DEPARTMENT OF THE INTERIOR

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PROBDIST : Probability Distributions for Modeling and Simulation in the Absence of Data

By

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Open-File Report 90-446

A -- Documentation (paper copy)
B -- Executable program (5.25" diskette)

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1. INTRODUCTION

PROBDIST is a general purpose computer-based methodology for helping geologists to appraise natural resources in terms of a selected probability distribution. The program supports 13 probability distribution models commonly used in making estimates of natural resources. PROBDIST starts with the parameters of a given probability distribution model and computes estimates consisting of the two moments, mean value and standard deviation, and seven fractiles (*F*100, *F*95, *F*75, *F*50, *F*25, *F*5, *F*0). Each fractile *Fi* is a value of the variable, with an *i*/100 probability that the value of the variable exceeds *Fi*.

Also, to aid the user in selecting an appropriate distribution model, PROBDIST can display a histogram graph of a typical distribution of each supported model; and, to aid the user in refining input data, PROBDIST can display a similar graph showing the appearance of the particular distribution based on the user's own input parameters.

The user selects a probability distribution from the following thirteen models:

1 7-fractile probability histogram, specified in terms of the fractiles *F*100, *F*95, *F*75, *F*50, *F*25, *F*5, and *F*0 of the variable. In this model the output distribution is already determined; even so, PROBDIST determines the mean and standard deviation of the distribution, and displays graphically the shape of the histogram. See Figure 2, page 15.

2 3-fractile probability histogram, specified in terms of a minimum (*F*100), median (*F*50), and maximum (*F*0) fractiles of the variable. See Figure 3, page 16.

3 Normal distribution, specified in terms of the minimum (*F*100) and maximum (*F*0) fractiles of the variable. See Figure 4, page 17.

4 Normal distribution, specified in terms of the mean and standard deviation of the variable. See Figure 5, page 18.

5 Truncated normal distribution, specified in terms of the minimum (*F*100), mean, maximum (*F*0), and standard deviation of the variable. See Figure 6, page 19.

6 Lognormal distribution, specified in terms of the minimum (*F*100), median (*F*50), and maximum (*F*0) of the variable. See Figure 7, page 20.

7 Truncated lognormal distribution, specified in terms of the minimum (*F*100), normal mean, maximum (*F*0), and standard deviation of the variable. See Figure 8, page 21.

8 Exponential distribution, specified in terms of the minimum (*F*100) and maximum (*F*0) of the variable. See Figure 9, page 22.

9 Truncated exponential distribution, specified in terms of the minimum ($F100$), maximum ($F0$), and shape parameter beta (β) of the variable. See Figure 10, page 23.

10 Pareto distribution, specified in terms of the minimum ($F100$) and maximum ($F0$) of the variable. See Figure 11, page 24.

11 Truncated Pareto distribution, specified in terms of the minimum ($F100$), maximum ($F0$), and shape parameter d of the variable. See Figure 12, page 25.

12 Uniform distribution, specified in terms of the minimum ($F100$) and maximum ($F0$) of the variable. See Figure 13, page 26.

13 Triangular distribution, specified in terms of the minimum ($F100$), mode, and maximum ($F0$) of the variable. See Figure 14, page 27.

The computer program named PROBDIST carries out natural resource estimation using these models. PROBDIST runs on microcomputers compatible with the IBM PC/XT/AT standard. An IBM-PC compatible 5.25" diskette containing the executable program and documentation files is distributed as a separate Open-File Report, number 90-446-B.

2. COMPUTER REQUIREMENTS

The computer hardware needed for running PROBDIST includes:

IBM PC/XT/AT or compatible computer;
Monochrome or color monitor;
2 diskette drives (5.25"), or 1 diskette and a hard disk;
256K memory;
Printer able to print 132 characters per line.

A mathematics coprocessor and graphics adapter are not required, but graphic capability is desirable for displaying a plot of the probability histogram of any variable, as in the sample shown in Figure 2 (page 15). If the computer used to run PROBDIST lacks a graphics adapter, the program displays an error message if the user gives the plot command. The program uses the Computer Graphics Adapter (CGA) graphics standard. PROBDIST graphic images can be printed only with a dot matrix or other graphics compatible printer.

Minor compatibility problems may occur in running PROBDIST on some display hardware, or with some older versions of the DOS operating system. The development system was MS-DOS 3.1.

Section 5 of this Open-File Report is a manual for installing and using the PROBDIST system.

3. METHODOLOGY

The philosophy of selecting a probability distribution in the absence of data can be found in Law and Kelton (1982), page 205:

> Let us assume that the random quantity of interest is a continuous random variable X. It will also be useful to think of this random variable as being the time to perform some task, e.g., the time required to construct part of a new ship, or as being the duration of an activity, e.g., the time to failure of a new computer component. The first step in using either heuristic approach [two heuristic procedures are given by the authors] is to identify an interval $[a, b]$ (a and b real numbers such that $a < b$) in which it is felt that X will lie with probability close to 1; that is, $P\{X < a \text{ or } X > b\} \approx 0$. In order to obtain subjective estimates of a and b, "experts" are asked for their most optimistic and pessimistic estimates, respectively, of the time to perform the task. Once an interval $[a, b]$ has been subjectively identified, the next step is to place a probability density function on $[a, b]$ which is thought to be representative of X.

The probability distribution models and corresponding input parameters which the user may select when using PROBDIST are shown in Figure 1 (page 4), which reproduces the menu shown on the computer screen when the program is executing.

The user needs a probability distribution to model a particular continuous random variable of interest (e.g., oil accumulation size, porosity, saturation, etc.). The thirteen available models consist of probability distributions with the corresponding parameters needed to specify the model. The various models require different numbers and types of parameters (e.g., minimum, average, maximum, shape or scale). From the type of distribution and its parameters, the methodology described in section 3.1 calculates missing fractiles to produce an output distribution for each model consisting of 7 fractiles of the estimate: $F100$, $F95$, $F75$, $F50$, $F25$, $F5$, $F0$, where, for example, the probability of more than $F95$ equals 0.95. The calculation of the approximate mean and standard deviation from the output fractiles is common to the first twelve models (not including the triangular distribution), and this is discussed in section 3.2.

3.1 Missing Fractile Calculation

The input into each model consists of a set of variables (e.g., petroleum resource estimates) and, for each variable a set of parameters appropriate to specifying a probability distribution for the chosen model.

Each of the 13 probability models supported by PROBDIST is discussed in sections 3.1.1 to 3.1.13, with the analytic formulas used in calculating the missing fractiles in the output distribution.

Select probability distribution model for TRIAL.DAT

	Model	Min			Ave			Max	Shape or Scale
1	Probability Histogram	F100	F95	F75	F50	F25	F5	F0	
2	Probability Histogram	F100			F50			F0	
3	Normal	F100						F0	
4	Normal				Mean				σ
5	Truncated Normal	F100			Mean			F0	σ
6	Lognormal	F100			F50			F0	
7	Truncated Lognormal	F100			Mean (normal)			F0	σ
8	Exponential	F100						F0	
9	Truncated Exponential	F100						F0	β
10	Pareto	F100						F0	
11	Truncated Pareto	F100						F0	d
12	Uniform	F100						F0	
13	Triangular	F100			Mode			F0	

NOTE: F50 = Median and P (X > F50) = 0.50

MOVE video bar to desired model. RETURN to select, CTRL-G to see sample graph.

Figure 1. Model selection menu.

3.1.1 7-Fractile Probability Histogram

For each variable the input requires seven parameters: the fractiles $F100$, $F95$, $F75$, $F50$, $F25$, $F5$, and $F0$, where, for example, the fractile $F75$ is the value of the variable with a probability of 0.75 that the variable exceeds $F75$. PROBDIST outputs these 7 fractiles together with the mean and standard deviation of the variable.

This model is useful in evaluating a choice of fractiles by viewing the shape of the distribution using the PROBDIST graphic display mode. The 7-fractile model could also be used to manually refine the 7-fractile output distribution of one of the other models.

3.1.2 3-Fractile Probability Histogram

For each variable the input requires three parameters: the minimum ($F100$), median ($F50$), and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows.

Given the parameters

$$a = \text{minimum} = F100, \quad b = \text{median} = F50, \quad c = \text{maximum} = F0$$

the seven fractiles $F100p$, for p = 1.0, 0.95, 0.75, 0.5, 0.25, 0.05 and 0.0, are computed for three different cases:

Case 1: The median approximately equals the midpoint of the minimum and maximum. This condition is expressed as

$$\left| b - \frac{a+c}{2} \right| < \frac{c-a}{20}$$

Then the four missing fractiles are given by

$$F95 = a + \frac{b - a}{3}$$

$$F75 = a + 2\frac{b - a}{3}$$

$$F25 = b + \frac{c - b}{3}$$

$$F5 = b + 2\frac{c - b}{3}$$

Case 2: The median is less than the midpoint of the minimum and maximum (positively skewed), that is

$$b < \frac{a + c}{2}$$

The four missing fractiles are given by

$$F95 = a + \frac{b - a}{3}$$

$$F75 = a + 2\frac{b - a}{3}$$

$$F25 = b + \frac{c - b}{4}$$

$$F5 = b + 2\frac{c - b}{3}$$

Case 3: The median exceeds the midpoint of the minimum and maximum (negatively skewed), that is

$$b > \frac{a + c}{2}$$

The four missing fractiles are given by

$$F95 = a + \frac{b - a}{3}$$

$$F75 = a + 3\frac{b - a}{4}$$

$$F25 = b + \frac{c - b}{3}$$

$$F5 = b + 2\frac{c - b}{3}$$

3.1.3 Normal Distribution, Given Minimum and Maximum

For each variable the input requires two parameters: the minimum ($F100$) and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100, \quad c = \text{maximum} = F0$$

then

$$\mu = \frac{a + c}{2}, \qquad \sigma = \frac{c - \mu}{3}$$

and the missing fractiles are

$$\begin{aligned} F95 &= \mu - 1.645\,\sigma \\ F75 &= \mu - 0.675\,\sigma \\ F50 &= \mu \\ F25 &= \mu + 0.675\,\sigma \\ F5 &= \mu + 1.645\,\sigma \end{aligned}$$

3.1.4 Normal Distribution, Given Mean and Standard Deviation

For each variable the input requires two parameters: the mean and standard deviation of the variable. PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$\text{Mean} = \mu, \qquad \text{Standard deviation} = \sigma$$

then

$$\begin{aligned} F100 &= \mu - 3\,\sigma \\ F0 &= \mu + 3\,\sigma \end{aligned}$$

The rest of the missing fractiles are computed by the same formulas as in 3.1.3.

3.1.5 Truncated Normal Distribution

For each variable the input requires four parameters: the minimum ($F100$), mean, maximum ($F0$), and standard deviation (the scale parameter). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

a = minimum = $F100$, $\qquad$ c = maximum = $F0$
mean = μ, $\qquad$ standard deviation = σ

compute the corresponding z-values by

$$z_a = \frac{a - \mu}{\sigma}, \qquad z_c = \frac{c - \mu}{\sigma}$$

These z-values are then interpolated in a z-table Φ (in which $\Phi(z) = P(Z < z) \equiv A$) to obtain areas A_a and A_c:

$$A_a = \Phi(z_a), \qquad A_c = \Phi(z_c)$$

Then for each p = 0.95, 0.75, 0.50, 0.25, 0.05, an area A_p is computed from

$$A_p = p\,A_a + (1 - p)\,A_c$$

Each of these areas is back-interpolated through the z-table to produce a corresponding value of z_p. These are used to compute the missing fractiles by the formula

$$F100p = \mu + z_p\,\sigma$$

3.1.6 Lognormal Distribution

For each variable the input requires three parameters: the minimum ($F100$), median ($F50$), and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100, \quad b = \text{median} = F50, \quad c = \text{maximum} = F0$$

then the mean μ and standard deviation σ are given by

$$\mu = \ln (b - a)$$

$$\sigma = \frac{1}{3} \ln \frac{c - a}{b - a}$$

The missing fractiles are

$$F95 = a + e^{\mu - 1.645\,\sigma}$$

$$F75 = a + e^{\mu - 0.675\,\sigma}$$

$$F25 = a + e^{\mu + 0.675\,\sigma}$$

$$F5 = a + e^{\mu + 1.645\,\sigma}$$

3.1.7 Truncated Lognormal Distribution

For each variable the input requires four parameters: the minimum ($F100$), normal mean μ, maximum ($F0$), and normal standard deviation σ (the shape parameter). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

a = minimum = $F100$, c = maximum = $F0$
normal mean = μ, normal standard deviation = $\sigma > 0$

then define new parameters

$$a' = \ln a, \quad c' = \ln c$$

Taken together with μ and σ, these new parameters are used to find truncated normal fractiles for each p = 0.95, 0.75, 0.50, 0.25, 0.05, using the method of 3.1.5:

$$y'_p = \mu + z_p \sigma$$

These truncated normal fractiles are converted to truncated lognormal fractiles by the formula

$$F100p = e^{y'_p}$$

3.1.8 Exponential Distribution

For each variable the input requires two parameters: the minimum ($F100$) and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100, \quad c = \text{maximum} = F0$$

the missing fractiles for each p = 0.95, 0.75, 0.50, 0.25, 0.05 are computed by the formula

$$F100p = a + \frac{\ln p}{\ln 0.001} (c - a)$$

3.1.9 Truncated Exponential Distribution

For each variable the input requires three parameters: the minimum ($F100$), maximum ($F0$), and scale parameter beta ($\beta > 0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100 \geq 0, \quad c = \text{maximum} = F0$$

the missing fractiles for each p = 0.95, 0.75, 0.50, 0.25, 0.05 are given by

$$F100p = a - \beta \ln\left[p + (1 - p)\, e^{-(c - a)/\beta}\right]$$

3.1.10 Pareto Distribution

For each variable the input requires two parameters: the minimum ($F100$) and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100 > 0, \quad c = \text{maximum} = F0$$

then for each p = 0.95, 0.75, 0.50, 0.25, 0.05, the missing fractile $F100p$ is given by

$$F100p = \frac{a}{p^r}, \qquad \text{where } r = \frac{\ln a - \ln c}{\ln 0.001}$$

3.1.11 Truncated Pareto Distribution

For each variable the input requires three parameters: the minimum ($F100$), maximum ($F0$), and shape parameter $d > 0$. PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100 > 0, \quad c = \text{maximum} = F0$$

then for each p = 0.95, 0.75, 0.50, 0.25, 0.05, the missing fractile $F100p$ is given by

$$F100p = \frac{a}{\sqrt[d]{1 - (1 - p)\left[1 - \left(\frac{a}{c}\right)^d\right]}}$$

3.1.12 Uniform Distribution

For each variable the input requires two parameters: the minimum ($F100$) and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given

$$a = \text{minimum} = F100, \qquad c = \text{maximum} = F0$$

the missing fractiles for each $p = 0.95, 0.75, 0.50, 0.25, 0.05$ are computed by the formula

$$F100p = a + (1 - p)(c - a)$$

3.1.13 Triangular Distribution

The triangular distribution differs from the first twelve models: its mean and standard deviation are computed by separate formulas. For each variable the input requires three parameters: the minimum ($F100$), mode, and maximum ($F0$). PROBDIST outputs a 7-fractile distribution by calculating the missing fractiles as follows. Given the parameters

$$a = \text{minimum} = F100, \quad b = \text{mode}, \quad c = \text{maximum} = F0$$

the mean μ and standard deviation σ are computed by the formulas

$$\mu = \frac{a + b + c}{3}$$

$$\sigma = \sqrt{\frac{a^2 + b^2 + c^2 - ab - ac - bc}{18}}$$

The seven fractiles $F100p$, for $p = 1.0, 0.95, 0.75, 0.5, 0.25, 0.05$ and 0.0, are computed for three different cases as follows.

