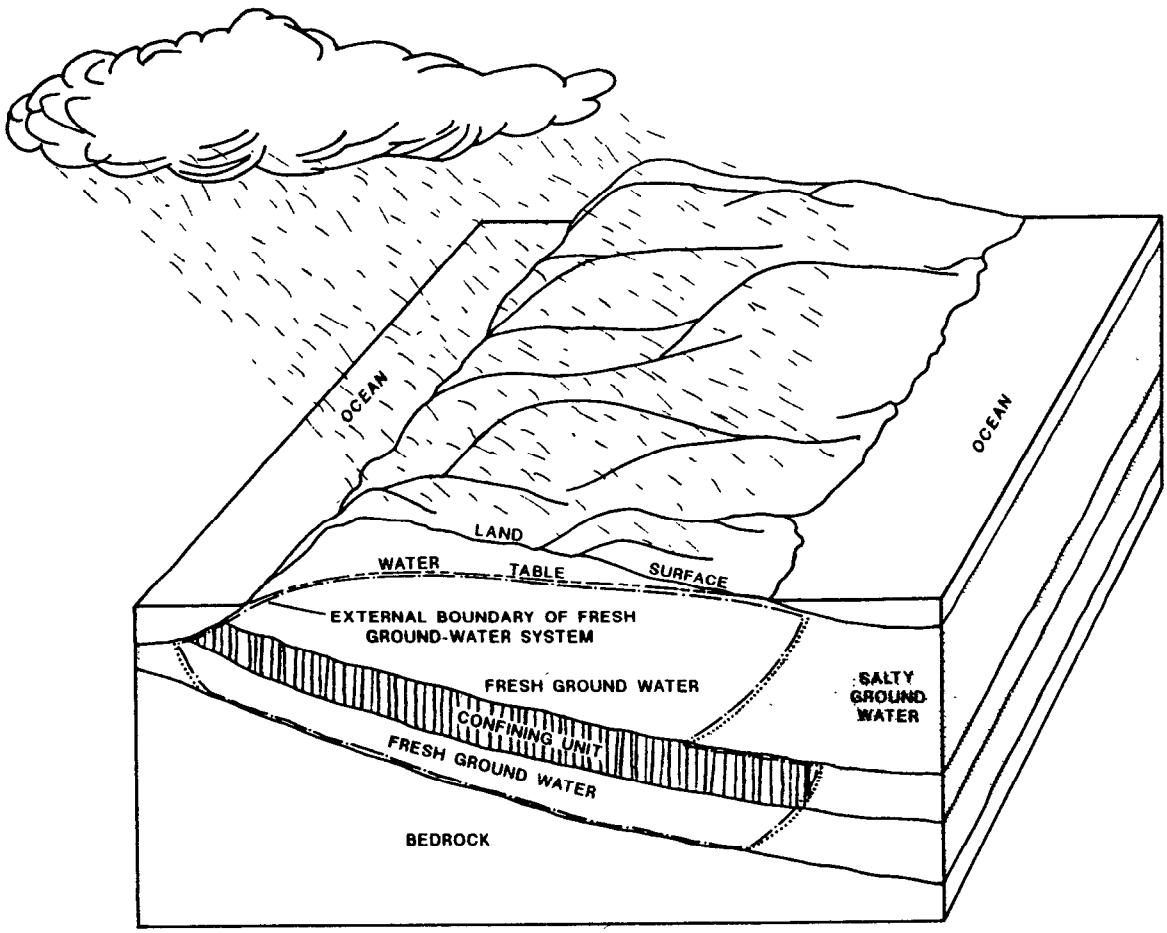
STUDY GUIDE FOR A BEGINNING COURSE IN GROUND-WATER HYDROLOGY: PART I -- COURSE PARTICIPANTS



U.S. GEOLOGICAL SURVEY
Open-File
Report 90-183

USGS
science for a changing world

Click here to return to USGS Publications

SECTION (5)--GROUND-WATER CONTAMINATION

The goal of this section of the course is to introduce the physical mechanisms of solute movement in ground water. Further treatment of the vast and rapidly developing area of science and technology related to ground-water contamination can be found in the extensive literature that is available or in additional training courses.

Background and Field Procedures Related to Ground-Water Contamination

Assignments

*Study Fetter (1988), p. 367-389, 406-442; Freeze and Cherry (1979), p. 384-457; or Todd (1980), p. 344-346.

The depth of topical coverage in this section of the course will depend primarily on the time available and the interests of the instructors and participants. A useful and readable discussion on the conceptualization and organization of a field study involving solute transport, along with a pertinent bibliography, is provided by Reilly and others (1987).

Physical Mechanisms of Solute Transport in Ground Water

Assignments

*Study Fetter (1988), p. 389-405.

*Study Note (5-1)--Physical mechanisms of solute transport in ground water

*Work Exercise (5-1)--Ground-water travel times in the flow system beneath a partially penetrating impermeable wall

*Work Exercise (5-2)--Advective movement and travel times in a hypothetical stream-aquifer system

*Study Note (5-2)--Analytical solutions for analysis of solute transport in ground water

*Work Exercise (5-3)--Application of the one-dimensional advective-dispersive equation

The background for this section is provided in Note (5-1), which is an introductory discussion of the basic physical mechanisms of solute movement--advection and dispersion. Exercises (5-1) and (5-2) consider only advective movement of ground water and involve calculation of travel times by using the average linear velocity (Darcy velocity divided by porosity). In Exercise (5-1) travel times are calculated in a vertical cross-section of a simple flow system, and in Exercise (5-2) travel times are calculated in plan view.

Comments on the field application of analytical solutions to the advective-dispersive differential equation are provided in Note (5-2), and Exercise (5-3) involves numerical calculations with one of the simplest analytical solutions.

Note (5-1).--Physical Mechanisms of Solute Transport in Ground Water[1]

The following section on physical mechanisms of solute transport in ground-water systems (1) defines and describes the two physical mechanisms advection and dispersion, (2) emphasizes the interdependence of these mechanisms and the implications of the scale of analysis in transport studies, and (3) addresses the primary goal of the study of physical mechanisms--to define a working approximation of the three-dimensional ground-water-flow velocity field affecting the contaminant plume, by building upon the information and knowledge gained in the hydraulic analysis and description of solute distribution.

Advection is the process by which solutes are transported by the bulk motion of the flowing ground water (Freeze and Cherry, 1979, p. 75). The bulk motion of the flowing ground water is characterized by the average linear velocity (v), which is defined as

$$v = -\frac{K}{n}\frac{dh}{dl}$$

where

K = hydraulic conductivity (L/T),
n = porosity (dimensionless),
h = hydraulic head (L), and
l = distance along a flowline (L).

The Darcian velocities developed by using a flow model differ from the actual velocities required for transport analysis in that the average linear velocity (v) is the Darcian velocity (q) divided by porosity (n); that is,

$$v = q/n.$$

Thus, a new, spatially varying parameter, the porosity (n) of the porous material in the neighborhood of the point at which velocity is calculated, is introduced. Errors in estimating the magnitude and distribution of porosity produce proportional errors in estimates of actual ground-water velocity.

---

[1] This note on the physical mechanisms of solute transport in ground-water systems is from Reilly and others (1987, p. 21-29).

