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Computer programs in HP 9845 BASIC for modeling and interpreting multiple-layer, plane-interface shallow refraction seismics

by

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ABSTRACT

This report presents computer programs in Hewlett Packard 9845 BASIC to model and interpret multiple-layer, plane-interface shallow refraction seismics. The programs handle up to four layers for both single-ended and reciprocal spreads. Model output includes tabulated values of intercept and reciprocal times, apparent velocities, crossover and critical distances, and time-distance plots showing direct and refracted arrival times along both reciprocal spreads. The interpretation program produces first approximations of layer thicknesses and velocities using data obtained from traveltime curves.

INTRODUCTION

Two seismic refraction computer programs are listed and discussed in this report:

1. A forward modeling program (file name: REFRPM) that requires all layers (a maximum of four) have constant velocities and planar interfaces and that the source points and detectors lie along the top of the upper layer at a constant elevation, and
2. An interpretation program (file name: REFRSC) that assumes the geologic section can be approximated by the same conditions as those imposed on the forward model.

Though simplistic, the programs are nevertheless useful in planning a seismic refraction survey and in making first estimates of layer thicknesses and velocities using data displayed on time-distance plots.

In the programs, SP A is the source point (SP) for the forward spread, and SP B is the source point for the reverse spread. If the far geophone of the spread from SP A is located at the SP B position and the far geophone from the spread from SP B is located at the SP A position, then the spreads are reciprocal. If the spreads used are not truly reciprocal (that is, the far geophones are not at SP positions), far-trace times must be forced to reciprocity by extrapolation before the interpretation program of this report can be used.

When data are taken with an isolated, single-ended spread, the results can be interpreted only as if the refracting horizons have zero dip and true velocities along them equal to those obtained from the traveltime curves. If upon inspection of the reciprocal traveltime curves the intercept times and slopes (for the same layer) for the forward and reverse SP's are equal, then the single-ended type of computations can be made. However, with few exceptions, these times and slopes are not equal, and reciprocal spread methods are selected.

The programs were developed for use by engineering geophysicists, and as such, units are used, that though non-standard are more applicable in shallow-depth studies. For example, arrival times are in milliseconds (ms), distances are in meters (m), and velocities are expressed as m/ms--the equivalent of km/s. Option is provided for those who prefer to work in the English system, in which case, distances are entered in feet and velocities as ft/ms.

Input to the forward modeling program consists of entry of the number, thickness, dip, and velocity of the layers and the distance to the far detector. Output of the modeling program includes tabulated values of intercept and reciprocal times, apparent velocities, crossover and critical distances, and traveltime curves showing direct and all refracted arrivals along both forward and reverse spreads.

Working from observed traveltime curves that have been approximated by straight line segments, input to the interpretation program consists of distance and first-arrival time at the far detector, an estimate of the upper layer velocity, and the intercept time for each layer. For the two-layer case, option is provided for the crossover distance to be included as an input. Output consists of a tabulation of velocities, depths, and dips of the layers.

In the following four introductory examples, the values used are those drawn from the results of a preliminary survey (Mark Vendl, 1988, written communication) made to investigate the feasibility of using seismic refraction to map a landfill and its environs. In this area, glacial till covers a dolomite whose upper surface may be weathered and fractured. The computed depth of the overburden was 10 m and its indicated P-wave velocity within the unsaturated zone was 0.6 m/ms. Assumed velocity of the saturated overburden is 1.8 m/ms; average water table depth at nearby wells is 5 m, and indicated velocity of the bedrock is approximately 5 m/ms.

TWO-LAYER, ZERO-DIP EXAMPLE

Using the above values and a distance of 36 m to the far detector, the two-layer, no-dip, forward-model results shown on Figure 1 were produced. Following is the order of entry and input data for this model--upper left column of Figure 1.

Number of layers (2),
Velocity within the first layer (0.6 m/ms),
Distance from SP A to SP B (36 m),
Vertical thickness of the first layer at SP A (10),
Dip of the second layer = base of the first layer (0).

Upon entry of the above quantities, the computed values for layers 1 and 2 are printed either on the screen or with a hard copy, and then the traveltime curves are plotted. The critical distances (2.4 m) from each SP are indicated on these plots with dotted circles. Note that only those segments of the traveltime curves that are first arrivals are drawn with solid lines. The crossover distance from each SP is 22.6 m.

The traveltime curves on Figure 1 show that spreads must be greater than 24 m before arrivals from the high speed layer (layer 2) will be first arrivals. With a geophone spacing of 3 m, this plot also shows that first arrivals from layer 2 would be observed only on five traces; therefore, in planning the survey, a SP-to-SP spacing greater than 36 m might be advisable.

On the upper right of Figure 1 are the inputs and the results produced by the interpretation program. All input values are taken from the traveltime curves. The interpretation program begins with a prompt to select the type of problem as revealed from inspection of the traveltime curves. Upon examination of the time-distance (T/X) plot of Figure 1 it is clear that reciprocal spreads have been used. However, since no dip of the second layer is indicated (slopes of the refracted arrivals and intercept times from both SP's are equal), computations proceed as if the data had been taken with a single-ended spread. Once the spread type and the apparent number of layers (two in this case) are entered, the course of the program is set.

Following is the order of entry and the input values for the interpretation program--right column of Figure 1:

Distance from SP to far detector (36 m),
Velocity within the upper layer (0.6 m/ms),
First arrival (first break, FB) time for refraction arrivals from the second layer at the far detector (40.3 ms),
Observed intercept time for the second layer (33.1 ms).

Computed values are printed below these entries.

INITIAL PARAMETERS

Number of layers =	2
Velocity within first layer =	.6
Velocity within second layer =	5.0
Distance from SP A to SP B =	36.0

PARAMETERS FOR FIRST LAYER

Vertical thickness, 1st lay, SP A =	10.0
Dip of 2nd layer, SP A to SP B =	0.0

COMPUTED VALUES FOR LAYERS 1 AND 2

Normal thickness, 1st lay, SP A =	10.0
Vertical thickness, 1st lay, SP B =	10.0
Depth to 2nd layer under SP B =	10.0
Normal thickness, 1st lay, SP B =	10.0
Reciprocal time, top of layer 2 =	40.3
Intercept time at SP A, layer 2 =	33.1
Intercept time at SP B, layer 2 =	33.1
Crossover dist from SP A, lay 1/2 =	22.6
Crossover dist from SP B, lay 1/2 =	22.6
Critical dist from SP A, layer 2 =	2.4
Critical dist from SP B, layer 2 =	2.4

LANDFILL PROBLEM (UNSATURATED OVERBURDEN)
TWO LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER

Velocity within upper layer =	.60
FB time, 2nd layer, far detector =	40.3
Observed 2nd layer intercept time =	33.1

COMPUTED VALUES FOR 1ST & 2ND LAYERS

Apparent velocity of 2nd layer =	5.00
1st & 2nd lay crossover distance =	22.6
Depth to top of 2nd layer =	10.0
Critical distance =	2.4

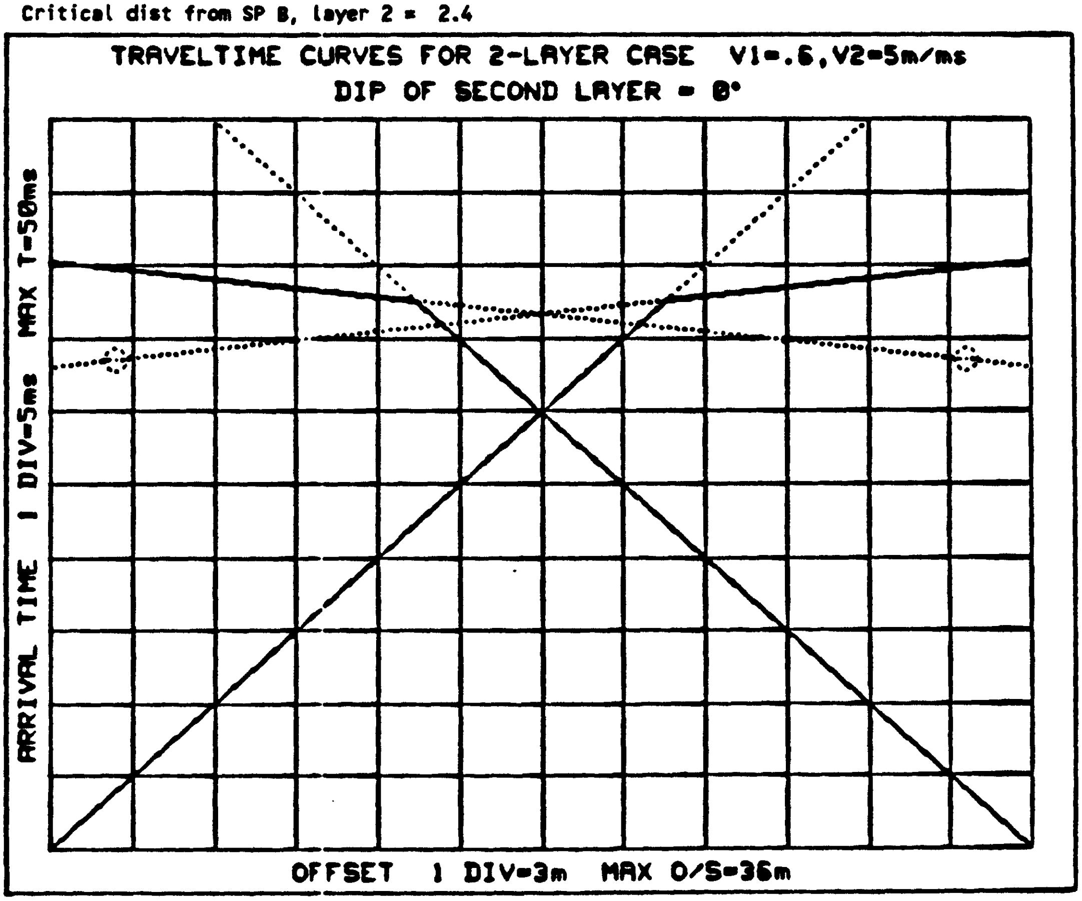


Figure 1. Input and computed values for modeling program (left column), traveltime curves, and results of interpretation program (right column) for two-layer, zero-dip case. First arrivals are connected by solid lines and critical distances are shown with dotted circles.

THREE-LAYER ZERO-DIP EXAMPLE

Figure 2a shows the forward model and interpreted results with the top half of the overburden unsaturated and the bottom half of the overburden saturated. The two-layer model of Figure 1 now becomes a three-layer model.

The input parameters and computed values for this three-layer model are tabulated in the left column of Figure 2a. At the upper right of Figure 2a are the interpreted values when the second layer is taken into account. However, note on the traveltime curve (Figure 2b) that the presence of the second layer (the water table horizon) is seen on only a single trace when the geophone spacing equals 3 m. It is quite possible with actual data that in drawing straight lines on the traveltime curve that the one-point, second-layer arrival would be ignored. The interpreted results that would be obtained if the second-layer arrival was missed are shown as the two-layer computation in the tabulation at the lower part of the right column. Omission of the second-layer arrivals results in a computed depth to the high speed layer of 6.6 m, an error of 34 per cent.

THREE-LAYER, ZERO-DIP, HIDDEN LAYER EXAMPLE

A "blind zone" (Sheriff, 1984) is "...a layer that cannot be detected by first-break refraction methods, also called hidden layer..." The blind zone may have a layer within a set of layers wherein the velocity of this layer is less than the layer above it (for example, an intercalated sandstone within a limestone sequence) and thus no refracted ray can return from it. Also, a blind zone may have a velocity intermediate between the layers which bound it but not have enough velocity difference or thickness to produce a first arrival. If in the model shown on Figure 2 the one-point arrival from the second layer had been ignored, then the second layer (the water table) would have been a hidden layer.

By increasing the thickness of the unsaturated portion of the overburden from 5 to 6 m and maintaining the depth to bedrock at 10 m, a hidden-layer is created. This model and the interpretation of results from it are shown in Figures 3a and 3b. Figure 3a lists the input parameters for both layers and the computed values for layers 2 and 3. Note in the tabulation of computed values that lines are printed stating that layer 2 is hidden when shot from SP A and from SP B. On the traveltime curve (Figure 3b) observe that at no distance along the line of refracted returns from layer 2 are any of the arrivals first arrivals--a solid line on the T/X plot.

Since the traveltime curve of Figure 3 indicates a two-layer case, input values are so entered. Although the second-layer velocity is correct, both the computed depth and critical distances are in error. The depth of 7.3 m is in error by 2.7 m (27 percent).

FOUR-LAYER, ZERO-DIP EXAMPLE

Let us now examine the effect of a fractured/weathered zone forming a transition layer on top of the dolomite. Assuming this layer has a velocity equal to half the dolomite velocity and a thickness of 1 m, the four-layer model with input values as shown in the left column on Figure 4a is given. As may be anticipated, layer 3 (the transition zone) is a hidden layer; therefore, the first arrivals on the traveltime curves give the appearance of a three-layer case. However, as shown by the tabulated results of computations (lower right of Figure 4a), the error in depth to the high speed layer is only 0.3 m. T/X plots for this four-layer case are shown on Figure 4b.

INITIAL PARAMETERS

Number of layers = 3
Velocity within first layer = .6
Velocity within second layer = 1.8
Velocity within third layer = 5.0
Distance from SP A to SP B = 36.0

PARAMETERS FOR FIRST LAYER

Vertical thickness, 1st lay, SP A = 5.0
Dip of 2nd layer, SP A to SP B = 0.0

COMPUTED VALUES FOR LAYERS 1 AND 2

Normal thickness, 1st lay, SP A = 5.0
Vertical thickness, 1st lay, SP B = 5.0
Depth to 2nd layer under SP B = 5.0
Normal thickness, 1st lay, SP B = 5.0
Reciprocal time, top of layer 2 = 35.7
Intercept time at SP A, layer 2 = 15.7
Intercept time at SP B, layer 2 = 15.7
Crossover dist from SP A, lay 1/2 = 14.1
Crossover dist from SP B, lay 1/2 = 14.1
Critical dist from SP A, layer 2 = 3.5
Critical dist from SP B, layer 2 = 3.5

PARAMETERS FOR SECOND LAYER

Vertical thickness, 2nd lay, SP A = 5.0
Depth to 3rd layer under SP A = 10.0
Dip, top of 3rd lay, SP A to SP B = 0.0

COMPUTED VALUES FOR LAYERS 2 AND 3

Normal thickness, 2nd lay, SP A = 5.0
Depth to 3rd layer under SP B = 10.0
Vertical thickness, 2nd lay, SP B = 5.0
Normal thickness, 2nd lay, SP B = 5.0
Reciprocal time, top of layer 3 = 28.9
Intercept time at SP A for lay 3 = 21.7
Intercept time at SP B for lay 3 = 21.7
Apparent vel from SP A of layer 3 = 5.00
Apparent vel from SP B of layer 3 = 5.00
Approx mean vel on top of layer 3 = 5.00
Crossover dist, SP A, layer 1 & 3 = 14.8
Crossover dist, SP A, layer 2 & 3 = 16.9
Crossover dist, SP B, layer 1 & 3 = 14.8
Crossover dist, SP B, layer 2 & 3 = 16.9
Critical dist from SP A, layer 3 = 5.1
Critical dist from SP B, layer 3 = 5.1

LANDFILL PROBLEM (TOP HALF UNSATURATED)
THREE LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER

Velocity within upper layer = .60
FB time, 2nd layer, far detector = 35.7
Observed 2nd layer intercept time = 15.7

COMPUTED VALUES FOR 1ST & 2ND LAYERS

Apparent velocity of 2nd layer = 1.80
1st & 2nd lay crossover distance = 14.1
Depth to top of 2nd layer = 5.0
Critical distance = 3.5

INPUT VALUES FOR THIRD LAYER

FB time, 3rd layer, far detector = 28.9
Observed 3rd layer intercept time = 21.7

COMPUTED VALUES FOR 2ND & 3RD LAYERS

Apparent velocity of 3rd layer = 5.00
2nd & 3rd lay crossover distance = 16.9
Thickness of second layer = 5.0
Depth to top of third layer = 10.0
Critical distance for 3rd layer = 5.1

LANDFILL PROBLEM (TOP HALF UNSATURATED)
TWO LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER

Velocity within upper layer = .60
FB time, 2nd layer, far detector = 28.9
Observed 2nd layer intercept time = 21.7

COMPUTED VALUES FOR 1ST & 2ND LAYERS

Apparent velocity of 2nd layer = 5.00
1st & 2nd lay crossover distance = 14.8
Depth to top of 2nd layer = 6.6
Critical distance = 1.6

Figure 2a. Input and computed values for modeling program (left column) and results from interpretation program (upper right column) for three-layer, zero-dip case. Lower right column shows interpretation if traveltime curves had been forced to a two-layer case.

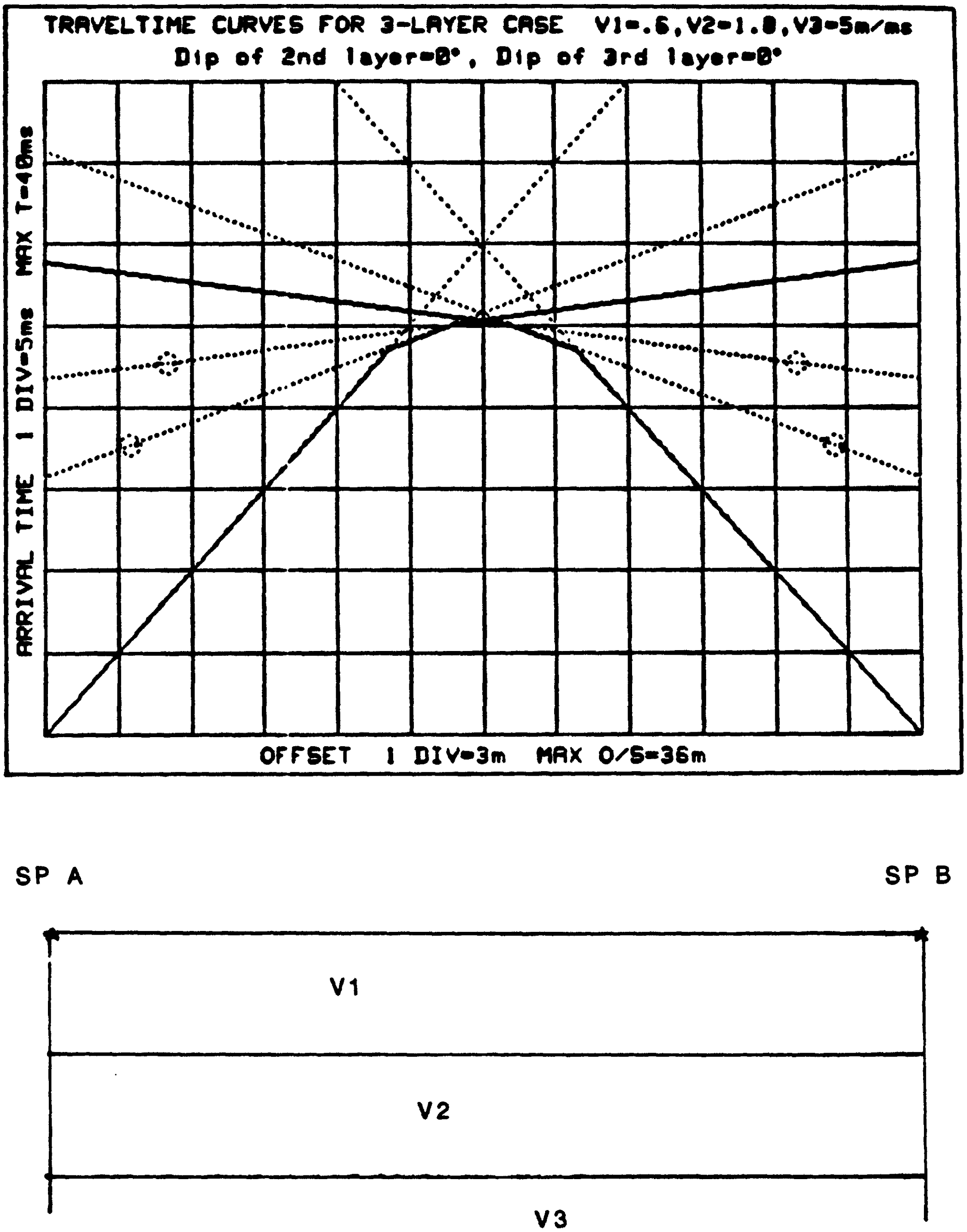


Figure 2b. Traveltime curves for three-layer, zero-dip case with model quantities as listed in left column on Figure 2a. First arrivals are connected with solid lines and critical distances are shown with dotted circles. True scale sketch beneath the T/X plot is not produced by the computer program. It has been added to draw attention to the relation between the cross section and the traveltime curves.