Case 1: $a < b < c$

If $p \geq \frac{c - b}{c - a}$ then

$$F100p = a + \sqrt{(1 - p)(c - a)(b - a)}$$

Otherwise,

$$F100p = c - \sqrt{p(c - a)(c - b)}$$

Case 2: $a = b < c$

$$F100p = c - \sqrt{p}\ (c - a)$$

Case 3: $a < b = c$

$$F100p = a + \sqrt{1 - p}\ (c - a)$$

3.2 Mean and Standard Deviation Calculation

For the first 12 models above (not including the triangular distribution), after the seven fractiles $F100$, $F95$, $F75$, $F50$, $F25$, $F5$, and $F0$ have been determined, the approximate mean μ, variance σ^2 and standard deviation σ of the probability distribution are computed by the formulas

$$\mu = 0.025\ (F100 + F0) + 0.125\ (F95 + F5) + 0.225\ (F75 + F25) + 0.25\ (F50)$$

$$\begin{aligned}\sigma^2 = \frac{1}{3}[\ & 0.05\ (F100^2 + F100\,F95 + F5\,F0 + F0^2) \\ & + 0.25\ (F95^2 + F75\,F50 + F50\,F25 + F5^2) \\ & + 0.20\ (F95\,F75 + F25\,F5) \\ & + 0.45\ (F75^2 + F25^2) \\ & + 0.5\ (F50^2)\ ] \\ & - \mu^2\end{aligned}$$

$$\sigma = \sqrt{\sigma^2}$$

3.3 Typical Sample Graphs

When selecting a probability distribution model, the user views a menu of the models available, and has an opportunity to display graphically on the computer screen the shape of a typical distribution generated by that model and the input parameters necessary to specify the model. The computer must have CGA graphics compatibility to provide this preview. Reproductions of the typical graphs are shown in Figures 2 through 14.

3.4 Sample Data Examples

Examples of the output of PROBDIST for sample data input to the thirteen models are shown in Figures 15 through 27. The text part of the output shows the content of the listing file generated by the PROBDIST estimator module, and the graph shows a screen print taken during data entry and editing.

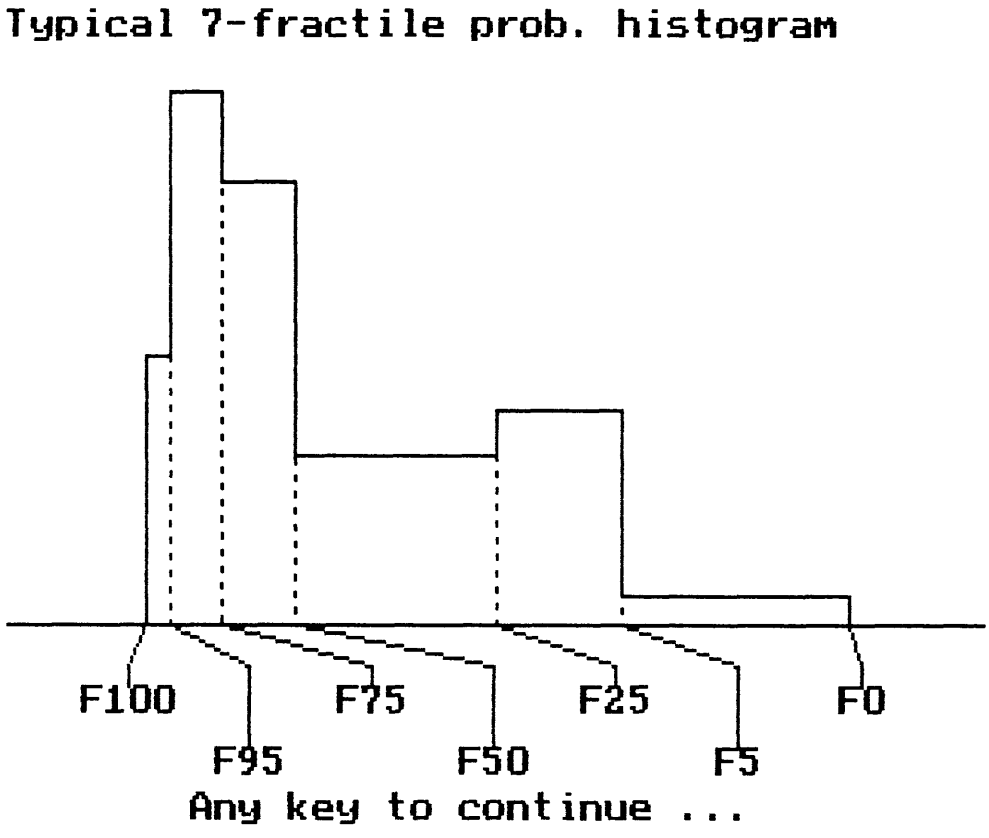

Figure 2. Typical 7-fractile probability histogram.

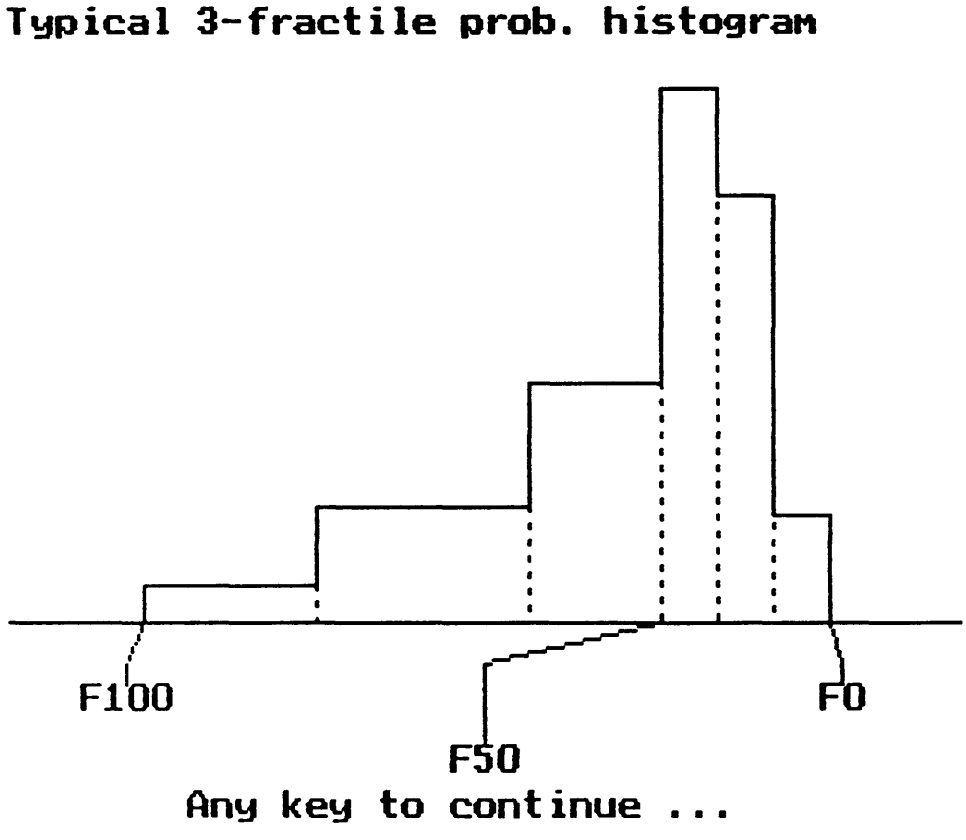

Figure 3. Typical 3-fractile probability histogram.

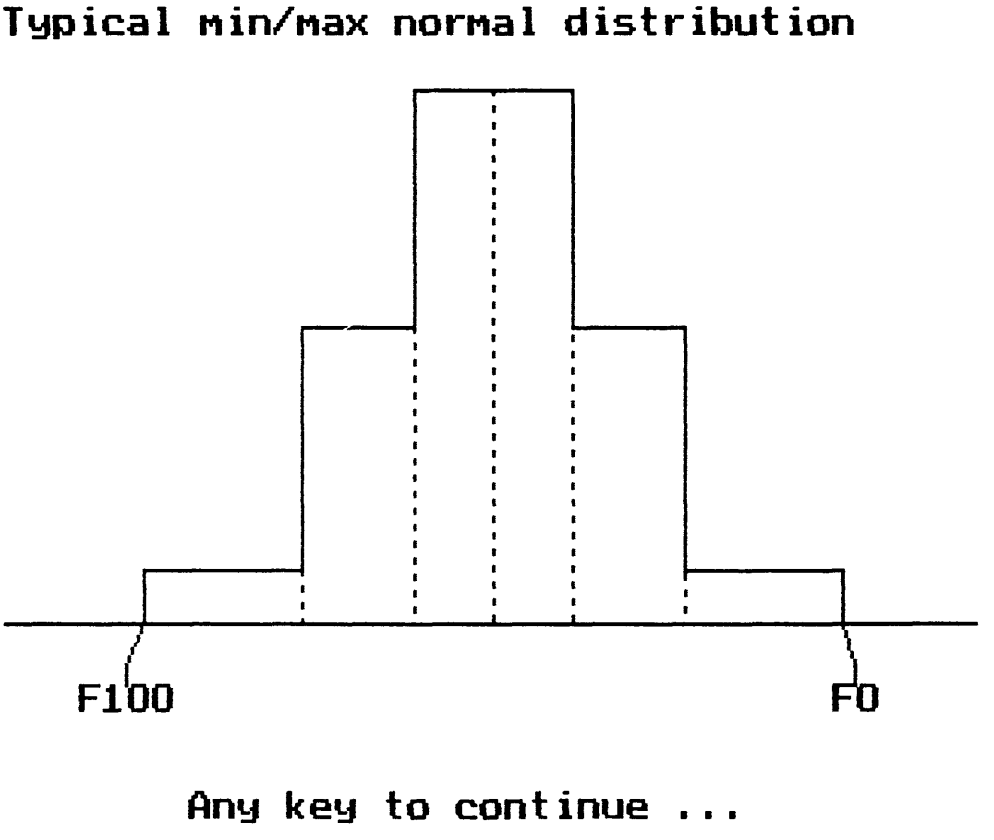

Figure 4. Typical minimum/maximum normal distribution.

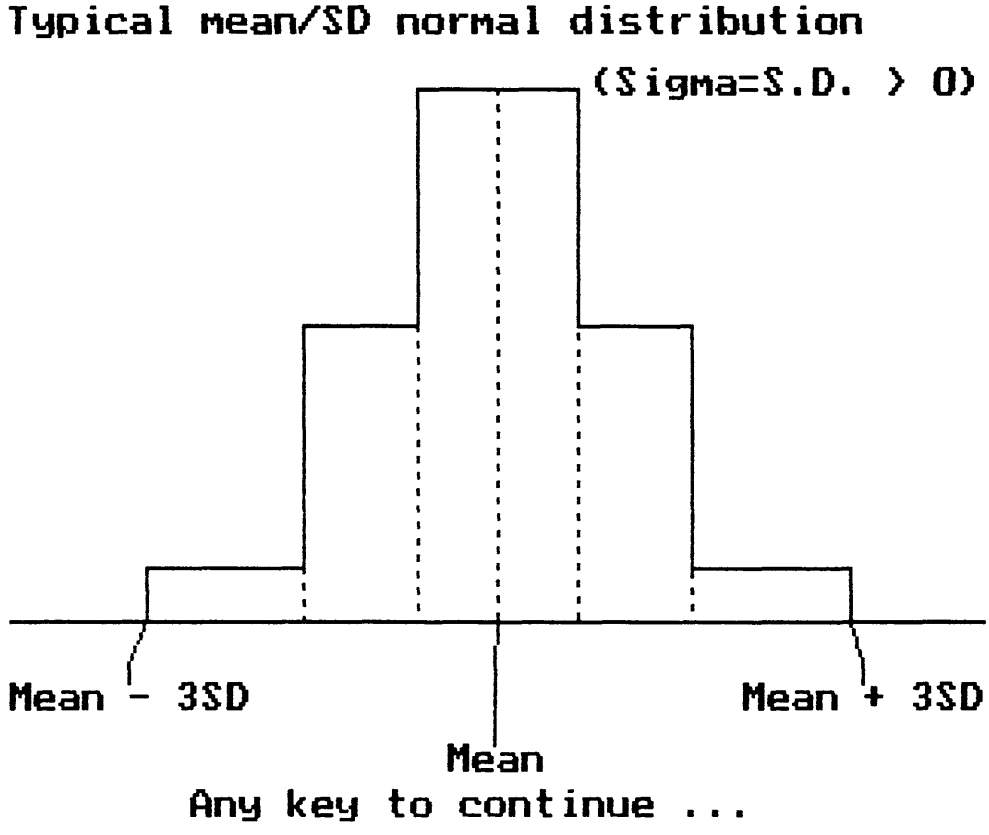

Figure 5. Typical mean/standard deviation normal distribution.

