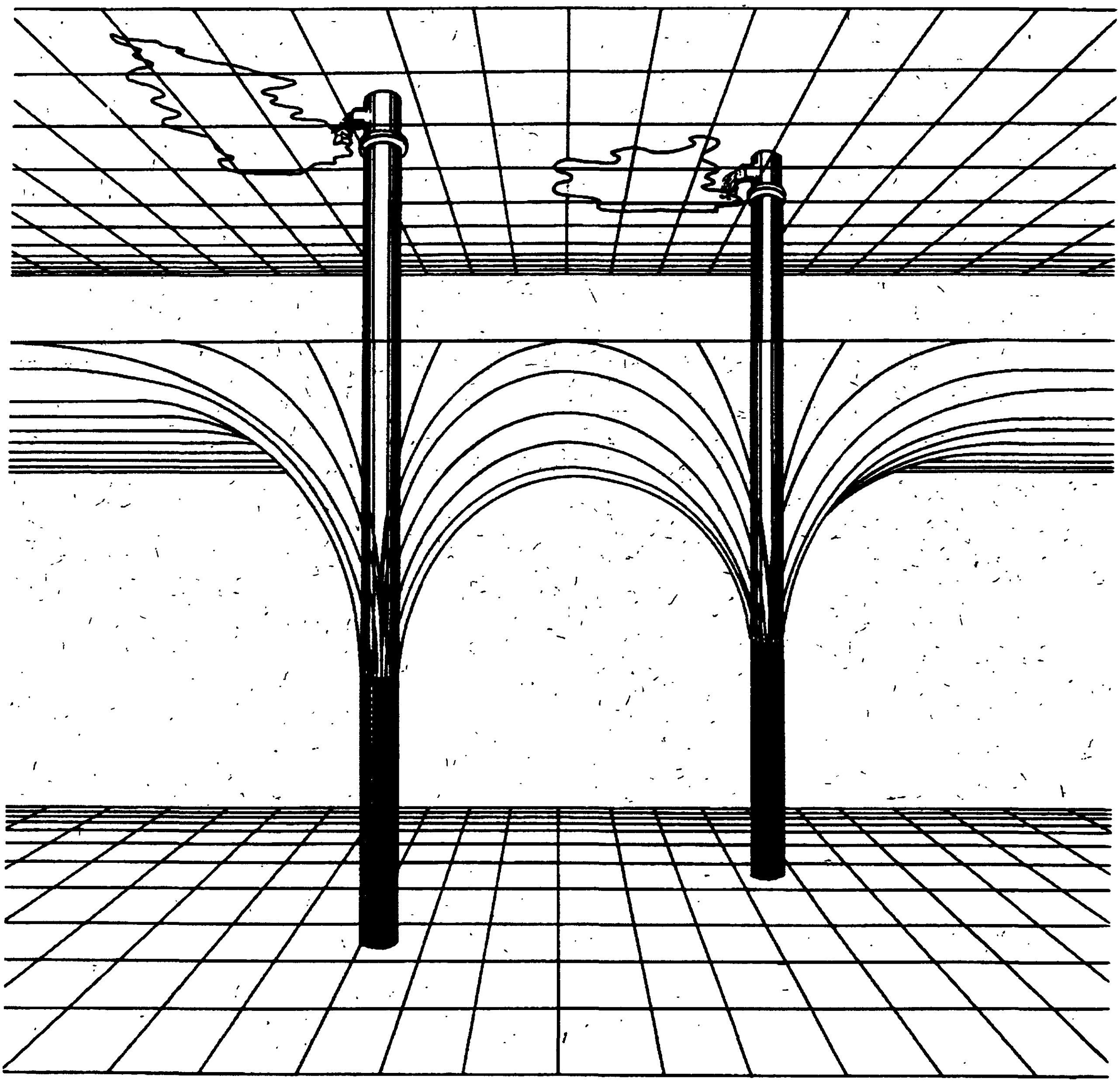
Documentation of a Computer Program to Estimate the Head in a Well of Finite Radius Using the U.S. Geological Survey Modular Finite Difference Ground-Water Flow Model



