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CALIBRATION OF A LINEAR SPRING-SUPPORTED, VERTICAL-COMPONENT MOVING-COIL SEISMOMETER BY MEANS OF DAMPING TEST AND/OR A CURRENT RELEASE TEST

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INTRODUCTION

The most commonly used seismometer in U. S. regional networks today is the Mark Products L4-C vertical-component moving-coil seismometer. This seismometer is convenient because of its low natural frequency (1 hz), its high output level (1 volt/cm/sec across an appropriate load resistor), its small size (about 3" diameter and 7" height), and its rugged waterproof construction. Because its case is sealed, however, tests to determine its operating parameters must be carried out by means of electrical tests applied to the seismometer's output leads. From such tests and an a priori knowledge (from the manufacturer) of the mass of the moving system, one can determine the free period, motor constant, and open circuit damping of the seismometer and can calculate values of two external resistors (T=series and S=shunt) required to attain the desired damping and sensitivity.

In practice, two sets of tests are carried out: 1) before installation, the coil resistance, free period, open circuit damping, and motor constant are determined in the laboratory and the required values of the T and S resistors are calculated. This procedure detects seismometers whose parameters (usually free period) are outside the range of acceptable values and permits the seismometer to be set up with standard sensitivity and damping when installed; 2)during normal operation in the field, an electronic step test and a current release test are applied periodically (once per day) to the seismic amplifier and the seismometer, respectively. The purpose of these tests is to provide a basis for monitoring the free period, damping, and motor constant remotely in order to detect seismometers whose operating parameters drift outside the range of acceptable

values. Both sets are interpreted in terms of particular solutions of the differential equations of motion of the seismometer. Accordingly, we shall derive the equation of motion in terms of the physical properties and operating partameters of the seismometer and shall obtain solutions corresponding to the particular tests to be applied.

TRANSDUCER

First, we shall examine the moving coil (velocity) transducer and the interactions between currents and emf's in the coil and forces on, and velocity of, the seismometer coil, which is an integral part of the moving system.

The L4-C, figure 1, employs a cylindrical magnet 36 mm high with inside and outside diameters of 18 mm and 38 mm, respectively, and a permeable cylindar 42 mm high with inside and outside diameters of 55 mm amd 58 mm, repectively, as the magnetic flux return path. Permeable 3-mm thick disc-shaped pole pieces 45 mm in diameter are mounted on the ends of the magnet and form 5 mm thick air gaps between the edges of the pole pieces and the encircling flux path. Two circular coils, each with an inside diameter of about 46 mm, width of 12 mm, and thickness of 2 mm, are centered in the air gaps at the ends of the magnet and are mounted on thin (ca 0.5 mm) aluminum sleeves attached to the moving system. From the space occupied by the coil, and its resistance, it appears that the 5000-ohm L4-C coil consists of about 8000 turms of #40 copper magnet wire.

The strong magnetic induction in the air gaps is perpendicular to the axis of the coils and is narrow (3-mm pole pieces) compared to the width of the coils (12 mm). The force on one turn of the coil winding due to the interaction of a current $\vec{\imath}$ through the coil and an induction $\vec{\mathcal{B}}$ in the air gap can be written

$$1)\quad \vec{F} = \oint i\,d\vec{\ell}\times\vec{B} = iB\int_0^{2\pi} r\,d\theta = 2\pi r B i = \ell B i$$

where r and ℓ are the radius and circumferenc of the coil and $\mathcal{B}$ is the average induction in the region occupied by the coil. For n turns

2) $\vec{F} = n\ell\mathcal{B}i$

$\vec{F}$ is parallel to the axis of the seismometer, i.e., along the vertical axis, and its sense depends on both the direction of induction in the air gap and the direction of current in the coil. The two coils are connected in such a way that the current is counterclockwise in one coil and clockwise in the other so that the forces (or emf's) produced by interaction of induction and current (or velocity) in the coils at opposite ends of the magnet are additive.

In terms of the seismometer motor constant

3) $G = n\ell\mathcal{B}$

the force on the moving system due to the current through its coil can be written

4) $F = Gi$

When the coil moves relative to the magnet, an emf is generated in the coil

5) $\mathcal{E} = -\frac{d\Phi}{dt}$, where $\frac{d\Phi}{dt}$ is the rate of change of flux through the coil.

From the construction of the transducer outlined above

$d\Phi = n\ell\mathcal{B}dz$ so;

6) $\mathcal{E}(t) = -G\dot{z}(t)$

Such an emf causes a current $i(t)$ to flow in the coil and associated circuit

$i(t) = \mathcal{E}(t)/(R+S)$, where R is the coil resistance

and S is the resistance external to the seismometer. Thus,

$$7)\quad i(t) = -\frac{G\,\dot{z}(t)}{R+S}$$

Such a current through the coil, in turn, interacts with the induction in the region occupied by the coil to produce a force on the coil to oppose its motion

$$8)\quad F_d(t) = G\,i(t) = -\frac{G^2\,\dot{z}(t)}{R+S}$$

$F_d(t)$ is an electrodynamic damping force that retards the coil's motion.

The form of the foregoing equations in B and G (i.e., having the constant of proportionality between F and the product iB equal to 1 in equation 1) requires that we use the absolute electromagnetic system of units. The relationships between these units and practical units are:

10 amperes = 1 emu, 1 volt = 10^8 emu, and 1 ohm = 10^9 emu.

EQUATIONS OF MOTION

Consider a linear, spring-supported, vertical-component, moving-coil electromagnetic seismometer like that indicated in figure 2. Let $\mathcal{Y}$ represent the position of the earth (and seismometer frame) with respect to a Newtonian framework $\mathcal{Y}'$, and let z represent the position of the seismometer moving system with respect to $\mathcal{Y}$. Let the parameters of the seismometer be specified as follows:

M = mass of seismometer moving system,
k = spring stiffness,
l_o = zero length of spring (i.e., spring exerts no restoring force at length l_o),

ℓ = working length of spring,
g = acceleration due to gravity (980 cm/sec/sec),
R = resistance of seismometer coil,
L = inductance of seismometer coil,
S = resistance external to seismometer coil,
G = seismometer motor constant (electrodynamic constant),
E = emf generated across coil leads (open circuit) due to relative motion of seismometer moving system and frame,
I = current flowing in seismometer coil.

We shall neglect the influence of coil inductance, L, on the seismometer current, I, as is traditional. This omission should have little effect at the relatively low frequencies normally encountered in earthquakes (< 30 hz). The effect of inductance during the current release test will be discussed below.

For a quiet earth, the forces acting on the seismometer moving system are:

a) Gravity: $-Mg$

b) Spring: $+[(\ell-\ell_0)-z]u \qquad (= Mg - zu)$

c) Damping:

open circuit $-\Delta \dot{z}$

electromagnetic $-GI \qquad \left(= -\frac{G^2 \dot{z}}{R+S}\right)$

total damping $-\left(\Delta + \frac{G^2}{R+S}\right)\dot{z}$

d) Inertia: $-M\ddot{z}$

For a moving earth, we must also add a virtual force to account for the acceleration of the seismometer reference framework $\mathfrak{z}$ with respect to the Newtonian framework $\mathfrak{z}'$.

e) Virtual force: $+M\ddot{\zeta}$

Summing these forces we have:

$$-M\ddot{z}-\left(\Delta+\frac{G^2}{R+S}\right)\dot{z}-Mg-zU+Mg+M\ddot{\zeta}=0$$

Simplifying and dividing by M :

$$\ddot{z}+\frac{1}{M}\left(\Delta+\frac{G^2}{R+S}\right)\dot{z}+\frac{U}{M}z-\ddot{\zeta}=0$$

Let $\frac{U}{M}=\Omega_0^2$, $\frac{1}{M}\left(\Delta+\frac{G^2}{R+S}\right)=2\beta\Omega_0$

Let $\beta=\beta_0+\beta_1$, where $\frac{\Delta}{M}=2\beta_0\Omega_0$ and $\frac{1}{M}\frac{G^2}{R+S}=2\beta_1\Omega_0$

Note that $\Omega_0=2\pi f_0=2\pi/T_0$, where f_0 and T_0 are the natural frequency and free period of the seismometer, respectively.