INITIAL PARAMETERS

Number of layers =	3
Velocity within first layer =	.6
Velocity within second layer =	1.8
Velocity within third layer =	5.0
Distance from SP A to SP B =	36.0

PARAMETERS FOR FIRST LAYER

Vertical thickness, 1st lay, SP A =	6.0
Dip of 2nd layer, SP A to SP B =	0.0

COMPUTED VALUES FOR LAYERS 1 AND 2

Normal thickness, 1st lay, SP A =	6.0
Vertical thickness, 1st lay, SP B =	6.0
Depth to 2nd layer under SP B =	6.0
Normal thickness, 1st lay, SP B =	6.0
Reciprocal time, top of layer 2 =	38.9
Intercept time at SP A, layer 2 =	18.9
Intercept time at SP B, layer 2 =	18.9
Crossover dist from SP A, lay 1/2 =	17.0
Crossover dist from SP B, lay 1/2 =	17.0
Critical dist from SP A, layer 2 =	4.2
Critical dist from SP B, layer 2 =	4.2

PARAMETERS FOR SECOND LAYER

Vertical thickness, 2nd lay, SP A =	4.0
Depth to 3rd layer under SP A =	10.0
Dip, top of 3rd lay, SP A to SP B =	0.0

COMPUTED VALUES FOR LAYERS 2 AND 3

Normal thickness, 2nd lay, SP A =	4.0
Depth to 3rd layer under SP B =	10.0
Vertical thickness, 2nd lay, SP B =	4.0
Normal thickness, 2nd lay, SP B =	4.0
Reciprocal time, top of layer 3 =	31.2
Intercept time at SP A for lay 3 =	24.0
Intercept time at SP B for lay 3 =	24.0
Apparent vel from SP A of layer 3 =	5.00
Apparent vel from SP B of layer 3 =	5.00
Approx mean vel on top of layer 3 =	5.00
Crossover dist, SP A, layer 1 & 3 =	16.4
Crossover dist, SP A, layer 2 & 3 =	14.5

Layer 2 is hidden when shot from SP A

Crossover dist, SP B, layer 1 & 3 =	16.4
Crossover dist, SP B, layer 2 & 3 =	14.5

Layer 2 is hidden when shot from SP B

Critical dist from SP A, layer 3 =	4.5
Critical dist from SP B, layer 3 =	4.5

LANDFILL PROBLEM (TOP 60% UNSATURATED)

TWO LAYERS, SINGLE ENDED, MAX O/S = 36

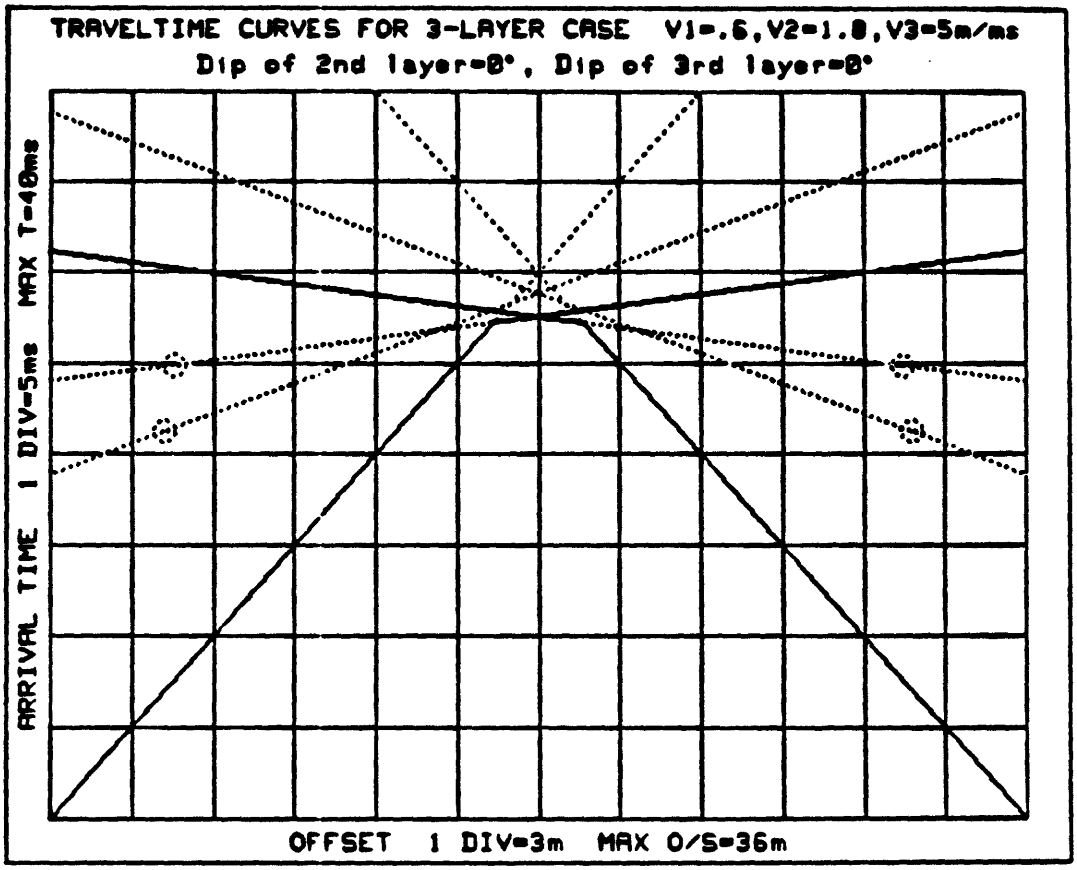
INPUT VALUES FOR 1ST AND 2ND LAYER

Velocity within upper layer =	.60
FB time, 2nd layer, far detector =	31.2
Observed 2nd layer intercept time =	24.0

COMPUTED VALUES FOR 1ST & 2ND LAYERS

Apparent velocity of 2nd layer =	5.00
1st & 2nd lay crossover distance =	16.4
Depth to top of 2nd layer =	7.3
Critical distance =	1.8

Figure 3a. Input and computed values for modeling program (left column) and results from interpretation program (right column) for three-layer, zero-dip case with second layer hidden. Note: since traveltime curves (Figure 3b) indicate a two-layer case with no dip, interpretation was made on that basis.



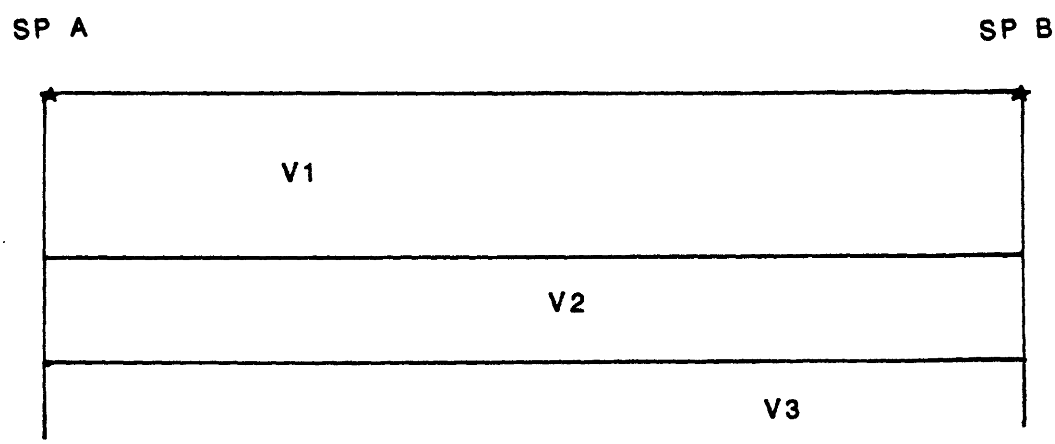


Figure 3b. Traveltime curves for three-layer, zero-dip case with model quantities as listed in the left column on Figure 3a. Note that the presence of the second layer is indicated only by the dotted line T/X plot and as such it is a hidden layer. True scale sketch beneath the T/X plot is not produced by the computer program. It has been added to draw attention to the relation between the cross section and the traveltime curves.

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INITIAL PARAMETERS
                     Number of layers =  4
         Velocity within first layer =   .6
        Velocity within second layer =  1.8
         Velocity within third layer =  2.5
        Velocity within fourth layer =  5.0
          Distance from SP A to SP B = 36.0
PARAMETERS FOR FIRST LAYER
Vertical thickness, 1st lay, SP A =  5.0
  Dip of 2nd layer,  SP A to SP B =  0.0
COMPUTED VALUES FOR LAYERS 1 AND 2
  Normal thickness, 1st lay, SP A =  5.0
Vertical thickness, 1st lay, SP B =  5.0
    Depth to 2nd layer under SP B =  5.0
  Normal thickness, 1st lay, SP B =  5.0
  Reciprocal time, top of layer 2 = 35.7
  Intercept time at SP A, layer 2 = 15.7
  Intercept time at SP B, layer 2 = 15.7
Crossover dist from SP A, lay 1/2 = 14.1
Crossover dist from SP B, lay 1/2 = 14.1
  Critical dist from SP A, layer 2 =  3.5
  Criticsl dist from SP B, layer 2 =  3.5
PARAMETERS FOR SECOND LAYER
Vertical thickness, 2nd lay, SP A =  5.0
   Depth to 3rd layer under SP A = 10.0
Dip, top of 3rd lay, SP A to SP B =  0.0
COMPUTED VALUES FOR LAYERS 2 AND 3
  Normal thickness, 2nd lay, SP A =  5.0
   Depth to 3rd layer under SP B = 10.0
Vertical thickness, 2nd lay, SP B =  5.0
  Normal thickness, 2nd lay, SP B =  5.0
  Reciprocal time, top of layer 3 = 34.4
 Intercept time at SP A for lay 3 = 20.0
 Intercept time at SP B for lay 3 = 20.0
Apparent vel from SP A of layer 3 =  2.50
Apparent vel from SP B of layer 3 =  2.50
Approx mean vel on top of layer 3 =  2.50
Crossover dist, SP A, layer 1 & 3 = 15.8
Crossover dist, SP A, layer 2 & 3 = 27.8
Crossover dist, SP B, layer 1 & 3 = 15.8
Crossover dist, SP B, layer 2 & 3 = 27.8
 Critical dist from SP A, layer 3 = 12.8
 Critical dist from SP B, layer 3 = 12.8

PARAMETERS FOR THIRD LAYER
Vertical thickness, 3rd lay, SP A =  1.0
    Depth to 4th layer under SP A = 11.0
Dip, top of 4th lay, SP A to SP B =  0.0
COMPUTED VALUES FOR LAYERS 3 AND 4
  Normal thickness, 3rd lay, SP A =  1.0
    Depth to 4th layer under SP B = 11.0
Vertical thickness, 3rd lay, SP B =  1.0
  Normal thickness, 3rd lay, SP B =  1.0
  Reciprocal time, top of layer 4 = 29.6
 Intercept time at SP A for lay 4 = 22.4
 Intercept time at SP B for lay 4 = 22.4
Apparent vel from SP A of layer 4 =  5.00
Apparent vel from SP B of layer 4 =  5.00
Approx mean vel on top of layer 4 =  5.00
Crossover dist, SP A, layer 1 & 4 = 15.3
Crossover dist, SP A, layer 2 & 4 = 18.9
Crossover dist, SP A, layer 3 & 4 = 11.9
Layer 3 is hidden when shot from SP A
Crossover dist, SP B, layer 1 & 4 = 15.3
Crossover dist, SP B, layer 2 & 4 = 18.9
Crossover dist, SP B, layer 3 & 4 = 11.9
Layer 3 is hidden when shot from SP B
 Critical dist from SP A, layer 4 =  6.2
 Critical dist from SP B, layer 4 =  6.2

LANDFILL PROBLEM (TRANSITION AT LAYER 3)
THREE LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
       Velocity within upper layer =   .60
 FB time, 2nd layer, far detector = 35.7
Observed 2nd layer intercept time = 15.7
COMPUTED VALUES FOR 1ST & 2ND LAYERS
    Apparent velocity of 2nd layer =  1.80
  1st & 2nd lay crossover distance = 14.1
          Depth to top of 2nd layer =  5.0
                  Critical distance =  3.5

INPUT VALUES FOR THIRD LAYER
 FB time, 3rd layer, far detector = 29.6
Observed 3rd layer intercept time = 22.4
COMPUTED VALUES FOR 2ND & 3RD LAYERS
    Apparent velocity of 3rd layer =  5.00
  2nd & 3rd lay crossover distance = 18.8
         Thickness of second layer =  5.7
        Depth to top of third layer = 10.7
    Critical distance for 3rd layer =  5.6
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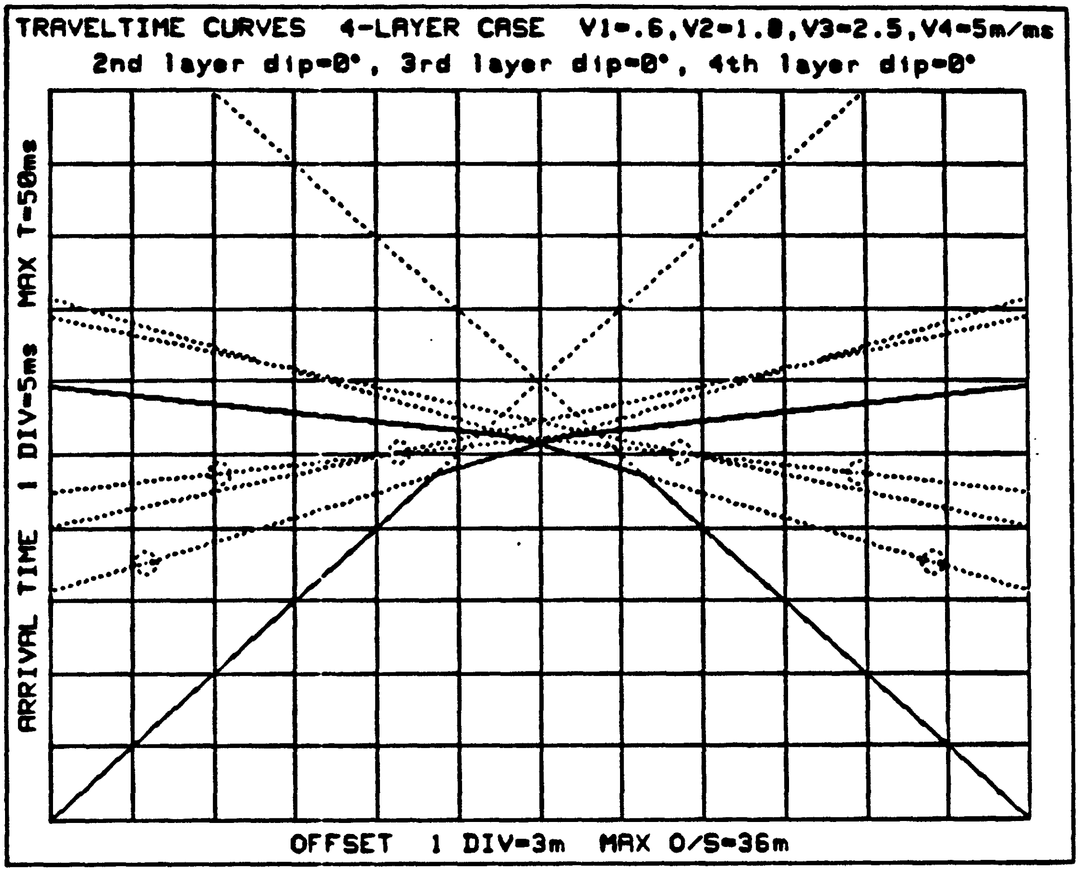
Figure 4a. Input and computed values (layer-by-layer) for modeling programs (left and upper right columns) and results of interpretation program (lower right column) for four-layer, zero-dip case. Note that layer 3 is a hidden layer.



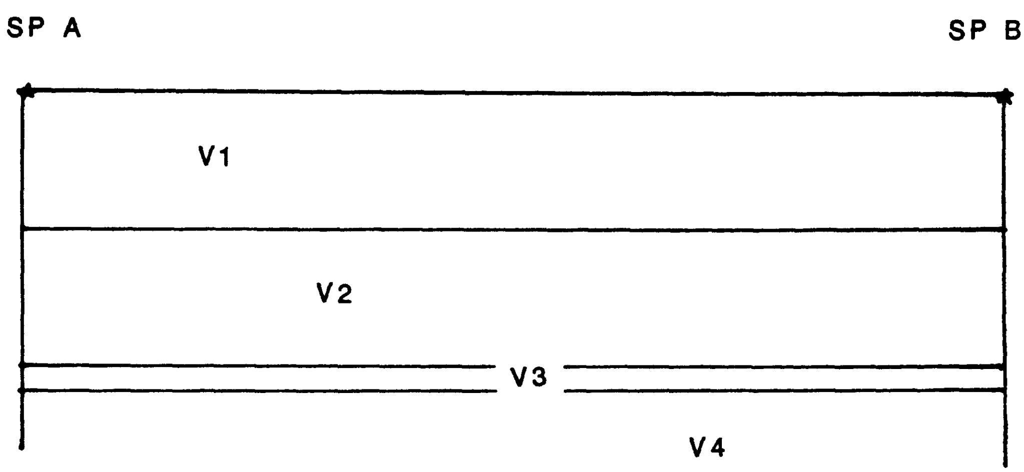


Figure 4b. Traveltime curves for four-layer, zero-dip case with model quantities as listed on Figure 4a. Note that refracted returns from layer 3 only appear along a dotted line indicating that layer 3 is a hidden layer. True scale sketch beneath the T/X plot is not produced by the computer program. It has been added to draw attention to the relation between the cross section and the traveltime curves.

DEVELOPMENT FOR THE MODELING AND INTERPRETATION PROGRAMS

In this section, the symbols and equations used in the development of the modeling and interpretation programs are given. If one is interested only in using the programs without regard to their development, then this section may be skipped. Having been so alerted, let us continue.

The development of the forward modeling and interpretation programs follows essentially parallel paths with emphasis being placed on the use of intercept and delay times. Neither of these concepts is new. They date back more than a half century to the earliest days of refraction seismics. Suggested references among the many available include Palmer (1980 and 1986), Sheriff (1984), and Sjögren (1984).

Rather than stating equations in their more customary mathematical form, the equations in the text are written using the symbols and BASIC operators as they appear in the programs. By writing the equations this way it is easier to tie them to their appearance in the programs. An attempt was made to develop a systematic set of symbols, some of which are shown on Figure 5 on which rays refracting from the third layer and arriving at station C at the surface are sketched. Source point A (SP A) is off to the left side of the figure; SP B if off to the right. Down dip from SP A toward SP B is taken as positive, thus both dip angles A2 and A3 on Figure 5 are positive.

SYSTEM OF SYMBOLS

The numbers in a symbol refer to the layer in which the quantity either resides or is referred; for example, the constant velocity in layer 1 is V1 and the dip at the top of layer 2 is A2. In the procedures of this report, the ground surface is level; that is, A1 = 0.

All angles of rays at interfaces are identified by a double-number indicating the layers upon whose velocities these angles depend. For example, the angle I23 is the critical angle between layers 2 and 3. The angles A13 and B13 are what I call the "dip critical angles" because their values depend on the dip of the layers in addition to their velocity ratios. The leading upper-case letter indicates the SP at which the rays originate, thus A13 is related to SP A. In subsequent developments, rays coming from the left side carry the letter A; those coming from the right side, the letter B. Apparent velocities are identified by a digit-letter combination showing respectively the high speed layer along which the refracted ray has traveled and the SP at which the rays originated. For example, the apparent velocity for the ray along layer 3 coming from SP A is symbolized by V3a.

The first letter of a symbol indicates (within reason) the nature of the variable (Dt for delay time, T for arrival time, X for offset distance, Z for depth) followed by a single or pair of numbers related to layer numbering. Next is a letter (a or b) indicating the SP for reciprocal spreads; no letter in this position indicates either a zero-dip or single-ended-spread case. The last letters indicate the nature of the variable; for example, i for intercept, c for crossover, cd for critical distance. The following examples illustrate the type of symbols used:

T3ai =	Intercept time for ray along layer 3 from SP A
T3i =	Intercept time along layer 3, single-ended spread
T23aco =	Time at the crossover point for layers 2 and 3 from SP A
X23bco =	Crossover distance for layers 2 and 3 from SP B
X4acd =	Critical distance for layer 4 from SP A
Z3a =	Depth to layer 3 at SP A
L2b =	Vertical thickness of layer 2 at SP B
N2a =	Normal thickness of layer 2 relative to SP A

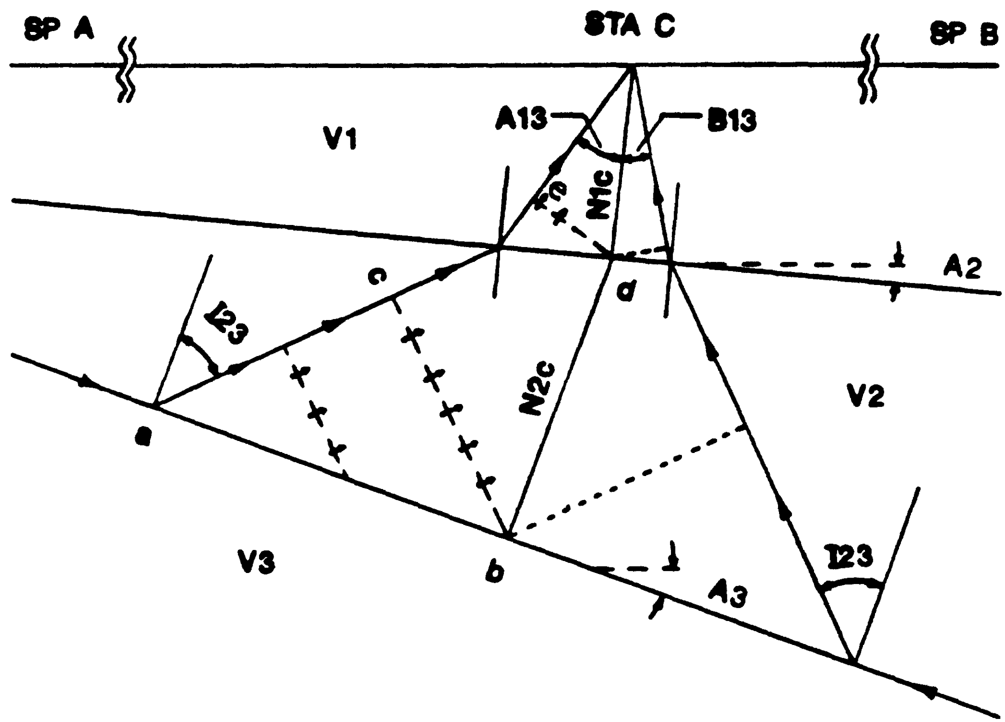


Figure 5. Diagram illustrating some of the symbols used in the development of the modeling and interpretation programs. The arrowed wave fronts are drawn to illustrate the use of delay times in the development. Note that the time for the wave front to move from the "c" to "e" positions equals the time to move from positions "b" to "d".

DELAY TIMES

Using the well-known delay time concept, delay time for layer 3 from SP A to station C is

$$Dt3a(c) = N1c*COS(A13)/V1 + N2c*COS(I23)/V2, \quad (1)$$

and the delay time for layer 3 from SP B to station C is

$$Dt3b(c) = N1c*COS(B13)/V1 + N2c*COS(I23)/V2, \quad (2)$$

where A13 and B13 are dip critical angles (to be discussed later), and N1c and N2c are normal thicknesses of layers 1 and 2, respectively, see Figure 5. Since at SP A the ray can be considered as entering from the right side (the "B" side) , delay time at SP A from layer 3 is

$$Dt3a = N1a*COS(B13)/V1 + N2a*COS(I23)/V2, \quad (3)$$

and similarly, since at SP B the ray can be considered as entering from the left side (the "A" side), delay time at SP B from layer 3 is

$$Dt3b = N1b*COS(A13)/V1 + N2b*COS(I23)/V2. \quad (4)$$

For the two-layer case, the delay time at SP A is

$$Dt2a = N1a*COS(I12)/V1. \quad (5)$$

For the four-layer case, the delay time at SP A is

$$Dt4a=N1a*COS(B14)/V1+N2a*COS(B24)/V2+N3a*COS(I34)/V3. \quad (6)$$

REFRACTION TIME FROM SP A TO SP B, THE RECIPROCAL TIME

The reciprocal time is defined as the time from SP A to SP B, which from inspection of the geometry of Figure 6 equals the time from SP B to SP A. Letting the SP-to-SP distance be Xc, and using the delay time concept as an aid to developing refraction times with planar interfaces, the reciprocal time for the third layer is

$$T3rt = D3/V3 + Dt3a + Dt3b, \quad (7)$$

where Dt3a and Dt3b are the third-layer delay times at SP's A and B, respectively, and D3 (see Figure 6) is the distance along the top of the third layer and is

$$D3 = Xc*COS(A2)*COS(A3-A2). \quad (8)$$

For the two-layer case, the reciprocal time is

$$T2rt = D2/V2 + Dt2a + Dt2b, \quad (9)$$

where D2 = Xc*COS(A2).

