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# A Statistical Representation of Results of the U.S. Geological Survey's 1995 National Assessment of Conventional Oil and Gas Resources

by

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This report is preliminary and has not been reviewed for conformity with U.S. Geological Survey editorial standards and stratigraphic nomenclature.

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## ABSTRACT

The 20/80 law is a heuristic law that has evolved over the years into the following rule of thumb for many populations: 20% of the population accounts for 80% of the total value. This law states quantitatively that often a relatively small portion of a population accounts for most of the total value of the population. This principle is certainly applicable in the case of conventional oil and nonassociated-gas resources in the United States where the few largest plays have an overwhelming amount of the total resources.

The general $p100/q100$ law in statistical form is defined with the statistic $q$ as a function of $p$ where $p$ is the population proportion and $q$ is the proportion of total value. The general $p100/q100$ law in probabilistic form is defined with the parameter $q$ as a function of $p$ for any probability distribution that models the population distribution. Using the lognormal distribution, the $p100/q100$ law in lognormal form is derived with the lognormal $q$ being a fractal, where $q$ possesses the scale invariance property.

The $p100/q100$ law in lognormal form was applied to data on technically recoverable resources in oil and nonassociated-gas (conventional) plays in the onshore areas and adjoining State waters of the United States. These data were generated as part of the U.S. Geological Survey's 1995 National Assessment of United States oil and gas resources. The theoretical percentages of total resources using the lognormal $q$ are extremely close to the empirical percentages from the oil and nonassociated-gas data using the statistic $q$. For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal $q$, or for 75.52% if we use the statistic $q$; 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal $q$, or for 78.87% if we use the statistic $q$.

## INTRODUCTION

Vilfredo Pareto (1848-1923), an Italian engineer turned economist and sociologist, was a pioneer in econometrics, applying the statistical methods of the physical sciences to the social sciences. Pareto discovered that, generally, a relatively small portion of a population accounts for most of the total value of the population. That is, in any set of elements, the critical elements usually constitute a minority of the set (Douglass and Douglass, 1993). Pareto's principle is certainly applicable in the case of technically recoverable petroleum resources in oil and nonassociated-gas (conventional) plays in the United States, where the few largest plays have an overwhelming amount of the total resources (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995).

As originally proposed, Pareto's law (Pareto, 1897; Turla and Hawkins, 1983) was an empirical relation describing the distribution of income among the population of a country; viz.,

20 percent of the people in Italy owned 80 percent of the wealth.

From this principle, the heuristic "20/80 law" has evolved over the years into the following simple rule of thumb, which is applicable to many populations:

20% of the population accounts for 80% of the total value.

Numerous applications of this law exist in the management sciences. For example,

20% of the customers account for 80% of the sales;

20% of the employees account for 80% of the work; and,

20% of the components account for 80% of the cost.

The principle is observed in other disciplines as well. The objective of the present paper is to generalize "the 20/80 law" using the lognormal distribution and apply the generalization to data on oil and nonassociated-gas resources to obtain for each: $p100\%$ of the plays account for $q100\%$ ($q$ is a function of $p$) of the total resources of the plays. Fractal lognormal percentage theory is developed first; the oil data are analyzed next; and, finally, the theory is applied to the nonassociated-gas data.

## FRACTAL LOGNORMAL PERCENTAGE THEORY

The material of this section was first presented in Crovelli (1995) and is reproduced here for completeness.

### THE $p100/q100$ LAW IN STATISTICAL FORM

Consider a population of $N$ values for a random variable $X$ arranged in increasing order of magnitude:

$$x_1, x_2, \ldots, x_N.$$

The total value of the population is:

$$\sum_{i=1}^{N} x_i .$$

Given a proportion $p$ $(0 \le p \le 1)$ of largest values of the population, the partial sum of the $p100\%$ of largest values of the population is:

$$\sum_{i=[(1-p)N]+1}^{N} x_i \ ,$$

where $[\cdot]$ is the greatest integer value function.

Let the proportion of the total value of the population accounted for by $p100\%$ of the population be given by $q$ $(0 \le q \le 1)$, where the statistic $q$ is defined as:

$$q = \sum_{i=[(1-p)N]+1}^{N} x_i \Big/ \sum_{i=1}^{N} x_i .$$

An alternative form is:

$$q = \frac{p \sum_{i=[(1-p)N]+1}^{N} x_i \Big/ pN}{\sum_{i=1}^{N} x_i \Big/ N} .$$

Summarizing, $p$ is a proportion (or fraction) of the population, and $p100\%$ is a percentage of the population; whereas, $q$ is a proportion (or fraction) of the total value, $q100\%$ is a percentage of the total value, and $p100\%$ of the population accounts for $q100\%$ of the total value.

## THE $p100/q100$ LAW IN PROBABILISTIC FORM

Given a population of values of a random variable $X$ having a probability distribution that models the population distribution, then $p100\%$ of the population values account for $q100\%$ of the total value, where the parameter $q$ is called the proportion of total value and defined in terms of conditional expectation, $E(X|\cdot)$, as follows.

$$q = \frac{pE(X \mid X > x_p)}{E(X)}, \quad 0 \le p \le 1,$$

with $p = P(X > x_p)$ and $x_p$ is called the $p100$th fractile.

Various probability distributions could be used as models for the population distribution. We will consider the lognormal probability distribution. The percentage theory based on the Pareto distribution has been developed and applied to petroleum field size data (Crovelli, 1995).

## THE LOGNORMAL PROBABILITY DISTRIBUTION

The lognormal distribution is discussed in detail in Aitchison and Brown (1957), Johnson and others (1994), and Crow and Shimizu (1988). A nonnegative random variable $X$ has a lognormal distribution if the random variable $Y = \ln X$ has a normal distribution.

The lognormal probability density function of $X$ is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma x} \exp \frac{-(\ln x - \mu)^2}{2\sigma^2}, \qquad x > 0$$

where $\sigma > 0$ is a shape parameter and $\mu$ is a scale parameter (Law and Kelton, 1991, p. 337).

The expected value or mean of $X$ is

$$E(X) = e^{\mu + \sigma^2/2}$$

The variance of $X$ is

$$V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

The scale parameter $\mu$ and the shape parameter $\sigma$ of the lognormal distribution are computed from the following formulas:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} \ln x_i$$

and

$$\sigma = \left[ \frac{N\sum_{i=1}^{N} (\ln x_i)^2 - \left( \sum_{i=1}^{N} \ln x_i \right)^2}{N^2} \right]^{1/2}.$$

The following notation will be used throughout the remainder of the paper:

$X \sim$ lognormal (shape parameter $\sigma$, scale parameter $\mu$),

where the symbol ~ is read "is distributed as."

## THE LOGNORMAL DISTRIBUTION HAS A FRACTAL PROPERTY

If $X$ ~ lognormal (shape parameter $\sigma$, scale parameter $\mu$), let

$$X' = cX, \qquad c > 0.$$

The fractal property of $X$ under a positive multiplicative constant results in:

$X'$ ~ lognormal (shape parameter $\sigma$, scale parameter $\mu + \ln c$ ).

Note that the distributions of $X$ and $X'$ are identical (with the same shape parameter) except for scale. A probabilistic fractal is a probability distribution that is invariant except for scale (scale invariant or self-similar) under a given transformation (Mandelbrot, 1983, p. 343). Under a positive multiplicative constant, the lognormal distribution remains a lognormal distribution with fixed $\sigma$ and changed $\mu$.

## THE *p*100/*q*100 LAW IN LOGNORMAL FORM

Consider the $p100/q100$ law in probabilistic form when the population distribution is modeled as a lognormal distribution; i.e.,

$$q = \frac{pE(X|X > x_p)}{E(X)}, \quad 0 \le p \le 1$$

when $X$ ~ lognormal (shape parameter $\sigma$, scale parameter $\mu$).

Recall that the expected value of $X$ is

$$E(X) = e^{\mu+\sigma^2/2}.$$

It can be shown (Johnson and others, 1994) that the conditional expectation is given by

$$E(X|X > x_p) = e^{\mu+\sigma^2/2} \frac{R((\ln x_p - \mu - \sigma^2)/\sigma)}{R((\ln x_p - \mu)/\sigma)}$$

where $R$ denotes the complementary cumulative distribution function of a standard normal random variable $Z$, i.e., $R(z) \equiv P(Z > z)$.

We define the following notation:

$$R(z_p) = P(Z > z_p) = p$$

where $z_p$ is called the $p100$th fractile of $Z$.

Because

$$R((\ln x_p - \mu)/\sigma) = R(z_p) = p$$

and

$$R((\ln x_p - \mu - \sigma^2)/\sigma) = R(z_p - \sigma)$$

we have

$$E(X \mid X > x_p) = e^{\mu+\sigma^2/2} \frac{R(z_p - \sigma)}{p}$$

Substituting into the general formula for $q$ and simplifying, we get

$$q = R(z_p - \sigma), \quad 0 < p < 1$$

or

$$q = 1 - \Phi(z_p - \sigma), \quad 0 < p < 1$$

where $\Phi$ denotes the standard normal cumulative distribution function.

Observe that if $p \to 0$, then $q \to 0$, and if $p \to 1$, then $q \to 1$.Thus we define $q = 0$ if $p = 0$, and $q = 1$ if $p = 1$.The special case when $p = 0.5$ and $z_p = 0$ yields

$$q = R(-\sigma) = 1 - \Phi(-\sigma)$$

The parameter $q$ has the very remarkable property of being scale-free; i.e., $q$ does not depend upon the scale parameter $\mu$. Because the parameter $q$ is scale-free, $q$ stays the same for all lognormal distributions with the same shape parameter $\sigma$, as in the case of the fractal property of the lognormal distribution under a positive multiplicative constant. Because it is scale invariant, the proportion $q$ can be considered to be a parametric fractal.

The proportion $q$ of the $p100/q100$ law in statistical form is called the statistic $q$, in probabilistic form it is called the parameter $q$, and in lognormal form it is called the lognormal $q$.

## OIL AND NONASSOCIATED-GAS DATA

The U.S. Geological Survey periodically makes appraisals of the oil and gas resources of the Nation. In its 1995 National Assessment (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995), the onshore areas and adjoining State waters of the Nation were assessed. The basic assessment unit was the play.

A conventional accumulation of the 1995 National Assessment is one in which oil or gas is trapped as a discrete deposit (field) that is usually bounded by a downdip water contact. A conventional play is a set of known and (or) postulated conventional accumulations sharing similar geologic, geographic, and temporal properties. Conventional plays were assessed as described by Gautier and Dolton (1995).

For purposes of this report, terms such as oil resources, oil data, and oil plays refer to conventional oil plays of the 1995 National Assessment; terms involving nonassociated-gas refer to conventional nonassociated-gas plays of the 1995 National Assessment. An accumulation with a gas-oil ratio in excess of 20,000 is considered a nonassociated-gas accumulation.

As part of the 1995 National Assessment, 274 conventional oil plays were assessed. Estimates of mean, undiscovered, technically recoverable resources for these plays total 30.25 billion barrels of oil (U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995). The mean, undiscovered, technically recoverable oil resources estimated for each of the 274 conventional oil plays constitute the data set used in the following section on Application of Theory to Conventional Oil Resources.

Estimates of mean, undiscovered, technically recoverable resources for the 239 conventional nonassociated-gas plays assessed as part of the 1995 National Assessment total 258.69 trillion cubic feet of gas (U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995). The mean, undiscovered, technically recoverable gas resources estimated for each of the 239 conventional nonassociated-gas plays constitute the data set used in the section on Application of Theory to Conventional Nonassociated-Gas Resources.

## APPLICATION OF THEORY TO CONVENTIONAL OIL RESOURCES

The fractal lognormal percentage theory is applied in this section to the oil data. This new probabilistic methodology for percentage assessment lends itself as an ideal spreadsheet software application.