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Documentation of a Computer Program to Estimate the Head in a Well of Finite Radius Using the U.S. Geological Survey Modular Finite Difference Ground-Water Flow Model

By Michael Planert

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Abstract

The **hdpw** (head-in-a-pumping-well) program described in this report is a post-processor that calculates the head in a pumping well based on the simulated head at a finite-difference model cell that contains the well. The calculations are based on the Thiem equation. The **hdpw** code works with the U.S. Geological Survey modular finite-difference ground-water flow model, which is commonly called MODFLOW. The **hdpw** code is a complete program that has incorporated many of MODFLOW subroutines to read data. Code was added to the well package to calculate the head and drawdown in a fully penetrating well of finite radius.

Introduction

A finite-difference model does not calculate an accurate value for a head in a pumping well when the grid dimension is larger than the well diameter. The model-calculated value of head is usually higher than the actual value for a pumping well and lower than the actual value for an injection well. Prickett (1967) has shown that the head in a square model cell containing a pumping well could be related to a radius (termed effective radius by Prickett) of approximately 0.21 times the cell dimension in which a well resides. The values of the head calculated for the cell and the effective radius for the cell can be used in the Thiem equation to estimate what the head would be in an actual well having a much smaller diameter.

The program described in this report can be used with MODFLOW to compute the head and drawdown in a well of a finite radius. Calculating an approximate head for a pumping well allows a better evaluation of the production capabilities of the aquifer and of designs to remediate ground-water contamination. Also, the program might allow better evaluation for areal studies when only pumping levels are available for production wells that would be the calibration criteria for these flow models.

Copies of the computer program are available through the World-Wide Web at address:

http://h2o.usgs.gov/software/

Copies of the computer program are also available on diskette for the cost of processing from:

U.S. Geological Survey
NWIS Program Office
437 National Center
Reston, VA 20192
Telephone (703)648-5695

Theoretical Development of Computation of Head in a Pumping Well of Finite Radius

The following discussion is based on pages 8 to 10 of Trescott and others (1976). The hydraulic head computed at a cell containing a well represents the average hydraulic head for the entire cell and is not the head in the well. Prickett (1967) has shown that the effective radius, r_e, for the average head for a model cell can be determined from the cell dimensions, when $\Delta x = \Delta y$, by the equation

$$r_e = r_1/4.81, \qquad (1)$$

where $r_1 = \Delta x = \Delta y$.

The routine to calculate the head in a well is based on the Thiem (1906) equation which assumes steady flow, no stress term other than the well discharge, and that the area around the well is isotropic and homogeneous. The derivation of equation 1 is from the combination of equations written for planar flow to a model cell with a pumping well and radial flow to a well. Figure 1 depicts a model cell with a pumping well in planar and radial coordinates. The equation can be simplified by considering only two dimensions where the cell i,j is surrounded by four cells. Assuming that these cells have equal head values, all sides of the model cell will receive the same discharge. Figure 1a depicts one-quarter of the discharge to the well node i,j computed by the model as

$$Q_{w(i,j)}/4 = \Delta x_j \, T_{i,j} \, (\Delta h/\Delta y), \qquad (2)$$

where $\Delta h = h_{i-1,j} - h_{i,j}$ and

$T_{i,j} = T_{xx(i,j)} = T_{yy(i,j)}$.

An equivalent equation can be written for radial flow to a well using the Thiem (1906) equation, as shown in figure 1b:

$$Q_{w(i,j)}/4 = [\pi T_{i,j}/2] \, [\Delta h/\ln(r_1/r_e)]. \qquad (3)$$

Equation 1 can be obtained from equating the discharges in equations 2 and 3.

