U.S. DEPARTMENT OF THE INTERIOR

U.S. GEOLOGICAL SURVEY

Analysis of Errors in HEM Bird Calibration

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Open-File Report 97-742

17 December 1997

Introduction

With all geophysical measurements, calibration is a fundamental and necessary part of good measurement technique. Calibration is especially important for helicopter electromagnetic (HEM) systems owing to their inherently high (parts per million) sensitivity, which makes these systems vulnerable to errors introduced by minor geometrical uncertainties in the coil construction, size, and relative placement in the instrument pod, commonly referred to as the "bird." With the increasing use of quantitative interpretation and inversion of HEM data these errors take on greater importance than in the past. To overcome these problems and realize the full potential of the HEM method, calibration procedures have been developed whereby a calibration coil, called the Q-coil, is placed near the HEM bird while the bird is in the proximity of the ground. Based on calculations, the coil is designed to produce a known signal. These calculations have heretofore been made assuming that the ground below the Q-coil and bird is nonconducting, and that the Q-coil is properly adjusted and positioned. Some or all of these assumptions may be invalid.

To understand the above mentioned effects, I analyze the Q-coil response of a DIGHEM^V bird in the presence of a conducting ground. In addition to showing the effect of ground conductivity, errors in Q-coil position are analyzed. Finally, the actual steps used in calibrating an HEM bird are scrutinized step by step to determine their combined effect on the calibration procedure.

In this report responses are reported as true parts per million (ppm) of the primary signal. The convention used by DIGHEM is to normalize the horizontal coplanar and vertical coaxial responses in ppm to be the same when the secondary field intensities at the coaxial and coplanar receiver coils are the same. To convert the true coplanar coil response in ppm to the DIGHEM convention ppm, divide the true ppm values by 2. The coaxial values are the same for the true and DIGHEM conventions.

The errors analyzed here are not unique to the DIGHEM^V bird. Nevertheless, the DIGHEM^V bird was chosen for analysis because of my familiarity with it and the cooperation of Geoterrex-Dighem in providing necessary details required for the analysis. Other birds could be easily analyzed knowing bird geometry, operating frequencies, and Q-coil parameters.

Q-coil Response Near a Conducting Ground

In the theoretical formulation which follows, I generalize the analysis of Black (1996) for the response for a set of coils in free space to include the effect of a conducting ground. This is done by adopting a mutual impedance formulation which specifies the voltage induced in one coil due to a current flowing in another coil.

Mutual Coupling Formulation

The relationship between the voltage V_{ba} in coil B due to current I_a in coil A is expressed using the mutual impedance Z_{ba} (see Appendix) as

$$V_b = Z_{ba} I_a . \tag{1}$$

Coil Configuration

We will consider a coil configuration consisting of a transmitter coil, a receiver coil, a bucking coil, and a Q-coil as shown in Figure 1. The various coils are designated by subscripts T, R, B, and Q respectively. Thus the distance between the transmitter and Q-coil is given by r_{QT}. The analysis is done first for horizontal coplanar coils, and then vertical coaxial coils with the plane of the Q-coil oriented horizontally and vertically, respectively.

When the Q-coil is not present, we consider only the signal produced in the receiver and bucking coils by the transmitter and the ground. The voltage induced in the bucking coil is subtracted from the voltage induced in the receiver coil and recorded. Using a bucking coil increases the dynamic range of the instrument. Introduction of the Q-coil into the system creates a secondary source which is activated by currents induced in it by the transmitter and the ground. The Q-coil produces signals in the receiver and bucking coils.

Signal Produced Without a Q-coil

Let V_n be the signal at the receiver when the Q-coil is not present, namely

$$V_n = V_{RT} - V_{BT} \tag{2}$$

Expanding (2) using (1) gives

$$V_n = [Z_{RT} - Z_{BT}] I_T . \tag{3}$$

Signal Produced by the Q-coil

Introducing the Q-coil, the voltage at the receiver becomes

$$V_q = V_n + [Z_{RQ} - Z_{BQ}] I_Q \tag{4}$$

where I_Q is the current induced in the Q-coil. I_Q can be written as

$$I_Q = \frac{V_{QT}}{z_Q} = \frac{Z_{QT} I_T}{z_Q} \tag{5}$$

where $z_Q = R_Q + j\,\omega\,L_Q$ is the impedance of the Q-coil

Using the definition of $Q = \omega \, L_Q / R_Q$ we write

$$\frac{1}{z_Q} = \frac{1}{R_Q + j\,\omega\, L_Q} = \frac{1}{j\,\omega\, L_Q}\,\frac{1}{1/j\,Q + 1} = \frac{1}{j\,\omega\, L_Q}\,\frac{Q^2 + j\,Q}{1 + Q^2}\,. \tag{6}$$