Substituting the new coefficients of $\dot{z}$ and z :

9) $$\ddot{z}+2\beta\Omega_0\dot{z}+\Omega_0^2 z=\ddot{\zeta}$$

Applying the Laplace transform to equation 9 and rearranging terms:

10) $$\Phi(\tau)=\frac{\mathcal{L}[\ddot{\zeta}]+(\tau+2\beta\Omega_0)z(0)+\dot{z}(0)}{[\tau^2+2\beta\Omega_0\tau+\Omega_0^2]}$$

and

11) $$\Phi(\tau)=\frac{\mathcal{L}[\ddot{\zeta}]}{(\tau-\tau_1)(\tau-\tau_2)}+\frac{(\tau+2\beta\Omega_0)z(0)}{(\tau-\tau_1)(\tau-\tau_2)}+\frac{\dot{z}(0)}{(\tau-\tau_1)(\tau-\tau_2)}$$

where $$\tau_1=\Omega_0\left(-\beta+\sqrt{\beta^2-1}\right)$$

$$\tau_2=\Omega_0\left(-\beta-\sqrt{\beta^2-1}\right)$$

In equation 11 the three terms on the right of the equal sign correspond, respectively, to:

first - response of the seismometer to earth motion $\xi(t)$

second - response of the seismometer to an ititial displacement $z(0)$

third - response of the seismometer to an initial velocity $\dot{z}(0)$.

The specific earth motion we will consider is that in which the earth begins with simple harmonic motion beginning at $t=0$:

$$\xi(t) = A \sin \omega t \qquad t \geq 0$$

$$= 0 \qquad t < 0$$

$$\mathcal{L}[\xi(t)] = \frac{A\omega}{\tau^2+\omega^2}$$

$$\mathcal{L}[\ddot{\xi}(t)] = \frac{\tau^2 A\omega}{\tau^2+\omega^2} + \tau\,\xi(0) + \dot{\xi}(0)$$

For the case considered here $\xi(0)=0$, but $\dot{\xi}(0) \neq 0$. Equation 11 becomes:

$$12)\quad \Phi(\tau) = \frac{\tau^2}{\tau^2+\omega^2}\frac{A\omega}{(\tau-\tau_1)(\tau-\tau_2)} + \frac{\dot{\xi}(0)}{(\tau-\tau_1)(\tau-\tau_2)} + \frac{(\tau+2\beta-\Omega_0)z(0)}{(\tau-\tau_1)(\tau-\tau_2)} + \frac{\dot{z}(0)}{(\tau-\tau_1)(\tau-\tau_2)}$$

where τ_1 and τ_2 are as defined above.

Each term on the right hand side of equation 12 is of the form $H(\tau) = p(\tau)/q(\tau)$ where $p(\tau)$ and $q(\tau)$ are polynomials and the degree of $q(\tau)$ is greater than the degree of $p(\tau)$. Furthermore, $q(\tau)$ can be written in terms of its linear factors

$$q(\tau) = (\tau-\tau_1)(\tau-\tau_2)\cdots(\tau-\tau_n)$$

For the case of most interest to us, $\beta < 1$, and all of the roots are different. For the first term on the right the roots of

$q(\tau)$ are $+i\omega$, $-i\omega$, τ_1 , and τ_2 .

For the remaining terms the roots are τ_1 and τ_2 .

Under these conditions the inverse of $H(\tau)$ is

$$\mathcal{L}^{-1}[H(\tau)] = h(t) = \sum_{j=1}^{n} \frac{p(\tau_j) e^{\tau_j t}}{q'(\tau_j)}$$

We shall next apply this rule to evaluate equation 12 to obtain equations for the seismometer displacement, $z(t)$, for the following cases:

1) harmonic response: (for $y(t) = A \sin \omega t$ for t large)

2) release test: $z(0) \neq 0$, $\dot{z}(0) = 0$, $y(t) = 0$, $\dot{y}(0) = 0$

3) tapping test: $\dot{z}(0) \neq 0$, $z(0) = 0$, $y(t) = 0$, $\dot{y}(0) = 0$

HARMONIC RESPONSE

After inversion of equation 12 and manipulation of the results, the response of the seismometer to a long-sustained earth motion $y(t) = A \sin \omega t$ is:

$$z(t) = \frac{A \omega^2}{\left[(\Omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2 \Omega_0^2\right]^{1/2}} \sin\left[\omega t - \tan^{-1}\left(\frac{2\beta\omega\Omega_0}{\Omega_0^2 - \omega^2}\right)\right]$$

This motion induces an emf $E(t) = G\dot{z}(t)$ in the seismometer coil and an emf $E_s(t) = \frac{S}{R+S} G \dot{z}(t)$ across the external resistor S (figure 3).

$$13)\quad E_s(t) = \frac{S}{R+S} G \frac{A\omega^3 \cos\left[\omega t - \tan^{-1}\left(\frac{2\beta\omega\Omega_o}{\Omega_o^2-\omega^2}\right)\right]}{\left[(\Omega_o^2-\omega^2)^2 + 4\beta^2\omega^2\Omega_o^2\right]^{1/2}}$$

RELEASE TEST

After inversion of equation 12 and manipulation of the results, the response of the seismometer to an initial displacement $Z(0)$ is:

$$Z(t) = Z(0)\frac{e^{-\beta\Omega_o t}}{\sqrt{1-\beta^2}} \sin\left(\sqrt{1-\beta^2}\,\Omega_o t + \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}\right)$$

The emf generated by this motion across the external resistor S is:

$$14)\quad E_s(t) = \frac{S}{R+S} G \frac{Z(0)\Omega_o e^{-\beta\Omega_o t}}{\sqrt{1-\beta^2}} \sin\sqrt{1-\beta^2}\,\Omega_o t$$

The first maximum in $E_s(t)$ occurs when $\tan\sqrt{1-\beta^2}\,\Omega_o t = \frac{\sqrt{1-\beta^2}}{\beta}$

$$15)\quad E_{s\max} = \frac{S}{R+S} \frac{G Z(0)\Omega_o}{\sqrt{1-\beta^2}} e^{-\frac{\beta}{\sqrt{1-\beta^2}}\tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}}$$

To use these equations in a practical calibration procedure we must be able to impart an initial offset $Z(0)$ to the mass and to determine that offset with precision. Weight lift and current release tests for this purpose are discussed below.

TAPPING TEST

After inversion of equation 12 and manipulation of the results, the response of the seismometer to an initial velocity $\dot{Z}(0)$ is:

$$Z(t) = \frac{\dot{z}(0)\, e^{-\beta\Omega_0 t}}{\Omega_0\sqrt{1-\beta^2}} \sin\sqrt{1-\beta^2}\,\Omega_0 t$$

The emf generated by this motion across an external resistor S is:

$$16)\quad E_S(t) = \frac{S}{R+S}\,\frac{G\,\dot{z}(0)\, e^{-\beta\Omega_0 t}}{\sqrt{1-\beta^2}} \sin\left(\sqrt{1-\beta^2}\,\Omega_0 t - \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}\right)$$

To use this equation in a practical calibration procedure we must be able to impart an initial velocity to the seismometer mass and determine that velocity with precision. Mechanical (tapping) and electrical (current pulse) means of imparting an impulse of momentum are discussed below.

TAPPING TEST (pellet drop)

An important requirement of the tapping test is that the duration of the pulse that imparts an initial velocity to the mass be very short compared to the free period of the seismometer. A mechanical excitation in which an elastic pellet is dropped from a known height, attaining a velocity v_0 at the time of impact and rebounding with a velocity $-v_0$ after impact, can be envisaged for a laboratory-type seismometer. A suitable horizontal component of velocity would insure that the pellet would not fall back and hit the mass a second time. The momentum imparted to the seismometer in such an experiment would be $2mv_0$, where m is the mass of the pellet, and the initial velocity of the seismometer could be computed from the principal of the conservation of momentum. If the initial velocity of the seismometer were zero, its velocity after impact would be

$$\dot{z}(\epsilon) = -2mv_0/M$$

Such an experiment is impractical for a small sealed seismometer like the L4-C, however.

