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AFCMODEL

**A Microsoft Excel Workspace For Modeling Combined
Assimilation-Fractional Crystallization In Igneous Systems**

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Abstract

Combined assimilation-fractional crystallization (AFC) models were developed by DePaolo (1981) to explain trace element and isotopic variations in igneous systems. This report describes a Microsoft Excel spreadsheet that uses DePaolo's equations in an interactive manner. The spreadsheet (AFCMOD2.XLS) is linked to an Excel macro (AFCMODEL.XLM) that creates the three diagrams: ϵ_{Nd} vs Nd, $^{87}\text{Sr}/^{86}\text{Sr}$ vs Sr, ϵ_{Nd} vs $^{87}\text{Sr}/^{86}\text{Sr}$. The spreadsheet and macro are combined into an Excel workspace file (AFCMODEL.XLW) that automatically loads the spreadsheet and macro. The workspace greatly facilitates the modeling of combined assimilation-fractional crystallization in igneous systems.

INTRODUCTION

DePaolo (1981) derived the equation:

$$C_m/C_m^0 = F^{-Z} + (r/r-1) C_a/z C_m^0 (1-F^{-Z}) \quad [1]$$

to explain the variation of a trace element when both fractional crystallization and assimilation take place. The variables in this equation are:

- C_m = concentration of the element in residual magma
- C_m^0 = concentration of the element in the original magma
- C_a = concentration of the element in the assimilant
- r = ratio of the mass of the assimilant to the mass crystallized, i.e. M_a/M_c
- D = bulk distribution coefficient for trace element to be modeled
- $z = (r + D - 1)/(r - 1)$

The equation above applies when M_a is not equal to M_c , i.e. when r does not equal 1.

If $r + D = 1$ then $z = 0$. And as $z > 0$, $F^{-Z} > 1 - z \ln F$ and the above equation becomes:

$$C_m/C_m^0 = 1 + (r/r-1) C_a/C_m^0 \ln F \quad [2]$$

Other special cases are described in DePaolo (1981).

For combined assimilation and fractional crystallization involving isotopic systems DePaolo (1981) derived:

$$\epsilon_m = (r/r-1) C_a/z (1-F^{-Z}) \epsilon_a + C_m^0 F^{-Z} \epsilon_m^0 / [(r/r-1) C_a/z (1-F^{-Z}) + C_m^0 F^{-Z}] \quad [3]$$

where ϵ_m = isotope ratio or normalized value for the magma (e.g., $^{87}\text{Sr}/^{86}\text{Sr}$ or ϵ_{Sr}), ϵ_a = isotope ratio of the assimilant, and ϵ_m^0 = isotope ratio of the original magma. All other variables were defined above. This equation applies to the case of constant r and r not equal to 1.

Because crustal assimilation can take place at various depths and under different conditions, a wide range in r values can be used in AFC models. For lower crustal assimilation, r may approach 1 (DePaolo, 1981; Cameron and Cameron, 1985). For upper crustal contamination, r should be much less than 1 ($r < 1$).

Examples of the use of AFC modelling from the literature illustrates the range in r values. Clausen and Holm (1990) concluded that if AFC contributed to the variation in Sr-isotopic systematics of the Tolfa volcanic rocks of Italy, then r must have been less than 0.2. On the other hand, Wittke and others (1989) needed r values greater than 0.5 to explain the Sr and Nd-isotopic systematics of high Sr-basaltic rocks of the Black Hills, central Arizona.

Because igneous systems can have a wide range in composition, the bulk distribution coefficients for Sr and Nd can show a wide range. For lower crustal assimilation, D_{Sr} will typically be much less than 1. But during upper crustal assimilation, D_{Sr} may be close to 1 or even greater than 1. In most cases, D_{Nd} will be much less than 1. Because D_{Sr} and D_{Nd} are model dependent, they should be calculated for each case using standard methods.

The Excel spreadsheet (AFCMOD2.XLS) uses equations 1 and 3 above to model AFC in igneous systems. Two tests of the spreadsheet were performed. First it was tested using Sr isotopic data from DePaolo (1981) and compared with figure 3(a) in his paper. The tests proved that the spreadsheet can not be used when $r=1$ or when $r > 1$. In both cases, equation 1 should be replaced by the appropriate equations for special cases of AFC, as defined by DePaolo (1981). The spreadsheet was also tested with data from Cameron and Cameron (1985).

RUNNING THE WORKSPACE

Calculating combined assimilation-fractional crystallization models involves first creating the workspace AFCMODEL.XLW in your Excel subdirectory. Load AFCMOD2.XLS and AFCMODEL.XLM into Excel. These should be the only files that are loaded in Excel. Choose File Save Workspace, and type 'AFCMODEL.XLW' for the filename. Choose OK to save the workspace.

Running AFCMOD2.XLS with Nd-isotopic data is simple. Begin by copying the worksheet and the macro to your hard disk and saving them as a workspace. You enter your data only in certain cells. Each time a value is entered, the spreadsheet will recalculate itself. With a 386 or 486 computer, these recalculations will only appear as momentary flickers of the monitor. Obviously the recalculations can be suppressed by setting the spreadsheet to recalculate only on-demand. A value for r must be entered into cell C3. Concentrations of Nd in the original magma and in the assimilant must be entered in cells C4 and C5, respectively. Nd-isotopic ratios of the original magma and the assimilant must be entered in cells C7 and C6, respectively. A bulk distribution coefficient for Nd must be entered in cell C8. The value of z will then be calculated and placed in cell C9.

