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Uranium Resources/Reserves Lecture

by

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These lecture notes were prepared for the participants who attended the Regional Training Course on Uranium Resource Inventories and Ore Reserve Calculations in Changsha, China, October 6-17, 1997. The course was essentially intended for countries of West Asia that are involved in exploration and development of uranium deposits. The topics covered in this set of lectures includes the concepts of resources versus reserves, the principles of resource/reserve classification, national uranium resource assessments, methods of estimating the undiscovered uranium endowment of large regions, and the economic evaluation of uranium deposits. These lecture notes are by no means exhaustive nor do they represent original research. In large part, these notes are based on existing sources of material that can be found in the literature. It is hoped that these notes will serve as a focal point for those who are engaged in assessing the uranium resources, potential or otherwise, in their respective countries.

Uranium as an energy fuel

Uranium is a slightly radioactive metal that occurs throughout the earth's crust. It is about 500 times more abundant than gold and about as common as tin. It is present in most rocks and soils as well as in many rivers and in sea water. It is, for example, found in concentrations of about four parts per million (ppm) in granite, which makes up 60% of the earth's crust. In fertilisers, uranium concentration can be as high as 400 ppm (0.04%), and some coal deposits contain uranium at concentrations greater than 100 ppm (0.01%). There are a number of areas around the world where the concentration of uranium in the ground is sufficiently high that extraction for use as nuclear fuel is economically feasible.

Electricity is produced from uranium through the nuclear fuel cycle. Electricity is created by using the heat generated in a nuclear reactor to produce steam and drive a turbine connected to a generator. Fuel removed from a reactor, after it has reached the end of its useful life, can be reprocessed to produce new fuel. The various activities associated with the production of electricity from nuclear reactions are referred to collectively as the nuclear fuel cycle. The nuclear fuel cycle starts with the mining of uranium and ends with the disposal of nuclear waste as shown in figure 1 [1].

National uranium resource assessments

Assessment of the uranium resources of a nation involves the evaluation or estimation of its ultimate uranium resources, and, therefore, requires an estimation of both identified resources and undiscovered resources. Whereas the identified resources can be assessed with an acceptable degree of accuracy, the evaluation of potential or undiscovered resources is controversial, because it is based on speculative concepts and on more or less incomplete data on the geological environments of the uranium deposits. In spite of this uncertainty, it is worthwhile to evaluate the potential of yet-to-be discovered resources as an approximation that can be progressively refined as knowledge of the uranium deposits and their geologic settings becomes more complete.

The importance of an assessment of the uranium resources is twofold, on the one hand for the country itself, because this knowledge is essential for planning long range development programmes, and on the other hand for the world as a whole, because few countries are entirely self-sufficient in the uranium needed for economic development [2].

The needs for estimating undiscovered uranium resources are many. For a nation, knowledge of uranium resources can play an important role in establishing an energy policy and optimizing the use of public lands. For an exploration project, an initial estimate of undiscovered resources can be useful in guiding the collection of new data and the direction of exploration; subsequent estimates can aid in focusing exploration in the best areas for future discoveries of reserves. The method of estimation used will depend on the size of the area to be evaluated and on available information, but the goal remains to make the best estimate based on the available data.

Red Book

In the mid-1960's, the OECD Nuclear Energy Agency (NEA) and the International Atomic Energy Agency (IAEA) began the publication of a report titled: "Uranium Resources—Production and Demand." The report, commonly known as the **Red Book**, has been published at two-year intervals and has become widely recognized in the international nuclear community as a primary reference document on world uranium supply [3].

Each edition of the **Red Book** contains estimates of uranium resources in several categories of assurance of existence and economic attractiveness, and projections of production capability, installed nuclear capacity, and related reactor requirements.

Definitions and terminology

Only minor changes have been made to the NEA/IAEA resource terminology and definitions since the modifications that were introduced in the December 1983 edition of the Red Book, except for those relating to the introduction of a new lower-cost category, that is, resources recoverable at $40/kg U or less.

Resource estimates

Resource estimates are divided into separate categories reflecting different levels of confidence in the quantities reported. The resources are further separated into categories based on the cost of production. All resource estimates are expressed in terms of metric tons (tonnes) of recoverable uranium (U) rather than uranium oxide (U_3O_8). Estimates refer to quantities of uranium recoverable from mineable ore, unless otherwise noted.

Definitions of resource categories

Reasonably Assured Resources (RAR)

Uranium that occurs in known mineral deposits of delineated size, grade and configuration such that the quantities which could be recovered within the given production cost ranges with currently proven mining and processing technology, can be specified. Estimates of tonnage and grade are based on specific sample data and measurements of the deposits and on knowledge of deposit characteristics. Reasonably Assured Resources have a high assurance of existence.

Estimated Additional Resources —Category I (EAR-I)

Uranium in addition to RAR that is inferred to occur, mostly on the basis of direct geological evidence, in extensions of well-explored deposits, or in deposits in which geological continuity has been established but where specific data, including measurements of the deposits, and knowledge of the deposits' characteristics are considered to be inadequate to classify the resource as RAR. Estimates of tonnage, grade and cost of further delineation and recovery are based on such sampling as is available and on knowledge of the deposit characteristics as determined in the best known parts of the deposit or in similar deposits. Less reliance can be placed on the estimates in this category than on those for RAR.

Estimated Additional Resources —Category II (EAR-II)

Uranium in addition to EAR-I that is expected to occur in deposits for which the evidence is mainly indirect and which are believed to exist in well-defined geological trends or areas of mineralization with known deposits. Estimates of tonnage, grade and cost of discovery, delineation and recovery are based primarily on knowledge of deposit characteristics in known deposits within the respective trends or areas and on such sampling, geological, geophysical or geochemical evidence as may be available. Less reliance can be placed on the estimates in this category than on those for EAR-I.

Speculative Resources (SR)

Uranium, in addition to Estimated Additional Resources—Category II, that is thought to exist, mostly on the basis of indirect evidence and geological extrapolations, in deposits discoverable with existing exploration techniques. The location of deposits envisaged in this category could generally be specified only as being somewhere within a given region or geological trend. As the term implies, the existence and size of such resources are speculative.

Cost categories

The cost categories are defined as: $40/kg U or less; $80/kg U or less; $130/kg U or less; and $260/kg U or less.

NOTE: It is not intended that the cost categories should follow fluctuations in market conditions.

To convert from costs expressed in $/lb U_3O_8 to $/kg U, a factor of 2.6 has been used (that is, $15/lb U_3O_8 = $40/kg U; $30/lb U_3O_8 = $80/kg U; and $50/lb U_3O_8 = $130/kg U).

Conversion from other currencies into US$ should be done using the exchange rates of the year of the most recent Red Book. All resource categories are defined in terms of costs of uranium recovered at the ore processing plant.

When estimating the cost of production for assigning resources within these cost categories, account has been taken of the following costs:

- the direct costs of mining, transporting and processing the uranium ore;
- the costs of associated environmental and waste management;
- the costs of maintaining non-operating production units where applicable;
- in the case of ongoing projects, those capital costs which remain unamortized;
- the capital cost of providing new production units where applicable, including the cost of financing;
- indirect costs such as office overheads, taxes and royalties where applicable;
- future exploration and development costs wherever required for further ore delineation to the stage where it is ready to be mined.

Sunk costs are not normally taken into consideration.

