

**A NUMERICAL PROGRAM FOR STEADY-STATE
FLOW OF HAWAIIAN MAGMA-GAS MIXTURES
THROUGH VERTICAL ERUPTIVE CONDUITS**

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LIST OF VARIABLES USED IN TEXT

Variable	Definition	Units
A	cross-sectional area	m ²
c	sound speed	m/s
c _p	specific heat at constant pressure	J/(kg K)
c _v	specific heat at constant volume	J/(kg K)
D	conduit diameter	m
g	gravitational acceleration	m/s ²
h	specific enthalpy	J/kg
K	bulk modulus	MPa
N	constant used in viscosity eq. 18	dimensionless
n	moles exsolved gas per kg mixture	mol/kg
p	pressure	MPa
p _{H2O}	partial pressure of water in melt	MPa
p _o	reservoir pressure for ideal gas	MPa
R	Universal Gas Constant	J/(mole K)
r	conduit radius	m
Re	Reynolds number	dimensionless
s	empirically-derived constant for Henry's Law exsolution (eq. 34)	Pa ^{-0.7}
T	temperature	°C
T	absolute temperature	K
v	velocity	m/s
v _{max}	maximum theoretical velocity	m/s
v	volume	m ³
W _{CO2,e}	weight percent exsolved CO ₂	wt. %
W _{CO2,m}	weight percent dissolved CO ₂	wt. %
W _{CO2,*}	total weight percent CO ₂	wt. %
W _{H2O,e}	weight percent exsolved H ₂ O	wt. %
W _{H2O,m}	weight percent dissolved H ₂ O	wt. %
W _{H2O,*}	total weight percent H ₂ O	wt. %
W _{s,e}	weight percent exsolved S species	wt. %
W _{s,m}	weight percent dissolved S species	wt. %
W _{s,*}	total weight percent S species	wt. %
z	vertical position above base of conduit	m
β	empirically-derived constant for Henry's Law exsolution	dimensionless
η	viscosity of erupting mixture	Pa s
η ₁	bulk viscosity calculated from eq. 16	Pa s
η ₂	bulk viscosity calculated from eq. 17	Pa s
φ	volume fraction gas	dimensionless
ρ	mixture density	kg/m ³

Subscript	Definition
cr	country rock
e	value after equilibrating to 1 atm pressure
f	final value in conduit
g	gas
m	magma
i	incremental
ideal	value assuming ideal pseudogas behavior
o	reservoir value (for ideal pseudogases) or value at base of conduit
s	constant entropy conditions
w	water

INTRODUCTION

In many volcanic studies, estimates must be made of the changes that magma and its associated gases experience when traveling through an eruptive conduit to the surface. Exsolution of magmatic gas, acceleration, changes in pressure and temperature, depth of fragmentation, and final exit velocities affect such features as lava fountain heights, spatial distribution of eruptive products, and the degree to which water can enter the conduit during eruptive activity. Most of these quantities cannot be easily estimated without some sort of numerical model.

This report presents a model that calculates flow properties (pressure, vesicularity, and some 35 other parameters) as a function of vertical position within a volcanic conduit during a steady-state eruption. It uses temperature-viscosity relationships and gas solubility properties that are characteristic of Kilauean basalt. However it can also be applied to most other basaltic volcanoes. With some modifications to certain subroutines, the program can calculate flow properties in conduits for intermediate and silicic magmas as well. The model approximates the magma and gas in the conduit as a homogeneous mixture, and calculates processes such as gas exsolution under the assumption of equilibrium conditions. These are the same assumptions on which classic conduit models (e.g. Wilson and Head, 1981) have been based. They are most appropriate when applied to eruptions of rapidly-ascending magma (for example, basaltic lava-fountain eruptions, and Plinian or sub-Plinian eruptions of silicic magmas).

The original purpose of this report was to make the model available for scrutiny so that the results of studies that use it (Mastin, 1994, and future papers) can be independently verified. A second purpose is to provide a user's guide to investigators who may wish to apply the program to study eruptive dynamics for their own purposes. If you are interested in such a project, I invite you to contact me. More sophisticated versions of this program are currently being developed that may be useful (though at this time those versions are not sufficiently free of bugs to present publicly).

SYSTEM REQUIREMENTS AND INSTALLATION

The DOS-formatted disk that accompanies this report contains the following files:

File name	Size (kb)	Description
HICON.FOR	37	FORTRAN 77 source code (ASCII)
HICON.EXE	122	executable program (binary).
HICIN	2	example input file (ASCII)
HICOUT	8	example output file (ASCII)
DOSXMSF.EXE	393	file called by HICON.EXE (binary)
README.TXT	?	contains update information (ASCII)

The source code file, HICON.FOR, is written in ANSI FORTRAN 77 and can be compiled using any FORTRAN 77 compiler. For simplicity, no graphic output has been supplied; flow properties are written to output files and must be plotted using some other

software. This makes the program somewhat less user-friendly, but also makes it possible to compile and use it on any computer platform, with any associated hardware.

The executable file, HICON.EXE, will run on any DOS-based computer containing an INTEL® 80386 or later processor. The executable file may be copied from diskette to a hard disk using the *copy* command in DOS, or may be used while resident on the floppy disk. The input and output files are supplied as read-only files so that you don't inadvertently write over them before copying them to another place. You will need to explicitly change their read-only status to modify them.

The time (real, not CPU) required for a typical model run using HICON ranges from a few seconds or less (on a 60 MHz or faster Pentium®-based computer) to a few minutes (on a Data General AViiON® 300-series UNIX workstation¹). Different runs, of course, vary in time depending on the number of iterations required to reach a solution.

MODEL OVERVIEW

In this model, the calculation of flow properties in an eruptive conduit is fundamentally the same as the calculation of flow in a pipe (Fig. 1). That is, magma is injected into the base of the conduit under conditions that are specified as input into the program. The required input conditions include the pressure, velocity, temperature, magma density, and weight percent of the three main volatile components: CO₂, H₂O, and sulfur species (H₂S and SO₂). Also given as input are the conduit length, diameter, and a roughness term, f_0 .

The program then calculates other properties, including the weight percent of exsolved gas, vesicularity, bulk density of the magma/gas mixture, viscosity, Reynolds number, and friction factor, f , (which determines frictional pressure losses) at the base of the conduit. It then moves up the conduit, calculating other flow properties as it goes.

The model can calculate flow properties in either of two different ways. One option is to specify a conduit of constant diameter and solve for the pressure and other flow properties as a function of depth (Fig. 1, left side). Under that option, the program uses the momentum equation (presented in a later section) to calculate the pressure gradient at the initial depth and to extrapolate a new pressure at a slightly higher level in the conduit (Fig. 2, left side). Using that new pressure and a variety of constitutive relations (presented later), the amount of exsolved gas is calculated at the new depth, as well as the vesicularity, viscosity, bulk density, and other properties of the magma/gas mixture. Using the continuity equation, a new velocity is calculated. The program then calculates a new pressure gradient at the new depth, and the computations are repeated at successively higher levels to the surface.

A second option is to specify a pressure gradient in the conduit and calculate the vertical gradient in the conduit's cross-sectional area required to produce that pressure gradient. Under this scheme, the program begins again at the base of the conduit, and uses a rearranged version of the momentum equation to solve for the gradient in the conduit's cross-sectional area (Fig. 2, right side). A new cross-sectional area is then computed at a slightly higher level in the conduit, and new flow properties are calculated

¹ Use of trade names is for identification purposes only and does not imply endorsement by the U.S. Geological Survey.

at that depth using the continuity equation and constitutive relations. Then a new cross-sectional area gradient is calculated, and the computations are repeated to the top of the conduit.

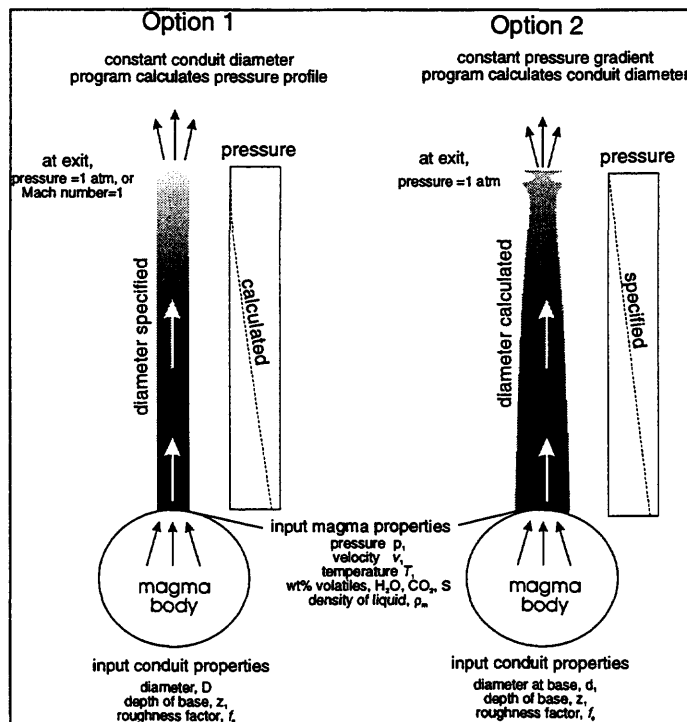


Figure 1: Illustration of the input variables required to the program HICON, and the two options available for calculating flow properties as a function of depth.

In option 1, the erupting mixture must satisfy one of two conditions: (1) if the exit velocity is less than its sonic velocity, the exit pressure must equal 1 atmosphere (atm). Alternatively, (2) the exit velocity must equal the sonic velocity. The latter boundary condition results from the fact that, in a conduit of constant cross-sectional area, the velocity of the mixture can never exceed its sonic velocity. This is a basic tenet of compressible fluid dynamics and is explained in a number of texts (e.g. Saad, 1985). Thus if the input pressure at the base of the conduit is raised above a certain threshold value, the erupting mixture will not be able to equilibrate to 1 atm pressure by the time it reaches the surface. The exit conditions will vary according to the input pressure, as shown in the table below:

Input pressure	Exit velocity	Exit pressure
\leq weight of magma column	0 (no eruption)	
slightly greater than weight of magma	subsonic	1 atm
much greater than weight of magma	sonic	> 1 atm

For Kilauea magmas in lava-fountain eruptions, the sonic velocity is typically 40-60 meters per second (m/s), which is roughly equal to exit velocities estimated from videos

and heights of lava fountains (Mangan and Cashman, in press). It is therefore likely that sonic conditions exist in many lava-fountain eruptions.

In order to match the exit conditions with the required boundary conditions, the program makes successive runs, adjusting the input velocity after each one, until one of the two boundary conditions is satisfied. In option 2, successive runs are not necessary--an output pressure of 1 atm can be achieved during a single iteration by calculating a conduit geometry that gives the specified pressure gradient. The sonic boundary condition does not apply because the variable conduit geometry allows the erupting mixture to accelerate to supersonic velocities.

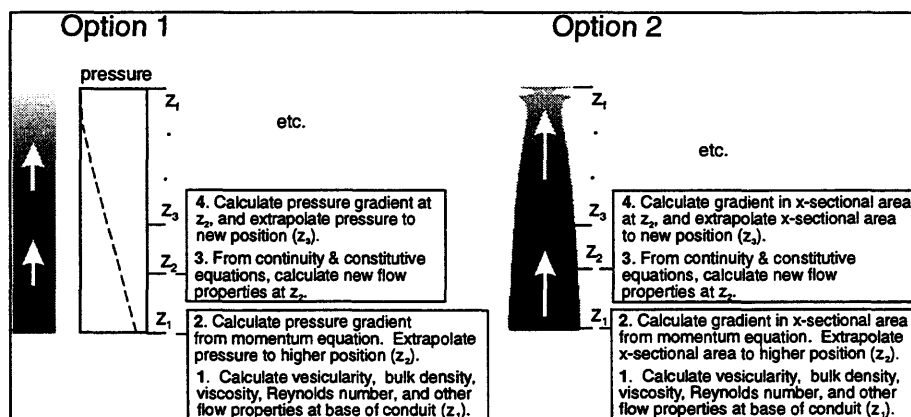


Figure 2: Schematic illustration of the sequence of steps used to calculate flow properties from the base to the top of a conduit, under option 1 (left side) and option 2 (right side).

MODEL ASSUMPTIONS AND LIMITATIONS

The model makes the following assumptions:

1. Flow of magma and exsolved gases is homogenous. That is, there is no relative movement between the gas and liquid phases as they ascend the conduit. This assumption allows the mixture to be treated as a single fluid phase whose density, viscosity, and other properties are bulk values for the mixture. The homogeneous-flow assumption is used by most modellers of volcanic eruptions, both mafic and silicic (e.g. Wilson et al., 1980; Wilson and Head, 1981; Head and Wilson, 1987; Buresti and Casarosa, 1989; Gilberti and Wilson, 1990), although its validity has been challenged for certain types of basaltic eruptions (Vergnolle and Jaupart, 1986; Dobran, 1992).

A few considerations are in order when evaluating this assumption in Kilauean eruptions. Typical Kilauean basalts contain about 0.27 weight percent (wt.%) water, 0.015-0.05 wt.% CO_2 , and 0.07-0.12 wt.% sulfur. After equilibrating to surface conditions, more than 80% by volume of the exsolved gas is water vapor, which doesn't begin to come out of solution until the magma is about 100-200 m from the surface (Gerlach, 1986). Estimated ascent rates range from 0.01-0.1 m/s for especially slow effusive magmas (Greenland et al., 1988) to tens of meter per second for lava fountains at the surface (Mangan and Cashman, in press). Thus the time available for nucleation and

growth of H₂O vesicles ranges from several seconds for lava-fountain eruptions (Mangan and Cashman, in press) to a few minutes for effusive eruptions (Mangan et al., 1993).

Whether the gas separates from the magma and rises at a different velocity depends largely on the size of individual bubbles, and on the opportunity for bubbles to coalesce into larger ones that rise more rapidly. Bubble sizes in Kilauean basalts are typically 0.1-1 millimeters (mm) for lava-fountain tephra, and 1-10 mm for effusive lava samples collected at the vent (Mangan et al., 1993; Mangan and Cashman, in press). Using the Stokes-flow equation for bubble rise (Bird et al., 1960, p. 182), ascent rates for bubbles of this size should be 10^{-7} - 10^{-5} m/s during lava-fountain eruptions, and $\sim 10^{-5}$ - 10^{-3} m/s in effusive eruptions.

In vigorous lava-fountain eruptions, the rise velocity of bubbles in magma is so small relative to the ascent velocity of the magma that both the gas and magma may be regarded as a single, homogeneous fluid. A homogeneous-flow program is probably appropriate for modelling such eruptions.² For effusive eruptions, the homogeneous-flow assumption may not be appropriate, depending on the magma ascent rate.

In Strombolian eruptions, the assumption of homogeneous flow is clearly inappropriate (Vergnolle and Jaupart, 1986). Such eruptions are produced when rising bubbles coalesce to produce gas slugs up to meters in diameter, that rise through the shallow conduit and produce bursts of spatter at the surface.