TAPPING TEST (current pulse)

A more practical means of imparting an initial velocity to the mass is to "fire" the charge of a small capacitor through the seismometer coil at $t=0$. The current through the coil exerts a force $f(t) = G\,i(t)$ on the seismometer moving system of mass M . The acceleration of the moving system due to this force is .

$$17) \quad \ddot{z}(t) = \frac{G}{M}\, i(t)$$

If appreciable current flows through the coil for a short interval of time, $0<t<\varepsilon$,
then the velocity of the mass at time ε can be calculated:

$$18) \quad \dot{z}(\varepsilon) = \frac{G}{M}\int_0^{\varepsilon} i(t)\,dt$$

where the integral is the portion of the charge on the capacitor that traverses the coil.

The emf across an external resistor S in response to this test is:

$$19) \quad E_S(t) = \frac{S}{R+S}\,\frac{G^2}{M}\left(\int_0^{\varepsilon} i(t)dt\right)\frac{e^{-\beta\Omega_0 t}}{\sqrt{1-\beta^2}}\sin\left(\sqrt{1-\beta^2}\,\Omega_0 t - \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}\right)$$

RELEASE TEST (weight lift)

Returning to figure 2 and the definition of Ω_0, at equilibrium the restoring force due to the stretched spring balances the gravitational pull on the mass:

$$(l-l_0)\,U = M g$$

If an additional mass m is placed on the mass, the spring stretches an additional amount $z(0)$.

$$(\ell + z(0) - \ell_0) = (M+m)\,g$$

$z(0) = mg/U$, but $U = M\Omega_0^2$

$z(0) = mg/M\Omega_0^2$ (z measured downward)

20) $z(0) = -mg/M\Omega_0^2$ (z measured upward)

Remove the weight and record the emf generated across the external resistor S :

$$21)\quad E_S(t) = -\frac{S}{R+S}\,G\,\frac{mg}{M\Omega_0}\,\frac{e^{-\beta\Omega_0 t}}{\sqrt{1-\beta^2}}\,\sin\sqrt{1-\beta^2}\,\Omega_0 t$$

RELEASE TEST (current release)

Send a current I through the coil and wait until the seismometer has come to rest. The current in the coil interacting with the magnetic induction in the transducer produces a force GI on the mass which is balanced by a change in spring force $-z(0)U$ resulting from the displacement of the mass $z(0)$.

$$U\,z(0) = GI$$

$$22)\quad z(0) = GI/U = G\,I/M\Omega_0^2$$

Cut off the current and record the emf generated across the external resistor S :

$$23)\quad E_S(t) = \frac{S}{R+S}\,\frac{G^2 I}{M\Omega_0}\,\frac{e^{-\beta\Omega_0 t}}{\sqrt{1-\beta^2}}\,\sin\sqrt{1-\beta^2}\,\Omega_0 t$$

The first maximum (when $\tan\sqrt{1-\beta^2}\,\Omega_0 t = \sqrt{1-\beta^2}/\beta$) is:

$$24)\quad E_{S\max} = \frac{S}{R+S}\,\frac{G^2 I}{M\Omega_0}\,e^{-\frac{\beta}{\sqrt{1-\beta^2}}\tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}}$$

This treatment neglects the transient perturbation caused by the pulse of current through the coil when the source is switched off. Experiments show, for the L4-C, that the duration of this current pulse is short (< 5 ms) and that the product

$G\int i\,dt$ is quite small.

EFFECT OF COIL INDUCTANCE ON THE CURRENT RELEASE TEST

A simple experiment outlined by the circuit in figure 4 and illustrated by the graphs of voltage versus time in figure 5 was performed to evaluate the effect of coil inductance on the current release test. A reed relay $RR1$ controlled by a low-frequency square wave from a signal generator alternately applied and removed a voltage, supplied by a Hg-cell and 2000-ohm series resistor, across a circuit consisting of the seismometer and series resistor T in parallel with the shunt resistor S. The current $i(t)$ through the seismometer coil was measured with a high-quality oscilloscope by means of the voltage drop across resistor T. In the experiment, a current release test was repeated about once every 10 seconds when the current through the seismometer coil was interrupted by the opening of $RR1$. The experiment was performed both with the seismometer upright (live release test) and inverted (circuit response test). Widely different horizontal trace speeds were required to reveal different aspects of the test.

Figure 5a, at 0.2 sec/inch, shows the release test preceded by a sudden drop of $i(t)$ to near zero at $t \approx 0$. Figure 5a, at 2ms/inch, also shows the decay of current from i_0 (at $t = 0$) to a minimum of about 2 microamps (5 mv/3000 ohms) at about 3 ms.

Figure 5b, at 0.1 ms/inch and 1.0 ms/inch, shows the initial decay of current with the seismometer inverted, i.e., with the release test suppressed. After a very short transient, during which the current drops to about 1/4 i_0 and then rebounds to

about 2/3 i_o in the first 25 microseconds, the current declines steadily to zero by about 4 ms. The initial sharp decline in current through T may result from the capacitance of the coil: when the external voltage source is cut off, the coil capacitance is "charged" and tends to drive a reverse current through the external circuit until it is discharged or overwhelmed by the larger effect of coil inductance, which tends to sustain a foreward current through the coil.

For the case in which the seismometer is inverted, as shown in figure 5b, theory predicts that the current in the circuit will decline when the external voltage is removed according to:

$$25)\quad i(t) = i_o e^{-\frac{R'}{L}t} \quad \text{, where } R' = R_s + T + S.$$

The total quantity of charge that flows through the coil during the inductance-sustained decline of current is:

$$26)\quad \int_0^\infty i_o e^{-\frac{R'}{L}t}\,dt = i_o \frac{L}{R'} \quad \text{, and this charge passes through the}$$

coil in only a few milliseconds. This charge produces an adventitious "tapping test" that accompanies the intended release test.

In the experiment illustrated in figures 4 and 5b, $R' = 13,300$ ohms, $i_o = 0.100$ ma, and the area under the decay curve in figure 5b represents about 0.0238×10^{-6} amp-sec. Interpreting these values in terms of equation 25, we find

$$L \approx 3.2 \text{ hy.}$$

Finally, we can write down the system response to the current release test by combining the current release test, driven by a current i_o, and the adventitious tapping test, driven by the current pulse due to inductance, $i_o L/R'$.

$$27)\quad E_s(t) = \frac{S}{R+S}\frac{G^2 i_o e^{-\beta\Omega_o t}}{M\sqrt{1-\beta^2}}\left[\frac{1}{\Omega_o}\sin\sqrt{1-\beta^2}\,\Omega_o t - \frac{L}{R+S}\sin\left(\sqrt{1-\beta^2}\,\Omega_o t - \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}\right)\right]$$

Comparing coefficients of the trigonometric factors for the experimental values shown above:

$$\frac{1}{\Omega_0} = 0.16 \qquad L/R+S = 0.00024$$

$$\frac{1}{\Omega_0} \Big/ \frac{L}{R+S} = 667$$

Thus, the adventitious tapping test component of the overall response is only 0.15% as large as the proper release test component itself; so we shall neglect the effect of inductance altogether.

SIGN CONVENTIONS

Determining the proper sign to attach to specific terms in many of the foregoing equations is a confusing task.
To be complete we should know the direction of magnetization of the seismometer magnet and the sense of winding in the upper and lower coils. Failing that, we should know which lead goes positive with respect to the other for a sharp upward displace ment of the seismometer. To test the signs of various terms in the equations for consistency, however, it is sufficient to stipulate the direction of inductance and sense of current flow, for a positive voltage across the seismometer leads, in the upper coil: the direction of induction and current flow will be opposite in the lower coil.

Consider the diagram in figure 1 to represent a view from above of the upper L4-C coil and the indicated current direction (counterclockwise) to result from a positive voltage across the seismometer leads. The indicated sense of magnetic induction is radially outwards. With this diagram for reference, let us go through the current release test step-by-step.

1) Apply a positive voltage V_0 to the seismometer leads to establish a current i_0 in the coil. The force on the coil is downward as indicated by the vector product $i\vec{dl} \times \vec{B}$ of the

current in a coil element of length $d\vec{\ell}$ and the induction $\vec{B}$. At equilibrium the coil is depressed an amount $-z(0)$, where z is positive upward.

