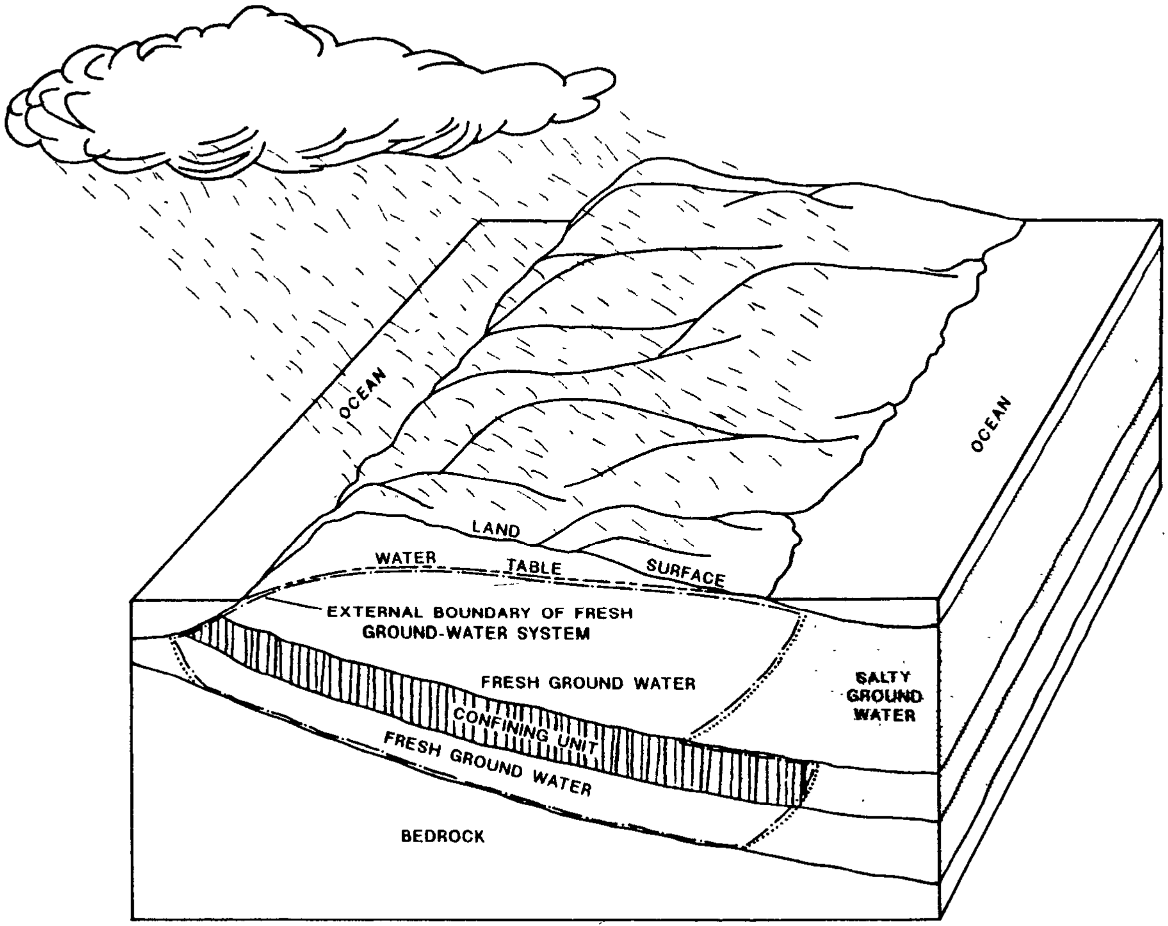
STUDY GUIDE FOR A BEGINNING COURSE IN GROUND-WATER HYDROLOGY: PART II -- INSTRUCTOR'S GUIDE



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Answer to the Third Unnumbered Assignment under "Preliminary Conceptualization of a Ground-Water System"

In figure 1-7 of Note (1-1) (p. 22 in Part I of the Study Guide), observation-well pair (a) indicates a downward component of the head gradient; pair (b) indicates neither a downward nor an upward component, implying nearly horizontal head gradients; and pair (c) indicates an upward component of the head gradient. Consider the longest and several shorter streamlines in any ground-water system. The head relation in observation-well pair (a) would be found where the head is high near the starting point of a streamline, which corresponds to an area of recharge in the system; the relation in pair (b) is typical of the "middle" part of the flow system, where streamlines in aquifers tend to be nearly horizontal; and the relation in pair (c) corresponds to the downgradient discharge part of the flow system. These relations are found at all scales. Refer to the shallow, local flow system depicted in figure 1-13 of Exercise (1-6) (p. 36) for an example of conditions (b) and (c) and to the entire ground-water system depicted in figure 3-7 of Exercise (3-1) (p. 84) for an example of all three conditions.

Analysis of Ground-Water Systems Through Use of Flow Nets

Assignments

*Study Fetter (1988), p. 137-141, 218-229; Freeze and Cherry (1979), p. 168-185; or Todd (1980), p. 83-93.

*Study Note (3-4)--Introducton to discretization.

*Work Exercise (3-2)--Flow net beneath an impermeable wall.

*Study Note (3-5)--Examples of flow nets.

Flow nets depict a selected number of accurately located flowlines and equipotential lines in the flow system, which together provide a quantitatively useful, graphical representation of the ground-water flow field. In fact, problems that involve ground-water flow often can be considered as solved if an accurate flow net is developed. Flow nets can be applied conveniently only in two-dimensional flow problems, and the technique is particularly useful in analyzing vertical sections of flow systems that are oriented along a regional "streamline" (actually, stream surface).

Comments

Flow beneath an impermeable wall (Exercise (3-2)) is the second ground-water system that is analyzed in detail in this course. The instructor's discussion of this system can be enhanced by explicit reference to table 3-1 in Note (3-2) and table 3-2 in Note (3-3). The format suggested at the beginning of Note (3-5) for analyzing flow nets is a repetition of parts of these two tables. Asking the participants to denote the boundary conditions of the flow nets in Note (3-5), and following this exercise with a complete review in class, is highly recommended.

Answers to Exercise (3-2)--Flow Net Beneath an Impermeable Wall

The following comments can be made by the instructor as a part of a class discussion before participants begin work on the flow net:

(1) Review the concept of two-dimensional flow. In this exercise we assume that the flow pattern is replicated exactly in planes parallel to the plane of the figures illustrating the impermeable-wall ground-water system. It is convenient to consider the plane of the figures as the x-z plane. A velocity vector at any point in the flow domain in the y-coordinate direction, which is perpendicular to the x-z plane or the plane of the figures, is equal to the velocity vector with the same (x,z) coordinate in the plane depicted in the figures.

(2) Outline the external geometry of the flow system, which is the boundary of the fine sand.

(3) Because the flow medium is assumed to be isotropic and homogeneous, no layering or internal geometry is present in this system, and hydraulic conductivity is constant throughout the system.

(4) With the assistance of the class, locate and draw the extents of the four boundaries. Participants sometimes designate the upper right-hand boundary, the discharge boundary, as a constant-flux boundary. In principle, this boundary could be designated as a constant-flux boundary if the flow through the system were known. Generally, however, this flow is not known, and one reason for performing an analysis of the system is to determine this flow. Furthermore, if some depth of standing water were present above this boundary--even a small depth--most hydrologists would conceptualize this boundary as a constant-head boundary because, if the system were stressed (not an issue in this exercise), the response of the system with a constant-head boundary would differ markedly from its response with a constant-flux boundary.

(5) Identify where water enters the system (upper left-hand constant-head boundary) and discharges from the system (upper right-hand constant-head boundary). Two bounding flowlines (the outer one along the impermeable sides and bottom, and the inner one along the impermeable wall) connect the inflow and outflow boundaries. Sketch several internal flowlines and equipotential lines. The purpose of this demonstration is to emphasize that, given the external geometry and boundary conditions, we can conceptualize the approximate flow pattern within a ground-water system without detailed data or analysis.

(6) The previous comments relate to the ground-water system depicted in figure 3-19 of Exercise (3-2). A comparison of the system depicted in figure 3-19 with similar real ground-water systems, however, indicates that the position, and possibly the type, of boundaries ST and VU usually are arbitrary instead of actual defined impermeable boundaries. In nature the flow system may extend laterally for a considerable distance. The purpose of simulation in this type of problem is to achieve realistic heads and flows in the vicinity of the engineering structure. A logical approach to simulation of this type of system is to perform a "sensitivity analysis" on the position of boundaries ST and VU--that is, to execute a series of

simulations in which the distance of these two vertical boundaries from the impermeable wall increases continuously until two successive simulations exhibit negligible differences in heads and flows near the wall. The question of assigning a boundary condition to these two vertical boundaries still remains. Possibilities include (a) constant-head, (b) constant-flux, and (c) flowline boundaries. Our reasonable concept of the flow pattern in this system envisions flowlines starting at the upper left-hand constant-head boundary and flowing beneath the wall. Because head is dissipated along flowlines, a vertical constant-head boundary near the wall is not appropriate. Wherever the position of an approximate vertical boundary ST is established, we are likely to neglect a small quantity of lateral inflow. Thus, a lateral constant flux boundary along ST is physically reasonable. A realistic estimate of this flux, however, would require a simulation whose lateral boundary was positioned considerably farther from the impermeable wall than the proposed constant-flux boundary. For this reason, the simplest and usual approach is to treat these lateral boundaries as flowlines in this problem type. The previous considerations did not play a role in positioning the lateral boundaries ST and VU in this exercise because the lateral no-flow impermeable boundaries were specified in the problem definition. However, in most problems of this type, these boundaries would be placed farther from the wall to perform a quantitative engineering analysis if no physical impermeable boundary were present.

