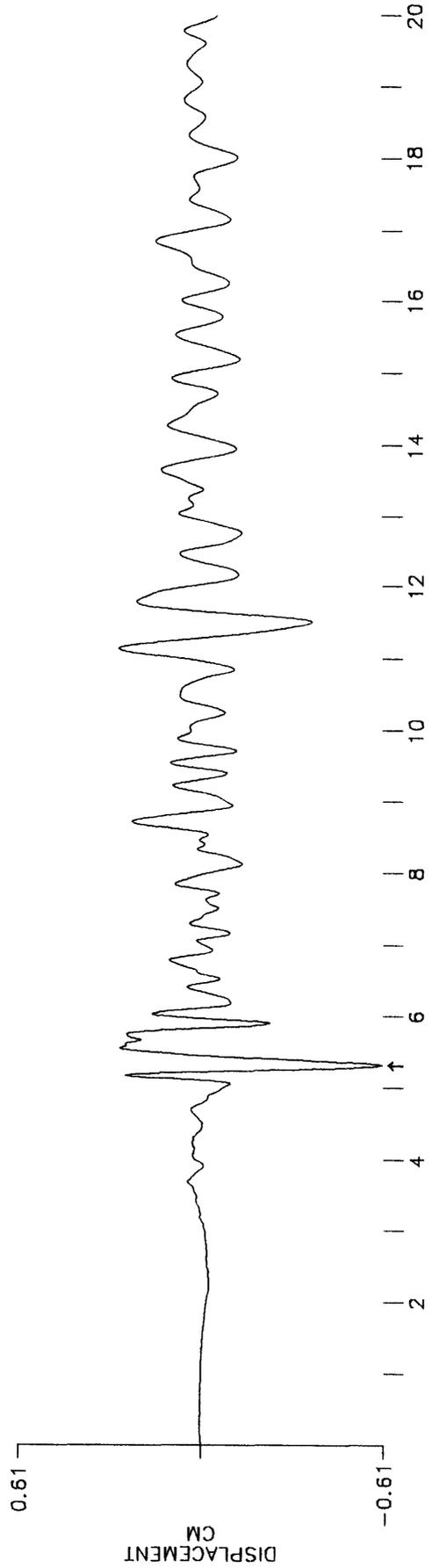
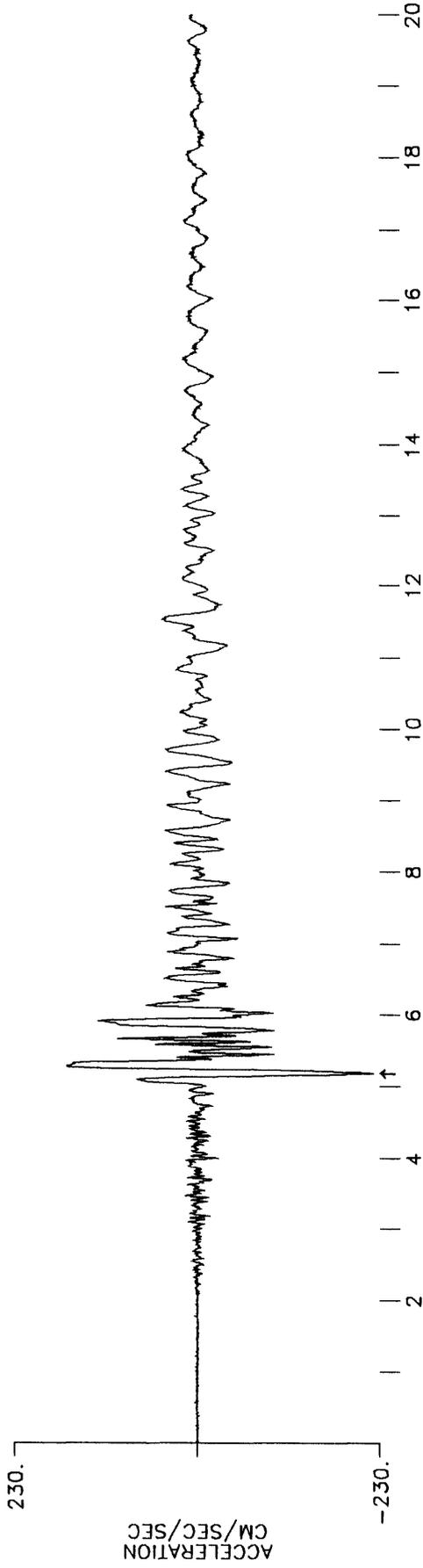


Fig 3

HOLLISTER, DIFFERENTIAL ARRAY NO 6 - NO 1, ROTATED  
018 DEGREES  
EARTHQUAKE OF FEBRUARY 20, 1988, 0839:57.5 GMT  
BUTTERWORTH AT .2 HZ, ORDER 2, BASELINE CORR: 0 TO 2 SEC  
PEAK VALUES (CM/SEC/SEC): -222.51, -0.61



SECONDS

Fig 4

HOLLISTER, DIFFERENTIAL ARRAY NO 5 - NO 1, ROTATED  
018 DEGREES  
EARTHQUAKE OF FEBRUARY 20, 1988, 0839:57.5 GMT  
BUTTERWORTH AT .2 HZ, ORDER 2, BASELINE CORR: 0 TO 2 SEC  
PEAK VALUES (CM/SEC/SEC): -162.94, -0.22

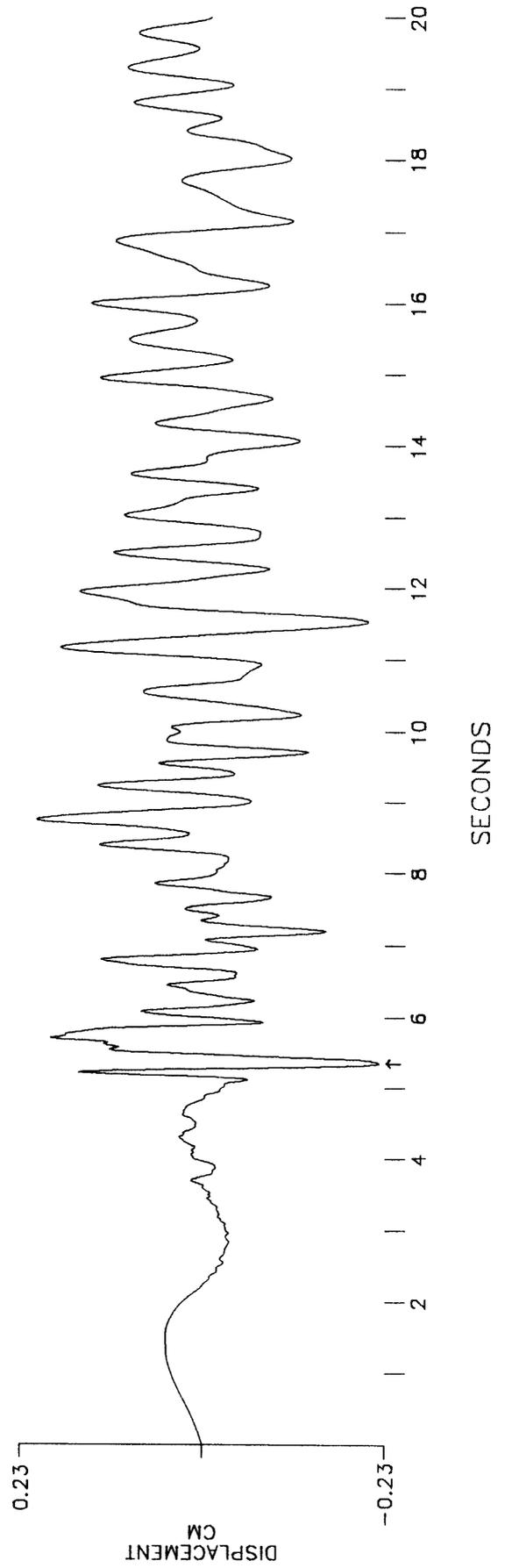
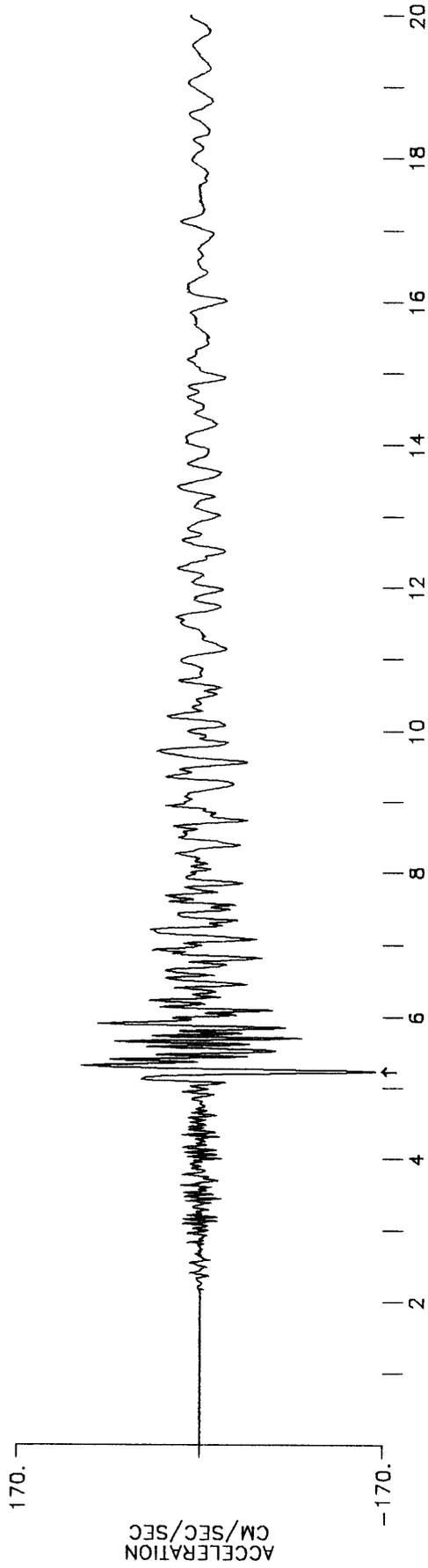


Fig 5

HOLLISTER, DIFFERENTIAL ARRAY NO 6 - NO 5, ROTATED  
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PEAK VALUES (CM/SEC/SEC): -238.60, -0.46