A more subtle difference between the velocity field developed by using a flow model designed for basic hydraulic analysis and the velocity field required for transport analysis is the scale at which the physical processes are considered. In the analysis of ground-water flow, the flow field usually is studied at a scale that is much larger in area than the area of a contaminant plume, because an accurate definition of boundary conditions is required to achieve a physically reasonable simulation. At this regional scale, the properties of the porous medium and variations in velocity are averaged. In the analysis of the velocity field for transport analysis, however, a more detailed scale is required. This finer scale permits the representation of local variations in hydraulic conductivity resulting from the heterogeneous nature of the porous media to be represented if possible. It also permits greater resolution in describing changes in velocity (both magnitude and direction) due to the three-dimensional movement of the ground water in response to local conditions.

Regardless of the degree of detail that is included in the representation of the flow field used to calculate the ground-water velocities, however, variations between actual and calculated velocities remain that cannot be accounted for explicitly. In any calculation of advective transport, whether by numerical model or by using an analytical solution, we assume that the velocity is uniform or varies in a simple way over specified regions of the flow field. For example, suppose a uniform flow in the x direction is simulated using the array of model nodes shown in figure 5-1. In calculations of solute transport using numerical models, velocity in the x direction is assumed to be uniform or to vary in a simple way (such as bilinear interpolation) in both magnitude and direction over the rectangular region R, which extends between adjacent nodes in the x direction. This uniformity is vertical as well as areal--that is, within the area R, velocity is assumed to be constant over the vertical depth interval represented by the simulation. By contrast, the actual ground-water velocity in the block of aquifer represented by R would exhibit different spatial variations depending on the scale at which the velocity is considered.

At the microscopic (pore) scale, velocity varies from a maximum along the centerline of each pore to zero along the pore walls, as shown in figure 5-2(A); both the centerline velocity and the velocity distribution differ in pores of different size. In addition, flow direction changes as the fluid moves through the tortuous paths of the interconnecting pore structure, as shown in figure 5-2(B).

At a larger (macroscopic) scale, local heterogeneity in the aquifer causes both the magnitude and direction of velocity to vary as the flow concentrates along zones of greater permeability or diverges around pockets of lesser permeability. In this discussion, the term "macroscopic heterogeneity" is used to suggest variations in features large enough to be readily discernible in surface exposures or test wells, but too small to map (or to represent in a mathematical model) at the scale at which we are working. For example, in a typical problem involving transport away from a landfill or waste lagoon, macroscopic heterogeneities might range from the size of a baseball to the size of a building.

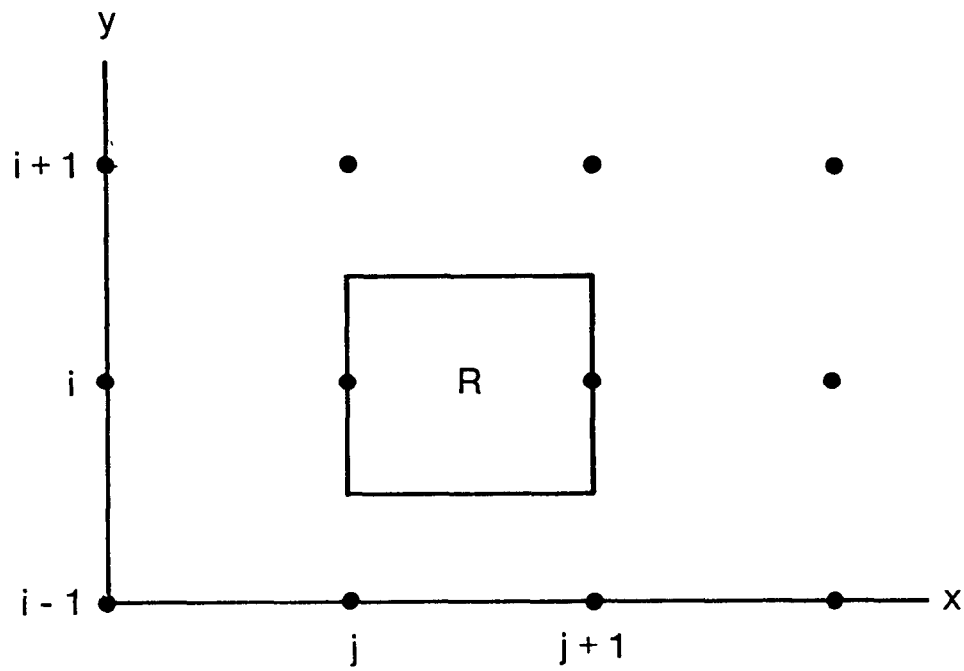


Figure 5-1.--Array of model nodes with region R between two representative nodes (i,j and i,j+1).

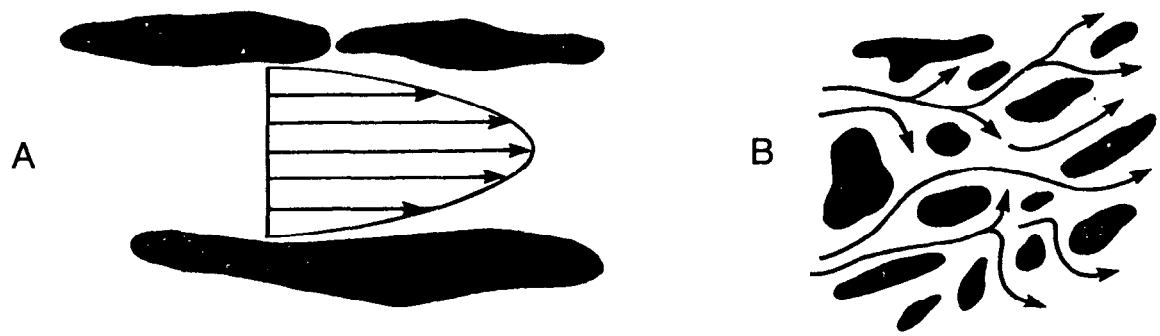


Figure 5-2.--(A) Approximate fluid velocity distribution in a single pore, and (B) tortuous paths of fluid movement in an unconsolidated porous medium.

Figure 5-3, which shows some results of laboratory tracer experiments in heterogeneous media by Skibitzke and Robinson (1963), illustrates the effects of macroscopic heterogeneity. The net effect is to increase the spreading of the solute in the system. This effect tends to increase progressively with the scale of the heterogeneity. At a still larger scale, we can envision heterogeneities that could be mapped at the scale at which we are working, and which could be taken into account in our calculations of advective transport, but which simply have not been recognized in the field or accounted for in simulation. Mercado (1967; 1984) showed the results of this effect in an analysis of the spreading of injected water that was caused by stratified layers of different permeabilities.

The velocity variations described for these three scales share certain characteristics:

(1) they may occur both areally and vertically over the region R (fig. 5-1);

(2) they influence the distribution of ions or tracers moving through the system; and

(3) they are not represented in calculations of advective solute movement through the region R that are made using the uniform model velocity.