For the four-layer case, the "D" distance is

$$D4 = Xc*COS(A2)*COS(A3-A2)*COS(A4-A3), \quad (10)$$

and the reciprocal time for the refracted ray along layer 4 is

$$T4rt = D4/V4 + Dt4a + Dt4b. \quad (11)$$

From the preceding developments, a pattern emerges in the progression from three- to four-layer cases, a pattern that can be used for extension beyond four layers. For example, for a five-layer case, the "D" distance becomes

$$D5 = Xc*COS(A2)*COS(A3-A2)*COS(A4-A3)*COS(A5-A4), \quad (12)$$

and the reciprocal times for the refracted ray along layer 5 is

$$T5rt = D5/V5 + Dt5a + Dt5b, \quad (13)$$

where the delay time at SP A is

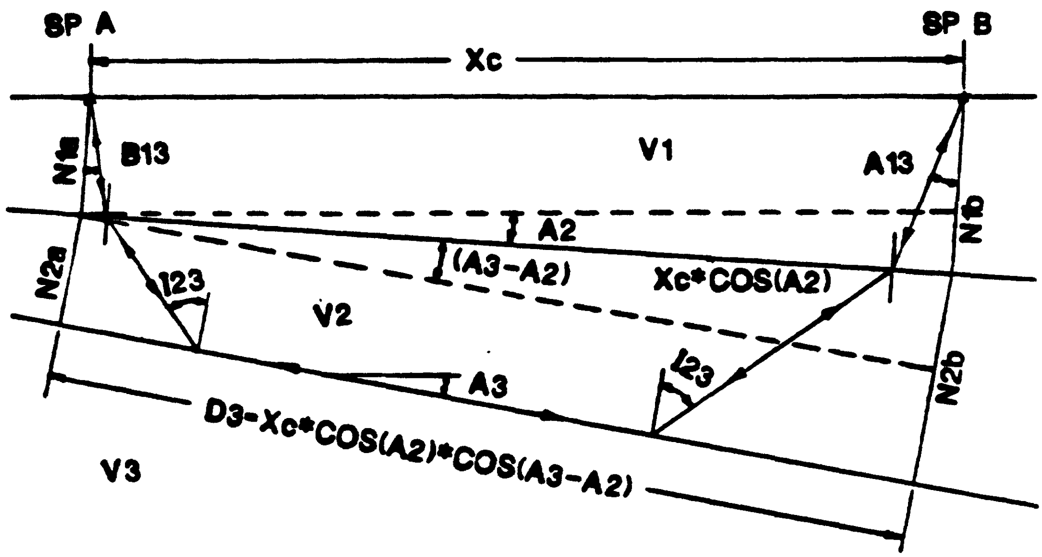


Figure 6. Diagram to illustrate symbols used in developing the reciprocal time with the use of delay times. Symbols used are identical to their appearance in the programs.

Dt5a = N1a*COS(B15)/V1 + N2a*COS(B25)/V2 + N3a*COS(B35)/V3 + N4a*COS(I45)/V4, (14)

and the delay time at SP B is

Dt5b = N1b*COS(A15)/V1 + N2b*COS(A25)/V2 + N3b*COS(A35)/V3 + N4b*COS(I45)/V4. (15)

As indicated by the progressive expansion of terms in the above expressions, it is not mathematically challenging to generalize any of the previous relations to "N" layers. However, for most engineering problems, four layers usually are enough.

REFRACTION TIME FROM SP TO DETECTOR

For the three-layer case (Figure 7), the refraction time from SP A to a detector at station C at an offset of Xc is

T3ra(c) = D3/V3 + Dt3a + Dt3ac; where D3=Xc*COS(A2)*COS(A3-A2). (16)

For future use in the development of the expression for the intercept time, it is better to write the SP-to-detector time as a function of offset and values associated with SP A. On Figure 7, note that the normal thickness distance for the first layer for station C equals the normal thickness distance at SP A plus a distance D1 (N1c = D1 + N1a), and similarly, N2c = D2 + N2a. Both D1 and D2 can be expressed in terms of the offset distance (Xc) and the interface dip angles, as follows:

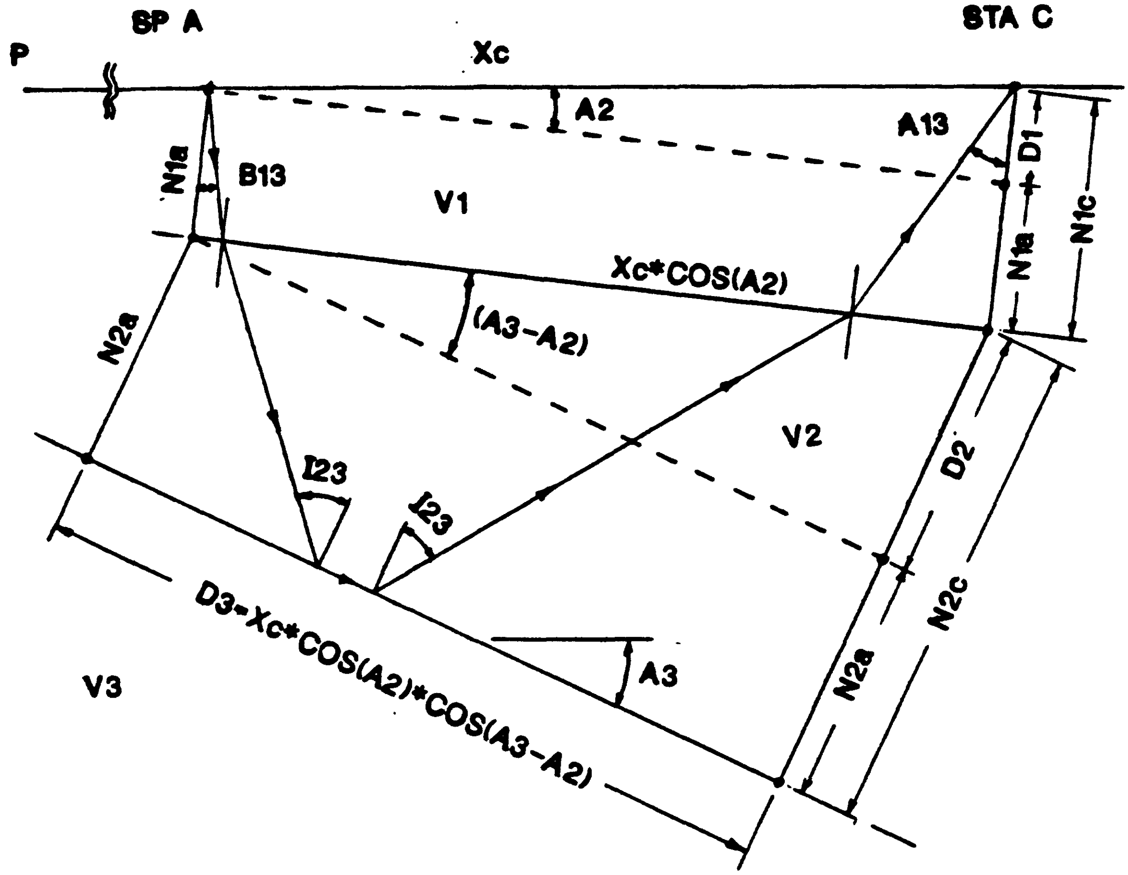


Figure 7. Diagram to illustrate symbols used in developing the refraction time from SP A to a detector at station C. Note that normal distances beneath station C (N1c and N2c) are expressed in terms of the normal distances (N1a and N2a) beneath SP A.

$$D1 = Xc*SIN(A2), \text{ and} \tag{17}$$

$$D2 = Xc*COS(A2)*SIN(A3-A2). \tag{18}$$

The delay time at station C is then

$$Dt3a(c) = (N1a+D1)*COS(A13)/V1 + (N2a+D2)*COS(I23)/V2. \tag{19}$$

Upon gathering and rearranging terms, the refraction time from SP A to station C is

$$Tra(c) = N1a*(COS(B13)+COS(A13))/V1 + 2*N2a*COS(I23)/V2 + Xc*(COS(A2)*COS(A3-A2))/V3 + Xc*(SIN(A2)*COS(A13)/V1 + COS(A2)*SIN(A3-A2)*COS(I23)/V2)) \quad (20)$$

Using a similar development, and remembering the sign convention for dip angle on the interface (down dip from SP A to a station to the right becomes an up dip from SP A to a station to the left for the same horizon), the refraction time from SP A to a detector at station P positioned to the left of the SP A position is

$$Tra(p) = N1a*(COS(B13)+COS(A13))/V1 + 2*N2a*COS(I23)/V2 + Xc*(COS(A2)*COS(A3-A2))/V3 + Xc*(SIN(-A2)*cos(A13)/V1 + COS(A2)*SIN(A2-A3)*COS(I23)/V2)) \quad (21)$$

INTERCEPT TIME

When the offset (SP to detector distance) term equals zero, the refraction time becomes the intercept time; therefore, the intercept time for layer 3 arrivals from SP A to station C is

$$T3ai(c) = N1a*(COS(A13)+COS(B13))/V1 + 2*N2a*COS(I23)/V2, \quad (22)$$

and the intercept time for layer 3 arrivals from SP A to station P is

$$T3ai(p) = N1a*(COS(A13)+COS(B13))/V1 + 2*N2a*COS(I23)/V2. \quad (23)$$

Note that these two intercept times are equal, thus station identifiers (c) and (p) can be dropped, and the intercept time at SP A becomes simply T3ai; that is

$$T3ai = N1a*(COS(A13)+COS(B13))/V1+2*N2a*COS(I23)/V2. \quad (24)$$

By similar development, the intercept time for the arrivals from layer 3 at SP B is

$$T3bi = N1b*(COS(A13)+COS(B13))/V1+2*N2b*COS(I23)/V2. \quad (25)$$

The significance of the observation of the equality of intercept times regardless of whether backwards or forwards from the SP is that when drawing straight lines on the traveltime curves, lines containing arrival times from the same layer when extrapolated back to X=0 should intersect.

RELATION BETWEEN DELAY TIMES AND INTERCEPT TIMES

The delay time at SP A for arrivals from the left (the back or reverse spread) is

$$Dt3a(left) = N1a*COS(A13)/V1 + N2a*COS(I23)/V2, \quad (26)$$

and the delay time at SP A for arrivals from the right (the forward spread) is

$$Dt3a(right) = N1a*COS(B13)/V1 + N2a*COS(I23)/V2. \quad (27)$$

The sum of these delay times is

$$SUM=N1a*(COS(A13)+COS(B13))/V1 + 2*N2a*COS(I23)/V2. \quad (28)$$

Note that the right side of this equation is the same as that for (24); therefore, it can be stated that the intercept time at any SP position equals the sum of the left and right delay times. For the two-layer case, since the left and right delay times are equal, the intercept time is equal to twice the delay time, therefore

$$T2ai = 2*Dt2a = 2*N1a*COS(I12)/V1. \quad (29)$$

DIP CRITICAL ANGLES

Dip critical angles are functions of layer velocities and dips. Figure 8 shows the quantities used to determine the dip critical angles A13 and B13. Starting with the interface between the highest speed layer and the lower speed layer above it (layers 3 and 2), from Snell's law,

$$SIN (I23)/V2 = SIN (90)/V3, \quad (30)$$

therefore, with ASN representing the arc sine,

$$I23 = ASN (V2/V3). \quad (31)$$

Working back to the interface between the second and first and layer (layers 2 and 1), again from Snell's law,

$$SIN (A13)/V1 = SIN (I23+(A3-A2))/V2, \quad (32)$$

which upon solution for A13 through the arc sine gives

$$A13 = ASN (V1*SIN(I23+(A3-A2))/V2). \quad (33)$$

Similarly, from inspection of Figure 8,

$$B13 = ASN (V1*SIN(I23-(A3-A2))/V2). \quad (34)$$

For the four-layer case, the needed dip critical angles are

$$A24 = ASN(V2*SIN(I34+(A4-A3)/V3), \quad (35)$$

$$B24 = ASN(V2*SIN(I34-(A4-A3)/V3), \quad (36)$$

$$A14 = ASN(V1*SIN(A24+(A3-A2)/V2), \quad (37)$$

$$B14 = ASN(V1*SIN(B24-(A3-A2)/V2). \quad (38)$$

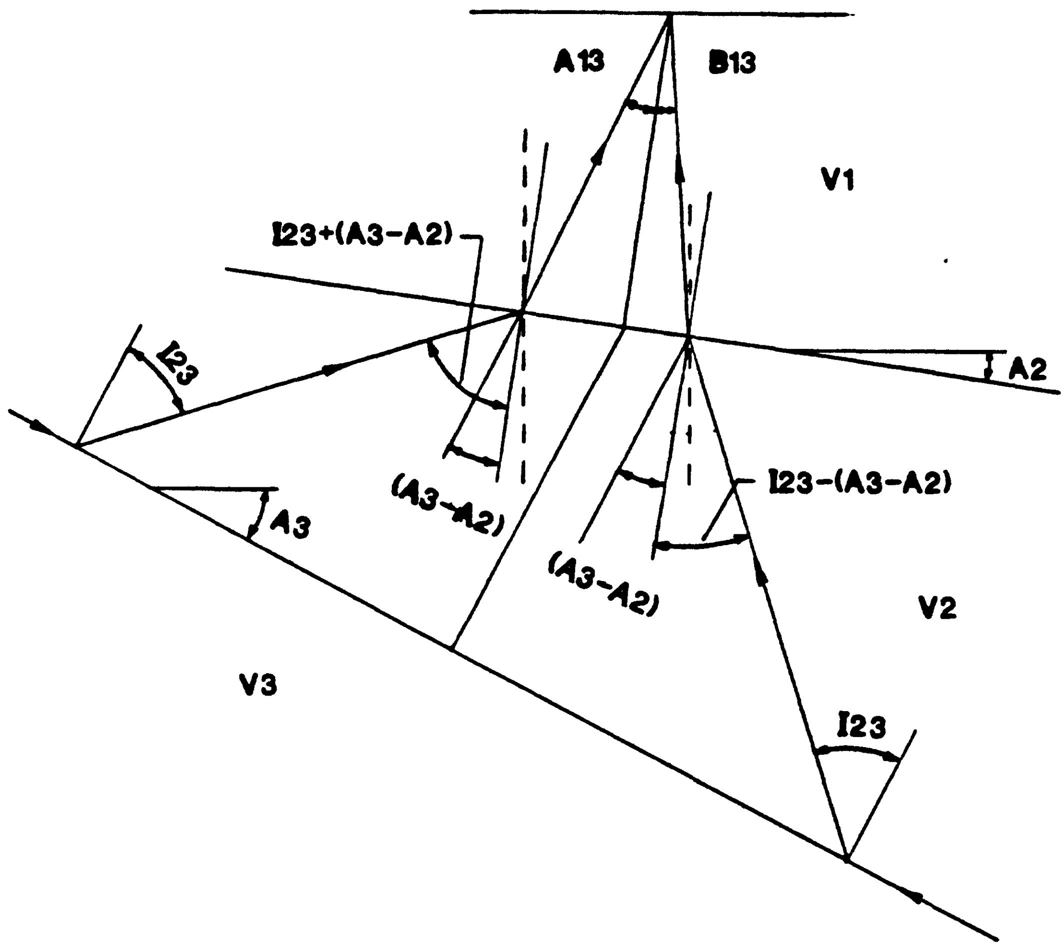


Figure 8. Diagram illustrating the quantities used in the development of expressions for the dip critical angles (A13 and B13) as functions of layer velocities and dips.

CROSSOVER DISTANCE

It is necessary when planning a seismic refraction survey to estimate the spread length required to obtain first arrivals from the target layer. The distance at which segments on a traveltime curves cross each other is called the crossover distance. Solutions for these distances derive from the point-slope equation (y = mx + b) with the intercept equal to the intercept time and the slope equal to the reciprocal of the apparent velocity. Considering the crossover distance from SP A for layers 2 and 3 (X23aco), the refraction time at the crossover distance from layers 2 and 3 is

$$T23aco = X23aco/V2a + T2ai = X23aco/V3a + T3ai, \tag{39}$$

where V2a and V3a are the apparent velocities of layers 2 and 3 from SP A. Solving for this particular crossover distance,

X23aco = V2a*V3a*(T3ai-T2ai)/(V3a-V2a). (40)

Using a similar development, crossover distance from SP B is

X23bco = V2b*V3b*(T3bi-T2bi)/(V3b-V2b). (41)

For the two-layer case, the crossover distance for SP A is

X12aco = V1*V2a*T2ai/(V2a-V1), (42)

and for SP B, the crossover distance is

X12bco = V1*V2b*T2bi/(V2b-V1). (43)

In the special two-layer case in which the dip of the second layer is zero, the depth to the second layer can be obtained from

Z2 = 0.5*X12co*SQR((V2-V1)/(V2+V1)), (44)

where SQR is the symbol in BASIC for square root.

For the four-layer case, the crossover distances for the third and fourth layers from SP A and SP B, respectively, are

X34aco = V3a*V4a*(T4ai-T3ai)/(V4a-V3a), (45)

X34bco = V3b*V4b*(T4bi-T3bi)/(V4b-V3b). (46)

CRITICAL DISTANCE

The critical distance is the offset at which reflection and refraction times are equal. Shown on figure 9 are critical distances X2acd and X3acd from SP A for layers 2 and 3, respectively. No refraction returns can occur inside the critical distances. On the traveltime curves produced by the modeling program, the critical distances are indicated by open circles.

For the two-layer case (top sketch on Figure 9), computation of X2acd is straightforward through use of the Law of Sines and the image point. With reference to Figure 9 (top), note that

X2acd/SIN(I12) = 2*N1A/SIN(90-(I12+A2)), (47)

which upon solution for X2acd yields

X2acd = 2*N1a*SIN(I12)/COS(I12+A2). (48)

By a similarly development,

X2bcd= 2*N1b*SIN(I12)/COS(I12-A2). (49)

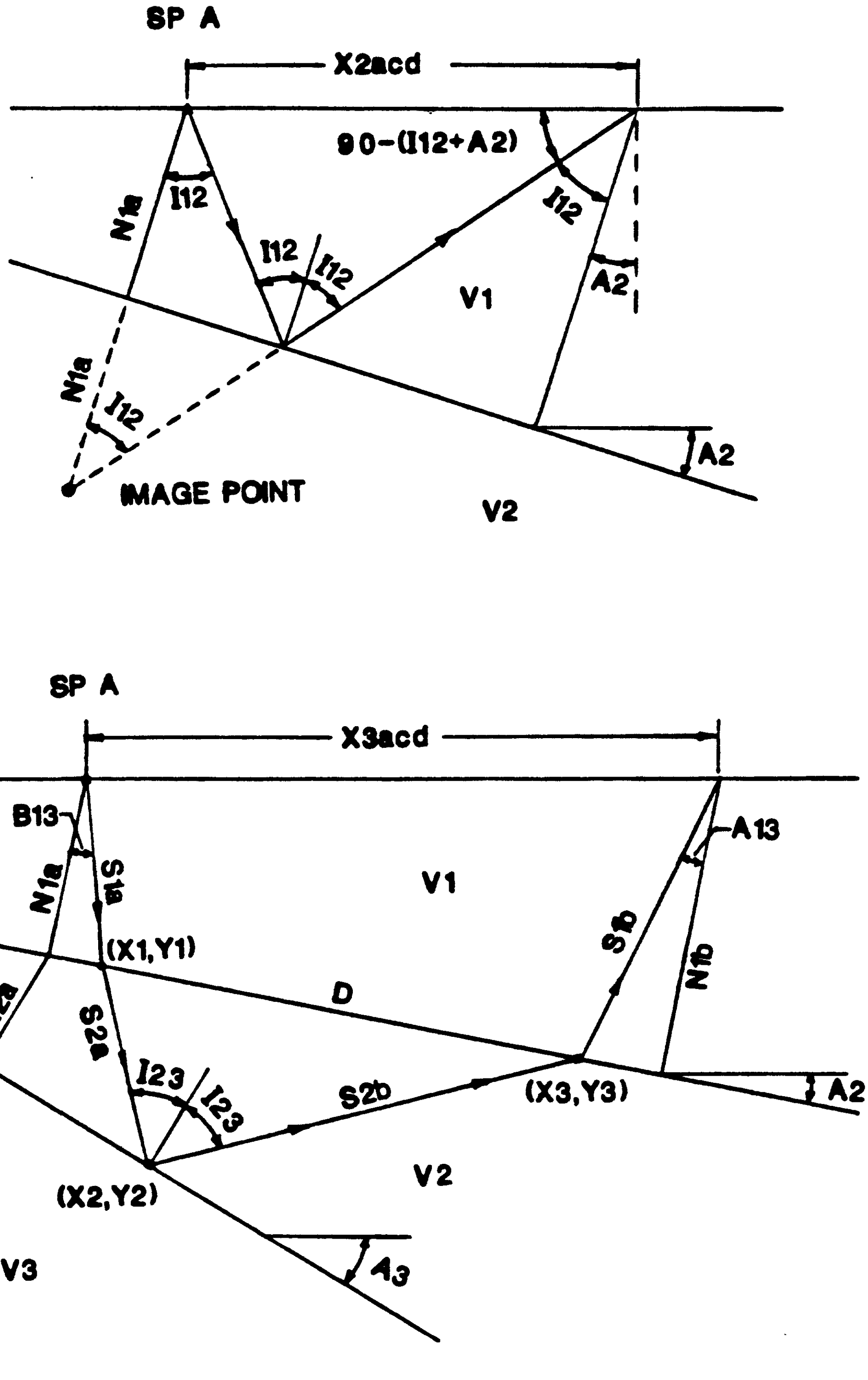


Figure 9. Diagrams illustrating the quantities used in developing expressions of critical distances for the two-layer case (upper) and the three-layer case (lower).

I know no way except brute force to compute critical distances when the number of dipping layers is more than one. As an example of this type of computation, let us trace through the solution for X3acd using the points, angles, and distances identified on Figure 9.