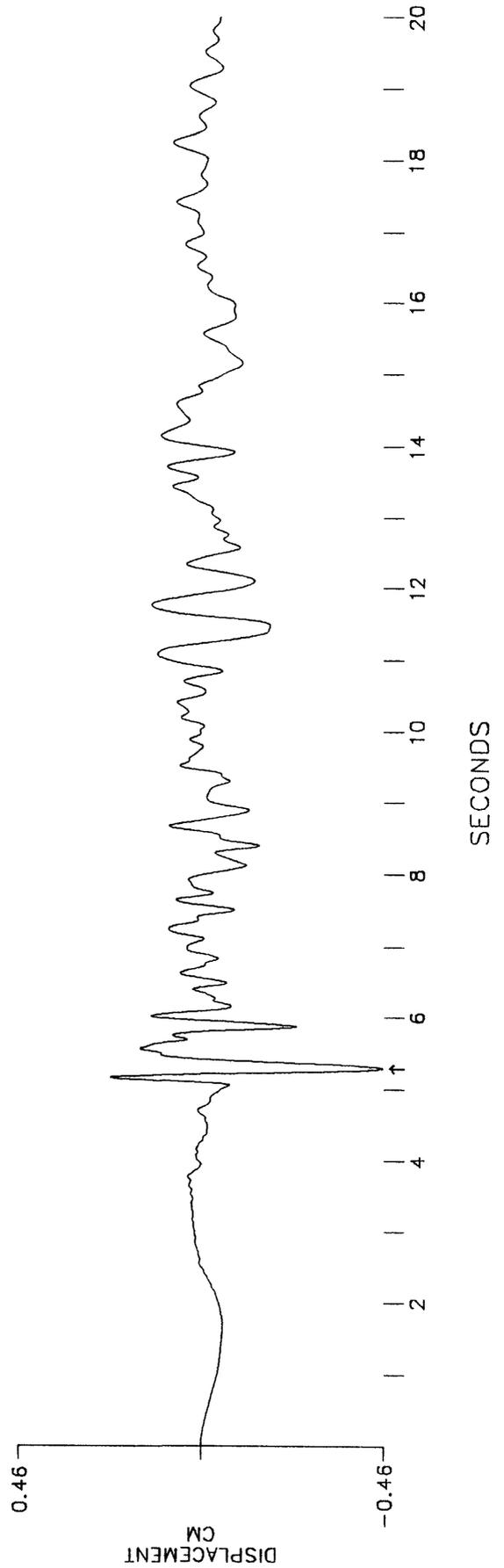
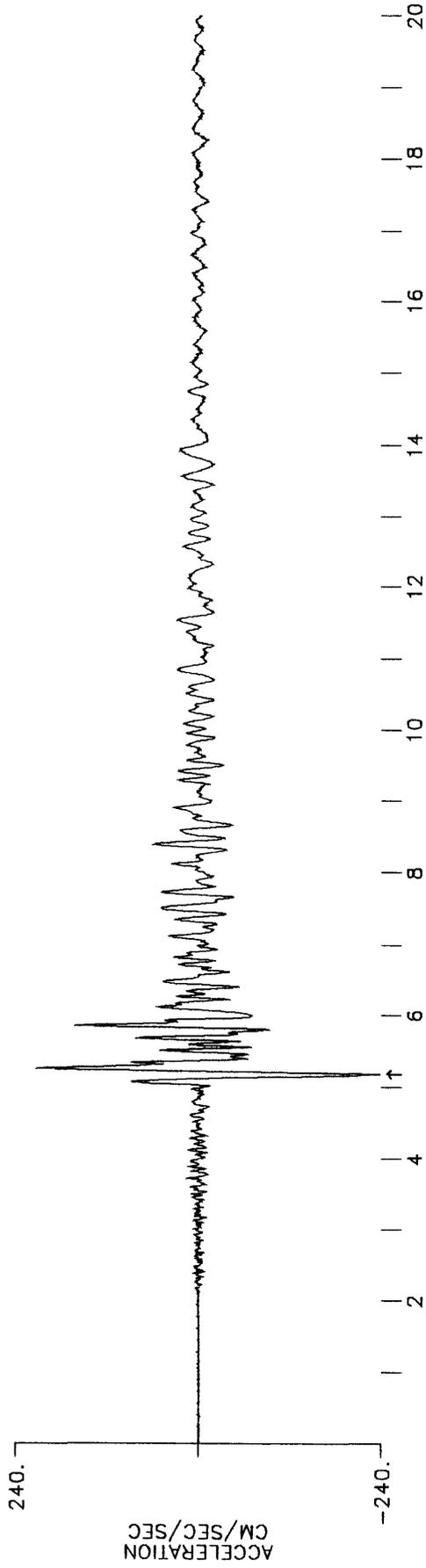
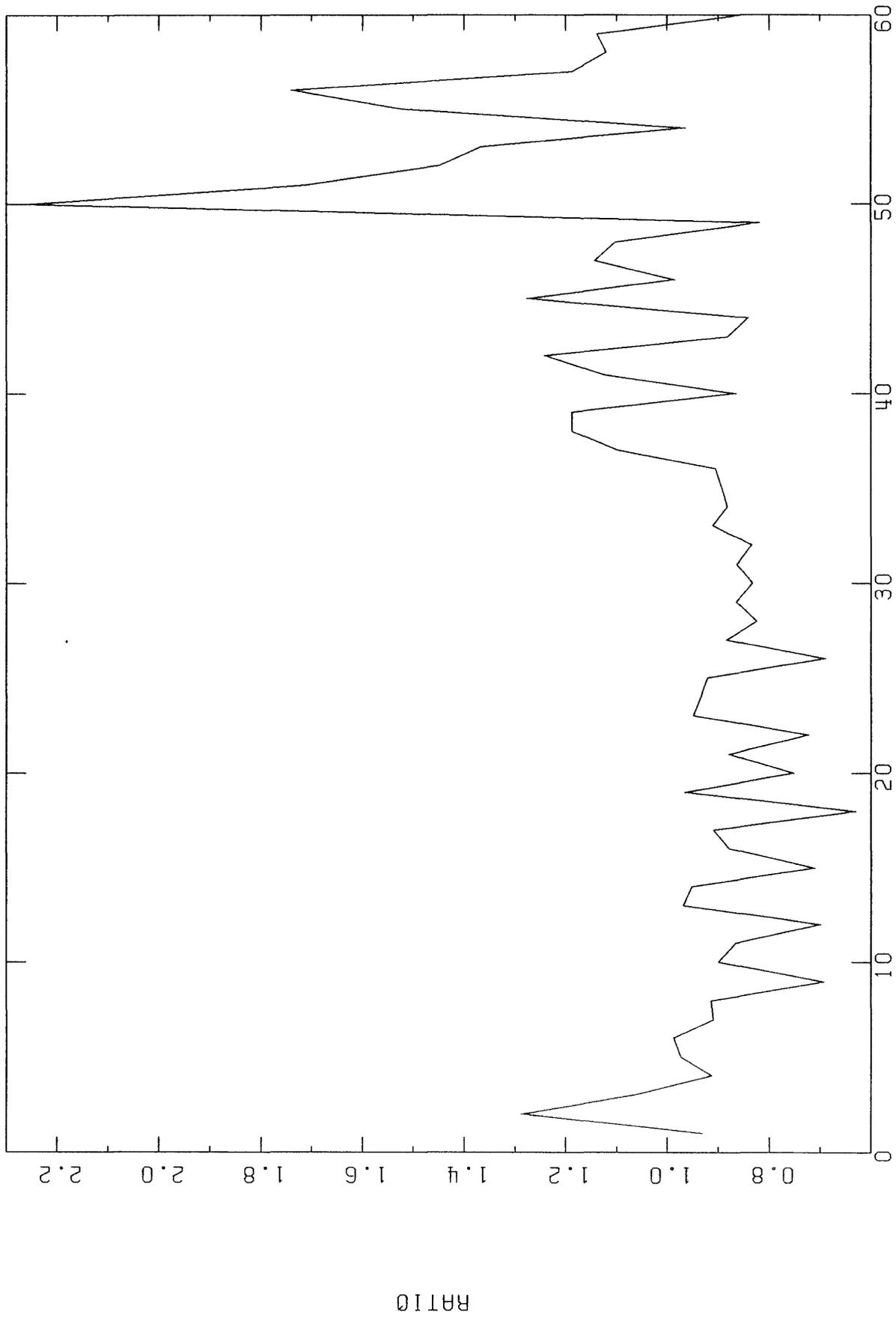


Fig 6

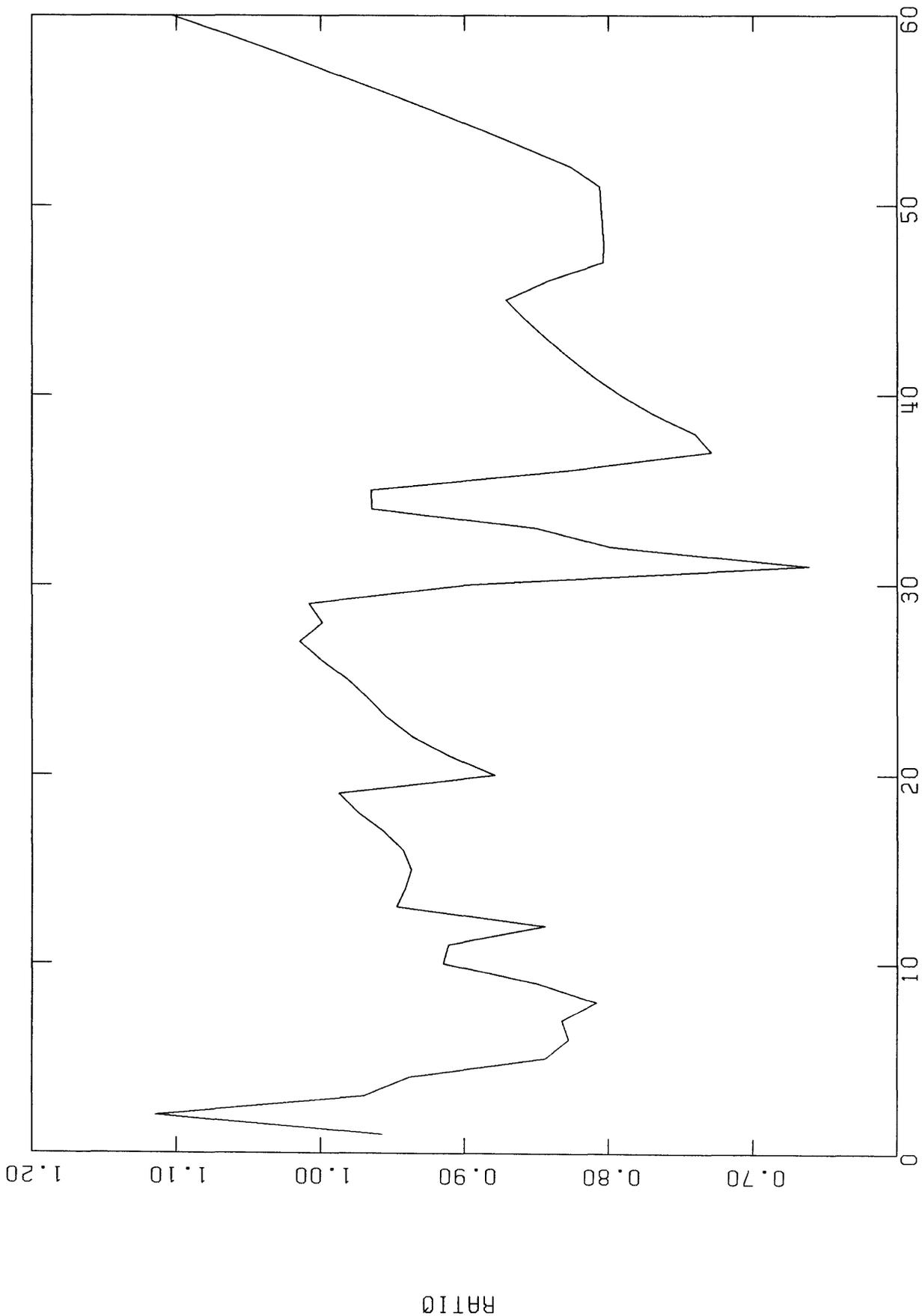
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OMEGA 1

Fig 7

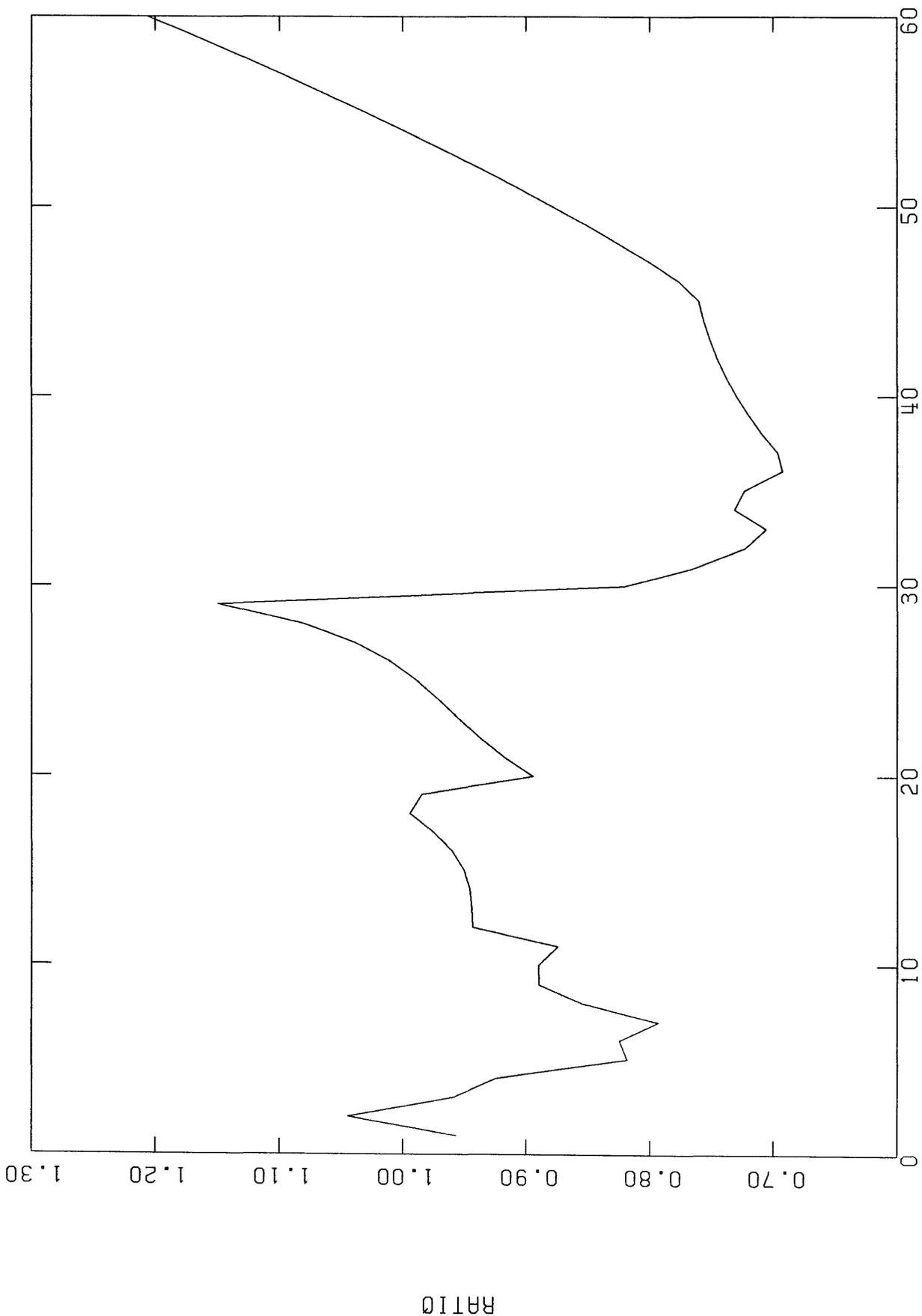
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OMEGA 1