The following comments can be made by the instructor as part of a class discussion after participants have completed work on the flow net:

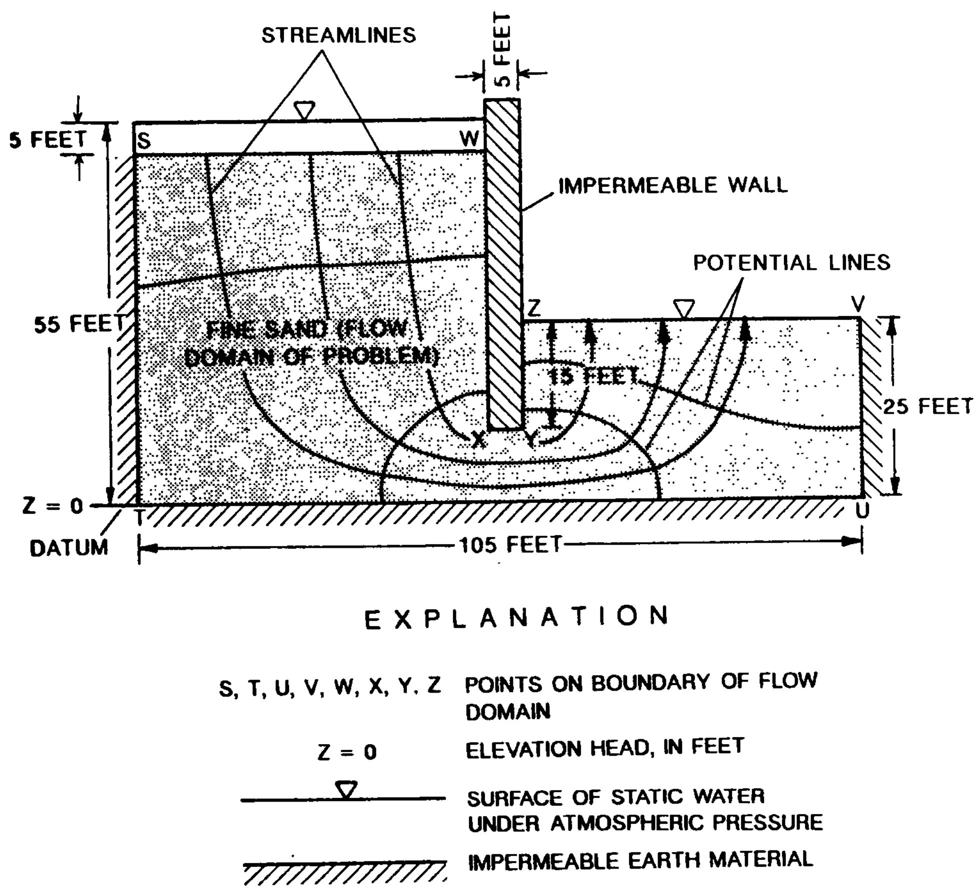
(1) According to Darcy's law, head is dissipated along flowlines. Thirty feet of head must be dissipated between the two ends of all flowlines in this system. The lengths of flowlines in this system vary continuously from a maximum for the vertical left-hand, horizontal bottom, vertical right-hand streamline to a minimum for the streamline that extends along the sides and bottom of the impermeable wall. Thus, the <u>average</u> distance between intersections of equipotential lines and flowlines decreases toward the impermeable wall.

(2) The actual distance between intersections of equipotential lines with any flowline varies widely. In this system, head dissipation is concentrated near the bottom of the wall--that is, equipotential lines are spaced most closely there. If the impermeable wall were deeper, the equipotential lines would be spaced even more closely in this region, and the opposite would be true if the wall were less deep. It is sometimes simpler to think in terms of "resistance to flow" instead of "relative ease of flow" in a ground-water system. The greatest "resistance to flow" in this system is found beneath the impermeable wall, where the area of flow is smallest.

(3) The spacing of flowlines that bound flow tubes containing equal proportions of total flow in the system is related to the pattern of head dissipation. The widths of the five flow tubes along the upper left-hand constant-head boundary decrease slightly, but continuously, from left to right toward the impermeable wall. The heads in the row of heads immediately below the upper left-hand constant-head boundary all must be equal in order for the widths of flow tubes along this boundary to be equal. In fact, heads in this row decrease from left to right toward the impermeable wall. On the discharge side of the impermeable wall, the widths of flow tubes along the upper right-hand boundary decrease markedly from right to left toward the impermeable wall, corresponding to a sharply increasing vertical gradient from right to left along this boundary.

(4) The spacing between equipotential lines and flowlines generally varies in a continuous and orderly manner in flow nets for isotropic and homogeneous media.

The aquifer blocks used in the calculation of block conductances and flows in the impermeable wall problem are shown in figure 3-20. The resultant fully-constructed flow net is shown in figure 3-21, based on the completed calculations in table 3-4.



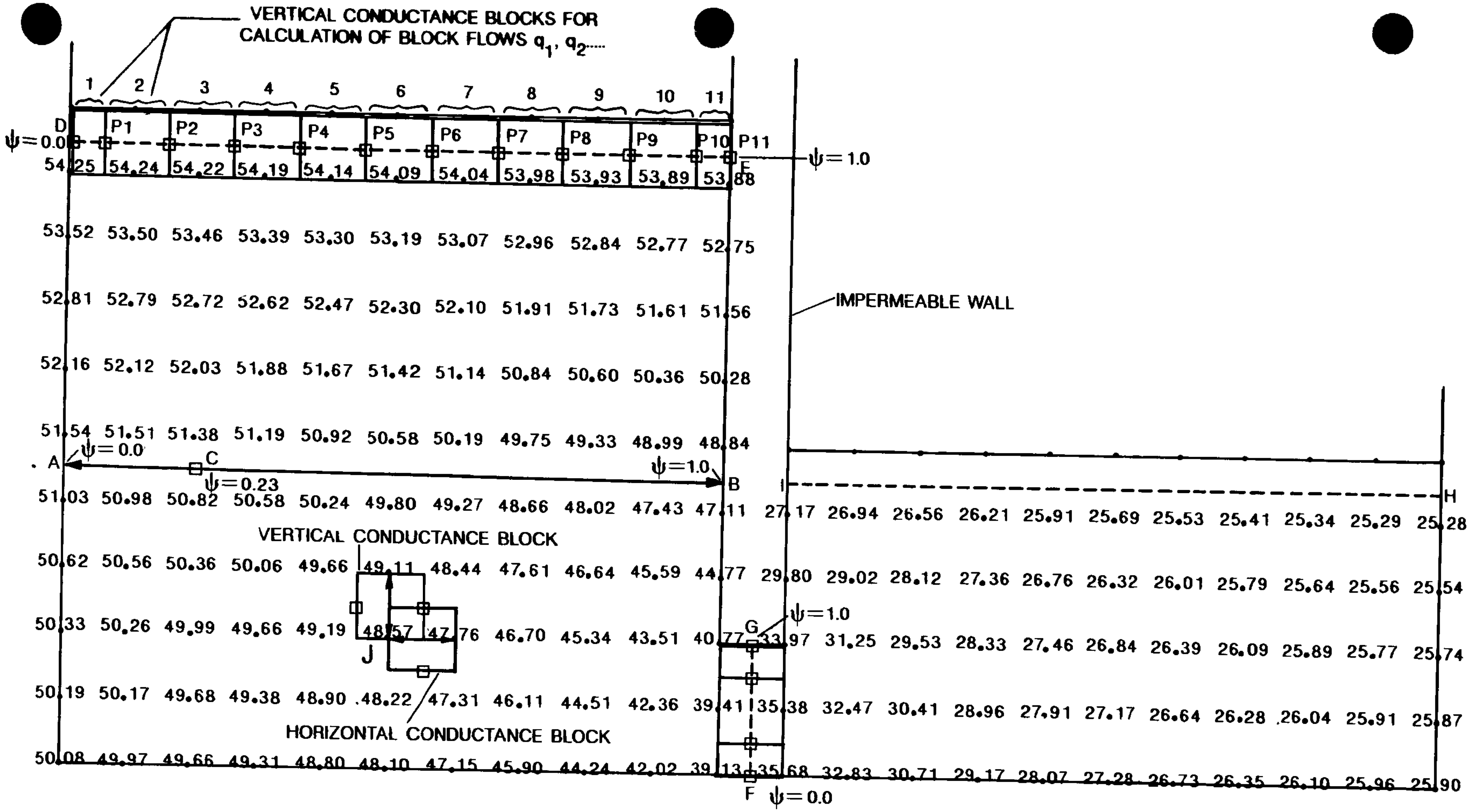
Boundaries

SW and ZV are constant-head boundaries

STUV and WXYZ are streamline, or no-flow boundaries.

The flow field has unit thickness perpendicular to the page.

Figure 9-19.--Vertical section through a ground-water flow system near a partially penetrating impermeable wall showing diagrammatic sketch of flow pattern.