Starting from the SP A, the origin of coordinates, the slant distance (S1a) from SP A to point (X1,Y1) is

$$\text{S1a} = \text{N1a/COS(B13).} \tag{50}$$

The coordinates of point (X1,Y1) are then found using

$$\text{X1=S1a*SIN(B13-A2)} \quad \text{and} \quad \text{Y1=S1a*COS(B13-A2).} \tag{51}$$

The slant distance (S2a) along the ray in the second layer is then computed using the relation:

$$\text{S2a} = \text{(N2a+S1a*SIN(B13)*SIN(A3-A2))/COS(I23).} \tag{52}$$

Next, the distance D from point (X1,Y1) to (X3,Y3) is determined from

$$\text{D} = \text{S2b*SIN(2*I23)/SIN(90-(A3-A2)-I23).} \tag{53}$$

The coordinates of point (X3,Y3) are then computed using

$$\text{X3=X1+D*COS(A2)} \quad \text{and} \quad \text{Y3=Y1+D*SIN(A2).} \tag{54}$$

Finally, the critical distance is computed from

$$\text{X3acd} = \text{X3+Y3*TAN(A13+A2).} \tag{55}$$

Arrival time at this distance is found with use of the intercept time and the apparent velocity from SP A using

$$\text{T3acd} = \text{X3acd/V3a + T3ai.} \tag{56}$$

INTERPRETATION ALGORITHMS

The interpretation program (REFRSC) is the inverse of the forward model program (REFRPM). Whereas the T/X plot is the end product of the modeling program, it is the starting point for the interpretation program. The following input information is derived from the traveltime curves:

1. Whether the data were taken with single-ended or reciprocal spreads,
2. Maximum offset for a single-ended spread or SP-to-SP distance for reciprocal spreads,
3. Apparent number of layers as indicated by straight-line segments,
4. First arrival times to the far detector or reciprocal times for each layer,
5. Intercept times for each layer at each SP,
6. Velocity of the first layer, and optionally,
7. Crossover distances for two-layer cases.

APPARENT VELOCITIES

Apparent velocities, the velocities at which refracted wavefronts sweep across the spread, are determined using the maximum offset and the reciprocal and intercept times for each layer for each SP; for example, the apparent velocity from SP A for the refraction from layer 3 is

$$V3a = Xc / (T3rt - T3ai). \quad (57)$$

If the slope on the traveltime curve (measured outward from the SP) approaches zero or is negative, the program assigns a flag value of 99.99 for the apparent velocity.

TRUE VELOCITY AND DIP

For single-ended spreads, the information obtainable from the traveltime curves from a single SP is insufficient to compute either the true velocity or dip of a refracting layer. Therefore the dip is taken to be zero and the true velocity is assumed equal to the apparent velocity.

Central to the development of the velocity and dip algorithms is the relationship between apparent velocity and the angle of emergence of the refracted wave. Referring to Figure 8, note that the angle of emergence for a refracted plane wave coming in from the right side (SP B) is B13-A2 and for one from the left side (SP A) is A13+A2. Therefore, the apparent velocities for SP A and SP B, respectively, can be expressed as

$$V3a = V1 / SIN (B13 + A2), \text{ and } V3b = V1 / SIN (A13 - A2). \quad (58)$$

The sines of the emergence angles for the three-layer case are therefore

$$SIN (B13 + A2) = V1/V3a, \text{ and } SIN (A13 - A2) = V1/V3b. \quad (59)$$

Upon transposing terms in (33) and (34), as developed for the three-layer case,

$$I23 + A3 - A2 = ASN (V2 * SIN (A13) / V1), \text{ and} \quad (60)$$

$$I23 - A3 + A2 = ASN (V2 * SIN (B13) / V1). \quad (61)$$

Solution of the above simultaneous equations yields

$$A3 = 0.5 * (ASN (V2 * SIN (A13)/V1) - ASN (V2 * SIN (B13)/V1)) + A2, \quad (62)$$

the dip of the third layer, and

$$I23 = 0.5 * (ASN (V2 * SIN (A13)/V1 + ASN (V2 * SIN (B13)/V1)), \quad (63)$$

the critical angle for the second and third layers. From Snell's law,
SIN (I23) = V2/V3, therefore, the true velocity of the third layer is

$$V3 = V2 / SIN (I23). \quad (64)$$

Development of true velocities and dips of the second and forth layers follows essentially the same course. Note in the above development that solution for layer velocity and dip requires use of velocity and dip of the overlying layer.

DEPTHS AND NORMAL THICKNESSES

Computation of depths and normal thicknesses also proceeds progressively from the upper to the lower layers. Reliance in placed primarily on the use of intercept times for computation of depths to the third and forth layers. Option is provided to compute depths of the second layer through the use of the crossover distances.

For the three-layer case with dip, the equations used to compute thicknesses are as follows (see Figure 6 for symbols):

$$N2a = 0.5 * (T3ai - N1a * (COS(A13)/V1b + COS(B13)/V1a))*V2/COS(I23), \qquad (65)$$

the normal thickness of layer 2 at SP A. The normal thickness of layer 2 at SP B is obtained from

$$N2b = 0.5 * (T3bi - N1b * (COS(A13)/V1b + COS(B13)/V1a))*V2/COS(I23), \qquad (66)$$

where N1a and N1b are the normal thicknesses of layer 1 at SP's A and B, respectively, and V1a and V1b are the first-layer velocities near SP's A and B, respectively.

The vertical thickness of layer 2 at SP A (symbolized by L2a) is determined using

$$L2a = (N2a + L1a * SIN (A2) * SIN (A3-A2)) / COS (A3), \qquad (67)$$

where L1a is the vertical thickness of layer 1 at SP A. The vertical thickness of layer 2 at SP B is

$$L2b = (N2b + L1b * SIN (A2) * SIN (A3-A2)) / COS (A3), \qquad (68)$$

where L1b is the vertical thickness of layer 1 at SP B. The depth to layer 3 is simply the sum of the vertical thicknesses of layers 1 and 2 at the specified SP's.

ADDITIONAL PROGRAM APPLICATIONS

Examples were given in the introduction to illustrate the basics of the operation of the programs. These examples were limited to zero-dip cases for which a single-ended spread interpretation was sufficient. In this section of the report, operation of the programs for two- and three-layer cases with dip is described and a brief discussion of the results is given.

EXAMPLE OF PROGRAM OPERATION WITH TWO-LAYER CASE WITH DIP:

Figure 10a lists model parameters (left column) and computed results (upper right column) for a simple two-layer case with a dipping second layer and with data obtained along reciprocal spreads. Comparing input values for this model to those for the model of Figure 1 (the two-layer case with zero dip) note that initial and first-layer parameters are similar with the exception of entering amount of dip at the top of the second layer from SP A to SP B (5 deg). The prompt for entry of the amount of dip contains a reminder that down dip from SP A to SP B is positive. The traveltime curves for this model are shown in Figure 10b.

Once the distance to the far detector (36 m) and the estimated upper layer velocity (1 m/ms) are entered, the program calls for entry of the reciprocal time (31.0 ms), and the observed intercept times at SP A (10.4 ms) and SP B (15.8 ms).

```
INITIAL PARAMETERS
                        Number of layers =  2
           Velocity within first layer =  1.0
          Velocity within second layer =  2.0
            Distance from SP A to SP B = 36.0
PARAMETERS FOR FIRST LAYER
Vertical thickness, 1st lay, SP A =  6.0
  Dip of 2nd layer,   SP A to SP B =  5.0
COMPUTED VALUES FOR LAYERS 1 AND 2
  Normal thickness, 1st lay, SP A =  6.0
Vertical thickness, 1st lay, SP B =  9.1
    Depth to 2nd layer under SP B =  9.1
  Normal thickness, 1st lay, SP B =  9.1
  Reciprocal time, top of layer 2 = 31.0
  Intercept time at SP A, layer 2 = 10.4
  Intercept time at SP B, layer 2 = 15.8
Apparent vel from SP A of layer 2 =  1.74
Apparent vel from SP B of layer 2 =  2.37
Approx mean vel on top of layer 2 =  2.00
Crossover dist from SP A, lay 1/2 = 24.3
Crossover dist from SP B, lay 1/2 = 27.3
 Critical dist from SP A, layer 2 =  7.3
 Critical dist from SP B, layer 2 = 10.1
```

```
SAMPLE PROBLEM #5  DIPPING HORIZON
TWO LAYERS, RECIPROCAL, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
Avg velocity, upper layer at SP A =  1.00
Avg velocity, upper layer at SP B =  1.00
  Average velocity of upper layer =  1.00
     Reciprocal time for 2nd layer = 31.0
  Observed intercept time at SP A = 10.4
  Observed intercept time at SP B = 15.8

COMPUTED VALUES FOR 1ST & 2ND LAYERS
 Apparent 2nd-layer vel from SP A =  1.75
 Apparent 2nd layer vel from SP B =  2.37
          Dip from SP A toward SP B =  5.0
            Velocity of second layer =  2.00
Computed crossover dist from SP A = 24.3
Computed crossover dist from SP B = 27.3
Normal thickness of lay 1 at SP A =  6.0
 Depth to second layer under SP A =  6.0
Normal thickness of lay 1 at SP B =  9.1
 Depth to second layer under SP B =  9.2
SAMPLE PROBLEM #5  DATA FROM SP A
TWO LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
        Velocity within upper layer =  1.00
  FB time, 2nd layer, far detector = 31.0
Observed 2nd layer intercept time = 10.4
COMPUTED VALUES FOR 1ST & 2ND LAYERS
    Apparent velocity of 2nd layer =  1.75
  1st & 2nd lay crossover distance = 24.3
            Depth to top of 2nd layer =  6.3
                    Critical distance =  8.8
SAMPLE PROBLEM #5  DATA FROM SP B
TWO LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
        Velocity within upper layer =  1.00
  FB time, 2nd layer, far detector = 31.0
Observed 2nd layer intercept time = 15.8
COMPUTED VALUES FOR 1ST & 2ND LAYERS
    Apparent velocity of 2nd layer =  2.37
  1st & 2nd lay crossover distance = 27.3
            Depth to top of 2nd layer =  8.7
                    Critical distance =  8.1
```

Figure 10a. Model parameters and values for two-layer case with dipping second layer. Shown in the right column are interpretations using reciprocal spread data (upper), using data only from SP A (middle), and using data only from SP B (lower).

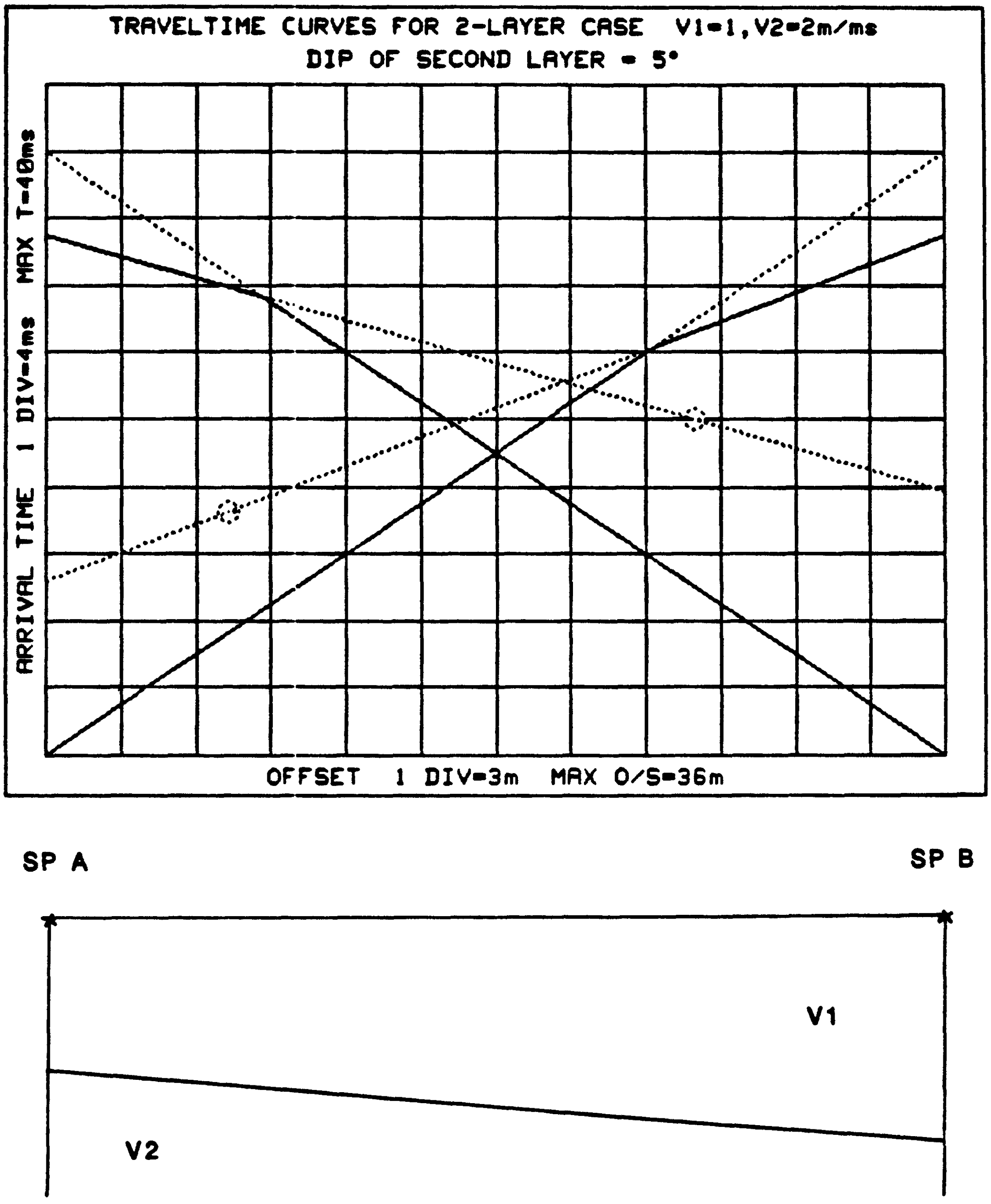


Figure 10b. Traveltime curves for two-layer case with dipping second layer using model quantities as listed in Figure 10a. True-scale sketch beneath the X/T plots is not produced by the computer program. It has been added to draw attention to the relation between the cross section and the traveltime curves.

DISCUSSION OF TWO-LAYER CASE WITH DIP

To illustrate the use of the programs of this report as a learning device, let us now examine interpreted results using the traveltime data shown on Figure 10b as if they had been obtained in only one direction. Inputs to and results from the interpretation program for the single-ended-spread data set from SP A are shown in the middle box in the right column of Figure 10a. Note that the computed depth is 6.3 m, a value within five percent of the 6-m true depth at SP A. Since the dip is taken to be zero with the single-ended spread calculation, the error at the far trace is much greater--a computed depth of 6.3 m rather than a true depth of 9.1 m, an error of 31 percent. The error in velocity is 12.5 percent, a velocity of 1.75 m/ms rather than the true velocity of 2 m/ms.

Inputs and results for the single-ended-spread data set from SP B are shown in the lower box in the right column of Figure 10a. Here, error in depth at SP B is only 4.4 percent, but error in velocity is 18.5 percent. Error at the far trace (a depth of 8.7 m rather than the true depth of 6 m) is 31 per cent.

The amount of error in depth to the high speed layer at the active SP produced by taking data in one direction when horizon dip (A2) is not zero SP can be calculated using

Error = Z2a * (1 - COS(A2)*COS(I12)/COS(Ac12)), (69)

where Z2a is the true depth at the active SP, Ac12 is the critical angle using observed velocities, Ac12=ASN(V1/V2a), and I12 is the critical angle using true velocities, I12=ASN(V1/V2.

The apparent velocity (V2a) in terms of horizon dip (A2) and velocities within the upper (V1) and lower (V2) layers is

V2a = 1 / (COS(A2)/V2 + SIN(A2)*COS(I12)/V1), (70)

where the sign of A2 is positive for down dip and negative for up dip.

The conclusion from this simple study is that although the error in depth <u>at the active SP</u> may not be great for moderate dips, the error in velocity is significant if dip of the planar interface is much above a few degrees. Therefore, using the velocity obtained from a single-ended spread as an indicator of rock properties is not advised. Arguing to the absurd, if the dip of the second layer is 30 degrees and the ratio of first to second layer velocity is 0.5, then the updip apparent velocity would be infinite.

EXAMPLE OF PROGRAM OPERATION WITH THREE-LAYER CASE WITH DIP

As a final example of use of the programs, let us consider the three-layer case with input parameters as listed (layer by layer) in the left column of Figure 11a. Results of the interpretation using layer-by-layer input data taken from the model are listed in the right column of Figure 11a. The traveltime curve derived from the model program and containing data used as input to the interpretation program is shown in Figure 11b. The true-scale sketch drawn beneath the X/T plot is included for instructional purposes only, and is not part of the program.

INITIAL PARAMETERS

Number of layers =	3
Velocity within first layer =	1.0
Velocity within second layer =	2.0
Velocity within third layer =	4.0
Distance from SP A to SP B =	36.0

PARAMETERS FOR FIRST LAYER

Vertical thickness, 1st lay, SP A =	2.0
Dip of 2nd layer, SP A to SP B =	2.0

COMPUTED VALUES FOR LAYERS 1 AND 2

Normal thickness, 1st lay, SP A =	2.0
Vertical thickness, 1st lay, SP B =	3.3
Depth to 2nd layer under SP B =	3.3
Normal thickness, 1st lay, SP B =	3.3
Reciprocal time, top of layer 2 =	22.5
Intercept time at SP A, layer 2 =	3.5
Intercept time at SP B, layer 2 =	5.6
Apparent vel from SP A of layer 2 =	1.89
Apparent vel from SP B of layer 2 =	2.13
Approx mean vel on top of layer 2 =	2.00
Crossover dist from SP A, lay 1/2 =	7.4
Crossover dist from SP B, lay 1/2 =	10.6
Critical dist from SP A, layer 2 =	2.4
Critical dist from SP B, layer 2 =	3.7

PARAMETERS FOR SECOND LAYER

Vertical thickness, 2nd lay, SP A =	4.0
Depth to 3rd layer under SP A =	6.0
Dip, top of 3rd lay, SP A to SP B =	4.0

COMPUTED VALUES FOR LAYERS 2 AND 3

Normal thickness, 2nd lay, SP A =	4.0
Depth to 3rd layer under SP B =	8.5
Vertical thickness, 2nd lay, SP B =	5.3
Normal thickness, 2nd lay, SP B =	5.2
Reciprocal time, top of layer 3 =	18.1
Intercept time at SP A for lay 3 =	7.3
Intercept time at SP B for lay 3 =	10.8
Apparent vel from SP A of layer 3 =	3.35
Apparent vel from SP B of layer 3 =	4.98
Approx mean vel on top of layer 3 =	4.00
Crossover dist, SP A, layer 1 & 3 =	10.4
Crossover dist, SP A, layer 2 & 3 =	16.7
Crossover dist, SP B, layer 1 & 3 =	13.6
Crossover dist, SP B, layer 2 & 3 =	19.4
Critical dist from SP A, layer 3 =	5.8
Critical dist from SP B, layer 3 =	7.6

SAMPLE PROBLEM #6 DIPPING LAYERS

THREE LAYERS, RECIPROCAL, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER

Avg velocity, upper layer at SP A =	1.00
Avg velocity, upper layer at SP B =	1.00
Average velocity of upper layer =	1.00
Reciprocal time for 2nd layer =	22.5
Observed intercept time at SP A =	3.5
Observed intercept time at SP B =	5.6

COMPUTED VALUES FOR 1ST & 2ND LAYERS

Apparent 2nd-layer vel from SP A =	1.89
Apparent 2nd layer vel from SP B =	2.13
Dip from SP A toward SP B =	1.9
Velocity of second layer =	2.00
Computed crossover dist from SP A =	7.4
Computed crossover dist from SP B =	10.6
Normal thickness of lay 1 at SP A =	2.0
Depth to second layer under SP A =	2.0
Normal thickness of lay 1 at SP B =	3.2
Depth to second layer under SP B =	3.2

INPUT VALUES FOR THIRD LAYER

Observed 3rd lay reciprocal time =	18.1
Observed 3rd layer intercept,SP A =	7.3
Observed 3rd layer intercept,SP B =	10.8

COMPUTED VALUES FOR 2ND & 3RD LAYERS

Apparent vel of 3rd lay, SP A =	3.33
Apparent vel of 3rd lay, SP B =	4.93
Computed dip of 3rd layer =	4.1
Computed velocity of 3rd layer =	3.97
Computed lay 2 & 3 crossover SP A =	16.7
Computed lay 2 & 3 crossover SP B =	19.5
Normal thickness of lay 2 at SP A =	3.9
Vert. thickness of lay 2 at SP A =	4.0
Depth to top of layer 3 at SP A =	6.0
Normal thickness of lay 2 at SP B =	5.3
Vert. thickness of lay 2 at SP B =	5.3
Depth to top of layer 3 at SP B =	8.5

Figure 11a. Model parameters and values for three-layer case with dipping second and third layer. Shown in the right column are interpretations using reciprocal spread data.