In cases where flow separation does occur, it tends to increase the density of the magma-gas mixture in the conduit (due to gas escape), increase gas velocities relative to those for homogeneous flow, decrease magma velocities, and (due to the higher average density of the degassed mixture) increase the vertical pressure gradient in the conduit (Vergnolle and Jaupart, 1986; Dobran, 1992). Under separated-flow conditions, the subsurface pressure required to sustain an eruption of a given magma flux rate would be higher than under homogeneous flow.

2. Gas exsolution maintains equilibrium with pressure in the conduit up to the point of fragmentation. This assumption has been made in all other models of conduit flow (Wilson et al., 1980; Wilson and Head, 1981; Gilberti and Wilson, 1990; Dobran, 1992). There is some evidence (Mangan and Cashman, in press) that rates of exsolution cannot keep pace with rates of pressure drop. For this reason, models cited above have arbitrarily shut off additional exsolution once a vesicularity of $\sim 75\%$ (implying magma fragmentation) has been reached. HICON offers the option of shutting off further gas exsolution once vesicularity reaches 75%, or allowing it to continue, at the discretion of the user.

3. At any given depth, flow properties can be averaged across the entire cross-sectional area of the conduit. This assumption simplifies the problem to a one-dimensional one.

² Vergnolle and Jaupart (1986) argue that Kilauean lava-fountain eruptions involve separated flow and therefore cannot be modeled using homogeneous models. Their argument, however, is based on an assertion that the eruptions are driven by CO₂ gas that occupies the center of the conduit and entrains an annular ring of liquid magma. The eruptions, they argue, are caused when CO₂ gas escapes from the magma chamber, in volumes several times greater than the volume of gas exsolved from the magma ejected during the eruptions. Most other researchers (e.g. Greenland, 1988; Head and Wilson, 1987; Parfitt and Wilson, 1994) do not accept this as a mechanism for driving Hawaiian lava-fountain eruptions.

4. The conduit is vertical. If one is modeling eruptions on Kilauea's flank, this assumption obviously limits the applicability of this model to the shallow section of the conduit.

5. Flow is steady state. Hawaiian lava-fountain eruptions commonly continue for minutes or, in some cases, hours, without perceptible changes in activity. Therefore this assumption should be adequate to model typical lava-fountain eruptions.

6. No heat is transferred across the conduit walls during the eruption. This assumption has been used in most previous eruption models. Eruption scenarios that most closely approximate this condition will be those that erupt through vents (like Pu'u O'o) that have become established with months or years of flow through them. Kilauean lavas that flow through surface tubes (Cashman et al., 1994) show less than ten degrees cooling through several kilometers of tube length. The assumption of no heat loss in well-established vertical conduits is therefore probably not bad.

7. The gas phase behaves essentially as an ideal gas. Extensive experiments on H₂O and CO₂ gas (e.g. Haar et al., 1984) have documented that, at temperatures and pressures appropriate for this model, this assumption is reasonable.

8. There is no migration of gas out through the conduit walls. This assumption limits applicability of the model to cases where gas generation is sufficiently rapid that bubbles cannot migrate to the margin of the conduit before they are released at the surface. It is probably appropriate for lava-fountain eruptions, where vesicle residence times are less than a minute. In slowly fed eruptions, gas escape through the conduit walls may reduce the vesicularity of the erupted magma significantly, resulting in the effusion of lava flows rather than highly fragmented pyroclastic debris (Eichelberger et al., 1986; Woods, 1995).

MODEL SETUP

The following section presents the constitutive and governing equations on which the computations are based. In addition to presenting the equations, I attempt to explain their meaning in physical terms so that the reader can understand their implications a little more fully.

Governing Equations

Using the assumptions described earlier, we can write equations for conservation of mass

$$\frac{dp}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0 \quad \text{eq. 1}$$

and of momentum

$$-\frac{dp}{dz} = \rho g + \rho v^2 \frac{f}{r} + \rho v \frac{dv}{dz} \quad \text{eq. 2}$$

of the erupting mixture. The variables ρ , v , and p are the density, velocity, and pressure of the mixture in the conduit, and A is the conduit's cross-sectional area (Fig. 3). f is a friction factor whose value controls frictional pressure loss in the vent³ (Bird et al., 1960), r is the radius of the conduit, and z is vertical position (upwards being positive).

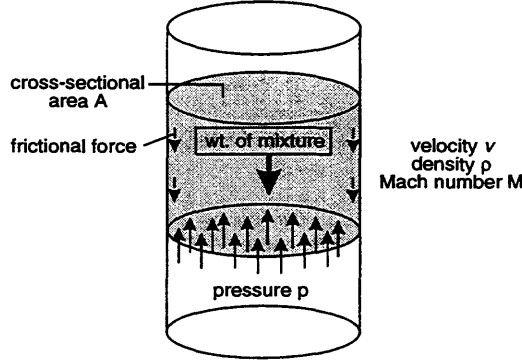


Figure 3: Profile illustrating the forces driving the movement of magma in a conduit, and some of the properties of the erupting mixture.

Equation 1 states simply that an expansion of the erupting mixture must be accompanied by acceleration, or by an increase in cross-sectional area within the vent in order to avoid movement of material into a space already occupied. It is derived from the postulate that the mass flux, $\dot{m} = \rho v A$, is constant at all points in the conduit. Equation 2 indicates that pressure variations within the vent are due to (1) the weight of the mixture (first term on the right side), (2) the frictional pressure loss associated with flow (middle term), and (3) changes in kinetic energy of the erupting mixture (right term). By rearranging eq. 1 as $dv = -v (dp/\rho + dA/A)$, substituting it into the right-hand term on the right side of eq. 2, and rearranging, the following new equation is obtained.

$$-\frac{dp}{dz} = \rho g + \rho v^2 \frac{f}{r} - \frac{\rho v^2}{A} \frac{dA}{dz} - v^2 \frac{dp}{dz} \quad \text{eq. 3}$$

This equation can be made more tractable by assuming that the right-hand term, dp/dz , is approximately equal to the product $dp/dz \approx (\partial \rho / \partial p)_s (dp/dz)$. The term $(\partial \rho / \partial p)_s$ is the partial of density with pressure under constant entropy for the gas/magma mixture. For homogeneous mixtures of gas dispersed in liquid (or vice versa), it can easily be calculated. Just as important, this quantity is the squared reciprocal of sound speed of the mixture, c (Liepmann and Roshko, 1957, p. 50). The equation can therefore be rewritten as

$$-\frac{dp}{dz} \left(1 - \frac{v^2}{c^2} \right) = \rho g + \rho v^2 \frac{f}{r} - \rho \frac{v^2}{A} \frac{dA}{dz} \quad \text{eq. 4}$$

³The friction factor defined by Bird et al. (1960), used here, differs by a factor of four from that defined by Schlichting (1955, p. 86) and used by Wilson et al. (1980). Therefore the second term on the right-hand side of eq. 2 also differs from the corresponding term in eq. 1 of Wilson et al. (1980).

or,

$$-\frac{dp}{dz} = \frac{\rho g + \rho v^2 \frac{f}{r} - \frac{\rho v^2}{A} \frac{dA}{dz}}{1 - M^2} \quad \text{eq. 5}$$

where M is the Mach number of the mixture, i.e. its velocity divided by its sonic velocity.

Equation 5 is used to calculate the pressure and pressure gradient in the conduit. It reveals some fundamental properties of the pressure at various states of flow. Under static conditions, $v=0$ and $M=0$, and the pressure gradient is simply $-dp/dz=\rho g$, or the weight of the magma column. If magma is flowing, but at a velocity that is small relative to its sonic velocity, $M \sim 0$ and the pressure gradient is a function of the weight of the magma column, frictional pressure losses (i.e. the first and second terms in the numerator on the right side of eq. 5), and changes in conduit geometry (the third term). As M approaches 1, the numerator on the right hand side of eq. 5 must approach zero in order to avoid a singular solution. Setting $A=\pi r^2$, the numerator on the right side of eq. 5 must satisfy the following equality in order to be equal to zero:

$$\rho g + \frac{f \rho v^2}{r} = \frac{\rho v^2}{\pi r^2} \frac{2 \pi r dr}{dz} \quad \text{eq. 6}$$

Rearranging leads to the following equation.

$$\frac{dr}{dz} = \frac{1}{2} \left(\frac{rg}{v^2} + f \right) \quad \text{eq. 7}$$

Because the two terms on the right hand side of eq. 7 are always positive, the vent must be slightly widening in the upward direction in order for the sonic velocity to be reached. In a constant-area duct, velocities can never reach $M=1$, regardless of the driving pressure at the base of the conduit (though from computational experience they can come extremely close). An increase in pressure at the base of the conduit will result in an increase in pressure at the conduit exit and an increase in mass flux (due to greater density of the mixture at the exit). It will not, however, result in an increase in the Mach number of the erupting mixture beyond $M=1$. The escaping gas/magma mixture will equilibrate with atmospheric pressure abruptly above the exit, through a series of shock waves (Liepmann and Roshko, 1957; Kieffer, 1984).

In a gradually flaring conduit, If $M < 1$ at the point where dr/dz satisfies eq. 7, and the conduit continues to diverge, the mixture will decelerate with increasing z and the pressure drop will be relatively modest. If, on the other hand, $M=1$ is achieved in this critical section, and the conduit continues to diverge, then the fluid will accelerate to supersonic velocity and the pressure will drop significantly with increasing z . At this stage, depending on the conduit geometry, the pressure can drop below $p=1$ atm prior to reaching the conduit exit. If this is the case, a stationary shock wave will develop within

the diverging section of the conduit, through which velocities of the erupting mixture will drop abruptly to a subsonic value and pressure will rise to a value that allows the mixture to reach 1 atm at the conduit exit (Saad, 1985, p. 158).

In a vent containing a constant pressure gradient, eq. 5 is rearranged to isolate the variable dA/dz as follows:

$$\frac{dA}{dz} = \frac{A}{\rho v^2} \left[\frac{dp}{dz} (1 - M^2) + \rho g + \frac{f \rho v^2}{r} \right] \quad \text{eq. 8}$$

This equation is used to calculate changes in cross-sectional area for model runs in which the pressure gradient is specified.

Constitutive Relationships

The following constitutive relationships are used to evaluate the terms on the right-hand side of equations 5 and 8.

Density

The density (ρ) of a magma/gas mixture is a function of the volume fractions and densities of the two phases, gas and magma. The amount of gas present in turn is a function of pressure and of the amounts of the main volatile components in the melt; H_2O , CO_2 , and the sulfur species, SO_2 and H_2S . Calculation of density therefore requires three steps: (1) calculating the amount of the main exsolved gases; (2) calculating the specific volume and density these gases; and (3) combining gas with magma volumes to determine an overall bulk density of the mixture.

The amount of exsolved gas and the percentage of the main gas species in the melt are determined using solubility relationships for Kilauean basalt described by Gerlach (1986) and explained in Appendix B. For a given pressure, p , and weight percentage ($W_{H_2O,*}$, $W_{CO_2,*}$, $W_{s,*}$) of H_2O , CO_2 and S in the melt/gas mixture, these relationships return the weight percentage of exsolved species ($W_{H_2O,e}$, $W_{CO_2,e}$, $W_{s,e}$), the mass fraction of gas, m_g , the mass fraction magma, m_m , and the number of moles of exsolved gas per kilogram of gas/magma mixture, n . Using those values, and assuming that the gases behave as ideal gases, the gas density is:

$$\rho_g = m_g \frac{p}{nRT} \quad \text{eq. 9}$$

and the ratio of gas volume (v_g) to magma volume (v_m) is given by

$$\frac{v_g}{v_m} = \frac{m_g \rho_m}{m_m \rho_g} \quad \text{eq. 10}$$

where ρ_m is the magma density, R is the Universal Gas Constant (in Joules per mole per degree Kelvin, $J/(mol \text{ K})$), and T is temperature, in Kelvin. The gas and magma are assumed to maintain thermal equilibrium with one another while cooling adiabatically. The algorithm for calculating adiabatic cooling is explained in Appendix C. It slightly

overestimates the amount of adiabatic cooling, but its results are acceptably close to the correct values (probably within a small fraction of a degree over the length of the conduit). Erupting mixtures that contain the amount of exsolved gas typical of Kilauea magmas (<~0.4 wt.%) normally do not cool adiabatically by more than about fifteen degrees Celsius during decompression (Mastin, 1995).

From the ratio v_g/v_m , the volume fraction gas (ϕ) is:

$$\phi = \frac{(v_g / v_m)}{(v_g / v_m) + 1} \quad \text{eq. 11}$$

and the bulk density of the mixture, ρ , is

$$\frac{1}{\rho} = \frac{m_g}{\rho_g} + \frac{m_m}{\rho_m} \quad \text{eq. 12}$$

Friction factor

The frictional resistance experienced by one-phase fluids flowing in circular conduits is well known from experimental data (Bird et al., 1960, p. 186). The frictional resistance is generally expressed as a friction factor, f , defined as the frictional force resisting flow through a unit length of a conduit, normalized to the surface area of the conduit in that path length and to the kinetic energy per unit volume of the flowing mixture (Bird et al., 1960, p. 181). Following previous eruption modellers (Wilson et al., 1980; Gilbert and Wilson, 1990; Dobran, 1992), we calculate f from the following equation:

$$f = \frac{16}{\text{Re}} + f_o = \frac{16\eta}{\rho v D} + f_o \quad \text{eq. 13}$$

where D is the conduit diameter and η is the viscosity of the erupting mixture. The variable f_o is an empirically-derived factor related to the roughness of the conduit walls.

For laminar flow conditions ($\text{Re} < \sim 2000$), which characterize all but the upper several tens of meters of the eruptive conduit, the left-hand term on the right side dominates the equation. The value of ρ can be evaluated at any point as described earlier as long as the pressure and volatile content of the magma are known. The velocity v can be determined with knowledge of ρ and A using the continuity equation (eq. 1), and D can be calculated or specified. The viscosity (η) of the mixture varies greatly during ascent due to vesiculation and fragmentation. The liquid phase is assumed to have a viscosity (η_m) that depends on temperature in a manner shown empirically by Ryan and Blevins (1987):

$$\log(\eta_m) = -10.737 + 1.8183 \left(\frac{10,000}{T} \right) \quad \text{eq. 14}$$

where viscosity is in Pascal seconds and temperature is in Kelvin. Kilauean basalts with temperatures of 1150-1200°C have viscosities of 40-110 Pascal-seconds (Pa s) using this relationship.