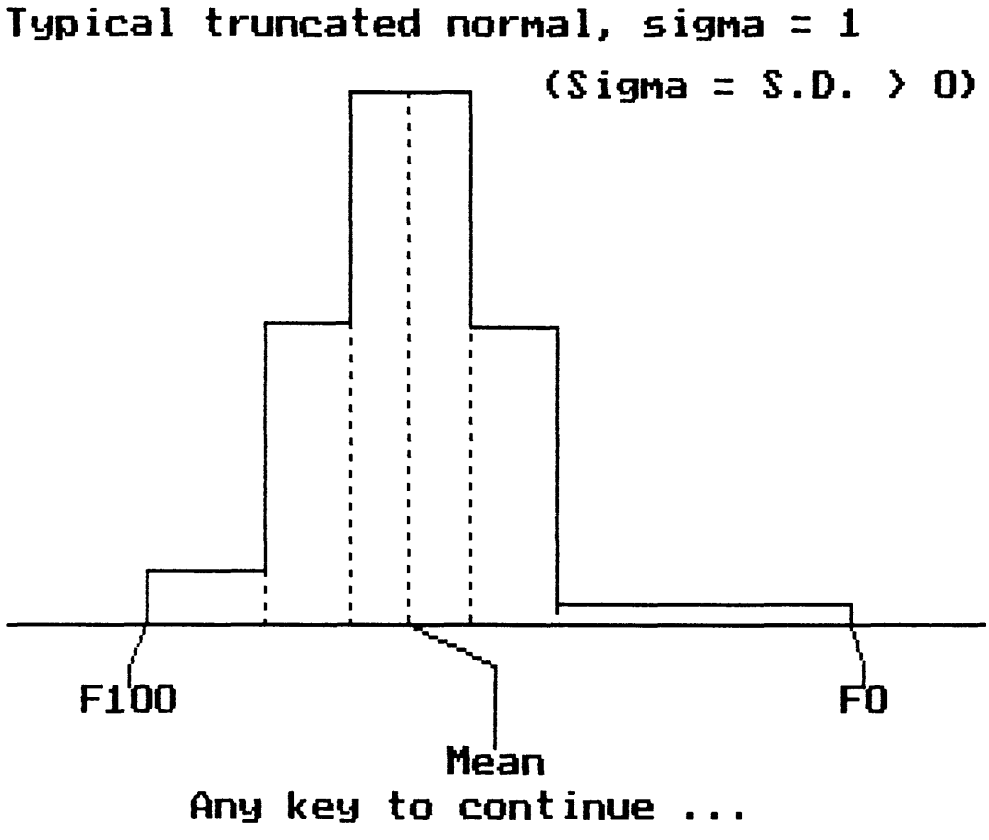

Figure 6. Typical truncated normal distribution.

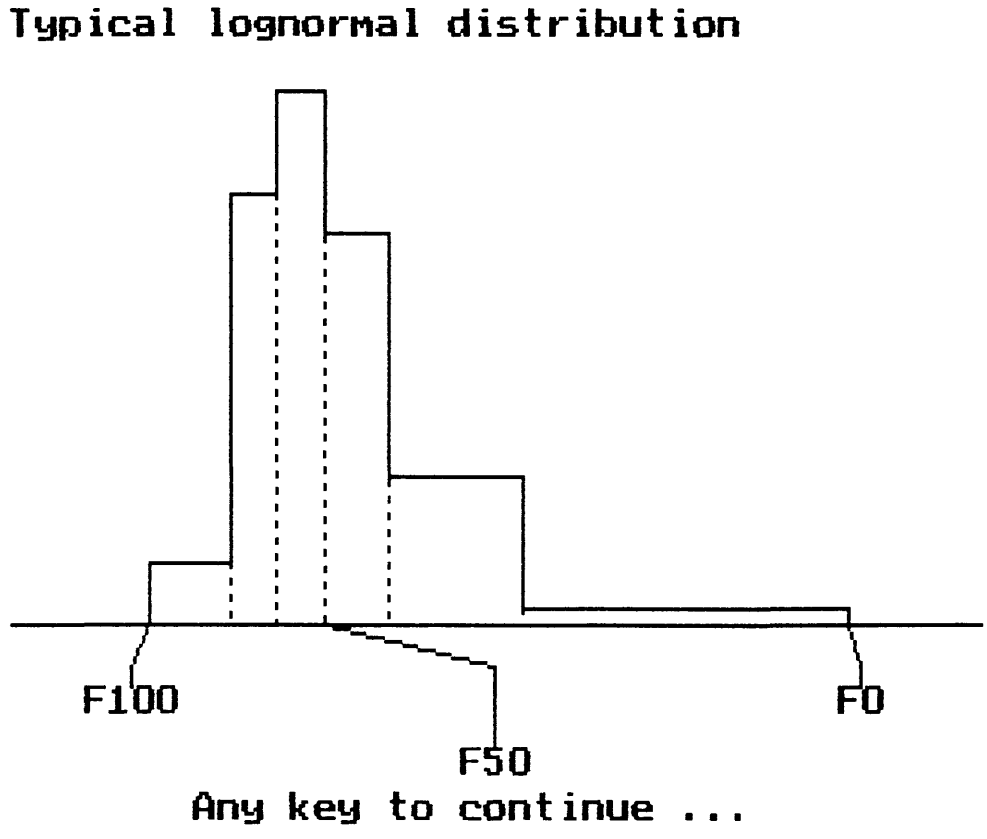

Figure 7. Typical lognormal distribution.

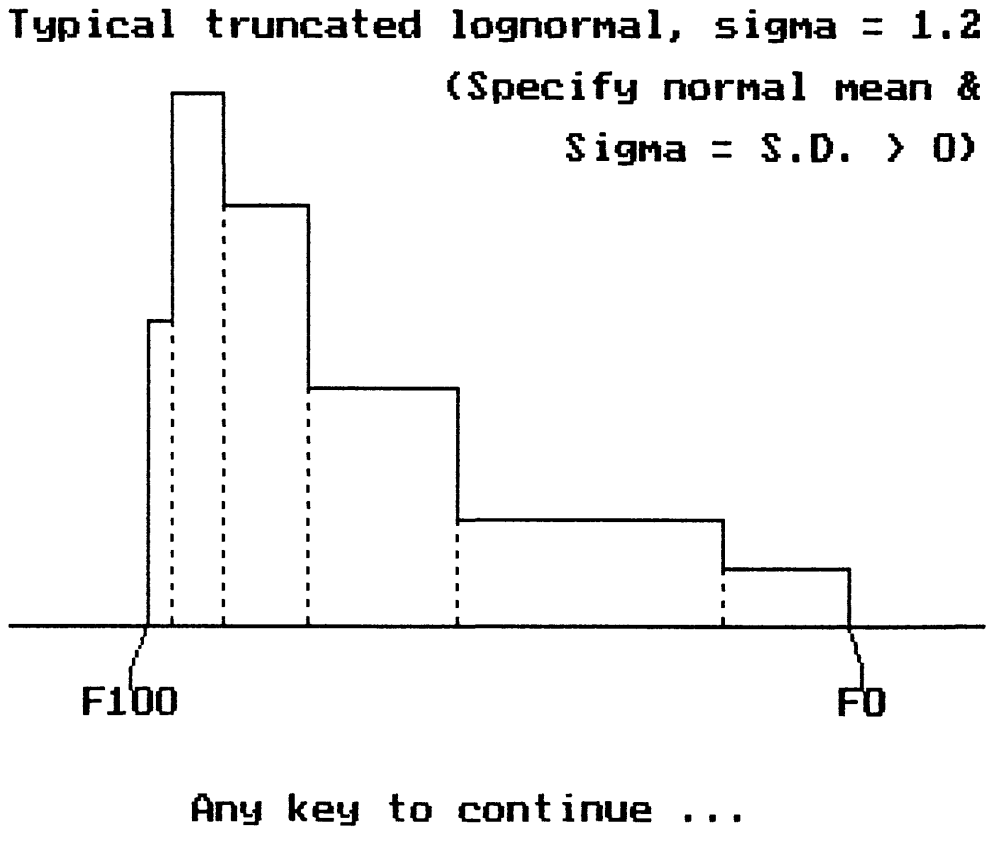

Figure 8. Typical truncated lognormal distribution.

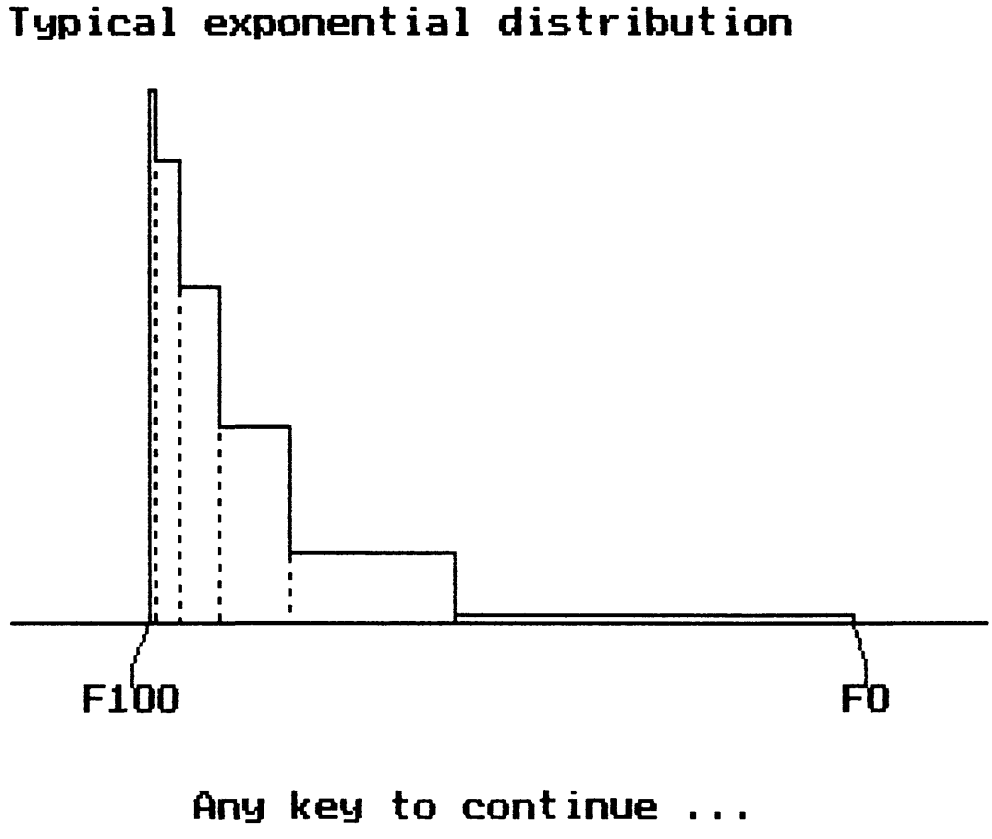

Figure 9. Typical exponential distribution.

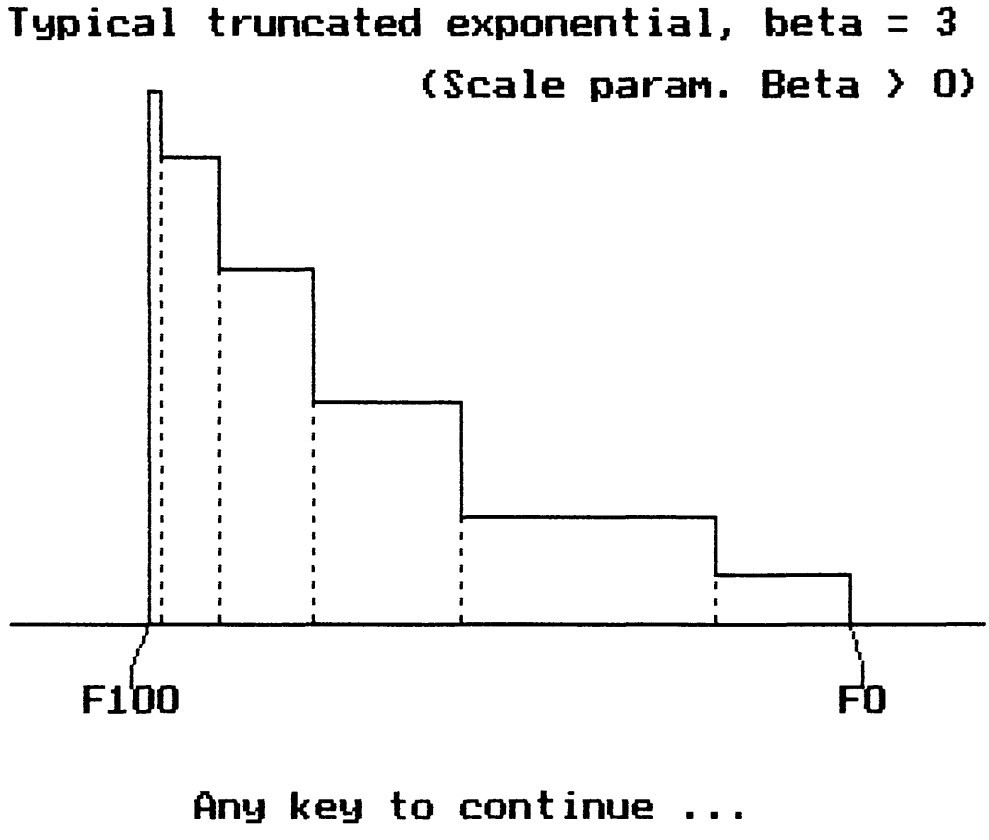

Figure 10. Typical truncated exponential distribution.