Using the velocity from the model, a tracer front introduced at the left side of region R would be predicted to traverse R as a sharp front moving with the average linear velocity of the water. In reality, however, a tracer front becomes progressively more irregular and diffuse as it moves through a porous medium. If we consider a vertical plane through the aquifer at the left edge of region R, the actual velocity varies in both magnitude and direction from one point to another; the same is true in the flow direction. Thus each tracer particle enters R at a velocity that generally is different from that of its neighbors, and each particle experiences a different sequence of velocities as it crosses R from left to right. Instead of a sharp front of advancing tracer as shown in figure 5-4(A), we see an irregular advance as in 5-4(B), with the forward part of the tracer distribution becoming broader and more diffuse with time. The pore-scale or microscopic velocity variations contribute only slightly to this overall dispersion; macroscopic variations contribute more significantly, whereas "mappable" variations generally have the largest effect.

If it were possible to generate a model or a computation that could account for all of the variations in velocity in natural aquifers, dispersive transport would not have to be considered (except for molecular diffusion); sufficiently detailed calculations of advective transport theoretically could duplicate the irregular tracer advance observed in the field. In practice, however, such calculations are impossible. Field data at the macroscopic scale never are available in sufficient detail, information at the "mappable" scale rarely is complete, and descriptions of microscopic scale variations are impossible except in a statistical sense. Even if complete data were available, however, an unreasonable computational effort would be required to define completely the natural velocity variations in an aquifer.

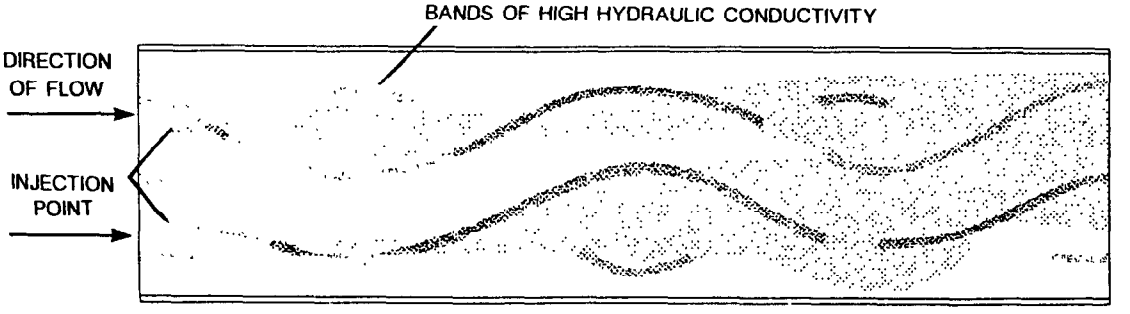


Figure 5-3.--Results of a laboratory experiment to determine the effects of macroscopic heterogeneity on a tracer. (Modified from Skibitzke and Robinson, 1963.)

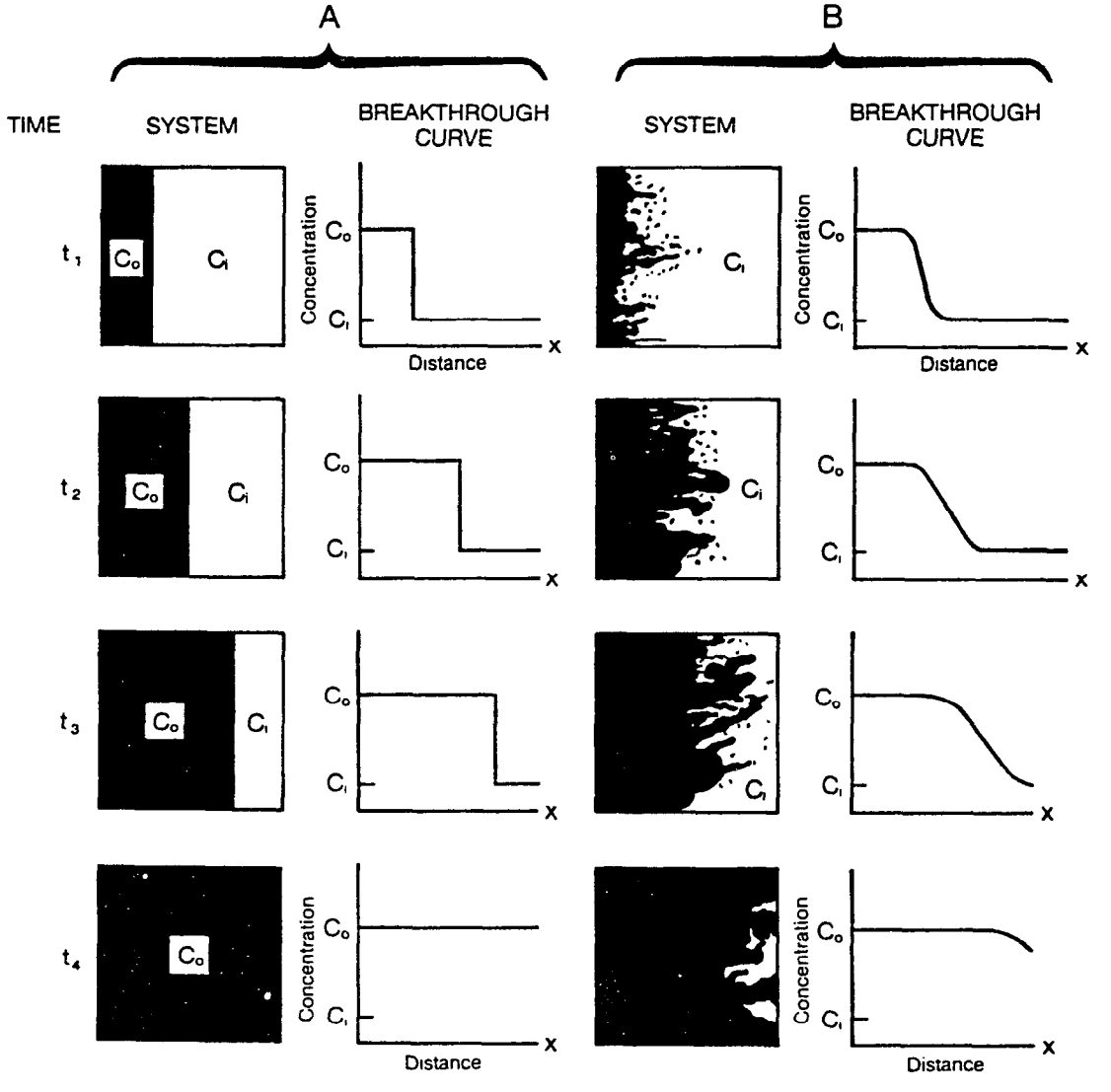


Figure 5-4.--Advance of a tracer for (A) a sharp front and (B) an irregular front.

The more closely we represent the actual permeability distribution of an aquifer, the more closely our calculations of advective transport match reality; the finer the scale of simulation, the greater is the opportunity to match natural permeability variations. In most situations, however, when both data collection and computational capacity have been extended to their practical limits, calculations of advective transport fail to match field observation; therefore, we must find a tractable method of adjusting or correcting such calculations.