The Thiem equation is further used to extrapolate from the head, $h_{i,j}$, for the model cell at the effective radius, r_e, to the head, h_w, at the desired well radius, r_w. The equation is written

$$h_w = h_{i,j} - [Q_{w(i,j)}/2\pi \, T_{i,j}] \ln (r_e/r_w). \qquad (4)$$

Assumptions for equation 4 are:

1. The aquifer is confined.
2. The well causes radially symmetric drawdown.
3. The well causes no vertical flow in the aquifer containing the well or from units above and below the aquifer.
4. The transmissivity is uniform and isotropic in the cell containing the well and the four neighboring cells.
5. The grid dimensions for the cell containing the well and the four neighboring cells are uniform.
6. The well is pumping under steady-state conditions.
7. There are no head-dependent conditions nearby.
8. The well is 100 percent efficient.

The analogous equation for an unconfined aquifer is written as:

$$H_w = \sqrt{H^2_{i,j} - [Q_{w(i,j)}/\pi K_{i,j}] \ln (r_e/r_w)}, \qquad (5)$$

where

$H_{i,j} = h_{i,j}$ - BOTTOM (I,J)	is the saturated thickness of the aquifer at radius r_e (L);
H_w	is the saturated thickness of the aquifer at the well (L);
$K_{i,j} = Kxx_{(i,j)} = Kyy_{(i,j)}$;	is the hydraulic conductivity for the cell;
BOTTOM (I,J)	is the elevation of the bottom of the aquifer. (The uppercase letters indicate that this parameter is identical to that used in the model.)

Additional assumptions for unconfined conditions are:

1. Rather than the aquifer having uniform transmissivity, the aquifer bottom is flat in the region and the hydraulic conductivity is uniform and isotropic.
2. There are no other stresses in the cell containing the well or in the four neighboring cells.
3. The saturated thickness of the aquifer is virtually equal at the cell containing the well and in the four neighboring cells.

There is a possibility that the aquifer may become dewatered at the well, r_w, even though it is not dewatered at the effective radius, r_e, for the cell. This condition is indicated in equation 5 when the value calculated beneath the square root symbol is negative. For this condition, the output in the well table is "WELL IS DRY." If this occurs while actual measurements are being simulated, a review of the transmissive properties and radius used for the well should be made, as most wells are not 100 percent efficient and the aquifer should not go dry. If a well goes dry while determining prospective rates for a pumping system then, obviously, the rates should be lowered.

