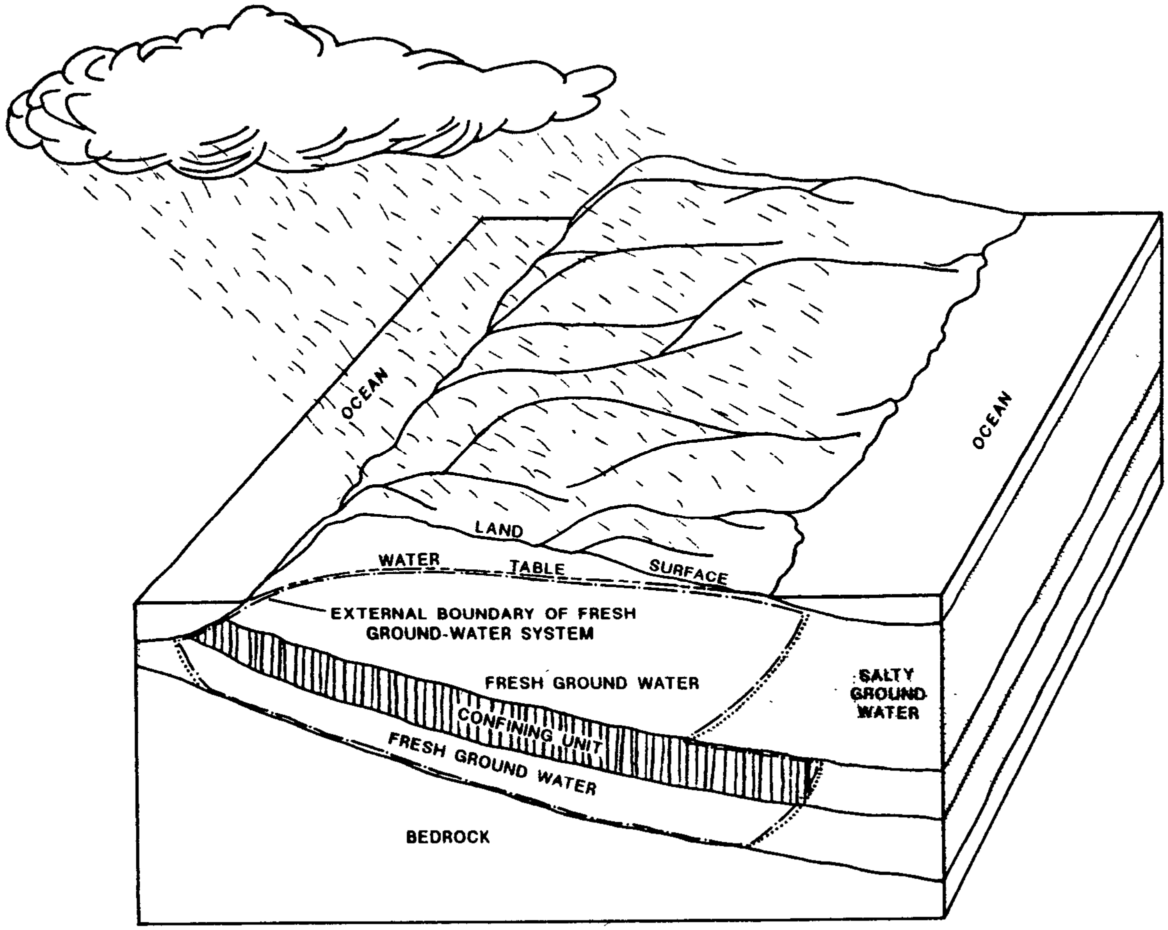
STUDY GUIDE FOR A BEGINNING COURSE IN GROUND-WATER HYDROLOGY: PART I -- COURSE PARTICIPANTS



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STUDY GUIDE FOR A BEGINNING COURSE IN GROUND-WATER HYDROLOGY: PART I--COURSE PARTICIPANTS

by O. Lehn Franke, Thomas E. Reilly, Ralph J. Haefner, and Dale L. Simmons

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U.S. GEOLOGICAL SURVEY

Open-File Report 90-183

Reston, Virginia

1990

DEPARTMENT OF THE INTERIOR

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CONVERSION FACTORS AND ABBREVIATIONS

Multiply inch-pound units	by	To obtain SI (metric units)
inch (in.)	25.4	millimeter (mm)
inch per year (in/yr)	25.4	millimeter per year (mm/yr)
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
square mile (mi^2)	2.59	square kilometer (km^2)
foot squared per day (ft^2/d)	0.0929	meter squared per day (m^2/d)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
gallon per minute (gal/min)	0.06309	liter per second (L/s)
million gallons per day (Mgal/d)	0.04381	cubic meter per second (m^3/s)
foot per year per square mile [$(ft/yr)/mi^2$]	0.7894	meter per year per square kilometer [$(m/yr)/km^2$]

PREFACE

The principal purpose of this study guide is to provide a broad selection of study materials that comprise a beginning course in ground-water hydrology. These study materials consist primarily of notes and exercises. The notes are designed to emphasize ideas and to clarify technical points that commonly cause difficulty and confusion to inexperienced hydrologists and may not receive adequate treatment in standard textbooks. Some of the exercises are more extensive than those usually found in textbooks to provide an additional level of detail and to focus on concepts that we consider to be particularly important. Detailed answers to exercises with explanatory comments are available in a companion publication.

The most important and unique technical feature of this course is the emphasis on the concept of a ground-water system. Generally, this concept is first developed extensively in a more advanced rather than a beginning course in ground-water hydrology. We believe that it is highly desirable to introduce this concept early in a hydrologist's education because it provides the best possible conceptual framework for analyzing and guiding all phases of any investigation related to ground water.

The study guide is divided into five sections: (1) Fundamental concepts and definitions, (2) Principles of ground-water flow and storage, (3) Description and analysis of ground-water systems, (4) Ground-water flow to wells, and (5) Ground-water contamination. Each section is subdivided into a number of subtopics, and each subtopic is followed by an appropriate "assignment" and comments on the topic or study materials. The "assignment" consists of a list, in preferred order of study, of readings in Applied Hydrogeology (Fetter, 1988), or readings in either Freeze and Cherry (1979) or Todd (1980), specially prepared notes, and exercises. The notes and exercises are numbered separately and sequentially in each major section of the study guide and are found immediately after the assignment and comments in the subsection in which they are listed.

If the user of this guide is participating in an intensive, short-term workshop, the material in the readings should be covered in lectures and discussion. In this case the readings can function as preparation for the workshop or a review and extended coverage of material afterwards. If a person is engaged in self-study, the readings are an essential part of the study sequence.

The ideal minimum technical background for users of this study guide is (a) 1 year of basic college physics, (b) 1 year of calculus, and (c) one semester of physical geology. Of course, additional background in any or all of these subject areas is highly desirable. A person with a technical background in a subject other than geology will benefit greatly from reading selected parts of basic texts in physical geology, stratigraphy, and structural geology. In addition, although we do not attempt to cover this subject area in the outline, basic chemistry and geochemistry are fundamental to the broad field of ground-water hydrology. Finally, we have taught beginning ground-water hydrology successfully to individuals with less technical background than that outlined above--perhaps the most important prerequisite for learning a new subject is the motivation of the prospective learner.

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INTRODUCTION

Background

Professional expertise in ground-water hydrology is required by a number of Federal agencies. Because such expertise is now generally in short supply, these agencies are faced with the prospect of providing basic training in this discipline to current employees with diverse academic backgrounds. Recognizing that appropriate courses commonly are either not available or inconveniently scheduled at local schools, one possible training option is to use the most knowledgeable in-house ground-water professionals as course instructors.

Purpose and Scope

The purposes of this study guide are to (1) provide a broad selection of study materials that comprise a beginning course in ground-water hydrology and (2) support in-house training by assembling these materials in a form that can be used easily by competent ground-water professionals who may not be experienced teachers to provide technically sound instruction in ground-water hydrology with a minimum of preparation.

The study guide for a beginning course in ground-water hydrology consists of two parts under separate cover. The first part is for course participants and consists of specially prepared notes and exercises, instructions on how to proceed and comments on the material, as well as appropriate readings keyed to three well-known textbooks in ground-water hydrology--Applied Hydrogeology (Second Edition) by C. W. Fetter (1988), Groundwater by R. A. Freeze and J. A. Cherry (1979) and Ground-Water Hydrology by D. K. Todd (1980). Any one of these three textbooks, as well as other available textbooks, are appropriate to use with this study guide. However, for continuity and because of the specific content and manner of presentation, particularly in the introductory chapters, we have adapted the notations and equations used in the book by Fetter (1988) in this study guide.

As implied in the previous paragraph, this study guide is not designed to stand alone, but is designed to be used in conjunction with a textbook in ground-water hydrology. However, for the most part, the notes and exercises in the study guide do stand alone and may be used to advantage individually in training courses without reference to the study guide.

The second part of the study guide under separate cover, "A Study Guide for a Beginning Course in Ground-Water Hydrology: Part II--Instructor's Manual," is for course instructors. It provides completely worked answers to problems, additional comments on the course materials, and additional references keyed to the specific topics in the outline.

The study guide is designed primarily for an intensive 1-week course or workshop (minimum 40 hours); between 30 and 50 percent of this time will be devoted to exercises and the remainder to lectures or reading by participants. Because all the material in the study guide probably cannot be covered in one week, the instructors will be required to make a discretionary selection of material. Additionally, this study guide can provide the basis for longer or shorter workshops, and the instructor can emphasize further an existing topic or add other specialized topics if desired. The study guide also is appropriate for self-paced instruction by highly motivated individuals with a minimum of assistance from a knowledgeable ground-water professional.

Two additional features of this study guide are the notes and exercises. The notes are designed to emphasize ideas and to clarify technical points that frequently cause difficulty and confusion to inexperienced hydrologists and may not receive adequate treatment in standard textbooks. Some of the exercises are more extensive than those usually found in textbooks to provide an additional level of detail and to focus on concepts that we consider to be particularly important.

The most important and unique technical feature of this study guide is the emphasis on the concept of a ground-water system. Generally, this concept is first developed extensively in a more advanced rather than a beginning course in ground-water hydrology. We believe that it is highly desirable to introduce this concept early in a hydrologist's education because it provides the best possible conceptual framework for analyzing and guiding all phases of any investigation related to ground water.

Technical Qualifications for Users of the Study Guide

The ideal minimum technical background for users of this study guide is (a) 1 year of basic college physics, (b) 1 year of calculus, and (c) one semester of physical geology. Of course, additional background in any or all of these subject areas is highly desirable. A person with a technical background in a subject other than geology will benefit greatly from reading selected parts of basic texts in physical geology, stratigraphy, and structural geology. In addition, although we do not attempt to cover this subject area in the outline, basic chemistry and geochemistry are fundamental to the broad field of ground-water hydrology. Finally, we have taught beginning ground-water hydrology successfully to individuals with less technical background than that outlined above--perhaps the most important prerequisite for learning a new subject is the motivation of the prospective learner.

ANNOTATED LIST OF SELECTED REFERENCES IN GROUND-WATER HYDROLOGY

Professionals beginning their career in ground-water hydrology generally are interested in starting their own technical library. The available literature in hydrology is overwhelming in volume and scope. The annotated list below consists of three well-known textbooks, and several publications produced by Federal agencies, primarily the U.S. Geological Survey. These publications are characterized by their technical relevance and generally high technical quality, modest cost, and ready availability.

Textbooks

Fetter, C. W., 1988, Applied hydrogeology: Columbus, Ohio, Merrill Publishing Company, 592 p.

Freeze, R. A., and Cherry J. A., 1979, Groundwater: Englewood Cliffs, New Jersey, Prentice-Hall, 604 p.

Todd, D. K., 1980, Ground-water hydrology: New York, John Wiley and Sons, 535 p.

Although there is a large measure of overlap in these three textbooks, as would be expected, the texts complement each other in their coverage of technical topics. In general, the treatment of solute transport and geochemistry in the text by Freeze and Cherry is more extensive than in the other two texts.

Federal Publications

Bennett, G. D., 1976, Introduction to ground-water hydraulics: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 3, Chapter B2, 172 p.

An excellent introduction to basic mechanics of ground-water flow; ideal for supplementary study in conjunction with this study guide.

Bureau of Reclamation, 1977, Ground water manual: U.S. Department of the Interior, Bureau of Reclamation, 480 p.

Useful as a source of information on field studies--their design, field measurements and procedures.

Environmental Protection Agency, 1987, Handbook of ground water: EPA/625/6-87/016, 212 p.

Useful compilation of ground-water information, particularly information related to ground-water contamination; not organized as a textbook in ground-water hydrology.

Ferris, J. G., Knowles, D. B., Brown, R. H., and Stallman, R. W., 1962, Theory of aquifer tests: U.S. Geological Survey Water-Supply Paper 1536-E, 174 p.

Authoritative introduction to theory and application of aquifer tests and image-well theory.

Franke, O. L., Reilly, T. E., and Bennett, G. D., 1987, Definition of boundary and initial conditions in the analysis of saturated ground-water flow systems--an introduction: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 3, Chapter B5, 15 p.

A concise introduction to boundary conditions used in ground-water hydrology; essential reading for anyone involved in computer simulation.

Haeni, F. P., 1988, Application of seismic-refraction techniques to hydrologic studies: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 2, Chapter D2, 86 p.

Heath, R. C., 1983, Basic ground-water hydrology: U.S. Geological Survey Water-Supply Paper 2220, 84 p.

Concise explanations of and figures illustrating basic ground-water concepts; useful supplementary source of information for this course.

Heath, R. C., 1984, Ground-water regions of the United States: U.S. Geological Survey Water-Supply Paper 2242, 78 p.

Concise overview of ground-water "regions", based on regional geology, in the United States.

Hem, J. D., 1985, Study and interpretation of the chemical characteristics of natural water: U.S. Geological Survey Water-Supply Paper 2254, 264 p.

An indispensable reference for all ground-water hydrologists; included in this list even though geochemistry is not discussed in this beginning course in ground-water hydrology.

Keys, W. S. 1988, Borehole geophysics applied to ground-water hydrology: U.S. Geological Survey Open-File Report 87-539, 305 p.

Lohman, S. W., 1972a, Ground-water hydraulics: U.S. Geological Survey Professional Paper 708, 70 p.

Useful as a reference, particularly for radial-flow problems.

Lohman, S. W. (editor), 1972b, Definitions of selected ground-water terms--revisions and conceptual refinements: U.S. Geological Survey Water-Supply Paper 1988, 21 p.

The most authoritative glossary of ground-water terms that is available.

Rantz, S. E., 1982, Measurement and computation of streamflow: Volume 1. Measurement of stage and discharge: U.S. Geological Survey Water-Supply Paper 2175, 284 p.

Reed, J. E., 1980, Type curves for selected problems of flow to wells in confined aquifers: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 3, Chapter B3, 106 p.

A well documented compilation of analytical solutions for confined radial-flow problems, with associated tables of function values, plotted type curves, and computer programs for calculating function values.

Reilly, T. E., Franke, O. L., and Bennett, G. D., 1987, The principle of superposition and its application in ground-water hydraulics: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 3, Chapter B6, 28 p.

Concept of superposition simply and thoroughly explained; clear discussion of the applications and advantages of using superposition in the simulation of ground-water systems.

Reilly, T. E., Franke, O. L., Buxton, H. T., and Bennett, G. D., 1987, A conceptual framework for ground-water solute-transport studies with emphasis on physical mechanisms of solute movement: U.S. Geological Survey Water-Resources Investigations Report 87-4191, 44 p.

A practical and readable discussion on how to approach and design a field study involving solute transport.

Shuter, E., and Teasdale, W. E., 1989, Application of drilling, coring, and sampling techniques to test holes and wells: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 2, Chapter F1, 97 p.

Stallman, R. W., 1971, Aquifer-test design, observation, and data analysis: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 3, Chapter B1, 26 p.

Basic reference on aquifer-test design.

Zohdy, A. A. R, Eaton, G. P., and Mabey, D. R., 1974, Application of surface geophysics to ground-water investigations: Techniques of Water-Resources Investigations of the U.S. Geological Survey, Book 2, Chapter D1, 116 p.

DETAILED OUTLINE WITH NOTES AND EXERCISES

The study guide is divided into five major sections (see Contents, p. 1): (1) Fundamental concepts and definitions, (2) Principles of ground-water flow and storage, (3) Description and analysis of ground-water systems, (4) Ground-water flow to wells, and (5) Ground-water contamination. Each section is subdivided into a number of subtopics, and each subtopic is followed by an appropriate "assignment" and comments on the topic or study materials. The "assignment" consists of a list in preferred order of study of readings in Applied Hydrogeology (Fetter, 1988), or readings in either Freeze and Cherry (1979) or Todd (1980), specially prepared notes, and exercises. The notes and exercises are numbered separately and sequentially in each major section of the study guide and are found immediately after the assignment and comments in the subsection in which they are listed.

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SECTION (1)--FUNDAMENTAL CONCEPTS AND DEFINITIONS

This section of the study guide provides a background in earth materials, selected hydrologic concepts and features, and physical principles that are sufficient to begin the quantitative study of ground-water hydrology in Section (2).

Dimensions and Conversion of Units

Assignment

*Work Exercise (1-1)--Dimensions and conversion of units.

Conversion of units is a painful necessity in everyday technical life. Tables of conversion factors for common hydrologic variables are found in Fetter (1988), both in the inside cover and several appendixes; Freeze and Cherry (1979), p. 22-23, 29, 526-530, and front inside cover; or Todd (1980), p. 521-525, and back inside cover.

Exercise (1-1)--Dimensions and Conversion of Units

The capability of executing unit conversions accurately, both within the inch-pound system of units and between the inch-pound and metric systems, is a necessity for any professional in a technical field. The purpose of this exercise is simply to serve as a reminder of this fact. Most beginning textbooks in any technical field address this topic. In addition, all engineering handbooks include extensive treatments of dimensions, units, and unit conversions.

Ground-water hydraulics is a specialty within the general field of mechanics. Variables in mechanics possess some combination of three fundamental dimensions--mass (M), length (L), and time (T). Careful analysis of dimensions is a valuable first step in becoming acquainted with unfamiliar variables and (or) formulas.

Below is a list of several conversions to be calculated. Before performing the calculations, test whether the two sets of units are dimensionally compatible. (One or more examples are not compatible.) To perform this test, write a general dimensional formula for each set of units in terms of mass (M), length (L), and time (T). For example, velocity has a general dimensional formula of (LT^{-1}), and force has a general dimensional formula of (MLT^{-2}). As part of the calculations, write out all conversion factors.

(1) 15 ft/d (feet per day) to (a) in/hr (inches per hour), (b) cm/s (centimeters per second)

(2) 200 gal/min (gallons per minute) to (a) ft^3/d, (b) cm^3/s

(3) 500 gal/d•ft^2 to (a) ft^2/d, (b) m^2/d

(4) 250 ft^2/d to (a) gal/d•ft, (b) cm^2/s

(5) 500,000 gal/d•mi^2 to (a) in/yr, (b) cm/d.

Water Budgets

Assignments

*Study Fetter (1988), p. 1-12, 15-24, 446-448; Freeze and Cherry (1979), p. 203-207, 364-367; or Todd (1980), p. 353-358.

*Work Exercise (1-2)--Water budgets and the hydrologic equation.

The preparation of an approximate water budget is an important first step in many hydrologic investigations. Unfortunately, the only two budget components that we can measure directly and do measure routinely are precipitation and streamflow. Evapotranspiration, the "great unknown" in hydrology, can be estimated by various indirect means, and estimates of subsurface flows also usually are subject to considerable uncertainty. The reasons for the uncertainty in subsurface-flow estimates are addressed later in this course.

In Exercise (1-2) and the accompanying discussion on water budgets, the following points are emphasized: (a) the differentiation between inflows and outflows from a basin as a whole and flows within the basin, (b) the possible specific inflow and outflow components of the saturated ground-water part of the hydrologic system, and (c) the necessity of defining clearly a reference volume when a water budget that focuses on the saturated ground-water part of the system is undertaken. This reference volume will be discussed again in later sections of the report that focus on the development of concepts specifically related to ground-water systems.

Exercise (1-2)--Water Budgets and the Hydrologic Equation

The following notes and problems assume previous reading and (or) discussion on the continuity principle, as represented in hydrology by the "hydrologic equation" or "water-budget equation"--that is,

$$\text{Inflow} = \text{Outflow} \pm \Delta \text{ Storage}$$

(where Δ means "change in")--and the various components of the hydrologic cycle. The continuity principle will be encountered again later in this course as the starting point for developing the basic differential equation of ground-water flow. The focus here is the application of this principle to the preparation of water budgets for hydrologic systems. It is conceptually useful to note, however, that the continuity principle is applicable at all physical scales, not only in hydrology, but also in other fields of science and technology.

The purpose of a water-budget analysis is to quantify, to the extent that data and time permit, the various fluxes[1] to, from, and within the hydrologic system. Many of the budget components or fluxes can be only roughly approximated in most systems (for example, ground-water evapotranspiration), and even the best "estimates" (for example, ground-water contribution to streamflow, which involves a base-flow separation) may involve considerable uncertainty. The best results are achieved by estimating each component in as many different ways as possible and, by continuous checking and comparisons, establishing ranges of uncertainty for each estimate and making certain that the estimates for the different components are consistent with one another. Usually, average water budgets for several years are prepared, as opposed to a water budget for a single year, so that changes in storage in the hydrologic system are small relative to other budget components and need not be considered. This approach implies use of the steady-state or equilibrium form of the hydrologic equation,

$$\text{Inflow} = \text{Outflow}.$$

Flow diagrams for the hydrologic system of central and eastern Long Island, New York under predevelopment and developed conditions are shown in figures 1-1 and 1-2, respectively. Although these diagrams were prepared for Long Island, they can be modified easily to accommodate local conditions. In these diagrams the "boxes" (atmosphere, land surface, zone of aeration) represent sites within or components of the hydrologic system, and the lines with arrows between boxes represent some of the major flow paths of water between the various sites. Most of these sites represented by boxes are also hydrologic storage sites; that is, some quantity of water is nearly always in storage at these sites. This quantity of stored water changes with time.

In figures 1-1 and 1-2, a larger rectangle encloses a number of boxes and flow lines. This larger rectangle represents the boundaries of the hydrologic system that has been isolated for study. In map view, boundaries of hydrologic systems usually are defined by the topographic drainage areas of streamflow-measuring stations. In terms of water budgets, a distinction is made between budgets for a river basin "as a whole" and water-budget components within the river-basin hydrologic system. In water budgets for the basin "as a whole", inflow components generally consist of precipitation, and outflow includes total evapotranspiration, surface-water outflow, and

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[1] The term flux refers to the rate of flow or transfer of some entity such as water, heat, electricity, mass, number of particles, and so on; more specifically, it is the quantity that crosses a unit area of a given surface in a unit of time. For example, heat flux might have the units of calories/$cm^2 \cdot s$ (calories per square centimeter per second); mass flux might have the units of $g/m^2 \cdot d$ (grams per square meter per day). In hydrology we often refer to the transfer or movement of water as a flux. A flux of water can be expressed as a volume flux with possible units, for example, of $ft^3/ft^2 \cdot d$ (cubic feet per square foot per day) or $m^3/m^2 \cdot s$ (cubic meters per square meter per second. Thus, a volume flux has the units of length divided by time. Sometimes, we refer loosely to a volumetric flow rate with units of volume divided by time as a flux. Often, in such cases, an area across which this volumetric flow rate is transferred is implied but not defined or taken into account explicitly.

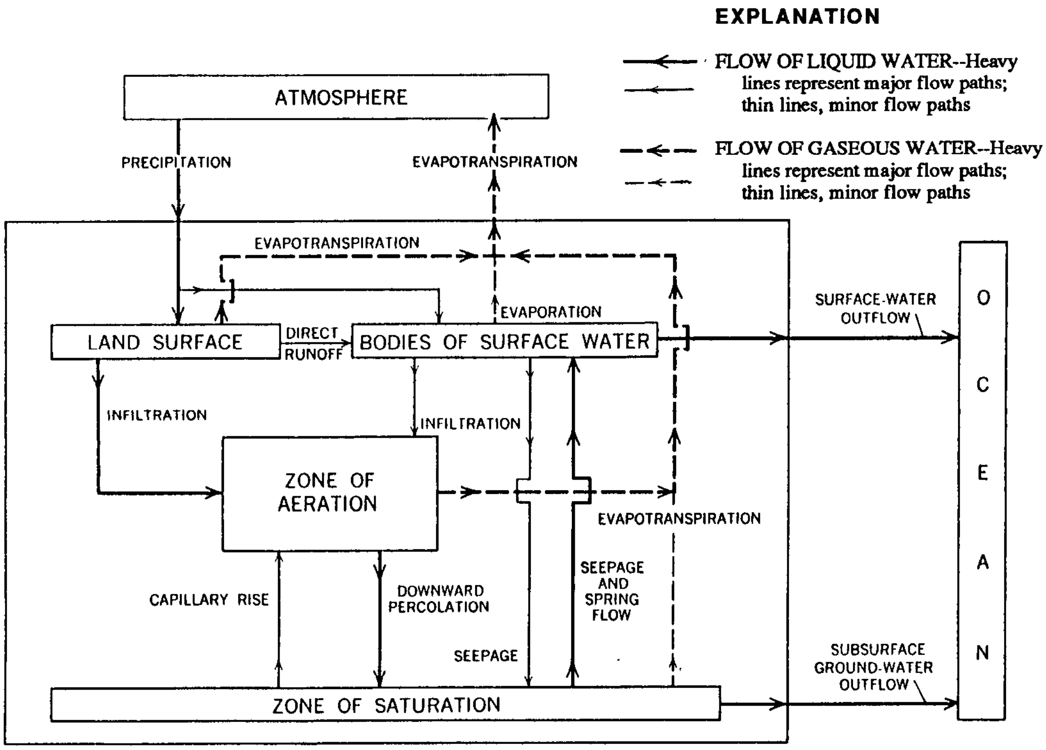

Figure 1-1.--Flow diagram of the hydrologic system, Nassau and Suffolk Counties, Long Island, New York, under predevelopment conditions. (From Franke and McClymonds, 1972, fig. 13.)

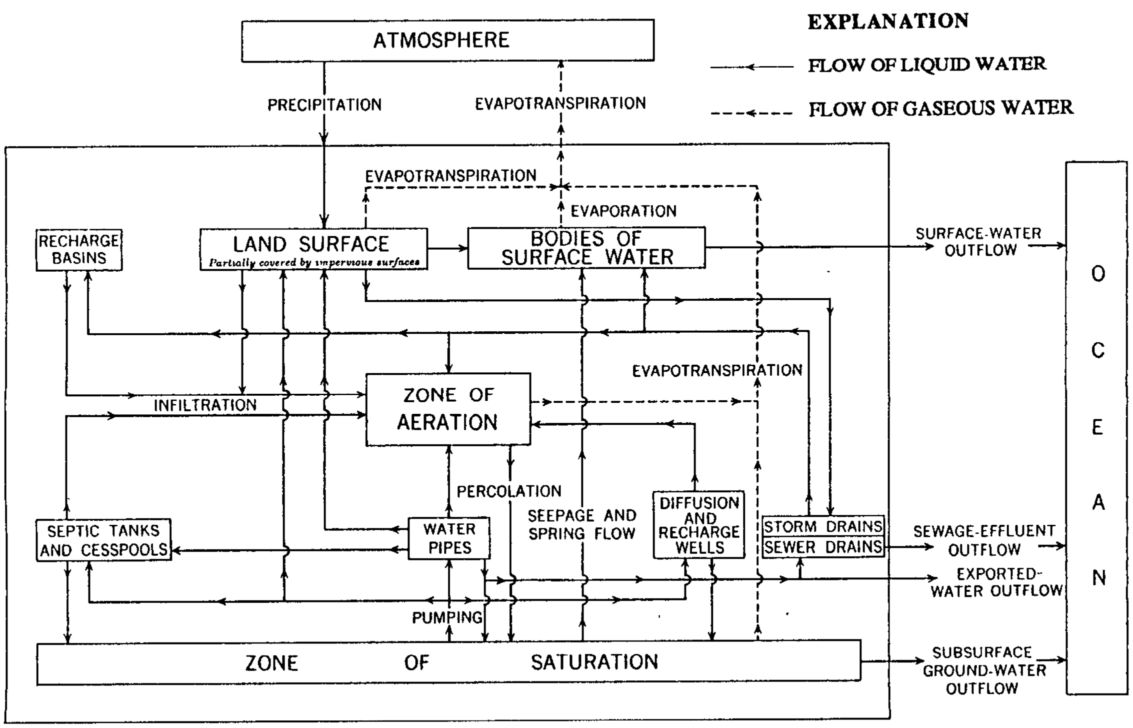

Figure 1-2.--Flow diagram of the hydrologic system, Nassau and Suffolk Counties, Long Island, New York, after noticeable influence from human activities. (From Franke and McClymonds, 1972, fig. 33.)

subsurface ground-water outflow (fig. 1-1), whereas ground-water flow to surface-water bodies, for example, occurs within the boundaries of the water-budget reference volume. This internal contribution to surface water becomes a part of the total measured surface-water outflow from the basin "as a whole" (fig. 1-1).

As implied in the previous discussion, preparation of a water budget requires the careful definition of a reference volume. Inflows and outflows of water occur across the surfaces of this reference volume and changes in storage occur within it. As noted above, however, in many basin studies water budgets are related to the area of the basin. This approach is valid only when most of the ground-water inflow occurs locally as recharge from precipitation, when ground-water outflow occurs as local discharge to streams, and when both flows occur within the basin boundaries. An explicit reference volume must be defined as the initial step in a water-budget analysis whenever (1) inflow and outflow of deeper percolating ground water represent a significant quantity of water relative to other water-budget components, or (2) the ground-water system is the focus of the investigation. In ground-water studies, delineation of an appropriate volume of saturated earth material for study (the ground-water system) is required not only for the preparation of water budgets but also for carrying out additional study elements including computer simulation. How the boundaries of this volume of saturated earth material are delineated comprises one of the most important decisions in the entire investigation.

To this point we have considered long-term average water budgets for which changes in storage between the beginning and end of the water-budget period are so small relative to total inflow and outflow for the water-budget period that we may assume Inflow = Outflow. Generally, however, inflow will not equal outflow in drainage-basin water budgets except fortuitously for water-budget periods of 1 year or less. The transient water-budget equation is written conveniently in the form

$$\text{Inflow} - \text{Outflow} = \pm \Delta \text{ Storage}$$

We will clarify the meaning of + or -change in storage on the right-hand side of the budget equation by means of a hypothetical numerical example. Let us assume that inflow = 10 units and outflow = 8 units, that is, inflow is greater than outflow. Then 10 units -8 units = +2 units change in storage. If we remember that estimates of water-budget components relate to a reference volume,

Inflow -----> Reference Volume -----> Outflow

then we can interpret the +2 units change in storage as an increase in water storage within the reference volume. Similarly, as long as the change in storage term is written on the right-hand side of the budget equation, a minus (-) change in storage means a decrease in water storage within the reference volume, that is, inflow is less than outflow.

Because the focus of this study guide is ground water, possible fluxes to and from the saturated zone under natural conditions are summarized in table 1-1 for reference.

Table 1-1.--Summary of possible fluxes to and from the saturated zone under natural conditions

Inflow ----------------------->	Saturated Zone[1]	Outflow ----------------------->
(1) From unsaturated zone--through-flow of "gravity" water to water table (intermittent areal recharge) (2) From bodies of surface water-- (a) recharge from losing streams (b) recharge from surface water bodies in flood stage (increase in bank storage)		(1) To bodies of surface water--streams, lakes, or saltwater bodies (bays, estuaries, or oceans) and springs (a) steady release of ground water in (relatively) long-term storage (b) relatively rapid, short-term release of ground water in bank storage caused by rapid fluctuations in stage of surface-water bodies (2) To atmosphere--ground-water evapotranspiration (plants derive moisture from capillary fringe)

[1] Changes in storage in the saturated zone are manifested by changes in ground-water levels. See later section on ground-water storage.

Exercise on Water Budgets

The following data refer to a hypothetical river basin in a coastal plain with a drainage area of 250 mi^2 (square miles). The data represent long-term average annual values and are assumed to be exact (never the case in the real world). Before answering the questions below, enter the known budget values next to the appropriate "flow" line between boxes on figure 1-1. If a water budget doesn't balance, what missing information might account for the discrepancy?

Data

Precipitation, 45 in/yr (inches/year); ground-water recharge, 20 in/yr; direct runoff, 1 in/yr; subsurface outflow, 8 in/yr; total evapotranspiration, 25 in/yr; streamflow, 12 in/yr; ground-water contribution to streams (base flow), 11 in/yr.

Questions

(1) Prepare a water budget for the basin as a whole using the principal inflow and outflow components. List inflow and outflow components in separate columns accompanied by long-term average quantities of water.

(2) Prepare a water budget for the streams (surface-water bodies). What possible budget components are neglected?

(3) Prepare a water budget for the ground-water reservoir (saturated zone). What possible budget components are neglected?

(4) What is the average volume of precipitation during one year over the whole basin, in ft^3 (cubic feet)?

(5) Express the average volume of precipitation for one year in (4) as a rate, in ft^3/s (cubic feet per second) and as a rate per unit area, in Mgal/d (millions of gallons per day) per square mile.

(6) For the same basin in a given year the following water budget figures are assumed to be correct: Precipitation, 35 in.; total evapotranspiration, 20 in.; streamflow, 10 in.; subsurface outflow, 7 in.. Write a formal water-budget equation using these figures. Write the water-budget equation in words and then a second time using the available numbers. What is the probable cause of the discrepancy and how must this factor be included in the water-budget equation?

Characteristics of Earth Materials Related to Hydrogeology

Assignments

*Study Fetter (1988), p. 63-73; Freeze and Cherry (1979), p. 29, 36-38; or Todd (1980, p. 25-31, 37-39.

*Look up and write the definitions of the following terms describing the flow medium in Fetter (1988), both in the glossary and in the index--isotropic, anisotropic, homogeneous, and heterogeneous.

In considering earth materials from the hydrogeologic viewpoint, the first level of differentiation generally is between consolidated and unconsolidated earth materials. In many ground-water studies, the thickness of the unconsolidated materials above bedrock defines the most permeable part of the ground-water system.

Relevant characteristics of earth materials from the hydrogeologic viewpoint include (a) mineralogy, (b) grain-size distribution of unconsolidated materials, (c) size and geometry of openings in consolidated rocks, (d) porosity, (e) permeability (hydraulic conductivity), and (f) specific yield.

Mineralogy is included in this list because it is one of the principal bases for the geologic classification of consolidated rocks, and it exerts a significant influence on the geochemical evolution of ground water, a topic which is not discussed in this course. Permeability and specific yield, included here to make the list of relevant characteristics more complete, will be defined and discussed later in the course.

Occurrence of Subsurface Water

Assignments

*Study Fetter (1988), p. 85-95, 99-101; Freeze and Cherry (1979), p. 38-41; or Todd (1980), p. 31-36.

Subsurface water generally is considered to occur in three zones--(a) the unsaturated zone, (b) the capillary or tension saturated zone, and (c) the saturated zone. The water table in coarse earth materials may be defined approximately as the upper bounding surface of the saturated zone. The main focus in this study guide is the saturated zone; however, hydrologic processes in the shallow saturated zone are controlled largely by physical processes in the overlying unsaturated zone. For example, most recharge to the water table must traverse some thickness of the unsaturated zone.

Pressure and Hydraulic Head

Assignments

*Work Exercise (1-3)--Hydrostatic pressure.

*Study Fetter (1988), p. 115-122; Freeze and Cherry (1979), p. 18-22; or Todd (1980), p. 65, 434-436.

*Study Note (1-1)--Piezometers and measurement of pressure and head.

*Work Exercise (1-4)--Hydraulic head.

Hydraulic head[1] is one of the key concepts in ground-water hydrology. However, it is a concept that remains confusing to many practitioners. Working with the concept will increase understanding.

The first assignment in this section is a review of hydrostatic pressure (Exercise (1-3)). This review provides background for the head concept which is developed in the reading from Fetter (1988). These concepts are developed further in Note (1-1) on the measurement of pressure and head in piezometers and wells. Practice in differentiating between the two components of hydraulic head--pressure head and elevation head, is provided in Exercise (1-4).

Exercise (1-3)--Hydrostatic Pressure

This exercise reviews the calculation of static fluid pressure, particularly the pressure exerted by a column of liquid whose upper surface is subject to atmospheric pressure. Further treatment of this topic may be found in any basic text on college physics or fluid mechanics.

The following definitions are provided for reference:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad [ML^{-1}T^{-2}]$$

$$\text{Hydrostatic pressure} = \frac{\text{Weight of fluid column}}{\text{Area}}$$

$$\text{By definition } \gamma = \rho g \quad \frac{\text{mass}}{\text{volume}} \bullet g = \frac{\text{weight}}{\text{volume}}$$

[1] Synonymous terms include "ground-water head," "total head," and "potentiometric head." We recommend and use in this course "hydraulic head," or simply "head."

where γ is weight density or specific weight $\left(\frac{\text{weight}}{\text{volume}}\right)$,

ρ is mass density $\left(\frac{\text{mass}}{\text{volume}}\right)$, and

g is acceleration due to gravity.

A reference prism of liquid that extends to a depth l in a larger body of static liquid bounded above by a free surface (a surface subject to atmospheric pressure) is shown in figure 1-3. The total force acting on the bottom face of the prism with area A is the sum of the forces exerted by atmospheric pressure on the top area of the prism plus the weight of the fluid column bounded by the prism (weight of fluid column = $\gamma l A$) or

$$P_t A = P_a A + \gamma_f l A \tag{1}$$

where P_t is the total pressure exerted at depth l in the liquid, P_a is atmospheric pressure, and γ_f is the weight density of the liquid. Dividing by A we have

$$P_t = P_a + \gamma_f l. \tag{2}$$

By convention, in hydraulics and fluid mechanics we generally do not work with total pressure, but with "gage" pressure--that is, the pressure exerted by the static liquid alone. Atmospheric pressure is regarded as an environmental constant that need not be taken into account explicitly. From (2) the pressure exerted by the static column of liquid P_f is

$$P_f = \gamma_f l. \tag{3}$$

From (3) the length of the fluid column l may be expressed as

$$l = \frac{P_f}{\gamma_f}. \tag{4}$$

In most developments of these relations the letter h is used instead of l to designate the vertical length of the fluid column under consideration. We use l because h generally is used to designate head in ground-water hydraulics (see later discussion on head).

Questions:

(1) Assuming that l is 12 feet and the liquid is fresh water (fig. 1-3), what is the water pressure acting at depth l in (a) lbs/ft^2 (pounds per square foot) and (b) lbs/in^2 (pounds per square inch)? (c) What is the "total" pressure acting at depth l?

(2) If the liquid is normal sea water, what is the fluid pressure acting at l = 12 ft, in lbs/in^2?

Constants for calculations:

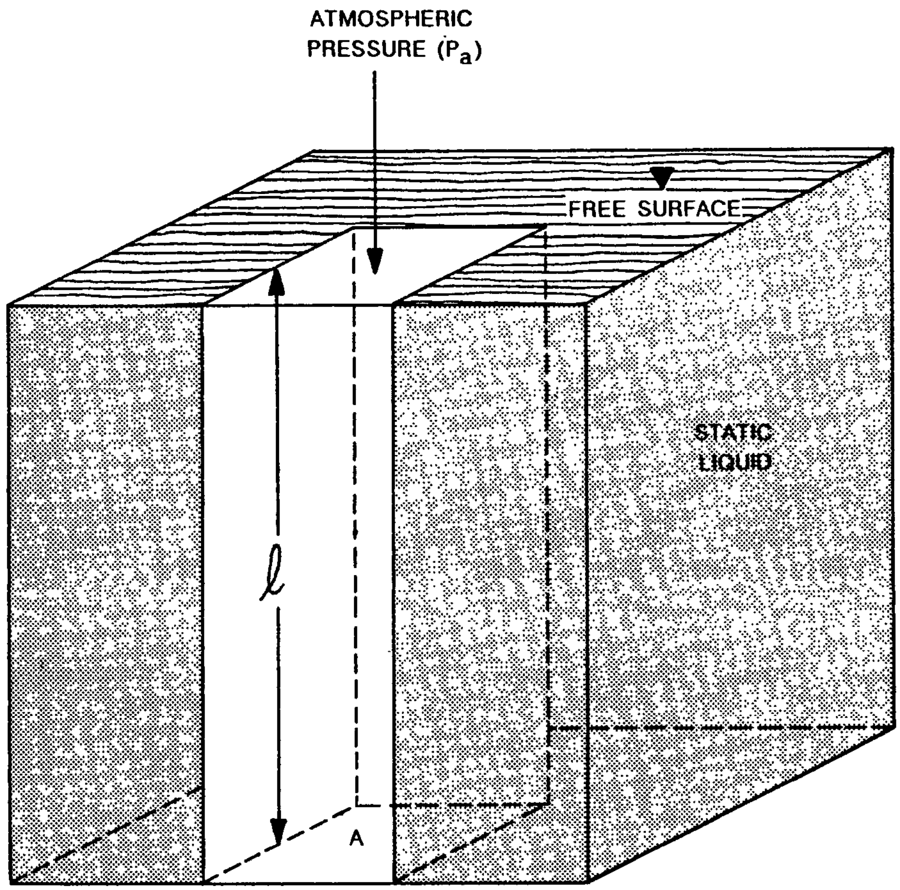

Figure 1-3.--Vertical reference prism of liquid that extends to a depth l in a larger body of static liquid bounded above by a free surface.

$\gamma_{\text{fresh water}} = 62.4 \text{ lbs/ft}^3 = 9.8 \times 10^3 \text{ N/m}^3$ (Newtons per cubic meter)

$\rho_{\text{fresh water}} = 1{,}000 \text{ kg/m}^3$ (kilograms per cubic meter)

$\rho_{\text{sea water}} = 1{,}025 \text{ kg/m}^3$

$P_{\text{atmospheric}} \approx 14.7 \text{ lbs/in}^2$

Note (1-1)--Piezometers and Measurement of Pressure and Head.

In hydraulics a piezometer is a pressure-measuring device consisting of a tube, one end of which taps the fluid system and the other end of which is open to the atmosphere (fig. 1-4). Pressure is measured at the point where the piezometer taps the fluid system (fig. 1-4)[1] and is proportional to the vertical height ℓ of the fluid column in the piezometer above the measuring point. With reference to the preceeding discussion of hydrostatic pressure, the pressure at the point of measurement is calculated using the formula

$$P_{\text{point of pressure measurement}} = \gamma_{\text{fluid}}\,\ell. \qquad (1)$$

In ground-water hydraulics a piezometer is a tightly cased well, usually of small diameter (4 in. or less), with a single, short (generally, 10 ft or less in length) well screen (fig. 1-5). For this discussion, we arbitrarily assume that the point of pressure measurement of the piezometer is at the midpoint of the screened interval. Usually, piezometers are installed for the specific purpose of measuring pressure and head at the piezometer's point of measurement.

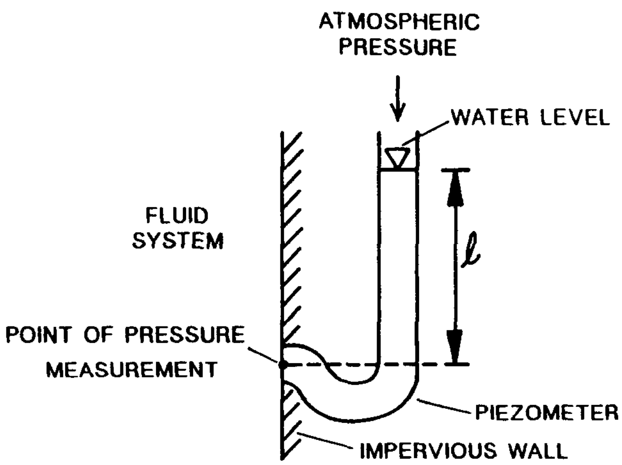

Figure 1-4.--A typical piezometer in a hydraulic system.

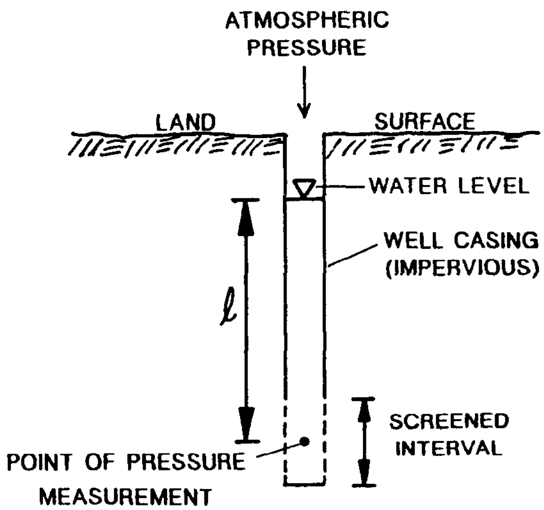

Figure 1-5.--A typical piezometer in a ground-water system.

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[1] In hydraulics other conventions sometimes are used for convenience; for example, a piezometer that taps a pipe flowing full generally is assumed to measure static pressure at the centerline of the pipe.

Wells installed for pumping ground water commonly have larger diameters and longer screened intervals than piezometers. Heads measured in wells with long screens effectively represent average heads in the aquifer opposite the screened interval. Generally, wells that are screened in more than one interval are not suitable for head measurements. The designation "observation well" is used widely. An observation well is used primarily to measure head and may be either a piezometer or a well as defined above.

Field measurements of head in a piezometer or well involve measurement of a depth to water (fig. 1-6 and following discussion) and thus require the identification and description of a fixed reference point or "depth-to-water measuring point" to which all field measurements are referred. This depth-to-water measuring point usually is a point at the top of the well casing, well cap, or access hole (fig. 1-6). As will become evident in the following discussion, an "accurate" determination of head requires that the altitude of the depth-to-water measuring point be accurately surveyed. Considerably less accurate determinations of the altitude of the depth-to-water "measuring point" (and also of head) are obtained by estimating

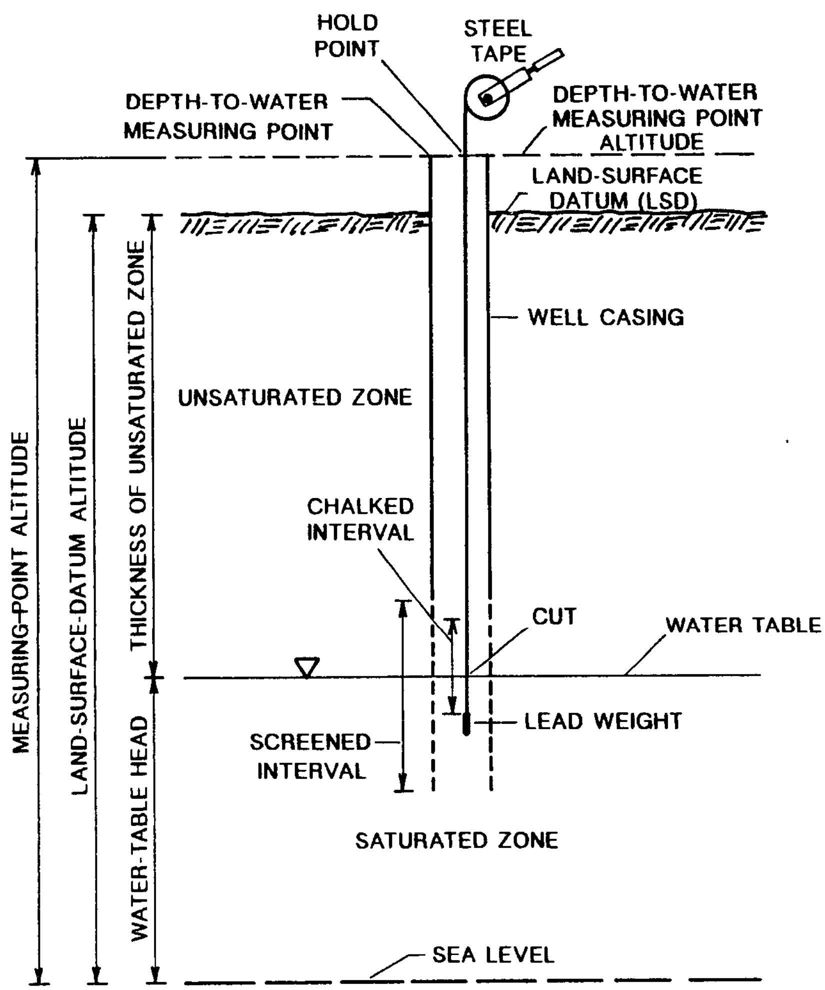

Figure 1-6.--Measurement of head in a well.

the land-surface altitude at the well from topographic maps and adding the measured vertical distance from the depth-to-water measuring point to the land-surface altitude (see following section).

Procedure for Making Accurate Head Measurements

The most used and generally most accurate method for making a head measurement utilizes a graduated steel tape with a weight attached to its end. The first step is to cover the bottom several feet of the tape with blue carpenter's chalk. Place the tape into the opening of the well and pull the first few feet of tape out by hand; then use the crank to lower the tape slowly down the well. When the desired level has been reached, hold the tape to the nearest whole number of feet at the depth-to-water measuring point (MP) of the well (fig. 1-6), making sure that the tape goes straight down from the MP and is not bent over the lip of the well. This number is the "hold" value.

While holding the tape firmly, slowly back away from the well 1 or 2 feet, and then slowly wind up the tape until the water mark or "cut" is visible. In standard practice, the "cut" value is read and recorded to the nearest hundredth of a foot (about 0.3 cm). If the wet mark cannot be read clearly, dry the tape and repeat the process. After recording both the "hold" and "cut" values, calculate the depth to water (DTW) (see sample calculations below). Repeat the process to insure accuracy, but this time extend the "hold" value an additional foot and determine whether the two DTW values are within an acceptable range of one another. The value of measured head is determined by subtracting the DTW from the MP elevation. The preceeding discussion indicates that field measurement of head is in essence a measurement of depth to water in a well from the depth-to-water measuring point.

In figure 1-6 the screened interval of the observation well intersects the water table and extends only a few feet below it. Thus, in this special case the head measurement in the observation well equals the adjacent altitude of the water table, and, therefore, represents the actual top of the saturated ground-water system.

Sample Head-Measurement Calculations:

Given well information:	Depth-to-water measuring point (MP) altitude	= 100.00 ft
	Land-surface datum (LSD)	= 97.00 ft

To determine depth to water (DTW) inside well casing:

Hold value = 75.00 ft

Cut value = 3.25 ft

DTW = (75.00 ft) - (3.25 ft) = 71.75 ft

To determine head:

MP altitude = 100.00 ft above sea level

DTW = 71.75 ft

Head = (100.00 ft) - (71.75 ft) =

28.25 ft above sea level = altitude of water table

To determine unsaturated-zone thickness (depth to water table below land surface):

LSD = 97.00 ft

Water-table altitude = 28.25 ft

Unsaturated zone thickness = (97.00 ft) - (28.25 ft) =

68.75 ft

Variability of Head with Depth

Three pairs of observation wells are shown in figure 1-7. Each pair consists of one shallow observation well whose screened interval intersects the water table as shown in figure 1-6 and one deeper observation well whose screened interval is a considerable depth below the water table. In figures 1-7(A), (B), and (C) the heads in the deeper wells are less than, equal to, and greater than the heads in the immediately adjacent shallow observation wells, respectively. The water levels in the casings of the deeper observation wells do not represent the position of the top of the saturated deposits, but do represent hydraulic heads at the point of pressure measurement of these observation wells.

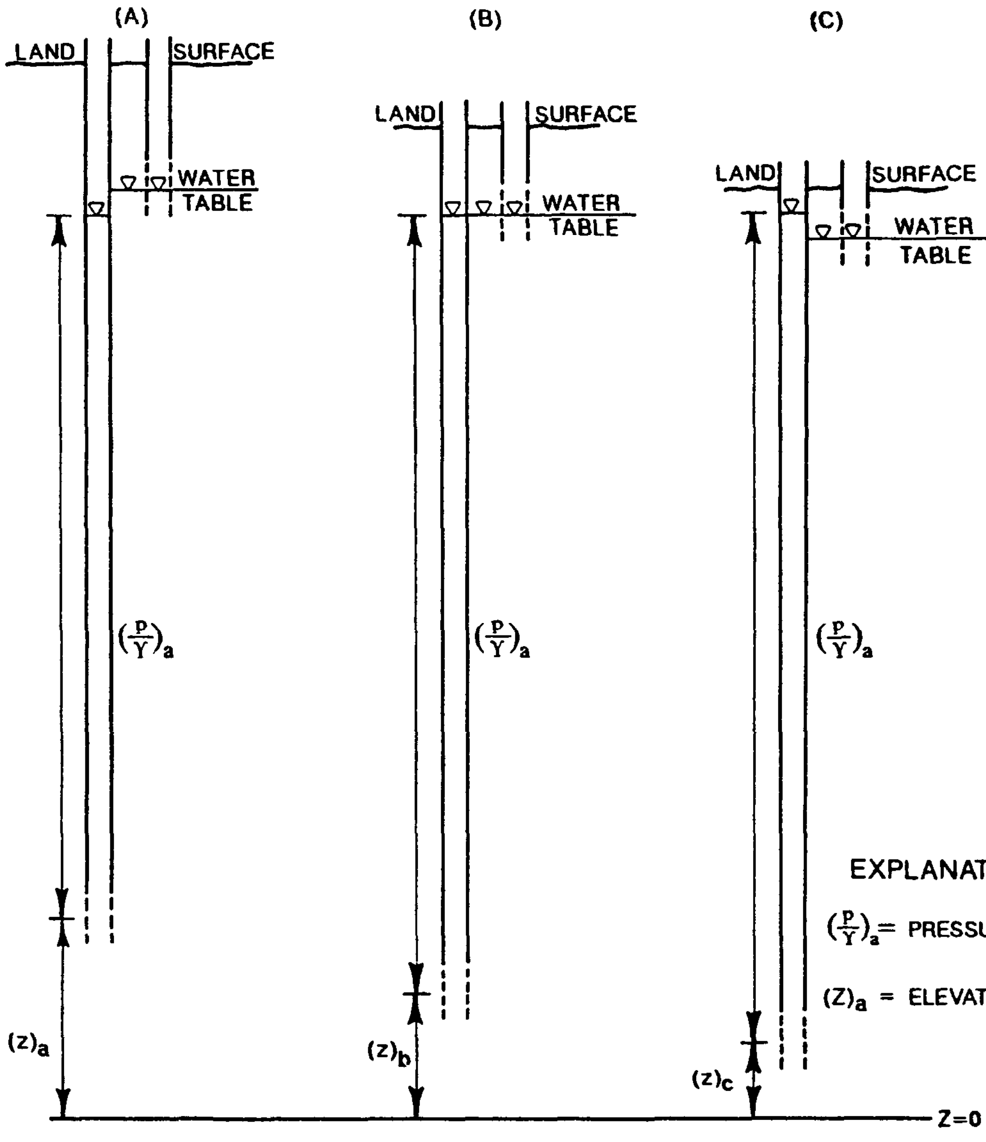

Figure 1-7.--Three pairs of observation wells in a hypothetical ground-water system; in each pair one observation well is screened at the water table and one is screened at some depth below the water table. In pair (A) head at the water table is greater than head in the deeper observation well; in pair (B) head at the water table equals head in the deeper observation well; in pair (C) head at the water table is less than head in the deeper observation well.

The relation between heads at the water table and heads in adjacent wells whose screened intervals lie at some depth below the water table depends on the position of the observation-well pair in the associated ground-water system. A general interpretation of the head relations depicted in figure 1-7 must wait for a more comprehensive discussion of ground-water systems in Section (3) of this course. The purpose of presenting figure 1-7 at this time is to emphasize that, in general, hydraulic head in ground-water systems varies not only with geographic location but also with depth.

Exercise (1-4)--Hydraulic Head

The purpose of this exercise is to provide practice in differentiating between the two components of head--pressure head and elevation head. The elevation head at a point in a ground-water system is arbitrary in that it depends on the altitude of an arbitrary datum. Sea level generally is used as head datum, the same datum used for land-surface topographic maps. However, the pressure head at a given point and a given time is not arbitrary, but is a physical quantity that can be measured directly. It is directly proportional to the height of the fluid column above the point of pressure measurement in a piezometer or observation well.

The data below are available for three closely spaced (in map view) observation wells with short well screens.

(1) Determine the missing entries in table 1-2.

(2) Make a careful sketch of each observation well on the accompanying worksheet (fig. 1-8). Plot and designate on each sketch the pressure head, elevation head, and total hydraulic head.

Table 1-2.--Head data for three closely spaced observation wells

Well	Land-surface altitude (feet above sea level)	Depth of top of screen below land surface (feet)	Depth to water (feet)	Altitude of water-level surface in well[1] (feet above sea level)	Pressure head (p/γ) (feet)	Elevation head (z) (feet)
1	50	25	15			
2	45	90	9			
3	51	350	13			

[1] Altitude of water-level surface in observation well equals hydraulic head at point of pressure measurement of observation well.

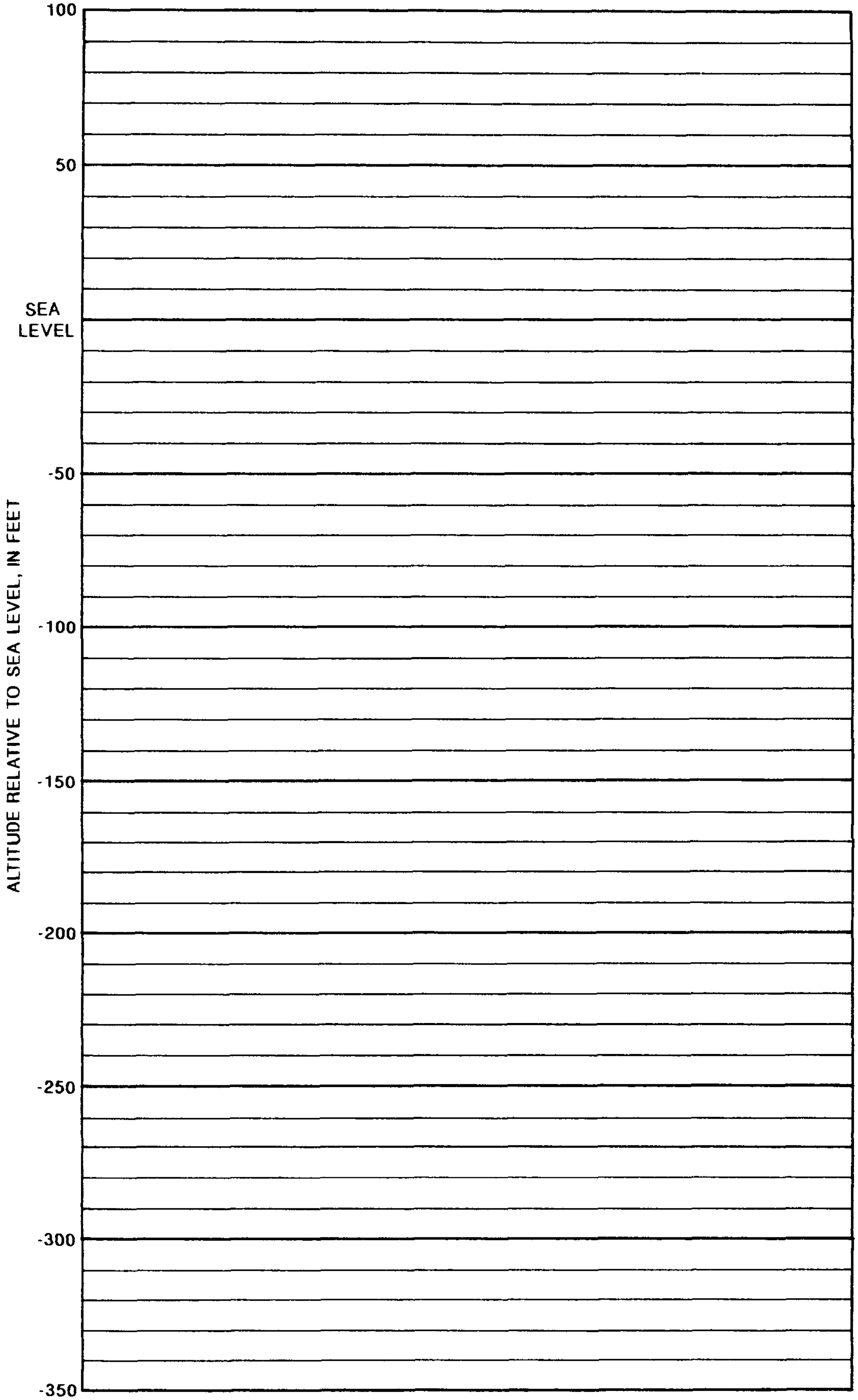

Figure 1-8.--Worksheet for head exercise.

An important rule concerning the value of hydraulic head in a stationary fluid body is that within such a body the *hydraulic head* is a constant at every point, including points along any boundary surface, regardless of the boundary-surface configuration. To visualize this concept, consider piezometers at various depths in a body of stationary fluid (fig. 1-9). At the surface of the fluid body (piezometer A), where the fluid is in contact with the atmosphere, $h = z$ because $p/\gamma = 0$. As one moves the piezometer downward from the fluid surface (piezometers B and C), the increase in pressure head (p/γ) is exactly balanced by a decrease in elevation head (z); thus, h remains constant. This relation will be useful when we consider physical boundaries between saturated ground water and surface-water bodies (for example, the streambed of a gaining stream); it indicates that the hydraulic head acting on such boundaries is equal to the water-level altitude of the surface-water body above the boundary, regardless of the surface configuration of this boundary.

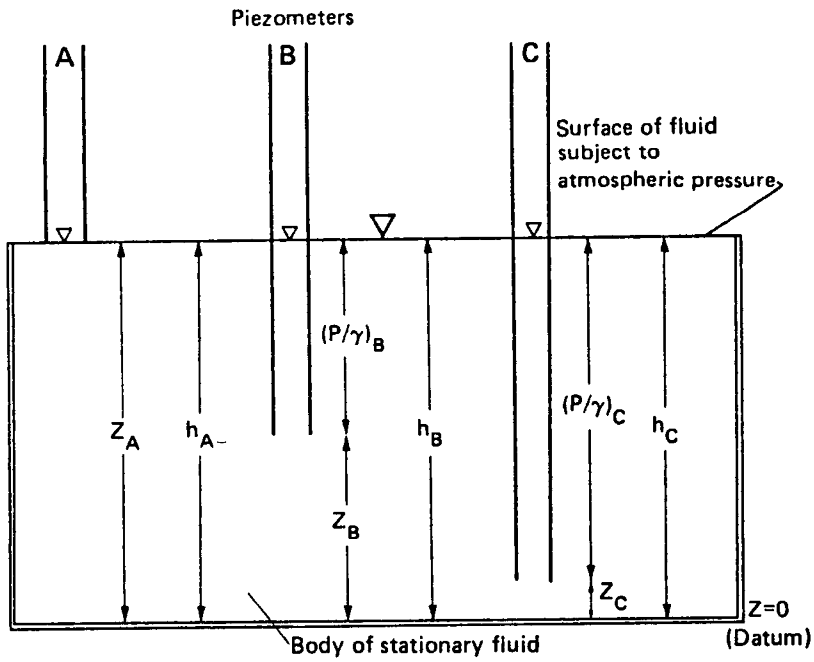

Figure 1-9.--Piezometers at three different depths, demonstrating that the total head at all depths in a continuous body of stationary fluid is constant. (From Franke and others, 1987, fig. 2.)

Preparation and Interpretation of Water-Table Maps

Assignments

*Study Fetter (1988), p. 136-137; Freeze and Cherry (1979), p. 45; or Todd (1980), p. 42-43, 85-88.

*Work Exercise (1-5)--Head gradients and the direction of ground-water flow.

The concept and procedure of contouring point data are familiar to geologists, meteorologists, and other scientists. At any given time the water table may be regarded as a topographic surface that lies for the most part below the land surface, the most familiar topographic surface. We measure water-table altitudes in shallow wells. The locations of the wells are plotted accurately on a map along with their associated water-table elevations. The objective is to develop the best possible representation of the water-table surface based on a few scattered water-table measurements at points. A water-table map is constructed by drawing contour lines of equal water-table elevation (equipotential lines or head contours)[1] at convenient intervals, using approximate linear interpolation between point measurements.

Head gradients commonly are estimated from water-table maps as demonstrated in Exercise (1-5). These gradient estimates necessarily are based on a two-dimensional representation of the equipotential surface. In nature, however, equipotential surfaces are inherently three-dimensional. Although "two-dimensional" gradients are adequate for many purposes, their use occasionally may lead to significant errors.

Exercise (1-5)--Head Gradients and the Direction of Ground-Water Flow

The purpose of this exercise is to gain familiarity with the concept of a head gradient and related direction of ground-water flow. We assume previous reading and (or) discussion of head-contour maps. Head-contour lines commonly are referred to as potential lines (lines of equal potential) or equipotential lines. We often forget that contour lines of equal head on a map are projections in map view of three-dimensional surfaces of equal head.

A gradient is the rate of change of a spatially continuous variable per unit distance in the direction of its maximum rate of change. We are concerned with head gradients. The spatially continuous variable is head measured in piezometers or observation wells.

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[1] In ground-water hydraulics the terms potential line, equipotential line, line of constant head, and head contour are used interchangeably. These terms also apply to surfaces of constant head or constant potential; for example, equipotential surface.

A formula for average head gradient (i) has the form $i = \frac{h_1 - h_2}{l}$ or $\frac{\Delta h}{l}$ [the symbol Δ (delta) means "change in"], where h_1 and h_2 are heads at a distance l apart, and the distance l is in the direction of the maximum rate of change of head. Because both the numerator and denominator of i have the dimensions of length, i is dimensionless.

In plan view, based on the assumption that ground-water flowlines or streamlines[1] are perpendicular to head contour lines[2], an average gradient can be calculated between any two points on the same streamline at which heads are known. Usually, however, average gradients are most useful when the length of streamline (l) between the points of known (or estimated) heads is small relative to the scale of the ground-water system under study.

Until now, we have discussed only average head gradients. In the following questions, bear in mind the difference between the average gradient between two points on a streamline and the gradient "at a point" on a streamline.

Three head-contour maps illustrating different contour patterns are shown in figure 1-10. With reference to this figure, answer the following questions.

(1) (a) With reference to figure 1-10(A), on the graph opposite figure 1-10(A) plot a topographic profile of the equipotential surface in the neighborhood of point A.

(b) Describe the pattern of head-contour lines in figure 1-10(A).

(c) Determine the gradient (maximum slope of the equipotential surface) at A. In this case the average head gradient in the neighborhood of A and the gradient at point A are ______________________________.

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[1] In ground-water hydraulics the terms "flowline" and "streamline" are used interchangeably. They mean the smoothed or average path of water particles between two points in the ground-water flow field. A more formal definition of streamline is "a line drawn in the fluid so that its tangent at each point is in the direction of the fluid velocity at that point" (Milne-Thomson, 1955, p. 5).

[2] In this discussion, we assume the rule that ground-water streamlines are perpendicular to equipotential lines. This rule, which is strictly true only if the flow medium (earth material) is isotropic and homogeneous, often is utilized as a reasonable approximation when the aquifer material is fairly homogeneous, and the aquifer and ground-water streamlines within the aquifer are nearly horizontal.

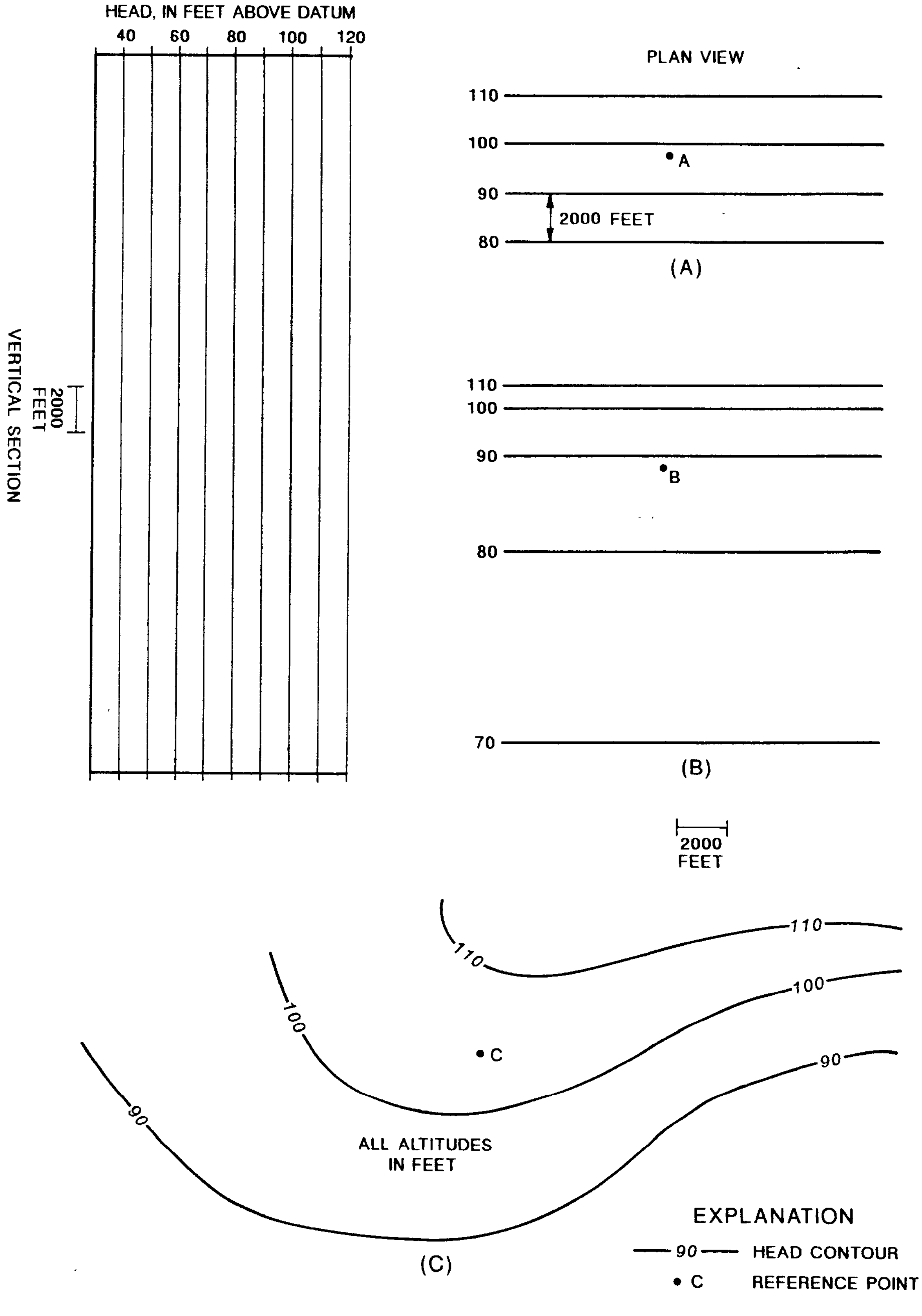


Figure 1-10.--Maps of ground-water head illustrating three different contour patterns.

(2) (a) With reference to figure 1-10(B), plot a topographic profile of the equipotential surface in the neighborhood of point B.

(b) Describe the pattern of head-contour lines in figure 1-10(B).

(c) Approximate an average gradient in the neighborhood of point B.

(3) Describe carefully a procedure for determining an average gradient in the neighborhood of point C in figure 1-10(C).

(4) Refer to the previous exercise, Exercise (1-4)--Hydraulic Head. Calculate the vertical component of the gradient between wells 1 and 2, which are located adjacent to one another. How is l in the gradient formula defined in this situation?

The following excerpt from Heath (1983, p. 10-11) provides additional discussion on head gradients and direction of ground-water flow.

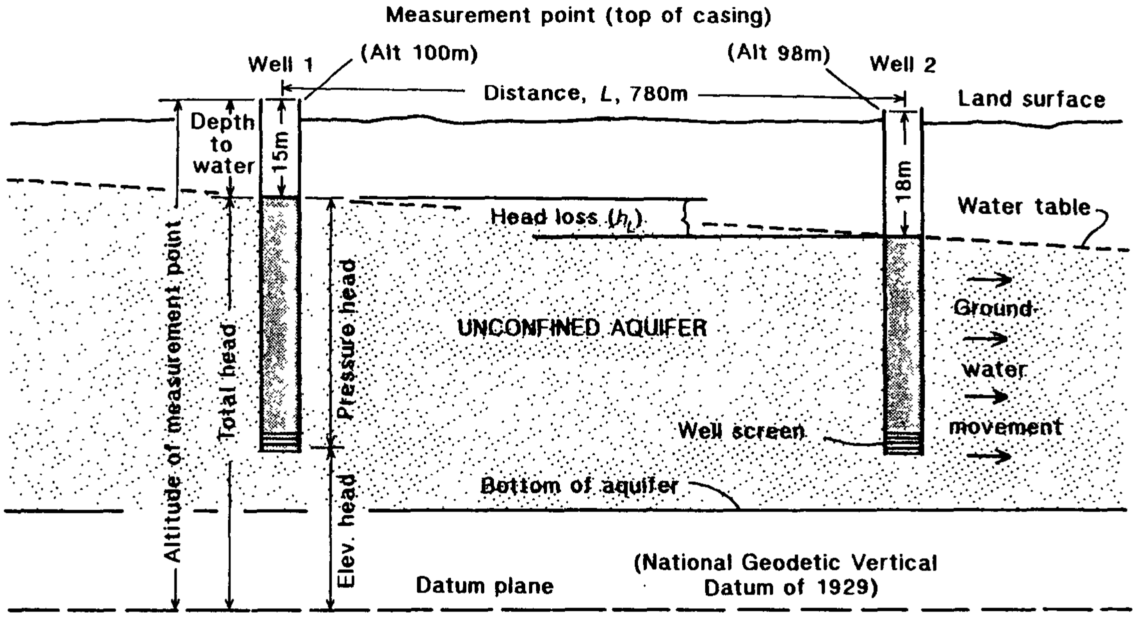
Following the directions provided in the foregoing discussion by Heath (1983), determine the approximate direction of ground-water flow and the head gradient using the data provided in question 5.