Running AFCMOD2.XLS with Sr-isotopic data is just as easy. An r value must be entered in cell C12. You must enter the concentrations of Sr in cells C13 and C14, and $^{87}\text{Sr}/^{86}\text{Sr}$ in cells C15 and C16 for the parent and assimilant, respectively. The bulk distribution coefficient for Sr must be entered in cell C17. A z value for the Sr model will then be calculated and placed in cell C18.

After entering and double checking your data entries, a set of plots can be created by holding down the Ctrl key and pressing A. Ctrl+A starts the macro AFCMOD.XLM. Excel 3.0 users can use the "Plot Data" button to launch the plotting macro. The plots created by the macro are placed on the screen along with AFCMOD2.XLS so that you can change variables in the model and instantaneously view the results in three chart windows. The macro

creates plots of $^{87}\text{Sr}/^{86}\text{Sr}$ vs. Sr, ϵNd vs. Nd, and ϵNd vs. $^{87}\text{Sr}/^{86}\text{Sr}$ (figs. 1-3). A change in values in the worksheet will cause changes in two or all three diagrams. In figures 1-3, the X- and Y-axis scales were increased and appropriate labels were placed on the X- and Y-axes.

Table 1. Parameters used in AFC models in figures 1-3.

Nd Model	
$r=$	0.05
$\text{Co}=$	21 ppm
$\text{Ca}=$	48 ppm
$\epsilon\text{Nd}_a=$	-11.50
$\epsilon\text{Nd}_o=$	0.00
$\text{DNd}=$	0.25
$Z=$	0.7368
Sr Model	
$r=$	0.2
$\text{C}_o\text{Sr}=$	400 ppm
$\text{C}_a\text{Sr}=$	100 ppm
$^{87}\text{Sr}/^{86}\text{Sr}_a=$	0.72
$^{87}\text{Sr}/^{86}\text{Sr}_o=$	0.7038
$\text{DSr}=$	0.9
$Z=$	-0.125

SPECIAL ENHANCEMENTS

AFCMOD2.XLS computes two Nd and two Sr models. The two pairs of models allow families of curves to be created with programs like Micrografx Charisma. Charisma reads Excel spreadsheets directly and allows data in them to be overlaid on the same diagram. It also allows the user to reformat diagrams in ways that Excel 2.1 doesn't (e.g. rotating the Y-axis title 90 degrees; Note: this feature is built into Excel 3.0 and 4.0). But the current version of Charisma lacks the macro capabilities of Excel. Figure 4 shows pairs of curves for AFC models, along with data for hypothetical samples.

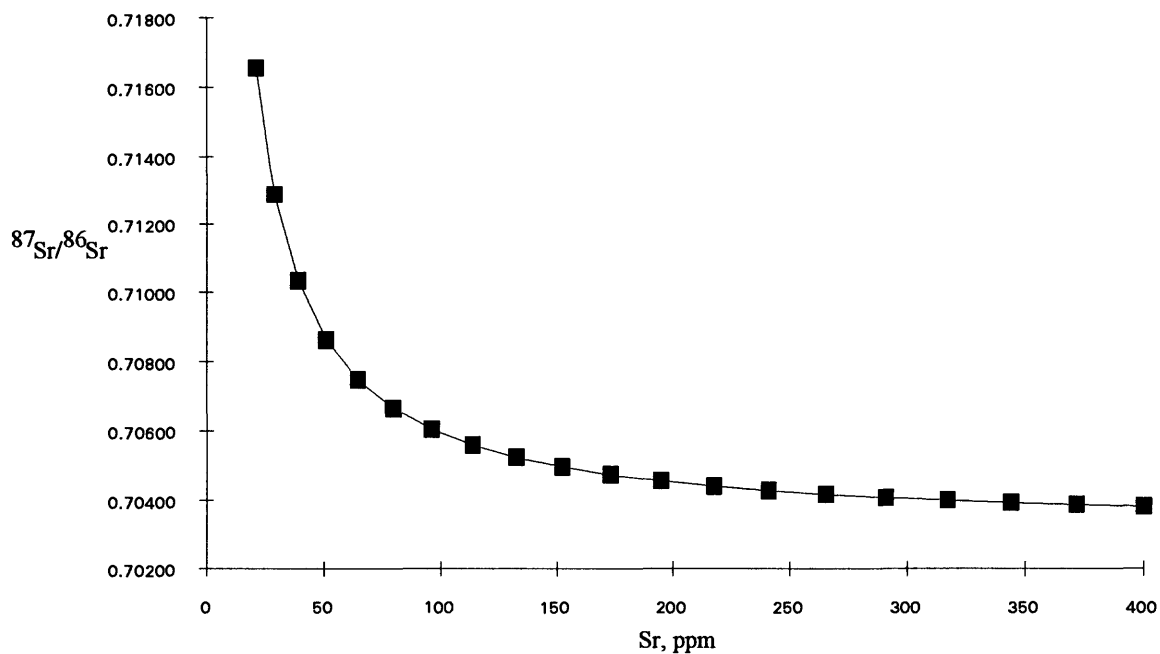


Figure 1. $^{87}\text{Sr}/^{86}\text{Sr}$ versus Sr variation diagram for a hypothetical magma affected by combined assimilation and fractional crystallization using AFCMOD2.XLW. Model values are given in table 1.