Historically, the effort to develop such a method of correction followed the diffusion model. Diffusion had been analyzed successfully as a process of random particle movement which, in the presence of concentration change, results in a net transport proportional to the concentration gradient in the direction of decreasing concentration. In the case of a moving fluid, the random movement ascribed to diffusion was viewed as superimposed on the motion caused by the fluid velocity. Thus, the net movement of any solute particle could be regarded as the vector sum of an advective component and a random diffusive component.

By analogy, it was assumed that solute transport through porous media could be viewed in the same way--as the sum of an advective component in which solutes move with the average linear velocity of the fluid, and a random "dispersive" component superimposed on the advective motion (Saffman, 1959). In effect, dispersion was seen as the net transport with respect to a point moving with the average linear velocity of the fluid. Because the dispersive motion of solute particles was assumed to be random, the flux was taken to be proportional to the concentration gradient.

While many difficulties have been perceived with the concentration-gradient approach, no satisfactory alternative has yet been found. Currently, we know that some method is required to adjust and correct the results of advective-transport calculations. The method commonly employed is to postulate an additional transport that is proportional to the concentration gradient in the direction of decreasing concentration; however, the coefficient of proportionality is treated as a function of the average flow velocity.

This approach can be derived or justified mathematically if assumptions similar to those used in the analysis of molecular diffusion in moving liquids are made--that is, if the actual velocity of particles through the system can be described as the sum of two components: (1) the average velocity used in advective calculation, and (2) a random deviation from the average velocity. To the extent that scale variations in velocity represent random deviation from the velocity used in advective transport calculation, and to the extent that these variations occur on a scale which is significantly smaller than the size of the region used for advective calculation (for example, region R of figure 5-1), dispersion theory may describe adequately the differences between advective calculation and field observation. However, if the velocity variations are not random, or if they are large relative to the region used for advective calculation, the suitability of the dispersion approach is questionable. Moreover, even when this approach appears to be justified, determination of the necessary coefficients usually must be approached empirically (for example, through model calibration). The range of validity of the quantities determined in this manner is uncertain.

Variations in velocity most often are caused by variations in the permeability and effective porosity of the porous medium on all three of the relevant scales. In theory, therefore, it should be possible to describe the dispersive-transport process through statistical analysis of variations in aquifer permeability. Gelhar and Axness (1983) have attempted to do this by using a stochastic analysis of permeability variation at the macroscopic scale to generate dispersivity values. The utility of this approach currently is limited by the difficulty in obtaining the necessary data on the statistics of permeability variation. However, Gelhar has demonstrated that in the limit, as distances of transport become large, a concentration-gradient approach is justified on theoretical grounds.

Because dispersive transport actually represents an aggregate of the deviations of actual particle velocities from the velocity used in advective-transport calculation, coefficients of dispersion must vary as the overall velocity of flow varies in order to create agreement between computed and observed results. As overall flow velocities in the system increase, the magnitude of velocity deviations from the average velocity used in advective-transport calculation must increase as well; therefore, dispersive transport is dependent on average flow velocity.

The description of dispersion in terms of velocity variation implies that problem scale must be a factor in any calculation of dispersive effects. As the size of the region used in advective-transport calculation (for example, region R in figure 5-1) increases, more heterogeneities are included in that region. If a small region of calculation is chosen (for example, corresponding to the size of a laboratory column), the dominant heterogeneities within it are those at the pore scale; dispersive effects and dispersion coefficients are correspondingly small. As the region R becomes larger, macroscopic and ultimately "mappable" heterogeneities dominate. Thus, as larger regions of calculation are taken, the dispersive effects tend to increase in magnitude, the determination of the coefficients required for their description becomes more difficult, and the applicability of the conventional concentration-gradient approach becomes questionable. In general, the scale at which advective-transport calculations are made (for example, the scale of discretization in a model analysis) ideally reflects the existing level of knowledge of heterogeneities in the system. The scale is chosen to be fine enough so that the effects of all recognized heterogeneities can be accounted for by advective transport, yet coarse enough so that individual regions of advective-transport calculation are large with respect to their unknown internal heterogeneities, which must be described by dispersive terms. Thus, in any calculation of the physical mechanisms of solute transport, advection and dispersion are interrelated, and the appropriate values of dispersion depend on the scale at which the advective field is quantified.

Exercise (5-1)--Ground-Water Travel Times in the Flow System Beneath a Partially Penetrating Impermeable Wall

Our goal in this exercise is to estimate travel times in the ground-water system beneath the partially penetrating impermeable wall (fig. 5-5) for which we developed a flow net in Exercise (3-2). First, we must make an assumption concerning the movement of the "tracer water" through the system.

Assume that at some instant of time (t=0, or reference time in this problem), water of different quality enters the flow field at the upper left inflow boundary and moves through the system. We assume that the "new" water moves by piston flow or plugflow, completely displacing the "old" water. Because we assume there is no mixing of the two waters--that is the processes of dispersion and diffusion are not acting--a sharp boundary or "front" exists between the two fluids as the "new" water advances through the system. From Darcy's law the specific discharge, or Darcy velocity q, is given by

$$q = \frac{K\Delta h}{L}$$

where L is the distance between two points on the same streamline at which head values h_1 and h_2 are known and $h_2 - h_1 = \Delta h$. The "actual" or average linear velocity v is given by

$$v = \frac{q}{n} = \frac{K\Delta h}{nL}$$

where n is the porosity of the earth material. Remembering that distance of travel (L) = velocity x time or L = vt, then v = L/t. Substituting for v and rearranging, we obtain