2) When the voltage is suddenly cut off (as in the experiment illustrated in figure 4), the coil inductance provides a positive emf that tends to sustain i in the counterclockwise direction. This diminishing current continues to interact with the induction to exert a downward force on the coil. The integral of the force due to this diminishing current over the brief interval that it is significant delivers a small downward pulse (of momentum) to the coil and imparts a small downward increment of velocity to it. Because i begins to diminish as soon as the external voltage is cut off, in spite of inductance, the net force on the coil due to the upward directed restoring force of the spring and the downward directed force of the dying current is upward. Thus, the effect of coil inductance is to retard the return of the mass to its rest position by a very small amount. Taken by itself, however, the force due to the inductance-driven current is downward and produces a small downward "tap" on the coil. The downward-directed component of initial velocity from this tap interacts with the induction to produce a negative emf opposing the direction of i .

3) As the seismometer is pulled upward towards its rest position when the voltage is cut off, the movement of the coil through the induction in the gap produces an emf in the coil given by $\mathcal{E}(t) = -G\dot{z}(t)$. The sense of this emf is given by the vector product $d\ell\,\vec{v}\times\vec{B}$, where $\vec{v}$ is the coil velocity and $d\ell$ is a small section of coil winding. Referring to figure 1, $\vec{v}$ is upward and $\vec{B}$ is outward; so $d\vec{E} = d\ell\,\vec{v}\times\vec{B}$ is in the direction of the initial current i_0 and is positive. The positive voltage so produced generates a positive current (in the direction of i_0) which interacts with the inductance to produce a downward force that opposes the action of the spring that is pulling the mass back towards its rest position.

If the seismometer described above is subjected to a sharp

upward displacement, the coil moves downward relative to the magnet and a clockwise emf (and current) is induced in the coil. The relationship between sense of ground motion and "polarity" of the seismometer leads can be switched simply by exchanging leads. In the release test, however, the polarity of the current produced as the seismometer returns to zero remains the same as that of the initial deflecting current.

DETERMINATION OF THE SEISMOMETER DAMPING AND MOTOR CONSTANT FROM DAMPING TESTS

If we differentiate equation 23 for the current release test with respect to time and recombine terms:

$$28)\quad \frac{dEs(t)}{dt} = \frac{S}{R+S}\,\frac{G^2 I}{M}\,\frac{e^{-\beta\Omega_o t}}{\sqrt{1-\beta^2}}\,\sin\left(\sqrt{1-\beta^2}\,\Omega_o t - \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta}\right)$$

From equations 23 and 28, the zeroes and extrema of $Es(t)$ occur for

$$\sqrt{1-\beta^2}\,\Omega_o t = n\pi \qquad n = 1, 2, 3, \cdots \quad : \text{zeroes}$$

$$\sqrt{1-\beta^2}\,\Omega_o t - \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta} = m\pi \qquad m = 1, 2, 3, \cdots \quad : \text{extrema}$$

Successive zeroes as well as successive extrema follow one another at intervals $t = T_o/2\sqrt{1-\beta^2}$, where T_o is the undamped free period of the seismometer. Thus,

$$29)\quad T_o = \sqrt{1-\beta^2}\,T$$

, where T is the damped period of the seismometer.

The ratio of successive extrema of $Es(t)$ is:

$$30)\quad \frac{Es_i}{Es_{i+1}} = \frac{e^{-\beta(\Omega_o t)_i}}{e^{-\beta(\Omega_o t)_{i+1}}} = -e^{+\pi\frac{\beta}{\sqrt{1-\beta^2}}}$$

Let $\lambda = \ln\left|\frac{E_{Si}}{E_{S_{i+1}}}\right|$, where λ is the logarithmic decrement.

Then

$$31)\quad \beta = \left(\frac{\pi^2}{\lambda^2}+1\right)^{-1/2}$$

Recall, from the definition of β , that

$$\beta = \beta_0+\beta_1 = \frac{1}{2M\Omega_0}\left(\Delta + \frac{G^2}{R+S}\right)$$, where

$\beta_0 = \Delta/2M\Omega_0$, and $\beta_1 = \frac{1}{2M\Omega_0}\frac{G^2}{R+S}$

Let

$$32)\quad \Gamma = \frac{G^2}{2M\Omega_0} = \beta - \beta_0(R+S)$$

Then

$$33)\quad G = \left[2M\Omega_0\Gamma\right]^{1/2} \text{ cgs and emu.}$$

Note that the expressions for β and G contain no term in deflecting current or voltage. In fact, it does not matter how the seismometer is set in motion so long as its motion is free after some arbitrary time preceding the first extremum used in determining β .

Note, also, that the equation for G contains the seismometer mass M . We have no practical means for measuring M without dismantling the seismometer. For this reason the manufacturer is required to supply a measured value of M for each seismometer.

The free period can be determined by measuring the damped period (preferably for the open circuit test) and correcting it for damping according to equation 29. Because the open circuit damping of the L4-C is relatively high (ca 0.3), only a few extrema of the release test can be measured. Also, it is simple

to determine T_o by a separate experiment. One such method puts the seismometer in series with a load resistor and a sine-wave generator. The voltage across the resistor is displayed on the vertical axis of an oscilloscope and the voltage across the seismometer is displayed on the horizontal axis. The frequency of the sine wave is then varied until the voltage across the seismometer and the current through it (measured as voltage across the series resistor) are 90° out of phase, which occurs when the driving frequency matches the natural frequency of the seismometer. This experiment is facilitated if a generator with adjustable phase reference is available.

In the determination of G by the damping test it is desirable to repeat the experiment with several different values of the shunt in order to get some idea of the precision of the method. Figure 6 shows a circuit that can be used for the damping test, as well as a diagram of the expected response indicating the measurements to be made. Shunt values should range from ∞, open circuit, through values from about 10 to 2 times the seismometer resistance.

The accompanying table summarizes damping tests on one L4-C seismometer.

R = 5580 ohms
M = 980.0 grams
T_o = 1.039 seconds

S (ohms)	β	$\beta-\beta_o$	$R+S$ (ohms)	Γ	dev (%)
∞	0.271	0.000	∞	—	—
20155.	0.507	0.236	25695.	6064.	+0.6
14999.	0.561	0.290	20579.	5968.	-1.0
13280.	0.592	0.321	18860.	6054.	+0.4
			average =	6029.	

From equation 33 and the experimental values determined

above:

$$G = 2.672 \times 10^{8} \text{ emu/cm/sec}$$

$$G = 2.672 \text{ volts/cm/sec}$$

To attain 0.80 critical damping of the seismometer we can calculate the required seismometer load (total external resistance) D from equation 32:

34) $$D = \frac{G^2}{2M\Omega_0}\left(\frac{1}{0.8-\beta_0}\right) - R$$

D = 5817 ohms for the case above.

With the values of G and D determined above, we can turn to appendix 1 to calculate the series and shunt resistors, T and S, respectively, that will provide the required 0.80 critical damping and will adjust the seismometer output to the standard 1.0 v/cm/sec across a 10,000 ohm amplifier input resistance.

When M, Ω_0, I, and β are known, G can be determined from the first maximum of the current release test, equation 24, to corroborate the value determined from the damping tests.

ACKNOWLEDGEMENT

John Vanschaack facilitated this work through his help in setting up the experiment to evaluate the effect of coil inductance on the current release test and through discussions of the characteristics of the L4-C seismometer and the electronics by which it is recorded.

FIGURE CAPTIONS

Figure 1 Schematic drawing, from above, of the upper half of the L4-C transducer.

Figure 2 Schematic drawing of a spring-supported, vertical-component seismometer showing spring length ℓ,

seismometer moving system coordinates ξ , and earth coordinates ζ .

Figure 3 Diagram of a moving coil seismometer circuit under working conditions.