Substituting (5) and (6) into (4) gives the general expression below

$$V_q = V_n + \left[Z_{RQ} - Z_{BQ}\right] Z_{QT}\,\frac{I_T}{j\,\omega\, L_Q}\,\frac{Q^2 + j\,Q}{1 + Q^2}\,. \tag{7}$$

The signal produced by the Q-coil is normalized by the signal produced in free space by the transmitter with no bucking coil present, namely

$$V_0 = Z_{0\,RT}\; I_T \tag{8}$$

to obtain

$$\frac{\Delta V}{V_0} = \frac{V_q - V_n}{V_0} = \left[Z_{RQ} - Z_{BQ}\right]\frac{Z_{QT}}{Z_{0\,RT}}\,\frac{1}{j\,\omega\, L_Q}\,\frac{Q^2 + j\,Q}{1 + Q^2}\,. \tag{9}$$

To convert the values of $\Delta V / V_0$ to parts per million (ppm), multiply by 10^6.

Equation (9) is a very general expression in that there are no restrictions on the placement or orientation of the coils. We will now evaluate it for the two coil-pair geometries in the bird. First let us look at the situation where the transmitter, receiver, and bucking coils are horizontal and in the same plane. At first we will impose no restrictions on the Q-coil other than it be horizontal. From (A13) and (A16) we obtain the required expressions to substitute into (9), and asuuming that the ratio of the moments of the bucking coil (S_B) to the receiver coil (S_R) are related to their respective distances from the transmitter coil by $S_B / S_R = r_{BT}^3 / r_{RT}^3$ gives

$$\frac{\Delta V}{V_0} = \frac{\mu_0}{4\,\pi}\,\frac{S_Q^2\, r_{RT}^3}{L_Q}\left\{\left[\frac{3(z_R - h_Q)^2}{R_{RQ}^5} - \frac{1}{R_{RQ}^3} - \frac{T_{0\,RQ}}{\delta^3}\right] - \frac{S_B}{S_R}\left[\frac{3(z_B - h_Q)^2}{R_{BQ}^5} - \frac{1}{R_{BQ}^3} - \frac{T_{0\,BQ}}{\delta^3}\right]\right\}$$

$$\times\left[\frac{3(z_Q - h_T)^2}{R_{QT}^5} - \frac{1}{R_{QT}^3} - \frac{T_{0\,QT}}{\delta^3}\right]\frac{Q^2 + j\,Q}{1 + Q^2}\,. \tag{10}$$

When all of the coils are in the same plane ($z_b = h_a$ and $R_{ba} = r_{ba}$), and (10) simplifies to

$$\frac{\Delta V}{V_0} = \frac{\mu_0}{4\,\pi}\,\frac{S_Q^2\, r_{RT}^3}{L_Q}\left\{\left[\frac{1}{r_{RQ}^3} + \frac{T_{0\,RQ}}{\delta^3}\right] - \frac{S_B}{S_R}\left[\frac{1}{r_{BQ}^3} + \frac{T_{0\,BQ}}{\delta^3}\right]\right\}\left[\frac{1}{r_{QT}^3} + \frac{T_{0\,QT}}{\delta^3}\right]\frac{Q^2 + j\,Q}{1 + Q^2}\,. \tag{11}$$

It is important to remember that the integrals represented by T_0 depend upon the specific coil pair being considered. For free space these terms become zero, and the normalized response is given by

$$\frac{\Delta V}{V_0} = \frac{\mu_0}{4\pi}\,\frac{S_Q^2}{L_Q}\,\frac{r_{RT}^3}{r_{QT}^3}\left[\frac{1}{r_{RQ}^3} - \frac{S_B}{S_R}\,\frac{1}{r_{BQ}^3}\right]\frac{Q^2 + jQ}{1 + Q^2}\ . \tag{12}$$

Notice that when Q=1, the inphase and quadrature responses in (12) are equal.