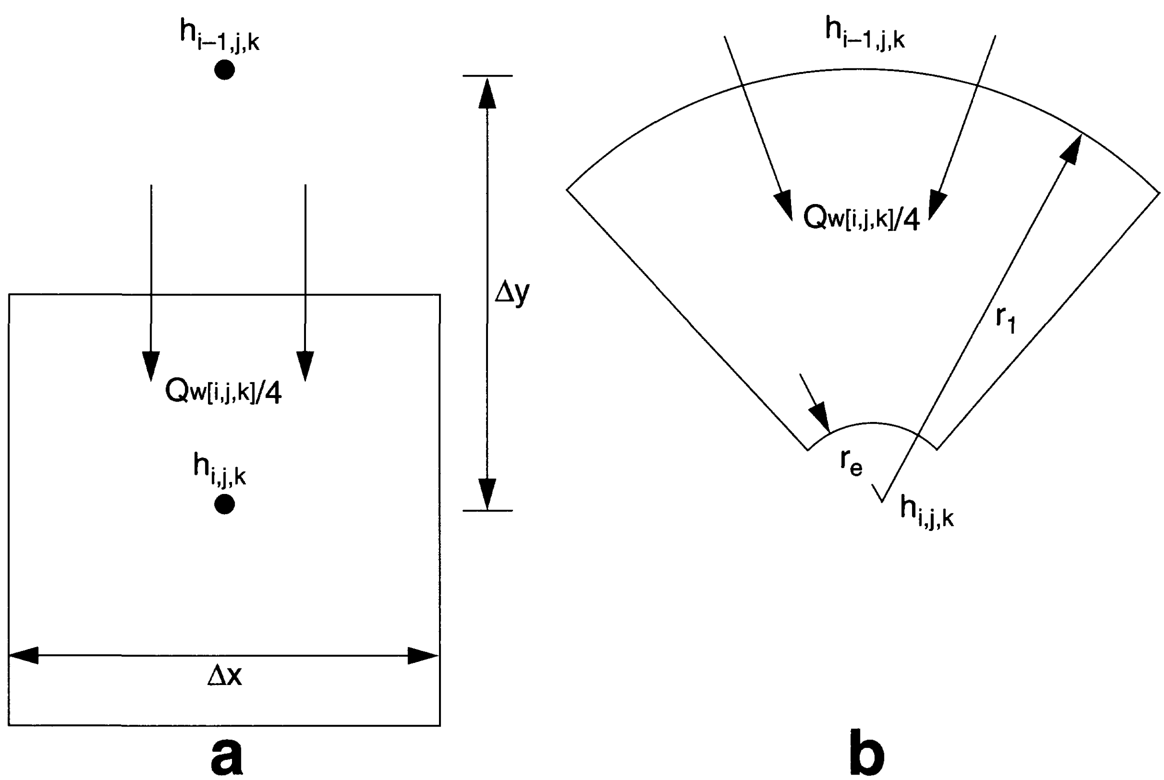


Figure 1. Flow from cell (i–1,j) to cell (i,j) (a) and equivalent radial flow to well (i,j) with radius r, (b).(From Trescott and others, 1976.)

Comparison of Simulated and Analytical Results

To test the accuracy of equation 4, model runs were made to compare to an analytical solution using the Theis equation (1935) assuming steady conditions were attained near the well at the end of the time period simulated. Two models, one using a transmissivity of 500 ft^2/d and the other a transmissivity of 5,000 ft^2/d, were constructed with a pumping well at the center of the grid, a storage coefficient of 0.0001, and a pumping rate of 10,200 ft^3/d. The test actually compares drawdowns, so the initial head was set to zero and the heads calculated were negative and equivalent to the absolute values of the analytical drawdowns. The derivation of equation 4 is based on square grid cells but the code is written to accommodate rectangular grid cells. Different grid dimensions were used to test the error that rectangular grid cells might introduce into the calculated head. As cell dimensions were changed from 100 by 100 ft cells to 150 by 100 ft cells to 200 by 100 ft cells to 500 by 100 ft cells, the grid dimensions changed with the number of rows ranging from 301 to 201 to 151 to 61 on a side. There were always 301 columns. The model was run for 1 day. Analytical results for the five timesteps used in the model are presented in table 1. Two transmissivities were used to compare errors for values of large and small drawdown. Results for the model runs using the different combinations of cell dimensions and transmissivities are presented in table 2. Only the two extremes of cell dimensions are presented for a transmissivity of 5,000 ft^2/d.

Table 1. Analytical results for pumping well used to test equation 4

		T = 500 ft^2/d			T = 5,000 ft^2/d		
Timestep	Time, in days	u	W(u)	s	u	W(u)	s
1	0.075829	$6.59E^{-7}$	13.6553	22.17	$6.59E^{-8}$	15.9579	2.59
2	0.189573	$2.64E^{-8}$	14.5703	23.65	$2.64E^{-8}$	16.8729	2.74
3	0.360190	$1.39E^{-7}$	15.2118	24.69	$1.39E^{-8}$	17.5144	2.84
4	0.616114	$8.12E^{-8}$	15.7491	25.57	$8.12E^{-9}$	18.0517	2.93
5	1.000000	$5.00E^{-8}$	16.2340	26.35	$5.00E^{-9}$	18.5366	3.01

The results show that equation 4 gives very good results for estimating the analytical solution to the problem. For the lower value of transmissivity and square cell dimensions, equation 4 comes within 0.06 ft of the 26.35 ft of drawdown and the maximum error was 0.65 ft (0.02 percent) for a cell dimension of 500 by 100 ft. The error for the higher transmissivity was 0.00 for the square cell dimensions and 0.08 (0.03 percent) for the cell dimensions of 500 by 100 ft.