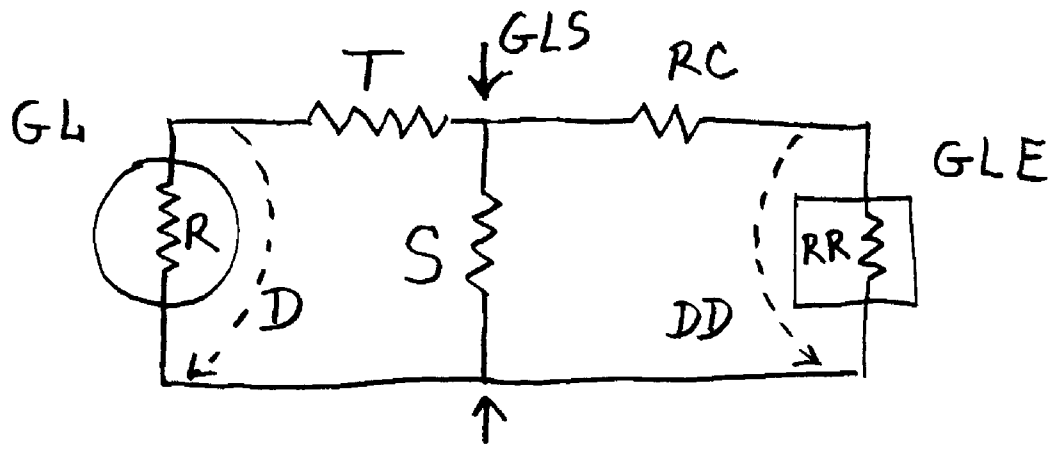
Figure 4 Diagram of the experimental circuit employed to evaluate the effect of coil inductance on the current release test.

Figure 5 Experimental results from the circuit shown in figure 4: a) current release test shown with slow time axis; b) circuit response (current release test with the seismometer inverted) with a fast time axis.

Figure 6 Test circuit and typical seismometer response for a damping test driven by a current release.

Appendix 1

L - pad attenuator to damp seismometer and to adjust its output to a desired level



R = seismometer resistance

D = required external seismometer resistance

RR = preamp input impedance (resistive)

RC = resistance of line between shunt and preamp

T = seis series resistance

S = seis shunt resistance

GL = emf induced in the seis coil

GLE = resulting emf across preamp input

GLS = resulting emf across S (in parallel with $RC + RR$)

R' = $S(RC+RR)/[S+(RC+RR)]$ resistance of S and $(RC+RR)$ in parallel

f = GLS/GL

F = $GLE/GL = f\dfrac{RR}{RR+RC}$

For given values of R , RR , RC , and GL we wish to select T and S to provide the correct external seismometer damping resistance, D , and to adjust the seismometer output (sensed across RR) to a prescribed value GLE . Our basic relationships are:

$$D = T + \frac{S(RC+RR)}{S+(RC+RR)}$$

$$F = \frac{GLE}{GL} = \frac{S(RR+RC)/(S+RR+RC)}{R+T+S(RR+RC)/(S+RR+RC)} \times \frac{RR}{(RR+RC)}$$

The second equation reduces to

$$F = \frac{S \times RR/(S+RR+RC)}{R+D} .$$

Solving for S,

$$(1) \quad S = \frac{F(R+D)(RR+RC)}{RR-F(R+D)}$$

Solving for T from the first equation

$$(2) \quad T = D - \frac{S(RR+RC)}{S+(RR+RC)}$$

I. If $D \geq (RR+RC)$

$$T_{min} = D-(RR+RC)$$
$$F_{max} = RR/(R+D)$$
$$GEM = GL \times RR/(R+D) \quad , \text{ where } GEM = GLE_{max}$$
$$S = \infty$$

II. If $D < (RR+RC)$

$$S_{max} = \frac{D(RR+RC)}{(RR+RC)-D}$$

$$T_{min} = 0$$

$$F_{max} = \frac{RR}{(RR+RC)} * \frac{D}{(R+D)}$$

$$GEM = \frac{D}{R+D} * \frac{RR}{(RR+RC)} * GL$$

To calculate S and T (for given R, RR, RC, D, and GL) to attain a desired value of GLE :

(a) Calculate $F = \frac{GLE}{GL}$

(b) Test F for admissability

i.e., $F \leq \frac{RR}{R+D}$ if $D \geq (RR+RC)$

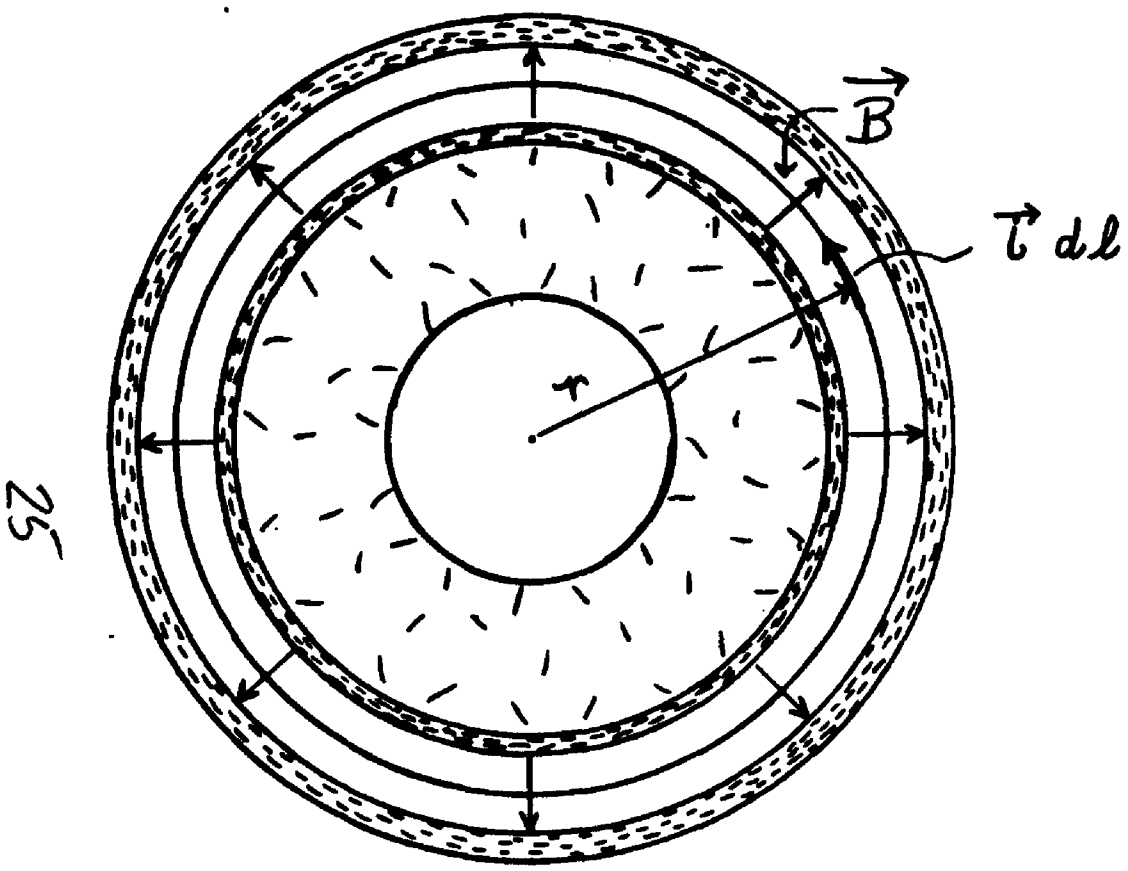
$F \leq \frac{D}{R+D} * \frac{RR}{(RR+RC)}$ if $D < (RR+RC)$

(c) Calculate S from (1)

(d) Calculate T from (2)