To obtain the result for vertical coaxial coils we again start with (9), but use expressions (A18) and (A21) and make a similar assumption about S_B and S_R as done previously. Requiring only that the plane of the Q-coil be vertical we obtain

$$\begin{aligned}\frac{\Delta V}{V_0} = \frac{-\mu_0}{8\pi}\,\frac{S_Q^2\, r_{RT}^3}{L_Q} &\left\langle \left\{\frac{3x^2}{R_{RQ}^5} - \frac{1}{R_{RQ}^3} + \frac{x^2}{\delta^3 r_{RQ}^2}\left[\left(1 - \frac{y^2}{x^2}\right)\frac{T_{2RQ}}{B_{RQ}} - T_{0RQ}\right]\right\}\right. \\ &\left. - \frac{S_B}{S_R}\left\{\frac{3x^2}{R_{BQ}^5} - \frac{1}{R_{BQ}^3} + \frac{x^2}{\delta^3 r_{BQ}^2}\left[\left(1 - \frac{y^2}{x^2}\right)\frac{T_{2BQ}}{B_{BQ}} - T_{0BQ}\right]\right\}\right\rangle \\ &\times\left\{\frac{3x^2}{R_{QT}^5} - \frac{1}{R_{QT}^3} + \frac{x^2}{\delta^3 r_{QT}^2}\left[\left(1 - \frac{y^2}{x^2}\right)\frac{T_{2QT}}{B_{QT}} - T_{0QT}\right]\right\}\frac{Q^2 + jQ}{1 + Q^2}\ .\end{aligned} \tag{13}$$

When all of the coils are coaxial $y = 0$, $z_b = h_a$, and $R_{ba} = r_{ba} = x$, and (13) simplifies to

$$\begin{aligned}\frac{\Delta V}{V_0} = \frac{-\mu_0}{8\pi}\,\frac{S_Q^2\, r_{RT}^3}{L_Q} &\left\{\left[\frac{2}{r_{RQ}^3} + \frac{1}{\delta^3}\left(\frac{T_{2RQ}}{B_{RQ}} - T_{0RQ}\right)\right] - \frac{S_B}{S_R}\left[\frac{2}{r_{BQ}^3} + \frac{1}{\delta^3}\left(\frac{T_{2BQ}}{B_{BQ}} - T_{0BQ}\right)\right]\right\} \\ &\times\left[\frac{2}{r_{QT}^3} + \frac{1}{\delta^3}\left(\frac{T_{2QT}}{B_{QT}} - T_{0QT}\right)\right]\frac{Q^2 + jQ}{1 + Q^2}\ .\end{aligned} \tag{14}$$

For free space the Hankel transform terms T_0 and T_2 become zero, and the expression becomes

$$\frac{\Delta V}{V_0} = \frac{-\mu_0}{2\pi}\,\frac{S_Q^2\, r_{RT}^3}{L_Q\, r_{TQ}^3}\left[\frac{1}{r_{RQ}^3} - \frac{S_B}{S_R}\,\frac{1}{r_{BQ}^3}\right]\frac{Q^2 + jQ}{1 + Q^2}\ . \tag{15}$$

Notice that this expression is -2 times the value for the horizontal coplanar coils given in (12). The negative signs in front of (13), (14), and (15) are ignored in the results presented as we only care about the absolute value.

Calculated Q-Coil Response

We now look at the calculated Q-coil response for a DIGHEM^{V} bird. The bird's geometric parameters are given in Table 1 and the Q-coil calibration parameters are given in Table 2. Before we look at the relative importance of these parameters and possible errors which can be introduced from variations in the calibration geometry, let us look at the magnitude of the free-space, Q-coil response.

Table 1 Bird parameters for a typical DIGHEM^{V} bird.

