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**Comments on the "three-step" method for quantification of
undiscovered mineral resources**

*A paper presented to the Panel for Evaluation of U.S. Geological Survey Mineral
Resources Assessment Methodology, August 6, 1992*

by

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PREFACE

This report is a position paper intended to promote discussion and debate about USGS mineral-resource estimation techniques, and was presented on August 6, 1992, to the Panel for Evaluation of USGS Mineral-Resource Assessment Methodology. Although there is substantial unresolved disagreement between the authors and two of the three technical reviewers of this report, all reviewers approved the manuscript for publication and it is being released in open-file format to facilitate rapid and thorough debate about this area of research. The work by the panel comprises another in a continuing series of scientific reviews and evaluations of USGS resource assessment methodologies that periodically are contracted for, or convened by, the USGS. The most recent previous example is an evaluation by the National Research Council of the Department of the Interior's 1989 assessment procedures for undiscovered oil and gas resources.

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Introduction

For over fifteen years, the U. S. Geological Survey has devoted considerable attention to the possibility of quantifying undiscovered mineral resources. The question is an important one and worthy of effort. Many of the products of this quest are valuable contributions to the geologic literature (for example, Singer, 1975; Singer and others, 1975; Cox and Singer, 1986; Orris and Bliss, 1991).

The U. S. Geological Survey Office of Mineral Resources (USGS-OMR) now requires its mineral resource assessments to include quantitative estimates of undiscovered resources (Glenn Allcott, U.S. Geological Survey, written commun., June 19, 1990; William C. Bagby, U.S. Geological Survey, written commun., July 5, 1991). This paper is in response to this mandatory use and addresses the adequacy of the current state of the art. We would like to promote some substantive debate on this issue in the hope that it will lead to improvements in quantitative mineral resource assessment.

The method the USGS-OMR has developed, called the three-step or three-part method (henceforth referred to as the OMR method in this report), is based on: classification of mineral deposits into types based on mineral deposit models; the delineation of tracts that are permissive for the occurrence of mineral deposits for each mineral deposit model; estimates, commonly by a panel of experts, of the number of undiscovered deposits for each selected deposit model; and combination of the number of deposits estimates with grade and tonnage estimates in a simulation procedure to produce the required estimates of contained metal (Singer and Cox, 1988). The OMR method is a common-sense way of using the known to predict the unknown, and thus worthy of test.

Requirements as a hypothesis

The currently (1991) employed OMR method primarily results from an evolutionary process that came about because of a mandate from the managerial hierarchy of USGS-OMR, as has been detailed by Drew (1990). Historically, such mandates have often led to science of inadequate rigor. For example, an official estimate of the ultimate amount of petroleum that would be produced from the lower 48 states and adjacent continental shelves was made by the U.S. Geological Survey (Zapp, 1962) in response to a presidential directive of March 4, 1961. This estimate, presented to a National Academy of Science Committee, was the most optimistic ever made, forecasting that 590 billion barrels of petroleum remained to be discovered in the lower 48 states. The estimate was based on the "Zapp hypothesis", which assumes that a constant amount of petroleum is discovered for each foot of exploration well drilled. That hypothesis was shown to be incorrect (beginning with Hubbert, 1969), and estimates made using it were the source of much controversy. The three-step method uses some basic assumptions that are not unlike that of the Zapp hypothesis. These include: 1) grade and tonnage attributes of undiscovered deposits will be identical to those of previously discovered deposits, 2) statistics describing deposits are stationary over space and time, and 3) deposits in small regions contain the same diversity as deposits world-wide. These and other assumptions used in the OMR method require that the OMR method be subject to careful scrutiny. Critical examination of the OMR method touches on policy issues; however, in this paper we restrict ourselves to a discussion of the rigor and accuracy

of the OMR method.

All scientific hypotheses, regardless of subject, must pass through two stages before their validity can be established. First, the hypothesis itself must be clearly presented. Second, the hypothesis must be tested. Though these two requirements seem rather clear, the method of quantitative mineral resource assessment being adopted by USGS-OMR has thus far satisfied neither.

Presentation of the OMR method

The overall OMR method has been published mostly as undocumented claims of success, in which the logic is poorly presented. The work actually proceeds on the basis of some in-house manuals that have not been circulated to all members of the mineral-resource assessment teams and that have not been subject to public scrutiny, internal USGS review, or external review. The most recent comprehensive description of the OMR method is an extended abstract by Menzie and Singer (1990); however, this document lacks clarity and is too brief to serve as the required presentation of the overall method of assessment. The simulation procedure has recently been described by Root and others (1992), but a detailed exposition of logic used in all parts of the OMR method is still lacking. Also, to the extent that the OMR method is variable, depending on circumstance, this variability should be described.

Testing of the OMR method

A hypothesis that is not tested remains merely a hypothesis. Attempts to test the ability of experts to estimate numbers of undiscovered deposits, the main geologic hypotheses embodied in the OMR method, apparently have been limited to a single unpublished study utilizing porphyry copper deposits that may or may not be present in 13 selected areas (J.A. Briskey, Jr., U.S. Geological Survey, written commun., 1991). Even though this test has been repeated frequently, it still constitutes a single test. Clearly, several blind tests using well known areas are required to validate the hypothesis for each deposit type. For the claimed general applicability to all deposit types, such tests are required for each deposit type. In addition, the tests themselves must be subject to peer review and public scrutiny before their validity can be established.

We emphasize here the great differences among types of mineral deposits. Mineral deposit types can be as different from each other as are the different disciplines of geology, including their overlaps into adjacent sciences. A test for one is not a test of another. A Lake Superior-type iron formation, for example, is as different from a porphyry copper deposit as, say, a trout from a deer. Would one try to use concepts and equipment suitably tested for one to catch the other? A test for one type of hydrothermal mineral deposit is obviously not a test for such a genetically unrelated type of mineral deposit; nor, given the great differences in process and environment of deposition, is it a test even of other types of hydrothermal deposits.

We maintain that the choice of porphyry copper deposits for a test enhanced the chances of a favorable result. No other type of ore deposit model is more highly correlated to geologic and geophysical data as expressed on regional scale maps than are porphyry copper deposits. Also, the necessary condition of the presence of an associated intrusion and the large size of these deposits puts strict bounds on the lower and upper limits of their numbers in a region. Even given these facts, only one half of the geologists tested were

"right" (J.A. Briskey, Jr., U.S. Geological Survey, written commun., 1991). The test itself is to some extent invalid since the actual number of geologic deposits that can be described by the porphyry copper model in the test area can not be determined.

Many mineral deposit models are uncorrelated to geologic and geophysical data at regional scales. In similar terranes, they may be either abundant or non-existent. Only local detailed studies can identify their numbers with any degree of certainty. While it is questionable that the testing of this method to date can validate the OMR methodology even for porphyry copper deposits, it most certainly does not validate the OMR methodology for polymetallic vein deposits, polymetallic replacement deposits, hot springs gold-silver deposits, and many other relatively important types of deposits, especially those that tend to be areally restricted, and/or with few features that appear on topical maps.

Analysis of errors in the OMR method

Any hypothesis must be subjected to an analysis of the errors and uncertainties inherent in the hypothesis and their effects on the predictions of the hypothesis. This analysis of errors should focus on the accuracy and precision of results that a procedure generates. The final result can be displayed with no more precision than the least precise parameter used to generate the result. In general, the final precision will be less than that of the input data and parameters due to propagation of errors that occur in the mathematical manipulation of the parameters by the procedure. It is in fact the analysis of errors that provides a test of the hypothesis. If the uncertainties are large, the hypothesis is not verified even if it fits the data perfectly (Jeffreys, 1973, Chapter IV).

There has to date been no analysis of the effects of the uncertainties in the various parameters in the OMR method upon the final predicted mineral endowment. We show in the section below on analysis of errors that the uncertainties in the statistical techniques alone are of the same order or larger than the estimated values. Even conservative estimates of the errors inherent in the subjective estimate of the number of undiscovered deposits indicate they are at least as large as the statistical uncertainties. These two error sources multiply, yielding an endowment estimate which is in no sense robust. In most cases tested, endowment estimates were not accurate to an order of magnitude, although they may be reasonably presented as precise to one significant digit, commonly, presentation is to 2 to 4 significant figures. The publication of estimates with no consideration of their reliability renders those estimates as mere guesses.

A discussion of the assumptions and procedures of the OMR method

The OMR method consists of three steps in each of the publications describing it. However, individual papers differ in what the steps actually are (table 1). The actual sequence of steps is the same in all these lists however, and for the purpose of this report we have divided this sequence into the following steps without implying that they are "correct": (1) classification of mineral deposits into types based on mineral deposit models and the delineation of tracts that are permissive for the occurrence of mineral deposits for each mineral deposit model, (2) estimates, by experts, of the number of undiscovered deposits for each deposit

model, and (3) the estimates obtained in step two are then combined with grade and tonnage estimates of the deposit model to produce the required (by USGS-OMR) estimates of contained metal. Recently this combination has been accomplished through the use of a Monte Carlo simulation procedure. This portion of the report will address problems associated with each of the steps of the OMR method.

The problems associated with step one, the identification of mineral deposit models in a region, center around finding a model that is truly representative of the kinds of mineral deposits that need to be estimated in a given region. While the second part of step one, the delineation of tracts that are permissive for the occurrence of mineral deposits for each mineral deposit model, is not problem free, the problems are not discussed here. Many USGS-OMR geologists feel comfortable delineating tracts that are permissive for the occurrence of ore deposits that are described by a well-defined mineral deposit model. This includes labelling the parts of the permissive tracts as having moderate potential or high potential for the occurrence of the mineral deposits in question. The quality of this part of the assessment is highly dependent on the the clarity and correctness of the mineral deposit models used, the expertise of the scientists performing the mineral resource assessment, and the existence and availability of diagnostic data.

Problems associated with step two, the estimation of number of deposits, involve the interrelation between geologic and statistical variables. Problems that are associated with step three, including the simulation procedure, are based on the assumptions that must be made when using grade and tonnage distributions.

Mineral deposit models as a basis for mineral resource assessment

The division of deposits into distinct types is done with mineral deposit models. The basic document used by USGS is its Bulletin 1693 (Cox and Singer, 1986), which includes both descriptive models and, in most cases, grade and tonnage models for each deposit type. This section focuses on the descriptive models; the grade and tonnage part of the respective models are discussed later.

The problems associated with using mineral deposit models as a basis for mineral resource assessment include: incomplete deposit models, under-representation of unconventional deposit models, arbitrary boundaries between models, omission of some deposit models in most assessments, and necessary but not sufficient conditions for ore deposit occurrence represented by the current deposit models.

Adequacy of mineral deposit models

Some deposit models have been published since Cox and Singer (1986), but the OMR method generally uses models exclusively from that Bulletin, even where all parties are agreed that the model therein is inadequate. This is due to the grade and tonnage models contained in Cox and Singer (1986) that are needed by the OMR method. Cox and Singer (1986) is a valuable addition to the literature of mineral deposits, especially for rapid familiarization with the great variety of mineral deposit types. We maintain, however, that the models, as described in the document, are not suited to some of the purposes for which they are being used. Introductory sections of the volume show that the editors were well aware of its limitations; Barton, in Cox and Singer (1986, p. III-IV), clearly describes the experimental and incomplete nature of the model compilation.

Many models are incomplete or unclear in their descriptions of deposit types; factors necessary for deposit formation may or may not be described in a discriminating manner. For example, the model for ocean-floor massive sulfide deposits (highlighted by Drew, 1990) mentions their preferential occurrence in Ordovician and Cretaceous sequences. If this had been phrased as a preference for anoxic bottom waters on rapidly spreading ridges, the exploration implications would be clarified for sequences of other ages.

Representation of unconventional deposits

Unconventional deposits are under-represented in the OMR method. Authors of Bulletin 1693 were asked to describe specific deposit types, not to provide all the models necessary to represent total resources in a particular field. If a given type of mineral deposit (known or unknown) lacks a descriptive mineral deposit model, and the accompanying grade and tonnage models, that deposit type will not contribute to the quantitative estimate in the OMR method. This problem will become worse as new deposit types are discovered.

Arbitrary boundaries between deposit models

Many boundaries between deposit types, especially in the hydrothermal deposits of most interest to resource assessors, are rather arbitrary. This would be of little importance if deposit boundaries did not correspond with different grade-tonnage models. We feel that the deposit models under-represent the importance of the environment of mineralization; given the same source and process of mineralization, different environments will preserve slightly different deposit types. Lack of appreciation of environmental factors will lead to errors in resource estimates. This topic will be discussed further as spatial and temporal variation in mineral deposits in the section entitled "Geologic problems associated with the estimation of the number of undiscovered deposits".

Consideration of all possible mineral deposit models

In an area-specific resource evaluation, all deposit types are not considered for various reasons. Those types for which some local information exists are typically the only ones considered, and this information commonly takes the form of existing prospects of those types. In some cases, deposit types that are locally unknown are not evaluated, even if the geology is indicative of their occurrence. Often, deposit types are not considered due to the unavailability of the appropriate grade and tonnage model, even if a deposit type would contribute substantially to the estimate of contained metal in a region. The more unconventional deposit types are systematically ignored under this system, because the evaluator and the prospector "guiding" him probably lack the pertinent knowledge.

Necessary and sufficient conditions for the formation of ore deposits

Last, and probably most important, characteristics listed in the deposit models generally represent necessary but not sufficient conditions for deposit formation. That is, even where all the factors listed in the model are present, a deposit need not occur. Thus the models by themselves cannot be used as a basis upon which to predict occurrence versus non-occurrence of deposits within terranes delineated as permissive by the model.

Uncertainties surrounding the estimation of undiscovered deposits

Step two of the OMR method attempts to estimate the number of undiscovered deposits in a region. Two types of problems plague these estimations. The first type of problem is geologic in nature and impacts severely upon a geologist's ability to reliably estimate the occurrence of ore deposits. The second type of problem is statistical in nature and is associated with the ability of statistics to describe and/or predict the number of ore deposits that remain to be discovered.

Geologic problems associated with the estimation of the number of undiscovered deposits

The geologic weaknesses in attempts to predict the number of undiscovered deposits in poorly known or covered areas are: the lack of correspondence between known processes of mineral deposition and the data sets used to predict the occurrence of mineral deposits, improper structure of the estimation process, and spatial and temporal variations and clustering in mineral deposits.

The imprint of actual mineralization processes on geologic data sets

Economically viable accumulations of minerals are present where several physical and chemical factors and processes have occurred in successive order. This generalization is valid across the broad spectrum of genetically unrelated deposit types, because a deposit in which such a succession of factors is incomplete will be rendered an occurrence of no economic interest by one in which the required succession of factors has taken place. Four types of factors are generally involved -- source, transport, entrapment, and preservation.

Unless deposits of a given type can fortuitously form by different factor pathways, the occurrence of an undiscovered mineral deposit requires all the same favorable factors in the same sequence as does a deposit of the same type in a similar well-known terrane. Prediction of the undiscovered deposit then requires knowledge of the factors and sequence that permitted the known deposit. These factors must either be gleaned from available data sets or the personal experience of an evaluator.

Although many types of data can enter into a prediction, the most important type is areal data, i.e. geologic and other thematic maps. Even in exceptionally favorable circumstances (i.e. a geologically well-mapped area with densely spaced geophysical and geochemical data) it is unlikely that all the requisite processes will be recorded or imprinted on regional scale data sets, as some of the critical factors are likely to be subtle variables not generally shown on maps, such as fracture sets or oxidation interfaces. Nor are the ages of structural features generally shown on these maps. In the poorly known and/or concealed areas where prediction with a higher level of resolution is desired, most of the required factors are not imprinted on the data sets that form the basis for prediction. Where such data are missing, the possibility of useful prediction vanishes.

The OMR method optimizes the personal-experience avenue by involving a panel of experts. Usually, however, a single person knows a given tract far better than the others, and because of group dynamics and respect for "one who has seen the ground", that person has a great influence on the outcome. A single person cannot cover all deposit types adequately. In poorly known areas, the panel is expert only in the interpretation of poor data sets. Even where personal experience with the tract being evaluated is great, subtle variables necessary to predict deposit formation are commonly missed. In the rare case where data sets are so good that

most necessary factors are represented on them, and a local expert is able to point out all the remaining subtle variables, deposit density can be accurately predicted, but so can deposit location. In other words, accurate expert prediction of deposit density in a tract requires as much knowledge as advanced stages of exploration.

In areas with poorly known geology, geologic analogy takes over as the prime method for estimating the number of undiscovered deposits. Assessors who use geologic analogy assume that the numbers (and sizes) of mineral deposits are similar in areas with similar geology. However, the analogies used may or may not embody the sufficient as well as the necessary critical factors for ore deposit formation. Even when the accepted critical factors are known to be present in a given area, ore deposits may or may not occur. Geologic analogy has been criticized often (National Academy of Sciences, 1975, p. 136; Steinhart and Bultman, 1983, p. 44; Hubbert, 1978, p. 122), even where the analogies are based on several specific geologic factors.

Structure of the estimation process

It is our observation that the estimation of the number of undiscovered deposits in the OMR method follows an informal free-form discussion of geologic factors. The lack of structure discourages discussion of the required factors in sequential order. This in turn prevents full and critical utilization of the pertinent geologic information available.