(5) Three piezometers are screened in the same horizontal aquifer. Piezometer A is 750 m (meters) due south of piezometer B and piezometer C is 1,000 m due east of piezometer A. The surface elevations of A, B, and C are 292 m, 284 m, and 288 m, respectively. The depth to water is 8 m in A, 4 m in B, and 6 m in C. Determine the direction of ground-water flow through the triangle ABC and estimate graphically the hydraulic gradient. The first step in solving this problem is to draw an accurate location map of the three points on the attached worksheet (fig. 1-11).

Comment 1: From geometry we know that the elevations of three points on a plane that are not in a straight line uniquely determine the position of the plane in space.

Comment 2: Course participants with a background in geology will recognize this problem as exactly the same as a "three-point problem" to determine the strike and dip of a plane. A line in the direction of the strike is a line of equal elevation (a contour line). The dip direction is perpendicular to the strike and parallel to the topographic gradient of the inclined plane.

Comment 3: In this problem we assume that the equipotential surface may be approximated locally by a sloping plane in space.



(1)

"The depth to the water table has an important effect on use of the land surface and on the development of water supplies from unconfined aquifers (1). Where the water table is at a shallow depth, the land may become "waterlogged" during wet weather and unsuitable for residential and many other uses. Where the water table is at great depth, the cost of constructing wells and pumping water for domestic needs may be prohibitively expensive.

The direction of the slope of the water table is also important because it indicates the direction of ground-water movement (1). The position and the slope of the water table (or of the potentiometric surface of a confined aquifer) is determined by measuring the position of the water level in wells from a fixed point (a measuring point) (1).To utilize these measurements to determine the slope of the water table, the position of the water table at each well must be determined relative to a *datum plane* that is common to all the wells. The datum plane most widely used is the National Geodetic Vertical Datum of 1929 (also commonly referred to as "sea level") (1).

If the depth to water in a nonflowing well is subtracted from the altitude of the measuring point, the result is the *total head* at the well. Total head, as defined in fluid mechanics, is composed of *elevation head*, *pressure head*, and *velocity head*. Because ground water moves relatively slowly, velocity head can be ignored. Therefore, the total head at an observation well involves only two components: elevation head and pressure head (1). Ground water moves in the direction of decreasing total head, which may or may not be in the direction of decreasing pressure head.

The equation for total head (h_t) is

$$h_t = z + h_p$$

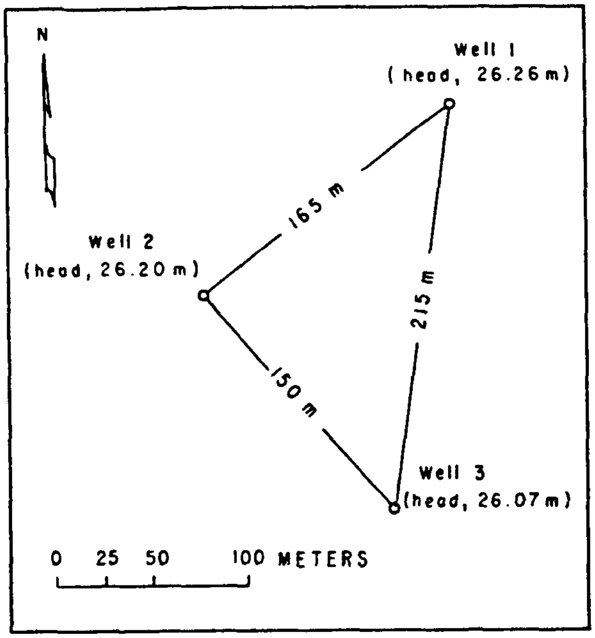
where z is elevation head and is the distance from the datum plane to the point where the pressure head h_p is determined.

All other factors being constant, the rate of ground-water movement depends on the *hydraulic gradient*. The hydraulic gradient is the change in head per unit of distance in a given direction. If the direction is not specified, it is understood to be in the direction in which the maximum rate of decrease in head occurs.

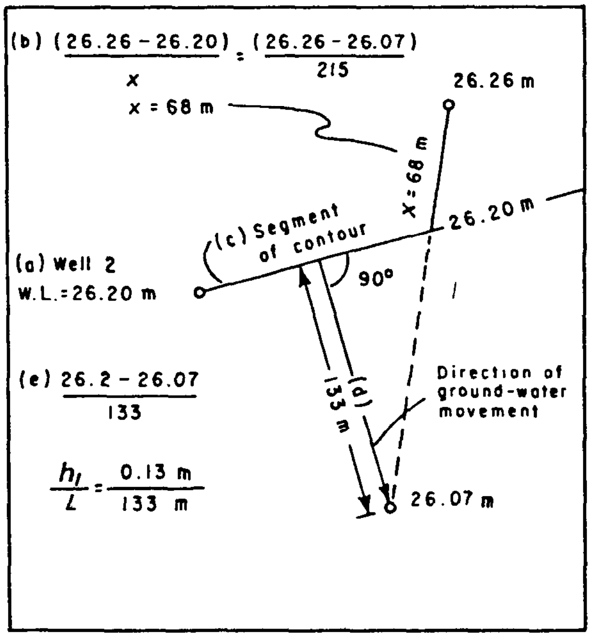
If the movement of ground water is assumed to be in the plane of sketch 1—in other words, if it moves from well 1 to well 2—the hydraulic gradient can be calculated from the information given on the drawing. The hydraulic gradient is h_L/L, where h_L is the head loss between wells 1 and 2 and L is the horizontal distance between them, or

$$\frac{h_L}{L} = \frac{(100\text{ m} - 15\text{ m}) - (98\text{ m} - 18\text{ m})}{780\text{ m}} = \frac{85\text{ m} - 80\text{ m}}{780\text{ m}} = \frac{5\text{ m}}{780\text{ m}}$$

When the hydraulic gradient is expressed in consistent units, as it is in the above example in which both the numerator and the denominator are in meters, any other consistent units of length can be substituted without changing the value of the gradient. Thus, a gradient of 5 ft/780 ft is the same as a gradient of 5 m/780 m. It is also relatively common to express hydraulic gradients in inconsistent units such as meters per



(2)



(3)

kilometer or feet per mile. A gradient of 5 m/780 m can be converted to meters per kilometer as follows:

$$\left(\frac{5\text{ m}}{780\text{ m}}\right) \times \left(\frac{1{,}000\text{ m}}{\text{km}}\right) = 6.4\text{ m km}^{-1}$$

Both the direction of ground-water movement and the hydraulic gradient can be determined if the following data are available for three wells located in any triangular arrangement such as that shown on sketch 2:

1. The relative geographic position of the wells.
2. The distance between the wells.
3. The total head at each well.

Steps in the solution are outlined below and illustrated in sketch 3:

a. Identify the well that has the intermediate water level (that is, neither the highest head nor the lowest head).
b. Calculate the position between the well having the highest head and the well having the lowest head at which the head is the same as that in the intermediate well.
c. Draw a straight line between the intermediate well and the point identified in step *b* as being between the well having the highest head and that having the lowest head. This line represents a segment of the water-level contour along which the total head is the same as that in the intermediate well.
d. Draw a line perpendicular to the water-level contour and through either the well with the highest head or the well with the lowest head. This line parallels the direction of ground-water movement.
e. Divide the difference between the head of the well and that of the contour by the distance between the well and the contour. The answer is the hydraulic gradient."

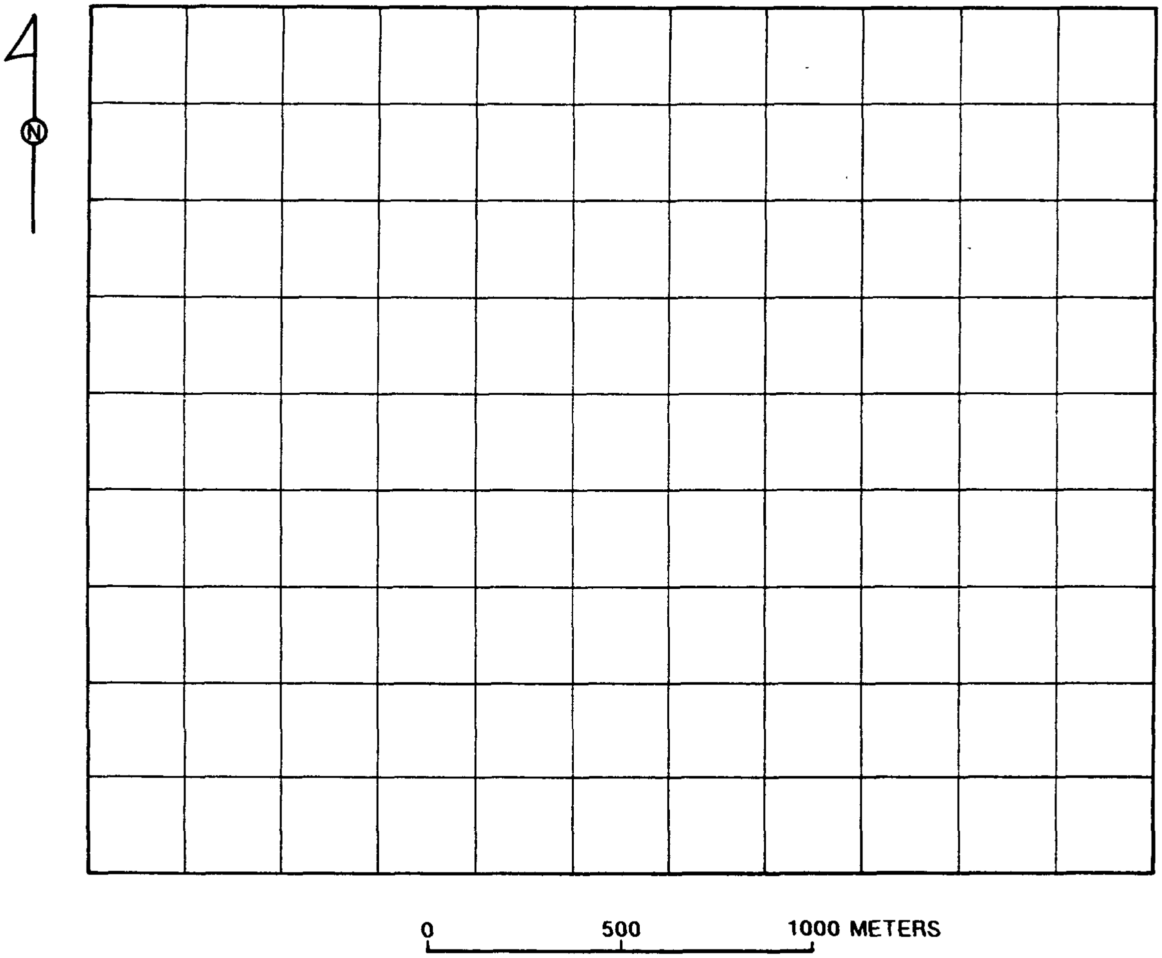


Figure 1-11.--Worksheet for the "three-point" head-gradient problem.

Ground-Water/Surface-Water Relations

Assignments

*Study Fetter (1988), p. 37-48; Freeze and Cherry (1979), p. 208-211, 217-221, 225-229; or Todd (1980), p. 222-230.

*Work Exercise (1-6)--Ground-water flow pattern near gaining streams.

*Sketch several water-table contour lines near a losing stream.

The relation between shallow aquifers and streams is of great importance in both ground-water and surface-water hydrology. The bed and banks of a gaining stream are an area of discharge for shallow ground water and this discharge is one of the principal outflow components from many ground-water systems. This water is usually a major part of the base flow of streams, which is the principal component of streamflow during dry periods. In many areas base flow is critical for water supply and maintenance of stream water quality.

In a gaining stream a "hydraulic connection" exists between the shallow aquifer and the stream--that is, the earth material beneath the streambed is continuously saturated, and saturated ground-water flow occurs between the aquifer and the stream. A losing reach of a stream may exhibit either (a) hydraulic connection between stream and aquifer or (b) no hydraulic connection. The absence of a hydraulic connection implies the presence of some thickness of unsaturated earth material below the streambed--that is, the stream is recharging the shallow aquifer through an unsaturated zone. Losing streams may be important sources of recharge to shallow ground-water systems.

Exercise (1-6)--Ground-Water Flow Pattern Near Gaining Streams

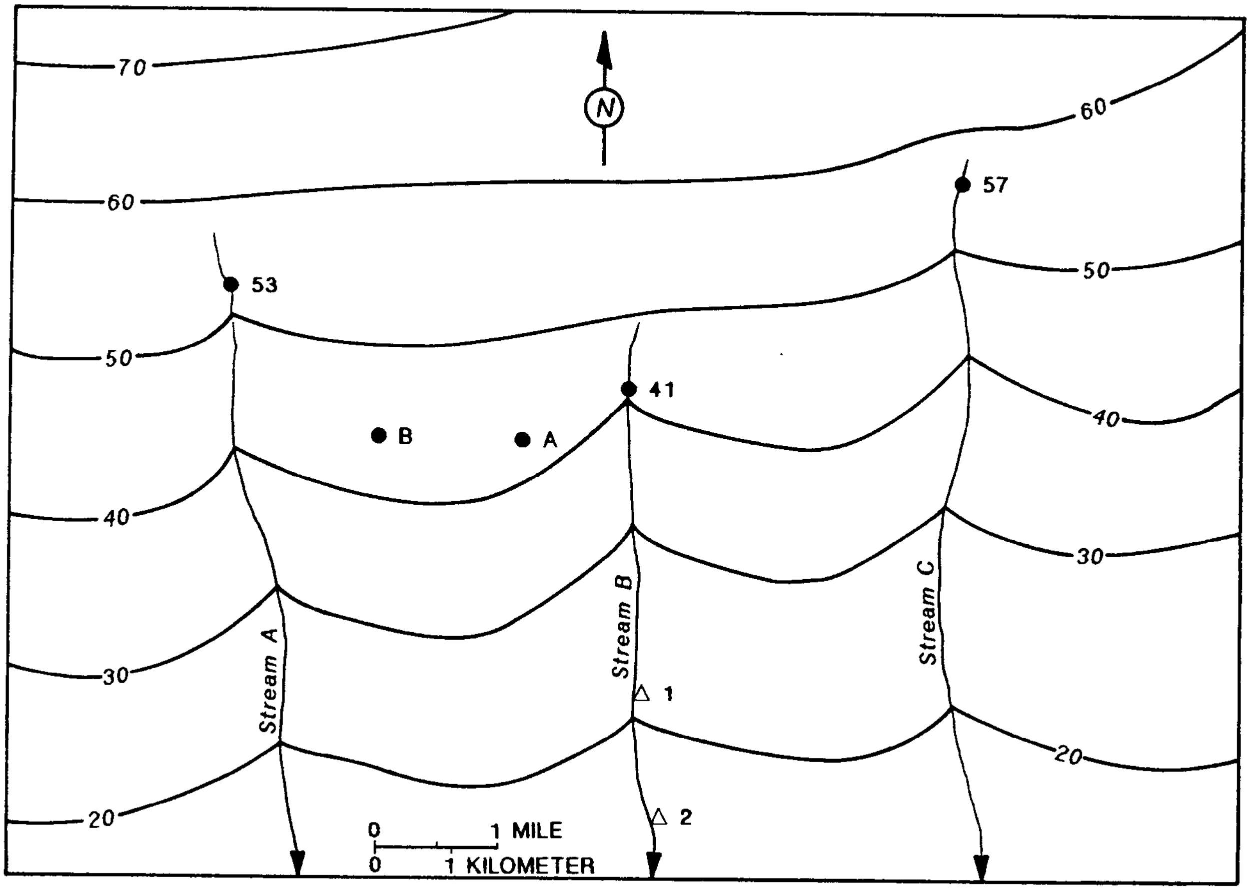
The following exercise assumes some previous discussion on the preparation of water-table maps. The basic assumption in this exercise is that ground-water flowlines, or streamlines, are perpendicular to water-table contour lines.

With reference to the attached hypothetical water-table map (fig. 1-12), answer the following questions. Assume the hydraulic conductivity of the water-table aquifer equals 125 ft/day and its porosity (n) equals 33 percent.

(1) Estimate the hydraulic gradient in the neighborhood of point B.

(2) Draw flowlines from points A and B to their points of discharge into a stream.

(3) Why do the lengths of the two flow paths differ significantly? Relate your explanation to the local configuration of the water-table contour lines, not to the observable fact that point B is further from the nearest stream than is point A.



EXPLANATION

—20— WATER-TABLE CONTOUR -- Shows altitude of water table. Contour interval 10 feet. Datum is sea level

● 41 LOCATION OF START OF FLOW OF STREAM -- Number is altitude of stream, in feet above sea level

△ 2 LOCATION AND NUMBER OF STREAM DISCHARGE MEASUREMENT POINT

Figure 1-12.--Hypothetical water-table map of an area underlain by permeable deposits in a humid climate.

(4) Considering that the particle of water at point A travels to stream B and the particle of water at point B travels to stream A, what hydrologic feature must exist between points A and B? Is the position of this feature fixed in space and time?

(5) Draw roughly north-south-trending ground-water divides between streams A and B and between streams B and C. Sketch streamlines from points 1 and 2 on stream B to the two lateral ground-water divides. Given that these are gaining streams, what does the area bounded by the four streamlines and the two lateral ground-water divides represent in relation to the stream reach between points 1 and 2 on stream B?

(6) Assume that the long-term average increase in discharge (stream "pick-up") between points 1 and 2 due to discharge of ground water into the stream is known. Using the information at your disposal and assuming the ground-water contributing area you have sketched is valid, what potentially useful hydrologic parameter can you now estimate?

(7) What are some of the problems and pitfalls involved in estimating the ground-water drainage area of an entire stream, particularly in its upper reaches?

Our interpretation of the water-table map in figure 1-12 assumes horizontal, or almost horizontal, flow in the water-table aquifer. Now, we will consider the field-measured head distribution in vertical section near a gaining stream on Long Island, New York (fig. 1-13). The field procedure for obtaining these head values is described in Prince and others (1988).

(8) Contour the head values in figure 1-13 at a contour interval of 0.20 feet. Draw contour lines for 26.20, 26.40 ... 27.20 ft.

(9) How does the ground-water flow pattern in figure 1-13 differ from the flow pattern in figure 1-12?

(10) At what distance from the streambank in figure 1-13 are the head contour lines almost vertical? What does this observation suggest about the direction of ground-water flow at this distance from the streambank?

(11) In figure 1-13, at the center of the stream, a head value of 26.70 ft was measured at about 3 ft below the streambed. Calculate an average vertical gradient beneath the streambed at the center of the stream. Find the ratio of this vertical gradient to the horizontal gradient calculated in question (1).

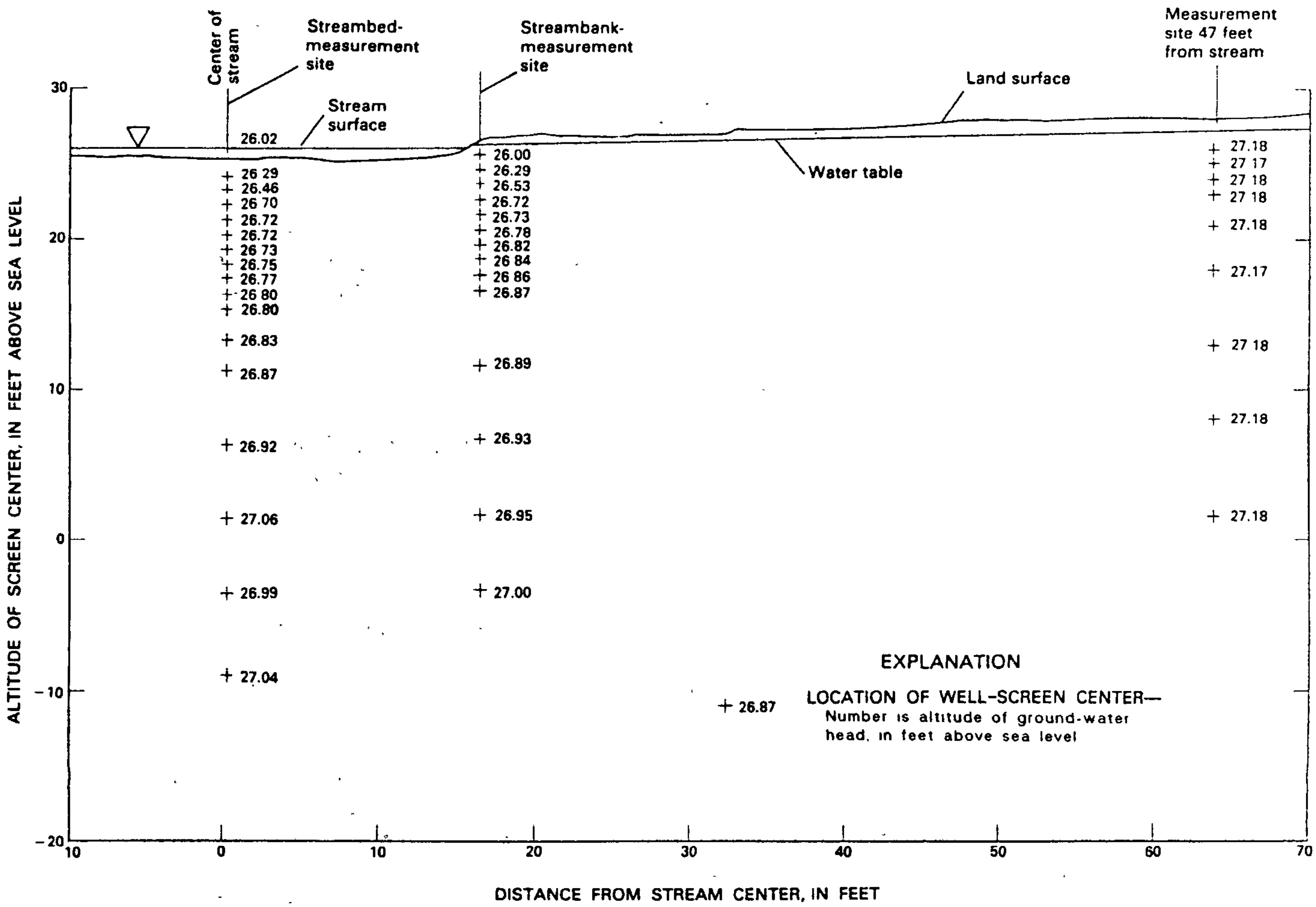


Figure 1-13.--Head measurements near Connetquot Brook, Long Island, New York, during a 3-day period in October 1978. (Modified from Prince and others, 1988, fig. 10.)

SECTION (2)--PRINCIPLES OF GROUND-WATER FLOW AND STORAGE

The keystone of this section and the entire course is Darcy's law, which provides the basis for quantitative analysis of ground-water flow. The final outcome of this section, after establishing the necessary supporting relationships, is a simplified development of the ground-water flow equation.

Darcy's Law

Assignments

*Study Fetter (1988), p. 75-85, 123-131; Freeze and Cherry (1979), Darcy's law--p. 15-18, 34-35, 72-73; physical content of permeability--p. 26-30; Darcy velocity and average linear velocity--p. 69-71; or Todd (1980), p. 64-74.

*Work Exercise (2-1)--Darcy's law.

*Define the following terms, using the glossary in Fetter (1988), an unabridged dictionary, or other available sources--steady state, unsteady state, transient, equilibrium, nonequilibrium.

*Study Note (2-1)--Dimensionality of a ground-water flow field.

The importance of Darcy's law to ground-water hydrology cannot be overstated; it provides the basis for quantitative analysis of ground-water flow. Several important points related to Darcy's law that are covered in Fetter (1988) are emphasized below.

(1) The physical content of hydraulic conductivity. The reason for the statement by some writers that hydraulic conductivity is a coefficient of proportionality in Darcy's experiment is demonstrated in the first part of Exercise (2-1). Theory and experiment indicate that the coefficient of hydraulic conductivity represents the combined properties of both the flowing fluid (ground water) and the porous medium. The physical content of hydraulic conductivity is developed in connection with equations (4-8) and (4-9) in Fetter (1988). The term "intrinsic permeability" designates the parameter that describes only the properties of the porous medium, irrespective of the flowing fluid. Explicit use of fluid properties and intrinsic permeability instead of hydraulic conductivity is required in analyzing density-dependent flows (for example, movement of water with variable density in fresh ground water-salty ground water problems) or flows that involve more than one phase or more than one fluid, as occurs in the unsaturated zone, in petroleum reservoirs, and in many situations that involve contaminated ground water.

(2) The Darcy velocity (or specific discharge) and the average linear velocity. The Darcy velocity (equation (5-24) in Fetter, 1988) is an apparent average velocity that is derived directly from Darcy's law. The average linear velocity (equation (5-25), the Darcy velocity divided by the porosity (n), is a better approximation of the actual average velocity of flow in the openings within the solid earth material. In most practical problems, particularly those involving movement of contaminants, the average linear velocity is applicable.

(3) Dimensionality of flow fields. Flow patterns in real ground-water systems are inherently three dimensional. Often, hydrologists analyze ground-water flow patterns in two or even one dimension. The purpose of Note (2-1) is to introduce the concept of flow system dimensionality. The hydrologist must differentiate between the actual ground-water flow patterns that occur in a real ground-water system and what is assumed about these flow patterns as an approximation in order to simplify their quantitative analysis.

Exercise (2-1)--Darcy's Law

The purpose of this exercise is to develop an increased familiarity with Darcy's experiment and to practice using Darcy's law in some typical problems.

A sketch of a laboratory seepage system is shown in figure 2-1. The "seepage system" may be thought of as a steady flow of water through the square prism of fine sand. The system input is the volume of water flowing through any cross-section of the sand prism per unit of time (Q). The system response is the hydraulic gradient in the sand prism, defined as the difference in head (Δh) between the two piezometers divided by the distance between them (l). The water input may be changed by adjusting the control valve. This water input, which equals the discharge from the system, is measured at the downstream end of the experimental apparatus.

The results of a series of hypothetical experiments on this flow system are listed in table 2-1. For each experiment, Q is changed by adjusting the control valve, and both Q and Δh are measured. The most convenient way to consider these data is to make a graphical plot. We will assume that we already know, based on previous experiments, that Q is proportional to A, the cross-sectional area of the sand prism. In other words, if all other experimental conditions are the same, doubling the prism cross-sectional area A will double Q. In our experiments A equals 1.21 ft^2.

Complete the entries in table 2-1 and make a plot of $\Delta h/l$ (y-axis) against Q/A (x-axis) on the worksheet provided (fig. 2-2). After you have prepared the graph, answer the following questions:

(1) How would you describe or characterize the relationship between the two variables $\Delta h/l$ and Q/A?

(2) Assuming that we are dealing with a linear relationship, write an equation for this relationship between the two variables. The first step is to recall the basic form of a linear equation in terms of x and y.

(3) Make a graphical determination of the slope of the "experimental" curve (in this case, straight line).

(4) Express the relationship in (2) in terms of Q; that is, Q = ?. This is a form of Darcy's law that we see frequently, except that the slope of the "experimental" curve is in the denominator.

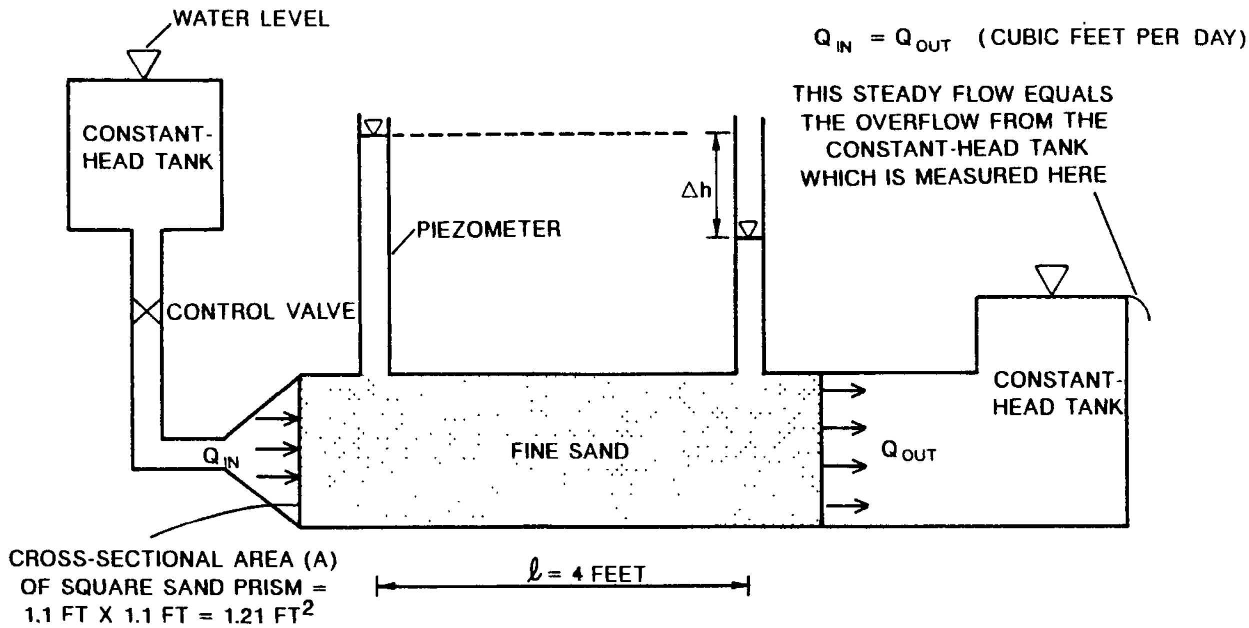


Figure 2-1.--Sketch of laboratory seepage system.

Table 2-1.--Data from hypothetical experiments with the laboratory seepage system

Test number	Q[1] (cubic feet per day)	Δh (feet)	Δh/l	Q/A (feet/day)
1	2.2	0.11		
2	3.3	.17		
3	4.6	.23		
4	5.4	.26		
5	6.7	.34		
6	7.3	.38		
7	7.9	.40		

[1] Q is steady flow through sand prism, Δh is head difference between two piezometers; l is distance between two piezometers; A is constant cross-sectional area of sand prism (fig. 2-1).

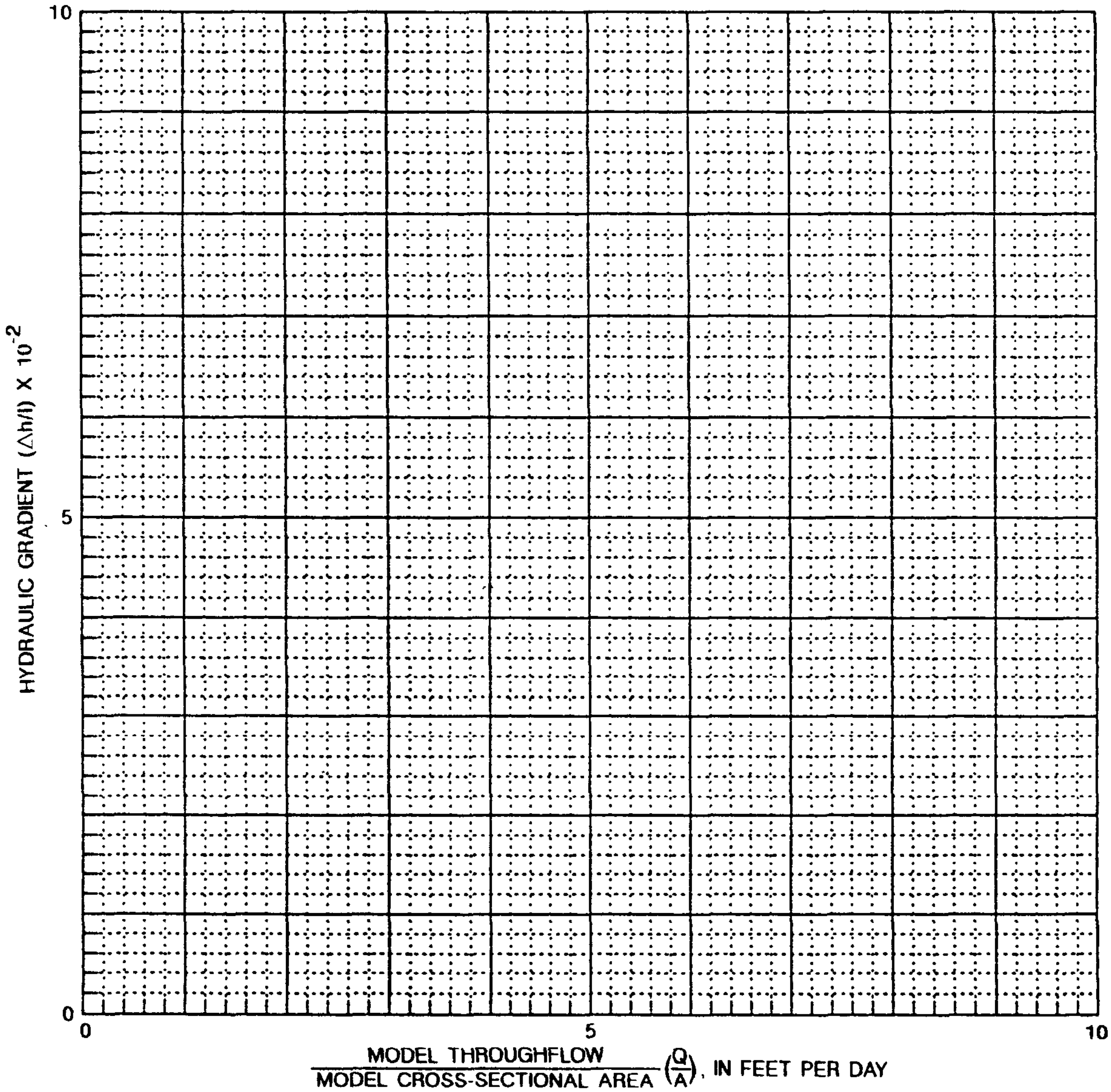


Figure 2-2.--Worksheet for plotting data from hypothetical experiments with the laboratory seepage system.

(5) If we express Darcy's law in the form $Q = K \bullet \Delta h / l \bullet A$ or $Q = K \bullet i \bullet A$, where K is the hydraulic conductivity of the sand, what is the numerical value of K in this "experiment"?

(6) On the sketch of the laboratory seepage system describe the conditions of head and flow at the boundaries of the sand prism (upstream end, downstream end, and walls of prism); that is, describe the BOUNDARY CONDITIONS of the sand prism. Also sketch some typical streamlines and equipotential lines within the sand prism.

(7) Solve problems 6 and 7, p. 159 in Applied Hydrogeology by C. W. Fetter (1988). Make a sketch of the stated problem, and write the appropriate formulas before performing numerical calculations.

Note (2-1).--Dimensionality of a Ground-Water Flow Field

Ground-water velocity at a point in a ground-water flow field is a vector; that is, it possesses both magnitude and direction. In general, the magnitude and direction of the velocity vector is a function of location in the flow field.

Several examples of velocity fields, in which velocity vectors are drawn at selected points in the flow field, are shown in figure 2-3. Also shown in figure 2-3 are x and y Cartesian coordinate axes. These axes are mutually perpendicular and located in the plane of the figure. The third dimension is represented conceptually by the z coordinate axis (not shown) which is oriented perpendicular to the plane of the figure. In figure 2-3(A) all the velocity vectors are parallel to one another, equal in magnitude, and oriented parallel to the x coordinate axis. In figure 2-3(B) the same conditions apply except that the velocity vectors are not equal in magnitude.

In both figures 2-3(A) and 2-3(B) we assume that the illustrated velocity vectors, which are drawn parallel to the x coordinate axis, are replicated exactly in the y and z coordinate directions. In other words, in this special situation, if the velocity vectors are known or defined by an equation at all points in the x coordinate direction, the velocity vectors are also known at any point in the x-y plane, the x-z plane, and the y-z plane. Thus, in this special situation, the velocity distribution in the ground-water flow field is completely described if it is defined only in the x coordinate direction, or, more formally, velocity (v) = f(x). Such a velocity field is termed "one-dimensional."

Examples of two-dimensional velocity fields are shown in figures 2-3(C) and 2-3(D). Again, in these examples we assume that the velocity field is replicated exactly perpendicular to the plane of the figures in the z coordinate direction. In these cases, because velocity varies from point to point in the two-dimensional x-y plane, two coordinates in the plane are required to specify the velocity field, or v = f(x,y). Similarly, the concept of flow-field dimensionality is extended to three dimensions; that is, velocity varies from point to point in three-dimensional x-y-z space, or v = f(x,y,z).

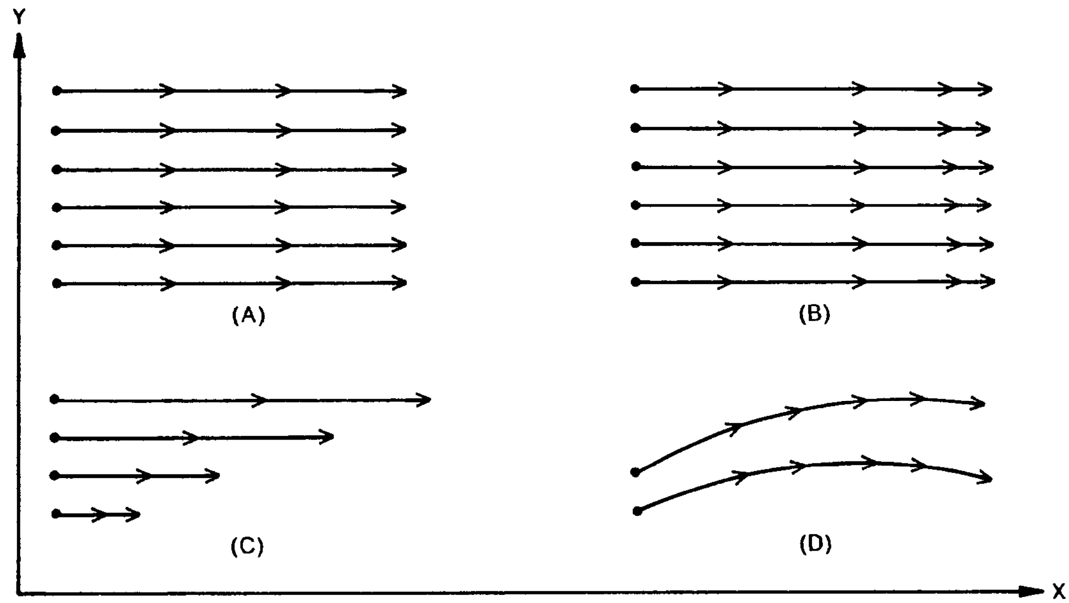


Figure 2-3.--Examples of ground-water flow fields depicted by velocity vectors at selected points: (A) and (B) are one-dimensional flow fields; (C) and (D) are two-dimensional flow fields.

In the Darcy experiment the earth material in the prism in which flow and head differences are measured is assumed to be isotropic and homogeneous. This assumption, together with measured linear head drops in the prism, result in equally spaced head contours (potential lines) and streamlines that are perpendicular to them as shown in figure 2-4. This pattern of potential lines and streamlines represents a flow field in which the velocity vectors are parallel and equal everywhere in the prism as in figure 2-3(A). Therefore, the flow in an ideal Darcy prism is an example of one-dimensional flow.

The Darcy prism in figure 2-4 is not horizontal as is the prism in figure 2-3. If the same hydraulic conditions at the boundaries of the prism are maintained during the experiment, the results of a Darcy experiment are independent of the orientation of the sand prism--that is, the prism can be horizontal, vertical, or tipped at any intermediate angle. This is one of the conclusions drawn by Fetter (1988) in his discussion of figure 5.3 on page 57.

In real ground-water systems, ground-water flow fields are always three-dimensional. However, in order to simplify problem analysis, we often assume as an approximation that the flow field is two-dimensional, or sometimes even one-dimensional. Problem solutions of acceptable accuracy sometimes can be obtained by using such simplifying assumptions regarding the flow field; however, in other situations the results obtained by employing such simplifications may be grossly in error.

Transmissivity

Assignments

*Study Fetter (1988), p. 105, 108-111; Freeze and Cherry (1979), p. 30-34, 59-62; or Todd (1980), p. 69, 78-81.

*Work Exercise (2-2)--Transmissivity and equivalent vertical hydraulic conductivity in a layered sequence.

Transmissivity is a convenient composite variable that applies only to horizontal or nearly horizontal hydrogeologic units. In order to analyze vertical ground-water flow, we must use values of hydraulic conductivity that are appropriate to the vertical direction. Exercise (2-2) provides practice in the use of formulas (4-16), (4-17), (4-22), and (4-23) in Fetter (1988).

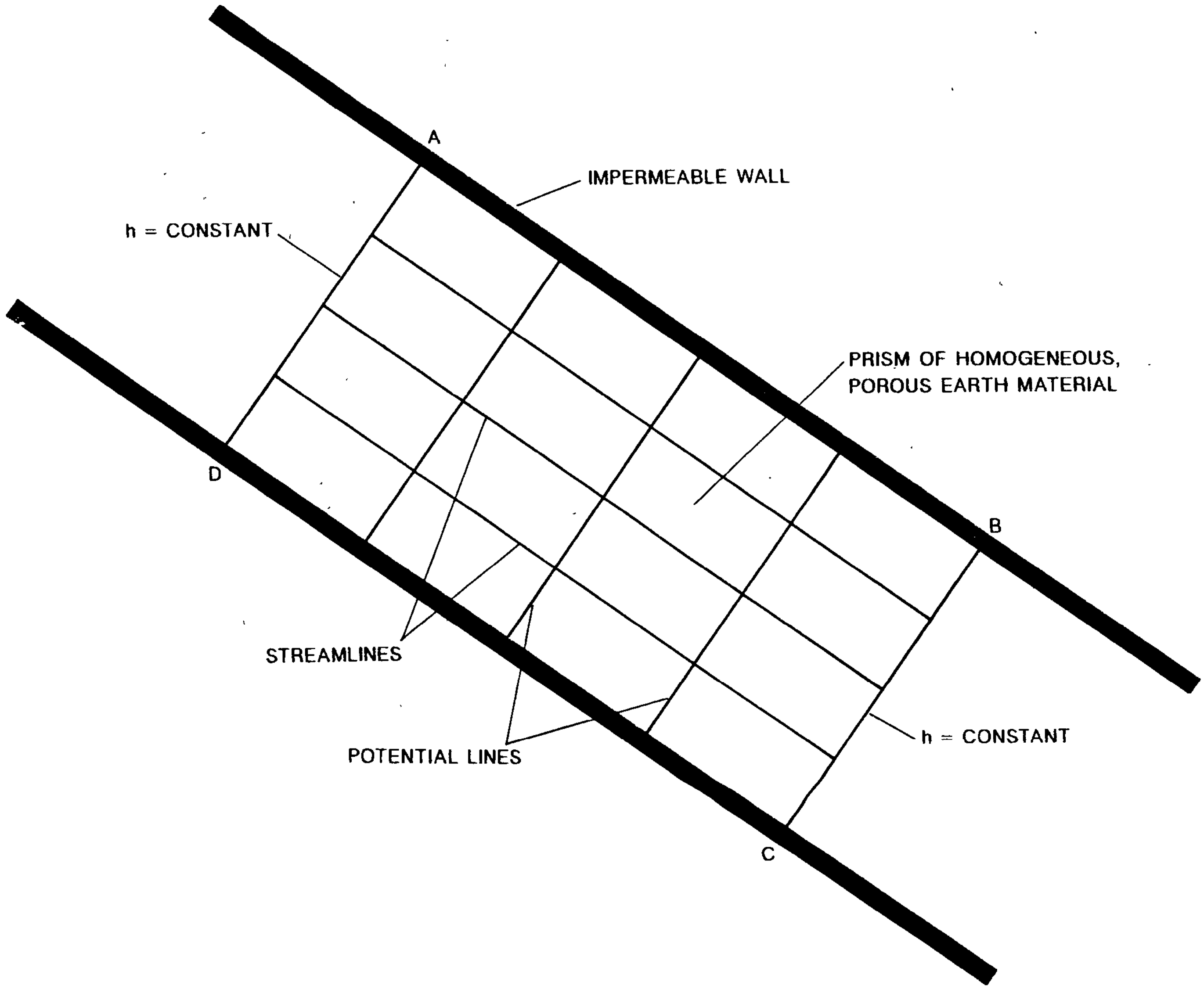


Figure 2-4.--Idealized flow pattern in a Darcy prism, ABCD, composed of homogeneous, porous earth material.

Exercise (2-2)--Transmissivity and Equivalent Vertical Hydraulic Conductivity in a Layered Sequence

The definitions of transmissivity and equivalent vertical hydraulic conductivity and the relevant formulas for their calculation can be found in any standard textbook on ground-water hydrology (see Fetter, 1988, p. 105, 110, 111).

The four horizontal beds described in the table below are assumed to be isotropic and homogeneous (not a generally realistic assumption for a field situation).

(1) Calculate the equivalent horizontal hydraulic conductivity (K_x) and the transmissivity (T) for the four layers.

(2) Calculate the equivalent vertical hydraulic conductivity (K_z) for the four layers.

(3) In these calculations (a) what bed or beds exert the greatest control on the equivalent horizontal hydraulic conductivity and transmissivity, and (b) what bed or beds exert the greatest control on the equivalent vertical hydraulic conductivity?

Bed number	Bed thickness (feet)	Bed hydraulic conductivity (K) (feet/day)
1	25	10
2	30	100
3	20	0.001
4	50	50

Aquifers, Confining Layers, Unconfined and Confined Flow

Assignment

*Study Fetter (1988), p. 101-105; Freeze and Cherry (1979), p. 47-49; or Todd (1980), p. 25-26, 37-45.

The physical mechanisms by which ground-water storage in saturated aquifers or parts of aquifers is increased or decreased (described in the next section of the outline) are determined by the hydraulic conditions under which the ground water occurs. In nature, ground water in the saturated zone occurs in unconfined aquifers and confined aquifers. The upper bounding surface of an unconfined aquifer is a water table, which is overlain by an unsaturated zone and is subject to atmospheric pressure, whereas confined aquifers are overlain and underlain by confining beds. A confining bed has a relatively low hydraulic conductivity compared to that of the adjacent aquifer. Ratios of hydraulic conductivity generally are at least 1,000 (aquifer) to 1 (confining bed), and commonly are much larger. In addition, the head at the top of the confined aquifer always is higher than the bottom of the overlying confining bed. This means that the entire thickness of the confined aquifer is fully saturated.

Ground-Water Storage

Assignments

*Study Fetter (1988), p. 73-76, 105-107; Freeze and Cherry (1979), p. 51-62; or Todd (1980), p. 36-37, 45-46.

*Study Note (2-2)--Ground-water storage.

*Work Exercise (2-3)--Specific yield.

Hydraulic parameters for earth materials may be divided into (a) transmitting parameters and (b) storage parameters. We already have encountered the principal transmitting parameters, hydraulic conductivity (K) or intrinsic permeability (k), and transmissivity (T). In this section the principal storage parameters, storage coefficient (S), specific storage (S_s), and specific yield (S_y), are introduced.

The physical mechanisms involved in unconfined storage and confined storage are different. A change in storage in an unconfined aquifer involves a physical dewatering of the earth materials; that is, earth materials that previously were saturated become unsaturated. When a change in storage takes place in a confined aquifer, the earth materials in the confined aquifer remain saturated.

Note (2-2).--Ground-Water Storage, by Gordon D. Bennett[1]

Originally it was thought that a porous medium acted only as a conduit--in other words, that it simply transmitted water according to Darcy's law. Approximately 50 years ago, hydrologists recognized that a porous medium could also act as a storage reservoir--that water could be accumulated in an aquifer, retained for a certain time, and then released.

In problems of steady-state ground-water flow, the inflow to a unit volume of aquifer always balances the outflow. When inflow equals outflow in this manner, no water is accumulating in the system, and storage need not be considered. In general, however, inflow and outflow are not in balance. The

[1] U.S. Geological Survey, Reston, Virginia.

general equation for systems in which storage is a factor is termed the equation of continuity, and may be written as follows:

$$\text{Inflow} - \text{Outflow} = \text{Rate of accumulation.} \qquad (1)$$

Steady-state flow refers to the special case where the rate of accumulation is zero.

The general equation indicated above could be applied to a water tank which is being filled at a rate Q_1, and drained simultaneously at a rate Q_2. The rate of accumulation in the tank (fig. 2-5) is $Q_1 - Q_2$. Negative accumulation, or depletion, occurs if Q_2 exceeds Q_1. If, for example, water is flowing in at 5 cubic feet per second, and flowing out at 6 cubic feet per second, the volume of water in the tank will diminish at a rate of 1 cubic foot per second; and if the area of the bottom of the tank is 10 square feet, the water level in the tank will fall at a rate of 0.1 feet per second. In a tank, therefore, the factor that relates the rate of change of water level to the rate of accumulation of fluid is simply the base area of the tank, A. If V is the volume of water in the tank and h is the water level, then

$$V = Ah, \qquad (2)$$

and if we add a volume of water ΔV to the tank, the water level rises by an increment Δh such that

$$\Delta V = A \Delta h. \qquad (3)$$

Now suppose a length of time Δt is required to add the volume of water ΔV. Dividing the above equation by this time interval Δt gives

$$\frac{\Delta V}{\Delta t} = A \frac{\Delta h}{\Delta t} \qquad (4)$$

where $\frac{\Delta V}{\Delta t}$ is the rate at which water is added to the tank which may be expressed, for example, in cubic feet per second; and

$\frac{\Delta h}{\Delta t}$ is the rate at which the water level rises in the tank--expressed, for example, in feet per second.

In the customary notation of differential calculus, we would use $\frac{dV}{dt}$ in place of $\frac{\Delta V}{\Delta t}$ for the rate at which fluid is added to the tank, or taken into storage; and $\frac{dh}{dt}$ in place of $\frac{\Delta h}{\Delta t}$ for the rate at which the fluid level rises in the tank.

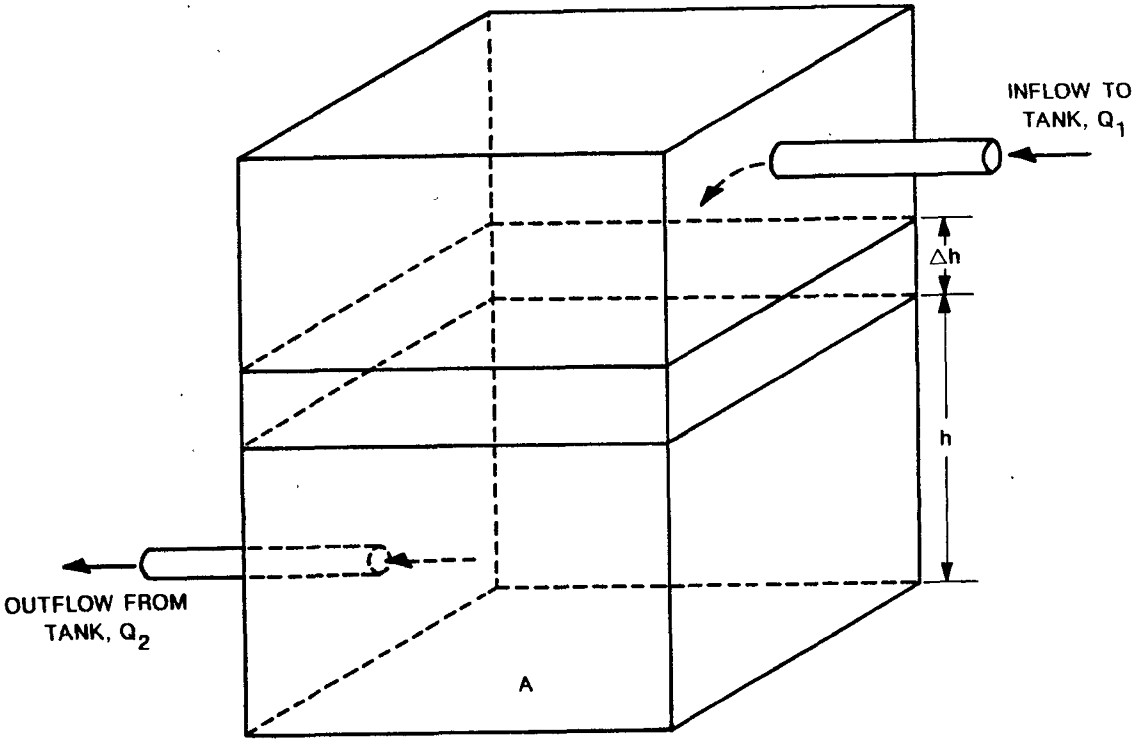


Figure 2-5.--Inflow to and outflow from a tank.

When $\frac{dh}{dt}$ is zero, $\frac{dV}{dt}$ must also be zero, and inflow must equal outflow. Because h is not changing with time, the system is said to be at equilibrium, or to be in the steady state. When $\frac{dh}{dt}$ differs from zero, inflow and outflow are out of balance; accumulation or depletion is occurring, and the system is described as nonequilibrium.

The mechanism of storage in an unconfined aquifer is essentially similar to that of storage in an open tank. Consider a prism (as shown in fig. 2-6) through an unconfined aquifer which is bounded below by an impervious layer. Let the base area of the prism be A, and the porosity be n. If the prism is saturated to a height h above the base, the volume of water contained in the prism is (n A h); and if the water level is falling at a rate $\frac{dh}{dt}$, the volume of water in the prism is decreasing at a rate

$$\frac{dV}{dt} = \frac{d}{dt}(n\,A\,h) = n\,A\,\frac{dh}{dt}, \tag{5}$$

assuming that the sand is fully drained as the water level falls.

In general, however, a certain fraction of the water is retained in the pores by capillary forces as the water level falls. When the water level is lowered a distance Δh, therefore, the volume of water removed will not be $nA\Delta h$, but rather $\alpha n \bullet A\Delta h$, where α is the percentage of the water, expressed as a fraction, that can be drained by gravity. The fraction that is retained by capillary forces is $(1-\alpha)$. In this case, then,

$$\Delta V = \alpha n A \Delta h \tag{6}$$

is the volume of water removed and

$$\frac{dV}{dt} = \alpha n A \frac{dh}{dt} \tag{7}$$

is the volumetric rate of removal of water from the prism. The quantity αn is called the specific yield[1] or storage coefficient of the aquifer, and is usually denoted as S. Using this notation, the expression for ΔV, the volume of water that must be removed to achieve a drop in water level of Δh, is

$$\Delta V = SA\Delta h. \tag{8}$$

[1] The specific yield of an unconfined aquifer is often denoted by Sy or S.Y. The storage coefficient S and specific yield Sy of an unconfined aquifer are approximately equivalent (see Fetter, 1988, p. 107, eqn. 4-20). The storage coefficient S is used to describe storage properties of both unconfined and confined aquifers.

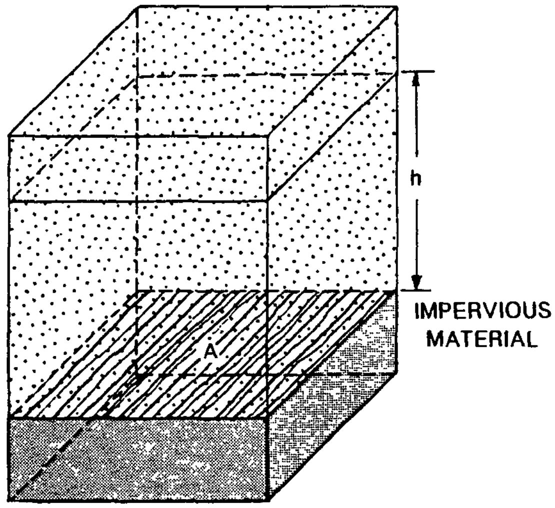


Figure 2-6.--Reference prism in an unconfined aquifer bounded below by an impervious layer.

Therefore,

$$\frac{\Delta V}{\Delta h} = SA \tag{9}$$

or, expressed as a derivative,

$$\frac{dV}{dh} = SA \tag{10}$$

Storage coefficient, for the unconfined case, therefore is given by the equation

$$S = \frac{1}{A} \cdot \frac{dV}{dh}. \tag{11}$$

This equation states that storage coefficient is the volume of water released per unit decline in head per unit surface area of aquifer. The expression given previously for the volumetric rate of removal of water from the prism was

$$\frac{dV}{dt} = \alpha nA \frac{dh}{dt} \tag{12}$$

or, in terms of storage coefficient,

$$\frac{dV}{dt} = SA \frac{dh}{dt}. \tag{13}$$

The same result can be obtained from the general rules that govern derivatives, as follows:

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = SA \frac{dh}{dt}. \tag{14}$$

In the given prism through the unconfined aquifer, therefore, it is not necessary for inflow to equal outflow. If the inflow to the prism exceeds outflow, water will accumulate in the prism at a rate equal to the difference in flow, and a rise in water level with time will be observed. If outflow exceeds inflow, water is being depleted within the prism, and a fall of water level with time will be observed. Thus, any record of water level versus time in an aquifer is essentially a record of water taken into storage or released from storage in the vicinity of the recording station.

The property of storage is observed in confined aquifers as well as in unconfined aquifers. The mechanism of confined storage depends, at least in part, on compression and expansion of the water itself and of the porous framework of the aquifer; for this reason confined storage sometimes is referred to as "compressive storage." In this discussion we do not attempt to analyze of the mechanisms of confined storage, but concentrate instead on developing a mathematical description of its effects that is suitable for hydrologic calculations. In order to describe the effects of confined storage, we will consider an imaginary experiment.

Consider a vertical prism of unit cross-sectional area, cut from a horizontal confined aquifer and extending the full thickness of the aquifer (fig. 2-7). Assume that water is pumped into the prism through a pipe and that the sides of the prism are sealed so that all of the water that is pumped in must accumulate in storage within the prism. Assume that the prism is saturated to begin with, and that as additional water is pumped in, the pressure within the prism, as measured by the height of water in the measuring piezometer, increases. Assume further that the pressure increase is observed to occur in the following way: a constant incremental change (or increase), Δh, occurs in the water level in the piezometer tube for each constant incremental change (or increase), ΔV, in fluid volume injected into the prism. Thus, if the total injected volume, V, is plotted against the water level in the piezometer, the graph will be a straight line with slope $\frac{dV}{dh} = \frac{\Delta V}{\Delta h}$.

Finally, assume that the following is observed: if the cross-sectional area of the prism is doubled, twice as much water must be injected in order to produce the increase in water level Δh; if the area is tripled, three times as much water must be injected in order to produce the incremental increase Δh; and so on. Thus, if the area of the prism is allowed to vary, the quantity $\frac{dV}{dh}$ will not be the same in each case, but the quantity $\frac{dV/dh}{A}$ will be the same, where A is the base area of the prism. This latter quantity, then, is a constant, independent of the area of the prism under consideration, as well as of V and h. It is presumably a function only of the properties of the aquifer material and the thickness of the aquifer. If the aquifer is homogeneous and of uniform thickness, the quantity $\frac{dV/dh}{A}$ will have the same value for any prism through the aquifer, and may be considered to be a constant for the aquifer. It is denoted as the storage coefficient, S--that is, $S = \frac{dV/dh}{A}$.

Figure 2-8 illustrates the relation between V, h, and the cross-sectional area A for the prism of aquifer we have considered. For any given base area, the ratio of the increment of injected water to the resulting head increment is a constant, $\frac{\Delta V}{\Delta h}$. Thus, a plot of V against h will have a constant slope. However, if the base area is varied, the slope of the graph must vary proportionately. Thus, if we are given three prisms for which

$$A_1 = \frac{1}{2} A_2 = \frac{1}{3} A_3 \tag{15}$$

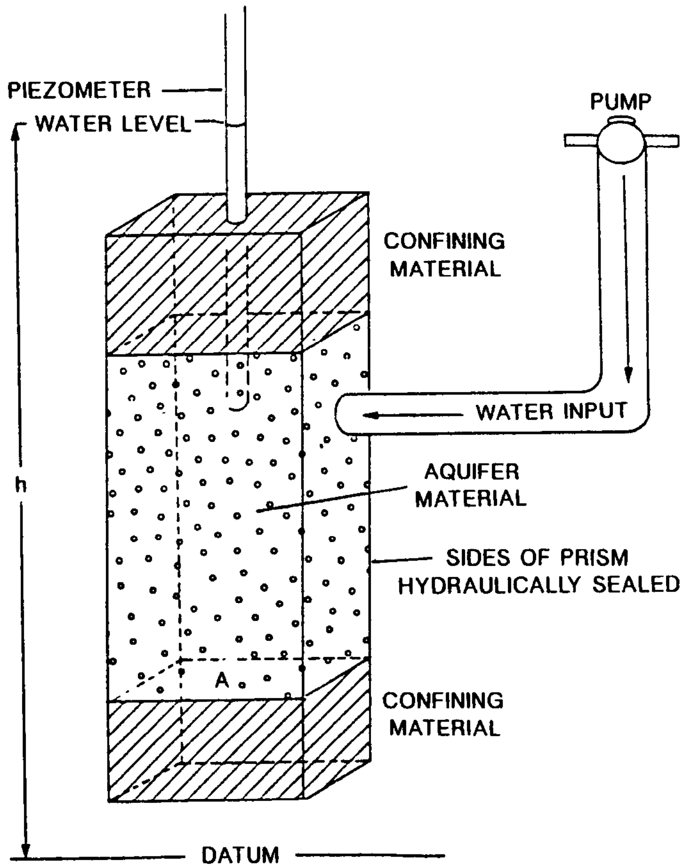


Figure 2-7.--Vertical prism in a confined aquifer bounded above and below by confining material and laterally by walls that are hydraulically sealed. (Modified from Bennett, 1976, p. 55.)

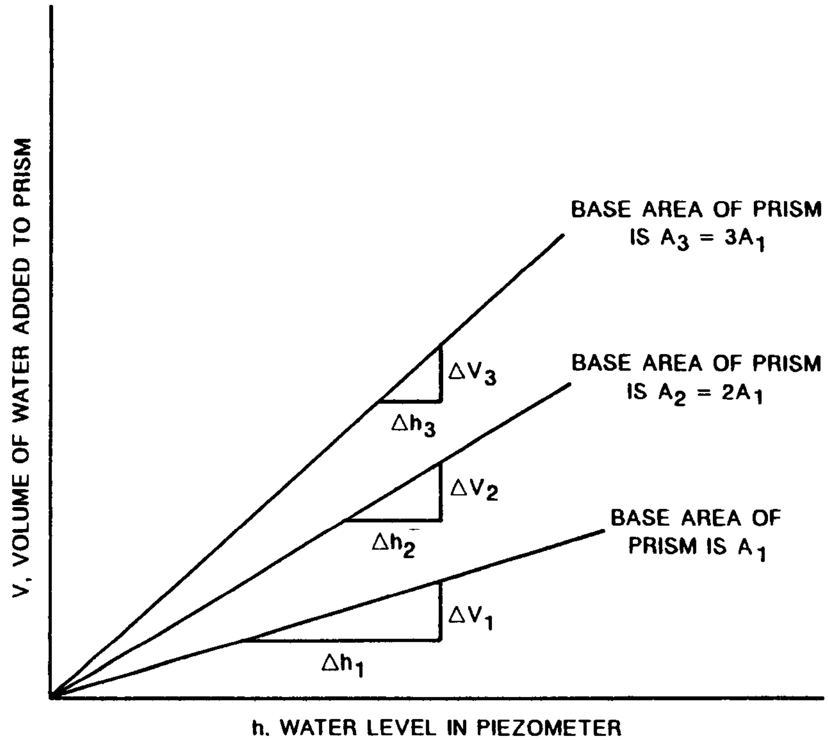


Figure 2-8.--Relation between volume of water injected into a prism of earth material that extends the full thickness of a confined aquifer and head in the prism for different values of prism base area.

we will observe that

$$\frac{\Delta V_1}{\Delta h_1} = \frac{1}{2}\frac{\Delta V_2}{\Delta h_2} = \frac{1}{3}\frac{\Delta V_3}{\Delta h_3} \tag{16}$$

as indicated in figure 4. Therefore,

$$\frac{1}{A_1}\frac{(\Delta V_1)}{\Delta h_1} = \frac{1}{A_2}\frac{(\Delta V_2)}{\Delta h_2} = \frac{1}{A_3}\frac{(\Delta V_3)}{\Delta h_3} = \frac{\frac{dV}{dh}}{A} = S. \tag{17}$$

Storage coefficient in the confined case is similar to that in the unconfined case in that it is the volume of water taken into storage per unit increase in head per unit surface area of aquifer, or the volume released from storage per unit decline in head per unit surface area of aquifer. Storage coefficient in a confined aquifer is thus a measure of the capacity of the aquifer to absorb water under pressure, and to release water in response to pressure drop, where pressure is measured in feet of water. If head is observed to be increasing at a rate $\frac{dh}{dt}$ in the prism of aquifer under consideration, water is being taken into storage in the prism at a volumetric rate $\frac{dV}{dt}$ where

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = S\,A\,\frac{dh}{dt}. \tag{18}$$

Although the definition of confined storage coefficient by the equation

$$S = \frac{1}{A}\;\frac{dV}{dh} \tag{19}$$

gives no information about the reasons for storage, it describes accurately the effects of storage, and is therefore an adequate definition for the purpose of engineering calculations. As in the case of an unconfined aquifer, if head in the given prism of confined aquifer is increasing at the rate $\frac{dh}{dt}$, water is being taken into storage in the prism at a rate $\frac{dV}{dt}$ where

$$\frac{dV}{dt} = \frac{dV}{dh}\;\frac{dh}{dt} = S\,A\,\frac{dh}{dt}. \tag{20}$$

In either the unconfined or the confined case, therefore, the equation

$$\text{Inflow} - \text{Outflow} = \text{Rate of Accumulation}, \frac{dV}{dt} \tag{21}$$

becomes

$$\text{Inflow} - \text{Outflow} = S\, A\, \frac{dh}{dt} \tag{22}$$

when applied to a prism of aquifer that has a base area A. If $\frac{dh}{dt} = 0$, inflow to the prism equals outflow.

Exercise (2-3)--Specific Yield

A rectangular prism whose base is a square with sides equal to 1.5 ft and height equal to 6 ft is filled with fine sand whose pores are saturated with water. The porosity (n) of the sand equals 34 percent. The prism is drained by opening a drainage hole in the bottom and 2.43 ft^3 of water is collected. Calculate the following quantities:

total volume of prism	
volume of sand grains in prism	
total volume of water in prism before drainage	
volume of water drained by gravity	2.43 ft^3
volume of water retained in prism (not drained by gravity)	
specific yield	
specific retention	

(1) Assuming the value of specific yield determined above, what volume of water, in ft^3, is lost from ground-water storage per mi^2 for an average 1-ft decline in the water table?

Express this volume as a rate for 1 day in ft^3/sec.

Express this volume as depth of water in inches over the mi^2.

(2) Assuming the value of specific yield determined above, a volume of water added as recharge at the water table that is equal to (a) 1 in. and (b) 4.8 in. per unit area would represent what average change in ground-water levels, expressed in feet?

Ground-Water Flow Equation

Assignments

*Study Fetter (1988), p. 131-136; Freeze and Cherry (1979), p. 63-66, 174-178, 531-533; or Todd (1980), p. 99-101.

*Study Note (2-3)--Ground-water flow equation.

*Define the following terms by referring to any available mathematics text that covers differential equations--independent variable, dependent variable, order, degree, linear, nonlinear.

A differential equation that describes or "governs" ground-water flow under a particular set of physical circumstances may be regarded as a kind of mathematical model. In ground-water flow equations head generally is the dependent variable. If the flow equation is solved, either analytically or numerically, values of head can be calculated as a function of position in space in the ground-water reservoir (coordinates x, y, and z) and time (t). The differential equation provides a general rule that describes how head must vary in the neighborhood of any and all points within the flow domain (ground-water flow system). Numerical algorithms that are amenable to solution by digital computers (for example, the finite-difference approximation of a differential equation) may be developed directly from the differential equation.

The ground-water flow equation developed in Note (2-3) is widely applicable. Note that the steady-state form of this equation represents the mathematical combination of (a) the equation of continuity and (b) Darcy's law.

Note (2-3).--Ground-Water Flow Equation--A Simplified Development,

by Thomas E. Reilly[1]

The following development of the ground-water flow equation is simplified in that it (1) employs an intuitive and physical rather than a mathematical approach and (2) implicitly makes the assumption that the flowing fluid (water) is incompressible. This assumption will be discussed further during the course of the presentation.

The continuity principle often is expressed by the "hydrologic equation" as:

$$\text{Inflow (of water)} = \text{Outflow (of water)} \pm \Delta \text{ Storage (of water)} \qquad (1)$$

(The symbol Δ (delta) means "change in".) The units of (1), because we are dealing with liquid water, are L^3/T, the same units as discharge Q.

---

[1] U.S. Geological Survey, Reston, Virginia

Let us first consider the steady-state form of (1) in relation to a hypothetical rectangular block of aquifer material (fig. 2-9). The steady-state form of (1)

$$\text{Inflow} = \text{Outflow} \tag{2}$$

may be written

$$\text{Inflow} - \text{Outflow} = 0,$$

or expressed equivalently in different words:
Quantity (of water) in (to block) - Quantity (of water) out (from block) = 0.

Let us define flow into the block as positive and out of the block as negative. Then, with reference to figure 2-9 the previous equation may be written:

$$Qx_{left} + Qy_{front} + Qz_{top} + Qx_{right} + Qy_{back} + Qz_{bottom} = 0 \tag{3}$$

Let: $\Delta Qx = Qx_{left} + Qx_{right}$ = Quantity of flow gained or lost in the x direction

$\Delta Qy = Qy_{front} + Qy_{back}$ = Quantity of flow gained or lost in the y direction; and

$\Delta Qz = Qz_{top} + Qz_{bottom}$ = Quantity of flow gained or lost in the z direction.

Then:

$$\Delta Qx + \Delta Qy + \Delta Qz = 0. \tag{4}$$

Equation (4), which is expressed in terms of gains or losses in flow in the three coordinate directions relative to the aquifer block rather than absolute flow magnitudes at the faces of the block, is a convenient form of the steady-state continuity equation (2) and (3) for our purposes.

Equation (4) is specific in that we are dealing with the flow of water relative to a block of aquifer material. However, it is too general and, therefore, useless in practical applications unless we have a specific rule for calculating Qy_{front}, Qz_{top}, and so forth in (3) or ΔQx, and so forth in (4). The rule that relates (4) specifically to ground-water flow is Darcy's law. Relative to the block of aquifer material in figure 2-9, Darcy's law may be written:

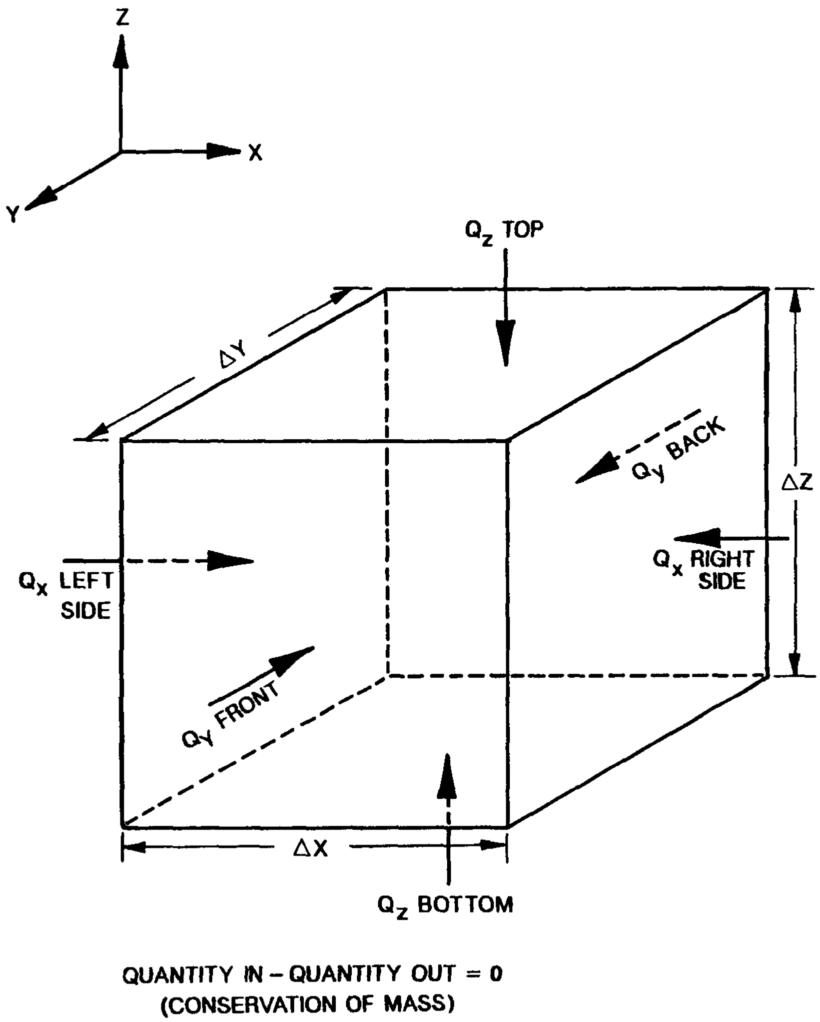


Figure 2-9.--Steady-state (equilibrium) flow of ground water through a typical block of aquifer material.

$$Qx = K_x \, \Delta y \, \Delta z \, \frac{\Delta h}{\Delta x}$$

$$Qy = K_y \, \Delta x \, \Delta z \, \frac{\Delta h}{\Delta y} \qquad (5)$$

$$Qz = K_z \, \Delta x \, \Delta y \, \frac{\Delta h}{\Delta z}$$

where K_x, K_y, and K_z are values of hydraulic conductivity in the respective coordinate directions, and Δh is the change in head between respective pairs of block centers.[2] We assume that the hydraulic conductivity remains constant within the aquifer block in each coordinate direction. Expressed more formally, we assume that the aquifer block is anisotropic and homogeneous with respect to hydraulic conductivity. Note that Darcy's law in equation (5) calculates the flow across each of the six block faces in the three coordinate directions x, y, and z and does <u>not</u> calculate the change in flow, ΔQx, ΔQy, or ΔQz in each of the three coordinate directions as expressed in equation 4. For example, the first equation in (5) $Qx = K_x \, \Delta y \, \Delta z \, \frac{\Delta h}{\Delta x}$ would be used to calculate Qx_{left} and Qx_{right}, and ΔQx is the algebraic sum of these two across-face flows, as expressed in equation (4).