Coil Pair	Bird Geometry[1]	Freq [Hz]	r_{RT} [m]	r_{BT} [m]	S_B/S_R
1	HCP	867	7.976	2.659	0.0370510
2	HCP	7221	7.976	2.659	0.0370510
3	HCP	56653	6.325	2.108	0.0370195
4	VCX	1023	7.976	2.659	0.0370510
5	VCX	6504	7.976	2.659	0.0370510

[1]Bird coil orientations: HCP= horizontal coplanar, VCX=vertical coaxial

Table 2 Calibration and Q-coil parameters for a DIGHEM^{V} bird.

Coil Pair	Q-coil Geometry[2]	r_{RQ} [m]	r_{QT} [m]	r_{BQ} [m]	n_Q [turns]	a_Q [m]	L_Q [mH]
1	in-line	1.501	9.477	6.818	250	0.232	69.900
2	in-line	1.501	9.477	6.818	124	0.230	17.600
3	offset	1.613	6.527	4.515	10	0.229	0.127
4	in-line	2.217	10.193	7.534	250	0.232	69.900
5	in-line	2.217	10.193	7.534	124	0.230	17.600

[2]Refers to Q-coil location relative to bird axis (see Figure 1).

Table 3 gives the calculated, free-space calibration signal for the five coil-pair-frequency configurations. Notice that the values are higher than their nominal values of 200 ppm and 100 ppm for the horizontal coplanar and vertical coaxial geometries respectively. This discrepancy is due to an error in Dighem's formulation of the calibration calculations arising from the difference in the response of the bird to a source in the near and far fields. When the bird is at a nominal altitude of 30 m, the effect of currents flowing in the ground on the receiver and bucking coils will be proportional to their moments. Furthermore the instrument will record the receiver coil signal less the bucking coil signal. Because the ratio of the coil moments is $S_B/S_R = r_{BT}^3/r_{RT}^3$ the measured signal will be reduced by a factor equal to $B_f = 1 - \frac{r_{BT}^3}{r_{RT}^3}$. For the DIGHEM^{V} system the bucking coil is located one-third of the receiver-transmitter distance from the transmitter coil making this factor be 0.963. When the bird is on the ground and the Q-coil is located nearby, the reduction of the measured signal by the bucking coil is insignificant. As one would like the instrument to read a true signal when at altitude, the gain of the instrument must be set too high by a factor of $1/B_f$ to compensate. This can be done by setting the Q-coil at a distance from the receiver coil such that it produces a signal of 200 B_f ppm (or 100 B_f ppm), but calling this signal 200 ppm (or 100 ppm) on the output chart.

Table 3 Q-coil response in free space.

Coil Pair	Bird Geometry	Freq[1] [Hz]	Inphase and Quad Q Signal [ppm]
1	HCP	900	225.2
2	HCP	7200	212.6
3	HCP	56000	231.3
4	VCX	900	112.3
5	VCX	7200	106.0

[1]These are nominal frequencies. Actual values used are given in Table 1.

Because Dighem had been using a Q-coil signal of $200/B_f$ ppm (or $100/B_f$ ppm) rather than $200\,B_f$ ppm (or $100\,B_f$ ppm), the gain has been set too low by a factor of $B_f^2 = 0.927$ or 7.3 percent for those surveys flown prior to this study (November 1997).

Effect of Conductivity

Figure 2 shows the calibration response over a conducting ground. First consider the horizontal coplanar (HCP) coil response (Figure 2a). The curves start at the free-space values, which are equal to the 10k ohm-m half-space values. Decreasing the half-space resistivity increases the Q-coil quadrature response. As the resistivity decreases further, the response goes through a maximum and then decreases. This behavior becomes more severe as the transmitter frequency increases, that is, the departure from the free-space response occurs at a higher resistivity and is of greater magnitude.

The inphase response decreases slightly and then increases sharply as the resistivity decreases. The 7200-Hz inphase response shows a significant increase below 5 ohm-m, while the 900-Hz inphase response has only a hint of increasing at the resistivity levels shown.

The vertical coaxial (VCX) coils behave differently (see Figure 2b). Moderate decreases in resistivity produce no effect. Below 10 ohm-m the inphase responses decrease with the 7200-Hz being rather dramatic. The quadrature responses show a very small decrease before increasing rapidly as resistivity decreases below 10 ohm-m.