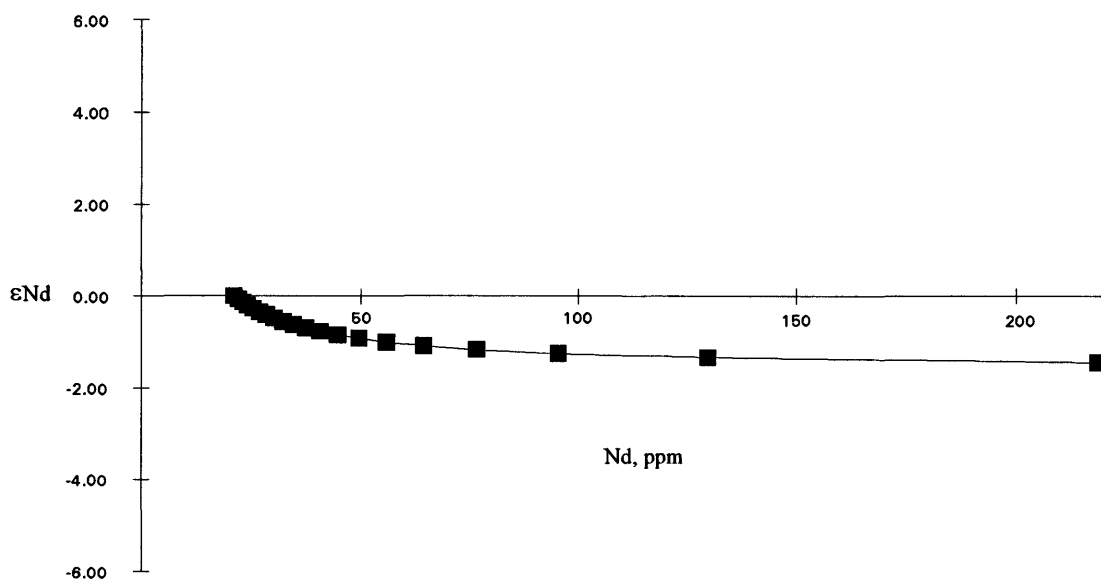


Figure 2. ϵNd versus Nd variation diagram for a hypothetical magma affected by combined assimilation and fractional crystallization using AFCMOD2.XLW. Model values are given in table 1.

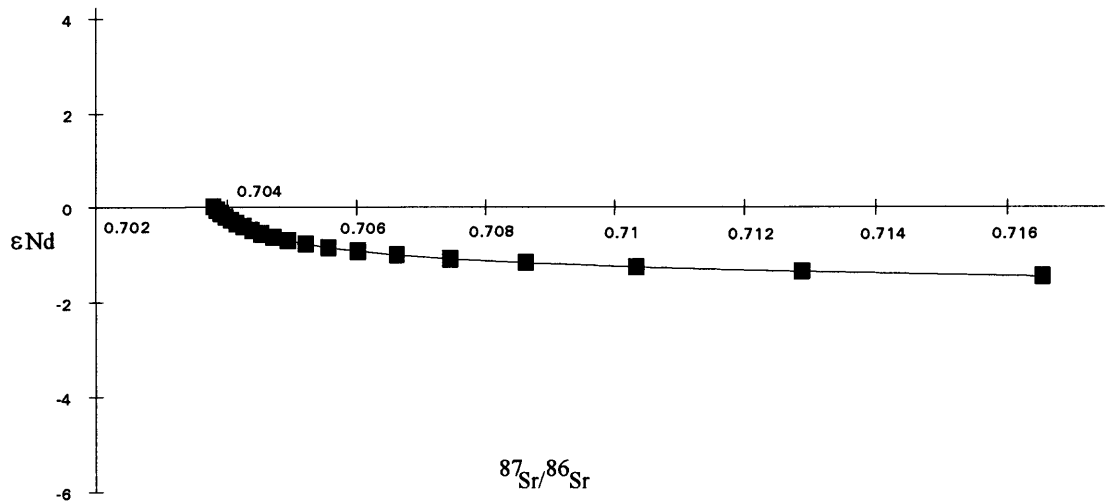


Figure 3. ϵ_{Nd} versus $^{87}Sr/^{86}Sr$ variation diagram for a hypothetical magma affected by combined assimilation and fractional crystallization using AFCMOD2.XLW. Model values are given in table 1.