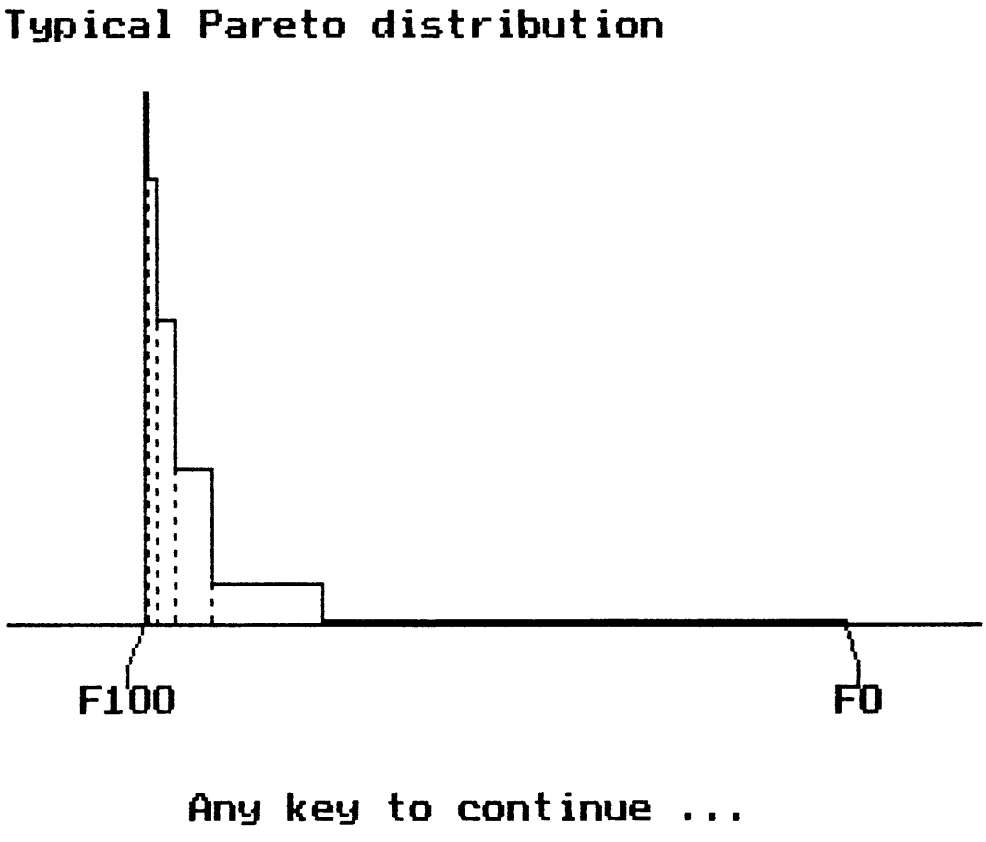

Figure 11. Typical Pareto distribution.

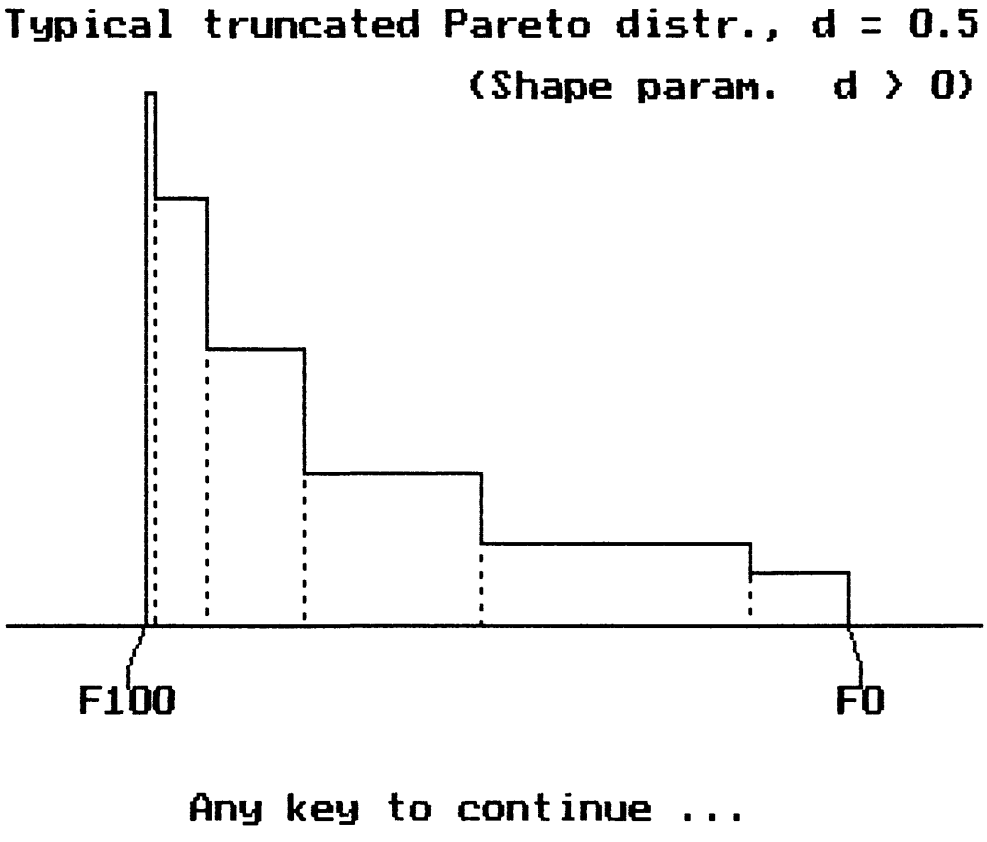

Figure 12. Typical truncated Pareto distribution.

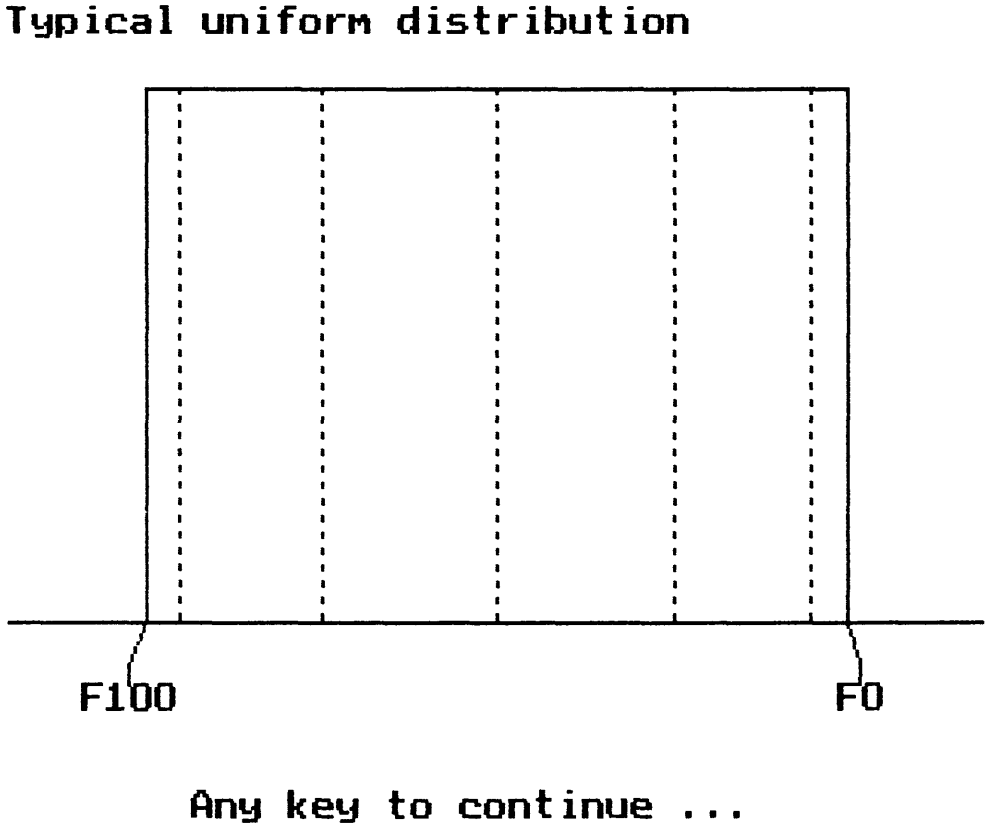

Figure 13. Typical uniform distribution.

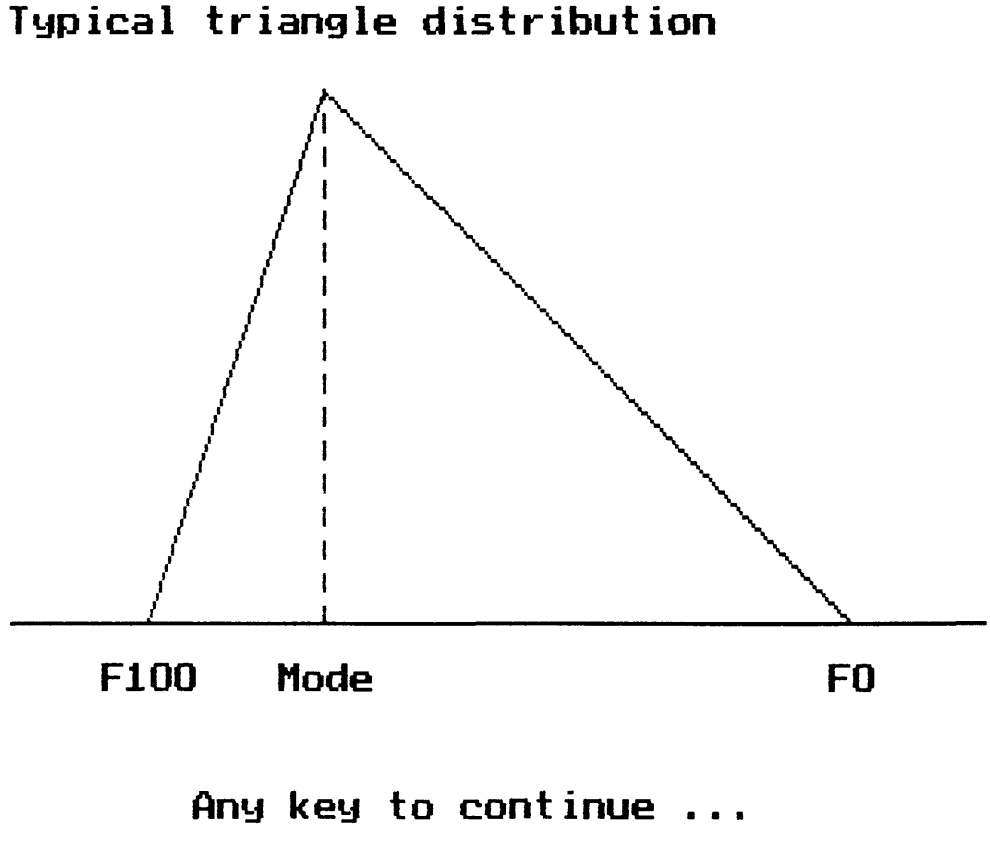

Figure 14. Typical triangular distribution.

PROBDIST 89.11 FIG15.PDL 14:38:33 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : 7-fractile Probability Histogram

INPUT: PARAMETERS

VARIABLE NAME	Min F100	F95	F75	Median F50	F25	F5	Max F0
Sample data	0.00000	1.00000	3.00000	6.00000	14.0000	19.0000	28.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	8.52500	6.52745	0.00000	1.00000	3.00000	6.00000	14.0000	19.0000	28.0000

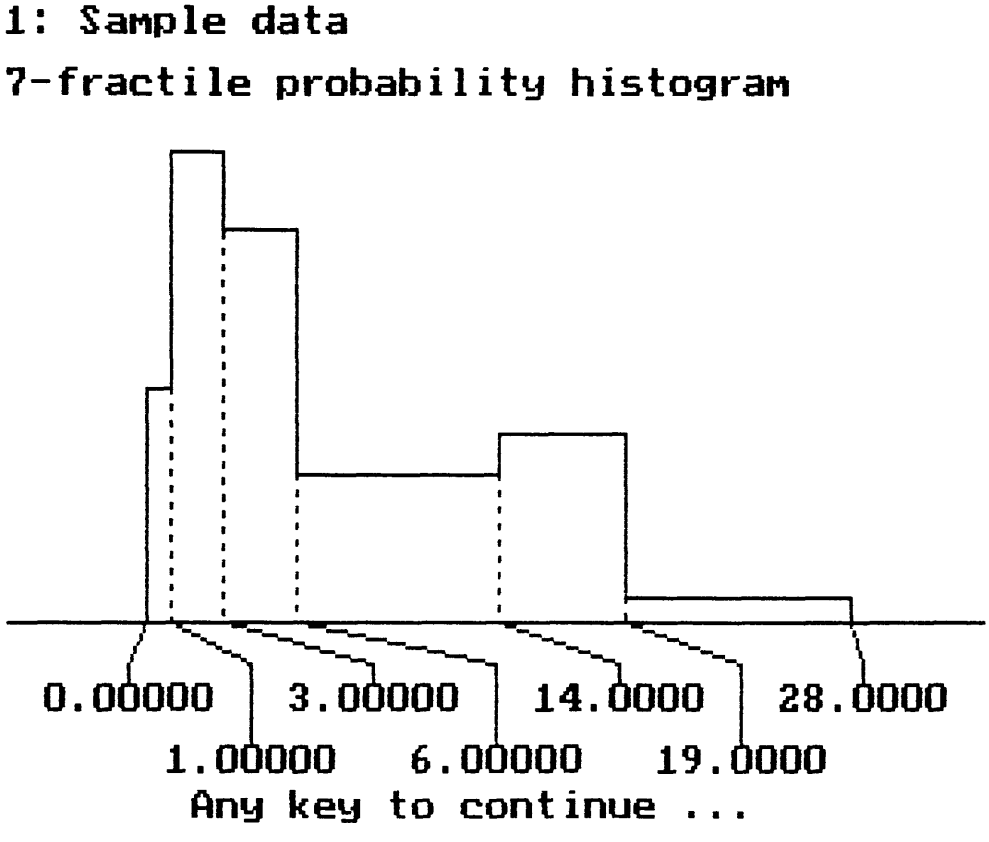

Figure 15. Output of PROBDIST for the 7-fractile histogram model.