Relationship between resource categories

Figure 2 illustrates the relations among the different resource categories. The horizontal axis expresses the level of assurance about the actual existence of given tonnages based on varying degrees of geologic knowledge and the vertical axis expresses the economic feasibility of exploitation by the division into cost categories.

The dashed lines between RAR, EAR-I, EAR-II and SR in the highest cost category indicate that the distinctions of level of confidence are not always clear. The shaded area indicates that known resources (that is, RAR plus

EAR-I) recoverable at costs of $80 kg/U or less are distinctly important because they support most of the world's EXISTING and COMMITTED production centres. RAR at prevailing market prices are commonly defined as "Reserves." Because resources in the EAR-II and SR categories are undiscovered, the information on them is such that it is not always possible to divide them into different cost categories and this is indicated by the horizontal dashed lines between the different cost categories.

Recoverable resources

Resource estimates are expressed in terms of recoverable tonnes of uranium, that is, quantities of uranium recoverable from mineable ore, as opposed to quantities contained in mineable ore, or quantities in situ. Therefore both expected mining and ore processing losses have been deducted in most cases. Deviations from this practice are indicated in the tables.

Types of resources

The major uranium resources of the world can be assigned on the basis of their geological setting to the following types:

- Unconformity-related deposits
- Sandstone deposits
- Quartz-pebble conglomerate deposits
- Vein deposits
- Breccia complex deposits;
- Intrusive deposits
- Phosphoric deposits
- Collapse breccia pipe deposits
- Volcanic deposits
- Surficial deposits

- Metasomatite deposits
- Metamorphite deposits
- Lignite
- Black shale deposits
- Other types of deposits

Conventional resources

Conventional resources are those that have an established history of production where uranium is either a primary product, co-product or an important by-product (for example, gold).

Unconventional resources

Very low grade resources, which are not now economic or from which uranium is only recoverable as a minor by-product are considered unconventional resources (that is, phosphates, monazite, coal, lignites, black shales, and so forth).

Units of measure

Metric units are used in all tabulations and statements. Resources and production quantities are expressed in terms of metric tons (tonnes) contained uranium (U) rather than uranium oxide (U_3O_8).

1 short ton U_3O_8 = 0.769 tonnes U
$1/lb U_3O_8 = $2.6/kg U

Glossary of terms

Uranium occurrence—A naturally occurring anomalous concentration of uranium.

Uranium deposit—A mass of naturally occurring mineral material from which uranium could be exploited at present or in the future.

Resource classification—San Juan Basin, New Mexico, USA—An example

During the period between 1970 and 1990, the San Juan basin was the principal source of supply for uranium for the United States [4]. By far the most important host for uranium deposits in the basin was the Late Jurassic Westwater Canyon Member of the Morrison Formation. The Westwater Canyon was estimated to contain more than 90 percent of the discovered uranium resources in the basin. Consequently, most of the geologic studies and exploration activities of the Grants uranium region, the area's principal producing area, have focused on the Westwater Canyon. In the San Juan basin, other host rocks are known to contain uranium or else are known to have a potential for uranium deposits. Figure 3 illustrates the geologic setting in the San Juan basin in which the presence and amount of undiscovered resources can be predicted with varying degrees of confidence.

In the main producing region, the Grants uranium region, the uranium resources can be classified as RAR or EAR-I depending whether the uranium occurs in a known deposit or is inferred to occur in extensions of known deposits. In the case of the latter, the estimates of the uranium resources that are made are based on direct geological evidence such as extensions of well-explored deposits, or in deposits in which geological continuity has been established but where specific data and knowledge of the deposits' characteristics are considered to be inadequate to classify the resource as RAR.

In the area to the north and west, the uranium resources can be classified as EAR-II. The presence of uranium in the Westwater Canyon can be anticipated because of the many large deposits in the Grants mineral region to the south. In this area, however, the evidence is largely indirect, in that no uranium deposits are known to exist and thus less reliance can be placed on the estimates that are made of the undiscovered uranium resources.

In the two areas to the north and east, uranium mineralization is expected to occur in upper Cretaceous and Tertiary sandstones based on indirect evidence and geological extrapolations from geologic settings outside these areas. In this case, estimates that are made of the undiscovered resources would be classified as SR because the location of deposits envisaged in this category can be specified only as being somewhere within the given areas.

Methods for estimation of undiscovered resources

The process of estimating undiscovered uranium resources consists of three basic steps, namely: the determination (ranking) of geologic favourability within the region being assessed, the probabilistic estimation of the undiscovered endowment of uranium deposits in the region, and the economic evaluation of the undiscovered endowment [5].

Determination of geologic favourability is based upon the analogy of the geologic setting of the area being assessed to areas that contain uranium deposits of the expected type. In this process, areas are judged to be favourable or unfavourable. Areas that are known insufficiently may be designated as having uncertain favourability. The probabilistic estimates of the undiscovered endowment within a study area are derived primarily by comparison to the known deposits in what are referred to as "control areas." Finally, the undiscovered uranium endowment is evaluated in terms of economics based on cost models using factors, such as grade cut-off, thickness, depth, and current mining and milling costs. Depending upon the knowledge base, the resources of the area are classified and divided into one of the resource and cost or price categories.

Delineation of favourable areas

Estimating uranium resources proceeds from identifying a permissive area based on general favourability for occurrence of a class of uranium deposit to specific outlining of favourable areas. General permissive favourability is based on key recognition criteria, such as presence of favourable regional geology, favourable host rock, and any known uranium anomalies, for a suspected class of deposit. Delineation and actual outlining of the area favourable for uranium deposits of a specific deposit type is based on the geographic or spatial (vertical) distribution of key recognition criteria. Key factors include: outcrop of favourable host rock and extensions downdip; distribution of a distinctly favourable lithology or facies; area lacking post host-rock faults (preservation factor); and distribution of uranium anomalies. Favourability maps would result from such an analysis, and overlay intersections or overlaps would identify the favourable area for that specific deposit type. A different outline for a different deposit type might exist within the permissive area. For purposes of national resource assessments, favourable areas are delineated on an appropriate quadrangle map series. In the United States, 1° by 2° quadrangle maps at a scale of 1:250,000 were used. For other purposes, larger scale maps may be more useful; for example, exploration requires maps at scales of 1:24,000 and larger.

Study for favourability commonly exposes the lack of certain data required to draw acceptable boundaries, in which case the favourable area will have approximate outlines. Such a situation will guide geologic studies for later improvements in delineation of favourable areas. The study of favourability also allows for ranking of areas of different favourability.

Control areas

The principal purpose of control areas is to support the estimation of uranium resources in a favourable area. A control area is a specific geographic area for which geologic characteristics (recognition criteria), potential resources, uranium reserves, and production are known. Well-established control areas for deposits of many kinds and from around the world were defined during the U. S. Department of Energy's National Uranium Resource Evaluation (NURE) programme [6]. A completely described control area of the NURE programme consisted of the following: a set of geologic recognition criteria; a map of the outcrop of the host formation within its downdip extent of thoroughly explored land, and locations of mines and reserve blocks; size, thickness, and grade distribution for mining properties (a mining property commonly is not a single geologic deposit as one property might cover a whole deposit, parts of several, or only part of one deposit); distribution curves for the in-place inventory of U by grade; range in depth to ore; and a set (lower limit, most likely, and upper limit) of values for the *A*, *F*, *T*, and *G* factors in the standard NURE equation of estimating the undiscovered uranium endowment [5].