Fig 8

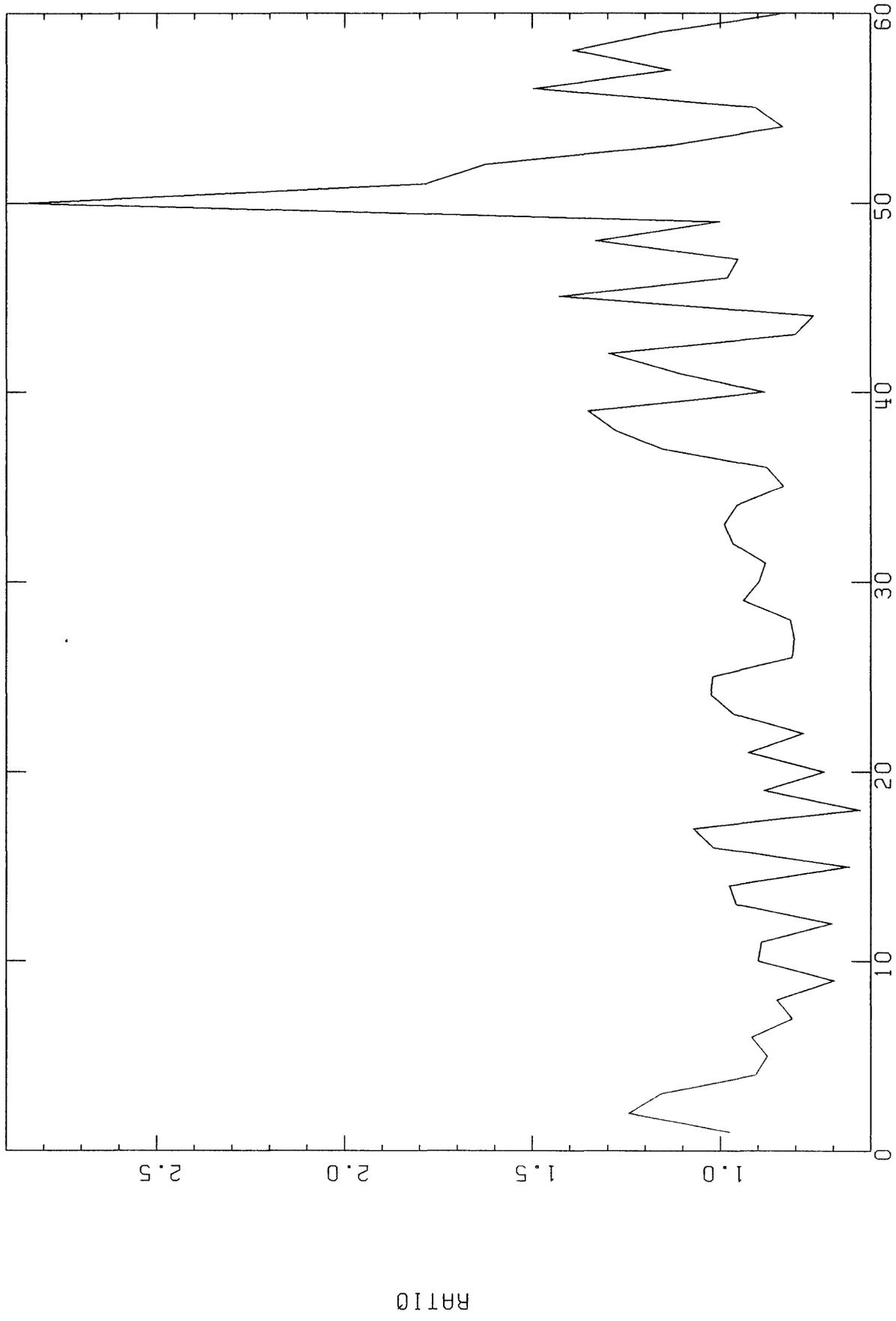
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OMEGA 1

Fig 9

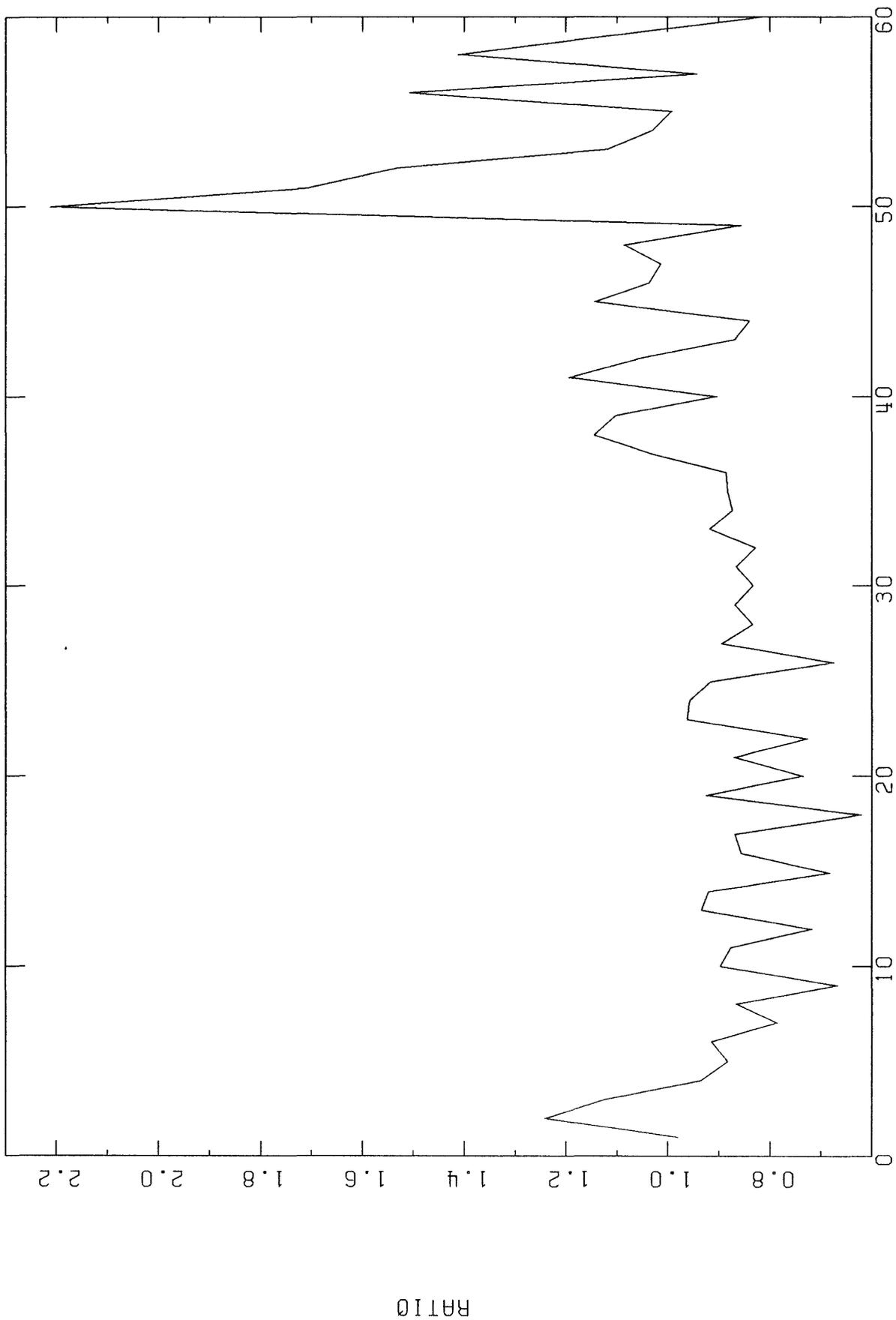
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OMEGA 1

Fig 10

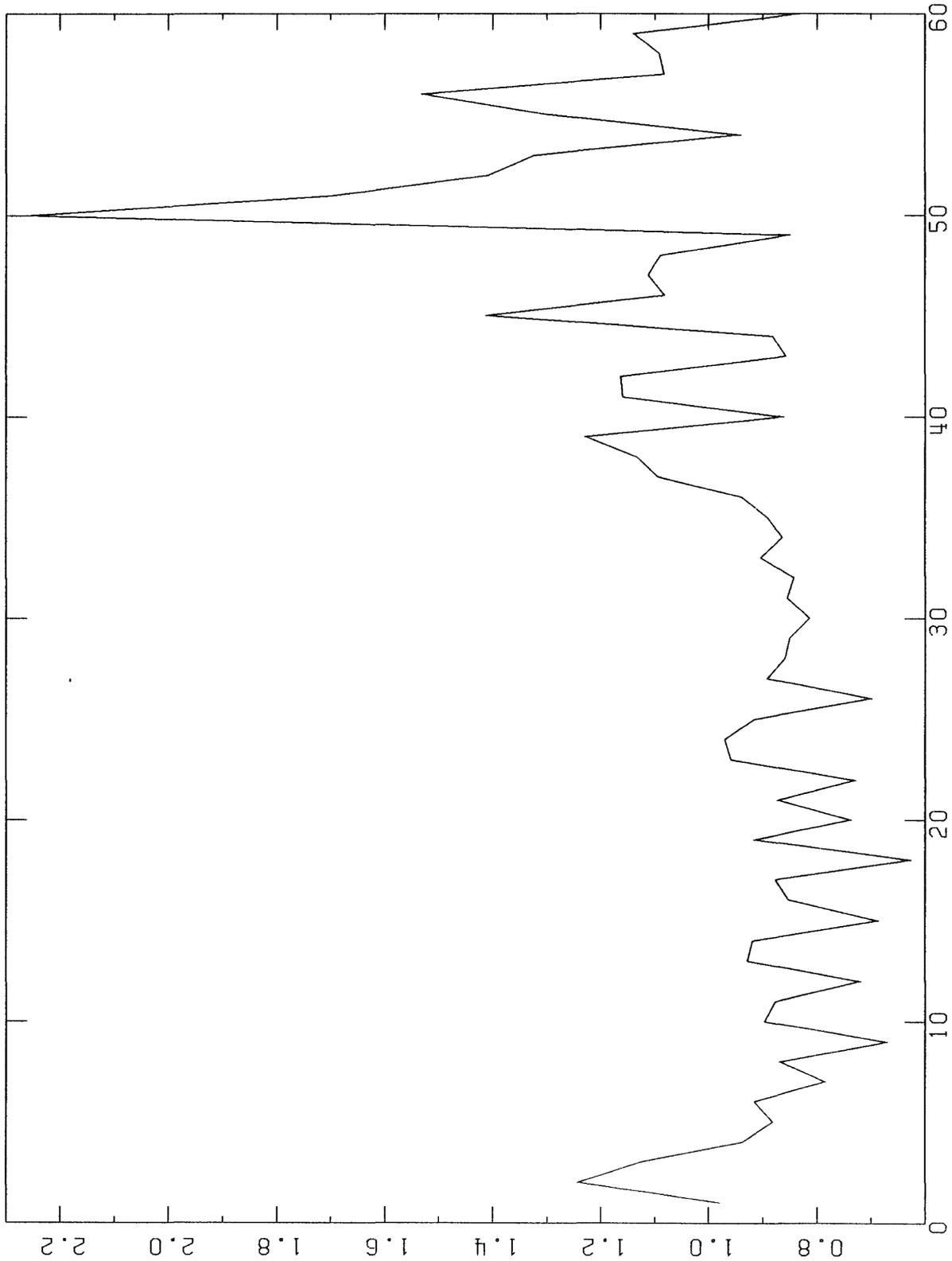
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OMEGA 1

Fig 11

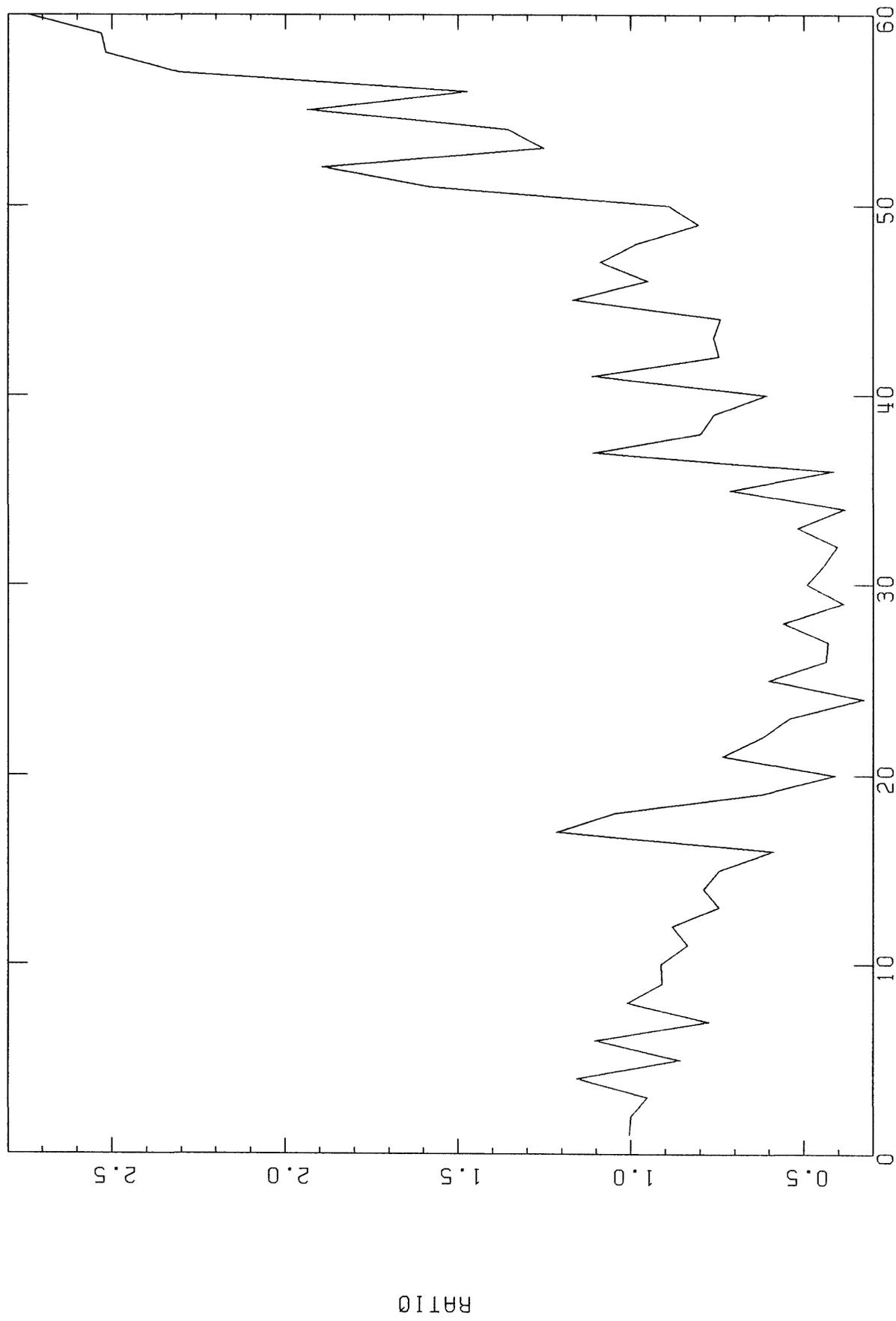
HDIFF2-HDIFF1, OMEGA 2 = 50.0 DAMPING = 0.0, LENGTH = 30. SEC, CHANNEL 03