A frequency distribution of the 274 oil plays (in million barrels mean oil) is displayed in Figure 1. Because the plotted random variable is the logarithm of mean oil, Ln (million barrels mean oil), the bell-shaped distribution suggests that the oil resources are approximately distributed as a lognormal distribution.

Spreadsheet software was used to make various calculations involving the oil data. Table 1 is a spreadsheet with 274 rows corresponding to the 274 oil plays. Each oil play of Table 1 is identified by the play number that was used in the 1995 National Assessment (Gautier and others, 1995). The data are listed and several calculations are made, including those necessary for the statistic $q$. The lognormal parameters were computed: scale parameter $\mu = 3.3669$ and shape parameter $\sigma = 1.4559$.

Table 2 is a spreadsheet of the calculations from the oil data that are necessary for the lognormal $q$ as a function of various values of $p$. The formula for the lognormal $q$ in the case of the oil data is the following:

$$q = R(z_p - 1.4559) = 1 - \Phi(z_p - 1.4559), \quad 0 < p < 1$$

Therefore, for any specified value of $p$, we can obtain: $p100\%$ of the oil plays account for $q100\%$ of the total oil resources of the plays. The fractal lognormal percentage assessment of conventional oil resources is summarized in Table 3. The corresponding graph of the summary is given in Figure 2.

Note that in Table 3 and Figure 2, the theoretical percentages of total oil resources using the lognormal $q$ are extremely close to the empirical percentages from the oil data using the statistic $q$. For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal $q$, or for 75.52% if we use the statistic $q$. Plays presumably vary in their degree of heterogeneity, and this can be quantitatively captured by this method. The method also forms a basis for comparing plays of different geologic styles, and similar geologic styles.

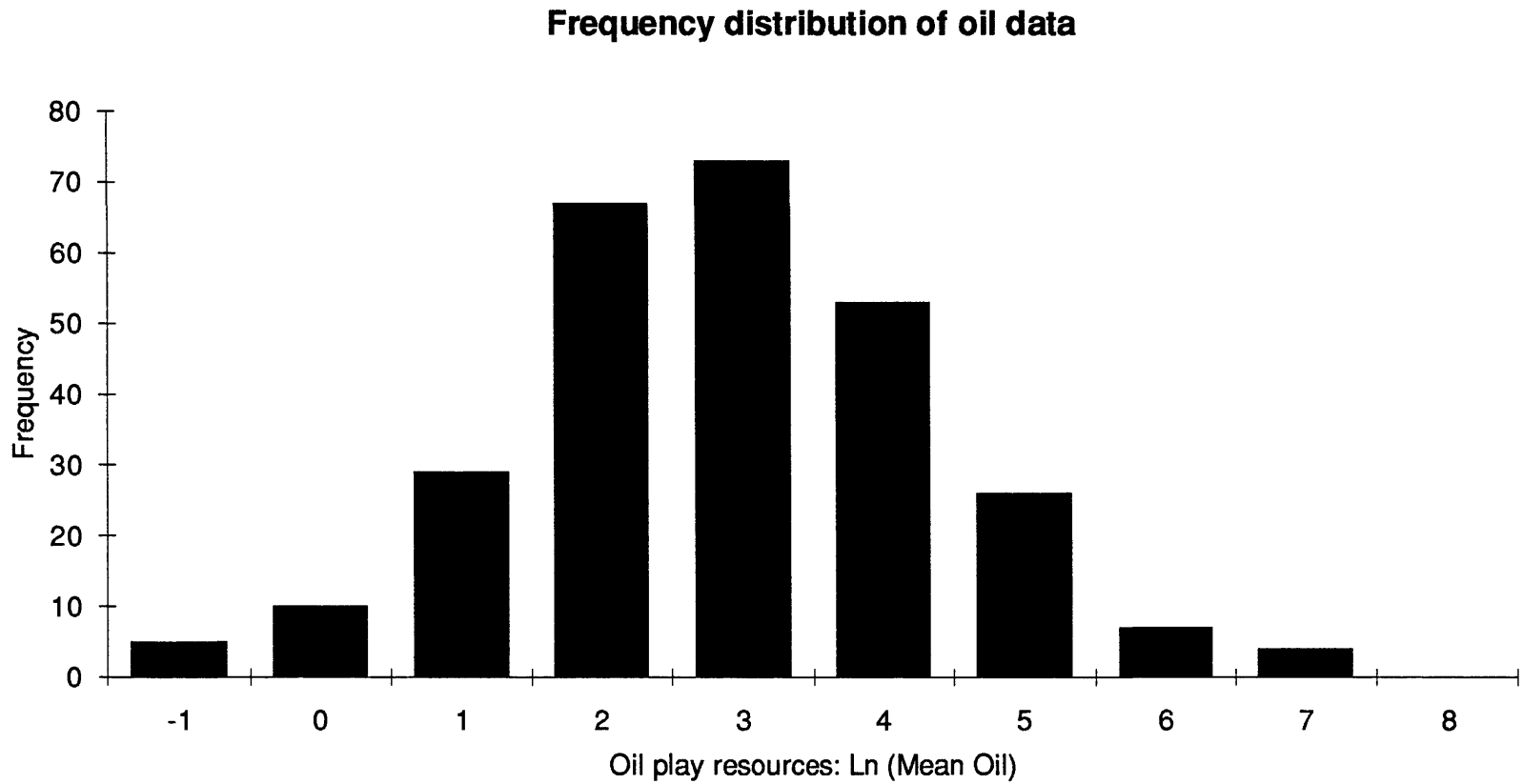


Figure 1. Frequency distribution of 274 oil plays (in million barrels mean oil). Data and bin frequencies are included with Table 1.

Table 1. Spreadsheet of oil data and calculations for the statistic *q*.

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 1 | 101 | 1564.4 | 7.3553 | 1592.8 | 1592.800 | 0.06645 |
| 2 | 102 | 1254.3 | 7.1343 | 1564.4 | 3157.200 | 0.13172 |
| 3 | 103 | 1489.2 | 7.3060 | 1489.2 | 4646.400 | 0.19385 |
| 4 | 104 | 493.7 | 6.2019 | 1254.3 | 5900.700 | 0.24619 |
| 5 | 109 | 617.3 | 6.4254 | 809.2 | 6709.900 | 0.27995 |
| 6 | 110 | 226.9 | 5.4245 | 646.8 | 7356.700 | 0.30693 |
| 7 | 111 | 1592.8 | 7.3732 | 617.3 | 7974.000 | 0.33269 |
| 8 | 205 | 61.2 | 4.1141 | 583 | 8557.000 | 0.35701 |
| 9 | 301 | 52.1 | 3.9532 | 522.3 | 9079.300 | 0.37880 |
| 10 | 302 | 9.1 | 2.2083 | 507 | 9586.300 | 0.39995 |
| 11 | 304 | 646.8 | 6.4720 | 493.7 | 10080.000 | 0.42055 |
| 12 | 308 | 172.6 | 5.1510 | 402.3 | 10482.300 | 0.43734 |
| 13 | 309 | 57.2 | 4.0466 | 321.2 | 10803.500 | 0.45074 |
| 14 | 405 | 18.2 | 2.9014 | 298.9 | 11102.400 | 0.46321 |
| 15 | 406 | 1.1 | 0.0953 | 296.1 | 11398.500 | 0.47556 |
| 16 | 703 | 20.9 | 3.0397 | 264.9 | 11663.400 | 0.48661 |
| 17 | 801 | 3.7 | 1.3083 | 253.7 | 11917.100 | 0.49720 |
| 18 | 1002 | 85.2 | 4.4450 | 252.3 | 12169.400 | 0.50772 |
| 19 | 1003 | 62 | 4.1271 | 251.1 | 12420.500 | 0.51820 |
| 20 | 1004 | 583 | 6.3682 | 249.6 | 12670.100 | 0.52861 |
| 21 | 1005 | 238.8 | 5.4756 | 248.9 | 12919.000 | 0.53900 |
| 22 | 1006 | 11.7 | 2.4596 | 238.8 | 13157.800 | 0.54896 |
| 23 | 1008 | 35.9 | 3.5807 | 230.5 | 13388.300 | 0.55858 |
| 24 | 1009 | 9.7 | 2.2721 | 229.1 | 13617.400 | 0.56814 |
| 25 | 1010 | 50 | 3.9120 | 226.9 | 13844.300 | 0.57760 |
| 26 | 1101 | 8.8 | 2.1748 | 223.6 | 14067.900 | 0.58693 |
| 27 | 1102 | 21 | 3.0445 | 214.2 | 14282.100 | 0.59587 |
| 28 | 1103 | 6.6 | 1.8871 | 213 | 14495.100 | 0.60476 |
| 29 | 1104 | 52.4 | 3.9589 | 203.6 | 14698.700 | 0.61325 |
| 30 | 1105 | 14.3 | 2.6603 | 194.6 | 14893.300 | 0.62137 |
| 31 | 1106 | 223.6 | 5.4099 | 194.6 | 15087.900 | 0.62949 |
| 32 | 1107 | 59 | 4.0775 | 192.4 | 15280.300 | 0.63752 |
| 33 | 1109 | 6.6 | 1.8871 | 182.1 | 15462.400 | 0.64511 |
| 34 | 1201 | 33.8 | 3.5205 | 172.6 | 15635.000 | 0.65231 |
| 35 | 1202 | 19.4 | 2.9653 | 168.5 | 15803.500 | 0.65934 |
| 36 | 1204 | 16.3 | 2.7912 | 152.4 | 15955.900 | 0.66570 |
| 37 | 1211 | 103.9 | 4.6434 | 152 | 16107.900 | 0.67204 |
| 38 | 1301 | 126.2 | 4.8379 | 142.8 | 16250.700 | 0.67800 |
| 39 | 1302 | 251.1 | 5.5259 | 142 | 16392.700 | 0.68393 |
| 40 | 1311 | 296.1 | 5.6907 | 132 | 16524.700 | 0.68943 |
| 41 | 1312 | 249.6 | 5.5199 | 127.2 | 16651.900 | 0.69474 |
| 42 | 1401 | 214.2 | 5.3669 | 126.2 | 16778.100 | 0.70001 |
| 43 | 1402 | 142.8 | 4.9614 | 126.2 | 16904.300 | 0.70527 |
| 44 | 1403 | 264.9 | 5.5794 | 124.7 | 17029.000 | 0.71047 |
| 45 | 1404 | 89.8 | 4.4976 | 109.3 | 17138.300 | 0.71503 |
| 46 | 1405 | 109.3 | 4.6941 | 106.6 | 17244.900 | 0.71948 |
| 47 | 1406 | 30.8 | 3.4275 | 106.4 | 17351.300 | 0.72392 |
| 48 | 1407 | 18.3 | 2.9069 | 103.9 | 17455.200 | 0.72826 |

| *Bin* | *Frequency* |
|---|---|
| –1 | 5 |
| 0 | 10 |
| 1 | 29 |
| 2 | 67 |
| 3 | 73 |
| 4 | 53 |
| 5 | 26 |
| 6 | 7 |
| 7 | 4 |
| 8 | 0 |

Table 1. (cont.)