(e) Calculate

$$DD = RC + \frac{S(T+R)}{S+(T+R)}$$



25

Fig 1

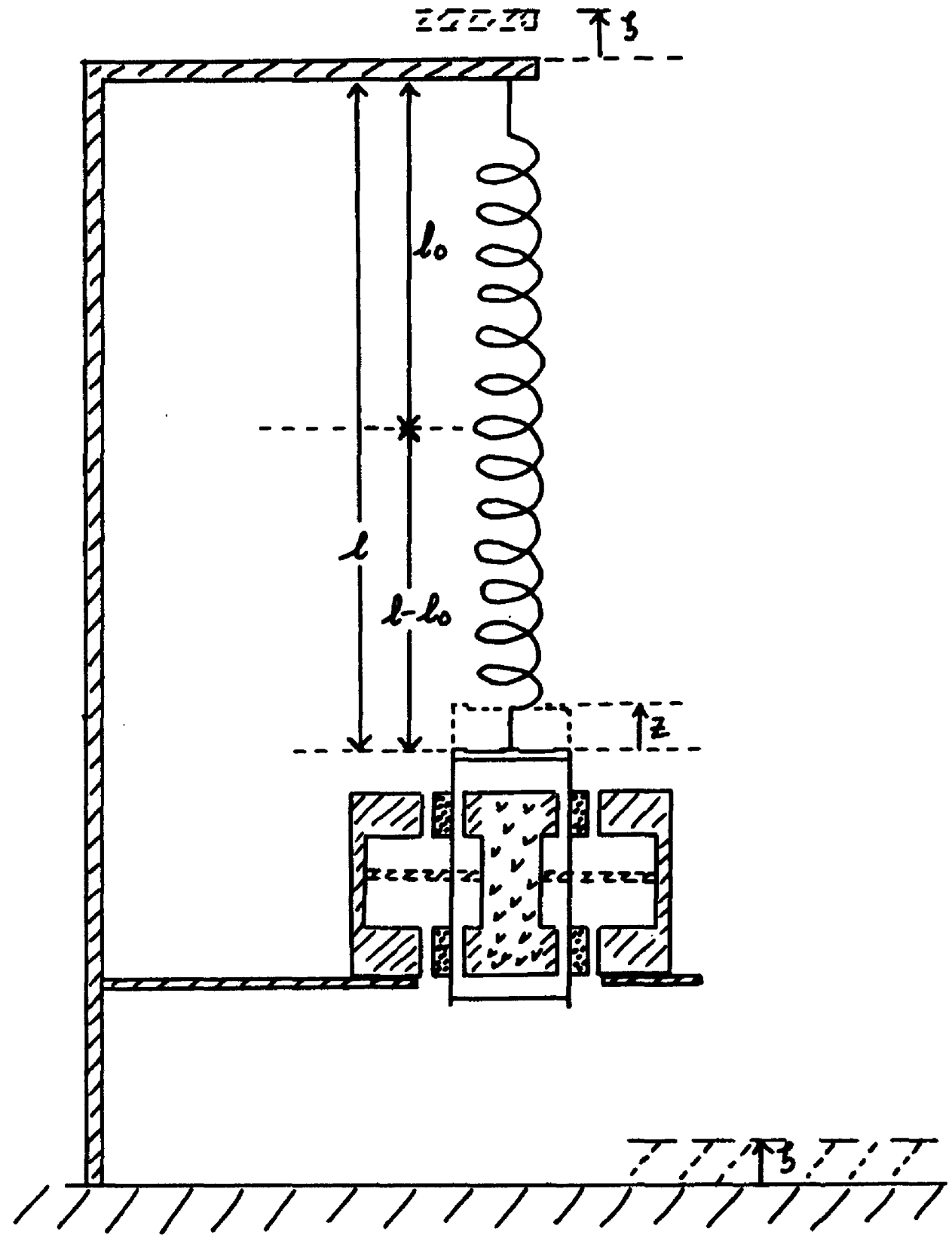
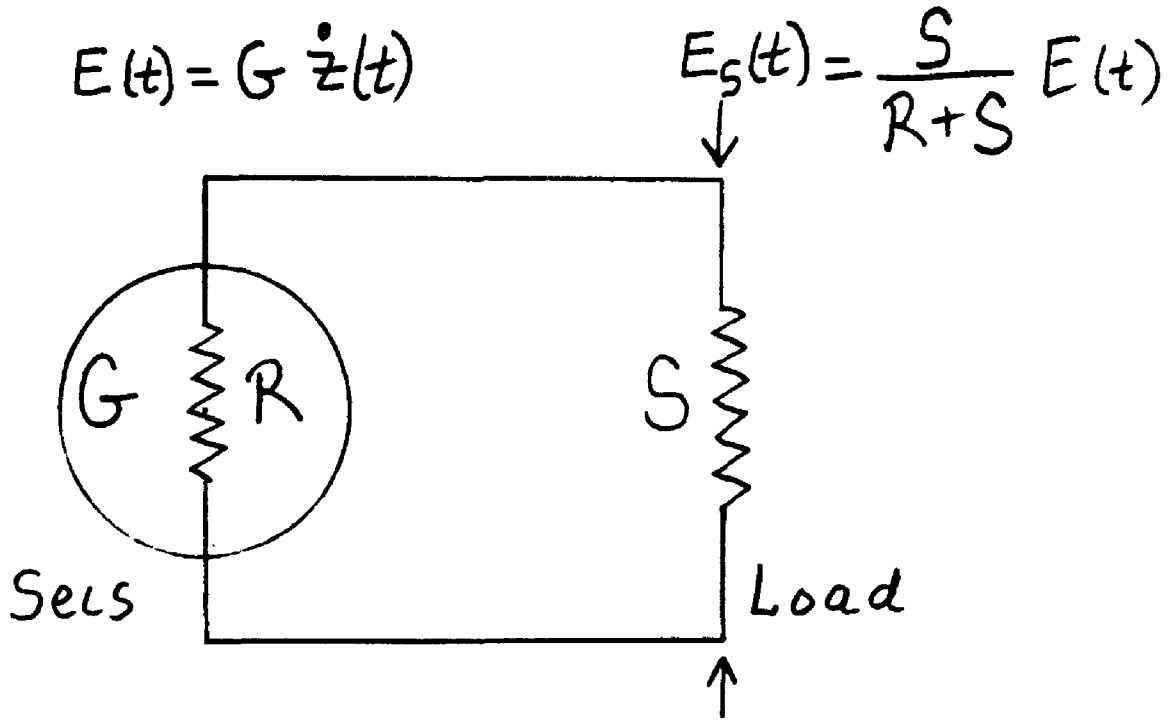