OMEGA 1

Fig 12

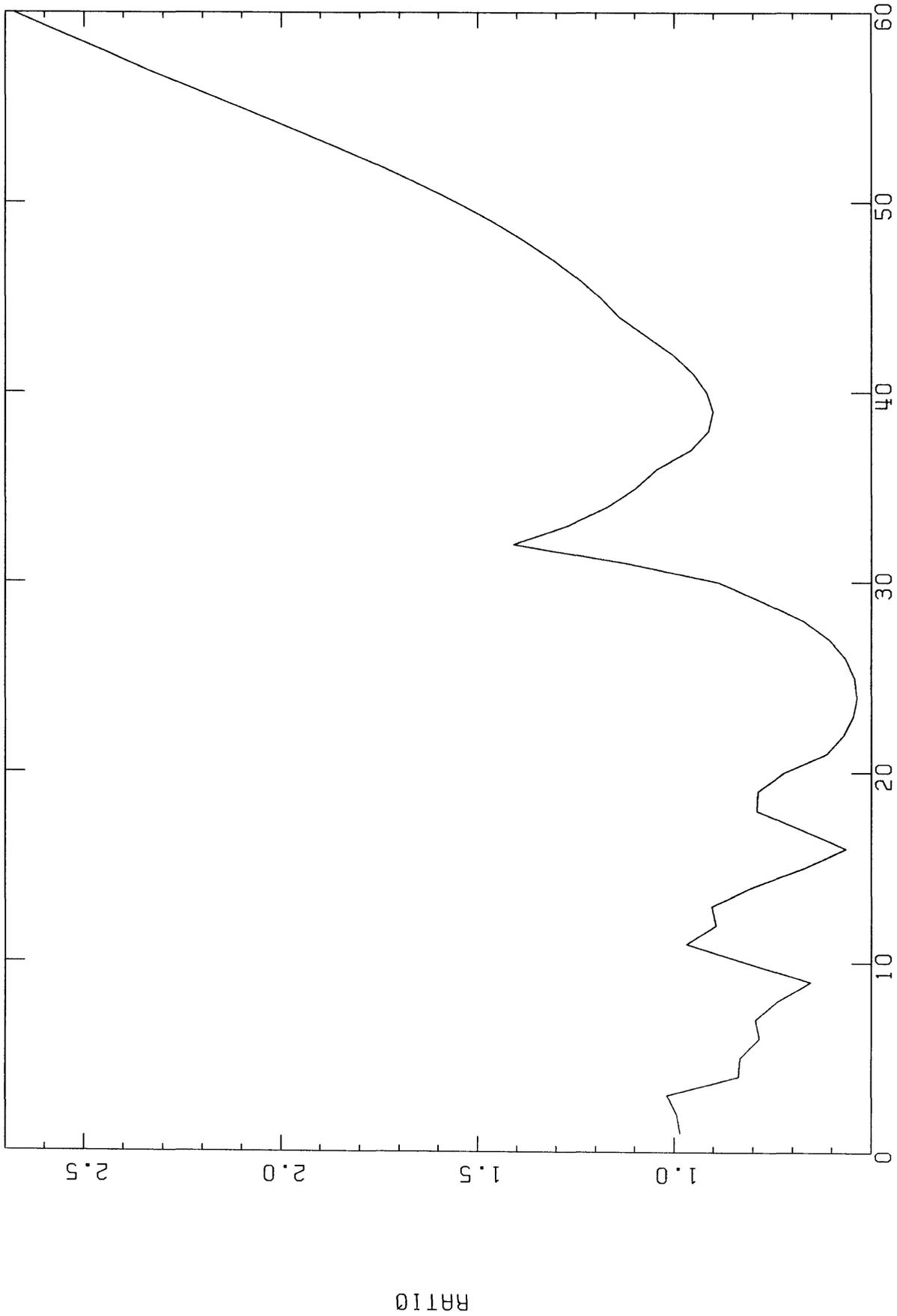
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OMEGA 1

Fig 13

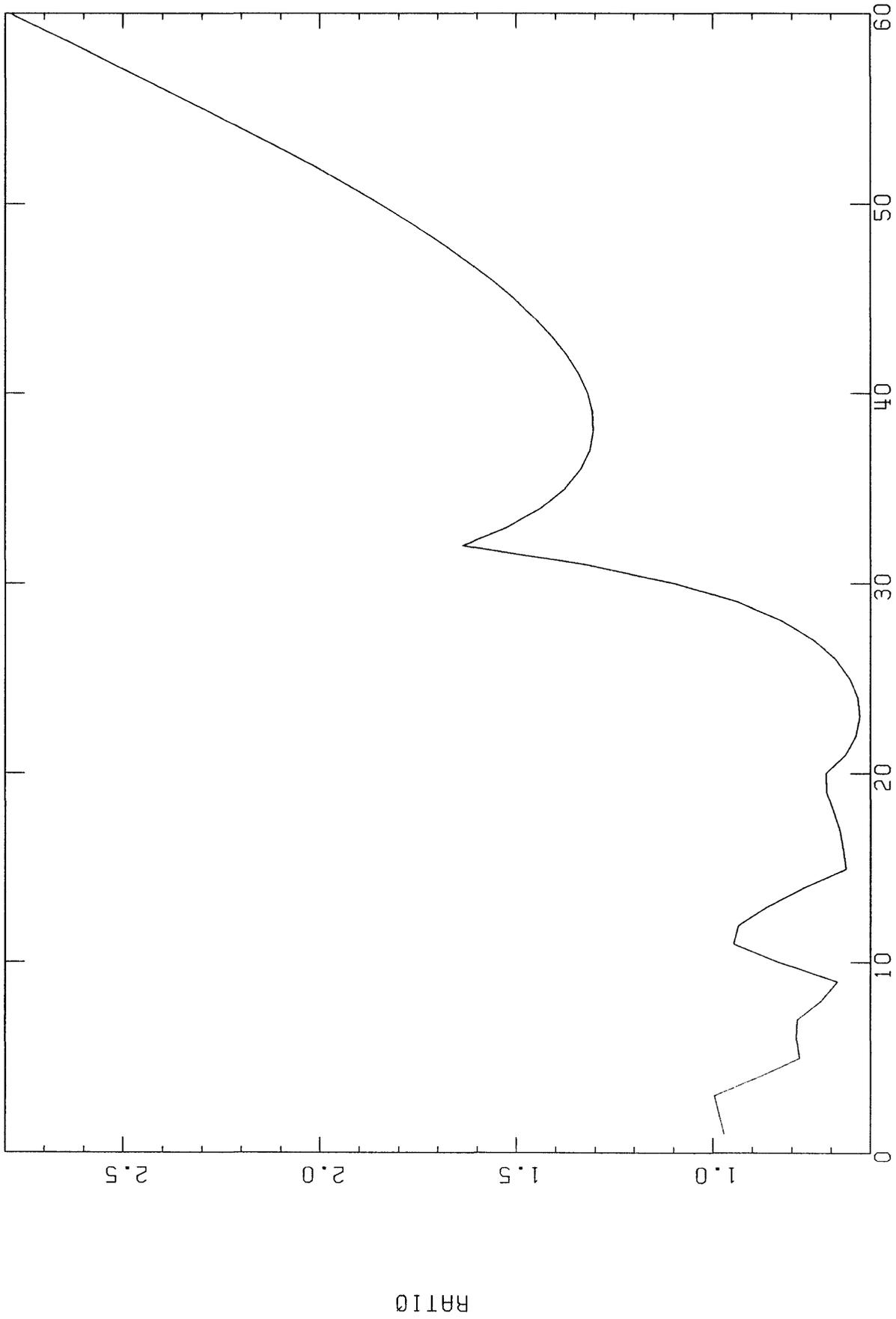
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OMEGA 1

Fig 14

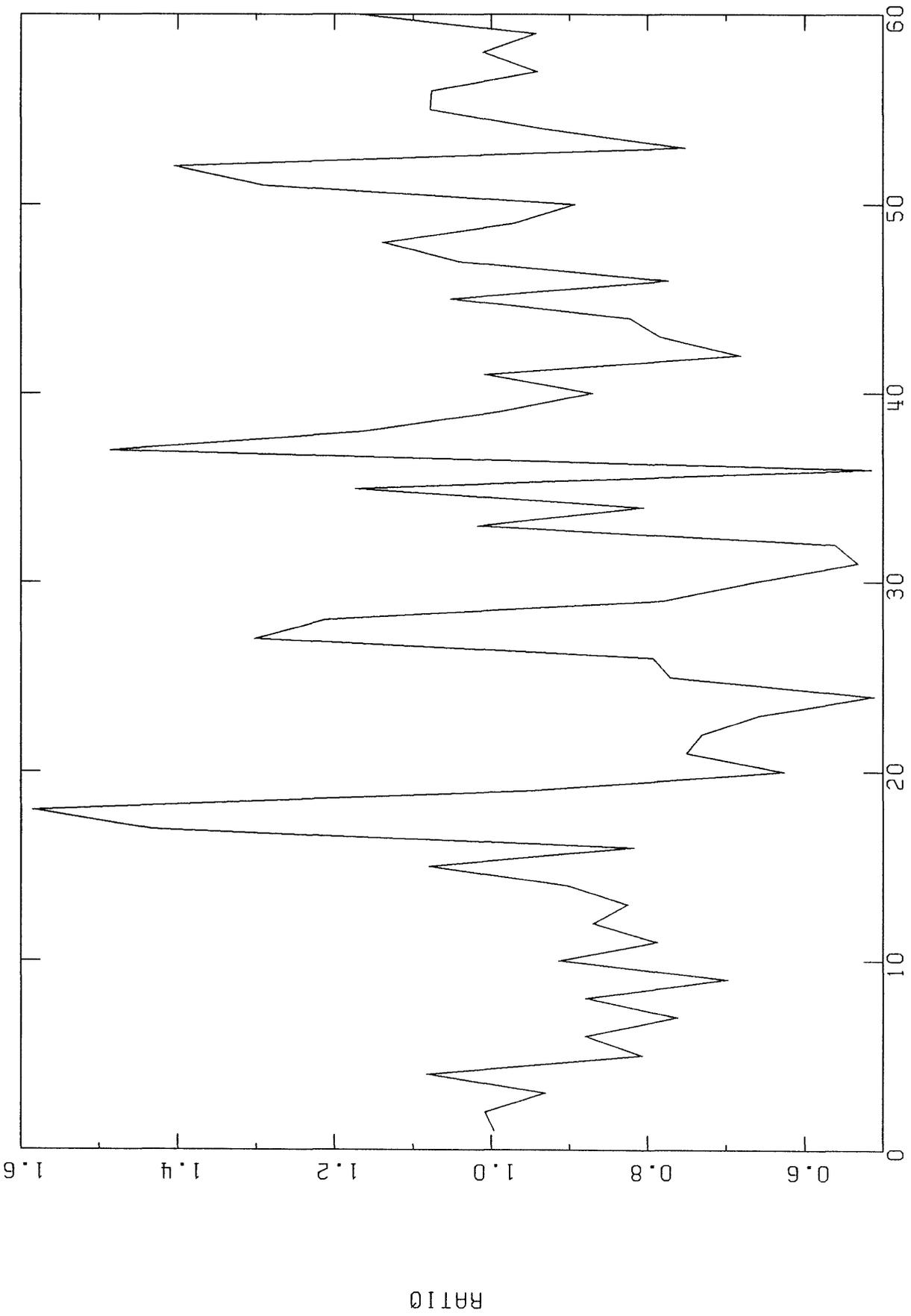
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OMEGA 1

