Table 2. Drainage-basin parameters used in logistic regression

Variable	Variable name	Approximate[1] probability distribution	Range in values	Units
Drainage-basin area	AREA	lognormal	-1.3 to 3.8	km^2
Height above river of:				
- headwaters	HEADHT	normal	402 to 2207	m
- Hermit Shale	HERMHT	bimodal normal	0 to 1488	m
- Supai Group	SUPHT	bimodal normal	0 to 1134	m
- Muav Limestone	MUHT	bimodal normal	0 to 890	m
Inverse of channel length to:				
- headwaters	HEADD	lognormal	-1.8 to 0.3	$\log (m)^{-1}$
- Hermit Shale	HERMD	bimodal lognormal	-3.0 to 0.7	$\log (m)^{-1}$
- Supai Group	SUPD	bimodal lognormal	-3.0 to 1.0	$\log (m)^{-1}$
- Muav Limestone	MUD	bimodal lognormal	-3.0 to 1.0	$\log (m)^{-1}$
Channel gradient to:				
- headwaters	HEADG	lognormal	-1.8 to 0.3	none
- Hermit Shale	HERMG	bimodal lognormal	-3.0 to 0.7	none
- Supai Group	SUPG	bimodal lognormal	-3.0 to 0	none
- Muav Limestone	MUG	bimodal lognormal	-3.0 to -0.2	none
Elevation of:				
- headwaters	HEADEL	normal	1061 to 2804	m
- Hermit Shale	HERMEL	bimodal normal	0 to 2073	m
- Supai Group	SUPEL	bimodal normal	0 to 1951	m
- Muav Limestone	MUEL	bimodal normal	0 to 1707	m
Tributary aspect	TASP	uniform	0 to 1.0	none
River aspect	RASP	uniform	0 to 1.0	none
Log of total length of faults	FAULT	normal	-2 to 2.3	log (km)
River kilometer	RKM	uniform	4.5 to 395.9	km

[1] Bimodal normal, the distribution is normal except for zero values. Bimodal lognormal, the distribution is lognormal except for transformed zero values.

$$\Theta = |\cos(\Phi - 45°)|. \tag{4}$$

The influence of geologic structure in each drainage was evaluated as the linear sum of all surface faults delineated on geologic maps of the area (Haynes and Hackman, 1978; Huntoon and others, 1981; Huntoon and Billingsley, 1983; Huntoon and others, 1986). One important difficulty with these data is that geologic map coverage of the study area is not at a uniform scale: map scales ranged from 1:250,000 to 1:48,000. Thus, on the basis of scale variation alone, apparent fault density may differ from one area to another depending on the map used. In this case, fault density may increase artificially from east to west, because map scales increased in that direction.

We also included a measure of river kilometer, the distance in kilometers along the river from Lees Ferry to the confluence with each tributary. This variable is intended to reflect any ordered spatial variation in debris-flow frequency along the river corridor that is not accounted for by the other variables.

All drainage-basin data were measured from USGS 7.5’ topographic maps (1:24,000 scale) and various geologic maps (Haynes and Hackman, 1978; Huntoon and others, 1981; Huntoon and Billingsley, 1983; Huntoon and others, 1986). For source lithologies, elevations, heights above and channel distance from the river, we averaged the largest and smallest values measured to the bottom of the units. We calculated an inverse-channel distance variable, which is the reciprocal of channel distance. For lithologic strata that are not present in a given basin, zero values were entered for their variables. Gradient was calculated simply as height above river divided by channel distance, and so is also a mean value. Drainage-basin boundaries were drawn by hand on topographic maps, digitized, and entered into a GIS, which calculated drainage areas and centroids.

Statistical Procedures

Because the dependent variable, debris-flow frequency, is binomial, we chose logistic regression for modeling the relation of drainage-basin variables to debris-flow frequency. Where linear regression returns a continuous value for the dependent variable, logistic regression returns the probability of a positive binomial outcome (in this case, debris-flow occurrence during the last century). Logistic regression is commonly used in medical and biological studies where the dependent variable is the presence or absence of a given illness or disease, and independent variables the presumed controlling factors. In medical research, logistic regression is used to analyze the statistical significance of certain factors in relation to diseases, as well as for modeling the probability of contracting the disease on the basis of the significant controlling factors (Hosmer and Lemeshow, 1989).

For logistic regression, the conditional mean probability, ($\pi(\mathbf{x})$) is:

$$\pi(x) = e^{g(\mathbf{x})} / [1 + e^{g(\mathbf{x})}], \qquad (5)$$

where

$$g(\mathbf{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_i x_i \qquad (6)$$

and i = number of variables. The coefficients (β_v) are estimated by the method of maximum likelihood, where coefficients with the highest probability of returning the observed values are selected. Maximum likelihood is determined using the likelihood function, which expresses the probability of the observed data as a function of the unknown coefficients (Hosmer and Lemeshow, 1989). Those coefficients that maximize the likelihood function are thus the coefficients with the greatest probability of returning the observed values. SAS statistical software was used to calculate these model coefficients as well as various measures of their significance (SAS, 1990). After the coefficients are estimated using the calibration set, and verified as reasonable using the verification set, we calculated the probabilities of debris-flow occurrence for all 600 tributaries using equations (5) and (6).