Fig 2

Fig 3


$E(t) = G\,\dot{z}(t)$
$E_S(t) = \frac{S}{R+S} E(t)$
G
R
S
Seis
Load

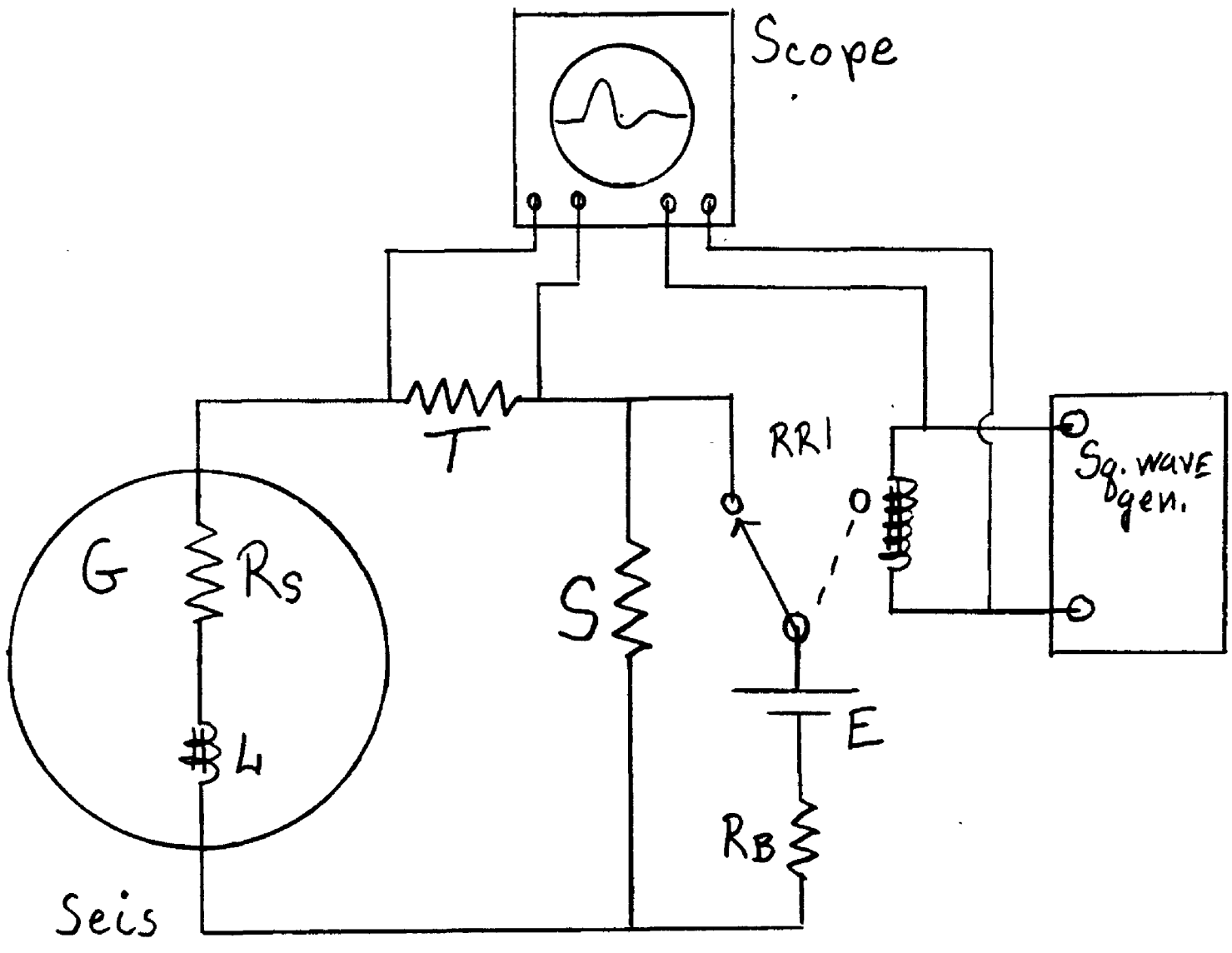

Scope
T
RR1
Sq. wave gen.
G
R_S
L
S
E
R_B
Seis

Fig 4

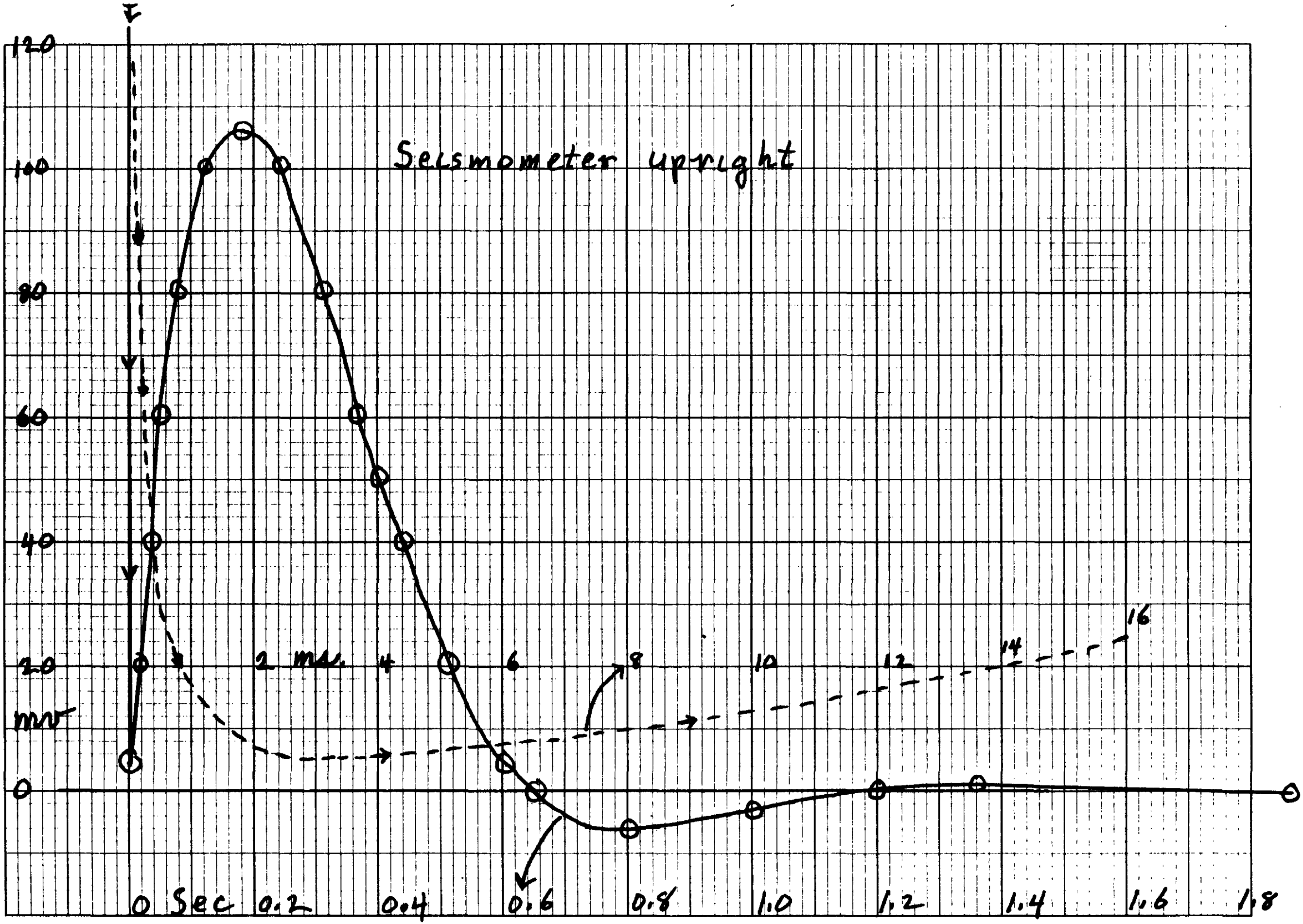


Fig 5a

46 0780

K&E 10 X 10 TO THE INCH • 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.