To confirm that the relation of the effective radius to cell dimension presented by Prickett (1967) is appropriate, the drawdown at the cell center was used in the Theis equation to calculate the appropriate analytical radius. For the cell dimensions of 100 by 100 and a transmissivity of 500 ft^2/d, the analytical radius was 21.21 ft. The effective radius, r_e, from equation 1 was 20.79. The ratio of the effective radius to the analytical radius is 0.98 (20.79/21.21) whereas the ratio of the simulated drawdown to the analytical drawdown is 1.00 (26.29/26.35). For cell dimensions of 500 by 100 ft and a transmissivity of 500 ft^2/d, the analytical radius was 76.55 ft and the effective radius was 62.37 ft. The ratio of the effective radius to the analytical radius was 0.81 (62.37/76.55) whereas the ratio of the simulated drawdown to the analytical drawdown was 0.97 (25.69/ 26.35). These results show that, although the error in effective radius may grow as the cell dimensions are exaggerated, the relative error in calculated head remains small. This result is not to say that cell dimensions are not important. The cell dimensions tested were uniform in each direction. Care is still needed in a truly variable grid system where dimensions are being changed in each direction.

Program Design

The program code consists of routines adapted from MODFLOW that provide the proper input of grid dimension, transmissivity data, pumping data, and output data. Program code from Trescott and others (1976) was added to calculate the head in a well. The output from the program lists much of the basic package information to identify the problem being modeled, output control flags for each timestep, and a table listing the well location, the head and drawdown in the well, and the radius of the well. If an actual head is available for the final timestep in a stress period, this head value and the difference between the actual and simulated heads will be printed. If starting heads are not saved (ISTRT= 0), all drawdowns are set to zero in the tables.

Table 2. Comparison of analytical results to simulated results using equation 4 for varying grid dimensions and two values of transmissivity

	Head at time-step 1	Head at time-step 2	Head at time-step 3	Head at time-step 4	Head at time-step 5	Mass-balance error	Cell head, Time step 5	Draw down at boundary
				Model with T = 500 ft^2/d				
Analytical	-22.17	-23.65	-24.69	-25.57	-26.35	----	----	----
100 x 100	-21.35	-23.35	-24.54	-25.47	-26.29	-0.07	16.44	0.00
150 x 100	-21.27	-23.28	-24.47	-25.41	-26.23	-0.07	15.65	0.00
200 x 100	-21.15	-23.17	-24.36	-25.30	-26.12	-0.06	14.95	0.00
500 x 100	-20.61	-22.70	-23.92	-24.87	-25.69	-0.06	12.27	0.00
				Model with T = 5,000 ft^2/d				
Analytical	-2.59	-2.74	-2.84	-2.93	- 3.01			
100 x 100	-2.51	-2.71	-2.83	-2.92	-3.01	-0.51	2.02	0.06
500 x 100	-2.45	-2.65	-2.77	-2.86	-2.95	-0.31	1.61	0.06

The head in a pumping well (**hdpw**) code is designed to be a post-processor that may be executed by itself or within the same runfile as MODFLOW. If executed within the same runfile as MODFLOW, output tables may be written to the output listing of the modular model or to a separate file. The only changes to the MODFLOW input data is that a radius value is placed in columns 51-60 of the individual well record (record 3) for a head and drawdown to be calculated for a particular well and that a value of head for an actual measurement is placed in columns 61-70 for a comparison to be made between the calculated head and the measured head. If no radius is input (radius = 0), no calculation will be made for that well. If no actual head is input (ACHD=0), no comparison to the value of simulated head will be made. For a transient simulation, heads must be saved for each time period heads and drawdowns are to be calculated.