EXPLANATION

A, B, C ... REFERENCE POINTS

ψ =1.0 VALUE OF STREAM FUNCTION

HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY

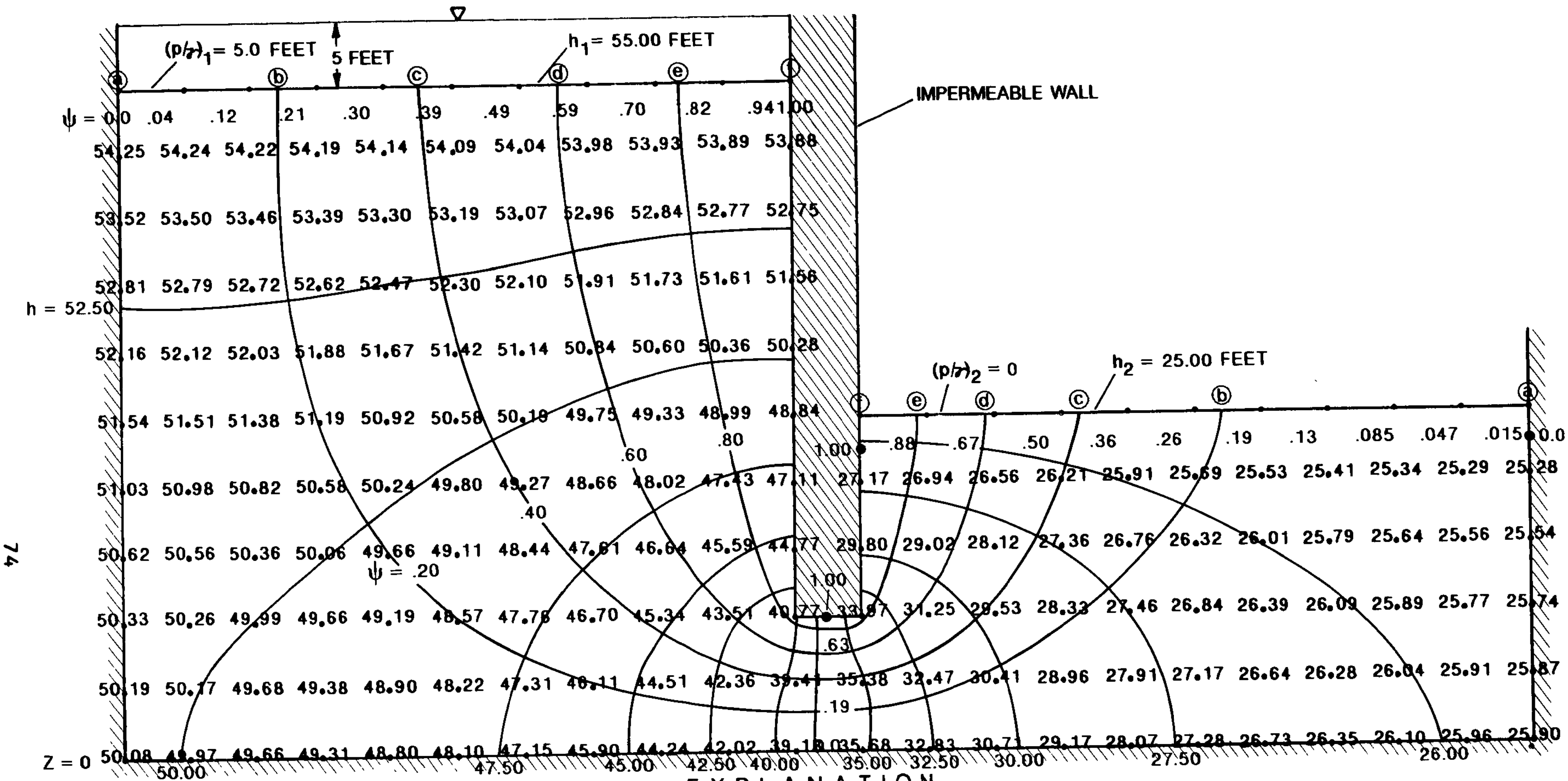
DISTANCE BETWEEN NODES = 5 FEET

------ LINE OF TRAVERSE FOR CALCULATION OF STREAM FUNCTIONS

□ PLOTTING POSITION FOR STREAM FUNCTION

P1, P2 ... NUMBERED PLOTTING POSITIONS FOR TRAVERSE D E

Figure 9-20.--Aquifer blocks for calculating block conductances and block flows, and for plotting positions of calculated values of stream functions.



EXPLANATION

- ▽ SURFACE OF STATIC WATER UNDER ATMOSPHERIC PRESSURE
- IMPERMEABLE MATERIAL
- • NODE IN DISCRETIZED SYSTEM AND DECIMAL POINT OF HEAD VALUE AT NODE
- 53.39 HEAD AT NODE, IN FEET
- Z IS ELEVATION HEAD, p/γ IS PRESSURE HEAD, h IS TOTAL HEAD, IN FEET
- DISTANCE BETWEEN NODES = 5 FEET
- HYDRAULIC CONDUCTIVITY = 45 FEET PER DAY
- $\psi = .20$ VALUE OF STREAM FUNCTION
- (b) FLOWLINE

Figure 9-21.--Flow net for a ground-water system near an impermeable wall.

Table 3-4.--Format for calculation of stream functions in impermeable wall problem (page 1 of 2)

[For locations of numbered blocks, traverse DE, and plotting positions for stream functions p1, p2, ..., see figure 3-20; C_{block} is hydraulic conductance of discretized block, which equals KA/L, where K = hydraulic conductivity of earth material in block, A = cross-sectional area of block perpendicular to direction of ground-water flow, and L = length of block; h_1 and h_2 are head values at nodes located at ends of block; $\Delta h = h_1 - h_2$; q_{block} = flow through a single block; Σq_{block} = flow in a numbered block plus the flows through all lower-numbered blocks (cumulative sum of block flows in traverse); Q_{total} = total flow through the ground-water system beneath the impermeable wall; ft= feet; ft^2/d = square feet per day; ft^3/d = cubic feet per day; Ψ = stream function]

	BLOCK NUMBER	$C_{block} =$ KA/L (ft^2/d)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	$q_{block} = C\Delta h$ (ft^3/d)	Σq_{block} (ft^3/d)	$\Psi = \frac{\Sigma q_{block}}{Q_{total}}$	PLOTTING POSITION FOR STREAM FUNCTIONS
INFLOW BOUNDARY TRAVERSE DE	1	22.5	55.00	54.25	0.75	16.88	16.88	0.04	P1
	2	45.0	55.00	54.24	0.76	34.20	51.08	0.12	P2
	3	45.0	55.00	54.22	0.78	35.10	86.18	0.21	P3
	4	45.0	55.00	54.19	0.81	36.45	122.63	0.30	P4
	5	45.0	55.00	54.14	0.86	38.70	161.33	0.39	P5
	6	45.0	55.00	54.09	0.91	40.95	202.28	0.49	P6
	7	45.0	55.00	54.04	0.96	43.20	245.48	0.59	P7
	8	45.0	55.00	53.98	1.02	45.90	291.38	0.70	P8
	9	45.0	55.00	53.93	1.07	48.15	339.53	0.82	P9
	10	45.0	55.00	53.89	1.11	49.95	389.48	0.94	P10
	11	22.5	55.00	53.88	1.12	25.20	414.68	1.00	P11
BELOW WALL TRAVERSE F G	1	22.5	39.13	35.68	3.45	77.63	77.63	0.19	
	2	45.0	39.41	35.38	4.03	181.35	258.98	0.63	
	3	22.5	40.77	33.97	6.80	153.00	411.98	1.00	

Answers to Exercise (8-2) (continued)

Table 8-4.--Format for calculation of stream functions in impermeable wall problem (page 2 of 2)

	BLOCK NUMBER	C_{block} = KA/L (ft^2/d)	h_1 (ft)	h_2 (ft)	Δh_{block} (ft)	q_{block} = $C \Delta h$ (ft^3/d)	Σq_{block} (ft^3/d)	$\Psi = \frac{\Sigma q_{block}}{Q_{total}}$
OUTFLOW BOUNDARY	1	22.5	25.28	25.00	0.28	6.30	6.30	0.015
	2	45.0	25.29	25.00	0.29	13.05	19.35	0.05
	3	45.0	25.34	25.00	0.34	15.30	34.65	0.085
TRAVERSE H I	4	45.0	25.41	25.00	0.41	18.45	53.10	0.13
	5	45.0	25.53	25.00	0.53	23.85	76.95	0.19
	6	45.0	25.69	25.00	0.69	31.05	108.00	0.26
	7	45.0	25.91	25.00	0.91	40.95	148.95	0.36
	8	45.0	26.21	25.00	1.21	54.45	203.40	0.50
	9	45.0	26.56	25.00	1.56	70.20	273.60	0.67
	10	45.0	26.94	25.00	1.94	87.30	360.90	0.88
	11	22.5	27.17	25.00	2.17	48.83	409.73	1.00

Regional Ground-Water Flow and Depiction of Ground-Water Systems by Means of Hydrogeologic Maps and Sections

Assignments

*Study Fetter (1988), p. 230-258; or Freeze and Cherry (1979), p. 253.

*Study Note (3-6)--Examples of hydrogeologic maps and sections.

A comprehensive introduction to many of the most areally extensive regional aquifer systems in the United States is provided in U.S. Geological Survey Circular 1002 (Sun, 1986).

Common types of hydrogeologic maps and sections include (1) structure-contour maps that depict the topographic surfaces corresponding to the tops and bottoms of hydrogeologic units; (2) isopach (thickness) maps, which can be regarded as difference maps between two selected structure-contour maps; (3) sections that depict hydrogeologic units--sections sometimes show actual lithologic or borehole geophysical logs; (4) fence diagrams and block diagrams, which extend the geometric representation of hydrogeologic units to three dimensions; (5) head maps of a single hydrogeologic unit; and (6) sections showing both hydrogeologic units and head information. Examples of some of these types of hydrogeologic illustrations are given in Note (3-6).

Reference

Heath and Trainer (1968), p. 183-203.

Comments

Participants with a technical background in a subject other than geology will require a more detailed review of hydrogeologic illustrations than geologists. Instructors may wish to substitute their favorite examples of hydrogeologic illustrations in place of the examples in Note (3-6).

Geology and the Occurrence of Ground Water

Assignment

*Study Fetter (1988), p. 259-324; Freeze and Cherry (1979), p. 144-166; or Todd (1980), p. 37-42.

Much has been written about the effect of rock type, depositional environment of sediments, geologic structure, and climate on the occurrence of ground water. The reading assignment listed above deals with these aspects of ground-water hydrology in sufficient detail for the purposes of this course.

References

Davis and DeWiest (1966), p. 318-444.
Meinzer (1923), p. 102-192.

Comments

Time constraints generally dictate that participants acquire needed knowledge of this topic from their own reading; however, if the participants are especially interested in the geology of a particular geographic area and the associated occurrence of ground water, the instructor may lead a profitable discussion to meet this interest.

Description of a Real Ground-Water System

We suggest that at this point in the course the instructor or someone else make a formal presentation that describes in detail the operation of a real ground-water system, preferably one that is of particular interest to the participants. Some of the information that such a presentation might contain is listed below. Of particular importance in the context of this course is a clear conceptualization of the natural system, which includes a careful description of the system's physical boundary conditions (items (2) and (3) in the following list).

(1) Location of study area, geography, and climate.

(2) Geologic framework--pertinent features but not lengthy stratigraphic descriptions.

(3) Natural hydrologic system--how the system operates; inputs and locations; areas of discharge; head maps for pertinent hydrogeologic units; careful designation of boundaries and boundary conditions of natural hydrologic system; data available, and methods to estimate distribution of hydraulic properties.

(4) Human effects on hydrologic system--brief historical survey.

(5) If the presentation includes discussion of a model simulation, reason for developing model or definition of problem to be solved with model.

(6) Description of model--areal extent; areal discretization scheme; number of model layers; careful designation of model boundaries and boundary conditions; comparison with boundaries in (3) and justification of any differences; definition of initial conditions; time-discretization scheme if unsteady model; superposition versus absolute heads; preliminary model runs and what can be learned from them; calibration procedures; and subjective evaluation of validity of final simulation results to solve the problem posed.

Comments

As one might expect from previous exercises and comments, the list of requested information for describing a ground-water system is closely related to table 3-1 of Note (3-2) and table 3-2 of Note (3-3). Continued reference to these tables is appropriate. If a transient model simulation is described in the presentation, the instructor is obliged to discuss initial conditions, a topic not covered thus far in this course. Sufficient information on initial conditions for such an introductory discussion is provided by Franke and others (1987, p. 11-13).

Source of Water to a Pumped Well

Assignment

*Work Exercise (3-3)--Source of water to a pumped well.

What is the source of water to a pumped well placed at various locations within the ground-water system? Answering this question qualitatively in the early part of a ground-water investigation can be a productive part of the conceptualization of a ground-water system. As some thought about the question may suggest, the response of a system to stress ultimately must depend on that system's physical boundary conditions.

Reference and Comments

We recommend Theis's consideration of the source of water to a pumped well, as outlined in the first part of Exercise (3-3), as a conceptually useful way of evaluating the effects of stress on a ground-water system. The value of this approach lies in relating the effects of stress directly to the ground-water system's physical boundary conditions. The best additional reference is Theis's original paper (Theis, 1940).