The selection of expert panels is a potential problem. First, the qualifications of experts are not defined. Expertise can be based on regional experience, experience with ore deposit models, experience with a particular deposit type, experience with the simulation procedure used by the OMR method, or some other criteria. In some cases, the "expertise" of a panel member consists largely of a willingness to make the estimates of the number of undiscovered deposits. Often, these are individuals who are very familiar with the statistics of the OMR method and believe they can manipulate the estimates of undiscovered deposits to compensate for weaknesses in the OMR method. Second, group dynamics may allow one panel member to significantly influence the conclusions of the panel. Given such variability of panel makeup, it is required that the repeatability of the results of one panel by another panel, or even by the original panel, be determined.

Perhaps most importantly, the quality of the estimate can be improved if the estimating panel follows a procedure that mimics the formation of the type of mineral deposit in question to the extent that the available information allows. The factors involved in mineral deposit formation should be addressed sequentially. Where too little is known to evaluate a factor, estimates should be made by other methods and a substantial error margin should be assigned to that factor and combined with other errors inherent in the OMR method described herein.

Spatial and temporal variations and clustering in mineral deposition

Attributes of a mineral deposit may differ from attributes of other deposits belonging to the same deposit model as a function of environmental factors at scales ranging from host lithology to crustal blocks. Variation at the latter scale produces great differences in deposit types from one part of the earth to another. Most economic geologists embody this factor in their reasoning, and we have seen ample evidence for it in our work, which encompasses in-depth studies ranging from those focused on individual ore deposits to regional mineral resource studies throughout the United States and in many other countries. Mineral deposit models by nature tend to ignore this factor, necessarily resulting in (1) mixed populations within a given

grade-tonnage relation, (2) intergrading of deposit types presented as distinct, and/or (3) omission of some facies of some deposit types. In the statistical stationarity section below, we discuss wide differences in a deposit type (precious metal veins) between similar but distinct terranes. It should be noted that Cox and Singer (1986) have implicitly acknowledged a spatial component of mineral deposit occurrence by naming some mineral deposit models after the location of their most representative deposit.

In addition, some basic earth processes have varied from time to time as a result of repartitioning of heat energy from one form of release to another (Larson, 1991). Resulting temporal differences in magmatism, deformation, oceanography, and climate have influenced the types and locales of mineral deposition. Mineral deposit models, on the other hand, are basically time-independent, and thus take account of few temporal changes.

The OMR method treats the occurrence of mineral deposits as independent events with a random spatial distribution. This is necessary for a well-known area to function as an accurate analog for a similar less-known area being evaluated. However, mineral deposits actually tend to cluster spatially (Slichter, 1962; Mandelbrot, 1962 (implied), 1983; Harris, 1984 (chapter 7); Carlson, 1991). This fact is a necessary corollary of their occurrence where several factors have been favorable in a certain order; in a small area where this rare condition has been satisfied in one place, it may easily be satisfied in several others. Clustering of mineral deposits is further discussed under the section "Statistical problems associated with the estimation of the number of undiscovered deposits" that follows.

The net effect of the spatial and temporal variations of mineral deposits is a great limitation on the usefulness of mineral deposit models and derivative predictions. In exploration, a too-strict adherence to mineral deposit models has commonly led to deposits being missed, only to be discovered later by a less-literate geoscientist. A similar effect must occur in mineral assessment.

Statistical problems associated with the estimation of the number of undiscovered deposits

Some previously discussed geologic attributes of mineral deposits translate to specific statistical problems associated with the estimation of undiscovered deposits in the OMR method. In addition, some problems associated with the use of statistics as an estimator for natural phenomena and confusion over what is actually being estimated are discussed below. These problems include the definition of the unit of measure used by the OMR method, the interpretation of the predicted estimate of the number of undiscovered deposits, the ability of the OMR method to model theoretical distributions of numbers of undiscovered deposits, the spatial distribution of mineral deposits and implications for the use of statistical modelling, modelling a particular sequence of geologic factors, the influence of the estimation procedure on significant figures for the estimate of contained metal, and accounting for exploration intensity.

The unit of measure in the OMR method

Grade and tonnage models are the only available tools used for visualization of an ore deposit. An assessor must summarize the grade and tonnage models into the unit of measure in the OMR method, a deposit. This task is extremely difficult since contained metal can vary by 4 to 5 orders of magnitude within ore deposits representative of a given mineral deposit model and there is no way to reduce the data within these models to something that is easy to comprehend or to use. A mean ore deposit (in terms of contained

metal) may be biased by a few very large deposits while a median ore deposit may not reflect the skewness of the tonnage (and grade) distribution. Every geologist carries a different picture in his/her mind about what a "typical" ore deposit looks like for a given deposit model, but in general this picture contains very little quantitative data on contained metal. For the OMR method to be valid, the undiscovered ore deposit being estimated must be represented by the grade and tonnage models of the mineral deposit model. Numerous cases are known of production from discovered deposits within a permissive terrane not having the same grade and tonnage attributes as the models used in the assessment procedure, thus the utility of the OMR method can be questioned.

Interpretation of the estimate of the number of undiscovered deposits by the simulation procedure

In any unexplored area described as permissive for the occurrence of a given mineral deposit model the function describing the number of undiscovered deposits is a delta function with an integer value at the number of deposits that actually exist. The delta function is dependent on the definition of deposit, but that issue is not addressed here. As exploration proceeds, this delta function moves towards smaller numbers, arriving at zero when the last undiscovered deposit has been found. In practice, however, the exact number of undiscovered deposits is not known. Therefore, in the OMR method this delta function of undiscovered deposits is approximated through the use of a subjectively estimated probability mass function (PMF), the discrete version of a probability density function. This PMF presents a subjective estimate of the probability that any given integer, within the range of integers considered, is the actual number of undiscovered deposits. Ideally, in any given area, the PMF should have a mode at or near the actual number of undiscovered deposits in that area.

The simulation procedure used in the OMR method produces a PMF based on assessors subjective probabilistic estimates of undiscovered deposits. For each iteration of the simulation, a numbers of undiscovered deposits estimate is entered into the simulator based on the probability allocated to each integer in the PMF. Thus the shape of the PMF created by the simulation procedure is very important to the final result of contained metal produced by the OMR method. For large tracts in unexplored areas, a monomodal or uniform shape of the PMF may be quite reasonable. In small to moderate size tracts in well explored areas (such as exposed bedrock in the conterminous United States), many geologists feel that the PMF of undiscovered deposits contains monotonically decreasing probabilities for increasing number of deposits. This would give the estimate of zero deposits the largest probability of occurrence.

The method used to build the PMF in the OMR simulation procedure is based on estimates of undiscovered deposits at given probabilities. Each estimate is based on the occurrence of "at least" a given number of deposits at a given probability or more (after Root and others, 1992, p. 130). Most assessments made using the OMR method are based on estimates of numbers of deposits at the 10-, 50-, and 90-percent quantiles or more of the PMF. Recently, the 5-, and 1-percent quantiles have been added. The PMF built from an the "at least" estimate requires that at least 90-percent of the total area beneath the PMF must be at numbers of deposits equal to or greater than the 90-percent estimate. Similar relationships must hold for the 50- and 10-percent estimates.

Since there are an infinite number of of PMFs that fit the above set of criteria, it is necessary to look at

the specific set of assumptions used to build PMFs from the probabilistic estimates of the numbers of undiscovered deposits by the OMR simulation procedure. These rules are described in Root and others (1992) and the following paragraph is taken directly from that manuscript.

"The allocation of the unit probability among the nonnegative integers that defines the default distribution of the number of deposits is described as follows. The three numbers, $N(.9)$, $N(.5)$, and $N(.1)$ (*$N(.1)$, $N(.5)$, and $N(.9)$ refer to the 10-, 50-, and 90-percent estimates*), divide the non-negative integers into four intervals: 0 to $N(.9)$, $N(.9)$ to $N(.5)$, $N(.5)$ to $N(.1)$, and $N(.1)$ to infinity. Integers in these four intervals receive 10%, 40%, 40%, and 10%, respectively, of the unit probability. In allocating probability the numbers $N(.9)$ and $N(.5)$ lie half in each of the two intervals of which they are endpoints, and they receive probability from each but only half of what the interior points get. $N(.1)$ receives half of what an interior point to the interval $N(.5)$ to $N(.1)$ receives plus 0.1. The largest number that is given a non-zero probability is $N(.1)$." (italics added) .

While the method used by the OMR simulation procedure may represent one valid interpretation of the subjective probabilistic estimation of the number of undiscovered deposits, it is the authors' opinion that assessors need experience with the technique used to produce PMFs before they can accurately convey information to the simulation procedure. It is imperative that all assessors be able to manipulate the shape and skewness of the PMF created by the OMR simulation procedure. Presently, there are no visualization tools available to assessors (that would allow them to see the PMFs created by their estimates) and there has been no required training for geoscientists involved in quantitative assessments.

All of the authors have been involved or observed quantitative mineral assessments. They have personally observed confusion over what the probabilistic estimates of the number of undiscovered deposits actually mean. Few mineral resource assessors that the authors have communicated with have understood the relationship between the probabilistic estimate of the number of undiscovered deposits and the method used by the simulation procedure to construct PMFs. While our conclusion is based only on our own experience, it seems that not all assessors do understand how to manipulate the shape of the PMF.

Adding to the confusion are the occurrence of imprecise and/or inconsistent statements explaining exactly what the resource assessors have estimated in published mineral resource assessments. For example, Reed and others (1989) state " we estimated the number of deposits of each type at a 90-, 50-, and 10-percent confidence level." In tables within the same document, these estimates are labelled "Percent chance that at least the indicated number of deposits are present". These are two different estimates and probability mass functions of the number of undiscovered deposits built from the two statements should be different. Hodges and Ludington (1991) also display a lack of consistency in dealing with estimates of numbers of undiscovered deposits. That report states "For each of the deposit types listed below, the greatest number of deposits in the EMNSA (specifically in those tracts of land designated permissive) is estimated for which there is a 90-, 50-, 10-, 5-, and 1-percent chance or greater of occurring." That report then goes on to say, in reference to a table, "The level of confidence that the stated number of undiscovered deposits exists is indicated by the probabilities, in percent, heading each column." Again, PMFs built from each of these statements should differ.

Statements in the literature, such as " we estimated the number of deposits of each type at a 90-, 50-, and 10-percent confidence levels", Reed and others (1989), may give an assessor the impression that they are

making point estimates of the number of undiscovered deposits at 10-, 50-, and 90-percent confidence levels. In these cases, an assessor who gives a high probability (the 90-percent estimate) to a small number of deposits expects there to be a high likelihood of a small number of deposits and expects the PMF generated by the simulation procedure to portray this accurately. Harris (1984, p. 369) observed a similar effect for subjective estimates of the probability of at least one mineral deposit of at least a given size in his review of the National Uranium Resource Evaluation program (NURE). In this case, the individuals eliciting the estimates asked for the probability of one deposit instead of at least one deposit. The errors in the NURE study are blamed (by Harris) on a breakdown in communication. The problem we observe in the OMR method may be blamed, in part, on communication, but, it is also due to the lack of experience on the part of assessors in using the rules to make the probabilistic estimates of undiscovered deposits. It is not intuitively clear how to control the shape of a PMF based on estimates of at least a given number of deposits at probabilities of 10-percent or more, 50-percent or more, and 90-percent or more.

The OMR simulation procedure allocates only 10-percent of the total probability over the zero deposits to the 90-percent estimate interval ($N(.9)$), so there is generally only a small probability of occurrence (for each number of deposits estimate possible) in the interval associated with the 90-percent estimate. Figure 1 displays the PMF for an estimate of at least 2, 5, and 8 deposits at 10-, 50-, and 90-percent confidence levels, or more, respectively. Figure 2 displays regions where monotonically decreasing probabilities for increasing number of deposits are possible and impossible for given values of $N(.9)$ and $N(.5)$ (the 90- and 50-percent or more estimates respectively). The regions in figure 2 are based on the following assumptions: (1) $N(.5) - N(.9) \geq N(.1) - N(.5)$, (2) the last 0.1 of the allocated probability is ignored or spread over large deposit values, and (3) $P(.9) > 0$. For most cases, when $N(.9) > 0$, the PMFs created by the OMR simulation procedure have shapes that can be described as uniform, having a central mode, or as negatively skewed. This is because only 10-percent of the probability is allocated to integers in the interval from 0 to $N(.9)$ while 40-percent of the probability is allocated to integers in the interval from $N(.9)$ to $N(.5)$. Estimates of the number of undiscovered deposits that lie along the line presented in figure 2 represent PMFs that are uniform (if the probability from the 10-percent quantile to the 0-percent quantile is ignored). Near this line, small changes in the number of deposits estimate can change the form of the PMF from monotonically decreasing probabilities for increasing number of deposits to uniform or to monotonically increasing probabilities for increasing number of deposits. This is demonstrated in figures 3a, 3b, and 3c.

While the OMR simulation procedure can generate some PMFs that are monotonically decreasing with increasing deposit numbers, it requires that $N(.5)$ be much larger than $N(.9)$ or that $N(.9)$ be zero. If small tonnage deposits (possibly based on those expected in a local terrane) are used in an assessment over a large region, there may be little chance of zero deposits occurring at a 90- or 50-percent level. Also, estimates for many tracts are generally summed for a region before a simulation is run. Thus, in many assessments, it may be virtually impossible to generate a PMF that has monotonically decreasing probability for increasing numbers of deposits when an "at least" scenario of estimation is used.

According to the rules used in the OMR simulation procedure to allocate probability, the use of zero as an estimate of the number of undiscovered deposits will allocate the interval of probability for the given estimate to zero deposits. For example, by setting the $N(.9)$ and the $N(.5)$ estimate to zero, an assessor allocates 0.5 of the total probability on zero (along with another portion of probability which is defined by the $N(.1)$ estimate). The remaining probability is allocated in the interval from 1 to $N(.1)$. This situation differs

little from an option in OMR simulation procedure that allows assessors to make a point estimate of the probability of zero deposits ($p(0)$) along with the probabilistic estimates. It is assumed that this procedure simply sets the probability of zero deposits to the estimate given and then normalizes the rest of the estimates so that they sum to $1-p(0)$. Once the point estimate of zero deposits is made, the estimates at $N(.9)$, $N(.5)$, and $N(.1)$ become invalid in that they only retain relative relationships. This option is seldom used in the OMR method.

Recently, subjective estimates of the number of undiscovered deposits, have included 5- and 1-percent estimates (Hodges and Ludington, 1991, and McCammon and others, 1991). These additional estimates were presumably incorporated in order to apply the OMR method to small areas, where the 90-, 50-, and 10-percent estimates of at least a given number of undiscovered deposits may be zero. The effect of estimates of zero deposits at 90-, 50, and 10-percent probability levels is to allocate approximately 90-percent of the probability on zero deposits. Thus, with the 2 added estimates, a very large amount of the total probability can be allocated on the zero deposits estimate and it becomes easy to produce PMFs with monotonically decreasing probability for increasing deposit number. It should be noted, that in response to subjective estimates of continuous variables, Harris (1984, p. 369) states "... But, even the well informed person tends to underestimate the spread between the 5th and 95th percentile. Under the best of circumstances, the extreme values are not unbiased estimates."

The authors question the use of assessments based on estimates of at least a given number of deposits. While these type of estimates make it relatively easy to construct a PMF from the estimates, it means that an assessor must know how to manipulate the estimates to produce the PMF he/she wishes to be used in the simulation. Also, estimates that are theoretically unbounded to large numbers of deposits are difficult for many assessors to deal with. Many other methods can be envisioned, methods that may directly relate to the structure of ore deposit formation or to methods that would put physical bounds on numbers of undiscovered deposits. If it is decided that the at least method is to be retained, visualization tools (that graphically portray the PMF) should be used so that assessors can concentrate on the estimate of the number of deposits and not on the interpretation of their estimates by the OMR simulation procedure. Also, to increase the flexibility of the procedure used to build PMFs, it may be necessary to include estimates at alternative and/or additional probability levels.

An examination of the OMR simulation procedure's ability to construct PMFs that model three theoretical distributions

In this section, the ability of the OMR simulation procedure to construct PMFs that model three simple theoretical distributions is examined. PMFs are constructed from estimates of the numbers of undiscovered deposits derived from the complementary cumulative probability function at the 0.9, 0.5, and 0.1 quantiles (or greater) for the three theoretical distributions (more recently, estimates at the 0.05 and 0.01 quantiles have been added to the OMR method but are not dealt with here). As explained earlier, these estimates are obtained from answers to the questions "At least how many events are predicted at the 90-percent or more, 50-percent or more, and 10-percent or more probability levels?"

Construction of the probability mass function was accomplished using the code presented by Root and others, (1992). The algorithm used in the code is derived by approximating the complementary cumulative probability by straight line segments connecting the 1.0 probability at zero deposits with the 0.9, 0.5 and 0.1

estimates. The remaining 0.1 probability is allocated to the highest number of deposits in the code of Root and others (1991). Other schemes are possible; the results obtained here are not particularly sensitive to how the upper 0.1 probability is allocated.