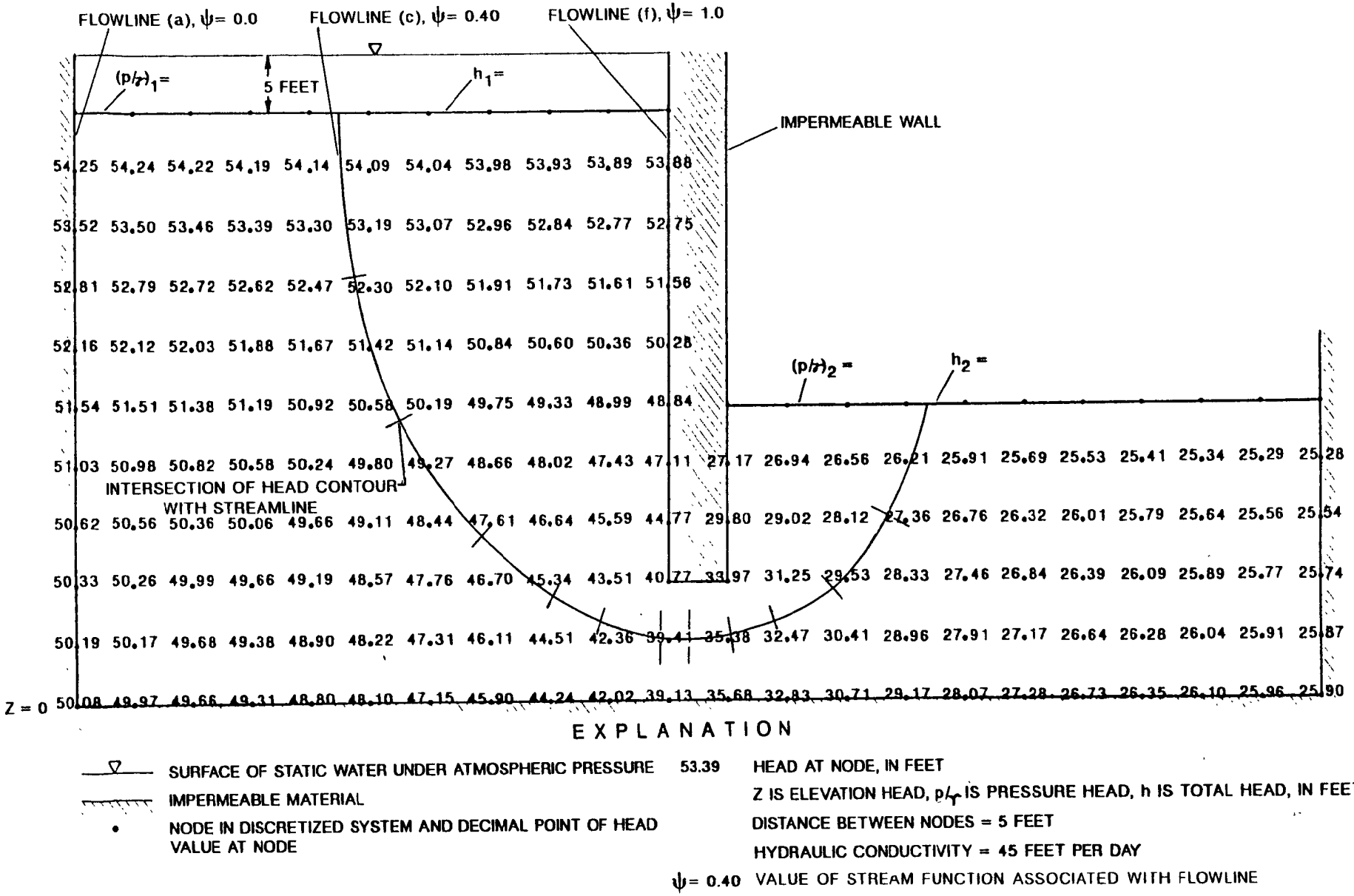
$$t = \frac{L}{\frac{K\Delta h}{nL}} = \frac{nL^2}{K\Delta h}.$$

This is the basic formula for calculating the time of travel between two points on a streamline that are a distance L apart.

Given that K = 45 ft/d and n = 0.30, the formula for time of travel between two points on a streamline in the impermeable-wall problem is

$$t = \frac{6.67 \times 10^{-3} L^2}{\Delta h}$$

where t is in days.



EXPLANATION

SURFACE OF STATIC WATER UNDER ATMOSPHERIC PRESSURE

IMPERMEABLE MATERIAL

NODE IN DISCRETIZED SYSTEM AND DECIMAL POINT OF HEAD VALUE AT NODE

53.39 HEAD AT NODE, IN FEET

Z IS ELEVATION HEAD, p/γ IS PRESSURE HEAD, h IS TOTAL HEAD, IN FEET

DISTANCE BETWEEN NODES = 5 FEET

HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY

ψ= 0.40 VALUE OF STREAM FUNCTION ASSOCIATED WITH FLOWLINE

Figure 5-5.--Head "map" of a ground-water system with an impermeable wall for plotting cumulative time of travel from the system inflow boundary along two bounding flowlines and an internal flowline.

1. Using the format in table 5-1, calculate times of travel from node to node along the two bounding streamlines (streamlines "a" and "f" on figure 5-5) of the flow system. For these two streamlines, because we are calculating travel times between nodes, L is constant and equals 5 feet. Thus, for these two streamlines only,

$$t = \frac{.167}{\Delta h} .$$

Our main interest in this problem is not the travel times between points on the streamlines, but the total time of travel from the upper left-hand inflow boundary to the point in question. The value of "Σt" in table 5-1 represents this total calculated travel time along the given streamline from the inflow boundary to the given point on the streamline. Plot the values of Σt at the appropriate points on figure 5-5.

2. An internal streamline (flowline (c), fig. 5-5) from the original flow net beneath the impermeable wall (Exercise 3-2) has been traced onto figure 5-5. Intersections of the potential lines from the original flow net are marked on this internal streamline. Calculate travel times along this internal streamline between intersection points of potential lines. Note that in this case Δh is constant and L varies. Calculate and plot Σt at appropriate points on figure 5-5 as before.

3. Contour Σt values for Σt equal to 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 5.0, and 10.0 days. The contour lines represent calculated positions of the sharp front between "new" and "old" water at successive times after introduction of the "new" water at the inflow boundary.

What time is required for "new" water to reach the discharge boundary? What time is required for "new" water to completely fill the flow system? At the end of this analysis, recall that we assumed piston flow in our time calculations, and that our calculations are only approximate, even for this assumption. However, this approach gives useful order-of-magnitude estimates of travel times in ground-water flow systems.

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 1 of 3)

[h is head at a node or other point in flow system; L is distance between two points on a flowline at which head is known; Δh is difference in head between two points on a flowline; t is time of travel between two points on a flowline; Σt is time of travel from inflow boundary to point on flowline]

h (feet)	L (feet)	Δh (feet)	$t \text{ (days)} = \frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines in impermeable-wall problem (page 2 of 3)

h (feet)	L (feet)	Δh (feet)	$t \text{ (days)} = \frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Table 5-1.--Format for calculation of time of travel along selected flowlines impermeable-wall problem (page 3 of 3)

h (feet)	L (feet)	Δh (feet)	$t \text{ (days)} = \frac{6.67 \times 10^{-3} L^2}{\Delta h}$	Σt (days)

Exercise (5-2)--Advective Movement and Travel Times in a Hypothetical Stream-Aquifer System

In Exercise (1-6), the approximate positions of flowlines from points A and B (fig. 1-12) to streams in an areal flow system were drawn. These flowlines are given in figure 5-6. Assuming a uniform gradient, a hydraulic conductivity (K) of 125 ft/d, and a porosity of 0.33, calculate the approximate time for a contaminant placed at points A and B to move advectively to a stream.