Once gases begin to exsolve, the rheological properties are much more difficult to evaluate. At this point, the magma becomes non-Newtonian (Bagdassarov and Dingwell, 1992; Stein and Spera, 1992), and no constitutive law relating rheology to vesicularity of silicates currently exists. Constitutive relations derived from studies of non-silicate emulsions containing rigid inclusions (Eilers, 1943; Mooney, 1951; Roscoe, 1952; Pal and Rhodes, 1989) suggest that viscosity increases substantially with bubble content. One example is by Eilers (1943):

$$\eta = \eta_m \exp \left[2 \ln \left(\frac{1 + 1.25\phi}{1 - 1.3\phi} \right) \right] \quad \text{eq. 15}$$

This relationship gives a viscosity of the mixture (η) that is about three orders of magnitude greater than that of the fluid alone (η_m) at a volume-fraction of inclusions (ϕ) approaching 0.7 (Fig. 4). The viscosity increase given by this relationship is probably much greater than actually exists in vesicular magmas, because the bubbles in magma can deform to accommodate shear strains.

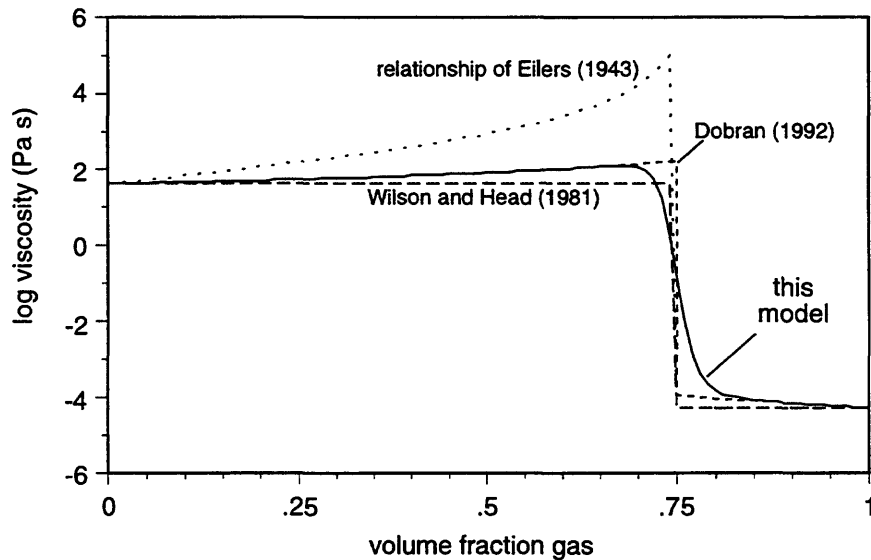


Figure 4: Log viscosity (Pa s) versus volume fraction gas using the relationships of Eilers (1943) (dotted line), Dobran (1992) (short-dashed line), Wilson and Head (1981) (long-dashed line), and that used in HICON (solid line).

In their models of conduit flow, Wilson et al. (1980), Wilson and Head (1981), and Gilberti and Wilson (1990) assumed no change in viscosity with vesicle content up to the point of fragmentation, and viscosity equal to that of the gas phase (η_g) above that point. Dobran (1992) used the following relationship to model conduit flow (simplified for the case where $\eta_g \ll \eta_m$):

$$\eta = \frac{\eta_m}{1 - \phi}, \quad \phi \leq 0.75 \quad \text{eq. 16}$$

$$\eta = \eta_g \left[1 - \left(\frac{1 - \phi}{0.62} \right) \right]^{-1.56} \quad \phi > 0.75 \quad \text{eq. 17}$$

This relationship gives a four-fold increase in viscosity as vesicularity increases from zero to 0.75, and a viscosity slightly above the gas viscosity at $\phi > 0.75$ (Fig. 4). The output of runs was compared using each of these three relationships. The relationships of Wilson and Head (1981) and Dobran (1992) result in nearly identical pressure profiles for both small diameter (1 m, Fig. 5A) and large diameter (10 m, Fig. 5B) conduits. Only the Eilers relationship produces significantly different pressure profiles, and those differences are large only in small-diameter conduits. The lack of any significant difference in pressure profiles resulting calculated using the Dobran (1992) and the Wilson and Head (1981) relationships suggests that the exact law for viscosity as a function of vesicularity is not critical in these models. The constitutive relations of Dobran (1992) are used in HICON, with a gradual transition between pre-fragmentation and post-fragmentation viscosities between about $\phi = 0.7$ and $\phi = 0.8$ (Fig. 4). The gradual transition was mathematically created using the following equation:

$$\log(\eta) = 2^{-N} \log(\eta_1) + 2^N \log(\eta_2) \quad \text{eq. 18}$$

where

$$N = \left(\frac{\phi}{0.75} \right)^{40}$$

and η_1 and η_2 are the viscosities calculated in equations 16 and 17, respectively.

At Reynolds numbers typical for turbulent flow (the upper tens to hundred of meters of the conduit), the friction factor f is determined primarily by the right-hand term, f_o , in eq. 13. Experimental values of f_o range from about 0.001 to 0.02; values of around 0.0025 are commonly used to model flow in rough-walled eruptive conduits (Wilson et al., 1980; Gilberti and Wilson, 1990), and we use that value here. As shown in Fig. 5C, variations in f_o between 0.002 and 0.02 have an insignificant effect on conduit pressures for typical Kilauea magma and conduit conditions.

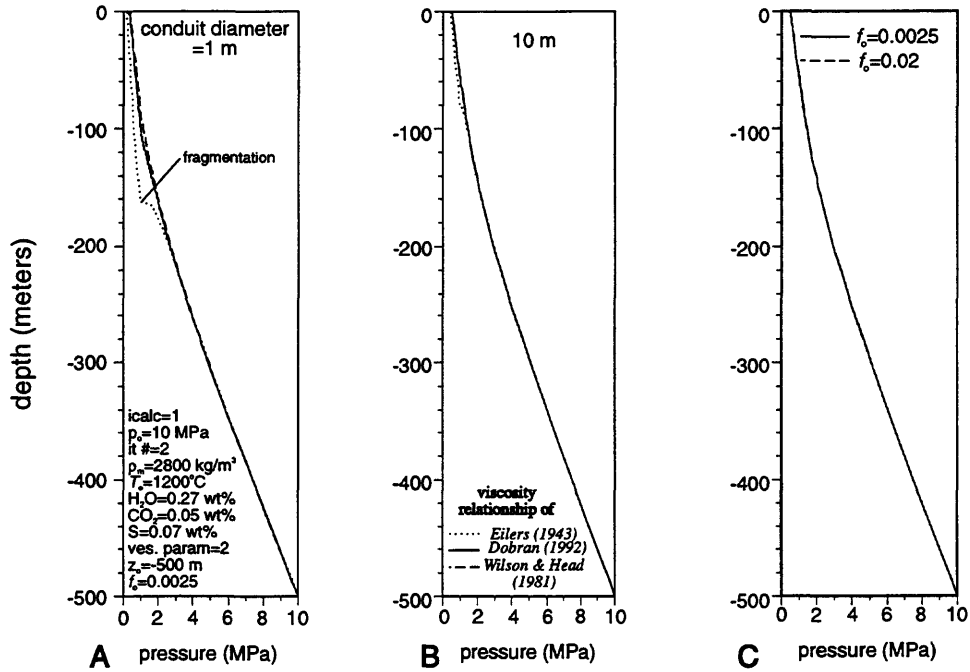


Figure 5: A) Pressure profile in a 500-m-long conduit, 1 meter in diameter throughout, calculated using the input parameters shown, with the viscosity/vesicularity relationships of Eilers (1943) (dotted line), of Wilson and Head (1981) (dashed line), and that from eq. 18 of the text (solid line). The pressure discontinuity in the Eilers model at 160 m depth is due to the abrupt change in viscosity at the point of fragmentation (assumed where vesicularity=75%). B) The same comparison for a 10-meter diameter conduit. Curves using relationships from eq. 18 and Wilson and Head are indistinguishable. C) Comparison of the pressure profile in a 10-m diameter conduit for the same input conditions as A and B, using the viscosity-vesicularity relationship from eq. 18. Two lines are plotted that represent results using roughness factors (f_s) of 0.0025 (solid line) and 0.02 (dashed line). The lines are indistinguishable.

Mach number

The Mach number of the mixture is its velocity divided by the mixture's (approximate) sonic velocity (c). The latter is defined as

$$c^2 = \left(\frac{\partial p}{\partial \rho} \right)_s \quad \text{eq. 19}$$

where the subscript s indicates constant entropy conditions. This equation can also be written in terms analogous to seismic velocity equations, as

$$c^2 = \frac{K}{\rho} \quad \text{eq. 20}$$

where K is the bulk modulus of the mixture. For a dispersed mixture of particles in gas, the bulk modulus is:

$$\frac{1}{K} = \frac{1 - \phi}{K_m} + \frac{\phi}{K_g} \quad \text{eq. 21}$$

where ϕ is the volume fraction gas, and K_m , and K_g are the bulk moduli of liquid magma and gas, respectively. The bulk modulus of unvesiculated magma, like rock (Jaeger and Cooke, 1979), is probably of the order 10^5 megapascals (MPa), while bulk moduli of the gas phase can be calculated from ideal gas relations:

$$B_g = \rho_g \left(\frac{\partial p}{\partial \rho} \right)_s = \rho_g \gamma_g \frac{n}{m_g} RT \quad \text{eq. 22}$$

where γ_g is the ratio of specific heat at constant pressure ($c_{p,g}$) to specific heat at constant volume ($c_{v,g}$) of the gas phase, n is the number of moles of gas per kilogram of magma/gas mixture (presented in Appendix B), and m_g is the mass fraction gas in the mixture. The value of c_p for each gas species is calculated using empirical relations from Moran and Shapiro (1992, Appendix A-15) given in Appendix C. The values of c_v are calculated from the ideal gas relationship (Moran and Shapiro, 1992, p. 97). For CO_2 , this relationship is expressed as:

$$c_{v,\text{CO}_2} = c_{p,\text{CO}_2} - \frac{R}{M_{\text{CO}_2}} \quad \text{eq. 23}$$

where M_{CO_2} is the molar weight of the gas species, in kilograms (kg) per mole. The relationship is analogous for the other gas species. The values of $c_{p,g}$ and $c_{v,g}$ for the gas phase are calculated as follows:

$$c_{p,g} = \frac{W_{\text{CO}_2,e} c_{p,\text{CO}_2} + W_{\text{H}_2\text{O},e} c_{p,\text{H}_2\text{O}} + W_{s,e} c_{p,s}}{W_{\text{CO}_2,e} + W_{\text{H}_2\text{O},e} + W_{s,e}} \quad \text{eq. 24}$$

$$c_{v,g} = \frac{W_{\text{CO}_2,e} c_{v,\text{CO}_2} + W_{\text{H}_2\text{O},e} c_{v,\text{H}_2\text{O}} + W_{s,e} c_{v,s}}{W_{\text{CO}_2,e} + W_{\text{H}_2\text{O},e} + W_{s,e}} \quad \text{eq. 25}$$

Numerical Procedure

For the case of constant cross sectional area in the conduit, all terms on the right-hand side of eq. 5 can be determined as long as the pressure and velocity at the base of the conduit are specified. By calculating dp/dz from eq. 5, a new pressure can be extrapolated to a higher point in the conduit. The continuity equation, eq. 1, as well as the constitutive relations in equations 9-25 and the appendices, can be used to evaluate density, velocity, friction factor, and Mach number at this new depth. Using these values, a new dp/dz can be evaluated using eq. 5, and the procedure is repeated from the base to the top of the conduit. For the case of constant pressure gradient, the procedure is the

same except that a new gradient in cross-sectional area is evaluated at each depth using eq. 8, rather than a new pressure gradient using eq. 5.

The integration was carried out with a standard fourth-order Runge-Kutta method, using a subroutine (named "RK4") in Press et al. (1986, p. 550). A second subroutine ("RKQC") from Press et al. (1986, p. 554), was used to automatically adjust the vertical step size (δz) throughout the conduit, to concentrate calculations at points where properties are changing most rapidly. The details of those subroutines are described in Press et al.

TESTING THE MODEL

In a strict sense, it is not possible to conclusively demonstrate the validity of any geophysical model, given the uncertainty in natural conditions that exist within the earth (Oreskes et al., 1994). In practice, however, one can generally develop confidence that a numerical model is accurately simulating a particular phenomenon by comparing the model's calculations with the observations of controlled experiments. For engineering purposes, numerous experiments of critical, two-phase flow in conduits have been carried out and compared with various model results (Wallis, 1980). Unfortunately, few if any experiments have attempted to model the quantitative aspects of two-phase critical flow of magma with exsolving volatile species. Moreover, the scale-dependent aspects of this phenomenon make it very difficult to construct such experiments and maintain dynamic similarity.

The testing of the model HICON is therefore done in a somewhat less rigorous manner; by comparing the model's results with certain special cases where the flow properties can be calculated using independently derived formulas or procedures. The comparisons will be made as follows:

First, I compare the results of HICON with the simplest form of conduit flow: laminar flow of a single-phase, incompressible Newtonian fluid in a vent of constant cross-sectional area, under flow velocities approaching $M=0$. Under those conditions, an analytical solution exists that relates pressure to velocity, viscosity, conduit radius, and conduit length.

Second, using a mixture with very high mass fraction of gas, I compare the results of a modified version of HICON with analytical solutions for flow of an ideal gas through a frictionless nozzle.

Third, I compare the results of HICON with published results from the conduit model of Wilson and Head (1981), using similar input conditions.

Steady Flow of Incompressible Fluid in a Conduit of Constant Cross-sectional Area

The continuity equation (eq. 1) for this case reduces to $dp=0$. Equation 5 reduces to:

$$-\frac{dp}{dz} = \rho g + \frac{f \rho v^2}{r} \quad \text{eq. 26}$$

Substituting $f=16/Re$ (where Re is the Reynolds number), and considering that $Re=2\rho vr/\eta$, the equation can be rewritten as follows:

$$-\frac{dp}{dz} = \rho g + \frac{8\eta v}{r^2} \quad \text{eq. 27}$$

This is easily integrated to give:

$$p_f - p_o = -\left(\rho g + \frac{8\eta v}{r^2}\right)(z_f - z_o) \quad \text{eq.28}$$

where the subscripts f and o refer to the final and initial values, respectively, of p and z . Figure 6 (top) shows $(p_o - p_f)$ versus $(z_f - z_o)$ calculated for conduit flow with a volatile-free magma at 1200°C (40.36 Pa s viscosity), flowing at 1 m/s. The results given by the program (all plotted symbols, except for the dark rectangles) match the analytical solutions (solid lines) more or less exactly, except for the smallest conduit ($r=0.1$ m), where they underestimate $(p_o - p_f)$ at high values of $(z_f - z_o)$. The discrepancy is due to viscous heating of the magma, which raises its temperature to 1210° after 1000 m of flow (middle plot), and hence decreases its viscosity to about 33 Pa s (lower plot). If the program constrains the viscosity to remain constant at 40.36 Pa s, regardless of temperature, its results (dark rectangles in top plot, Fig. 6) match the analytical solution. Frictional heating in conduits larger than 1 m radius is insignificant. Moreover, because of the small role played by friction, the pressure gradient in conduits of 1 m radius or larger is nearly identical to that from the weight of the magma alone (dashed line on upper plot). This is also true for vesiculating magma.