The three theoretical distributions used were the uniform (constant) distribution, the linearly decreasing distribution, and the exponentially decreasing distribution. In all cases, the single parameter defining the distribution was N , the number of events such that $p(n \geq N) = 0$, or arbitrarily nearly so in the case of the exponential distribution. Thus, N represents the estimate of the maximum number of deposits possible in a region.

The uniform probability distribution

Let N be the number of deposits such that the probability density function (PDF) for the continuous variable $n \geq N$ is zero. Then the continuous probability density function $p(n)$ is

$$\begin{aligned} p(n) &= b & 0 \leq n < N, \text{ where } b \text{ is a constant} \\ &= 0 & n \geq N. \end{aligned} \tag{1}$$

The requirement that the integral of the PDF be unity gives

$$\int_0^N p(n) \, dn = bN = 1 \text{ so } b = 1/N \tag{2}$$

The probability of at least M deposits is the complement of the cumulative probability function

$$\int_M^N dn/N = 1 - M/N \equiv P(M). \tag{3}$$

To apply this to the case of integer numbers of deposits, the discrete form of these equations must be calculated. Formally, the discrete probability mass function (PMF) is obtained by calculating the expectation value of the pdf for the half-open interval $[i, i+1)$. For i and N integer, the PMF is thus

$$\begin{aligned} p(i) &= b & 0 \leq i < N, \text{ where } b \text{ is a constant} \\ &= 0 & i \geq N. \end{aligned} \tag{4}$$

and there are N states $0, 1, 2, \dots, N-1$ with nonzero probability function b . Since the sum of the PMF for all states is unity,

$$\sum_{i=0}^{N-1} p(i) = 1 = bN \text{ and } p(i) = 1/N \tag{5}$$

The complement of the cumulative probability function in the discrete case is

$$P(M) = 1 - \sum_{i=0}^M p(i) = 1 - M/N \quad (6)$$

where M is the state label, here, the number of deposits. Note that $P(M)$ is a linearly decreasing function of M , zero at N deposits.

The linearly decreasing probability distribution

Let the continuous PDF be defined as

$$p(n) = mn + b, \quad p(n \geq N) = 0, \quad p(0) = b. \quad (7)$$

A development similar to the uniform case above yields

$$p(n) = 2(1 - n/N)/N \quad \text{and} \quad P(M) = (M/N - 1)^2 \quad (8)$$

for the continuous PDF and complement of the cumulative probability, respectively. The expectation value of $p(n)$ in the half open interval $[i, i+1)$ yields the discrete PMF

$$p(i) = (2N - 1 - 2i)/N^2 \quad (9)$$

Summing the PMF and evaluating the resulting arithmetic series yields for the complement of the cumulative probability

$$P(M) = 1 - [M(2N - M)/N^2] \quad (10)$$

which behaves as a quadratic in M , decreasing to zero for $M=N$.

The exponentially decreasing probability distribution

The continuous PDF has the form

$$p(n) = A e^{-bn} \quad \text{with} \quad p(0) = A \quad (11)$$

Since the exponential is only zero as $n \rightarrow \infty$, the condition $p(N) = 0$ must be relaxed to $p(N) = \epsilon$ where ϵ is chosen arbitrarily small. The condition that the integral of $p(n)$ for all n must be unity requires that $A = b$. For $n = N$,

$$p(N) = b e^{-bN} = \epsilon \quad (12)$$

and thus

$$\ln \epsilon - \ln b + bN = 0 \quad (13)$$

This transcendental equation can be easily solved numerically (for example, by bisection) to determine b given N and ϵ . The complement of the cumulative probability function is

$$P(M) = e^{-bM} \quad (14)$$

Calculation of the expectation value for $[i, i+1)$ yields the discrete PMF

$$p(i) = e^{-bi} (1 - e^{-b}) \quad (15)$$

Summing $p(i)$ yields a geometric series which can be evaluated to give for the discrete form of the complementary cumulative probability

$$P(M) = e^{-bM} \quad (16)$$

Plots of the probability mass functions for the case $N = 10$ for the uniform (eq. 5), linearly decreasing (eq. 9) and exponentially decreasing (eq. 15) cases are shown in figure 4, and the corresponding complementary cumulative probabilities (eqs. 6, 10, and 16) are shown in figure 5. Both figures 4 and 5 are for the discrete case and thus, the lines connecting successive points are only a visual aid; the functions are not defined between the integer values representing the number of deposits. For each distribution, the 90-, 50-, and 10-percent or greater estimates forming the input for the OMR simulation procedure are given by the number of deposits corresponding to the intersection of the 0.9, 0.5, and 0.1 probability values with the probability curve of figure 5 for that distribution. The value of $N = 10$ was chosen because it is typical of many estimates, but is arbitrary. The parameter ϵ , the probability mass function value for the exponentially decreasing distribution at N , was chosen as 0.001.

An evaluation of the OMR simulation procedure PMF algorithm

The resulting 90-, 50-, and 10-percent estimates from figure 5 are 1, 5, and 9 for the uniform PMF case, 0, 2, and 6 for the linearly decreasing case, and 0, 1, and 3 for the exponentially decreasing case. The PMFs produced by the code given by Root and others (1992) for estimates are shown in figure 4 (solid symbols) with the true distributions (open symbols). We note that estimates of at least zero deposits can only occur at 1.0 probability, since the estimate comprises all possibilities and any algorithm constructing a PMF must use zero only at the 100th percentile.

Except for the linear distribution, it is apparent that the approximate PMF constructed from the OMR simulation program algorithm is biased toward large numbers of deposits (the "spike" at the high number of deposits end of the PMF is clearly also a source of bias to large number of deposits, but it is ignored in this discussion because of the recent addition of 5- and 1-percent estimates to the simulation procedure, or several other schemes, can eliminate it). The bias is the most pronounced in the case of the exponentially decreasing distribution (Slichter and others, 1962), but occurs even in the very optimistic case of the uniform distribution

which predicts that it is as likely that there are large number of undiscovered deposits as none. The large number of deposits end of the distribution is only well approximated in the uniform case. Although the mean number of deposits is not too different for the constructed PMFs from the test distributions, at high or low number of deposits the estimates are biased. This bias will influence the estimation of contained metal. Study of figure 3, and other examples not included here, reveals that the algorithm is very sensitive to a change of unity in any of the estimates, especially the 0.9 and 0.5 values. As discussed elsewhere, this sensitivity leads to the conclusion that the algorithm is not a stable estimator in the sense that it can change the shape of the PMF radically for a small change in its parameters. This suggests that extreme caution should be used in the estimation process to insure that the resulting PMF reflects the opinion of the estimators. To make the procedure less sensitive, estimates of numbers of deposits will need to be made at more probability levels and it will be required that assessors have a thorough understanding of the the procedure used to construct the PMF . Viewing the constructed PMF at the time the estimates are made is necessary.

By adding probabilities from the PMFs constructed by the OMR simulation procedure, the cumulative probability distribution estimates can be calculated, thus testing the consistency of the method to recover its input data. These results are plotted in figure 5. It is evident that the probabilities are systematically high, except for zero deposits. This probably occurs because the algorithm distributes the last 10-percent of the probability over the range of 0 to 10-percent part of the curve. These probability values disagree by as much as 0.18 for the case discussed.

The spatial distribution of mineral deposits and its relationship to the estimate of undiscovered deposits

Slichter and others, (1962), have shown that the spatial distribution of mines and, presumably, mineral deposits is clustered. Parenthetically, the work of Slichter and others (1962) probably constitutes the first realization that the spatial distribution of mineral deposits is fractal. Models of spatial distribution of ore deposits were proposed by Mandelbrot (1962; 1983) and showed that the distribution of mines is indeed clustered. This has also been shown for the spatial distribution of a subset of hydrothermal precious metal deposits in the Basin and Range province (Carlson, 1991).

The occurrence of a mineral deposit is a deterministic process; mathematically, this is described as the intersection of the set of required geologic factors and their sequence in time and space being non-empty. This is not a statistical process; it is instead a process dependent on the intersection of the set of conditions caused as the result of deterministic physical law. If the geologic environment determines that these conditions are met in one location, they may also be easily met in nearby locations. Further attempts to model spatial distributions of mineral deposits based on fractal geometry, Krigging methods (Journel and Huijbregts 1978), multidimensional spaces, and cellular automata techniques (Chase and Woodward, 1990) should be researched.

Clearly, mineral deposits can not be treated as independent events. The reason deposits occur in clusters is precisely because they are not independent events. Assessors must understand the clustered nature of mineral deposits when making probabilistic assessments. Analogy tools based on areal deposit density may not convey information on clustering of deposits and should not be used, except possibly in clusters.

Modelling mineral deposit formation and its implications for estimation of undiscovered deposits

The actual *formation* of a mineral deposit is deterministic as described above, occurring only where and when the sequential conditions for ore genesis are met. Modelling this situation statistically is a response to some ignorance of what all the factors are and some ignorance of whether or where they occur in the evaluated area. Clearly a model based as far as possible on the set of intersection of physical and chemical factors is desirable. Statistics should be used for residual factors. This statistical analysis should itself mimic the sequence of factors by 1) dealing with the permutational intersection of factor sets and 2) using statistics that are suitable for sparse data sets, non-linear processes, clustering, etc. Attempts to model the spatial distribution based on fractal geometry, spatial geostatistics, and non-linear models should be researched.

The fundamental issue here is whether the spatial distribution of mineral deposits or the PMF of estimates of the number of undiscovered deposits can be modelled statistically. Admittedly, any two variables can be modelled statistically, for example, spatial positions and mineral deposit occurrence, or size of galaxies and miles of yellow paint stripes on federal highways. Even those relationships showing good statistical correlation may be physically meaningless, as in Bertrand Russell's famous example of the schedules of factory whistles and the movements of workers in other provinces. Statistics is a way of empirically describing observations; predictions made from parameters that describe observations only reflect those observations. If there is not some evidence of a relationship of the variables through physical law, the model is not likely to prove very useful, and in many cases may even be misleading.

Influence of estimation procedure on significant figures for the estimate of contained metal

In an areal evaluation, estimators are asked to estimate "at least" a given number of deposits of each type present at several fixed confidence levels (confusion over what is being estimated has already been discussed). For many deposit models used in small or moderate sized areas, the critical decision is at which confidence level one deposit is present. Thus, at most confidence levels, the pertinent choice is between zero and one -- essentially a presence or absence decision.

In presentation of the final result, the OMR method does not adequately consider precision (significant digits) even though the OMR method uses data of the highest accuracy possible. The precision of the final result must be based on the accuracy of the parameters that generated it and on errors introduced by the OMR method; the result can never have a higher precision than the least precise parameter used in its derivation. The number of significant figures in the final result of the OMR method is therefore certainly less than one, because of ubiquity of presence-or-absence decisions; however it is commonly presented with two to four significant figures of precision.

Accounting for exploration intensity

An estimate of the number of undiscovered deposits in a region is quite dependent on the level of exploration in the region for those types of deposits. Any exploration in a region exhausts the region to some degree for undiscovered mineral deposits making it less likely that a given number of mineral deposits will be discovered. An area with discovered deposits has obviously been somewhat explored. The discovered deposits may imply permissive terrain to the assessor, but it also implies partial exhaustion of ground for hosting mineral deposits; thus the estimates should somehow account for the previously discovered deposits.

If the level of exploration activity cannot be entered quantitatively, the assessment of remaining resources cannot be quantitative, either. In the absence of other guidelines, this factor is frequently evaluated qualitatively based on rumor and anecdotes.

This problem is exacerbated if the only deposit types evaluated are those with existing prospects. Under these conditions, all evaluations are compromised by partial exploration and hence partial exhaustion.

Other methods of estimating the number of undiscovered deposits

Other approaches in estimating the number of undiscovered deposits have tried to relate geological, geochemical, and geophysical data to the occurrence of ore deposits. Agterberg and Cabilio (1969) used multiple linear regression to estimate gold occurrences in the Greenbelt of western Ontario with little success. Chung and Agterberg (1980) applied Poisson regression with somewhat better results. Many authors have made attempts to produce such relationships, but a review of that work is beyond the scope of this report. To our knowledge, there is no published literature that demonstrates success for any deterministic model attempting to estimate undiscovered mineral resources for a wide variety of mineral deposit models. There is a need for more work in this area. Currently, some of the most interesting work is being performed by Liu and others (1991), Harris and Pan (1987, 1990), and Pan and Harris (1991).

Conclusions on the estimation of the number of undiscovered deposits

An assessment that used a technique similar to the OMR method demonstrates some of the large discrepancies that are possible from individual geologists who provide estimates of the number of undiscovered deposits. This study, initiated by the Grand Junction Office of the Energy Research and Development Administration (Harris, 1984, p. 330) in 1975, called on thirty-six geologists to estimate the uranium endowment of the state of New Mexico. The Grand Junction assessment used geologists selected from industry, government, and academia. In the study, the state of New Mexico was divided into 62 cells, each with an area of approximately 1,970 square miles. The geologists used subjective probability estimates of numbers of uranium deposits that remain to be discovered and subjective estimates of grade and tonnage relationships for these deposits in each cell. The assessment was done using the modified Delphi procedure (Harris, 1984 p. 335), which allows geologists to learn from each other and adjust their estimates of the number of undiscovered deposits. The estimates were then combined in a Monte Carlo procedure to create estimates of the tons of U_3O_8 that remain to be discovered in New Mexico.

The Grand Junction assessment used estimates of numbers of deposits which were combined with estimated grades and tonnages in a simulation procedure. There was no training given to the assessors, and, in many cases, the selection of assessors can be questioned. The Grand Junction assessment was not based on permissive terranes but on simple grid cells.

The results of this study are reported on a geologist by geologist level. Before the Delphi procedure, estimates in cell 20 among all geologists ranged from a low of 2.73 mean undiscovered deposits to a high of 356.17 mean deposits (Harris, 1984, p. 339). The sorted estimates of the geologists are shown in figure 6. Most estimates were scattered throughout the range, a few are clustered together. They indicate that there can be a very large difference of opinion among geologists who subjectively estimated numbers of undiscovered deposits.

We have outlined a number of problems that demonstrate that the determination of the number of undiscovered deposits holds little resemblance to any accepted scientific method and in fact amounts to an educated guess. Though the estimation of the number of undiscovered deposits in a region is a potentially fruitful area of research, results that cannot scientifically predict uncertainty and that can not be tested should be re-examined. Classical statistical modelling of the occurrence of undiscovered mineral deposits within terranes is problematic at best, and consequently estimates of contained metals based upon them have such large uncertainty as to be of little use. The further use of complex simulations to estimate total metals present seems unjustified at this time.

The use of the number of undiscovered deposits as the basic estimator removes spatial information from mineral resource assessments done using the OMR method. While most published assessments present maps displaying tracts that are permissive for the occurrence of a mineral deposit, there is no attempt to indicate which regions in the tracts have a higher likelihood for the occurrence of a deposit. That is, spatial information portraying favorability of the permissive tracts is lost in the OMR method. Permissive tracts only delineate regions that can host a mineral deposit, however, it is possible that only a small portion of a tract is important in a mineral resource potential sense. Geologic, geochemical, and geophysical data that are presently used in quantitative mineral resource assessments could be used to delineate regions that have a higher potential for mineral occurrence than simply being permissive.

Considerations on the use of grade and tonnages models and simulation procedures in estimating the amount of contained metal in undiscovered deposits

The OMR method employs statistical models of mineral deposit grades and tonnages along with a simulation procedure to estimate the contained metal in undiscovered deposits. USGS Bulletin 1693 (Cox and Singer, 1986) contains grade and tonnage models for most of the descriptive models; these are used in the simulation procedure. These statistical models are built from worldwide grade and tonnage data from individual deposits and in some cases from mineral districts. Tonnage estimates are based on "lowest cutoff grades" (Cox and Singer, 1986, p. 7). The contained metal models constructed from grade and tonnage models for a region being assessed suffer from three sources of error that are present in the grade and tonnage models. These are: lack of statistical stationarity of deposits' tonnages and grades, model biases, and model mis-specification. Following these sections, the results of two simple simulations are presented to test some of the problems associated with model biases and with model mis-specification. Also, several additional problems are discussed.

Model biases and model mis-specification have been addressed by Singer (1990), but have never been explicitly dealt with in respect to the OMR method. As shown below, the errors generated from these effects can be sufficiently large that the use of statistics as a predictive tool is not meaningful.

The lack of statistical stationarity in grade and tonnage distributions

The use of grade and tonnage models derived from ore deposits worldwide to make quantitative estimates leads to errors in the resource estimates for a given geologic terrane. The size of an ore deposit is controlled to a large degree by the geologic features found in the immediate area. In differing crustal

environments, differing components of mineralization are supplied, trapped, and preserved. For a given deposit type, using grade and tonnage estimates derived from deposits which are found in many distant analogous terranes leads to large biases. For example, the size characteristics, grades and tonnages, and amounts of metals produced from precious metal vein deposits within the volcanic rocks (terrane) of four large volcanic fields are compared in table 2. Clearly, large differences exist between precious metal vein deposits from these terranes. A grade and tonnage model built on the basis of one terrane would not accurately predict the grade and tonnage in another terrane. A grade and tonnage model built on all terranes would not accurately predict deposit attributes in any of the terranes. Therefore, to best estimate resources within a given terrane, it is necessary to determine and use models of the grades and tonnages of deposits within the geologic terrane under consideration for every mineral deposit model in the estimation procedure. This is seldom done in the OMR method.