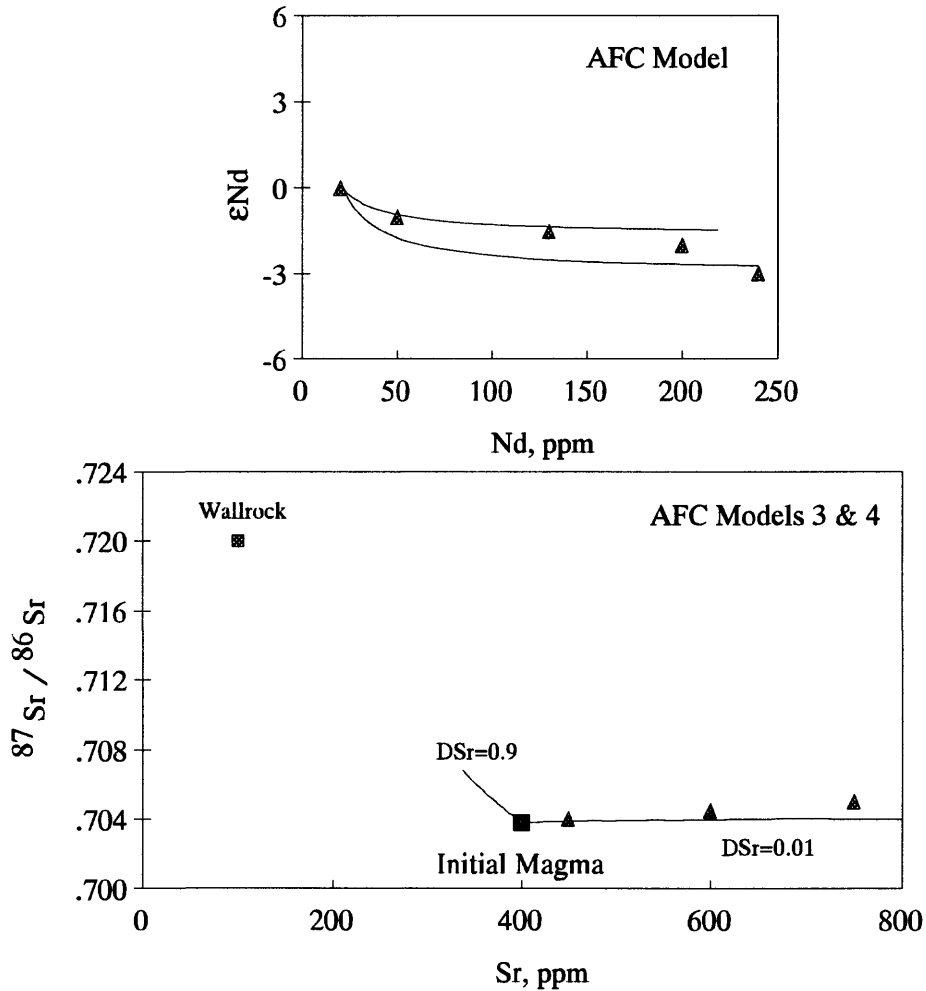


Figure 4. ϵ_{Nd} versus Nd and $^{87}Sr/^{86}Sr$ versus Sr diagrams showing AFC models, as redrawn and annotated with the program Charisma. Hypothetical samples are represented by filled triangles.

MORE ABOUT AFCMOD2.XLS

AFCMOD2.XLS contains a matrix of values used in the calculating AFC models. F-values for 1 to 0.05 are stored in columns E and K. F-values are incremented units of 0.05. Some users may want to change the incrementing value, causing the isotope plots to read easier. To do so, select the appropriate cells starting with column E, select cells to column R, and delete them. The macro AFCMOD.XLM should also be modified.

Calculated concentrations of Nd in the residual magma are held in columns F and H for models 1 and 2, respectively. Equations for calculating Nd take the form " $=Co*(E4^{(-Z)} + (Rr/(Rr-1))*(Ca/(Z*Co))*(1-E4^{(-Z)}))$ " for Nd model 1 where E4 is the reference to cell E4 and F=1. They take the form " $=Co*(E4^{(-Zz2)} + (Rr2/(Rr2-1))*(Ca/(Zz2*Co))*(1-E4^{(-Zz2)}))$ " for model 2.

Calculated concentrations of Sr in the residual magma are held in columns L and N for models 3 and 4, respectively. Equations for Sr take the form " $=CoSr*(K4^{(-Zz3)} + (Rr3/(Rr3-1))*(CaSr/(Zz3*CoSr))*(1-K4^{(-Zz3)}))$ " for model 3, and " $=CoSr*(K4^{(-Zz4)} + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr))*(1-K4^{(-Zz4)}))$ " for model 4.

Columns G and I hold the calculated values of the Nd isotopic ratios for models 1 and 2, respectively. The Nd isotopic ratio is calculated from " $=((Rr/(Rr-1))*(Ca/Z)*(1-E4^{(-Z)})*Epa) + (Co*E4^{(-Z)}*Epo))/(Rr/(Rr-1)*Ca/Z*(1-E4^{(-Z)}) + Co*E4^{(-Z)})$ " for model 1 and from " $=((Rr2/(Rr2-1))*(Ca/Zz2)*(1-E4^{(-Zz2)})*Epa) + (Co*E4^{(-Zz2)}*Epo))/(Rr2/(Rr2-1)*Ca/Zz2*(1-E4^{(-Zz2)}) + Co*E4^{(-Zz2)})$ " for model 2.