For advective flow, the average linear velocity (Fetter (1988), p. 391) is

$$v = \frac{K}{n}\frac{dh}{dl}.$$

1. Measure the total length (L) of each streamline.

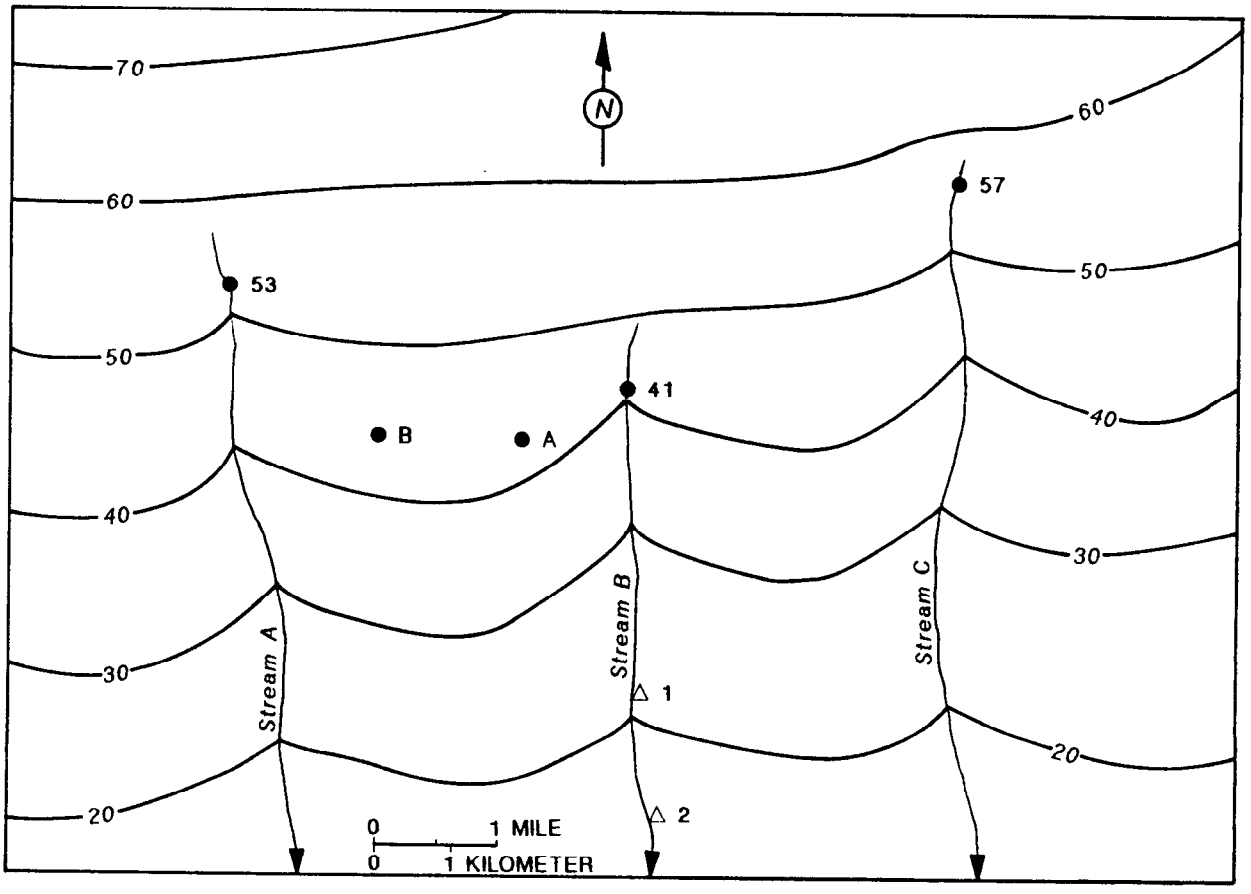
2. Calculate the "average" velocity along each streamline, assuming a constant gradient.

3. Calculate the time required to travel the distance (L) to the stream by dividing the distance by the average velocity.

Simple calculations of this type are extremely useful in understanding contaminant behavior. Franke and Cohen (1972) estimated the positions of flowlines (fig. 5-7) in a stream-aquifer system. These flowlines, when used in conjunction with estimated hydraulic conductivities, porosities, and gradients, enabled the estimation of travel times (fig. 5-8) for the entire stream basin. These time-of-travel estimates can then be used to predict the movement and persistence of contaminants in the shallow ground-water system.

Note (5-2).--Analytical Solutions for Analysis of Solute Transport in Ground Water

As discussed in Note (4-2), an analytical solution, which is a formal, closed-form mathematical solution to a boundary-value problem, simulates ground-water systems that are highly idealized and generally simple relative to the usual complexity of natural systems. For example, in these systems the external geometry usually is simple (squares, rectangles, and circles or three-dimensional equivalents), and the flow medium is at least homogeneous, if not isotropic and homogeneous, so that the properties of the flow medium are specified easily. In view of this inherent simplicity, the similarity between the system represented in the mathematical solution and the natural system never is exact and often is poor. However, valuable qualitative insight into the real system often can be gained through easily executed numerical experimentation with similar hypothetical systems. In general, however, considerable care is required to relate one or more of the available mathematical solutions to the natural system under study.



EXPLANATION

—20— WATER-TABLE CONTOUR -- Shows altitude of water table. Contour interval 10 feet Datum is sea level

● 41 LOCATION OF START OF FLOW OF STREAM -- Number is altitude of stream, in feet above sea level

△ 2 LOCATION AND NUMBER OF STREAM DISCHARGE MEASUREMENT POINT

Figure 5-6.--Hypothetical water-table map of an area underlain by permeable deposits in a humid climate.