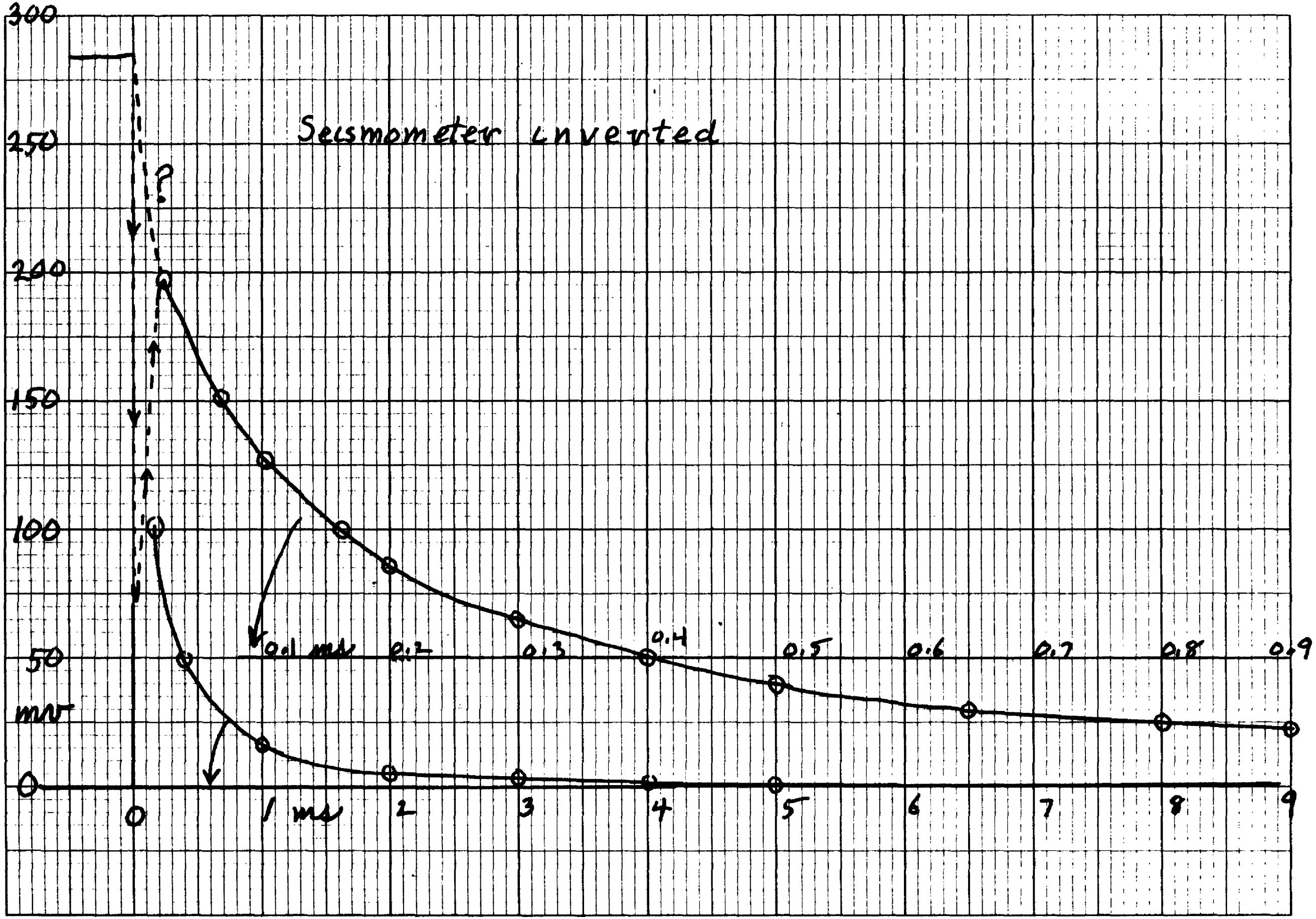
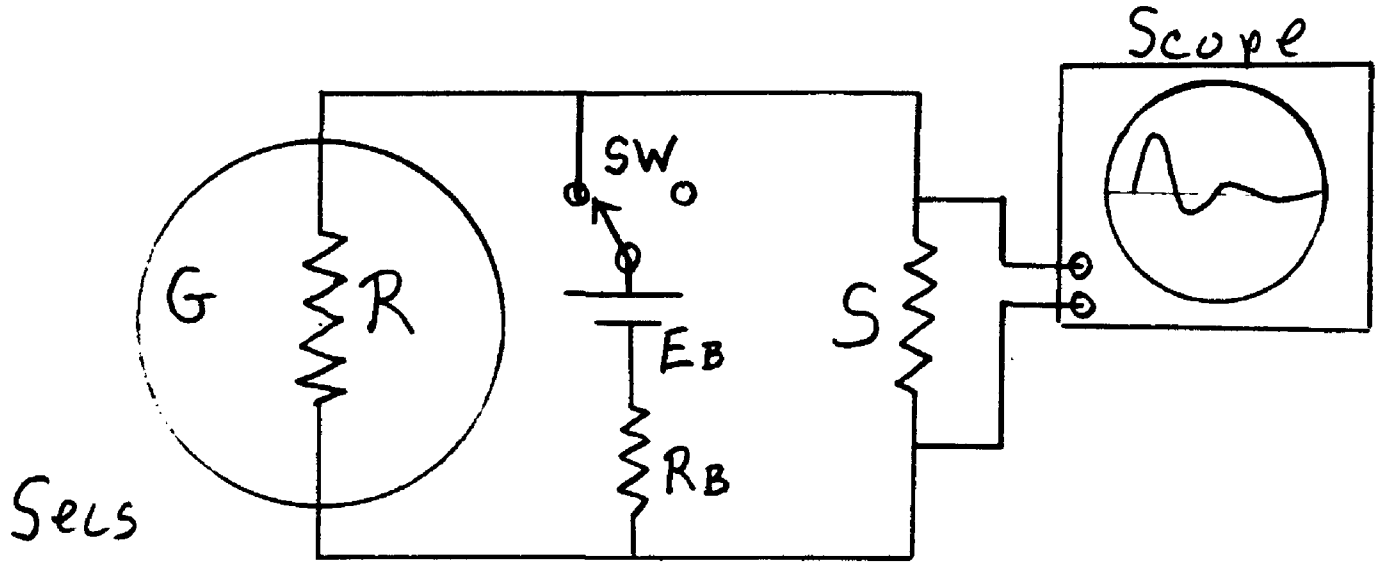


Fig 5 b

K&E 10 X 10 TO THE INCH • 7 X 10 INCHES
KEUFFEL & ESSER CO MADE IN U.S.A.
46 0780



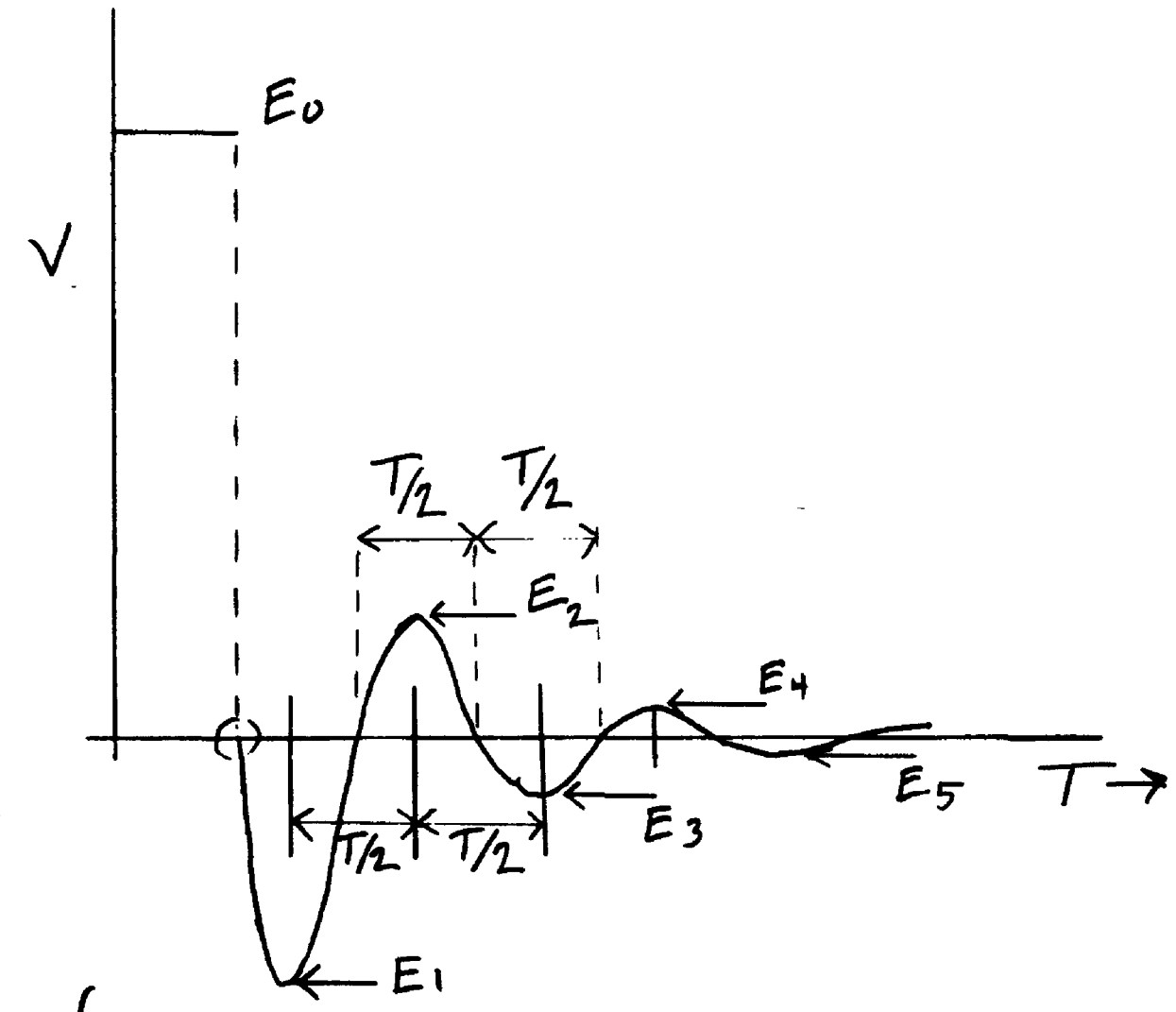


Fig 6