Substituting equations (5) into (4) and dividing all terms by $\Delta x \, \Delta y \, \Delta z$ we obtain:

$$\frac{\Delta\left(K_x \, \Delta y \, \Delta z \, \frac{\Delta h}{\Delta x}\right)}{\Delta x \, \Delta y \, \Delta z} + \frac{\Delta\left(K_y \, \Delta x \, \Delta z \, \frac{\Delta h}{\Delta y}\right)}{\Delta x \, \Delta y \, \Delta z} + \frac{\Delta\left(K_z \, \Delta x \, \Delta y \, \frac{\Delta h}{\Delta z}\right)}{\Delta x \, \Delta y \, \Delta z} = 0$$

or

$$\frac{\Delta}{\Delta x}\left(K_x \frac{\Delta h}{\Delta x}\right) + \frac{\Delta}{\Delta y}\left(K_y \frac{\Delta h}{\Delta y}\right) + \frac{\Delta}{\Delta z}\left(K_z \frac{\Delta h}{\Delta z}\right) = 0. \qquad (6)$$

If we take the limit of this equation as the block of aquifer material becomes smaller and smaller (Δx, Δy, and Δz approach zero), we may write (6) in terms of partial derivatives

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0. \qquad (7)$$

---

[2] Here and in the immediately succeeding discussion, reference is made to the six aquifer blocks and heads at their respective centers that are located adjacent to the six faces of the reference block of the aquifer depicted in figure 2-9.

This equation is related to the widely utilized Laplace equation[3] but differs from it in that this equation accounts for differences in hydraulic conductivity in the three coordinate directions (K_x, K_y, and K_z).

Let us now add storage to the basic steady-state flow equations (2), (4), (6), and (7). With reference to equation (4), we can write:

$$\Delta Qz + \Delta Qy + \Delta Qx = \frac{\Delta V}{\Delta t} \qquad (8)$$

as a form of equation (1) which includes changes in storage. The right-hand side of (8) symbolizes that the volume of water in our hypothetical block of aquifer material (V) (fig. 2-9) changes (ΔV) during a specified time interval (Δt). This symbolic representation of a rate of change in storage $\frac{\Delta V}{\Delta t}$ is general and non-specific in the sense that it could represent types and configurations of flow other than ground-water flow. Our problem now is to obtain an expression equivalent to $\frac{\Delta V}{\Delta t}$ that relates specifically to ground-water flow--that is, to our (or any) block of aquifer material.

At this point review (if necessary) the previous note on ground-water storage by G. D. Bennett, which provides a physical discussion of the *following required relationships:*

$$\Delta V = S\ A\ \Delta h = S\ \Delta x\ \Delta y\ \Delta h \qquad (9)$$

and

$$\frac{\Delta V}{\Delta t} = S\ \Delta x\ \Delta y\ \frac{\Delta h}{\Delta t} \qquad (10)$$

---

[3] Laplace's equation, a second-order partial-differential equation that is used in diverse fields of science, may be written as a three-dimensional ground-water flow equation in the form

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial h}{\partial z}\right) = 0$$

or

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0.$$

Because hydraulic conductivity K does not appear in this equation *explicitly, it must be constant in all directions.* In other words, this equation implies that the flow medium is isotropic and homogeneous with respect to hydraulic conductivity.

where S is the storage coefficient (dimensionless) and Δh is the change in head as measured in a tightly cased well (piezometer) in the block of aquifer material during the time interval Δt.

In reviewing the discussion leading to equations (9) and (10) in the note on storage by Bennett, we recall that the storage coefficient S is defined with reference to a vertical prism of earth material that extends completely through any given aquifer. Thus, the storage coefficient S is a storage parameter that relates to representations of ground-water flow systems in which the entire aquifer thickness is treated as a single unit or layer.[4] The appropriate storage parameter for more fully three-dimensional flow is the coefficient of specific storage S_s. This coefficient is equivalent to S/b where b is the aquifer thickness. Thus, specific storage represents the volume of water released from or taken into storage per unit volume (instead of "per unit area" as for storage coefficient S) of the porous medium per unit change in head. Specific storage S_s has units of L^{-1}.

We see from the preceeding definition that specific storage S_s is a storage parameter that relates to a unit volume of aquifer material; that is, a unit area of aquifer in the xy plane times a unit thickness of aquifer. Thus, in equations (9) and (10), we can express S, relative to the original reference block of aquifer material with dimensions of Δx, Δy, and Δz, as $S_s\Delta z$. Therefore, we can express equation (10) as

$$\frac{\Delta V}{\Delta t} = S_s \Delta z \Delta x \Delta y \frac{\Delta h}{\Delta t}. \tag{10a}$$

To add storage to the continuity equation that describes the flow through a block of aquifer material, we substitute Darcy's law and equation (10a) into the transient continuity equation (8), which becomes

$$\Delta \left(K_x \Delta y \Delta z \frac{\Delta h}{\Delta x}\right) + \Delta \left(K_y \Delta x \Delta z \frac{\Delta h}{\Delta y}\right) + \Delta \left(K_z \Delta x \Delta y \frac{\Delta h}{\Delta z}\right) = S_s \Delta z \Delta x \Delta y \frac{\Delta h}{\Delta t}. \tag{11}$$

By dividing all terms in (11) by $\Delta x \Delta y \Delta z$, we obtain

$$\frac{\Delta}{\Delta x}\left(K_x \frac{\Delta h}{\Delta x}\right) + \frac{\Delta}{\Delta y}\left(K_y \frac{\Delta h}{\Delta y}\right) + \frac{\Delta}{\Delta z}\left(K_z \frac{\Delta h}{\Delta z}\right) = S_s \frac{\Delta h}{\Delta t}. \tag{12}$$

Taking the limit of this equation (Δx, Δy, Δz, and Δt approach zero), we may write equation (12) in terms of partial derivatives:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t}. \tag{13}$$

---

[4] As a result of its definition, the storage coefficient S is generally used in flow equations that represent two-dimensional nearly horizontal flows.

Equation (13) is a widely applicable form of a transient, three-dimensional ground-water flow equation. One- and two-dimensional simplifications of this equation can be found in most ground-water texts.

To review, a steady-state form of the ground-water flow equation always involves the combination of two rules or principles:

(1) a statement of the continuity principle in the form of an equation which is appropriate to the problem we are trying to solve, and

(2) Darcy's law, which provides a specific rule for calculating the system flux; in our case this flux involves the flow of liquid water through porous earth materials.

If our problem is a transient one, we must add a storage term to the equation, which represents a change in the dependent variable (in our problems h or head) as a function of time. In the preceeding development this storage term was represented in equation (10):

$$\frac{\Delta V}{\Delta t} = S \, \Delta x \, \Delta y \, \frac{\Delta h}{\Delta t}.$$

In more formal developments of the ground-water flow equation the continuity equation is written in terms of <u>mass</u> flux instead of <u>volume</u> flux. Instead of equation (2) relative to our block of aquifer material

$$\text{Quantity of water in} - \text{Quantity of water out} = 0, \tag{2}$$

we express the continuity equation in more general terms as:

$$\text{Mass of fluid in} - \text{Mass of fluid out} = 0. \tag{14}$$

More specific to our problem, the steady-state continuity equation that involves fluid mass is expressed in terms of ρ (fluid density) and the fluid velocity components v_x, v_y, and v_z as:

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0. \tag{15}$$

Note that (ρv) has the units of mass flux, M/L^2T.

Writing Darcy's law in the form

$$v_x = K_x \frac{\partial h}{\partial x} \tag{16}$$

for one coordinate direction and substituting (16) in the first term of (15) we obtain

$$\frac{\partial}{\partial x}\left(\rho \; K_x \frac{\partial h}{\partial x}\right) = 0. \tag{17}$$

Assuming that ρ is constant we can write

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) = 0, \tag{18}$$

which is exactly the same as the first term in (7). This last discussion indicates that our simplified approach to developing the flow equation involved the implicit assumption that the fluid density remains constant throughout the flow field. This is equivalent to the assumption that liquid water is incompressible. Physically meaningful solutions to many practical problems in ground-water hydrology employ this assumption.

SECTION (3)--DESCRIPTION AND ANALYSIS OF GROUND-WATER SYSTEMS

The first three subsections below--system concept, information required to describe a ground-water system, and preliminary conceptualization of a ground-water system--introduce the system concept as it is applied to ground-water systems. The system concept is exceedingly useful in ground-water hydrology. It provides an organized and technically sound framework for thinking about and executing any type of ground-water investigation and is the basis for numerical simulation of ground-water systems, the most powerful investigative tool that is available. Although the system concept usually is not developed in a beginning course in ground-water hydrology to the extent that it is here, its fundamental importance, particularly as a framework for thinking about a ground-water problem, warrants this emphasis.

An example of the need for "system thinking" in practical problems is the "site" investigations of ground-water contamination from point sources, a major activity of hydrogeologists at this time. Many of these studies suffer irreparably from the investigators' failure to apply "system thinking" by not placing and studying the local "site" in the context of the larger ground-water system of which the "site" is only a small part.

System Concept

Assignment

*Study Note (3-1)--System concept as applied to ground-water systems.

In Note (3-1), attend particularly to the list of features that characterize a ground-water system. Although these features may seem to be abstract at this time, the reasons for this formulation will become evident as we proceed.

Note (3-1).--System Concept as Applied to Ground-Water Systems

The word system occurs frequently in ground-water literature in combinations such as hydrologic system and ground-water system. The following comments on the system concept and a distillation of those aspects of the concept relevant to a definition of a ground-water system may be used to establish a general framework for ground-water resource evaluation.

A general definition of a system is an orderly combination or arrangement of parts or elements into a whole, especially such combination according to some rational principle giving it unity and completeness. In thermodynamics, a system is a portion of the universe defined by a closed mathematical surface. The rest of the universe is referred to as the surroundings or the environment of the system. To be useful, this definition must be supplemented by additional information describing the physical properties of the enclosing surface (the walls or boundaries of the system)--whether these boundaries are impermeable, permeable, or selectively permeable to the flow of matter and (or) energy across them. These additional considerations lead to the following definitions:

open system--system constantly exchanges both matter and energy with its environment and is maintained by this exchange;

relatively closed system--system constantly exchanges energy but not matter with its environment;

absolutely closed (isolated) system--system exchanges neither energy nor matter with its environment.

In general, ground-water systems are open systems because they exchange both matter (water) and heat energy obtained from the sun or the interior of the earth with their surrounding environment. A simple, schematic representation of a system with its accompanying input and output is shown below.

Flow of matter
and (or) energy
and (or) information

Input (or stress) -----> System -----> Output (or response)

Depending on the investigator's objectives and point of view, a natural ground-water flow system may be defined in various ways. In this discussion, the term "flow system" refers to the part of the ground-water regime that has been isolated for study, and implies the following:

1. the three-dimensional body of earth material under consideration is saturated with flowing ground water;

2. the moving ground water is bounded by a closed surface--the <u>boundary surface</u> of the flow system;

3. under natural conditions, heads and flows associated with specific locations or parts of the system vary with time, normally oscillating around a mean condition;

4. for the flow system to operate continuously through time, water input (continuous or intermittent) to the system and water output (continuous) from the system must occur through at least part of the boundary surface.

A common example of an input to (or stress on) the ground-water system is areal recharge to the water table, which ultimately is derived from precipitation. Short-term fluctuations or longer-term changes in natural recharge due to variations in precipitation or other climatic variables also may be regarded as a stress on the ground-water system. Other stresses on a ground-water system include pumping of ground water, any other mechanism of artificial withdrawal such as an infiltration gallery, and any form of artificial recharge--for example, injection wells, spreading basins and leaking pipe networks.

The most commonly measured response to stress is a change in head in one or more aquifers comprising the ground-water system. These changes in head are an indirect manifestation of changes in flow (as well as changes in storage) either into or out of various parts of the system. Changes in flow in the ground-water system sometimes can be measured directly by monitoring through time increases or decreases in base flow in selected reaches of a stream.

Information Required to Describe a Ground-Water System

Assignments

*Study Fetter (1988), p. 533-534; or Freeze and Cherry (1979), p. 67-69, 534-535.

*Study Note (3-2)--Information necessary to describe a ground-water system.

In these and other study assignments concentrate particularly on all the available information about the boundary conditions used in ground-water hydrology (name, properties, and physical occurrence in real ground-water systems). This is the most important new information in this section and also the most difficult to apply to specific problems.

Note (3-2).--Information Necessary to Describe a Ground-Water System

Quantitative analysis or simulation of a ground-water system entails the solution of a boundary-value problem--a type of mathematical problem which has been studied extensively and which has applications in many areas of science and technology. The flow of ground water in the general case is described by partial differential equations. A ground-water problem is "defined" by establishing the appropriate boundary-value problem; solving the problem involves solving the governing partial differential equation in the flow domain while at the same time satisfying the specified boundary and initial conditions. In ground-water problems, the solution usually is expressed in terms of head (h); that is, head usually is the dependent variable in the governing partial differential equation. The solution to a simple boundary-value problem in ground-water flow is given by Franke and others (1987, Appendix 1).

The information necessary to describe a ground-water system is summarized in table 3-1. This information is in fact the same information that is needed to formulate a boundary-value problem expressed in ground-water hydraulic terminology. Of the four types of information listed, we will consider at this time only (1) external and internal geometry of system, (2) boundary conditions, and (4) distribution of hydraulic conducting and storage parameters. Item (3), initial conditions, will not be discussed in this course.

Table S-1.--Information necessary for quantitative definition of a ground-water flow system in context of a general system concept

Input ---------------->>	System -------------->>	Output
Input or stress applied to ground-water system	Factors that define the ground-water system	Output or response of ground-water system
(1) Stress to be analyzed: - expressed as volumes of water added or withdrawn - defined as function of space and time	(1) External and internal geometry of system (geologic framework) - defined in space (2) Boundary conditions -defined with respect to heads and flows as a function of location and time on boundary surface (3) Initial conditions -defined in terms of heads and flows as a function of space (4) Distribution of hydraulic conducting and storage parameters - defined in space	(1) Heads, drawdowns, or pressures[1] -defined as function of space and time

[1] Flows or changes in flow within parts of the ground-water system or across its boundaries sometimes also may be regarded as a dependent variable. However, the dependent variable in the differential equations governing ground-water flow generally is expressed in terms of either head, drawdown, or pressure. Simulated flows across any reference surface can be calculated when the governing equations are solved for one of these variables, and flows in real systems can be measured directly or estimated from field observations.

The list of factors in table 3-1 may be clarified by considering the specific example of an island hydrologic system illustrated in figure 3-1. We already have encountered the idea of defining a specific volume of saturated earth material or reference volume for study in previous discussions of water budgets and the system concept. The external geometry of the ground-water system is the position in space of the outer bounding surface of this reference volume. In figure 3-1, the trace of this outer bounding surface is depicted by a broken line and dots. Note that this line in figure 3-1 defines a continuous closed curve.

The internal system geometry or geologic framework refers to lithologic units or combinations of units within the reference volume that can be differentiated in the subsurface and which often exhibit marked differences in hydraulic conductivity. When contrasts in hydraulic conductivity are large (several orders of magnitude), and the higher conductivity units can provide water to wells, we distinguish between aquifers and confining units. In figure 3-1 units (1) and (3) are aquifers, and unit (2) is a confining unit.

Often, some part of the boundary surface of a ground-water system corresponds to identifiable hydrologic features at which some characteristic of ground-water flow is described easily in hydraulic terms, such as heads acting on or flows through that part of the surface; examples are a body of surface water, an almost impermeable surface, a water table, and so on.

First, consider the contact surface between a fresh surface-water body such as a lake or stream and the saturated ground-water reservoir (for example, the streambed of a gaining stream). From a previous discussion (see last part of exercise on ground-water head) we know that the hydraulic head acting on this contact surface is equal to the water-level elevation of the surface-water body above it, irrespective of the configuration of the contact surface. The usual way of defining the "boundary condition" of the ground-water system along such a contact surface is as a "constant-head" boundary condition. If the surface elevation of the surface-water body changes with location, as would generally be the case for streams, then the hydraulic head acting on the contact surface would also change as a function of location.

Second, consider the contact surface between nearly impermeable bedrock and an overlying hydrogeologic unit whose hydraulic conductivity is several orders of magnitude larger (for example, the surface between unit (3) and the underlying bedrock represented by line FG in figure 3-1). If we assume that the bedrock is effectively impermeable compared to the overlying units, then the contact surface between the underlying effectively impermeable unit and the overlying relatively permeable unit represents hydraulically a stream surface; that is, ground-water flowlines move parallel to this surface but not across it. This type of boundary is known as a "no-flow" or streamline boundary.

Third, consider a water table (line AB in figure 3-1) that is subject to intermittent areal recharge through the unsaturated zone by water derived from precipitation at the land surface. Under these hydraulic conditions this upper boundary of the saturated ground-water system often is considered to be a "flux" boundary; that is, the volume of water entering the saturated ground-water system at the water table per unit area per unit time is

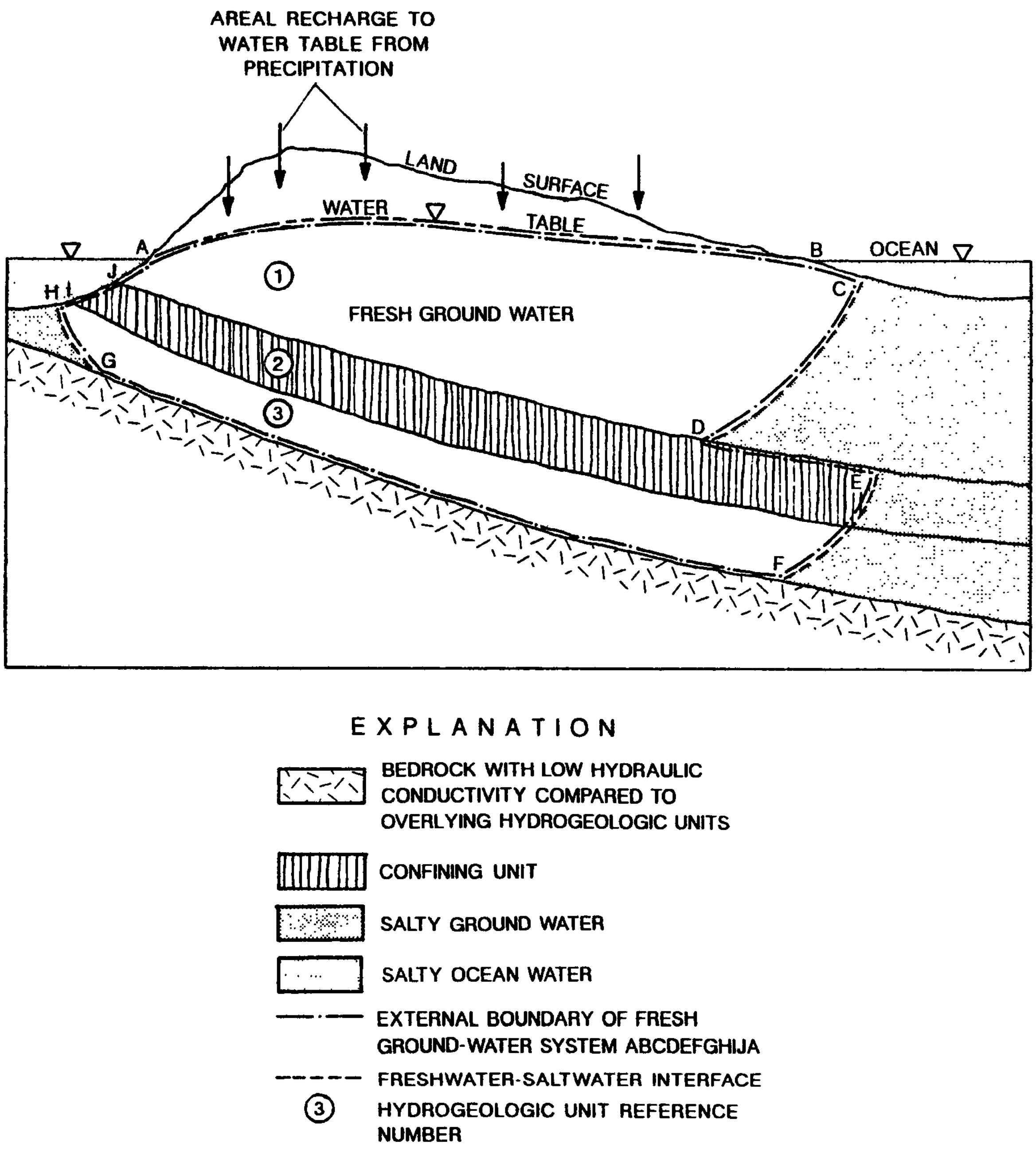


Figure 9-1.--Vertical section through an island ground-water system surrounded by salty surface-water bodies and underlain by nearly impermeable bedrock.

specified. This value of specified flux may vary as a function of location and time.

The preceeding three examples represent common hydrologic features of ground-water systems, and their representations in terms of boundary conditions--(1) constant head, (2) streamline or "no flow", and (3) specified flux--are equally common. Further discussion of these and other boundary conditions utilized in analyzing ground-water systems is provided by Franke and others (1987).

Before concluding this brief introduction to boundary conditions, two additional comments are appropriate. (1) As noted previously, the broken line with dots in figure 1 designates the external bounding surface of the ground-water system. Specifying the boundary conditions of the ground-water flow system, as required to define a ground-water-system boundary-value problem (table 3-1), means assigning a boundary type (for example, one of the types discussed above) to every point on the boundary surface. (2) The "mathematical" boundary conditions that are used to describe hydraulic conditions in terms of heads and flows at the bounding surface of the ground-water system generally are greatly simplified representations of the hydraulic conditions that actually exist in nature at these boundaries.

A principal activity of ground-water hydrologists is to determine by various techniques the spatial distribution of hydraulic properties of earth materials which include both conductivity (horizontal and vertical hydraulic conductivity and transmissivity) and storage (storage coefficient, coefficient of specific storage) parameters (item (4) in table 3-1). A ground-water system such as the one depicted in figure 3-1 would exhibit not only possibly significant differences in "average" hydraulic properties of the three designated hydrogeologic units (two aquifers separated by a confining unit), but also possibly significant areal variations in hydraulic properties within each unit.

A clear understanding of the factors that define a ground-water system as listed in table 3-1 is an essential prerequisite for both the descriptive and quantitative study of these systems.

Preliminary Conceptualization of a Ground-Water System

<u>Assignments</u>

*Study Note (3-3)--Preliminary conceptualization of a ground-water system.

*Work Exercise (3-1)--Refining the conceptualization of a ground-water flow system from head maps and hydrogeologic cross sections.

*Refer to figure 1-7 of Note (1-1) on head in which three pairs of observation wells are depicted, each pair showing a different relationship between shallow heads and deeper heads. Based on your study of the ground-water system in Exercise (3-1), where would you expect to find each pair of observation wells in a "typical" ground-water system, irrespective of the scale of that system?

After finishing the assignments, note that (a) our conceptualization of a ground-water system is based on what we know about that system at any particular time and must be revised continually as new information becomes available; and (b) a system conceptualization that bears little resemblance to the real system under study may lead to quantitative analyses of that system that are grossly in error because, in essence, the "wrong" system is being analyzed.

Note (3-3).--Preliminary Conceptualization of a Ground-Water System

A conceptual model of a ground-water system is a clear, qualitative, physical representation of how the system operates. A hydrologist's conceptual model of the ground-water system under study at any specific time determines the direction, focus, and specific content of the progressing investigation. If the hydrologist's operating conceptual model bears little resemblance to the operation of the natural ground-water system, then the results of the investigation will be at best misleading and at worst grossly in error. Steps for developing a conceptual model, which in essence describe a sequential thought process, are listed in table 3-2.

The first four steps listed in table 3-2 were mentioned previously in notes and exercises on water budgets and the systems concept as applied to ground-water systems. Thus, table 3-2 is both a summary to this point and a qualitative extension of the system concept that may be used to guide the study of any ground-water system. In the context of this course, table 3-2 serves as a guide to the detailed study of two simple ground-water systems described in subsequent exercises.

With reference to item (5) in table 3-2, ground-water flowlines must join boundary areas of inflow to boundary areas of outflow, assuming that no internal sources or sinks exist within the ground-water system; in addition, "no-flow" system boundaries define the location of locally bounding system flowlines. Coupling this information with the fact that flowlines cannot cross one another often enables the investigator to sketch a crude but conceptually useful flow pattern within the ground-water system. The inclusion of geologic framework information, particularly the spatial distribution of aquifers and confining layers, and heads depicted on head maps and vertical cross sections, further refines the resulting physical picture. As part of this process, it is also useful to think about not only the general pattern of flow within the ground-water system but also the gross distribution of flow in the various parts of the system, to the extent that available information permits.

The response of a ground-water system to a "large" stress ultimately depends on the type and areal extent of system boundaries and their location relative to the stress (item (6) in table 3-2). Further discussion of item (6) follows the exercise on the source of water to a pumping well (Exercise 3-3).

Table 8-2.--Steps in developing a conceptual model of a ground-water system

(1) Isolate for study an appropriate three-dimensional body of saturated earth material (ground-water system), which is equivalent to the reference volume for the water budget of the ground-water system.

(2) Delineate areas where water enters (recharges) and leaves (discharges from) the boundary surface of the reference volume defined in (1), as well as boundary areas across which little or no flow occurs.

(3) Describe the hydraulic conditions in terms of heads and flows on the various boundary surface areas delineated in (2). Assign mathematical boundary conditions to these areas based on their associated hydraulic descriptions.

(4) Depict the spatial distribution and hydraulic characteristics of the principal aquifers and confining beds in the ground-water system by means of hydrogeologic maps and sections.

(5) Conceptualize an approximate ground-water flow pattern within the ground-water system on the basis of the results obtained in (2), (3), and (4), associated head maps, and related information.

(6) Evaluate qualitatively the response of the ground-water system to (further) development--that is, to stresses of different magnitude and location within the system.

Exercise (S-1)--Refining the Conceptualization of a Ground-Water Flow System from Head Maps and Hydrogeologic Sections

by Herbert T. Buxton and Debra E. Bohn[1]

The first stage of any ground-water-system analysis includes the development of a clear concept of the structure and operation of the system under investigation. This concept includes an evaluation of (1) the system's hydrogeologic framework (geometry), (2) its hydrologic boundaries, and (3) the distribution of water-transmitting properties within the system. Early in the investigation of actual ground-water systems this concept often is highly oversimplified because of a lack of hydrologic data. Continuing analysis accompanied by additional hydrologic data gradually increases the understanding of the ground-water system and refines the initial concept.

Flow patterns in natural ground-water systems are characteristically three-dimensional and complex at the local scale. The pattern of flowlines through a system is controlled by the distribution of flow entering and leaving the system and the distribution of water-transmitting properties within the system. Recharge to and discharge from a ground-water system is controlled by the location and characteristics of its natural hydrologic boundaries, which often are conceptualized in a mathematical sense by identifying an associated boundary condition. The distribution of water-transmitting properties at the system scale is defined by its internal hydrogeologic geometry, which usually is depicted as a sequence of aquifers and confining units, each with distinct hydrologic properties.

The hydrologist often approximates ground-water flow patterns from the distribution of hydraulic head throughout the ground-water system. In general, hydraulic head varies in three dimensions throughout the flow domain. Practical problems arise, however, in the attempt to depict equipotential surfaces and hydraulic gradients in three dimensions. Head distributions typically are represented on plan-view maps and (or) cross sections which must be constructed in a manner that 1) properly represents our concept of the structure and operation of the ground-water system, and 2) accurately depicts the three-dimensional features of the ground-water system and the head distribution within it.

The purpose of this exercise is to demonstrate (1) how an accurate depiction of the three-dimensional distribution of hydraulic head in a system can be described on a series of maps and sections derived from (a) pertinent hydrologic data, (b) knowledge of the physics of ground-water flow, and (c) a preliminary concept of the structure and operation of a ground-water system, and (2) how this depiction of the head distribution confirms and (or) modifies our initial conceptualization of the system.

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A description of a hypothetical ground-water system is provided, followed by data from a synoptic water-level measurement of a ground-water observation-well network in that system. A step-by-step procedure for mapping the three-dimensional distribution of hydraulic head in the system is presented. A similar procedure is used to depict the change in head distribution caused by (1) the introduction of a pumped well to the system and (2) a hypothetical change in the internal geometry of the system. The "observed" data presented for each of these exercises were obtained from numerical models of these hypothetical systems.

Description of Hypothetical Ground-Water System

The hypothetical ground-water system represented in figure 3-2(A) shows a rectangular glacial valley approximately 56,000 ft wide (east-west) by 40,000 ft long (north-south). The valley was eroded in consolidated bedrock and subsequently filled with alluvium. A large lake lies south of the valley. A small tributary stream drains the valley and flows into the lake.

Sections A-A' and B-B' (figs. 3-2(B) and 3-2(C)) depict the hydrogeologic framework of the ground-water system. An initial concept of the hydrogeologic framework of this system is provided to facilitate the head-mapping exercise. In an actual field investigation, undoubtedly, development of this initial concept would require data collection and analysis.

The system is bounded laterally on three sides and on the bottom by bedrock and on the top by the water table. The hydraulic conductivity of the bedrock is assumed to be negligible compared to that of the aquifer units in the alluvium. To the south the water-table aquifer is bounded by the lake. The lower aquifer, which initial hydrogeologic data indicate is confined, appears to pinch out abruptly several miles offshore, south of the map and sections shown in figure 3-2.

The water table, generally, is a subdued replica of the valley topography. Inflow to the ground-water system occurs as areal recharge at the water table derived from precipitation; and ground water discharges to the stream and lake at the stream bed and lake bottom, respectively. The water table configuration approaches and intersects the downstream portion of the stream surface as shown in figure 3-2(C). Along this reach ground-water seeps into the channel, which acts as a ground-water drain. Discharge measurements verify that this stream is a gaining stream; its base flow increases continuously downstream. Seasonal and annual changes in areal recharge affect the altitude and configuration of the water table, thereby changing the quantity and temporal distribution of base flow in the stream and the point at which flow in the stream channel begins (start-of-flow).

The internal geometry of the system consists of two aquifers separated by an approximately 40-ft-thick confining unit (fig. 3-2(B) and 3-2(C)). Data available from previous studies indicate that the water-transmitting properties of the hydrogeologic units are approximately as follows: the water-table aquifer has a hydraulic conductivity of about 300 ft/d and a horizontal to vertical anisotropy of about 100:1; the confining unit has a vertical hydraulic conductivity of approximately 0.002 ft/d, and the confined aquifer has a hydraulic conductivity of about 100 ft/d (no data on its anisotropy are available.

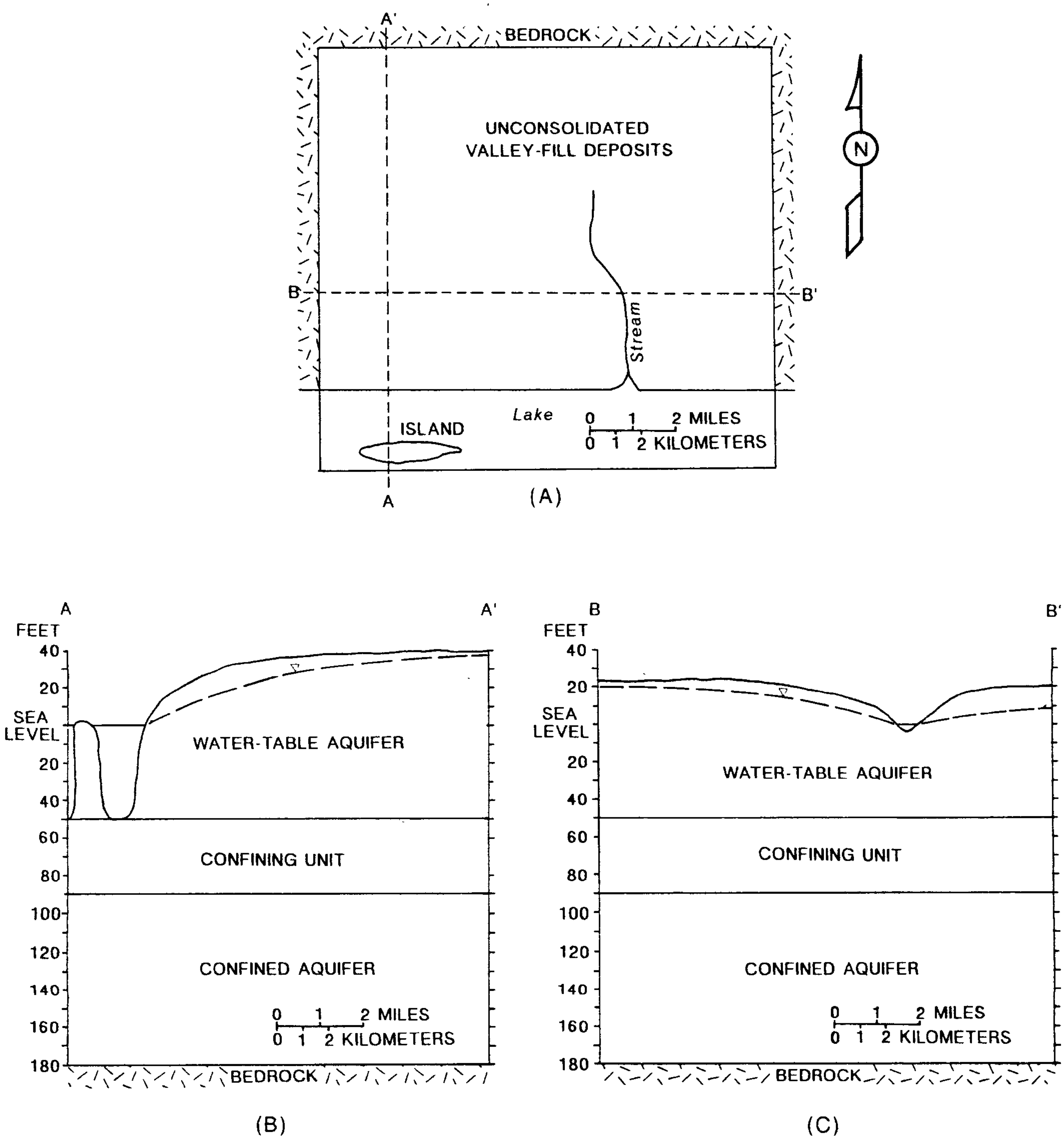


Figure 3-2.--Hydrogeologic framework of hypothetical ground-water system: (A) plan view, (B) north-south-trending section, (C) east-west trending section.

Question 1.--From the preceding description, assign a physically reasonable boundary condition to each of the system boundaries shown on figures 3-2(A), 3-2(B), and 3-2(C) (e.g. constant-head, no-flow, and so on.) A discussion of boundary conditions in ground-water systems is provided by Franke and others (1987).

Observation-Well-Network Design in Hypothetical Ground-Water System

An observation-well network installed to monitor hydrologic conditions in this system is illustrated in figure 3-3. Doublet and triplet wells indicated in figure 3-3 mark sites where multiple wells were installed and screened at different altitudes. This observation-well network was designed to permit the definition of the three-dimensional head distribution using a limited number of point observations. Observation wells are piezometers that measure hydraulic head at the point of opening to the system (well screen). One-foot-long screens were installed in these wells so that subsequent measurements could be assigned to a unique location in space.

In order to define the head distribution within the ground-water system, well locations must be chosen with a measure of hydrologic insight--for example, anticipating areas of large hydraulic gradients and selecting surfaces on which head maps or sections will be constructed. Some wells, screened at or near the water table, are intended to be used primarily for construction of maps that describe the configuration of the water-table surface. Other wells, screened near the base of the water-table aquifer, are intended to indicate the magnitude of the vertical component of head gradients within the aquifer. The remaining wells were screened in the confined aquifer and are intended to be used for construction of maps of the potentiometric surface of the confined aquifer.

The numbers marked along the tributary stream in figure 3-3 indicate streambed altitudes, taken from topographic maps, which closely approximate the actual stage in this shallow stream.

Mapping Hydraulic Head in a Layered Ground-Water System: Water-Table Map

Head and related data from a synoptic measurement of the observation-well network are plotted on several maps and cross sections. Water-table altitude, streambed altitudes, and the start-of-flow point in the tributary stream are plotted in figure 3-4. The start-of-flow point of this ground-water-fed stream marks the upstream limit of intersection between the water-table surface and the stream-channel surface. Downstream from the start-of-flow point the stream surface defines the water-table altitude; upstream from the start-of-flow point the streambed is dry because the water-table altitude is below the streambed altitude. Because the water table is depressed near a gaining stream, equipotential lines form "V"s that point upstream, indicating that ground water flows toward the stream channel.

A number of additional hydrologic facts and principles are useful in constructing head maps of this system. The head distribution within a ground-water system is a continuum in three-dimensional space; therefore, contour lines should be smooth and subparallel to one another. The shape and changes in spacing of contour lines indicate changes in hydraulic gradients. Kinks or jogs in contour lines indicate changes in direction of flow (usually

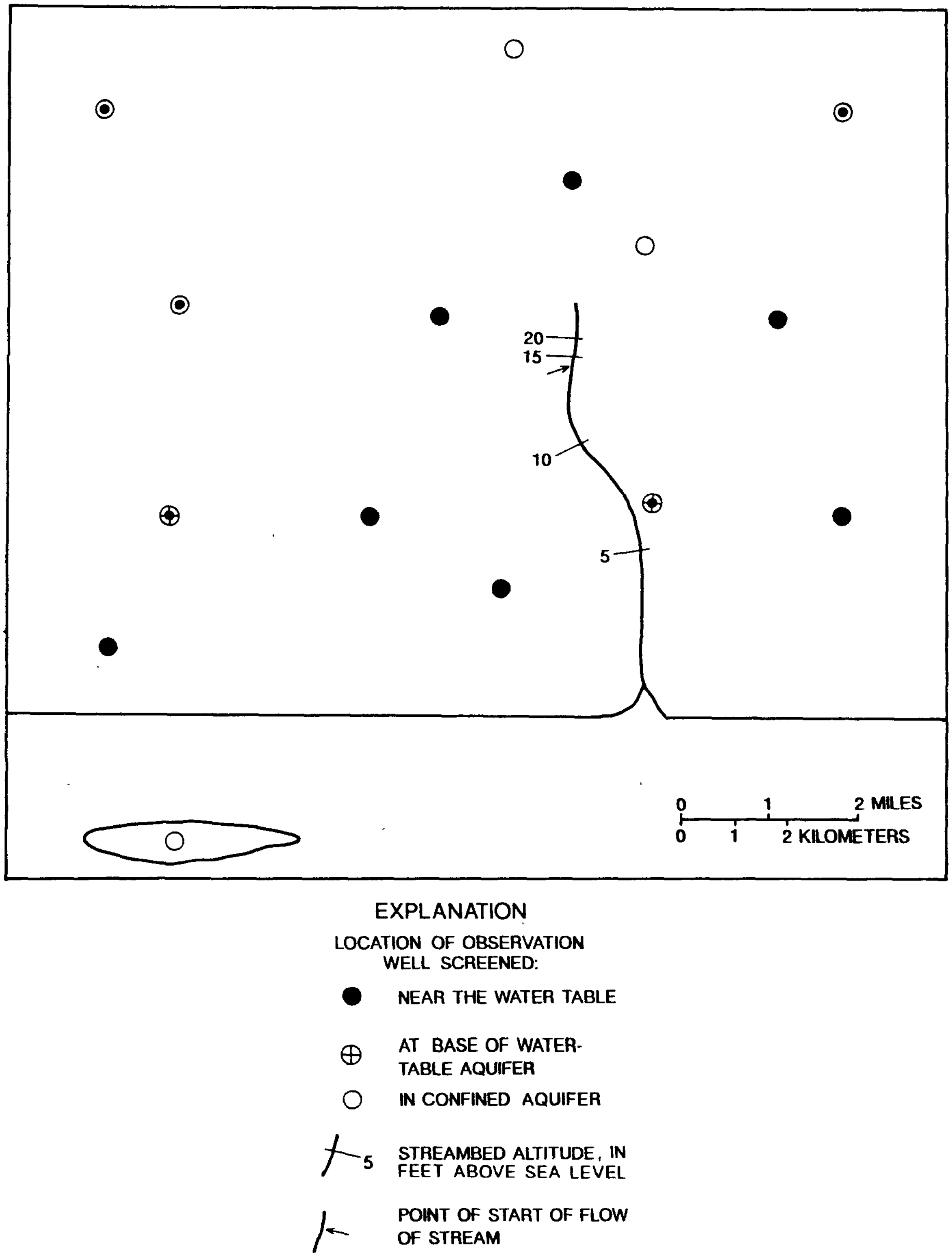


Figure 3-3.--Observation-well network and streambed altitudes for hypothetical ground-water system.

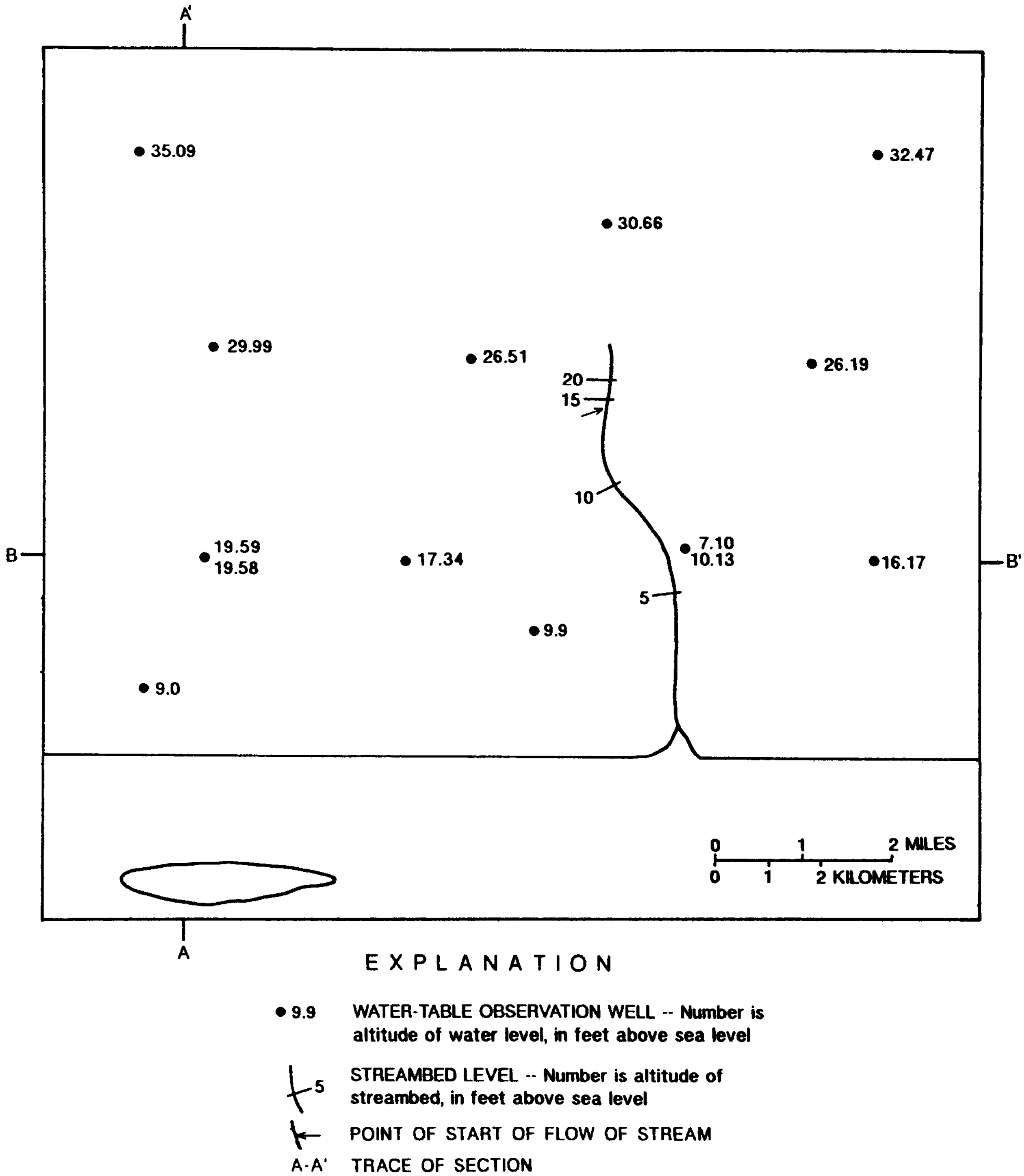


Figure 8-4.--Water-table heads obtained during a synoptic measurement of water levels in observation wells.

convergent or divergent flow); irregular spacing indicates changes in gradients caused either by changes in flow rates or variations in transmissivity.

The shape of equipotential lines near hydrologic boundaries is affected by the boundary type. The configuration of equipotential lines in the vicinity of impermeable, constant-head, and water-table boundaries in a homogeneous and isotropic system is shown in figure 3-5. Equipotential lines typically are perpendicular to no-flow boundaries, are parallel to constant-head boundaries, and represent the altitude of the water-table surface. Freeze and Cherry (1979, p. 168-170) describe these features in greater detail.

Recharge enters this ground-water system at the water table and moves generally toward the discharge boundary (lake shore), resulting in a progressively greater volume of water flowing through the system as one approaches the discharge boundary. In addition, because this aquifer is unconfined, its saturated thickness decreases toward the shoreline. Both of these reasons cause a continuous increase in the hydraulic gradient toward the shore; equipotential lines, therefore, become progressively closer to one another in this direction.

Question 2.--Based on your knowledge of the operation of this system and an understanding of the physics of ground-water flow, draw a contour map of the water-table surface using the data set plotted in figure 3-4 and taking into account the hydrologic principles discussed above. Use a 5-ft contour interval.

Mapping Hydraulic Head in a Layered Ground-Water System: Potentiometric-Surface Map

A potentiometric-surface map depicts the distribution of hydraulic head throughout a confined aquifer. We typically assume (not always correctly) that vertical gradients within the confined aquifer are negligible. Therefore, the head in a well screened in a confined aquifer defines a point on the potentiometric surface of that aquifer.

The confined aquifer shown in figure 3-6 is recharged by slow downward leakage through the confining unit. Heads in tightly cased wells screened in this aquifer are above the altitude of the top of the aquifer.

The potentiometric surface of the confined aquifer in this system is a subdued replica of the water-table configuration. Intuitively, we know that ground water must flow downward from the water-table aquifer into the confined aquifer in the north, and upward out of the confined aquifer to the water-table aquifer in the south, where heads in the water-table aquifer decrease rapidly near the lake and stream.

Question 3.--Use your present knowledge of the ground-water system under consideration to draw the potentiometric surface of the confined aquifer. Use the map and data presented in figure 3-6. Keep in mind the same hydrologic factors that were used to construct the water-table map, and the additional factors noted above. Use a 5-ft contour interval. Overlay this map on the water-table map using a light table. Changes in spacing between nearby equal-valued contours in these two maps indicate changes in vertical

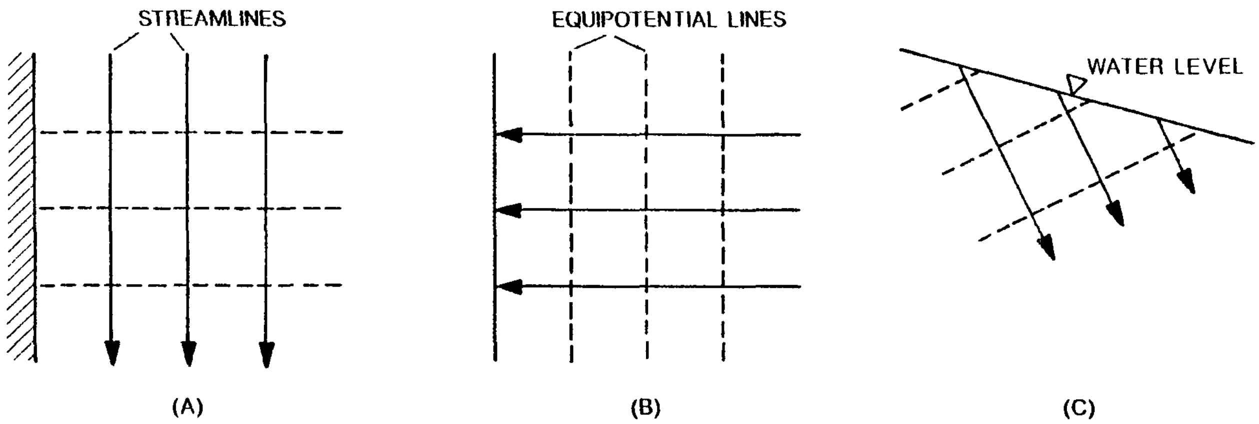
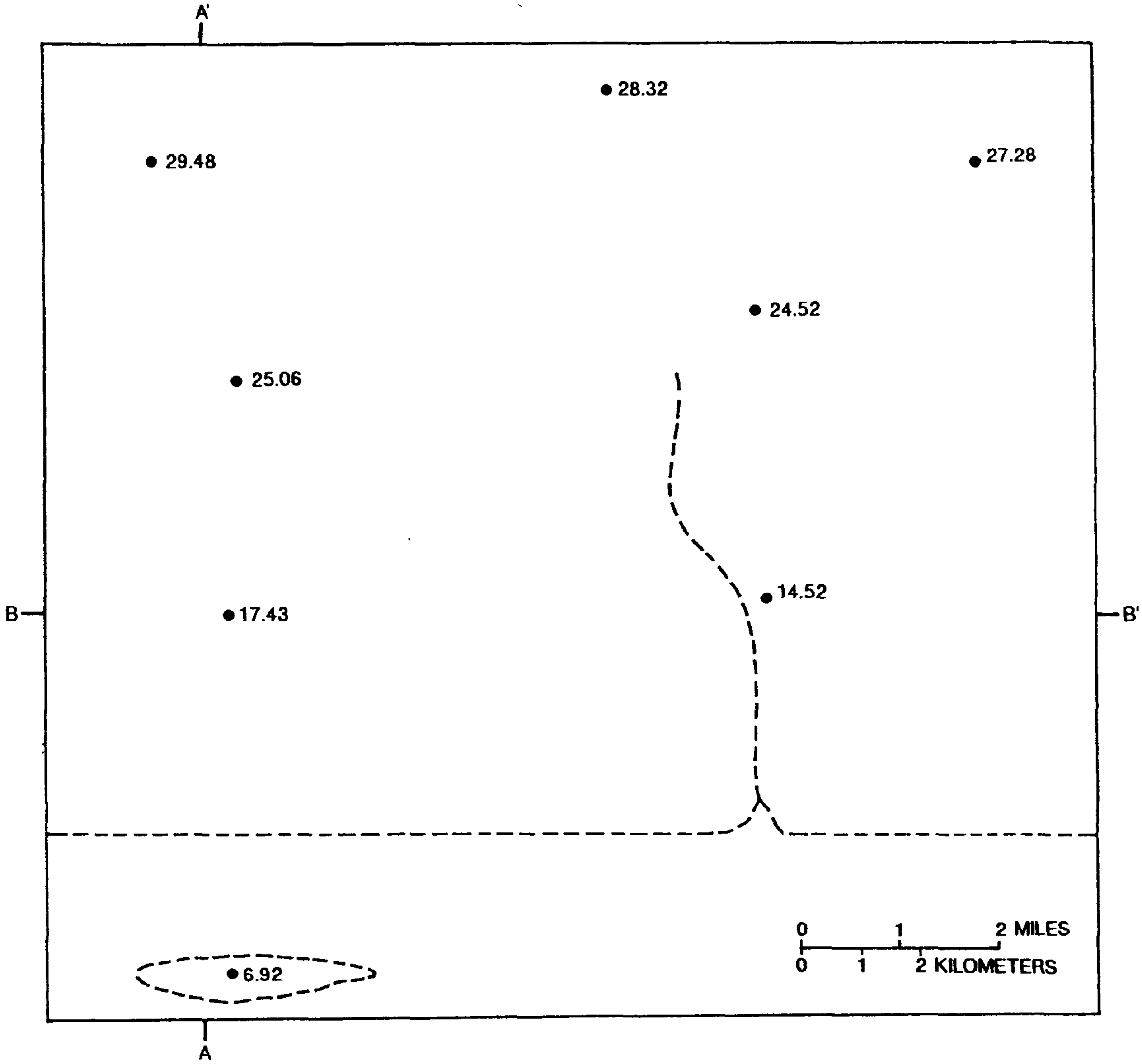


Figure 8-5.--Ground-water-flow pattern in the vicinity of (A) an impermeable boundary, (B) a constant-head boundary, and (C) a water-table boundary. (From Freeze and Cherry, 1979, fig. 5.1.)



EXPLANATION

• 6.92 OBSERVATION WELL SCREENED IN THE CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure 9-6.--Heads in the confined aquifer obtained during a synoptic measurement of water levels in observation wells.

gradients. Modify either or both contour maps if inconsistencies in the magnitude and areal distribution of vertical gradients occur. Together these maps provide a picture of the continuous distribution of hydraulic head in the system.

Mapping Hydraulic Head in a Layered Ground-Water System: Hydrogeologic Sections

Hydrogeologic sections typically depict the vertical distribution of aquifers and confining units along with vertical variations in head within the ground-water system. They allow correlation between heterogeneities in hydrogeologic geometry and the vertical distributions of hydraulic head and flow. Measurements of head at the well screens are plotted for wells along sections A-A' and B-B' in figures 3-7 and 3-8. To review: an observation well is a piezometer--a pressure- and head-measuring device. The well's measuring point is the midpoint of the well screen, which represents the point in three-dimensional space at which the pressure and head is observed. The assumption is that the well screen is sufficiently short to preclude connecting volumes of aquifer with significantly different heads and, therefore, does not transmit a volume of water through the screen and well bore that is sufficient to affect heads locally in the flow system.

All the hydrologic factors discussed previously that are considered in the construction of both water-table and potentiometric-surface maps also are useful in drawing head contours in section. For example, rules that govern the shape of equipotential lines near hydrologic boundaries such as constant-head, streamline, and the water-table boundaries in maps also are true in section (fig. 3-5). The distribution of head in hydrologic sections of natural systems, however, generally is more complicated than in head maps, because the distribution in sections is more likely to reflect the heterogeneity and anisotropy of the aquifers and confining units in the ground-water system.

Heterogeneous Systems.--Vertical heterogeneities (sequences of aquifers and confining units) often depicted in hydrogeologic sections include stratigraphic boundaries between units which exhibit large contrasts in hydraulic conductivity. Flow and equipotential lines refract in a predictable manner as they cross these boundaries (fig. 3-9). The angle of refraction at a boundary for both sets of lines can be calculated from the angle of incidence and the ratio between the hydraulic conductivities of both units (Fetter, 1988, p. 139-141). In an intuitive way, we can consider the changes in the ground-water flow pattern that are necessary to maintain flow from a more permeable to a less permeable region. Given that the flow in any stream tube (the flow conduit between any two streamlines) in figure 3-9 remains constant, it is evident that: (1) streamlines refract toward the vertical when entering a less permeable region, thereby increasing the cross-sectional area of flow within stream tubes; and (2) equipotential lines refract away from the vertical and decrease their spacing, thereby increasing the hydraulic gradient within stream tubes. In accordance with these principles vertical gradients within aquifers often are small in comparison to vertical gradients across confining units, resulting in equipotential lines that are nearly vertical in aquifers and nearly horizontal in confining units.

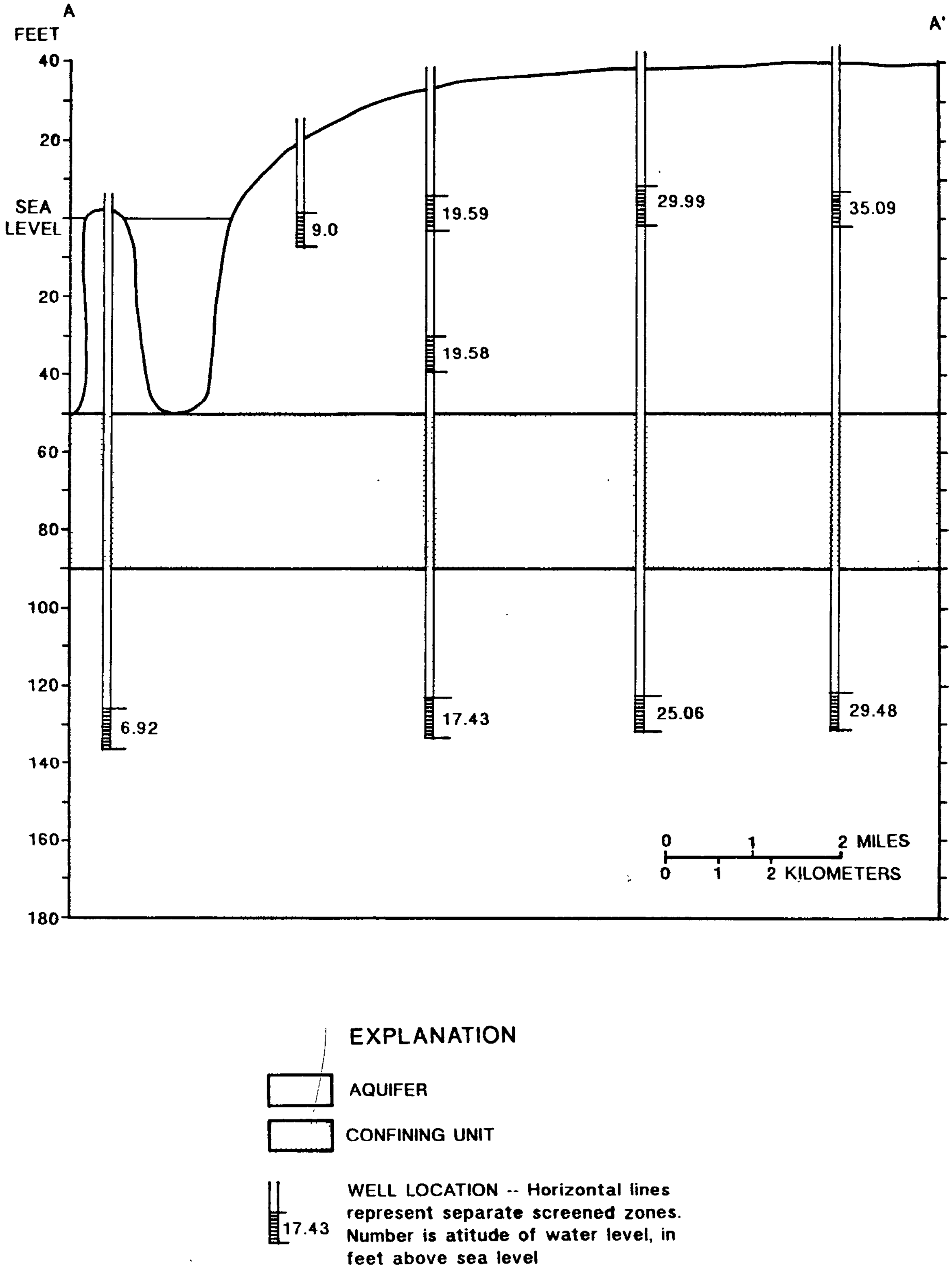


Figure 3-7.--North-south-trending hydrogeologic section showing heads obtained during synoptic measurement of water levels in observation wells. (*Location of section A-A'* is shown in fig. 3-2.)

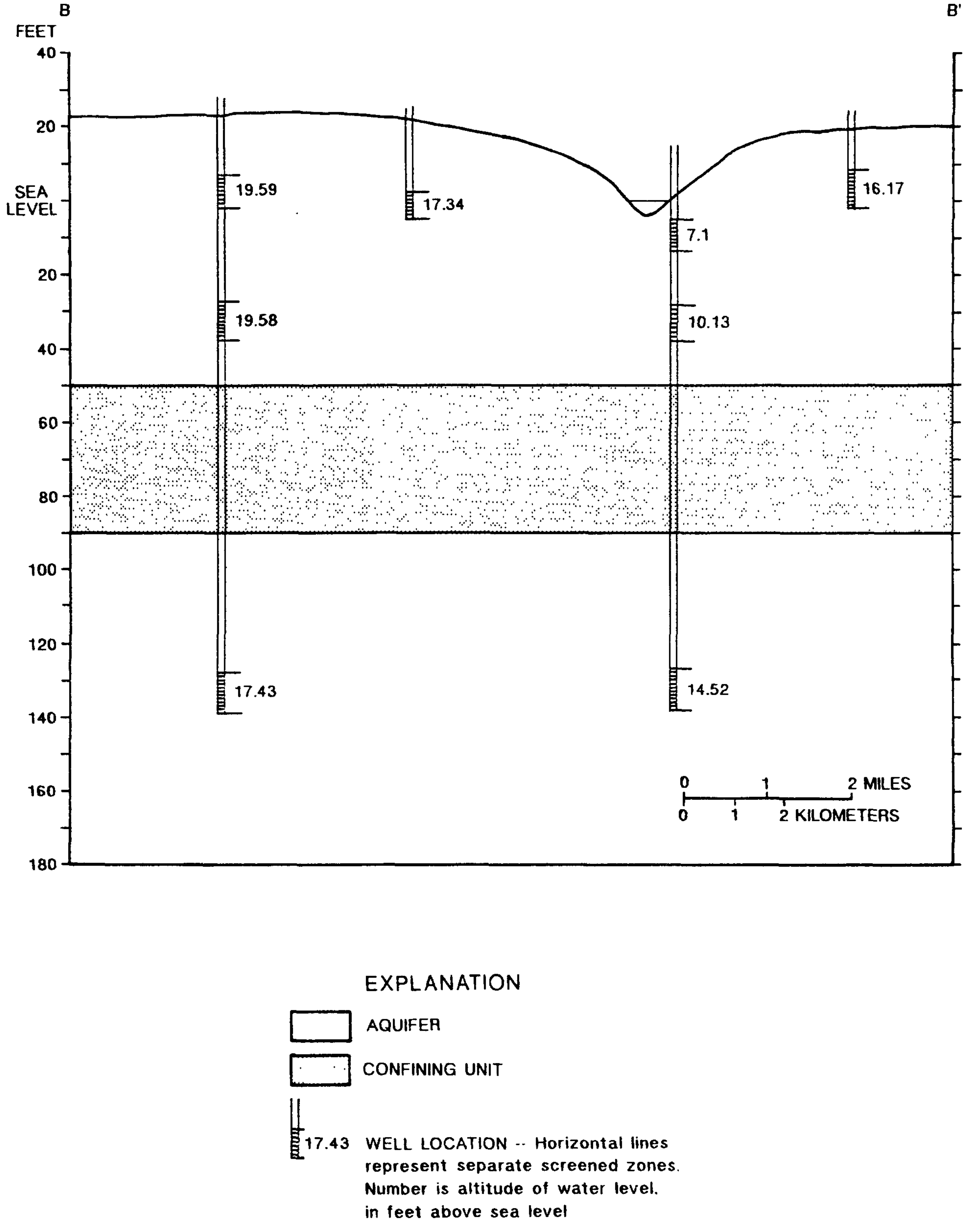


Figure 3-8.--East-west-trending hydrogeologic section showing heads obtained during synoptic measurement of water levels in observation wells. (Location of section B-B' is shown in fig. 3-2.)

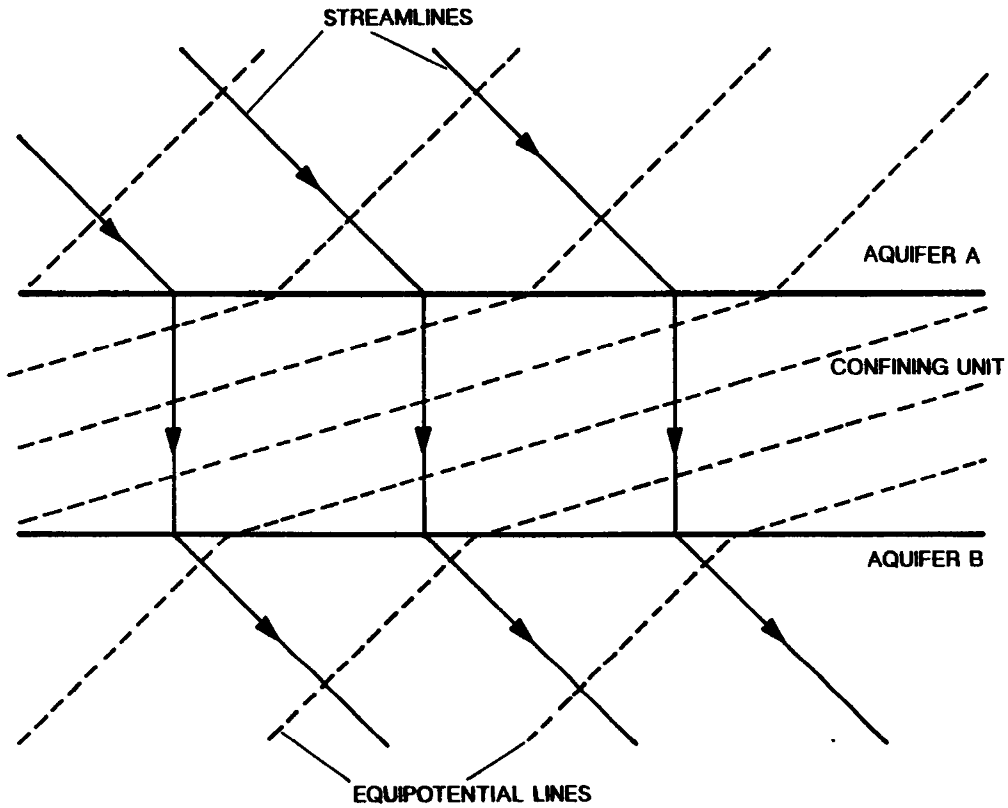


Figure 3-9.--Pattern of streamlines (flowlines) and equipotential lines passing through units with highly contrasting isotropic hydraulic conductivities.

Alignment of Flowlines and Equipotential Lines.--Although many hydrogeologic units are effectively isotropic in plan view, the layered deposition of geologic strata causes most hydrogeologic units to exhibit different values of hydraulic conductivity in directions normal and parallel to planes of deposition. In general, most hydrogeologic units are deposited in nearly horizontal beds, and as a result, horizontal hydraulic conductivity is usually greater than vertical hydraulic conductivity. The most significant result of vertical anisotropy in ground-water systems is its effect on the vertical distribution of flow within a system. In anisotropic systems, flowlines and equipotential lines are not orthogonal; their angle of intersection depends on the relation between the direction of the hydraulic gradient and the orientation of the axes of maximum and minimum hydraulic conductivity. Thus, vertical anisotropy also affects the distribution of head within a system and complicates the interpolation of head contours between points of measured head in a vertical section despite a general knowledge of the ground-water flow pattern. A more detailed discussion of this topic is found in Freeze and Cherry (1979, p. 174-178).

The relation between flowlines and equipotential lines is additionally complicated by the fact that hydrogeologic sections usually are constructed with a vertical exaggeration that may be as high as several hundred. Vertical exaggeration skews or distorts both flowlines and equipotential lines in the vertical axis direction and changes their apparent angle of intersection. As a consequence, although equipotential lines may appear to be vertical on a section, indicating horizontal flow, significant vertical flow components may be present.

Question 4.--Use your present knowledge of this ground-water system and the hydrologic factors discussed above to draw equipotential lines on both sections A-A' and B-B' (figs. 3-7 and 3-8). A good starting point is to locate the position of each contour line on the section from both the water-table and potentiometric-surface maps. Plot the water-table surface on both sections using these data. Remember that contour lines on the water-table map show the altitude of the water-table surface and should be marked on the section at the water-table surface. Assume that vertical gradients within the confined aquifer are negligible; that is, contour lines are effectively vertical in that aquifer.

Constructing hydrogeologic sections can result in an improved understanding of the head distribution and general flow pattern in the ground-water system. While contouring, either the water-table or the potentiometric-surface map can be refined to reflect an improved concept of the system. Upon completion, the maps and sections should represent a consistent picture of the three-dimensional distribution of hydraulic head within the system.

Sketch arrows across equipotential lines to indicate general flow directions based on observed gradients. Keep in mind that flowlines and equipotential lines are perpendicular only in isotropic systems, and that even in isotropic systems they do not appear to be perpendicular on sections in which the vertical dimension is exaggerated.

Question 5.--Use the completed set of maps and sections produced in Questions 2, 3, and 4 to delineate the regions in which ground water flows upward and downward between aquifers. Draw a dashed line on the water-table map (fig. 3-3) that marks the transition between these regions. Again, use a light table for an accurate comparison of the head in each aquifer. Also mark the position of this transition on both sections (figs. 3-7 and 3-8). If this demarcation line on the water-table map is not a smooth curve or its pattern is not consistent with the concept of the flow system developed previously, then changes should be made in the position of head contours on those maps and cross sections that will improve the configuration of this demarcation line. Our concept dictates that vertical flow is generally downward in the north and reverses to upward in the south, near the lake. The line of demarcation also is affected to some extent by the gaining tributary stream, which has depressed the water-table surface locally.

Mapping Hydraulic Head in a Layered Ground-Water System with a Pumping Well

The ground-water system discussed in this section is identical to the one discussed previously. However, a pumping well has been introduced to the system, and ground-water levels have achieved a new equilibrium condition in response to pumping. The well is located in the northwest corner of the system, 17,500 ft south of the northern boundary of the ground-water system and 12,500 ft east of the western boundary of the system. It is screened in the bottom 25 ft of the water-table aquifer.

Although the major features of the system are the same, hydrologic conditions in the system have changed, requiring a revision to our initial concept of the flow system's operation before additional analysis. The well is pumped at a rate of approximately 1.66 Mgal/d (million gallons per day). The recharge rate for this ground-water system is estimated to be approximately 0.475 ft/yr (feet per year). To indicate the magnitude of the pumping stress relative to total inflow to the system, calculate the total rate of recharge to the system, in million gallons per day, and the percent of the total flow in the system that is pumped. In a budget sense, how will the pumping affect the inflow and outflow at system boundaries?

The pumping well affects this system by "rearranging" ground-water flow paths to divert a fraction of the flow through the system to the well. The areal extent of the cone of depression (or the area in which head changes) in response to the discharging well extends to the boundaries of the system. The area of flow diversion (or the area in which all flowlines terminate at the well, also known as the capture area) is such that the recharge entering this area equals the quantity of water discharged by the well. Head data obtained during a synoptic measurement under steady pumping conditions are shown in figures 3-10, 3-11, and 3-12. These data indicate that ground water still discharges to the lake and stream, and no inflow from the lake has been induced.

Keep in mind that measured heads in observation wells reflect the effect of the pumping stress on the head distribution at known points within a system, and that the location of the stress actually is the pumping-well screen. Because the pumping-well screen is the destination of flow in the area that surrounds the well, it is, locally, the point of lowest head. Mark the location of the pumping well on figure 3-10 and sketch in the pumping well and screen on section A-A' (fig. 3-12) before the mapping exercise.