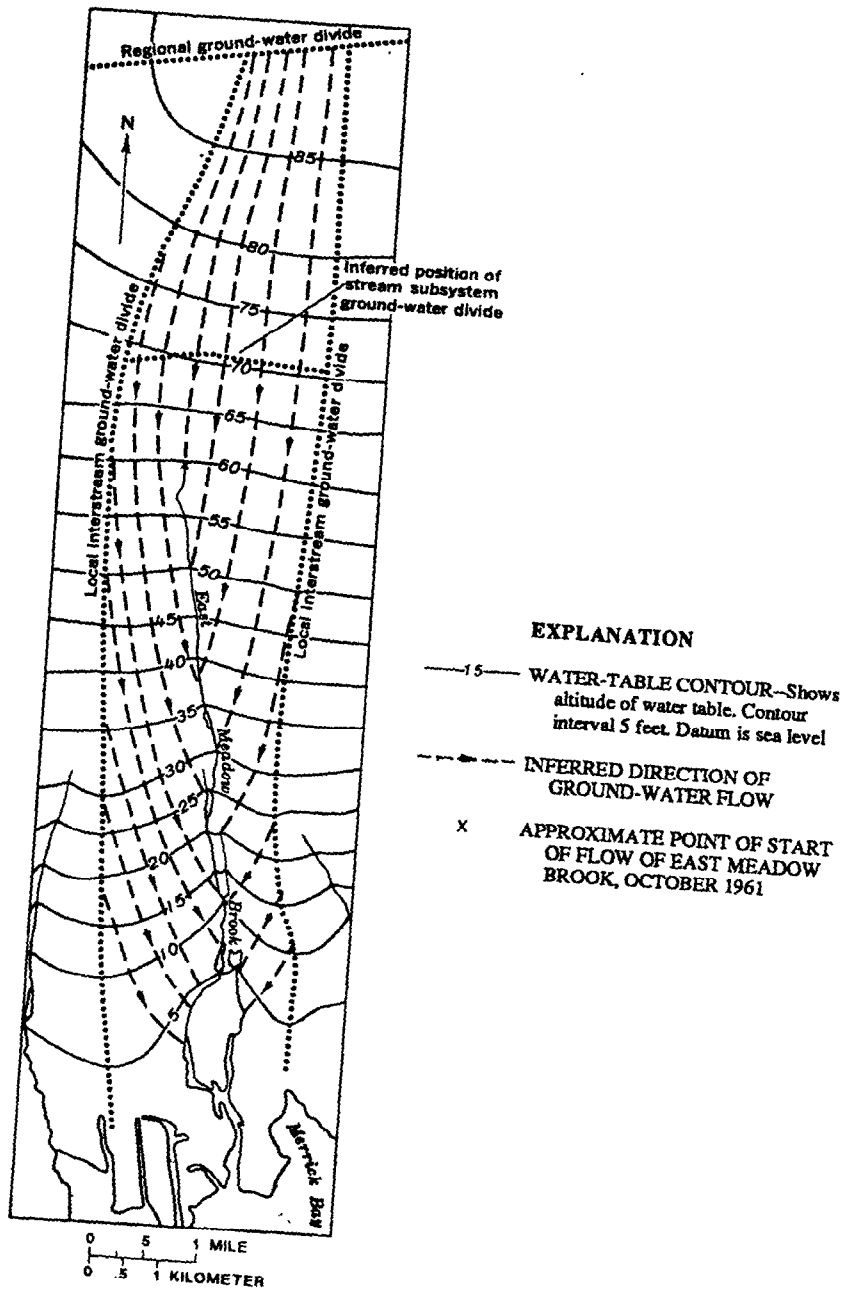


Figure 5-7.--Ground-water flow net in the vicinity of East Meadow Brook, Long Island, New York, in October 1961. (From Franke and Cohen, 1972.)

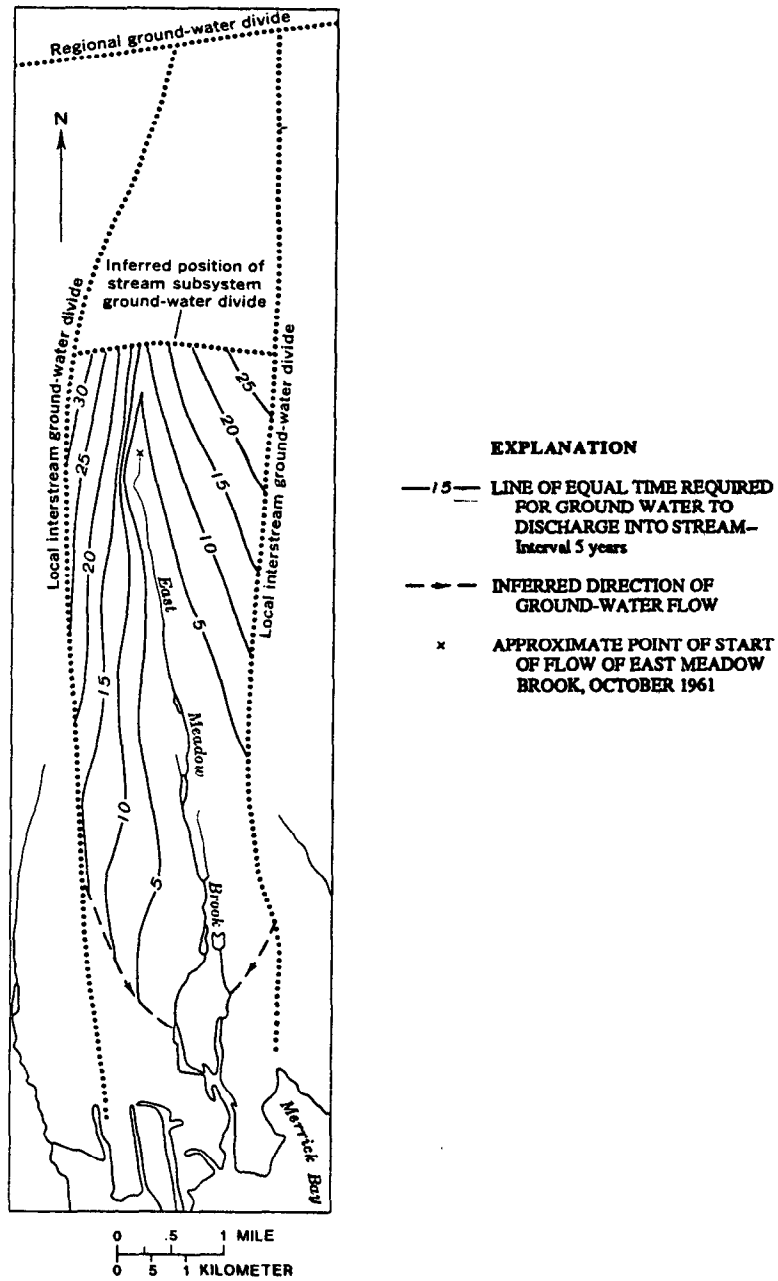


Figure 5-8.--Approximate time required for a particle of water in the shallow ground-water subsystem to discharge into East Meadow Brook, Long Island, New York, under conditions similar to those in October 1961. (From Franke and Cohen, 1972.)

As discussed in Reilly and others (1987), boundary conditions are a key feature to consider in selecting a mathematical solution as a surrogate for the natural system and in evaluating the degree of correspondence between the two systems. The value of applying analytical solutions to a field situation often lies in using them to define limiting cases and then comparing the results of the analytical solution with field data. For example, an analytical solution might represent advective and dispersive transport of a conservative solute in a highly idealized flow field. By judicious selection of the parameters for several cases, the results from a series of solutions to this hypothetical problem may bracket the distribution of a conservative constituent in the field problem. If this bracketing does not occur, some process in the natural system requires further explanation. Some of the available analytical solutions for solute-transport problems are given by Bear (1972), Bear (1979), Freeze and Cherry (1979), and Javandel and others (1984). In addition, Wexler (1989) compiled nine analytical solutions for one-, two-, and three-dimensional solute transport problems and provides computer programs to facilitate their use.

Fetter (1988, p. 393-394) gives the governing one-dimensional differential equation for advection and dispersion as

$$D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} = \frac{\partial c}{\partial t} \qquad (1)$$

where

D is the longitudinal dispersion coefficient (L^2/T),
c is the solute concentration (M/L^3),
v is the average linear velocity in the x-direction (L/T), and
t is the time since start of solute invasion (T).

The analytical solution to this governing differential equation gives the concentration, c, at some distance, L, from the source, whose concentration is c_0, at time, t, as

$$c = \frac{c_0}{2} \left[\operatorname{erfc} \left(\frac{L-vt}{2\sqrt{Dt}} \right) + \exp \left(\frac{vL}{D} \right) \operatorname{erfc} \left(\frac{L+vt}{2\sqrt{Dt}} \right) \right] \qquad (2)$$

For conditions in which the dispersion coefficient is small or L or t is large, the second term is negligible and the equation reduces to

$$c = \frac{c_0}{2} \operatorname{erfc} \frac{L-vt}{2\sqrt{Dt}} . \qquad (3)$$

Exercise (5-3)--Application of the One-Dimensional Advective-Dispersive Equation

The one-dimensional advective-dispersive equation, given in Note (5-2) and in Fetter (1988, p. 394), may be used to develop an estimate of the transport and distribution of solutes in a three-dimensional natural system. After reading sections 10.6.4, 10.6.5, and 10.6.6 in Fetter (1988, p. 391-397), do the example problem in Fetter (1988) on page 395 for practice in using the equation.

Note that step 2 of the sample problem, "Determine the longitudinal dispersion coefficient," is not accurate. This method of estimating the dispersion coefficient is used simply to facilitate the calculation. Actually, as discussed in Note (5-1), the value of the dispersion coefficient usually is determined by history-matching in a numerical simulation. Active research whose goal is to determine the dispersion coefficient based on the distribution of hydraulic conductivity at the local scale is in progress.