PROBDIST 89.11 FIG16.PDL 14:39:16 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : 3-fractile Probability Histogram

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Median F50	Max F0
Sample data	2.00000	8.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	7.37916	1.75230	2.00000	4.00000	6.50000	8.00000	8.66666	9.33333	10.0000

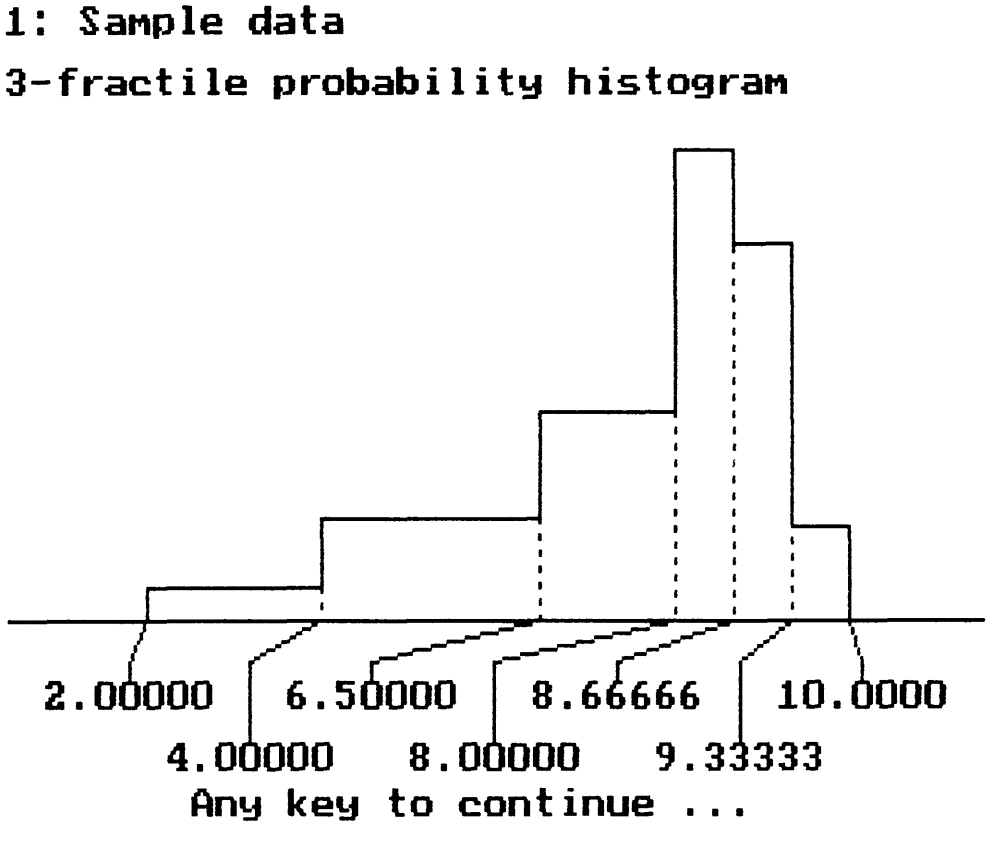

Figure 16. Output of PROBDIST for the 3-fractile histogram model.

PROBDIST 89.11 FIG17.PDL 14:39:43 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Min/max Normal Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min P100	Max P0
Sample data	2.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	P100	P95	P75	P50	P25	P5	P0
Sample data	6.00000	1.46074	2.00000	3.80666	5.10000	6.00000	6.90000	8.19333	10.0000

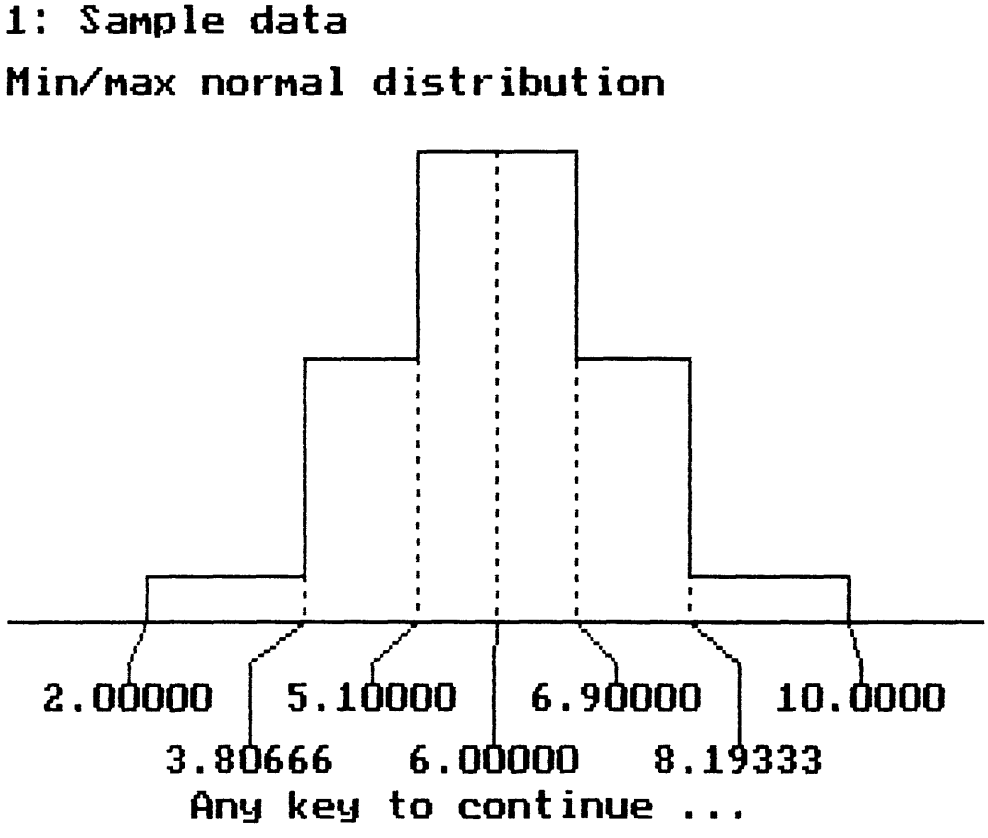

Figure 17. Output of PROBDIST for the minimum/maximum normal distribution model.

PROBDIST 89.11 FIG18.PDL 14:40:08 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Mean/SD Normal Distribution

INPUT: PARAMETERS

VARIABLE NAME	Mean (Mu)	S.D. (Sigma)
Sample data	5.00000	1.00000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	5.00000	1.09555	2.00000	3.35500	4.32500	5.00000	5.67500	6.64500	8.00000

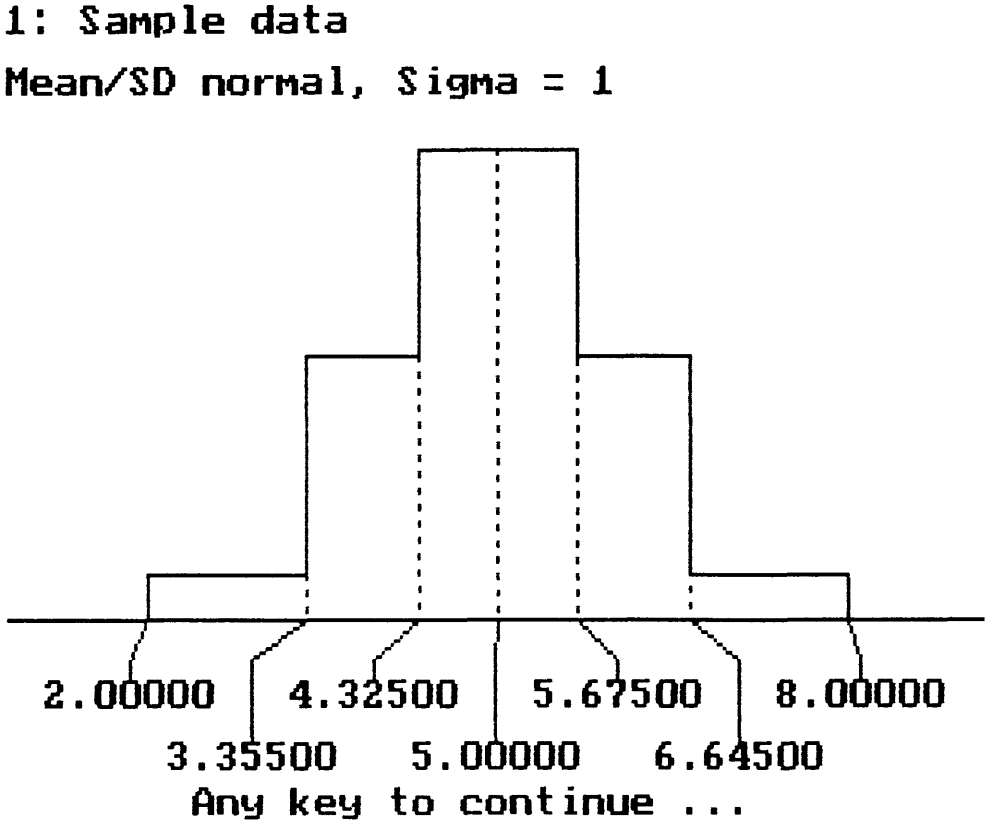

Figure 18. Output of PROBDIST for the mean/standard deviation normal distribution model.

PROBDIST 89.11 FIG19.PDL 14:40:33 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Truncated Normal Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min (F100)	Mean (Mu)	Max (F0)	S.D. (Sigma)
Sample data	2.00000	5.00000	10.0000	1.00000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	5.05290	1.23119	2.00000	3.36504	4.32778	5.00163	5.67623	6.64769	10.0000

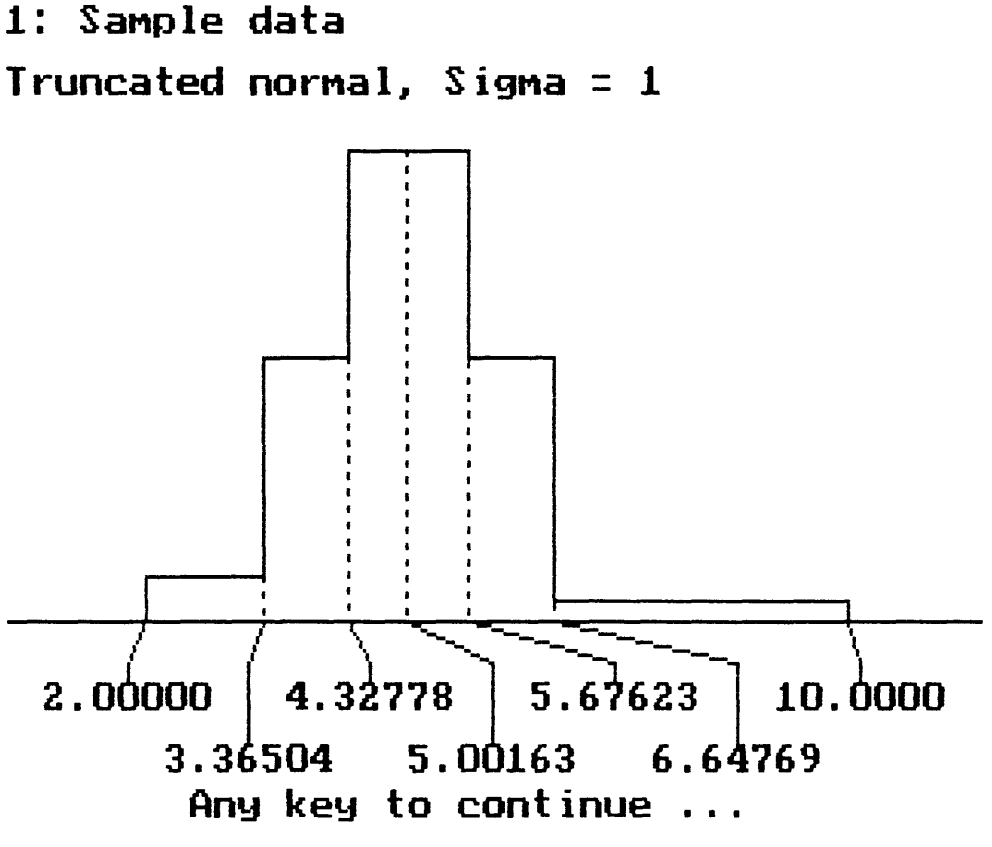

Figure 19. Output of PROBDIST for the truncated normal distribution model.