Fig 15

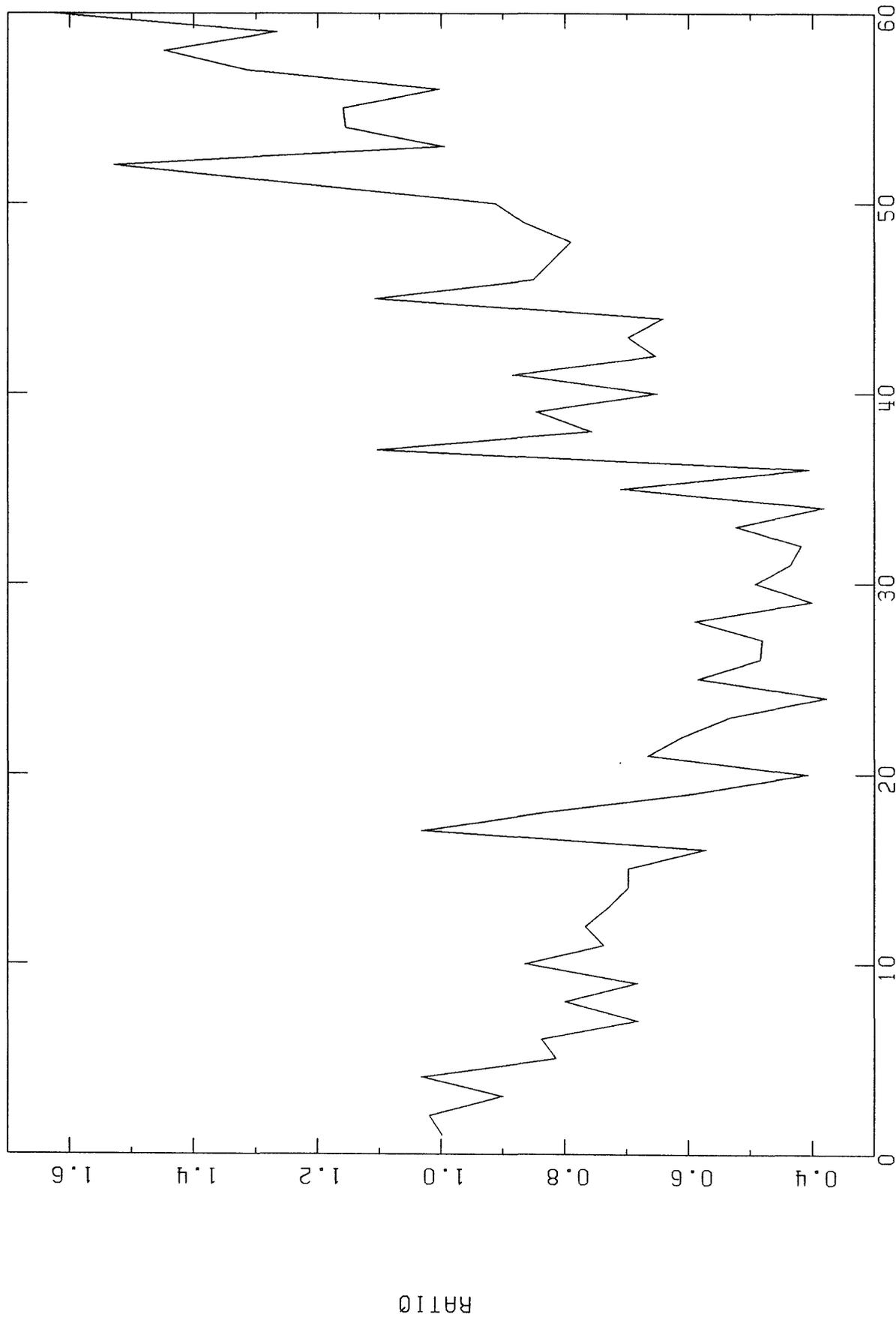
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OMEGA 1

Fig 16

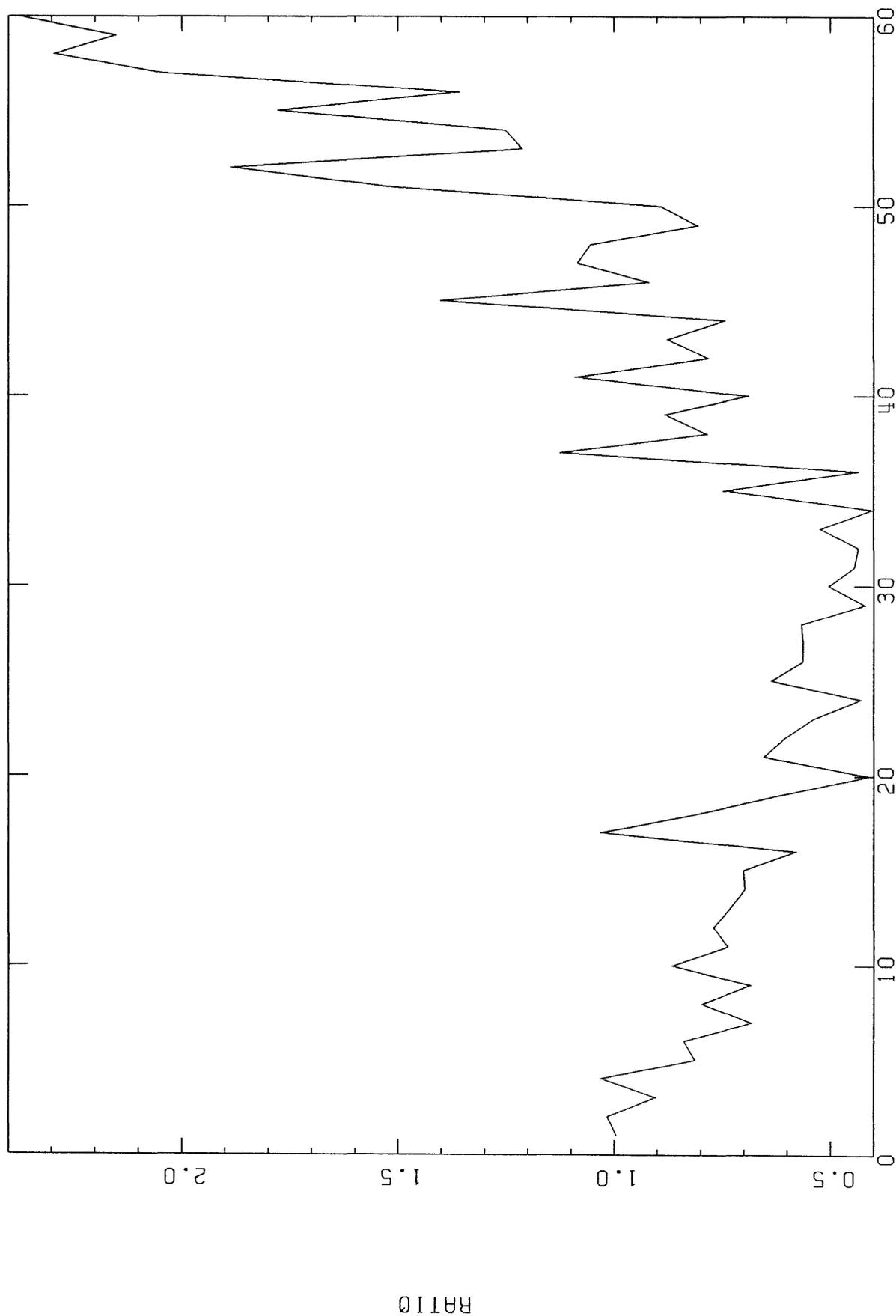
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OMEGA 1

Fig 17

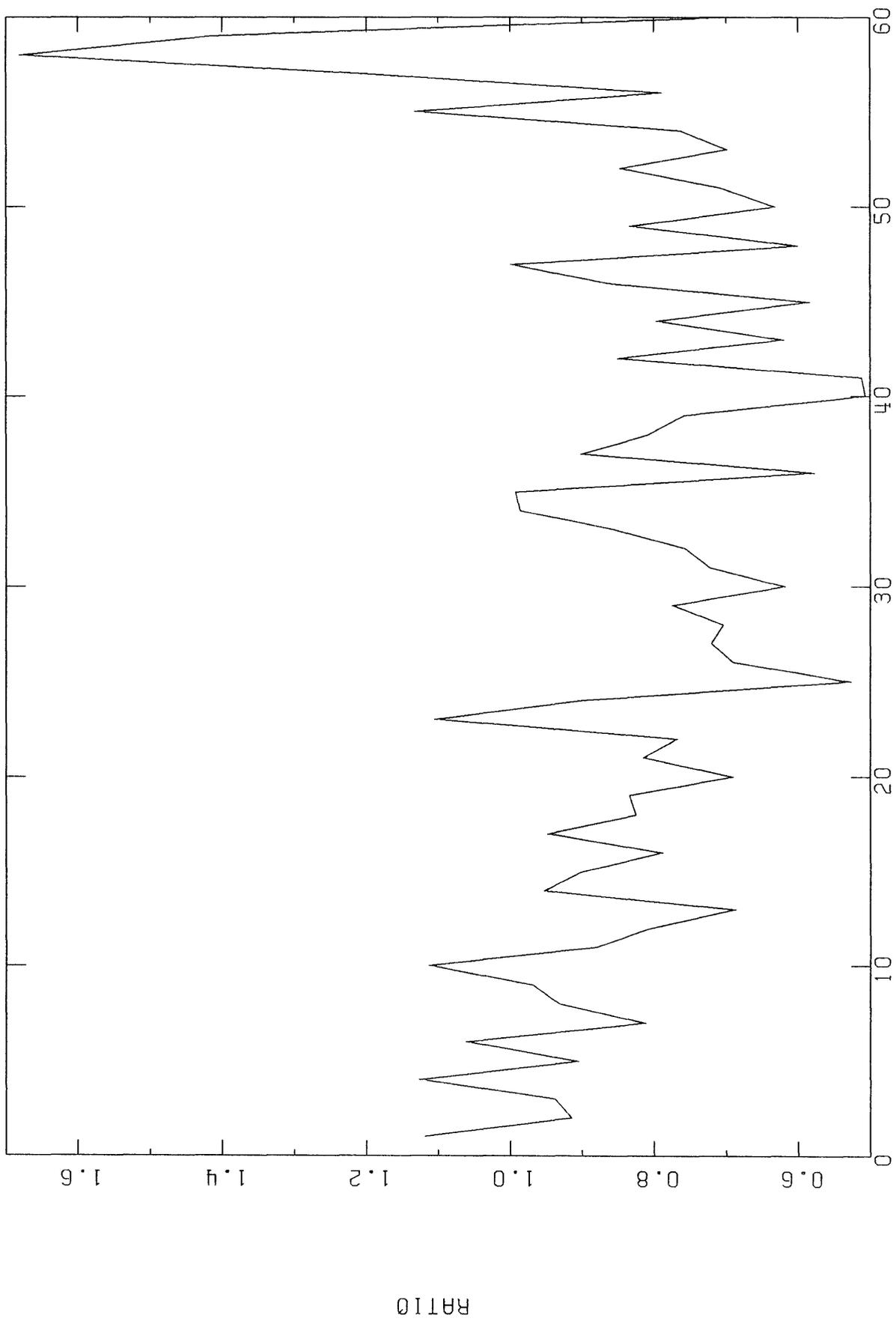
RHOIFF6-RHOIFF1, OMEGA 2 = 50.0 DAMPING = 0.0, LENGTH = 30. SEC, CHANNEL 03



OMEGA 1

Fig 18

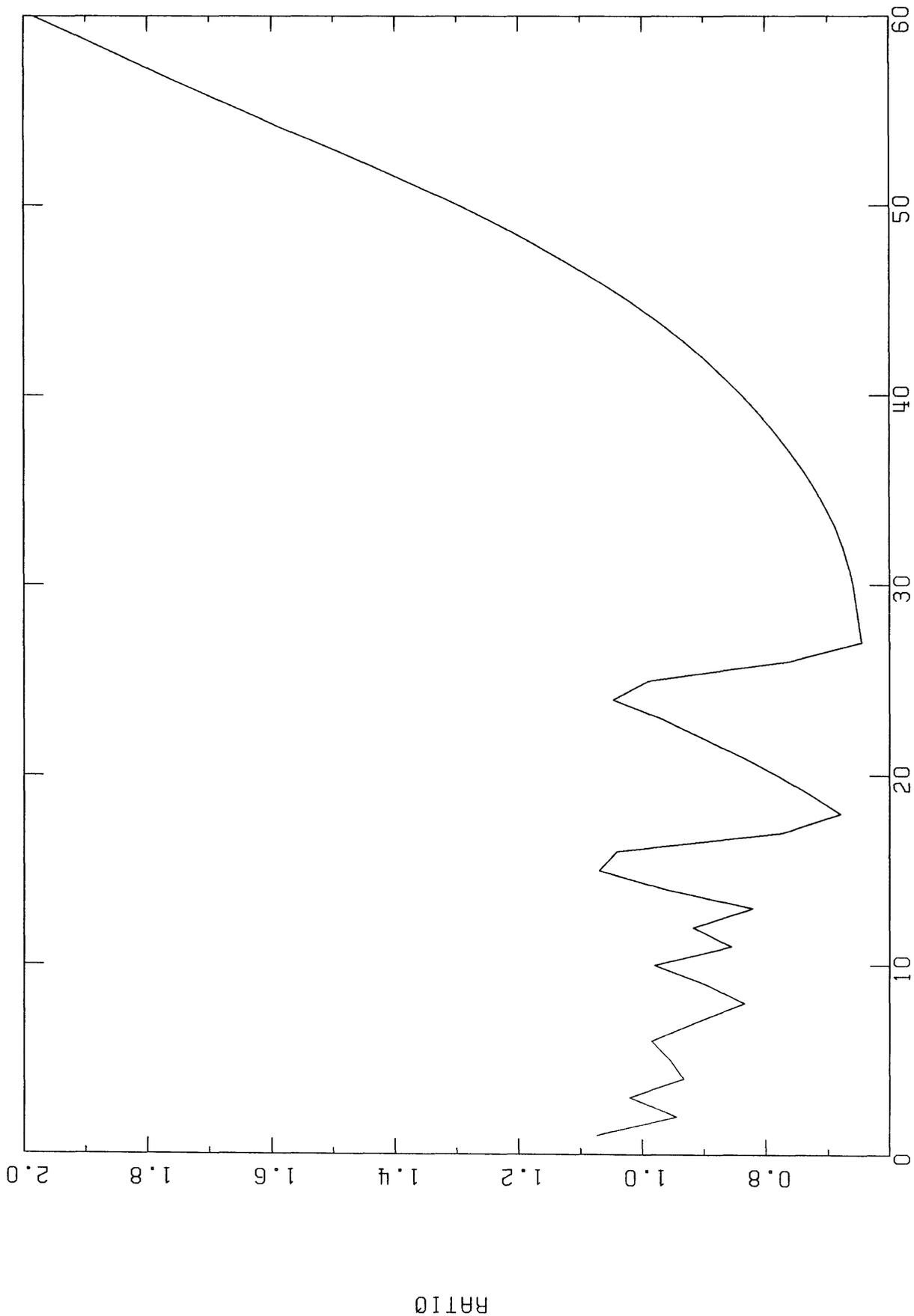
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OMEGA 1