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 49 | 1701 | 0.9 | –0.1054 | 103.8 | 17559.000 | 0.73259 |
| 50 | 1803 | 0.6 | –0.5108 | 103.5 | 17662.500 | 0.73690 |
| 51 | 1901 | 321.2 | 5.7721 | 95 | 17757.500 | 0.74087 |
| 52 | 1902 | 15.4 | 2.7344 | 92.6 | 17850.100 | 0.74473 |
| 53 | 1903 | 24 | 3.1781 | 90.7 | 17940.800 | 0.74852 |
| 54 | 1905 | 4.5 | 1.5041 | 89.8 | 18030.600 | 0.75226 |
| 55 | 1906 | 6.7 | 1.9021 | 87.7 | 18118.300 | 0.75592 |
| 56 | 1907 | 11 | 2.3979 | 86.9 | 18205.200 | 0.75955 |
| 57 | 2002 | 40.5 | 3.7013 | 85.2 | 18290.400 | 0.76310 |
| 58 | 2004 | 3.4 | 1.2238 | 83.7 | 18374.100 | 0.76659 |
| 59 | 2005 | 142 | 4.9558 | 83.4 | 18457.500 | 0.77007 |
| 60 | 2014 | 2.7 | 0.9933 | 81.7 | 18539.200 | 0.77348 |
| 61 | 2101 | 40.1 | 3.6914 | 81.4 | 18620.600 | 0.77688 |
| 62 | 2102 | 152 | 5.0239 | 80.2 | 18700.800 | 0.78022 |
| 63 | 2104 | 2.3 | 0.8329 | 79.2 | 18780.000 | 0.78353 |
| 64 | 2105 | 18.1 | 2.8959 | 75 | 18855.000 | 0.78666 |
| 65 | 2106 | 20.8 | 3.0350 | 75 | 18930.000 | 0.78979 |
| 66 | 2204 | 21.3 | 3.0587 | 71.3 | 19001.300 | 0.79276 |
| 67 | 2206 | 30.5 | 3.4177 | 70.6 | 19071.900 | 0.79571 |
| 68 | 2207 | 31.4 | 3.4468 | 67.8 | 19139.700 | 0.79854 |
| 69 | 2210 | 7.8 | 2.0541 | 66.7 | 19206.400 | 0.80132 |
| 70 | 2301 | 7 | 1.9459 | 66.2 | 19272.600 | 0.80408 |
| 71 | 2302 | 2.4 | 0.8755 | 63.2 | 19335.800 | 0.80672 |
| 72 | 2305 | 22.5 | 3.1135 | 63.1 | 19398.900 | 0.80935 |
| 73 | 2402 | 9 | 2.1972 | 63.1 | 19462.000 | 0.81198 |
| 74 | 2403 | 33.5 | 3.5115 | 62.4 | 19524.400 | 0.81459 |
| 75 | 2404 | 2.7 | 0.9933 | 62 | 19586.400 | 0.81717 |
| 76 | 2504 | 18.7 | 2.9285 | 61.4 | 19647.800 | 0.81973 |
| 77 | 2705 | 1.1 | 0.0953 | 61.2 | 19709.000 | 0.82229 |
| 78 | 2706 | 0.9 | –0.1054 | 60.8 | 19769.800 | 0.82482 |
| 79 | 2707 | 1.3 | 0.2624 | 59.1 | 19828.900 | 0.82729 |
| 80 | 2802 | 1.4 | 0.3365 | 59 | 19887.900 | 0.82975 |
| 81 | 2803 | 3.7 | 1.3083 | 58.6 | 19946.500 | 0.83220 |
| 82 | 2805 | 57.8 | 4.0570 | 58.1 | 20004.600 | 0.83462 |
| 83 | 2806 | 43.7 | 3.7773 | 58.1 | 20062.700 | 0.83704 |
| 84 | 2807 | 11.4 | 2.4336 | 58 | 20120.700 | 0.83946 |
| 85 | 2808 | 40 | 3.6889 | 57.8 | 20178.500 | 0.84188 |
| 86 | 2903 | 3.1 | 1.1314 | 57.8 | 20236.300 | 0.84429 |
| 87 | 2904 | 5.1 | 1.6292 | 57.2 | 20293.500 | 0.84667 |
| 88 | 2906 | 2.7 | 0.9933 | 56.1 | 20349.600 | 0.84901 |
| 89 | 2907 | 0.8 | –0.2231 | 55.3 | 20404.900 | 0.85132 |
| 90 | 2910 | 10.8 | 2.3795 | 55.1 | 20460.000 | 0.85362 |
| 91 | 3101 | 168.5 | 5.1269 | 53.8 | 20513.800 | 0.85586 |
| 92 | 3102 | 53.5 | 3.9797 | 53.5 | 20567.300 | 0.85810 |
| 93 | 3103 | 55.3 | 4.0128 | 52.4 | 20619.700 | 0.86028 |
| 94 | 3105 | 49.3 | 3.8979 | 52.1 | 20671.800 | 0.86246 |
| 95 | 3106 | 17.7 | 2.8736 | 50.6 | 20722.400 | 0.86457 |
| 96 | 3301 | 20.1 | 3.0007 | 50 | 20772.400 | 0.86665 |
| 97 | 3302 | 6.8 | 1.9169 | 49.5 | 20821.900 | 0.86872 |

Table 1. (cont.)

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 98 | 3303 | 81.4 | 4.3994 | 49.3 | 20871.200 | 0.87078 |
| 99 | 3304 | 522.3 | 6.2582 | 48.3 | 20919.500 | 0.87279 |
| 100 | 3305 | 53.8 | 3.9853 | 48 | 20967.500 | 0.87479 |
| 101 | 3306 | 192.4 | 5.2596 | 47.7 | 21015.200 | 0.87678 |
| 102 | 3307 | 59.1 | 4.0792 | 46.9 | 21062.100 | 0.87874 |
| 103 | 3309 | 48.3 | 3.8774 | 46.5 | 21108.600 | 0.88068 |
| 104 | 3310 | 21.1 | 3.0493 | 45.4 | 21154.000 | 0.88258 |
| 105 | 3312 | 67.8 | 4.2166 | 43.7 | 21197.700 | 0.88440 |
| 106 | 3313 | 58.1 | 4.0622 | 42.6 | 21240.300 | 0.88618 |
| 107 | 3401 | 22 | 3.0910 | 41.3 | 21281.600 | 0.88790 |
| 108 | 3402 | 24.7 | 3.2068 | 40.5 | 21322.100 | 0.88959 |
| 109 | 3405 | 229.1 | 5.4342 | 40.4 | 21362.500 | 0.89127 |
| 110 | 3406 | 18.1 | 2.8959 | 40.1 | 21402.600 | 0.89295 |
| 111 | 3407 | 28.3 | 3.3429 | 40 | 21442.600 | 0.89462 |
| 112 | 3408 | 6.1 | 1.8083 | 39.3 | 21481.900 | 0.89626 |
| 113 | 3501 | 31.1 | 3.4372 | 39.2 | 21521.100 | 0.89789 |
| 114 | 3502 | 19 | 2.9444 | 38.7 | 21559.800 | 0.89951 |
| 115 | 3503 | 8.3 | 2.1163 | 38.1 | 21597.900 | 0.90110 |
| 116 | 3504 | 58.6 | 4.0707 | 35.9 | 21633.800 | 0.90259 |
| 117 | 3506 | 11.7 | 2.4596 | 34.6 | 21668.400 | 0.90404 |
| 118 | 3603 | 230.5 | 5.4403 | 34.6 | 21703.000 | 0.90548 |
| 119 | 3604 | 248.9 | 5.5171 | 34.3 | 21737.300 | 0.90691 |
| 120 | 3606 | 63.1 | 4.1447 | 33.9 | 21771.200 | 0.90833 |
| 121 | 3607 | 49.5 | 3.9020 | 33.8 | 21805.000 | 0.90974 |
| 122 | 3701 | 41.3 | 3.7209 | 33.5 | 21838.500 | 0.91113 |
| 123 | 3703 | 12.6 | 2.5337 | 32.7 | 21871.200 | 0.91250 |
| 124 | 3704 | 15.8 | 2.7600 | 32.5 | 21903.700 | 0.91385 |
| 125 | 3705 | 18.9 | 2.9392 | 31.5 | 21935.200 | 0.91517 |
| 126 | 3706 | 18.5 | 2.9178 | 31.4 | 21966.600 | 0.91648 |
| 127 | 3707 | 34.6 | 3.5439 | 31.1 | 21997.700 | 0.91778 |
| 128 | 3708 | 6.2 | 1.8245 | 30.8 | 22028.500 | 0.91906 |
| 129 | 3801 | 11.9 | 2.4765 | 30.5 | 22059.000 | 0.92033 |
| 130 | 3802 | 6.5 | 1.8718 | 30.5 | 22089.500 | 0.92161 |
| 131 | 3901 | 7.1 | 1.9601 | 30.5 | 22120.000 | 0.92288 |
| 132 | 3905 | 61.4 | 4.1174 | 30.1 | 22150.100 | 0.92413 |
| 133 | 3907 | 14.5 | 2.6741 | 29.6 | 22179.700 | 0.92537 |
| 134 | 3908 | 34.3 | 3.5351 | 29.2 | 22208.900 | 0.92659 |
| 135 | 4001 | 4.1 | 1.4110 | 29.2 | 22238.100 | 0.92781 |
| 136 | 4004 | 95 | 4.5539 | 28.8 | 22266.900 | 0.92901 |
| 137 | 4005 | 18 | 2.8904 | 28.3 | 22295.200 | 0.93019 |
| 138 | 4301 | 32.7 | 3.4874 | 28.3 | 22323.500 | 0.93137 |
| 139 | 4402 | 126.2 | 4.8379 | 27.3 | 22350.800 | 0.93251 |
| 140 | 4403 | 252.3 | 5.5306 | 27 | 22377.800 | 0.93363 |
| 141 | 4404 | 39.2 | 3.6687 | 26.6 | 22404.400 | 0.93474 |
| 142 | 4405 | 30.5 | 3.4177 | 26.4 | 22430.800 | 0.93584 |
| 143 | 4406 | 507 | 6.2285 | 26.2 | 22457.000 | 0.93694 |
| 144 | 4407 | 80.2 | 4.3845 | 26 | 22483.000 | 0.93802 |
| 145 | 4408 | 8.2 | 2.1041 | 25 | 22508.000 | 0.93907 |
| 146 | 4409 | 63.2 | 4.1463 | 24.8 | 22532.800 | 0.94010 |

Table 1. (cont.)