Distributions for control areas

Uranium grade and tonnage distributions developed from the set of known deposits of a control area are extremely important in the estimation of undiscovered endowment, economic resources, and discoverable economic resources of the favourable area. There are different kinds of grade and tonnage distributions, each kind being useful in different ways. To distinguish between them, it is useful to adopt different terms. Histograms or tables that depict the distribution of deposit size (ore tonnage) and deposit average grade are referred to as deposit tonnage (size) and grade distributions or models. These histograms aid in the estimation of endowment by serving as initial descriptions of possible size and grades of the unknown deposits of the favourable area. Thus, when converted to relative frequency histograms, these distributions serve as *a priori* estimates of the probability distributions for deposit size and grade that can be modified to reflect differences in the geology of the favourable and control areas.

To clarify the nature of grade-tonnage curves, consider an orebody of one million tons for which the grades of mining blocks have been estimated and the ore in all blocks has been classified into five grade intervals, as shown in table 1. Multiplication of the midpoint grade (column 2) and tonnage of ore (column 3) for an interval gives amount of metal in the interval (column 4). Dividing cumulative metal (column 6) by cumulative ore (column 5) gives cumulative average (column 8).

Table 1. Grade tonnage calculations and relations.

Grade interval (proportion) q	Grade midpoint	Ore (short tons) ($x10^6$)	Metal[b] (short tons) ($x10^6$)[a]	Cumulative ore (short tons) ($x10^6$)	Cumulative metal (short tons) ($x10^6$)	Cut-off grade (percent)	Cumulative average grade[c] (percent)
$.015 \leq q < .025$	0.02	0.10	0.002	1.00	0.0365	0.015	0.0365
$.025 \leq q < .035$	0.03	0.40	0.012	0.90	0.0345	0.025	0.0383
$.035 \leq q < .045$	0.04	0.30	0.012	0.50	0.0225	0.035	0.0450
$.045 \leq q < .055$	0.05	0.15	0.0075	0.20	0.0105	0.045	0.0525
$.055 \leq q < .065$	0.06	0.05	0.003	0.06	0.003	0.055	0.0600

[a] 1 short ton = 2000 lb: 1 lb = 0.4536 kg

[b] Column 2 times column 3

[c] Column 6 divided by column 5

Plotting of cumulative ore (column 5) against cut-off grade (column 7) gives a curve that is useful in depicting the amount of ore having grades above a specified cut-off grade. Plotting of cumulative average grade (column 8) against cut-off grade (column 7) gives a curve that depicts the average grade of all ore having grades above a specified cut-off grade. Together, these two curves facilitate the description of the amount and average grade of an orebody for a specified cut-off grade. Typically, such grade-tonnage curves were constructed by the U.S. Department of Energy for each analogue of a control area.

The usefulness of tonnage-grade distributions developed on deposit data from explored areas is predicated upon the membership of the explored areas and the appraisal region in the same statistical and geologic populations. In actual practice, definition of deposit populations often is problematic, because each deposit may be unique in some regard. Selection of those criteria which subsume some particulars and at the same time heed those features that truly define populations in some larger, meaningful sense is difficult and never totally acceptable to all experts and critics. Nevertheless, in order to make the scarce data that are available useful to the geologist who is attempting an even more difficult task, that is, the estimation of unknown mineral endowment, useful populations must be defined and available data must be thus organized and provided to those making resource appraisals.

When data on control areas exist, one useful procedure is to prepare relative frequency tables for deposit size and average grade for each analogue area and for as large a composite(s) as is (are) useful and at the same time compatible with geologic considerations, e.g., deposit type and geologic environment. Everything else being equal, histograms and relative frequency tables are more acceptable as approximations to probability distribution when they are developed on a large number of samples, provided of course, that all samples are from the same population. Furthermore, when these distributions have been developed on acceptable control areas, they can be accepted as approximations to the desired probability distributions unless, and only unless, there are compelling geological differences between the appraisal region and the analogues, and these differences give reason to expect different deposit sizes or grades. That situation would occur when, conditional upon a deposit occurring in the favourable area, the probability that the deposit will have a tonnage and average grade within specified intervals is believed on geological grounds to differ from the relative frequencies developed on the control areas. When this is the case the *a priori* distributions should be modified accordingly.

In favourable areas where no grade-tonnage data are available and where the available data are not suitable (for example, where mining property information does not correspond to a single deposit as discussed above), new grade-tonnage distributions will need to be developed. For new favourable areas, grade-tonnage data may be developed in two ways: from a single well-developed deposit in the control area or from available production and reserve data on all known deposits in the well-explored part of a control area. In either case, the average grade and tonnes of contained U at selected cut-off grades are tabulated in table 2. From these data, grade and tonnage curves are constructed as shown in figure 4.

Table 2. Average grades and tonnes of ore and U at various cut-off grades [6]

Cut-off grade % U	Average grade % U	Tonnes of ore x 10^6	Tonnes of U
0.01	0.06	472	283,000
0.02	0.07	381	267,000
0.03	0.09	273	246,000
0.04	0.11	198	218,000
0.05	0.13	153	200,000
0.06	0.14	130	183,000
0.07	0.15	112	168,000
0.08	0.17	92	156,000
0.09	0.18	79	143,000
0.10	0.20	67	133,000
0.11	0.22	56	124,000
0.12	0.23	49	112,000
0.13	0.24	44	106,000
0.14	0.26	37	97,000

The control area selected for use in a given favourable area must be described in terms of factors needed for the appraisal method being used. The geology of the favourable area must match closely the selected control area. Ideally, the control area should be totally explored so that endowment factors represent all of the uranium that occurs, but seldom is this the case. Where exploration is incomplete, information on exploration density in the control area is useful to adjust some endowment factors, either subjectively or by formal analyses [7].

Probability distribution for endowment of a region

Probabilistic descriptions of endowment descriptors per se, that is the number of deposits, deposit size and grade, and depth to deposit, usually are of little use to decision makers and land managers. Of greater use to them are measures derived from these descriptors. Typically, these measures have been of stocks, such as the total uranium present in occurrences meeting the endowment cut-offs, the total amount of uranium present in endowment occurrences that could be economically exploited if they were known (resources that meet specified economic conditions), and the total uranium present in endowment occurrences that could be discovered and economically produced (potential supply) for specified economic conditions.

Any analysis and description of stocks is complicated, because the basic endowment descriptors are random variables described by probability distributions. Consequently, the descriptors must be combined in ways that honor probability and statistical laws and that produce estimates that meet the specified conditions. The descriptions that follow will proceed from the simplest of cases to those that are more complex. In all cases, number of deposits is considered to be independent of deposit size and grade.

Simplest case-A single region

This case is the simplest because we do not concern ourselves with deriving a probability distribution for endowment for the aggregate of several regions. Let us consider first the case in which endowment descriptors have been described by probability tables, not by mathematical models. Suppose that probabilities for the descriptors number of deposits, deposit size, and deposit grade are described by a one-dimensional table for number of deposits (table 3) and a two-dimensional table for size and grade (table 4).