PROBDIST 89.11 FIG20.PDL 14:40:57 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Lognormal Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min P100	Median P50	Max P0
Sample data	2.00000	4.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	P100	P95	P75	P50	P25	P5	P0
Sample data	4.29568	1.28129	2.00000	2.93519	3.46408	4.00000	4.73208	6.27719	10.0000

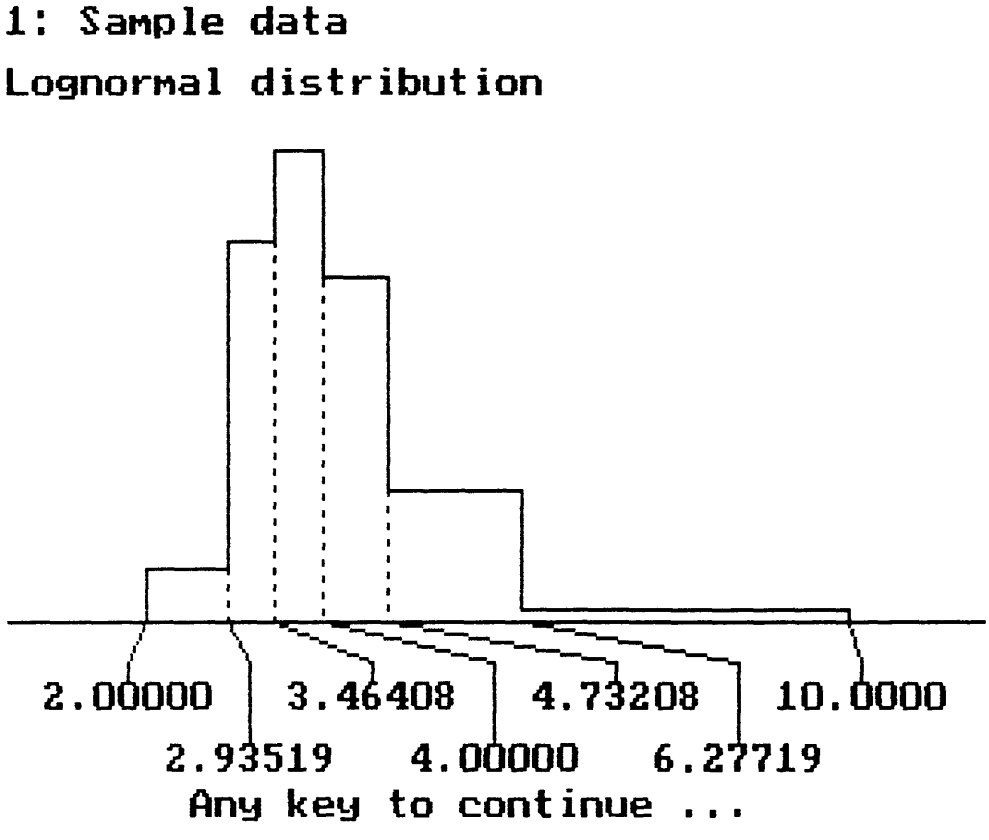

Figure 20. Output of PROBDIST for the lognormal distribution model.

PROBDIST 89.11 FIG21.PDL 14:41:21 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Truncated Lognormal Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min Normal Mean F100	Mean (Mu)	Max F0	S.D. (Sigma)
Sample data	2.00000	5.00000	500.000	1.20000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	158.630	125.516	2.00000	18.7198	56.6487	117.261	223.345	411.409	500.000

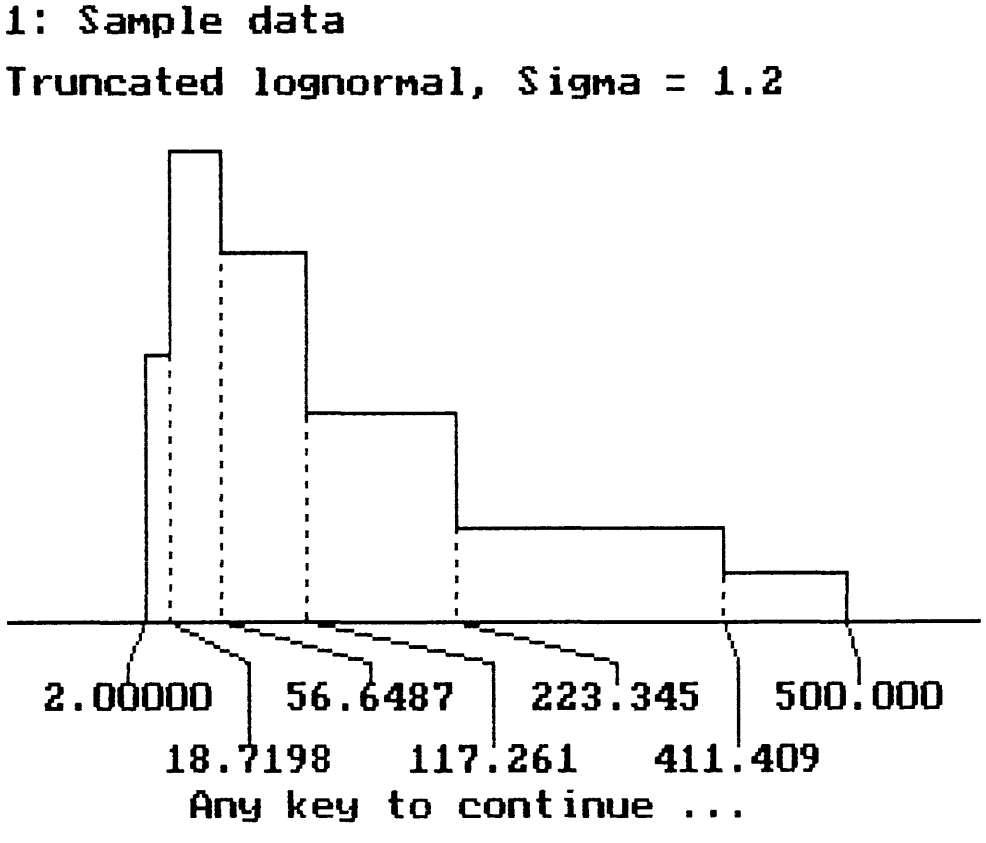

Figure 21. Output of PROBDIST for the truncated lognormal distribution model.

PROBDIST 89.11 FIG22.PDL 14:41:42 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Exponential Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Max F0
Sample data	2.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	3.27798	1.38289	2.00000	2.05940	2.33316	2.80274	3.60549	5.46941	10.0000

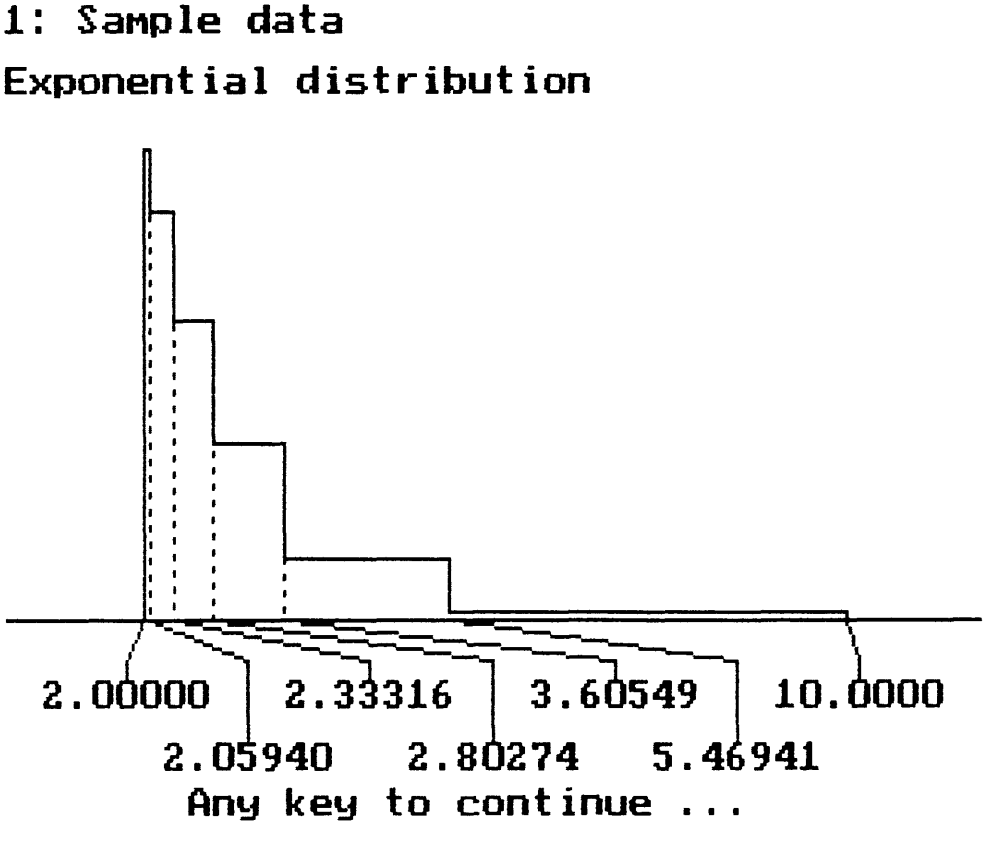

Figure 22. Output of PROBDIST for the exponential distribution model.

PROBDIST 89.11 FIG23.PDL 14:42:06 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Truncated Exponential Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Max F0	Beta
Sample data	2.00000	10.0000	3.00000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	4.48180	2.02684	2.00000	2.14292	2.79435	3.87791	5.59086	8.46225	10.0000

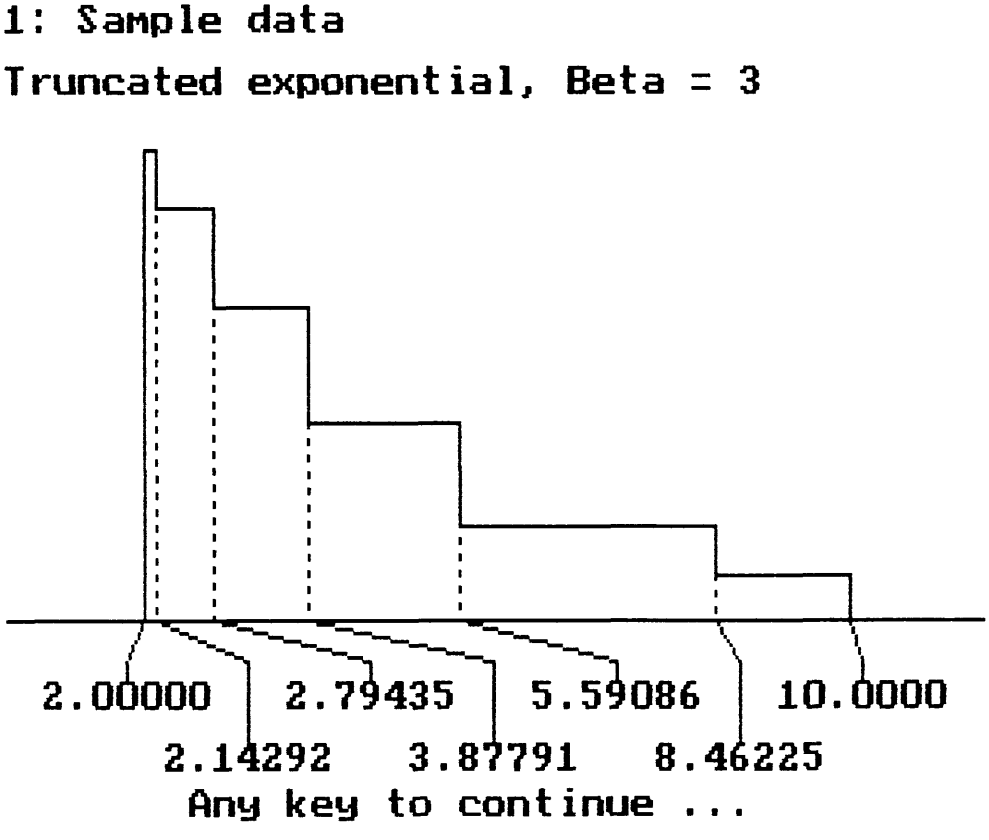

Figure 23. Output of PROBDIST for the truncated exponential distribution model.

PROBDIST 89.11 FIG24.PDL 14:42:35 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Pareto Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Max F0
Sample data	2.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	2.74582	1.16589	2.00000	2.02404	2.13864	2.35053	2.76251	4.01936	10.0000

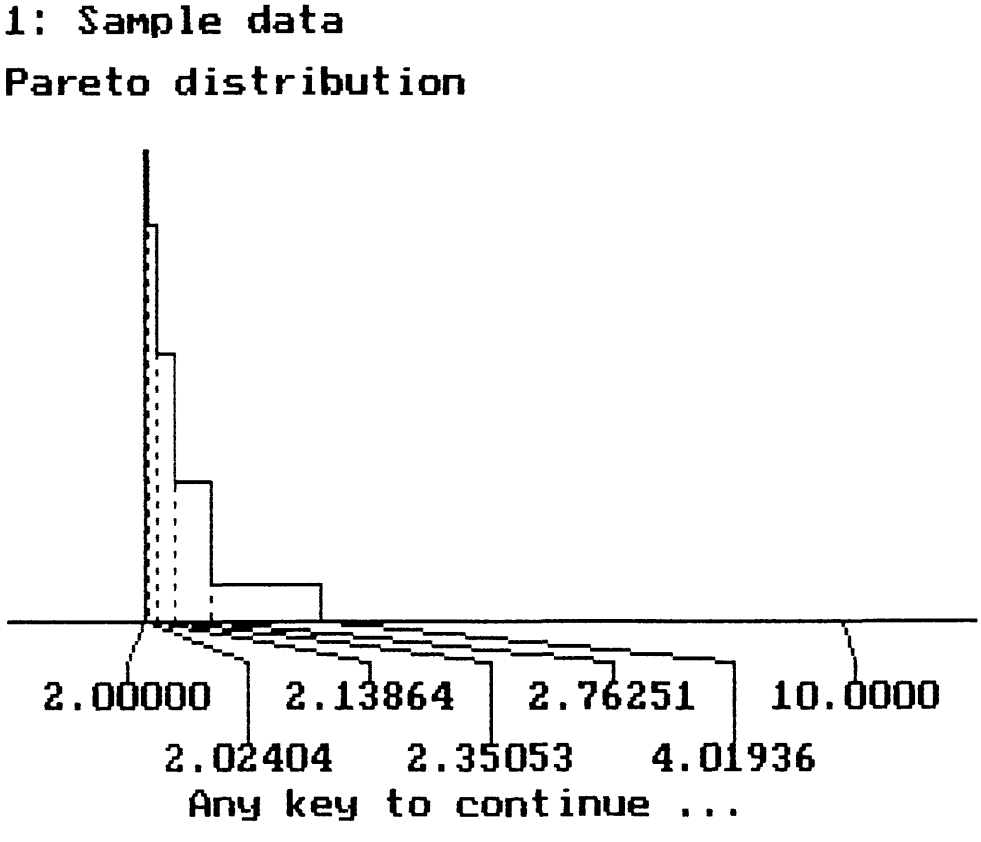

Figure 24. Output of PROBDIST for the Pareto distribution model.