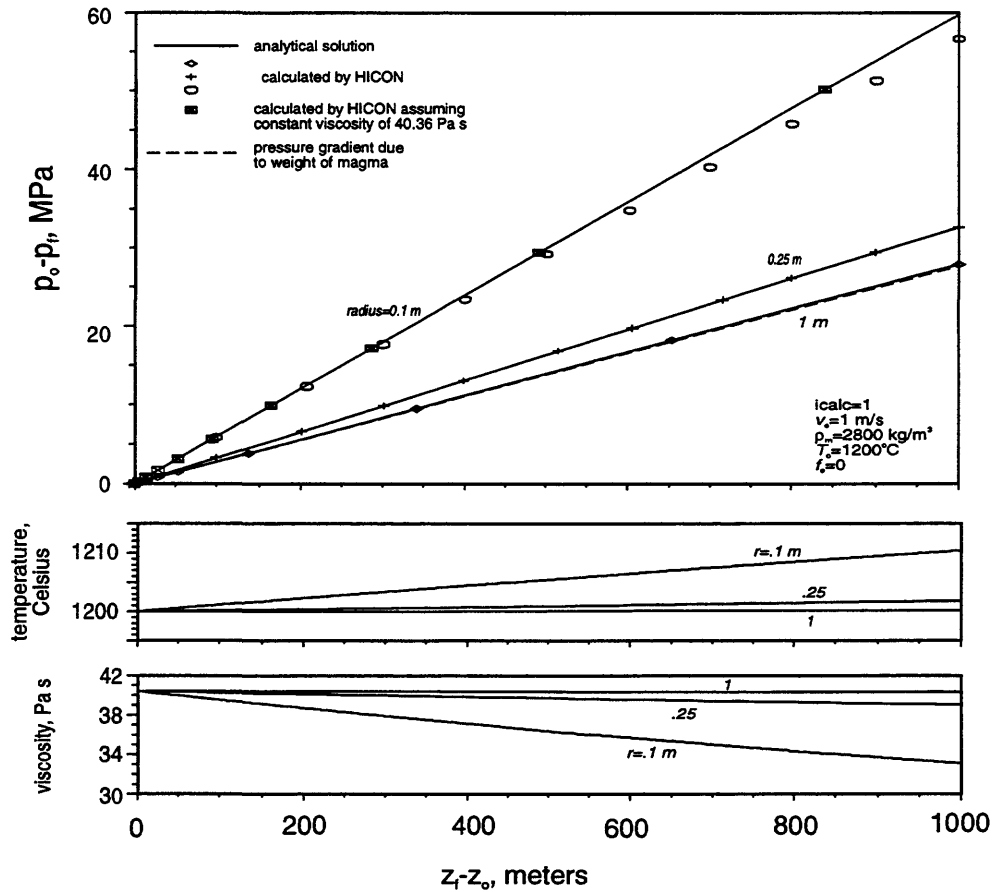


Figure 6: (top) Pressure drop ($p_r - p_o$) as a function of distance travelled ($z_o - z_r$) up a conduit of constant cross-sectional area, for a magma under laminar flow with no exsolved volatiles, and with other input conditions as listed. Details are explained in the text.

Choked Flow of a Gaseous Mixture Through a Nozzle

For an ideal gas with values of c_p and c_v that do not change with temperature, relationships between pressure, temperature, density, Mach number, and other variables for one-dimensional, frictionless flow through nozzles and diffusers are well developed (e.g. Liepmann and Roshko, 1957; Saad, 1985). Those relationships ignore the weight of the fluid (i.e. they assume there is no “pg” term in eqs. 5 and 8). Because those relationships assume ideal gas behavior, they also assume that no new gas is being generated (for example, by exsolution) during flow. Dilute gas/particle mixtures in volcanic eruptions have been occasionally modeled as frictionless, weightless ideal gases (Kieffer, 1981, 1984; Turcotte et al., 1990). These models assume that such mixtures roughly obey the $p v = n R T$ relationship. The assumption of ideal gas behavior tends to be more valid as the volume fraction (or mass fraction) of gas in the mixture increases.

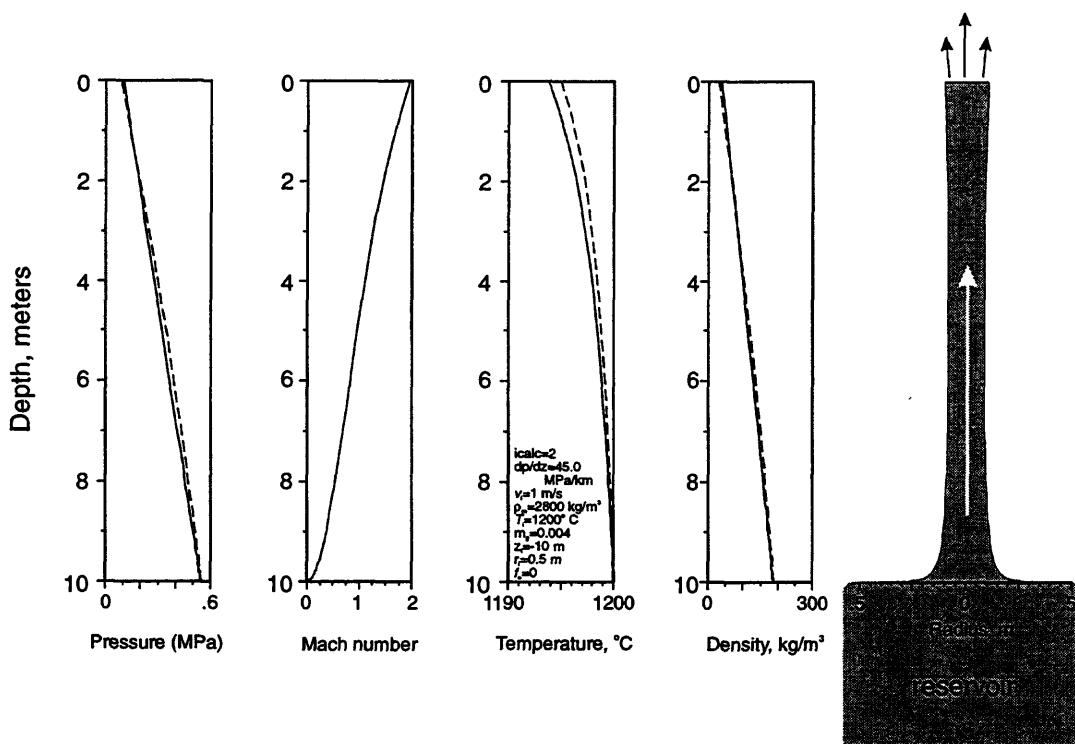


Figure 7: Flow properties for choked flow of fragmented magma and gas, calculated from a modified version of HICON, as explained in the text. The input conditions are shown in the plot. Solid lines are results calculated by HICON. Dashed lines are calculated for ideal pseudogases using eqs. 31-33.

Using these assumptions, pressure-velocity relationships of adiabatically decompressing ideal pseudogases follow the relationship (Kieffer, 1984):

$$pv^\gamma = \text{constant} \quad \text{eq. 29}$$

where γ is the ratio c_p/c_v of the gas/particulate mixture. For air, $\gamma=1.4$. For gas/particulate mixtures, the parameter γ is calculated from the following formula:

$$\gamma = \frac{c_p}{c_v} = \frac{m_g c_{p,g} + m_m c_m}{m_g c_{v,g} + m_m c_m} \quad \text{eq. 30}$$

For Kilauean basalt, m_m is about 0.996 ($m_g \approx 0.004$), and γ is only about 1.001838.

By combining eq. 29 with the continuity and momentum equations for an ideal gas, one obtains the following relationships between pressure (p_{ideal}), density (ρ_{ideal}), temperature (T_{ideal}), and Mach number for flow within a nozzle (Saad, 1985, p. 85-88):

$$\frac{T_0}{T_{\text{ideal}}} = 1 + \frac{\gamma - 1}{2} M^2 \quad \text{eq. 31}$$

$$\frac{p_o}{p_{ideal}} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \quad \text{eq. 32}$$

$$\frac{\rho_o}{\rho_{ideal}} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} \quad \text{eq. 33}$$

where T_o , p_o , and ρ_o are the temperature, pressure, and density of the mixture in an upstream reservoir where the velocity is negligible. If T_o , p_o , and ρ_o are known, and the Mach number at a particular point in the nozzle is known, then the temperature, pressure, and density at those points can be calculated.

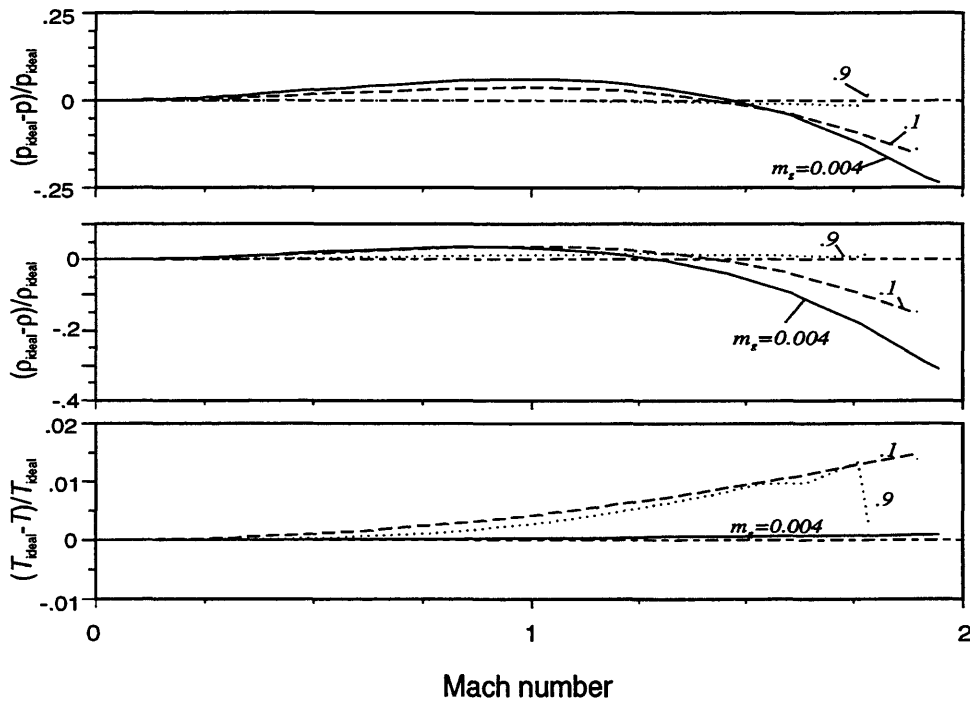


Figure 8: (top) Difference between pressure at a given Mach number calculated for choked flow of an ideal pseudogas (p_{ideal}) and pressure (p) calculated at the same Mach number using a modified version of the program HICON, normalized to p_{ideal} . Middle and lower plots show similar relationships for mixture density (middle plot) and for absolute temperature (lower plot). Curves are plotted for three values of mass fraction gas (m_g): 0.004 (solid line), 0.1 (long-dashed line), and 0.9 (dotted line). The long-dashed, short-dashed line is the zero line.

An ideal gas/particulate mixture can be approximated in the program HICON by making the following changes: (1) Disable the subroutine that calculates gas exsolution, so that the mass fraction gas remains constant throughout the conduit. To simplify calculation of c_p , c_v , γ , and molar weights, the gas phase is assumed to be entirely H_2O . With these changes, the mass fraction of the gas phase (m_g) is given as input to the program, rather than initial weight percent of H_2O , CO_2 , and S. (2) Set f_o to 0, to approximate frictionless conditions. The friction factor also depends on the Reynolds

number, but Reynolds numbers used in these runs typically exceed 10^6 , making the first term in eq. 13 insignificant. (3) Remove the “ ρg ” term from the momentum equation, so that the model is not calculating pressure change due to weight of the mixture.

Using these modifications, flow through the conduit is calculated by setting a constant pressure gradient (icalc=2) and having the program calculate the cross-sectional profile. The model then calculates the Mach number, temperature, density, and pressure at each point. Those properties are plotted (solid lines) as a function of conduit position in Fig. 7 for a mass fraction gas of 0.004, with other input conditions given in the figure. At each depth, using the Mach number calculated by HICON, the ideal pseudogas values of density, pressure, and temperature were calculated using eqs. 31-33. Those values are plotted in dashed lines.

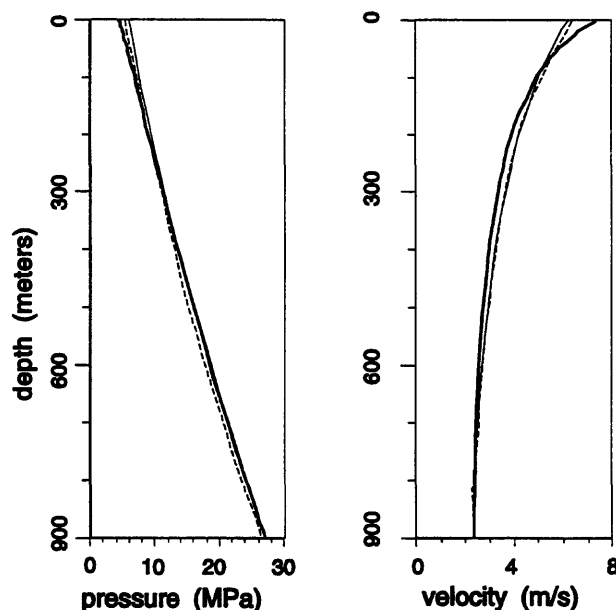


Figure 9: Comparison of model results of Wilson and Head (1981) for flow up a conduit with constant cross-sectional area (light dashed lines) with a version of HICON, modified as explained in the text (light solid lines). Results from Wilson and Head (1981) were obtained by digitizing lines from Figures 3 and 5 in their paper. The heavy solid lines give the result using the unmodified version of HICON, with comparable input conditions (Table 1, right column).

In Fig. 8 (top), the pressure calculated by HICON at each point in that run was subtracted from the pressure (p_{ideal}) calculated using eq. 32 for the same Mach number and a gamma value of 1.001838. That difference ($p_{ideal}-p$), normalized to p_{ideal} , is plotted (solid line) as a function of Mach number on the top plot. Similar calculations were made for $m_g=0.1$ (dashed line) and 0.9 (dotted line). The long-dashed, short-dashed line represents $(p_{ideal}-p)/p_{ideal}=0$.