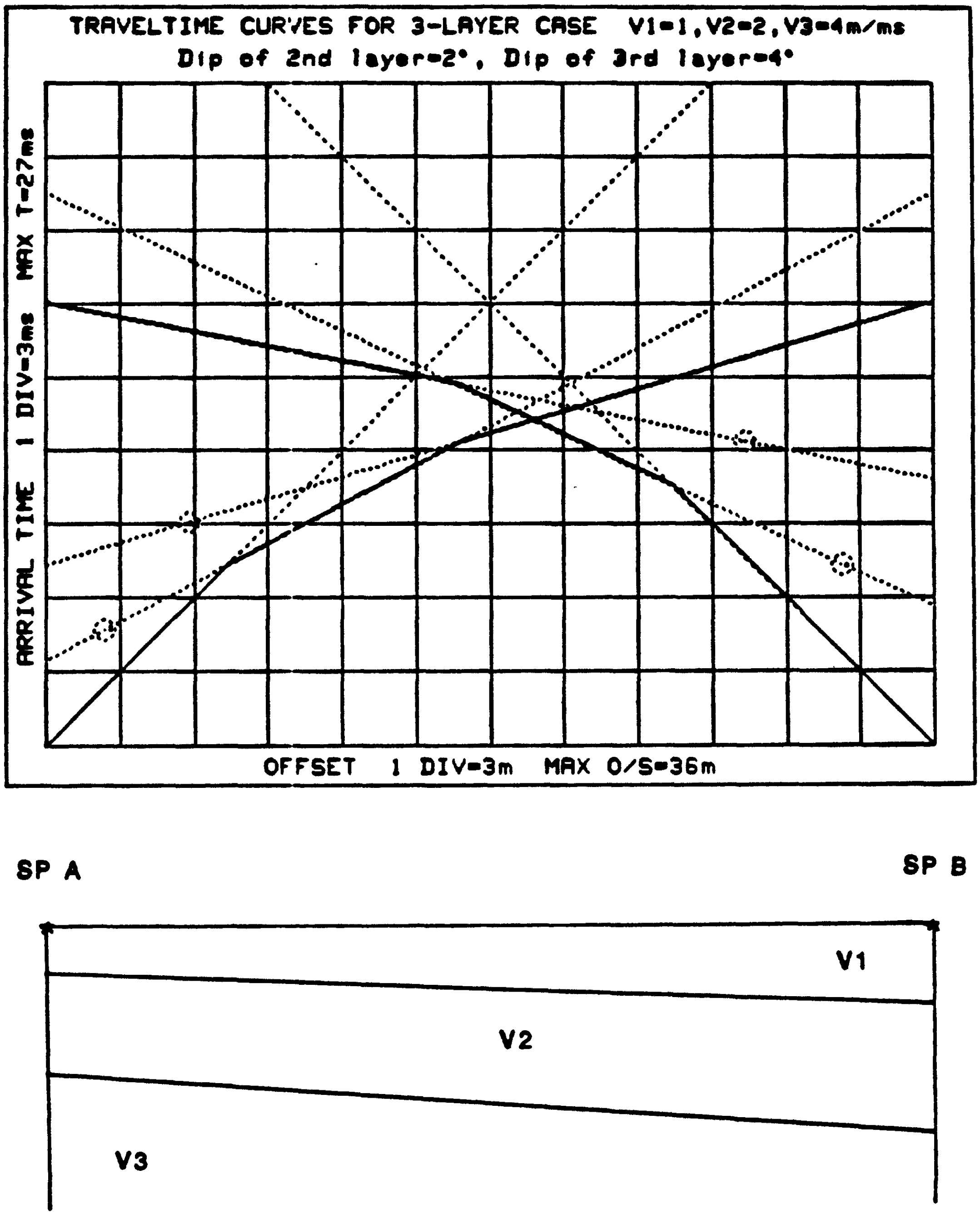


Figure 11b. Traveltime curves for three-layer case with dipping second and third layer using model quantities as listed in Figure 11a. True-scale sketch beneath the X/T plots is not produced by the computer program. It has been added to draw attention to the relation between the cross section and the traveltime curves.

```
SAMPLE PROBLEM #6  DATA FROM SP A
THREE LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
     Velocity within upper layer =  1.00
  FB time, 2nd layer, far detector = 22.5
Observed 2nd layer intercept time =  3.5

COMPUTED VALUES FOR 1ST & 2ND LAYERS
   Apparent velocity of 2nd layer =  1.89
  1st & 2nd lay crossover distance =  7.4
        Depth to top of 2nd layer =  2.1
                Critical distance =  2.6

INPUT VALUES FOR THIRD LAYER
  FB time, 3rd layer, far detector = 18.1
Observed 3rd layer intercept time =  7.3

COMPUTED VALUES FOR 2ND & 3RD LAYERS
   Apparent velocity of 3rd layer =  3.33
  2nd & 3rd lay crossover distance = 16.7
         Thickness of second layer =  3.9
       Depth to top of third layer =  5.9
    Critical distance for 3rd layer =  6.7
```

```
SAMPLE PROBLEM #6  DATA FROM SP B
THREE LAYERS, SINGLE ENDED, MAX O/S = 36

INPUT VALUES FOR 1ST AND 2ND LAYER
     Velocity within upper layer =  1.00
  FB time, 2nd layer, far detector = 22.5
Observed 2nd layer intercept time =  5.6

COMPUTED VALUES FOR 1ST & 2ND LAYERS
   Apparent velocity of 2nd layer =  2.13
  1st & 2nd lay crossover distance = 10.6
        Depth to top of 2nd layer =  3.2
                Critical distance =  3.4

INPUT VALUES FOR THIRD LAYER
  FB time, 3rd layer, far detector = 18.1
Observed 3rd layer intercept time = 10.8

COMPUTED VALUES FOR 2ND & 3RD LAYERS
   Apparent velocity of 3rd layer =  4.93
  2nd & 3rd lay crossover distance = 19.5
         Thickness of second layer =  5.4
       Depth to top of third layer =  8.6
    Critical distance for 3rd layer =  6.5
```

Figure 11c. Interpretation using the values shown on the traveltime curves of Figure 11b, but with single-ended-spread interpretation based on using only data from SP A (left column) and SP B (right column).

DISCUSSION OF THREE-LAYER CASE WITH DIP

The results of assuming that data were taken in only one direction; that is, with a single-ended spread, are shown in Figure 11c. Note that the near SP depths are close to being correct (within 0.1 m) and that the error in apparent velocity is significant even though the dips are relatively small. For example, the error in the third layer velocity from SP B is 23 percent for a refracting horizon with a dip of only 4 degrees.

Crossover distances define the detectable regions for the different layers. Thus, for the forward spread (SP A), spreads must be greater than 8.5 m and less than 16 m to use refracted returns from the top of layer 2, and they must be greater than 16 m to show a return from layer 3. For second layer detection, offsets ranging from 9 to 15 m would be required.

SUMMARY AND CONCLUSIONS

This report contains two useful shallow-depth, seismic refraction computer programs. Both programs operate on the assumption that the layers (a maximum of four) have constant velocities and planar interfaces. In the modeling program (file name: REFRPM), information about the layers is entered and the output is a set of traveltime curves plus a tabulation of the quantities observable on these plots. The principal use of this program is in initial planning of the survey. In the interpretation program (file name: REFRSC), the input data are taken from the traveltime curves and the output is a tabulation of computed results. The principal use of this program is in making first estimates of layer thicknesses and velocities.

NOTICE

Although the development of the procedures described in this report have been partially supported by the United States Environmental Protection Agency through Interagency Agreement Number DW14933103-01 to the United States Geological Survey, it has not been subjected to Agency review and therefore does not necessarily reflect the views of the Agency and no official endorsement should be inferred. Mention of trade names or commercial products does not constitute endorsement or recommendation for use.

REFERENCES

Palmer, Derecke, 1980, The generalized reciprocal method of seismic refraction interpretation: Soc. Exploration Geophysicist, Tulsa, Okla.

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Sheriff, R. E., 1984, Encyclopedic dictionary of exploration geophysics (2nd ed), Soc. Exploration Geophysicist, Tulsa, Okla.

Sjögren, Bengt, 1984, Shallow refraction seismics, Chapman and Hall, New York.

APPENDIX

Further examples of the traveltime curves producible by the REFRPM program are shown in Figures A1, A2, and A3. These results also form a catalog for study of 15 different T/X plots produced by cases in which the total vertical thickness of a multi-layered overburden is held to 6 m as measured at the left side of each traverse. The layer velocities and dips remain fixed, and only the thicknesses of the layers within the overburden are varied. The layer velocities and dips are as follows:

Layer 1: velocity = 0.5 m/ms,
Layer 2: velocity = 1 m/ms and dip = 5 deg,
Layer 3: velocity = 2 m/ms and dip = 10 deg,
Layer 4: velocity = 4 m/ms and dip = 15 deg.

Bold numbers near the lower middle of each plot show the thickness in meters of progressive layers. For example, for the traveltime curve at the upper left on Figure A3, the thicknesses of the first, second, and third layers are 1, 2, and 3 m, respectively. On each of the 15 plots, the maximum arrival time is 50 ms with a grid interval of 5 ms and the maximum offset is 36 m with a grid spacing of 3 m. All X/T plots are presented as if data had been acquired using reciprocal spreads.

Sufficient information is displayed on the traveltime curves to allow both reconstruction of the T/X plots with the REFRPM program and interpretation with the REFRSC program.

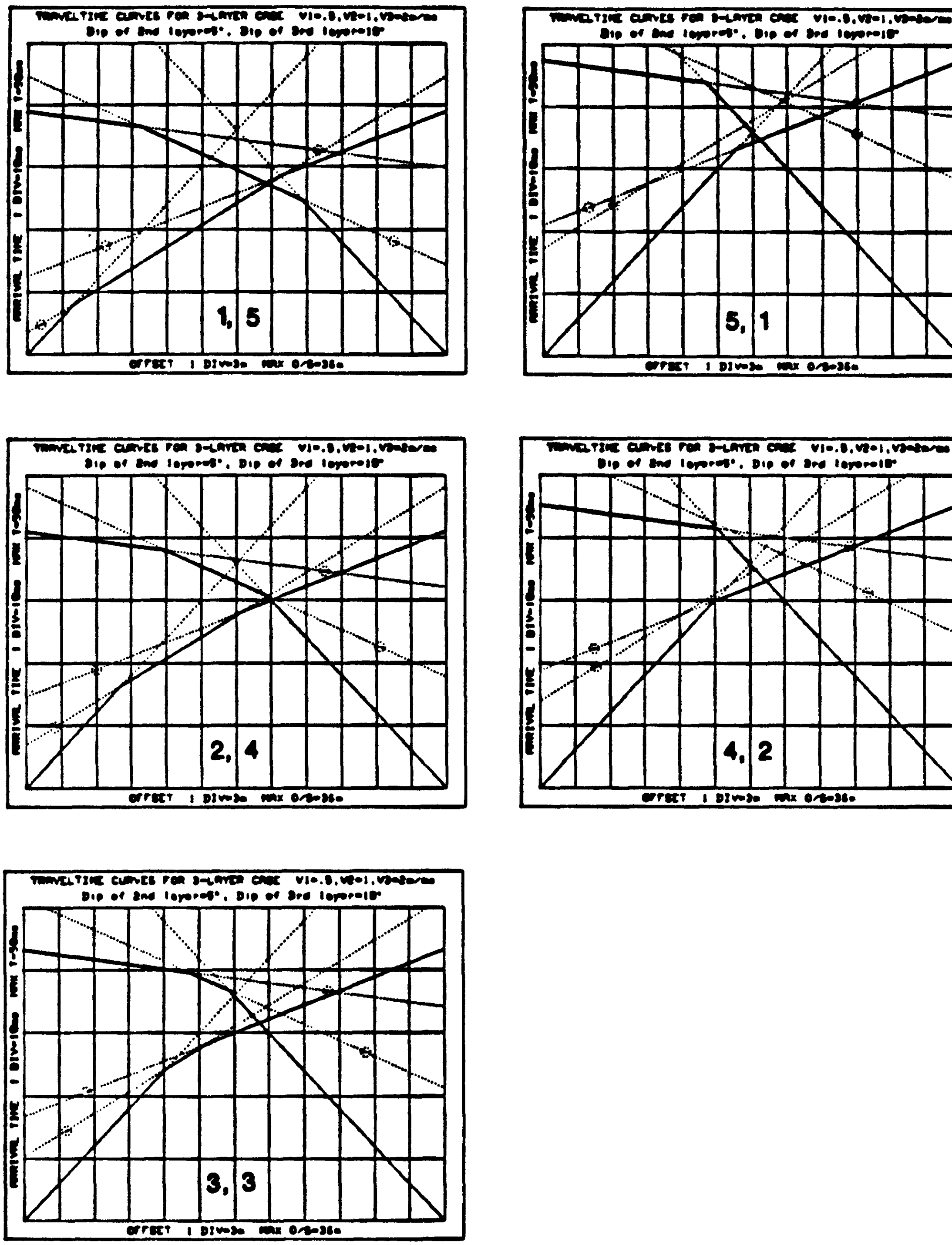

Figure A1. Traveltime curves for three-layer cases with velocities and dips as shown on the top label of each plot. Bold numbers near the lower middle of each plot represent the depth (m) to the top of the second and third layers, respectively, at the left side of the plot. Sum of depths to bedrock equals 6 m in each case.

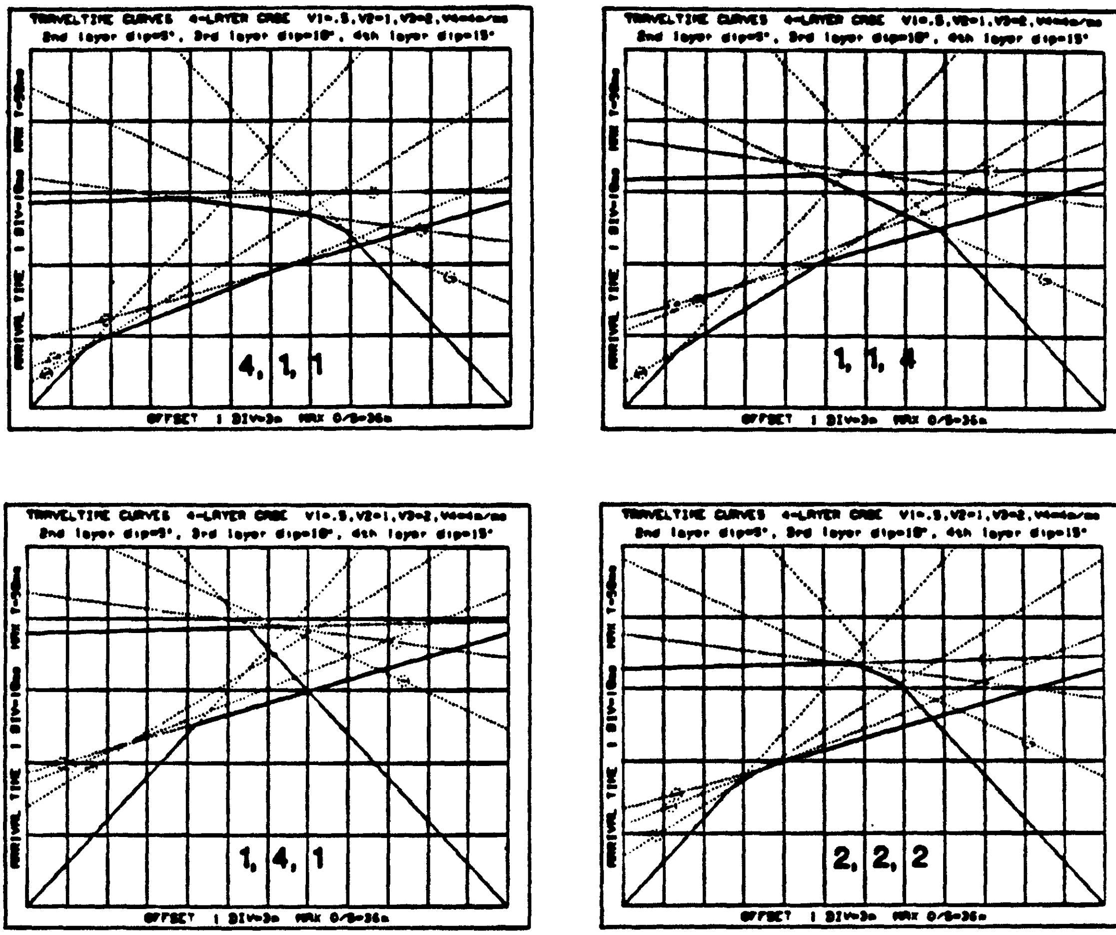

Figure A2. Traveltime curves for four-layer cases with velocities and dips as shown on the top label of each plot. Numbers near the lower middle of each plot represent the depth to the top of the second, third, and fourth layers, respectively, at the left side of the plot. Sum of depths equals 6 m in all cases.