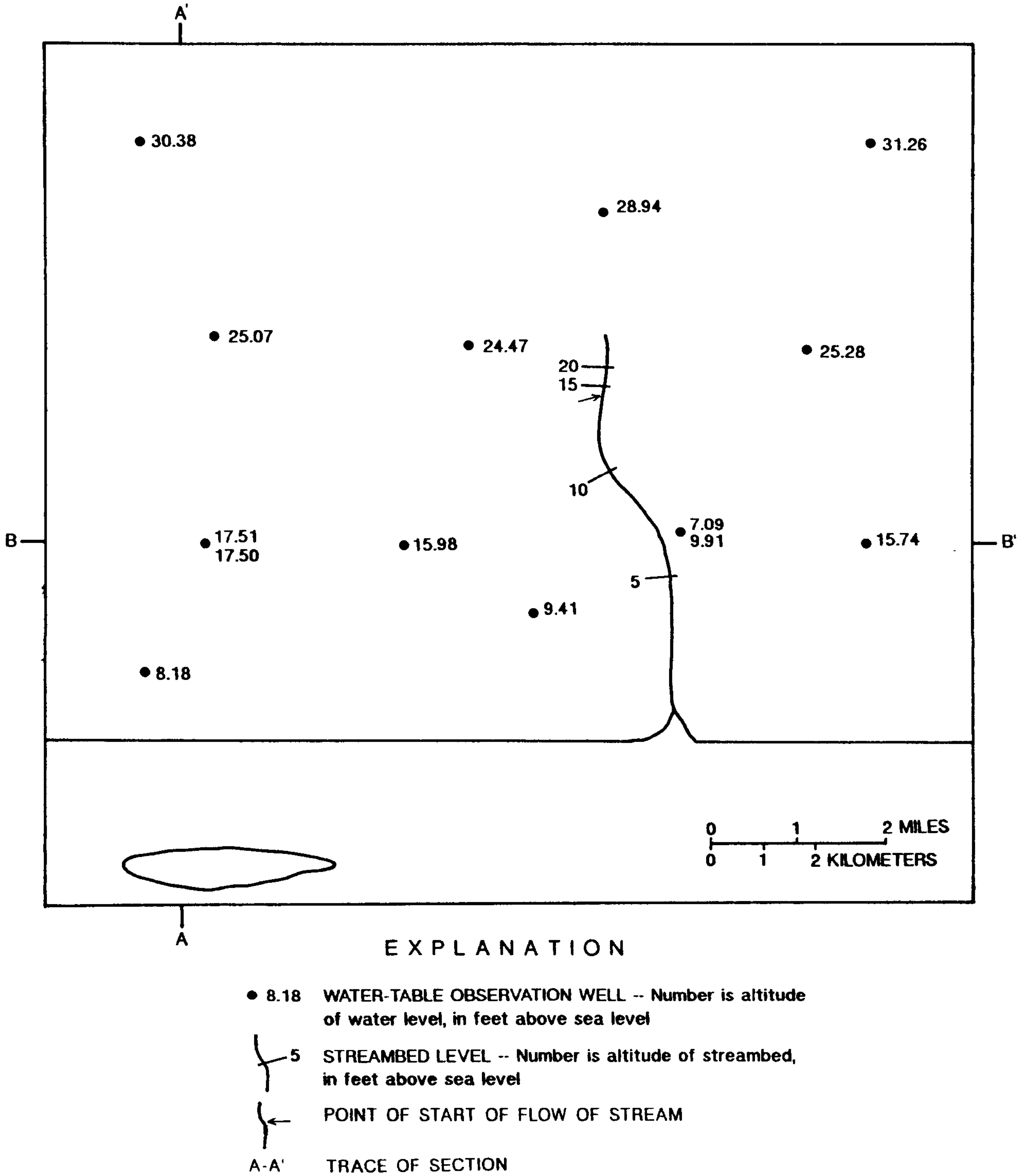
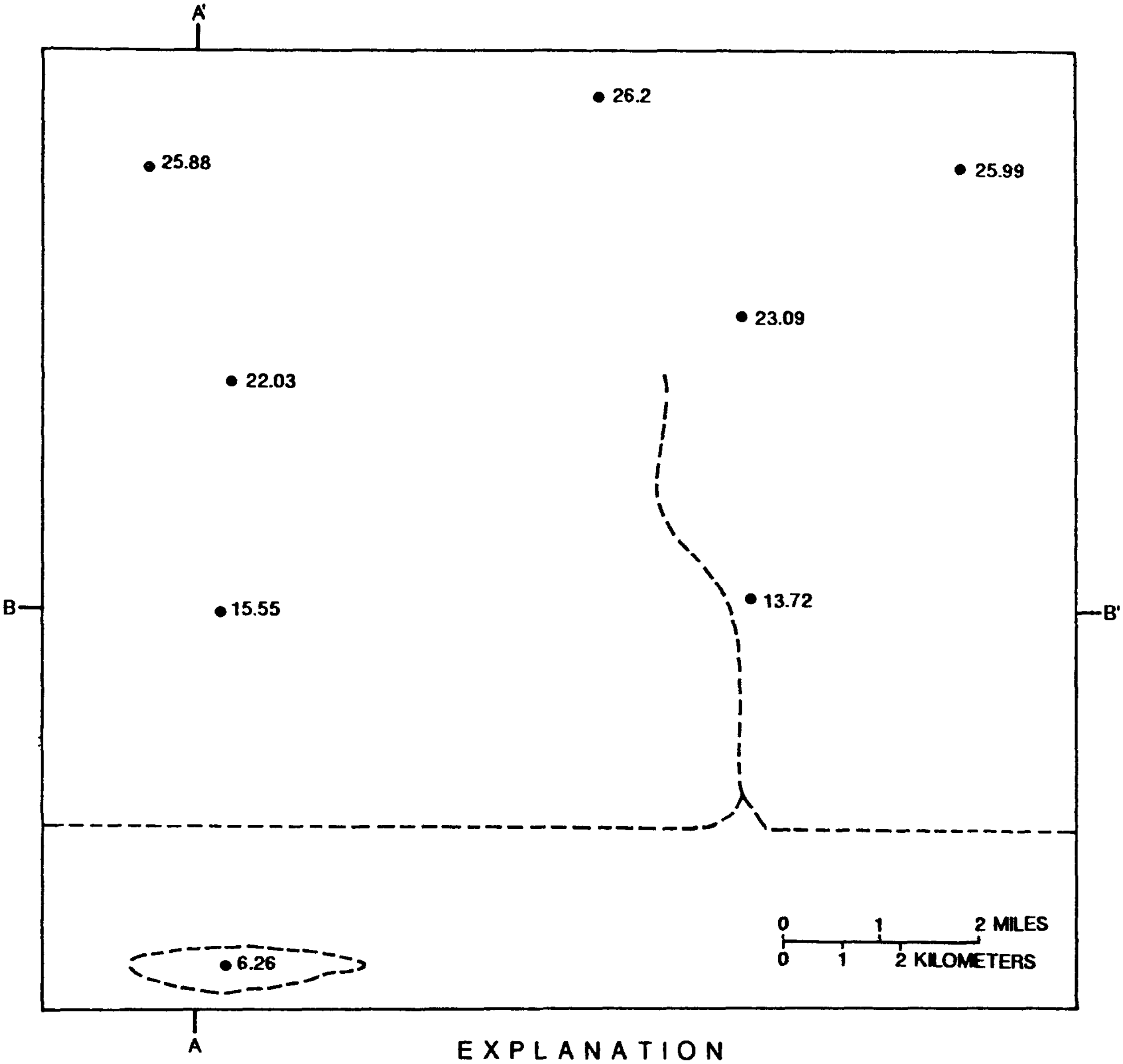


Figure 9-10.--Measured heads in the water-table aquifer in response to steady pumping from a well screened in the lower part of the water-table aquifer.



EXPLANATION

● 6.26 OBSERVATION WELL SCREENED IN CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure 3-11.--Measured heads in the confined aquifer in response to steady pumping from a well screened in the lower part of the water-table aquifer.

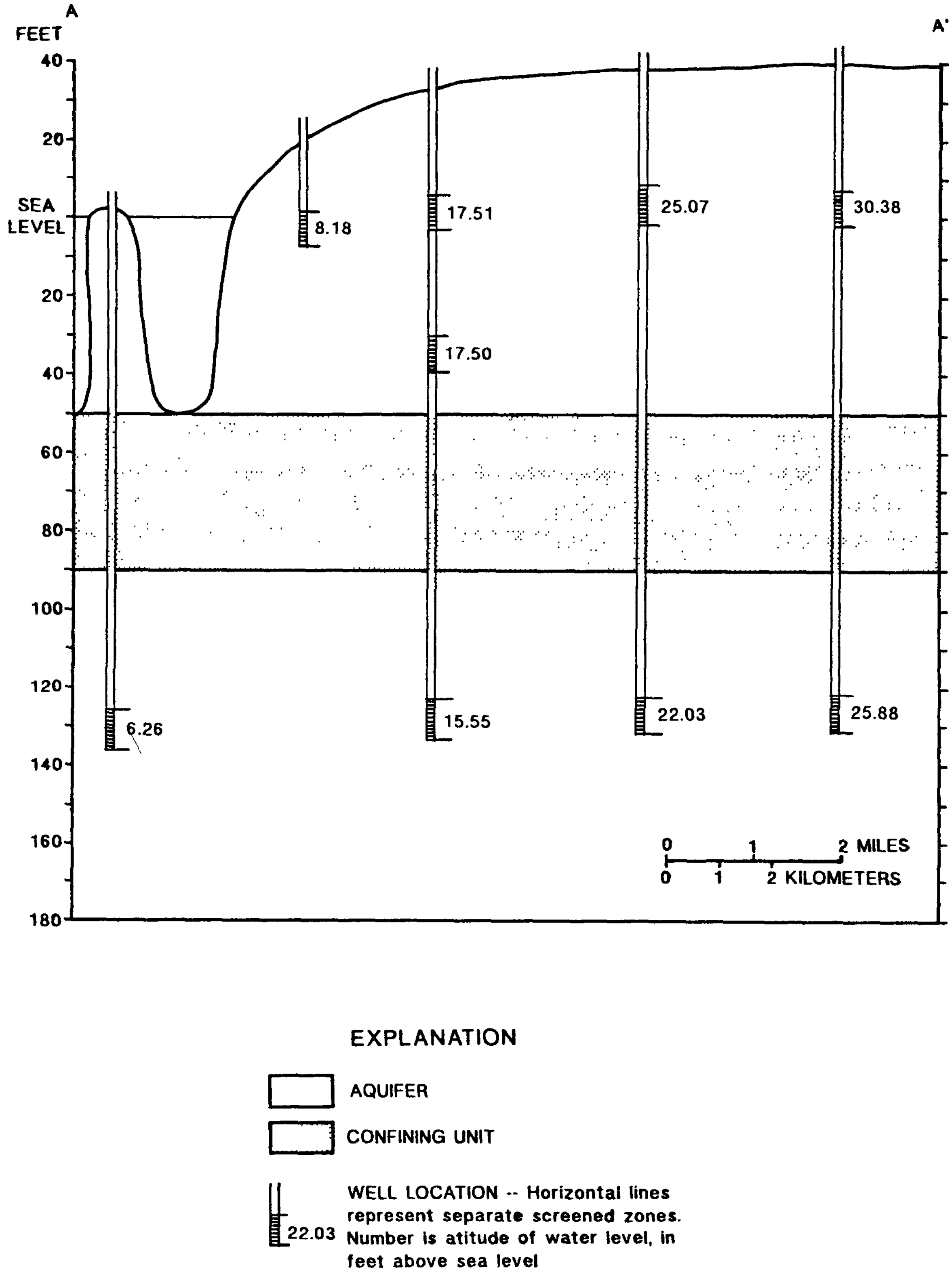


Figure 3-12.--North-south-trending hydrogeologic section showing measured heads in response to steady pumping. (Location of section A-A' is shown in fig. 3-2.)

The mapped distribution of head in the unstressed system may be used as a guide to mapping head in the stressed system. Because the key hydrologic features, recharge and lake stage, have not changed, the head at any point in the stressed system is measurably less than or almost equal to the corresponding head in the unstressed system. Furthermore, the difference between the two head distributions is equal to the drawdown caused by pumping, about which some general deductions may be made. The cone of depression is greatest at the pumping-well screen. The cone of depression is assymmetric because of its proximity to impermeable boundaries to the west and north and to constant-head boundaries to the south and east; the drawdown at a given distance from the pumping well is greater near the impermeable boundaries than near the constant-head boundaries. (See Freeze and Cherry, 1979, p. 330 for a discussion of bounded aquifers.)

Question 6.--Using your knowledge of this system and its operation under pumping conditions, construct a set of maps and sections that depict the three-dimensional distribution of head in the system. First construct potential maps using 5-ft contour intervals of the water table and confined aquifers on figures 3-10 and 3-11. Draw equipotential lines at 5-ft intervals on section A-A' (fig. 3-12). Draw a dashed line on figure 3-12 to indicate the water-table surface under unstressed conditions, then include the water-table surface under stressed conditions. Insert small arrows on the completed section to indicate approximate directions of flow. Locate the line of transition between regions of downward and upward flow between the aquifers. Draw a dashed line on figure 3-10 that indicates the transition line under unstressed conditions. A small zone of upward flow may exist in the immediate area of the pumping well, but the data are insufficient to verify this possibility. Compare the line of transition for the stressed conditions with that for the unstressed conditions. Describe the differences, and what this means in terms of the flow within the system.

Mapping Hydraulic Head in a Layered Ground-Water System with a Discontinuous Confining Unit

The head data presented in this section (figs. 3-13 and 3-14) were obtained during a synoptic measurement of observation wells in a ground-water system similar to the one described previously in this exercise. Although initial concepts of the geologic framework and hydrologic features of these two ground-water systems may have been identical, observed head data indicate some difference between these systems. Compare the data set presented in figures 3-13 and 3-14 in this section with that presented earlier in figures 3-4 and 3-6.

The major difference between the two data sets is the absence of vertical gradients between water-table and confined aquifers in the northwest part of the system. The difference in head at the well doublet in the extreme northwest is only 0.01 ft (33.77 ft minus 33.76 ft), and the difference in head at the doublet directly to the south is 0.04 ft (28.82 ft minus 28.78 ft); both differences are much smaller than the corresponding head differences in the original system. In other areas vertical gradients are considerably larger than in the original system; for example, the well doublet in the northeast corner of the modified system shows a 6-ft head difference. Hypothesize a modified concept of the ground-water system that would explain these new observed head data.

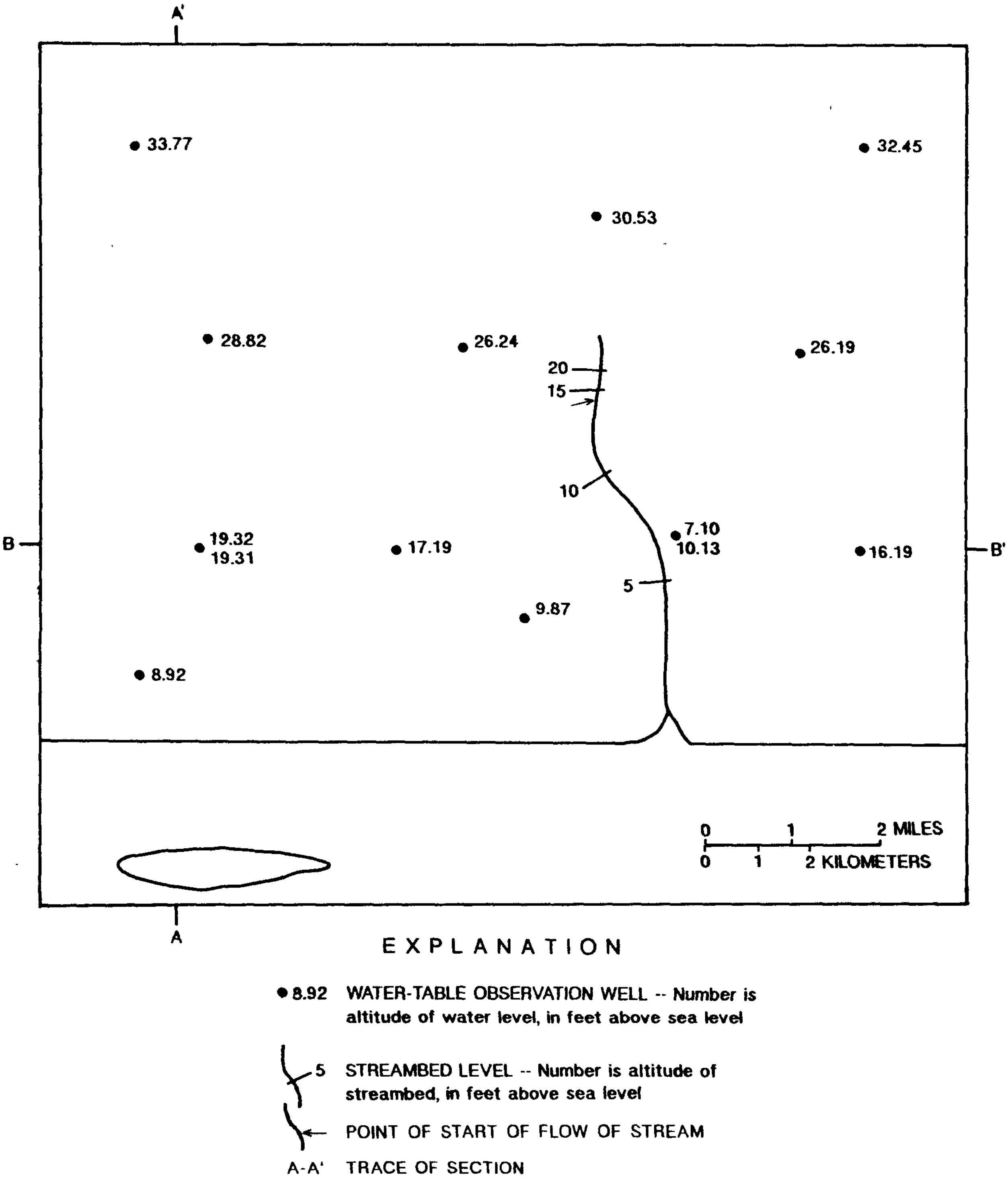
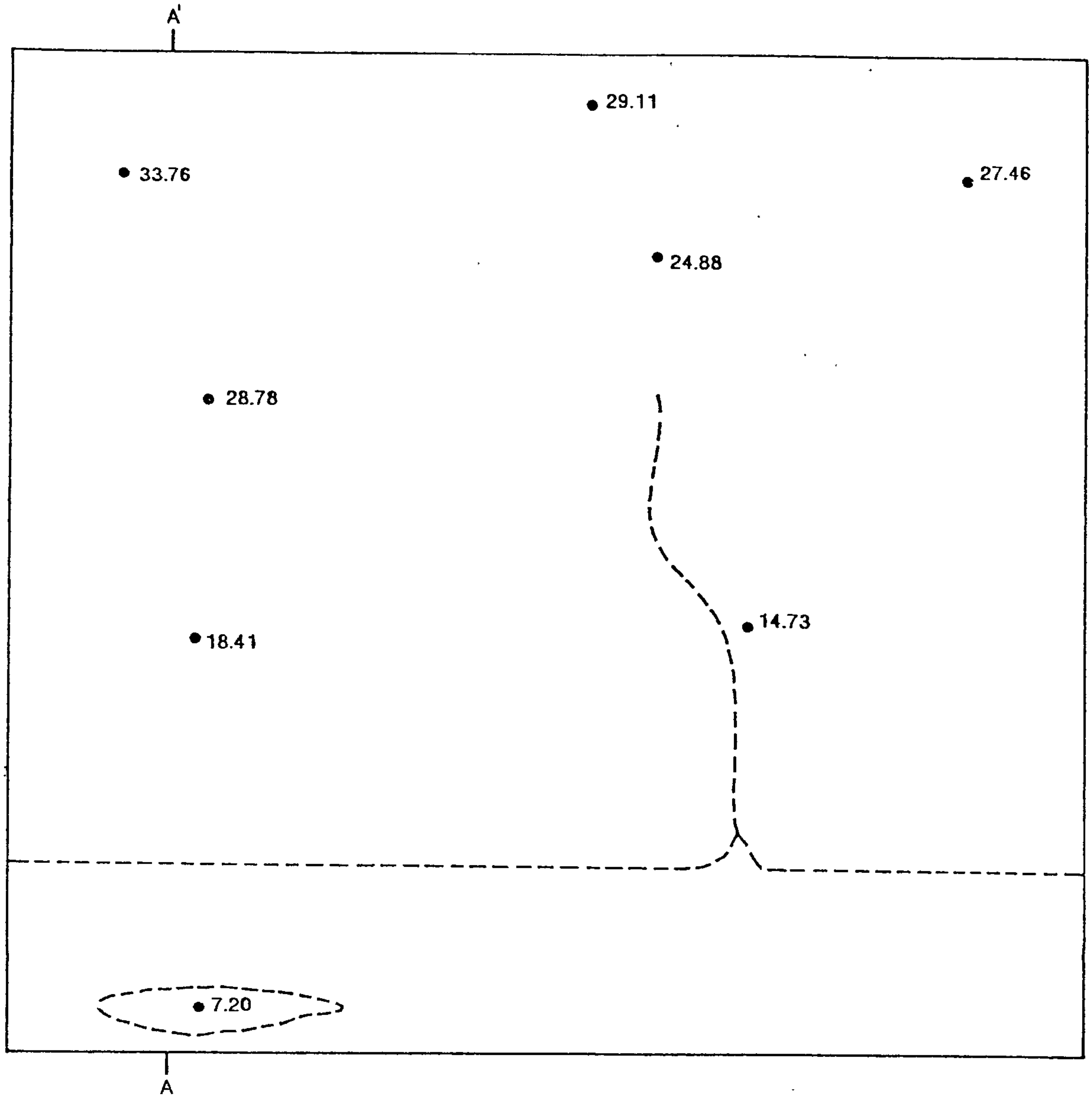


Figure 3-13.--Measured heads in the water-table aquifer in a ground-water system with a discontinuous confining unit.



EXPLANATION

● 14.73 OBSERVATION WELL SCREENED IN THE CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure 3-14.--Measured heads in the confined aquifer in a ground-water system with a discontinuous confining unit.

Further field investigation of this phenomenon reveals an irregular hole in the confining unit in the northwest corner of the area (fig. 3-15). This hole results in a direct hydraulic connection locally between the water-table and confined aquifers. Although the vertical gradient between the aquifers is least in the area of the hole, the flux between the aquifers probably is greatest there. An additional effect of this hole on the flow system might be that, to a limited degree, water converges above the hole in the water-table aquifer, flows through the hole, and disperses (flowlines diverge) within the confined aquifer.

Question 7.--Using your current understanding of the structure and operation of this ground-water system, construct a set of maps that depict its three-dimensional head distribution. Construct maps for the water-table and potentiometric surfaces and section A-A' (figs. 3-13, 3-15, and 3-16. Draw equipotential lines at 5-ft intervals.

Locate the line of transition between regions of downward and upward flow between aquifers, and mark it as a dashed line on figure 3-15. Compare this transition line with the one determined in question 5 (for unstressed conditions with continuous confining unit); also compare the head maps constructed for both scenarios. What does this comparison indicate about the effect of the hole in the confining unit on the operation of this system?

Analysis of Ground-Water Systems Using Flow Nets

Assignments

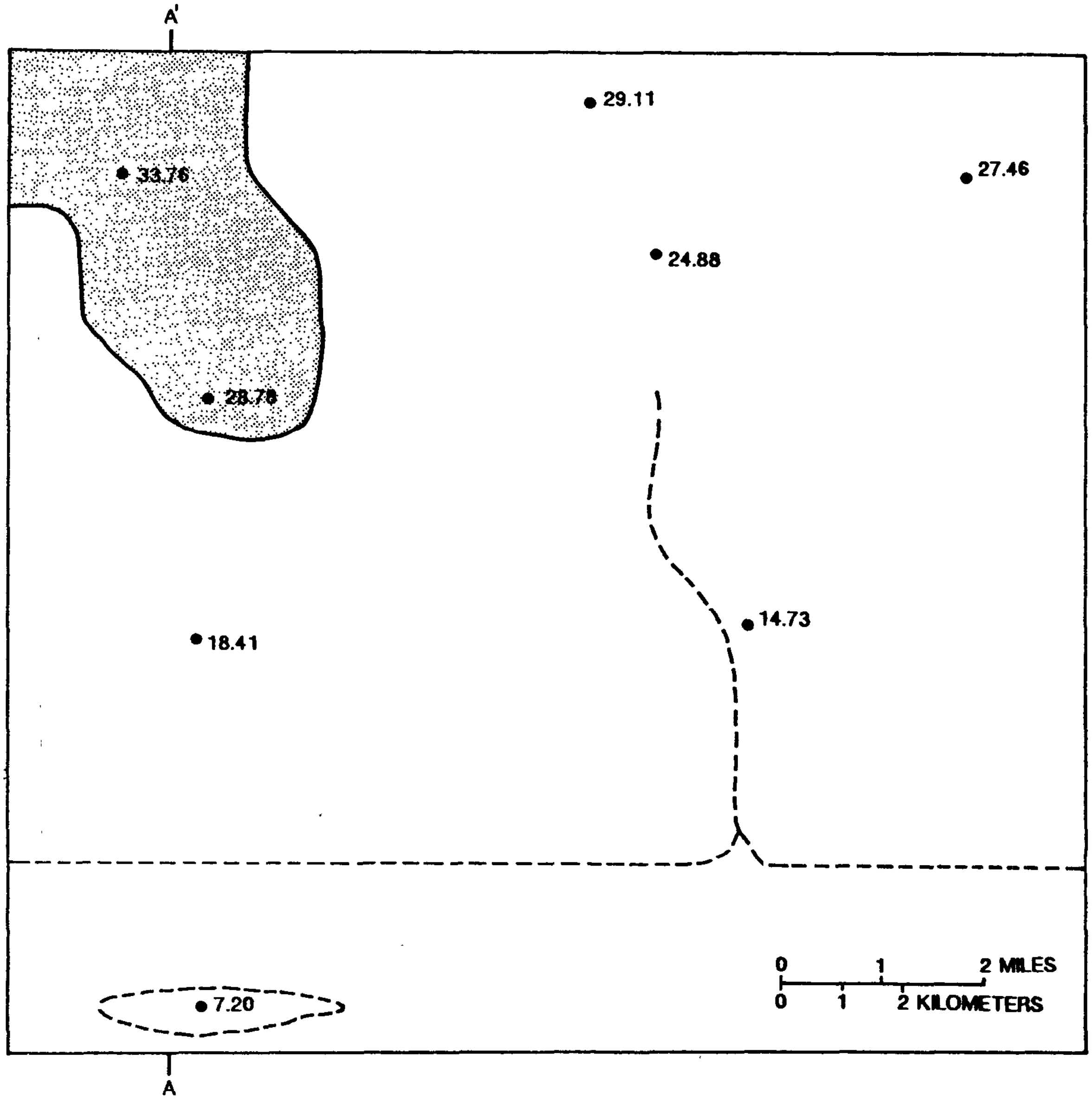
*Study Fetter (1988), p. 137-141, 218-229; Freeze and Cherry (1979), p. 168-185; or Todd (1980), p. 83-93.

*Study Note (3-4)--Introducton to discretization.

*Work Exercise (3-2)--Flow net beneath an impermeable wall.

*Study Note (3-5)--Examples of flow nets.

Flow nets depict a selected number of accurately located flowlines and equipotential lines in the flow system, which provide in total a quantitatively useful, graphical representation of the ground-water flow field. In fact, problems that involve ground-water flow often can be considered as solved if an accurate flow net is developed. Flow nets can be applied conveniently only in two-dimensional flow problems, and the technique is particularly useful in analyzing vertical sections of flow systems that are oriented along a regional "streamline" (actually, stream surface).



EXPLANATION

AREA OF HOLE IN CONFINING UNIT

• 14.73 OBSERVATION WELL SCREENED IN THE CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure S-15.--Measured heads in the confined aquifer and location of hole in the overlying confining unit.

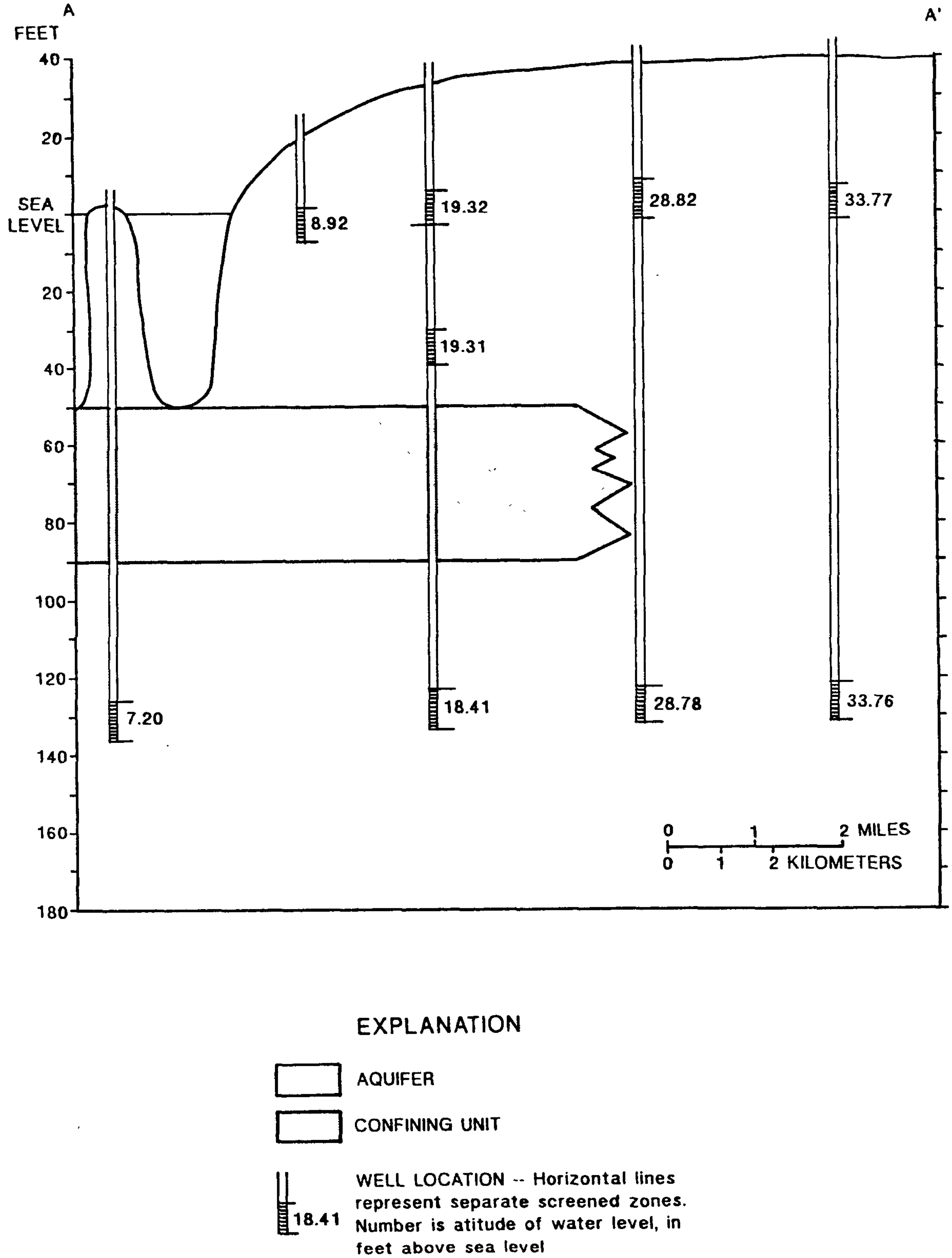


Figure 3-16.--North-south-trending hydrogeologic section showing heads measured in a ground-water system with a discontinuous confining unit. (Location of section A-A' is shown in fig. 3-2.)

Note (S-4).--Introduction to Discretization

The purpose of this note is to provide an introduction to "square-mesh" finite-difference discretization that is sufficient to permit the calculation of flows in the subsequent impermeable wall flow-net exercise (Exercise 3-2). First, we will review some of the important concepts related to discretization.

1. What is discretization?

Discretization is the breaking up of a continuous system into blocks or lumped "discrete" elements. A map of a space-discretized system consists of a network of lines (branches) which intersect at points (junctions or nodes) (fig. 3-17). Flow in a discretized system can occur only along branches. Head values in a discretized system can be measured or calculated only at nodes.

2. Why discretize?

Differential equations describe ground-water flow in continuous space. These equations cannot be solved directly for complicated field problems. Discretization (in this case, the finite-difference method of discretization) allows the use of a set of linear algebraic equations to represent the continuous differential equation that governs a specific problem in ground-water flow. For each node in a discretized system, one algebraic equation expresses the principle of continuity in the vicinity of that node. For a system with n nodes, n simultaneous linear equations are solved to obtain a solution, which consists of a calculated head value at each node.

3. How is a system discretized?

A system can be discretized in space by the finite-difference method using either uniform-grid spacing, in which network branches form squares (fig. 3-17), or variable-grid spacing, in which network branches form rectangles.

The purpose of the following paragraphs is to describe the procedure for calculating flows through branches in a square finite-difference network. The first step is to determine the top or map area (vector area) associated with each branch (fig. 3-18), which along with the "thickness" defines the block of aquifer material associated with each branch. As will be seen later, this area associated with the branch is <u>not</u> the area needed to calculate the branch flow.

Square-mesh networks (fig. 3-18) are a special case of rectangular-mesh networks. The following procedure for determining the top or map area associated with branches in rectangular networks is applicable to both network types and is illustrated in figure 3-18. As an example in applying the following instructions, refer to branch AB and start at node B.

1. Starting at a node at either end of the branch under consideration, draw a line that is perpendicular to the branch halfway to the next node. This line is coincident with another branch.

2. From this point, draw a line equal in length and parallel to the branch under consideration.

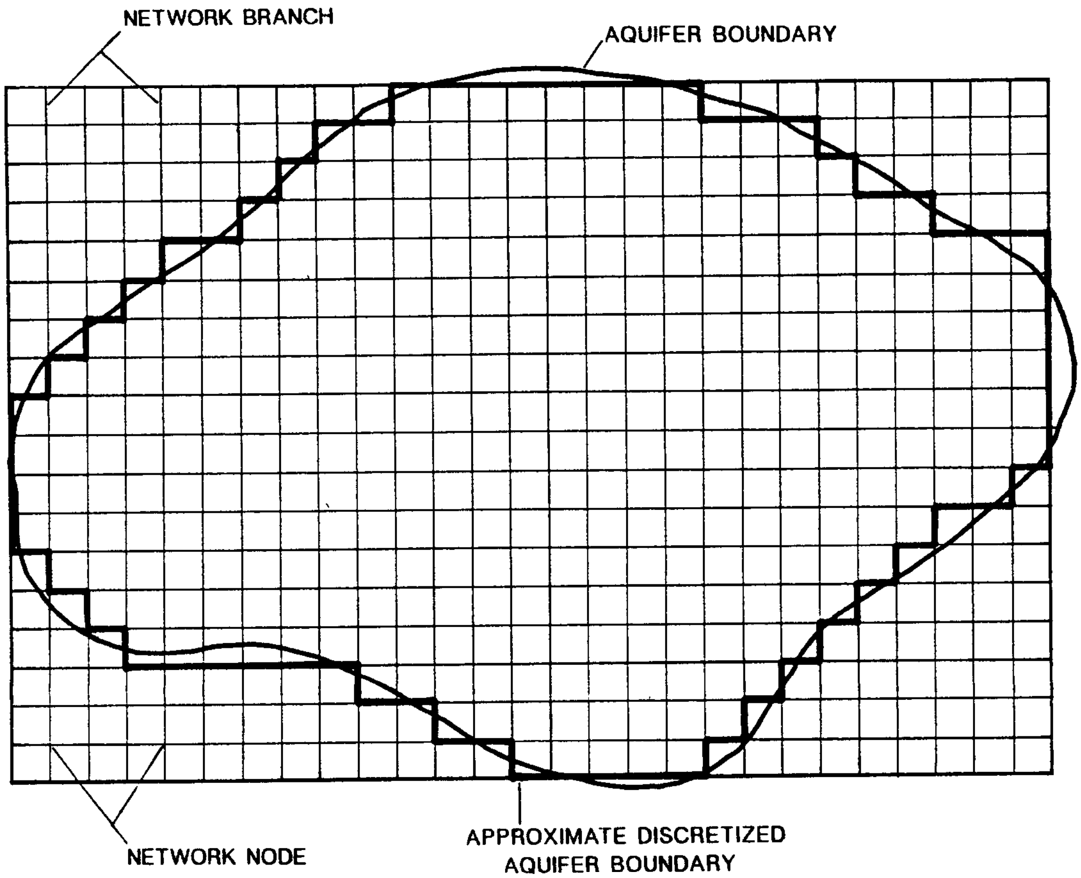
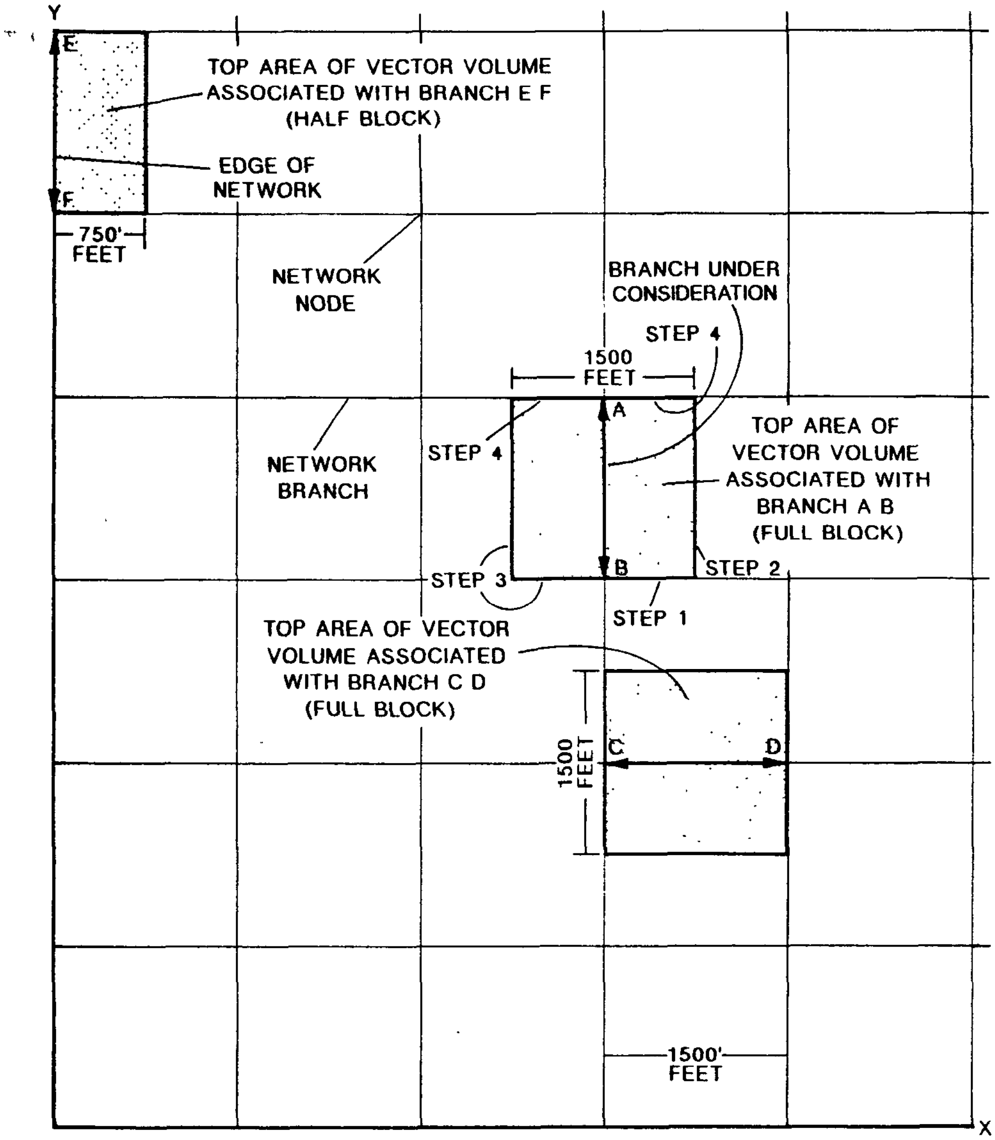


Figure 3-17.--Plan view of a square-mesh finite-difference grid over a map of an aquifer system. (From Prickett and Lonnquist, 1971, fig. 7.)



EXPLANATION

A, B, C, D, E, F -- JUNCTIONS OR NODES IN SQUARE-MESH NETWORK

A B -- INTERIOR NETWORK BRANCH IN Y DIRECTION

C D -- INTERIOR NETWORK BRANCH IN X DIRECTION

E F -- BRANCH AT EDGE OF NETWORK IN Y DIRECTION

K = HYDRAULIC CONDUCTIVITY

K_x = 40 FEET PER DAY

K_y = 60 FEET PER DAY

b = THICKNESS OF AQUIFER MATERIAL ASSOCIATED WITH A BLOCK = 35 FEET

STEP 1, 2, 3, 4 -- SEE ACCOMPANYING TEXT

Figure 3-18.--Vector areas associated with branches in a square-mesh finite-difference grid.

3. Return to the starting node and repeat steps 1 and 2 in the opposite direction. 4. Three sides of a square are drawn in steps 1-3. Complete the square. The fourth side of the square is perpendicular to the branch under consideration (fig. 3-18).

The resulting square (fig. 3-18) represents the vector area (area on network "map" of discretized system) associated with the branch. The vector volume of earth material associated with the branch is obtained by multiplying this area by the aquifer thickness.

Calculations of the areas and volumes associated with branches for the examples shown in figure 3-18 are given in table 3-3. Note the "half block" associated with branch EF at the boundary of the network.

The blocks of earth material whose volumes are calculated in table 3-3 can be thought of as Darcy prisms that are represented by branches between nodes in the network map where head is measured. Darcy's law can be written

$$Q = KA \frac{\Delta h}{L} = \frac{KA}{L} \Delta h \qquad (1)$$

where Δh is the difference in head at the two ends of the prism. The combination of parameters KA/L is called the hydraulic conductance. Note particularly that the area A in this combination (and in Darcy's law) does <u>not</u> refer to the map area associated with the branch, but to the area of the vertical prism face perpendicular to the branch and perpendicular to the plane of the network map. Values of hydraulic conductance for the same network branches shown in figure 3-18 also are listed in table 3-3. The hydraulic conductance is a "lumped" coefficient, obtained directly from Darcy's law, that represents the transmitting capability of a block of earth material.

In summary, application of Darcy's law as written in equation (1) to a discretized finite-difference network permits calculation of the flow through any branch in the network.

Table 3-3.--Areas, volumes, and hydraulic conductances associated with network branches in figure 3-18

Branch	Area associated with branch (square feet)	Volume associated with branch (cubic feet)	Hydraulic conductance associated with branch (feet squared per day)
AB	(750+750)(1,500) = 2,250,000	(1,500)(1,500)(35) = 78,750,000	$\frac{60[35(750+750)]}{1,500} = 2,100$
CD	(750+750)(1,500) = 2,250,000	(1,500)(1,500)(35) = 78,750,000	$\frac{40[35(750+750)]}{1,500} = 1,400$
EF	(750)(1,500) = 1,125,000	(750)(1,500)(35) = 39,375,000	$\frac{60[35(750)]}{1,500} = 1,050$

Exercise (3-2)--Flow Net Beneath an Impermeable Wall

A cross section of a ground-water flow system near a partially penetrating impermeable wall is shown in figure 3-19. This section depicts a two-dimensional flow field. Flow is assumed to occur only in the plane of the figure; that is, there is no flow perpendicular to the plane of the figure. The flow field has unit thickness--that is, the thickness of the flow system perpendicular to the page is 1 ft. The wall is impermeable, as are the bottom and lateral boundaries. The "top" of the ground-water flow system to the left of the impermeable wall lies 5 ft beneath a standing body of water whose surface elevation remains constant at 55 ft above the impermeable bottom boundary (datum). To the right of the impermeable wall the surface of the aquifer material is at an elevation of 25 ft above datum; ground water discharges at this surface to nearby surface drains and by evaporation. The earth material near the impermeable wall is fine sand, which is assumed to be isotropic and homogeneous.

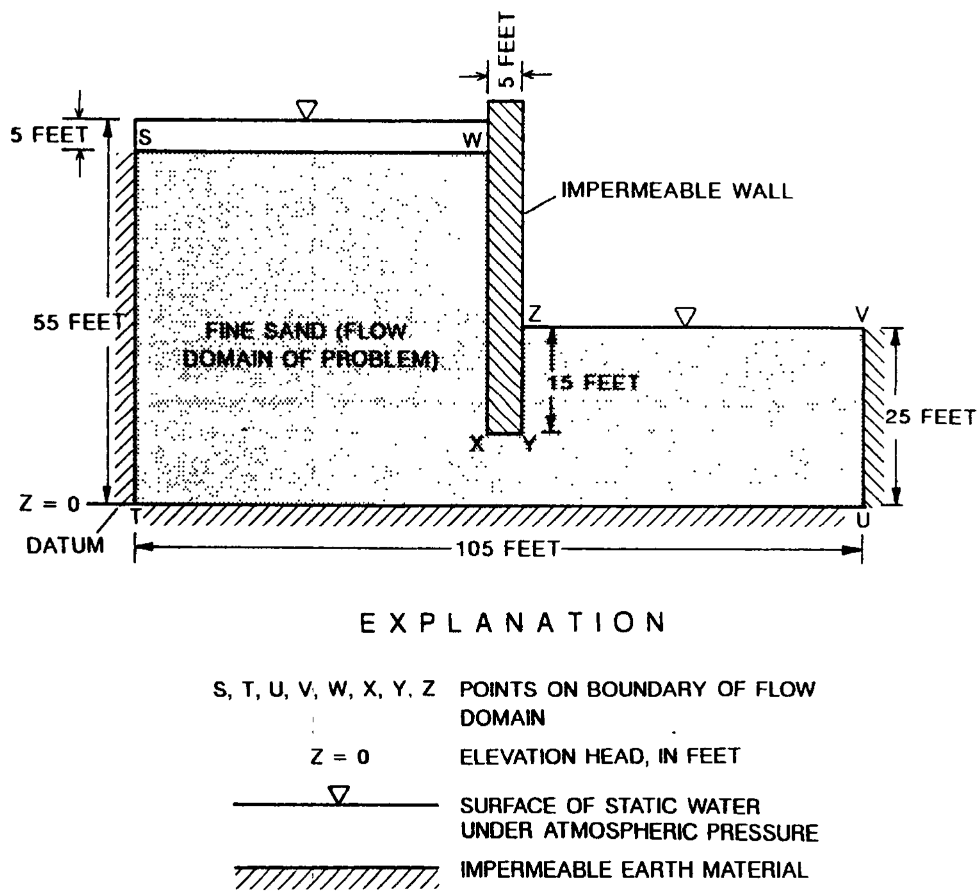


Figure 3-19.--Vertical section through a ground-water flow system near a partially penetrating impermeable wall.

The head distribution in this cross section, obtained by numerical simulation, is shown on figures 3-20 and 3-21. The "node" at which each head value applies is located at the decimal point of the head value. All head values are in feet above datum. The nodes form a square discretization grid with an equal 5-ft spacing between nodes.

The head values on figures 3-20 and 3-21 represent the standard output of a digital simulation of this problem. Often, or perhaps usually, these head data can be contoured to improve insight into the flow pattern. In this exercise, we will use these head data as the starting point for calculating the position of streamlines--an essential step in developing a flow net for this system. Approximate times of travel and residence times within the flow system will be calculated from these head data in a later exercise.

The first step in analyzing any ground-water problem is to develop a simple (compared to the complexity of the real system) conceptual "picture" or model of the operation of the ground-water system. To attain a reasonable conceptual model of the flow system, the minimum required information is (1) the shape (geometry) of the flow system and (2) the boundary conditions. The geometry of the flow system already has been defined in figure 3-19. The next step is to define the boundary conditions of the problem.

Delineate carefully with colored pencils, the extent and type of the boundaries in the impermeable wall problem in figure 3-19. You will find four boundaries and two different boundary conditions. Remember that your designations of the boundaries by means of colored pencils must result in a loop or closed curve without gaps in color. A "gap" without color would represent a portion of the boundary surface for which you have not defined the governing boundary condition.

Where does ground water enter the system? Where does ground water discharge from the system? Sketch the approximate pattern of several flowlines and equipotential lines on figure 3-19. Does your conceptual model of the flow system "make sense"?

Make a table of p/γ, z, and h values for the upper left and upper right horizontal boundaries. What is the total head drop (Δh) in the ground-water system? Is this information consistent with your concept of the flow system?

Flow Net

1. Our goal in this exercise is to construct a fairly accurate flow net from the head data shown on figure 3-21. It is advisable to make copies of this worksheet before you begin in case you make errors. Contour the head data using a contour interval of 2.5 ft--that is, draw contour lines for 52.5, 50, 47.5, 45, . . . 27.5 ft. The contour lines should be smooth curves that intersect streamline boundaries at right angles. Draw all contours in pencil so that corrections and improvements can be made easily. Draw these contour lines carefully because later work depends on their position.

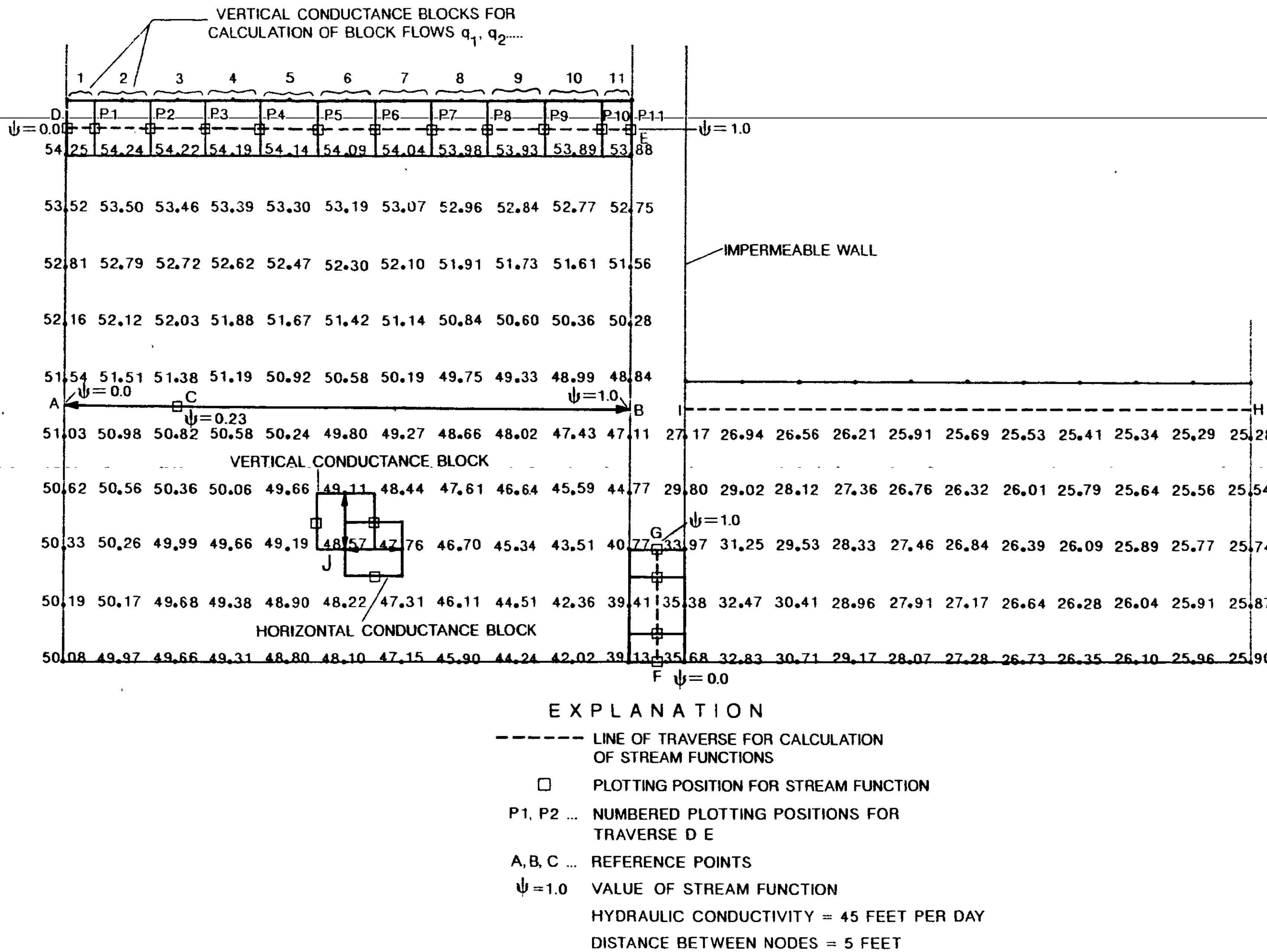
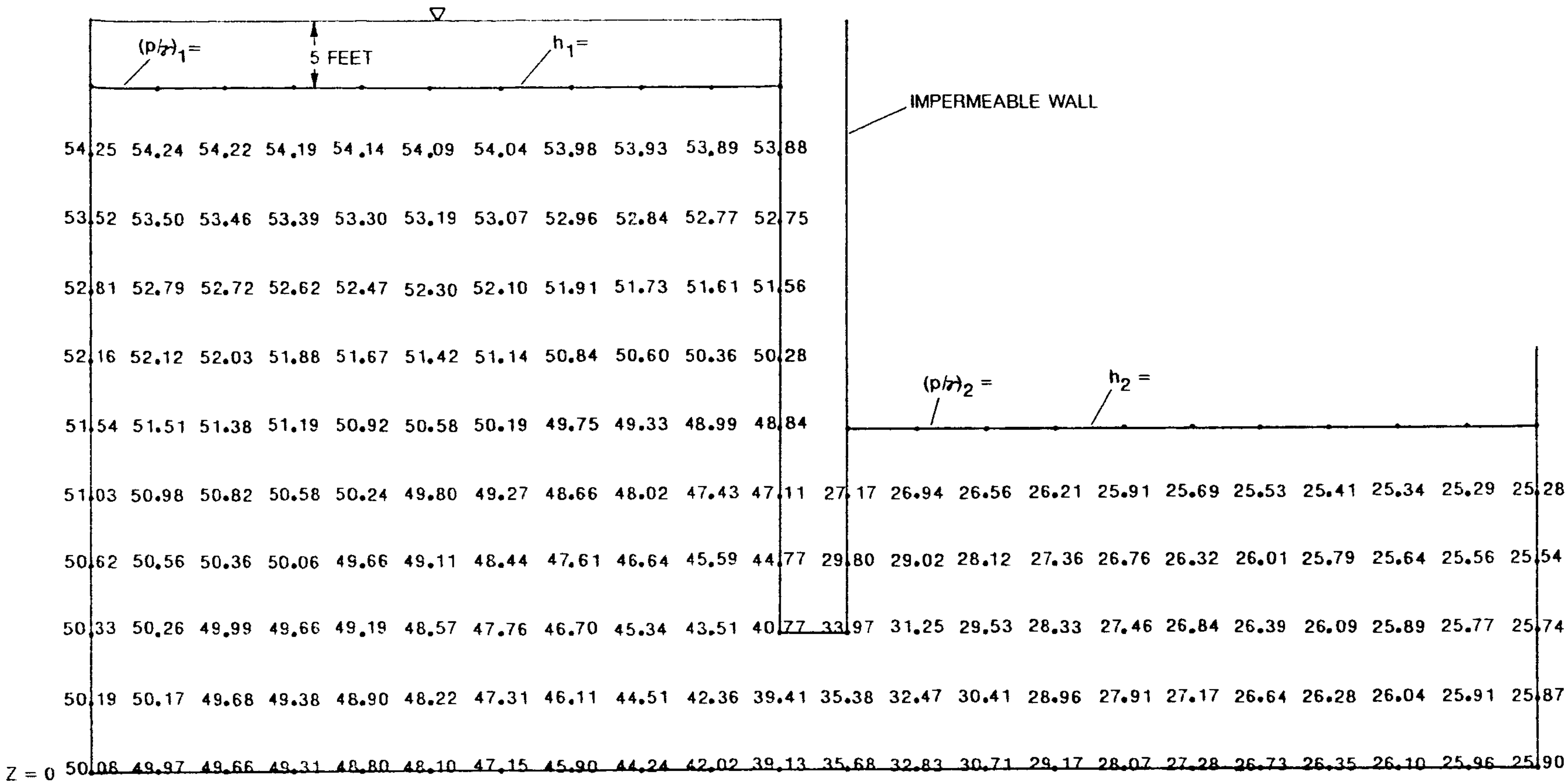


Figure 8-20.--Aquifer blocks for calculating block conductances and block flows, and for plotting positions of calculated values of stream functions.



EXPLANATION

▽ SURFACE OF STATIC WATER UNDER ATMOSPHERIC PRESSURE

IMPERMEABLE MATERIAL

• NODE IN DISCRETIZED SYSTEM AND DECIMAL POINT OF HEAD VALUE AT NODE

53.39 HEAD AT NODE, IN FEET

Z IS ELEVATION HEAD, p/γ IS PRESSURE HEAD, h IS TOTAL HEAD, IN FEET

DISTANCE BETWEEN NODES = 5 FEET

HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY

Figure 8-21.--Worksheet for preparation of a flow net for a ground-water system near an impermeable wall.

2. The next step is to determine the position of several interior streamlines in the flow system. These streamlines intersect the head contours at right angles, and generally are constructed so that the flows between adjacent streamlines are equal. (Two adjacent streamlines define a flow tube.)

To begin, identify the two bounding streamlines in the system. Next, we have decided arbitrarily to draw four interior streamlines, so that the system is divided into five flow tubes. Thus, the internal streamlines must be positioned so that one-fifth of the total flow Q beneath the impermeable wall or 0.20Q is transmitted through each flow tube.

In order to locate the four internal streamlines, we will calculate stream functions along selected traverses across the flow field. However, before considering the procedure for calculating stream functions, we will discuss what the stream function represents.

Assume that our flow system is the original continuous system composed of fine sand--that is, we have not yet discretized the system for the purpose of obtaining a numerical solution for head values at nodes. Also assume that we know the total flow through the system. Now, make an arbitrary traverse from one bounding streamline to the other bounding streamline. To do this, designate a point on one bounding streamline as the starting point of the traverse. All traverses across the system must begin on the same bounding streamline. For example, let a traverse start at A on the outside bounding streamline and traverse the system to point B on the other bounding streamline, as shown in figure 3-20. Even though the direction of ground-water flow may not be perpendicular to the traverse line at any given point, we must, nevertheless, intersect the total flow through the system along the traverse from A to B.

Assume further that we measure each increment of flow as we proceed along the traverse. Because we know the total flow, we can assign to any point on the traverse the <u>proportion</u> of the total flow that we have encountered to that point. This proportion is equal to the stream function, Ψ. For example, at point C, assuming that we started at point A, we have encountered 0.23Q, where Q is the total flow--that is, 0.23Q is behind us on the traverse and 0.77Q remains in front of us on the traverse. Clearly, at point A we have intersected none of the flow and the stream function $\Psi = 0$, (Ψ is the Greek letter psi used as a symbol for stream function.) At B, we have intersected the total or 100 percent of the flow, and $\Psi = 1.0$.[1]

---

[1] The stream function is actually the total flow traversed to a given point on a traverse line such as point C on traverse AB. We have defined a dimensionless proportion-of-total-flow function which is the stream function divided by a constant, the total flow in the system. For convenience, we will refer to this ratio simply as the stream function.

The stream function is constant along a streamline. Consider a number of closely spaced traverses through the flow field similar to AB and assume that we know the value of the stream function at every point on the traverses. By connecting points of equal stream function--for example, $\Psi_1 = 0.40$ and $\Psi_2 = 0.60$--we are drawing a flow tube bounded by the streamlines $\Psi_1 = 0.40$ and $\Psi_2 = 0.60$ such that 20 percent (0.20 Q) of the total flow is found within this flow tube ($\Psi_2 - \Psi_1 = 0.60 - 0.40 = 0.20$). The stream function is a scalar[2] function of position, just as head is a scalar function of position. A unique value of the stream function may be defined for every point in a continuous flow field. We could write the ground-water flow equations using stream functions instead of head as the dependent variable, although this is seldom done.

Next, we will develop a procedure for calculating stream functions in the discretized impermeable wall problem (fig. 3-20) along three traverses--DE (near the upper left constant-head boundary), FG (beneath the impermeable wall), and HI (near the upper right constant-head boundary). The calculation of stream functions is facilitated by using the format in table 3-4. We will begin with traverse DE (fig. 3-20). Note that blocks 1 and 11 are "half" blocks. Calculate the conductance of the blocks on the traverse using the formula C = KA/L. Determine the flow through each block using the head differences across the blocks. Next, calculate the cumulative flow for the blocks along the traverse from D to E (see format in table 3-4). Divide the cumulative flow at the right-hand edge of each block by the total flow. This calculated value is the stream function at the right-hand edge of that particular block--that is, the percent of the total flow across line DE at the right-hand edge of that particular block on the traverse. Note that the plotting positions of the stream functions are at the right-hand edges of the blocks. For example, the stream function along traverse DE for block 1 is plotted at "p1" (fig. 3-20); the stream function for block 11 is plotted at "p11," the boundary of the flow system.

This choice of plotting positions permits a unique value of the stream function to be plotted on the discretized grid no matter how we make a traverse across the flow field. Compare the two plotting positions of the stream functions at two edges of a typical vertical conductance block and the two plotting positions at two edges of an overlapping horizontal conductance block near J in figure 3-20. One plotting position is shared by both blocks. By extension of this pattern, the stream function plotting positions form a square array of points throughout the flow domain that is offset from the square array of points that constitutes the head nodes. Complete the stream function calculations for traverses FG and HI in table 3-4.

---

[2] A scalar quantity can be identified by a single number and has no implied direction; a scalar may be contrasted to a vector quantity, which has direction and requires more than one number for its description.

Table 3-4.--Format for calculation of stream functions in impermeable wall problem (page 1 of 2).

[For locations of numbered blocks, traverse DE, and plotting positions for stream functions p1, p2, ..., see figure 2; C_{block} is hydraulic conductance of discretized block which equals KA/L, where K = hydraulic conductivity of earth material in block, A = cross-sectional area of block perpendicular to direction of ground-water flow, and L = length of block; h_1 and h_2 are head values at nodes located at ends of block; $\Delta h = h_1 - h_2$; q_{block} = flow through a single block; Σq_{block} = flow in a numbered block plus the flows through all lower-numbered blocks (cumulative sum of block flows in traverse); Q_{total} = total flow through the ground-water system beneath the impermeable wall; ft= feet; ft^2 = square feet; ft^3 = cubic feet; Ψ = stream function]

TRAVERSE D E	BLOCK NUMBER	C_{block} = KA/L (ft^2/day)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	$q_{block} = C\Delta h$ (ft^3/day)	Σq_{block} (ft^3/day)	$\Psi = \frac{\Sigma q_{block}}{Q_{total}}$	PLOTTING POSITION FOR STREAM FUNCTIONS
	1	22.5	55.00	54.25	.75	16.88	16.88		P1
	2								P2
	3								P3
	4								P4
	5								P5
	6								P6
	7								P7
	8								P8
	9								P9
	10								P10
	11							1.00	P11

Table 3-4.--Format for calculation of stream functions in impermeable wall problem (page 2 of 2).

BLOCK NUMBER	$C_{block} =$ KA/L (ft^2/day)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	$q_{block} =$ $C \Delta h$ (ft^3/day)	Σq_{block} (ft^3/day)	$\Psi = \frac{\Sigma q_{block}}{Q_{total}}$

3. The procedure for completing the flow net is the following. Plot the individual stream function values on figure 3-21 at the appropriate points. By interpolation mark on each traverse line the position of the stream functions $\Psi = 0.20$, $\Psi = 0.40$, $\Psi = 0.60$, and $\Psi = 0.80$. After completing this, you have established three points on the four streamlines that you wish to draw. Now sketch the four streamlines on figure 3-21, being careful to draw the streamlines perpendicular to the already existing equipotential lines. Starting at the left end of the upper-left horizontal boundary (fig. 3-21), label the streamlines "a" through "f" (the designations on the two streamline boundaries). The result should be a respectable flow net. Of course, you can improve the flow net by calculating additional values of stream functions along additional traverses through the flow system and refining, thereby, the positions of the four internal streamlines.

Note (3-5).--Examples of Flow Nets

Study of flow nets provides valuable information about and insight into typical flow patterns in ground-water systems. It is advisable to study systematically all the flow nets that you encounter. To obtain the greatest possible return from studying flow nets, the following sequence of steps is suggested: (1) differentiate between the equipotential lines and flowlines, (2) identify exactly where water enters the ground-water system and where water leaves the system, (3) designate the boundary conditions of the ground-water system, first for the inflow and outflow boundaries in (2), and then for the remainder of the system boundaries, and (4) study the actual pattern of flowlines and equipotential lines, asking questions such as (a) where ground-water velocities are greatest and least, (b) where "resistance" to flow in the system is greatest, which corresponds to where head drops (head dissipation) in the system are concentrated, and (c) how flowlines refract at boundaries between layers within the system that have different values of hydraulic conductivity.

Examples of flow nets are given in figures 3-22, 3-23, and 3-24. Go through the thought sequence above for each of these flow nets and for the flow net beneath the impermeable wall in Exercise (3-2).

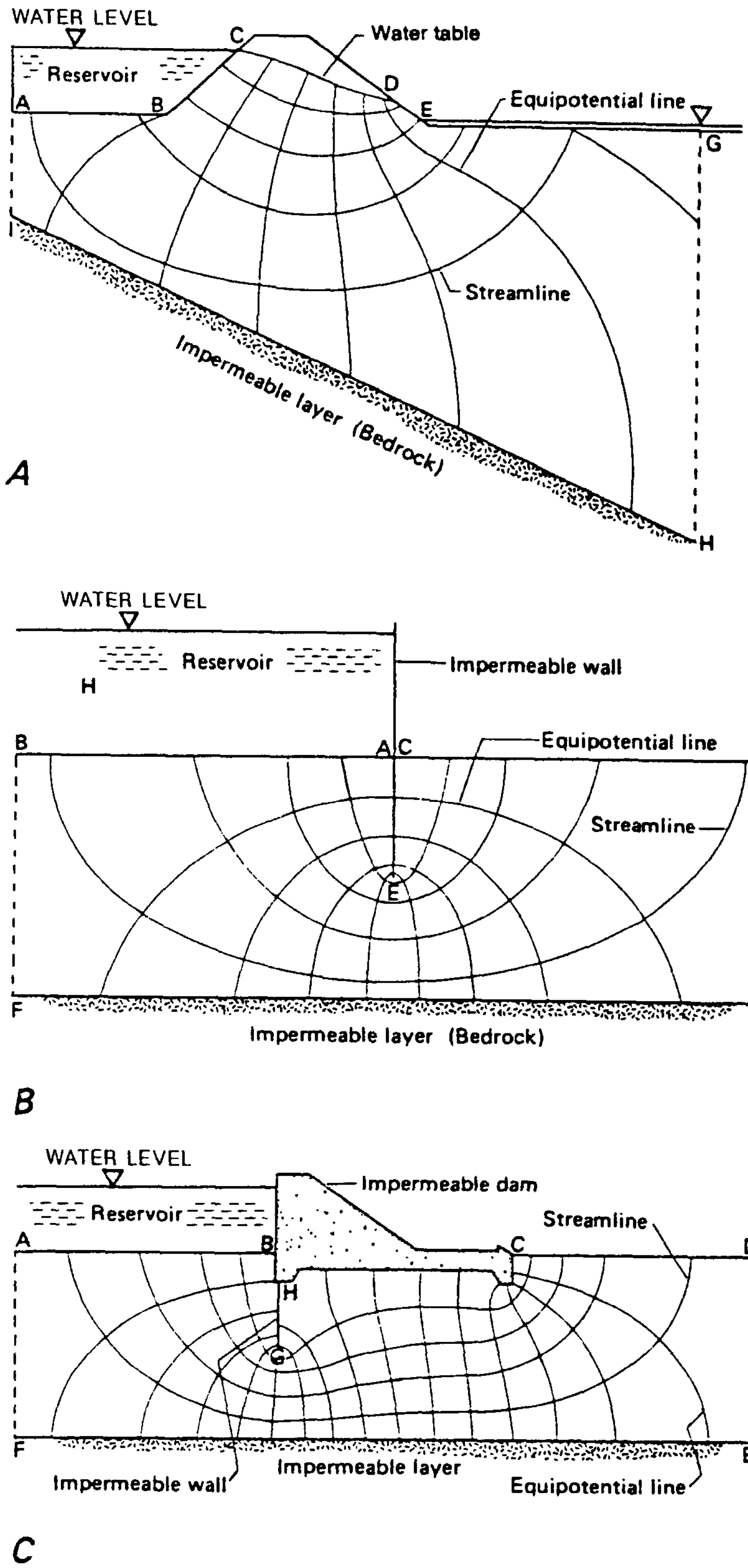


Figure 3-22.--Flow net within three different hydraulic settings: A, through and beneath an earth dam underlain by sloping bedrock; B, beneath a vertical impermeable wall; and C, beneath an impermeable dam and a vertical impermeable wall. (From Franke and others, 1987.)

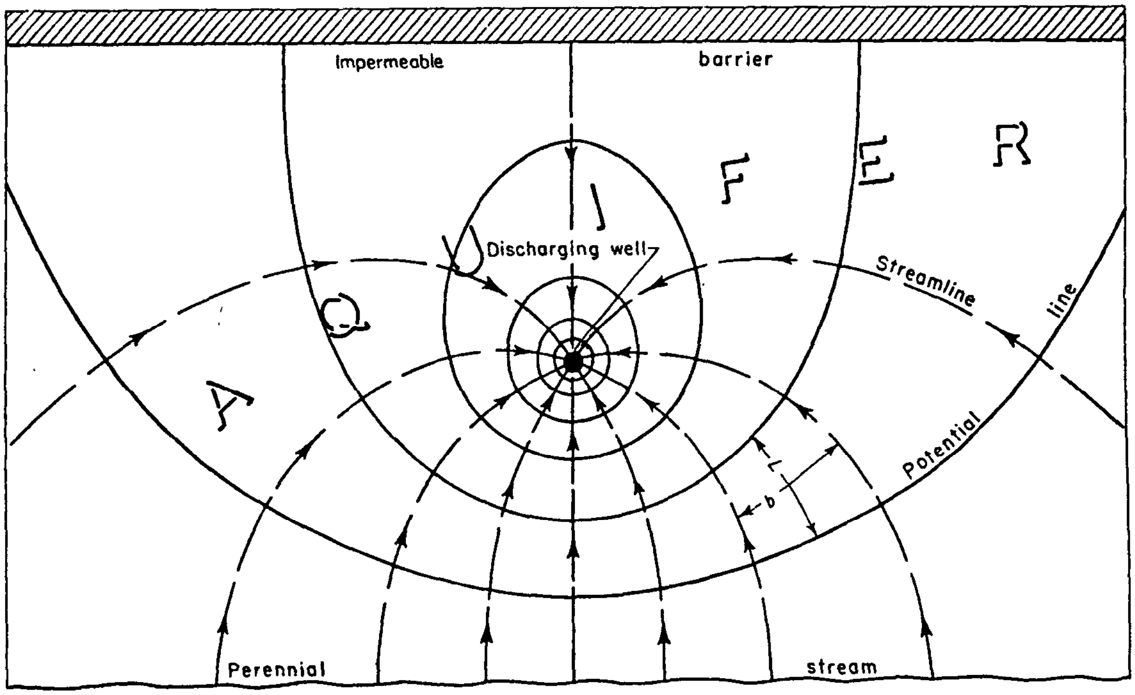


Figure 3-23.--Flow net for a discharging well in an aquifer bounded by a perennial stream parallel to an impermeable barrier. (From Ferris and others, 1962, fig. 3.)

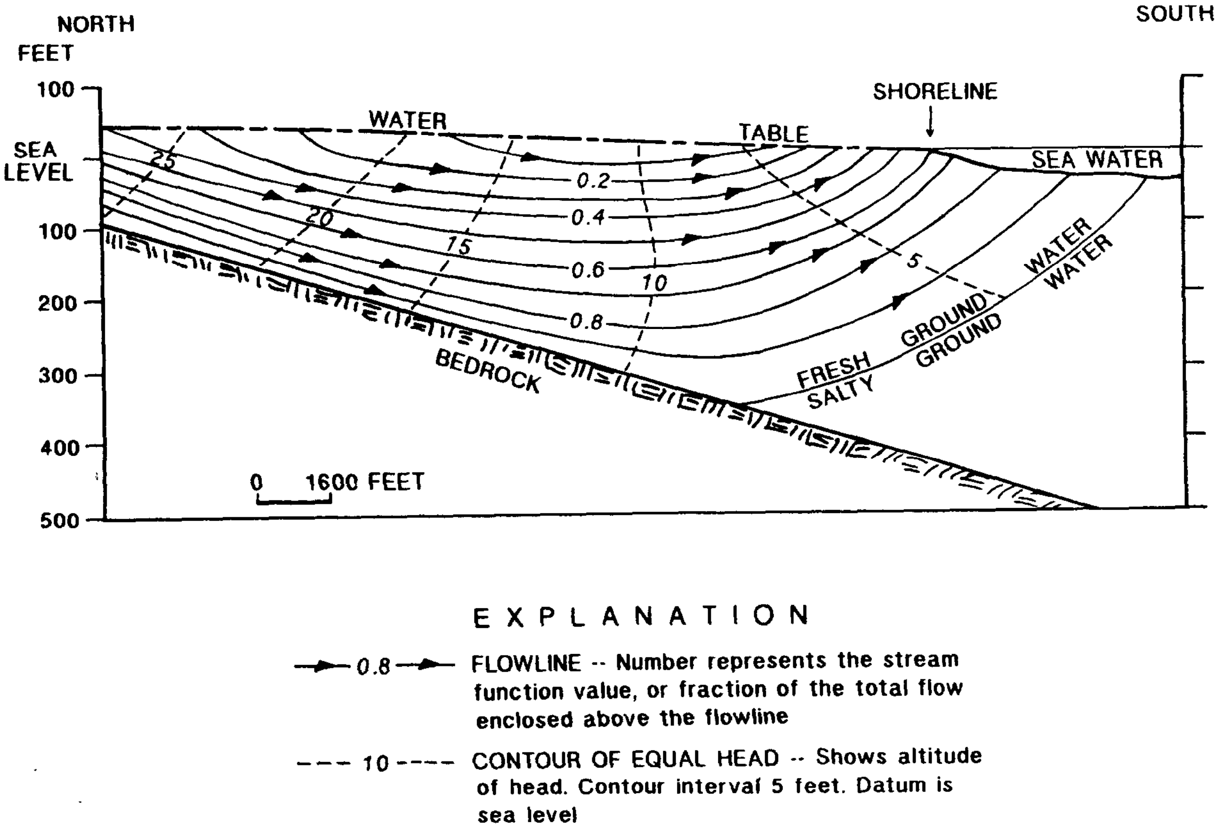


Figure 3-24.--Analog simulation of coastal ground-water flow pattern near Ponce, Puerto Rico, assuming a homogeneous ground-water reservoir. (From Bennett and Giusti, 1971.)

Regional Ground-Water Flow and Depiction of Ground-Water Systems Using Hydrogeologic Maps and Sections

Assignments

*Study Fetter (1988), p. 230-258; or Freeze and Cherry (1979), p. 253.

*Study Note (3-6)--Examples of hydrogeologic maps and sections.

A comprehensive introduction to many of the most areally extensive regional aquifer systems in the United States is provided in U.S. Geological Survey Circular 1002 edited by Sun (1986).

Common types of hydrogeologic maps and sections include (a) structure-contour maps that depict the topographic surfaces corresponding to the tops and bottoms of hydrogeologic units; (b) isopach (thickness) maps, which can be regarded as difference maps between two selected structure contour maps; (c) cross sections that depict hydrogeologic units--sometimes cross sections show actual lithologic or borehole geophysical logs; (d) fence diagrams and block diagrams, which extend the geometric representation of hydrogeologic units to three dimensions; (e) head maps of a single hydrogeologic unit; and (f) cross sections showing both hydrogeologic units and head information. Examples of some of these types of hydrogeologic illustrations are given in Note (3-6), figures 3-25 through 3-32.

Note (3-6).--Examples of Hydrogeologic Maps and Sections

The following series of figures, figures 3-25 through 3-32, is a collection of representative hydrogeologic maps and sections that depict hydrogeologic features of the ground-water system beneath Long Island, New York. This ground-water system is used as an example primarily because it has been studied intensively and a great deal of hydrogeologic information about this system is available.

Ideally, an intensive study of a ground-water system includes preparation of a series of internally consistent hydrogeologic maps and cross sections. As a start, this series might include (a) structure-contour maps on tops of all hydrogeologic units, (b) isopach maps for all hydrogeologic units, (c) head maps of all aquifers for predevelopment conditions and at subsequent times, (d) transmissivity maps for all aquifers, and (e) selected hydrogeologic sections showing the geologic framework and associated equipotential lines.