Table 3. Probability for number of deposits

Number of deposits	Probability
$n = 0$	0.2
$1 \leq n < 4$	0.4
$4 \leq n < 7$	0.3
$7 \leq n < 10$	0.1

Table 4. Probability for deposit size and grade

	Average grade (Q) interval (%U)		
Size (T) interval (10^3 tonnes ore)	0.005 - 0.01 (0.0075)[a]	0.01 - 0.10 (0.055)	0.10 - 0.4 (0.25)
50 - 100 (75)[a]	0.15	0.20	0.10
100 - 300 (200)	0.20	0.10	0.05
300 - 500 (400)	0.10	0.075	0.025

[a]Numbers within parentheses are midpoints of intervals

Knowing that our objective is a probability distribution for endowment, a new table is needed, one that shows probabilities for quantities of metal (m) per deposit. Since metal is the product of grade and ore tonnage, divided by 100, the information from table 4 can be used to construct table 5. In this approximation, the midpoints of size and grade intervals for each element in table 4 are multiplied and divided by 100 to give m for each element in table 5.

Table 5. Quantity of metal (m) for tonnage and grade intervals and associated probabilities

Probability		
(0.15)[a]	(0.20)	(0.1)
5.625	41.25	187.5
(0.20)	(0.10)	(0.05)
15	110	500
(0.10)	(0.075)	(0.025)
30	220	1000

[a]Directly over each quantity of metal (tonnes) is the probability (within parentheses) for its combination of size and quantity; these are simply reproduced from table 4, because all of the tonnage and grade combinations yield different amounts of metal.

The most complete description of total metal endowment requires the probability distribution for total endowment (E), where $E = N \cdot M$. Thus, it is necessary to create a new random variable E using the distributions for random variables N and M. Because there are 4 values that N can take and 9 values that M can take, there are 36 possible values for E. Furthermore, since N and M are assumed to be independent, the probability for each combination is simply the product of the probabilities of the associated states of N and M. These 36 values for E are listed in table 6, along with the probability for each combination. Column 3 of table 6 shows the value of endowment (E) obtained by multiplying number of deposits (N) by metal per deposit (M), columns 1 and 2, respectively. Column 4, titled probability, is the product of the probability for N and the probability for M: $P(N) \cdot P(M)$. Finally, column 5 shows the product of endowment and probability. The expectation for endowment is the sum of these products, which is 536.106 tonnes.

Having computed probabilities for combinations of endowment, it is now necessary to gather or group common events and sum their probabilities, because different combinations of ore tonnage and grade give the same quantity of metal. Intervals of endowment are useful for this gathering process; otherwise the resulting endowment distribution would be excessively ragged. In order to regularize the distribution of endowment, it is useful to define eight intervals for endowment such that they cover the range of nonzero endowments in table 6. Such intervals are shown in column one of table 7. Each row of column 2 of this table shows the sum of probabilities of those events listed in t able 6 that fall within that interval. For example, the probability that E is greater than 80 but less than 160 is the sum of probability for E = 82.5 (5th quantity from top of column 3) and the probability for E = 112.5:

$$P(80 \leq E < 160) = P(E = 82.5) + P(E = 112.5) = 0.08 + 0.06 = 0.14.$$

Table 6. Combinations of E = N · M and probabilities

N	M	E	Probability	E · Probability
0	5.62	0	(0.2) · (0.15) = 0.03	0.0
0	41.25	0	(0.2) · (0.2) = 0.04	0.0
0	187.50	0	(0.2) · (0.10) = 0.02	0.0
2	5.62	112.50	(0.4) · (0.15) = 0.06	0.675
2	41.25	82.50	(0.4) · (0.20) = 0.08	6.6
2	187.50	375.00	(0.4) · (0.10) = 0.04	15.0
5	5.62	281.25	(0.3) · (0.15) = 0.045	1.266
5	41.25	206.25	(0.3) · (0.20) = 0.06	12.375
5	187.50	937.50	(0.3) · (0.10) = 0.03	28.125
8	5.62	450	(0.1) · (0.15) = 0.015	0.675
8	41.25	330	(0.1) · (0.20) = 0.02	6.6
8	187.50	1500	(0.1) · (0.10) = 0.01	15.0
0	150	0	(0.2) · (0.20) = 0.04	0.0
0	110	0	(0.2) · (0.10) = 0.02	0.0
0	500	0	(0.2) · (0.05) = 0.01	0.0
2	150	300	(0.4) · (0.20) = 0.08	2.4
2	110	220	(0.4) · (0.10) = 0.04	8.8
2	500	1000	(0.4) · (0.05) = 0.02	20.0
5	150	750	(0.3) · (0.20) = 0.06	4.5
5	110	550	(0.3) · (0.10) = 0.03	16.5
5	500	2500	(0.3) · (0.05) = 0.015	37.5
8	150	1200	(0.1) · (0.20) = 0.02	2.4
8	110	880	(0.2) · (0.10) = 0.01	8.8
8	500	4000	(0.1) · (0.05) = 0.005	20.0
0	300	0	(0.2) · (0.10) = 0.02	0.0
0	220	0	(0.2) · (0.075)=0.015	0.0
0	1000	0	(0.2) · (0.025)=0.005	0.0
2	300	600	(0.4) · (0.10) = 0.04	2.4
2	220	440	(0.4) · (0.075)=0.03	13.2
2	1000	2000	(0.4) · (0.025 = 0.01	20.0
5	300	1500	(0.3) · (0.10) = 0.03	4.5
5	220	1100	(0.3) · (0.075)=0.0225	24.75
5	1000	5000	(0.3) · (0.025)=0.0075	37.5
8	300	2400	(0.1) · (0.10) = 0.01	2.4
8	220	1760	(0.1) · (0.075)=0.0075	13.2
8	1000	8000	(0.1) · (0.025)=0.0025	20.0
			TOTAL	345.166

Table 7. Probability distributions for endowment

Endowment (E)	P(E)	P(E | N > 0) = P(E | E > 0)
0	0.2	
80 - 160	0.14	0.175
160 - 300	0.145	0.18125
300 - 500	0.185	0.23125
500 - 800	0.130	0.1625
800 - 1300	0.1025	0.128125
1300 - 2000	0.0575	0.071875
2000 - 4500	0.025	0.03125
4500 - 8500	0.010	0.0125

Note that 0.14 is the probability in row 2 of column 2 of table 7. Continuing in this fashion yields the probabilities shown in column 2 of table 7 for all intervals.

Columns 1 and 2 of table 7 represent a generalization of the computed probabilities to a histogram-like form. Table 7 has a 3rd column that can be considered the conditional probability distribution for endowment, conditional upon the presence of at least one deposit. Because all tonnages and grades are positive and not zero, the only way that an endowment of zero can occur is for zero deposits to occur. An alternative representation is to decompose the probability distribution into two distributions: one distribution describes the probability for E, given that E > 0, and the other describes P (E = 0) and P (E > 0). As indicated above, P (E = 0) = P (N = 0) and P(E > 0) = P (N > 0). The probability distribution for E conditional upon E > 0 is given in column 3. This distribution is obtained by removing zero from the domain of E and normalizing the new probability space to sum to one by dividing each probability by 0.8 = 1 - P (E = 0).

Economic potential estimation

A quantity of resources of U consists of U in those mineral occurrences of the endowment which, if they were discovered, could be exploited profitably for the specified product price and technologies of production. This means that price covers all operating costs, returns invested capital, and yields a normal return on capital (cost for use of capital). In principle, comprehensive resource appraisal is very similar to the economic evaluation of an identified prospect. This is difficult to conceive of when considering only a total quantity of uranium endowment. But, suppose that in addition to the endowment quantity, the population of deposits that comprise the endowment were described by their major features, such as size, grade, and depth. Then, resource appraisal could be achieved by evaluating each hypothetical deposit, one at a time, just as though it were a known prospect.