A general flow chart of the program is diagramed in figure 2. Data files for the MODFLOW modules of the BASIC, BLOCK-CENTERED FLOW, OUTPUT CONTROL, and WELL packages are read to provide input for the **hdpw** program. The basic package is read in its entirety to define the model dimensions and time parameters. The block-centered flow package has been truncated and modified to allow the transmissivity of grid cells to be calculated and stored. No aquifer coefficients (CC and CR) are calculated. The output control package is read in its entirety to mark times when heads are saved to disk. The time periods when heads are saved signals to the program that heads for wells will be calculated for that time period. The well package is read in its entirety to define the wells that have finite radii and a subroutine (WELHPW) has been added to the MODFLOW well package that calculates the head and drawdown for a well.

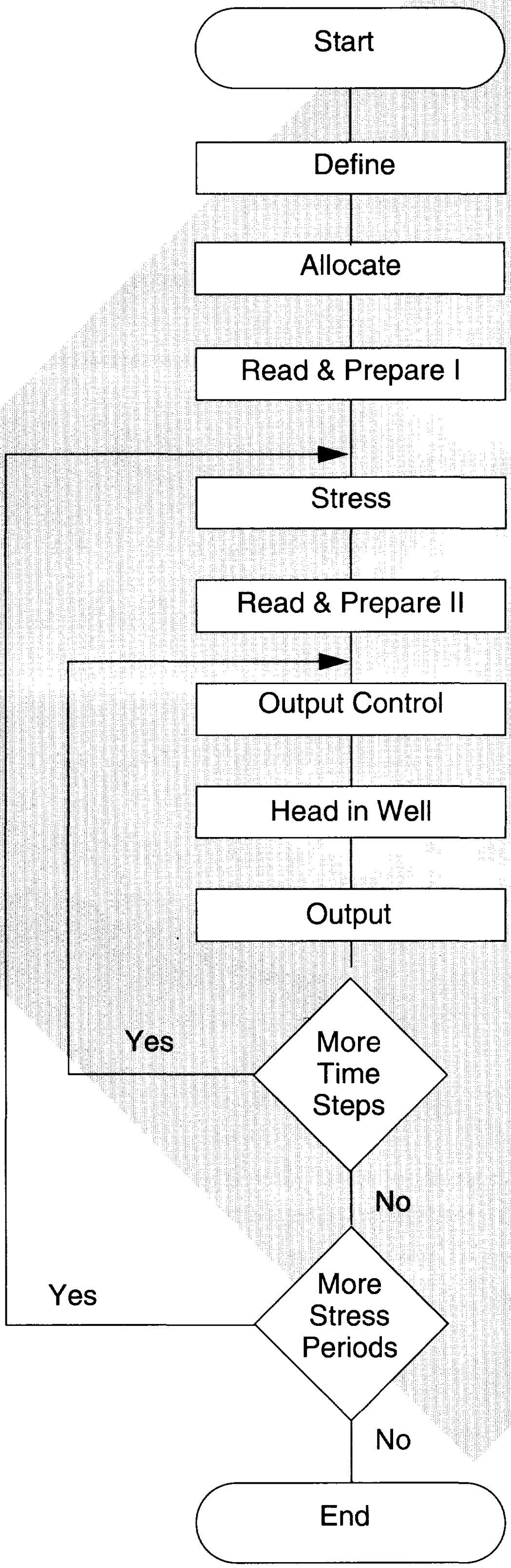
Description of Subroutine WELHPW

The subroutine for calculating the head in a well (WELHPW) contains the following steps:

1. Reads unformatted record containing heads saved for appropriate time period.
2. Reads well records to determine if any wells have a finite radius.
3. For wells with a finite radius, computes effective radius for cell -- r_e= (DELC(I)+DELR(J))/9.62.