Calibration over a conducting ground will change the calibration response from the free-space values. The coplanar coils are influenced more by a conducting ground than the coaxial coils. For the coplanar coils resistivities greater than 100 ohm-m have no significant effect except for the quadrature channels. For the 56-kHz quadrature response, a 10 percent change in response is seen at resistivities below 40 ohm-m. For the 7200-Hz quadrature response the 10 percent change is reached below 10 ohm-m. For the inphase response, the change exceeds 10 percent for resistivities of less than 6 ohm-m for the 56-kHz coil pairs, and less than 2 ohm-m for the 7200-Hz coil pairs respectively. For the coaxial coils a 10 percent change is seen in the 7200-Hz quadrature response when the resistivity drops below 1.3 ohm-m, while for the 7200-Hz inphase response a 10 percent changes does occur until resistivities are less than 2 ohm-m.

Effect of Q-Coil Positioning Error

We now turn our attention to the effect of Q-coil positioning error on the calibration response (see Figure 3). Shown are the inphase response of the coplanar and coaxial coil pairs for a 10k-ohm-m half-space as a function of the positioning error of the Q-coil. For the 900-Hz and 7200-Hz coils only positioning errors in the in-line direction (x) are significant, while for the 56-kHz coil the largest errors are produced by y-direction, and to a lesser degree, x-direction displacement of the Q-coil. The 56-kHz coplanar coil behavior is different because its Q-coil is located perpendicular to the axis of the bird (see Figure 1b).

Table 4 Average effect of Q-coil positioning error on calibration signal expressed as percent error per centimeter averaged over ±5 cm for various half-space resistivities. Centers of coils are nominally 1.23 m above the half-space.

Resistivity [ohm-m]	Coil Pair	X I [%/cm]	X Q [%/cm]	Y I [%/cm]	Y Q [%/cm]	Z I [%/cm]	Z Q [%/cm]
10k	900 HCP	2.33	2.33	0.03	0.03	0.10	0.10
	7200 HCP	2.33	2.33	0.03	0.03	0.10	0.10
	56k HCP	0.44	0.44	1.98	1.98	0.09	0.09
	900 VCX	1.65	1.65	0.03	0.03	0.03	0.03
	7200 VCX	1.65	1.65	0.03	0.03	0.03	0.03
50	900 HCP	2.33	2.33	0.03	0.03	0.10	0.10
	7200 VCP	2.33	2.32	0.03	0.03	0.10	0.10
	56k HCP	0.45	0.43	1.99	1.97	0.09	0.09
	900 VCX	1.65	1.65	0.03	0.03	0.03	0.03
	7200 VCX	1.65	1.66	0.03	0.03	0.03	0.03
1	900 HCP	2.33	2.31	0.03	0.03	0.10	0.10
	7200 HCP	2.27	2.32	0.03	0.03	0.10	0.10
	56k HCP	0.41	0.64	1.96	1.91	0.09	0.17
	900 VCX	1.66	1.65	0.03	0.03	0.03	0.03
	7200 VCX	1.67	1.59	0.03	0.03	0.04	0.05

Table 4 presents the average percent change in Q-coil response per centimeter of positioning error averaged over the range of -5 cm to +5 cm. The 900-Hz and 7200-Hz coplanar coils (2.3 %/cm) are about 50 percent more sensitive to inline (x-direction) positioning errors than the corresponding coaxial coils (1.7 %/cm). Positioning errors in the y- and z-directions are much smaller (0.3-0.6 %/cm) for all coil pairs except the y-direction for the 56-kHz coil.

Decreasing the half-space resistivity to 50 ohm-m (Figure 4) produces similar results with the exception of changing the relative amplitude of the curves. This effect is the most pronounced in the quadrature components. The percentage change per unit distance (Table 4), however, remains virtually the same as for the 10k ohm-m half-space. Further reduction of the half-space resistivity to 1 ohm-m (Figure 5) causes even larger changes in the relative amplitude of the curves, especially at the higher frequencies. The percentage change in response due to positioning error only shows minor differences in the 56-kHz behavior from the results for the more resistive half-space.

Because of these magnitude of these errors, extreme care should be taken during HEM calibration to insure that the Q-coil is properly positioned with respect to the bird. In particular changes in position of the Q-coil in the horizontal plane produce the largest effect.