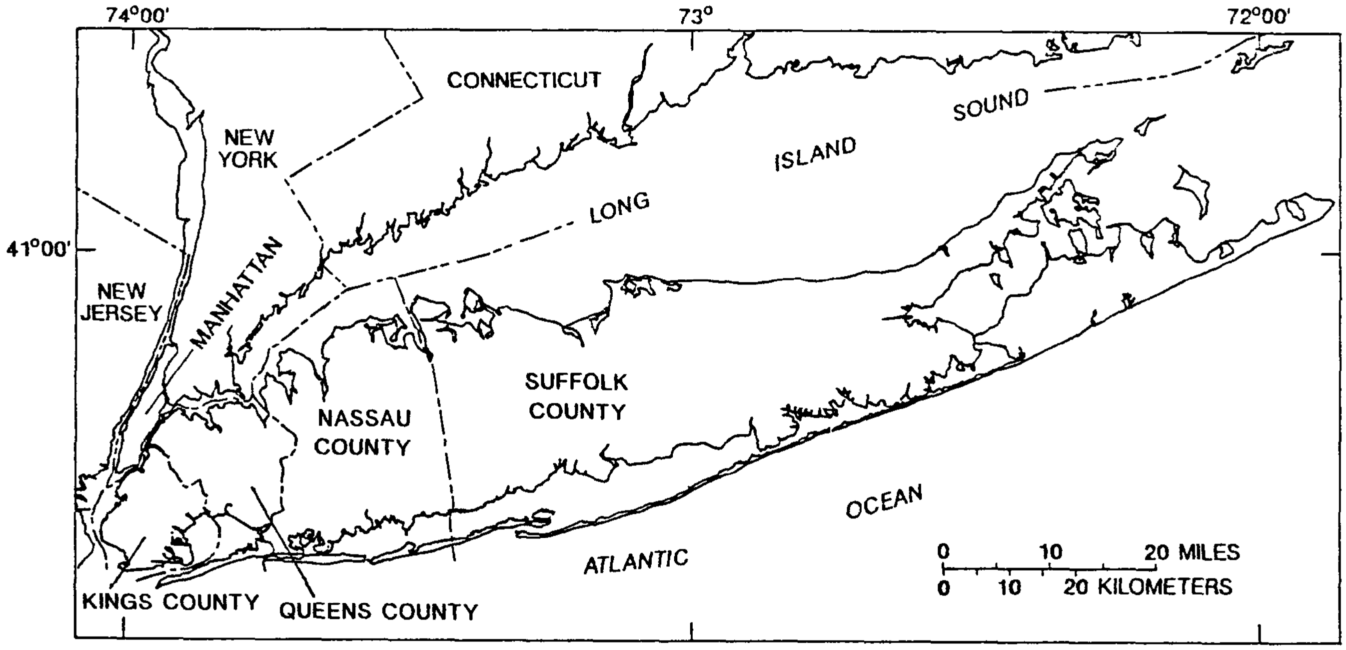


Figure 3-25.--Location and general geographic features of Long Island, New York. (From Franke and McClymonds, 1972, fig. 1.)

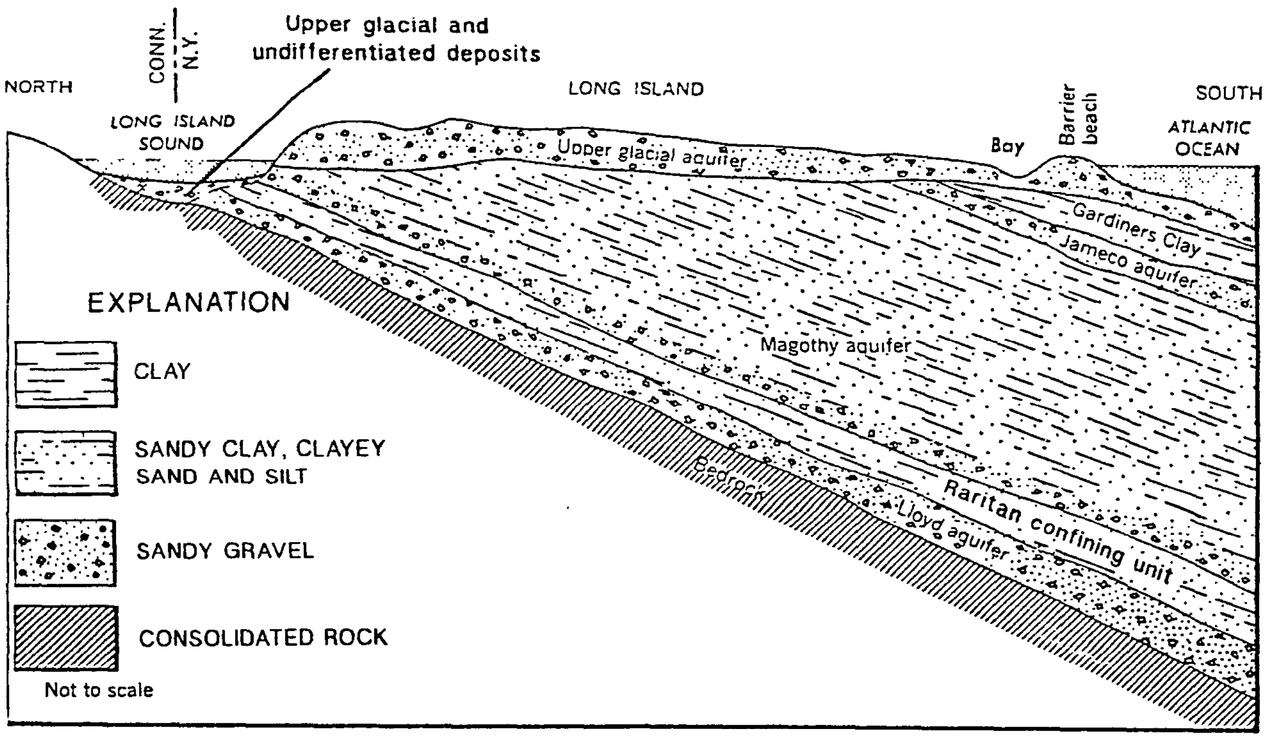
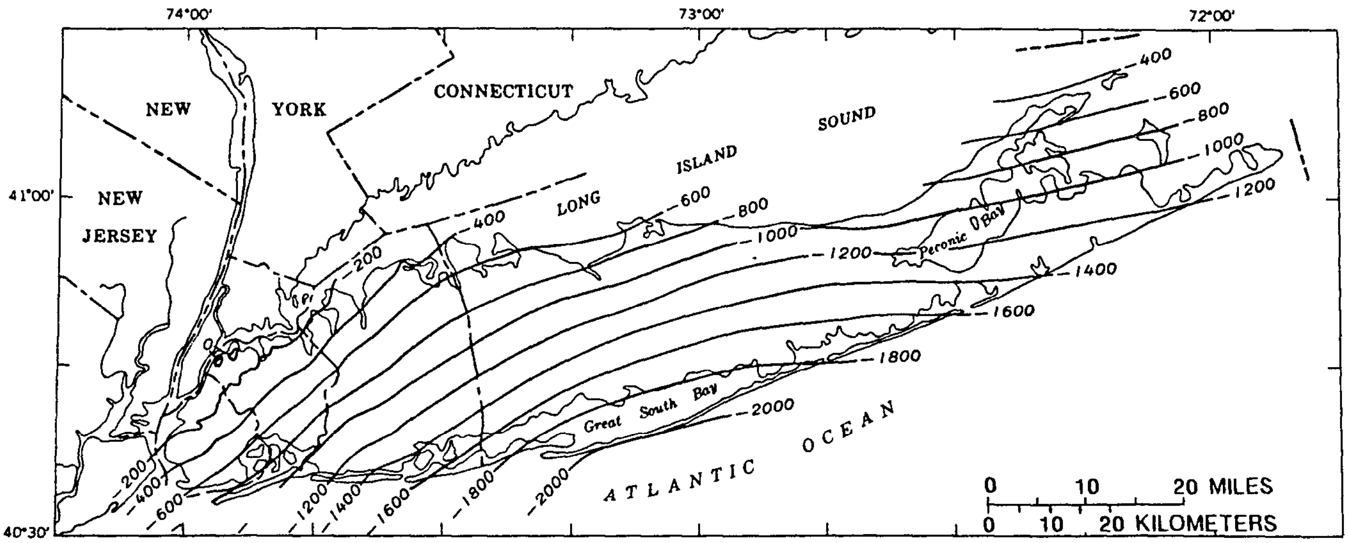


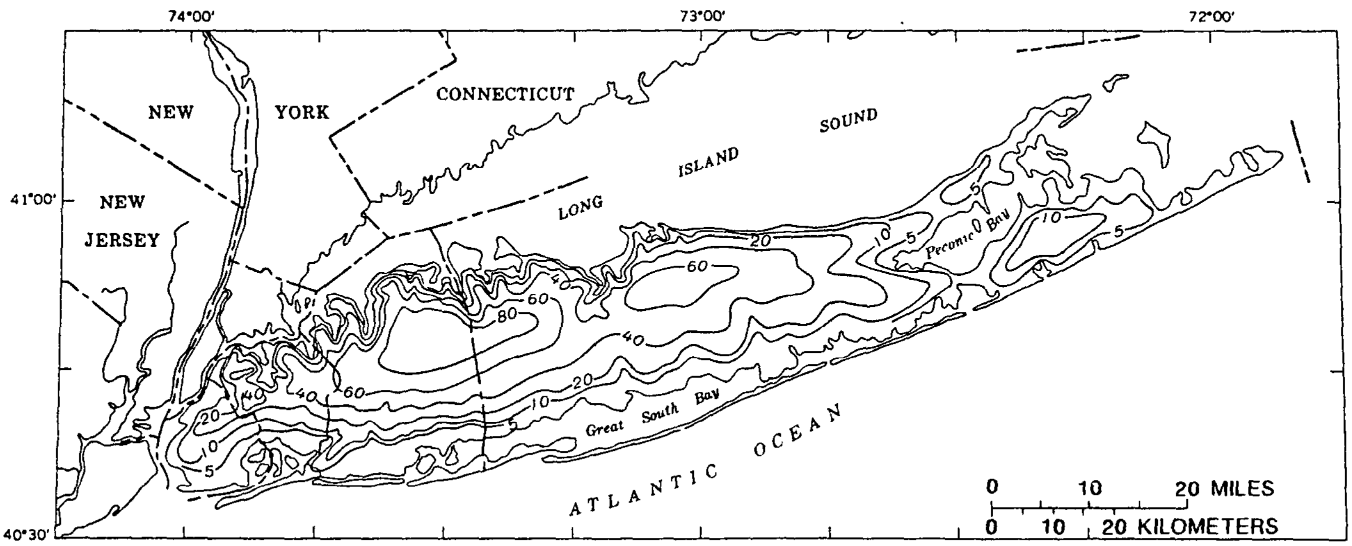
Figure 3-26.--Geologic features of the Long Island ground-water reservoir. (From Franke and McClymonds, 1972, fig. 8.)



EXPLANATION

—— -200 —— BEDROCK CONTOUR -- Shows altitude of bedrock surface. Contour interval 200 feet. Datum is sea level

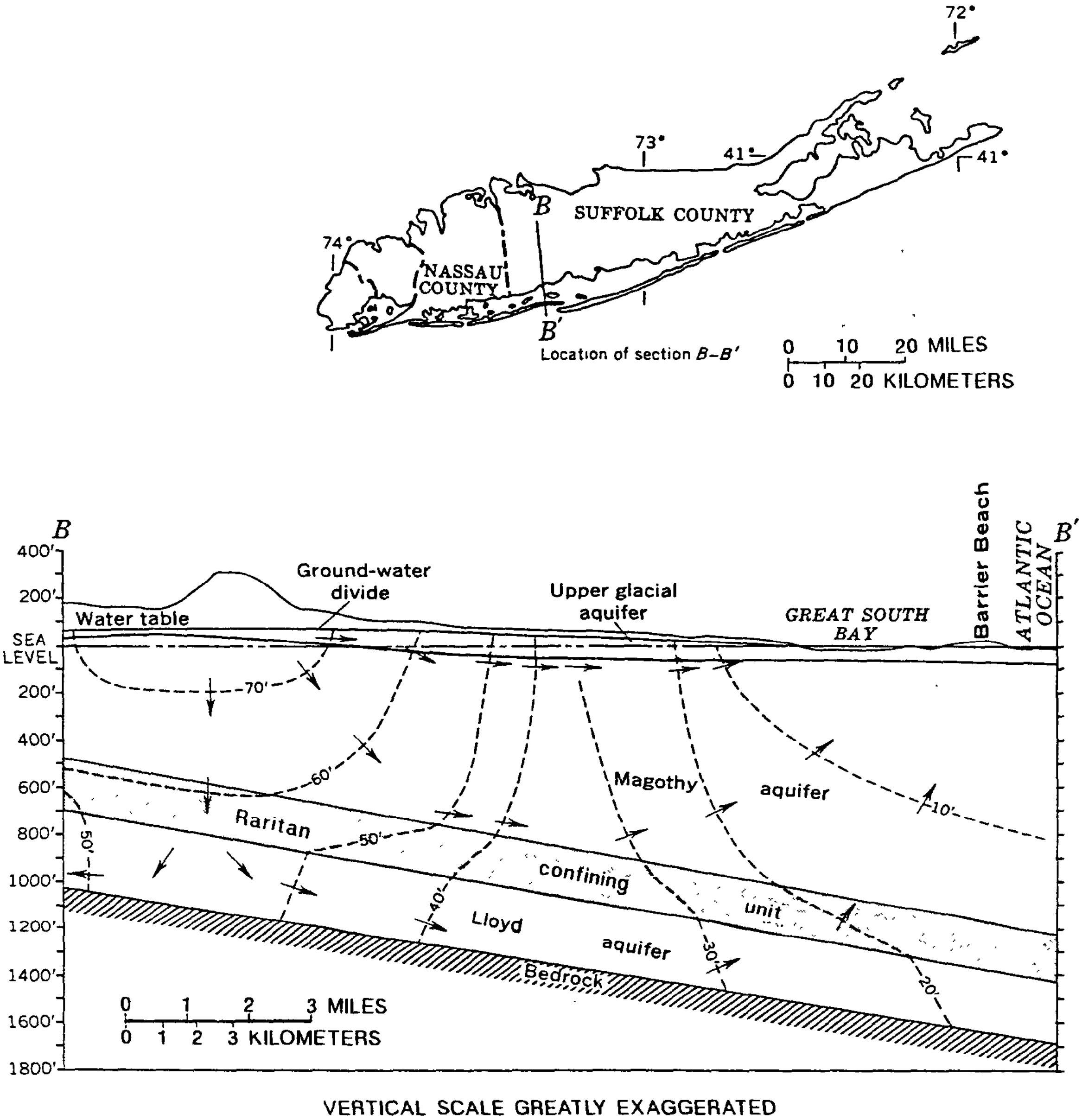
Figure 8-27.--Contour map of the bedrock surface, Long Island, New York. (From Franke and McClymonds, 1972, fig. 11.)



EXPLANATION

—— 20 —— WATER-TABLE CONTOUR -- Shows estimated altitude of water table under natural conditions. Contour intervals 5, 10, and 20 feet. Datum is sea level

Figure 8-28.--Estimated average position of the water table under natural conditions. (From Franke and McClymonds, 1972, fig. 9.)



EXPLANATION

---40--- POTENTIOMETRIC CONTOUR -- Shows altitude of equipotential surface. Contour interval 10 feet. Datum is sea level

→ DIRECTION OF GROUND-WATER FLOW

——— HYDROGEOLOGIC BOUNDARY

Figure 3-29.--Geohydrologic section of the ground-water reservoir in southwestern Suffolk County, Long Island, New York, in October 1960. (From Franke and McClymonds, 1972, fig. 21.)

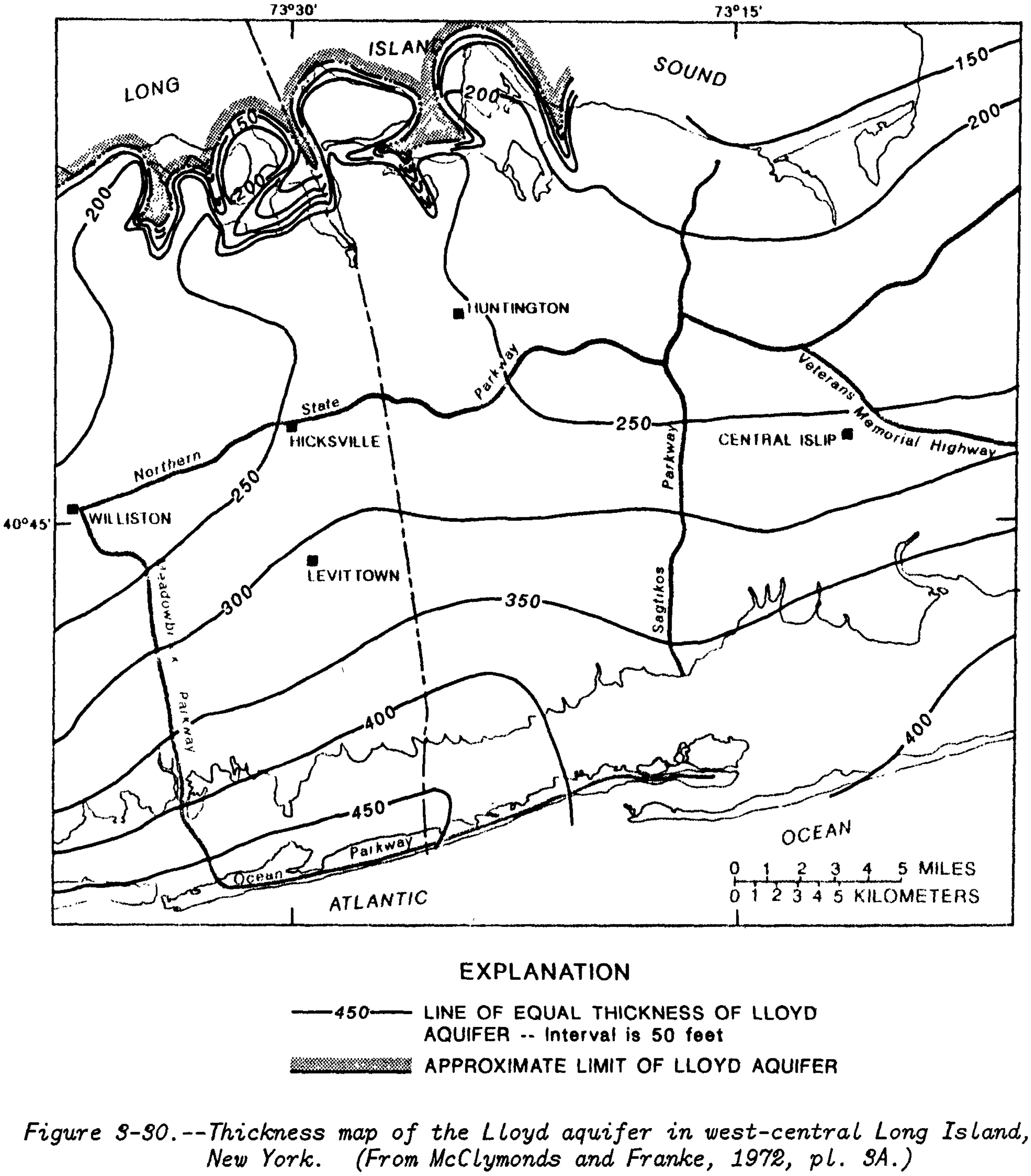


Figure 3-30.--Thickness map of the Lloyd aquifer in west-central Long Island, New York. (From McClymonds and Franke, 1972, pl. 3A.)

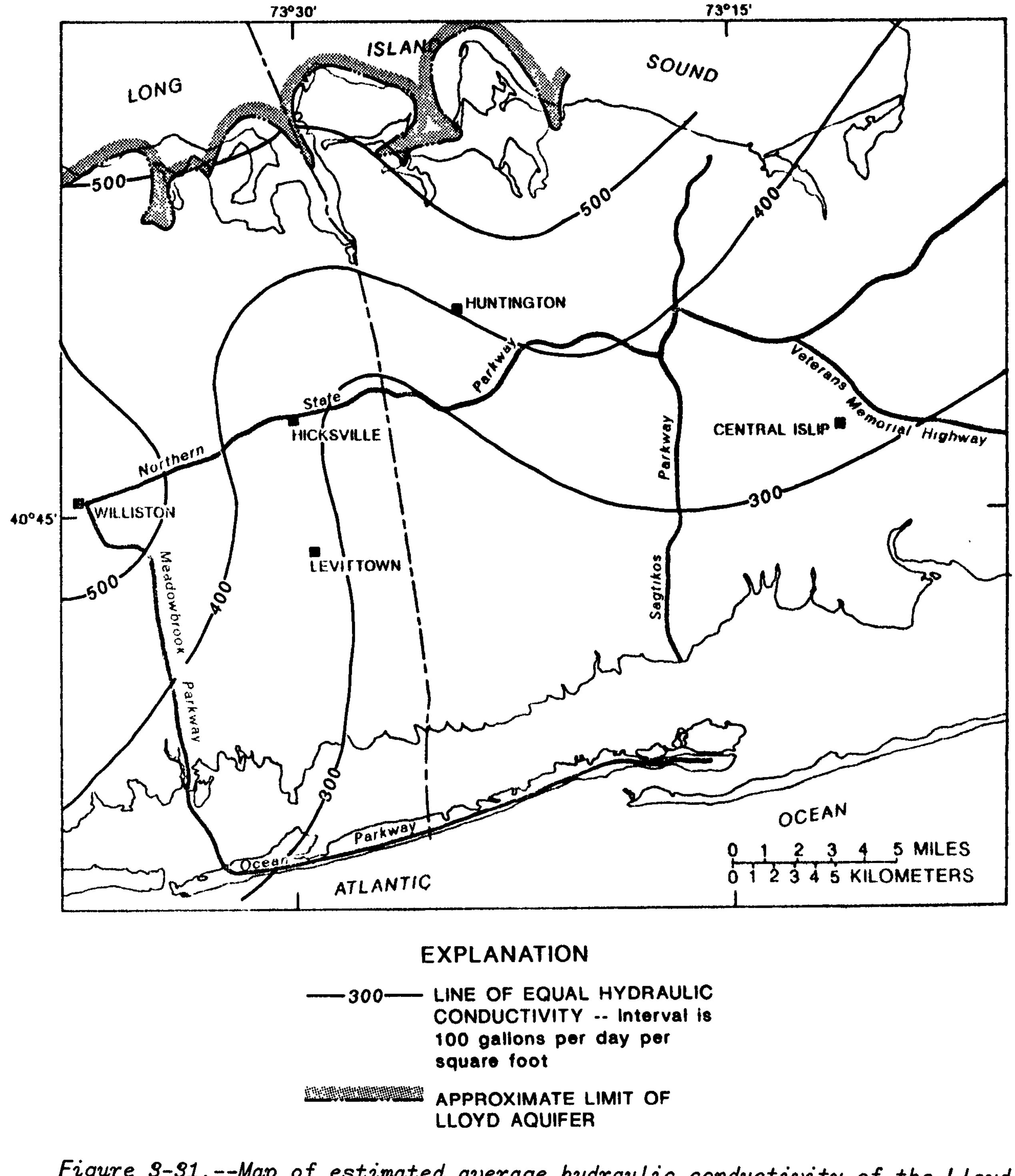
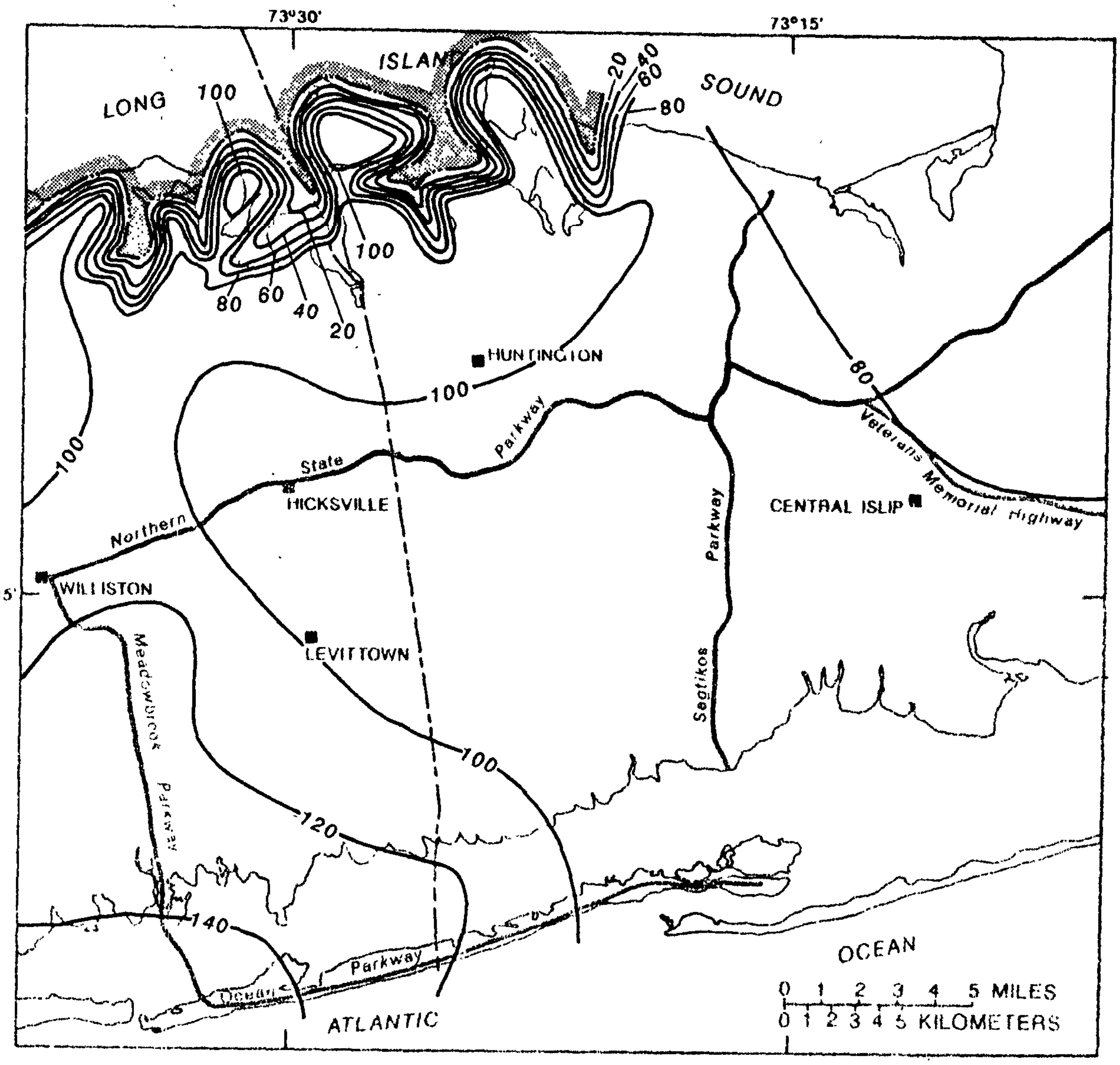


Figure 3-31.--Map of estimated average hydraulic conductivity of the Lloyd aquifer in west-central Long Island, New York. (From McClymonds and Franke, 1972, pl. 3B.)



EXPLANATION

—140— LINE OF EQUAL TRANSMISSIVITY IN THOUSANDS -- Interval is 20,000 gallons per day per foot

APPROXIMATE LIMIT OF LLOYD AQUIFER

Figure 3-32.--Map of estimated transmissivity of the Lloyd aquifer in western Long Island, New York. (From McClymonds and Franke, 1972, pl. 3C.)

Geology and the Occurrence of Ground Water

Assignment

*Study Fetter (1988), p. 259-324; Freeze and Cherry (1979), p. 144-166; or Todd (1980) p. 37-42.

Much has been written about the role of rock type, depositional environment of sediments, geologic structure and climate on the occurrence of ground water. The reading assignment listed above deals with these aspects of ground-water hydrology in sufficient detail for the purposes of this course.

Description of a Real Ground-Water System

At this point in the course we suggest that the instructor or someone else make a formal presentation that describes in detail the operation of a real ground-water system, preferably one that is of particular interest to the participants. Some of the information that such a presentation might contain is listed below. Of particular importance in the context of this course is a clear conceptualization of the natural system, which includes a careful description of the system's physical boundary conditions (items (2) and (3) in the following list).

(1) Location of study area, geography, and climate.

(2) Geologic framework--pertinent features but not lengthy stratigraphic descriptions.

(3) Natural hydrologic system--how the system operates; inputs and locations; areas of discharge; head maps for pertinent hydrogeologic units; careful designation of boundaries and boundary conditions of natural hydrologic system; data available, and methods to estimate distribution of hydraulic properties.

(4) Human effects on hydrologic system--brief historical survey.

(5) If the presentation includes discussion of a model simulation, reason for developing model or definition of problem to be solved by model.

(6) Description of model--areal extent; areal discretization scheme; number of model layers; careful designation of model boundaries and boundary conditions; compare with boundaries in (3) and justify any differences; definition of initial conditions; time-discretization scheme if unsteady model; superposition versus absolute heads; preliminary model runs and what one might learn from them; calibration procedures; and subjective evaluation of reliability of final model results to solve the problem posed.

Source of Water to a Pumping Well

Assignment

*Work Exercise (3-3)--Source of water to a pumping well.

What is the source of water to a pumping well placed at different locations within the ground-water system? Answering this question qualitatively in the early part of a ground-water investigation can be a productive part of the conceptualization of a ground-water system. As some thought about the question may suggest, the response of a system to stress ultimately must depend on that system's physical boundary conditions.

Exercise (3-3)--Source of Water to a Pumping Well

The points made by C. V. Theis (1940) in his paper "The source of water derived from wells--essential factors controlling the response of an aquifer to development" may be summarized and extended as follows. Consideration of the hydrologic equation Inflow = Outflow $\pm$ ΔStorage suggests that, in principle, there are three possible sources of water to a pumping well--a decrease in ground-water storage, an increase in inflow to the ground-water system, or a decrease in outflow from the ground-water system. This abstract statement of principle can be clarified by application to a concrete example.

Consider a simple hydrologic system under predevelopment conditions in a state of dynamic equilibrium for which inflow = outflow (fig. 3-33(A)). When a well is added to the system and pumping starts at a rate Q_1, initially water is withdrawn only from storage. As water levels continue to fall and hydraulic gradients are reduced in areas of natural discharge, natural discharge is reduced (fig. 3-33(B)). These processes reduce the amount of water that must come from storage--in effect, flow is rerouted from the original discharge area, the stream, to the pumping well. As the rate of storage depletion decreases, the rate of water-level decline slows and the system approaches a new equilibrium (fig. 3-33(C)).

At a later time the equilibrium condition depicted in figure 3-33(C) is further disturbed by a higher rate of pumpage (Q_2). After an initial removal of ground water from storage accompanied by a further decline in water levels, in contrast to the situation depicted in figure 3-33(C) in which a water-table divide exists between the well and the stream, the new equilibrium condition exhibits no divide; that is, a hydraulic gradient exists between the stream and the pumping well (fig. 3-33(D)). This condition induces movement of water from the stream into the aquifer. Thus, the stream, which formerly was a gaining stream under natural conditions and a lesser rate of pumpage Q_1 (fig. 3-33(C)), is now locally a losing stream (fig. 3-33(D)).

In summary, the source of water to the well at the initial rate of pumpage Q_1, after a new equilibrium condition had been achieved, was reduced *outflow* of ground water to the stream. However, in contrast, the source of water to the well at the higher rate of pumpage Q_2 includes both reduced outflow to the stream and induced inflow from the stream to the aquifer.

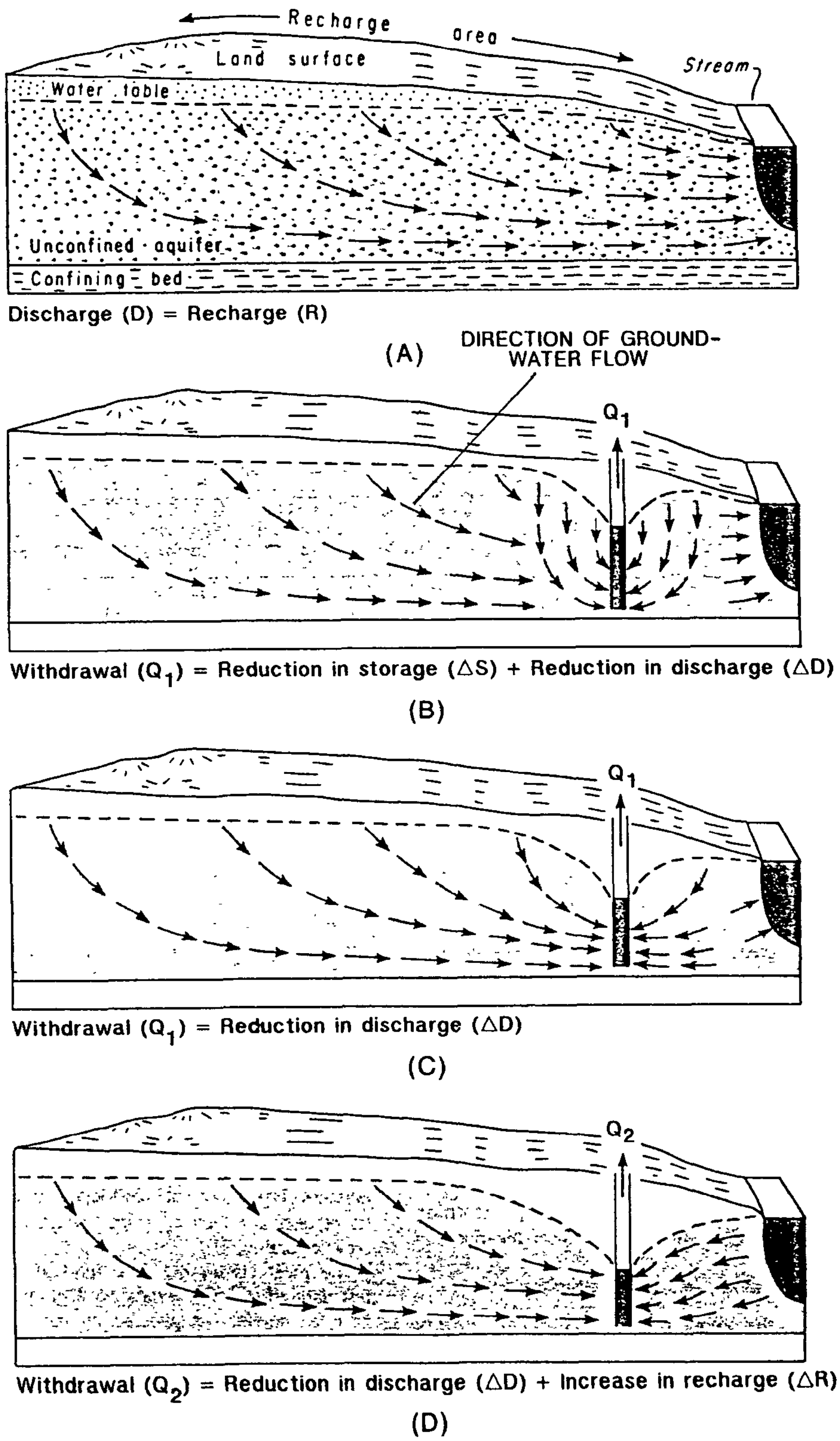


Figure 3-33.--Ground-water flow patterns in a hypothetical system (A) under natural conditions and (B, C, and D) in response to different levels of stress resulting from local pumping of ground water. (Modified from Heath, 1983, p. 33.)

In some cases, the pumpage may exceed the increases in recharge and decreases in natural discharge that can be induced. In these cases, withdrawal from storage continues until falling water levels or exhaustion of the supply force a reduction in the pumping rate. A new equilibrium is then attained in which the reduced pumping rate equals the increases in recharge and decreases in discharge that have been achieved.

If pumpage is not held constant, but rather is increased from year to year, new periods of withdrawal from storage accompany each increase in pumpage.

The following exercise will help to clarify some of these concepts. In this exercise only equilibrium (steady-state) states of the system will be considered--that is, transient conditions in the system, in which some of the water pumped is obtained from ground-water storage, will not be analyzed.

A square confined aquifer with a uniform transmissivity is shown in figures 3-34(A) and 3-34(B). The aquifer is bounded laterally by two impermeable rock walls and two surface-water bodies. The earth materials above and below the aquifer are assumed to be impermeable. The surface-water bodies are a river and a reservoir whose stages remain constant. Thus, a constant head is exerted by the surface-water bodies on their surfaces of contact with the aquifer. The natural head distribution with the river stage at zero altitude and the reservoir stage at 200 ft is a straight line in cross section, as shown in figure 3-35(B). The long-term average increase in flow in the river due to inflow of ground water from the aquifer is 3.1 ft^3/s.

A steady-state simulation of the system with a well (figs. 3-34(B), 3-35(A)) that is pumped at 3.1 ft^3/s (discharge of well is equal to the natural steady-state flow through the aquifer before pumping) using a numerical model solved by a digital computer resulted in the head distribution shown in figure 3-35(A). The steady-state increase in flow in the river opposite the aquifer with this steady rate of pumpage is decreased to 2.0 ft^3/s from its original value of 3.1 ft^3/s.

(1) What is the transmissivity of the aquifer, in ft^2/s?

(2) Contour the head values in figure 3-35(A) using a 20-ft contour interval.

(3) Draw a head profile along AC on figure 3-35(B).

(4) What hydrologic feature may be observed at point B on the head profile?

(5) Draw two streamlines (perpendicular to contours of equal head) from point B to the reservoir on figure 3-35(A).

(a) What hydrologic feature is represented by these two streamlines?

(b) What hydrologic feature is represented by the area enclosed by the two streamlines and the reservoir?

(6) Using the information given above, what must be the total inflow to the aquifer from the reservoir when the well is being pumped?

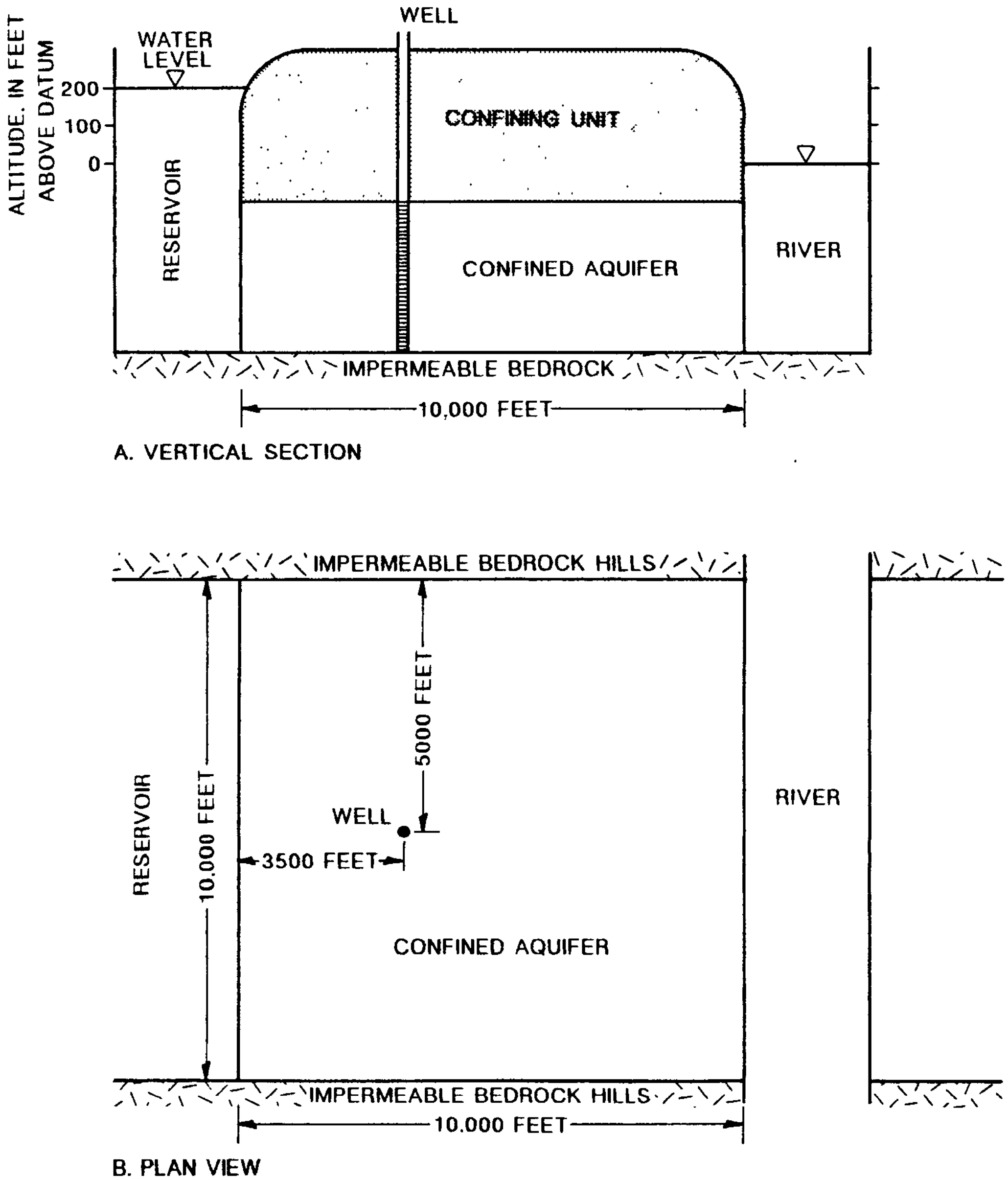


Figure 3-34.--A hypothetical aquifer system in (A) vertical section and (B) plan view.

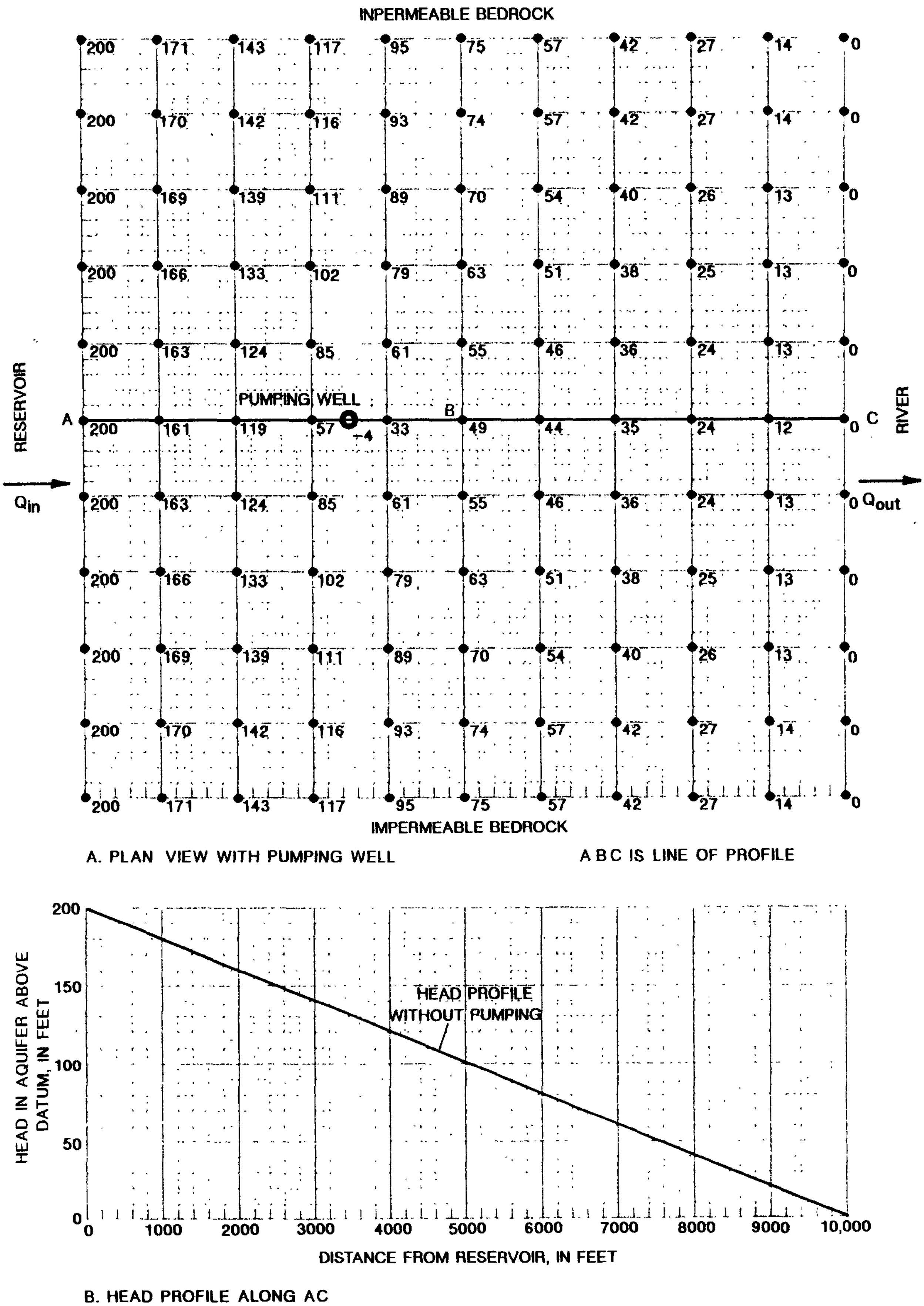


Figure 3-35.--(A) Heads in the stressed aquifer determined by numerical simulation when the pumping rate of the well is 3.1 cubic feet per second. (B) Graph for plotting head profile using data from (A).

(7) Applying the Theis concepts to this situation, what is the "source" of the water to the pumping well, in terms of increased inflow (or recharge) to the aquifer and decreased outflow from the aquifer?

(8) The pumping rate of the well is increased significantly. The resulting head profile along section AC is shown in figure 3-36.

(a) How does the head profile in figure 3-36 differ from the head profile in figure 3-35(B)?

(b) In terms of the Theis concepts, what are the three sources of water to the pumping well in figure 3-36?

(9) The following questions involve qualitative comparisons between the ground-water system described in this exercise and depicted in figure 3-34, designated for convenience as system (a), and the system depicted in figure 3-2 of Exercise (3-1), designated as system (b). Refer to item (6) in table 3-2 of Note (3-3).

(a) After reviewing the boundary conditions of both systems, list the differences in the two sets of boundary conditions.

(b) Place a hypothetical pumping well at two or more locations in both systems in order for (i) the drawdowns caused by the pumping well to be a minimum and (ii) the drawdowns caused by the well to be a maximum.

(c) List the probable sources of water to the pumping well at each location in (b).

(d) A pumping stress interacts with two boundaries in system (a). What is the corresponding situation in system (b)?

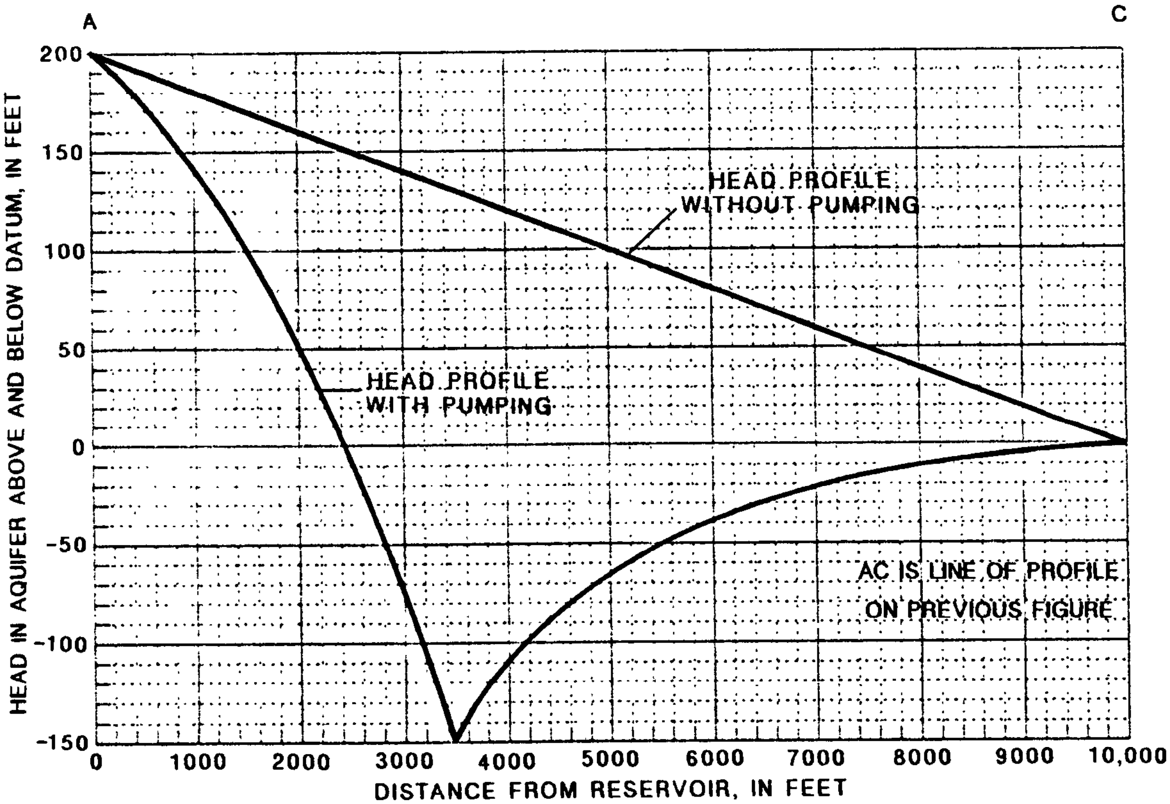


Figure 3-36.--Head profile in the aquifer when the pumping rate of the well is much greater than 3.1 cubic feet per second.

Role of Numerical Simulation in Analyzing Ground-Water Systems

Assignments

*Study Fetter (1988), p. 525-548; Freeze and Cherry (1979), p. 352-364, 540-541; or Todd (1980), p. 384-408.

*Study Note (3-7)--Role of numerical simulation in analyzing ground-water systems.

The most powerful quantitative tool that is available to the hydrologist is numerical simulation. An example of a well documented, general purpose three-dimensional numerical model for ground-water flow simulation is the U.S. Geological Survey Modular Model (McDonald and Harbaugh, 1988). The purpose of the brief comments in Note (3-7) is to suggest a number of ways in which this tool can be effectively utilized.

Simulation, however, can only be effectively utilized in the hands of a knowledgeable hydrologist. The authors have observed instances in which simulation was incorrectly applied. Unfortunately, although the results of these simulations are incorrect and misleading, the conceptual errors leading to these incorrect results may be difficult to identify, and the results may be perceived as correct because of their source.

Note (3-7).--Role of Numerical Simulation in Analyzing Ground-Water Systems

The following statement on the role of simulation in analyzing ground-water systems is an excerpt from an unpublished manuscript by Gordon D. Bennett (U.S. Geological Survey, written commun., 1983). We wish to emphasize two ideas expressed in this excerpt--(1) the importance of simulation as an investigative tool to increase our understanding of the functioning of the ground-water system, as opposed to the usual emphasis on using simulation for prediction, and (2) the idea that several different models of varying type and complexity can be used profitably in parallel early in an investigation to study specific features of the ground-water system.

> Simulation is the central activity in a modern ground-water resource evaluation. It is used ultimately in the predictive phases of the investigation to evaluate the effects of various proposed courses of development. More importantly, however, it is used throughout the study as an investigative tool to develop concepts and test hypotheses, to determine the sensitivity of the system to various parameters, to obtain estimates of parameters by inverse techniques, and to guide the collection and analysis of new data.

As working hypotheses are developed regarding system boundaries and parameter ranges, simulations are designed to test those hypotheses. The head values and ground-water flows obtained in the simulations are compared with corresponding observed heads or flow estimates, and the working hypotheses are modified as necessary. The simulations may be cross-sectional, areal, or three-dimensional, and may represent an original undisturbed equilibrium, a transient response to development, or a new equilibrium achieved after adjustment to development.

This use of simulation as an investigative tool should begin early in the investigation, in parallel with other project activities. Ground-water systems are always three-dimensional. In these early simulations, however, it is often preferable to focus first on individual aspects of the system which can be represented approximately through two-dimensional analysis. In general, both areal and cross-sectional models should be employed, and both steady-state and transient analyses should be made. This phase of the work should be carried on in a parallel, rather than a sequential mode; that is, it is usually a mistake to try to complete all areal simulations before undertaking cross-sectional simulations, or all steady-state analyses before undertaking transient analyses...

The general objectives of simulation remain the testing of hypotheses, establishment of parameter ranges, identification of sensitive parameters, and general insight into the operation of the ground-water system.

SECTION (4)--GROUND-WATER FLOW TO WELLS

Wells are our direct means of access or "window" to the subsurface environment. Uses of wells include pumping water for water supply, measuring pressures and heads, obtaining ground-water samples for chemical analysis, acting as an access hole for borehole geophysical logs, and direct sampling of earth materials for geologic description and laboratory analysis, primarily during the process of drilling the wells. Hydrogeologic investigations are based on these potential sources of well-related information.

Concept of Ground-Water Flow to Wells

Assignments

*Look up and write the definitions of the following terms relating to radial flow and wells in Fetter (1988), both in the glossary and in the index--drawdown, specific capacity of well, completely penetrating well, partially penetrating well, leaky confined aquifer, leaky artesian aquifer, semiconfined aquifer, and leaky confining bed or layer.

*Study Note (4-1)--Concept of ground-water flow to wells.

The general laws (Darcy's law and the principle of continuity) that govern ground-water flow to wells are the same as those that govern regional ground-water flow. The system concept is equally valid--we are still concerned with system geometry, both external and internal, boundary conditions, initial conditions, and spatial distribution of hydraulic parameters as outlined in table 1 of Note (3-2). However, the process of removing water from a vertical well imposes a particular geometry on the ground-water flow pattern in the vicinity of the well, which is called radial flow. Radial flow to a pumping well is a strongly converging flow whose geometry may be described by a particular family of differential equations that utilize cylindrical coordinates (r,z) instead of cartesian coordinates (x,y,z). A large number of analytical solutions to these differential equations with different boundary conditions describe the distribution of head near a pumping well.

Note (4-1).--Concept of Ground-Water Flow to Wells

As has been noted previously, ground-water flow in real systems is three-dimensional. To obtain water from the ground-water system, wells are installed and pumped. Water pumped from the well lowers the water level in the well, thereby establishing a head gradient from the aquifer toward the well. As a result, water moves from the surrounding aquifer into the well. As pumping proceeds, a decline in head or drawdown propagates away from the well as water continues to move from areas of higher head to areas of lower head and is pumped out of the well.

Ground-water flow to a pumping well can be viewed as occurring through a series of concentric vertical cylinders with the center of the well at the central vertical axis (fig. 4-1). If the aquifer properties (hydraulic conductivity (K) and storage coefficient (S)) are symmetric around the well, then the hydraulic head (or change in hydraulic head) and the flow of water (or change in flow) also will be symmetric. This symmetry enables us to simplify the analysis of a general three-dimensional flow system to a two- or one-dimensional system using cylindrical (or radial) coordinates.

Further consideration of figure 4-1 indicates that flow to a well, or radial flow is a converging flow, because the areas of the concentric cylinders ($A = 2\pi rb$), which are perpendicular to the direction of ground-water flow, decrease continuously toward the well as the radial distance r from the center of the well decreases while the aquifer thickness b remains constant. If we apply Darcy's law conceptually to this flow system,

$$Q = KA \frac{h_1 - h_2}{l}$$

where $A = 2\pi rb$, and assume that Q and K are constant (that is, the flow system is in steady state and the aquifer is homogeneous), we can write

$$\frac{Q}{K} = A \frac{h_1 - h_2}{l} = \text{constant}$$

We have seen that the area perpendicular to ground-water flow decreases toward the well. Thus, for the above relation to be true, the head gradient $\frac{h_1 - h_2}{l}$ must increase continuously toward the well. This qualitative inference from Darcy's law is a general and characteristic feature of flow to wells. Equations that define this increase in gradient toward the pumping well are developed in subsequent notes and exercises.

Because pumping wells may be located in diverse hydrogeologic environments, quantitative analysis of the associated radial-flow systems requires the use of simplifying assumptions. Our conceptualization of the system, based on the distribution of the transmitting and storage properties of the aquifer and the boundary conditions of the radial section under study, determines how the radial flow system is simplified and formulated for analysis. For example, transmitting and storage properties vary depending on whether the aquifer in question is one homogeneous aquifer, a heterogeneous layered aquifer, or a complex aquifer system. Boundary conditions, such as a partially penetrating well that causes significant vertical movement, or an impermeable top and bottom of the aquifer as opposed to a "leaky" top and bottom, also affect the complexity of the system to be analyzed.

All radial flow systems can be conceptualized in a variety of ways, each of which leads to a different simplification that is incorporated into a mathematical description of the system. As shown in figure 4-2(A), a well pumping in a multi-aquifer system could be studied in the context of the entire aquifer system, and the head and flow throughout the system could be

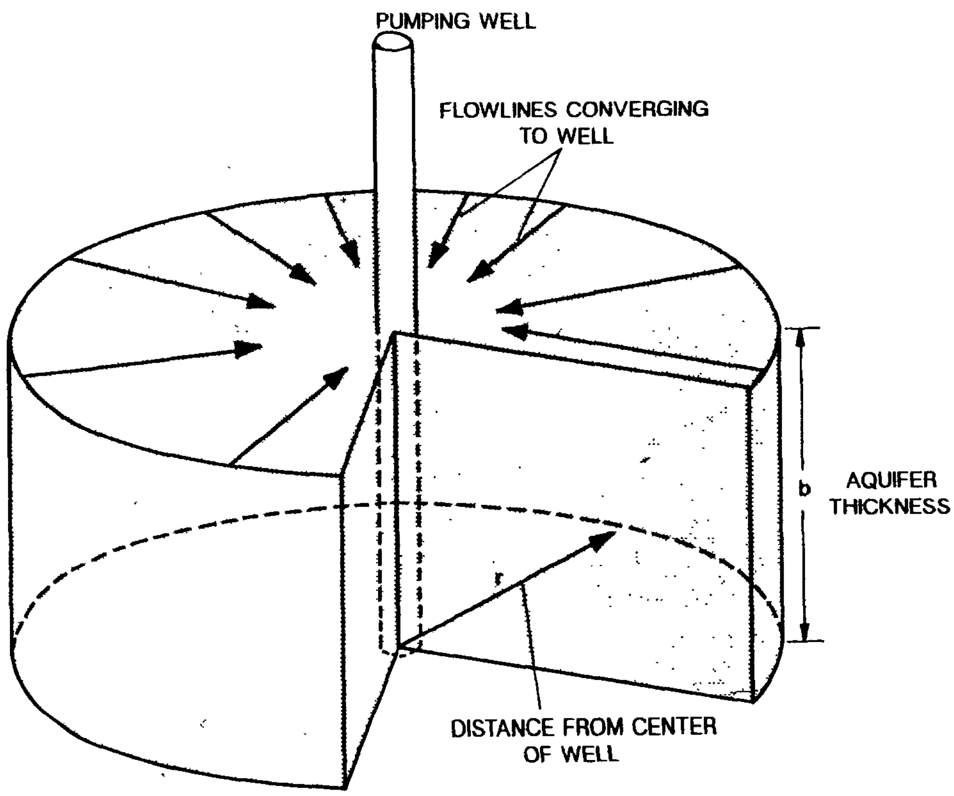


Figure 4-1.--Flow to a well viewed as radial flow through a continuous series of concentric vertical cylindrical sections with the axis of the well at the center.

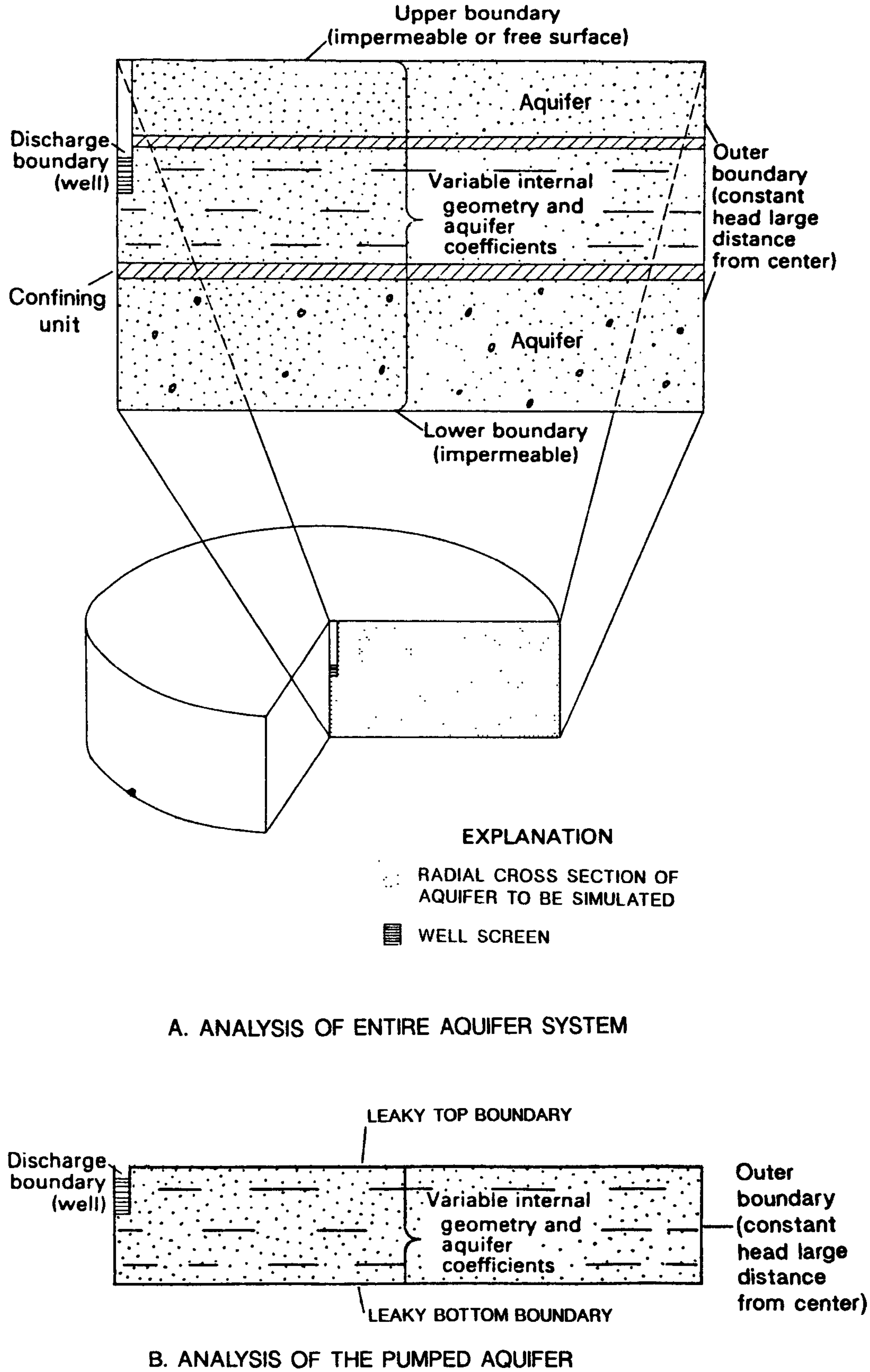


Figure 4-2.--Two conceptualizations of the same ground-water system.

simulated quantitatively by means of a numerical model. Or, the problem could be conceptualized by assuming that the pumping would not affect significantly the aquifer system above and below the extensive confining units. In this situation the flow system could be analyzed as a single aquifer that receives leakage from the overlying and underlying aquifers through the confining beds (fig. 4-2(B)). In case A, the entire multi-aquifer system is defined as the system under study, while in case B, only the aquifer being pumped is analyzed.

The notes and problems that follow describe the use of mathematical solutions for different radial flow conceptualizations. Keep in mind the internal characteristics and boundary conditions of the various radial-flow models and solutions that are discussed; these features determine the degree to which the mathematical representation of the flow system corresponds to the real, physical flow system.

Analysis of Flow to a Well--Introduction to Basic Analytical Solutions

Assignments

*Study Note (4-2)--Analytical solutions to the differential equations governing ground-water flow.

*Study Fetter (1988), p. 143, 199-201; Freeze and Cherry (1979), p. 188-189, 314-319; or Todd (1980), p. 112-113, 115-119, 123-124.

*Study Note (4-3)--Derivation of the Thiem equation for confined radial flow.

*Work Exercise (4-1)--Derivation of the Dupuit-Thiem equation for unconfined radial flow.

*Study Fetter (1988), p. 161-169.

*Study Note (4-4)--Additional analytical equations for well-hydraulic problems.

This subsection is primarily a study section that provides an introduction to some of the simplest and most widely applied radial-flow equations. We focus on three such equations--(1) the Thiem equation for steady-state confined flow, (2) the Dupuit-Thiem equation for steady-state unconfined flow, and (3) the Theis equation for unsteady confined flow. These and all other radial-flow equations relate to specific, highly idealized ground-water flow systems. We cannot overemphasize the importance of learning the key features of the individual flow systems to which each equation applies. These key features relate in large part to the boundary conditions that are assumed in the derivation of a given equation.

Note (4-2).--Analytical Solutions to the Differential Equations Governing Ground-Water Flow

This note reviews and extends some of the ideas discussed in a previous note on the information required to describe a ground-water system (Note 3-2).

Quantitative analysis of a ground-water flow problem involves the definition of an appropriate boundary-value problem. Definition of a boundary-value problem requires the specification of the governing differential equation and the initial and boundary conditions applicable to the specific problem under study. The governing differential equation is a mathematical model that describes ground-water flow in the flow domain. The information needed to define a boundary-value problem involving ground-water flow is shown in table 4-1 (reproduced from table 3-1, Note 3-2) in the context of a simple system diagram.

Solution of a boundary-value problem involves solving the governing differential equation (generally a partial-differential equation in ground-water flow problems) for the initial and boundary conditions that apply to the problem. Today, complex boundary-value problems generally are solved by numerical methods with the assistance of a digital computer. However, many useful analytical solutions to boundary-value problems representing simple systems are available.

An analytical solution to a ground-water problem is an exact mathematical solution to a specific boundary-value problem that is relevant to ground-water studies. We may think of an analytical solution as a "formula" amenable to calculation that relates the dependent variable in the differential equation (generally head (h) or drawdown (s) in ground-water problems) to the independent variable(s) in the differential equation (coordinates of position (x,y,z) and time (t)). Thus, an analytical solution may be represented in a general way as

$$h = f(x,y,z,t).$$

An analytical solution is an exact solution to the governing differential equation, and provides a formula that permits calculation of the dependent variable in continuous space and time. However, analytical solutions usually are available only for highly idealized conceptualizations of ground-water systems. Thus, the solution to the idealized mathematical representation (governing differential equation and boundary conditions) is exact, but the mathematical representation rarely corresponds closely to hydrogeologic conditions in the real system.

Some of the typical simplifying assumptions used in the mathematical model to develop analytical solutions are (a) flow medium (earth material) is isotropic and homogeneous, (b) the aquifer is confined, (c) the aquifer is unbounded laterally (infinite areal extent), and so on. Furthermore, the geometry of the flow system generally is simple--for example, the flow system is bounded by a rectangular or circular prism, the aquifer is horizontal and of constant thickness, the well completely penetrates the aquifer, and so on. Finally, boundary conditions usually are simple (constant head, no-flow, and constant flux are common boundary conditions).

Table 4-1.--Information necessary for quantitative definition of a ground-water flow system in context of a general system concept

Input ---------------->>	System -------------->>	Output
Input or stress applied to ground-water system	Factors that define the ground-water system	Output or response of ground-water system
(1) Stress to be analyzed: - expressed as volumes of water added or withdrawn - defined as function of space and time	(1) External and internal geometry of system (geologic framework) - defined in space (2) Boundary conditions -defined with respect to heads and flows as a function of location and time on boundary surface (3) Initial conditions -defined in terms of heads and flows as a function of space (4) Distribution of hydraulic conducting and storage parameters - defined in space	(1) Heads, drawdowns, or pressures[1] -defined as function of space and time

[1] Flows or changes in flow within parts of the ground-water system or across its boundaries sometimes may also be regarded as a dependent variable. However, the dependent variable in the differential equations governing ground-water flow generally is expressed in terms of either head, drawdown, or pressure. Simulated flows across any reference surface can be calculated when the governing equations are solved for one of these variables, and flows in real systems can be measured directly or estimated from field observations.

Even with all their simplifications, analytical solutions can provide invaluable hydrologic insight into idealized but nevertheless representative and relevant ground-water systems. Furthermore, they often can be used effectively in the quantitative analysis of ground-water problems (for example, analysis of aquifer tests). In general, no analytical solution corresponds exactly to a given field situation. Thus, a proper application of analytical solutions to field problems requires that the hydrologist have a detailed understanding of the physical system represented by the analytical solution and the assumptions (the most important assumptions often involve boundary conditions) that underlie the analytical solution.

Note (4-3).--Derivation of the Thiem Equation for Confined Radial Flow

Darcy's law describes the flow of water through a saturated porous medium and can be written as follows (Fetter, 1988, p. 123, equation 5-19):

$$Q = -KA\frac{dh}{dr}$$

where A is the cross-sectional area through which the water flows, r is distance along the ground-water flow path (in this case, radial distance), and the other terms are as previously defined. Steady flow to a well (fig. 4-3) in a confined aquifer bounded on top and bottom by impermeable units is radially convergent flow through a cylindrical area around the well. As shown in figure 4-3, the area (A) through which flow occurs is

$$A = 2\pi rb,$$

where b is the thickness of the completely confined aquifer. Substituting this expression for A into Darcy's law gives:

$$Q = -2\pi Kbr\frac{dh}{dr}.$$

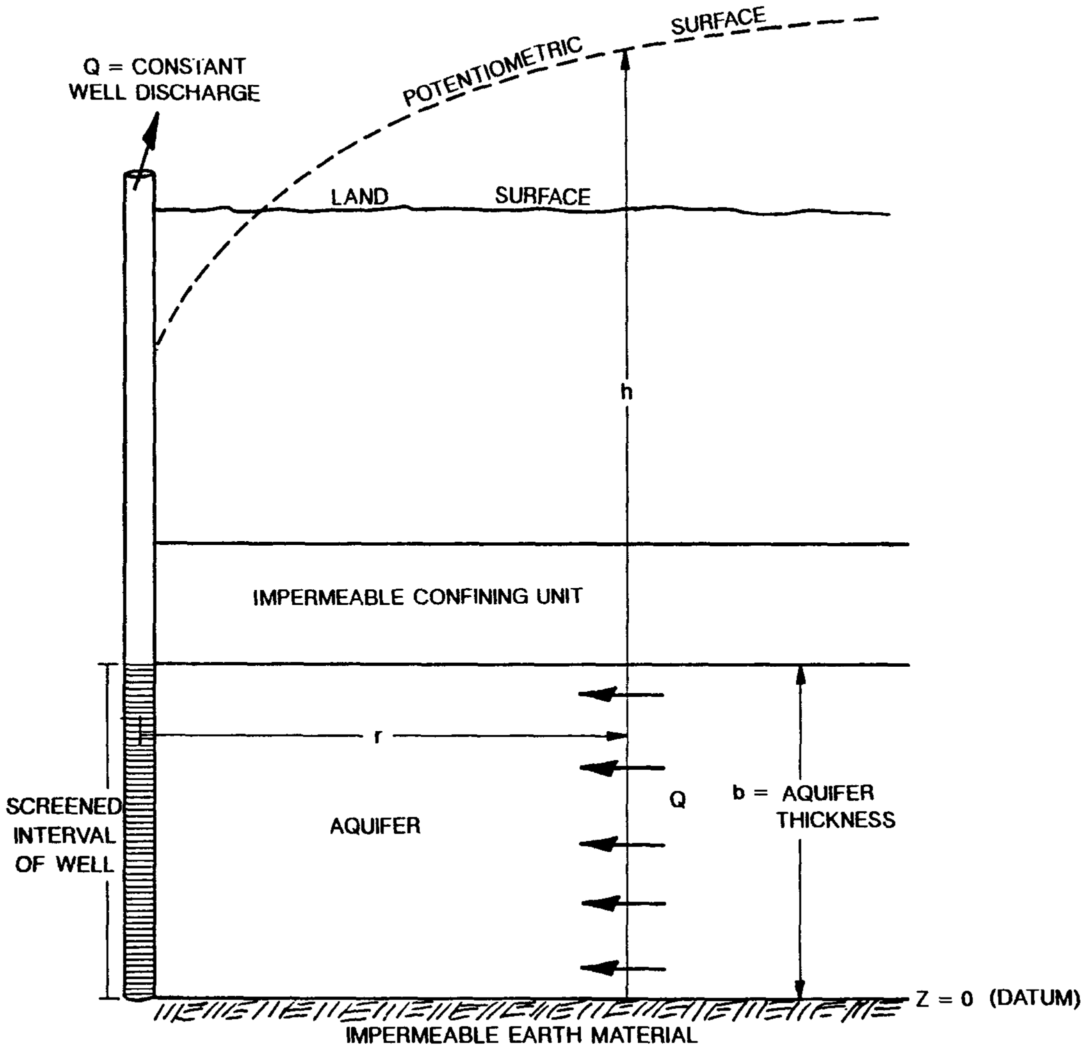
For steady flow, Q, the constant quantity of water pumped from the well, is also the flow rate through any cylindrical shell around the well.

This equation can be solved by separating variables and integrating both sides of the equation. Separation of variables gives:

$$\frac{1}{r}dr = -\frac{2\pi Kb}{Q}dh.$$

Integrating from r_2 to r_1, where the heads are h_2 and h_1, respectively,

$$\int_{r_1}^{r_2}\frac{dr}{r} = \int_{h_1}^{h_2} -\frac{2\pi Kb}{Q}dh$$



Note: h is head in aquifer above datum at radial distance r; Q is constant well discharge which equals constant radial flow in aquifer to well; r is radial distance from axis of well; Z is elevation head.