It is clear that, as m_g approaches 1, the difference between the pressure calculated by HICON and the ideal-gas solution approaches zero. The same is true for mixture density (middle plot). The one exception to this tendency is the temperature calculation (lower plot). As m_g increases, it tends to differ more from the ideal gas solution. Moreover, at $m_g=0.9$, it appears to become unstable at high Mach numbers. This tendency is due to the

approximate manner in which temperature is calculated, as described in Appendix C. Mixture temperatures are not calculated by iteration, hence the calculated temperatures tend to be accurate so long as the temperature does not change greatly within the conduit. Gas-poor mixtures do not expand or cool very much when they decompress; so their temperature calculations are fairly accurate. Gas-rich mixtures, on the other hand, expand greatly and therefore experience more adiabatic cooling. The errors caused by the approximate temperature calculations are apparently not great enough to offset the general tendency for pressure and density values to approach the ideal gas values as m_g approaches 1.

Comparison of Results with those of Wilson and Head (1981)

Like the model HICON, that of Wilson and Head (1981) calculates equilibrium frictional flow of a homogeneous magma/gas mixture in a vertical conduit. Their model differs from HICON in only a couple of respects: (1) Wilson and Head use a simpler gas exsolution law based on Henry's Law, with the following form:

$$m_g = s p^\beta \quad \text{eq. 34}$$

where s and β are empirically derived constants. For basalt, Wilson and Head use values of $6.8 \times 10^{-10} \text{ Pa}^{-0.7}$ and $\beta=0.7$. Wilson and Head also assume that the gas phase is composed entirely of H_2O . (2) Wilson and Head use a magma viscosity specified as an input value (rather than calculated from temperature), which doesn't change with vesicularity prior to fragmentation. Finally, (3) they use an analytical formula to calculate the pressure-depth curve below the depth of initial gas exsolution.

The program HICON was modified to incorporate these changes, then run using input values (Table 1) similar to those used to generate curves "D" in Figs. 3 and 5 of Wilson and Head (1981). The results, shown as light, solid lines in Fig. 9, are nearly identical to those of Wilson and Head (light dashed lines) obtained by digitizing their curves. Minor differences are probably due to slight variations in the numerical procedure, or to errors in digitizing lines from their plots. The unmodified version of HICON was also run (heavy, solid lines, Fig. 9) using similar input parameters (Table 1). The difference between the results of the unmodified version of HICON and those of Wilson and Head are primarily due to the different gas exsolution law. The former calculates somewhat lower velocities than those of Wilson and Head at depths of 100-600 m, but higher velocities (by up to ~10%) at shallower depths.

The fact that the current model agrees with that from Wilson and Head for this one particular case does not necessarily indicate that either model is "correct", in the sense that it accurately models the natural phenomenon of magmatic eruptions. However given the fact that the assumptions for both models are similar, one would expect them to produce similar results if there were no errors in the numerical code. The agreement between their results suggests that, for this one particular case at least, no such errors are apparent.

Table 1: Input values used to generate lines in Figure 9.

Parameter	Value		
	<i>W & H (1981)</i>	<i>modified version of HICON</i>	<i>unmodified version of HICON</i>
icalc	1	1	1
p_o	27 MPa ¹	27 MPa	27 MPa
v_o	2.36 m/s	2.36 m/s	2.36 m/s
iteration #	1	1	1
ρ_m	2800 kg/m ³	2800 kg/m ³	2800 kg/m ³
T_o	1200 °K	1200°K	1323° K ²
η	10 ³ Pa s	10 ³ Pa s	-- ³
H ₂ O	1.0 wt %	1.0 wt %	0.88 wt. %
CO ₂	0 wt % ⁴	0 wt %	0.05 wt. %
S	0 wt %	0 wt %	0.07 wt. %
z_o	-900 m	-900 m	-900 m
diameter	6.2 m	6.2 m	6.2 m
f_o	0.0025 ⁵	0.0025	0.0025

Notes

¹For run "D" in figures 3 and 5 of Wilson and Head (1981), they specified input pressure and velocity, and made no attempt to satisfy the exit conditions of $p=1$ atm or $M=1$.

²Temperature that corresponds to a viscosity of 10³ Pa s in Hawaiian basalts

³viscosity is not an input parameter in this program, but is calculated from temperature.

⁴solubility of CO₂ and sulfur species is not considered in these models.

⁵Wilson and Head use a different definition of f_o . Their value of $f_o=0.01$ is equivalent to $f_o=0.0025$ in this study.

INPUT TO THE MODEL

On the following page is a sample input file for the program HICON. The twelve lines following the first line of the file contain the input parameters on the left side. Those parameters are read using unformatted read statements, so they can be changed without worrying about column numbers or number of decimal places. Just be careful not to add or delete any lines while editing the file. All variables are double precision, real numbers, with the exceptions of icalc, the vesiculation parameter, and the iteration number, which are integers.

The right hand side of each line explains (briefly) what each parameter represents. Parameter explanations that require somewhat more information are followed by asterisks or “plus” signs, with supplemental information on following lines. Although most parameters are self-explanatory, a few could benefit from further information:

icalc

This parameter specifies which option to use when running the program. If icalc=1, the program assumes a constant conduit diameter and calculates a pressure profile. If icalc=2, constant pressure gradient is assumed and the program calculates the profile in cross-sectional area that would produce such a pressure gradient.

Pressure at base of conduit

This parameter is used only if icalc=1. There is no real upper limit to the maximum input pressure that can be used. However the lower limit is constrained by the weight of the magma in the conduit: if the input pressure is less than that weight, the magma will not erupt. In such a case, the model will reach $p=1$ atm at some depth below the surface. If the model is set to iterate until $p=1$ atm or $M=1$ at the surface, it will decrease the velocity at the base of the conduit and try another run. If, after several iterations, the initial velocity drops to 0.001 m/s and $p=1$ atm is still reached below the ground surface, the program returns the following message to the screen:

```
pressure insufficient to produce eruption
```

and writes the results of the last run (in which initial velocity=0.001 m/s) to the output file. The following table indicates the minimum pressures that will produce upflow for various conduit lengths, given other input parameters identical to those shown in the example input file:

depth at base of conduit	minimum pressure for upflow (MPa)
100 m	0.17
200	0.35
500	4.35
1000	17.1
3000	71.6

Sample input file for the program HICON

INPUT PARAMETERS:	PARAMETER EXPLANATIONS:
1	icalc+
74.0	pressure at base of conduit (MPa) (used only if icalc=1)
28.5	conduit pressure gradient, MPa/km (used only if icalc=2)
2	iteration number++
1.0	initial velocity (m/s)
2800.	unvesiculated magma density (kg/m3)
1200.	initial temperature (C)
0.27, 0.05, 0.07	initial h2o, co2, S content (wt%)
2	vesiculation parameter*
-3000.	initial depth (m)
10.0	conduit diameter (m)**
0.0025	fo (wall rock roughness term)

NOTES ON INPUT PARAMETERS:

+icalc= 1 if specifying conduit of constant x-sectional area, and having the program calculate the pressure profile,
 2 if specifying a constant pressure gradient in the conduit, and having the program calculate x-sectional area.

++iteration #=2 if the velocity is to be adjusted automatically to reach sonic velocities at the exit (valid only if icalc=1), or 1 if no adjustment is desired.

*vesiculation p.= 2 if gas exsolution is to stop after fragmentation 1 if not

**diameter = diameter throughout the conduit if icalc=1 diameter at base of conduit if icalc=2

OUTPUT PARAMETERS:

List of variables to be written out. Enter a number in the first column indicating the column # where this variable will be written in the output file. You can write out up to seven variables.

1	x-sectional area (m2)
5	Mach number
7	pressure (MPa)
	Reynolds number
	mixture density
2	time (seconds) since entering the conduit
4	velocity (m/s)
3	volume fraction gas
	log viscosity (Pa s)
1	z (depth, meters)
	dadz (change in x-sectional area with depth, m2/m)
	log(pressure)
	dpdz (pressure gradient, Pa/m)
	dz (vertical step size, meters)
	exco2 (wt % exsolved co2)
	exh2o (wt % exsolved h2o)
	exsulfur (wt % exsolved s)
	f (friction factor)
	gamma (Cp/Cv for gas phase)
	mf (mass fraction exsolved gas)
	mm (mass fraction exsolved magma)
	r (Universal Gas const. * n)
	rhof (gas density)
	sv (sonic velocity (m/s)
	xco2 (mole fraction co2 in gas)
	xh2c (mole fraction h2o in gas)
	xsulfur (mole fraction sulfur species in gas)
	cph2o (sp. heat at const. p. of H2O gas, kJ/kmol K)
	cpco2 (" " " " " " CO2 ")
	cps (" " " " " " S species)
	cvh2o (" " " " " " v. " H2O gas, kJ/kmol K)
	cvco2 (" " " " " " CO2 ")
	cvs (" " " " " " S species)
	temperature (C)
6	enthalpy of mixture (kJ/kg)
	cp (sp. heat at const. p) of gas phase (kJ/kg C)
	conduit radius (m)

In reality, significantly higher pressures would be necessary at these depths to drive eruptions, since gas escape at low magma velocities would densify the magma column and increase its weight.

Conduit pressure gradient

The conduit pressure gradient is specified in option 2 (icalc=2), and the conduit cross-sectional area adjusted, along with flow properties, to fit this gradient. Previous models of conduit flow (e.g. Wilson and Head, 1981; Dobran, 1992) generally assume that the pressure gradient driving magma flow is the gradient $p_{cr,g}$, determined by the country rock density, ρ_{cr} . In those programs, if a country rock density of 2300 kg/m^3 is used as input, the program calculates a pressure gradient of $p_{cr,g}=2.25 \times 10^4 \text{ Pa/m}$, and a pressure at the base of a 3-km-long conduit of $1.013 \times 10^5 \text{ Pa} + (3000\text{m})(2.25 \times 10^4 \text{ Pa/m}) = 6.78 \times 10^7 \text{ Pa}$, or 67.8 MPa. In fact, far-field horizontal stress gradients may be as important as the lithostatic pressure gradient in controlling the flow up the conduit. In the program HICON, a vertical pressure gradient is given directly as input to the program, rather than a rock density from which a pressure gradient is calculated.

There is one caveat when considering the input value for this parameter. If the pressure gradient in the conduit is less than that due to the weight of the magma/gas mixture at the base of the conduit, the magma will not flow upward. In that case, the following error message will appear:

```
Density of magma/gas mixture = 2840. kg/m3. (for example)
Thus its pressure gradient is 28.4 MPa/km.
This is greater than that specified for the conduit.
It must be LESS THAN that of the conduit
or else magma will not erupt.

program stopped.
```

If you receive this message, you will have to increase the pressure gradient and try again.

Iteration number

If this number is 2, the velocity is adjusted to match the exit boundary conditions (this only applies if icalc=2). That is, the program will iterate until either (1) the output pressure is between 0.1012 and 0.1014 MPa (1 atm= 0.1013 Mpa), or (2) until $M=1$ is reached (to double-precision accuracy) within 0.05 meters of the surface.

On a few occasions, the program may have some difficulty reaching a solution within the tolerance levels specified above. Sometimes this problem is due to the fact that final exit pressures or velocities are extremely sensitive to the input velocity, and very slight changes in input velocity (usually less than 10^{-4} m/s) cannot produce an acceptable result. In such a case, the program stops, writes out the results of its best run, and prints the following message to the screen:

```
limit of resolution reached
```

On more rare occasions, the program just won't converge at all. If this happens, a slight change to an input parameter will usually solve the problem.

If the iteration number is 1, the program calculates a single run up the conduit and writes out the results, regardless of what the output pressures and velocities are. If the velocity of the mixture reaches the sonic velocity (Mach number, M , =1) before the calculations reach the top of the conduit, the program stops at that point. The same is the case if the pressure drops below atmospheric before the calculations reach the surface.

Initial velocity

In option 1 (icalc=1), if the iteration number=1, the velocity is adjusted until the output pressure=1 atm or the output velocity=sonic velocity of the mixture. Under these circumstances, the initial input velocity is only the starting point of the iteration sequence. If icalc=2, or the iteration number=2, then the initial velocity is used for the final solution.

Unvesiculated magma density

This parameter is used to determine the bulk density of the magma/gas mixture.

Initial temperature

Used to calculate viscosity of magma, specific volume of the gas phase (using ideal gas relationships), and enthalpy of the magma/gas mixture. Kilauean magma temperatures are typically 1150-1200° C (Helz and Thornber, 1987), with the higher part of the temperature range corresponding to more primitive magmas.

Initial H₂O, CO₂, and S content

Used to calculate amount and species of exsolved gases, and vesicularity, at a given pressure. The program can handle a wide range of these variables. The only combination (found thus far) that it can't handle are zero-values of H₂O, and non-zero values of CO₂ and S. Under those circumstances, it returns with the following message and stops:

```
Sorry, I can't handle zero-values of H2O and  
non-zero values of CO2 and sulfur. Please try  
another combination.
```

In general, dissolved H₂O in Kilauean magmas is about 0.27 wt %. Dissolved CO₂ is about 0.02-0.05 wt.% in magmas that have equilibrated in the summit magma chamber, but could approach 1 wt.% if the magma comes directly from the upper mantle (Gerlach, 1986). Sulfur contents typically range around 0.07-0.12 wt.% (Gerlach, 1986).

Vesiculation parameter

In the uppermost several tens of meters of the conduit, when vesicularity and eruptive velocities are high, gas exsolution rates may not keep pace with the rate of depressurization. If the vesiculation parameter is set to 2, gas exsolution is no longer computed once the vesicularity reaches 75% (though gas expansion due to decompression is still calculated). If the vesiculation parameter=1, gas exsolution is also calculated at vesicularities above 75%.

Initial depth

The depth of the base of the conduit. The computer program considers elevations below the ground surface to be negative, and they are written out as such in the output file. If the input value is given as a positive number, it is converted to a negative number by the program (i.e. assumed to be below surface elevation). The program can handle any arbitrary starting depth, from several kilometers (or more), essentially up to the ground surface. If unusually shallow starting depths are used, the mixture will already be highly vesiculated. This will be reflected in the output data.

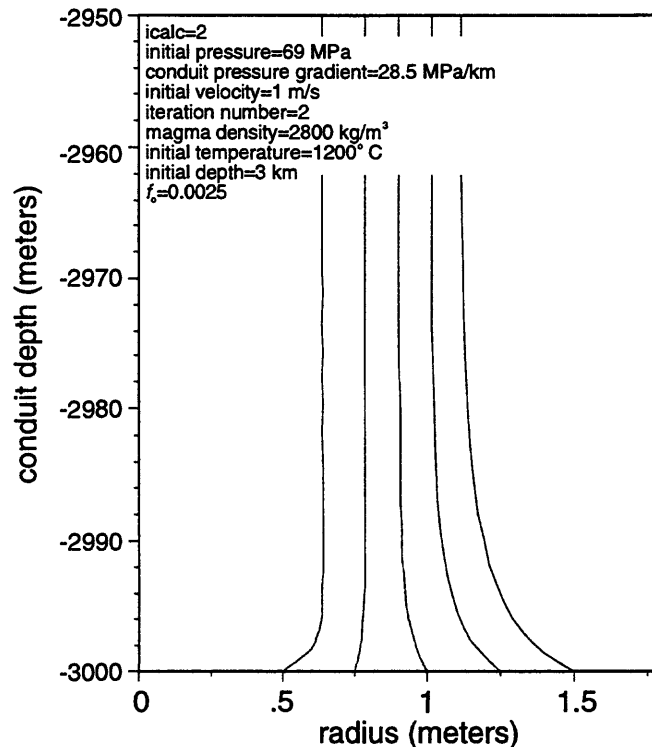


Figure 10: Changes in conduit radius in the lowermost 50 meters of an eruptive conduit, calculated using input parameters shown, using five input values for radius: 0.5, 0.75, 1, 1.25, and 1.5 meters. As the model calculates progressively higher up the conduit, radius values tend to migrate to an equilibrium, which may be greater or less than their initial value. Model runs made using initial radii that differed significantly from the equilibrium diameter require many more iterations to complete than those made where initial diameters were close to equilibrium values. In this case, for example, the run made using an initial radius of 0.5 m required more than three times the number of iterations as that made using an initial radius of 1 m.