Answers to Exercise (3-3)--Source of Water to a Pumped Well

(1) The aquifer in this hypothetical problem is a large rectangular prism of sand bounded on its sides, top, and bottom by impermeable surfaces and bounded on its ends by two constant-head boundaries. Thus, Darcy's law is directly applicable. Using the form of Darcy's law Q = TiL, where L is the "width" of the sand prism,

$$T = \frac{Q}{iL} = \frac{3.1\ ft^3/s}{\frac{200\ ft}{10{,}000\ ft} \cdot 10{,}000\ ft} = 1.55 \times 10^{-2}\ ft^2/s = 1{,}340\ ft^2/d\ \text{(rounded)}$$

If this approach is confusing to participants, one can <u>assume</u> any value for the aquifer thickness (b), solve for hydraulic conductivity (K) by using the usual form of Darcy's law, and then multiply K times b to obtain T.

For example, assume the aquifer thickness (b) = 50 ft.

$Q = KiA$

$$K = \frac{Q}{iA} = \frac{3.1\ ft^3/s}{\frac{200}{10{,}000} \cdot 50\ ft \cdot 10{,}000\ ft} = 0.00031\ ft/s$$

$T = .00031\ ft/s \times 50\ ft = 1.55 \times 10^{-2}\ ft^2/s.$

(2) See figure 3-35.

(3) See figure 3-35.

(4) A head divide; to the left of the divide, head gradients in the profile are toward the well; to the right of the divide, head gradients are toward the stream.

(5) See figure 3-35.

(a) These two streamlines are "bounding" streamlines that also represent a kind of "ground-water divide," but not a divide in which head gradients are in opposite directions (except at point B on the head profile along AC in question (4)). "Outside" the two bounding streamlines, all ground water flowing in the aquifer ultimately discharges into the stream; "inside" the two streamlines, all ground water flowing in the aquifer discharges at the well.

(b) The area in plan view bounded by the two streamlines and the reservoir is a contributing area of the pumped well for the specified pumping rate. This area is sometimes called the "area of diversion" of the pumped well. In our particular case, all the water discharged at the well is obtained from the reservoir (an aquifer boundary) between the two bounding streamlines.

(6) Outflow from aquifer during pumping:

2.0 ft^3/s to stream +
3.1 ft^3/s from well =
5.1 ft^3/s total.

For equilibrium to be maintained--Inflow from reservoir = 5.1 ft^3/s.

(7) Before pumping:

Inflow from reservoir = 3.1 ft^3/s.
Outflow to stream = 3.1 ft^3/s.

Pumping from the well has

(a) increased inflow from the reservoir by (5.1 - 3.1) = 2.0 ft^3/s, and

(b) decreased outflow to the stream by (3.1 - 2.0) = 1.1 ft^3/s.

The sum of increased inflow (2.0 ft^3/s) +
decreased outflow (1.1 ft^3/s) =
discharge of the well (3.1 ft^3/s).

(8)(a) One can see readily that the cone of depression in figure 3-36 is "deeper" than the cone in figure 3-35. The significant difference for this discussion, however, is that a head divide no longer exists between the well and the stream in figure 3-36. In other words, a head gradient exists along profile AC between the stream and the well.

(b) The head gradient between the stream and well along profile AC means that, at least for part of the area of contact between the stream and the aquifer, water is moving from the stream into the aquifer (induced inflow from the stream). Thus, in this more extreme case of stress on the aquifer, the source of water to the pumped well in terms of the Theis concepts has three components (rather than two components, as in the previous case)--namely,

(1) increased inflow from the reservoir,
(2) decreased outflow to the stream, and
(3) induced inflow from the stream.

(9)(a) Comparison of boundary conditions in the two systems indicates that (1) the "top" boundary surface is impermeable in system (a) and is a recharge boundary (water table) in system (b); (2) a vertical impermeable boundary in system (b) corresponds to one of the vertical constant-head boundaries in system (a); and (3) the one constant-head boundary in system (b) partially penetrates the ground-water system, whereas both constant-head boundaries in system (a) are completely penetrating.

(b) The presence of constant-head boundaries in a ground-water system reduces drawdowns in response to large pumping stresses in comparison to drawdowns in a system without them.

(i) In general, minimum drawdowns occur in response to pumping when the pumped well is placed adjacent to the constant-head boundary.

(ii) In general, maximum drawdowns caused by a pumped well are obtained at maximum distances from the constant head boundaries. In system (a), this maximum distance is found on a line halfway between the two constant-head boundaries. Maximum drawdowns on this line, corresponding to a minimum influence of the two constant-head boundaries, occur when the well is placed at the two extremities of the line. In system (b), the maximum distance from the single constant-head boundary is found along the "back" vertical impermeable boundary. Maximum drawdowns on this "back" boundary, corresponding to a minimum influence of the single constant-head boundary, occur for a well placed at the extremities of the back boundary or the two "back corners" of the ground-water system.

(c) As noted in previous questions, the possible sources of water in system (a) under assumed conditions of steady flow are reduced outflow to one constant-head boundary and increased inflow from two constant-head boundaries. In system (b), possible sources of water are reduced outflow to and increased inflow from the single constant-head boundary. In system (b), large stresses will cause the gaining stream to become dry.

(d) Previous discussion has developed the concept that hydraulic conditions near any constant-head boundary in hydraulic connection with the ground-water system will change in response to a pumping stress. In system (b), the quantity of recharge or flux at the water table is fixed and, therefore, is not affected directly by a pumping stress. If the pumping stress lowers heads in areas where the water table is shallow and evapotranspiration from the water table is active, however, the pumping stress may result in a decrease in evapotranspiration from the water table. This result of pumping is sometimes referred to as "evapotranspiration salvage."

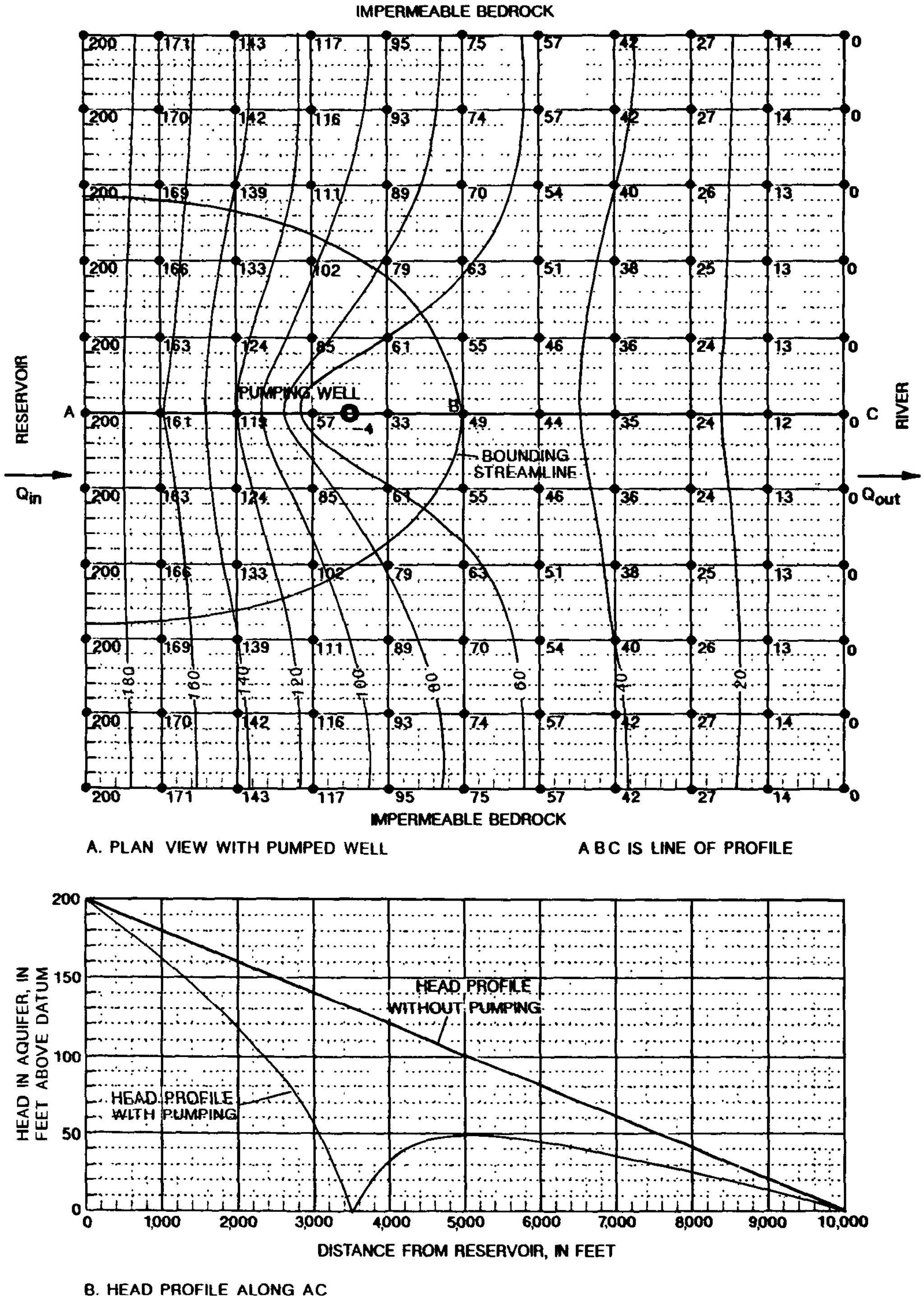


Figure 9-35.--(A) Head map for the stressed aquifer when the pumping rate of the well is 9.1 cubic feet per second with bounding flowlines delineating the area of diversion of the pumped well. (B) Head profile along section AC in (A).

Role of Numerical Simulation in Analyzing Ground-Water Systems

Assignments

*Study Fetter (1988), p. 525-548; Freeze and Cherry (1979), p. 352-364, 540-541; or Todd (1980), p. 384-408.

*Study Note (3-7)--Role of numerical simulation in analyzing ground-water systems.

Numerical simulation is the most powerful quantitative tool available to the hydrogeologist. One example of a well-documented, general-purpose three-dimensional numerical model for ground-water-flow simulation is the U.S. Geological Survey modular model (McDonald and Harbaugh, 1988). The purpose of the brief comments in Note (3-7) is to suggest a number of ways in which this tool can be used effectively.

Simulation, however, can only be used effectively by a knowledgeable hydrologist. The authors have observed instances in which simulation was applied incorrectly. Unfortunately, although the results of these simulations are incorrect and misleading, the conceptual errors leading to these incorrect results may be difficult to identify, and the results may be perceived as correct because they are results of a simulation.