Table 2 indicates that ore deposits are uniquely related to the terrane in which they occur, and statistics which describe populations of grades and tonnages of ore deposits can not be viewed as stationary over different geologic terranes. The OMR method, in general, uses grade and tonnage models based on worldwide data. To apply the OMR method to an area encompassing one terrane or even a small number of geologic terranes (such as a National Forest or Wilderness area) makes little sense. The mineralization within that area is constrained by the geologic environment unique to those terranes and cannot be accurately represented by a model derived from numerous geographically dispersed terranes.

The lack of statistical stationarity of ore deposit model grades and tonnages means the OMR method, *sensu stricto*, can not be used to assess the mineral resource potential of any region other than the region represented by the grade and tonnage model for the given mineral deposit model. As used now, the OMR method assigns a world-wide diversity of deposits to even small (100 square kilometers or less) permissive tracts in unique geologic environments. As indicated in table 2 and the discussion above, the results in these cases are meaningless. In addition to these errors, the resource estimate is subject to biases as discussed in the following section.

Grade and tonnage model biases

In order to estimate the mineral endowment of a region it is necessary to understand the geologic distribution of mineralization (geologic mineral deposits, not economic mineral deposits) in that region. The only information we have about the distribution of ore comes from discovered mineral deposits. There are a number of sampling biases that ensure the statistical measures obtained from discovered deposits do not reflect the the properties of the underlying population of geologic mineral deposits from which they were drawn. These sampling biases, discussed below, include: sampling proportional to size, economic truncation, translation, and censorship.

The result of sampling biases in the OMR method is a biased model of contained metal that may have little in common with the actual geologic occurrence of ore minerals remaining to be discovered in a region. This statement is especially true in areas where there has been moderate to intense exploration, a condition which holds true for the many of the exposed host lithologies found in the conterminous United States.

Sampling proportional to size

Sampling proportional to size results from larger tonnage deposits being found before smaller deposits. This occurs because they have a greater areal extent and, due to their size and their geologic, geochemical, and geophysical attributes, are easier to detect. Sampling proportional to size has been demonstrated for hydrocarbon accumulations and is the basis of some exploration process models in hydrocarbon resource appraisal (Schuenemeyer and Drew, 1983; Forman and Hinde; 1986, Drew and others, 1980; Meisner and Demirmen, 1981). It can also be demonstrated for some types of mineral deposits including the sediment-hosted gold deposits of the Carlin Trend in Nevada (Bultman, M. W., 1990, unpublished manuscript) and is discussed in Singer and Mosier (1981). Sampling proportional to size must be considered when building models from a region or assessing the mineral resource potential of a region that has had even a minimal amount of exploration; otherwise hypothetical deposits used to estimate contained metal will be biased to larger values. When dealing with a population of events that has a right hand tail whose values are orders of magnitude larger than the mean or median value of the data, such as ore deposit tonnages, the effect produced by sampling proportional to size can be enormous. This will be shown through a sensitivity analysis whose results are given in a following section.

Economic truncation

Some mineralized zones may not be developed to the point that they are announced as a deposit if they can not produce metals at a profit. This effect is termed economic truncation; it can occur for many reasons but is usually a result of the relationship between the local transportation-industrial infrastructure, the deposit grade and tonnage, and economies of scale. Many deposits that could be economic in the proper setting are not developed because they are in the wrong place. This component also has strong ties to current levels of technology, and technological, as well as economic, biases are inherent in grade and tonnage models. Since undeveloped deposits are commonly not well described in the literature, economic truncation means that smaller deposits are not accurately represented in statistics derived from discovered deposits. Thus, contained metal distributions that are built from many of the grade and tonnage models are biased to large sizes.

Translation

Translational biases occur because mineral producers generally develop only the highest grade part of a mineralized system and thus statistics from discovered deposits will tend to be higher in grade and lower in tonnage than the actual geologic resources of the mineralized system. Tonnage models in Cox and Singer (1986) are created using the lowest cutoff grades possible for a deposit, but still represent only a part of the geologic mineral deposit. The effects of translation on contained metal distributions are hard to predict due to a lack of information on the geologic mineral deposit.

Censorship

Censorship means that not every discovered deposit has published or available grades and tonnages. Many individuals, companies, and/or governments do not release data for a variety of reasons. Most often censorship will occur for small deposits, leading to contained metal distributions that are biased to large sizes.

Implications of the sampling biases

These sampling biases insure that any function used to model the sample of tonnages or grades of discovered deposits has a different functional form and/or a different parameterization than the underlying population of geologic mineral deposit sizes and grades. Sampling proportional to size, economic truncation, and censorship will all bias the tonnage models so that large deposits are over-represented in the distributions of contained metal created from these models. The effects of translation will depend on the amount of distortion in grades and tonnages from individual deposits and is difficult to predict. The effects of these biases can not be observed in the data if there is no correlation between grade and tonnage and if the functional form of the underlying population of mineral deposit is not known. For the population of geologic deposits to be truly understood, an exploration model which accurately describes the discovery of ore deposits (how the sample of discovered deposits is selected from the population of geologic deposits) would need to be developed. Using that model, a researcher could approximate the underlying geologic distribution of deposit grades and tonnages from the sample of discovered deposits. No such model has yet been developed.

Given the existence of these biases, and the effects from a lack of statistical stationarity of deposit tonnages and grades, practitioners of the OMR method have had to make a choice between estimating the chance of occurrence of events drawn from the population as described by the grade tonnage curve or events drawn from the natural population. The first choice gives internal consistency, but is a trivial academic exercise because of the magnitude of bias. The second choice strives for reality, but is impossible to validate. Most existing literature on the OMR method suggests that comparison is to the grade tonnage population (Cox and Singer, 1986, Menzie and Singer, 1990), but Reed and others (1989, p. 1942) and Menzie and Singer (1990) also claim that the undiscovered deposits are accurately modelled by the grade-tonnage models and thus both choices coincide. The above discussions of statistical stationarity and bias shows that this cannot be the case.

The cumulative effects of these sampling biases is to bias the hypothetical distribution of contained metals to large values, but no analysis has been done for the OMR method to determine the magnitude of these biases. Because we are dealing with distributions that have an extended tail to large values, the importance of these biases can not be overstated. In the porphyry copper tonnage model, roughly 80-percent of the total tonnage is contained in the largest 20-percent of the deposits.

Model mis-specification

The OMR simulation procedure uses a model to describe grade and tonnage data. The choice of the model that is used to describe the distribution of grades and tonnages is extremely important to the estimates of contained metal that are generated in the OMR method. This is true even if an assessor accepts that the assessment is based only on discovered deposits. The functional form of the model is used to generate a large number of hypothetical deposit grades and tonnages, which are combined with the PMF of undiscovered deposits in order to create a resource estimate. There is no published research demonstrating that any specific functional form applies to either the underlying population of geologic mineral deposits or to the sample of discovered mineral deposits for all deposit models.

Grade and tonnage models used in the OMR method were developed under the assumption that the logarithm of grade and tonnage of discovered deposits could be modelled by a normal distribution (Singer,

1990, p. 5). The choice for the use of the lognormal distribution is observational, that is, most grade and tonnage distributions are positively skewed and when transferred to logarithmic space, their distribution takes on a Gaussian appearance. For most deposit types, close examination of the plots in Cox and Singer (1986) show that the data do not appear to be lognormal and no statistical measure of the lognormality is available. The small size of many data sets in Cox and Singer (1986) precludes any statistical measure of distributional fit. Despite this fact, the belief that the distribution of grades and tonnages for a given deposit model should be lognormal is used to classify deposits into deposit models. Menzie and Singer (1990) state: "First, they (*grade and tonnage models*) can be used to help correctly classify known deposits in the area being assessed.". Singer (1990) states: ".. these (*grade and tonnage*) models play two roles: first, grade and tonnage models can help classify the deposits in a region." (italics added). Deposits with an incomplete geologic description may be assigned to a given deposit model based on its grade and tonnage. Also, a deposit with good geologic information may not be assigned to a deposit model due its grade and tonnage.

While the law of the proportionate effect (Aitchison and Brown, 1957) can be called upon to rationalize a lognormal distribution, there are no known applications of this law to the geological processes that account for the occurrence of mineralization. In fact, the Gaussian appearance of the logarithms of economic mineral deposit tonnages may only result from the aforementioned sampling biases. Several studies suggest most lognormal distributions in the earth sciences are the result of sampling biases (Mandelbrot, 1962; Zacek and Krivanek, 1991).

The assertion that the underlying distributions of mineral deposit grades and tonnages are mono-modal is questionable. Because of sampling biases, it is not clear that there is a peak in the underlying distribution; like the magnitudes and numbers of earthquakes (which are also clustered; see Mandelbrot, 1983), the distribution could represent decreasing likelihood of occurrence with increasing grade or tonnage. Some grade and/or tonnage distributions may even be multi-modal, as has been proposed for the distribution of element concentrations or grades in the earth's crust (Skinner, 1976).

In many of the grade and tonnages distributions presented in Cox and Singer (1986) it appears visually that a piecewise log-linear cumulative distribution would fit the data better than a lognormal distribution. This may imply that discrete sets of geologic processes may be responsible for each segment of the distribution and that, possibly, each segment represents one geographic or geologic area. This hypothesis is not testable, based on Cox and Singer (1986), because deposit names are not associated with grades and tonnages therein.

Any set of geologic data plotted cumulatively will have a sort of sigmoidal shape if it is bounded and contains sampling biases that restrict access to low data values. In general, the use of a cumulative plot to fit distributions amounts to applying a severe smoothing filter to the data before fitting it to a distributional model and thus does not utilize the full information content of the data. Conventional histograms of the grade and/or tonnage data in Cox and Singer (1986) would display the character of the data with much higher sensitivity (compare figs. 7 and 8).

Many of the models in Cox and Singer (1986) are based on very small data sets; most have less than 100 points, and many have 40 or less. Especially for these cases, no distribution can be safely assumed to be better than any other, and statistical estimation is inaccurate. The paradox is that only in well-explored domains can a statistical appraisal be made with any validity (assuming only local grade and tonnage models apply to the assessment), and in those cases, no appraisal is needed. Conversely, assuming a distribution type in an unexplored area is unsatisfactory because of the lack of statistical stationarity in grade and tonnage

models as discussed above. Statistical appraisal of mineral resources is really an ex-post facto method.

Singer (1990) states, "For the estimated number of deposits to be consistent with a grade and tonnage model, approximately half of the deposits estimated should have greater than the model's median tonnage or grade. Thus, the probability that an untested prospect represents a deposit from the population represented by the grade and tonnage model must be estimated with care. Until the proper statistics are gathered, this task is probably best performed by a geologist who has had extensive experience with this deposit type.". The major premise of this statement is misleading. To properly model any set of data it is required, at a minimum, that the data have a similar mean, variance, and range, as well as median, to the proposed mathematical distribution. This is especially true for mineral deposit tonnage models because of the large size of elements in the right-hand tail of their tonnage distribution and will be demonstrated in the section "A simple sensitivity analysis".

A demonstration of the importance of the choice of the functional form of the assumed distribution and the methodology used to fit the data to the functional form of the distribution, is provided by Mayer and others (1980). They modelled the sizes of oil fields in the Denver-Julesberg basin in Colorado. The authors fit two different theoretical distributions using three different methods to fit the data. They chose the lognormal and the gamma distribution to model the distribution of oil field sizes in this basin. The three methods used to estimate the theoretical distributions are: a) the method of moments, b) the maximum likelihood method (also used in the OMR method), and c) the Lorentz curve method. The method of moments sets the mean and variance of the data equal to the mean and variance of the theoretical distribution. The maximum likelihood method maximizes the probability that the data are a random sample from the theoretical distribution. This method gives more attention to the quality of fit in the right-hand tail of the distribution. The Lorentz curve method selects the distribution which yields a probability estimate that one would observe a field of less than a given size (Mayer and others, 1980). The results of this study are given in table 3, from which one can conclude that the estimates of expected field size are greatly affected by the choice of model and method of fitting the model. The largest estimate of the probability of discovery of a 10 million barrel field is 15 times the smallest, and the largest expected field size is over twice the smallest.

The OMR simulation procedure allows an assessor to choose between two different methods to model the grade and tonnage data (Root, 1992). One method models the data with a lognormal distribution, the other models the data as a set of piecewise lines (Root, 1992). The grade and tonnage data themselves are not used in the simulation, instead, the models are used. The OMR simulation procedure includes a mathematical procedure which minimizes errors between the model and the data. Theoretically, errors due to model mis-specification are minimized, but no published assessment contains information on errors due to models not fitting the data (Root, 1992).

There is one additional philosophical point to make. Modelling data is generally done for two reasons. One is to test the data against a model to see if the model accurately predicts the data. The second is to simplify the data for visualization purposes. The use of a functional form to model grade and tonnage data in the OMR simulation procedure implies that the functional forms used represent a model that conveys more information than the data itself. The hypothesis that either the piecewise linear distribution or the lognormal distribution actually contain more information than the data has not been demonstrated. Since any simulation procedure can sample actual data as easily as it can sample a model, the purpose of modelling the data is questionable.

A simple sensitivity analysis

In order to test the effect of sampling biases and model mis-specification in the estimation of the distribution of metal content of undiscovered mineral deposits, two experiments were performed. Data were digitized from the porphyry copper model and the tin veins model in Cox and Singer (1986). The digitized data were an accurate portrayal of the data used to create the graphs in Cox and Singer (1986) since the mean and variances of the digitized data were almost identical to mean and variance of the models presented therein. The porphyry copper model was chosen because it is modelled quite well by the lognormal distribution. The tin vein model was chosen because it is modelled poorly by the lognormal distribution. The porphyry copper model will be used to test the effects of sampling proportional to size while the tin vein model will be used to test the effects of model mis-specification. It should be noted that we were not able to test for effects due to correlations between grades and tonnages in our simulations due to the fact that grades and tonnages are not tied to deposit names in Cox and Singer (1986). Cox and Singer (1986) give no indication of any correlation in these deposit models (for copper and tin) and only correlations among the largest 20- to 30-percent of the deposits would have significant impact on the results.

Data from the porphyry copper model (Cox and Singer, 1986) are known to generally fit a lognormal model. A histogram of the tonnage data is shown in figure 7, and a fit of the tonnage model is shown in figure 8. While the model in figure 8 fits well visually, it is apparent from figure 7 that there are two large tonnage deposits that control much of the fit of the right hand portion of the distribution. The maximum likelihood method of fitting the distribution, used here and in the OMR method of assessment, will ensure the best fit in this right hand tail.

Even though the lognormal model fits the porphyry copper data quite well, there are other considerations to make when using the model. First, data in the extreme right-hand tail of the distribution may be true outliers, and deposits as large may not exist in other regions. In fact, the model dictates that approximately 4 in 1000 simulated deposits will be larger in tonnage than the largest actual data value. Second, discovery proportional to size greatly reduces the chance that any undiscovered deposits as large as deposits in the extreme right-hand tail will be found if any exploration has occurred in an area. These concepts, combined with the size of deposits in the extreme right-hand portion of the distribution compared to the size of the majority of deposits, indicate that the lognormal model can vastly overestimate tonnage.

In order to test the sensitivity of change in the right hand tail of the lognormal distributions of tonnage of porphyry copper deposits four methods were used to generate a distribution of hypothetical mineral deposits. The first method selected 10,000 random samples of grade and tonnage from the statistical (lognormal) models of grades and tonnages, where the model had the same mean and variance as the data, in order to generate a model database. A simulation procedure then selected a grade and a tonnage at random from this database, and multiplied them to create a hypothetical copper deposit. This was repeated 5000 times to generate a distribution of 5,000 hypothetical copper deposits. Method two sampled the actual grade and tonnage data randomly 5,000 times to create the 5,000 deposits. Method three sampled the actual data with the 2 largest deposits (tonnages) removed from the 208 actual tonnages and method 4 sampled the data with the 10 largest deposits (tonnages) removed from the 208 actual tonnages. The results are shown in table 4.

The mean amount of copper in the hypothetical distribution of deposits is largest under method 1. This is to be expected because this method uses a functional form which extends the distribution to the right of the largest data point. Since any increase in the size of a deposit in the simulation procedure is actually an

exponential increase in deposit size (the logs of tonnages are modelled), this effect is quite pronounced. When the 2 largest and 10 largest deposits are removed from the data, the sensitivity of this methodology to the fit of the right-hand tail is demonstrated. In the case of removal of just the 10 largest deposits (of 208), the mean hypothetical deposit contains only 46-percent of the copper as the mean hypothetical deposit using the lognormal approximation. Probably, no deposit as large as the 2 largest deposits in the porphyry copper model will be found in the continental United States; and it is likely that none as large as the 10 largest will be found, at least in an environment where the host rocks are exposed. The standard deviation, which represents the variability of the estimate and greatly affects the estimation of contained metal, is very large in method 1. This large standard deviation is attributable to the lognormal model of the right-hand tail of the distribution. It helps create a contained metal distribution which is further biased (beyond the effects of tonnage means) to large values because it represents a number of very large hypothetical deposits. The variability, as measured by the large standard deviation, is generally much larger than the estimate, which renders the validity of the OMR method doubtful.