Our 9,516 km^2 study area is too geomorphically diverse to be effectively treated in one model of Grand Canyon; drainage-basin lithology and morphology differ markedly over the length of the Colorado River. Instead, we separated the initial 164 drainages with known debris-flow frequencies into two distinct data sets, one each for eastern and western Grand Canyon. It is extremely difficult to identify a unique point of geomorphic transition between the eastern and western canyon as a variety of major structural and morphometric changes occur between Phantom Ranch (river mile 97.8) and Crystal Creek (river mile 98.2). The margin faults passing across Grand Canyon at Crystal Creek (Hunter and others, 1986) suggested that mile 98 was the approximate point of separation between eastern and western Grand Canyon. Splitting the data at river mile 98.0 (Crystal Creek) resulted in models that presented the best balance in terms of model fit and stability. Moving the point of separation east or west from this location excessively strengthened one model at the expense of the other. Separating the data at river mile 98.0 placed 78 drainages in eastern Grand Canyon (river mile 0 to 98.0) and 86 drainages in western Grand Canyon (river mile 98.1 to 248.3). These data, which we call the “calibration set,” do not represent a random sample of our population, based as they are on the historical photographic evidence available to us. Nevertheless, a comparison of the sample and population distributions of each drainage-basin variable indicates that the sample is statistically representative of the population of ungaged tributaries throughout Grand Canyon (fig. 6).

We used principal-component analysis to identify the drainage-basin variables that are statistically redundant. After a qualitative assessment of the redundant variables that contribute to the effectiveness of the model (usually