Figure 4-8.--Steady flow to a completely penetrating well in a confined aquifer.

gives

$$\ln r_2 - \ln r_1 = - \frac{2\pi Kb}{Q} (h_2 - h_1),$$

or

$$\ln \frac{r_2}{r_1} = - \frac{2\pi Kb}{Q} (h_2 - h_1).$$

Rearranging terms gives

$$Kb = \frac{-Q}{2\pi(h_2 - h_1)} \ln \frac{r_2}{r_1} .$$

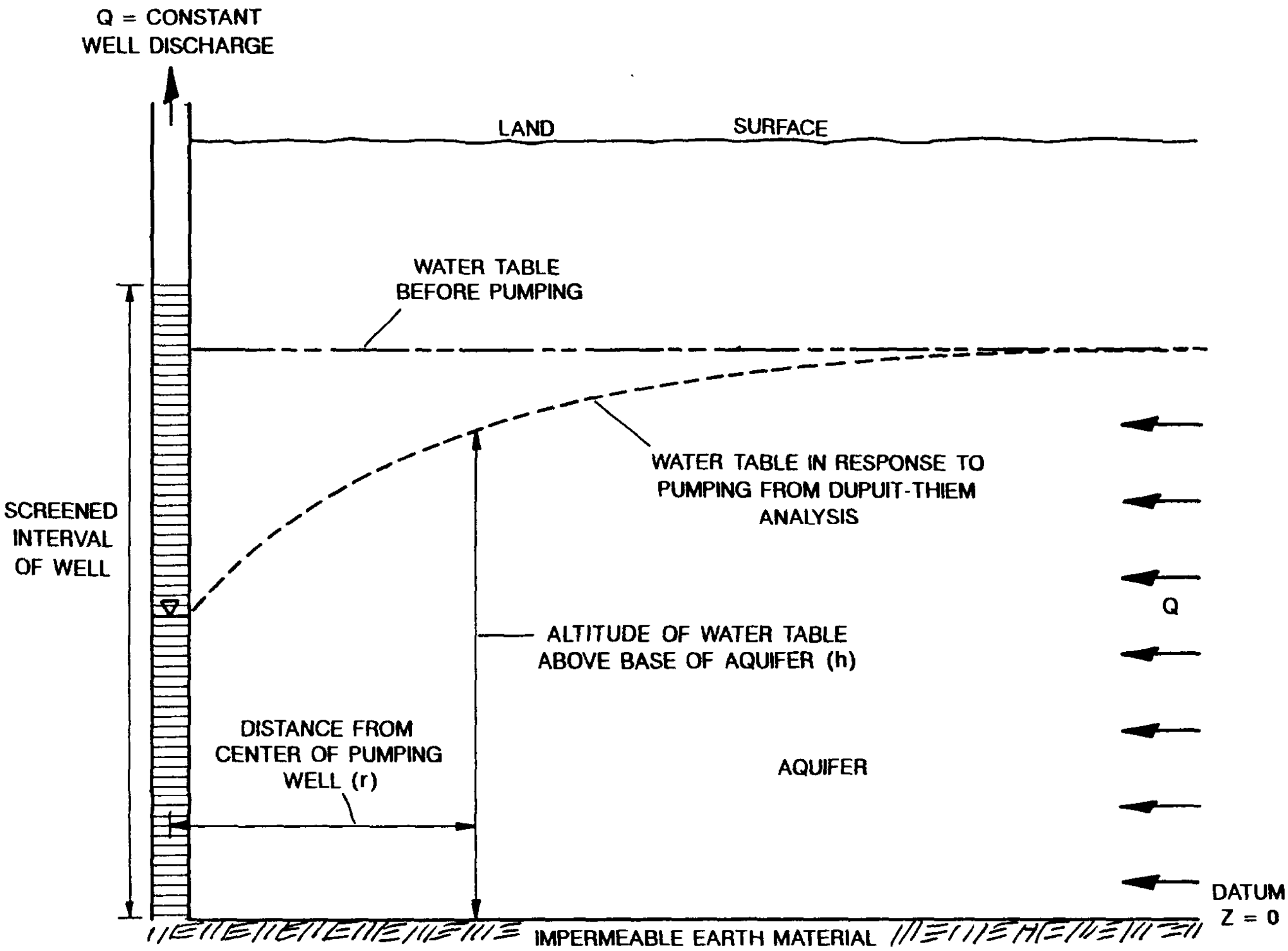
Because Kb equals transmissivity (T) and the pumping rate is defined as a positive number, the resulting equation is

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln \frac{r_2}{r_1} ,$$

which is the Thiem equation as given by Fetter (1988, p. 200, equation 6-56).

Exercise (4-1)--Derivation of the Dupuit-Thiem Equation for Unconfined Radial Flow

The Dupuit-Thiem equation (Fetter, 1988, p. 200, eqation 6-57) for unconfined radial flow is analogous hydrologically to the Thiem equation for confined radial flow (Note 4-3). Review Note 4-3 and derive the Dupuit-Thiem equation using a similar sequence of steps. The key difference between this derivation and the derivation of the Thiem equation lies in expressing the cylindrical area of flow around a pumping well in an unconfined aquifer as $A = 2\pi rh$ (fig. 4-4), as opposed to $A = 2\pi rb$ for confined flow, where h is the saturated thickness of the unconfined aquifer at a distance r from the pumping well. Expressed in another way, the datum or reference elevation for h is at the bottom of the unconfined aquifer, which is assumed to be an impermeable boundary (fig. 4-4).



Note: Q is constant well discharge which equals constant radial flow in aquifer to well; Z is elevation head

Figure 4-4.--Steady flow to a completely penetrating well in an unconfined aquifer as represented in a Dupuit-Thiem analysis.

Note (4-4).--Additional Analytical Equations for Well-Hydraulic Problems

As discussed previously, flow patterns in ground-water systems stressed by pumping from a well are three-dimensional. Furthermore, aquifer systems can have widely varying internal characteristics and boundary conditions. A different mathematical model and a corresponding different solution, either numerical or analytical, can be developed for each conceptualization of radial flow. We already have discussed the three simplest conceptualizations and their corresponding mathematical solutions. These three solutions are

1. steady-state, confined, one-dimensional radial flow--the Thiem equation;

2. steady-state, unconfined, one-dimensional radial flow--the Dupuit-Thiem equation; and

3. transient-state, confined, one-dimensional radial flow--the Theis equation.

However, many more complex hydrogeologic situations routinely exist in nature, and a number of additional analytical solutions are available for some of these situations. Many of these solutions are given in Lohman (1972a) and Fetter (1988). For instance, a solution is available for leaky, semiconfined aquifers, either with no storage in the leaky confining layer (Fetter, 1988, p. 178) or with storage in the leaky confining layer (Fetter, 1988, p. 179). Solutions also are available to represent the effect of partial penetration of wells or the response of an unconfined aquifer with vertical flow (Fetter, 1988, p. 189-195).

The appropriateness of any given solution depends on the degree of similarity between the real system under study and the mathematical model. As noted previously, analytical solutions usually are restricted to simplified hydrogeologic conditions, whereas numerical simulation allows the representation and solution of many different and more complex system conceptualizations.

Analysis of Flow to a Well--Applying Analytical Solutions to Specific Problems

Assignments

*Study Fetter (1988), p. 170-199; Freeze and Cherry (1979), p. 343-349; or Todd (1980), p. 125-134.

*Work Exercise (4-2)--Comparison of drawdown near a pumping well in confined and unconfined aquifers using the Thiem and Dupuit-Thiem equations.

*Work (a) the example problem in Fetter (1988), p. 165, and (b) using the same data as in (a), determine the radial distance at which the drawdown would be 0.30 meters after 1 day of pumping.

*Work Exercise (4-3)--Analysis of a hypothetical aquifer test using the Theis solution.

In this subsection we apply the analytical solutions introduced in the previous section to some typical problems. Additional problems, some that require other analytical solutions, are available in Fetter (1988) at the end of chapter 6.

Exercise (4-2)--Comparison of Drawdown Near a Pumping Well in Confined and Unconfined Aquifers Using the Thiem and Dupuit-Thiem Equations

The purpose of this exercise is to (1) become more closely acquainted with the Thiem and Dupuit-Thiem equations by using them in numerical calculations and (2) contrast the response to stress (pumping) of a linear (confined) ground-water system and a nonlinear (unconfined) ground-water system. The concept of system linearity or nonlinearity refers to the relationship between system stresses, such as changes in pumping or recharge, and system response, as measured by changes in heads or drawdowns. For example, in a linear system, doubling the pumping rate of a given well in steady-state conditions, doubles the drawdown at every point in the neighborhood of that well. The response of a ground-water system to stress is inherently nonlinear if the geometry of the system changes in response to the stress. Common examples of changes in system geometry in response to stress are (1) changes in the elevation of the water table, (2) changes in the position of a freshwater-saltwater interface, and (3) changes in the length of a stream in hydraulic connection with the ground-water system.

The explicit purpose of the numerical calculations below is to compare the steady-state drawdown at r = 100 ft (radial distance from the pumping well) due to pumping from a completely penetrating well at three rates (pumping rates and other parameters given below) in (a) a confined aquifer and (b) an unconfined aquifer. Make a sketch of the two cases. Plot calculated drawdowns (2 curves, 3 values on each curve) on the graph paper provided (fig. 4-5).

Pumping rates: $Q_1 = 25{,}920\ ft^3/d$, $Q_2 = 51{,}840\ ft^3/d$, $Q_3 = 103{,}680\ ft^3/d$.

Confined Case (Thiem equation)

K = 50 ft/day

b (aquifer thickness) = 75 ft

r_e ("radius of influence")[1] = 10,000 ft (assume the head is constant at this distance)

r (radial distance from pumping well at which calculations of head and drawdown will be made) = 100 ft

$h_{initial}$ (head in aquifer before pumping begins) = h_e = 200 ft

Unconfined Case (Dupuit-Thiem equation)

K = 50 ft/day

r_e = 10,000 ft

r = 100 ft

$h_{initial}$ (saturated thickness of unconfined aquifer before pumping begins) = 75 ft

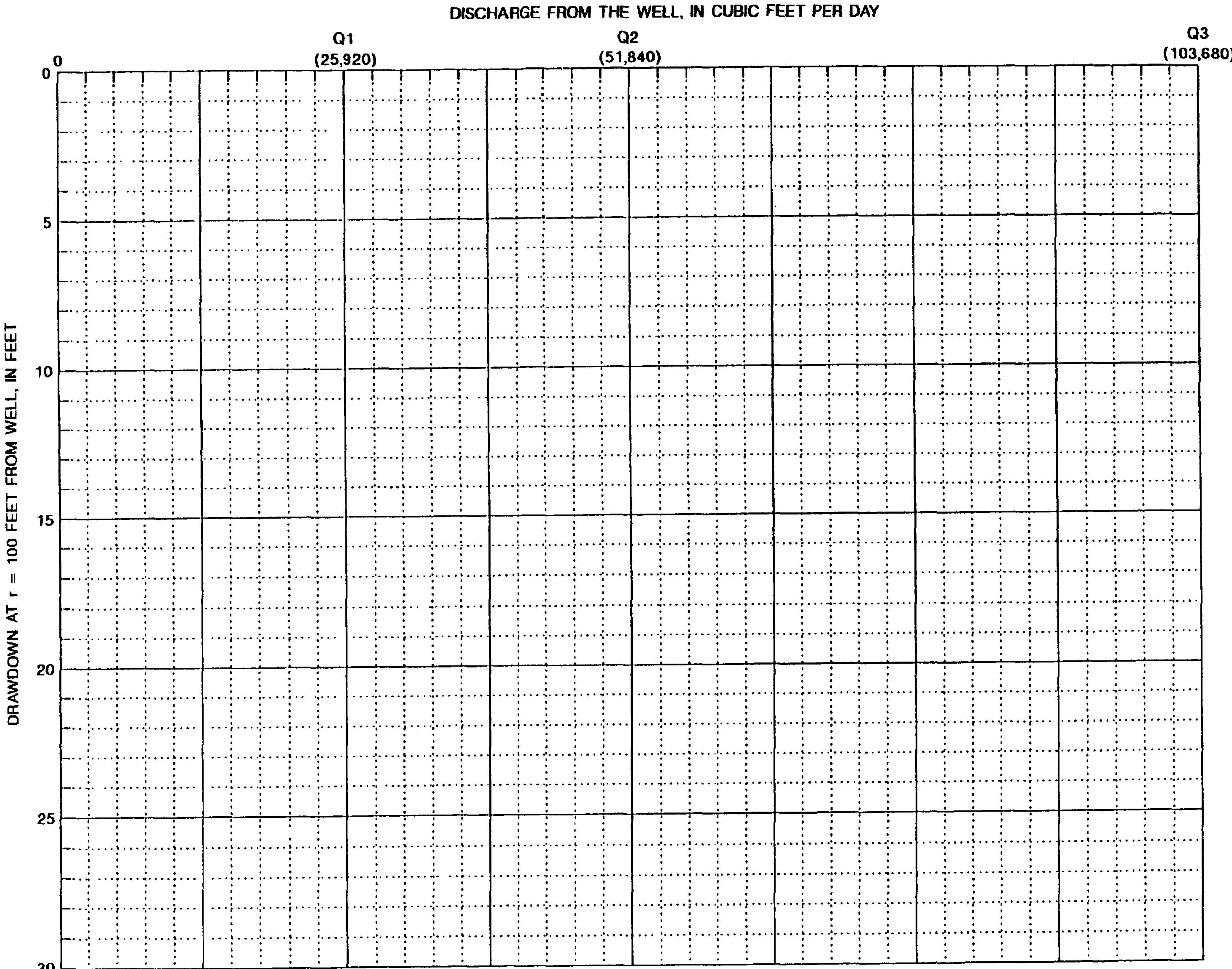
First, write the appropriate formula and solve algebraically for the unknown head before inserting numerical values. Then calculate drawdown.

(1) Suppose that the initial head in the confined aquifer is 500 ft instead of 200 ft. Would this change in initial head have any effect on the result of your calculation of drawdown?

(2) Write a careful description of the two curves plotted in figure 4-5.

---

[1] The phrase "radius of influence" of a pumping well is loosely defined, but implies a distance from the pumping well at which the head is constant in all radial directions or the drawdown in response to that particular stress either is so small that it cannot be measured or becomes impossible to distinguish from "background noise" in the aquifer. In calculations with the Thiem and Dupuit-Thiem equations, the "radius of influence" is the assumed or approximated distance from the pumping well at which head remains constant at the prepumping level.



Note: Q1, Q2 is well discharge, in cubic feet per day;
r is distance from well, in feet.

Figure 4-5.--Graph for plotting calculated drawdowns.

Exercise (4-3)--Analysis of a Hypothetical Aquifer Test Using the Theis Solution

The purpose of this exercise is to use the Theis solution to determine the aquifer properties T and S by curve-matching. Drawdowns at three wells spaced 200, 400, and 800 ft from a well pumping at a rate of 96,000 ft^3/d are listed in table 4-2 (from Lohman, 1972).

(1) Plot the aquifer-test data in table 4-2 on log-log paper in two ways--(a) drawdown (s) against time (t) using data from a single well (any one of the three observation wells) on figure 4-7, and (b) drawdown (s) against t/r^2 using data from all three observation wells on figure 4-8. In general, if data from more than one observation well are available, alternative (b) is preferable. Calculate t/r^2 by either taking the reciprocal of r^2/t in table 4-2 or performing the calculation directly from the data given.

(2) Overlay the Theis type curve (fig. 4-6) onto each plot of test data and determine a match point. Use the values obtained from the match point and equations 6-3 and 6-4 in Fetter (1988, p. 164) to determine the transmissivity (T) and storage coefficient (S) of the aquifer. To facilitate the calculations, equation 6-3 can be rearranged as

$$T = \frac{Q}{4\pi s} W(u)$$

and equation 6-4 as

$$S = 4Tu \cdot \frac{t}{r^2} .$$

Concept of Superposition and Its Application to Well-Hydraulic Problems

Assignments

*Study Fetter (1988), p. 201-204; Freeze and Cherry (1979), p. 327-332; or Todd (1980), p. 139-149.

*Study Note (4-5)--Application of superposition to well-hydraulic problems.

*Work Exercise (4-4)--Superposition of drawdowns caused by a pumping well on the pre-existing head distribution in an areal flow system.

Superposition is a concept that has many applications to ground-water hydrology as well as to other physical systems that are described by linear differential equations. We use superposition when we analyze (most) aquifer tests, perhaps without realizing this fact, and in the theory of images and image wells. Superposition also has applications to the numerical simulation of ground-water systems, a topic that is not discussed in this course.

Table 4-2.--Drawdown of water level in observation wells N-1, N-2, and N-3 at distance r from well being pumped at constant rate of 96,000 cubic feet per day

[From Lohman, 1972, table 6; r is distance from pumped well;

t is time since pumping began; ft = feet; min = minutes]

Time since pumping started, t (min)	N-1 (r = 200 ft)		N-2 (r = 400 ft)		N-3 (r = 800 ft)	
	Observed drawdown, s (ft)	r^2/t (ft²/day)	Observed drawdown, s (ft)	r^2/t (ft²/day)	Observed drawdown, s (ft)	r^2/t (ft²/day)
1.0	0.66	5.76×10^7	0.16	2.3×10^8	0.0046	9.23×10^8
1.5	.87	3.84×10^7	.27	1.53×10^8	.02	6.15×10^8
2.0	.99	2.88×10^7	.38	1.15×10^8	.04	4.6×10^8
2.5	1.11	2.30×10^7	.46	9.2×10^7	.07	3.7×10^8
3.0	1.21	1.92×10^7	.53	7.65×10^7	.09	3.1×10^8
4	1.36	1.44×10^7	.67	5.75×10^7	.16	2.3×10^8
5	1.49	1.15×10^7	.77	4.6×10^7	.22	1.85×10^8
6	1.59	9.6×10^6	.87	3.82×10^7	.27	1.54×10^8
8	1.75	7.2×10^6	.99	2.87×10^7	.37	1.15×10^8
10	1.86	5.76×10^6	1.12	2.3×10^7	.46	9.23×10^7
12	1.97	4.80×10^6	1.21	1.92×10^7	.53	7.7×10^7
14	2.08	4.1×10^6	1.26	1.75×10^7	.59	6.6×10^7
18	2.20	3.2×10^6	1.43	1.28×10^7	.72	5.1×10^7
24	2.36	2.4×10^6	1.58	9.6×10^6	.87	3.84×10^7
30	2.49	1.92×10^6	1.70	7.65×10^6	.95	3.1×10^7
40	2.65	1.44×10^6	1.88	5.75×10^6	1.12	2.3×10^7
50	2.78	1.15×10^6	2.00	4.6×10^6	1.23	1.85×10^7
60	2.88	9.6×10^5	2.11	3.82×10^6	1.32	1.54×10^7
80	3.04	7.2×10^5	2.24	2.87×10^6	1.49	1.15×10^7
100	3.16	5.76×10^5	2.38	2.3×10^6	1.62	9.23×10^6
120	3.28	4.8×10^5	2.49	1.92×10^6	1.70	7.7×10^6
150	3.42	3.84×10^5	2.62	1.53×10^6	1.83	6.15×10^6
180	3.51	3.2×10^5	2.72	1.28×10^6	1.94	5.1×10^6
210	3.61	2.74×10^5	2.81	1.1×10^6	2.03	4.4×10^6
240	3.67	2.5×10^5	2.88	9.6×10^5	2.11	3.84×10^6

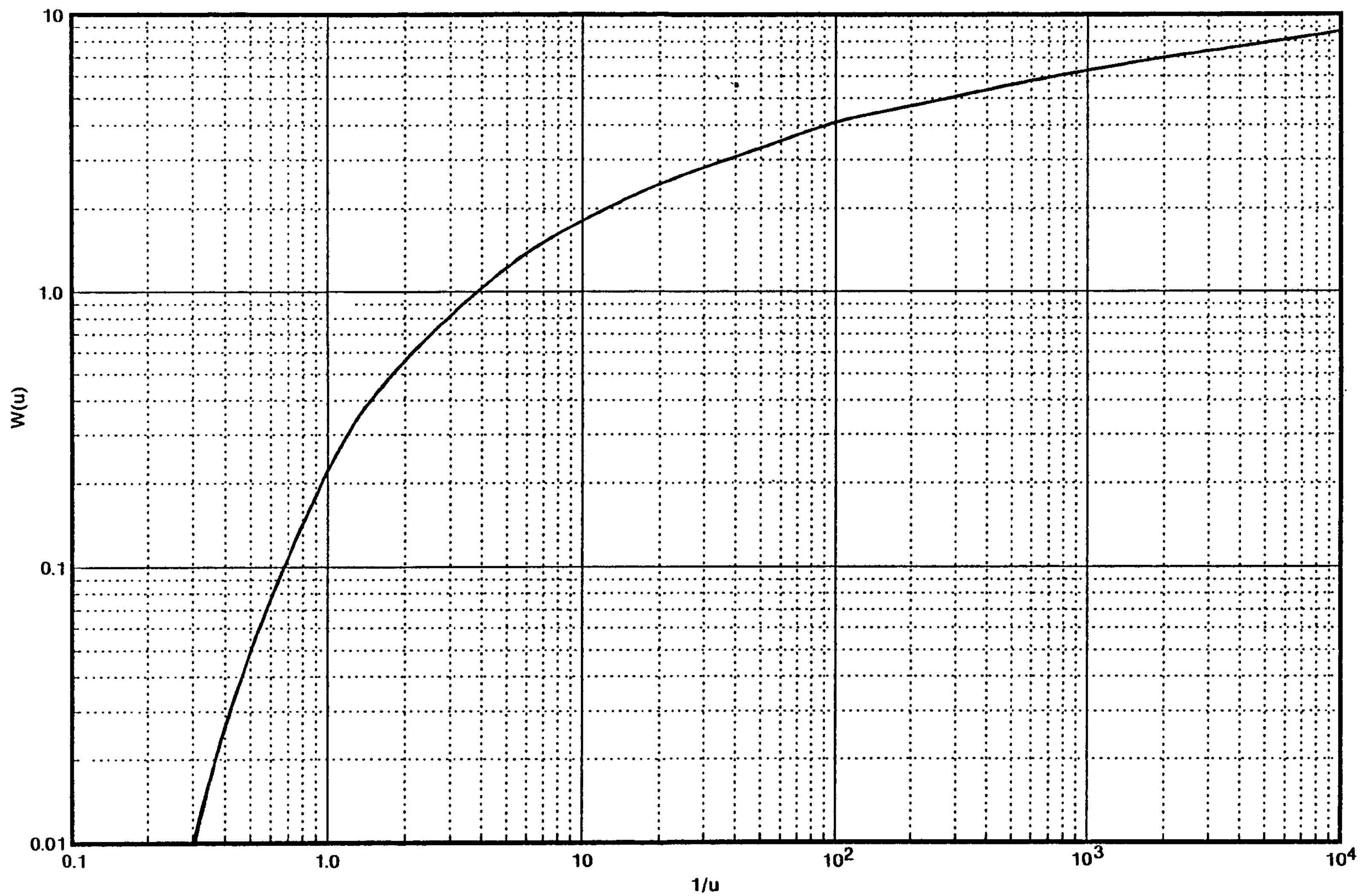


Figure 4-6.--Theis type-curve plot of parameters W(u) against 1/u.

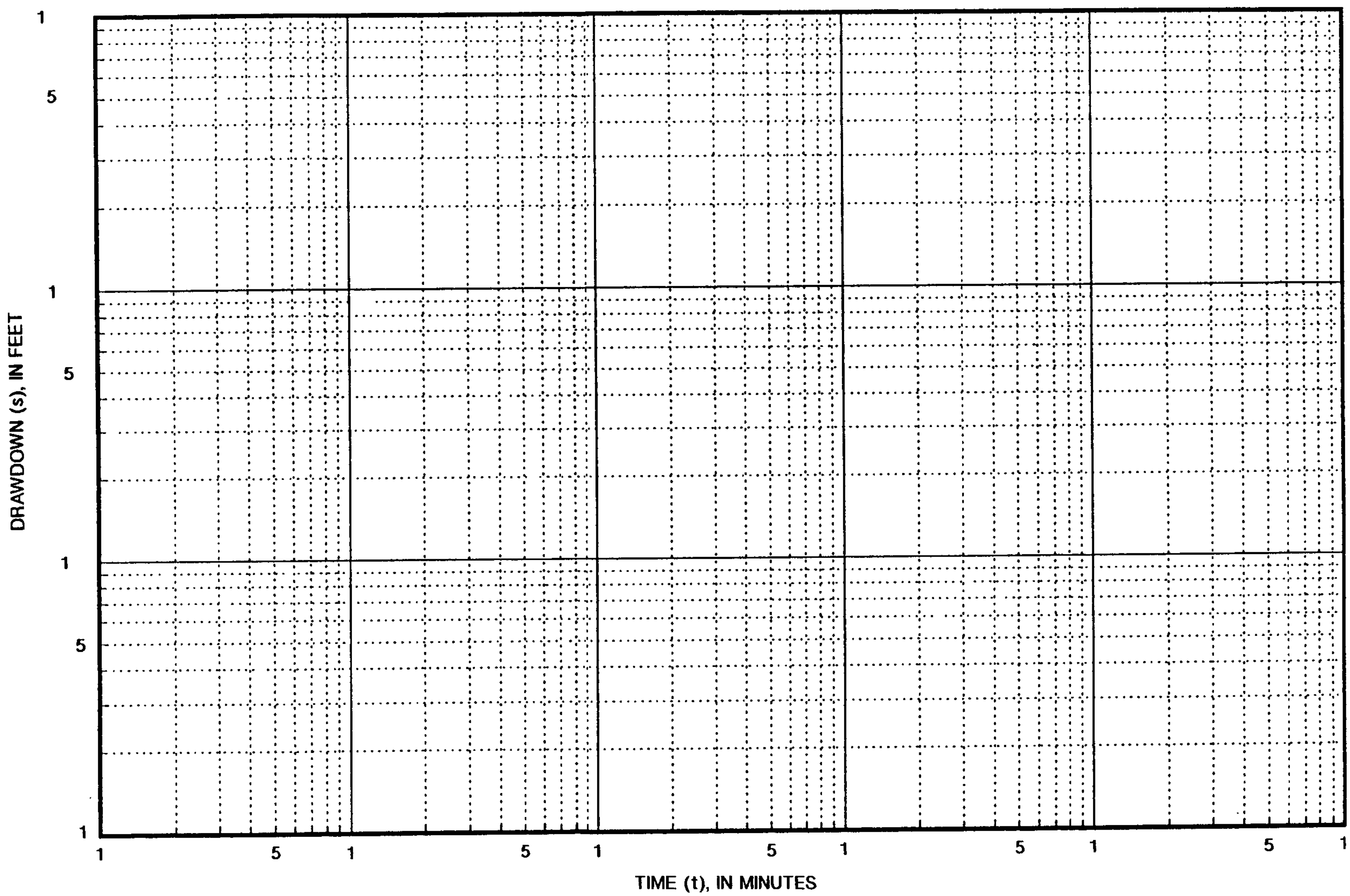


Figure 4-7.--Graph for plotting aquifer-test data from a single observation well in a Theis analysis (drawdown (s) against time (t)).

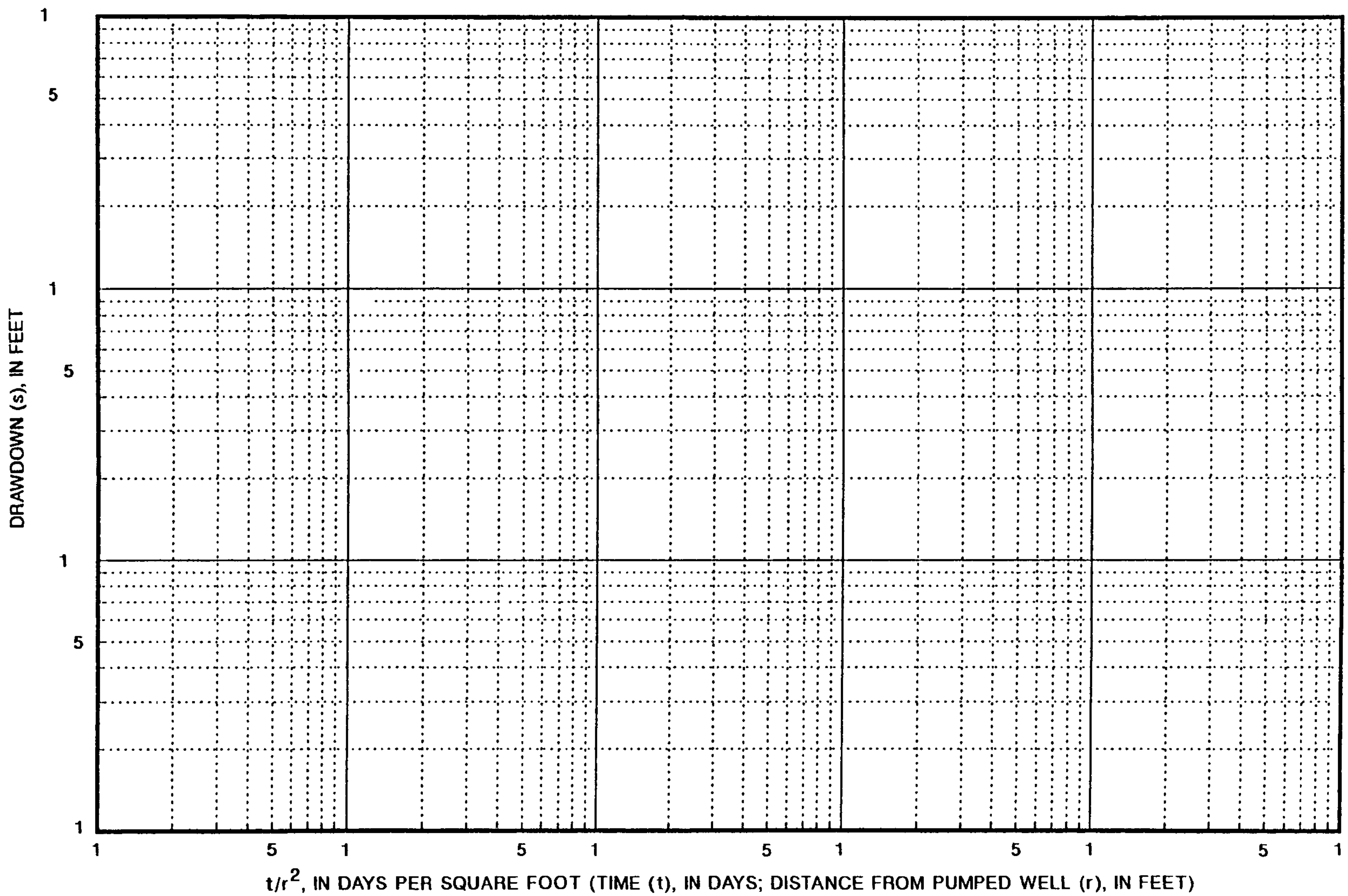


Figure 4-8.--Graph for plotting aquifer-test data from more than one observation well in a Theis analysis (drawdown (s) against t/rr).

Note (4-5).--Application of Superposition to Well-Hydraulic Problems

To simplify the analysis of ground-water flow to wells we have assumed until now that heads and flows around the well axis are radially symmetrical. However, heads and flows are not always symmetrical around the axis of a well. If a regional gradient exists, the head upgradient from the well is higher than the head downgradient from the well. However, the change in head (the drawdown) and the change in flow due to pumping still are symmetrical about the well.

Using the theory of superposition (Reilly and others, 1987), we can analyze most well-hydraulic problems in terms of drawdowns and changes in flow. The theory of superposition states that, for linear systems, the solution to a problem involving multiple inputs (or stresses) is equal to the sum of the solutions for each individual input or stress. A more formal definition of superposition is that if Y_1 and Y_2 are two solutions to a linear differential equation with linear boundary conditions, then $C_1Y_1 + C_2Y_2$ is also a solution, where C_1 and C_2 are constants.

Superposition allows us to avoid analyzing the actual heads and to analyze only the drawdown. The Theis solution given by Fetter (1988, p. 164, equation 6-3) is stated in terms of drawdown (h_0-h) as

$$h_0 - h = \frac{Q}{4\pi T} W(u).$$

This equation is a solution to the governing differential equation which is given in terms of head by Fetter (1988, p. 162, equation 6-1) as

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}.$$

The principle of superposition allows this equation to be written in terms of the changes in head (or drawdowns) that occur in the system as

$$\frac{\partial^2 (h_0 - h)}{\partial r^2} + \frac{1}{r}\frac{\partial (h_0 - h)}{\partial r} = \frac{S}{T}\frac{\partial (h_0 - h)}{\partial t},$$

or

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

where s is the drawdown. This formulation greatly simplifies the mathematical solution because the initial conditions[1] are a constant zero drawdown everywhere at the start of pumping, and the drawdown is radially symmetrical.

A more complete explanation of superposition and a set of problems is available in Reilly, Franke, and Bennett (1987).

Exercise (4-4)--Superposition of Drawdowns Caused by a Pumping Well on the Pre-Existing Head Distribution in an Areal Flow System

In Note 4-3, we derived the Thiem equation in terms of absolute head. In this form of the Thiem equation, the head at some radial distance from a pumping well must be the same in all directions. Through the use of superposition (Note 4-5), the Thiem equation can be applied to more general field situations in which drawdowns are radially symmetric even if absolute heads are not.

The Thiem equation from Note 4-3 is

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln (r_2/r_1).$$

We can represent the head at points 1 and 2 by

$$h_1 = h_0 - s_1, \text{ and}$$

$$h_2 = h_0 - s_2,$$

where h_0 is the original head before onset of pumping, and s is the drawdown. Substituting these equations into the Thiem equation gives

$$T = \frac{Q}{2\pi(s_1 - s_2)} \ln (r_2/r_1).$$

Assuming that the drawdown s_2 is negligible at some distance, r_e, from the pumping well and rearranging gives:

$$s_1 = \frac{Q}{2\pi T} \ln (r_e/r_1).$$

This form of the Thiem equation gives the drawdown, s_1, at any radial distance, r_1, from the pumping well.

---

[1] In analyses of ground-water systems, "initial conditions" means specifying the head distribution throughout the system at some particular time. These specified heads can be considered to be reference heads; calculated changes in head through time are relative to these given heads, and the time represented by these reference heads is the reference time. For further discussion of initial conditions see Franke, Reilly, and Bennett (1987).

A uniform head (potential) distribution in a hypothetical confined aquifer of uniform transmissivity, T, is shown in figure 4-9. Determine the future potential distribution under steady-state conditions in response to a pumping well centered in the figure at the square. Assume that there is no drawdown at a distance, r_e, of 5,000 ft, for a well pumping at 9,090 ft^3/d. The transmissivity of the aquifer is 1,000 ft^2/d.

We will calculate drawdowns and the predicted new heads at locations marked with circles and labeled with letters in figure 4-9. Perform the calculations and contour the new head distribution using the following sequence of steps:

1. Calculate drawdowns--Use table 4-3 to calculate the drawdowns at various distances from the pumping well. Note that the locations of all 30 reference points are defined by only six radial distances, r, from the pumping well.

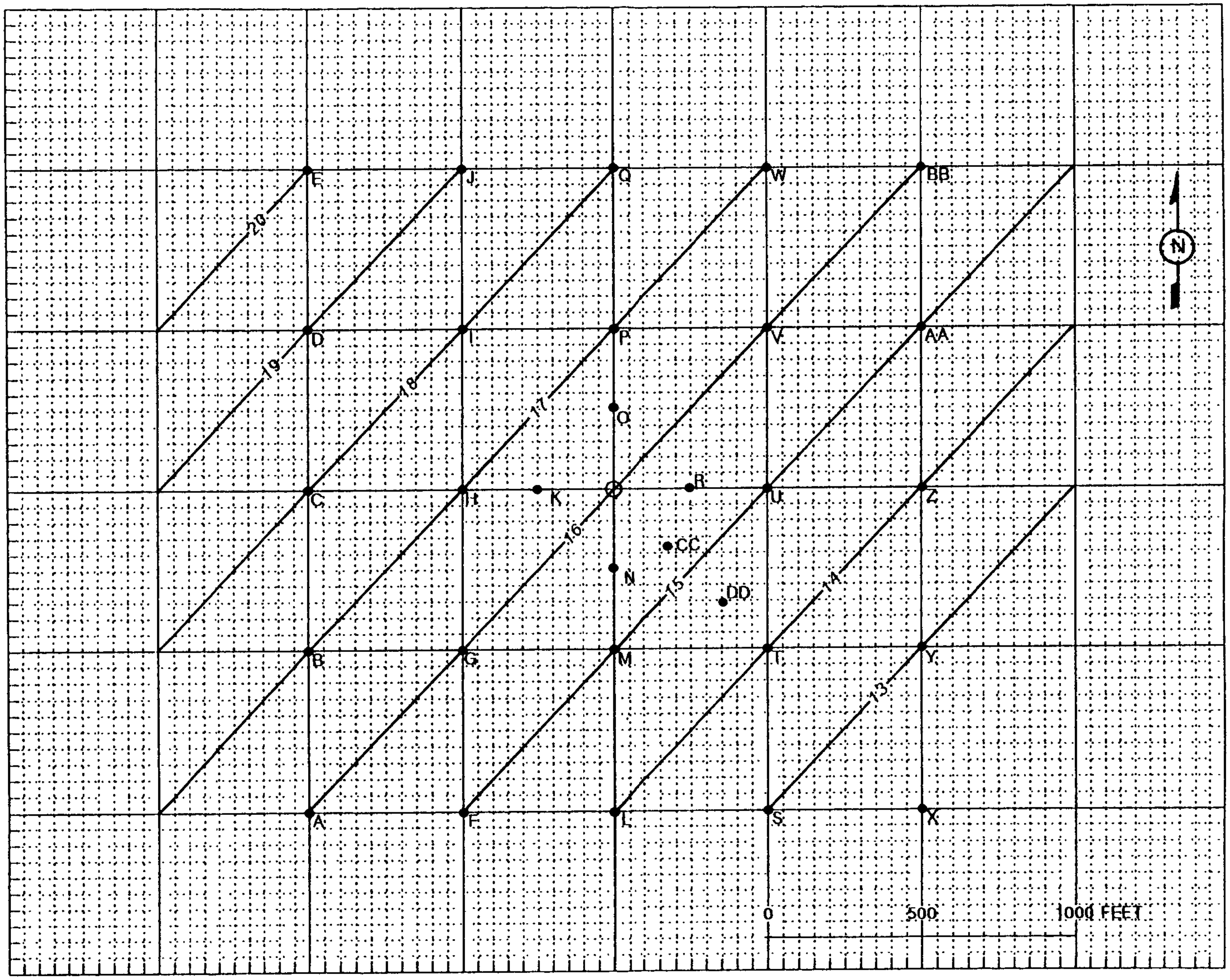
2. Calculate absolute heads--Use table 4-4 to calculate the new head at each reference point. Determine the initial prepumping head from the contour lines given in figure 4-9. Determine the distance of the observation point from the pumping well, and transfer the appropriate drawdown from table 4-3. Finally, subtract the drawdown from the initial head for each reference point.

3. Contour new potentiometric surface--Plot the new heads on figure 4-10 and contour, using a 1-ft contour interval.

As an aid in contouring the new potentiometric surface, consider the original potentiometric surface in figure 4-9 and draw a dashed line on figure 4-9 that is perpendicular to the head contour lines and passes through the location of the pumping well. Because these initial head contour lines are parallel straight lines, the new potentiometric surface resulting from steady pumping of the well will be symmetrical about the dashed line. Draw a dashed line at the same position on figure 4-10 and observe that the potentiometric surface being contoured is symmetric about this line. This new potentiometric surface shows the effect of the discharging well.

(1) Based on available head data, estimate the position of the ground-water divide on the dashed line in figure 4-10. Starting at this point on the divide, sketch two upgradient streamlines, one on each side of the well, making the assumption that these streamlines are perpendicular to the existing head contour lines. Sketch two or three additional streamlines between the first two streamlines and the well. What is the significance of the first two streamlines? What is the area upgradient from those first two streamlines called?

(2) The drawdown at reference point T is 2.84 ft. Is the direction of flow at T toward or away from the discharging well? Explain why the water at this point is flowing in a direction away from the discharging well despite significant drawdowns at wells X, T, S, and Y.



EXPLANATION

—14— CONTOUR OF EQUAL HEAD, IN FEET

○ LOCATION OF PUMPING WELL

•D REFERENCE POINT WITH IDENTIFICATION LETTER

Figure 4-9.--Prepumping potentiometric surface and location of pumping well and reference points.

Table 4-8.--Format for calculation of drawdowns at specified distances from the pumping well

[r_e is distance from pumping well at which drawdown is negligible; r_1 is distance from pumping well at which drawdown equals s_1; $\ln$ is natural logarithm; Q is pumping rate of well; T is transmissivity of aquifer]

Preliminary calculation: $\frac{-Q}{2\pi T}$ = constant =

r_1 (FEET)	$\ln(r_e/r_1)$	s_1 (FEET) = $-\frac{Q}{2\pi T}\ln(r_e/r_1)$
250		
500		
707		
1000		
1118		
1414		

Table 4-4.--Format for calculation of absolute heads at specified reference points

WELL IDENTIFICATION LETTER	INITIAL PREPUMPING HEAD (FEET)	DISTANCE FROM WELL (r), IN FEET	DRAWDOWN DUE TO PUMPING (FEET)	HEAD = INITIAL HEAD-DRAWDOWN, IN FEET
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				
K				
L				
M				
N				
O				
P				
Q				
R				
S				
T				
U				
V				
W				
X				
Y				
Z				
AA				
BB				
CC				
DD				

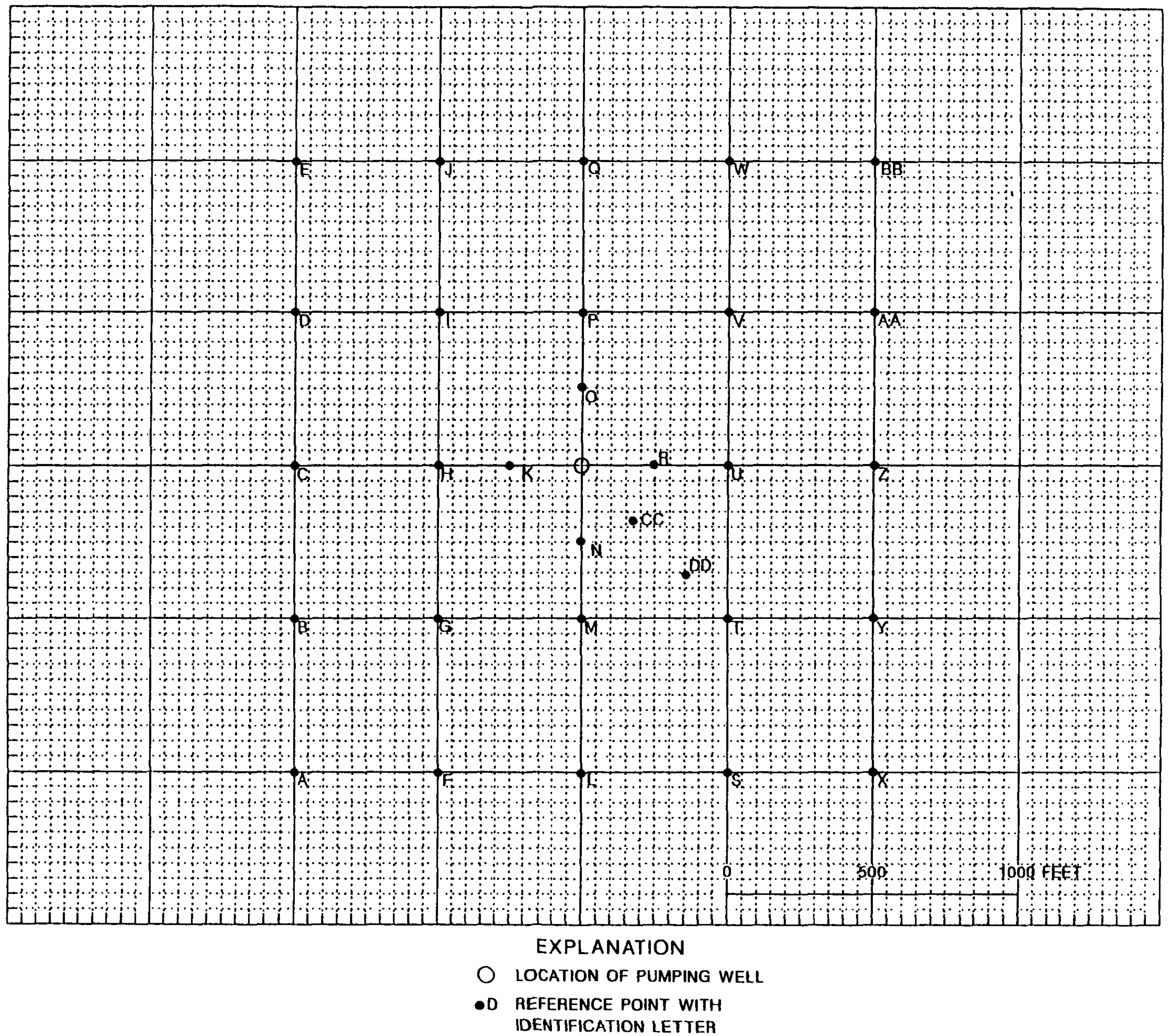


Figure 4-10.--Location map for plotting and contouring head distribution resulting from pumping.

Aquifer Tests

Assignments

*Study Fetter (1988), p. 204-209; Freeze and Cherry (1979), p. 335-343, 349-350; or Todd (1980), p. 45-46, 70-78.

*Study Note (4-6)--Aquifer tests.

One of the main activities of ground-water hydrologists is to estimate physically reasonable values of aquifer parameters for different parts of the ground-water system under study. The most powerful and direct field method for obtaining aquifer parameters is a carefully designed, executed, and analyzed aquifer test. Unfortunately, aquifer tests are labor- and time-intensive. Often, the most important decision in connection with an aquifer test is whether or not to perform one--in other words, whether the value of the test data equals the cost of obtaining those data. This generally is a difficult question to answer.

Note (4-6).--Aquifer Tests

An aquifer test is a controlled field experiment that is designed to determine the hydraulic properties of an aquifer and (or) associated confining beds. The most common type of aquifer test involves pumping a well at a constant rate to stress the aquifer, monitoring the drawdown response of the aquifer, and analyzing these data. The analysis usually assumes radial symmetry and uses either an analytical solution to the conceptually appropriate mathematical model or a mathematical-numerical model solved by computer.

Stallman (1971, p. 1-3) discusses the philosophy and general procedure of aquifer tests succinctly. He outlines the general procedure in terms of three phases--test design, field observations, and data analysis. It is critically important that the test be designed with an initial conceptualization of the system and a proposed method of analysis. The conceptualization of the system may change as analysis proceeds; then different methods of analysis may be required.

As noted previously, many analytical solutions to well-hydraulic problems exist. For example, Reed (1980) gives analytical solutions and type curves for 11 different cases of flow to wells in confined aquifers. However, analytical solutions tend to describe the response of simplified homogeneous systems. Therefore, numerical simulation sometimes is required to estimate the hydraulic properties of the aquifer being tested. Simulation usually is used in a trial-and-error manner, changing aquifer and confining bed coefficients in a systematic and physically reasonable way based on previous knowledge or simplified analyses (for example, analyses using analytical solutions), until an acceptable match between the observed response of the aquifer and the simulated response is achieved.

SECTION (5)--GROUND-WATER CONTAMINATION

The goal of this section of the course is to introduce the physical mechanisms of solute movement in ground water. Further treatment of the vast and rapidly developing area of science and technology related to ground-water contamination can be found in the extensive literature that is available or in additional training courses.

Background and Field Procedures Related to Ground-Water Contamination

Assignments

*Study Fetter (1988), p. 367-389, 406-442; Freeze and Cherry (1979), p. 384-457; or Todd (1980), p. 344-346.

The depth of topical coverage in this section of the course will depend primarily on the time available and the interests of the instructors and participants. A useful and readable discussion on the conceptualization and organization of a field study involving solute transport, along with a pertinent bibliography, is provided by Reilly and others (1987).

Physical Mechanisms of Solute Transport in Ground Water

Assignments

*Study Fetter (1988), p. 389-405.

*Study Note (5-1)--Physical mechanisms of solute transport in ground water

*Work Exercise (5-1)--Ground-water travel times in the flow system beneath a partially penetrating impermeable wall

*Work Exercise (5-2)--Advective movement and travel times in a hypothetical stream-aquifer system

*Study Note (5-2)--Analytical solutions for analysis of solute transport in ground water

*Work Exercise (5-3)--Application of the one-dimensional advective-dispersive equation

The background for this section is provided in Note (5-1), which is an introductory discussion of the basic physical mechanisms of solute movement--advection and dispersion. Exercises (5-1) and (5-2) consider only advective movement of ground water and involve calculation of travel times by using the average linear velocity (Darcy velocity divided by porosity). In Exercise (5-1) travel times are calculated in a vertical cross-section of a simple flow system, and in Exercise (5-2) travel times are calculated in plan view.

Comments on the field application of analytical solutions to the advective-dispersive differential equation are provided in Note (5-2), and Exercise (5-3) involves numerical calculations with one of the simplest analytical solutions.

Note (5-1).--Physical Mechanisms of Solute Transport in Ground Water[1]

The following section on physical mechanisms of solute transport in ground-water systems (1) defines and describes the two physical mechanisms advection and dispersion, (2) emphasizes the interdependence of these mechanisms and the implications of the scale of analysis in transport studies, and (3) addresses the primary goal of the study of physical mechanisms--to define a working approximation of the three-dimensional ground-water-flow velocity field affecting the contaminant plume, by building upon the information and knowledge gained in the hydraulic analysis and description of solute distribution.

Advection is the process by which solutes are transported by the bulk motion of the flowing ground water (Freeze and Cherry, 1979, p. 75). The bulk motion of the flowing ground water is characterized by the average linear velocity (v), which is defined as

$$v = -\frac{K}{n}\frac{dh}{d\ell}$$

where

K = hydraulic conductivity (L/T),
n = porosity (dimensionless),
h = hydraulic head (L), and
ℓ = distance along a flowline (L).

The Darcian velocities developed by using a flow model differ from the actual velocities required for transport analysis in that the average linear velocity (v) is the Darcian velocity (q) divided by porosity (n); that is,

$$v = q/n.$$

Thus, a new, spatially varying parameter, the porosity (n) of the porous material in the neighborhood of the point at which velocity is calculated, is introduced. Errors in estimating the magnitude and distribution of porosity produce proportional errors in estimates of actual ground-water velocity.

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[1] This note on the physical mechanisms of solute transport in ground-water systems is from Reilly and others (1987, p. 21-29).

A more subtle difference between the velocity field developed by using a flow model designed for basic hydraulic analysis and the velocity field required for transport analysis is the scale at which the physical processes are considered. In the analysis of ground-water flow, the flow field usually is studied at a scale that is much larger in area than the area of a contaminant plume, because an accurate definition of boundary conditions is required to achieve a physically reasonable simulation. At this regional scale, the properties of the porous medium and variations in velocity are averaged. In the analysis of the velocity field for transport analysis, however, a more detailed scale is required. This finer scale permits the representation of local variations in hydraulic conductivity resulting from the heterogeneous nature of the porous media to be represented if possible. It also permits greater resolution in describing changes in velocity (both magnitude and direction) due to the three-dimensional movement of the ground water in response to local conditions.

Regardless of the degree of detail that is included in the representation of the flow field used to calculate the ground-water velocities, however, variations between actual and calculated velocities remain that cannot be accounted for explicitly. In any calculation of advective transport, whether by numerical model or by using an analytical solution, we assume that the velocity is uniform or varies in a simple way over specified regions of the flow field. For example, suppose a uniform flow in the x direction is simulated using the array of model nodes shown in figure 5-1. In calculations of solute transport using numerical models, velocity in the x direction is assumed to be uniform or to vary in a simple way (such as bilinear interpolation) in both magnitude and direction over the rectangular region R, which extends between adjacent nodes in the x direction. This uniformity is vertical as well as areal--that is, within the area R, velocity is assumed to be constant over the vertical depth interval represented by the simulation. By contrast, the actual ground-water velocity in the block of aquifer represented by R would exhibit different spatial variations depending on the scale at which the velocity is considered.

At the microscopic (pore) scale, velocity varies from a maximum along the centerline of each pore to zero along the pore walls, as shown in figure 5-2(A); both the centerline velocity and the velocity distribution differ in pores of different size. In addition, flow direction changes as the fluid moves through the tortuous paths of the interconnecting pore structure, as shown in figure 5-2(B).

At a larger (macroscopic) scale, local heterogeneity in the aquifer causes both the magnitude and direction of velocity to vary as the flow concentrates along zones of greater permeability or diverges around pockets of lesser permeability. In this discussion, the term "macroscopic heterogeneity" is used to suggest variations in features large enough to be readily discernible in surface exposures or test wells, but too small to map (or to represent in a mathematical model) at the scale at which we are working. For example, in a typical problem involving transport away from a landfill or waste lagoon, macroscopic heterogeneities might range from the size of a baseball to the size of a building.

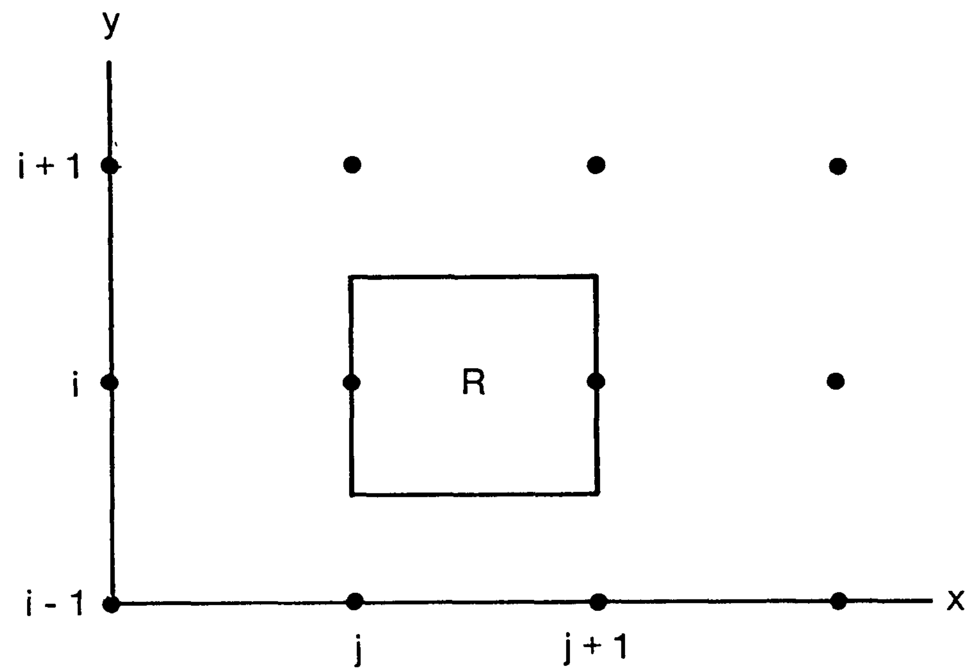


Figure 5-1.--Array of model nodes with region R between two representative nodes (i,j and i,j+1).

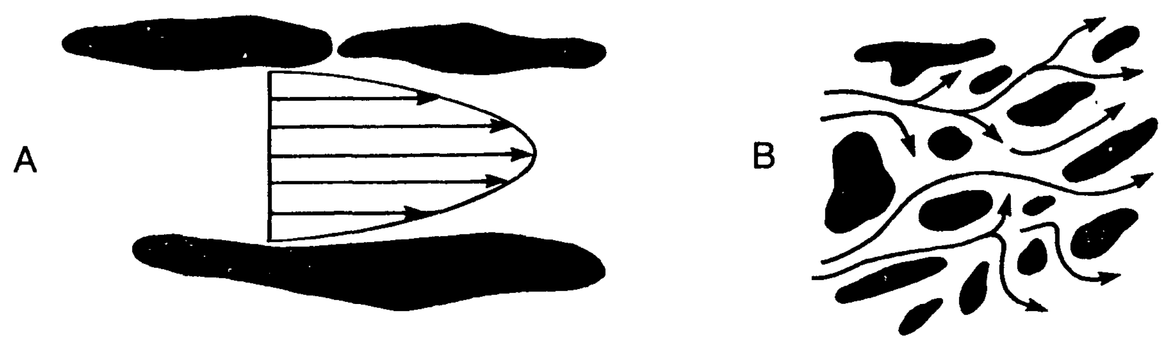


Figure 5-2.--(A) Approximate fluid velocity distribution in a single pore, and (B) tortuous paths of fluid movement in an unconsolidated porous medium.

Figure 5-3, which shows some results of laboratory tracer experiments in heterogeneous media by Skibitzke and Robinson (1963), illustrates the effects of macroscopic heterogeneity. The net effect is to increase the spreading of the solute in the system. This effect tends to increase progressively with the scale of the heterogeneity. At a still larger scale, we can envision heterogeneities that could be mapped at the scale at which we are working, and which could be taken into account in our calculations of advective transport, but which simply have not been recognized in the field or accounted for in simulation. Mercado (1967; 1984) showed the results of this effect in an analysis of the spreading of injected water that was caused by stratified layers of different permeabilities.

The velocity variations described for these three scales share certain characteristics:

(1) they may occur both areally and vertically over the region R (fig. 5-1);

(2) they influence the distribution of ions or tracers moving through the system; and

(3) they are not represented in calculations of advective solute movement through the region R that are made using the uniform model velocity.

Using the velocity from the model, a tracer front introduced at the left side of region R would be predicted to traverse R as a sharp front moving with the average linear velocity of the water. In reality, however, a tracer front becomes progressively more irregular and diffuse as it moves through a porous medium. If we consider a vertical plane through the aquifer at the left edge of region R, the actual velocity varies in both magnitude and direction from one point to another; the same is true in the flow direction. Thus each tracer particle enters R at a velocity that generally is different from that of its neighbors, and each particle experiences a different sequence of velocities as it crosses R from left to right. Instead of a sharp front of advancing tracer as shown in figure 5-4(A), we see an irregular advance as in 5-4(B), with the forward part of the tracer distribution becoming broader and more diffuse with time. The pore-scale or microscopic velocity variations contribute only slightly to this overall dispersion; macroscopic variations contribute more significantly, whereas "mappable" variations generally have the largest effect.

If it were possible to generate a model or a computation that could account for all of the variations in velocity in natural aquifers, dispersive transport would not have to be considered (except for molecular diffusion); sufficiently detailed calculations of advective transport theoretically could duplicate the irregular tracer advance observed in the field. In practice, however, such calculations are impossible. Field data at the macroscopic scale never are available in sufficient detail, information at the "mappable" scale rarely is complete, and descriptions of microscopic scale variations are impossible except in a statistical sense. Even if complete data were available, however, an unreasonable computational effort would be required to define completely the natural velocity variations in an aquifer.

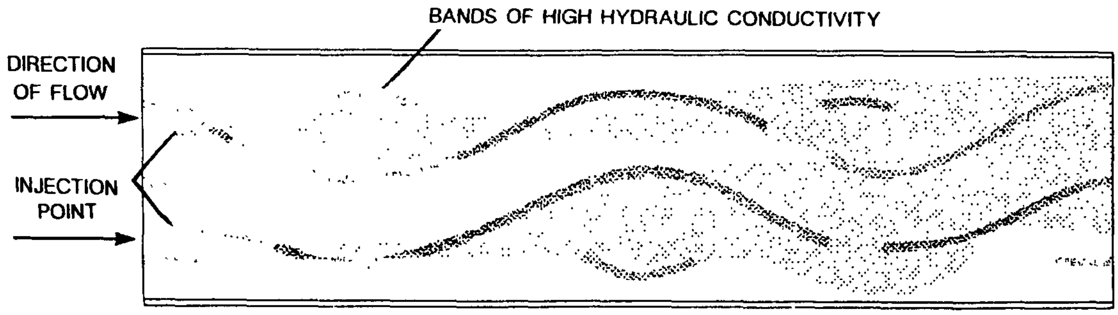


Figure 5-3.--Results of a laboratory experiment to determine the effects of macroscopic heterogeneity on a tracer. (Modified from Skibitzke and Robinson, 1963.)

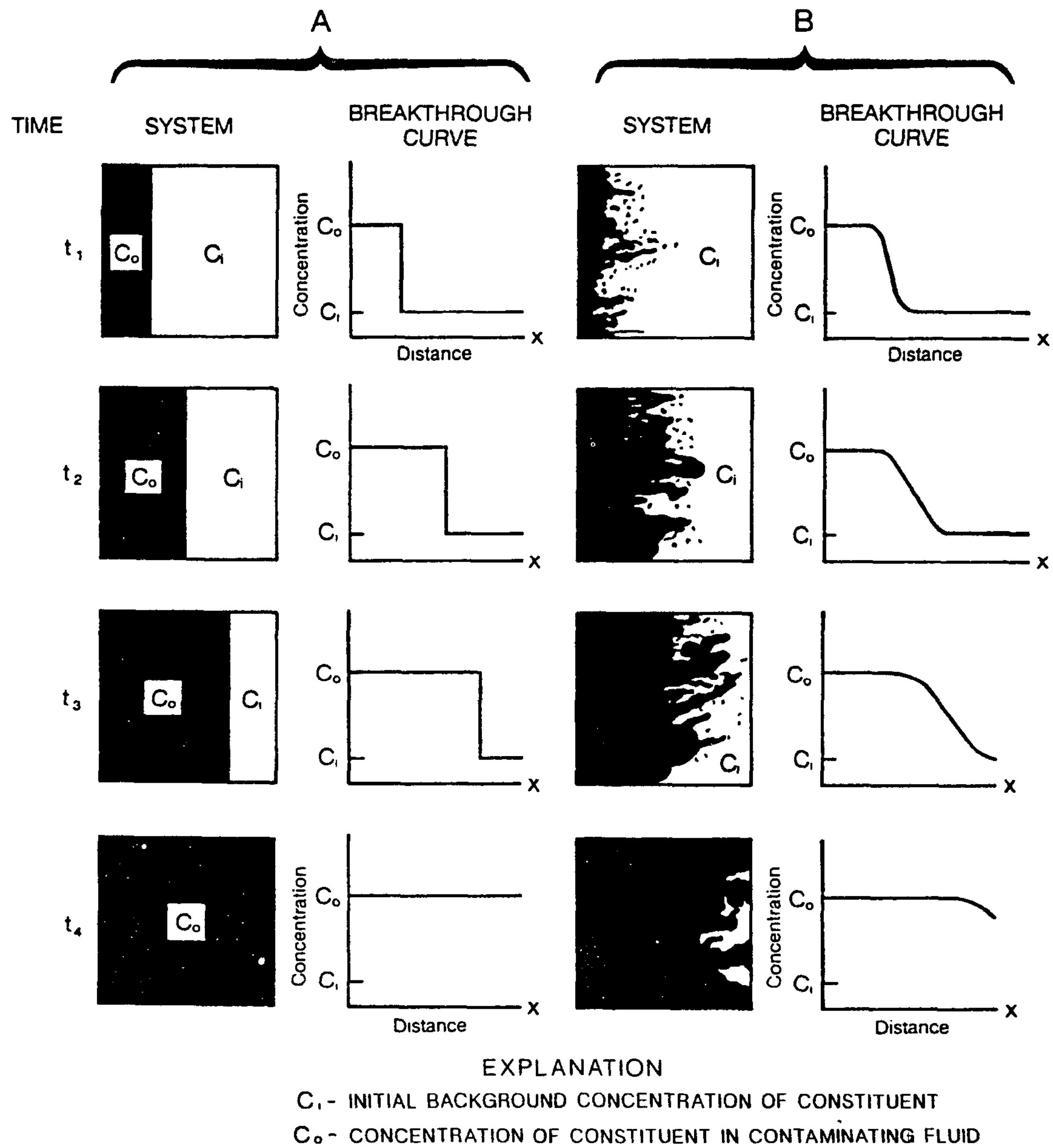


Figure 5-4.--Advance of a tracer for (A) a sharp front and (B) an irregular front.

The more closely we represent the actual permeability distribution of an aquifer, the more closely our calculations of advective transport match reality; the finer the scale of simulation, the greater is the opportunity to match natural permeability variations. In most situations, however, when both data collection and computational capacity have been extended to their practical limits, calculations of advective transport fail to match field observation; therefore, we must find a tractable method of adjusting or correcting such calculations.

Historically, the effort to develop such a method of correction followed the diffusion model. Diffusion had been analyzed successfully as a process of random particle movement which, in the presence of concentration change, results in a net transport proportional to the concentration gradient in the direction of decreasing concentration. In the case of a moving fluid, the random movement ascribed to diffusion was viewed as superimposed on the motion caused by the fluid velocity. Thus, the net movement of any solute particle could be regarded as the vector sum of an advective component and a random diffusive component.

By analogy, it was assumed that solute transport through porous media could be viewed in the same way--as the sum of an advective component in which solutes move with the average linear velocity of the fluid, and a random "dispersive" component superimposed on the advective motion (Saffman, 1959). In effect, dispersion was seen as the net transport with respect to a point moving with the average linear velocity of the fluid. Because the dispersive motion of solute particles was assumed to be random, the flux was taken to be proportional to the concentration gradient.

While many difficulties have been perceived with the concentration-gradient approach, no satisfactory alternative has yet been found. Currently, we know that some method is required to adjust and correct the results of advective-transport calculations. The method commonly employed is to postulate an additional transport that is proportional to the concentration gradient in the direction of decreasing concentration; however, the coefficient of proportionality is treated as a function of the average flow velocity.

This approach can be derived or justified mathematically if assumptions similar to those used in the analysis of molecular diffusion in moving liquids are made--that is, if the actual velocity of particles through the system can be described as the sum of two components: (1) the average velocity used in advective calculation, and (2) a random deviation from the average velocity. To the extent that scale variations in velocity represent random deviation from the velocity used in advective transport calculation, and to the extent that these variations occur on a scale which is significantly smaller than the size of the region used for advective calculation (for example, region R of figure 5-1), dispersion theory may describe adequately the differences between advective calculation and field observation. However, if the velocity variations are not random, or if they are large relative to the region used for advective calculation, the suitability of the dispersion approach is questionable. Moreover, even when this approach appears to be justified, determination of the necessary coefficients usually must be approached empirically (for example, through model calibration). The range of validity of the quantities determined in this manner is uncertain.

Variations in velocity most often are caused by variations in the permeability and effective porosity of the porous medium on all three of the relevant scales. In theory, therefore, it should be possible to describe the dispersive-transport process through statistical analysis of variations in aquifer permeability. Gelhar and Axness (1983) have attempted to do this by using a stochastic analysis of permeability variation at the macroscopic scale to generate dispersivity values. The utility of this approach currently is limited by the difficulty in obtaining the necessary data on the statistics of permeability variation. However, Gelhar has demonstrated that in the limit, as distances of transport become large, a concentration-gradient approach is justified on theoretical grounds.

Because dispersive transport actually represents an aggregate of the deviations of actual particle velocities from the velocity used in advective-transport calculation, coefficients of dispersion must vary as the overall velocity of flow varies in order to create agreement between computed and observed results. As overall flow velocities in the system increase, the magnitude of velocity deviations from the average velocity used in advective-transport calculation must increase as well; therefore, dispersive transport is dependent on average flow velocity.

The description of dispersion in terms of velocity variation implies that problem scale must be a factor in any calculation of dispersive effects. As the size of the region used in advective-transport calculation (for example, region R in figure 5-1) increases, more heterogeneities are included in that region. If a small region of calculation is chosen (for example, corresponding to the size of a laboratory column), the dominant heterogeneities within it are those at the pore scale; dispersive effects and dispersion coefficients are correspondingly small. As the region R becomes larger, macroscopic and ultimately "mappable" heterogeneities dominate. Thus, as larger regions of calculation are taken, the dispersive effects tend to increase in magnitude, the determination of the coefficients required for their description becomes more difficult, and the applicability of the conventional concentration-gradient approach becomes questionable. In general, the scale at which advective-transport calculations are made (for example, the scale of discretization in a model analysis) ideally reflects the existing level of knowledge of heterogeneities in the system. The scale is chosen to be fine enough so that the effects of all recognized heterogeneities can be accounted for by advective transport, yet coarse enough so that individual regions of advective-transport calculation are large with respect to their unknown internal heterogeneities, which must be described by dispersive terms. Thus, in any calculation of the physical mechanisms of solute transport, advection and dispersion are interrelated, and the appropriate values of dispersion depend on the scale at which the advective field is quantified.

Exercise (5-1)--Ground-Water Travel Times in the Flow System Beneath a Partially Penetrating Impermeable Wall

Our goal in this exercise is to estimate travel times in the ground-water system beneath the partially penetrating impermeable wall (fig. 5-5) for which we developed a flow net in Exercise (3-2). First, we must make an assumption concerning the movement of the "tracer water" through the system.

Assume that at some instant of time (t=0, or reference time in this problem), water of different quality enters the flow field at the upper left inflow boundary and moves through the system. We assume that the "new" water moves by piston flow or plugflow, completely displacing the "old" water. Because we assume there is no mixing of the two waters--that is the processes of dispersion and diffusion are not acting--a sharp boundary or "front" exists between the two fluids as the "new" water advances through the system. From Darcy's law the specific discharge, or Darcy velocity q, is given by

$$q = \frac{K\Delta h}{L}$$

where L is the distance between two points on the same streamline at which head values h_1 and h_2 are known and $h_2 - h_1 = \Delta h$. The "actual" or average linear velocity v is given by

$$v = \frac{q}{n} = \frac{K\Delta h}{nL}$$

where n is the porosity of the earth material. Remembering that distance of travel (L) = velocity x time or L = vt, then v = L/t. Substituting for v and rearranging, we obtain

$$t = \frac{L}{\frac{K\Delta h}{nL}} = \frac{nL^2}{K\Delta h}.$$

This is the basic formula for calculating the time of travel between two points on a streamline that are a distance L apart.

Given that K = 45 ft/d and n = 0.30, the formula for time of travel between two points on a streamline in the impermeable-wall problem is

$$t = \frac{6.67 \times 10^{-3} L^2}{\Delta h}$$

where t is in days.

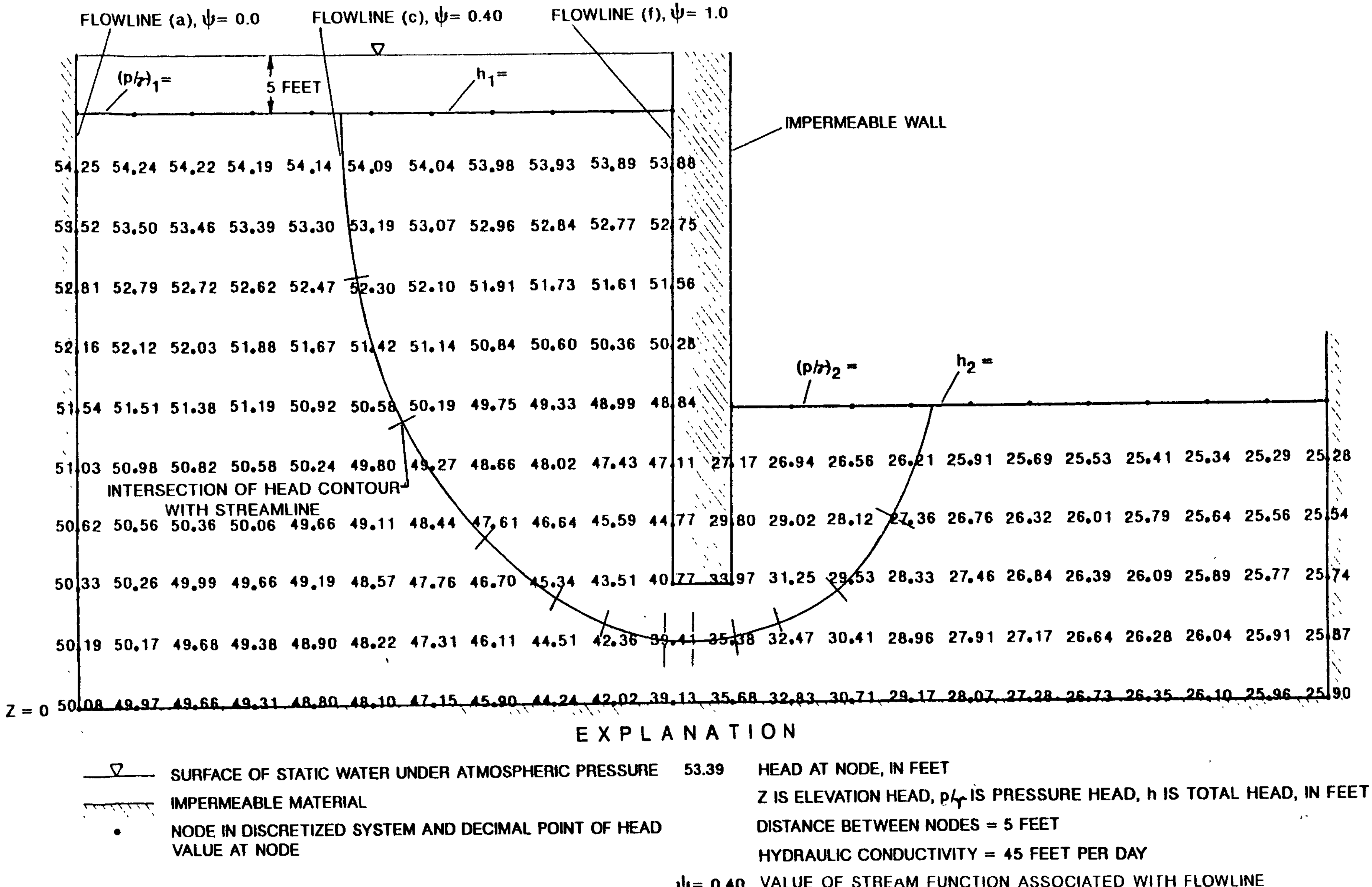


Figure 5-5.--Head "map" of a ground-water system with an impermeable wall for plotting cumulative time of travel from the system inflow boundary along two bounding flowlines and an internal flowline.

1. Using the format in table 5-1, calculate times of travel from node to node along the two bounding streamlines (streamlines "a" and "f" on figure 5-5) of the flow system. For these two streamlines, because we are calculating travel times between nodes, L is constant and equals 5 feet. Thus, for these two streamlines only,

$$t = \frac{.167}{\Delta h} .$$

Our main interest in this problem is not the travel times between points on the streamlines, but the total time of travel from the upper left-hand inflow boundary to the point in question. The value of "Σt" in table 5-1 represents this total calculated travel time along the given streamline from the inflow boundary to the given point on the streamline. Plot the values of Σt at the appropriate points on figure 5-5.