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 147 | 4410 | 132 | 4.8828 | 24.7 | 22557.500 | 0.94113 |
| 148 | 4411 | 809.2 | 6.6960 | 24 | 22581.500 | 0.94213 |
| 149 | 4412 | 194.6 | 5.2709 | 23.4 | 22604.900 | 0.94311 |
| 150 | 4501 | 26.2 | 3.2658 | 23.4 | 22628.300 | 0.94408 |
| 151 | 4502 | 50.6 | 3.9240 | 22.5 | 22650.800 | 0.94502 |
| 152 | 4504 | 45.4 | 3.8155 | 22 | 22672.800 | 0.94594 |
| 153 | 4505 | 152.4 | 5.0265 | 21.3 | 22694.100 | 0.94683 |
| 154 | 4506 | 106.4 | 4.6672 | 21.1 | 22715.200 | 0.94771 |
| 155 | 4601 | 16.2 | 2.7850 | 21.1 | 22736.300 | 0.94859 |
| 156 | 4701 | 58.1 | 4.0622 | 21 | 22757.300 | 0.94947 |
| 157 | 4704 | 26.6 | 3.2809 | 21 | 22778.300 | 0.95034 |
| 158 | 4705 | 127.2 | 4.8458 | 20.9 | 22799.200 | 0.95122 |
| 159 | 4708 | 213 | 5.3613 | 20.8 | 22820.000 | 0.95208 |
| 160 | 4710 | 298.9 | 5.7001 | 20.8 | 22840.800 | 0.95295 |
| 161 | 4715 | 66.7 | 4.2002 | 20.6 | 22861.400 | 0.95381 |
| 162 | 4716 | 203.6 | 5.3162 | 20.3 | 22881.700 | 0.95466 |
| 163 | 4719 | 42.6 | 3.7519 | 20.1 | 22901.800 | 0.95550 |
| 164 | 4722 | 106.6 | 4.6691 | 19.7 | 22921.500 | 0.95632 |
| 165 | 4724 | 20.3 | 3.0106 | 19.4 | 22940.900 | 0.95713 |
| 166 | 4725 | 20.6 | 3.0253 | 19 | 22959.900 | 0.95792 |
| 167 | 4726 | 32.5 | 3.4812 | 18.9 | 22978.800 | 0.95871 |
| 168 | 4728 | 26.4 | 3.2734 | 18.9 | 22997.700 | 0.95950 |
| 169 | 4733 | 81.7 | 4.4031 | 18.8 | 23016.500 | 0.96028 |
| 170 | 4734 | 34.6 | 3.5439 | 18.7 | 23035.200 | 0.96106 |
| 171 | 4735 | 56.1 | 4.0271 | 18.5 | 23053.700 | 0.96183 |
| 172 | 4737 | 15.4 | 2.7344 | 18.3 | 23072.000 | 0.96260 |
| 173 | 4738 | 30.1 | 3.4045 | 18.3 | 23090.300 | 0.96336 |
| 174 | 4739 | 26 | 3.2581 | 18.2 | 23108.500 | 0.96412 |
| 175 | 4740 | 71.3 | 4.2669 | 18.1 | 23126.600 | 0.96487 |
| 176 | 4742 | 29.2 | 3.3742 | 18.1 | 23144.700 | 0.96563 |
| 177 | 4743 | 31.5 | 3.4500 | 18 | 23162.700 | 0.96638 |
| 178 | 4744 | 103.8 | 4.6425 | 17.7 | 23180.400 | 0.96712 |
| 179 | 4745 | 63.1 | 4.1447 | 16.9 | 23197.300 | 0.96782 |
| 180 | 4901 | 55.1 | 4.0091 | 16.7 | 23214.000 | 0.96852 |
| 181 | 4902 | 25 | 3.2189 | 16.3 | 23230.300 | 0.96920 |
| 182 | 4905 | 83.4 | 4.4236 | 16.3 | 23246.600 | 0.96988 |
| 183 | 4906 | 75 | 4.3175 | 16.2 | 23262.800 | 0.97056 |
| 184 | 4910 | 90.7 | 4.5076 | 15.8 | 23278.600 | 0.97122 |
| 185 | 4911 | 124.7 | 4.8259 | 15.5 | 23294.100 | 0.97186 |
| 186 | 4912 | 103.5 | 4.6396 | 15.4 | 23309.500 | 0.97251 |
| 187 | 4916 | 46.5 | 3.8395 | 15.4 | 23324.900 | 0.97315 |
| 188 | 4917 | 87.7 | 4.4739 | 15.3 | 23340.200 | 0.97379 |
| 189 | 4918 | 57.8 | 4.0570 | 15.3 | 23355.500 | 0.97442 |
| 190 | 4919 | 194.6 | 5.2709 | 15 | 23370.500 | 0.97505 |
| 191 | 4921 | 38.1 | 3.6402 | 14.7 | 23385.200 | 0.97566 |
| 192 | 4925 | 24.8 | 3.2108 | 14.5 | 23399.700 | 0.97627 |
| 193 | 4928 | 28.8 | 3.3604 | 14.3 | 23414.000 | 0.97687 |
| 194 | 4930 | 21.1 | 3.0493 | 13.5 | 23427.500 | 0.97743 |
| 195 | 4932 | 70.6 | 4.2570 | 12.8 | 23440.300 | 0.97796 |

Table 1. (cont.)

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 196 | 4934 | 60.8 | 4.1076 | 12.6 | 23452.900 | 0.97849 |
| 197 | 4936 | 40.4 | 3.6988 | 12.2 | 23465.100 | 0.97900 |
| 198 | 4937 | 79.2 | 4.3720 | 11.9 | 23477.000 | 0.97949 |
| 199 | 4938 | 182.1 | 5.2046 | 11.7 | 23488.700 | 0.97998 |
| 200 | 4942 | 29.6 | 3.3878 | 11.7 | 23500.400 | 0.98047 |
| 201 | 4943 | 18.9 | 2.9392 | 11.5 | 23511.900 | 0.98095 |
| 202 | 4945 | 62.4 | 4.1336 | 11.4 | 23523.300 | 0.98143 |
| 203 | 5001 | 253.7 | 5.5362 | 11.3 | 23534.600 | 0.98190 |
| 204 | 5002 | 12.2 | 2.5014 | 11.2 | 23545.800 | 0.98236 |
| 205 | 5003 | 66.2 | 4.1927 | 11 | 23556.800 | 0.98282 |
| 206 | 5004 | 10.7 | 2.3702 | 10.8 | 23567.600 | 0.98327 |
| 207 | 5005 | 27.3 | 3.3069 | 10.8 | 23578.400 | 0.98372 |
| 208 | 5101 | 46.9 | 3.8480 | 10.8 | 23589.200 | 0.98418 |
| 209 | 5305 | 48 | 3.8712 | 10.7 | 23599.900 | 0.98462 |
| 210 | 5309 | 15 | 2.7081 | 10.1 | 23610.000 | 0.98504 |
| 211 | 5501 | 29.2 | 3.3742 | 9.9 | 23619.900 | 0.98546 |
| 212 | 5503 | 15.3 | 2.7279 | 9.7 | 23629.600 | 0.98586 |
| 213 | 5504 | 18.8 | 2.9339 | 9.6 | 23639.200 | 0.98626 |
| 214 | 5505 | 10.8 | 2.3795 | 9.3 | 23648.500 | 0.98665 |
| 215 | 5507 | 8.6 | 2.1518 | 9.3 | 23657.800 | 0.98704 |
| 216 | 5601 | 5.8 | 1.7579 | 9.1 | 23666.900 | 0.98742 |
| 217 | 5603 | 0.6 | –0.5108 | 9 | 23675.900 | 0.98779 |
| 218 | 5802 | 6.6 | 1.8871 | 8.9 | 23684.800 | 0.98816 |
| 219 | 5803 | 11.3 | 2.4248 | 8.8 | 23693.600 | 0.98853 |
| 220 | 5804 | 20.8 | 3.0350 | 8.6 | 23702.200 | 0.98889 |
| 221 | 5805 | 6 | 1.7918 | 8.5 | 23710.700 | 0.98924 |
| 222 | 5807 | 5.5 | 1.7047 | 8.4 | 23719.100 | 0.98959 |
| 223 | 5809 | 12.8 | 2.5494 | 8.3 | 23727.400 | 0.98994 |
| 224 | 5810 | 19.7 | 2.9806 | 8.3 | 23735.700 | 0.99029 |
| 225 | 5813 | 30.5 | 3.4177 | 8.3 | 23744.000 | 0.99063 |
| 226 | 5814 | 33.9 | 3.5234 | 8.2 | 23752.200 | 0.99098 |
| 227 | 5815 | 13.5 | 2.6027 | 8 | 23760.200 | 0.99131 |
| 228 | 5816 | 75 | 4.3175 | 7.8 | 23768.000 | 0.99163 |
| 229 | 5818 | 8.9 | 2.1861 | 7.7 | 23775.700 | 0.99196 |
| 230 | 5819 | 10.1 | 2.3125 | 7.6 | 23783.300 | 0.99227 |
| 231 | 5820 | 16.9 | 2.8273 | 7.1 | 23790.400 | 0.99257 |
| 232 | 5821 | 23.4 | 3.1527 | 7 | 23797.400 | 0.99286 |
| 233 | 5822 | 6.2 | 1.8245 | 6.9 | 23804.300 | 0.99315 |
| 234 | 5823 | 8.3 | 2.1163 | 6.8 | 23811.100 | 0.99343 |
| 235 | 5824 | 5.1 | 1.6292 | 6.8 | 23817.900 | 0.99372 |
| 236 | 5827 | 15.5 | 2.7408 | 6.7 | 23824.600 | 0.99400 |
| 237 | 5901 | 8 | 2.0794 | 6.7 | 23831.300 | 0.99428 |
| 238 | 5902 | 9.3 | 2.2300 | 6.6 | 23837.900 | 0.99455 |
| 239 | 5903 | 9.3 | 2.2300 | 6.6 | 23844.500 | 0.99483 |
| 240 | 6001 | 14.7 | 2.6878 | 6.6 | 23851.100 | 0.99510 |
| 241 | 6003 | 15.3 | 2.7279 | 6.5 | 23857.600 | 0.99537 |
| 242 | 6004 | 2 | 0.6931 | 6.2 | 23863.800 | 0.99563 |
| 243 | 6005 | 28.3 | 3.3429 | 6.2 | 23870.000 | 0.99589 |
| 244 | 6007 | 6.9 | 1.9315 | 6.1 | 23876.100 | 0.99614 |

Table 1. (cont.)

| *n* | Play No. | Mean Oil (MMBBL) | Ln of Mean Oil | Sorted Mean Oil | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 245 | 6102 | 86.9 | 4.4648 | 6 | 23882.100 | 0.99640 |
| 246 | 6103 | 47.7 | 3.8649 | 5.8 | 23887.900 | 0.99664 |
| 247 | 6104 | 16.7 | 2.8154 | 5.6 | 23893.500 | 0.99687 |
| 248 | 6105 | 5.3 | 1.6677 | 5.5 | 23899.000 | 0.99710 |
| 249 | 6107 | 9.9 | 2.2925 | 5.3 | 23904.300 | 0.99732 |
| 250 | 6109 | 5.6 | 1.7228 | 5.1 | 23909.400 | 0.99753 |
| 251 | 6110 | 8.3 | 2.1163 | 5.1 | 23914.500 | 0.99775 |
| 252 | 6111 | 10.8 | 2.3795 | 4.6 | 23919.100 | 0.99794 |
| 253 | 6201 | 3.4 | 1.2238 | 4.5 | 23923.600 | 0.99813 |
| 254 | 6301 | 16.3 | 2.7912 | 4.1 | 23927.700 | 0.99830 |
| 255 | 6304 | 38.7 | 3.6558 | 3.7 | 23931.400 | 0.99845 |
| 256 | 6306 | 39.3 | 3.6712 | 3.7 | 23935.100 | 0.99861 |
| 257 | 6307 | 83.7 | 4.4272 | 3.4 | 23938.500 | 0.99875 |
| 258 | 6308 | 23.4 | 3.1527 | 3.4 | 23941.900 | 0.99889 |
| 259 | 6309 | 402.3 | 5.9972 | 3.1 | 23945.000 | 0.99902 |
| 260 | 6310 | 18.3 | 2.9069 | 2.7 | 23947.700 | 0.99913 |
| 261 | 6311 | 92.6 | 4.5283 | 2.7 | 23950.400 | 0.99924 |
| 262 | 6313 | 21 | 3.0445 | 2.7 | 23953.100 | 0.99936 |
| 263 | 6401 | 7.7 | 2.0412 | 2.4 | 23955.500 | 0.99946 |
| 264 | 6402 | 11.2 | 2.4159 | 2.3 | 23957.800 | 0.99955 |
| 265 | 6403 | 11.5 | 2.4423 | 2 | 23959.800 | 0.99964 |
| 266 | 6404 | 4.6 | 1.5261 | 1.4 | 23961.200 | 0.99970 |
| 267 | 6501 | 6.8 | 1.9169 | 1.3 | 23962.500 | 0.99975 |
| 268 | 6502 | 7.6 | 2.0281 | 1.1 | 23963.600 | 0.99980 |
| 269 | 6601 | 8.5 | 2.1401 | 1.1 | 23964.700 | 0.99984 |
| 270 | 6602 | 8.4 | 2.1282 | 0.9 | 23965.600 | 0.99988 |
| 271 | 6703 | 58 | 4.0604 | 0.9 | 23966.500 | 0.99992 |
| 272 | 6706 | 27 | 3.2958 | 0.8 | 23967.300 | 0.99995 |
| 273 | 6725 | 9.6 | 2.2618 | 0.6 | 23967.900 | 0.99997 |
| 274 | 6732 | 6.7 | 1.9021 | 0.6 | 23968.500 | 1.00000 |

| | | |
|---|---|---|
| 0.6000 | –0.5108 | <== Min |
| 1592.8000 | 7.3732 | <== Max |
| 23968.5000 | 922.5384 | <== Total |
| 87.4763 | 3.3669 | <== Mean ($\mu$) |
| 202.3034 | 1.4559 | <== Standard deviation ($\sigma$) |