Accordingly, given expected characteristics of a deposit, determination of whether or not its uranium endowment is part of the region's resources consists of the following four general activities:

Activity I: Estimation of operating and capital costs: mine; mill; transportation and infrastructure

Activity II: Estimation of efficiency parameters: mine dilution and recovery; mill recovery; mine and mill lives

Activity III: Projection of annual cash flows: price, property, and severance taxes; royalties; depreciation schedule; depletion allowance; income or profit taxes

Activity IV: Calculation of net present value (NPV): discounting cash flows and capital expenditure by the rate of return required on capital

Probability distribution for resources using cost tables—A simplified numerical demonstration

The very simple number distribution of table 3 and the size-grade distribution of table 4 serve to demonstrate the calculations for a probability distribution for resources for a specified price/cost. The endowment population must be transformed to the resources population, which contains only those deposits that could be exploited profitably. As discussed above, such analysis done comprehensively requires estimation of the cost components, cash-flow accounting, and computation of net-present-value for each deposit of the endowment population. Subsequent to or concurrent with the economic evaluation, the probability distribution for the new population (resources) must be calculated. In order to demonstrate more clearly the interaction of economic and probability analyses, the economic analysis is simplified by using a cost table (table 8), which lists total costs of recoverable U for each deposit size and grade combination of table 4. In other words, each element of table 8 represents the result of economic analysis, estimation of cost components, cash flow analysis, and calculation of net present value.

Table 8. Total costs US$ per kg U for deposit sizes and grades

Size (10^3 tonnes of ore)	Average grade (%U)		
	0.005-0.01	0.01-0.10	0.10-0.40
50-100	100	40	15
100-300	80	30	12
300-500	50	20	10

Table 9. Overall mining and milling recoveries as a proportion

Size (10^3 tonnes ore)	Average grade (% U) 0.005-0.01	0.01-0.10	0.1-0.40
50-100	0.58	0.62	0.75
100-300	0.60	0.65	0.82
300-500	0.62	0.70	0.85

Table 10. Quantity (tonnes) of U in $20/kg resources

Size (10^3 tonnes ore)	Average grade (% U) 0.005-0.01	0.01-0.10	0.10-0.40
50-100	0	0	140.625 = (187.5)(0.75)
100-300	0	0	410.0 = (500.0)(0.82)
300-500	0	154 = (220)(0.70)	850.0 = (1000.0)(0.85)

Let us assume that the objective is a probability distribution for resources economically producible at a price of $20/kg U. On the assumption that price is competitively determined, product price = total unit cost. Thus, the maximum allowable cost is $20/kg U. At this cost, only four of the size-grade combinations of table 8 describe deposits that belong to the resources population.

The procedure for calculating the $20/kg U resources probability distribution is similar to that for endowment, except that the amount of U for any size-grade combination must first be reduced to that amount recovered overall by mining and milling. Suppose that such recoveries have been estimated and are tabulated by deposit size and grade, as shown in table 9. Multiplication of these fractions by corresponding elements of the $20/kg U-resources-per-deposit table, gives the element of table 10, recoverable $20/kg

U-resources-per-deposit. For example, the first element in the last column of table 10 shows 187.5 multiplied by 0.75. The number 187.5 is the product of the midpoint of the first ore tonnage (75,000 tonnes) and the midpoint of the last grade class (0.25) divided by 100: 187.5 = (75,000) (0.25)/100.

Given tables 3, 4, and 10, probabilities and recoverable $20/kg U-resources for size-grade combinations, the computation of the probability distribution for $20/kg U-resources parallels that for endowment in that the probability distribution for number of deposits must be integrated with the probability distribution for $20/kg U-resources by size and grade combination. This requires elaboration of all combinations, as was done in table 6; however, inasmuch as resources are zero for any combination in which either $20/kg U-resources-per-deposit is zero or for which number of deposits is zero, computations can be reduced considerably by elaborating only nonzero states, as is done in table 11.

Table 11. Nonzero combinations of number of deposits (N), amount of U per deposit ($\bar{M}$), and associated probabilities (P) and resources (R)

N[a]	$\bar{M}$[b]	R = N · M	P(N,$\bar{M}$)[c]	P(N,$\bar{M}$)R
2	154.0	308.0	0.03 = (0.4)·(0.075)	9.24
5	154.0	770.0	0.0225 = (0.3)·(0.075)	17.325
8	154.0	1232.0	0.0075 = (0.1)·(0.075)	9.24
2	140.625	281.25	0.04 = (0.4)·(0.1)	11.25
2	410.0	820.0	0.02 = (0.4)·(0.05)	16.4
2	850.0	1700.0	0.01 = (0.4)·(0.025)	17.0
5	140.625	703.125	0.03 = (0.3)·(0.1)	21.09375
5	410.0	2050.0	0.015 = (0.3)·(0.05)	30.75
5	850.0	4250.0	0.0075 = (0.3)·(0.025)	31.875
8	140.625	1125.0	0.01 = (0.1)·(0.1)	11.25
8	410.0	3280.0	0.005 = (0.1)·(0.05)	16.4
8	850.0	6800.0	0.0025 = (0.1)·(0.025)	17.0
			0.2	208.82375[d]

[a]From table 3
[b]From table 10
[c]From tables 4 and 4
[d]Expected (average) value for $20/kg U-resources

Table 12. Probability distribution for $20/kg U resources

Quantity of resources (R) (tonnes) $r_l \leq R < r_u$	$P(r_l \leq R < r_u)$	$P(r_l \leq R < r_u \mid r > 0)$
0.0	0.8	
0 < R < 200	0.0	0.0
200 ≤ R < 300	0.04	0.20
300 ≤ R < 500	0.03	0.15
500 ≤ R < 800	0.0525	0.2625
800 ≤ R < 1300	0.03	0.15
1300 ≤ R < 2000	0.0175	0.0875
2000 ≤ R < 4500	0.0275	0.1375
4500 ≤ R < 8500	0.0025	0.0125

The sum of the joint probabilities P (N,M) is 0.2. This is the probability that $20/kg U resources are greater than zero: P (R > 0) = 0.20. Accordingly, P ($R \leq 0$) = 1 - 0.20 = 0.80. The second column of table 12 is the probability distribution for $20/kg U resources. The third column is probability for $20/kg U resources, given that resources are greater than zero.

The foregoing example was highly simplified to facilitate demonstration of concepts and calculations. Accordingly, three continuous variables (t, q, and m) were represented by at most three intervals, and each interval was represented by its midpoint. When such a procedure is employed, it is necessary to use many more intervals than used in the example if the resulting distribution is to approximate closely that which would result from treating them properly as continuous variables. Clearly, the motivation to decrease interval size, hence increase numbers of intervals eventually is constrained by the large number of combinations that are created in the probability analysis. There are other methods that are useful in computing the probability distribution for uranium resources, depending upon the forms in which the probability distribution for number of deposits and deposit size and grade are described, such as functional and histogram, and the forms of the endowment transformation function and the cost analysis.

When probabilities for the random variables are described by fitted models, economic analysis is represented by a single continuous cost function, and endowment-resource transformation is described by a continuous and differentiable function, the probability density for resources per deposit at a cost of $kg/U can be determined mathematically.