Conduit diameter

If icalc=1, this is the diameter throughout the conduit. If icalc=2, it is the diameter at the base of the conduit. When using the icalc=2 option, for a given input velocity, there is a “natural” conduit diameter to which the program will migrate (Fig. 10). If the input diameter differs significantly from this natural diameter (or if the conduit is more than about 3 km long), the solution will require an unusually large number of iterations to complete. The program has been arbitrarily set to stop at 2000 iterations. If the top of

the conduit has not been reached by that time, the following message will appear on the terminal screen:

```
Number of iterations has exceeded the limit.
You should be able to perform this calculation in
fewer iterations. Check and see if the radius is
changing rapidly at the bottom of the conduit.
(if icalc=2). If so, try adjusting these
parameters and running it over again.
```

For input velocities of 0.5-2 m/s and conduit depths of 1-3 km, conduit diameters of 1-3 m generally work.

Wall rock roughness term

This term controls frictional pressure losses in the conduit when flow becomes turbulent (usually the uppermost 100 meters or so in lava-fountain eruptions). Experimental values of f_0 range from about 0.001 to 0.02 (Bird et al., 1960); values of around 0.0025 are commonly used to model flow in rough-walled eruptive conduits (Wilson et al., 1980⁴; Gilberti and Wilson, 1990). Variations in f_0 between 0.002 and 0.02 have a minor effect on conduit flow.

MODEL OUTPUT

The last 37 lines of the input file contain the names of variables that can be written to the output file for each depth. You may specify up to seven variables to be written out, by entering a number to the left of each variable, corresponding to the column in the output file where this variable will appear. When the program is executed and a final solution is reached, the program writes out a table containing flow properties at each depth calculated. A run using the example input file, for example, would produce an output file whose first eleven lines look like:

120	i	z (m)	time (s)	vfgas	vel (m/s)	mach #	h (kJ/kg)	p (MPa)
	1	-3000.0000	0.0000	0.0009	1.5321	0.0004	1499.6374	74.0000
	2	-2970.0000	19.5799	0.0009	1.5322	0.0004	1499.3419	73.1759
	3	-2919.0000	52.8623	0.0011	1.5324	0.0004	1498.8416	71.7752
	4	-2832.3000	109.4314	0.0014	1.5328	0.0004	1497.9911	69.3944
	5	-2532.0938	305.1825	0.0024	1.5344	0.0005	1495.0461	61.1557
	6	-2033.7189	629.5578	0.0052	1.5384	0.0008	1490.1570	47.5021
	7	-1695.9606	848.7734	0.0082	1.5431	0.0011	1486.8436	38.2747
	8	-1423.5309	1024.9729	0.0122	1.5492	0.0015	1484.1710	30.8577
	9	-1214.2431	1159.7379	0.0171	1.5568	0.0020	1482.1179	25.1846
	10	-1048.9164	1265.6162	0.0230	1.5662	0.0025	1480.4961	20.7273

The first line contains (at left) the number of iterations (120) calculated between the base and the top of the conduit, as well as the names of each output parameter. The following lines list (from left to right) the iteration number, depth, residence time of

⁴ The friction factor defined by Bird et al. (1960), used here, differs by a factor of four from that defined by Schlichting (1968, p. 86) and used by Wilson et al. (1980). Therefore the second term in eq. 2 also differs from the corresponding term in eq. 1 of Wilson et al. (1980).

magma in the conduit when it reaches that depth, volume fraction gas, velocity, Mach number, specific enthalpy, and pressure.

If, by some oversight, you specify more than seven variables, or you specify that more (or less) than one variable be written out to a particular column, the program will return with the following message:

```
output parameters incorrectly listed.  
program stopped.
```

MODEL EXECUTION

If you are using the executable file HICON on a DOS-based computer, the program can be executed simply by moving to the directory where it resides, and typing "HICON" on the DOS command line. If your computer uses Microsoft Windows®, you should exit Windows before executing the program. Also, if you move HICON.EXE to another drive or directory and intend to run it from there, be sure either to move the input file, HICIN, and the executable file, DOSXMSF.EXE, to the same directory, or modify your AUTOEXEC.BAT file to include their directory paths so the program can find them.

Two examples of program execution are given below: one using option 1 (constant conduit diameter), the other using option 2 (constant pressure gradient).

Example using option 1

Once the program is started, it will make the following request for a file name to which the output parameters will be written:

```
enter name of output file:
```

Enter whatever file name you wish, up to 40 characters in length. In this example, we'll call the output file name "outfile". Once you have entered a file name, the program will write out the input parameters that it has read from the input file, as follows:

```
INPUT VALUES:
```

```
input velocity = 1.0000 m/s  
magma density= 2800. kg/m3  
input temperature = 1200. degrees Celsius  
fo (wall roughness) = 0.0025  
initial dissolved h2o= 0.270 wt %  
initial dissolved co2= 0.050 wt %  
initial dissolved S= 0.070 wt %
```

```
assume constant conduit diameter:  
diameter = 10.000 meters  
input pressure = 74.00 MPa
```

```
automatic velocity adjustment
```

```
no exsolution after fragmentation
```

These are the same input parameters specified in the example input file above. For this run, the conduit diameter is taken to be constant (icalc=1) and the program is to adjust the input velocity until $M=1$ or $p=1$ atm at the surface.

Next, the program will begin a run, calculating flow properties from the bottom to the top of the conduit. The output to the screen during this run is:

```
STARTING RUN NUMBER 1:          mass flux= 0.2197E+06 kg/s

  i      z (m)  time (s)    vfgas vel (m/s)    mach #  h (kJ/kg)    p (MPa)
  1 -3000.000    0.000      0.001    1.000      0.000  1499.637    74.000
  80    0.000  2685.237      0.937    15.745      0.419  1470.083     0.197
exit pressure > 1 atm and M < 1
```

After writing out the mass flux calculation, the program then writes a line of output variables calculated at the bottom of the conduit, and one at the final depth (in this case, the surface). As the program notes on the last line, the final pressure is greater than 1 atm, and the final Mach number is less than 1. The program therefore increases the input velocity and computes a second run, writing the output as follows:

```
trying new input velocity 1.50000 m/s

STARTING RUN NUMBER 2:          mass flux= 0.3296E+06 kg/s

  i      z (m)  time (s)    vfgas vel (m/s)    mach #  h (kJ/kg)    p (MPa)
  1 -3000.000    0.000      0.001    1.500      0.000  1499.637    74.000
  89    0.000  1788.987      0.952    31.312      0.846  1469.717     0.147
exit pressure > 1 atm and M < 1
```

Again, the exit pressure exceeded 1 atm, and the Mach number was less than 1. After adjusting the input velocity again, a third run is attempted:

```
trying new input velocity 2.25000 m/s

STARTING RUN NUMBER 3:          mass flux= .4944E+06 kg/s

  i      z (m)  time (s)    vfgas vel (m/s)    mach #  h (kJ/kg)    p (MPa)
  1 -3000.000    .000      .001    2.250      .001  1499.639    74.000
 128 -46.759  1190.035      .940    37.450      1.000  1469.966     .186
mach number > 1. adjusting vi
```

This time, the sonic velocity was reached 47 meters below the surface. So now the model decreases the velocity and tries again:

```
trying new input velocity 2.21493 m/s

STARTING RUN NUMBER 4:          mass flux= .4867E+06 kg/s

  i      z (m)  time (s)    vfgas vel (m/s)    mach #  h (kJ/kg)    p (MPa)
  1 -3000.000    .000      .001    2.215      .001  1499.639    74.000
 123 -44.653  1208.994      .941    37.455      1.000  1469.945     .183
mach number > 1. adjusting vi
```

Again, the Mach number reached 1 before the surface was reached. A total of eight runs are required to match the output conditions. During the last run, the following information is written to the screen:

```
trying new input velocity 1.53189 m/s

STARTING RUN NUMBER 8:          mass flux= .3366E+06 kg/s

  i      z (m)  time (s)    vfgas vel (m/s)    mach #  h (kJ/kg)    p (MPa)
  1 -3000.000    .000      .001    1.532      .000  1499.637    74.000
 128  -.020  1751.659      .958    36.725      1.000  1469.533     .127

successful completion
```

```

AFTER ISENTROPIC EQUILIBRATION TO 1 ATM PRESSURE:
final temperature = 1192.53 deg. C
temperature change = .232 deg. K
enthalpy change = .2421E+03 J/kg
max. theoretical velocity = 58.73 m/s

```

```

maximum water table depth that will allow g.w. influx = -95.35 meters
(negative values are below ground surface, positive values are above)

```

This output shows that, during the last run, the Mach number reached 1 slightly before the mixture reached the surface, but it was within the 0.05 m considered acceptable.

In all runs where the Mach number=1 when the mixture exits the conduit, the pressure will be greater than atmospheric. After the mixture leaves the conduit, it will continue to accelerate and cool adiabatically as it drops to atmospheric pressure. If we assume that these processes take place isentropically (i.e. without friction), we can calculate a maximum theoretical velocity and a maximum amount of adiabatic cooling. These calculations are done by assuming that all excess enthalpy in the mixture is converted to kinetic energy during expansion (Mastin, 1995). Procedures for this calculation are explained in Appendix A. The output written above indicates that the velocity could theoretically accelerate from 36.7 m/s to 58.7 m/s after leaving the vent. The temperature at the exit (which is not listed in the output because we didn't request it in the input file) is 1192.77° C. It could theoretically cool to about 1192.53° during adiabatic expansion.

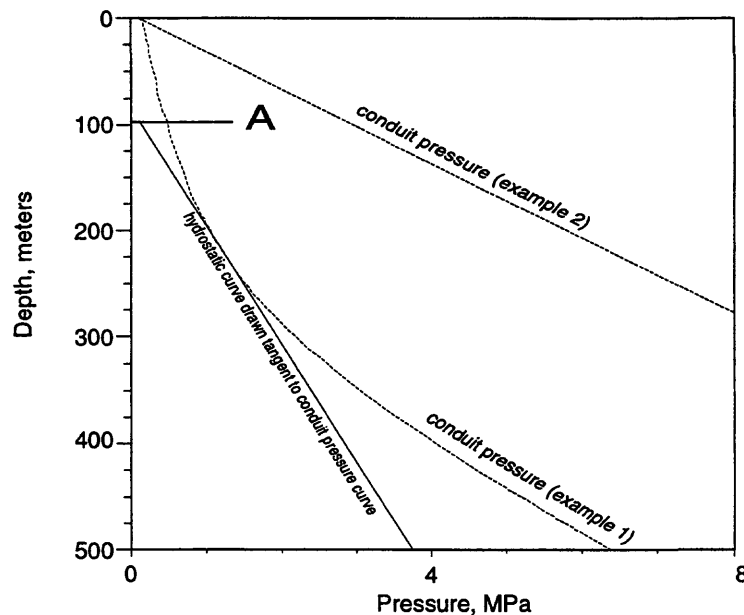


Figure 11: Pressure profile in uppermost 500 meters of eruptive conduit (dashed lines), from example runs 1 and 2. To calculate the maximum water table depth that will allow ground-water influx in example 1, a hydrostatic curve is numerically computed by the program (solid line) that is tangent to the conduit pressure curve. The depth at which this hydrostatic curve reaches 1 atm pressure (point A) gives the maximum water-table depth that will allow ground-water influx. In example 2, the conduit pressure curve is linear with a higher pressure gradient than the hydrostat, so the hydrostatic curve will not intersect the conduit pressure curve below the surface.

Program output. Once the program is completed, open the output file, `outfile`, and you will see the following table (already described):

•

•

•

For the second example, we've taken the sample input file and changed it slightly, so that icalc=2, the conduit diameter=1.0 m, and the conduit radius is written to column 6 in the output file instead of the mixture enthalpy (h). After typing HICON to start the program and entering the name of the output file, the following messages appear:

```
input velocity = 1.0000 m/s
magma density= 2800. kg/m3
input temperature = 1200. degrees Celsius
fo (wall roughness) = 0.0025
initial dissolved h2o= 0.270 wt %
initial dissolved co2= 0.050 wt %
initial dissolved S= 0.070 wt %

assume constant pressure gradient
pressure gradient = 28.5 MPa/km
initial conduit diameter = 1.000 meters

no velocity adjustment

no exsolution after fragmentation
```

This does not appear greatly different from the message list that came up when we ran the last example, except the program acknowledges that we are using a constant pressure gradient of 28.50 MPa/km. As is always the case when icalc=2, no velocity adjustment is made.

Messages that appear during the single run up the conduit are also similar to the last example:

```
STARTING RUN NUMBER 1:          mass flux= 0.2199E+04 kg/s

   i      z (m)   time (s)   vfgas vel (m/s)   mach #   radius   p (MPa)
   1 -3000.000    0.000    0.000    1.000    0.000    0.500    85.601
1515    0.000  2853.569    0.966    74.710    2.051    0.316    0.101

successful completion

AFTER ISENTROPIC EQUILIBRATION TO 1 ATM PRESSURE:
  final temperature = 1194.74 deg. C
  temperature change = 0.000 deg. K
  enthalpy change = 0.0000E+00 J/kg
  max. theoretical velocity = 74.71 m/s

maximum water table depth that will allow g.w. influx = 0.00 meters
(negative values are below ground surface, positive values are above)
```

The model required more than ten times as many iteration steps to make it up the conduit (a total of 1515, as seen under “i” on the left-hand side of the two tabulated lines of output). These runs typically require more iterations than those when icalc=1. Note also that the eruption Mach number (2.051) is much greater than 1, as it can be with a variable conduit geometry. Similarly, the exit velocity (74.71 m/s) is equal to the maximum theoretical velocity, because the erupting mixture has fully equilibrated with atmospheric pressure by the time it reaches the surface. The maximum water-table depth that will allow ground-water influx is zero, because the surface is the only place where the two pressure curves (the hydrostat and the conduit pressure curve) intersect (Fig. 11).