PROBDIST 89.11 FIG25.PDL 14:42:56 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Truncated Pareto Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Max F0	d
Sample data	2.00000	10.0000	0.50000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	4.54702	2.17003	2.00000	2.11531	2.69285	3.81966	5.83592	8.86975	10.0000

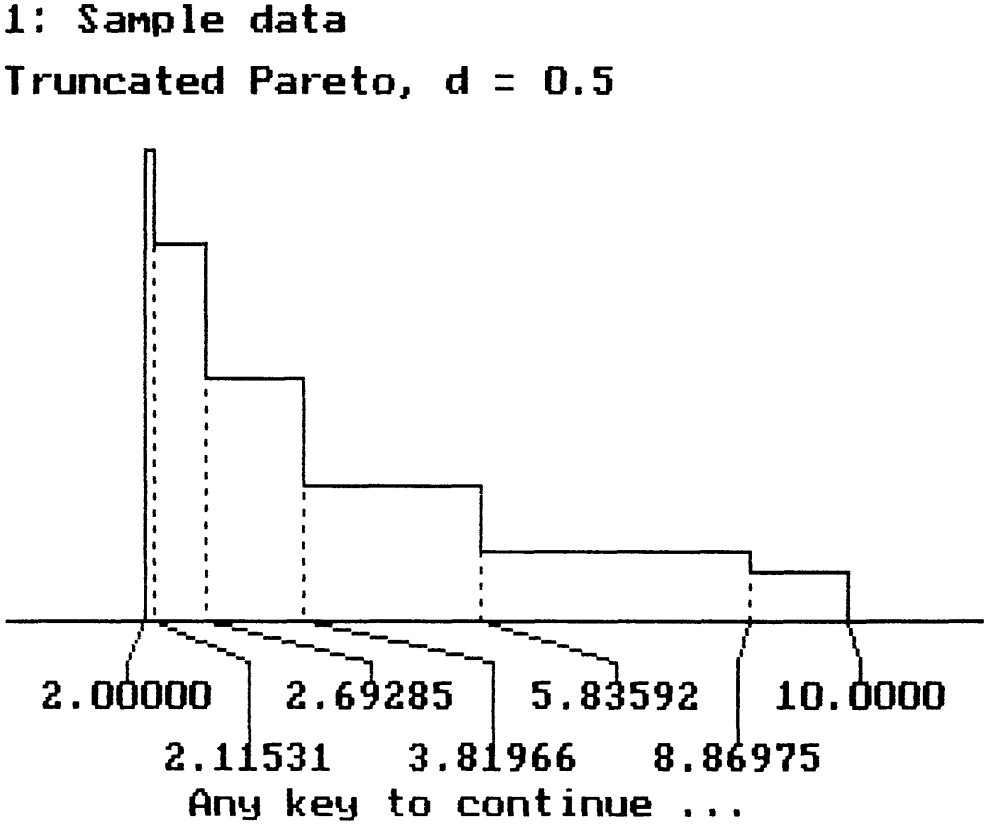

Figure 25. Output of PROBDIST for the truncated Pareto distribution model.

PROBDIST 89.11 FIG26.PDL 14:43:21 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Uniform Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Max F0
Sample data	2.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	6.00000	2.30940	2.00000	2.40000	4.00000	6.00000	8.00000	9.60000	10.0000

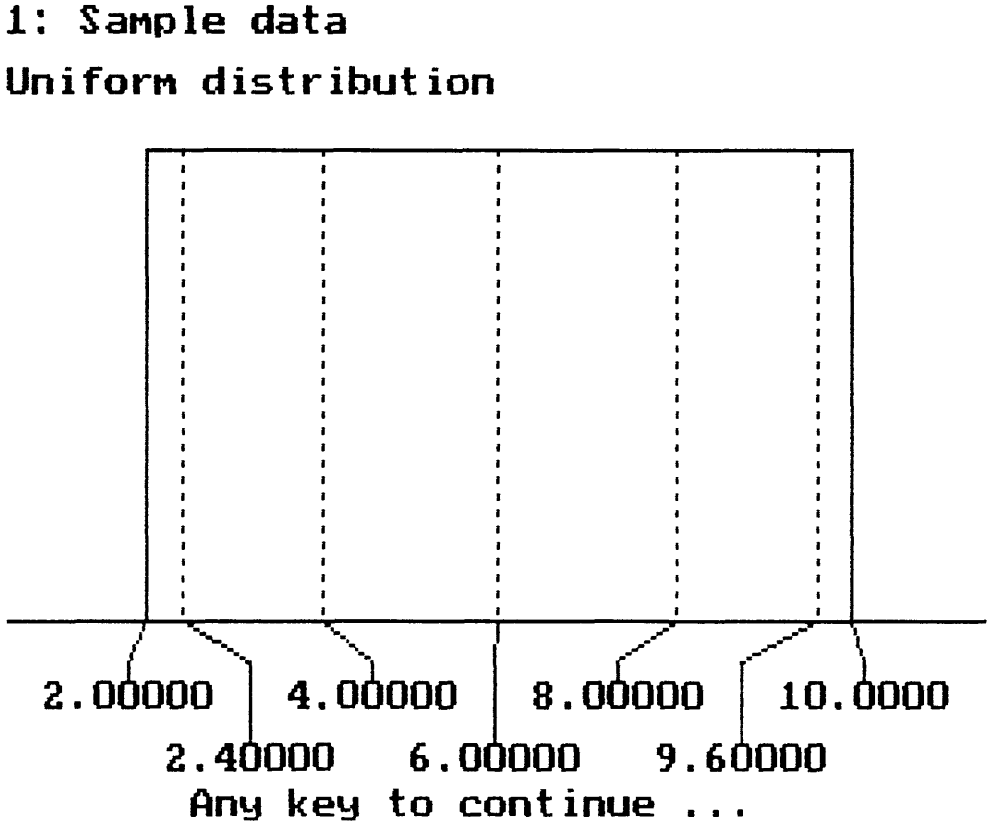

Figure 26. Output of PROBDIST for the uniform distribution model.

PROBDIST 89.11 FIG27.PDL 14:43:47 3-May-1990

Project name : Open File Report
Estimation name : Test data
Units : none
Model : Triangular Distribution

INPUT: PARAMETERS

VARIABLE NAME	Min F100	Mode	Max F0
Sample data	2.00000	4.00000	10.0000

OUTPUT: ESTIMATES

VARIABLE NAME	MEAN	S. D.	F100	F95	F75	F50	F25	F5	F0
Sample data	5.36398	1.78678	2.00000	2.89442	4.00000	5.10102	6.53589	8.45080	10.0000

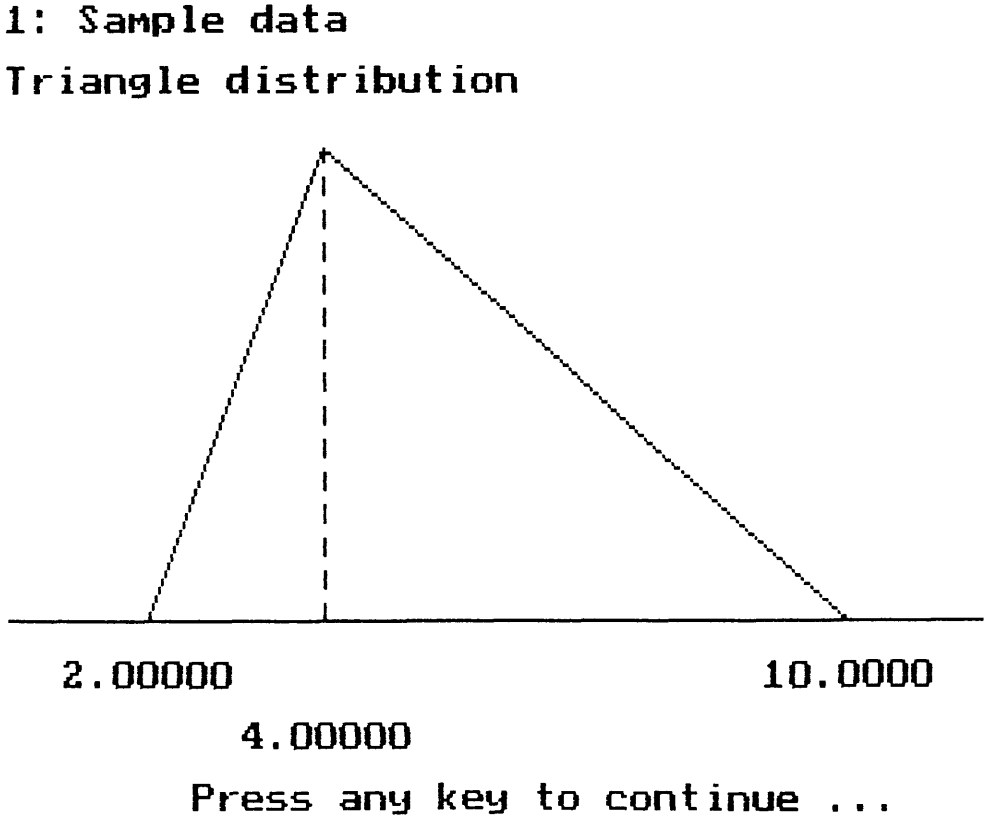

Figure 27. Output of PROBDIST for the triangular distribution model.

4. SOFTWARE INSTALLATION

This section covers the installation of the necessary executable files to generate a PROBDIST working system on either a dual 5.25" diskette computer or on a computer with a 5.25" diskette and a hard disk. Information on running the PROBDIST system after installation follows in the next section.

The files on the issue diskette are:

-README	The title page and disclaimer for the software release;
PROBDIST.EXE	The PROBDIST program;
PROBDIST.DOC	The documentation file for the above program;
INSTAL2D.BAT	The automatic installation batch file for a 2-diskette computer (5.25" diskette);
INSTALHD.BAT	The automatic installation batch file for a hard disk computer;
PDTEST.DAT	A file of hypothetical sample data for test-running the PROBDIST program.

The PROBDIST.DOC file contains most of the document you are now reading. It can be copied onto a printer using the DOS command

```
PRINT  disk:PROBDIST.DOC
```

where *disk* should be replaced by the drive letter of the disk drive containing the PROBDIST.DOC file; for example, if PROBDIST.DOC is in disk drive B, the command would be `PRINT B:PROBDIST.DOC`.

In all the instructions that follow, italics are used to indicate a part of a command which must be replaced with a phrase that fits your application.

4.1 Installation for a Diskette System

To install a PROBDIST system on a PC with two 5.25" diskette drives:

Format a new diskette. Use the standard form of the FORMAT command which includes the DOS operating system on the new diskette. For help in doing this, refer to the MS-DOS or PC-DOS manual for your machine. Label this diskette "PROBDIST SYSTEM."

Install PROBDIST. Insert the new PROBDIST system disk into drive A. Insert the master diskette on which you received PROBDIST into drive B. Now run the installation program for two diskettes by typing

```
B:INSTAL2D
```

This command takes care of copying the necessary files onto the new PROBDIST system disk.

Make a backup copy of the system diskette, and keep it in a secure place for recovery of the system in case the working disk becomes damaged. Refer to the MS-DOS or PC-DOS manual for your machine for help in copying a diskette.

Format another diskette to use as a PROBDIST data disk. Don't include the DOS operating system on this diskette. As PROBDIST runs, it generates additional files which are placed on the same disk with the data. Because of its limited free space, the PROBDIST system disk should not be used to store data files.

Enable graphics. If the computer has graphics capability, make the following arrangements to have the system load the DOS-supplied GRAPHICS program so copies of PROBDIST plots can be produced on a dot matrix printer. Use an ASCII text editor (such as EDLIN, Sidekick, or a word processor) to create an AUTOEXEC.BAT file on the PROBDIST system disk, and add the following line to the file:

```
GRAPHICS
```

Or add this line to AUTOEXEC.BAT if such a file already exists.

4.2 Installation for a Hard Disk System

To install a PROBDIST system on a PC with a hard disk and a diskette drive, use the following procedures.

The PROBDIST installation program will create a new directory on the hard disk, named \PROBDIST. In the unlikely case that such a directory already exists on the hard disk, the installation may fail. The existing \PROBDIST directory should be eliminated (refer to the RMDIR command in the MS-DOS or PC-DOS manual) before running the installation procedure.

Insert the master diskette on which you received PROBDIST into a diskette drive, and log onto this drive.