2. An internal streamline (flowline (c), fig. 5-5) from the original flow net beneath the impermeable wall (Exercise 3-2) has been traced onto figure 5-5. Intersections of the potential lines from the original flow net are marked on this internal streamline. Calculate travel times along this internal streamline between intersection points of potential lines. Note that in this case Δh is constant and L varies. Calculate and plot Σt at appropriate points on figure 5-5 as before.

3. Contour Σt values for Σt equal to 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 5.0, and 10.0 days. The contour lines represent calculated positions of the sharp front between "new" and "old" water at successive times after introduction of the "new" water at the inflow boundary.

What time is required for "new" water to reach the discharge boundary? What time is required for "new" water to completely fill the flow system? At the end of this analysis, recall that we assumed piston flow in our time calculations, and that our calculations are only approximate, even for this assumption. However, this approach gives useful order-of-magnitude estimates of travel times in ground-water flow systems.

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 1 of 3)

[h is head at a node or other point in flow system; L is distance between two points on a flowline at which head is known; Δh is difference in head between two points on a flowline; t is time of travel between two points on a flowline; Σt is time of travel from inflow boundary to point on flowline]

h (feet)	L (feet)	Δh (feet)	t (days) = $\frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 2 of 3)

h (feet)	L (feet)	Δh (feet)	t (days) = $\dfrac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines impermeable-wall problem (page 3 of 3)

h (feet)	L (feet)	Δh (feet)	t (days) = $\dfrac{6.67 \times 10^{-3}\ L^2}{\Delta h}$	Σt (days)

Exercise (5-2)--Advective Movement and Travel Times in a Hypothetical Stream-Aquifer System

In Exercise (1-6), the approximate positions of flowlines from points A and B (fig. 1-12) to streams in an areal flow system were drawn. These flowlines are given in figure 5-6. Assuming a uniform gradient, a hydraulic conductivity (K) of 125 ft/d, and a porosity of 0.33, calculate the approximate time for a contaminant placed at points A and B to move advectively to a stream.

For advective flow, the average linear velocity (Fetter (1988), p. 391) is

$$v = \frac{K}{n}\frac{dh}{dl}.$$

1. Measure the total length (L) of each streamline.

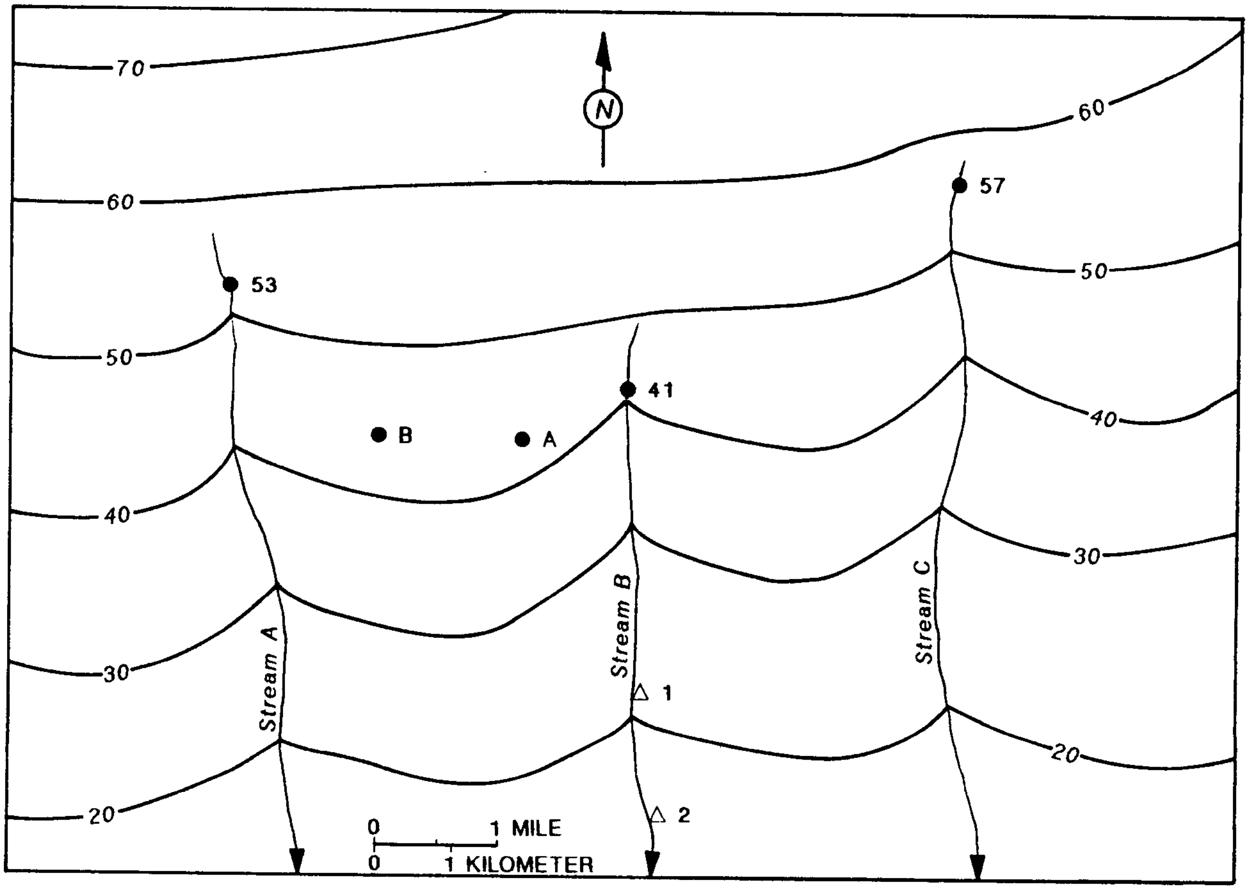
2. Calculate the "average" velocity along each streamline, assuming a constant gradient.

3. Calculate the time required to travel the distance (L) to the stream by dividing the distance by the average velocity.

Simple calculations of this type are extremely useful in understanding contaminant behavior. Franke and Cohen (1972) estimated the positions of flowlines (fig. 5-7) in a stream-aquifer system. These flowlines, when used in conjunction with estimated hydraulic conductivities, porosities, and gradients, enabled the estimation of travel times (fig. 5-8) for the entire stream basin. These time-of-travel estimates can then be used to predict the movement and persistence of contaminants in the shallow ground-water system.

Note (5-2).--Analytical Solutions for Analysis of Solute Transport in Ground Water

As discussed in Note (4-2), an analytical solution, which is a formal, closed-form mathematical solution to a boundary-value problem, simulates ground-water systems that are highly idealized and generally simple relative to the usual complexity of natural systems. For example, in these systems the external geometry usually is simple (squares, rectangles, and circles or three-dimensional equivalents), and the flow medium is at least homogeneous, if not isotropic and homogeneous, so that the properties of the flow medium are specified easily. In view of this inherent simplicity, the similarity between the system represented in the mathematical solution and the natural system never is exact and often is poor. However, valuable qualitative insight into the real system often can be gained through easily executed numerical experimentation with similar hypothetical systems. In general, however, considerable care is required to relate one or more of the available mathematical solutions to the natural system under study.



EXPLANATION

20 WATER-TABLE CONTOUR -- Shows altitude of water table. Contour interval 10 feet Datum is sea level

● 41 LOCATION OF START OF FLOW OF STREAM -- Number is altitude of stream, in feet above sea level

△ 2 LOCATION AND NUMBER OF STREAM DISCHARGE MEASUREMENT POINT

Figure 5-6.--Hypothetical water-table map of an area underlain by permeable deposits in a humid climate.

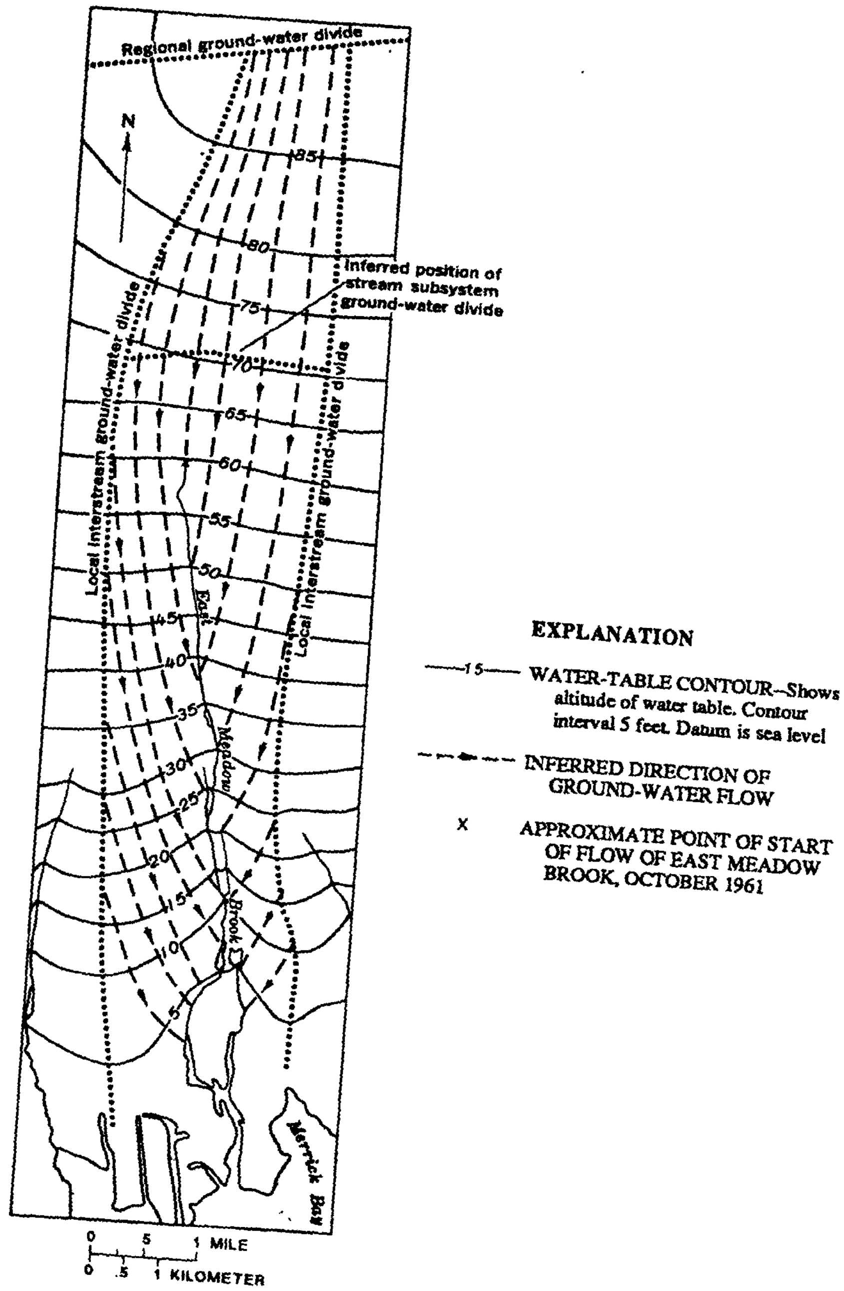


Figure 5-7.--Ground-water flow net in the vicinity of East Meadow Brook, Long Island, New York, in October 1961. (From Franke and Cohen, 1972.)

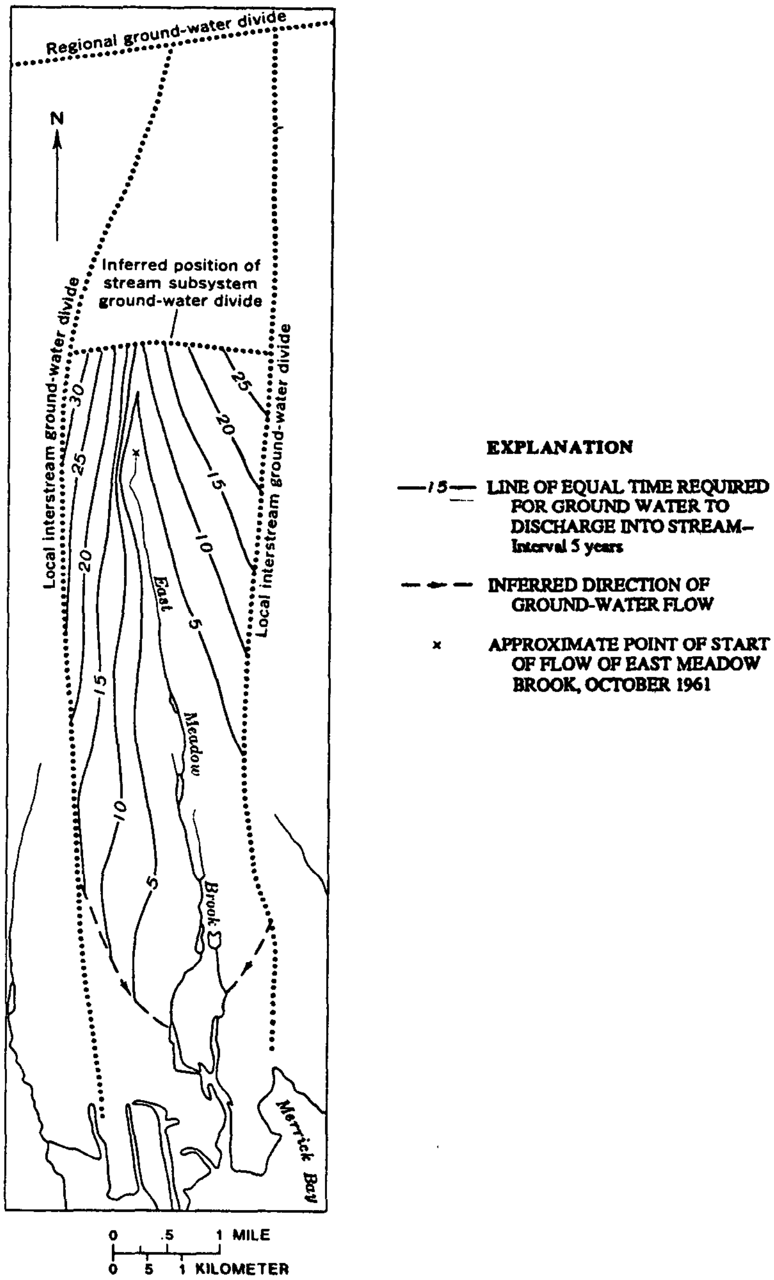


Figure 5-8.--Approximate time required for a particle of water in the shallow ground-water subsystem to discharge into East Meadow Brook, Long Island, New York, under conditions similar to those in October 1961. (From Franke and Cohen, 1972.)

As discussed in Reilly and others (1987), boundary conditions are a key feature to consider in selecting a mathematical solution as a surrogate for the natural system and in evaluating the degree of correspondence between the two systems. The value of applying analytical solutions to a field situation often lies in using them to define limiting cases and then comparing the results of the analytical solution with field data. For example, an analytical solution might represent advective and dispersive transport of a conservative solute in a highly idealized flow field. By judicious selection of the parameters for several cases, the results from a series of solutions to this hypothetical problem may bracket the distribution of a conservative constituent in the field problem. If this bracketing does not occur, some process in the natural system requires further explanation. Some of the available analytical solutions for solute-transport problems are given by Bear (1972), Bear (1979), Freeze and Cherry (1979), and Javandel and others (1984). In addition, Wexler (1989) compiled nine analytical solutions for one-, two-, and three-dimensional solute transport problems and provides computer programs to facilitate their use.

Fetter (1988, p. 393-394) gives the governing one-dimensional differential equation for advection and dispersion as

$$D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} = \frac{\partial c}{\partial t} \tag{1}$$

where

D is the longitudinal dispersion coefficient (L^2/T),
c is the solute concentration (M/L^3),
v is the average linear velocity in the x-direction (L/T), and
t is the time since start of solute invasion (T).

The analytical solution to this governing differential equation gives the concentration, c, at some distance, L, from the source, whose concentration is c_0, at time, t, as

$$c = \frac{c_0}{2} \left[\operatorname{erfc} \left(\frac{L-vt}{2\sqrt{Dt}} \right) + \exp \left(\frac{vL}{D} \right) \operatorname{erfc} \left(\frac{L+vt}{2\sqrt{Dt}} \right) \right] \tag{2}$$

For conditions in which the dispersion coefficient is small or L or t is large, the second term is negligible and the equation reduces to

$$c = \frac{c_0}{2} \operatorname{erfc} \frac{L-vt}{2\sqrt{Dt}} \, . \tag{3}$$

Exercise (5-8)--Application of the One-Dimensional Advective-Dispersive Equation

The one-dimensional advective-dispersive equation, given in Note (5-2) and in Fetter (1988, p. 394), may be used to develop an estimate of the transport and distribution of solutes in a three-dimensional natural system. After reading sections 10.6.4, 10.6.5, and 10.6.6 in Fetter (1988, p. 391-397), do the example problem in Fetter (1988) on page 395 for practice in using the equation.

Note that step 2 of the sample problem, "Determine the longitudinal dispersion coefficient," is not accurate. This method of estimating the dispersion coefficient is used simply to facilitate the calculation. Actually, as discussed in Note (5-1), the value of the dispersion coefficient usually is determined by history-matching in a numerical simulation. Active research whose goal is to determine the dispersion coefficient based on the distribution of hydraulic conductivity at the local scale is in progress.

Using the simplified one-dimensional analytical solution given in Note (5-2), calculate the solute concentrations at the intervals given in tables 5-2 and 5-3 for a dispersion coefficient, D, of 10 ft^2/d and 100 ft^2/d, and compare the results. Use the values

t = 1,000 days,
v = 2 ft/d, and
C_0 = 100 mg/L,

the two tables, and the values of the complementary error function (erfc) in Appendix 13 of Fetter (1988, p. 562). Plot the results of the calculations on figure 5-9 as a graph of relative concentration C/C_0 against distance from source L.

Answer the following questions:

1. What is the effect of the larger dispersion coefficient?

2. What distance would the solute have traveled under plug flow (purely advective movement, $x = vt$)? Draw a vertical line on figure 5-9 at this distance. What solute concentration is calculated at that distance for each dispersion coefficient?

Table 5-2.--Format for calculating solute concentrations when the dispersion coefficient D = 10 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, square feet per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

- C = concentration of solute at point in plume at specified time, in mg/L
- C_0 = solute concentration of source, in mg/L
- L = distance from source, in feet
- v = average linear velocity of ground water, in ft/d
- t = elapsed time since introduction of solute at source, in days
- D = dispersion coefficient, in ft^2/d
- erfc = complementary error function (see text)

Preliminary calculation:

For D = 10 ft^2/d, $\left(\frac{L - vt}{2\sqrt{Dt}}\right) =$

L (feet)	$\frac{L-2,000}{200}$	$\operatorname{erfc}\left(\frac{L-2,000}{200}\right)$ [1]	$C = 50 \text{ mg/L} \operatorname{erfc}\left(\frac{L-2,000}{200}\right)$
1,500			
1,600			
1,700			
1,800			
1,900			
2,000			
2,100			
2,200			
2,300			
2,400			

[1] $\operatorname{erfc}(-x) = 1 + \operatorname{erf}(x)$.

Table 5-8.--Format for calculating solute concentrations when the dispersion coefficient D = 100 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, feet squared per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

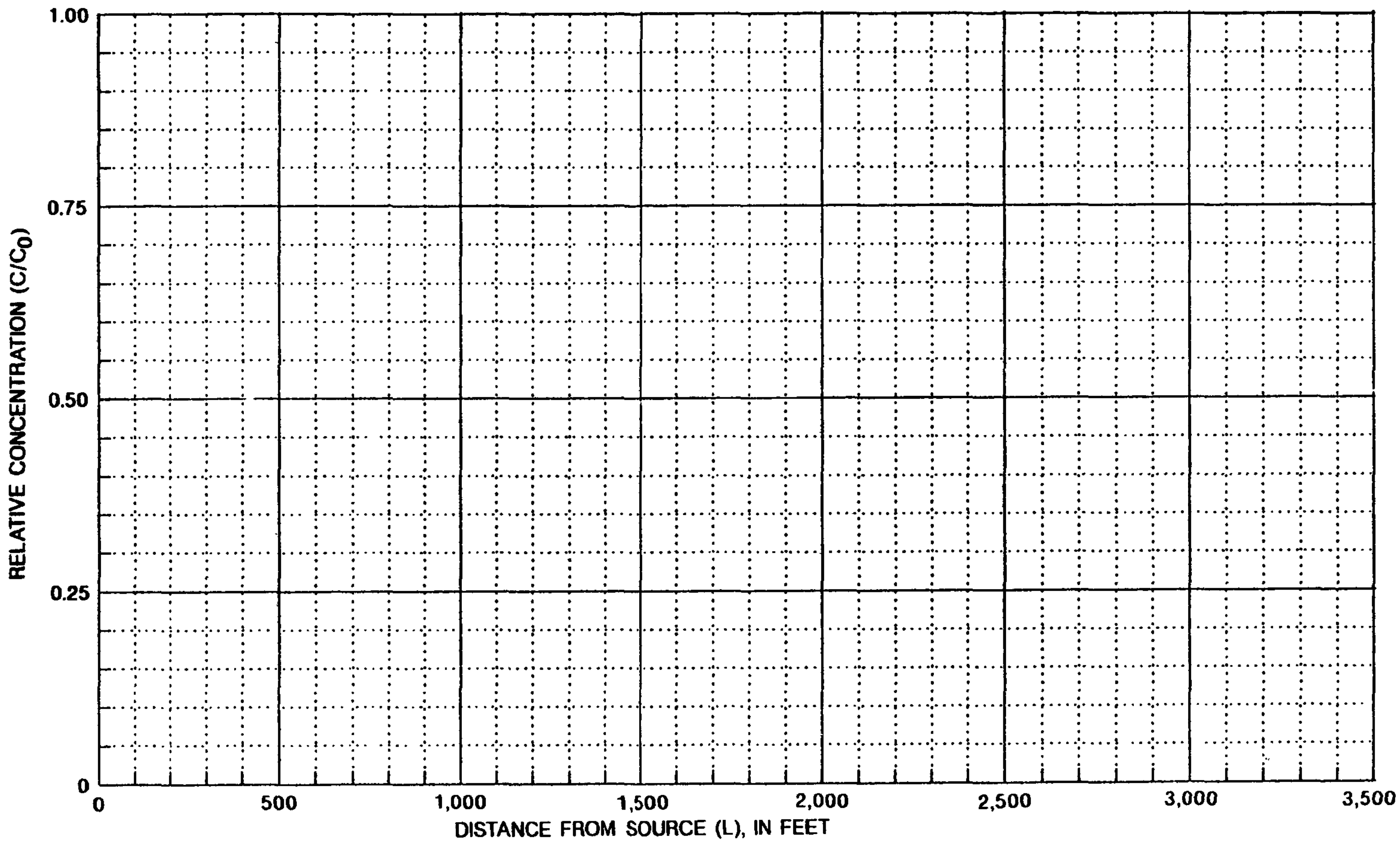
C = concentration of solute at point in plume at specified time, in mg/L
C_0 = solute concentration of source, in mg/L
L = distance from source, in feet
v = average linear velocity of ground water, in ft/d
t = elapsed time since introduction of solute at source, in days
D = dispersion coefficient, in ft^2/d
erfc = complementary error function (see text)

Preliminary calculation:

For D = 100 ft^2/d, $\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ =

L (feet)	$\frac{L-2,000}{632.5}$	$\operatorname{erfc}\left(\frac{L-2,000}{632.5}\right)$ [1]	C = 50 mg/L $\operatorname{erfc}\left(\frac{L-2,000}{632.5}\right)$
1,000			
1,250			
1,500			
1,750			
2,000			
2,250			
2,500			
2,750			
3,000			

[1] erfc(-x) = 1 + erf(x).



EXPLANATION

C_0 = SOLUTE CONCENTRATION OF SOURCE

C = CONCENTRATION OF SOLUTE AT POINT IN PLUME

Figure 5-9.--Graph for plotting relative concentration against distance from source for two values of the dispersion coefficient and an elapsed time of 1,000 days.

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EXTRA COPIES

OF

WORK SHEETS

FOR

PROBLEMS

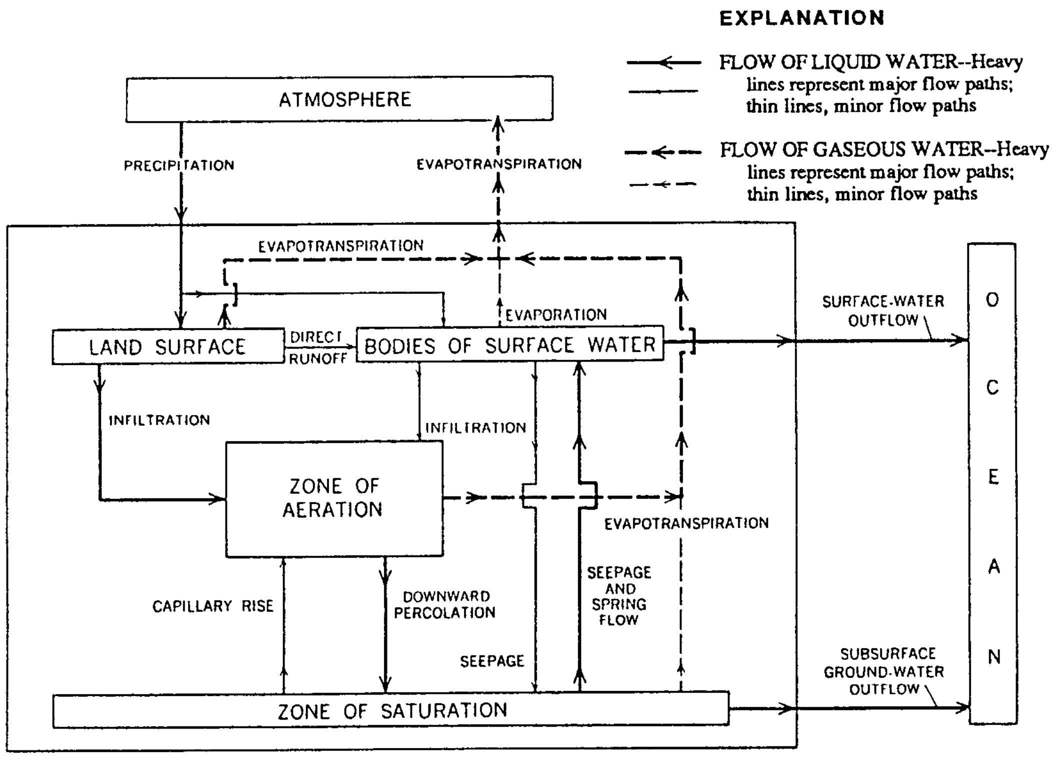


Figure 1-1.--Flow diagram of the hydrologic system, Nassau and Suffolk Counties, Long Island, New York, under predevelopment conditions. (From Franke and McClymonds, 1972, fig. 13.)

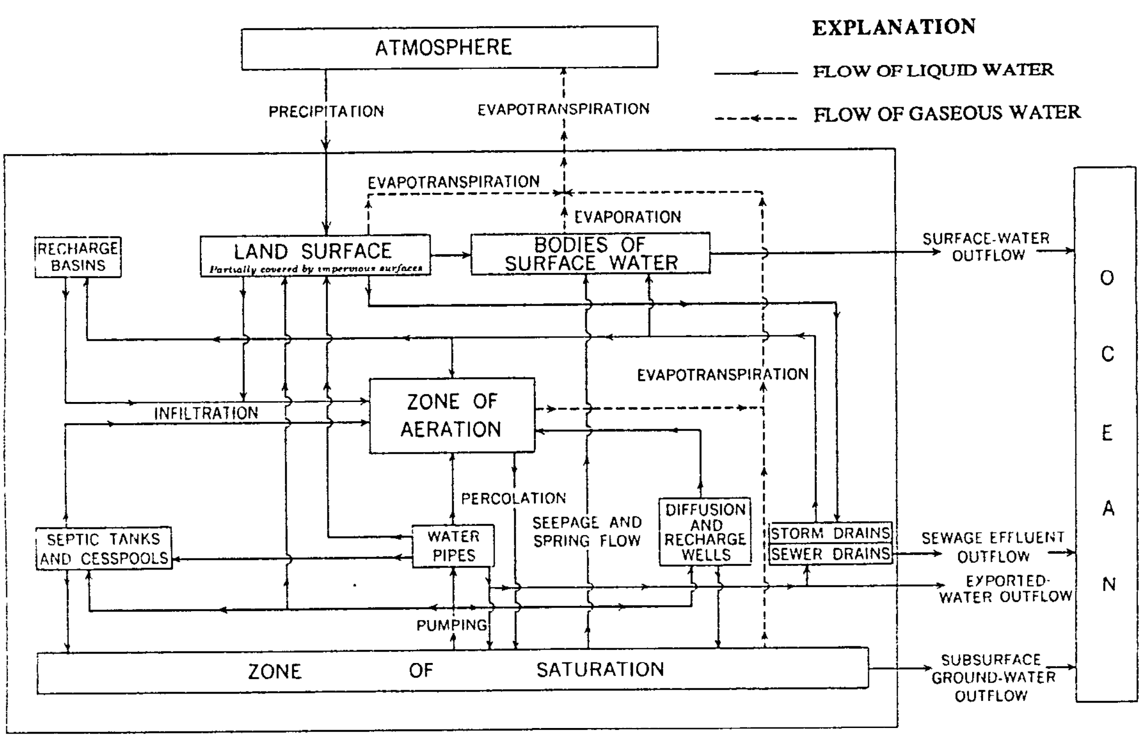


Figure 1-2.--Flow diagram of the hydrologic system, Nassau and Suffolk Counties, Long Island, New York, after noticeable influence from human activities. (From Franke and McClymonds, 1972, fig. 33.)

The relation between heads at the water table and heads in adjacent wells whose screened intervals lie at some depth below the water table depends on the position of the observation-well pair in the associated ground-water system. A general interpretation of the head relations depicted in figure 1-7 must wait for a more comprehensive discussion of ground-water systems in Section (3) of this course. The purpose of presenting figure 1-7 at this time is to emphasize that, in general, hydraulic head in ground-water systems varies not only with geographic location but also with depth.

Exercise (1-4)--Hydraulic Head

The purpose of this exercise is to provide practice in differentiating between the two components of head--pressure head and elevation head. The elevation head at a point in a ground-water system is arbitrary in that it depends on the altitude of an arbitrary datum. Sea level generally is used as head datum, the same datum used for land-surface topographic maps. However, the pressure head at a given point and a given time is not arbitrary, but is a physical quantity that can be measured directly. It is directly proportional to the height of the fluid column above the point of pressure measurement in a piezometer or observation well.

The data below are available for three closely spaced (in map view) observation wells with short well screens.

(1) Determine the missing entries in table 1-2.

(2) Make a careful sketch of each observation well on the accompanying worksheet (fig. 1-8). Plot and designate on each sketch the pressure head, elevation head, and total hydraulic head.

Table 1-2.--Head data for three closely spaced observation wells

Well	Land-surface altitude (feet above sea level)	Depth of top of screen below land surface (feet)	Depth to water (feet)	Altitude of water-level surface in well[1] (feet above sea level)	Pressure head (p/γ) (feet)	Elevation head (z) (feet)
1	50	25	15			
2	45	90	9			
3	51	350	13			

[1] Altitude of water-level surface in observation well equals hydraulic head at point of pressure measurement of observation well.

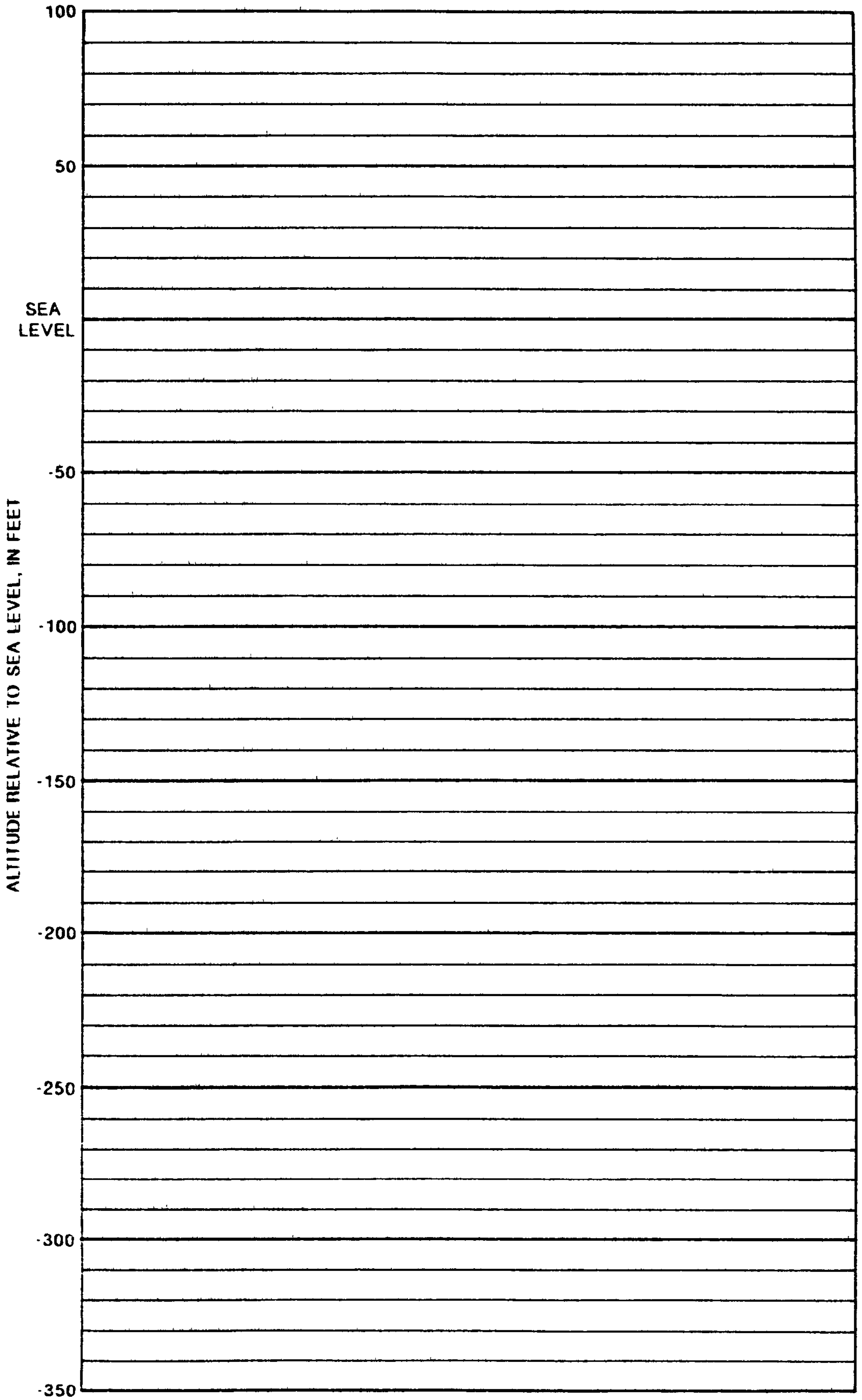


Figure 1-8.--Worksheet for head exercise.

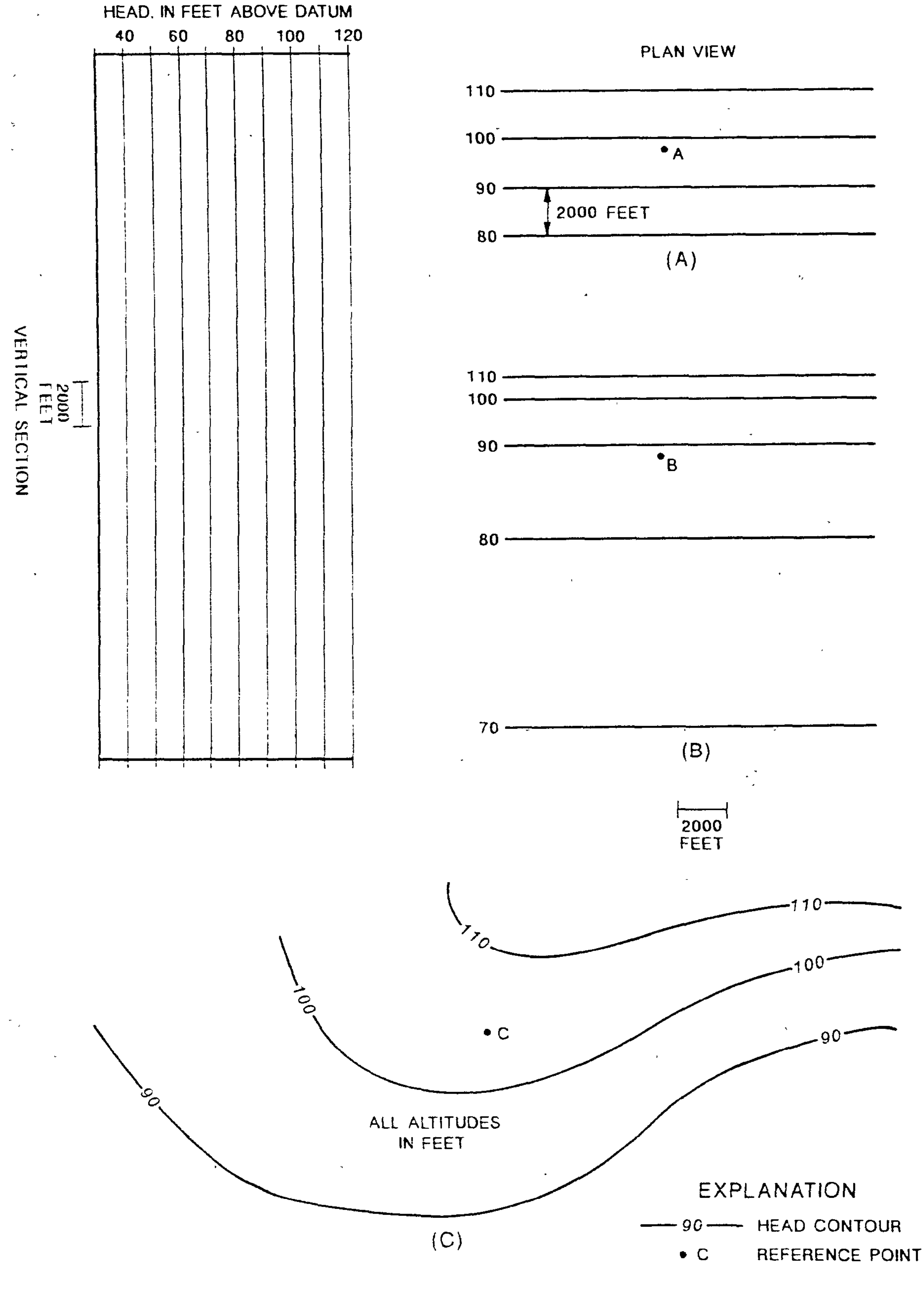


Figure 1-10.--Maps of ground-water head illustrating three different contour patterns.

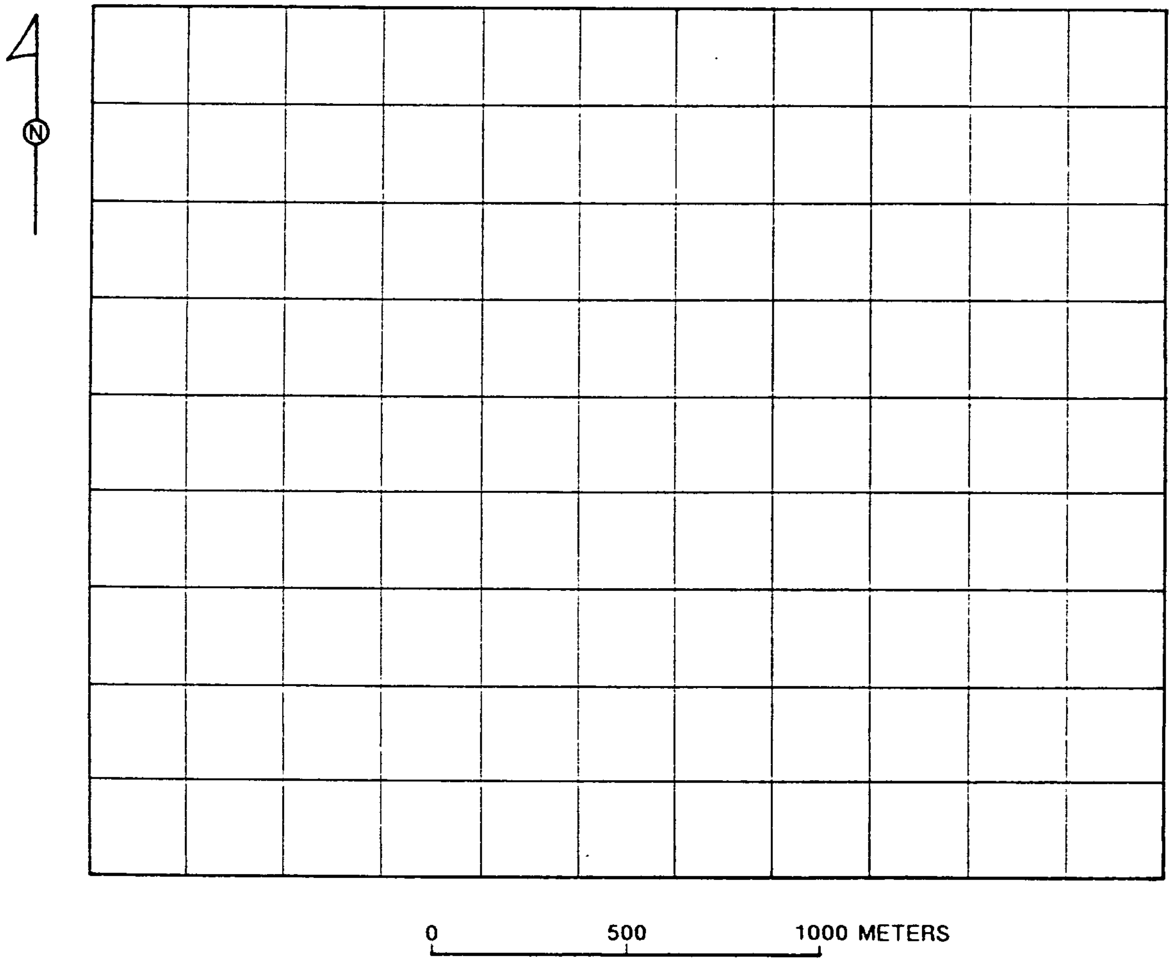
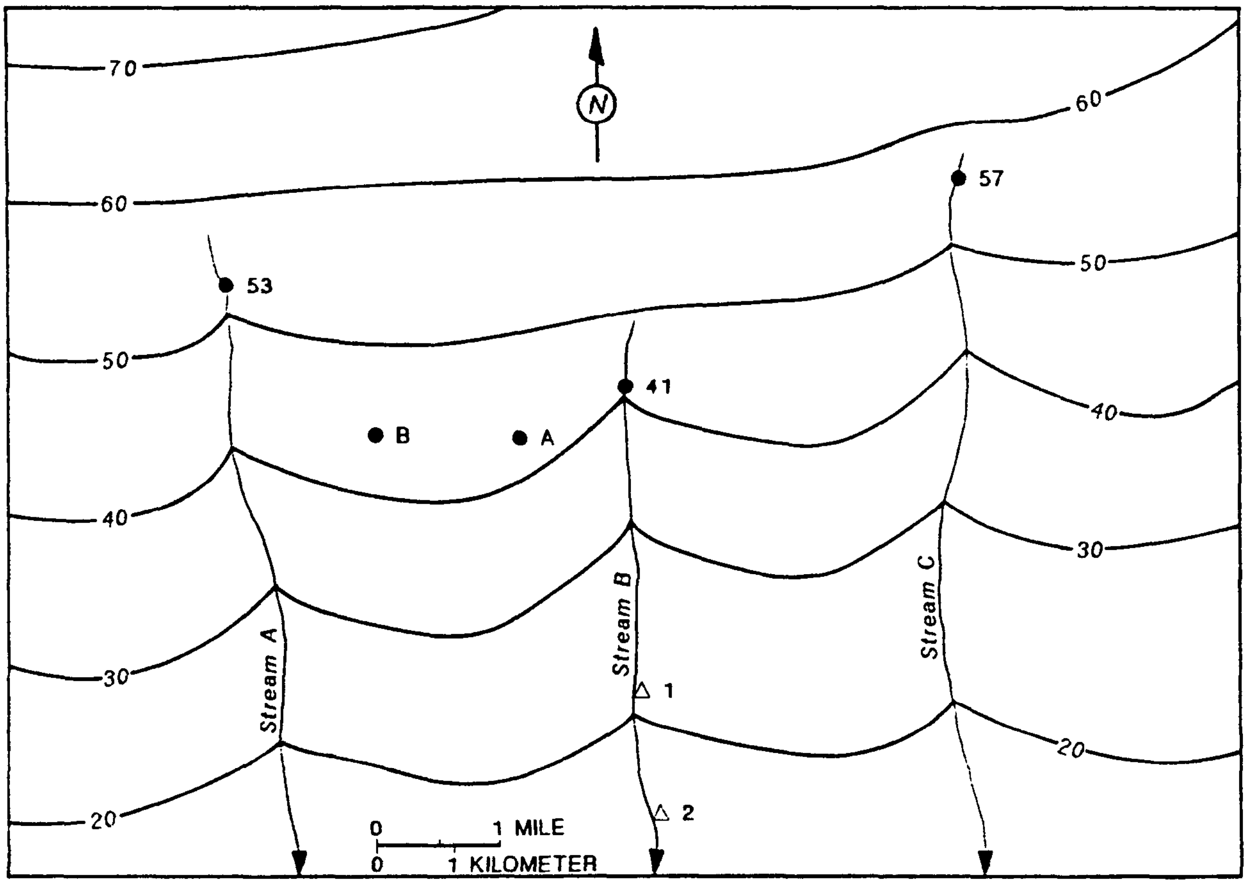


Figure 1-11.--Worksheet for the "three-point" head-gradient problem.



EXPLANATION

20 WATER-TABLE CONTOUR -- Shows altitude of water table. Contour interval 10 feet. Datum is sea level

● 41 LOCATION OF START OF FLOW OF STREAM -- Number is altitude of stream, in feet above sea level

△ 2 LOCATION AND NUMBER OF STREAM DISCHARGE MEASUREMENT POINT

Figure 1-12.--Hypothetical water-table map of an area underlain by permeable deposits in a humid climate.

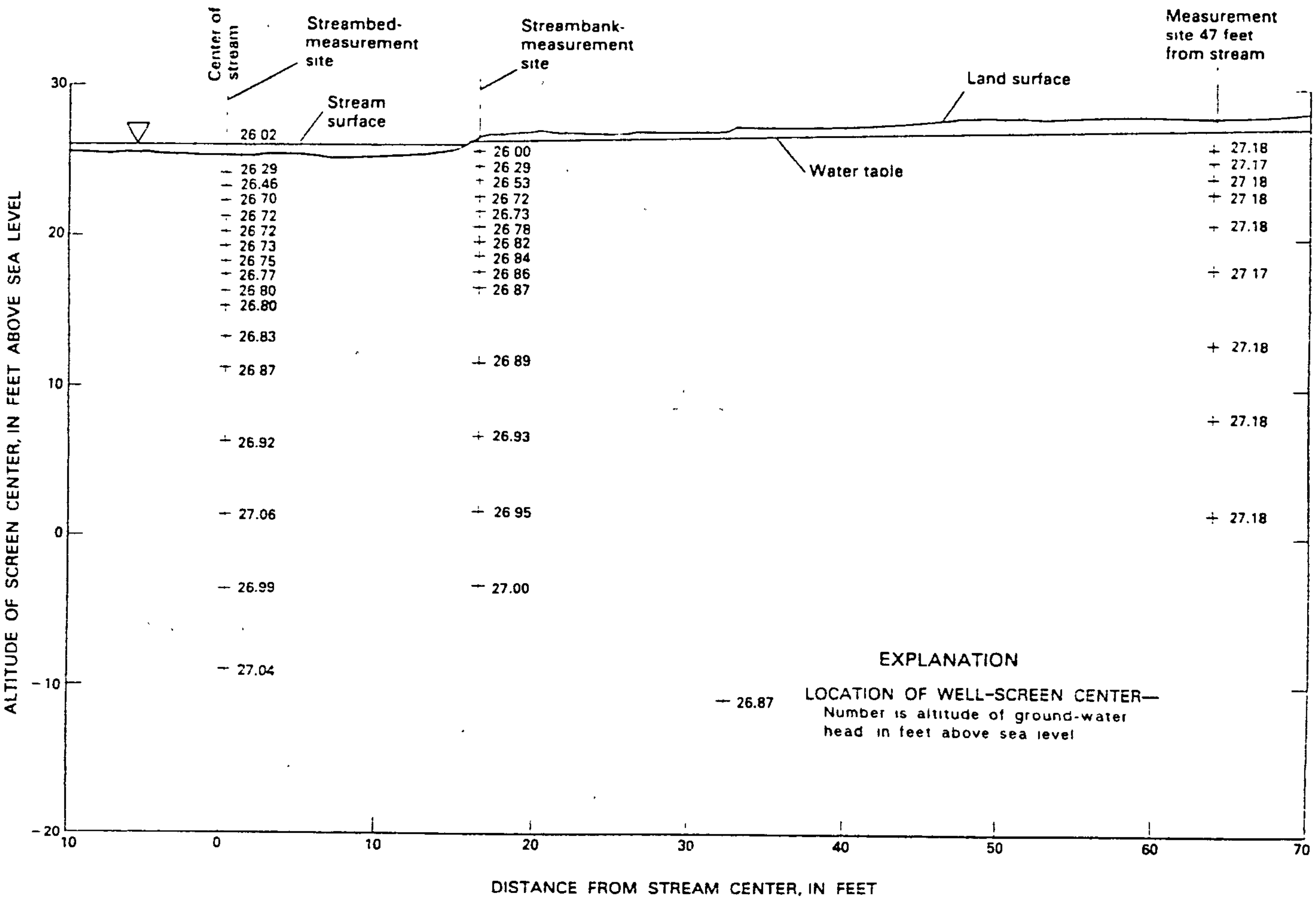


Figure 1-19.--Head measurements near Connetquot Brook, Long Island, New York, during a 3-day period in October 1978. (Modified from Prince and others, 1988, fig. 10.)

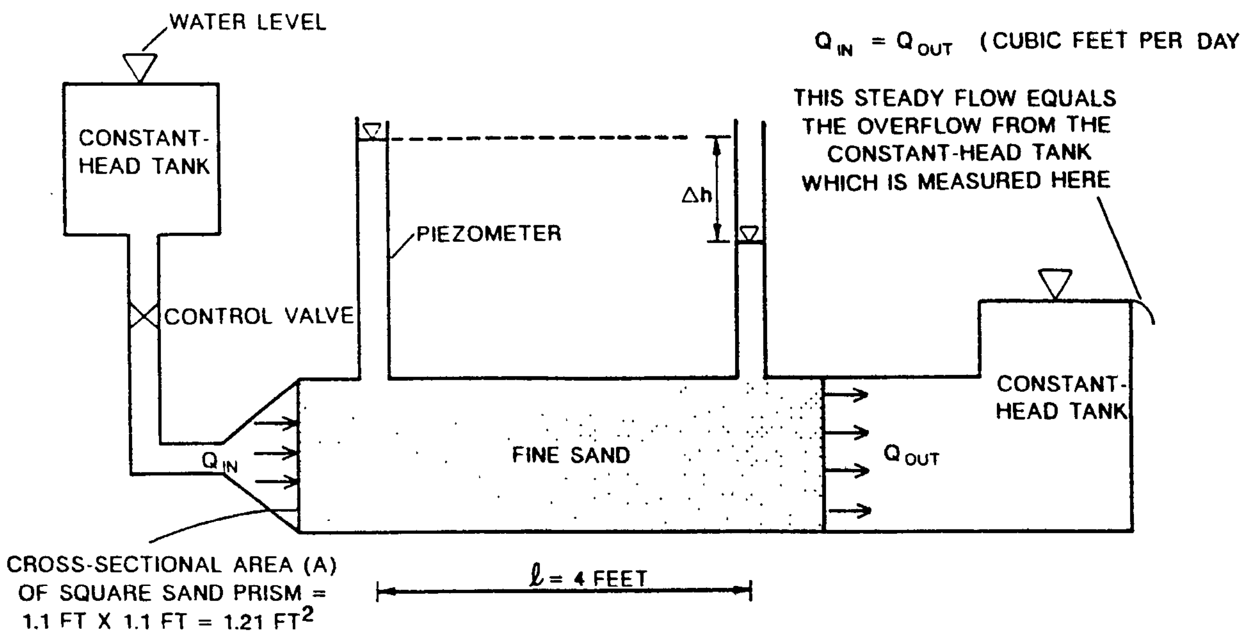


Figure 2-1.--Sketch of laboratory seepage system.

Table 2-1.--Data from hypothetical experiments with the laboratory seepage system

Test number	Q[1] (cubic feet per day)	Δh (feet)	$\Delta h/l$	Q/A (feet/day)
1	2.2	0.11		
2	3.3	.17		
3	4.6	.23		
4	5.4	.26		
5	6.7	.34		
6	7.3	.38		
7	7.9	.40		

[1] Q is steady flow through sand prism, Δh is head difference between two piezometers; l is distance between two piezometers; A is constant cross-sectional area of sand prism (fig. 2-1).

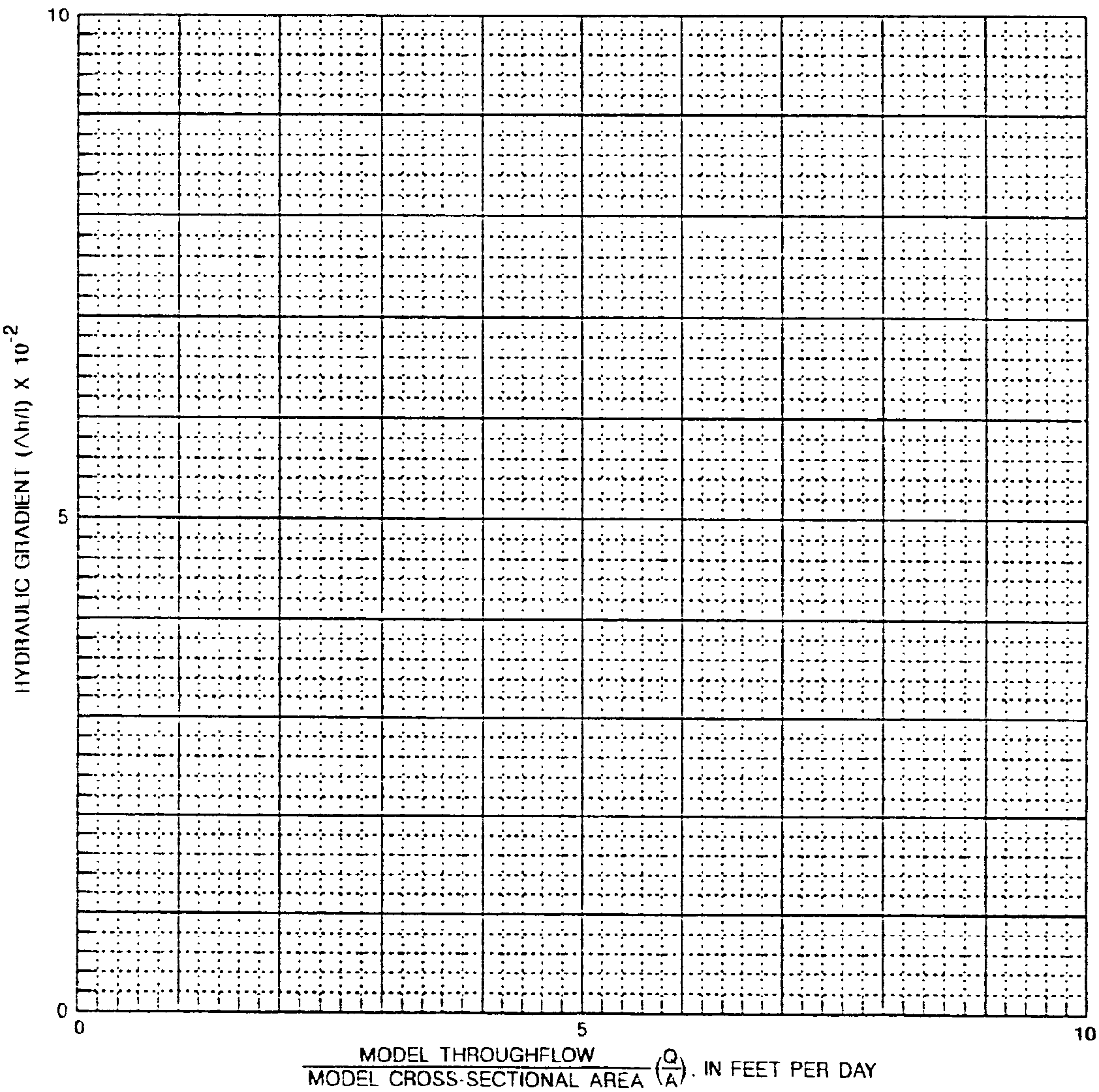


Figure 2-2.--Worksheet for plotting data from hypothetical experiments with the laboratory seepage system.

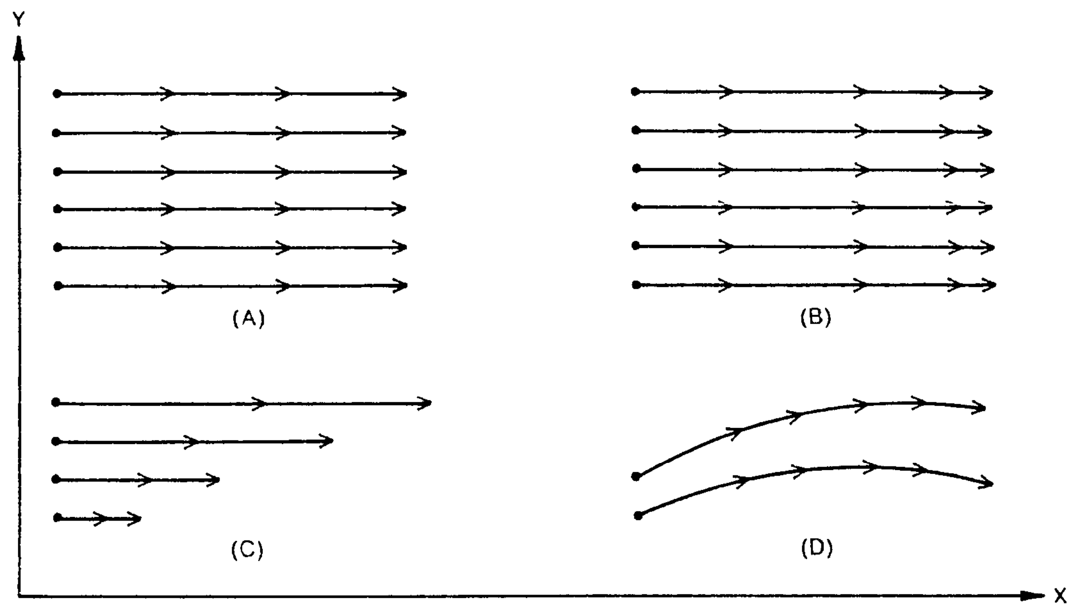


Figure 2-3.--Examples of ground-water flow fields depicted by velocity vectors at selected points: (A) and (B) are one-dimensional flow fields; (C) and (D) are two-dimensional flow fields.

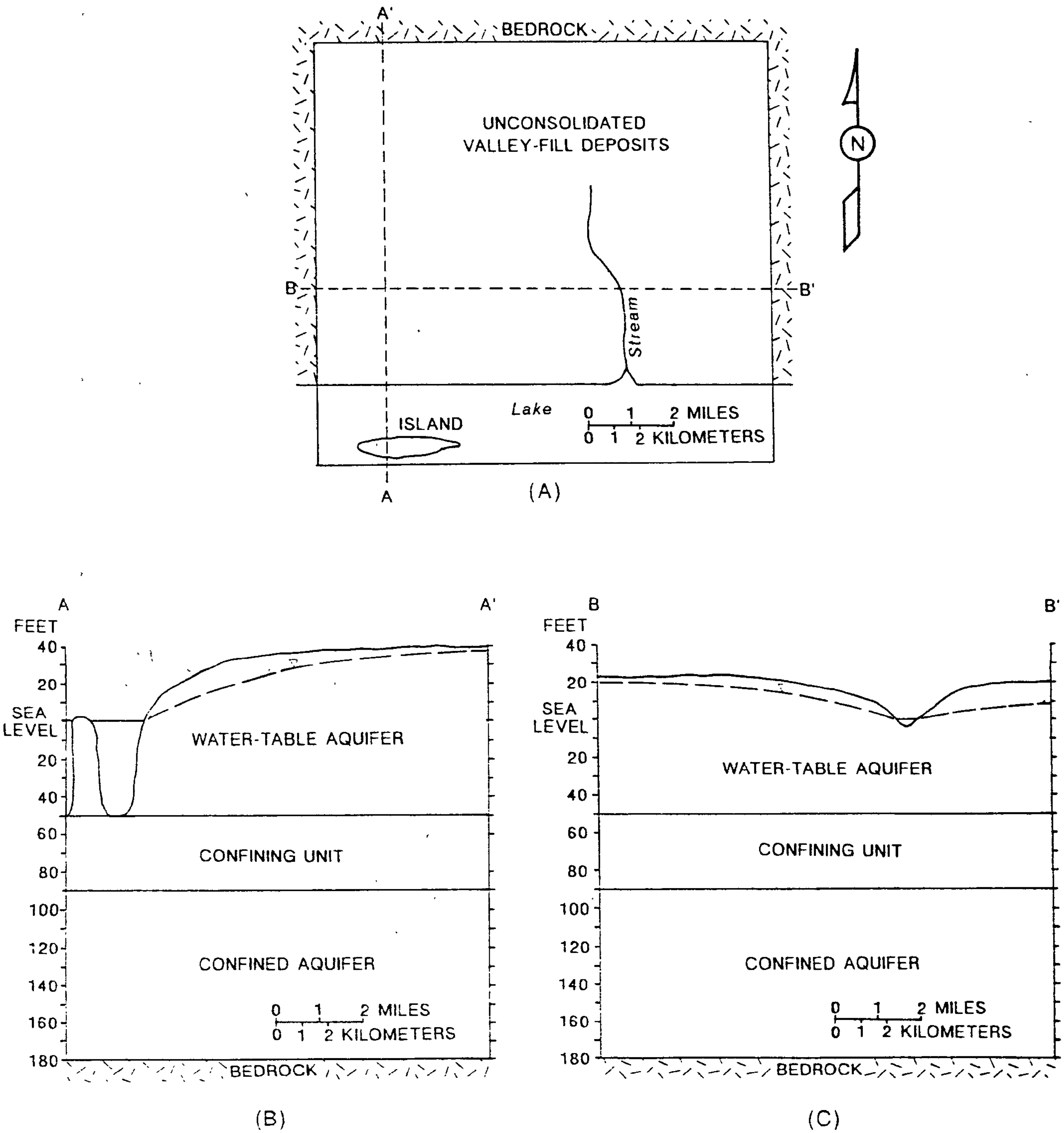


Figure 3-2.--Hydrogeologic framework of hypothetical ground-water system: (A) plan view, (B) north-south-trending section, (C) east-west trending section.

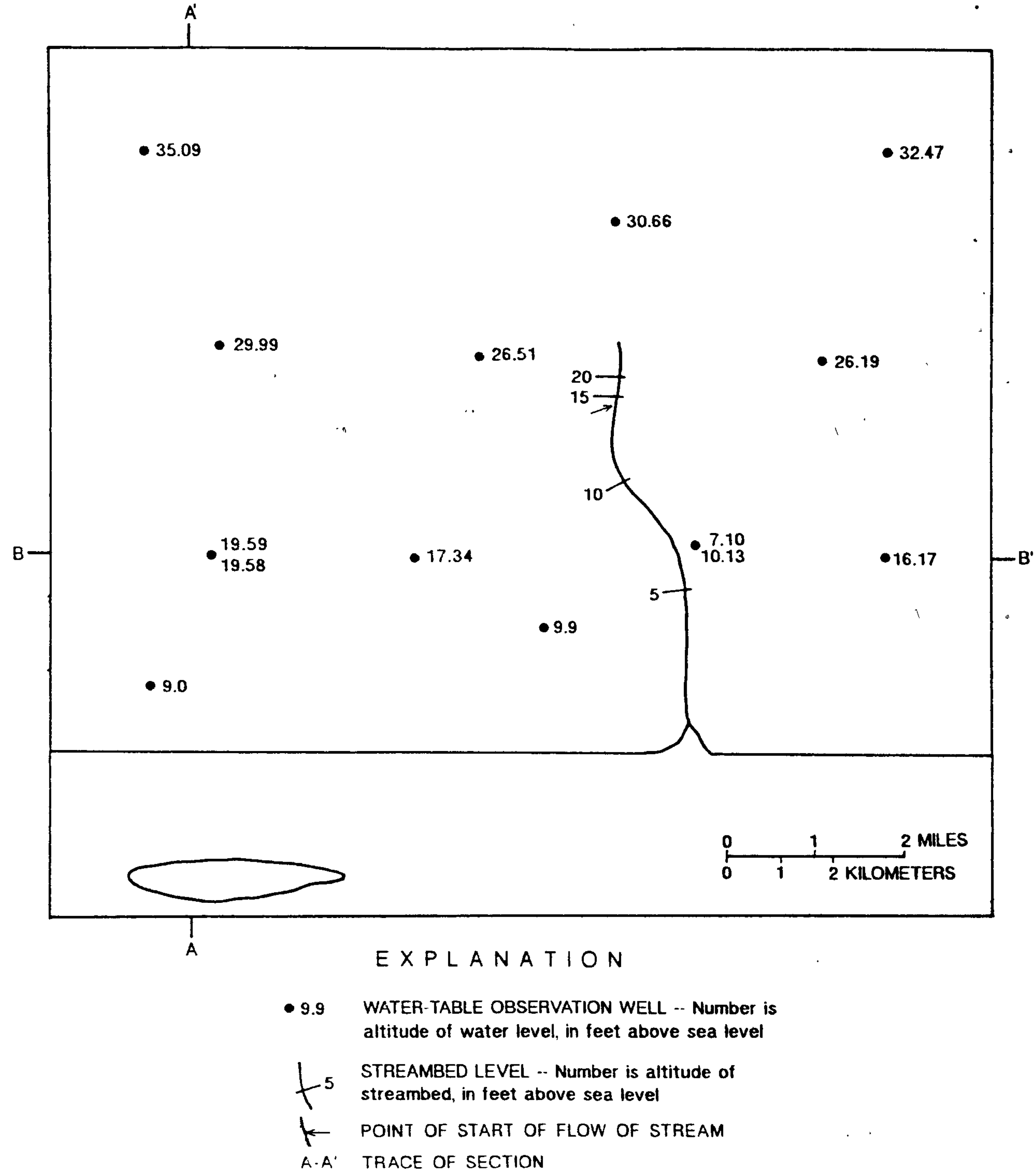
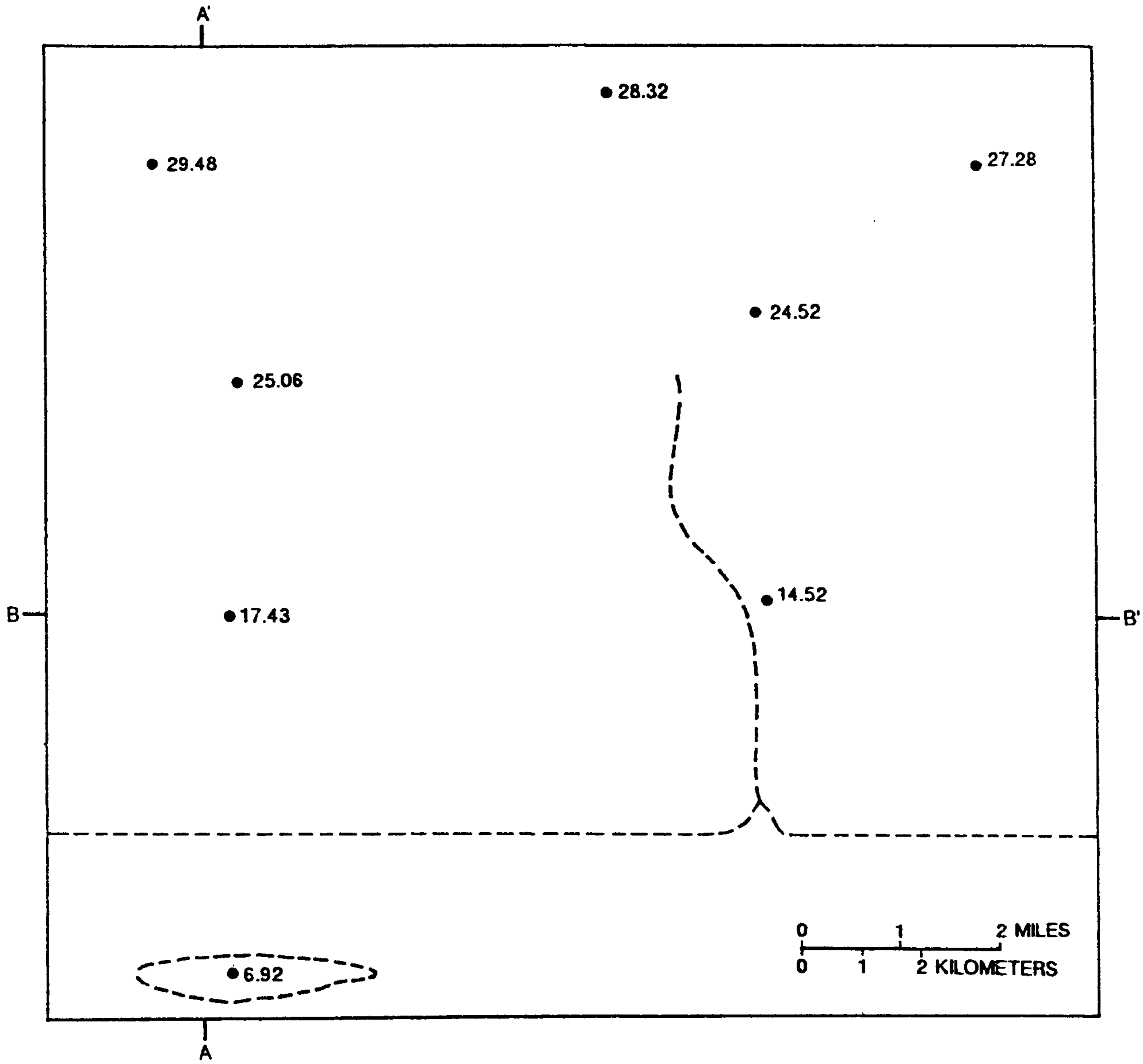


Figure S-4.--Water-table heads obtained during a synoptic measurement of water levels in observation wells.



EXPLANATION

● 6.92 OBSERVATION WELL SCREENED IN THE CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure S-6.--Heads in the confined aquifer obtained during a synoptic measurement of water levels in observation wells.

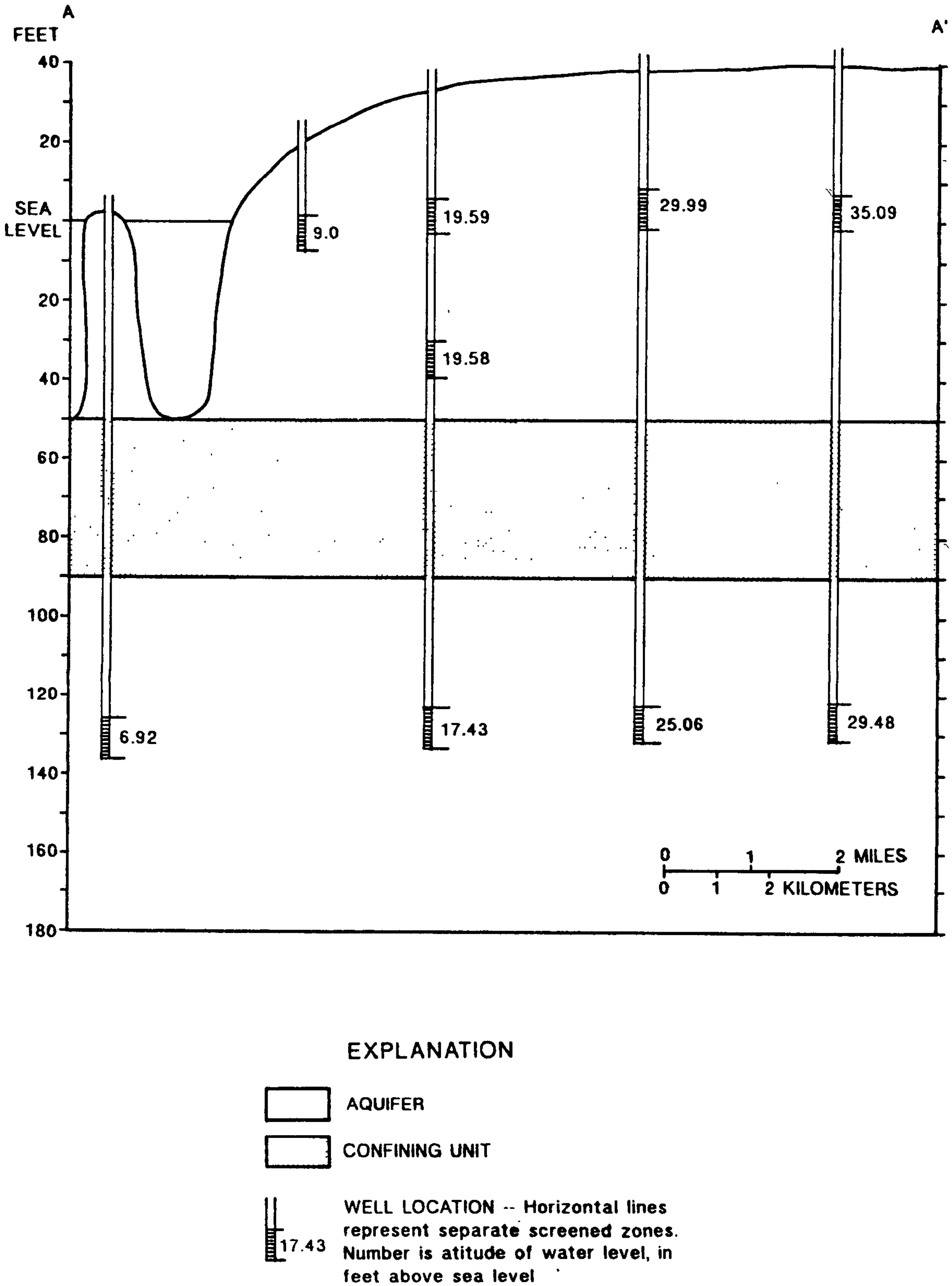


Figure 3-7.--North-south-trending hydrogeologic section showing heads obtained during synoptic measurement of water levels in observation wells. (Location of section A-A' is shown in fig. 3-2.)