Probability distribution for resources using mathematical analyses or computer simulation

The simplest way to estimate the probability distribution for uranium resources for a specified price or cost level, π, is by Monte Carlo simulation. The following simplified demonstration assumes that a probability distribution, F(M), for uranium endowment, M, has been estimated. Furthermore, probability distributions, G(t) and J(q), for deposit tonnage of ore (t) and average grade (q) respectively, have been constructed from control area information. Finally, production cost per kg U (c) is described by a cost function: $c\phi$ (t,q).

The first step is to simulate nature by sampling F(M) for an endowment. This is done by selecting a number, rn, at random on the interval [0,1] and locating it on the probability axis to obtain the associated value for M (see figure 5). The next step is to decompose M into deposits with ore tonnage and grade features. For this, we use M, G(t), and J(q). Select two random numbers on [0,1], rn_1, and rn_2. Using one of these numbers, for example, rn_1, sample G(t) for a deposit tonnage, t, and using rn_2 sample for deposit grade, q (see figure 6), we have

$$U_1 = t_1 \cdot q_1/100$$

Subtract U_1, from M to give M_1:

$$M_1 = M - U_1$$

Select two new random numbers and sample for deposit tonnage and grade, t_2, q_2. As before, subtract $U_2 = t_2 \cdot q_2/100$ from M_1 to give M_2:

$$M_2 = M_1 - U_2$$

Continue in this fashion until the nth sampling for which M_n=0. This procedure will have replaced endowment M by a set of n deposits, each described by tonnage and grade: $\{t_i, q_i\}$, i = 1, 2, ..., n. Evaluate each deposit for its cost:

$$c_i = \phi(t_i, q_i), i = 1,..., n.$$

Create a file of the k deposits for which $c_i \leq \pi$, where π is either price or maximum allowable cost per kg U; then, defining rs as quantity of U resources at price π, rs_1, the first simulated resources for price π, is determined by the following:

$$rs_1 = \sum_{i=1}^{k} t_i q_i / 100$$

This is an estimate of uranium resources, given the simulated value for M. In other words, this completes one iteration. Return to the beginning and sample for another M and create rs_2, just as was done for rs_1. Repeat this many (N) times, at least 100's of times, generating a file of N values for rs (rs_i, $i = 1, 2, ..., N$). The relative frequency histogram for this file is an approximation of the probability distribution for rs for price π. This entire process can be redone for different values of π, so as to show the effect of price on probability distribution of resources. The foregoing depicted Monte Carlo sampling graphically to facilitate understanding of the approach. In practice, all Monte Carlo analyses are done on computers. Accordingly, sampling uses computer subroutines in place of graphical interpretation.

Probability distributions created by Monte Carlo are at best an approximation to the actual distribution. This approximation improves as the number of iterations increases. When the number of random variables is small and their relations are not unduly complex, mathematical analysis not only is a feasible alternative to Monte Carlo but is preferred, because it is exact, not an approximation. In practice, probability distributions for uranium resources have been computed using Monte Carlo analysis [8] and by mathematical analysis [9].

Utilization of resource estimates

An estimate of undiscovered resources is the quantity of uranium expressed as tonnes U contained in the deposits thought to exist and thought to be exploitable with existing technology at the stated price or cost. By definition, undiscovered uranium resources are estimated to occur in unknown deposits. Such estimates are speculative as they are based on present geological-geochemical knowledge. Further research may provide information that justifies the modification of these estimates. Even if these resources were discovered, it is not certain that they could be made available as supply at the stated price, because of uncertainties about costs of producing deposits that have not been discovered. Constraints that could influence the discovery of these resources include many items, such as the incentive to conduct exploration, access to favourable areas, difficulties in raising the needed risk capital, and the available specialized human resources.

In view of these uncertainties, resource numbers must be used with utmost care and prudence. Some uses include economic planning, land management, legal requirements for export versus import, nuclear power planning, and determination of possible self-sufficiency for nuclear fuel. Resource estimates can be used by firms advantageously for the selection of areas for exploration. It is strongly recommended, however, that the estimated undiscovered resource numbers not be used without appropriate qualifiers or probability statements.

The transfer from undiscovered resources, which include both SR and EAR-II, to higher confidence resource categories, such as EAR-I, and RAR, occurs through exploration at various stages. Starting with SR believed to exist in geologically favourable areas, literature research and regional exploration may outline mineralized areas containing EAR-II, which through detailed follow-up work including close-spaced drilling will lead to the delineation of ore bodies containing RAR and EAR-I. This sequence shows that a number of exploration stages are needed and that these require expertise, human resources, capital, and time.

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Figures

Figure 1. Nuclear fuel cycle [1]

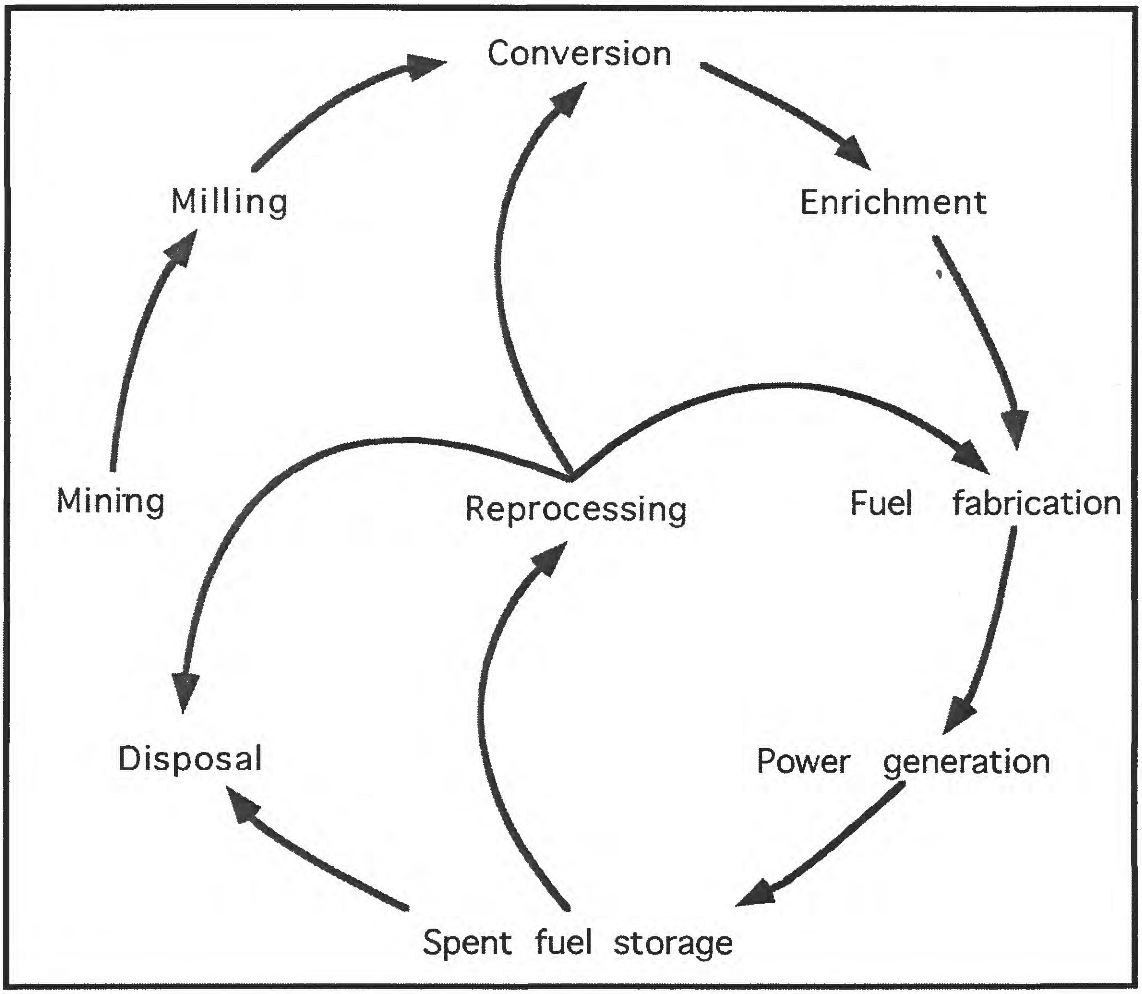
Figure 2. NEA/IAEA classification scheme for uranium resources [3]