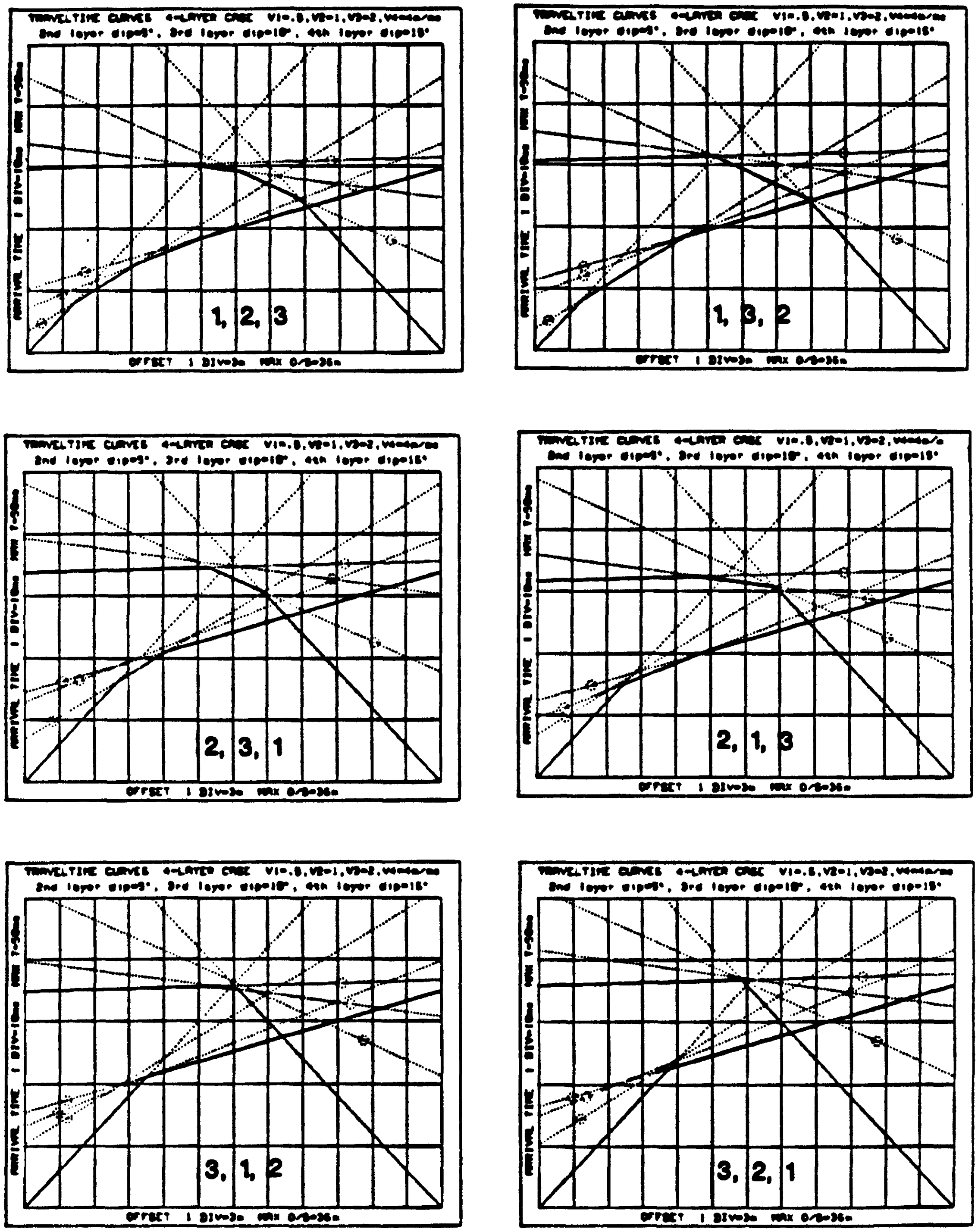

Figure A3. Traveltime curves for four-layer cases with velocities and dips as shown on the top label of each plot. Numbers near the lower middle of each plot represent the depth to the top of the second, third, and fourth layers, respectively, at the left side of the plot. Sum of depths to bedrock equals 6 m in all cases.

```
10    PRINT "SIMPLE PLANE LAYER FORWARD REFRACTION MODELS 'REFRPM' 22 JUNE 90"
20    OPTION BASE 1
30    PRINTER IS 16
40    DEG                               ! Computations use angles in degrees
50    !
60    PRINT "The following conditions apply in all computations:"
70    PRINT "   1. Maximum of 4 layers with plane boundaries"
80    PRINT "   2. All stations and SP's at constant surface elevation"
90    PRINT "   3. All layers have discrete velocities"
100   PRINT "   4. All times in milliseconds (ms)"
110   !
120   DIM Title$[80]
130   !
140   GOSUB Begin                       ! Initial questions
150   GOSUB Init_mod_param              ! Initial model parameters
160   GOSUB Paths                       ! Establish computational paths
170   GOSUB Plot                        ! Plot results
180   BEEP
190   OUTPUT 9;"R"
200   ENTER 9;T$
210   PRINT LIN(1);"COMPUTED: ";T$;", ";Y$;LIN(4)
220   PRINTER IS 16
230   DISP "PROGRAM COMPLETED"
240   END
250   !
260 Begin:                              ! Initial questions
270   PRINTER IS 16                     ! Printout on screen
280   Y$="1990"
290   INPUT "Year computations made (default is 1990):",Y$
300   Qu=1                              ! Qu=1; flag for metric units
310   G$="Y"
320   INPUT "Are units in metric? (Y/N--default is Y)",G$
330   IF G$="Y" THEN 360
340   PRINT "English units (ft and ft/ms)"
350   GOTO 440
360   PRINT "Metric units (m and m/ms)"
370   RETURN
380   !
390 Printout:                           ! Select printout mode
400   PRINTER IS 16                     ! Printout on screen
410   G$="N"
420   INPUT "Do you want thermal printout? (Y/N--default is N)",G$
430   IF G$="N" THEN 450
440   PRINTER IS 0                      ! Thermal printout on internal printer
450   RETURN
460   !
470 Init_mod_param:                     ! Initial model parameters
480   PRINTER IS 16                     ! Screen display
490   PRINT "ENTER INITIAL MODEL PARAMETERS"
```

```
500   GOSUB Printout                    ! Select printout mode
510   PRINT LIN(1);"INITIAL PARAMETERS"
520   Nl=2
530   INPUT "Number of layers (maximum=4; default is 2):",Nl
540   PRINT "                   Number of layers = ";Nl
550   INPUT "Velocity within first layer:",V1
560   V1$=VAL$(V1)
570   IMAGE "     Velocity within first layer = ",2D.D
580   PRINT USING 570;V1
590   INPUT "Velocity within second layer:",V2
600   V2$=VAL$(V2)
610   IMAGE "     Velocity within second layer = ",2D.D
620   PRINT USING 610;V2
630   I12=ASN(V1/V2)                    ! Critical angle, layers 1 and 2
640   C12=COS(I12)                      ! Cosine of critical angle
650   IF Nl=2 THEN 790
660   INPUT "Velocity of third layer:",V3
670   V3$=VAL$(V3)
680   IMAGE "     Velocity within third layer = ",2D.D
690   PRINT USING 680;V3
700   I23=ASN(V2/V3)                    ! Crit angle, lay 2 relative to lay 3
710   C23=COS(I23)                      ! Cosine of critical angle
720   IF Nl=3 THEN 790
730   INPUT "Velocity of fourth layer:",V4
740   V4$=VAL$(V4)
750   IMAGE "     Velocity within fourth layer = ",2D.D
760   PRINT USING 750;V4
770   I34=ASN(V3/V4)                    ! Crit angle, lay 3 relative to lay 4
780   C34=COS(I34)                      ! Cosine of critical angle
790   INPUT "Distance from SP A to SP B:",Xc
800   IMAGE "      Distance from SP A to SP B =",3D.D
810   PRINT USING 800;Xc
820   RETURN
830   !
840 Paths:                              ! Establish computation paths
850   ON Nl-1 GOTO 860,880,910          ! Select as function of number of layers
860   GOSUB Two_layers
870   GOTO 940
880   GOSUB Two_layers
890   GOSUB Three_layers
900   GOTO 940
910   GOSUB Two_layers
920   GOSUB Three_layers
930   GOSUB Four_layers
940   RETURN
950   !
960 Two_layers:                         ! Computations for two layers
970   PRINTER IS 16
980   PRINT "ENTER FIRST LAYER PARAMETERS  NOTE: DOWN DIP IS POSITIVE"
990   GOSUB Printout                    ! Select printout mode
```

```
1000  PRINT LIN(1);"PARAMETERS FOR FIRST LAYER"
1010  INPUT "Vertical thickness of first layer under SP A:",L1a
1020  IMAGE "Vertical thickness, 1st lay, SP A = ",2D.D
1030  PRINT USING 1020;L1a
1040  Z2a=L1a
1050  IMAGE "    Depth to 2nd layer under SP A =",3D.D
1060  PRINT USING 1050;Z2a
1070  A2=0                                                        ! DEFAULT
1080  INPUT "Dip of top of 2nd layer from SP A to SP B (default = 0):",A2
1090  IMAGE "  Dip of 2nd layer,  SP A to SP B = ",2D.D
1100  PRINT USING 1090;A2
1110  Title$="DIP OF SECOND LAYER = "&VAL$(A2)&CHR$(179)
1120  PRINT LIN(1);"COMPUTED VALUES FOR LAYERS 1 AND 2"
1130  N1a=L1a*COS(A2)                   ! Normal thickness, layer 1 at SP A
1140  IMAGE "  Normal thickness, 1st lay, SP A = ",2D.D
1150  PRINT USING 1140;N1a
1160  D1a=N1a*C12/V1                    ! Delay time for layer 1 at SP A
1170  L1b=L1a+Xc*TAN(A2)
1180  IMAGE "Vertical thickness, 1st lay, SP B = ",2D.D
1190  PRINT USING 1180;L1b
1200  Z2b=L1b                           ! Depth to 2nd layer at SP B
1210  IMAGE "    Depth to 2nd layer under SP B =",3D.D
1220  PRINT USING 1210;Z2b
1230  N1b=L1b*COS(A2)
1240  IMAGE "  Normal thickness, 1st lay, SP B = ",2D.D
1250  PRINT USING 1240;N1b
1260  D1b=N1b*C12/V1                    ! Delay time for layer 1 at SP B
1270  T2rt=Xc*COS(A2)/V2+D1a+D1b
1280  IMAGE "  Reciprocal time, top of layer 2 =",3D.D
1290  PRINT USING 1280;T2rt
1300  T2ai=2*D1a                        ! Intercept time at SP A
1310  IMAGE "  Intercept time at SP A, layer 2 =",3D.D
1320  PRINT USING 1310;T2ai
1330  T2bi=2*D1b                        ! Intercept time at SP B
1340  IMAGE "  Intercept time at SP B, layer 2 =",3D.D
1350  PRINT USING 1340;T2bi
1360  IF A2<>0 THEN 1400                 ! Branch; dip on layer 2
1370  V2a=V2
1380  V2b=V2
1390  GOTO 1570
1400  Tdiff=T2rt-T2ai
1410  IF Tdiff<>0 THEN 1440
1420  V2a=99.99                         ! Avoid division by zero, infinite Va2
1430  GOTO 1450
1440  V2a=Xc/Tdiff
1450  IMAGE "Apparent vel from SP A of layer 2 =",3D.2D
1460  PRINT USING 1450;V2a
1470  Tdiff=T2rt-T2bi
1480  IF Tdiff<>0 THEN 1510
1490  V2b=99.99                         ! Avoid division by zero, infinite Vb2
```

```
1500  GOTO 1520
1510  V2b=Xc/Tdiff
1520  IMAGE "Apparent vel from SP B of layer 2 =",3D.2D
1530  PRINT USING 1520;V2b
1540  V2m=2*COS(A2)*V2a*V2b/(V2a+V2b)
1550  IMAGE "Approx mean vel on top of layer 2 =",3D.2D
1560  PRINT USING 1550;V2m
1570  X12aco=V1*V2a*T2ai/(V2a-V1)
1580  IMAGE "Crossover dist from SP A, lay 1/2 =",3D.D
1590  PRINT USING 1580;X12aco
1600  T12aco=X12aco/V1
1610  IMAGE "Time at crossover,  SP A, lay 1/2 =",3D.D
1620  PRINT USING 1610;T12aco
1630  X12bco=V1*V2b*T2bi/(V2b-V1)
1640  IMAGE "Crossover dist from SP B, lay 1/2 =",3D.D
1650  PRINT USING 1640;X12bco
1660  T12bco=X12bco/V1
1670  IMAGE "Time at crossover,  SP B, lay 1/2 =",3D.D
1680  PRINT USING 1670;T12bco
1690  X2acd=2*L1a*SIN(I12)*COS(A2)/COS(A2+I12)       ! Critical dist from SP A
1700  IMAGE " Critical dist from SP A, layer 2 =",3D.D
1710  PRINT USING 1700;X2acd
1720  T2acd=T2ai+X2acd/V2a
1730  IMAGE " Time at crit dist, SP A, layer 2 =",3D.D
1740  PRINT USING 1730;T2acd
1750  X2bcd=2*L1b*SIN(I12)*COS(A2)/COS(I12-A2)       ! Critical dist from SP B
1760  IMAGE " Critical dist from SP B, layer 2 =",3D.D
1770  PRINT USING 1760;X2bcd
1780  T2bcd=T2bi+X2bcd/V2b
1790  IMAGE " Time at crit dist, SP B, layer 2 =",3D.D
1800  PRINT USING 1790;T2bcd
1810  RETURN
1820  !
1830 Three_layers:                   ! Computations for three layer cases
1840  PRINTER IS 16                  ! On screen display
1850  PRINT "ENTER SECOND LAYER PARAMETERS  NOTE: DOWN DIP IS POSITIVE"
1860  GOSUB Printout                 ! Select printout mode
1870  PRINT LIN(1);"PARAMETERS FOR SECOND LAYER"
1880  INPUT "Vertical thickness of 2nd layer under SP A:",L2a
1890  IMAGE "Vertical thickness, 2nd lay, SP A =",3D.D
1900  PRINT USING 1890;L2a
1910  Z3a=L1a+L2a
1920  IMAGE "    Depth to 3rd layer under SP A =",3D.D
1930  PRINT USING 1920;Z3a
1940  A3=0                                                     ! DEFAULT
1950  INPUT "Dip of top of 3rd layer from SP A to SP B (default = 0):",A3
1960  IMAGE "Dip, top of 3rd lay, SP A to SP B = ",2D.D
1970  PRINT USING 1960;A3
1980  Title$="Dip of 2nd layer="&VAL$(A2)&CHR$(179)&", Dip of 3rd layer="&VAL$(A3)&CHR$(179)
1990  PRINT LIN(1);"COMPUTED VALUES FOR LAYERS 2 AND 3"
```

```
2000 N2a=L2a*COS(A3)-L1a*SIN(A2)*SIN(A3-A2)
2010 IMAGE "  Normal thickness, 2nd lay, SP A = ",2D.D
2020 PRINT USING 2010;N2a
2030 Z3b=Z3a+Xc*TAN(A3)
2040 IMAGE "    Depth to 3rd layer under SP B =",3D.D
2050 PRINT USING 2040;Z3b
2060 L2b=Z3b-Z2b
2070 IMAGE "Vertical thickness, 2nd lay, SP B =",3D.D
2080 PRINT USING 2070;L2b
2090 N2b=L2b*COS(A3)-L1b*SIN(A2)*SIN(A3-A2)
2100 IMAGE "  Normal thickness, 2nd lay, SP B = ",2D.D
2110 PRINT USING 2100;N2b
2120 IF Z3b>=Z2b THEN 2160           ! Test for layer intersection
2130 BEEP
2140 PRINT "ERROR! TOP OF THIRD LAYER INTERSECTS TOP OF SECOND LAYER"
2150 GOTO 1880
2160 B13=ASN(V1*SIN(I23-A3+A2)/V2)   ! Dip crit angle (V1&V3) at SP A
2170 A13=ASN(V1*SIN(I23+A3-A2)/V2)   ! Dip crit angle (V1&V3) at SP B
2180 D2a=N1a*COS(B13)/V1+N2a*C23/V2  ! Delay time at SP A
2190 D2b=N1b*COS(A13)/V1+N2b*C23/V2  ! Delay time at SP B
2200 Fcos=COS(A2)*COS(A3-A2)         ! Function of cosines
2210 T3rt=Xc*Fcos/V3+D2a+D2b         ! Reciprocal time from third layer
2220 IMAGE "  Reciprocal time, top of layer 3 =",3D.D
2230 PRINT USING 2220;T3rt
2240 T3ai=N1a*(COS(A13)+COS(B13))/V1+2*N2a*C23/V2   ! Intercept time at SP A
2250 IMAGE " Intercept time at SP A for lay 3 =",3D.D
2260 PRINT USING 2250;T3ai
2270 T3bi=N1b*(COS(A13)+COS(B13))/V1+2*N2b*C23/V2   ! Intercept time at SP B
2280 IMAGE " Intercept time at SP B for lay 3 =",3D.D
2290 PRINT USING 2280;T3bi
2300 Tdiff=T3rt-T3ai
2310 IF Tdiff<>0 THEN 2340           ! Test on division by zero
2320 V3a=99.99
2330 GOTO 2380
2340 V3a=Xc/Tdiff                    ! Compute apparent vel, lay 3, SP A
2350 IF V3a>99 THEN V3a=99.99
2360 IF V3a<-99 THEN V3a=-99.99
2370 IMAGE "Apparent vel from SP A of layer 3 =",3D.2D
2380 PRINT USING 2370;V3a
2390 Tdiff=T3rt-T3bi
2400 IF Tdiff<>0 THEN 2440
2410 GOTO 3020
2420 V3b=99.99
2430 GOTO 2480
2440 V3b=Xc/Tdiff                    ! Compute apparent vel, lay 3, SP B
2450 IF V3a>99 THEN V3a=99.99
2460 IF V3a<-99 THEN V3a=-99.99
2470 IMAGE "Apparent vel from SP B of layer 3 =",3D.2D
2480 PRINT USING 2470;V3b
2490 V3m=2*COS(A3)*V3a*V3b/(V3a+V3b)
```

```
2500 IMAGE "Approx mean vel on top of layer 3 =",3D.2D
2510 PRINT USING 2500;V3m
2520 Ha23=0                          ! Compute crossover distances from SP A
2530 X13aco=V1*V3a*T3ai/(V3a-V1)
2540 IMAGE "Crossover dist, SP A, layer 1 & 3 =",3D.D
2550 PRINT USING 2540;X13aco
2560 T13aco=X13aco/V3a+T3ai
2570 IMAGE "Crossover time, SP A, layer 1 & 3 =",3D.D
2580 PRINT USING 2570;T13aco
2590 X23aco=V2a*V3a*(T3ai-T2ai)/(V3a-V2a)
2600 IMAGE "Crossover dist, SP A, layer 2 & 3 =",3D.D
2610 PRINT USING 2600;X23aco
2620 T23aco=X23aco/V3a+T3ai
2630 IMAGE "Crossover time, SP A, layer 2 & 3 =",3D.D
2640 PRINT USING 2630;T23aco
2650 IF (X23aco>X13aco) AND (X13aco>X12aco) THEN 2680
2660 Ha23=1                          ! Flag for hidden layer from SP A
2670 PRINT "Layer 2 is hidden when shot from SP A"
2680 Hb23=0                          ! Compute crossover distances from SP B
2690 X13bco=V1*V3b*T3bi/(V3b-V1)
2700 IMAGE "Crossover dist, SP B, layer 1 & 3 =",3D.D
2710 PRINT USING 2700;X13bco
2720 T13bco=X13bco/V3b+T3bi
2730 IMAGE "Crossover time, SP B, layer 1 & 3 =",3D.D
2740 PRINT USING 2730;T13bco
2750 X23bco=V2b*V3b*(T3bi-T2bi)/(V3b-V2b)
2760 IMAGE "Crossover dist, SP B, layer 2 & 3 =",3D.D
2770 PRINT USING 2760;X23bco
2780 T23bco=X23bco/V3b+T3bi
2790 IMAGE "Crossover time, SP B, layer 2 & 3 =",3D.D
2800 PRINT USING 2790;T23bco
2810 IF (X23bco>X13bco) AND (X13bco>X12bco) THEN 2840
2820 Hb23=1                          ! Flag for hidden layer from SP B
2830 PRINT "Layer 2 is hidden when shot from SP B"
2840 S1b=N1a/COS(B13)                ! Compute critical distance from SP A
2850 X1=S1b*SIN(B13-A2)              ! X coordinate, refr pt on layer 2
2860 Y1=S1b*COS(B13-A2)              ! Y coordinate, refr pt on layer 2
2870 S2b=(N2a+S1b*SIN(B13)*SIN(A3-A2))/C23   ! Slant dist, lay 2, from SP A
2880 D=S2b*SIN(2*I23)/SIN(90+A2-A3-I23)
2890 X3=D*COS(A2)+X1                 ! X coordinate along layer 2
2900 Y3=Y1+D*SIN(A2)                 ! Y coordinate along layer 2
2910 X3acd=X3+Y3*TAN(A13+A2)         ! Critical distance from SP A
2920 IMAGE " Critical dist from SP A, layer 3 =",3D.D
2930 PRINT USING 2920;X3acd
2940 T3acd=X3acd/V3a+T3ai            ! Time at critical dist from  SP A
2950 IMAGE " Time at crit dist, SP A, layer 3 =",3D.D
2960 PRINT USING 2950;T3acd
2970 S1a=N1b/COS(A13)                ! Compute critical distance from SP B
2980 X5=S1a*SIN(A13+A2)              ! X coordinate, refr pt on layer 2
2990 Y5=S1a*COS(A13+A2)              ! Y coorindate, refr pt on layer 2
```

```
3000  S2a=(N2b+S1a*SIN(A13)*SIN(A2-A3))/C23   ! Slant dist, lay 2, from SP B
3010  D=S2a*SIN(2*I23)/SIN(90-A2+A3-I23)
3020  X6=D*COS(-A2)+X5                    ! X coordinate along layer 2
3030  Y6=Y5+D*SIN(-A2)                    ! Y coordinate along layer 2
3040  X3bcd=X6+Y6*TAN(A13-A2)             ! Critical distance from SP B
3050  IMAGE " Critical dist from SP B, layer 3 =",3D.D
3060  PRINT USING 3050;X3bcd
3070  T3bcd=X3bcd/V3b+T3bi                ! Time at crit distance ffrom SP B
3080  IMAGE " Time at crit dist, SP B, layer 3 =",3D.D
3090  PRINT USING 3080;T3bcd
3100  RETURN
3110  !
3120 Four_layers:                         ! Computation for four-layer case
3130  PRINTER IS 16                       ! Screen display
3140  PRINT "ENTER VALUES FOR THIRD LAYER"
3150  GOSUB Printout                      ! Select printout mode
3160  PRINT LIN(1);"PARAMETERS FOR THIRD LAYER"
3170  INPUT "Vertical thickness of 3rd layer under SP A:",L3a
3180  IMAGE "Vertical thickness, 3rd lay, SP A =",3D.D
3190  PRINT USING 3180;L3a
3200  Z4a=Z3a+L3a
3210  IMAGE "    Depth to 4th layer under SP A =",3D.D
3220  PRINT USING 3210;Z4a
3230  A4=0                                                          ! DEFAULT
3240  INPUT "Dip of top of 4th layer from SP A to SP B (default = 0):",A4
3250  IMAGE "Dip, top of 4th lay, SP A to SP B = ",2D.D
3260  PRINT USING 3250;A4
3270  Title$="2nd layer dip="&VAL$(A2)&CHR$(179)&", 3rd layer dip="&VAL$(A3)&CHR$(179)&", 4th layer dip="&VAL$(A4)&CHR$(
179)
3280  PRINT LIN(1);"COMPUTED VALUES FOR LAYERS 3 AND 4"
3290  N3a=L3a*COS(A4)-(L1a*SIN(A2)*COS(A3-A2)+L2a*SIN(A3))*SIN(A4-A3)
3300  IMAGE "  Normal thickness, 3rd lay, SP A = ",2D.D
3310  PRINT USING 3300;N3a
3320  Z4b=Z4a+Xc*TAN(A4)
3330  IMAGE "    Depth to 4th layer under SP B =",3D.D
3340  PRINT USING 3330;Z4b
3350  L3b=Z4b-Z3b
3360  IMAGE "Vertical thickness, 3rd lay, SP B =",3D.D
3370  PRINT USING 3360;L3b
3380  N3b=L3b*COS(A4)-(L1b*SIN(A2)*COS(A3-A2)+L2b*SIN(A3))*SIN(A4-A3)
3390  IMAGE "  Normal thickness, 3rd lay, SP B = ",2D.D
3400  PRINT USING 3390;N3b
3410  IF Z4b>=Z3b THEN 3450               ! Test on layer intersection
3420  BEEP
3430  PRINT "ERROR! TOP OF FOURTH LAYER INTERSECTS TOP OF THIRD LAYER"
3440  GOTO 3170                           ! Re-select third layer parameters
3450  B24=ASN(V2*SIN(I34-(A4-A3))/V3)    ! Dip crit angle at SP A (V2 & V4)
3460  A24=ASN(V2*SIN(I34+(A4-A3))/V3)    ! Dip crit angle at SP B (V2 & V4)
3470  B14=ASN(V1*SIN(B24-(A3-A2))/V2)    ! Dip crit angle at SP A (V1 & V4)
3480  A14=ASN(V1*SIN(A24+(A3-A2))/V2)    ! Dip crit angle at SP B (V1 & V4)
```

```
3490  D3a=N1a*COS(B14)/V1+N2a*COS(B24)/V2+N3a*C34/V3   ! Delay time at SP A
3500  D3b=N1b*COS(A14)/V1+N2b*COS(A24)/V2+N3b*C34/V3   ! Delay time at SP B
3510  Fcos=Fcos*COS(A4-A3)              ! Funtion of cosines of dip angles
3520  T4rt=Xc*Fcos/V4+D3a+D3b           ! Reciprocal time from 4th layer
3530  IMAGE "  Reciprocal time, top of layer 4 =",3D.D
3540  PRINT USING 3530;T4rt
3550  T4ai=N1a*(COS(A14)+COS(B14))/V1+N2a*(COS(A24)+COS(B24))/V2+2*N3a*C34/V3
3560  IMAGE " Intercept time at SP A for lay 4 =",3D.D
3570  PRINT USING 3560;T4ai
3580  T4bi=N1b*(COS(A14)+COS(B14))/V1+N2b*(COS(A24)+COS(B24))/V2+2*N3b*C34/V3
3590  IMAGE " Intercept time at SP B for lay 4 =",3D.D
3600  PRINT USING 3590;T4bi
3610  Tdiff=T4rt-T4ai
3620  IF Tdiff<>0 THEN 3650
3630  V4a=99.99
3640  GOTO 3690
3650  V4a=Xc/Tdiff
3660  IF V4a>99 THEN V4a=99.99
3670  IF V4a<-99 THEN V4a=-99.99
3680  IMAGE "Apparent vel from SP A of layer 4 =",3D.2D
3690  PRINT USING 3680;V4a
3700  Tdiff=T4rt-T4bi
3710  IF Tdiff<>0 THEN 3740
3720  V4b=99.9
3730  GOTO 3780
3740  V4b=Xc/Tdiff
3750  IF V4b>99 THEN V4a=99.99
3760  IF V4b<-99 THEN V4a=-99.99
3770  IMAGE "Apparent vel from SP B of layer 4 =",3D.2D
3780  PRINT USING 3770;V4b
3790  V4m=2*COS(A4)*V4a*V4b/(V4a+V4b)  ! Arithmetic mean for vel of layer 4
3800  IMAGE "Approx mean vel on top of layer 4 =",3D.2D
3810  PRINT USING 3800;V4m
3820  Ha34=Ha24=0                       ! Compute crossover distances from SP A
3830  X14aco=V1*V4a*T4ai/(V4a-V1)
3840  IMAGE "Crossover dist, SP A, layer 1 & 4 =",3D.D
3850  PRINT USING 3840;X14aco
3860  T14aco=X14aco/V4a+T4ai
3870  IMAGE "Crossover time, SP A, layer 1 & 4 =",3D.D
3880  PRINT USING 3870;T14aco
3890  X24aco=V2a*V4a*(T4ai-T2ai)/(V4a-V2a)
3900  IMAGE "Crossover dist, SP A, layer 2 & 4 =",3D.D
3910  PRINT USING 3900;X24aco
3920  T24aco=X24aco/V4a+T4ai
3930  IMAGE "Crossover time, SP A, layer 2 & 4 =",3D.D