Table 2. Spreadsheet of calculations from oil data of Figure 1 for the lognormal $q$ as a function of various values of $p$. NORMSINV is the spreadsheet function that computes $z$; NORMSDIST is the function that computes $1 - q$.

| $p$ Pop. Prop. | $(p)(N)$ $(p)(274)$ | $1-p$ | $z$ NORMSINV | $z-\sigma$ $(z-1.4559)$ | $1-q$ NORMSDIST | lognormal $q$ (theoretical) | statistic $q$ (empirical) |
|---|---|---|---|---|---|---|---|
| 0.01 | 2.74 | 0.99 | 2.3263 | 0.8704 | 0.8080 | 0.1920 | 0.1777 |
| 0.025 | 6.85 | 0.975 | 1.9600 | 0.5041 | 0.6929 | 0.3071 | 0.3288 |
| 0.05 | 13.7 | 0.95 | 1.6449 | 0.1890 | 0.5749 | 0.4251 | 0.4595 |
| 0.1 | 27.4 | 0.9 | 1.2816 | –0.1743 | 0.4308 | 0.5692 | 0.5994 |
| 0.2 | 54.8 | 0.8 | 0.8416 | –0.6143 | 0.2695 | 0.7305 | 0.7552 |
| 0.3 | 82.2 | 0.7 | 0.5244 | –0.9315 | 0.1758 | 0.8242 | 0.8351 |
| 0.4 | 109.6 | 0.6 | 0.2533 | –1.2026 | 0.1146 | 0.8854 | 0.8923 |
| 0.5 | 137 | 0.5 | 0.0000 | –1.4559 | 0.0727 | 0.9273 | 0.9302 |
| 0.6 | 164.4 | 0.4 | –0.2533 | –1.7092 | 0.0437 | 0.9563 | 0.9566 |
| 0.7 | 191.8 | 0.3 | –0.5244 | –1.9803 | 0.0238 | 0.9762 | 0.9761 |
| 0.8 | 219.2 | 0.2 | –0.8416 | –2.2975 | 0.0108 | 0.9892 | 0.9886 |
| 0.9 | 246.6 | 0.1 | –1.2816 | –2.7375 | 0.0031 | 0.9969 | 0.9968 |

Table 3. Summary of fractal lognormal percentage assessment of conventional oil resources.

| Percentage of oil plays | Percentage of total oil resources $q100\%$ | |
|---|---|---|
| $p100\%$ | Theoretical (lognormal $q$) | Empirical (statistic $q$) |
| 0 | 0.00 | 0.00 |
| 1 | 19.20 | 17.77 |
| 2.5 | 30.71 | 32.88 |
| 5 | 42.51 | 45.95 |
| 10 | 56.92 | 59.94 |
| 20 | 73.05 | 75.52 |
| 30 | 82.42 | 83.51 |
| 40 | 88.54 | 89.23 |
| 50 | 92.73 | 93.02 |
| 60 | 95.63 | 95.66 |
| 70 | 97.62 | 97.61 |
| 80 | 98.92 | 98.86 |
| 90 | 99.69 | 99.68 |
| 100 | 100.00 | 100.00 |

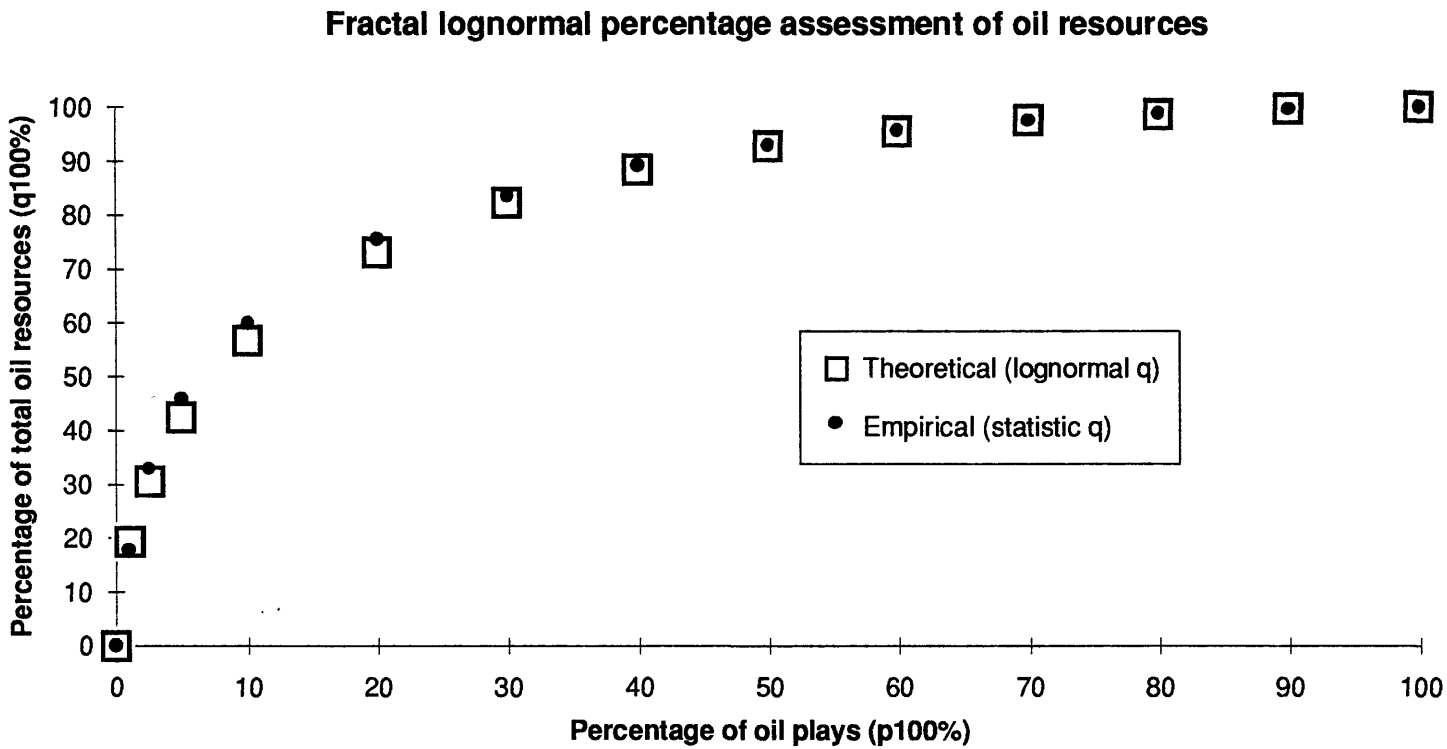


Figure 2. Theoretical percentage of total oil resources using the lognormal $q$ as a function of the percentage of oil plays, along with the corresponding empirical values using the statistic $q$. Data are from Table 3.

## APPLICATION OF THEORY TO CONVENTIONAL NONASSOCIATED-GAS RESOURCES

The fractal lognormal percentage theory is applied here to the nonassociated-gas data. A frequency distribution of the 239 nonassociated-gas plays (in billion cubic feet mean gas) is displayed in Figure 3. Because the plotted random variable is the logarithm of mean gas, Ln (billion cubic feet mean gas), the bell-shaped distribution suggests that the nonassociated-gas resources are approximately distributed as a lognormal distribution.

Table 4 is a spreadsheet with 239 rows corresponding to the 239 nonassociated-gas plays. Each gas play of Table 4 is identified by the play number that was used in the 1995 National Assessment (Gautier and others, 1995). The data are listed and several calculations are made, including those necessary for the statistic $q$. The lognormal parameters were computed: scale parameter $\mu$ = 5.3584 and shape parameter $\sigma$ = 1.5582.

Table 5 is a spreadsheet of the calculations from the nonassociated-gas data that are necessary for the lognormal $q$ as a function of various values of $p$. The formula for the lognormal $q$ in the case of the nonassociated-gas data is the following:

$$q = R(z_p - 1.5582) = 1 - \Phi(z_p - 1.5582), \quad 0 < p < 1$$

The fractal lognormal percentage assessment of conventional nonassociated-gas resources is summarized in Table 6. The corresponding graph of the summary is given in Figure 4.

Note that in Table 6 and Figure 4, the theoretical percentages of total nonassociated-gas resources using the lognormal $q$ are extremely close to the empirical percentages from the nonassociated-gas data using the statistic $q$. For example, 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal $q$, or for 78.87% if we use the statistic $q$.

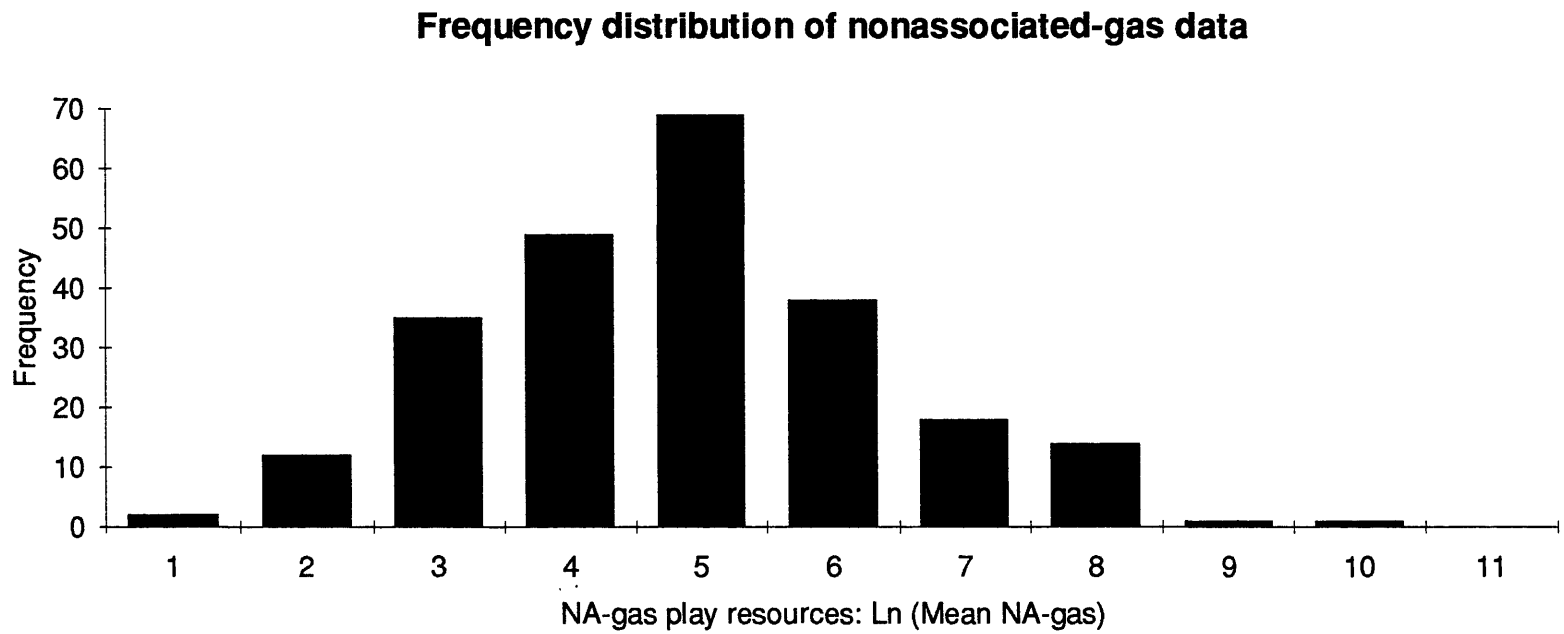


Figure 3. Frequency distribution of 239 nonassociated-gas plays (in billion cubic feet mean gas). Data and bin frequencies are included with Table 4. (NA means nonassociated.)