Using the simplified one-dimensional analytical solution given in Note (5-2), calculate the solute concentrations at the intervals given in tables 5-2 and 5-3 for a dispersion coefficient, D, of 10 ft^2/d and 100 ft^2/d, and compare the results. Use the values

t = 1,000 days,
v = 2 ft/d, and
C_0 = 100 mg/L,

the two tables, and the values of the complementary error function (erfc) in Appendix 13 of Fetter (1988, p. 562). Plot the results of the calculations on figure 5-9 as a graph of relative concentration C/C_0 against distance from source L.

Answer the following questions:

1. What is the effect of the larger dispersion coefficient?

2. What distance would the solute have traveled under plug flow (purely advective movement, $x = vt$)? Draw a vertical line on figure 5-9 at this distance. What solute concentration is calculated at that distance for each dispersion coefficient?

Table 5-2.--Format for calculating solute concentrations when the dispersion coefficient D = 10 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, square feet per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

C = concentration of solute at point in plume at specified time, in mg/L
C_0 = solute concentration of source, in mg/L
L = distance from source, in feet
v = average linear velocity of ground water, in ft/d
t = elapsed time since introduction of solute at source, in days
D = dispersion coefficient, in ft^2/d
erfc = complementary error function (see text)

Preliminary calculation:

For D = 10 ft^2/d, $\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ =

L (feet)	$\frac{L-2,000}{200}$	$\operatorname{erfc}\left(\frac{L-2,000}{200}\right)$ [1]	C = 50 mg/L $\operatorname{erfc}\left(\frac{L-2,000}{200}\right)$
1,500			
1,600			
1,700			
1,800			
1,900			
2,000			
2,100			
2,200			
2,300			
2,400			

[1] erfc(-x) = 1 + erf(x).

Table 5-8.--Format for calculating solute concentrations when the dispersion coefficient D = 100 square feet per day and the elapsed time t = 1,000 days

[ft^2/d, feet squared per day; mg/L, milligrams per liter]

Formula for calculations: $C = \frac{C_0}{2} \operatorname{erfc}\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ where

- C = concentration of solute at point in plume at specified time, in mg/L
- C_0 = solute concentration of source, in mg/L
- L = distance from source, in feet
- v = average linear velocity of ground water, in ft/d
- t = elapsed time since introduction of solute at source, in days
- D = dispersion coefficient, in ft^2/d
- erfc = complementary error function (see text)

Preliminary calculation:

For $D = 100\ ft^2/d$, $\left(\frac{L - vt}{2\sqrt{Dt}}\right)$ =

L (feet)	$\frac{L-2{,}000}{632.5}$	$\operatorname{erfc}\left(\frac{L-2{,}000}{632.5}\right)$[1]	$C = 50$ mg/L $\operatorname{erfc}\left(\frac{L-2{,}000}{632.5}\right)$
1,000			
1,250			
1,500			
1,750			
2,000			
2,250			
2,500			
2,750			
3,000			

[1] $\operatorname{erfc}(-x) = 1 + \operatorname{erf}(x)$.

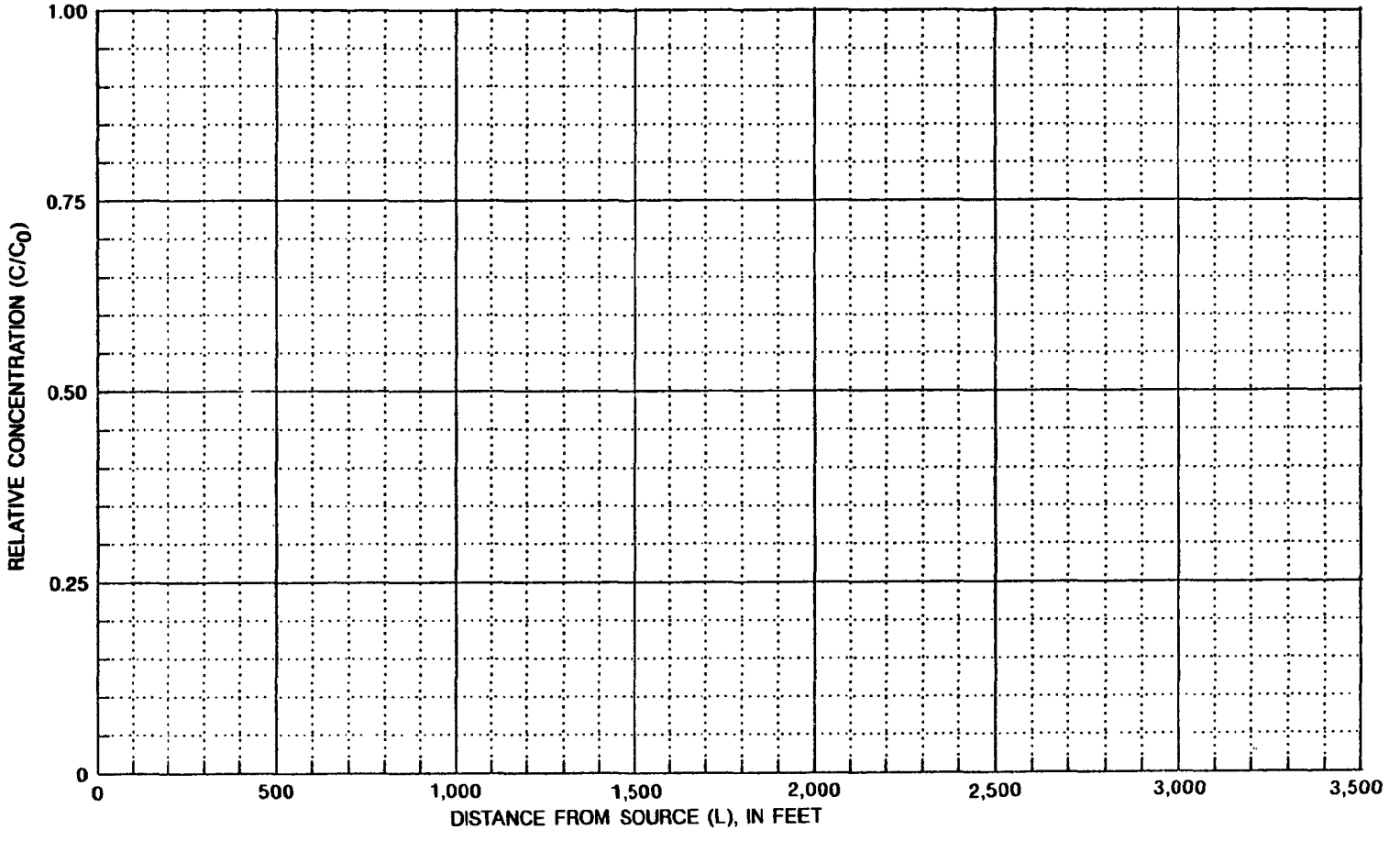


Figure 5-9.--Graph for plotting relative concentration against distance from source for two values of the dispersion coefficient and an elapsed time of 1,000 days.