7.19

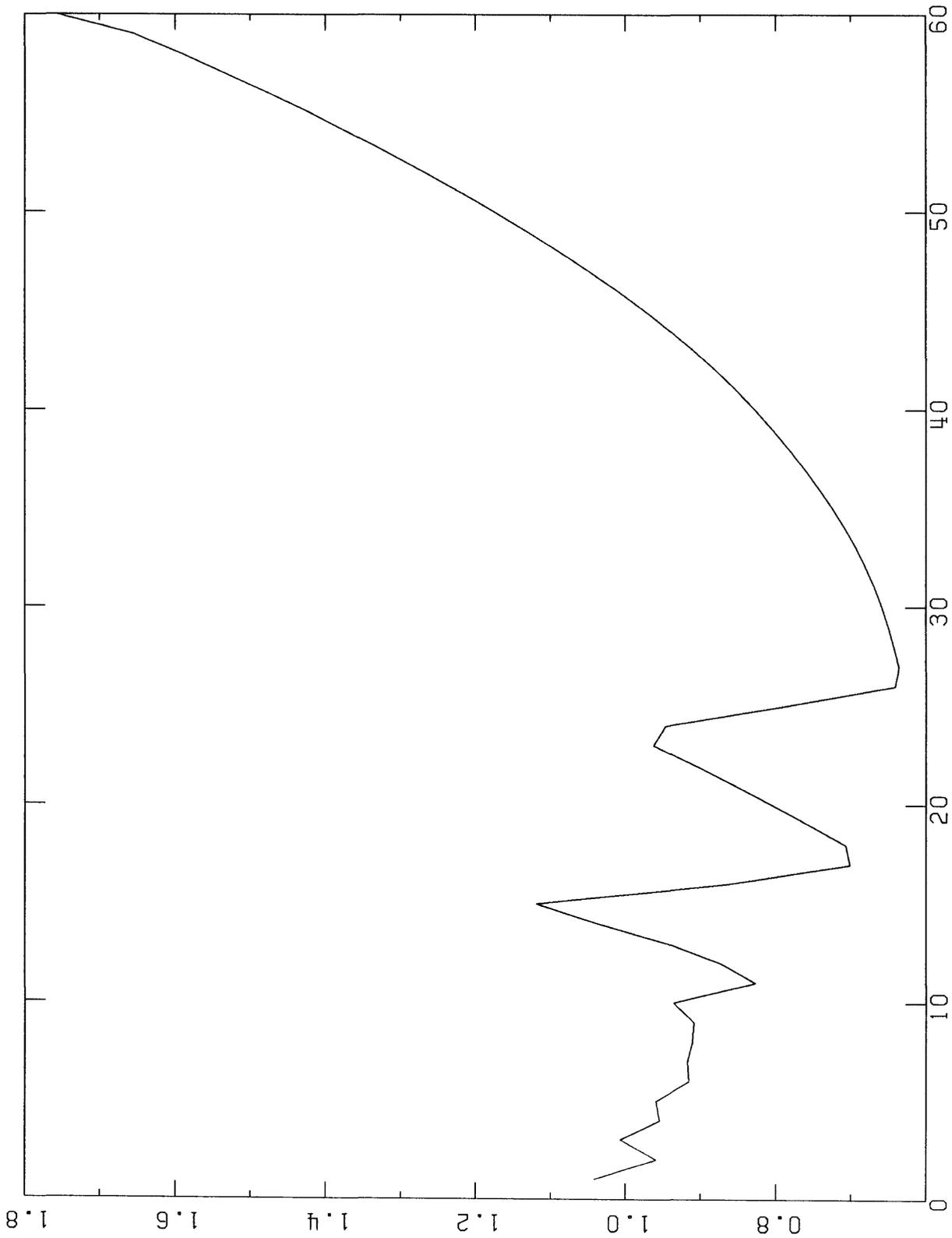
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OMEGA 1

Fig 20

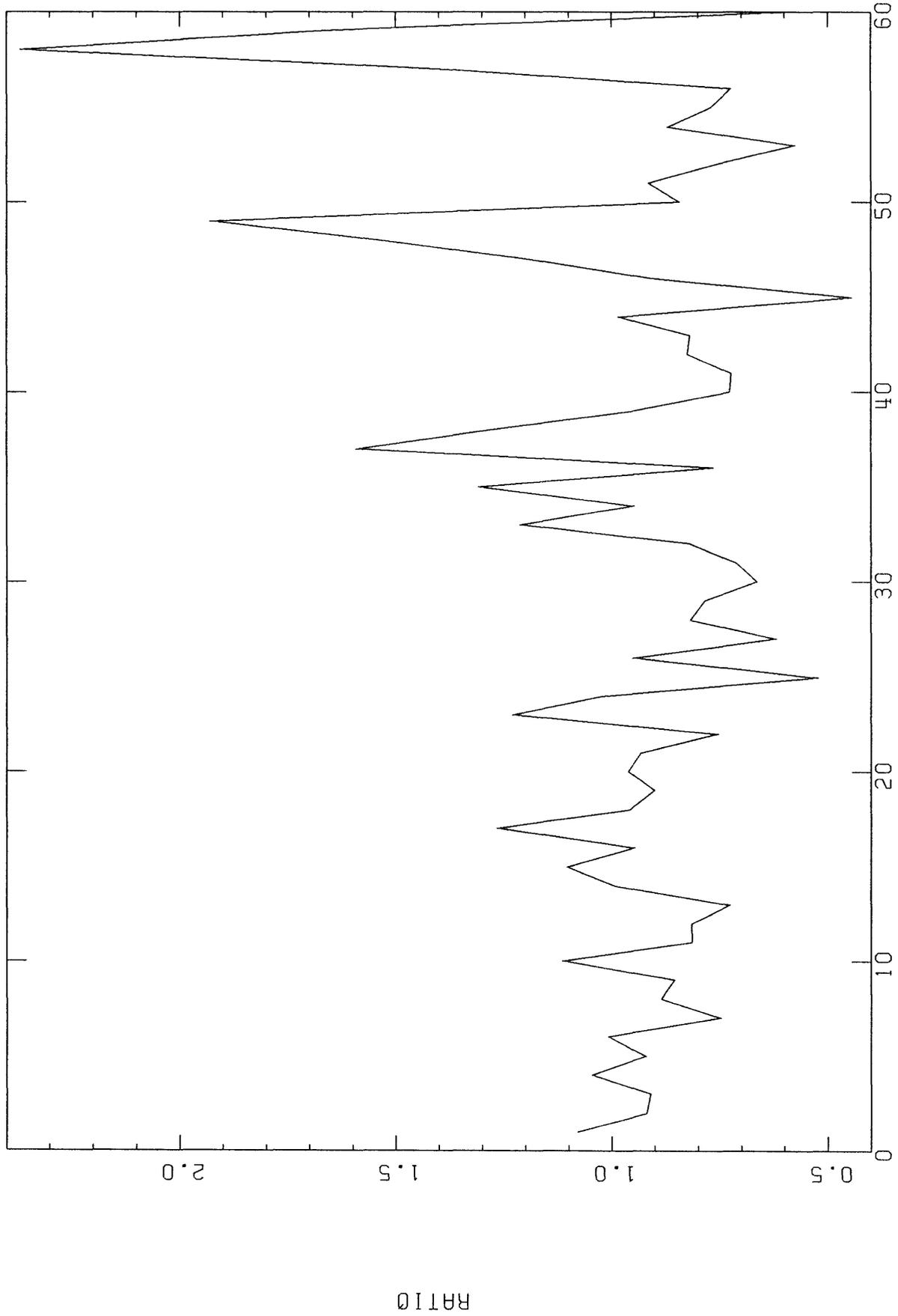
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OMEGA 1

Fig 21

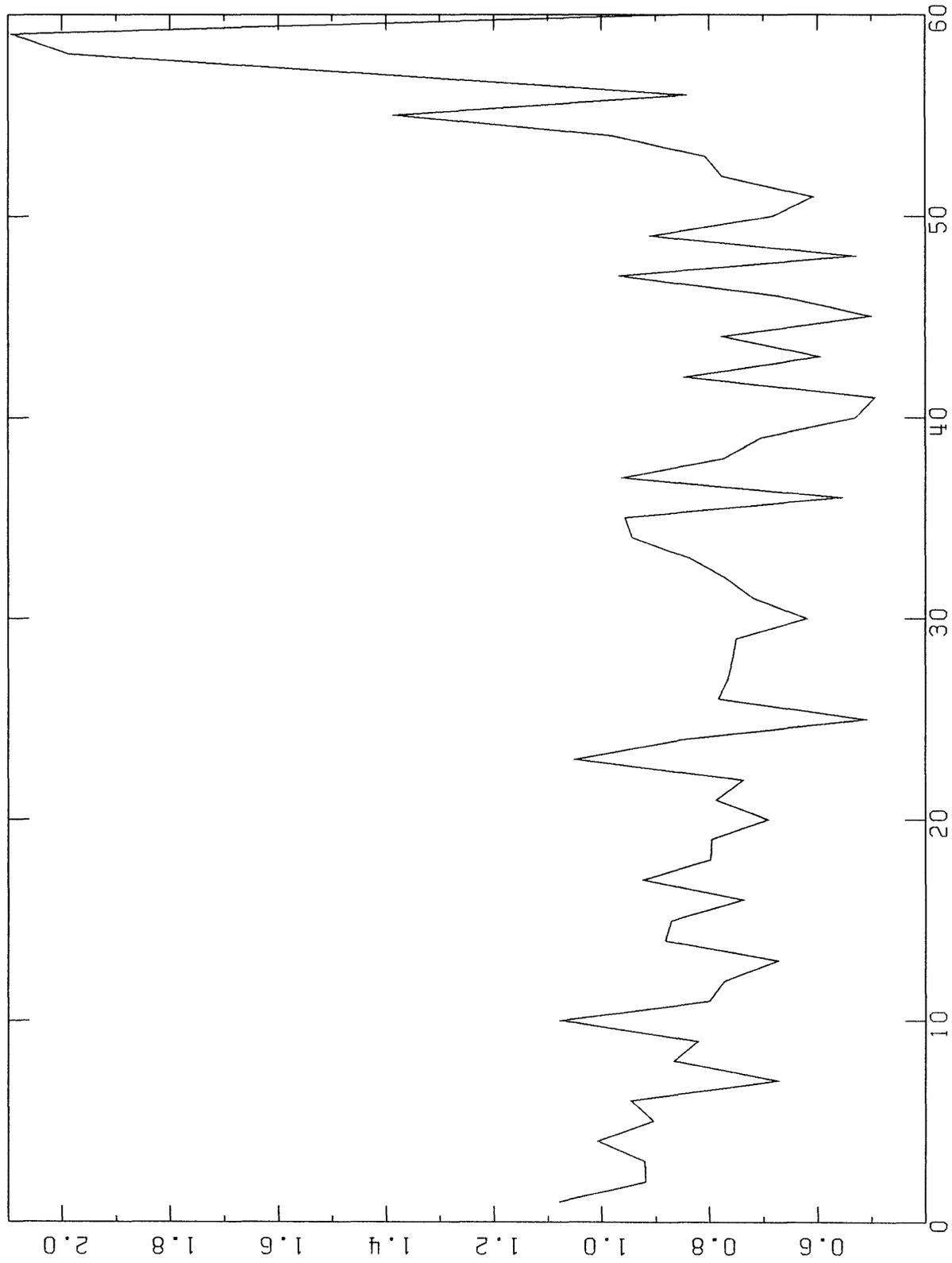
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OMEGA 1