Install PROBDIST. Run the installation program for hard disk systems by typing

```
INSTALHD  disk:
```

In place of *disk* substitute the drive letter of the hard disk where PROBDIST is to be installed (one of C, D, E, ...), and be sure to include the colon after the drive letter. An example of the install command is

```
INSTALHD  C:
```

Modify AUTOEXEC.BAT. Use an ASCII text editor (such as EDLIN, Sidekick, or a word processor) to modify the PATH command in the AUTOEXEC.BAT file in the root directory of the startup disk. At the end of the PATH command, add the text

```
;disk:\PROBDIST
```

where *disk* is the same as in the preceding paragraph. For example, if you are installing to disk C, append to the PATH command the phrase

```
;C:\PROBDIST
```

If the AUTOEXEC.BAT file doesn't have a PATH command in it, add a line at the end of the file which says

```
PATH disk:\PROBDIST
```

An example of this is `PATH C:\PROBDIST`.

If the startup disk doesn't have an AUTOEXEC.BAT file, use an ASCII text editor to create one, and put the above PATH command in it.

If the computer has graphics capability, arrange to have the system load the DOS-supplied GRAPHICS program so copies of PROBDIST plots can be produced on a dot matrix printer. To do this, include the line

```
GRAPHICS
```

in the AUTOEXEC.BAT file.

Execute AUTOEXEC.BAT. To make the changes in AUTOEXEC.BAT take effect before running the PROBDIST system for the first time, enter the command

```
\AUTOEXEC
```

This makes it possible to use PROBDIST immediately. This command only has to be done once. From then on, the modifications in AUTOEXEC.BAT will take effect automatically every time the computer is turned on.

Make a data directory. The data files can reside in the same directory with the system, but clustering the data files in a separate directory is preferable. The data may be kept on a diskette instead of the hard disk, but this is slower and less convenient.

To make a new directory for storage of PROBDIST files, enter the DOS command

```
MKDIR  \dirname
```

In place of *dirname*, substitute the actual name chosen for the new directory, 8 or fewer characters in length.

5. PROBDIST OPERATION GUIDE

Before running PROBDIST, it must be correctly installed on the computer. If this has not been done, complete the installation procedure described in section 4. After installing PROBDIST, proceed with the operation of the system as described below.

When PROBDIST generates its output file, the current time and date are included at the top of the output page. Time and date are taken from the computer's internal clock calendar. If the computer does not have a continuous clock with a battery backup feature, you must type the time and date every time the computer is turned on. If this entry is bypassed, the time and date printed by PROBDIST will be meaningless.

In most respects, operation of PROBDIST is the same whether the computer has a dual diskette or a hard disk. There are a few differences in the ways these two installations are started; both cases are covered separately below.

5.1 Starting PROBDIST - Diskette Computers

Insert disks. Put the PROBDIST system disk in drive A and a formatted data disk in drive B.

Start the program. After the DOS prompt A>, type

```
PROBDIST
```

Enter data file name. When PROBDIST asks for a data file name, type a legal file name in the form

```
B:datafile.DAT
```

where B: is the data disk drive designator, and *datafile* is replaced by the name chosen for your data file. An example of a file name is `B:FILE5.DAT`. The data file name must be 8 or fewer letters or digits, not counting the .DAT suffix. The .DAT is not actually required as part of the name, but it is recommended. Other suffixes are possible, but you must avoid the suffix .PDL as PROBDIST uses this for its own output files.

5.2 Starting PROBDIST - Hard Disk Computers

Log into the directory on the hard disk where data files are to be kept, or stay in the root directory if you prefer.

Start the program. After the DOS prompt, type

```
PROBDIST
```

Enter data file name. When PROBDIST asks for a data file name, type a legal file name in the form

```
datafile.DAT
```

For *datafile*, substitute the name of your data file. An example of a data file name is SOUTH.DAT. The data file name must be 8 or fewer letters or digits. The suffix .DAT is not actually required, but it is recommended. Other suffixes are possible, but you must avoid the suffix .PDL as PROBDIST uses this for its own output files.

The file name may include a directory path prefix if needed, for example \MYDATA\TESTRUN.DAT.

The PROBDIST editor uses the data file to store the parameters entered for the variables to be estimated. Then the PROBDIST estimator module reads the file and makes its assessment. The program leaves the data in the file directory for later editing and rerun.

5.3 Operation of PROBDIST - Both Diskette and Hard Disk Computers

After attaching a data file, PROBDIST handles both diskette and hard disk systems the same. A diskette system works a bit slower.

5.3.1 New files

If the named data file does not already exist, PROBDIST asks if you want to create a new file with that name. Press key Y or N; no carriage return is needed for this entry. The N response is provided as an escape in case you really wanted an existing file but typed its name incorrectly. If you are creating a new file, PROBDIST loads the data entry module so you can proceed to enter new data.

PROBDIST now requests the probability model to be used for all the variables in this estimate. It displays a menu of the thirteen models and the input parameters required for each. Using the up/down cursor keys, move the video bar on the screen to highlight the model you want to use. As an aid in selecting a suitable model, you can view a sample graph of a typical distribution for the highlighted model by keying CTRL-G on the keyboard: hold down on the CTRL key while striking the G key once. After viewing the graph, return to the model selection menu by striking any key. After deciding on a model, make sure it is highlighted and press the ENTER or RETURN key to select that model for your data.

5.3.2 Old files

If the named file already exists, then PROBDIST gives the options of editing the file or sending the file directly to the PROBDIST estimator module. The details of this are given in section 5.5 (page 49).

5.4 Operation of the Data Entry Editor

After choosing a probability model for a new file or choosing to edit an old file, the first data entry screen appears. It includes a bar at the top of the screen containing the name of the file being edited, and a bar at the bottom showing a menu of the control keys: the arrow keys, the RETURN and TAB keys, PgUp and PgDn. The details of working with the data input editor are described in this section.

PROBDIST has a forms-oriented data entry module. The program accepts keyboard entry of input data parameters through a series of several display screens. Each screen shows an entry form containing cells for entering a group of parameters.

Screen control keys. By using the cursor keypad and other control keys you can browse randomly through the cells on the visible screen, and through the adjacent screens, until you come to the cell where you want to enter or edit data. These are the control keys:

Up arrow key	This key jumps the cursor to the next line above the current line. If the cursor is already on the first line of one of the screens of variables, it jumps to the previous variable screen (if any). The up arrow key will not move off the first variable screen; use the PgUp key in that case.
Down arrow key	This key jumps the cursor to the next line below the current line. If the cursor is already on the last line of one of the screens of variables, it jumps to the next variable screen. The down arrow key will not move off screen 1; use the PgDn key in that case.
Right arrow key	This key jumps the cursor to the next cell to the right of the current cell, if one exists. If the cursor already rests on the last cell on the current line, the cursor jumps to the first cell on the next line below. If the cursor already rests on the last cell of the screen, the effect is the same as with the down arrow key.
Left arrow key	This key jumps the cursor to the next cell to the left of the current cell, if one exists. If the cursor already rests on the leftmost cell of the current line, there is no effect.
RETURN key TAB key	These two have the same effect as the right arrow key.

PgUp key	This key jumps to the previous screen. If the cursor already rests on screen 1, PROBDIST displays "No previous screen."
PgDn key	This key jumps to the next screen. If the cursor already rests on the last data screen (the one containing variable number 992), PROBDIST displays "No following screen."
ESC key	This key can be pressed at any time to escape from the data entry function. PROBDIST then gives the options of sending the data to the estimator module, to return for more editing on the data, or to quit.

You can step sequentially through the cells on a screen by striking the RETURN key after entering each value. The right arrow key and the TAB key have the same effect as RETURN.

For a new file, PROBDIST displays empty cells. You can step through the cells and substitute actual values for your application.

The size of each cell limits the amount of space available for that entry. If a cell completely fills with characters, the editor jumps to the next cell.

Correcting errors. PROBDIST checks for errors as values are entered. If a cell has an error in it, PROBDIST displays an error message on the screen and waits for the operator to retype correctly.

If the current cell has a typing error, you can backspace over the incorrect characters and then retype. Once the cursor moves off a cell, you can return to it and make a correction, but then the entire cell must be retyped.

Numeric entries. If a data entry requires a number, it can be entered either in fixed point notation (as in 3.1416) or in floating point scientific notation (as in 2.386E+3, which means 2.386×10^3). A number in scientific notation must have at least one digit ahead of the E, otherwise PROBDIST assumes it is zero. Numbers may have an algebraic sign prefix (+ or -). Integers or whole numbers are acceptable for any numeric cell. Numbers are never written with commas or any other punctuation except for a sign prefix or a decimal point.

5.4.1 Screen 1.

This has cells for 3 parameters:

Project name	This allows documenting program output by including a main title for the entire project (up to 60 characters).
Estimate name	This is a subtitle for giving a name to this individual estimate within the project (up to 35 characters).
Units	This defines the units in which the parameters of each variable are expressed (up to 20 characters).

5.4.2 Other screens.

These contain cells for recording parameters of as many as 992 variables. Each variable has cells for the name of the variable (35 characters), and numeric entries for a varying number of fractiles or other parameters depending on the probability model selected. Each numeric cell allows an entry of up to 10 characters.

Lines of data for variables. The cells for entry of variables on screen 2 and all following screens consist of a variable name (cell 1) and the parameters for that variable (remaining cells). The editor does not allow an incomplete line: if a variable name is entered in an empty cell, the system supplies tentative values (all zeros) for the parameters. You can then advance to the parameter cells and substitute other values. If the variable name cell is empty, the editor prevents an attempt to enter numbers in the parameter cells.

For model #1 (7-fractile probability histogram), each variable requires the entry of seven fractiles. Since the screen hasn't enough space to display all of these together with the variable name, PROBDIST shows only the cells for the variable names and keeps the fractile cells hidden off the right side of the screen. To scroll the parameters into view, press the right arrow key. While the parameters are in view, the variable names are hidden off the left side; to return to the variable name cells, press the left arrow key (more than once if necessary).

If any variables are entered with blank lines between them, the program will move the nonblank lines into consecutive locations when the editing ends and PROBDIST writes the file out into the directory.

Parameter limitations. The parameters entered on screen 2 and all following screens usually have certain constraints: some must be positive, and most sets of fractiles must be in strictly increasing order. If a constraint is violated, PROBDIST gives an error message. Due to the computer's magnitude limits on real numbers, no parameter may exceed 1.0E18 (which is 10^{18}) in absolute value.

Graphic display. While the cursor rests on any variable, the user may key CTRL-G to bring up a graphic display showing the shape of the probability distribution of that variable. The CTRL-G command is given by holding down on the CTRL key while striking the G key. See Figures 14-27 for examples.

If the computer has no CGA compatible graphics adapter, PROBDIST displays an error message at the CTRL-G command.

While the plot is displayed on the screen, you can print out a copy of it by keying SHIFT-PrtScrn on the keyboard; hold down on the SHIFT key while striking the PrtScrn (or PrintScrn) key. Only a graphics compatible dot matrix printer is able to copy the graphic image. Press any key to escape from the graph and return to data entry mode. If the GRAPHICS program has not been loaded prior to running PROBDIST, this SHIFT-PrtScrn command will not work correctly. See sections 4.1 and 4.2 above.

Editing existing cells. If the content of a cell on any screen needs to be changed, move the cursor to that cell and type the new entry to replace the old. There is no way to edit part of a cell; the entire cell must be retyped.

Deleting a variable. To erase an existing variable and its parameters, move to the screen displaying the variable, move the cursor to the variable name to be deleted, and press the space bar. The variable name will disappear, and when the cursor moves away from that line or the ESC key is used to escape from the editor, all the other entries for the line will vanish.

There is no way to move a variable to a different line of the screen, other than by deleting it as shown above and retyping it elsewhere.

When you have entered all the data on the input screens, press the ESC key. PROBDIST shows the DO WHAT menu:

Do what with this file?

E Return to editing
S Save file - Send to estimator
X Exit - save file
Q Quit without saving.

Press the letter key corresponding to the action wanted. Either upper or lower case letters are accepted, and no carriage return is needed.

The **E** command causes PROBDIST to go back to the data entry module for further review and editing.

The **S** command causes PROBDIST to save the file in the directory, then send the data directly to the estimator module. PROBDIST then terminates, keeping only the new version of the file (in place of the old version, if any).

The **X** command causes PROBDIST to save the data file in the directory and then quit. Estimates based on the data are not computed. Only the new version of the file is kept, and it replaces the old version, if any.

The **Q** command causes the system to terminate without saving the newly created (or edited) file; use this option if you realize the current edits are useless and you don't want to save this version of the data.

5.5 Computing the Estimates

The DO WHAT menu gives the option of sending the current data file to the PROBDIST estimator module. To do this, press the S key. The PROBDIST estimator reads the input data file, estimates the variables, and produces its output file, a listing which summarizes the input data and displays output estimates in terms of mean, standard deviation, and fractiles of each variable.

PROBDIST names the summary listing file to agree with the data file name, with the file name suffix replaced by .PDL. The .PDL suffix suggests "PROBDIST List." The summary file shows the current time and date, obtained from the computer system either through its clock calendar adapter (if it has one) or through manual entry when the computer was first turned on.

The running time varies with the speed of the host computer and the number of variables in the data file; most runs will finish within a few seconds. The program displays a bar showing the progress of the calculation. PROBDIST returns to DOS on termination. The .PDL output file, as well as the source data file, are kept in the file directory.

5.6 Printing the Output

If the printer normally prints less than 132 columns across the page, it must be adjusted manually, or by a mode setting utility program, to print 132 columns. Scroll the paper in the printer until the print head is positioned about 3 lines below the tearoff perforations.

Now enter the standard DOS command:

```
PRINT  testdata.PDL
```

In place of *testdata*, substitute the name given to the data file when it was originally created through the PROBDIST editor. Examples of this command are

```
PRINT TEST3.PDL   and   PRINT B:WESTERN.PDL
```

The output file is printed on one or more pages.

6. SELECTED REFERENCES

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