Table 4. Spreadsheet of nonassociated (NA)-gas data and calculations for the statistic *q*.

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 1 | 101 | 312.60 | 5.7449 | 23739.20 | 23739.200 | 0.13121 |
| 2 | 102 | 4469.50 | 8.4050 | 13296.70 | 37035.900 | 0.20470 |
| 3 | 103 | 2042.40 | 7.6219 | 6315.30 | 43351.200 | 0.23960 |
| 4 | 105 | 5626.90 | 8.6353 | 5709.80 | 49061.000 | 0.27116 |
| 5 | 106 | 3944.10 | 8.2800 | 5626.90 | 54687.900 | 0.30226 |
| 6 | 109 | 13296.70 | 9.4953 | 5259.60 | 59947.500 | 0.33133 |
| 7 | 110 | 1886.80 | 7.5426 | 5164.20 | 65111.700 | 0.35988 |
| 8 | 111 | 23739.20 | 10.0749 | 4469.50 | 69581.200 | 0.38458 |
| 9 | 201 | 2209.80 | 7.7007 | 3944.10 | 73525.300 | 0.40638 |
| 10 | 205 | 116.40 | 4.7570 | 3935.70 | 77461.000 | 0.42813 |
| 11 | 302 | 179.20 | 5.1885 | 3795.90 | 81256.900 | 0.44911 |
| 12 | 303 | 737.90 | 6.6038 | 3203.50 | 84460.400 | 0.46682 |
| 13 | 401 | 30.00 | 3.4012 | 3187.10 | 87647.500 | 0.48443 |
| 14 | 402 | 163.20 | 5.0950 | 3097.70 | 90745.200 | 0.50155 |
| 15 | 403 | 26.80 | 3.2884 | 3053.70 | 93798.900 | 0.51843 |
| 16 | 404 | 15.00 | 2.7081 | 3024.00 | 96822.900 | 0.53514 |
| 17 | 407 | 331.10 | 5.8024 | 2940.40 | 99763.300 | 0.55140 |
| 18 | 410 | 24.40 | 3.1946 | 2905.60 | 102668.900 | 0.56746 |
| 19 | 501 | 235.10 | 5.4600 | 2594.60 | 105263.500 | 0.58180 |
| 20 | 502 | 78.40 | 4.3618 | 2388.70 | 107652.200 | 0.59500 |
| 21 | 701 | 679.60 | 6.5215 | 2307.50 | 109959.700 | 0.60775 |
| 22 | 703 | 125.20 | 4.8299 | 2209.80 | 112169.500 | 0.61997 |
| 23 | 801 | 22.10 | 3.0956 | 2042.40 | 114211.900 | 0.63125 |
| 24 | 901 | 532.00 | 6.2766 | 1886.80 | 116098.700 | 0.64168 |
| 25 | 902 | 386.70 | 5.9576 | 1830.80 | 117929.500 | 0.65180 |
| 26 | 903 | 1209.70 | 7.0981 | 1559.40 | 119488.900 | 0.66042 |
| 27 | 1001 | 85.30 | 4.4462 | 1462.70 | 120951.600 | 0.66851 |
| 28 | 1007 | 174.60 | 5.1625 | 1362.20 | 122313.800 | 0.67603 |
| 29 | 1301 | 211.90 | 5.3561 | 1280.70 | 123594.500 | 0.68311 |
| 30 | 1311 | 487.70 | 6.1897 | 1209.70 | 124804.200 | 0.68980 |
| 31 | 1701 | 11.10 | 2.4069 | 1173.70 | 125977.900 | 0.69629 |
| 32 | 1803 | 4.30 | 1.4586 | 1172.10 | 127150.000 | 0.70276 |
| 33 | 1902 | 12.00 | 2.4849 | 1125.30 | 128275.300 | 0.70898 |
| 34 | 1905 | 30.80 | 3.4275 | 1119.00 | 129394.300 | 0.71517 |
| 35 | 1907 | 136.80 | 4.9185 | 1080.50 | 130474.800 | 0.72114 |
| 36 | 2001 | 1019.10 | 6.9267 | 1068.10 | 131542.900 | 0.72704 |
| 37 | 2002 | 133.40 | 4.8934 | 1066.90 | 132609.800 | 0.73294 |
| 38 | 2003 | 392.80 | 5.9733 | 1029.00 | 133638.800 | 0.73863 |
| 39 | 2004 | 190.30 | 5.2486 | 1019.10 | 134657.900 | 0.74426 |
| 40 | 2014 | 18.30 | 2.9069 | 1017.00 | 135674.900 | 0.74988 |
| 41 | 2101 | 155.20 | 5.0447 | 994.50 | 136669.400 | 0.75538 |
| 42 | 2102 | 102.40 | 4.6289 | 960.00 | 137629.400 | 0.76068 |
| 43 | 2104 | 49.20 | 3.8959 | 947.10 | 138576.500 | 0.76592 |
| 44 | 2105 | 351.10 | 5.8611 | 920.50 | 139497.000 | 0.77101 |

| *Bin* | *Frequency* |
|---|---|
| 1 | 2 |
| 2 | 12 |
| 3 | 35 |
| 4 | 49 |
| 5 | 69 |
| 6 | 38 |
| 7 | 18 |
| 8 | 14 |
| 9 | 1 |
| 10 | 1 |
| 11 | 0 |

Table 4. (cont.)

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 45 | 2107 | 58.30 | 4.0656 | 899.40 | 140396.400 | 0.77598 |
| 46 | 2206 | 29.60 | 3.3878 | 833.50 | 141229.900 | 0.78058 |
| 47 | 2207 | 93.10 | 4.5337 | 820.90 | 142050.800 | 0.78512 |
| 48 | 2212 | 261.60 | 5.5668 | 802.40 | 142853.200 | 0.78956 |
| 49 | 2301 | 197.50 | 5.2857 | 794.30 | 143647.500 | 0.79395 |
| 50 | 2403 | 104.60 | 4.6501 | 783.50 | 144431.000 | 0.79828 |
| 51 | 2504 | 191.40 | 5.2544 | 744.40 | 145175.400 | 0.80239 |
| 52 | 2701 | 1830.80 | 7.5125 | 737.90 | 145913.300 | 0.80647 |
| 53 | 2704 | 42.00 | 3.7377 | 724.30 | 146637.600 | 0.81047 |
| 54 | 2706 | 13.00 | 2.5649 | 679.60 | 147317.200 | 0.81423 |
| 55 | 2808 | 90.90 | 4.5098 | 651.60 | 147968.800 | 0.81783 |
| 56 | 2809 | 400.20 | 5.9920 | 639.70 | 148608.500 | 0.82137 |
| 57 | 2901 | 234.20 | 5.4562 | 573.60 | 149182.100 | 0.82454 |
| 58 | 2903 | 19.30 | 2.9601 | 563.70 | 149745.800 | 0.82765 |
| 59 | 2904 | 10.70 | 2.3702 | 548.40 | 150294.200 | 0.83068 |
| 60 | 2907 | 6.90 | 1.9315 | 539.60 | 150833.800 | 0.83367 |
| 61 | 3102 | 318.00 | 5.7621 | 532.00 | 151365.800 | 0.83661 |
| 62 | 3103 | 70.80 | 4.2599 | 530.50 | 151896.300 | 0.83954 |
| 63 | 3105 | 111.40 | 4.7131 | 517.40 | 152413.700 | 0.84240 |
| 64 | 3107 | 95.40 | 4.5581 | 511.60 | 152925.300 | 0.84523 |
| 65 | 3307 | 271.80 | 5.6051 | 487.70 | 153413.000 | 0.84792 |
| 66 | 3401 | 39.60 | 3.6788 | 473.20 | 153886.200 | 0.85054 |
| 67 | 3402 | 52.20 | 3.9551 | 459.00 | 154345.200 | 0.85307 |
| 68 | 3403 | 126.50 | 4.8402 | 447.00 | 154792.200 | 0.85554 |
| 69 | 3408 | 174.10 | 5.1596 | 444.50 | 155236.700 | 0.85800 |
| 70 | 3501 | 128.70 | 4.8575 | 424.40 | 155661.100 | 0.86035 |
| 71 | 3502 | 36.40 | 3.5946 | 409.70 | 156070.800 | 0.86261 |
| 72 | 3503 | 379.10 | 5.9378 | 407.90 | 156478.700 | 0.86487 |
| 73 | 3504 | 150.90 | 5.0166 | 402.40 | 156881.100 | 0.86709 |
| 74 | 3515 | 55.20 | 4.0110 | 400.20 | 157281.300 | 0.86930 |
| 75 | 3601 | 2905.60 | 7.9744 | 398.30 | 157679.600 | 0.87150 |
| 76 | 3602 | 365.90 | 5.9024 | 397.00 | 158076.600 | 0.87370 |
| 77 | 3603 | 3097.70 | 8.0384 | 395.80 | 158472.400 | 0.87588 |
| 78 | 3604 | 920.50 | 6.8249 | 392.80 | 158865.200 | 0.87806 |
| 79 | 3606 | 444.50 | 6.0970 | 388.60 | 159253.800 | 0.88020 |
| 80 | 3701 | 138.40 | 4.9301 | 387.40 | 159641.200 | 0.88234 |
| 81 | 3702 | 165.90 | 5.1114 | 386.70 | 160027.900 | 0.88448 |
| 82 | 3703 | 16.40 | 2.7973 | 379.10 | 160407.000 | 0.88658 |
| 83 | 3704 | 352.50 | 5.8651 | 370.20 | 160777.200 | 0.88862 |
| 84 | 3705 | 97.50 | 4.5799 | 365.90 | 161143.100 | 0.89065 |
| 85 | 3706 | 69.60 | 4.2428 | 359.50 | 161502.600 | 0.89263 |
| 86 | 3707 | 20.10 | 3.0007 | 352.50 | 161855.100 | 0.89458 |
| 87 | 3708 | 30.00 | 3.4012 | 351.10 | 162206.200 | 0.89652 |
| 88 | 3901 | 49.60 | 3.9040 | 348.20 | 162554.400 | 0.89845 |

Table 4. (cont.)