F 22

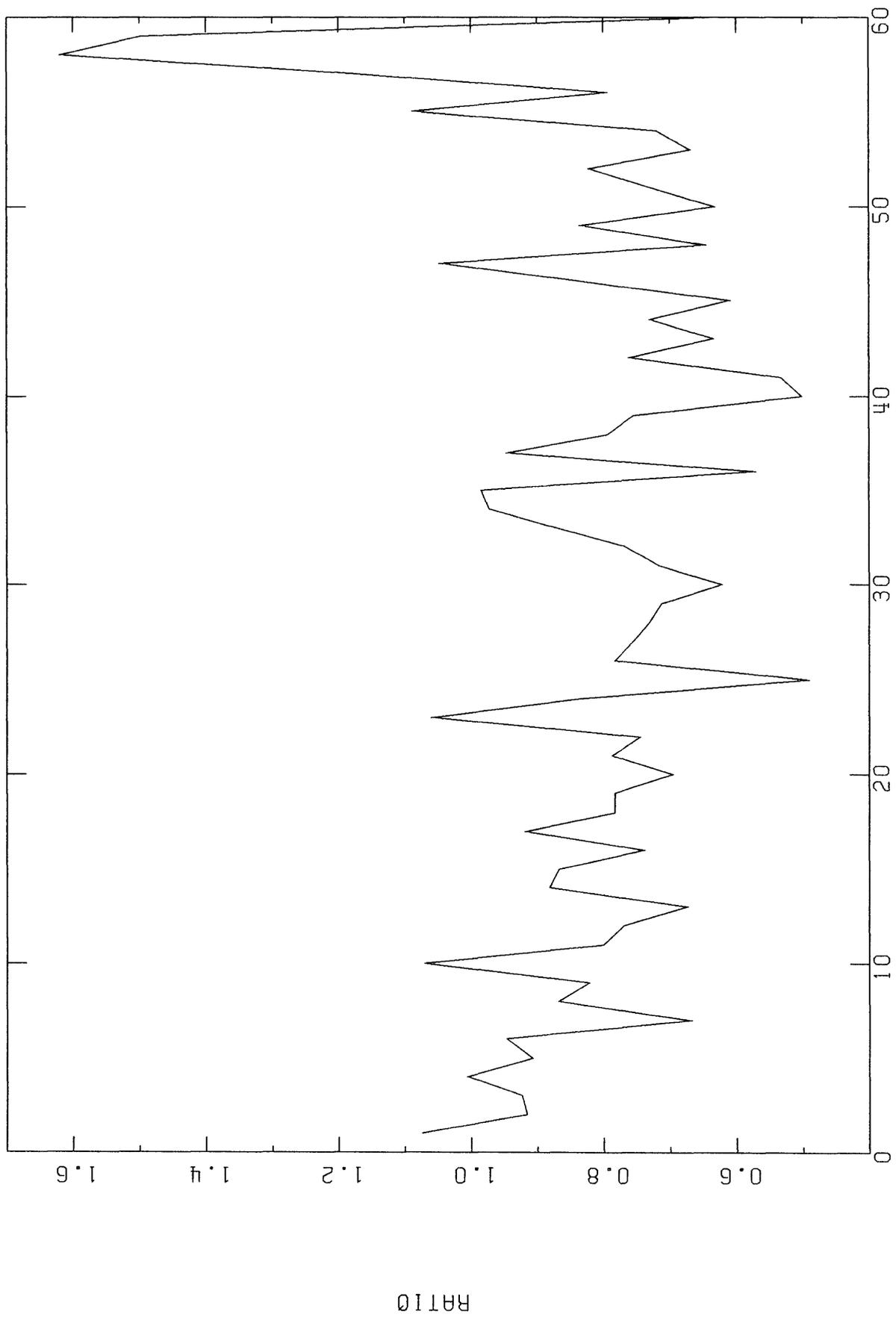
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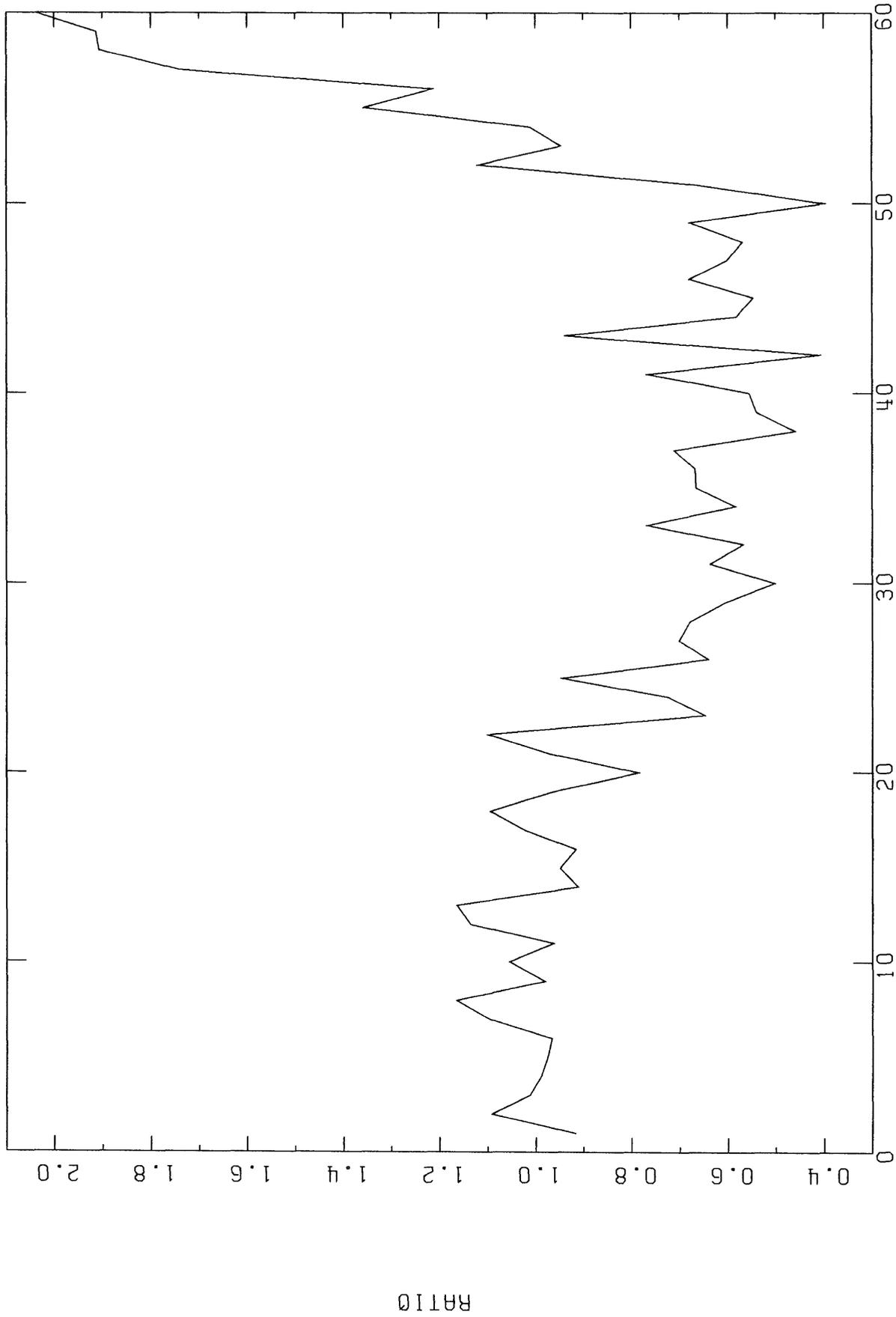
Fig 23

RHDIFFS-RHDIFF1, OMEGA 2 = 50.0 DAMPING = 0.0, LENGTH = 30. SEC, CHANNEL 03



OMEGA 1  
Fig 24

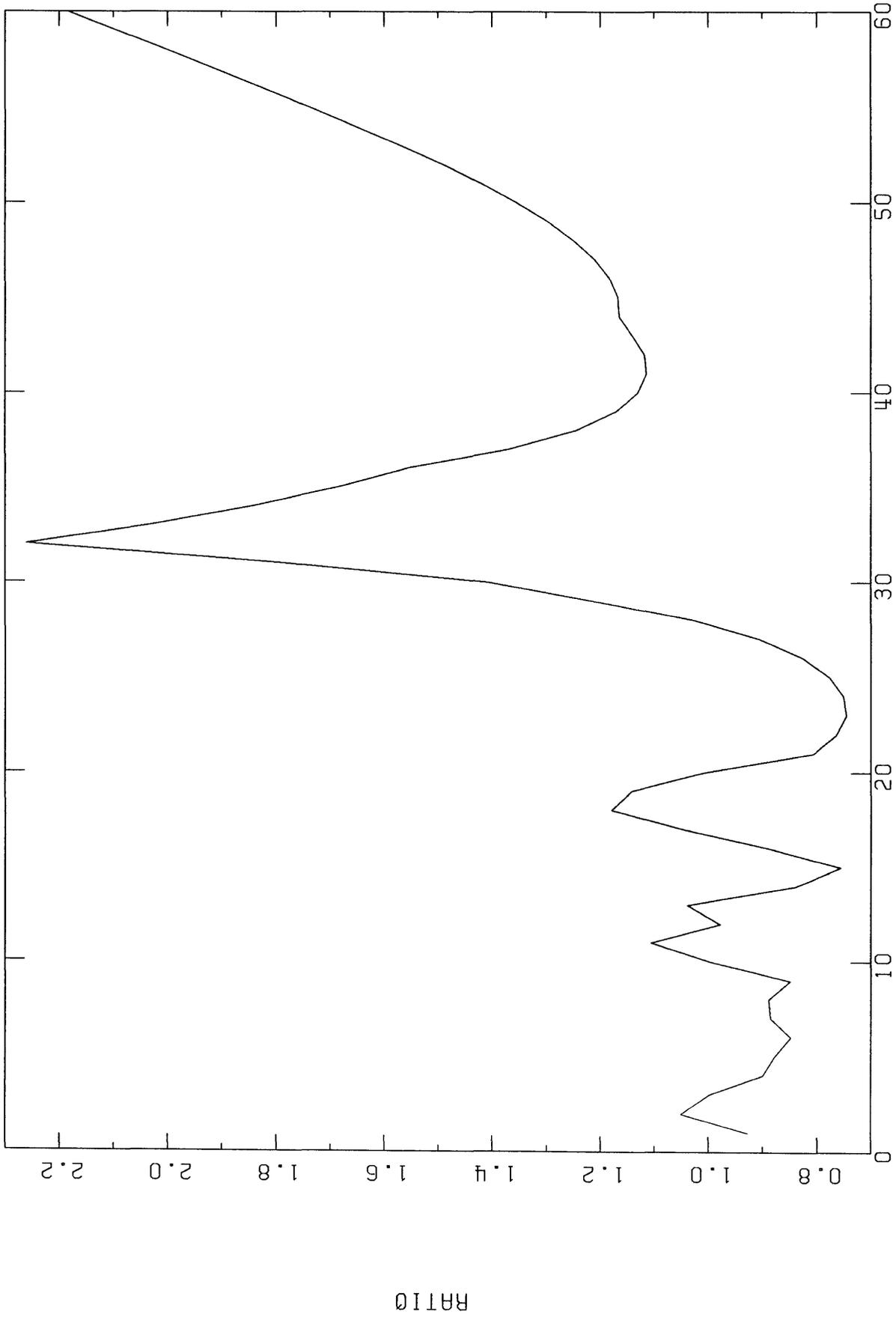
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OMEGA 1

F. 25

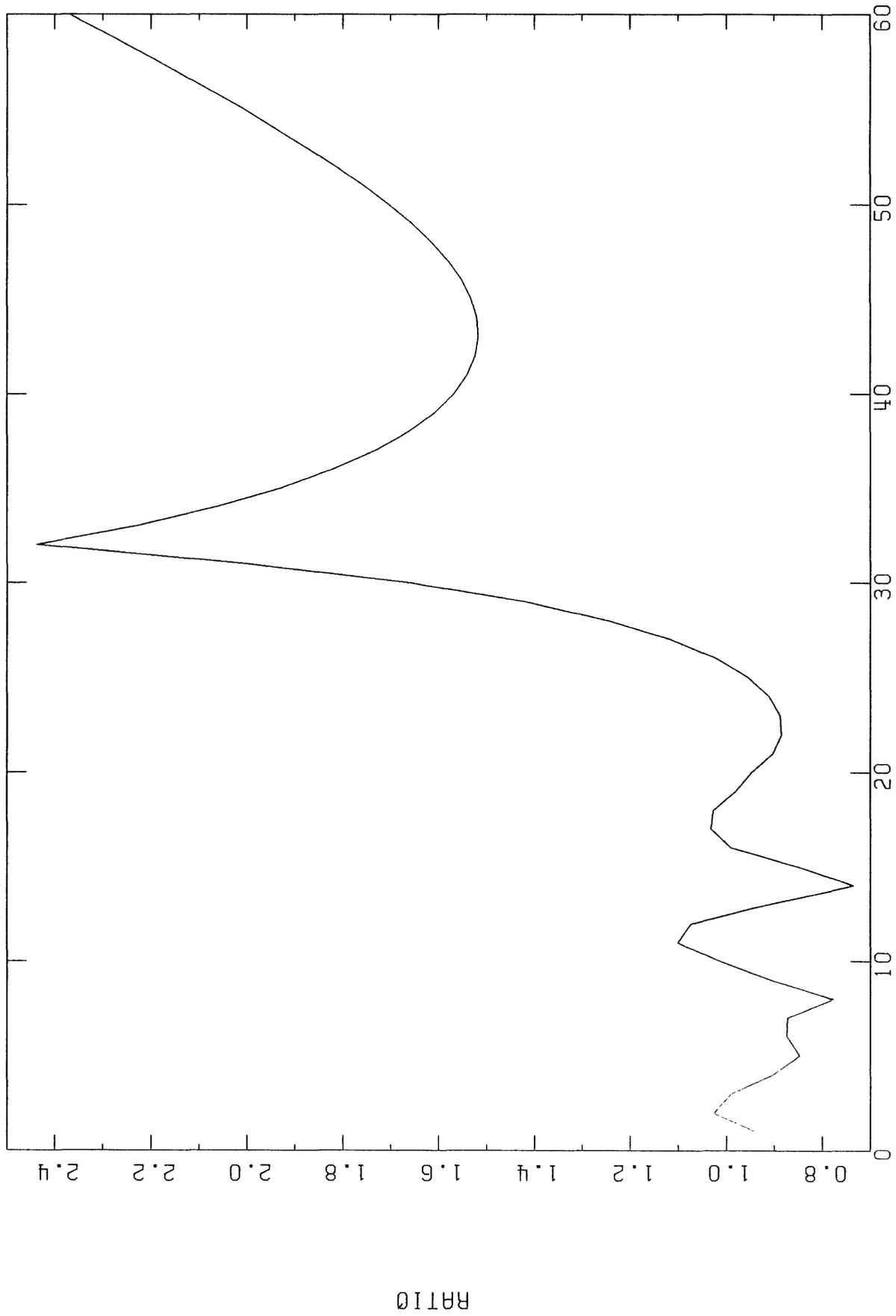
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OMEGA 1