Columns M and O hold the calculated values of the Sr isotopic ratios for models 3 and 4, respectively. The Sr isotopic ratio is calculated from " $=((Rr3/(Rr3-1))*(CaSr/Zz3)*(1-K4^{(-Zz3)})*-RSra) + (CoSr*K4^{(-Zz3)}*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K4^{(-Zz3)}) + CoSr*K4^{(-Zz3)})$ " for model 3 and from " $=((Rr4/(Rr4-1))*(CaSr/Zz4)*(1-K4^{(-Zz4)})*RSra) + (CoSr*K4^{(-Zz4)}*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K4^{(-Zz4)}) + CoSr*K4^{(-Zz4)})$ " for model 4.

Columns Q and R hold the calculated values of the Sr and Nd isotopic ratios for models 3 and 1, respectively. They allow the spreadsheet to display a composite $^{87}\text{Sr}/^{86}\text{Sr}$ - ϵNd assimilation-fractional crystallization diagram.

In each of the above equations, references to ϵNd , $^{87}\text{Sr}/^{86}\text{Sr}$, concentration of Sr and Nd in the parent magma, concentration of Sr and Nd in the assimilant were defined using Excel's definition feature. For example: Sr in the parent is represented by the definition "CoSr"; Sr in the assimilant is represented by "CaSr"; $^{87}\text{Sr}/^{86}\text{Sr}$ in the parent is represented by RSro; $^{87}\text{Sr}/^{86}\text{Sr}$ in the assimilant is represented by "RSra"; ϵNd in the parent is represented by "Epo" and ϵNd in the assimilant is represented by "Epa." These definitions are created in Excel 2.1 by selecting the appropriate cell and clicking on Formula. In the drop-down menu, select Define Name, typing in a name and clicking on OK.

Notice that Co, Ca, Ep(a) and Ep(o) do not need to be entered for model 2. Similarly, CoSr, CaSr, RSr(o) and RSr(a) do not need to be entered for model 4. The spreadsheet was designed to copy these values automatically from models 1 and 3 into models 2 and 4. If the user has cause to change these values from the keyboard, he/she should first make a copy of the original version of the spreadsheet, then save the revised spreadsheet to the filename "AFCMOD3.XLS".

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Appendices

Appendix 1. Listing for Excel worksheet AFCMOD2.XLS. Columns C and E contain test data.

Column	B	C	D	E
Nd	Model	1	2	
Enter r	Rr=	0.05	0.1	F
Enter Initial ppm of parent	Co=	21	=C4	1
Enter ppm of assimilant	Ca=	48	=C5	0.95
Enter epsilon of assimilant	Epa=	-11.5	=C6	0.9
Enter epsilon of parent	Epo	0	=C7	0.85
Enter bulk D	D=	0.25	=C8	0.8
	Z=	=(C3+C8-1)/(C3-1)	=(D3+D8-1)/(D3-1)	0.75
				0.7
Sr	Model	3	4	0.65
r from Nd model	Rr=	0.2	0.1	0.6
Enter initial Sr	CoSr=	400	=C13	0.55
Enter Sr of assimilant	CaSr=	100	=C14	0.5
Enter 87Sr/86Sr of assimilant	RSra	0.72	=C15	0.45
Enter 87Sr/86Sr of parent	RSro	0.7038	=C16	0.4
Enter bulk D	DSr=	0.9	0.01	0.35
	Z=	=(C12+C17-1)/(C12-1)	=(D12+D17-1)/(D12-1)	0.3
				0.25
				0.2
				0.15
				0.1
				0.05

F

Nd, ppm

$$\begin{aligned}
 &= \text{Co} * (\text{E4}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E4}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E5}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E5}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E6}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E6}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E7}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E7}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E8}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E8}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E9}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E9}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E10}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E10}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E11}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E11}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E12}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E12}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E13}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E13}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E14}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E14}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E15}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E15}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E16}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E16}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E17}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E17}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E18}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E18}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E19}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E19}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E20}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E20}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E21}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E21}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E22}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E22}^{\wedge}(-\text{Z}))) \\
 &= \text{Co} * (\text{E23}^{\wedge}(-\text{Z}) + (\text{Rr}/(\text{Rr}-1)) * (\text{Ca}/(\text{Z} * \text{Co})) * (1 - \text{E23}^{\wedge}(-\text{Z})))
 \end{aligned}$$