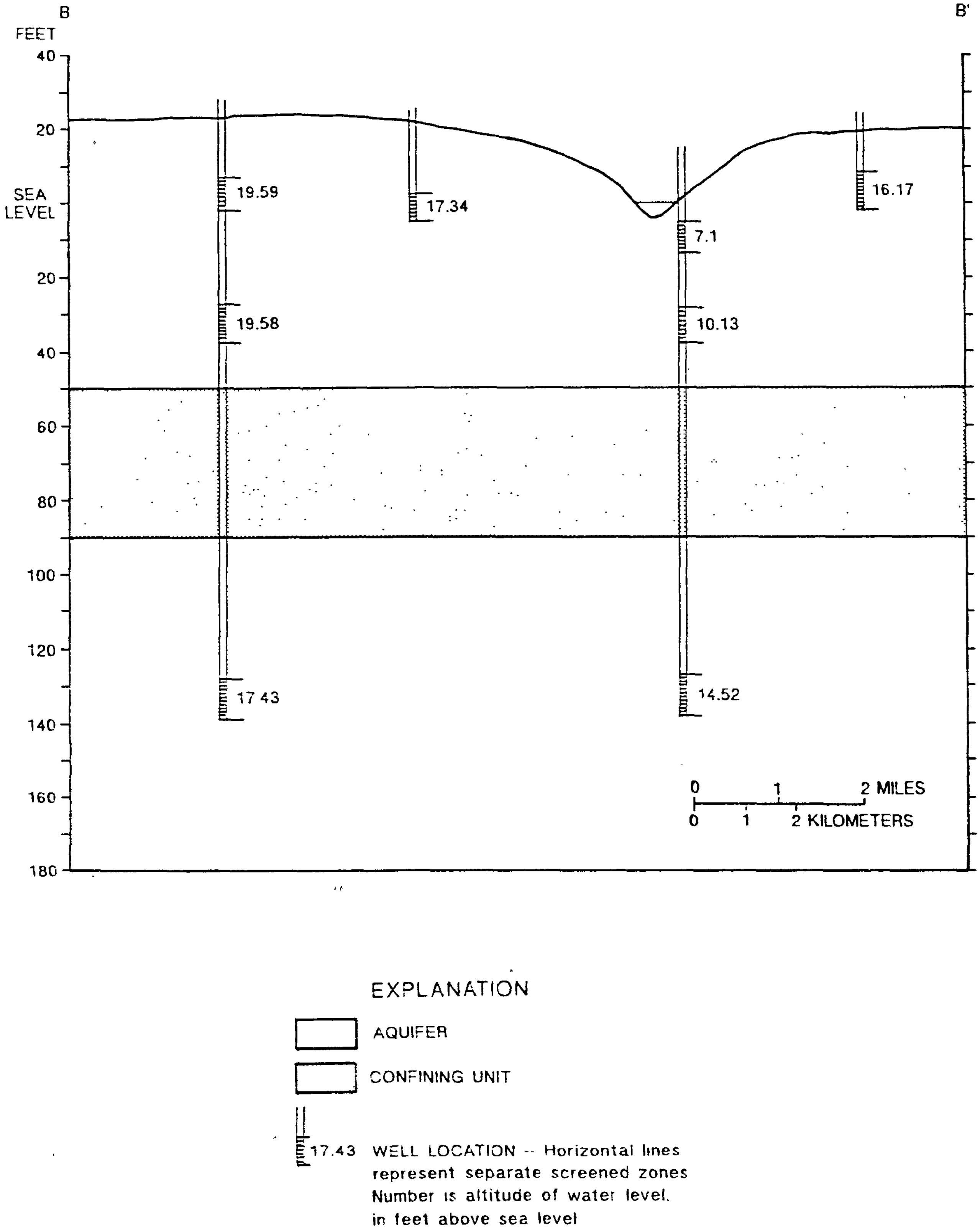
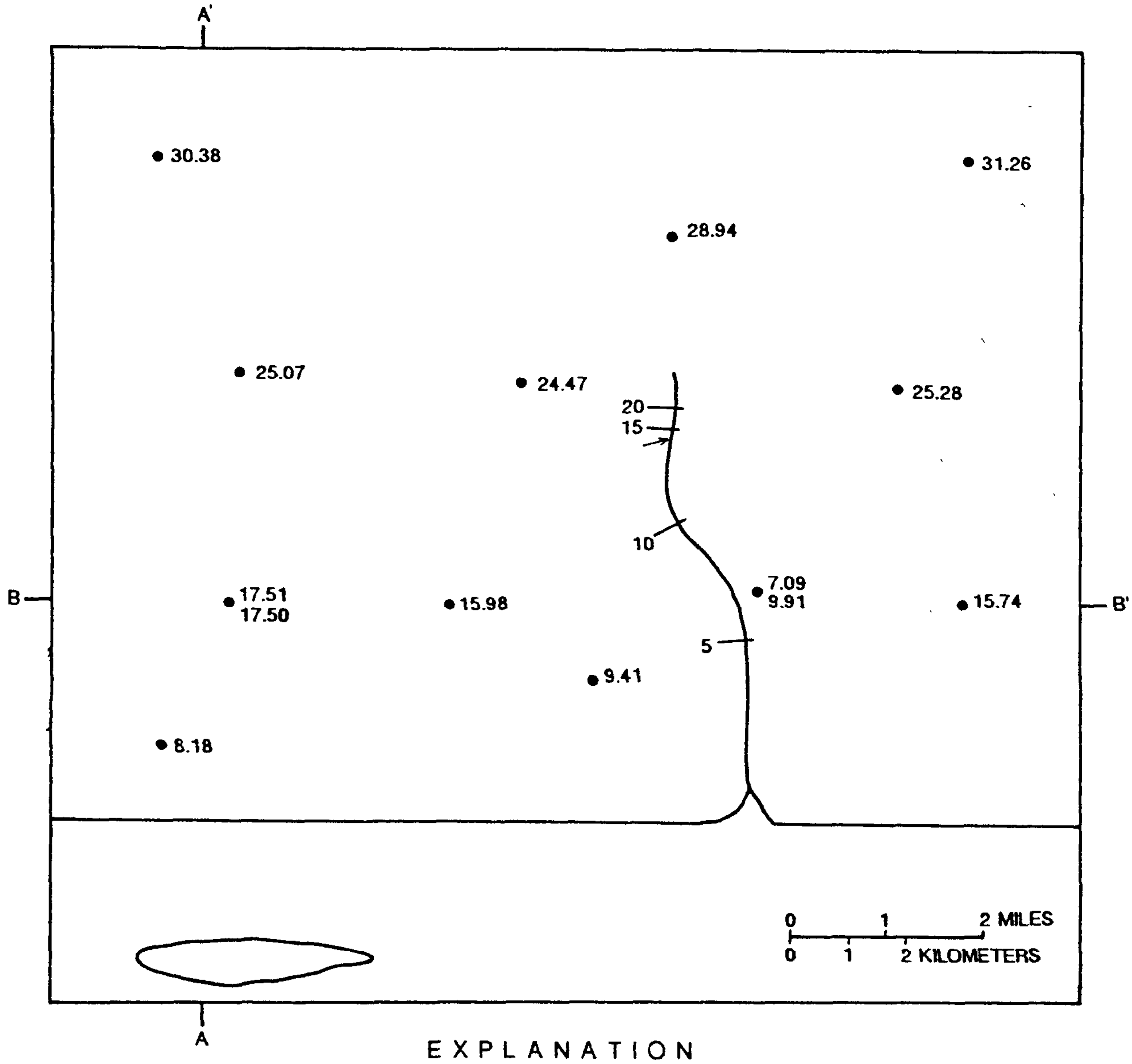


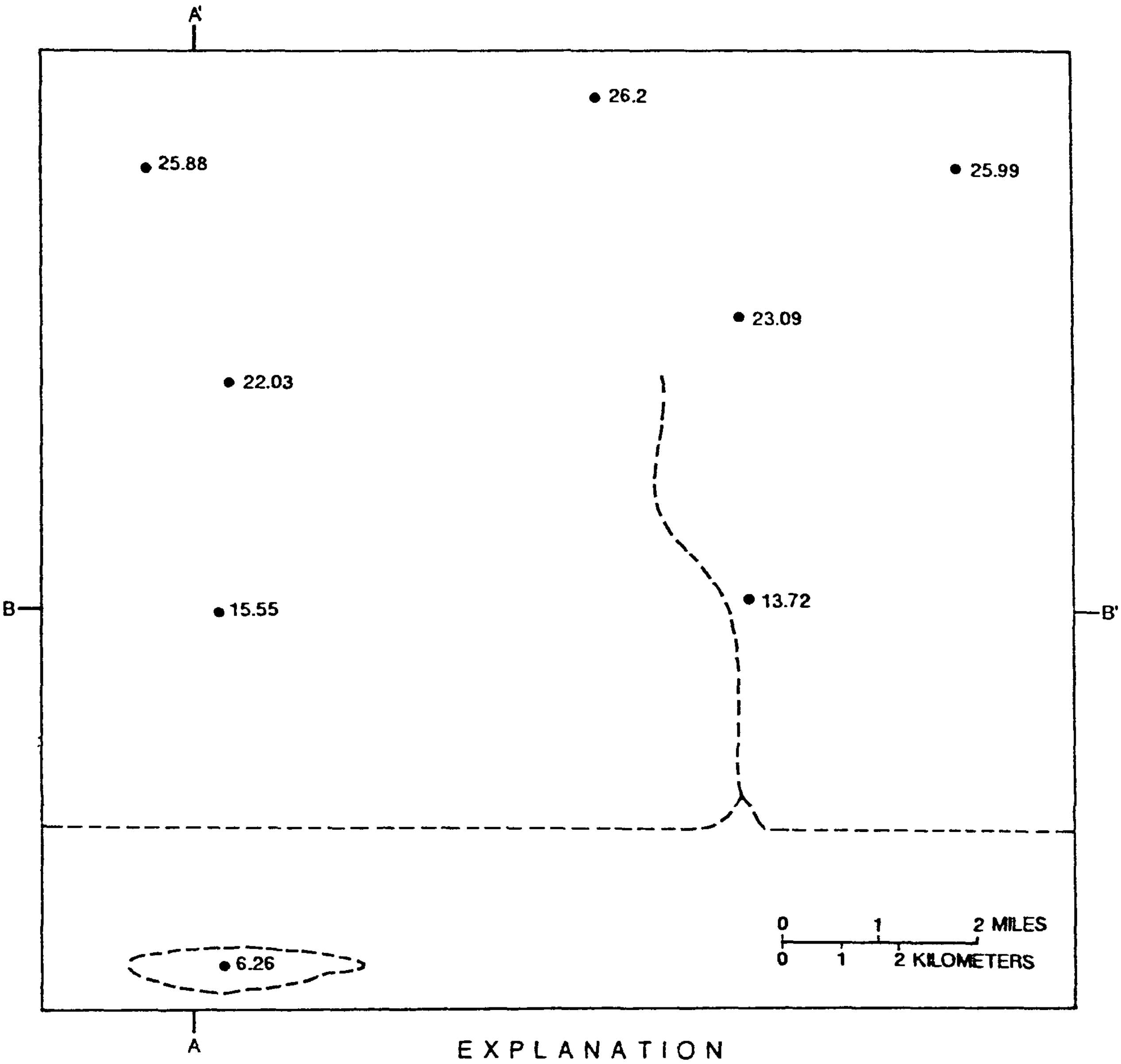
Figure 3-8.--East-west-trending hydrogeologic section showing heads obtained during synoptic measurement of water levels in observation wells. (Location of section B-B' is shown in fig. 3-2.)



EXPLANATION

- 8.18 WATER-TABLE OBSERVATION WELL -- Number is altitude of water level, in feet above sea level
- 5 STREAMBED LEVEL -- Number is altitude of streambed, in feet above sea level
- POINT OF START OF FLOW OF STREAM
- A-A' TRACE OF SECTION

Figure S-10.--Measured heads in the water-table aquifer in response to steady pumping from a well screened in the lower part of the water-table aquifer.



EXPLANATION

• 6.26 OBSERVATION WELL SCREENED IN CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure 9-11.--Measured heads in the confined aquifer in response to steady pumping from a well screened in the lower part of the water-table aquifer.

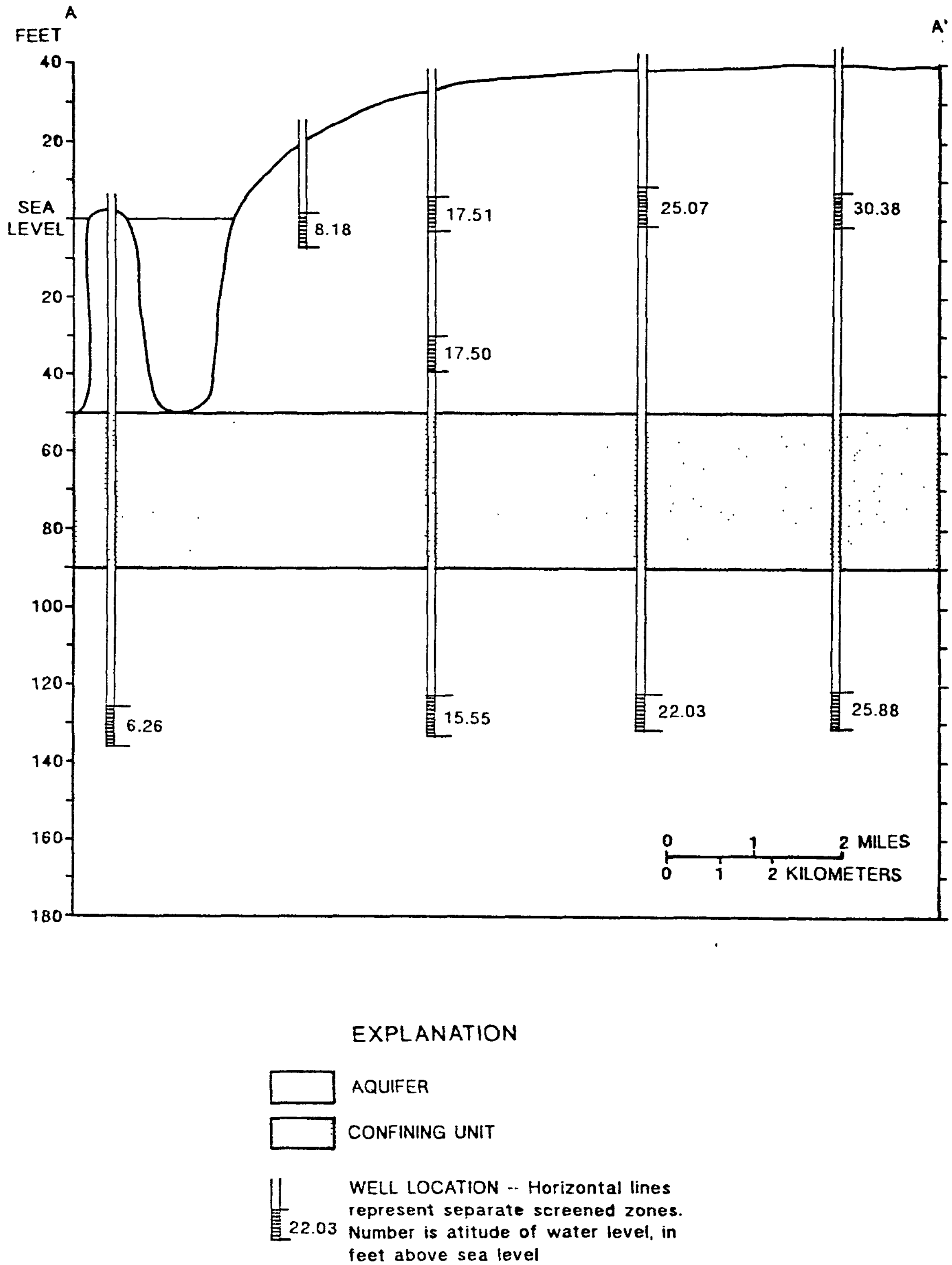


Figure S-12.--North-south-trending hydrogeologic section showing measured heads in response to steady pumping. (Location of section A-A' is shown in fig. S-2.)

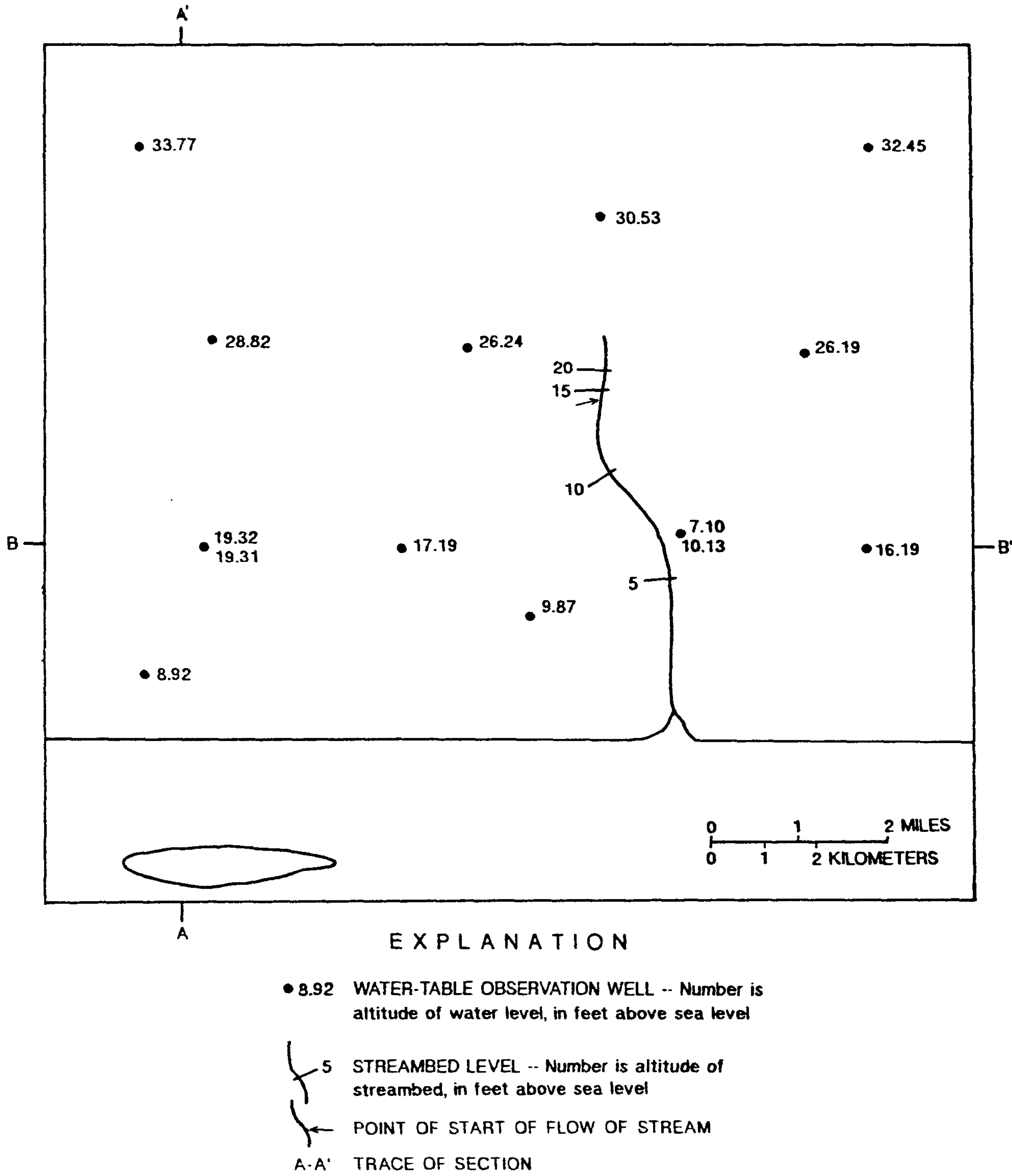
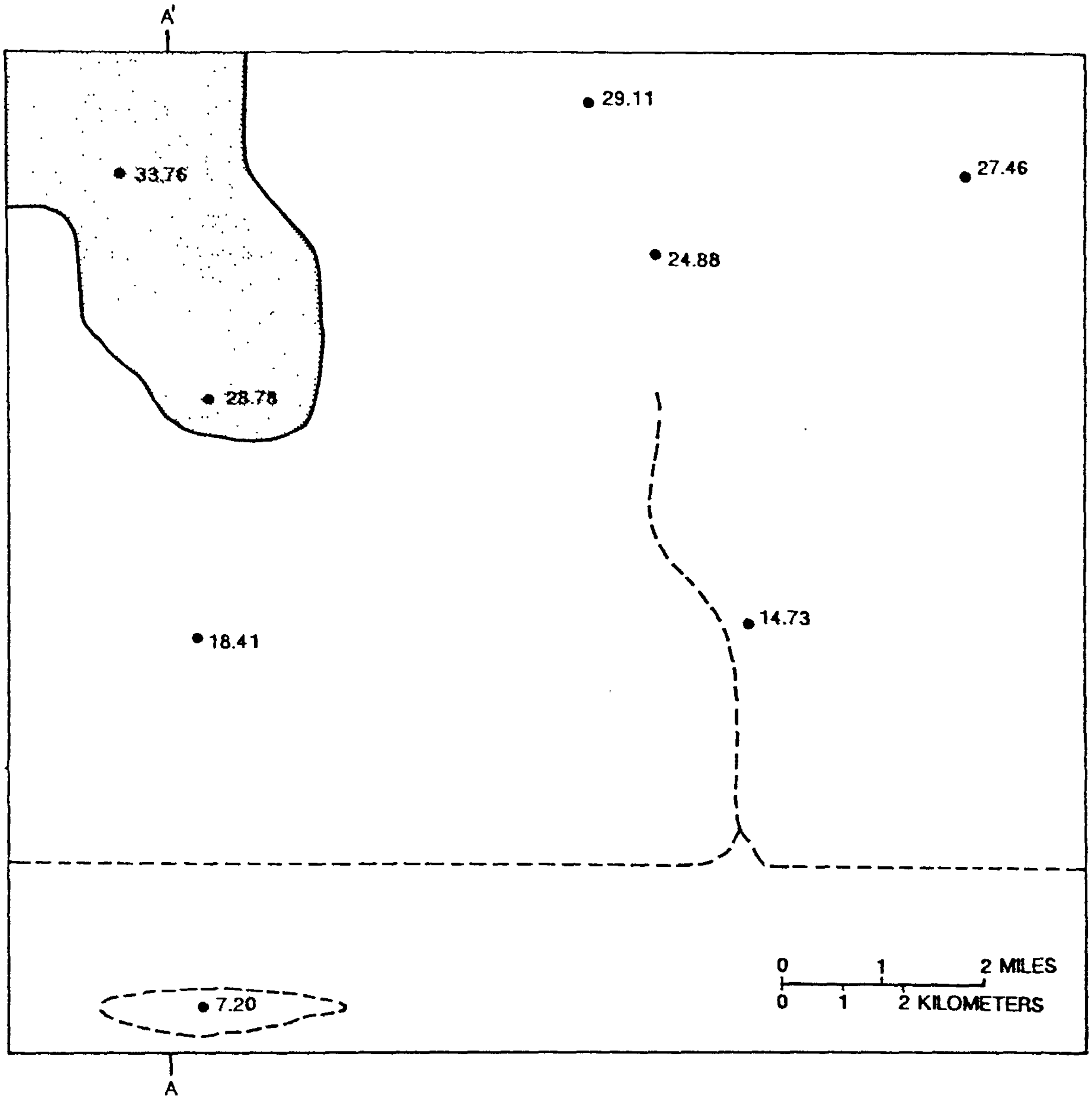


Figure S-13.--Measured heads in the water-table aquifer in a ground-water system with a discontinuous confining unit.



EXPLANATION

AREA OF HOLE IN CONFINING UNIT

• 14.73 OBSERVATION WELL SCREENED IN THE CONFINED AQUIFER -- Number is altitude of water level, in feet above sea level

A-A' TRACE OF SECTION

Figure 9-15.--Measured heads in the confined aquifer and location of hole in the overlying confining unit.

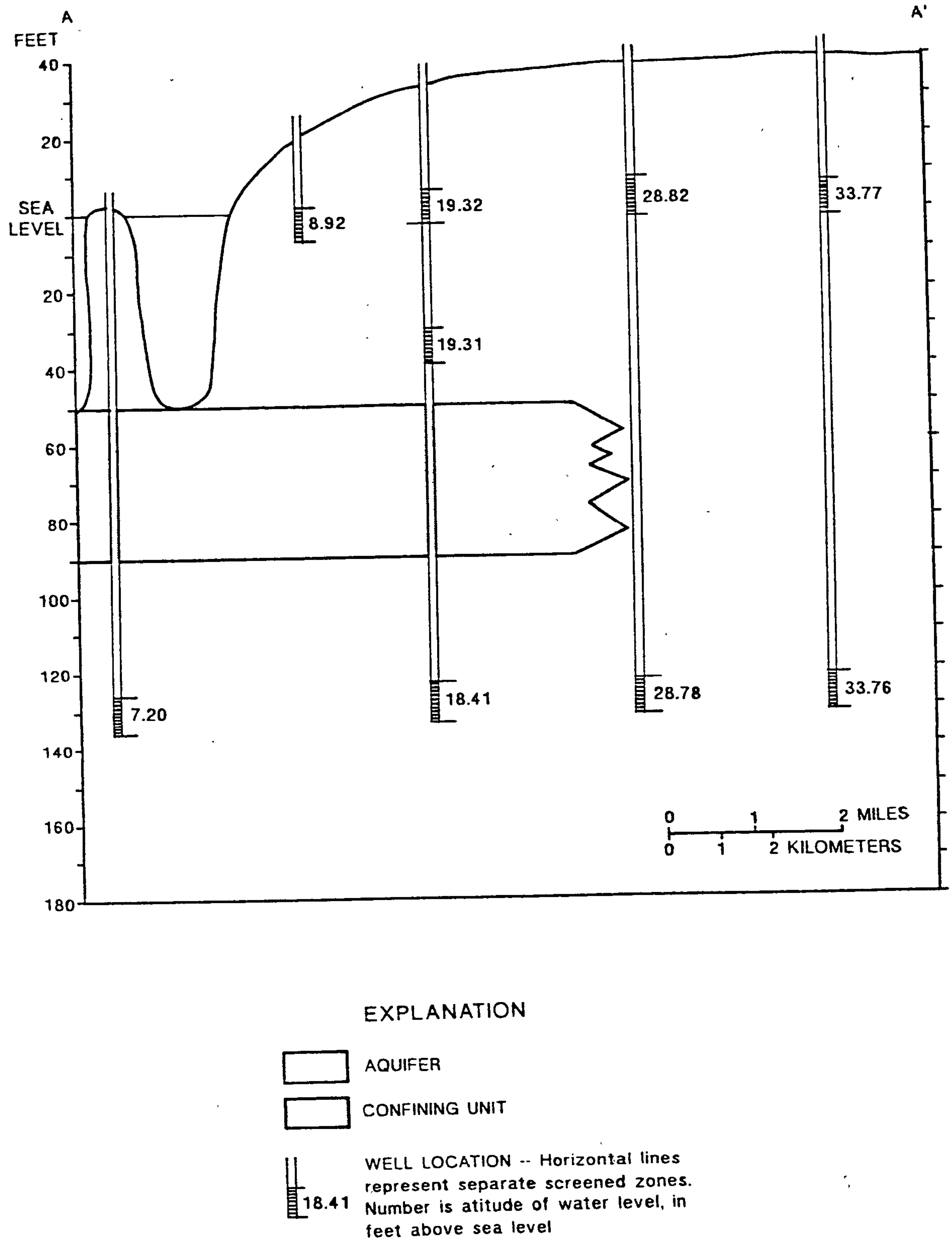


Figure 3-16.--North-south-trending hydrogeologic section showing heads measured in a ground-water system with a discontinuous confining unit. (*Location of section A-A'* is shown in fig. 3-2.)

Exercise (3-2)--Flow Net Beneath an Impermeable Wall

A cross section of a ground-water flow system near a partially penetrating impermeable wall is shown in figure 3-19. This section depicts a two-dimensional flow field. Flow is assumed to occur only in the plane of the figure; that is, there is no flow perpendicular to the plane of the figure. The flow field has unit thickness--that is, the thickness of the flow system perpendicular to the page is 1 ft. The wall is impermeable, as are the bottom and lateral boundaries. The "top" of the ground-water flow system to the left of the impermeable wall lies 5 ft beneath a standing body of water whose surface elevation remains constant at 55 ft above the impermeable bottom boundary (datum). To the right of the impermeable wall the surface of the aquifer material is at an elevation of 25 ft above datum; ground water discharges at this surface to nearby surface drains and by evaporation. The earth material near the impermeable wall is fine sand, which is assumed to be isotropic and homogeneous.

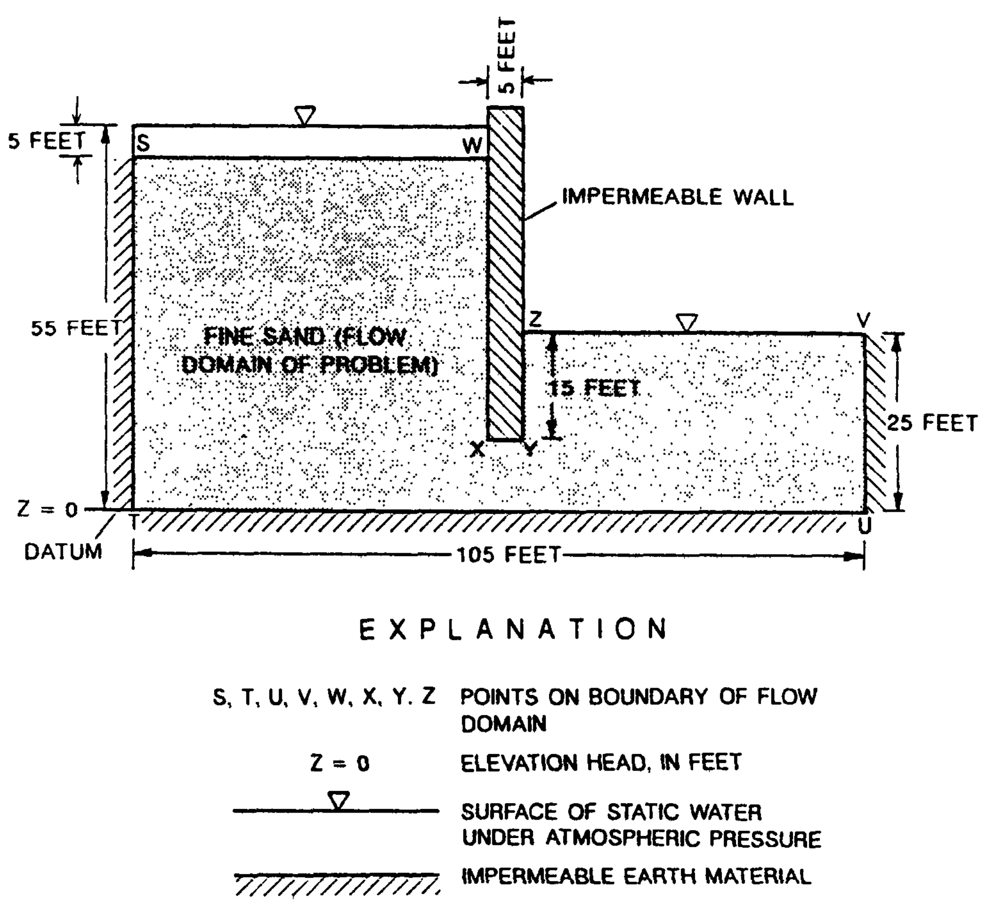
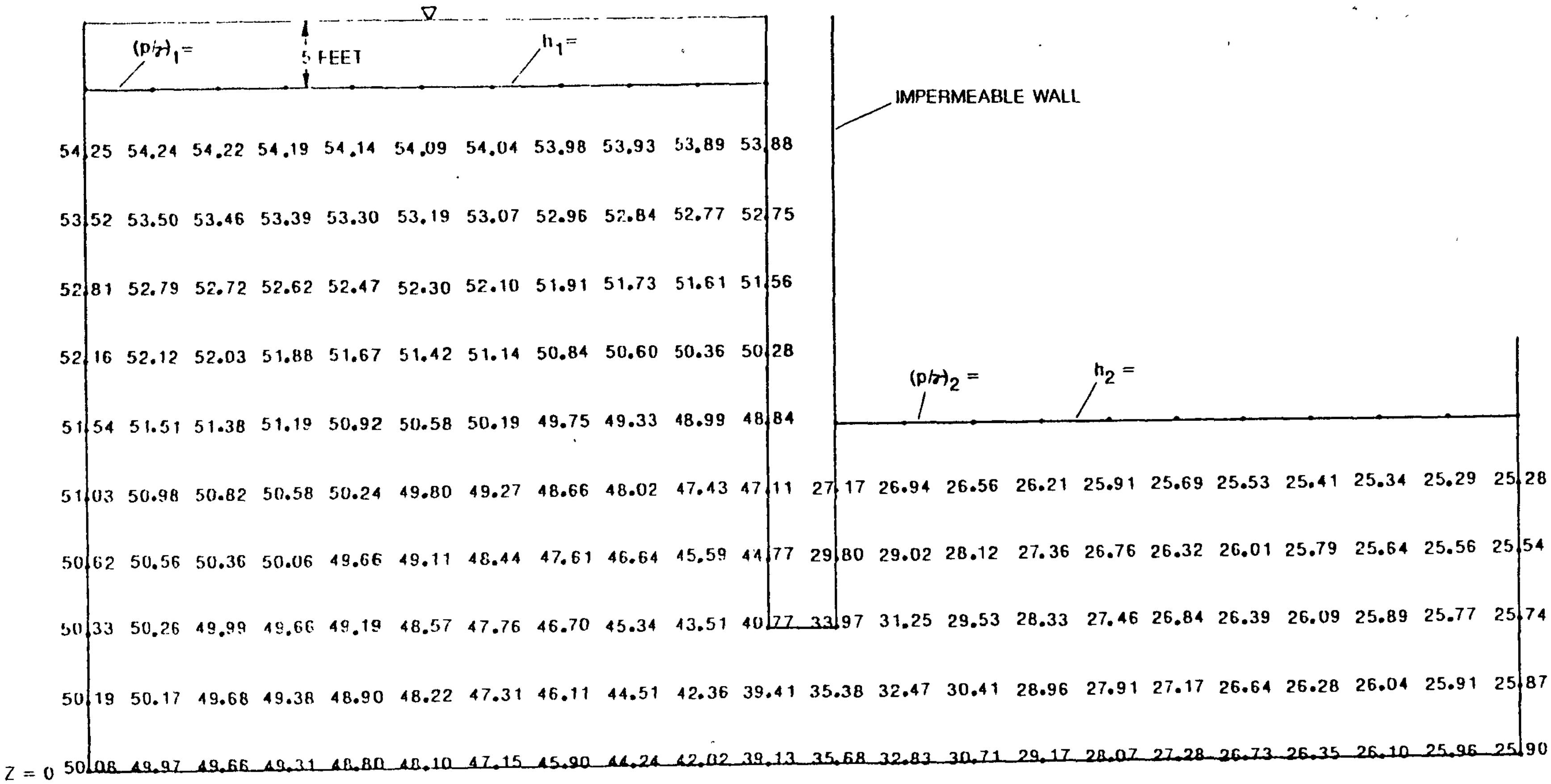


Figure 3-19.--Vertical section through a ground-water flow system near a partially penetrating impermeable wall.



EXPLANATION

- ▽ SURFACE OF STATIC WATER UNDER ATMOSPHERIC PRESSURE
- —— IMPERMEABLE MATERIAL
- • NODE IN DISCRETIZED SYSTEM AND DECIMAL POINT OF HEAD VALUE AT NODE
- 53.39 HEAD AT NODE, IN FEET

Z IS ELEVATION HEAD, p/γ IS PRESSURE HEAD, h IS TOTAL HEAD, IN FEET

DISTANCE BETWEEN NODES = 5 FEET

HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY

Figure 9-21.--Worksheet for preparation of a flow net for a ground-water system near an impermeable wall.

Table 3-4.--Format for calculation of stream functions in impermeable wall problem (page 1 of 2).

[For locations of numbered blocks, traverse DE, and plotting positions for stream functions p1, p2, ..., see figure 2; C_{block} is hydraulic conductance of discretized block which equals KA/L, where K = hydraulic conductivity of earth material in block, A = cross-sectional area of block perpendicular to direction of ground-water flow, and L = length of block; h_1 and h_2 are head values at nodes located at ends of block; $\Delta h = h_1 - h_2$; q_{block} = flow through a single block; Σq_{block} = flow in a numbered block plus the flows through all lower-numbered blocks (cumulative sum of block flows in traverse); Q_{total} = total flow through the ground-water system beneath the impermeable wall; ft= feet; ft^2 = square feet; ft^3 = cubic feet; Ψ = stream function]

TRAVERSE D E

BLOCK NUMBER	$C_{block} =$ KA/L (ft^2/day)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	$q_{block} = C\Delta h$ (ft^3/day)	Σq_{block} (ft^3/day)	$\Psi = \frac{\Sigma q_{block}}{Q_{total}}$	PLOTTING POSITION FOR STREAM FUNCTIONS
1	22.5	55.00	54.25	.75	16.88	16.88		P1
2								P2
3								P3
4								P4
5								P5
6								P6
7								P7
8								P8
9								P9
10								P10
11							1.00	P11

Table 3-4.--Format for calculation of stream functions in impermeable wall problem (page 2 of 2).

BLOCK NUMBER	C_{block} = KA/L (ft^2/day)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	q_{block} = $C\Delta h$ (ft^3/day)	Σq_{block} (ft^3/day)	$\Psi = \dfrac{\Sigma q_{block}}{Q_{total}}$

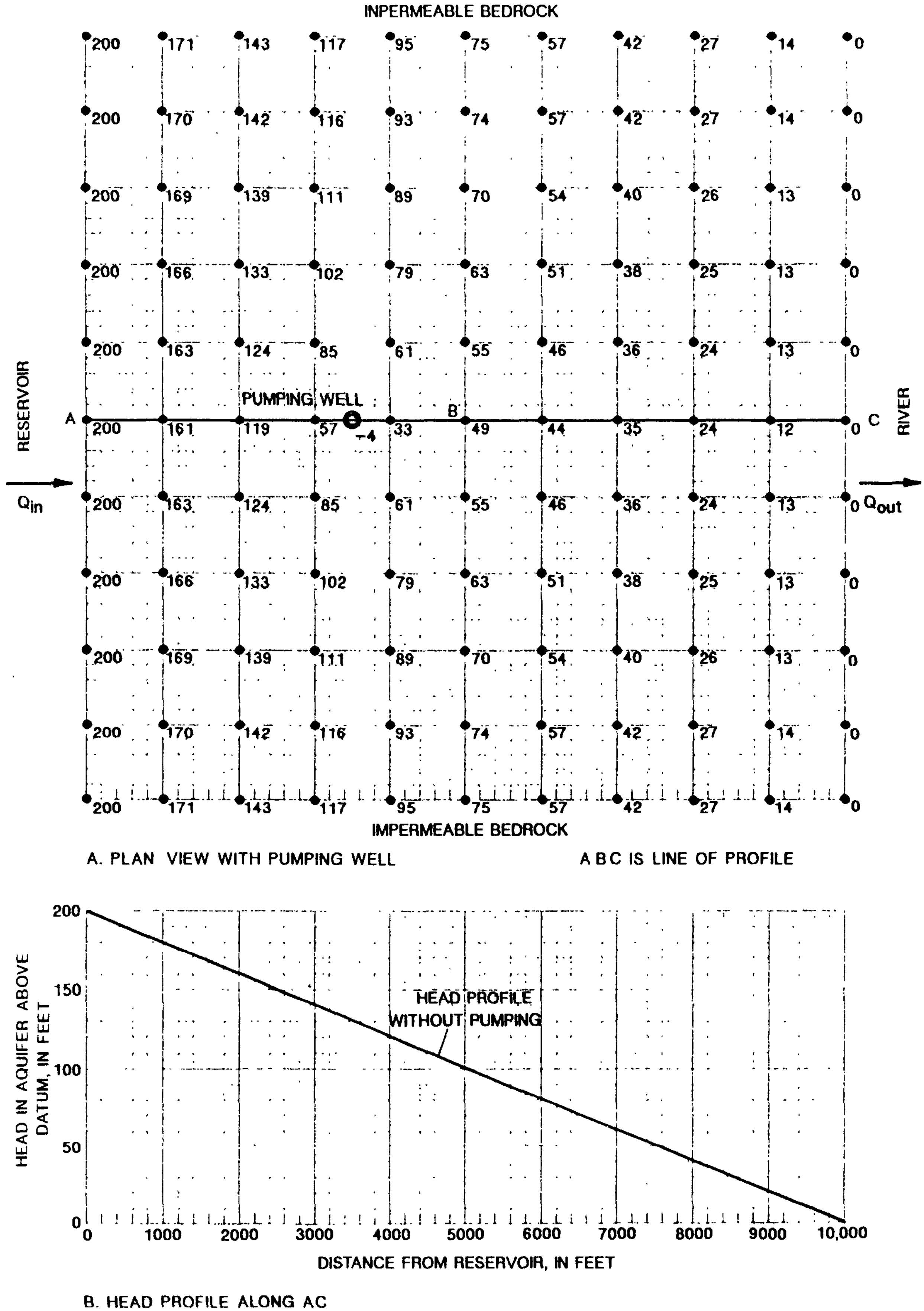
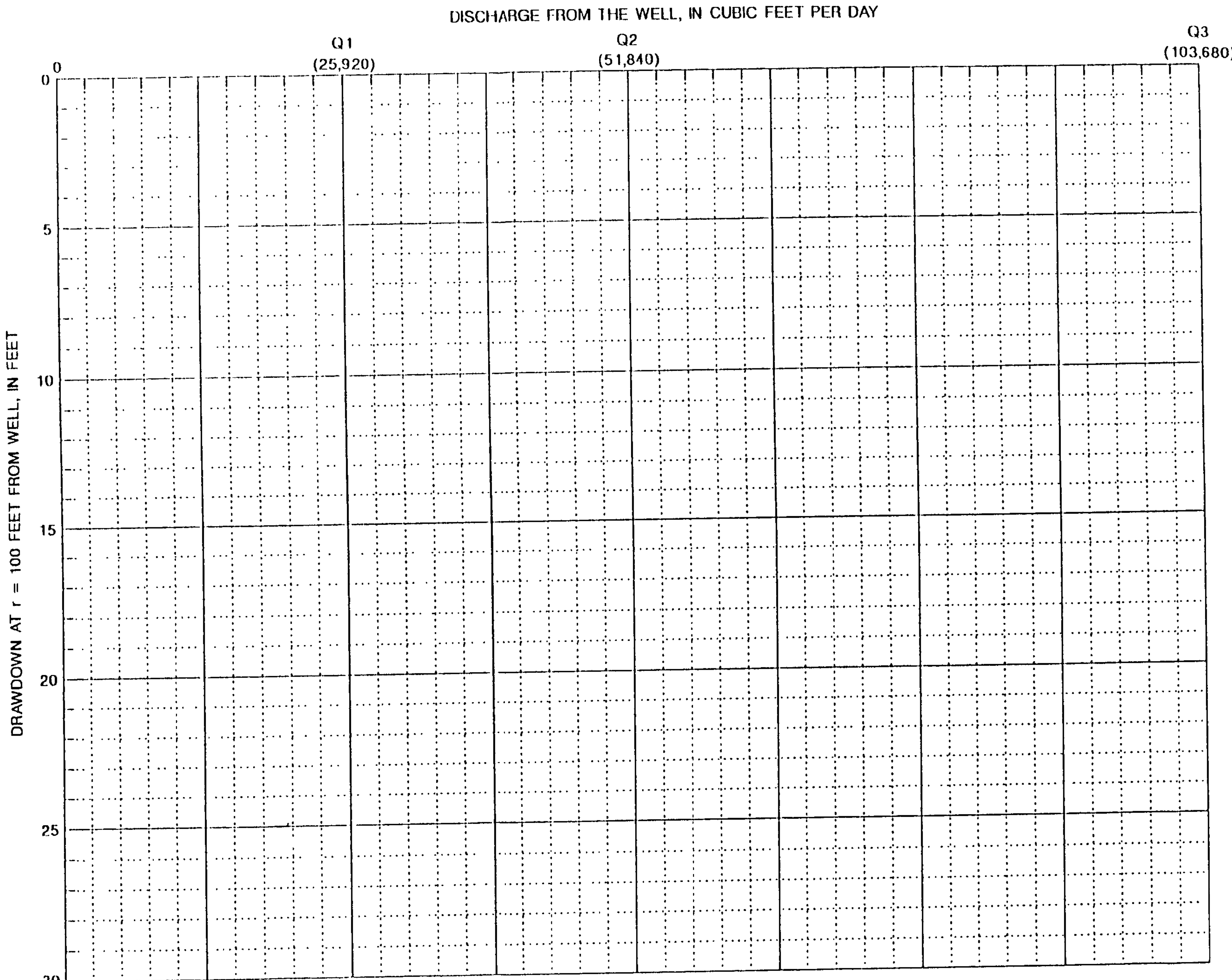


Figure 9-95.--(A) Heads in the stressed aquifer determined by numerical simulation when the pumping rate of the well is 9.1 cubic feet per second. (B) Graph for plotting head profile using data from (A).



Note: Q1, Q2 is well discharge, in cubic feet per day;
r is distance from well, in feet.

Figure 4-5.--Graph for plotting calculated drawdowns.

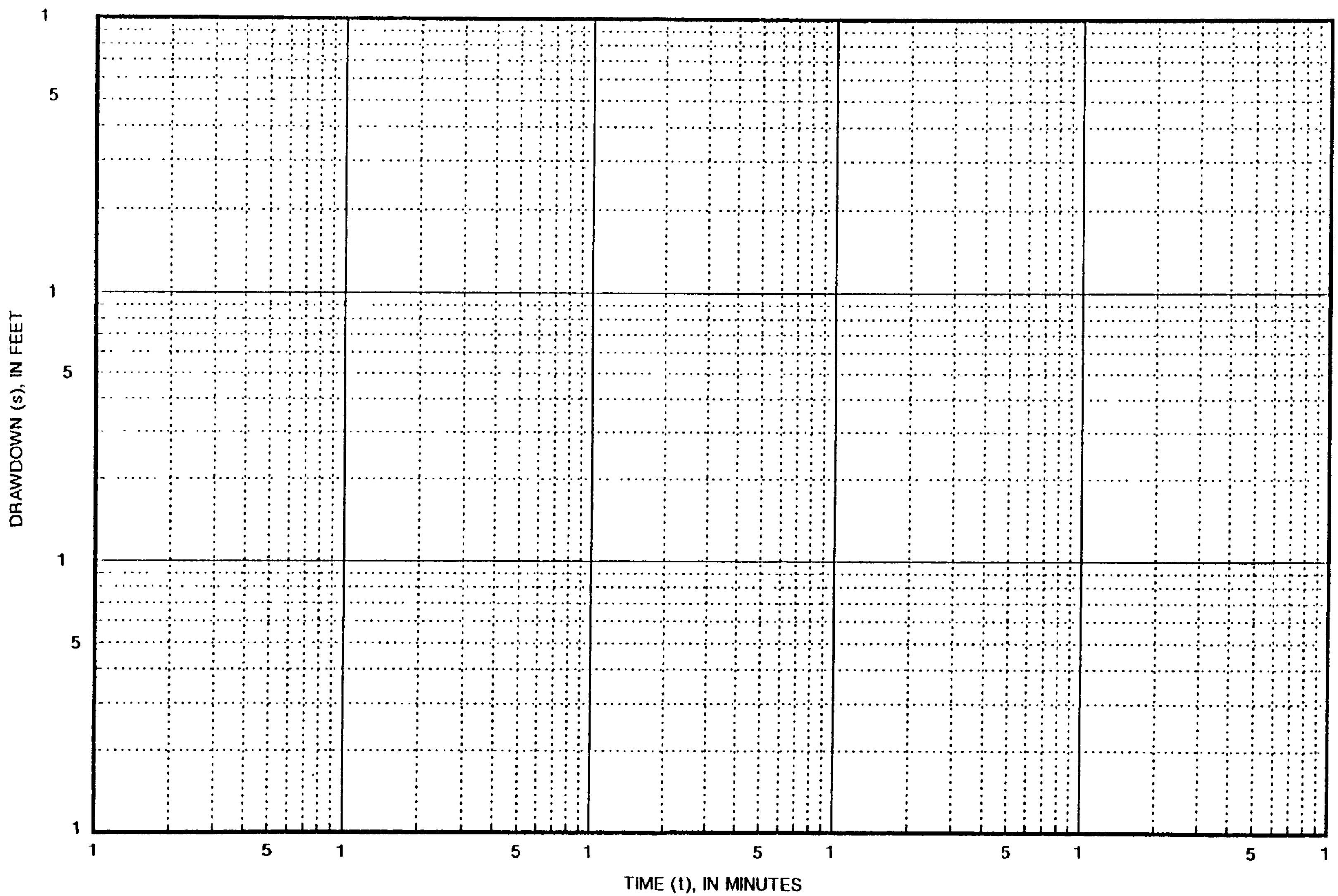


Figure 4-7.--Graph for plotting aquifer-test data from a single observation well in a Theis analysis (drawdown (s) against time (t)).

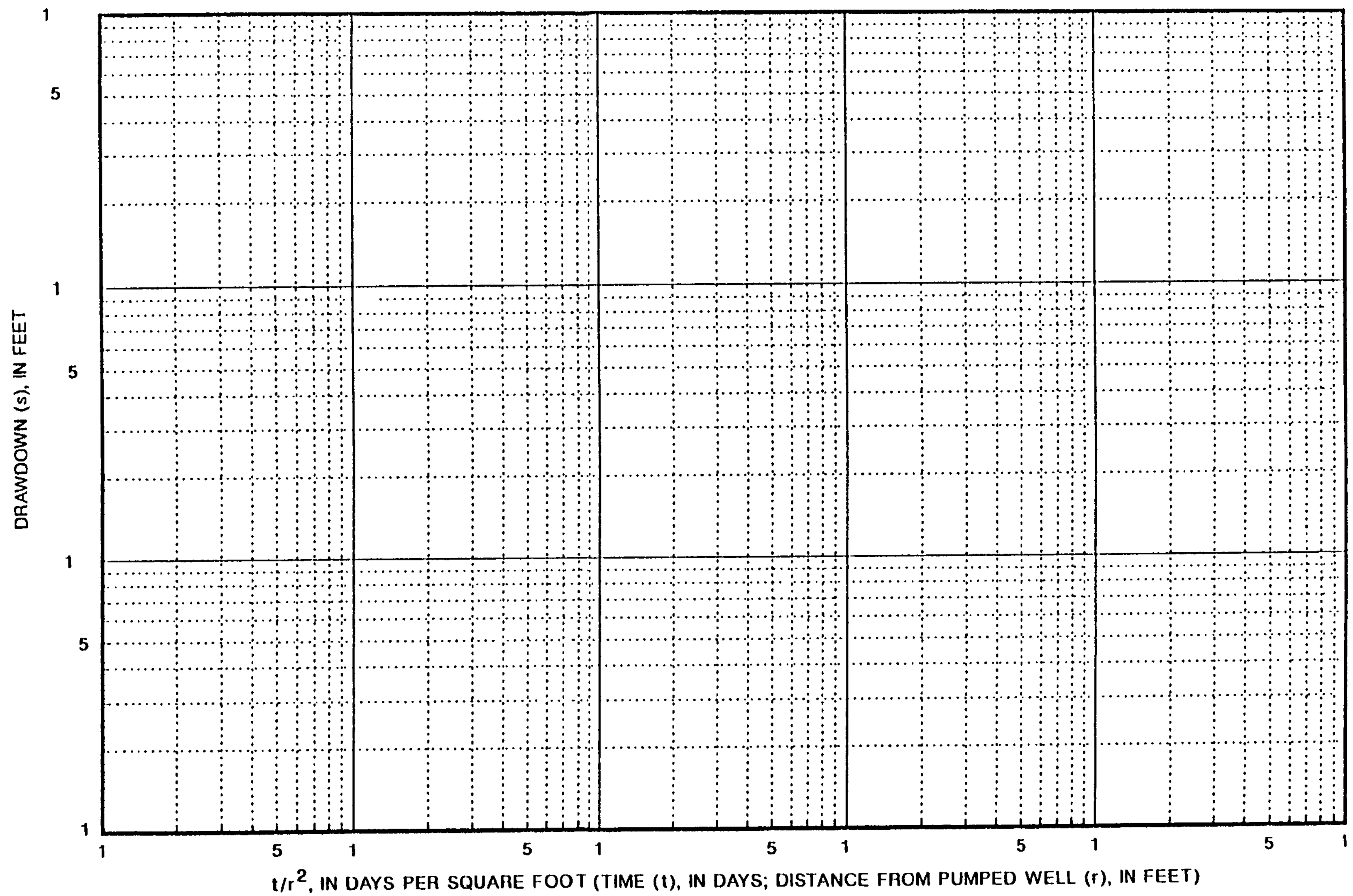


Figure 4-8.--Graph for plotting aquifer-test data from more than one observation well in a Theis analysis (drawdown (s) against t/rr).

Table 4-8.--Format for calculation of drawdowns at specified distances from the pumping well

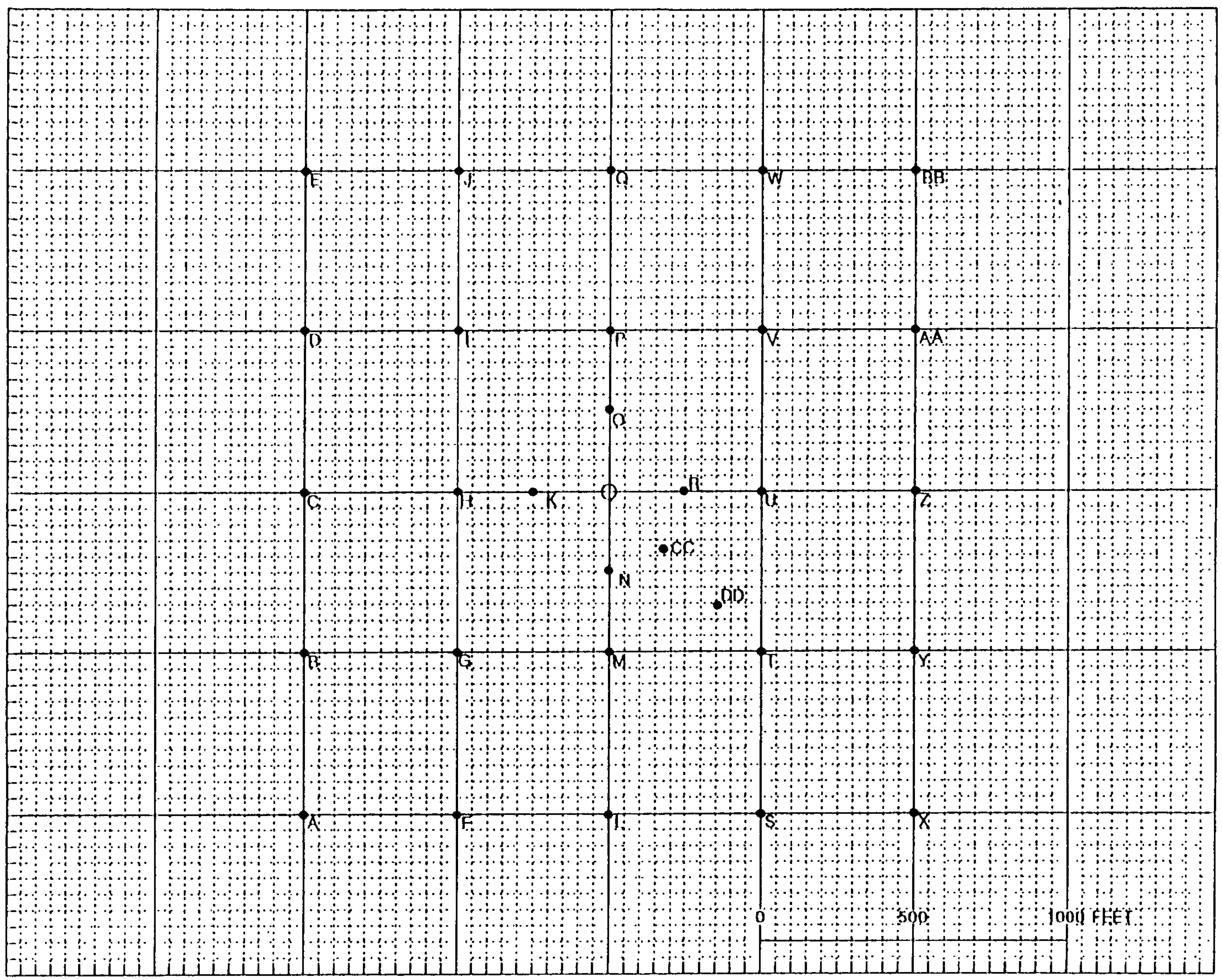
[r_e is distance from pumping well at which drawdown is negligible; r_1 is distance from pumping well at which drawdown equals s_1; $\ln$ is natural logarithm; Q is pumping rate of well; T is transmissivity of aquifer]

Preliminary calculation: $\frac{-Q}{2\pi T}$ = constant =

r_1 (FEET)	$\ln(r_e/r_1)$	S_1 (FEET) = $-\frac{Q}{2\pi T} \ln(r_e/r_1)$
250		
500		
707		
1000		
1118		
1414		

Table 4-4.--Format for calculation of absolute heads at specified reference points

WELL IDENTIFICATION LETTER	INITIAL PREPUMPING HEAD (FEET)	DISTANCE FROM WELL (r), IN FEET	DRAWDOWN DUE TO PUMPING (FEET)	HEAD = INITIAL HEAD-DRAWDOWN, IN FEET
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				
K				
L				
M				
N				
O				
P				
Q				
R				
S				
T				
U				
V				
W				
X				
Y				
Z				
AA				
BB				
CC				
DD				

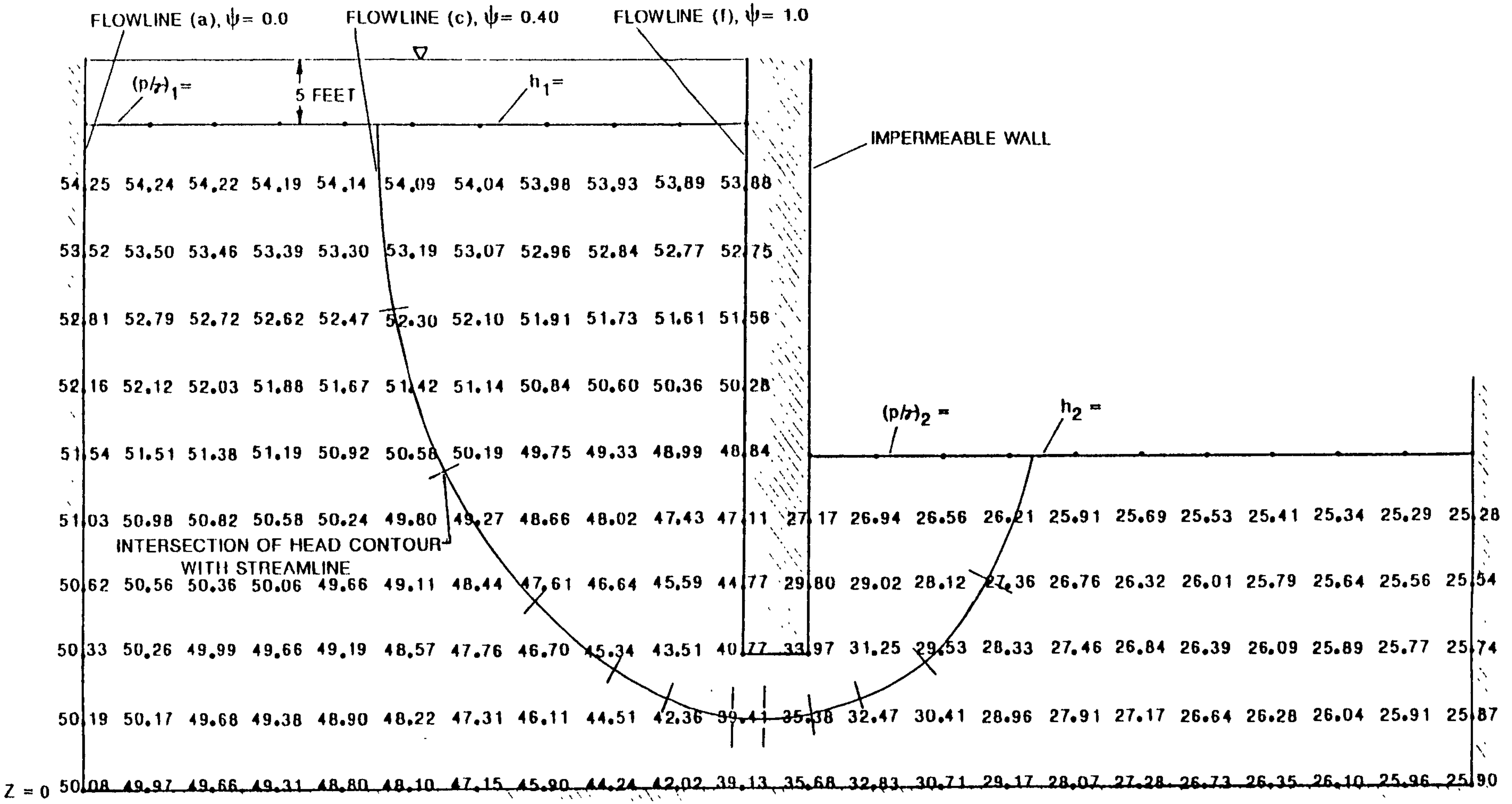


EXPLANATION

○ LOCATION OF PUMPING WELL

●D REFERENCE POINT WITH IDENTIFICATION LETTER

Figure 4-10.--Location map for plotting and contouring head distribution resulting from pumping.



EXPLANATION

- ▽ SURFACE OF STATIC WATER UNDER ATMOSPHERIC PRESSURE
- IMPERMEABLE MATERIAL
- • NODE IN DISCRETIZED SYSTEM AND DECIMAL POINT OF HEAD VALUE AT NODE

53.39 HEAD AT NODE, IN FEET

Z IS ELEVATION HEAD, p/γ IS PRESSURE HEAD, h IS TOTAL HEAD, IN FEET

DISTANCE BETWEEN NODES = 5 FEET

HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY

$\psi = 0.40$ VALUE OF STREAM FUNCTION ASSOCIATED WITH FLOWLINE

Figure 5-5.--Head "map" of a ground-water system with an impermeable wall for plotting cumulative time of travel from the system inflow boundary along two bounding flowlines and an internal flowline.

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 1 of 3)

[h is head at a node or other point in flow system; L is distance between two points on a flowline at which head is known; Δh is difference in head between two points on a flowline; t is time of travel between two points on a flowline; Σt is time of travel from inflow boundary to point on flowline]

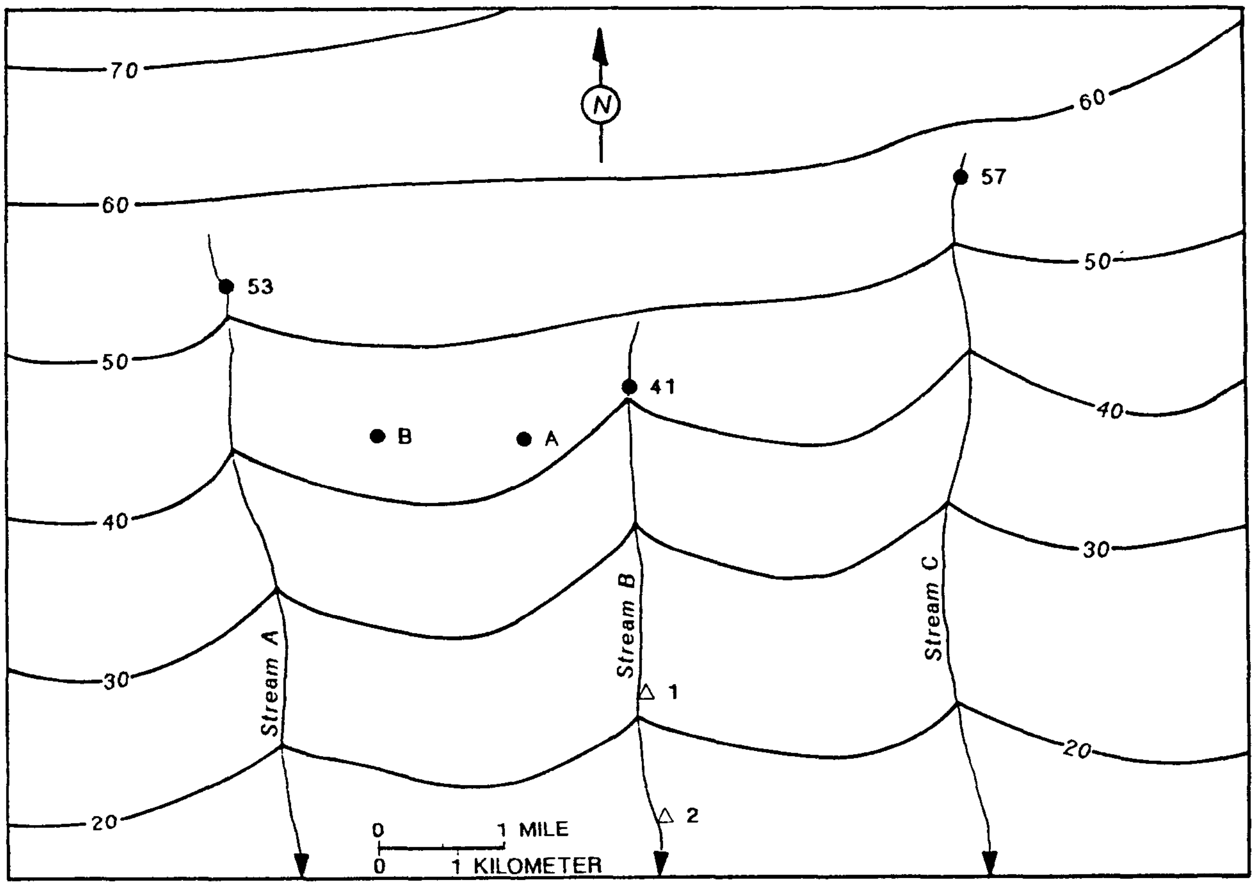
h (feet)	L (feet)	Δh (feet)	t (days) = $\frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 2 of 3)

h (feet)	L (feet)	Δh (feet)	t (days) = $\frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines impermeable-wall problem (page 3 of 3)

h (feet)	L (feet)	Δh (feet)	t (days) = $\dfrac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)



EXPLANATION

—20— WATER-TABLE CONTOUR -- Shows altitude of water table. Contour interval 10 feet Datum is sea level

● 41 LOCATION OF START OF FLOW OF STREAM -- Number is altitude of stream, in feet above sea level

△ 2 LOCATION AND NUMBER OF STREAM DISCHARGE MEASUREMENT POINT

Figure 5-6.--Hypothetical water-table map of an area underlain by permeable deposits in a humid climate.

Table 5-2.--Format for calculating solute concentrations when the dispersion coefficient D = 10 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, square feet per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

C = concentration of solute at point in plume at specified time, in mg/L
C_0 = solute concentration of source, in mg/L
L = distance from source, in feet
v = average linear velocity of ground water, in ft/d
t = elapsed time since introduction of solute at source, in days
D = dispersion coefficient, in ft^2/d
erfc = complementary error function (see text)

Preliminary calculation:

For D = 10 ft^2/d, $\left(\frac{L - vt}{2\sqrt{Dt}}\right) =$

L (feet)	$\frac{L-2,000}{200}$	$\operatorname{erfc}\left(\frac{L-2,000}{200}\right)$ [1]	C = 50 mg/L $\operatorname{erfc}\left(\frac{L-2,000}{200}\right)$
1,500			
1,600			
1,700			
1,800			
1,900			
2,000			
2,100			
2,200			
2,300			
2,400			

[1] erfc(-x) = 1 + erf(x).

Table 5-8.--Format for calculating solute concentrations when the dispersion coefficient D = 100 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, feet squared per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

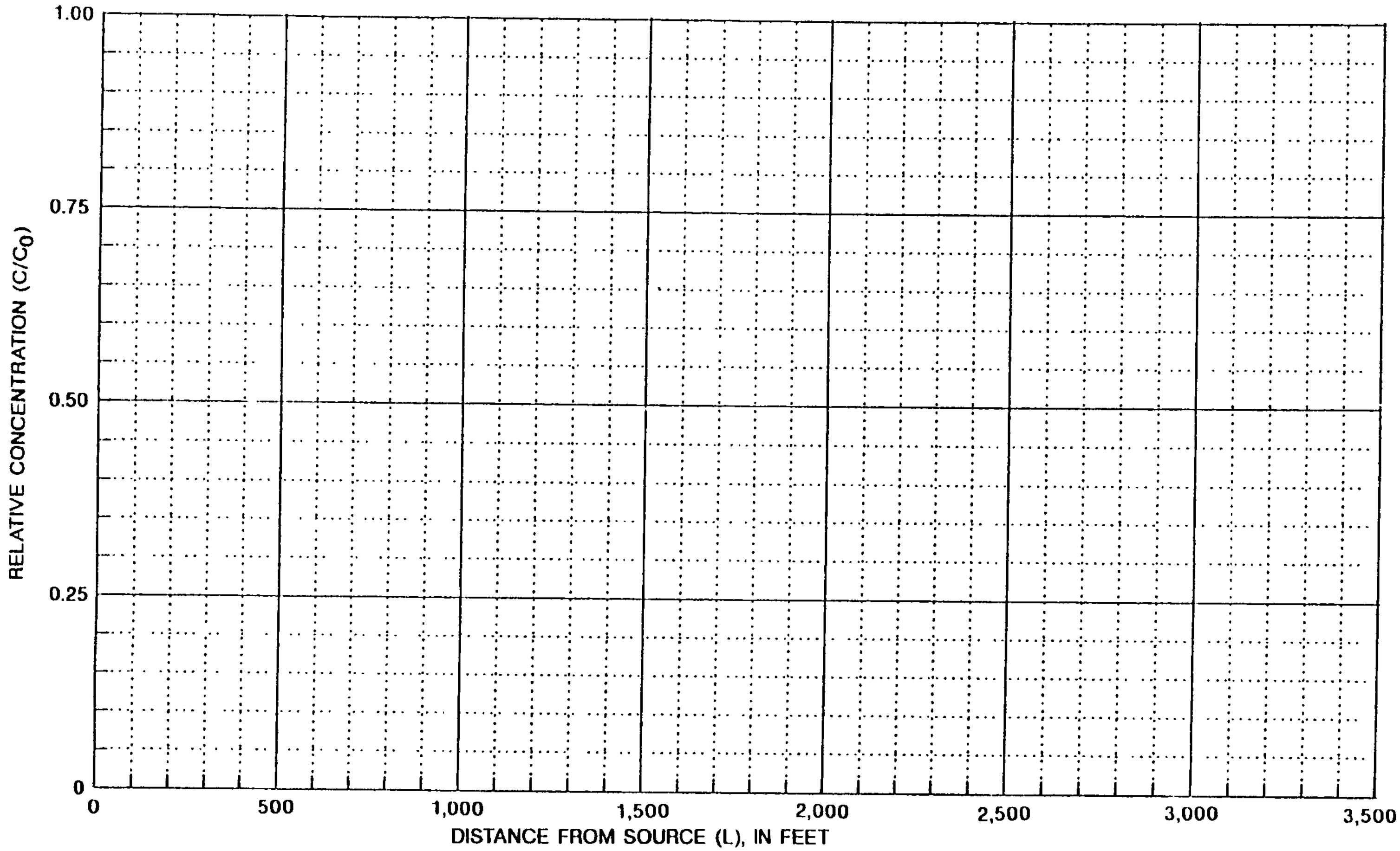
C = concentration of solute at point in plume at specified time, in mg/L
C_0 = solute concentration of source, in mg/L
L = distance from source, in feet
v = average linear velocity of ground water, in ft/d
t = elapsed time since introduction of solute at source, in days
D = dispersion coefficient, in ft^2/d
erfc = complementary error function (see text)

Preliminary calculation:

For D = 100 ft^2/d, $\left(\frac{L - vt}{2\sqrt{Dt}}\right) =$

L (feet)	$\frac{L-2,000}{632.5}$	$\operatorname{erfc}\left(\frac{L-2,000}{632.5}\right)$ [1]	C = 50 mg/L $\operatorname{erfc}\left(\frac{L-2,000}{632.5}\right)$
1,000			
1,250			
1,500			
1,750			
2,000			
2,250			
2,500			
2,750			
3,000			

[1] erfc(-x) = 1 + erf(x).



EXPLANATION

C_0 = SOLUTE CONCENTRATION OF SOURCE

C = CONCENTRATION OF SOLUTE AT POINT IN PLUME

Figure 5-9.--Graph for plotting relative concentration against distance from source for two values of the dispersion coefficient and an elapsed time of 1,000 days.