This test has only shown the effects of sampling proportional to size. If the other sampling biases, which would tend to increase the number of small deposits and thus decrease the likelihood of discovering a very large deposit, were included, the effects would be even more pronounced. When choosing an indicator of the central tendency of the distribution, the median is less sensitive to economic truncation than the mean because only a small number of very large deposits control the mean. Since the median may not reflect the skewness of the distribution, there is little theoretical or geological reason to choose the median as an estimator, other than it gives a more pleasing result.

The tin vein model (Figure 9 and 10; Cox and Singer, 1986) represents a model that has a poor fit to the lognormal model for both grade and tonnage. Two hypothetical distributions of contained metal were made for the tin vein model. The first employed a lognormal model of tonnage and grade obtained by the maximum likelihood procedure. The second used random samples from the actual data (table 5). The mean of the hypothetical distribution of deposits using raw data contains only 36-percent of the metal obtained by using the lognormal models. From figures 9 and 10, which respectively display the histogram of the data and the cumulative distribution of the data and the model, the discrepancy is easy to understand. The model overestimates the data in the right hand tail of the data. Reed and others (1989) used piecewise linear distributions to fit this data. This probably gave results closer to that of an estimate made using the data, but no analysis of errors from fitting the model to the data was included. Models that do not accurately reflect the data can create very large errors in a resource assessment, in those cases, sampling from the actual data is preferred. Any model should be subject to a thorough sensitivity analysis with the data set for which it is being considered. While errors between models and data have been minimized in the OMR method and are not likely to be as large as the errors shown here, the errors themselves are not given in published quantitative mineral assessments.

Additional problems in estimating quantities of contained metals in mineral deposit models

There are some tonnage and grade models in Cox and Singer (1986) that are built on districts (for example, polymetallic replacement and gold placers). In fact, some models mix deposits and districts (for example, copper skams). A problem exists when an area being evaluated contains, or could contain, *part* of a district. A district-based model requires that the assessors estimate the contained metal in a region by

estimating the number of undiscovered deposits in the region and dividing by the number of deposits in a district. District-based grade and tonnage models mean that it is impossible to have any comprehension of the size of the deposits that are being assessed. Statistics describing the distribution of deposits within a district have not been presented, thus assessments in these circumstances depend only on an assessor's experience with the districts in question. This is likely to lead to very large biases, especially since in some regions only portions of districts are all that remain to be discovered. For example, in Arizona, no porphyry copper deposit has ever been discovered which was not in an already known copper producing district (S. R. Titley, 1992, oral communication).

We have seen that the definition of a deposit is based on grade and tonnage models. Given that the tonnage and grade distributions presented in Cox and Singer (1986) are almost always unrepresentative of undiscovered deposits in a region being assessed, the question of what is being estimated needs to be addressed. If an estimator were to adjust a geologically based number of deposits estimate in order to compensate for grade and tonnage model problems or to fit a preconceived notion of the metal content, the estimate of the number of deposits is no longer valid. Since results of mineral assessments using the OMR method publish both number of deposit estimates and contained metal estimates, there could be inconsistencies if, in the future, another method is used to estimate contained metal from an existing estimate of the number of undiscovered deposits.

Conclusions on the use of grade and tonnage models in estimating quantities of contained metals in mineral deposit models

The OMR method is based entirely on the premise that grades and tonnages of undiscovered deposits will look like the deposits presented in the grade and tonnage models of Cox and Singer (1986). These models constrain the OMR method so that it can only estimate the amount of ore that is likely to be discovered based on deposits that have been exploited (or at least whose grades and tonnages have been established) in the past. Thus, the estimates are based entirely on past and current technology, discount rates, cost of capital, cost of energy, and metal demand. Changes in any of these parameters will greatly affect the resource estimate.

Any resource estimate that does not take into account the biases stated in the previous section is unlikely to be accurate. The likelihood that we have in the past consumed or are now consuming most of the large, high-grade ore deposits does not enter into the philosophy behind the OMR method. To base an assessment on the premise that newly discovered deposits will look like deposits that have already been exploited could be as misleading to mineral resource assessment as the Zapp hypothesis was to oil and gas resource assessment.

In general, combining grade and tonnage data to produce hypothetical distributions of metal contained in undiscovered deposits creates errors attributable to sampling biases and model mis-specification. Errors from the use of district-based distributions of grades and tonnages must be considered when such a model is used. Methods of mineral-resource assessment that employ these techniques and models must account for and present the errors involved in every mineral resource assessment. This must be done for each mineral deposit model and for each region in which it is being used.

Analysis of errors in three-step quantitative assessments

Many of the assumptions and procedures in the OMR method result in estimates of mineral endowment that are systematically too large or too small. Table 6 presents an analysis of the errors and uncertainties that the authors have discussed in this report. Where possible, subjective quantitative estimates of the errors are included.

The errors in table 6 are presented in two parts. Part one contains the errors identified in the estimation of the number of undiscovered deposits that are due to the use of mineral-deposit models as the basis of the assessment and due to uncertainties surrounding the estimation of undiscovered deposits. Where given, the quantitative error estimate addresses the error carried through to contained metal estimates. Part two presents the errors involved in the estimation of contained metal due to the use of grade and tonnage distributions and simulation in estimating the contained metal.

A complete analysis of the interactions of these errors is beyond the scope of this report. Clearly, errors within part one and within part two are additive. Errors between each of the parts multiply. Also, errors in part two occur for both grades and tonnages and these must be multiplied. A simulation-based sensitivity analysis is likely to be the only way that the total error can be estimated accurately. Based on table 6, it is easy to see that the possible errors in the OMR method can be larger than a factor of 10,000. Also, there is a systematic tendency for these estimates to be too large (table 6). Since the errors are many times larger than the estimates themselves, only order of magnitude estimates should be presented with their order of magnitude uncertainties. Any mineral resource assessment based on the present OMR method actually contains less information than would be implied by the terms low, moderate, or high mineral resource potential, if these terms are properly defined, since these terms imply a measure of favorability for mineralization by geographic location, information which is lost by the use of only permissive terranes in the OMR method.

Presentation of results and public reaction

Results of quantitative assessments created using the OMR method are generally presented at the 10-, 50-, and 90-percent quantiles of the hypothetical distribution of deposits. The 50-percent quantile is the median of the hypothetical distribution. In addition, a mean of the hypothetical distribution is usually presented. Given such an estimate, the mean value will usually be used by those who need to have an understanding of the mineral endowment of the region, because they would generally feel comfortable using a mean. However, these individuals may not comprehend the difference between an arithmetic mean and the mean of a lognormal distribution.

These individuals will, in general, have no idea about the subjectiveness of the decisions that went into the creation of this estimate. The number will be treated as if it were actually the mean of a set of observations of the contained metal in undiscovered deposits. It is not. No uncertainty estimates are generated in the OMR method, moreover, the 10-, 50-, and 90-percent quantiles do not represent uncertainty, but are separate, subjectively created, estimates of resource potential. The estimates stand as geologic judgements; they are unrepeatable, untested, and can only be accepted or rejected.

Recently, the estimation of gross in-place value (GIPV) of the estimated contained metal has been added to the OMR methodology. As used in Hodges and Ludington (1991), for example, this method uses 5 year average prices for metals to estimate a metal price and multiplies these prices by the estimates of contained metal to produce the GIPV. Given the amount of error in the contained metal estimate and assumptions that must be made about metal price (such as constant price), GIPV estimates have little or no utility, and numbers are presented with no mention of any uncertainty. A realistic estimation of the GIPV should consider the fact that mineral deposits can only be valued as a time series of production estimates. Thus any estimate must consider the time series of production values, metal prices, and future discount rates. Since these time series would amount to little more than guesses, the estimate of GIPV can be little more than a guess.

Moreover, the utility of GIPV is questionable since it does not take into account the cost of extraction (as well as many other factors) which in many cases may greatly exceed the GIPV; in those cases, the net value of the resource is negative. The GIPV estimate, as used by the OMR method, is not related to any geologic or economic constraints. The magnitude of any GIPV estimate, for a given number of undiscovered deposits, is controlled entirely by the grade and tonnage models used to make the estimate of contained metal. The lack of attention given to sampling biases and model mis-specification in grade and tonnage models mean that GIPV estimates are inherently optimistic. Finally, it is possible to calculate a GIPV based on the estimates of the crustal abundance of a given element for any given region, even though it is highly unlikely that humans will ever utilize even a small fraction of that resource.

The difficulties with the use of subjective estimates can be further illustrated with results obtained from petroleum resource estimates. An ambitious study by the Rand Corp. (Nehring, 1981) estimates that there is a 90-percent probability that there is less than 32 billion barrels of producible oil to be discovered in the United States. Dolton and others (1983) of the U.S. Geological Survey concluded that there is a 95-percent probability that more than 64 billion barrels of oil are yet to be found in the same region. Obviously, if these estimates were to meet formal statistical requirements, they would be mutually exclusive. Upon inspection of the reports it is revealed that the probabilities are based on subjectively generated point estimates that are then associated with a probability distribution. In fact, these probabilistic estimates carry no more information than the point estimate from which they were generated, and are really only educated guesses based on given premises.

The nature of subjective estimates has not escaped the attention of the outside world. The Economist refers to the Interior Department's (USGS's) subjective assessment of the Arctic National Wildlife Refuge (ANWR) as follows: "the Interior Department estimates that there is a 46-percent chance of finding at least 3.57 billion barrels of oil (whatever such absurd precision can possibly mean) at ANWR, ..." (July 20, 1991, p. 33).

Science News, in its August 24, 1991 edition (vol. 140, No. 8, pp. 113-138) reports on the conclusions of a National Research Council panel report which studied the use of computer models for decisions concerning changes in social and tax programs by Federal policy makers. The panel was chaired by Eric A. Hanushek of the University of Rochester who states " There is very little done to assess the validity of the estimates, the amount of uncertainty in the estimates and the options for improving them". The report cites the poor quality of available data and the lack of objective measures for assessing a computer model's reliability and validity. The same statements directly apply to the OMR method.

Recently, an article in the Denver Post (Oct. 12, 1991) questioned the results of a quantitative assessment

performed by the OMR method. In the opinion of the authors, this is the first in a large number of conflicts that will result from mineral-resource assessments that use the OMR method. Inflated results generated by the OMR method imply that the USGS-OMR is taking a position of advocacy for the minerals industry.

Conclusions and implications

We conclude that (1) the OMR method has not been adequately described in the published literature, (2) the hypothesis on which the OMR method is based is almost completely untested and does not consider error at any level, and (3) there is little support for either geologic or mathematical aspects of the hypothesis from known characteristics of mineralization. Geologically, the inadequacies of the method prevent it from proceeding beyond the guesswork stage. Mathematically, the weaknesses of the method are so severe that calling it a statistical-probabilistic method is false advertising. Our analysis of errors (table 6) present in the OMR method indicates that estimates contain uncertainties so large that these estimates are not even determinable to an order of magnitude. Therefore these estimates actually contains less information than would be provided by labelling the permissive tracts as low, moderate, or high mineral resource potential, since such labelling conveys spatial favorability information lost in the OMR method.

In reference to his review of the National Uranium Resource Evaluation Program (NURE), Harris, (1984, p. 371) states: "Many of the criticisms of the elicitation process made in this report presuppose that a geologist could estimate the variables of the endowment equation by the exercise of his science if he were properly prepared, sufficiently motivated, supported by relevant data, and given sufficient time for study and research. *It must be clearly understood that this presupposition is at present more an 'article of faith' than a demonstrated fact.*". That these estimates are in fact based more on faith than on science is not communicated in the presentation of the results of the OMR method. It must be.

Estimating the number of undiscovered deposits that exist in a terrane or region is the most important problem with the OMR method. In many cases, geologic data are not complete enough to determine even the necessary conditions needed for deposit formation. In these cases, methods used to predict numbers of undiscovered deposits, such as geologic analogy, have been shown to be unreliable. In areas with excellent geologic data, the necessary conditions for ore deposit formation may be determined from that data, however, for most mineral deposit models, the sufficient conditions needed for the occurrence of the deposit are not understood. Unless these facts are correctly incorporated into an assessor's knowledge base, this situation will lead to overestimation of the number of undiscovered deposits in a region. These problems make it meaningless to estimate the numbers of undiscovered deposits and impossible to attach any statement about error to the estimate. The estimation of the number of undiscovered deposits, in the present OMR method, represent an untestable hypothesis, and is contrary to the scientific method. Structuring the estimates of numbers of undiscovered deposits by mimicking the sequential factors of mineral deposit formation should make it possible to specify error. Alternative methods which do not require an estimate of the number of undiscovered deposits should be explored. For example, estimates based on exploration success would at least be testable.

Any quantitative assessment must include a realistic treatment of sources of error or uncertainty. Since

the OMR method is based only on a statistical treatment of known deposits, it can not deal with new deposit types or deposits which do not fit the published models and thus represents a closed system. The statistical approach discourages recognition of uniqueness within a deposit, district, or environment so that the development of new models and refinement or correction to existing models of ore genesis and occurrence are hampered.

At present the OMR method quantifies a guess. It is weakest where it is most desperately needed, in estimating potential resources from unconventional deposits and from areas where potential ore deposits are concealed. Until the OMR method can withstand the objections above, its results, in the authors' opinion, are invalid.

Suggested actions

In order to facilitate improvement of the OMR method discussed in this paper, we offer the following suggestions:

- When the USGS has sufficient confidence in a revised method, the entire methodology should be concisely described in one report that goes through standard USGS peer review and results in a substantive contribution to the published literature. This report should include all assumptions used in all phases of the OMR methodology and a complete analysis of the errors inherent in the OMR methodology.
- Testing of expert estimates of the number of undiscovered deposits should be conducted for a wide variety of deposit types, especially including: (1) areally restricted clustered deposit types, (2) types such as stratabound deposits for which homogeneous distributions cannot be assumed, and (3) syngenetic or other non-hydrothermal deposits.
- The OMR methodology for estimating the number of undiscovered deposits (the "at least a given number of deposits" method) and even the use of undiscovered deposits as an estimator should be reconsidered. Many other methods are possible. Continuation of the "at least" method necessitates either more training for assessors or visualization tools for assessors.
- The revised version of the OMR simulation procedure should be written in a computer language (such as "C") that can run on a variety of small computers. Also, there is no reason that the simulation could not be written using any of a number of commercially available statistical packages, possibly saving much programming time. By designing the simulation to run on small computers, it will be possible to display the PMF constructed from assessor's estimates in real-time and also to observe the quantitative estimates of contained metal during meetings where the number of deposits estimate are made.

- Guidelines regarding the choice of the experts used to make estimates of undiscovered deposits and training required should be established.
- The flow of logic used by the experts in estimating the number of undiscovered deposits should mimic the required sequential conditions for deposit formation unless a quantifiable relationship between geologic variables and occurrence of deposits can be proved. The precise method used to estimate the number of deposits should be communicated in the literature.
- Descriptive models in Cox and Singer (1986) should be revised to reflect: (1) geologic conditions that are actually sufficient for the formation of deposits, (2) better coverage of unconventional deposit types, and (3) the variation in deposit character, including grades and tonnages, with spatial and temporal variations in the geologic environment. A variable weight factor could be incorporated to reflect the environmental variables of mineral deposit formation.
- In those cases where deposit attributes in a region do not match the attributes of the grade and tonnage model in Cox and Singer (1986), estimates of numbers of undiscovered deposits should not be made based on the deposits as described in the model book. If an estimate of the number of deposits is made, it must include an exact description the deposits used as a basis for estimation. Eventually, the OMR method should be changed so that simulations are based on grade and tonnage estimates that are known to be associated with the geologic terrane that is being assessed.
- The OMR method must account for sampling biases in the grade and tonnage models or at least present an analysis of errors that are likely due to these biases.
- If it is decided that the OMR method should use models to fit the data in the grade and tonnage models from a given mineral deposit model, flexible nonparametric nonlinear models should be considered. Neither the lognormal or the piecewise linear models are acceptable as general models. Simulations based on the data are preferred over models. Also, data used to construct the grade and tonnage models in Cox and Singer (1986) should be presented. If proprietary aspects of the data prohibit this, the data should be presented in some way that ties together the location, grade, and tonnage for each deposit.
- The number of significant figures in the estimation procedure can be raised to one if the confidence level corresponding to a given number of deposits is estimated, rather than the reverse.
- Estimates of GIPV should be discontinued; econometric modelling inherent in these estimates is outside USGS purview.
- Work proposed by USGS-OMR personnel on non-discriminating variables and closest-analogue reasoning for mineral deposit models should proceed.

- Ways of quantifying favorability of terrane (rather than just outlining permissive terrane) should be researched.
- Development of alternative methods of mineral evaluation should be encouraged. Research on the nature of the spatial and temporal distribution of ore deposits should be emphasized. A research program to establish which, if any, characteristics of ore deposits can be forecast should be vigorously pursued.