CLOSING COMMENTS

This report is intended to give the user a concise summary of the underlying principles of this program, and of its potential applications. Like most programs that get used, this one will probably evolve with time into something more complicated and, hopefully, more realistic. If you intend to make extensive use of this program or would like to find out about new revisions, you are encouraged to contact the author at (360) 696-7518 (e-mail at lgmastin@usgs.gov).

APPENDIX A: CALCULATION OF MAXIMUM THEORETICAL VELOCITY AND TEMPERATURE AFTER ISENTROPIC EQUILIBRATION TO 1 ATM PRESSURE

If the erupting mixture reaches the conduit exit before the pressure has dropped to 1 atmosphere, it will abruptly expand to equilibrate with atmospheric pressure. This decompression will be accompanied by expansion and adiabatic cooling. If these processes take place without frictional dissipation of energy, the process is said to be isentropic, and maximum amounts of acceleration and cooling can be calculated. In this program the calculations are done with the assumption that the mixture acts as an ideal “pseudogas” (Kieffer, 1984). That is, the mixture’s bulk properties approximately follow the ideal gas relationship, $pV=nRT$. For ideal gases and pseudogases expanding under adiabatic, isentropic conditions, the pressure and temperature before and after decompression are related by the equation (Moran and Shapiro, 1992, p. 104):

$$\frac{T_e}{T_f} = \left(\frac{p_e}{p_f} \right)^{(\gamma-1)/\gamma} \quad \text{eq. 1A}$$

where the subscript f refers to the “final” value in the conduit, before decompression, and “e” refers to the value after decompression. The variable γ is the ratio c_p/c_v , where c_p and c_v are the specific heats at constant pressure and constant volume, respectively, of the magma/gas mixture. Those specific heats are given by the equations:

$$c_p = m_g c_{p,g} + m_m c_m \quad \text{eq. 2A}$$

$$c_v = m_g c_{v,g} + m_m c_m \quad \text{eq. 3A}$$

where $c_{p,g}$, $c_{v,g}$, are the specific heats at constant pressure and volume, respectively, of the gas phase, and c_m is the specific heat of the liquid magma. The variables m_g and m_m are the mass fractions of gas and liquid magma, respectively. Specific heats of the gas phase are calculated as explained in Appendix C. The specific heat of the liquid magma is taken to be approximately 1 kJ/(kg K). The mass fractions of exsolved gas (m_g) and of liquid magma (m_m) are calculated using using solubility relations from Gerlach (1986) described in Appendix B.

Once the adiabatic temperature change has been calculated, the change in specific enthalpy (h) of the mixture during decompression is computed from the following equation for ideal gases (Moran and Shapiro, 1992, p. 96):

$$h_e - h_f = c_p(T_e - T_f) \quad \text{eq. 4A}$$

In addition to assuming ideal gas behavior, this equation assumes that c_p is invariant over the range of temperatures experienced during decompression.

The maximum theoretical velocity is then calculated assuming that all of the change in enthalpy of the expanding mixture is transformed into kinetic energy. This implies that an insignificant amount of energy goes into lifting of the material, or to frictional heating. In such a case, the maximum theoretical velocity (v_{\max}) would be:

$$v_{\max} = \sqrt{v_f^2 + 2(h_f - h_e)} \quad \text{eq. 5A}$$

APPENDIX B: CALCULATION OF EXSOLVED VOLATILES USING EQUATIONS OF GERLACH (1986)

The amounts of exsolved gas components are calculated using relationships for volatile solubility in Kilauean magmas derived by Gerlach (1986). His equations 1, 6, 7, 9, and 13 were used to solve for weight percent dissolved and exsolved CO_2 ($W_{\text{CO}_2,m}$, $W_{\text{CO}_2,e}$), weight percent dissolved and exsolved H_2O ($W_{\text{H}_2\text{O},m}$, $W_{\text{H}_2\text{O},e}$) and weight percent dissolved and exsolved sulfur ($W_{s,m}$, $W_{s,e}$), as follows:

First, the weight percent dissolved CO_2 was calculated using his equation 7:

$$W_{\text{CO}_2,m} = 0.0005 + 5.9 \times 10^{-4} p \quad \text{eq. 1B}$$

where p is the pressure in MPa. If the original CO_2 content of the melt ($W_{\text{CO}_2,*}$) is known, $W_{\text{CO}_2,e}$ is calculated as

$$W_{\text{CO}_2,e} = W_{\text{CO}_2,*} - W_{\text{CO}_2,m} \quad \text{eq. 2B}$$

The weight percent water is somewhat trickier to calculate. Gerlach's equation 6 gives $W_{\text{H}_2\text{O},m}$ as:

$$W_{\text{H}_2\text{O},m} = \frac{1802}{8394.19 p_{\text{H}_2\text{O}}^{-0.9917} p^{0.5} - 356.98} \quad \text{eq. 3B}$$

where $p_{\text{H}_2\text{O}}$ is the partial pressure (in MPa) of water in the melt. The partial pressure of water in the melt is given as (eq. 13)

$$p_{\text{H}_2\text{O}} = \frac{p M_{\text{CO}_2} W_{\text{H}_2\text{O},e}}{1.17(M_{\text{CO}_2} W_{\text{H}_2\text{O},e} + M_{\text{H}_2\text{O}} W_{\text{CO}_2,e})} \quad \text{eq. 4B}$$

where M_{CO_2} and $M_{\text{H}_2\text{O}}$ are the molar weights of CO_2 and H_2O , respectively. $W_{\text{H}_2\text{O},e}$ must be found using Gerlach's equation 6 and the following equation:

$$W_{\text{H}_2\text{O},e} = W_{\text{H}_2\text{O},*} - W_{\text{H}_2\text{O},m} \quad \text{eq. 5B}$$

The problem, obviously, is that one must know $p_{\text{H}_2\text{O}}$ in order to calculate $W_{\text{H}_2\text{O},m}$, but one must know $W_{\text{H}_2\text{O},e}$ (and hence $W_{\text{H}_2\text{O},m}$) in order to calculate $p_{\text{H}_2\text{O}}$!

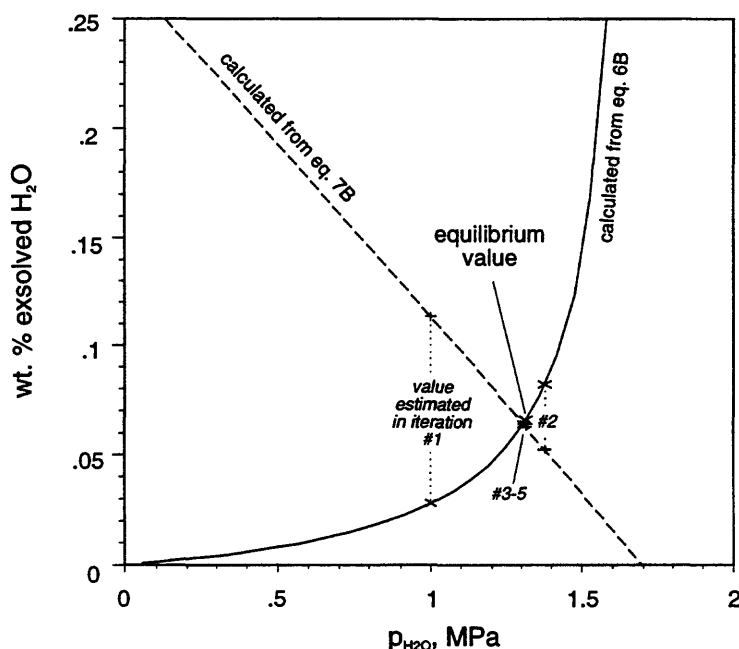


Figure 12: Weight percent exsolved H₂O calculated from equations 6B (solid line) and 7B (dashed line), versus hypothesized partial pressure of H₂O, for the conditions listed. The plotted symbols (“x”s and crosses) give the values of W_{H₂O,e} calculated iteratively by the program HICON en route to determining the final, equilibrium value of both variables.

The trick is to make a guess at p_{H_2O} and then solve for the correct value by successive approximations. I start by guessing that $p_{H_2O}=p/2$. By rearranging equations 3B and 4B, we can then calculate $W_{H_2O,e}$ from this value of p_{H_2O} using two different equations. Those equations are

$$W_{H_2O,e} = W_{H_2O,*} - \frac{1802}{8394.19 p_{H_2O}^{-0.9917} p^{0.5} - 356.98} \quad \text{eq. 6B}$$

and

$$W_{H_2O,e} = \frac{1.17 M_{H_2O} p_{H_2O} W_{CO_2,e}}{M_{CO_2} (p - 1.17 p_{H_2O})} \quad \text{eq. 7B}$$

Figure 12 shows $W_{H_2O,e}$ as a function of hypothesized values of p_{H_2O} using these two equations, for $W_{H_2O,*}=0.27$, $W_{CO_2,*}=0.05$, $p=2.0$ MPa. The point where the lines cross gives the equilibrium values of p_{H_2O} and $W_{H_2O,e}$. The program’s job is to find that point. To do this, the program calculates $W_{H_2O,e}$ using equations 6B and 7B, and if they’re not within 0.0001 of each other, it takes the first derivatives of the lines at those points. Those first derivatives define the slopes of lines that are tangent to those curves at our hypothesized value of p_{H_2O} . The value of p_{H_2O} where those two lines cross will be p_{H_2O} for the second iteration (labeled “#2”, Fig. 12). If $W_{H_2O,e}$ for equations 6B and 7B don’t

agree there either, it takes the derivatives of the functions at that value of p_{H_2O} and repeats the process until it has found the intersection. The case shown in Fig. 12 required five iterations to reach an acceptable solution. The values of $W_{H_2O,e}$ calculate during these iterations are shown in the figure.

Once p_{H_2O} and $W_{H_2O,e}$ have been found, the value of $W_{s,e}$ (i.e. exsolved sulfur), is determined using Gerlach's empirical equation 9:

$$W_{s,e} = 0.3025W_{H_2O,e} + 0.1238W_{CO_2,e} \quad \text{eq. 8B}$$

The mole fractions X_{CO_2} , X_{H_2O} , and X_s of the gas phase were determined using the following equations, from Gerlach:

$$X_{H_2O} = \frac{p_{H_2O}}{p} \quad \text{eq. 9B}$$

$$X_s = 0.145 \quad \text{eq. 10B}$$

$$X_{CO_2} = 0.855 - X_{H_2O} \quad \text{eq. 11B}$$

Gerlach's table 3 was reproduced using these calculations (Table 2).

Calculations of exsolved gas content are also used to determine other parameters used in the computer model. One of the more important of these is n , the number of moles of exsolved gas per kilogram melt:

$$n = \frac{100}{M_{H_2O} W_{H_2O,e} + M_{CO_2} W_{CO_2,e} + M_s W_{s,e}} \quad \text{eq. 12B}$$

where M_{H_2O} , M_{CO_2} , and M_s are the molar weights of H_2O , CO_2 , and S , respectively (kg/mole). Another is the mass fraction of exsolved gas in the melt:

$$m_g = \frac{W_{CO_2,e} + W_{H_2O,e} + W_{s,e}}{100} \quad \text{eq. 13B}$$

and its complement, the mass fraction magma (m_m):

$$m_m = 1 - m_g \quad \text{eq. 14B}$$

Table 2: Values of exsolved and dissolved gas species calculated using the algorithm described in the Appendix. The two depth values are for depths at the given pressure, assuming a rock density of 2300 kg/m³ (left side), and 2800 kg/m³ (right side). The term "extot" gives the total weight percent of all exsolved gas species. All values calculated for a reservoir-equilibrated magma initially containing 0.27 wt. % H₂O, 0.0195 wt. % CO₂, and 0.07 wt. % S.

P	depth	depth	v_g/v_m	$W_{H_2O,e}$	$W_{CO_2,e}$	$W_{S,e}$	extot	X_{H_2O}	X_{CO_2}
Mpa	m	m		wt %	wt %	wt %	wt %	mol	frac
32.2	1400.	1150.	0.00000	0.2700	0.0195	0.0700	0.0000	0.214	0.641
30.0	1304.	1071.	0.00051	0.2698	0.0182	0.0698	0.0017	0.221	0.634
25.0	1087.	893.	0.00206	0.2693	0.0153	0.0693	0.0057	0.242	0.613
20.0	870.	714.	0.00457	0.2686	0.0123	0.0687	0.0099	0.270	0.585
15.0	652.	536.	0.00923	0.2676	0.0093	0.0680	0.0145	0.310	0.545
10.0	435.	357.	0.02034	0.2658	0.0064	0.0671	0.0202	0.376	0.479
5.0	217.	179.	0.07109	0.2598	0.0035	0.0649	0.0313	0.519	0.336
3.0	130.	107.	0.20016	0.2488	0.0023	0.0614	0.0470	0.641	0.214
1.5	65.	54.	0.97614	0.2085	0.0014	0.0491	0.1005	0.763	0.092
1.0	43.	36.	2.15614	0.1760	0.0011	0.0393	0.1431	0.791	0.064
0.8	35.	29.	3.14977	0.1590	0.0010	0.0341	0.1655	0.800	0.055
0.6	26.	21.	4.91643	0.1388	0.0009	0.0280	0.1919	0.808	0.047
0.5	22.	18.	6.39520	0.1271	0.0008	0.0245	0.2071	0.811	0.044
0.4	17.	14.	8.68747	0.1141	0.0007	0.0205	0.2242	0.815	0.040
0.3	13.	11.	12.64312	0.0991	0.0007	0.0160	0.2437	0.818	0.037
0.2	9.	7.	20.86678	0.0813	0.0006	0.0106	0.2671	0.821	0.034

For comparison, the following is a table of the same parameters, taken from Gerlach (1986) (his Table 3)

TABLE 3. Calculated Exsolution of Volatiles From Reservoir-Equilibrated Magma Initially Containing 0.27 wt % H₂O, 0.0195 wt % CO₂, and 0.07 wt % S

P, MPa	D ₁ , m	D ₂ , m	$\frac{V_g}{V_m}$	$W_{H_2O,m}$, wt %	$W_{CO_2,m}$, wt %	$W_{S,m}$, wt %	W_{ex} , wt %	X_{H_2O} , mol frac	X_{CO_2} , mol frac
32.2	1401	1153	... ^a	0.2700	0.0195	0.0700	... ^a	... ^a	... ^a
30	1305	1074	0.0005	0.2698	0.0182	0.0698	0.0017	0.221	0.633
25	1088	895	0.002	0.2693	0.0152	0.0693	0.0057	0.242	0.613
20	870	716	0.005	0.2686	0.0123	0.0687	0.0099	0.270	0.585
15	653	537	0.009	0.2676	0.0093	0.0680	0.0145	0.309	0.545
10	435	358	0.02	0.2658	0.0064	0.0671	0.0202	0.376	0.479
5	218	179	0.07	0.2598	0.0034	0.0649	0.0313	0.518	0.337
3	131	107	0.2	0.2488	0.0023	0.0615	0.0470	0.640	0.215
1.5	65	54	1.0	0.2086	0.0014	0.0492	0.1003	0.760	0.095
1	44	36	2.2	0.1761	0.0011	0.0393	0.1430	0.790	0.065
0.8	35	29	3.2	0.1590	0.0010	0.0341	0.1654	0.800	0.055
0.6	26	22	4.9	0.1388	0.0008	0.0280	0.1918	0.800	0.055
0.5	22	18	6.4	0.1272	0.0008	0.0245	0.2070	0.812	0.043
0.4	17	14	8.7	0.1141	0.0007	0.0205	0.2241	0.815	0.040
0.3	13	11	12.6	0.0992	0.0007	0.0160	0.2436	0.817	0.038
0.2	9	7	20.9	0.0813	0.0006	0.0106	0.2670	0.820	0.035

^aThis solution gives the initial saturation pressure for magma containing 0.27 wt % H₂O, 0.0195 wt % CO₂, and 0.07 wt % S.