G

EpNd(m)1

$$\begin{aligned} &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E4^(-Z)) * Epa) + (Co * E4^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E4^(-Z)) + Co * E4^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E5^(-Z)) * Epa) + (Co * E5^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E5^(-Z)) + Co * E5^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E6^(-Z)) * Epa) + (Co * E6^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E6^(-Z)) + Co * E6^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E7^(-Z)) * Epa) + (Co * E7^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E7^(-Z)) + Co * E7^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E8^(-Z)) * Epa) + (Co * E8^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E8^(-Z)) + Co * E8^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E9^(-Z)) * Epa) + (Co * E9^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E9^(-Z)) + Co * E9^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E10^(-Z)) * Epa) + (Co * E10^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E10^(-Z)) + Co * E10^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E11^(-Z)) * Epa) + (Co * E11^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E11^(-Z)) + Co * E11^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E12^(-Z)) * Epa) + (Co * E12^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E12^(-Z)) + Co * E12^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E13^(-Z)) * Epa) + (Co * E13^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E13^(-Z)) + Co * E13^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E14^(-Z)) * Epa) + (Co * E14^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E14^(-Z)) + Co * E14^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E15^(-Z)) * Epa) + (Co * E15^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E15^(-Z)) + Co * E15^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E16^(-Z)) * Epa) + (Co * E16^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E16^(-Z)) + Co * E16^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E17^(-Z)) * Epa) + (Co * E17^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E17^(-Z)) + Co * E17^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E18^(-Z)) * Epa) + (Co * E18^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E18^(-Z)) + Co * E18^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E19^(-Z)) * Epa) + (Co * E19^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E19^(-Z)) + Co * E19^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E20^(-Z)) * Epa) + (Co * E20^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E20^(-Z)) + Co * E20^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E21^(-Z)) * Epa) + (Co * E21^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E21^(-Z)) + Co * E21^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E22^(-Z)) * Epa) + (Co * E22^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E22^(-Z)) + Co * E22^(-Z)) \\ &= ((Rr/(Rr-1)) * (Ca/Z) * (1-E23^(-Z)) * Epa) + (Co * E23^(-Z) * Epo)) / (Rr/(Rr-1) * Ca/Z * (1-E23^(-Z)) + Co * E23^(-Z)) \end{aligned}$$

H

Nd,ppm(2)

$$\begin{aligned} &= Co * (E4^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E4^(-Zz2))) \\ &= Co * (E5^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E5^(-Zz2))) \\ &= Co * (E6^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E6^(-Zz2))) \\ &= Co * (E7^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E7^(-Zz2))) \\ &= Co * (E8^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E8^(-Zz2))) \\ &= Co * (E9^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E9^(-Zz2))) \\ &= Co * (E10^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E10^(-Zz2))) \\ &= Co * (E11^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E11^(-Zz2))) \\ &= Co * (E12^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E12^(-Zz2))) \\ &= Co * (E13^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E13^(-Zz2))) \\ &= Co * (E14^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E14^(-Zz2))) \\ &= Co * (E15^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E15^(-Zz2))) \\ &= Co * (E16^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E16^(-Zz2))) \\ &= Co * (E17^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E17^(-Zz2))) \\ &= Co * (E18^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E18^(-Zz2))) \\ &= Co * (E19^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E19^(-Zz2))) \\ &= Co * (E20^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E20^(-Zz2))) \\ &= Co * (E21^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E21^(-Zz2))) \\ &= Co * (E22^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E22^(-Zz2))) \\ &= Co * (E23^(-Zz2) + (Rr2/(Rr2-1)) * (Ca/(Zz2 * Co)) * (1-E23^(-Zz2))) \end{aligned}$$

I

EpnD(m)2

$$\begin{aligned}
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E4^(-Zz2)) * Epa) + (Co * E4^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E4^(-Zz2)) + Co * E4^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E5^(-Zz2)) * Epa) + (Co * E5^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E5^(-Zz2)) + Co * E5^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E6^(-Zz2)) * Epa) + (Co * E6^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E6^(-Zz2)) + Co * E6^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E7^(-Zz2)) * Epa) + (Co * E7^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E7^(-Zz2)) + Co * E7^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E8^(-Zz2)) * Epa) + (Co * E8^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E8^(-Zz2)) + Co * E8^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E9^(-Zz2)) * Epa) + (Co * E9^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E9^(-Zz2)) + Co * E9^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E10^(-Zz2)) * Epa) + (Co * E10^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E10^(-Zz2)) + Co * E10^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E11^(-Zz2)) * Epa) + (Co * E11^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E11^(-Zz2)) + Co * E11^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E12^(-Zz2)) * Epa) + (Co * E12^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E12^(-Zz2)) + Co * E12^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E13^(-Zz2)) * Epa) + (Co * E13^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E13^(-Zz2)) + Co * E13^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E14^(-Zz2)) * Epa) + (Co * E14^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E14^(-Zz2)) + Co * E14^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E15^(-Zz2)) * Epa) + (Co * E15^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E15^(-Zz2)) + Co * E15^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E16^(-Zz2)) * Epa) + (Co * E16^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E16^(-Zz2)) + Co * E16^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E17^(-Zz2)) * Epa) + (Co * E17^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E17^(-Zz2)) + Co * E17^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E18^(-Zz2)) * Epa) + (Co * E18^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E18^(-Zz2)) + Co * E18^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E19^(-Zz2)) * Epa) + (Co * E19^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E19^(-Zz2)) + Co * E19^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E20^(-Zz2)) * Epa) + (Co * E20^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E20^(-Zz2)) + Co * E20^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E21^(-Zz2)) * Epa) + (Co * E21^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E21^(-Zz2)) + Co * E21^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E22^(-Zz2)) * Epa) + (Co * E22^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E22^(-Zz2)) + Co * E22^(-Zz2)) \\
&= ((Rr2/(Rr2-1)) * (Ca/Zz2) * (1-E23^(-Zz2)) * Epa) + (Co * E23^(-Zz2) * Epa)) / (Rr2/(Rr2-1) * Ca/Zz2 * (1-E23^(-Zz2)) + Co * E23^(-Zz2))
\end{aligned}$$