These suggestions are insufficient to rectify all the weaknesses of the OMR method, but we feel that they aim first-generation revisions of the OMR method in the proper direction.

Checklist

To prepare for the desired debate on the OMR method, we present the following list of *independent* points we have questioned. A valid method must be valid for each of the following issues.

1. Public presentation of the OMR method.
2. Testing of estimates of deposit number for a wide variety of deposit types.
3. Error listing and analysis.
4. Adequacy of models for discriminating and reliable prediction.
5. Adequacy of geologic data sets for discriminating and reliable prediction.
6. Adequacy of geologic analogy for discriminating and reliable prediction.
7. Estimation procedure in relation to mineralization process.
8. Assumptions about clustering and other spatial and temporal variations.
9. Specification of the variables being estimated.
10. Accounting for exploration intensity.
11. Dealing with bias in grade and tonnage models.
12. Matching mathematical models and data sets.
13. Method of dealing with secondary products.
14. Treatment of spatial mineral resource information.
15. Objective presentation of estimation results, together with uncertainties.

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Table 1. Different versions of the "three-step" method(s).

Reference	Step 1	Step 2	Step 3
Singer and Cox, 1988	Tract delineation	Number of deposits	Metal endowment
Drew, 1990: Menzie and Singer, 1990	Tract delineation	Grade-tonnage relation	Number of deposits
Drew and others, 1986	Data inspection	Number of Deposits	Metal endowment
Reed and others, 1989	Mineral deposit type identification	Geophysical characterization	Number of deposits

Table 2. Size characteristics, grades, tonnages, and production of precious metal veins from four volcanic fields, western United States

Volcanic field ¹ (terrane)	Strike length of veins	Width of veins	Continuity of vein down dip	Grades (Au) (Ag)	Tonnages of produced ore (in tons)	Metal production Au (Oz) Ag (Oz) base metals (lbs)
² Absaroka WY/MT	Up to 1000', mostly less than a few hundred feet	Some shear zones up to 30', mostly less than 1" to a few feet	discontinuous, up to several hundred feet	.00X to .XX .0X to XX	few hundred to a few thousand	< 100 < 1,000 < 10,000
³ Galileo AZ	up to a few 100', mostly less than 100'	sheared zones with veinlets that are mostly a few inches, occasionally a few feet	less than a few hundred feet	irregular < .00X to .X irregular .X to XX	few hundred	163 2,310 12,203
⁴ Challis ID	up to 1000' mostly a few 100'	few inches to a few feet mostly less than 2'	a few tens to several hundred feet, values diminish with depth	irregular < .00X to XX irregular .X to XXXX	few tons to several thousand	41,925 436,507 42,898
⁵ San Juan CO	up to 15,000' mostly several 100' to several 1000'	few inches to several tens of feet - commonly several feet	commonly continuous for several 1000' values diminish with depth	irregular .0X to X irregular X to XXX	more than 25 million	> 6 million > 134 million > 1.6 million

1) Excludes polymetallic veins and replacement deposits underlying volcanic rocks

2) Wilson, 1964; Fisher, 1972; Nelson, et al, 1980

3) Creasey, et al, 1981

4) Fisher and Johnson, 1987, p. 117-126 (data for Gravel Range, Parker Mt., and Yankee Fork districts)

5) Silver, 1957; p. 233 (data for late Tertiary volcanic rocks); Casadevall and Ohmoto, 1977; Mayor, 1978; Fisher, 1990

Table 3. Comparison of models and fitting methods for oil field sizes in the Denver-Julesburg basin.

Method used to fit data to distribution	Expected field size (million bbls)		Probability that size > 10 million bbls		Probability that size = 10 million bbls	
	l	g	l	g	l	g
MLE	12.9	8.2	0.16	0.25	0.029	0.002
Lorenz	9.8	6.8	0.2	0.16	0.014	0.008
MM	8.2	8.2	0.18	0.15	0.009	0.019

Key to abbreviations:

l : lognormal model

g : gamma model

MLE : maximum likelihood estimate

Lorenz : Lorentz curve estimate

MM : Method of moments estimate

Table 4. Statistics describing the simulated distribution of porphyry copper deposits using four methods.

Method	mean (millions of tonnes of copper)	standard deviation	quantiles of the distribution (millions of tonnes of copper)		
			10%	50%	90%
Method 1: Lognormal model of grade and tonnage.	3.07	8.60	0.09	0.75	6.58
Method 2: Random samples from actual data.	2.33	5.77	0.09	0.69	4.95
Method 3: Random samples from actual data with two largest (tonnage) deposits removed.	1.98	4.10	0.09	0.68	4.45
Method 4: Random sample from actual data with ten largest (tonnage) deposits removed.	1.42	2.07	0.09	0.68	3.70

Table 5. Statistics describing the hypothetical distribution of metal in tin vein deposits based on the lognormal model and on deposit grade and tonnage data.

Method used to create contained metal distribution	mean (millions of tonnes of tin)	standard deviation
Based on lognormal model.	7.3	79
Based on random samples from actual grade and tonnage data.	2.6	22

Table 6. An analysis of errors present in the three-step method of mineral resource assessment.

Part 1. Errors in the estimation of number of deposits based on mineral deposit models as the basis for the assessment and on procedures used to estimate undiscovered deposits.

Source of error	Over (+) or under (-) estimation of number of deposits	Error factor* of the estimate of contained metal	Justification
Lack of rigorous mineral deposit models	+/-	?	Could lead to large over or under estimation of deposits due to a poor understanding of the deposit model and its relationship to deposit occurrence. Insufficient data to attempt to quantify the error factor.
Arbitrary boundaries between deposit models	+/-	10?	May lead to estimation errors based on use of wrong model type. Similar models often display difference in tonnage of at least one order of magnitude.
The lack of sufficient conditions to predict the occurrence of ore deposits	+	10?	This will lead to an overestimate of the number of undiscovered deposits due to necessary conditions alone being chosen to indicate deposit occurrence. A maximum error factor of 10 is assumed.
The imprint of mineralization on geologic data sets	+/-	10?	May lead assessors to estimate deposits that are actually not there or to miss deposits that actually exist. A maximum error factor of 10 is assumed.
Clustering and the spatial distribution of mineral deposits	+	5?	The estimate of undiscovered deposits must take clustering into account. An overestimation of undiscovered deposits may occur by assuming a continuous areal density of undiscovered deposits in a given terrane. A maximum error factor of 5 is assumed.

* The error factor is based on the effect that errors in the number of deposits estimate have on the estimate of contained metal. It is presented as a multiplication factor for over estimation and as a division factor for under estimation.

Source of error	Over (+) or under (-) estimation of number of deposits	Error factor* of the estimate of contained metal	Justification
Spatial and temporal variations in mineral deposits	+/-	?	May lead to over or under estimation of deposit numbers due to difference in mineral deposition from one location to another or over time. Insufficient data to attempt to quantify the error factor.
Interpretation of the probabilistic estimate of the number of undiscovered deposits.	+	5?	Problems in the interpretation of the at least method may mean that assessors estimates are not being accurately communicated to the simulation procedure. This may lead to an error factor as large as 5.
Lack of ability to model ore deposit formation	+	?	As in lack of sufficient conditions for ore deposit formation, a set of requirements for ore deposit formation that is less restrictive than the actual requirements will be used to estimate the number of undiscovered deposits. This will lead to some unquantifiable level of error.
Lack of accounting for exploration intensity	+	10?	The lack of quantitative information about exploration could possibly lead to overestimation of the number of undiscovered deposits and to error factor of 10.

* The error factor is based on the effect that errors in the number of deposits estimate have on the estimate of contained metal. It is presented as a multiplication factor for over estimation and as a division factor for under estimation.

Part 2. Errors in the estimates of contained metal due to the use of mineral deposit models as the basis for the assessment and due to the use of grade and tonnage distributions and simulations for the assessment.

Source of error	Over (+) or under (-) estimation of contained metal	Error factor** of the estimate of contained metal	Justification
Under-representation of unconventional deposits	-	2-5	Unconventional deposits models are not well represented in Bulletin 1693 (Cox and Singer, 1986).
Lack of consideration of all known deposit models	-	?	Mineral deposits that are not considered cannot add to contained metals
Lack of statistical stationarity in grade and tonnage models	+/-	10,000 (t) 1,000(g)	From table 2, contained metal can vary by four orders of magnitude or more based on geologic terrane alone.
Sampling proportional to size	+	2(t)	Based on simulation results presented in table 4, estimates that are double the original estimate are easily possible.
Economic truncation	+	2(t)???	Impossible to estimate errors without knowledge of the geologic population of ore deposits.
Translation	?	?	It is impossible to tell what effects translation have on the data in a general sense.
Censorship	+	?	There is no way to estimate what pertinent data has not reported.
Model mis-specification	+/-	2(t)?	From table 5, estimates that are 2-3 times larger or smaller are possible.

** The error factor is presented as a multiplication factor for over estimation and as a division factor for under estimation. (t) refers to the error for tonnage models, (g) to the error for grade models.

Source of error	Over (+) or under (-) estimation of contained metal	Error factor** of the estimate of contained metal	Justification
Use of district based grade and tonnage models	+/-	5(t)	There is no published literature on the distribution of mines in districts. It is assumed that an estimate may be off by a factor of 5.
Conditioning estimates of metals on the presence of the metal	+ (for secondary metal estimates)	10(t)	Some secondary metals occur in as few as 1 in 10 deposits for a given mineral deposit model.

** The error factor is presented as a multiplication factor for over estimation and as a division factor for under estimation. (t) refers to the error for tonnage models, (g) to the error for grade models.

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Figure 1.

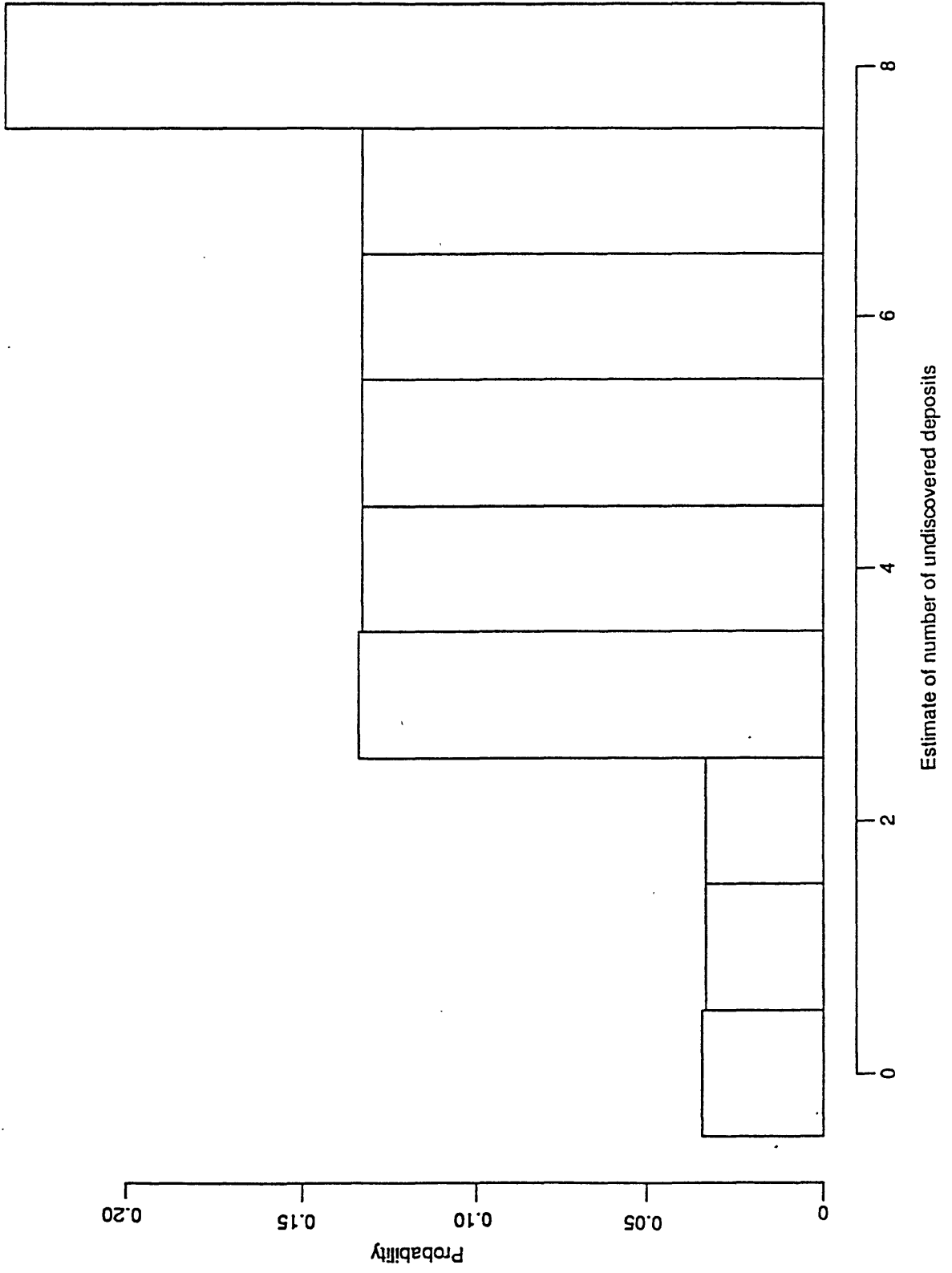


Figure 2.

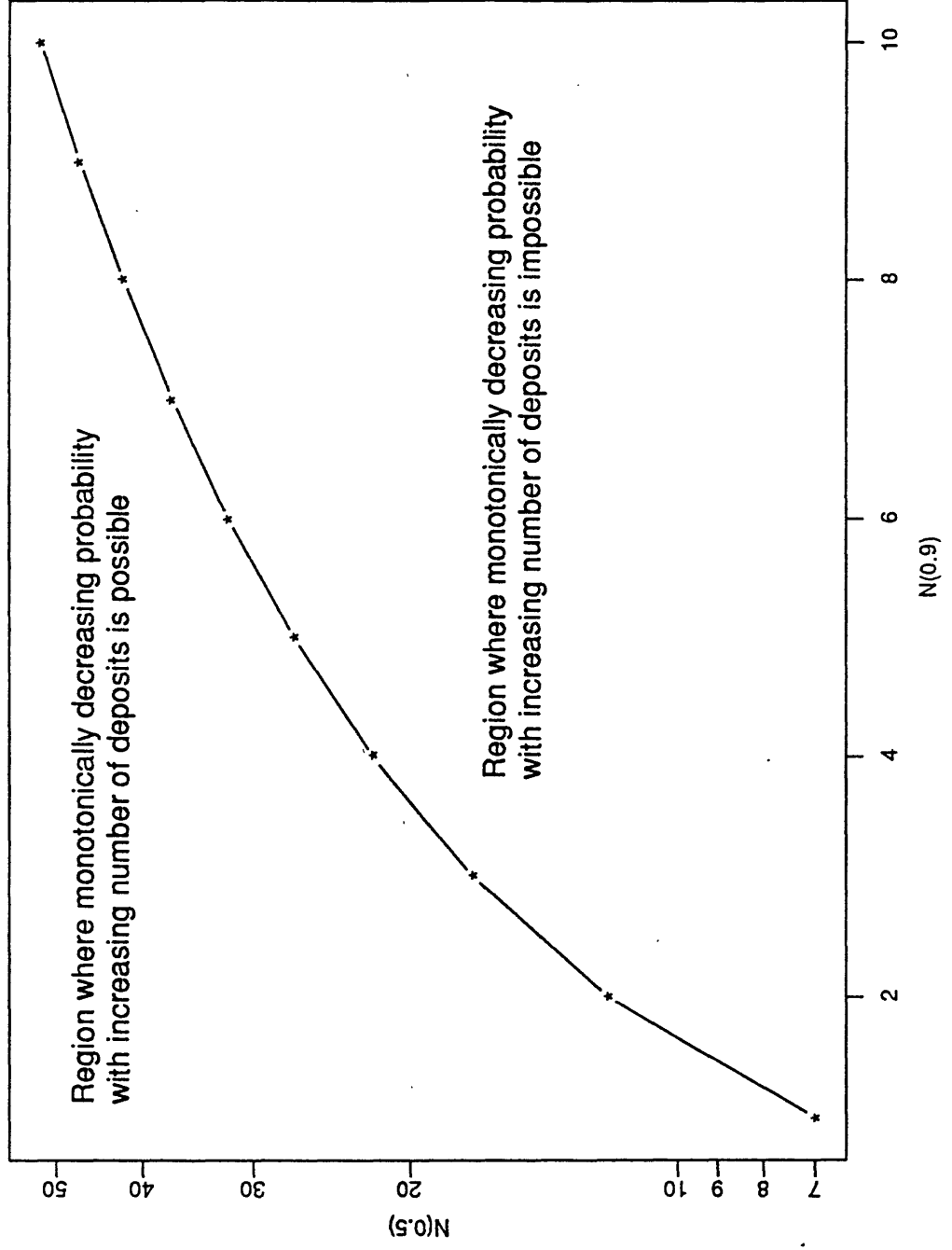


Figure 3a.