APPENDIX C: CALCULATION OF ADIABATIC TEMPERATURE CHANGE

Most eruption modellers (e.g. Wilson and Head, 1981; Gilberti and Wilson, 1990; Dobran, 1992) assume isothermal flow in their conduit models. In one model, by Buresti and Casarosa (1989), adiabatic temperature change was taken into account but gas exsolution was not calculated during magma ascent. Their model found relatively little temperature change of the magma/gas mixture during the eruption. Using some basic thermodynamic principles, a maximum temperature change for the erupting mixture can be calculated (Mastin, 1995) by assuming that the magma/gas mixture decompresses isentropically, so that no heat is generated by friction. The results of this calculation indicate that a typical Kilauean magma ($T=1200^{\circ}\text{C}$, volatile content ~ 0.40 wt.%) would cool less than about 15°C while decompressing from a few tens of MPa pressure to atmospheric pressure. Such a small temperature change would have a negligible effect on flow properties.

Nevertheless, for completeness, an approximate new temperature is calculated at each depth in the model. The temperature change is based on the thermodynamic principle (Moran and Shapiro, 1992, p. 128) that

$$\text{enthalpy of mixture} + \text{kinetic energy} + (\text{elevation}) \text{ potential energy} = \text{constant.}$$

For a unit mass of magma/gas mixture ascending the conduit, this equation is written in the following terms:

$$h_i + \frac{v_i^2}{2} + gz_i = h_o + \frac{v_o^2}{2} + gz_o \quad \text{eq. 1C}$$

where the subscripts i and o refer to properties at an arbitrary elevation in the conduit, z_i , and at the base of the conduit. The variable h is specific enthalpy of the mixture. The enthalpy is a function of temperature, so if we can solve for the enthalpy, we can solve for the temperature.

At each depth, the new vertical position (z) is known and the velocity (v) is calculated from the equation of continuity (eq. 1). Rearranging eq. 1C, the new enthalpy of the mixture is:

$$h_i = h_o - \frac{v_i^2 - v_o^2}{2} - g(z_i - z_o) \quad \text{eq. 2C}$$

The specific enthalpy is the sum of the specific enthalpies of the magma (h_m) and gaseous phases (h_g) times their respective mass fractions in the mixture (m_m and m_g):

$$h = m_g h_g + m_m h_m \quad \text{eq. 3C}$$

The specific enthalpies of the two phases are (Moran and Shapiro, 1992, p. 544):

$$h_g = c_{p,g} T \quad \text{eq. 4C}$$

$$h_m = c_m T + \frac{p}{\rho_m} \quad \text{eq. 5C}$$

where $c_{p,g}$, is the specific heat at constant pressure of the gas phase and c_m is the specific heat of magma (assumed incompressible), p is pressure, ρ_m is magma density, and T is temperature in Kelvin. Equation 4C assumes ideal gas behavior of the gas phase, and that the specific heat is invariant with temperature. The latter assumption is also made of the magma for eq. 4B. Rearranging and combining equations 3C, 4C, and 5C, we have the following equation for temperature:

$$T_i = \frac{h_i - \frac{m_m p}{\rho_m}}{m_g c_{p,g} + m_m c_m} \quad \text{eq. 6C}$$

The specific heat of the gas phase is calculated from the following equation:

$$c_{p,g} = \frac{W_{H_2O,e} c_{p,H_2O} + W_{CO_2,e} c_{p,CO_2} + W_{S,e} c_{p,S}}{W_{H_2O,e} + W_{CO_2,e} + W_{S,e}} \quad \text{eq. 7C}$$

where c_{p,CO_2} , c_{p,H_2O} , and $c_{p,S}$ are the specific heats at constant pressure of the three gas components. Specific heats of the gas components (in Joules/(kg K)) were taken from empirical formulas in Moran and Shapiro (1992, Appendix A-15):

$$c_{p,CO_2} = M_{CO_2} R (2.401 + 8.735 \times 10^{-3} T - 6.607 \times 10^{-6} T^2 + 2.002 \times 10^{-9} T^3)$$

$$c_{p,H_2O} = M_{H_2O} R (4.070 - 1.108 \times 10^{-3} T + 4.152 \times 10^{-6} T^2 - 2.964 \times 10^{-9} T^3 + 0.807 \times 10^{-12} T^4)$$

$$c_{p,S} \approx M_{SO_2} R (3.267 + 5.324 \times 10^{-3} T + 0.684 \times 10^{-6} T^2 - 5.281 \times 10^{-9} T^3 + 2.559 \times 10^{-12} T^4)$$

where M_{CO_2} , M_{H_2O} , and M_{SO_2} are the molar weights (in kg/mole) of CO_2 , H_2O , and SO_2 ; R is the Universal Gas Constant (in Joules/(mole K)); and T is temperature (in Kelvin). The specific heat of the sulfur species is approximated as that for SO_2 .

The temperature calculations are dependent on the pressure, velocity, and elevation of the erupting mixture at a given point in the conduit. Because these variables also depend on temperature, the problem should properly be solved by simultaneous solution of all variables, or by iterative recalculation of pressure, velocity, and temperature until all values converge on a final solution. This is not done in the program. I assume that the

pressure and velocity calculated at the new computed temperature are insignificantly different from those calculated at the former temperature. The difference in absolute temperature between adjacent depth intervals averages a few hundredths of a degree Kelvin, or 10^{-5} to 10^{-4} of the absolute temperature. Because density, velocity, and pressure are linearly related to absolute temperature, the error in temperature calculations at adjacent depth intervals is probably on the same order. The total error throughout the length of the conduit (about a hundred to a thousand vertical steps on average) is probably a small fraction of a degree Kelvin.

APPENDIX D: EXPLANATION OF VARIABLE NAMES IN PROGRAM

Name	Type	Description
area(i)	real*8	cross-sectional area of vent at z(i)
a1, a2	real*8	slopes of ph2o vs. exh2o lines calculated in subroutine EXSOLV
blkgas	real*8	bulk modulus of gas (Pa)
blkmag	real*8	bulk modulus of magma (Pa)
blkmix	real*8	bulk modulus of mixture (Pa)
b1, b2	real*8	intercepts of ph2o vs. exh2o lines calculated in subroutine EXSOLV
cm	real*8	sp. heat of magma (J/kg K)
co2	real*8	total CO ₂ content (wt.%)
cp	real*8	sp. heat at const. pressure of gas phase (J/kg K)
cpcO2	real*8	sp. heat at const. pressure of CO ₂ (J/mol K)
cph2o	real*8	sp. heat at const. pressure of H ₂ O (J/mol K)
cps	real*8	sp. heat at const. pressure of sulfur species (assumed SO ₂)
cv	real*8	specific heat of gas at constant volume (J/kg K)
cvco2	real*8	sp. heat at const. volume of CO ₂ (J/mol K)
cvh2o	real*8	sp. heat at const. volume of H ₂ O (J/mol K)
cvs	real*8	sp. heat at const. volume of sulfur species (assumed SO ₂)
dadz	real*8	gradient in cross-sectional area (m ² /m)
dco2	real*8	dissolved CO ₂ (wt.%)
deltah	real*8	change in enthalpy during isentropic decompression at vent
deltat	real*8	change in temperature during isentropic decompression at vent
dh2o	real*8	dissolved H ₂ O (wt.%)
diam	real*8	diameter of vent (m)
dpdz	real*8	pressure gradient (Pa/m)
dsulfur	real*8	dissolved sulfur (wt.%)
dz	real*8	current vertical step (m)
dznext	real*8	next vertical step (determined by subroutine rkqc) (m)

dzout	real*8	vertical step used by subroutine rkqc to extrapolate pressure or x-s area to new depth (may be smaller than dz if dz was too large to give an accurate extrapolation). (m)
eps	real*8	tolerance level for subroutine rkqc. The subroutine calculates pressure or cross-sectional area at the next depth using two methods. It then takes the difference between the two pressures or x-s areas, and normalizes the difference to the value of p or xsarea at the previous depth. If the resulting, normalized difference is greater than eps, subroutine rkqc reduces the dz step and tries again.
eta	real*8	temporary viscosity value used in subroutines
etagas	real*8	viscosity of dusty gas mixture (calculated using eq. 15) (Pa s).
etagaslog	real*8	log ₁₀ (etagas)
etamag	real*8	viscosity of vesicular magma (calculated using eq. 14) (Pa s).
etamaglg	real*8	log ₁₀ (etamag)
etamixlg	real*8	logarithm of viscosity (calculated using eq. 18).
exco2	real*8	exsolved CO ₂ in magma (wt.%)
exdif	real*8	exh2o1-exh2o2
exh2o	real*8	exsolved H ₂ O in melt (wt %)
exh2o1	real*8	exsolved H ₂ O in melt, calculated using eq. 6 from Gerlach
exh2o2	real*8	exsolved H ₂ O in melt, calculated using eq. 13 from Gerlach
exmol	real*8	moles of exsolved gas species (H ₂ O+CO ₂ +S) per kg melt
exsulfur	real*8	exsolved sulfur (wt %)
extotal	real*8	total exsolved gas (wt.%)
f	real*8	friction factor
fo	real*8	roughness factor
g	real*8	gravitational acceleration (9.81 m/s ²)
gamma	real*8	cp/cv for gas
gmix	real*8	gamma for mixture
grad	real*8	gradient calculate by subroutine derivs (either dadz, or dpdz).
h	real*8	enthalpy (J/kg)
ho	real*8	reservoir enthalpy
h2o	real*8	initial dissolved h2o in melt (wt.%)
icalc	int*4	=1 if specifying conduit of constant x-sectional area, and having the program calculate the pressure profile,2 if specifying a constant pressure grading in the conduit and having the program calculate x-sectional area.
iend	int*4	parameter used to determine whether all output values are to be printed.
ifinal	int*4	number of z-steps in a particular run
im	int*4	number of runs in which adjusted to reduce exit Mach #≥1 (used in subroutine ADJUST).
int	int*4	intercept of line of z1m vs. v1m, or z1p vs. v1p values, used in subroutine ADJUST to calculate new input velocity.
ip	int*4	number of iterations vi adjusted to increase exit pressure.

iruns	int*4	number of runs using a new input velocity.
ivar(15)	int*4	array of variables to be output from program.
ivars	int*4	number of variables to write out.
ives	int*4	vesiculation parameter: ives=1 if gas exsolution is to take place throughout eruption. ives=2 if exsolution is to stop when vesicularity reaches 75%.
ivt	int*4	iteration number. If ivt=1, the program is to calculate a single run up the conduit and stop. If ivt=2 (and icalc=1), multiplieruns are calculated until the exit conditions satisfy $M=1$ or $p=1$ atm.
m	real*8	current Mach number.
mach(i)	real*8	Mach number at $z(i)$.
mco2	real*8	molar weight of CO_2 (0.0440 kg/mole).
mdot	real*8	mass flux (kg/s).
mf	real*8	mass fraction gas.
mh2o	real*8	molar weight of H_2O (0.01801 kg/mole).
mm	real*8	mass fraction magma.
ms	real*8	molar weight of sulfur (0.032064 kg/mole).
output(10,400)	real*8	array of variables (area, mach #, pressure, etc.) to be written to output file.
p	real*8	current pressure (Pa).
pgrad	real*8	pressure gradient driving eruption (used only when icalc=2)
pi	real*8	3.14159.
pres(i)	real*8	pressure at $z(i)$.
pout	real*8	pressure at next z-step returned by subroutine rk4.
r	real*8	universal gas const.=8.314 (J/(mol*K)).
re()	real*8	Reynolds number at $z(i)$.
rey	real*8	current Reynolds number.
rho(i)	real*8	mixture density at $z(i)$.
rhof	real*8	fluid (gas) density (kg/m ³).
rhom	real*8	magma density (assumed constant), (kg/m ³).
rhomix	real*8	current density of mixture.
rmaxo	real*8	maximum initial velocity used in runs where the final mach # < 1 at the conduit exit. Used in subroutine ADJUST.
slope	real*8	slope of line of z_{1m} vs. v_{1m} , or z_{1p} vs. v_{1p} values, used in subroutine ADJUST to calculate new input pressure
sulfur	real*8	total sulfur content of magma (wt.%).
sv	real*8	sonic velocity of mixture (m/s).
t1	real*8	initial temperature in Celsius.
temp	real*8	current temperature of mixture (K).
temp2	real*8	absolute temperature after isentropic decompression to 1 atm.
temp2c	real*8	temperature (Celsius) after isentropic decompression to 1 atm.
time(i)	real*8	time at $z(i)$ since entry into conduit.
v	real*8	velocity (m/s).

vel(i)	real*8	velocity at z(i).
vesic(i)	real*8	volume fraction gas at z(i).
vfgas	real*8	volume fraction gas.
vfmag	real*8	volume fraction magma.
vgvm	real*8	volume of gas/volume of magma.
visc(i)	real*8	viscosity at z(i) (Pa s).
vmax	real*8	maximum theoretical velocity after isentropic decompression to 1 atm (m/s).
v1m()	real*8	initial velocities of iterations where final Mach number >1.
v1o()	real*8	initial velocities of iterations where final pressure > 1 atm and M<1.
v1p()	real*8	initial velocities of iterations where final pressure < 1 atm.
wtdepth	real*8	water table depth required for ground-water pressure to exceed the magma pressure at z(i).
wtmin	real*8	water table depth required for water to flow into the conduit. This is the minimum of all values of wtdepth calculated at all z values.
xco2	real*8	mole fraction CO ₂ in gas.
xh2o	real*8	mole fraction H ₂ O in gas.
xsulfur	real*8	mole fraction sulfur in gas.
xsarea	real*8	cross-sectional area of conduit (m ²).
yp	real*8	value of either xsarea or pressure sent to subroutine DERIVS.
z(i)	real*8	vertical position (m), upwards being positive, with z=0 at ground surface.
zin	real*8	current value of z, sent to subroutine RKQC.
znow	real*8	current value of z used in subroutine derivs (may not be one of the recorded z values).
zlm()	real*8	final z values of iterations where final Mach number >1. Used in subroutine ADJUST.
zlp()	real*8	final z values of iterations where final pressure < 1 atm. Used in subroutine ADJUST.

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