K

L

F

Sr, ppm (3)

1	= CoSr*(K4^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K4^(-Zz3)))
0.95	= CoSr*(K5^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K5^(-Zz3)))
0.9	= CoSr*(K6^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K6^(-Zz3)))
0.85	= CoSr*(K7^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K7^(-Zz3)))
0.8	= CoSr*(K8^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K8^(-Zz3)))
0.75	= CoSr*(K9^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K9^(-Zz3)))
0.7	= CoSr*(K10^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K10^(-Zz3)))
0.65	= CoSr*(K11^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K11^(-Zz3)))
0.6	= CoSr*(K12^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K12^(-Zz3)))
0.55	= CoSr*(K13^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K13^(-Zz3)))
0.5	= CoSr*(K14^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K14^(-Zz3)))
0.45	= CoSr*(K15^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K15^(-Zz3)))
0.4	= CoSr*(K16^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K16^(-Zz3)))
0.35	= CoSr*(K17^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K17^(-Zz3)))
0.3	= CoSr*(K18^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K18^(-Zz3)))
0.25	= CoSr*(K19^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K19^(-Zz3)))
0.2	= CoSr*(K20^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K20^(-Zz3)))
0.15	= CoSr*(K21^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K21^(-Zz3)))
0.1	= CoSr*(K22^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K22^(-Zz3)))
0.05	= CoSr*(K23^(-Zz3) + (Rr3/(Rr3-1)) * (CaSr/(Zz3*CoSr)) * (1-K23^(-Zz3)))

M

87Sr/86Sr(3)

$$\begin{aligned}
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K4^(-Zz3))*RSra) + (CoSr*K4^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K4^(-Zz3)) + CoSr*K4^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K5^(-Zz3))*RSra) + (CoSr*K5^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K5^(-Zz3)) + CoSr*K5^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K6^(-Zz3))*RSra) + (CoSr*K6^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K6^(-Zz3)) + CoSr*K6^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K7^(-Zz3))*RSra) + (CoSr*K7^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K7^(-Zz3)) + CoSr*K7^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K8^(-Zz3))*RSra) + (CoSr*K8^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K8^(-Zz3)) + CoSr*K8^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K9^(-Zz3))*RSra) + (CoSr*K9^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K9^(-Zz3)) + CoSr*K9^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K10^(-Zz3))*RSra) + (CoSr*K10^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K10^(-Zz3)) + CoSr*K10^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K11^(-Zz3))*RSra) + (CoSr*K11^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K11^(-Zz3)) + CoSr*K11^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K12^(-Zz3))*RSra) + (CoSr*K12^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K12^(-Zz3)) + CoSr*K12^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K13^(-Zz3))*RSra) + (CoSr*K13^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K13^(-Zz3)) + CoSr*K13^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K14^(-Zz3))*RSra) + (CoSr*K14^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K14^(-Zz3)) + CoSr*K14^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K15^(-Zz3))*RSra) + (CoSr*K15^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K15^(-Zz3)) + CoSr*K15^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K16^(-Zz3))*RSra) + (CoSr*K16^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K16^(-Zz3)) + CoSr*K16^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K17^(-Zz3))*RSra) + (CoSr*K17^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K17^(-Zz3)) + CoSr*K17^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K18^(-Zz3))*RSra) + (CoSr*K18^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K18^(-Zz3)) + CoSr*K18^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K19^(-Zz3))*RSra) + (CoSr*K19^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K19^(-Zz3)) + CoSr*K19^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K20^(-Zz3))*RSra) + (CoSr*K20^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K20^(-Zz3)) + CoSr*K20^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K21^(-Zz3))*RSra) + (CoSr*K21^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K21^(-Zz3)) + CoSr*K21^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K22^(-Zz3))*RSra) + (CoSr*K22^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K22^(-Zz3)) + CoSr*K22^(-Zz3)) \\
 &= ((Rr3/(Rr3-1)*(CaSr/Zz3)*(1-K23^(-Zz3))*RSra) + (CoSr*K23^(-Zz3)*RSro))/(Rr3/(Rr3-1)*CaSr/Zz3*(1-K23^(-Zz3)) + CoSr*K23^(-Zz3))
 \end{aligned}$$