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 89 | 3903 | 359.50 | 5.8847 | 345.40 | 162899.800 | 0.90036 |
| 90 | 3905 | 21.20 | 3.0540 | 338.30 | 163238.100 | 0.90222 |
| 91 | 4004 | 388.60 | 5.9626 | 331.10 | 163569.200 | 0.90405 |
| 92 | 4101 | 34.00 | 3.5264 | 326.60 | 163895.800 | 0.90586 |
| 93 | 4401 | 3935.70 | 8.2778 | 318.00 | 164213.800 | 0.90762 |
| 94 | 4402 | 144.40 | 4.9726 | 312.60 | 164526.400 | 0.90935 |
| 95 | 4403 | 135.60 | 4.9097 | 307.70 | 164834.100 | 0.91105 |
| 96 | 4404 | 3203.50 | 8.0720 | 303.50 | 165137.600 | 0.91272 |
| 97 | 4406 | 397.00 | 5.9839 | 303.00 | 165440.600 | 0.91440 |
| 98 | 4407 | 1172.10 | 7.0666 | 287.00 | 165727.600 | 0.91598 |
| 99 | 4410 | 188.10 | 5.2370 | 271.80 | 165999.400 | 0.91749 |
| 100 | 4411 | 398.30 | 5.9872 | 270.40 | 166269.800 | 0.91898 |
| 101 | 4412 | 303.50 | 5.7154 | 268.90 | 166538.700 | 0.92047 |
| 102 | 4502 | 87.50 | 4.4716 | 261.60 | 166800.300 | 0.92191 |
| 103 | 4504 | 724.30 | 6.5852 | 259.80 | 167060.100 | 0.92335 |
| 104 | 4505 | 217.80 | 5.3836 | 257.70 | 167317.800 | 0.92477 |
| 105 | 4601 | 99.10 | 4.5961 | 253.50 | 167571.300 | 0.92617 |
| 106 | 4701 | 833.50 | 6.7256 | 245.10 | 167816.400 | 0.92753 |
| 107 | 4702 | 548.40 | 6.3070 | 242.40 | 168058.800 | 0.92887 |
| 108 | 4703 | 1029.00 | 6.9363 | 240.10 | 168298.900 | 0.93020 |
| 109 | 4704 | 326.60 | 5.7887 | 239.20 | 168538.100 | 0.93152 |
| 110 | 4705 | 2307.50 | 7.7439 | 235.10 | 168773.200 | 0.93282 |
| 111 | 4709 | 6315.30 | 8.7507 | 234.20 | 169007.400 | 0.93411 |
| 112 | 4710 | 783.50 | 6.6638 | 228.40 | 169235.800 | 0.93537 |
| 113 | 4717 | 242.40 | 5.4906 | 227.50 | 169463.300 | 0.93663 |
| 114 | 4718 | 3187.10 | 8.0669 | 224.50 | 169687.800 | 0.93787 |
| 115 | 4719 | 563.70 | 6.3345 | 219.40 | 169907.200 | 0.93909 |
| 116 | 4720 | 3024.00 | 8.0143 | 219.30 | 170126.500 | 0.94030 |
| 117 | 4721 | 103.90 | 4.6434 | 218.00 | 170344.500 | 0.94150 |
| 118 | 4722 | 3795.90 | 8.2417 | 217.80 | 170562.300 | 0.94271 |
| 119 | 4723 | 5709.80 | 8.6499 | 217.40 | 170779.700 | 0.94391 |
| 120 | 4724 | 1125.30 | 7.0258 | 215.50 | 170995.200 | 0.94510 |
| 121 | 4725 | 303.00 | 5.7137 | 213.40 | 171208.600 | 0.94628 |
| 122 | 4726 | 539.60 | 6.2908 | 211.90 | 171420.500 | 0.94745 |
| 123 | 4727 | 5164.20 | 8.5495 | 208.40 | 171628.900 | 0.94860 |
| 124 | 4729 | 3053.70 | 8.0241 | 206.20 | 171835.100 | 0.94974 |
| 125 | 4730 | 88.50 | 4.4830 | 197.50 | 172032.600 | 0.95083 |
| 126 | 4731 | 1559.40 | 7.3521 | 191.40 | 172224.000 | 0.95189 |
| 127 | 4732 | 820.90 | 6.7104 | 190.30 | 172414.300 | 0.95294 |
| 128 | 4733 | 517.40 | 6.2488 | 188.10 | 172602.400 | 0.95398 |
| 129 | 4734 | 219.30 | 5.3904 | 180.20 | 172782.600 | 0.95498 |
| 130 | 4735 | 794.30 | 6.6775 | 179.20 | 172961.800 | 0.95597 |
| 131 | 4736 | 1462.70 | 7.2880 | 177.20 | 173139.000 | 0.95695 |
| 132 | 4737 | 395.80 | 5.9809 | 174.60 | 173313.600 | 0.95791 |

Table 4. (cont.)

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 133 | 4738 | 473.20 | 6.1595 | 174.10 | 173487.700 | 0.95887 |
| 134 | 4739 | 224.50 | 5.4139 | 173.30 | 173661.000 | 0.95983 |
| 135 | 4740 | 899.40 | 6.8017 | 171.30 | 173832.300 | 0.96078 |
| 136 | 4741 | 2940.40 | 7.9863 | 165.90 | 173998.200 | 0.96170 |
| 137 | 4742 | 744.40 | 6.6126 | 163.20 | 174161.400 | 0.96260 |
| 138 | 4743 | 345.40 | 5.8447 | 155.20 | 174316.600 | 0.96346 |
| 139 | 4744 | 99.80 | 4.6032 | 152.20 | 174468.800 | 0.96430 |
| 140 | 4745 | 253.50 | 5.5354 | 151.20 | 174620.000 | 0.96513 |
| 141 | 4901 | 651.60 | 6.4794 | 150.90 | 174770.900 | 0.96597 |
| 142 | 4903 | 5259.60 | 8.5678 | 144.40 | 174915.300 | 0.96677 |
| 143 | 4904 | 287.00 | 5.6595 | 138.40 | 175053.700 | 0.96753 |
| 144 | 4905 | 240.10 | 5.4811 | 138.00 | 175191.700 | 0.96829 |
| 145 | 4907 | 111.10 | 4.7104 | 136.80 | 175328.500 | 0.96905 |
| 146 | 4909 | 530.50 | 6.2738 | 135.60 | 175464.100 | 0.96980 |
| 147 | 4910 | 639.70 | 6.4610 | 133.40 | 175597.500 | 0.97054 |
| 148 | 4912 | 960.00 | 6.8669 | 128.70 | 175726.200 | 0.97125 |
| 149 | 4913 | 259.80 | 5.5599 | 127.90 | 175854.100 | 0.97195 |
| 150 | 4915 | 245.10 | 5.5017 | 126.50 | 175980.600 | 0.97265 |
| 151 | 4916 | 424.40 | 6.0507 | 125.20 | 176105.800 | 0.97335 |
| 152 | 4918 | 994.50 | 6.9022 | 122.70 | 176228.500 | 0.97402 |
| 153 | 4920 | 1119.00 | 7.0202 | 116.40 | 176344.900 | 0.97467 |
| 154 | 4922 | 407.90 | 6.0110 | 111.80 | 176456.700 | 0.97528 |
| 155 | 4923 | 409.70 | 6.0154 | 111.40 | 176568.100 | 0.97590 |
| 156 | 4924 | 1080.50 | 6.9852 | 111.10 | 176679.200 | 0.97651 |
| 157 | 4926 | 1280.70 | 7.1552 | 107.40 | 176786.600 | 0.97711 |
| 158 | 4927 | 511.60 | 6.2375 | 104.60 | 176891.200 | 0.97769 |
| 159 | 4929 | 270.40 | 5.5999 | 104.50 | 176995.700 | 0.97826 |
| 160 | 4930 | 151.20 | 5.0186 | 103.90 | 177099.600 | 0.97884 |
| 161 | 4931 | 1017.00 | 6.9246 | 102.40 | 177202.000 | 0.97940 |
| 162 | 4933 | 1068.10 | 6.9736 | 99.80 | 177301.800 | 0.97996 |
| 163 | 4935 | 257.70 | 5.5518 | 99.70 | 177401.500 | 0.98051 |
| 164 | 4937 | 40.60 | 3.7038 | 99.10 | 177500.600 | 0.98105 |
| 165 | 4938 | 90.00 | 4.4998 | 99.10 | 177599.700 | 0.98160 |
| 166 | 4941 | 459.00 | 6.1291 | 99.10 | 177698.800 | 0.98215 |
| 167 | 4946 | 60.10 | 4.0960 | 97.50 | 177796.300 | 0.98269 |
| 168 | 4947 | 180.20 | 5.1941 | 95.90 | 177892.200 | 0.98322 |
| 169 | 5101 | 307.70 | 5.7291 | 95.40 | 177987.600 | 0.98375 |
| 170 | 5303 | 53.30 | 3.9759 | 93.10 | 178080.700 | 0.98426 |
| 171 | 5501 | 35.40 | 3.5667 | 90.90 | 178171.600 | 0.98476 |
| 172 | 5504 | 52.60 | 3.9627 | 90.00 | 178261.600 | 0.98526 |
| 173 | 5801 | 947.10 | 6.8534 | 88.50 | 178350.100 | 0.98575 |
| 174 | 5802 | 45.80 | 3.8243 | 87.50 | 178437.600 | 0.98623 |
| 175 | 5803 | 67.50 | 4.2121 | 85.30 | 178522.900 | 0.98670 |
| 176 | 5804 | 370.20 | 5.9140 | 78.40 | 178601.300 | 0.98714 |

Table 4. (cont.)

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 177 | 5805 | 21.70 | 3.0773 | 76.30 | 178677.600 | 0.98756 |
| 178 | 5807 | 20.40 | 3.0155 | 70.80 | 178748.400 | 0.98795 |
| 179 | 5809 | 173.30 | 5.1550 | 70.80 | 178819.200 | 0.98834 |
| 180 | 5812 | 2594.60 | 7.8612 | 69.60 | 178888.800 | 0.98873 |
| 181 | 5813 | 228.40 | 5.4311 | 67.50 | 178956.300 | 0.98910 |
| 182 | 5814 | 218.00 | 5.3845 | 65.70 | 179022.000 | 0.98946 |
| 183 | 5815 | 219.40 | 5.3909 | 64.10 | 179086.100 | 0.98982 |
| 184 | 5816 | 1173.70 | 7.0679 | 60.10 | 179146.200 | 0.99015 |
| 185 | 5817 | 122.70 | 4.8097 | 58.30 | 179204.500 | 0.99047 |
| 186 | 5818 | 19.90 | 2.9907 | 57.70 | 179262.200 | 0.99079 |
| 187 | 5819 | 2388.70 | 7.7785 | 57.30 | 179319.500 | 0.99111 |
| 188 | 5820 | 48.00 | 3.8712 | 56.60 | 179376.100 | 0.99142 |
| 189 | 5821 | 177.20 | 5.1773 | 55.20 | 179431.300 | 0.99173 |
| 190 | 5822 | 24.20 | 3.1864 | 55.20 | 179486.500 | 0.99203 |
| 191 | 5823 | 227.50 | 5.4272 | 53.30 | 179539.800 | 0.99233 |
| 192 | 5824 | 65.70 | 4.1851 | 52.60 | 179592.400 | 0.99262 |
| 193 | 5825 | 12.60 | 2.5337 | 52.50 | 179644.900 | 0.99291 |
| 194 | 5827 | 387.40 | 5.9595 | 52.20 | 179697.100 | 0.99319 |
| 195 | 5902 | 19.30 | 2.9601 | 50.60 | 179747.700 | 0.99347 |
| 196 | 5903 | 45.30 | 3.8133 | 49.60 | 179797.300 | 0.99375 |
| 197 | 6001 | 22.20 | 3.1001 | 49.20 | 179846.500 | 0.99402 |
| 198 | 6005 | 24.70 | 3.2068 | 48.00 | 179894.500 | 0.99429 |
| 199 | 6101 | 215.50 | 5.3730 | 46.00 | 179940.500 | 0.99454 |
| 200 | 6104 | 33.80 | 3.5205 | 45.80 | 179986.300 | 0.99479 |
| 201 | 6107 | 99.70 | 4.6022 | 45.30 | 180031.600 | 0.99504 |
| 202 | 6108 | 56.60 | 4.0360 | 44.10 | 180075.700 | 0.99529 |
| 203 | 6111 | 29.10 | 3.3707 | 42.00 | 180117.700 | 0.99552 |
| 204 | 6202 | 206.20 | 5.3288 | 40.60 | 180158.300 | 0.99574 |
| 205 | 6203 | 57.30 | 4.0483 | 39.60 | 180197.900 | 0.99596 |
| 206 | 6204 | 76.30 | 4.3347 | 36.40 | 180234.300 | 0.99616 |
| 207 | 6205 | 573.60 | 6.3519 | 35.40 | 180269.700 | 0.99636 |
| 208 | 6206 | 1066.90 | 6.9725 | 34.00 | 180303.700 | 0.99655 |
| 209 | 6207 | 11.60 | 2.4510 | 33.80 | 180337.500 | 0.99673 |
| 210 | 6307 | 171.30 | 5.1434 | 30.80 | 180368.300 | 0.99690 |
| 211 | 6308 | 99.10 | 4.5961 | 30.00 | 180398.300 | 0.99707 |
| 212 | 6309 | 802.40 | 6.6876 | 30.00 | 180428.300 | 0.99724 |
| 213 | 6310 | 46.00 | 3.8286 | 29.60 | 180457.900 | 0.99740 |
| 214 | 6312 | 1362.20 | 7.2169 | 29.10 | 180487.000 | 0.99756 |
| 215 | 6313 | 55.20 | 4.0110 | 26.80 | 180513.800 | 0.99771 |
| 216 | 6318 | 107.40 | 4.6766 | 24.70 | 180538.500 | 0.99785 |
| 217 | 6405 | 268.90 | 5.5943 | 24.40 | 180562.900 | 0.99798 |
| 218 | 6501 | 152.20 | 5.0252 | 24.20 | 180587.100 | 0.99811 |
| 219 | 6502 | 402.40 | 5.9974 | 23.20 | 180610.300 | 0.99824 |
| 220 | 6503 | 447.00 | 6.1026 | 22.20 | 180632.500 | 0.99836 |