Figure 3. San Juan Basin, NW New Mexico, USA

Figure 4. Grade-tonnage curves for the Ambrosia Lake control area, New Mexico, USA [6]

Figure 5. Random sampling of the endowment distribution

Figure 6. Random sampling of the deposit tonnage and grade distribution



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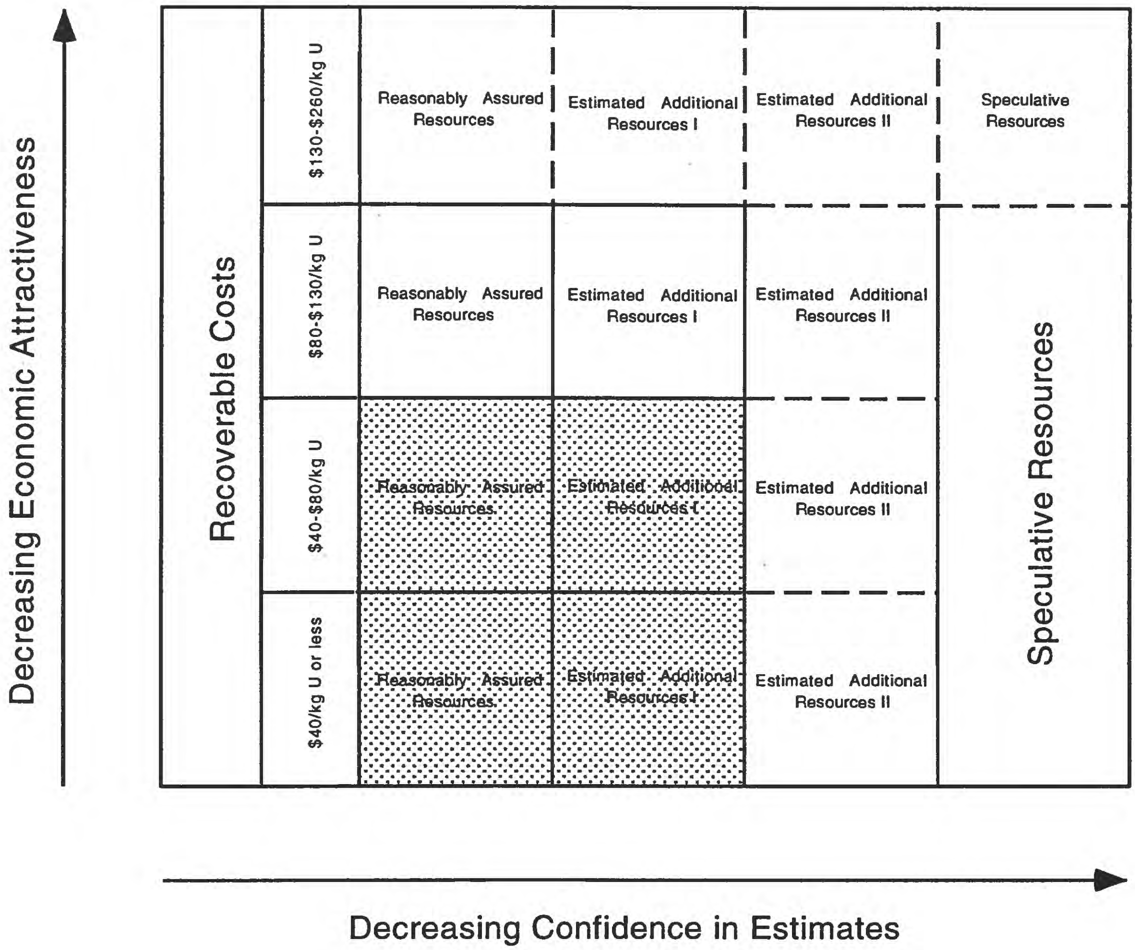


Figure 2. NEA/IAEA Classification Scheme for Uranium Resources [3]