Comments

Because digital computers with their ever-increasing computational efficiency are widely accessible, numerical simulation is now without question the simulation method of choice in ground-water studies. The outline for this course, however, does not include an introduction to numerical simulation. The purpose of Note (3-7) is to stimulate a discussion of the role of numerical simulation in ground-water studies. The emphasis in this course on the system concept and the treatment of the ground-water-flow equation provide the requisite background, and we recommend that an introduction to simulation be part of the next step in the ground-water education of course participants.

SECTION (4)--GROUND-WATER FLOW TO WELLS

Wells are our direct means of access, or "window," to the subsurface environment. Uses of wells include pumping water for water supply, measuring pressures and heads, obtaining ground-water samples for chemical analysis, acting as an access hole for borehole geophysical logs, and direct sampling of earth materials for geologic description and laboratory analysis, primarily during the process of drilling the wells. Hydrogeologic investigations are based on these potential sources of well-related information.

Concept of Ground-Water Flow to Wells

Assignment

*Look up in Fetter (1988), both in the glossary and in the index, and write the definitions of the following terms relating to radial flow and wells: drawdown, specific capacity of well, completely penetrating well, partially penetrating well, leaky confined aquifer, leaky artesian aquifer, semiconfined aquifer, and leaky confining unit or layer.

*Study Note (4-1)--Concept of ground-water flow to wells.

The general laws (Darcy's law and the principle of continuity) that govern ground-water flow to wells are the same as those that govern regional ground-water flow. The system concept is equally valid--we are still concerned with system geometry, both external and internal; boundary conditions; initial conditions; and spatial distribution of hydraulic characteristics, as outlined in table 1 of Note (3-2). The process of removing water from a vertical well, however, imposes a particular geometry on the ground-water flow pattern in the vicinity of the well that is called radial flow. Radial flow to a pumped well is a strongly converging flow whose geometry can be described by means of a particular family of differential equations that utilize cylindrical coordinates (r,z) instead of cartesian coordinates (x,y,z). A large number of analytical solutions to these differential equations with different boundary conditions describe the distribution of head near a pumped well.

Comments

Flow to wells, or radial flow, which includes aquifer testing by pumping a well, is a subspecialty in ground-water hydrology with a large and technically complex literature. In a 1- or 2-week workshop or a one-semester college course in introductory ground-water hydrology, time generally is insufficient to cover in detail even the material on radial flow in the keyed course textbooks. Given this time constraint, we have opted to include only an abbreviated list of possible topics in this outline that we consider essential to begin the study of radial flow, accompanied by an introduction to three widely applicable analytical solutions.

In the initial discussion of radial flow, participants will benefit from a review of polar coordinates in the horizontal plane (r, θ), radial or cylindrical coordinates in three dimensions (r, θ, z), and the concept of radial symmetry. When radial symmetry is assumed, the angular coordinate θ does not appear explicitly in the differential equation associated with an analytical solution to a radial flow problem. For example, the differential equation (6-1) in Fetter (1988, p. 162) assumes radial symmetry, a horizontal aquifer whose horizontal hydraulic conductivity is constant, and horizontal flowlines within the aquifer. The latter assumption implies that the pumped well completely penetrates the aquifer.

Analysis of Flow to a Well--Introduction to Basic Analytical Solutions

Assignments

*Study Note (4-2)--Analytical solutions to the differential equations governing ground-water flow.

*Study Fetter (1988), p. 143, 199-201; Freeze and Cherry (1979), p. 188-189, 314-319; or Todd (1980), p. 112-113, 115-119, 123-124.

*Study Note (4-3)--Derivation of the Thiem equation for confined radial flow.

*Work Exercise (4-1)--Derivation of the Dupuit-Thiem equation for unconfined radial flow.

*Study Fetter (1988), p. 161-169.

*Study Note (4-4)--Additional analytical equations for well-hydraulic problems.

This subsection is primarily a study section that provides an introduction to some of the simplest and most widely applied radial-flow equations. We focus on three such equations: (1) the Thiem equation for steady-state confined flow, (2) the Dupuit-Thiem equation for steady-state unconfined flow, and (3) the Theis equation for unsteady confined flow. These and all other radial-flow equations relate to specific, highly idealized ground-water flow systems. We cannot overemphasize the importance of learning the key features of the individual flow systems to which each equation applies. These key features relate in large part to the boundary conditions that are assumed in the derivation of a given equation.

References

Bennett, Reilly, and Hill (1990), p. 43-58.
Davis and DeWiest (1966), p. 183-186, 201-205.
Harr (1962), p. 40-42, 57-59.
Reed (1980).

Comments

The "radius of influence" of a pumped well, sometimes designated r_e or R, is a useful concept in connection with the Thiem and Dupuit-Thiem equations. This term is loosely defined, but implies a distance from the pumped well at which the drawdown in response to that particular stress either cannot be measured or becomes impossible to distinguish from "background noise" in the aquifer. With a steel tape we measure water levels in wells to 0.01 ft. Because natural logarithms of radial distances from the pumped well are present in these equations, an approximation of the distance at which a drawdown of 0.01 ft occurs is sufficient to define the radius of influence.

The "Dupuit assumptions" underlying the derivation of the Dupuit-Thiem equation and all other equations based on these assumptions require explanation. The explanation of these assumptions is more complete in Davis and DeWiest (1966) and Harr (1962) than in the keyed course textbooks. The Dupuit analysis assumes uniform horizontal flow and neglects vertical gradients and the presence of a seepage face at the pumped well and, as a result, the water-table profile calculated from the Dupuit analysis is always lower than the actual water-table profile in the vicinity of the well. Note that the water-table profile depicted in figure 4-4 of Exercise (4-1) is based on the Dupuit assumptions and does not represent an actual water-table profile. Because this is the first topic in this course to involve a seepage face, a general introduction to the seepage face as a boundary condition is appropriate at this time. (See Franke and others, 1987, p. 5-6).

The boundary conditions and other assumptions of the Theis solution merit class discussion because they define a hypothetical aquifer that can never be found in nature. Despite this limitation, the Theis solution is exceedingly useful in the transient analysis of aquifer tests to determine aquifer properties. As noted in Exercise (3-3), the possible sources of water to a pumped well are (1) increased inflow to the aquifer, (2) decreased outflow from the aquifer, and (3) removal of water from storage. After the Theis aquifer has been described, ask the class which possible source or sources contribute water to the well. (The answer is storage only, item (3).)

Answer to Exercise (4-1) Derivation of the Dupuit-Thiem Equation for Unconfined Radial Flow

As indicated in Note (4-3) on the derivation of the Thiem equation, Darcy's law can be written as follows (modified from Fetter, 1988, p. 123, equation 5-19):

$$Q = -KA\frac{dh}{dr}$$

where A is the cross-sectional area through which the water is flowing, r is distance (in this case, radial distance), h is head, K is hydraulic conductivity, and Q is volumetric flow rate. Steady flow to a well in a water-table or unconfined aquifer (an aquifer with a free surface as the top boundary) (fig. 4-4) is radially convergent flow through a cylindrical area around the well. As inferred from figure 4-4, the area A through which flow to the pumped well at any radial distance r occurs is

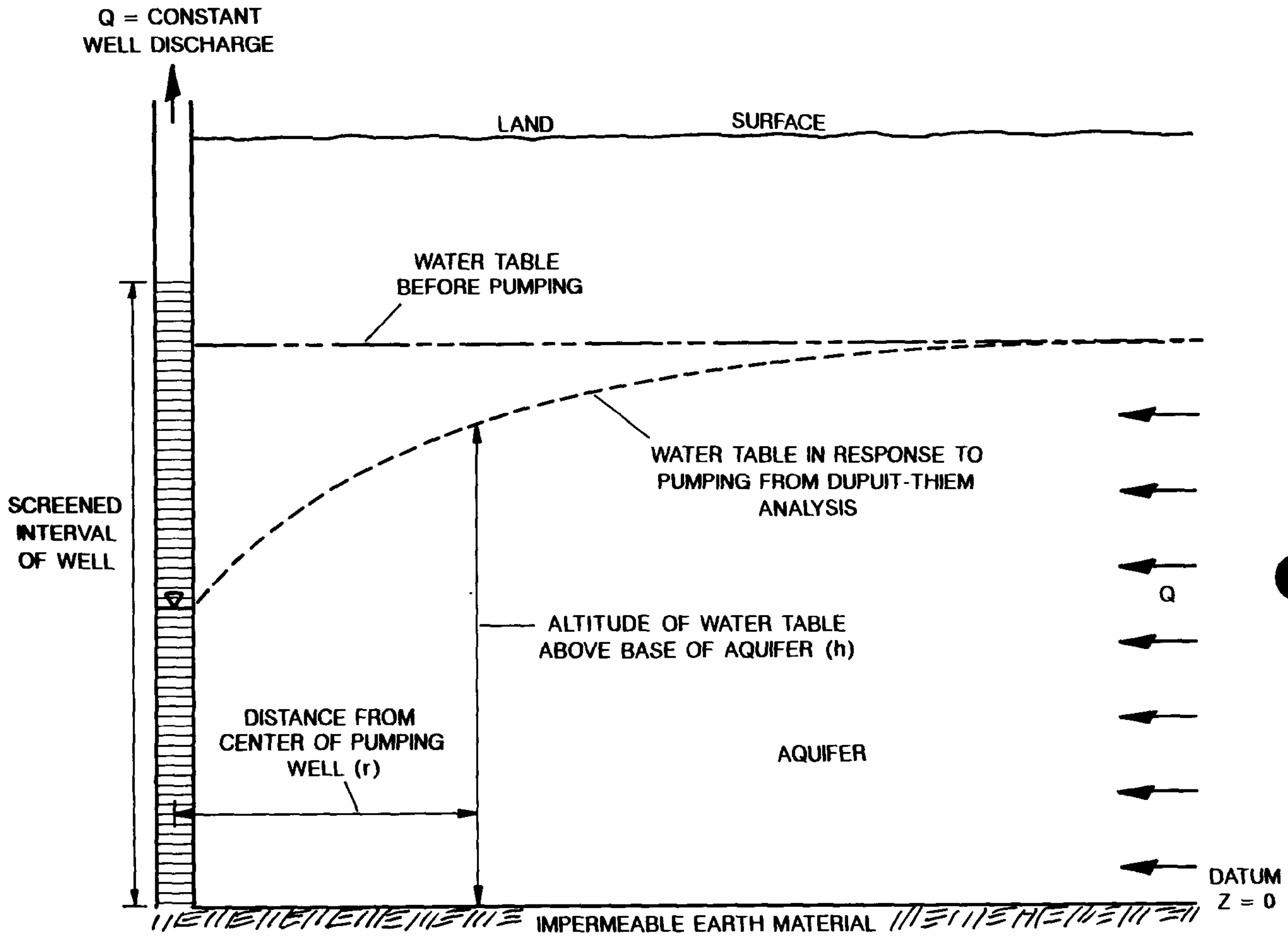
$$A = 2\pi rh,$$

where the head datum is set at a bottom impermeable horizontal bed so that h is the saturated thickness of the unconfined aquifer as well as the hydraulic head. Substituting into Darcy's law gives

$$Q = -2\pi rKh\frac{dh}{dr}.$$

For steady flow, the constant quantity of water pumped, Q, is also the flow rate through any cylindrical shell around the well. This governing differential equation can be solved by separating variables and integrating both sides of the equation. Separation of variables gives

$$\frac{1}{r}dr = -\frac{2\pi K}{Q}hdh.$$



Note: Q is constant well discharge which equals constant radial flow in aquifer to well; Z is elevation head

Figure 4-4.--Steady flow to a completely penetrating well in an unconfined aquifer as represented in a Dupuit-Thiem analysis.