3940  PRINT USING 3930;T24aco
3950  IF X24aco>X14aco THEN 3980
3960  Ha24=1
3970  PRINT "Layer 2 is hidden when shot from SP A"
3980  X34aco=V3a*V4a*(T4ai-T3ai)/(V4a-V3a)
```

```
3990 IMAGE "Crossover dist, SP A, layer 3 & 4 =",3D.D
4000 PRINT USING 3990;X34aco
4010 T34aco=X34aco/V4a+T4ai
4020 IMAGE "Crossover time, SP A, layer 3 & 4 =",3D.D
4030 PRINT USING 4020;T34aco
4040 IF (X34aco>X24aco) AND (X14aco>X13aco) THEN 4070
4050 Ha34=1
4060 PRINT "Layer 3 is hidden when shot from SP A"
4070 Hb34=Hb24=0                    ! Compute crossover distances from SP B
4080 X14bco=V1*V4b*T4bi/(V4b-V1)
4090 IMAGE "Crossover dist, SP B, layer 1 & 4 =",3D.D
4100 PRINT USING 4090;X14bco
4110 T14bco=X14bco/V4b+T4bi
4120 IMAGE "Crossover time, SP B, layer 1 & 4 =",3D.D
4130 PRINT USING 4120;T14bco
4140 X24bco=V2b*V4b*(T4bi-T2bi)/(V4b-V2b)
4150 IMAGE "Crossover dist, SP B, layer 2 & 4 =",3D.D
4160 PRINT USING 4150;X24bco
4170 T24bco=X24bco/V4b+T4bi
4180 IMAGE "Crossover time, SP B, layer 2 & 4 =",3D.D
4190 PRINT USING 4180;T24bco
4200 IF X24bco>X14bco THEN 4230
4210 Hb24=1
4220 PRINT "Layer 2 is hidden when shot from SP B"
4230 X34bco=V3b*V4b*(T4bi-T3bi)/(V4b-V3b)
4240 IMAGE "Crossover dist, SP B, layer 3 & 4 =",3D.D
4250 PRINT USING 4240;X34bco
4260 T34bco=X34bco/V4b+T4bi
4270 IMAGE "Crossover time, SP B, layer 3 & 4 =",3D.D
4280 PRINT USING 4270;T34bco
4290 IF (X34bco>X24bco) AND (X14bco>X13bco) THEN 4320
4300 Hb34=1
4310 PRINT "Layer 3 is hidden when shot from SP B"
4320 S1b=N1a/COS(B14)               ! Compute critical distance from SP A
4330 X1=S1b*SIN(B14-A2)
4340 Y1=S1b*COS(B14-A2)
4350 S2b=(N2a+S1b*SIN(B14)*SIN(A3-A2))/COS(B24)
4360 X2=X1+S2b*SIN(B24-A3)
4370 Y2=Y1+S2b*COS(B24-A3)
4380 S3b=(N3a+(S1b*SIN(B14)*COS(A3-A2)+S2b*SIN(B24))*SIN(A4-A3))/C34
4390 Ad1=90-(A4-A3)-I34
4400 D1=S3b*SIN(2*I34)/SIN(Ad1)
4410 X3=X2+D1*COS(A3)
4420 Y3=Y2+D1*SIN(A3)
4430 D2=Y3-Y1-(X3-X1)*TAN(A2)
4440 Ad2=90-(A3-A2)-A24
4450 S2a=D2*COS(A2)/SIN(Ad2)
4460 X4=X3+S2a*SIN(A24+A3)
4470 Y4=Y3-S2a*COS(A24+A3)
4480 S1a=Y4/COS(A14+A2)
```

```
4490  X4acd=X4+S1a*SIN(A14+A2)
4500  IMAGE " Critical dist from SP A, layer 4 =",3D.D
4510  PRINT USING 4500;X4acd
4520  T4acd=X4acd/V4a+T4ai
4530  IMAGE " Time at crit dist, SP A, layer 4 =",3D.2D
4540  PRINT USING 4530;T4acd
4550  S1a=N1b/COS(A14)               ! Compute critical distance from SP B
4560  X5=S1a*SIN(A14+A2)
4570  Y5=S1a*COS(A14+A2)
4580  S2a=(N2b-S1a*SIN(A14)*SIN(A3-A2))/COS(A24)
4590  X6=X5+S2a*SIN(A24+A3)
4600  Y6=Y5+S2a*COS(A24+A3)
4610  S3a=(N3b+(S1a*SIN(A14)*COS(A2-A3)+S2a*SIN(A24))*SIN(A3-A4))/C34
4620  Ad1=90+(A4-A3)-I34
4630  D1=S3a*SIN(2*I34)/SIN(Ad1)
4640  X7=X6+D1*COS(A3)
4650  Y7=Y6-D1*SIN(A3)
4660  D2=Y7-Y5+(X7-X5)*TAN(A2)
4670  Ad2=90+(A3-A2)-B24
4680  S2b=D2*SIN(90+A2)/SIN(Ad2)
4690  X8=X7+S2b*SIN(B24-A3)
4700  Y8=Y7-S2b*COS(B24-A3)
4710  S1b=Y8/COS(B14-A2)
4720  X4bcd=X8+S1b*SIN(B14-A2)
4730  IMAGE " Critical dist from SP B, layer 4 =",3D.D
4740  PRINT USING 4730;X4bcd
4750  T4bcd=X4bcd/V4b+T4bi
4760  IMAGE " Time at crit dist, SP B, layer 4 =",3D.2D
4770  PRINT USING 4760;T4bcd
4780  RETURN
4790  !
4800 Plot:                           ! One-page, quick plot of results
4810  PRINT USING "#,K";CHR$(27)&"&lOT"  ! Skip perforations
4820  PRINTER IS 16                  ! Printout on screen
4830  INPUT "Maximum arrival time on plot",Max_at
4840  INPUT "Grid spacing for arrival times:",Gy
4850  Gx=Xc/12                                                    ! DEFAULT
4860  INPUT "Grid spacing for spread offset (default=SP-SP dist/12):",Gx
4870  PRINTER IS 0
4880  PRINT LIN(1)
4890  PLOTTER IS 13,"GRAPHICS"
4900  GRAPHICS
4910  RESTORE 4920
4920  DATA 0,123,0,100
4930  READ B1,B2,B4,B5
4940  B3=B2-B1
4950  B6=B5-B4
4960  FRAME
4970  GOSUB Labels                   ! Print labels on graph
4980  LOCATE B1+5,B2-5,B4+5,B5-10    ! Plot border around results
```

```
4990  SCALE 0,Xc,0,Max_at
5000  FRAME
5010  GRID Gx,Gy                   ! Plot grid lines
5020                         ! PLOT DIRECT RAY ARRIVALS
5030  LINE TYPE 3                  ! Dotted line
5040  MOVE 0,0                     ! Plot direct-ray arrivals
5050  DRAW Xc,Xc/V1                ! Last value, direct ray from SP A
5060  MOVE 0,Xc/V1                 ! Last value, direct ray from SP B
5070  DRAW Xc,0                    ! Direct ray from SP B
5080                         ! PLOT ALL ARRIVALS FOR TWO LAYER CASE
5090  MOVE 0,T2ai                  ! Intercept time, SP A, layer 2
5100  DRAW X2acd,T2acd             ! Critical distance, SP A, layer 2
5110  POLYGON .4                   ! Draw cicle at critical distance
5120  MOVE X2acd,T2acd             ! Critical distance, SP A, layer 2
5130  DRAW Xc,T2rt                 ! Last value, refr from SP A, layer 2
5140  MOVE 0,T2rt                  ! Refracted ray from SP B, layer 2
5150  DRAW Xc-X2bcd,T2bcd          ! Critical distance, SP B, layer 2
5160  POLYGON .4                   ! Draw cicle at critical distance
5170  MOVE Xc-X2bcd,T2bcd          ! Critical distance, SP B, layer 2
5180  DRAW Xc,T2bi                 ! Intercept time, SP B, layer 2
5190  IF Nl=2 THEN 5470            ! Branch; plot FB's, 2 layer case
5200                         ! PLOT ALL ARRIVALS FOR 3 LAYER CASE
5210  LINE TYPE 3                  ! Dotted line
5220  MOVE 0,T3ai                  ! Intercept time, SP A, layer 3
5230  DRAW X3acd,T3acd             ! Critical distance, SP A, layer 3
5240  POLYGON .4                   ! Draw cicle at critical distance
5250  MOVE X3acd,T3acd             ! Critical distance, SP A, layer 3
5260  DRAW Xc,T3rt                 ! Last value, refr from SP A, layer 3
5270  MOVE 0,T3rt                  ! Last value, refr from SP B, layer 3
5280  DRAW Xc-X3bcd,T3bcd          ! Critical distance, SP B, layer 3
5290  POLYGON .4                   ! Draw cicle at critical distance
5300  MOVE Xc-X3bcd,T3bcd          ! Critical distance, SP B, layer 3
5310  DRAW Xc,T3bi                 ! Intercept time, SP B, layer 3
5320  IF Nl=3 THEN 5560            ! Branch; plot FB's, 3 layer case
5330                         ! PLOT ALL ARRIVALS FOR FOUR LAYER CASE
5340  LINE TYPE 3                  ! Dotted line
5350  MOVE 0,T4ai                  ! Intercept time, SP A, layer 4
5360  DRAW X4acd,T4acd             ! Critical distance, SP A, layer 4
5370  POLYGON .4                   ! Draw cicle at critical distance
5380  MOVE X4acd,T4acd             ! Critical distance, SP A, layer 4
5390  DRAW Xc,T4rt                 ! Last value, refr from SP A, layer 4
5400  MOVE 0,T4rt                  ! Last value, refr from SP B, layer 4
5410  DRAW Xc-X4bcd,T4bcd          ! Critical distance, SP B, layer 4
5420  POLYGON .4                   ! Draw cicle at critical distance
5430  MOVE Xc-X4bcd,T4bcd          ! Critical distance, SP B, layer 4
5440  DRAW Xc,T4bi                 ! Intercept time, SP B, layer 4
5450  GOTO 5740                    ! Branch; plot FB's, 4 layer case
5460                         ! PLOT FIRST ARRIVALS FOR TWO LAYER CASE
5470  LINE TYPE 1                  ! Solid line for FB's, 2 layer case
5480  MOVE 0,0                     ! Beginning value at SP A
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```
5490 DRAW X12aco,T12aco              ! DRAW to crossover point (layers 1 & 2)
5500 DRAW Xc,T2rt                    ! DRAW to last value from SP A
5510 MOVE Xc,0                       ! Beginning value at SP B
5520 DRAW Xc-X12bco,T12bco           ! DRAW to crossover point (layers 1 & 2)
5530 DRAW 0,T2rt                     ! DRAW to last value from SP B
5540 GOTO 6130                       ! Terminate plotting
5550                       ! PLOT FIRST ARRIVALS FOR THREE LAYER CASE
5560 LINE TYPE 1                     ! Solid line for FB's, 3 layer case
5570 MOVE 0,0                        ! Beginning velue at SP A
5580 IF Ha23=1 THEN 5620             ! Layer 2 is a hidden layer from SP A
5590 DRAW X12aco,T12aco              ! DRAW to crossover point (layers 1 & 2)
5600 DRAW X23aco,T23aco              ! DRAW to crossover point (layers 2 & 3)
5610 GOTO 5630
5620 DRAW X13aco,T13aco              ! DRAW to crossover point (layers 1 & 3)
5630 DRAW Xc,T3rt                    ! DRAW to last value from 3rd layer
5640 MOVE 0,T3rt                     ! MOVE to last value, 3rd layer, SP B
5650 MOVE Xc,0                       ! Beginning value at SP B
5660 IF Hb23=1 THEN 5700             ! Layer 2 is a hidden layer from SP B
5670 DRAW Xc-X12bco,T12bco           ! DRAW to crossover point (layers 1 & 2)
5680 DRAW Xc-X23bco,T23bco           ! DRAW to crossover point (layers 2 & 3)
5690 GOTO 5710
5700 DRAW Xc-X13bco,T13bco           ! DRAW to crossover point (layers 1 & 3)
5710 DRAW 0,T3rt                     ! DRAW to last value from SP B
5720 GOTO 6130                       ! Terminate plotting
5730                       ! PLOT FIRST ARRIVALS FOR FOUR LAYER CASE
5740 LINE TYPE 1                     ! Solid line for FB's, 4 layer case
5750 MOVE 0,0                        ! Beginning value at SP A
5760 IF (Ha23=0) AND (Ha34=0) THEN 5800   ! No hidden layers
5770 IF (Ha23=1) AND (Ha34=0) THEN 5840   ! Layer 2 is a hidden layer
5780 IF (Ha23=0) AND (Ha34=1) THEN 5870   ! Layer 3 is a hidden layer
5790 IF (Ha23=1) AND (Ha34=1) THEN 5900   ! Layers 2 & 3 are hidden layers
5800 DRAW X12aco,T12aco              ! DRAW to crossover point (layers 1 & 2)
5810 DRAW X23aco,T23aco              ! DRAW to crossover point (layers 2 & 3)
5820 DRAW X34aco,T34aco              ! DRAW to crossover point (layers 3 & 4)
5830 GOTO 5910
5840 DRAW X13aco,T13aco              ! DRAW to crossover point (layers 1 & 3)
5850 DRAW X34aco,T34aco              ! DRAW to crossover point (layers 3 & 4)
5860 GOTO 5910
5870 DRAW X12aco,T12aco              ! DRAW to crossover point (layers 1 & 2)
5880 DRAW X24aco,T24aco              ! DRAW to crossover point (layers 2 & 4)
5890 GOTO 5910
5900 DRAW X14aco,T14aco              ! DRAW to crossover point (layers 1 & 4)
5910 DRAW Xc,T4rt                    ! DRAW to last point from SP A
5920 IF Ha34=0 THEN 5940             ! Third layer not hidden from SP A
5930 GOTO 5950                       ! Skip
5940 DRAW X34aco,T34aco              ! DRAW to crossover point (layers 3 & 4)
5950 DRAW Xc,T4rt                    ! DRAW to last point from SP A
5960 MOVE Xc,0                       ! Beginning value at SP B
5970 IF (Hb23=0) AND (Hb34=0) THEN 6010   ! No hidden layers
5980 IF (Hb23=1) AND (Hb34=0) THEN 6050   ! Layer 2 is a hidden layer
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5990  IF (Hb23=0) AND (Hb34=1) THEN 6080    ! Layer 3 is a hidden layer
6000  IF (Hb23=1) AND (Hb34=1) THEN 6110    ! Layers 2 & 3 are hidden layers
6010  DRAW Xc-X12bco,T12bco             ! DRAW to crossover point (layers 1 & 2)
6020  DRAW Xc-X23bco,T23bco             ! DRAW to crossover point (layers 2 & 3)
6030  DRAW Xc-X34bco,T34bco             ! DRAW to crossover point (layers 3 & 4)
6040  GOTO 6120
6050  DRAW Xc-X13bco,T13bco             ! DRAW to crossover point (layers 1 & 3)
6060  DRAW Xc-X34bco,T34bco             ! DRAW to crossover point (layers 3 & 4)
6070  GOTO 6120
6080  DRAW Xc-X12bco,T12bco             ! DRAW to crossover point (layers 1 & 2)
6090  DRAW Xc-X24bco,T24bco             ! DRAW to crossover point (layers 2 & 4)
6100  GOTO 6120
6110  DRAW Xc-X14bco,T14bco             ! DRAW to crossover point (layers 1 & 4)
6120  DRAW 0,T4rt                       ! DRAW to last value from SP B
6130  DUMP GRAPHICS
6140  EXIT GRAPHICS
6150  RETURN
6160  !
6170 Labels:                            ! Print labels on plot
6180  LORG 5
6190  MOVE B3/2,B5-2.5
6200  ON Nl-1 GOTO 6210,6260,6310
6210  IF Qu=1 THEN 6240                 ! Metric units
6220  LABEL "TRAVELTIME CURVES FOR 2-LAYER CASE  V1="&V1$&",V2="&V2$&"ft/ms"
6230  GOTO 6350
6240  LABEL "TRAVELTIME CURVES FOR 2-LAYER CASE  V1="&V1$&",V2="&V2$&"m/ms"
6250  GOTO 6350
6260  IF Qu=1 THEN 6290                 ! Metric units
6270  LABEL "TRAVELTIME CURVES FOR 3-LAYER CASE  V1="&V1$&",V2="&V2$&",V3="&V3$&"ft/ms"
6280  GOTO 6350
6290  LABEL "TRAVELTIME CURVES FOR 3-LAYER CASE  V1="&V1$&",V2="&V2$&",V3="&V3$&"m/ms"
6300  GOTO 6350
6310  IF Qu=1 THEN 6340                 ! Metric units
6320  LABEL "TRAVELTIME CURVES  4-LAYER CASE  V1="&V1$&",V2="&V2$&",V3="&V3$&",V4="&V4$&"ft/ms"
6330  GOTO 6350
6340  LABEL "TRAVELTIME CURVES  4-LAYER CASE  V1="&V1$&",V2="&V2$&",V3="&V3$&",V4="&V4$&"m/ms"
6350  MOVE B3/2,B5-6.5
6360  LABEL Title$
6370  MOVE 2.5,B6/2
6380  LDIR 90
6390  LORG 5
6400  LABEL "ARRIVAL TIME  1 DIV="&VAL$(Gy)&"ms  MAX T="&VAL$(Max_at)&"ms"
6410  LDIR 0
6420  LORG 5
6430  MOVE B3/2,B1+2.5
6440  IF Qu=1 THEN 6470                 ! Metric units
6450  LABEL "OFFSET  1 DIV="&VAL$(Gx)&"m  MAX O/S="&VAL$(Xc)&"ft"
6460  GOTO 6480
6470  LABEL "OFFSET  1 DIV="&VAL$(Gx)&"m  MAX O/S="&VAL$(Xc)&"m"
6480  RETURN
```

```
10    PRINT "MAKE SIMPLE REFRACTION CALCULATIONS 'REFRSC' 17 AUGUST 1990"
20    OPTION BASE 1
30    PRINTER IS 16
40    PRINT "This program is used for first-approximation calculations"
50    PRINT "using first arrivals displayed on traveltime curves from"
60    PRINT "either single ended or reciprocal spreads.  The following"
70    PRINT "conditions apply:"
80    PRINT "   units may be metric or English, with times in millisec,"
90    PRINT "   constant velocity layers and planar interfaces,"
100   PRINT "   maximum of four layers with SP's on level surface,"
110   PRINT "Principal reliance is placed on the use of intercept times."
120   !
130   GOSUB Select                     ! Select type of computation wanted
140   !
150   BEEP
160   OUTPUT 9;"R"
170   ENTER 9;T$
180   PRINT LIN(1);"COMPUTED: ";T$;", ";Y$
190   PRINTER IS 16
200   DISP "PROGRAM COMPLETED"
210   END
220   !
230   DIM Name$[80],Title$(6)[80]
240   !
250 Printout:                          ! Select printout mode
260   PRINTER IS 16                    ! Print results on screen
270   G$="N"
280   INPUT "Do you want hard copy of results? (Y/N--default is N)",G$
290   IF G$="N" THEN 310
300   PRINTER IS 0                     ! Print results using internal printer
310   RETURN
320   !
330 Select:                            ! Select type of problem
340   DEG                              ! Computations use angles in degrees
350   PRINTER IS 16                    ! Screen display
360   Title$(1)="TWO LAYERS, SINGLE ENDED, "
370   Title$(2)="TWO LAYERS, RECIPROCAL, "
380   Title$(3)="THREE LAYERS, SINGLE ENDED, "
390   Title$(4)="THREE LAYERS, RECIPROCAL, "
400   Title$(5)="FOUR LAYERS, SINGLE ENDED, "
410   Title$(6)="FOUR LAYERS, RECIPROCAL, "
420   PRINT LIN(1);"LIST OF TYPE PROBLEMS"
430   PRINT "TWO-LAYER CASE"
440   PRINT "   1. Single ended spread data"
450   PRINT "   2. Reciprocal spread data"
460   PRINT "THREE-LAYER CASE"
470   PRINT "   3. Single ended spread data"
```

```
480   PRINT "   4. Reciprocal spread data"
490   PRINT "FOUR-LAYER CASE"
500   PRINT "   5. Single ended spread data"
510   PRINT "   6. Reciprocal spread data"
520   INPUT "Number of type problem from above list:",Type
530   ON Type GOTO 540,560,580,600,620,640
540   GOSUB Two_lay_single            ! Two-layer case, single ended spread
550   GOTO 650
560   GOSUB Two_lay_recip             ! Two-layer case, reciprocal spread
570   GOTO 650
580   GOSUB Three_lay_sing            ! Three-layer case, single ended spread
590   GOTO 650
600   GOSUB Three_lay_recip           ! Three-layer case, reciprocal spread
610   GOTO 650
620   GOSUB Four_lay_single           ! Four-layer case, single ended spread
630   GOTO 650
640   GOSUB Four_lay_recip            ! Four-layer case, reciprocal spread
650   RETURN
660   !
670 Common:                           ! Common quantities for all calculations
680   Y$="1990"                                                      ! DEFAULT
690   INPUT "Year computations made (default = 1990)",Y$
700   INPUT "Problem or area name (80 char, max):",Name$
710   PRINT LIN(2);Name$
720   Xc=36                                                          ! DEFAULT
730   INPUT "Distance from SP to far detector (default=36):",Xc
740   Xc$=VAL$(Xc)
750   Xc$="MAX O/S = "&Xc$
760   PRINT Title$(Type)&Xc$
770   RETURN
780   !
790 Two_lay_single:                   ! Computations for 2-layer case, single
800   GOSUB Printout                  ! Select printout mode
810   GOSUB Common                    ! Common quantities for all calculations
820   PRINT LIN(1);"INPUT VALUES FOR 1ST AND 2ND LAYER"
830   INPUT "Velocity within upper layer:",V1
840   IMAGE "      Velocity within upper layer =",3D.2D
850   PRINT USING 840;V1
860   INPUT "FB time, 2nd layer, at far detector:",T2far
870   IMAGE " FB time, 2nd layer, far detector =",3D.D
880   PRINT USING 870;T2far
890   INPUT "Observed 2nd layer intercept time:",T2i
900   IMAGE "Observed 2nd layer intercept time =",3D.D
910   PRINT USING 900;T2i
920   PRINT LIN(1);"COMPUTED VALUES FOR 1ST & 2ND LAYERS"
930   IF T2i<>T2far THEN 960
940   V2=99.99                        ! Infinite apparent velocity
950   GOTO 990
```

```
960  V2=Xc/(T2far-T2i)               ! Compute 2nd layer apparent velocity
970  IF (V2>99.99) OR (V2<-99.99) THEN V2=99.99
980  IMAGE "  Apparent velocity of 2nd layer = ",2D.2D
990  PRINT USING 980;V2
1000 I12=ASN(V1/V2)                  ! Critical angle, layer 1 & 2
1010 C12=COS(I12)                    ! Cosine of critical angle
1020 Z2=.5*T2i*V1/C12                ! Compute depth to second layer
1030 L1i=Z2                          ! Vertical thickness of 1st layer
1040 X12coc=V1*V2*T2i/(V2-V1)        ! Computed crossover distance
1050 IMAGE " 1st & 2nd lay crossover distance =",3D.D
1060 PRINT USING 1050;X12coc
1070 G$="N"
1080 INPUT "Do you also want to compute using crossover distance? (Y/N--default is N)",G$
1090 IF G$="N" THEN 1250
1100 INPUT "Observed crossover distance, layers 1&2:",X12co