Note that the effective radius is calculated using the individual row and column lengths so the potential is there to place a well in a rectangular cell.

4. Determines if layer is under confined or unconfined conditions.
5. Uses appropriate equation to calculate head and drawdown in a well.
6. For wells with an actual head to compare for the end of a stress period, computes difference between actual head and simulated head.
7. Prints well location, well radius, and head and drawdown for the well at each timestep heads are saved and also prints actual head and the difference between actual and simulated heads for the final timestep of the stress period.



DEFINE - read data specifying number of rows, columns, layers, stress periods, and major program options. (only BAS, BCF, OC, and WEL packages are read.)

ALLOCATE - allocate space to store data in the computer.

READ & PREPARE I - Read data which is constant throughout the simulation. Calculate transmissivities. (BAS1RP, BCF1RP)

STRESS - Read stress period information. (BAS1ST)

READ & PREPARE II - read data which may change each stress period. (WEL1RP)

OUTPUT CONTROL - determine whether results were saved on disk for this timestep.

HEAD IN WELL - calculate head and drawdown for wells that have finite radii.(WELHPW)

OUTPUT - print table of heads and drawdown for wells with finite radii.

Figure 2. Program structure.

Input for Well Package

FOR EACH SIMULATION

WEL1AL

1. Data:	MXWELL	IWELCB
Format:	I10	I10

FOR EACH STRESS PERIOD

WEL1RP

2. Data:	ITMP						
Format:	I10						
3. Data:	Layer	Row	Column	Q	(IFACE)	R	ACHD
Format:	I10	I10	I10	F10.0	I10	F10.0	F10.0

MXWELL -- is the maximum number of wells used for any stress period.

IWELCB -- is a flag or unit number for cell-by-cell terms.

ITMP -- is the number of wells active during the present stress period.

Q -- is the discharge rate for the well. The rate is negative for a pumping well and positive for an injection well.

IFACE -- used in program MODPATH to designate faces of model cell used in determining flow rates. (Only needed if MODPATH is being used in analysis.)

R -- is the radius of the well.

ACHD -- is an actual (measured) head to be compared with the simulated head at the end of the stress period.

SAMPLE INPUT

Following is the input needed to run the post-processor **hdpw**.

Basic package

```
TEST OF HDPW AGAINST THEIS ANALYTICAL SOLUTION --
   ONE WELL PUMPING 53 GPM FOR T OF 500 SQFT/D.
         1        301        301          1          4
 11 21  0  0  0  0  0  0  0  0  0 16 19  0  0  0  0  0  0  0  0
         0          0
         0          1              (40i2)          2
         0          0
         0          0                              9
         1          5       1.5
```

Block-centered flow package

```
         0         62
 0
         0          1
         0        100
         0        100
         0      .0001
         0        500
```

Output control package

```
         5          5         61         65
         1          1          1         62
         1          0          1          0
         1          1          1         62
         1          0          1          0
         1          1          1         62
         1          0          1          0
         1          1          1         62
         1          0          1          0
         1          1          1         62
         1          0          1          0
```

Well package

```
         1         62
         1
         1        151        151     -10200          6          1   -26.35
```