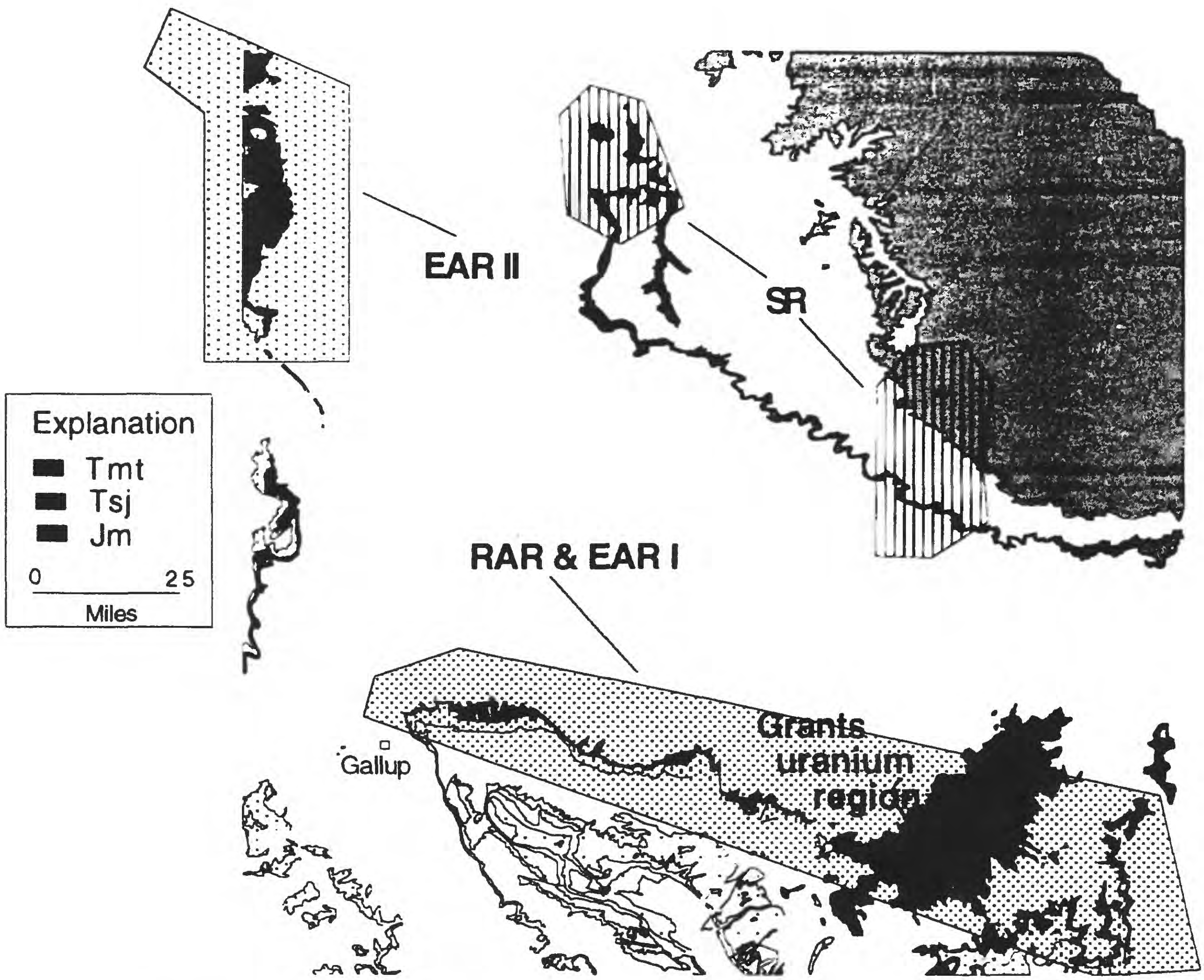
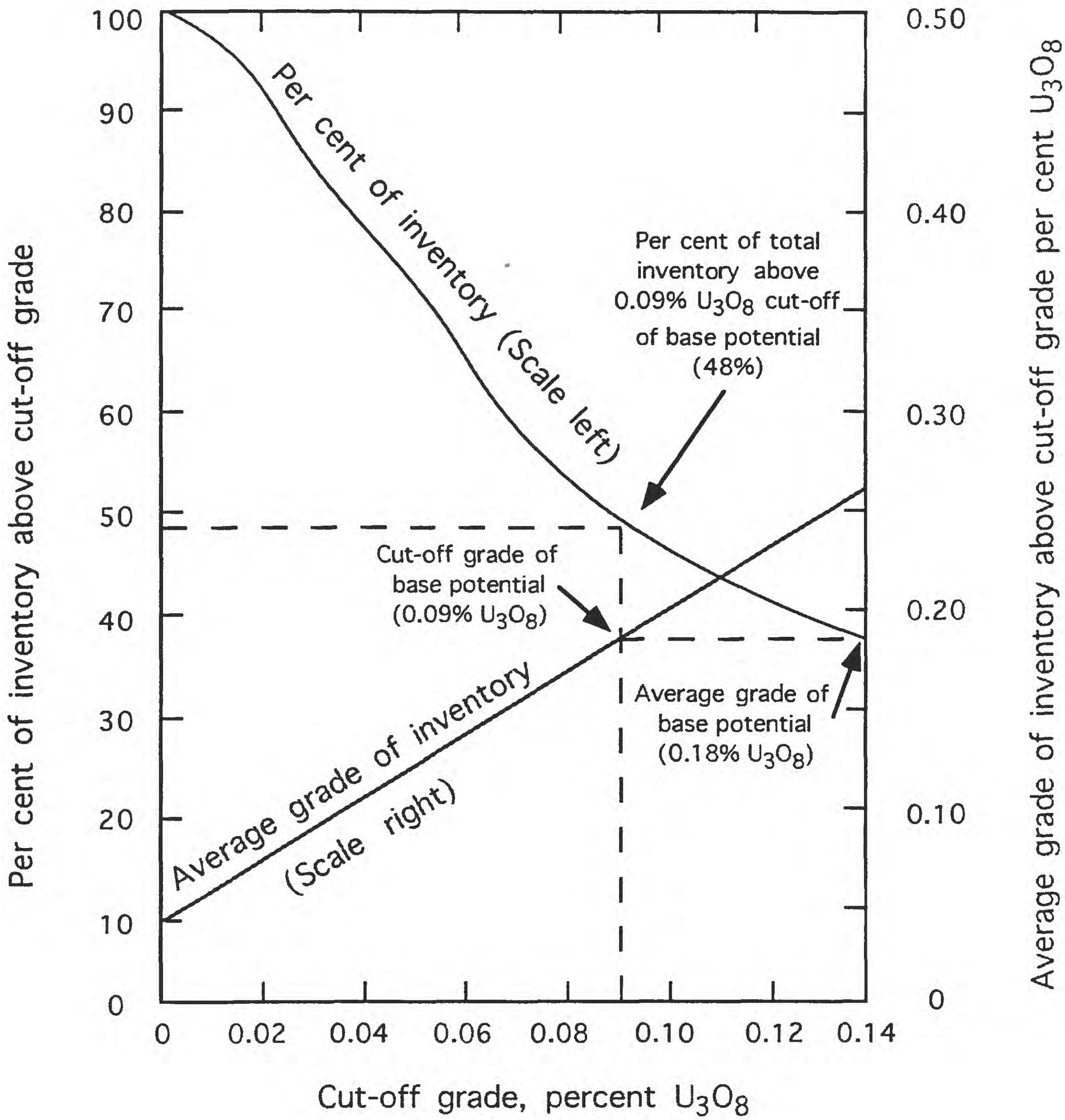


Figure 3. San Juan Basin, NW New Mexico, USA



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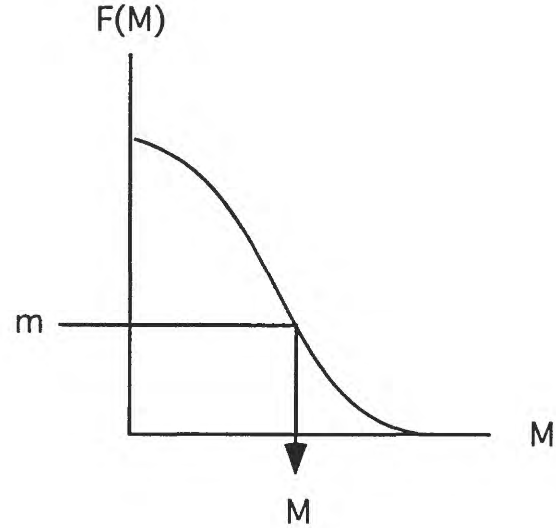

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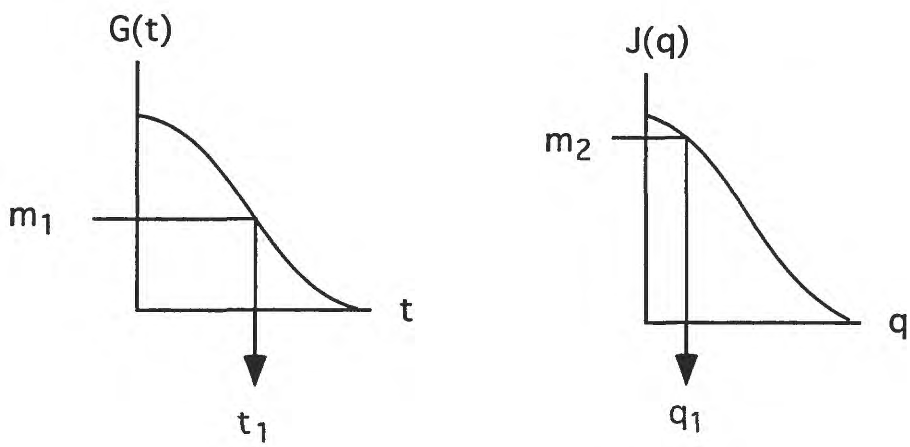

Figure 6. Random sampling of the deposit tonnage and grade distribution