N

Sr, ppm (4)

$$\begin{aligned}
 &= CoSr*(K4^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K4^(-Zz4))) \\
 &= CoSr*(K5^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K5^(-Zz4))) \\
 &= CoSr*(K6^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K6^(-Zz4))) \\
 &= CoSr*(K7^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K7^(-Zz4))) \\
 &= CoSr*(K8^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K8^(-Zz4))) \\
 &= CoSr*(K9^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K9^(-Zz4))) \\
 &= CoSr*(K10^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K10^(-Zz4))) \\
 &= CoSr*(K11^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K11^(-Zz4))) \\
 &= CoSr*(K12^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K12^(-Zz4))) \\
 &= CoSr*(K13^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K13^(-Zz4))) \\
 &= CoSr*(K14^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K14^(-Zz4))) \\
 &= CoSr*(K15^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K15^(-Zz4))) \\
 &= CoSr*(K16^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K16^(-Zz4))) \\
 &= CoSr*(K17^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K17^(-Zz4))) \\
 &= CoSr*(K18^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K18^(-Zz4))) \\
 &= CoSr*(K19^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K19^(-Zz4))) \\
 &= CoSr*(K20^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K20^(-Zz4))) \\
 &= CoSr*(K21^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K21^(-Zz4))) \\
 &= CoSr*(K22^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K22^(-Zz4))) \\
 &= CoSr*(K23^(-Zz4) + (Rr/(Rr-1))*(CaSr/(Zz4*CoSr)))*(1-K23^(-Zz4)))
 \end{aligned}$$

87Sr/86Sr(4)

$$\begin{aligned}
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K4^*(-Zz4))*RSra) + (CoSr*K4^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K4^*(-Zz4)) + CoSr*K4^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K5^*(-Zz4))*RSra) + (CoSr*K5^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K5^*(-Zz4)) + CoSr*K5^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K6^*(-Zz4))*RSra) + (CoSr*K6^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K6^*(-Zz4)) + CoSr*K6^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K7^*(-Zz4))*RSra) + (CoSr*K7^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K7^*(-Zz4)) + CoSr*K7^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K8^*(-Zz4))*RSra) + (CoSr*K8^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K8^*(-Zz4)) + CoSr*K8^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K9^*(-Zz4))*RSra) + (CoSr*K9^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K9^*(-Zz4)) + CoSr*K9^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K10^*(-Zz4))*RSra) + (CoSr*K10^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K10^*(-Zz4)) + CoSr*K10^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K11^*(-Zz4))*RSra) + (CoSr*K11^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K11^*(-Zz4)) + CoSr*K11^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K12^*(-Zz4))*RSra) + (CoSr*K12^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K12^*(-Zz4)) + CoSr*K12^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K13^*(-Zz4))*RSra) + (CoSr*K13^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K13^*(-Zz4)) + CoSr*K13^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K14^*(-Zz4))*RSra) + (CoSr*K14^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K14^*(-Zz4)) + CoSr*K14^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K15^*(-Zz4))*RSra) + (CoSr*K15^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K15^*(-Zz4)) + CoSr*K15^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K16^*(-Zz4))*RSra) + (CoSr*K16^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K16^*(-Zz4)) + CoSr*K16^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K17^*(-Zz4))*RSra) + (CoSr*K17^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K17^*(-Zz4)) + CoSr*K17^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K18^*(-Zz4))*RSra) + (CoSr*K18^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K18^*(-Zz4)) + CoSr*K18^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K19^*(-Zz4))*RSra) + (CoSr*K19^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K19^*(-Zz4)) + CoSr*K19^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K20^*(-Zz4))*RSra) + (CoSr*K20^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K20^*(-Zz4)) + CoSr*K20^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K21^*(-Zz4))*RSra) + (CoSr*K21^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K21^*(-Zz4)) + CoSr*K21^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K22^*(-Zz4))*RSra) + (CoSr*K22^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K22^*(-Zz4)) + CoSr*K22^*(-Zz4)) \\
&= ((Rr4/(Rr4-1)*(CaSr/Zz4)*(1-K23^*(-Zz4))*RSra) + (CoSr*K23^*(-Zz4)*RSro))/(Rr4/(Rr4-1)*CaSr/Zz4*(1-K23^*(-Zz4)) + CoSr*K23^*(-Zz4))
\end{aligned}$$

Q	R
MODEL 3	MODEL 1
87Sr/86Sr	Epsilon Nd
=M4	=G4
=M5	=G5
=M6	=G6
=M7	=G7
=M8	=G8
=M9	=G9
=M10	=G10
=M11	=G11
=M12	=G12
=M13	=G13
=M14	=G14
=M15	=G15
=M16	=G16
=M17	=G17
=M18	=G18
=M19	=G19
=M20	=G20
=M21	=G21
=M22	=G22
=M23	=G23

Appendix 2. Excel macro for creating AFC variation diagrams.

afcmmod
=ECHO(FALSE)
=ACTIVATE("AFCMOD.XLM")
=HIDE()
=SELECT("R3C6:R23C7")
=COPY()
=NEW(2)
=PASTE.SPECIAL(2,TRUE,TRUE,FALSE)
=GALLERY.SCATTER(2,TRUE)
=ACTIVATE("AFCMOD2.XLS")
=SELECT("R3C12:R23C13")
=COPY()
=NEW(2)
=PASTE.SPECIAL(2,TRUE,TRUE,FALSE)
=GALLERY.SCATTER(2,TRUE)
=ACTIVATE("AFCMOD2.XLS")
=SELECT("R3C17:R23C18")
=COPY()
=NEW(2)
=PASTE.SPECIAL(2,TRUE,TRUE,FALSE)
=GALLERY.SCATTER(2,TRUE)
=ARRANGE.ALL()
=SELECT("R2C2")
=RETURN()