Integrating from r_2 to r_1, where the heads are h_2 and h_1, respectively,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} - \frac{2\pi K}{Q} h dh,$$

leads to

$$\ln r_2 - \ln r_1 = - \frac{\pi K}{Q} (h_2^2 - h_1^2) \quad .$$

Rearranging terms gives

$$K = \frac{Q}{\pi(h_2^2 - h_1^2)} \ln \frac{r_2}{r_1}$$

where the pumping rate is defined as a positive number. This is the Dupuit-Thiem equation (Fetter, 1988, p. 200, equation 6-57), which Fetter identifies as the Thiem equation for an unconfined aquifer.

Analysis of Flow to a Well--Applying Analytical Solutions to Specific Problems

Assignments

*Study Fetter (1988), p. 170-199; Freeze and Cherry (1979), p. 343-349; or Todd (1980), p. 125-134.

*Work Exercise (4-2)--Comparison of drawdown near a pumped well in confined and unconfined aquifers by using the Thiem and Dupuit-Thiem equations.

*Work (a) the example problem in Fetter (1988), p. 165, and (b) by using the same data as in (a), determine the radial distance at which the drawdown would be 0.30 m after 1 d of pumping.

*Work Exercise (4-3)--Analysis of a hypothetical aquifer test by using the Theis solution.

In this subsection we apply the analytical solutions introduced in the previous section to some typical problems. Additional problems, some that require other analytical solutions, are available in Fetter (1988) at the end of chapter 6.

Reference

Heath and Trainer (1968), p. 108-119, 129.

Comments

Exercise (4-2) illustrates concepts of linearity and nonlinearity of ground-water systems. In the succeeding exercises most participants will be concerned primarily with mastering the mechanics of obtaining an answer from the Theis solution. The instructor's role is to help the participants master the mechanics and also to discuss hydrologic applications of, and hydrologic insights gained from, the Theis solution.

Answers to Exercise (4-2)--Comparison of Drawdown Near a Pumped Well in Confined and Unconfined Aquifers Through Use of the Thiem and Dupuit-Thiem Equations

The next pages contain (1) the appropriate formulas in a convenient form for calculation, (2) a listing of calculated answers in a table, (3) a plot of calculated drawdowns from the table as a function of well pumping rate, (4) answers to the final two questions in the exercise and brief remarks on linear and nonlinear equations and relations in ground-water flow, and (5) the answer to the problem based on an example problem in Fetter (1988), p. 165.

Formulas for calculation:

Confined case (Thiem equation)

$$Q = 2\pi Kb \frac{(h_{r_e} - h_r)}{\ln r_e/r}$$

$$h_r = h_{r_e} - \frac{Q}{2\pi Kb} \ln r_e/r$$

$$s = h_{r_e} - h_r$$

Unconfined case (Dupuit-Thiem equation)

$$Q = \frac{\pi K (h_{r_e}^2 - h_r^2)}{\ln r_e/r}$$

$$h_r^2 = h_{r_e}^2 - \frac{Q}{\pi K} \ln r_e/r$$

$$h_r = \sqrt{h_{r_e}^2 - \frac{Q}{\pi K} \ln r_e/r}$$

$$s = h_{r_e} - h_r$$

Results of calculations obtained by using the Thiem and Dupuit-Thiem equations

[ft^3/d = cubic feet per day; h_{r100} is head in feet at r = 100 feet from pumped well; Δh_{r100} is drawdown at r = 100 feet from pumped well; ft = feet; Q is pumping rate of well]

Pumping rate of well (ft^3/d)	Confined case (Thiem equation) h_{r100} (ft)	Confined case (Thiem equation) Δh_{r100} (ft)	Unconfined case (Dupuit-Thiem equation) h_{r100} (ft)	Unconfined case (Dupuit-Thiem equation) Δh_{r100} (ft)
Q_1 = 25,920 (0.3 ft^3/s)	194.93	5.07	69.75	5.25
Q_2 = 51,840 (0.6 ft^3/s)	189.87	10.13	64.07	10.93
Q_3 = 103,680 (1.2 ft^3/s)	179.74	20.26	50.84	24.16

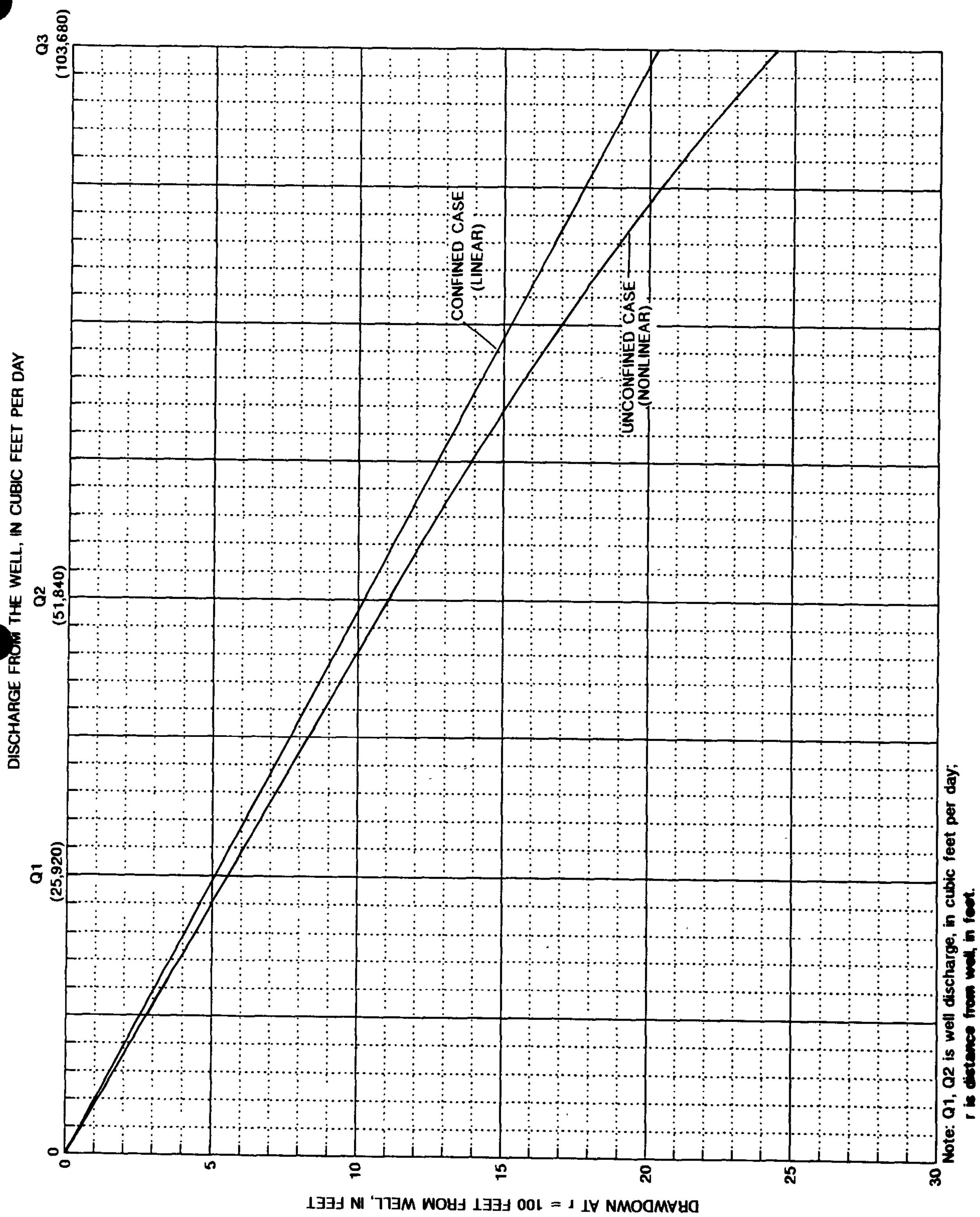


Figure 4-5.--Plot of calculated drawdowns obtained by using the Thiem (confined case) and Dupuit-Thiem (unconfined case) equations.

Answers to final two questions in Exercise (4-2), and additional remarks

(1) A change in initial head in the confined aquifer will not change the value of calculated drawdown. We see by inspection of the Thiem equation that, given values of Q, K, b, r_e, and r, the calculated value of drawdown $(h_{r_e} - h_r)$ is independent of the absolute values of the initial and pumping heads. Because of this fact, Q, the pumping rate of the well, and s, the drawdown in the well or at any other point in the ground-water system, are linearly related. On the other hand, inspection of the Dupuit-Thiem equation shows that the drawdown at any point in the system $(h_{r_e} - h_r)$ is a function of the absolute magnitude of the initial and pumping heads, as indicated by the term $(h_{r_e}^2 - h_r^2)$. In the unconfined case Q is linearly related to $(h_{r_e}^2 - h_r^2)$ but is not linearly related to $(h_{r_e} - h_r)$.

(2) The plot of calculated drawdowns in the previous table illustrates these points in a way that words cannot. In the confined case a linear relation exists between Q and s. In the unconfined case the relation between Q and s is nearly linear, and approaches the curve for the confined case for small drawdowns ("small" drawdown means a numerical value of drawdown that is small relative to the saturated thickness of the unconfined aquifer). As drawdown in the unconfined aquifer increases, however, the relation between Q and s becomes increasingly nonlinear, and the curve for the unconfined case deviates increasingly from the curve (straight line) for the confined case.