Fig 26

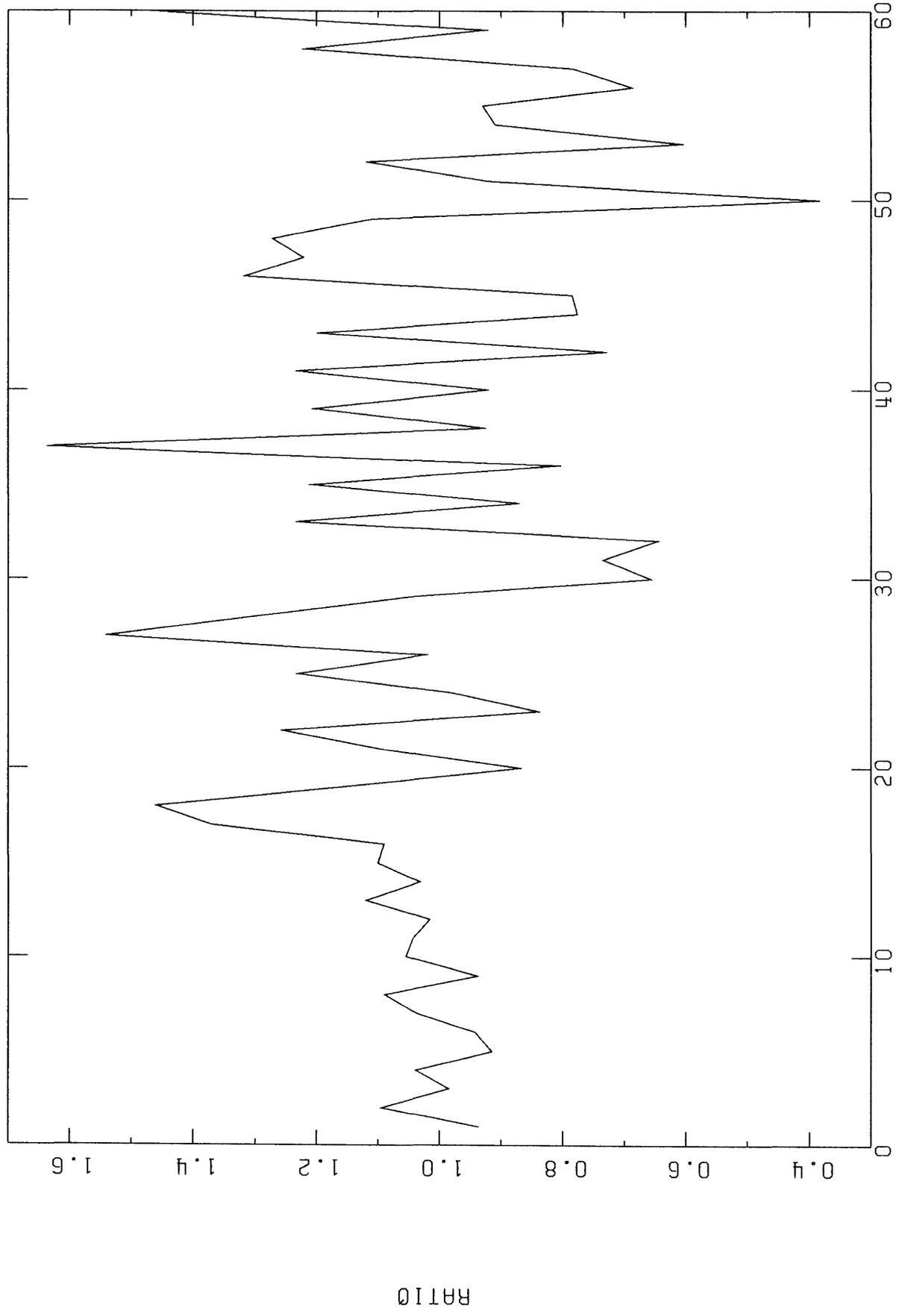
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OMEGA 1

Fig 27

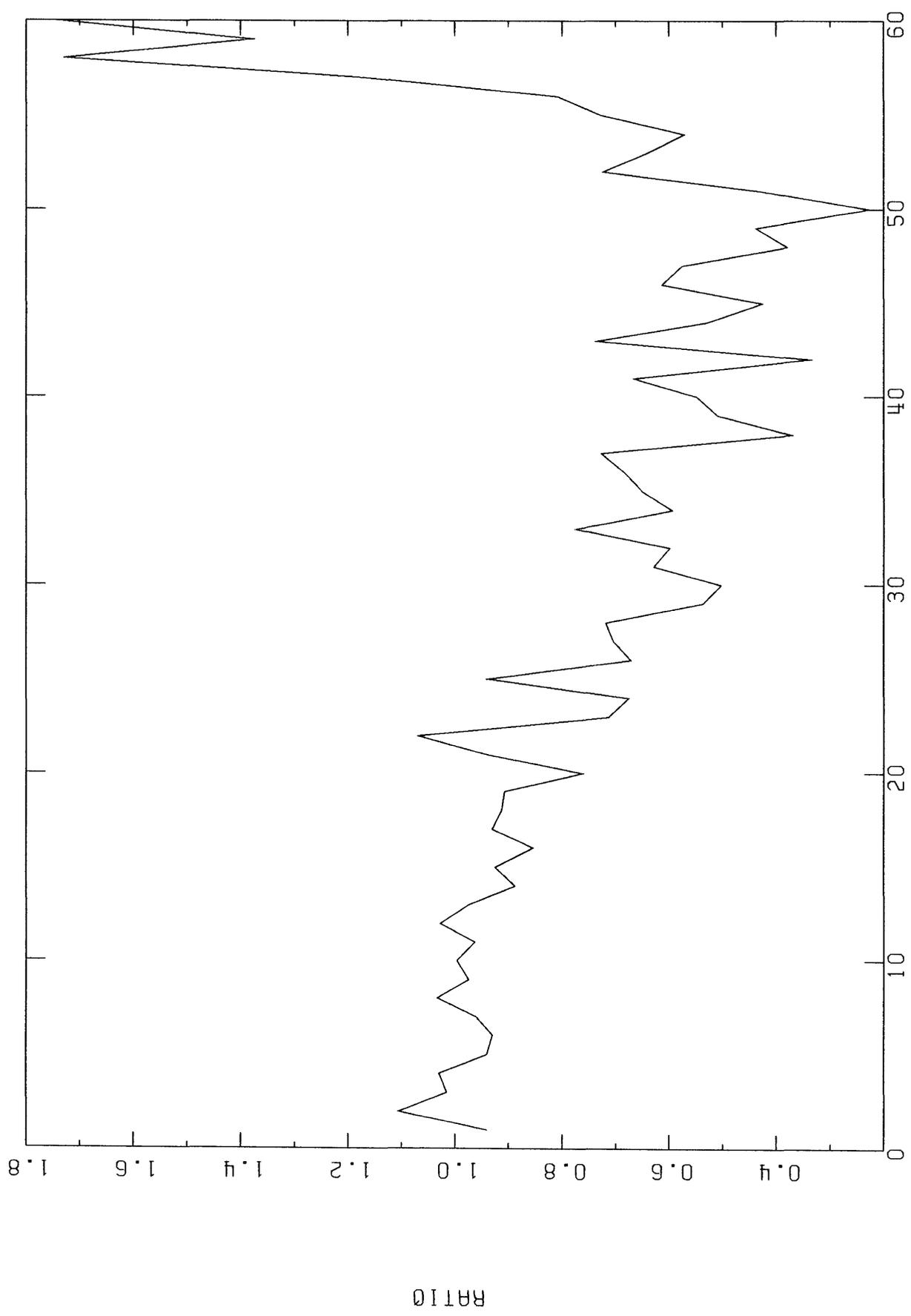
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OMEGA 1

Fig. 28

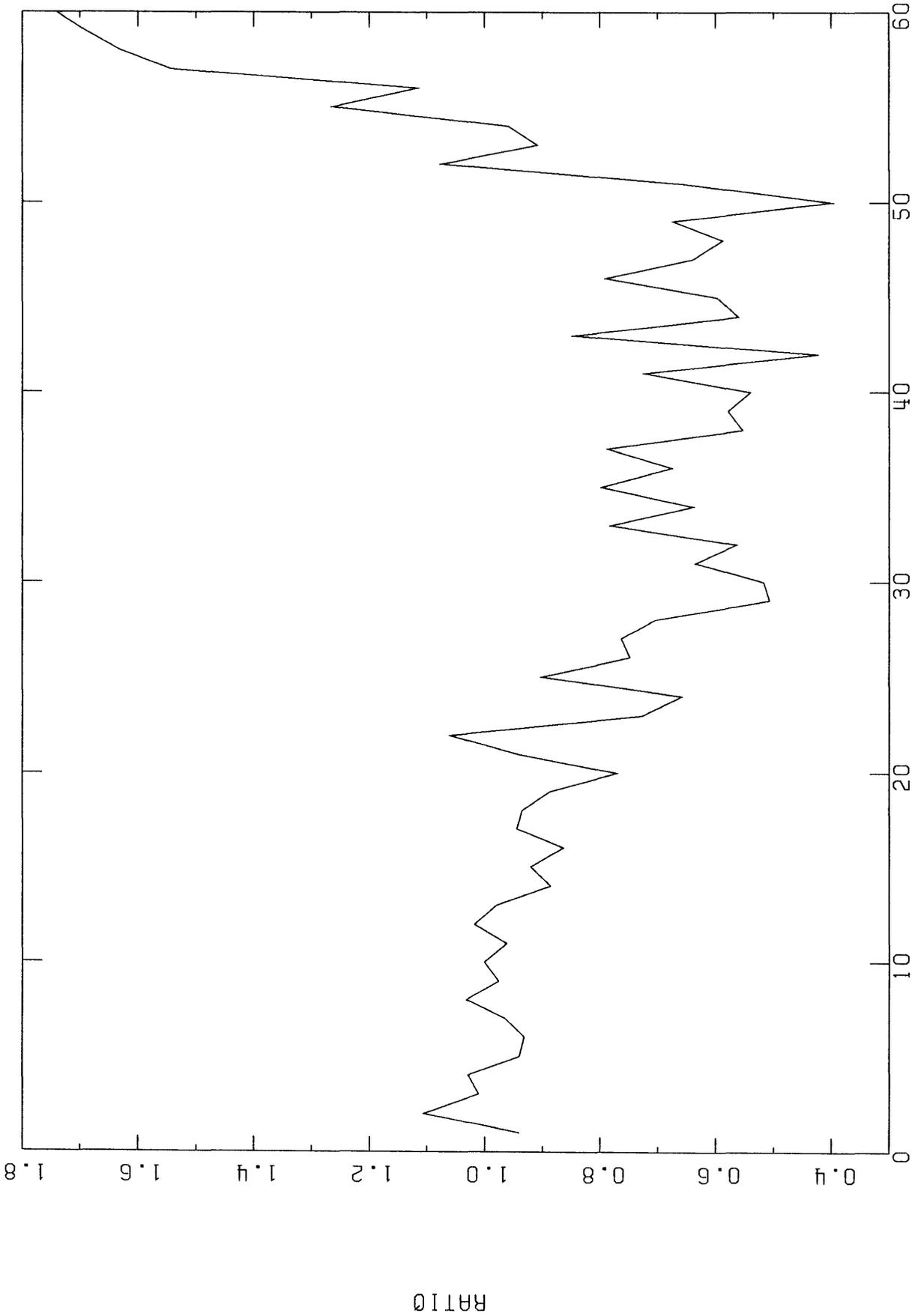
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OMEGA 1

Fig 29

RHDIFF6-RHDIFF5, OMEGA 2 = 50.0 DAMPING = 0.0, LENGTH = 30. SEC, CHANNEL 03



OMEGA 1

F<sub>1</sub> 30