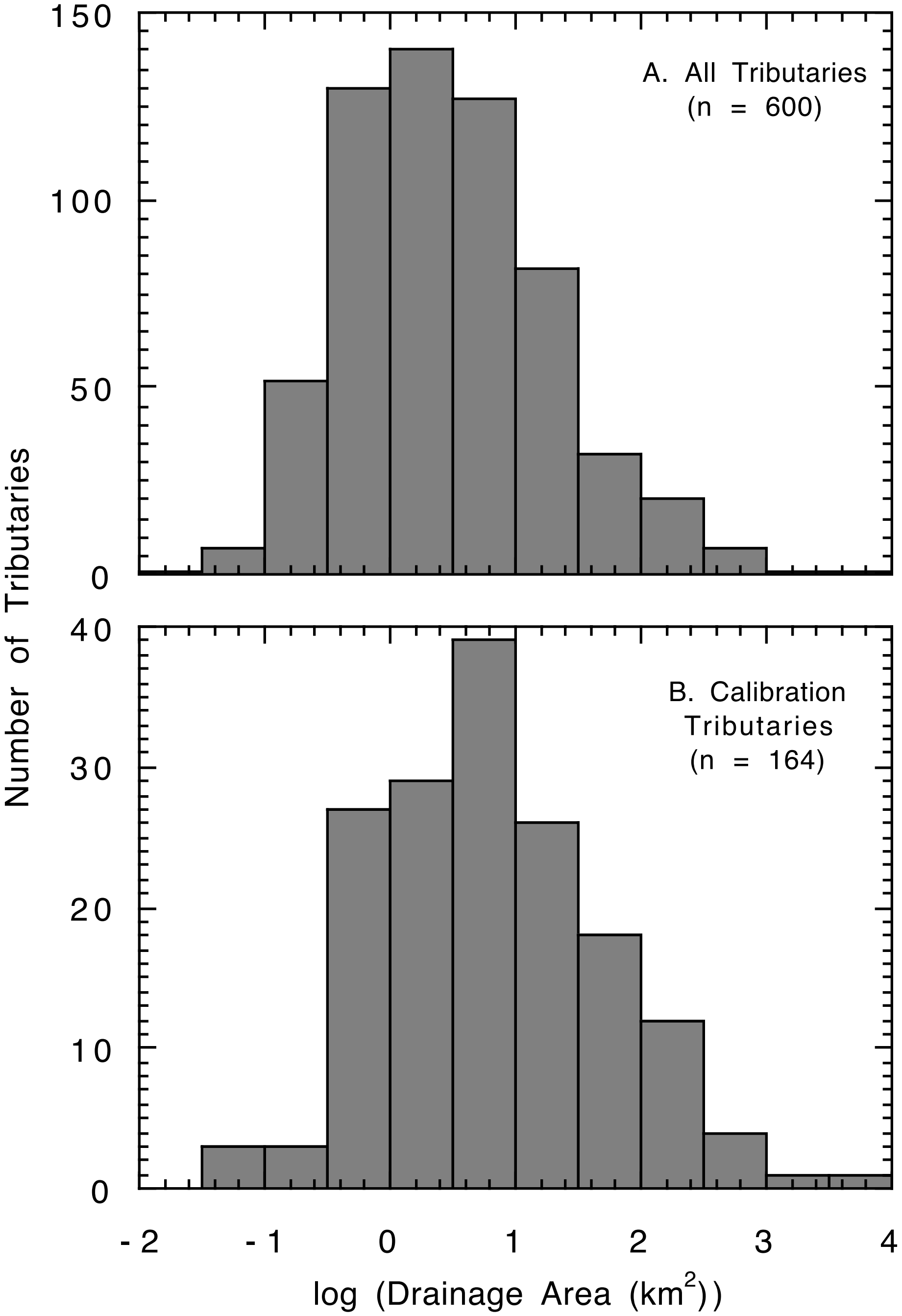

Figure 6. Histograms comparing the sample drainage-basin areas with the population drainage-basin areas of tributaries in Grand Canyon.

on the basis of increased information content), the extraneous variables were eliminated from further consideration. Variables measuring distance or area, such as drainage area, channel distances, channel gradient, and fault distance, are distributed logarithmically. Logistic regression is not dependent on normally distributed data, but log transformation of these variables reduces the redundancy identified by principal-component analysis. We therefore chose to use the log-transformed values in modeling the probability of debris-flow occurrence. To properly evaluate zero values in log-transformed data, zero values were replaced with a value one order of magnitude smaller than the smallest non-zero variable. For source-lithology channel distances and gradients this value was 0.001, resulting in a log-transformed value of -3. For fault lengths, these values were 0.01 and -2.

We used a step-backward elimination process in our logistic regression (SAS, 1990). Variables of least statistical significance are removed from the model until only variables with significance (ρ_v) less than a given threshold (0.10 in this study) remain. We used the χ^2 measure of the Wald statistic (Hosmer and Lemeshow, 1989; SAS, 1990) to evaluate variable significance. We employed several statistics to evaluate the quality of the resulting models. These statistics include measures of the overall significance of the final model compared with the model containing all initial variables (ρ_m); a percentage of accurately predicted debris-flow occurrence as a rough measure of model accuracy (α); and the Hosmer/Lemeshow model goodness-of-fit statistic (C), which can be expressed as a χ^2 significance measure (ρ_C) (Hosmer and Lemeshow, 1989).

We also calculated the odds ratio (Ψ) for each variable in the model. This statistic measures the change in the odds of outcome occurrence per unit increase of the variable. We evaluated how robust the models were by attempting to reproduce model results using larger data sets drawn from the same population of drainages. For this purpose, we determined debris-flow probability for the "verification set," an additional 50 drainages — 25 each in eastern and western Grand Canyon — using a variety of non-photographic methods, including radiometric dating, stratigraphic evidence, and other field evidence (Melis and others, 1994). Unfortunately, a verification set of only 25 observations is too small for reliable logistic regression modeling alone. We therefore added each set of 25 drainages to the original calibration data and formed two larger verification data sets (fig. 7). This overlap of calibration and verification data limits the usefulness of the model comparison.

INITIATION OF DEBRIS FLOWS

Debris flows in Grand Canyon are initiated by a combination of intense precipitation and subsequent slope failure. The intensity of rainfall necessary to initiate debris flows in Grand Canyon is poorly known because few climatic stations are in debris-flow producing tributaries. Previous studies have reported rainfall that initiates debris flows to have intensities greater than 25 mm/hr with a total rainfall of at least 16 to 50 mm (Webb and others, 1989; Melis and others, 1994). The recurrence interval of precipitation on the days when debris flows have occurred in Grand Canyon ranges from less than one year to more than sixty years (table 2). Multiday storms that precede debris flows had larger recurrence intervals, typically greater than 100 years.

Intense precipitation may occur in summer or winter throughout Grand Canyon. Three types of storms can cause floods in the southwestern United States: localized or widespread convective thunderstorms in summer, regional frontal systems in winter, and dissipating tropical cyclones in late summer and early fall (Hansen and Shwarz, 1981; Hirschboeck, 1985; Webb and Betancourt, 1992; Thomas and others, 1994). Most historic debris flows in Grand Canyon are associated with the intense precipitation of convective summer thunderstorms that affect only one or two drainages at a time. These storms are fed by large quantities of moisture, evaporated from the northern Pacific and Gulf of California by monsoonal circulation patterns. Debris flows also occur during prolonged precipitation produced in winter by regional frontal systems (Cooley and others, 1977). These widespread storms sweep across the Colorado Plateau from the west along the Pacific storm track, which is shifted south during the winter by the Aleutian Low in the North Pacific Ocean (Webb and Betancourt, 1992). Rain that can be both