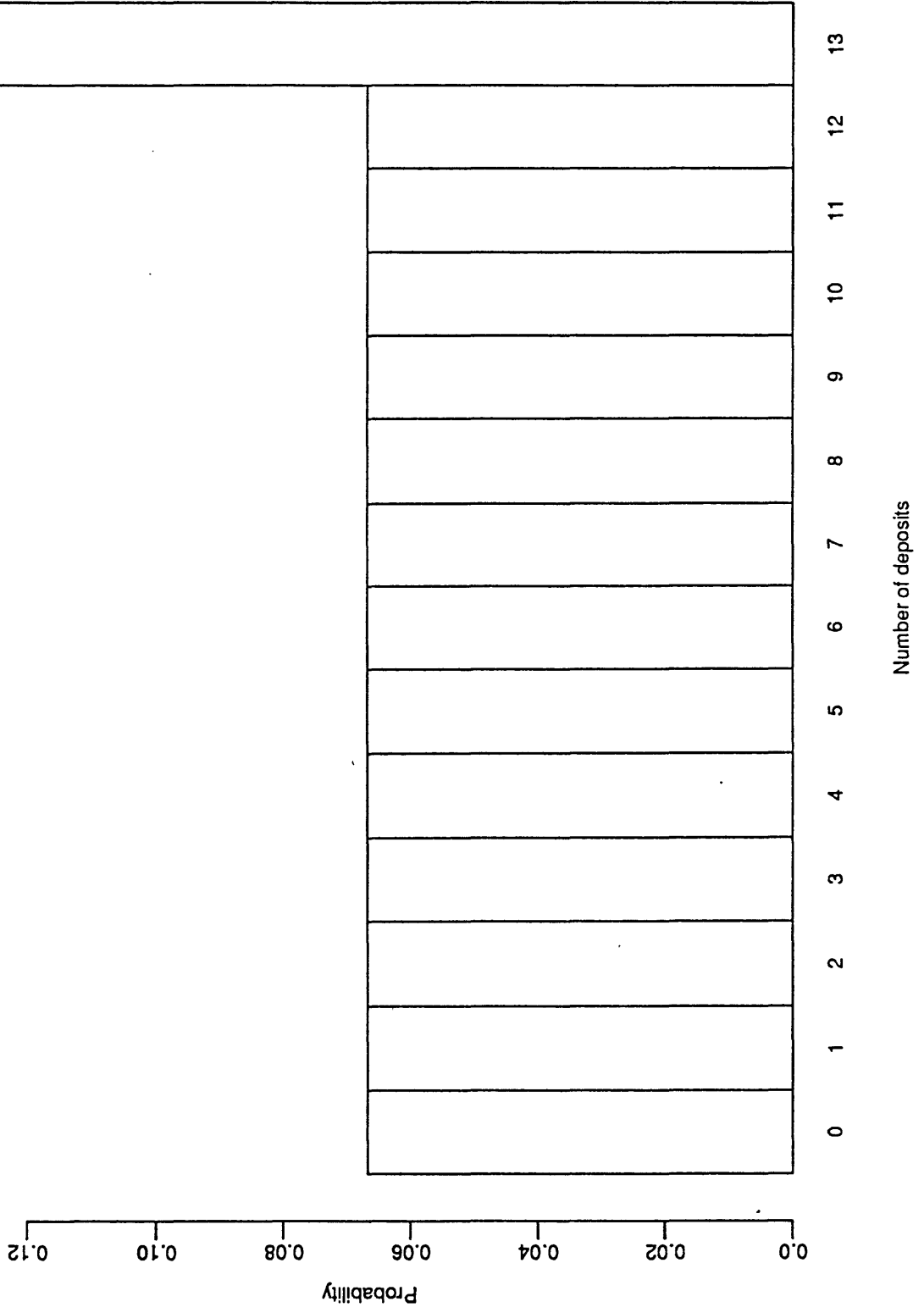


Figure 3b.

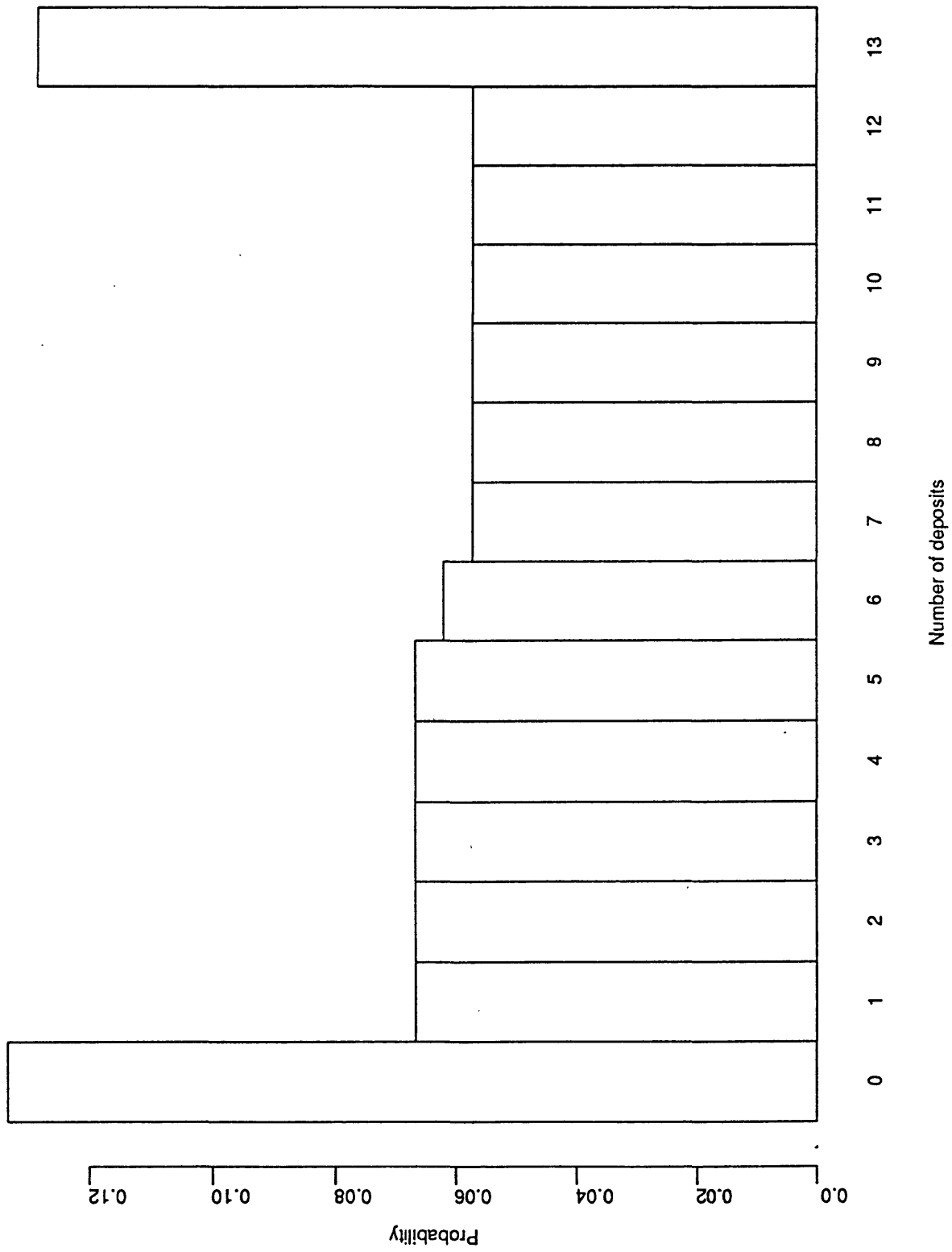


Figure 3c.

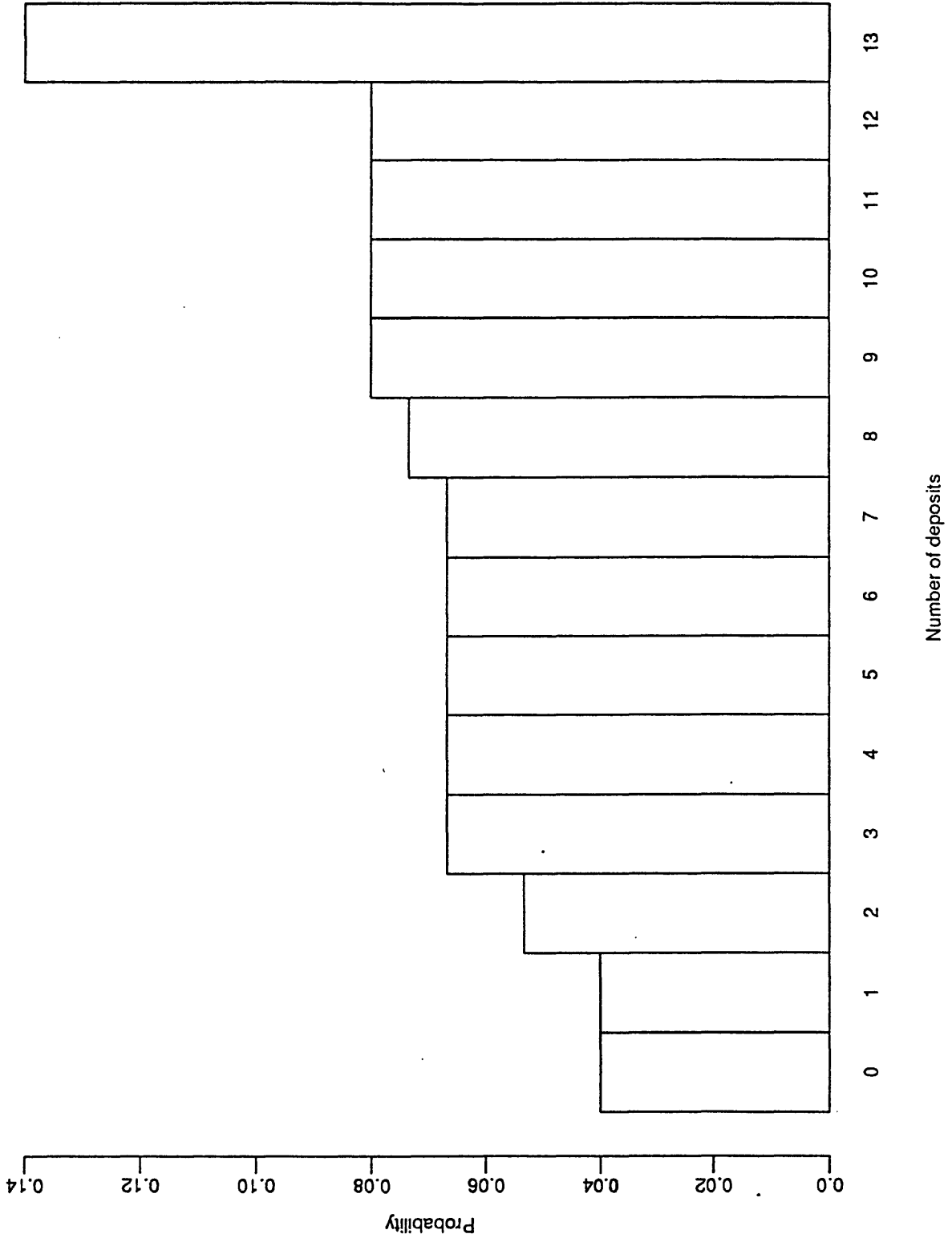
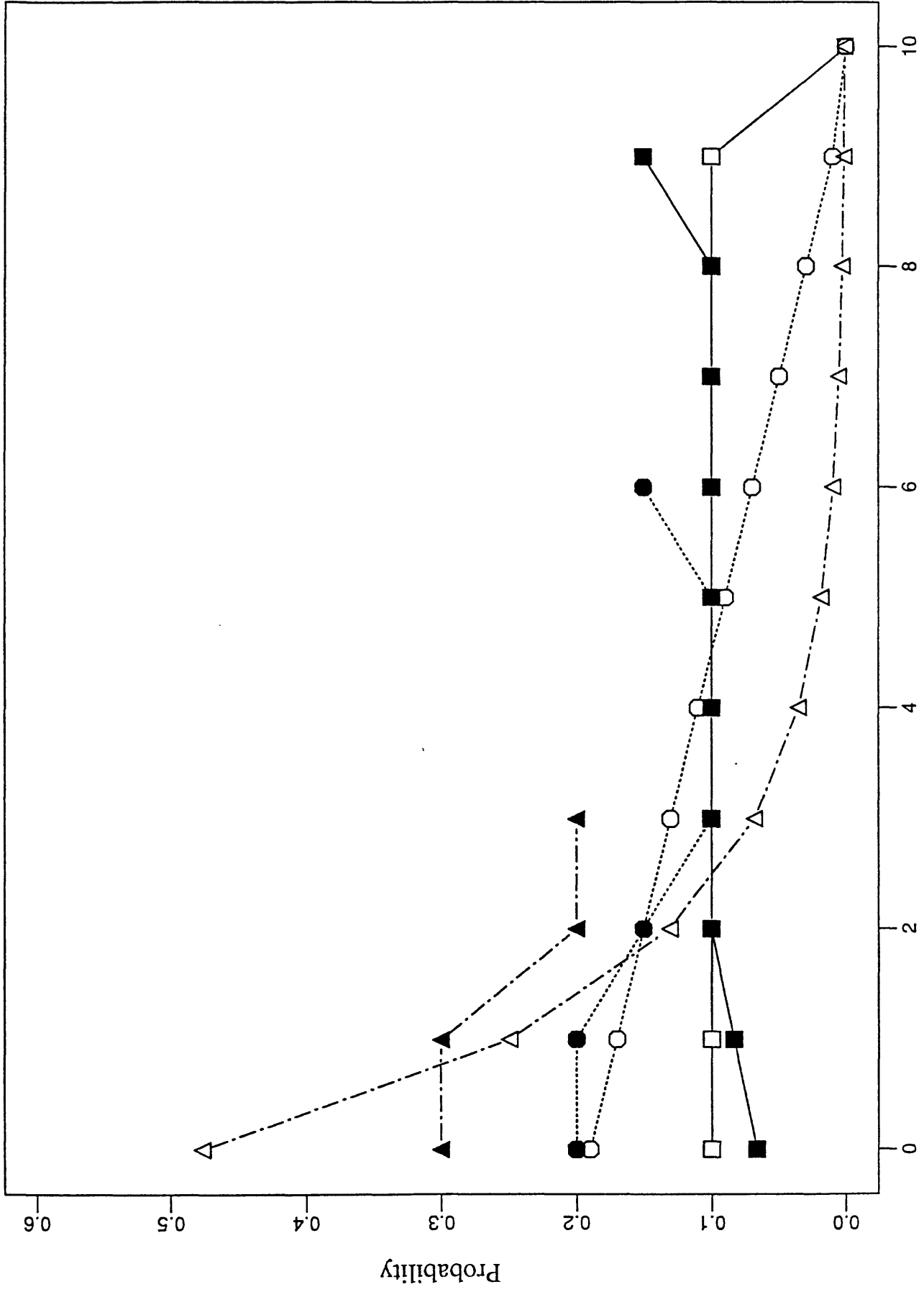