SAMPLE OUTPUT

```
********************************************************************************
********************************************************************************
 U.S. GEOLOGICAL SURVEY HEAD IN A WELL OF FINITE RADIUS POST-PROCESSING PROGRAM
********************************************************************************
********************************************************************************

TEST OF HDPW AGAINST THEIS ANALYTICAL SOLUTION --
   ONE WELL PUMPING 53 GPM FOR T OF 500 SQFT/D.

                       1 LAYERS       301 ROWS       301 COLUMNS

                           1 STRESS PERIOD(S) IN SIMULATION

                             MODEL TIME UNIT IS DAYS

 START HEAD WILL NOT BE SAVED -- DRAWDOWN CANNOT BE CALCULATED

     816034 ELEMENTS OF X ARRAY USED OUT OF   1400000


           WELL(S) IN CURRENT STRESS PERIOD =    1


     LAYER     ROW    COL    STRESS RATE     RADIUS        HEAD        WELL NO.
     ------------------------------------------------------------------------
        1      151    151       -10200.      1.0000     -26.350          1


********************************************************************************
OUTPUT FLAGS FOR EACH LAYER:
         HEAD    DRAWDOWN  HEAD  DRAWDOWN
 LAYER  PRINTOUT  PRINTOUT  SAVE    SAVE
 -----------------------------------------
    1       1         0       1       0



      HEAD AND DRAWDOWN IN PUMPING WELLS FOR STRESS PERIOD    1, TIMESTEP    1
                        TIME SIMULATED =    0.7582939E-01
     -------------------------------------------------------------------------


                  I    J      K    WELL RADIUS    HEAD      DRAWDOWN
                 --------------------------------------------------


                  151  151    1        1.00      -21.35        0.00
```

```
*******************************************************************************
OUTPUT FLAGS FOR EACH LAYER:
        HEAD    DRAWDOWN  HEAD  DRAWDOWN
 LAYER  PRINTOUT  PRINTOUT  SAVE    SAVE
 -----------------------------------------
   1       1         0       1       0


      HEAD AND DRAWDOWN IN PUMPING WELLS FOR STRESS PERIOD    1, TIMESTEP    2
                        TIME SIMULATED =    0.1895735
     -------------------------------------------------------------------------


                I    J      K    WELL RADIUS    HEAD       DRAWDOWN
               --------------------------------------------------

               151  151    1        1.00      -23.35        0.00

*******************************************************************************
OUTPUT FLAGS FOR EACH LAYER:
        HEAD    DRAWDOWN  HEAD  DRAWDOWN
 LAYER  PRINTOUT  PRINTOUT  SAVE    SAVE
 -----------------------------------------
   1       1         0       1       0


      HEAD AND DRAWDOWN IN PUMPING WELLS FOR STRESS PERIOD    1, TIMESTEP    3
                        TIME SIMULATED =    0.3601896
     -------------------------------------------------------------------------


                I    J      K    WELL RADIUS    HEAD       DRAWDOWN
               --------------------------------------------------

               151  151    1        1.00      -24.54        0.00

*******************************************************************************
OUTPUT FLAGS FOR EACH LAYER:
        HEAD    DRAWDOWN  HEAD  DRAWDOWN
 LAYER  PRINTOUT  PRINTOUT  SAVE    SAVE
 -----------------------------------------
   1       1         0       1       0


      HEAD AND DRAWDOWN IN PUMPING WELLS FOR STRESS PERIOD    1, TIMESTEP    4
                        TIME SIMULATED =    0.6161138
     -------------------------------------------------------------------------


                I    J      K    WELL RADIUS    HEAD       DRAWDOWN
               --------------------------------------------------

               151  151    1        1.00      -25.47        0.00
```

```
********************************************************************************
OUTPUT FLAGS FOR EACH LAYER:
          HEAD     DRAWDOWN  HEAD  DRAWDOWN
 LAYER  PRINTOUT  PRINTOUT  SAVE    SAVE
 -----------------------------------------
    1       1         0       1       0


      HEAD AND DRAWDOWN IN PUMPING WELLS FOR STRESS PERIOD    1, TIMESTEP    5
                        TIME SIMULATED =    1.000000
     -------------------------------------------------------------------------


     I   J    K    WELL RADIUS    HEAD       DRAWDOWN   ACTUAL HEAD   DIFFERENCE
     --------------------------------------------------------------------------

    151 151   1        1.00      -26.29        0.00      -26.35         0.06
```

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