1110 IMAGE " Observed crossover dist, lay 1&2 =",3D.D
1120 PRINT USING 1110;X12co
1130 L1c=.5*X12co*SQR((V2-V1)/(V2+V1))  ! Vertical thickness of 1st layer
1140 Z2=L1c                          ! Depth to top of 2nd layer
1150 T2farc=Xc/V2+2*L1c*C12/V1       ! Computed far trace time, 2nd layer
1160 IMAGE " Computed far trace time, layer 2 =",3D.D
1170 PRINT USING 1160;T2farc
1180 G$="Y"
1190 INPUT "Do far trace times sufficiently agree? (Y/N--default is Y)",G$
1200 IF G$="Y" THEN 1220             ! Proceed with computations
1210 GOTO 1100                       ! Re-enter crossover distance
1220 G$="Y"
1230 INPUT "Do you want to use values from intercept? (Y/N--default is Y)",G$
1240 IF G$="N" THEN 1270
1250 Z2=L1=L1i
1260 GOTO 1330
1270 G$="Y"
1280 INPUT "Do you want to use values from crossover? (Y/N--default is Y)",G$
1290 IF G$="N" THEN 1320
1300 Z2=L1=L1c
1310 GOTO 1330
1320 Z2=L1=.5*(L1i+L1c)              ! Average of results from int & cross
1330 IMAGE "        Depth to top of 2nd layer =",3D.D
1340 PRINT USING 1330;Z2
1350 X2cd=2*L1*TAN(I12)              ! Compute critical distance, 2nd layer
1360 IMAGE "                Critical distance =",3D.D
1370 PRINT USING 1360;X2cd
1380 RETURN
1390 !
1400 Two_lay_recip:                  ! Compute, 2-layer, reciprocal spread
1410 GOSUB Printout                  ! Select printout mode
1420 GOSUB Common                    ! Common quantities for all calculations
1430 PRINT LIN(1);"INPUT VALUES FOR 1ST AND 2ND LAYER"
```

```
1440  INPUT "Average velocity within upper layer at SP A:",V1a
1450  IMAGE "Avg velocity, upper layer at SP A =",3D.2D
1460  PRINT USING 1450;V1a
1470  INPUT "Average velocity within upper layer at SP B:",V1b
1480  IMAGE "Avg velocity, upper layer at SP B =",3D.2D
1490  PRINT USING 1480;V1b
1500  V1=.5*(V1a+V1b)                  ! V1=Average upper layer velocity
1510  IMAGE "  Average velocity of upper layer =",3D.2D
1520  PRINT USING 1510;V1
1530  INPUT "Reciprocal time for 2nd layer:",T2rt
1540  IMAGE "     Reciprocal time for 2nd layer =",3D.D
1550  PRINT USING 1540;T2rt
1560  INPUT "Observed Intercept time at SP A:",T2ai
1570  IMAGE "  Observed intercept time at SP A =",3D.D
1580  PRINT USING 1570;T2ai
1590  INPUT "Observed Intercept time at SP B:",T2bi
1600  IMAGE "  Observed intercept time at SP B =",3D.D
1610  PRINT USING 1600;T2bi
1620  PRINT LIN(1);"COMPUTED VALUES FOR 1ST & 2ND LAYERS"
1630  IF T2ai<>T2rt THEN 1660
1640  V2e=99.99                        ! Infinite apparent velocity
1650  GOTO 1690
1660  V2a=Xc/(T2rt-T2ai)               ! Apparent velocity from SP A
1670  IF (V2a>99.99) OR (V2a<-99.99) THEN V2a=99.99
1680  IMAGE " Apparent 2nd-layer vel from SP A =",3D.2D
1690  PRINT USING 1680;V2a
1700  IF T2bi<>T2rt THEN 1730
1710  V2b=99.99                        ! Infinite apparent velocity
1720  GOTO 1760
1730  V2b=Xc/(T2rt-T2bi)               ! Apparent velocity from SP B
1740  IF (V2b>99.99) OR (V2b<-99.99) THEN V2b=99.99
1750  IMAGE " Apparent 2nd layer vel from SP B =",3D.2D
1760  PRINT USING 1750;V2b
1770  A2=.5*(ASN(V1/V2a)-ASN(V1/V2b))  ! Avg dip angle for top of layer 2
1780  IMAGE "         Dip from SP A toward SP B =",3D.D
1790  PRINT USING 1780;A2
1800  I12=.5*(ASN(V1/V2a)+ASN(V1/V2b)) ! Average critical angle, layer 1 & 2
1810  V2=V1/SIN(I12)                   ! V2=Velocity of second layer
1820  C12=COS(I12)                     ! Cosine of critical angle
1830  IMAGE "          Velocity of second layer = ",2D.2D
1840  PRINT USING 1830;V2
1850  N1ai=.5*V1a*T2ai/C12             ! Normal thickness of layer 1 at SP A
1860  N1bi=.5*V1b*T2bi/C12             ! Normal thickness of layer 1 at SP B
1870  X12acoc=V1a*V2a*T2ai/(V2a-V1a)   ! Compute crossover distance from SP A
1880  IMAGE "Computed crossover dist from SP A =",3D.D
1890  PRINT USING 1880;X12acoc
1900  X12bcoc=V1b*V2b*T2bi/(V2b-V1b)   ! Compute crossover distance from SP B
1910  IMAGE "Computed crossover dist from SP B =",3D.D
```

```
1920 PRINT USING 1910;X12bcoc
1930 G$="N"
1940 INPUT "Do you also want to compute using crossover distance? (Y/N--default is N)",G$
1950 IF G$="N" THEN 2200
1960 INPUT "Crossover distance from SP A (if none, then hit CONT):",X12aco
1970 IMAGE "Observed crossover dist from SP A =",3D.D
1980 PRINT USING 1970;X12aco
1990 INPUT "Crossover distance from SP B (if none, then hit CONT):",X12bco
2000 IMAGE "Observed crossover dist from SP B =",3D.D
2010 PRINT USING 2000;X12bco
2020 IF X12aco=0 THEN 2060           ! Branch; no crossover from SP A
2030 L1a=.5*X12aco*(1-V1a*COS(A2)/V2-SIN(A2)*C12)/(COS(A2)*C12)
2040 Z2a=L1a                         ! Depth to top of layer 2 at SP A
2050 N1ac=L1a*COS(A2)                ! Normal thickness of layer 1 at SP A
2060 IF X12bco=0 THEN 2100           ! Branch; no crossover from SP B
2070 L1b=.5*X12bco*(1-V1b*COS(A2)/V2+SIN(A2)*C12)/(COS(A2)*C12)
2080 Z2b=L1b                         ! Depth to top of layer 2 at SP B
2090 N1bc=L1b*COS(A2)                ! Normal thickness of layer 1 at SP B
2100 T2rtc=Xc*COS(A2)/V2+(N1ac/V1a+N1bc/V1b)*C12  ! Computed reciprocal time
2110 IMAGE " Computed 2nd lay reciprocal time =",3D.D
2120 PRINT USING 2110;T2rtc
2130 G$="Y"
2140 INPUT "Do reciprocal times sufficiently agree? (Y/N--default is Y)",G$
2150 IF G$="N" THEN 2170
2160 GOTO 2200                       ! Use computed values using intercept
2170 G$="Y"
2180 INPUT "Do you want to use values from intercept? (Y/N--default is Y)",G$
2190 IF G$="N" THEN 2230
2200 N1a=N1ai
2210 N1b=N1bi
2220 GOTO 2310
2230 G$="Y"
2240 INPUT "Do you want to use values from crossover? (Y/N--default is Y)",G$
2250 IF G$="N" THEN 2290
2260 N1a=N1ac
2270 N1b=N1bc
2280 GOTO 2310
2290 N1a=.5*(N1ai+N1ac)
2300 N1b=.5*(N1bi+N1bc)
2310 IMAGE "Normal thickness of lay 1 at SP A =",3D.D
2320 PRINT USING 2310;N1a
2330 L1a=N1a/COS(A2)                 ! Vertical thickness of layer 1 at  SP A
2340 Z2a=L1a                         ! Depth to 2nd layer at SP A
2350 IMAGE " Depth to second layer under SP A =",3D.D
2360 PRINT USING 2350;Z2a
2370 IMAGE "Normal thickness of lay 1 at SP B =",3D.D
2380 PRINT USING 2370;N1b
2390 L1b=N1b/COS(A2)                 ! Vertical thickness of layer 1 at SP B
```

```
2400  Z2b=L1b                          ! Depth to 2nd layer at SP B
2410  IMAGE " Depth to second layer under SP B =",3D.D
2420  PRINT USING 2410;Z2b
2430  Q1=1                             ! Q1=1 is flag for no min depth calc
2440  IF Type>2 THEN 2660              ! Not a two-later case
2450  G$="N"                           ! Compute minimum depth to 3rd layer
2460  INPUT "Do you want minimum depth to 3rd layer? (Y/N--default is N)",G$
2470  IF G$="N" THEN 2660
2480  Q1=2                             ! Q1=2 is flag for min depth calculation
2490  PRINT LIN(1);"COMPUTATION OF MINIMUM DEPTH TO THIRD LAYER"
2500  INPUT "Estimated third layer velocity:",V3
2510  IMAGE "   Estimated third layer velocity =",3D.D
2520  PRINT USING 2510;V3
2530  A3=0                                                          ! DEFAULT
2540  INPUT "Estimated third layer dip (default=0):",A3
2550  IMAGE "        Estimated third layer dip =",3D.D
2560  PRINT USING 2550;A3
2570  I23=ASN(V2/V3)
2580  A13=ASN(V1*SIN(I23+(A3-A2))/V2)
2590  B13=ASN(V1*SIN(I23-(A3-A2))/V2)
2600  V3a=V1/SIN(A13+A2)
2610  V3b=V1/SIN(B13-A2)
2620  T3rt=T2rt
2630  T3ai=T3rt-Xc/V3a
2640  T3bi=T3rt-Xc/V3b
2650  GOTO 3490                        ! Complete comp with 3-layer computation
2660  RETURN
2670  !
2680 Three_lay_sing:                   ! Compute three-layer, zero dip
2690  IF Type=5 THEN 2710              ! Branch; skip repeat of two layer
2700  GOSUB Two_lay_single             ! Compute second layer values
2710  PRINT LIN(1);"INPUT VALUES FOR THIRD LAYER"
2720  INPUT "FB time, 3rd layer, at far detector:",T3far
2730  IMAGE " FB time, 3rd layer, far detector =",3D.D
2740  PRINT USING 2730;T3far
2750  INPUT "Observed 3rd layer intercept time:",T3i
2760  IMAGE "Observed 3rd layer intercept time = ",2D.D
2770  PRINT USING 2760;T3i
2780  PRINT LIN(1);"COMPUTED VALUES FOR 2ND & 3RD LAYERS"
2790  IF T3i<>T3far THEN 2820
2800  V3=99.99                         ! Infinite apparent velocity
2810  GOTO 2850
2820  V3=Xc/(T3far-T3i)                ! Apparent velocity of third layer
2830  IF (V3>99.99) OR (V3<-99.99) THEN V3=99.99
2840  IMAGE "   Apparent velocity of 3rd layer = ",2D.2D
2850  PRINT USING 2840;V3
2860  I23=ASN(V2/V3)                   ! Crit angle, layers 2 and 3
2870  C23=COS(I23)                     ! Cosine of crit angle, layers 2 and 3
```

```
2880  I13=ASN(V1/V3)                   ! Crit angle, layers 1 and 3
2890  C13=COS(I13)                     ! Cosine of crit angle, layers 1 and 3
2900  L2=.5*V2*(T3i-2*L1*C13/V1)/C23   ! Comp thickness of second layer
2910  X23coc=2*V2*V3*(L1*C13/V1+L2*C23/V2-L1*C12/V1)/(V3-V2)  ! Comp lay 2&3 co
2920  IMAGE " 2nd & 3rd lay crossover distance =",3D.D
2930  PRINT USING 2920;X23coc
2940  IMAGE "       Thickness of second layer =",3D.D
2950  PRINT USING 2940;L2
2960  Z3=Z2+L2
2970  IMAGE "      Depth to top of third layer =",3D.D
2980  PRINT USING 2970;Z3
2990  X3cd=2*L1*TAN(I13)+2*L2*TAN(I23)   ! Comp critical distance, 3rd layer
3000  IMAGE "  Critical distance for 3rd layer =",3D.D
3010  PRINT USING 3000;X3cd
3020  RETURN
3030  !
3040 Three_lay_recip:                  ! Comp three lay case, reciporcal spd
3050  IF Type=6 THEN 3070              ! Branch; skip repeat of two layer
3060  GOSUB Two_lay_recip              ! Compute second layer values
3070  PRINT LIN(1);"INPUT VALUES FOR THIRD LAYER"
3080  INPUT "Observed third layer reciprocal time:",T3rt
3090  IMAGE " Observed 3rd lay reciprocal time =",3D.D
3100  PRINT USING 3090;T3rt
3110  INPUT "Observed 3rd layer intercept time at SP A:",T3ai
3120  IMAGE "Observed 3rd layer intercept,SP A =",3D.D
3130  PRINT USING 3120;T3ai
3140  INPUT "Observed 3rd layer intercept time at SP B:",T3bi
3150  IMAGE "Observed 3rd layer intercept,SP B =",3D.D
3160  PRINT USING 3150;T3bi
3170  PRINT LIN(1);"COMPUTED VALUES FOR 2ND & 3RD LAYERS"
3180  IF T3rt<>T3ai THEN 3210
3190  V3a=99.99                        ! Inifinite apparent velocity
3200  GOTO 3240
3210  V3a=Xc/(T3rt-T3ai)               ! Apparent velocity from SP A
3220  IF (V3a>99.99) OR (V3a<-99.99) THEN V3a=99.99
3230  IMAGE "    Apparent vel of 3rd lay, SP A =",3D.2D
3240  PRINT USING 3230;V3a
3250  IF T3rt<>T3bi THEN 3280
3260  V3b=99.99                        ! Infinite apparent velocity
3270  GOTO 3310
3280  V3b=Xc/(T3rt-T3bi)               ! Apparent velocity from SP B
3290  IF (V3b>99.99) OR (V3b<-99.99) THEN V3b=99.99
3300  IMAGE "    Apparent vel of 3rd lay, SP B =",3D.2D
3310  PRINT USING 3300;V3b
3320  A13=ASN(V1/V3b)+A2               ! Dip crit angle at SP B from SP A
3330  B13=ASN(V1/V3a)-A2               ! Dip crit angle at SP A from SP B
3340  A=ASN(SIN(A13)/SIN(I12))
3350  B=ASN(SIN(B13)/SIN(I12))
```

```
3360 I23=.5*(A+B)                    ! Critical angle, layer 2 and 3
3370 A3=.5*(B-A)+A2                  ! Dip of 3rd layer from SP A to SP B
3380 IMAGE "        Computed dip of 3rd layer = ",2D.D
3390 PRINT USING 3380;A3
3400 V3=V2/SIN(I23)
3410 IMAGE "   Computed velocity of 3rd layer = ",2D.2D
3420 PRINT USING 3410;V3
3430 X23aco=V2a*V3a*(T3ai-T2ai)/(V3a-V2a)
3440 IMAGE "Computed lay 2 & 3 crossover SP A =",3D.D
3450 PRINT USING 3440;X23aco
3460 X23bco=V2b*V3b*(T3bi-T2bi)/(V3b-V2b)
3470 IMAGE "Computed lay 2 & 3 crossover SP B =",3D.D
3480 PRINT USING 3470;X23bco
3490 C23=COS(I23)                    ! Cosine of crit angle, layers 2 and 3
3500 N2a=.5*(T3ai-N1a*(COS(A13)/V1b+COS(B13)/V1a))*V2/C23
3510 IMAGE "Normal thickness of lay 2 at SP A =",3D.D
3520 PRINT USING 3510;N2a
3530 S=SIN(A2)*SIN(A3-A2)
3540 L2a=(N2a+L1a*S)/COS(A3)         ! Vertical thickness of lay 2 at SP A
3550 IMAGE " Vert. thickness of lay 2 at SP A =",3D.D
3560 PRINT USING 3550;L2a
3570 Z3a=Z2a+L2a                     ! Depth to top of lay 3 at SP A
3580 IMAGE "  Depth to top of layer 3 at SP A =",3D.D
3590 PRINT USING 3580;Z3a
3600 N2b=.5*(T3bi-N1b*(COS(A13)/V1b+COS(B13)/V1a))*V2/C23
3610 IMAGE "Normal thickness of lay 2 at SP B =",3D.D
3620 PRINT USING 3610;N2b
3630 L2b=(N2b+L1b*S)/COS(A3)         ! Vertical thickness of lay 2 at SP B
3640 IMAGE " Vert. thickness of lay 2 at SP B =",3D.D
3650 PRINT USING 3640;L2b
3660 Z3b=Z2b+L2b                     ! Depth to top of lay 3 at SP B
3670 IMAGE "  Depth to top of layer 3 at SP B =",3D.D
3680 PRINT USING 3670;Z3b
3690 RETURN
3700 !
3710 Four_lay_single:                ! Compute for four layer, single ended
3720 GOSUB Two_lay_single            ! Compute two layer values
3730 GOSUB Three_lay_sing            ! Compute three layer values
3740 PRINT LIN(1);"INPUT VALUES FOR FOURTH LAYER"
3750 INPUT "FB time, 4th layer, at far detector:",T4far
3760 IMAGE " FB time, 4th layer, far detector =",3D.D
3770 PRINT USING 3760;T4far
3780 INPUT "Observed 4th layer intercept time:",T4i
3790 IMAGE "Observed 4th layer intercept time = ",2D.D
3800 PRINT USING 3790;T4i
3810 PRINT LIN(1);"COMPUTED VALUES FOR 3RD & 4TH LAYERS"
3820 IF T4i<>T4far THEN 3850
3830 V4=99.99                        ! Inifinite apparent velocity
```

```
3840  GOTO 3880
3850  V4=Xc/(T4far-T4i)               ! Velocity of 4th layer
3860  IF (V4>99.99) OR (V4<-99.99) THEN V4=99.99
3870  IMAGE "   Computed velocity of 4th layer = ",2D.2D
3880  PRINT USING 3870;V4
3890  I14=ASN(V1/V4)                  ! Crit angle, layers 1 and 4
3900  C14=COS(I14)                    ! Cosine of crit angle, layers 1 and 4
3910  I24=ASN(V2/V4)                  ! Crit angle, layers 2 and 4
3920  C24=COS(I24)                    ! Cosine of crit angle, layers 2 and 4
3930  I34=ASN(V3/V4)                  ! Crit angle, layers 3 and 4
3940  C34=COS(I34)                    ! Cosine of crit angle, layers 3 and 4
3950  L3=.5*V3*(T4i-2*L1*C14/V1-2*L2*C24/V2)/C34      ! Thickness of 3rd layer
3960  X34coc=2*V3*V4*(L1*(C14-C13)/V1+L2*(C24-C23)/V2+L3*C34/V3)/(V4-V3)
3970  IMAGE " 3rd & 4th lay crossover distance =",3D.D
3980  PRINT USING 3970;X34coc
3990  IMAGE "         Thickness of third layer =",3D.D
4000  PRINT USING 3990;L3
4010  Z4=Z3+L3
4020  IMAGE "     Depth to top of fourth layer =",3D.D
4030  PRINT USING 4020;Z4
4040  X4cd=2*L1*TAN(I14)+2*L2*TAN(I24)+2*L3*TAN(I34)  ! Crit dist, 4th layer
4050  IMAGE "  Critical distance for 4th layer =",3D.D
4060  PRINT USING 3000;X4cd
4070  RETURN
4080  !
4090 Four_lay_recip:                  ! Compute for four-layer, reciprocal spd
4100  GOSUB Two_lay_recip             ! Compute two layer values
4110  GOSUB Three_lay_recip           ! Compute third layer values
4120  PRINT LIN(1);"INPUT VALUES FOR FOURTH LAYER"
4130  INPUT "Observed 4th layer reciprocal time:",T4rt
4140  IMAGE " Observed 4th lay reciprocal time =",3D.D
4150  PRINT USING 4140;T4rt
4160  INPUT "Observed 4th layer intercept time at SP A:",T4ai
4170  IMAGE "Observed 4th layer intercept,SP A =",3D.D
4180  PRINT USING 4170;T4ai
4190  INPUT "Observed 4th layer intercept time at SP B:",T4bi
4200  IMAGE "Observed 4th layer intercept,SP B =",3D.D
4210  PRINT USING 4200;T4bi
4220  IF T4ai<>T4rt THEN 4250
4230  V4a=99.99                       ! Infinite apparent velocity
4240  GOTO 4280
4250  V4a=Xc/(T4rt-T4ai)              ! Apparent velocity from SP A
4260  IF (V4a>99.99) OR (V4a<-99.99) THEN V4a=99.99
4270  IMAGE "    Apparent vel of 4th lay, SP A =",3D.2D
4280  PRINT USING 4270;V4a
4290  IF T4bi<>T4rt THEN 4320
4300  V4b=99.99                       ! Infinite apparent velocity
4310  GOTO 4350
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4320 V4b=Xc/(T4rt-T4bi)              ! Apparent velocity from SP B
4330 IF (V4b>99.99) OR (V4b<-99.99) THEN V4b=99.99
4340 IMAGE "    Apparent vel of 4th lay, SP B =",3D.2D
4350 PRINT USING 4340;V4b
4360 PRINT LIN(1);"COMPUTED VALUES FOR 3RD & 4TH LAYERS"
4370 A14=ASN(V1/V4b)+A2              ! Dip critical angle, SP A, at surface
4380 B14=ASN(V1/V4a)-A2              ! Dip critical angle, SP B, at surface
4390 A24=ASN(SIN(A14)/SIN(I12))+A3-A2 ! Dip crit angle, SP A, base layer 1
4400 B24=ASN(SIN(B14)/SIN(I12))+A2-A3 ! Dip crit angle, SP B, base layer 1
4410 A=ASN(SIN(A24)/SIN(I23))
4420 B=ASN(SIN(B24)/SIN(I23))
4430 I34=.5*(A+B)                    ! Critical angle, layer 3 and 4
4440 C34=COS(I34)                    ! Cosine of crit angle, layers 3 and 4
4450 A4=.5*(B-A)+A3                  ! Dip of 4th layer from SP A to SP B
4460 IMAGE "       Computed dip of 4th layer = ",2D.D
4470 PRINT USING 4460;A4
4480 V4=V3/SIN(I34)
4490 IMAGE "  Computed velocity of 4th layer =",3D.2D
4500 PRINT USING 4490;V4
4510 X34aco=V3a*V4a*(T4ai-T3ai)/(V4a-V3a)
4520 IMAGE "Computed lay 3 & 4 crossover SP A =",3D.D
4530 PRINT USING 3440;X34aco
4540 X34bco=V3b*V4b*(T4bi-T3bi)/(V4b-V3b)
4550 IMAGE "Computed lay 3 & 4 crossover SP B =",3D.D
4560 PRINT USING 3470;X34bco
4570 N3a=.5*V3*(T4ai-N1a*(COS(A14)/V1b+COS(B14)/V1a)-N2a*(COS(A24)+COS(B24))/V2)/C34
4580 IMAGE "Normal thickness of lay 3 at SP A =",3D.D
4590 PRINT USING 4580;N3a
4600 S=(N1a*SIN(A2)+N2a*SIN(A3))/COS(A3)
4610 L3a=(N3a+S*SIN(A4-A3))/COS(A4)
4620 IMAGE " Vert. thickness of lay 3 at SP A =",3D.D
4630 PRINT USING 4620;L3a
4640 Z4a=Z3a+L3a                     ! Depth to top of lay 4 at SP A
4650 IMAGE "  Depth to top of layer 4 at SP A =",3D.D
4660 PRINT USING 4650;Z4a
4670 N3b=.5*V3*(T4bi-N1b*(COS(A14)/V1b+COS(B14)/V1a)-N2b*(COS(A24)+COS(B24))/V2)/C34
4680 IMAGE "Normal thickness of lay 3 at SP B =",3D.D
4690 PRINT USING 4680;N3b
4700 L3b=(N3b+(N1b*SIN(A2)*COS(A3-A2)+L2b*SIN(A3))*SIN(A4-A3))/COS(A4)
4710 IMAGE " Vert. thickness of lay 3 at SP B =",3D.D
4720 PRINT USING 4710;L3b
4730 Z4b=Z3b+L3b                     ! Depth to top of lay 4 at SP B
4740 IMAGE "  Depth to top of layer 4 at SP B =",3D.D
4750 PRINT USING 4740;Z4b
4760 RETURN
4770 !
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