Table 4. (cont.)

| *n* | Play No. | Mean NAgas (BCF) | Ln of Mean NAgas | Sorted Mean NAgas | Partial sum Sorted | Statistic q Partial/Total |
|---|---|---|---|---|---|---|
| 221 | 6505 | 52.50 | 3.9608 | 22.10 | 180654.600 | 0.99849 |
| 222 | 6701 | 95.90 | 4.5633 | 21.70 | 180676.300 | 0.99861 |
| 223 | 6702 | 213.40 | 5.3632 | 21.20 | 180697.500 | 0.99872 |
| 224 | 6703 | 208.40 | 5.3395 | 20.40 | 180717.900 | 0.99884 |
| 225 | 6704 | 217.40 | 5.3817 | 20.10 | 180738.000 | 0.99895 |
| 226 | 6706 | 111.80 | 4.7167 | 19.90 | 180757.900 | 0.99906 |
| 227 | 6708 | 138.00 | 4.9273 | 19.30 | 180777.200 | 0.99916 |
| 228 | 6714 | 64.10 | 4.1604 | 19.30 | 180796.500 | 0.99927 |
| 229 | 6716 | 44.10 | 3.7865 | 18.30 | 180814.800 | 0.99937 |
| 230 | 6717 | 99.10 | 4.5961 | 16.40 | 180831.200 | 0.99946 |
| 231 | 6718 | 70.80 | 4.2599 | 15.00 | 180846.200 | 0.99955 |
| 232 | 6719 | 104.50 | 4.6492 | 13.00 | 180859.200 | 0.99962 |
| 233 | 6720 | 338.30 | 5.8239 | 12.60 | 180871.800 | 0.99969 |
| 234 | 6721 | 50.60 | 3.9240 | 12.00 | 180883.800 | 0.99975 |
| 235 | 6725 | 57.70 | 4.0553 | 11.60 | 180895.400 | 0.99982 |
| 236 | 6727 | 127.90 | 4.8512 | 11.10 | 180906.500 | 0.99988 |
| 237 | 6732 | 239.20 | 5.4773 | 10.70 | 180917.200 | 0.99994 |
| 238 | 6802 | 23.20 | 3.1442 | 6.90 | 180924.100 | 0.99998 |
| 239 | 6901 | 348.20 | 5.8528 | 4.30 | 180928.400 | 1.00000 |
| | | 4.3000 | 1.4586 | <== Min | | |
| | | 23739.2000 | 10.0749 | <== Max | | |
| | | 180928.4000 | 1280.6593 | <== Total | | |
| | | 757.0226 | 5.3584 | <== Mean ($\mu$) | | |
| | | 2012.8771 | 1.5582 | <== Standard deviation ($\sigma$) | | |

Table 5. Spreadsheet of calculations from nonassociated-gas data of Figure 3 for the lognormal $q$ as a function of various values of $p$. NORMSINV is the spreadsheet function that computes $z$; NORMSDIST is the function that computes $1 - q$.

| $p$ Pop. Prop. | $(p)(N)$ $(p)(239)$ | $1-p$ | $z$ NORMSINV | $z-\sigma$ $(z-1.5582)$ | $1-q$ NORMSDIST | lognormal $q$ (theoretical) | statistic $q$ (empirical) |
|---|---|---|---|---|---|---|---|
| 0.1 | 23.9 | 0.9 | 1.2816 | –0.2766 | 0.3910 | 0.6090 | 0.6406 |
| 0.2 | 47.8 | 0.8 | 0.8416 | –0.7166 | 0.2368 | 0.7632 | 0.7887 |
| 0.3 | 71.7 | 0.7 | 0.5244 | –1.0338 | 0.1506 | 0.8494 | 0.8642 |
| 0.4 | 95.6 | 0.6 | 0.2533 | –1.3049 | 0.0960 | 0.9040 | 0.9121 |
| 0.5 | 119.5 | 0.5 | 0.0000 | –1.5582 | 0.0596 | 0.9404 | 0.9445 |
| 0.6 | 143.4 | 0.4 | –0.2533 | –1.8115 | 0.0350 | 0.9650 | 0.9678 |
| 0.7 | 167.3 | 0.3 | –0.5244 | –2.0826 | 0.0186 | 0.9814 | 0.9828 |
| 0.8 | 191.2 | 0.2 | –0.8416 | –2.3998 | 0.0082 | 0.9918 | 0.9924 |
| 0.9 | 215.1 | 0.1 | –1.2816 | –2.8398 | 0.0023 | 0.9977 | 0.9977 |
| 0.05 | 11.95 | 0.95 | 1.6449 | 0.0867 | 0.5345 | 0.4655 | 0.4659 |
| 0.025 | 5.975 | 0.975 | 1.9600 | 0.4018 | 0.6561 | 0.3439 | 0.3306 |
| 0.01 | 2.39 | 0.99 | 2.3263 | 0.7681 | 0.7788 | 0.2212 | 0.2183 |

Table 6. Summary of fractal lognormal percentage assessment of conventional nonassociated-gas resources. (NA means nonassociated.)

| Percentage of NA gas plays | Percentage of total NA gas resources $q100\%$ | |
|---|---|---|
| | Theoretical | Empirical |
| $p100\%$ | (lognormal $q$) | (statistic $q$) |
| 0 | 0.00 | 0.00 |
| 1 | 22.12 | 21.83 |
| 2.5 | 34.39 | 33.06 |
| 5 | 46.55 | 46.59 |
| 10 | 60.90 | 64.06 |
| 20 | 76.32 | 78.87 |
| 30 | 84.94 | 86.42 |
| 40 | 90.40 | 91.21 |
| 50 | 94.04 | 94.45 |
| 60 | 96.50 | 96.78 |
| 70 | 98.14 | 98.28 |
| 80 | 99.18 | 99.24 |
| 90 | 99.77 | 99.77 |
| 100 | 100.00 | 100.00 |

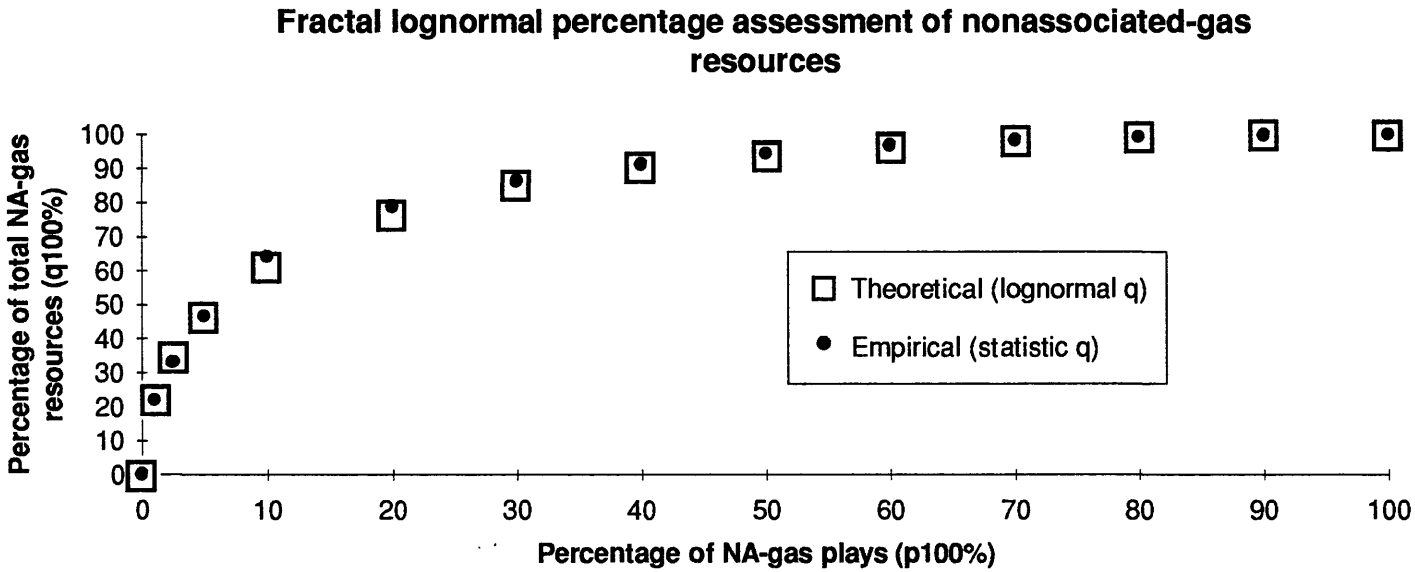


Figure 4. Theoretical percentage of total nonassociated-gas resources using the lognormal $q$ as a function of the percentage of nonassociated-gas plays, along with the corresponding empirical values using the statistic $q$. Data are from Table 6. (NA means nonassociated.)

## SUMMARY AND CONCLUSIONS

1. "The 20/80 law" is generalized as the $p100/q100$ law in probabilistic form for any probability distribution that models the population distribution. The parameter $q$ is called the proportion of total value and defined in terms of the population proportion $p$ and conditional expectation.

2. When the population distribution is modeled with the lognormal distribution, the $p100/q100$ law in probabilistic form produces the $p100/q100$ law in lognormal form. The proportion of total value is derived, and the parameter $q$ is a function only of $p$ and the shape parameter $\sigma$. The parameter $q$ is scale-free; i.e., it does not depend upon the lognormal scale parameter $\mu$. Because it is scale invariant, the proportion $q$ is a parametric fractal.

3. The $p100/q100$ law in lognormal form can be applied to conventional petroleum resources in oil and nonassociated-gas plays in the United States. The lognormal $q$ is a function of $p$ for each commodity such that $p100\%$ of the plays account for $q100\%$ of the total resources of the plays. In both cases the theoretical percentages of total resources using the lognormal $q$ are extremely close to the empirical percentages from the data using the statistic $q$. For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal $q$, or for 75.52% if we use the statistic $q$; 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal $q$, or for 78.87% if we use the statistic $q$.

4. This new probabilistic methodology for percentage assessment of resources lends itself as an ideal application for spreadsheet software.

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