Figure 4



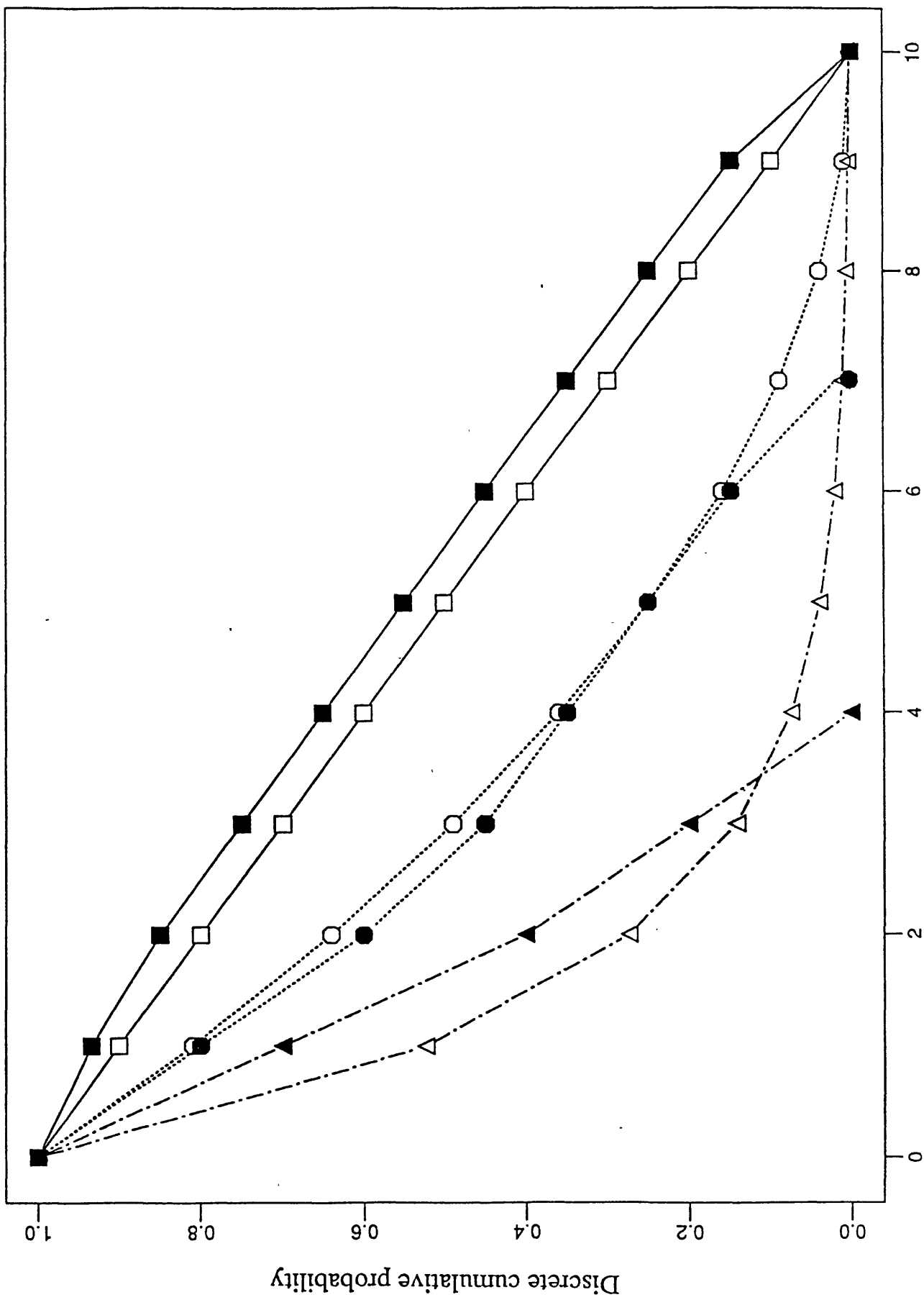
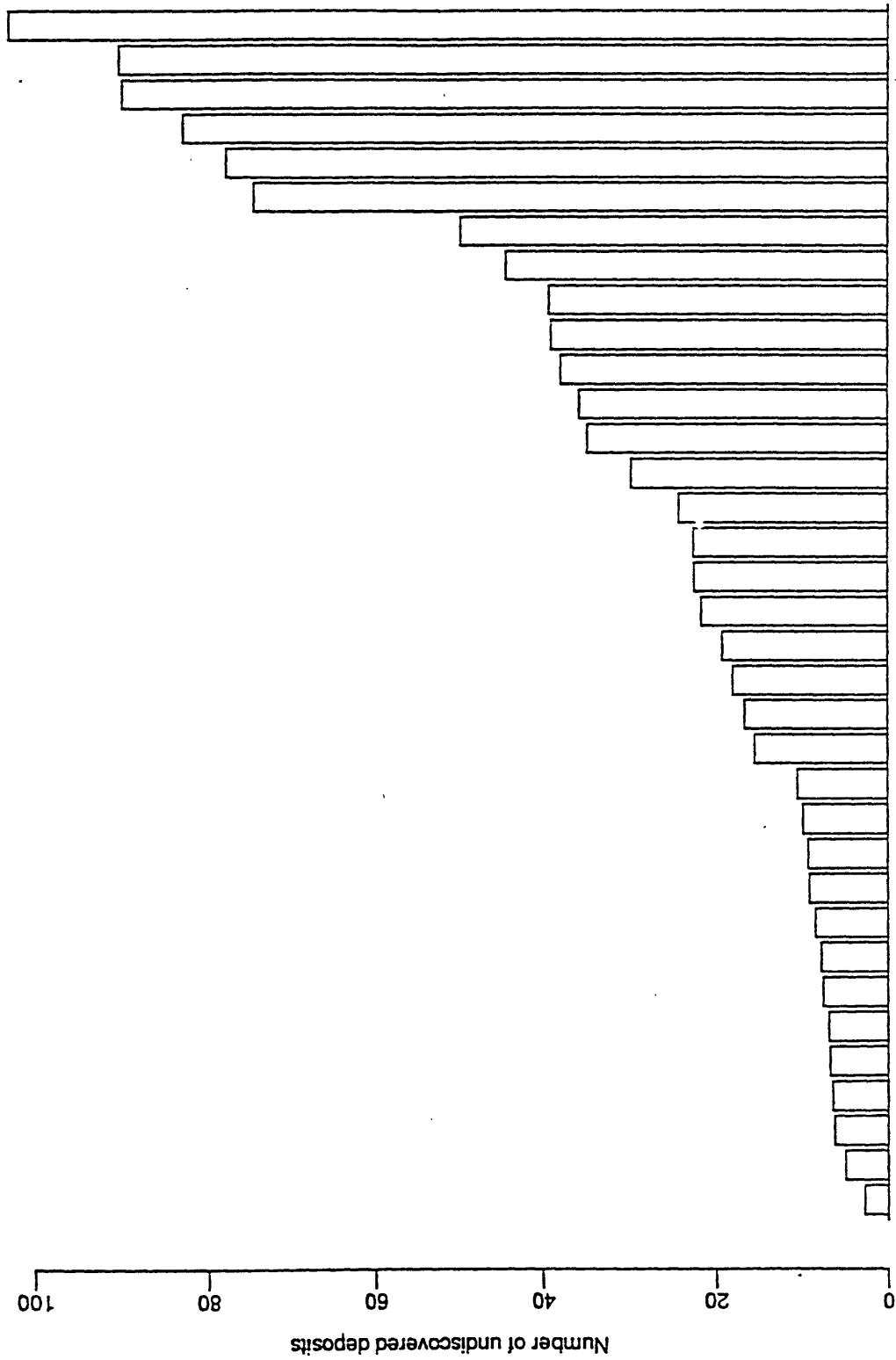


Figure 5

Figure 6.



Sorted estimates by each geologist

Figure 7.

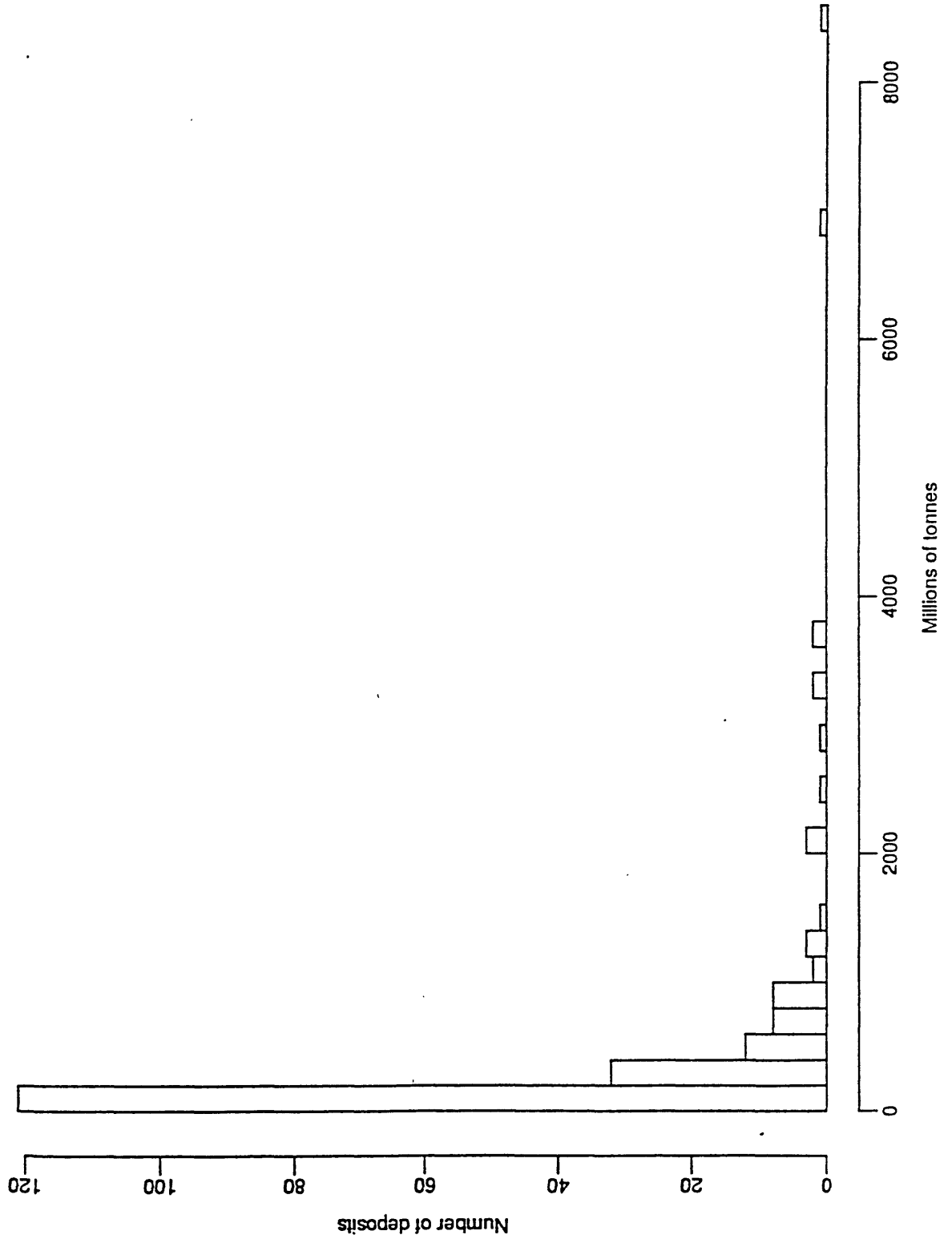


Figure 8.

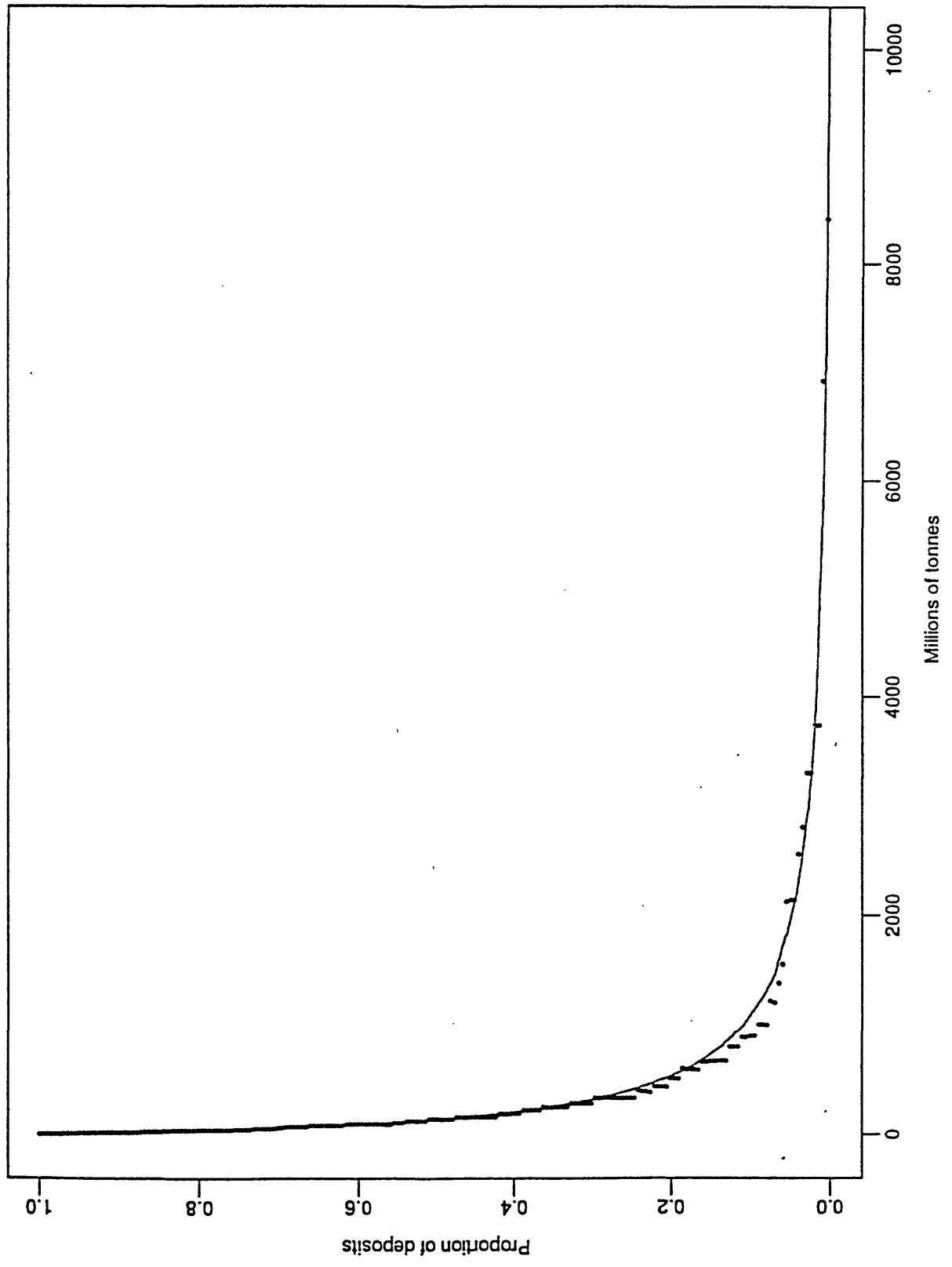


Figure 9.

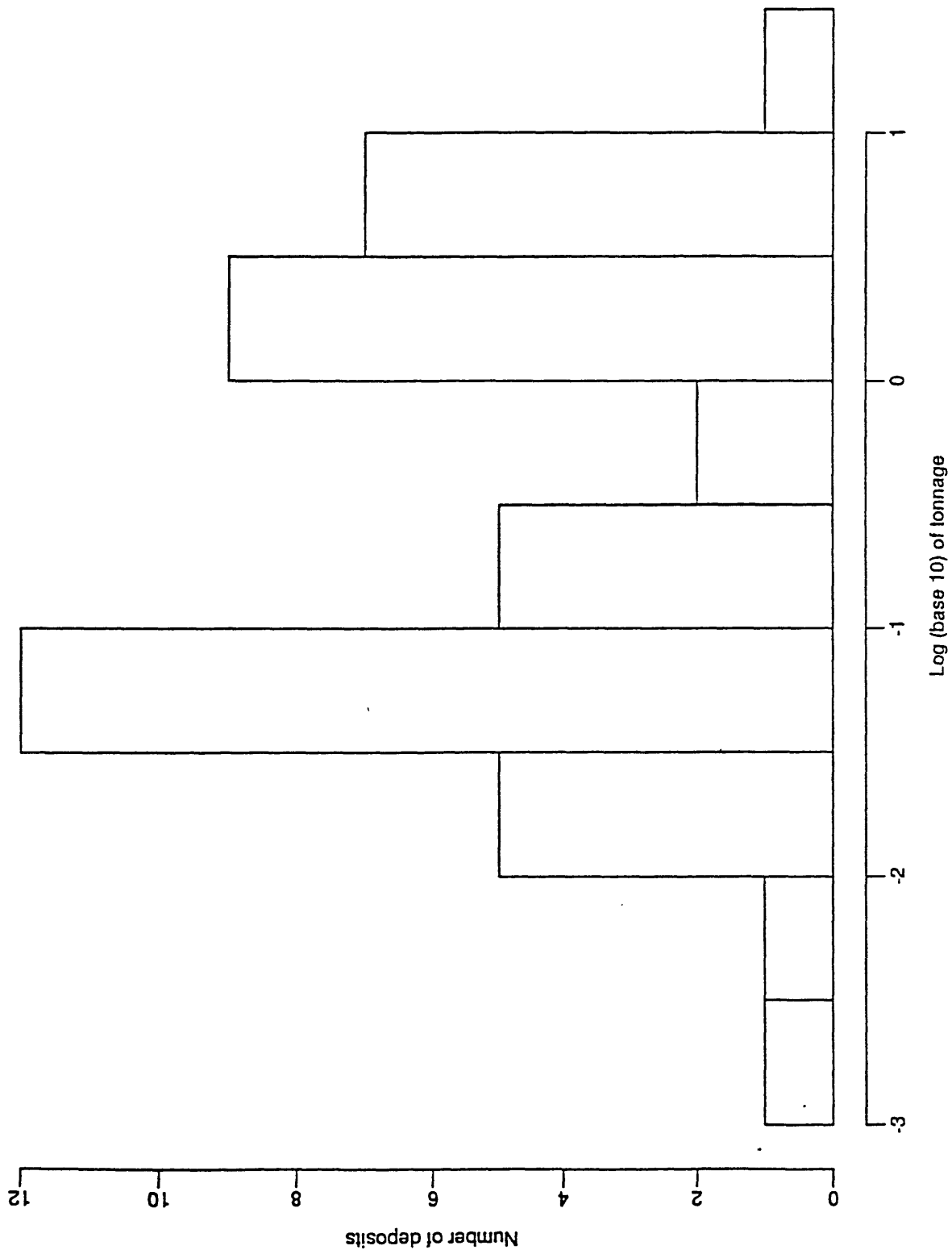


Figure 10.