Note that for similar hydrologic conditions (that is, the initial saturated thicknesses are equal) the drawdown in the unconfined case is always greater than the corresponding drawdown in the confined case in the graph under consideration. This statement also is true in general. Because drawdowns in the unconfined case involve aquifer dewatering and, thus, a decrease in the saturated thickness in which flow can occur (which is equivalent to a decrease in transmissivity of the unconfined aquifer), the "resistance to flow" must increase as drawdowns increase. This increase in flow resistance in the unconfined case causes an additional increment of drawdown in comparison to the confined case in which no dewatering occurs, and the saturated thickness of the aquifer and associated "resistance to flow" remain constant.

This fundamental difference between confined and unconfined flow is reflected in the differential equations that were solved to derive the Thiem and Dupuit-Thiem equations. The relevant terms to compare in the two differential equations are (a) $b \frac{dh}{dr}$ for the confined case and (b) $h \frac{dh}{dr}$ for the unconfined case. In (a) the aquifer thickness b is a constant and, thus, does not enter into the integration of this term. In (b) the changing thickness of the unconfined aquifer h is substituted for the constant aquifer thickness b in the confined case. The term $h \frac{dh}{dr}$ is a nonlinear term in h and its integration is a nonlinear solution, the Dupuit-Thiem equation, as we have seen.

In general, flow in confined aquifers is governed by linear differential equations (for example, the flow equation derived in Note (2-3)) and is inherently linear as long as the system boundary conditions are linear. A moving freshwater-saltwater interface is an example of a nonlinear boundary. In contrast, because of changes in the saturated thickness of unconfined aquifers under different hydrologic conditions, flow in unconfined aquifers is inherently nonlinear.

As noted subsequently in this section, the principle of superposition applies to linear systems with linear boundary conditions. See Note (4-5) and Exercise (4-4).

Answers to Unnumbered Example Problem

(a) See Fetter (1988), p. 165.

(b) $$h_0 - h = \frac{Q}{4\pi T} W(u)$$

$$.30\text{m} = W(u) \frac{2{,}725 \text{ m}^3/\text{d}}{4\pi \times 300 \text{ m}^2/\text{d}}$$

$$W(u) = \frac{.3 \times 4\pi \times 300}{2{,}725} = 0.4150$$

From the table of W(u) and u (Fetter, 1988, p. 550), for W(u) = 0.4150,

$$u = 6.5 \times 10^{-1}$$

$$u = \frac{r^2 S}{4Tt}$$

$$6.5 \times 10^{-1} = \frac{r^2 (.005)}{4 \times 300 \times 1}$$

$$r = \sqrt{\frac{6.5 \times 10^{-1} \times 4 \times 300}{.005}}$$

$$r = 395 \text{ m}$$

Answers to Exercise (4-3)--Analysis of a Hypothetical Aquifer Test by Using the Theis Solution

The next pages contain (1) individual plots of s against t for the three observation wells in table 4-2, (2) an analysis for T and S with match point and complete calculations obtained by using the data plot for observation well N-3 (r = 800 ft), (3) a plot of s against t/r^2 obtained by using data from all three observation wells, and (4) an analysis for T and S obtained by using the plot in (3) with match point and complete calculations.

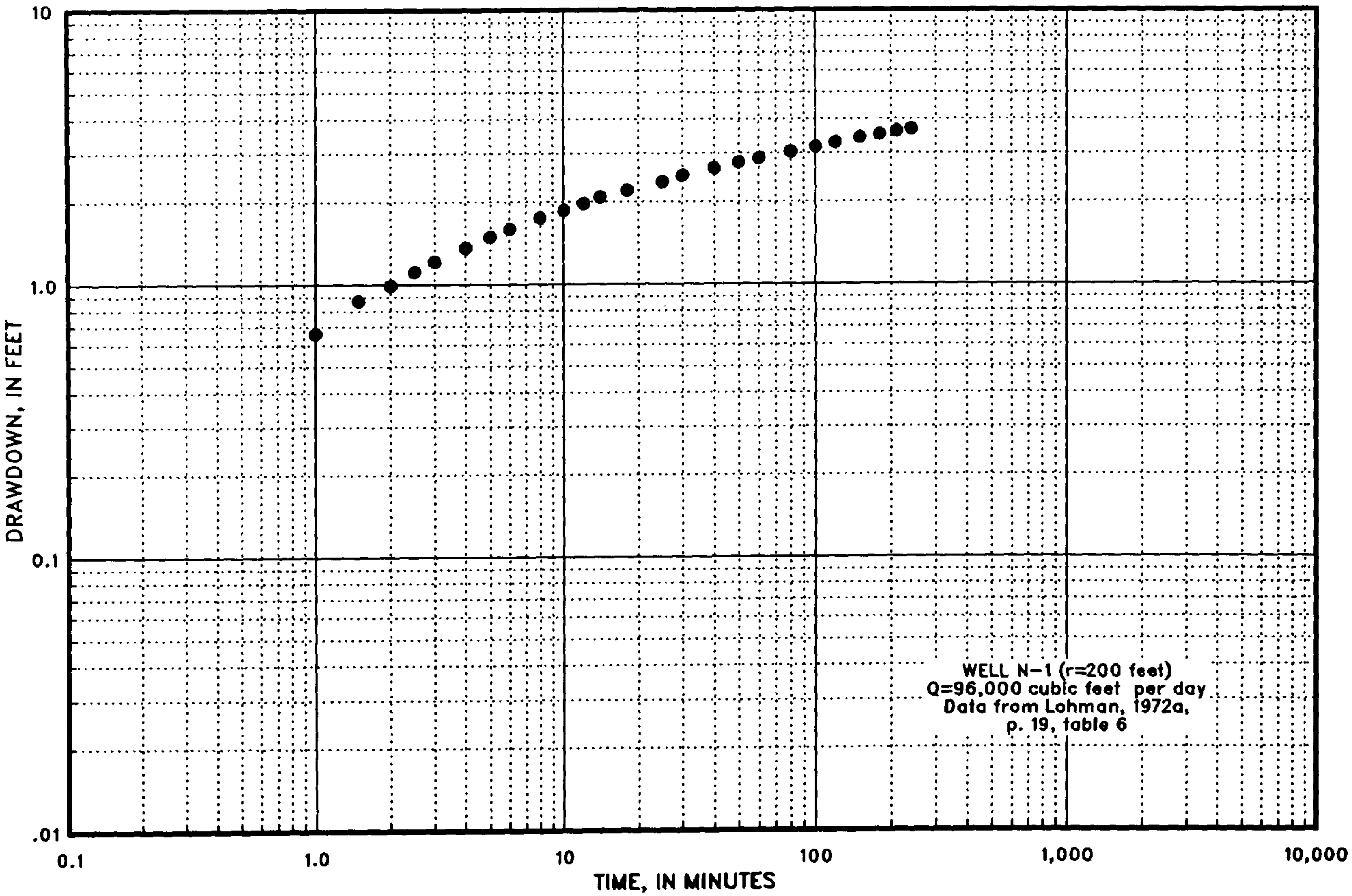


Figure 4-7(a).--Double logarithmic plot of drawdown against time for well N-1 (r = 200 feet). (From Lohman, 1972, table 6.)

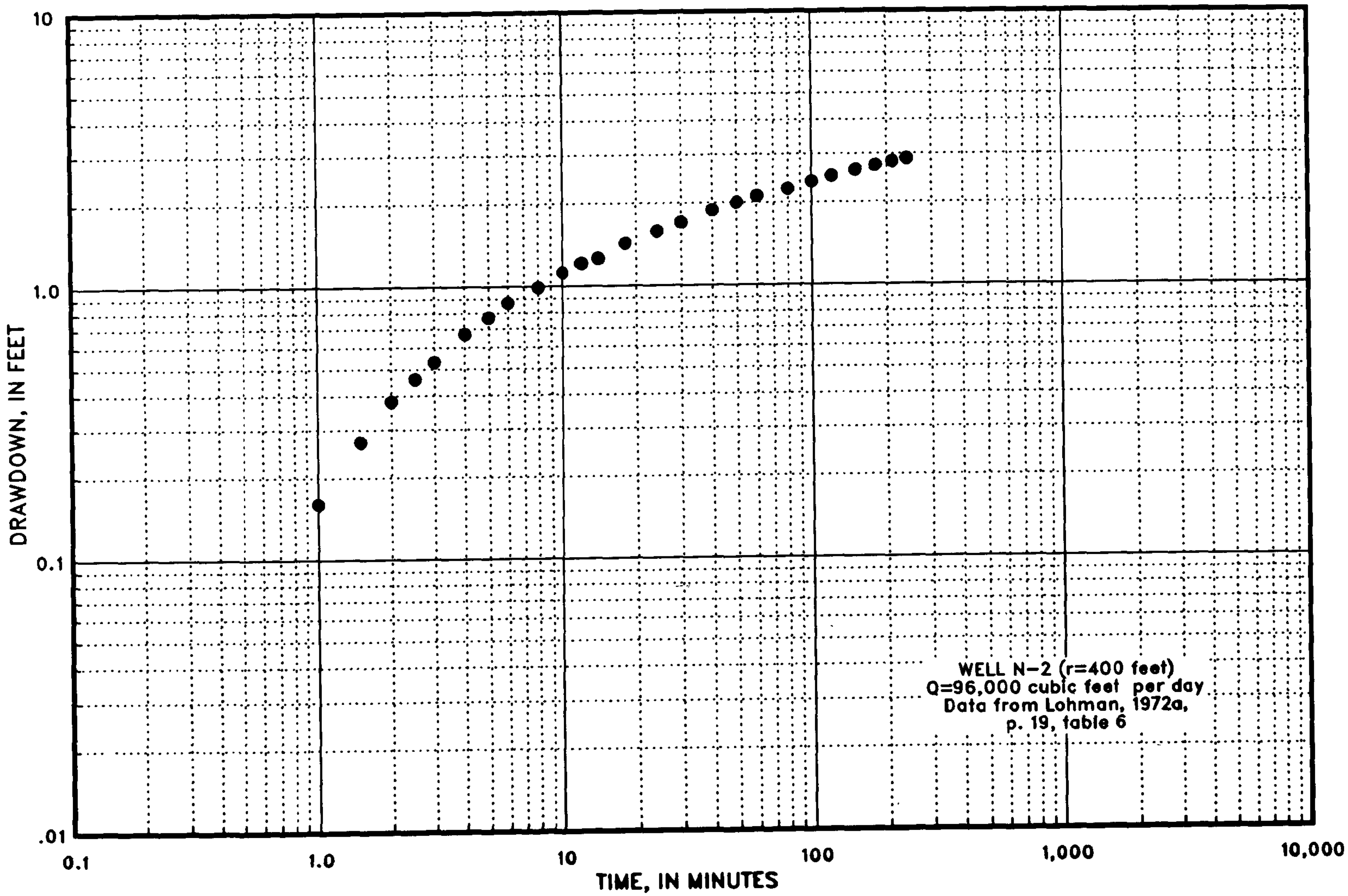


*Figure 4-7(b).--**Double logarithmic plot of drawdown against time for well N-2 (r = 400 feet).***
(From Lohman, 1972, table 6.)

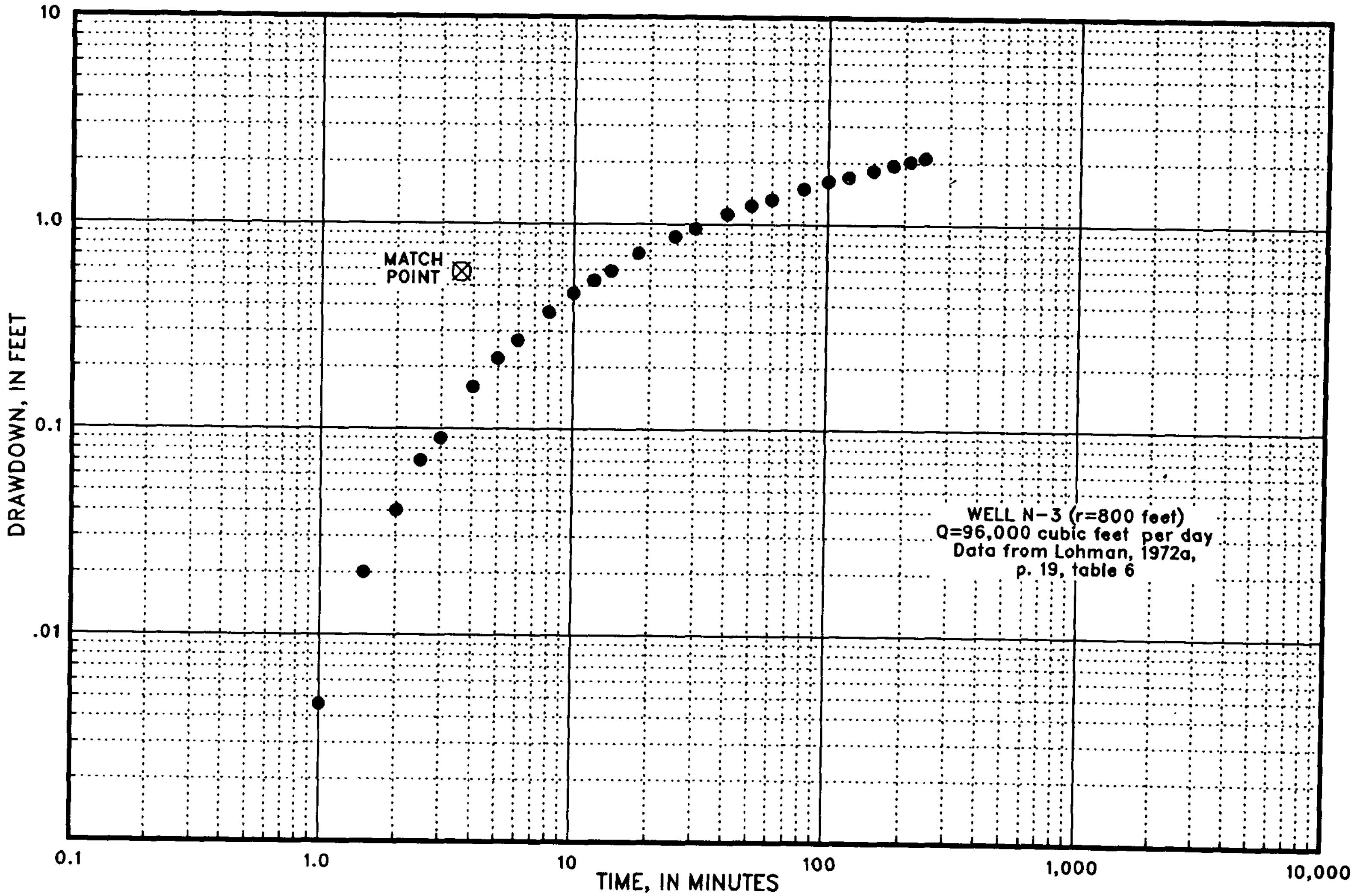


Figure 4-7(c).--Double logarithmic plot of drawdown against time for well N-3 (r = 800 feet). (From Lohman, 1972, table 6.

Analysis of plot of s against t

Data from Lohman (1972a, p. 19, table 6)--Well N-3 (r = 800 ft);

$Q = 96{,}000\ ft^3/d$

Match point

$W(u) = 1.0;\ \frac{1}{u} = 1.0;\ s = 0.58\ ft;\ t = 3.55\ min$

$$t = \frac{3.55}{1{,}440} = 2.465 \times 10^{-3}\ d$$

$$(1)\ T = \frac{Q \cdot W(u)}{4\pi s} = \frac{96{,}000\ ft^3/d \cdot 1.0}{4 \cdot \pi \cdot .58\ ft} = 13{,}170\ ft^2/d$$

Lohman's values: $s = 0.56$ ft, $T = 13{,}700\ ft^2/d$

$$(2)\ \frac{1}{u} = \frac{4Tt}{r^2 S}$$

$$S = \frac{4Ttu}{r^2} = \frac{4 \cdot 13{,}200\ ft^2/d \cdot 2.465 \times 10^{-3}\ d \cdot 1}{(800)^2\ ft^2} = 2.03 \times 10^{-4} \approx 2 \times 10^{-4}$$

Lohman's value: $S = 2 \times 10^{-4}$

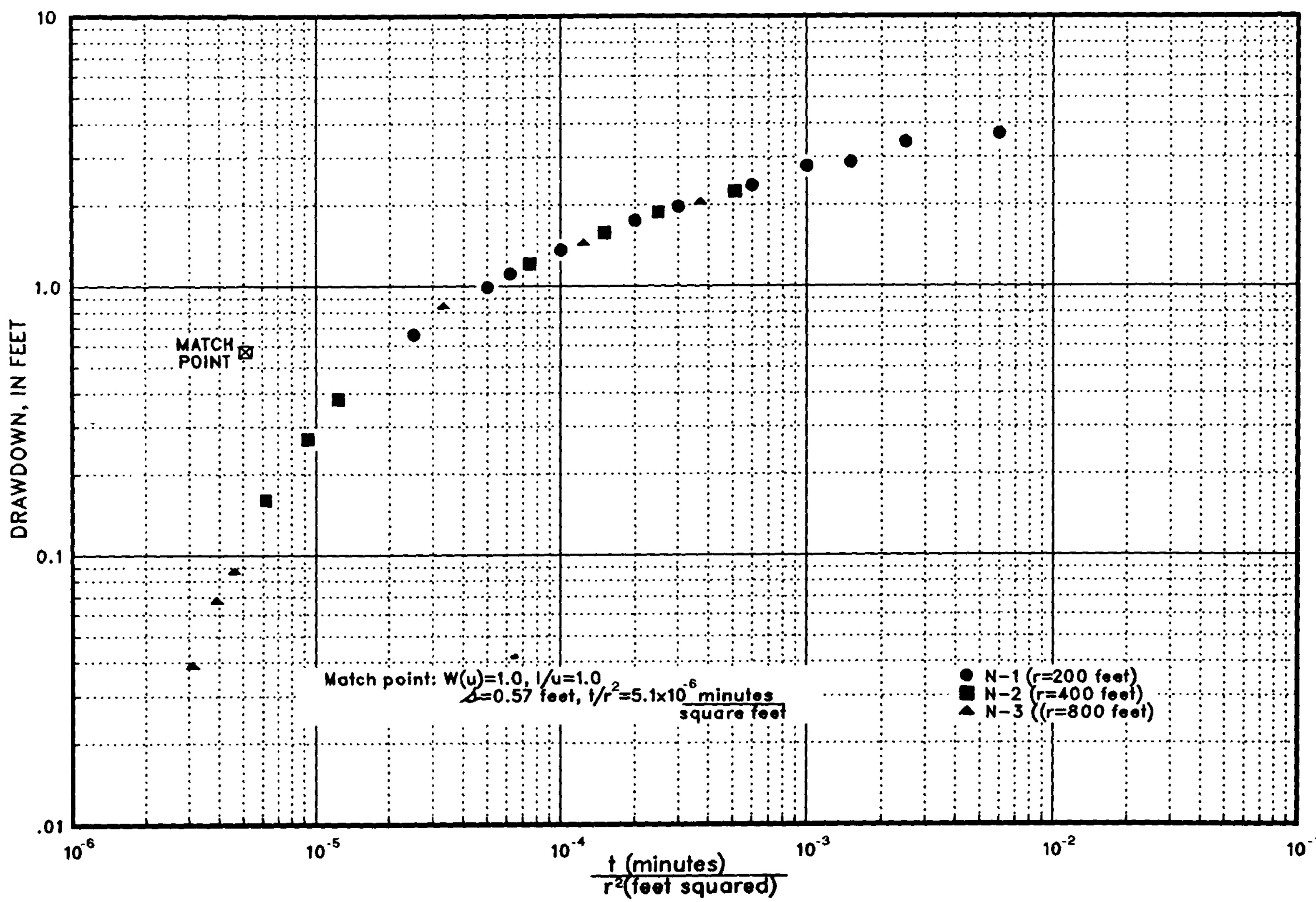


Figure 4-8.--Double logarithmic plot of selected values of drawdown against time/radius squared for wells N-1, N-2, and N-3. (From Lohman, 1972, table 6.)

Analysis of plot of s against t/r^2

Data from Lohman (1972a, p. 19, table 6)--selected drawdown data for wells N-1, N-2, N-3; $Q = 96,000\ ft^3/d$

Match point

$$W(u) = 1.0;\quad \frac{1}{u} = 1.0;\quad s = 0.57\ ft;\quad t/r^2 = 5.1 \times 10^{-6}\ min/ft^2$$

$$(1)\quad T = \frac{Q \cdot W(u)}{4\pi s} = \frac{96,000\ ft^3/d \times 1.0}{4\pi \times .57\ ft} = 13,400\ ft^2/d$$

(Lohman's value: $T = 13,700\ ft^2/d$)

$$(2)\quad \frac{1}{u} = \frac{4Tt}{r^2 S} = 1.0$$

$$S = 4T\frac{tu}{r^2} = 4 \times 13,400\ \frac{ft^2/d}{day}\ \frac{1}{1,440}\ \frac{d}{min} \times 5.1 \times 10^{-6}\ \frac{min \cdot 1}{ft^2} = 1.90 \times 10^{-4}$$

(Lohman's value: $S = 2 \times 10^{-4}$)