Effect of Q-Coil Tilt

The effect of tilting the Q-coil from its nominal horizontal or vertical position has not been analyzed.

Effect of Q Value

The Q of the Q-coil is nominally 1, and is adjustable with a trim pot. Adjustment is necessary for initial setting in the laboratory and may need to be adjusted in the field if the transmitter frequency is changed; thus misadjustment of Q is possible. A possible cause of misadjustment in the laboratory would be measuring the Q of the Q-coil when it is near a conductive sheet such as a metal desktop.

From the definition of Q it is clear that the relative change in Q is related to the relative change in R by $\delta Q/Q = -\delta R_Q/R_Q$. Thus a 10 percent increase in the Q-coil resistance will produce a 10 percent decrease in the Q. Figure 6 shows the inphase and quadrature parts of the expression $\frac{Q^2 + jQ}{1 + Q^2}$. When properly adjusted, the relative response of both components is 0.5, and the phase is 45°. The inphase response is more sensitive to variations in Q than the quadrature response is because of the squared dependence on Q.

HEM Bird Calibration Procedure

So far we have looked at calibration error caused by the conductivity of the ground, mispositioning of the Q-coil, and misadjustment of the Q value. We will now look at the overall calibration procedure where these various errors can interact.

HEM bird calibration consists of three steps: phase adjustment, Q-coil adjustment, and gain adjustment. Each of these steps, while designed to properly calibrate the system, can introduce errors in the bird calibration. Let us look at what these three calibration steps are designed to do and what the influence of a conducting earth has on them.

Phase Adjustment

The purpose of phase adjustment is to remove any phase shift between the inphase measurement channel and the transmitter inphase signal. The procedure consists of placing a ferrite rod near the receiver coil and aligning the rod perpendicular to the primary magnetic field direction. In this orientation a minimal signal is produced. The rod is rotated parallel to the primary magnetic field direction to enhance coupling with the primary magnetic field. Because of the ferrimagnetic properties of the rod, the local magnetic field is reduced in the primary field direction thereby introducing a negative inphase response and no quadrature response. The receiver phase is adjusted electronically so that rotation of the ferrite rod only produces a change in the inphase signal. After this adjustment is made, we refer to the signals recorded by the receiver as being in the phased, receiver-reference frame.

There are two possible errors which can be introduced in this step. First, if the ferrite bar is conductive, it will produce a quadrature response in addition to the inphase response. This behavior should be more noticeable at higher frequencies. In general ferrites are poor conductors, however, the actual conductivity of the bar would need to be known at the operating frequencies before the magnitude of this error could be assessed. We will assume that this effect is not significant.

Table 5 Effect of half-space resistivity on phase ϕ_n of total magnetic field. Centers of coils are 1.23 m above the half-space.

Resistivity [ohm-m]	HCP 900 Hz phase [deg]	HCP 7200 Hz phase [deg]	HCP 56 kHz phase [deg]	VCX 900 Hz phase [deg]	VCX 7200 Hz phase [deg]
10000	0.00	0.00	0.02	0.00	0.00
5000	0.00	0.01	0.05	0.00	0.00
2000	0.00	0.02	0.11	0.00	0.00
1000	0.01	0.05	0.21	0.00	0.00
500	0.01	0.09	0.41	0.00	-0.01
200	0.03	0.22	0.92	0.00	-0.02
100	0.06	0.42	1.63	-0.01	-0.04
50	0.11	0.79	2.72	-0.01	-0.06
20	0.26	1.70	4.54	-0.03	-0.09
10	0.50	2.82	5.36	-0.05	-0.07
5	0.93	4.21	4.28	-0.08	0.14
2	1.96	5.24	-1.80	-0.09	1.34
1	3.18	3.57	-9.11	0.03	3.88

The second error results from a phase shift caused by the non-zero conductivity of the ground, which in turn produces a secondary magnetic field. Let us assume that the magnetic field in the absence of a conducting earth (primary field) produces a signal V_0 , which is purely real, i.e., inphase with the transmitter signal. When a conducting ground is introduced, the signal produced is $V_n = V'_0 e^{j\phi_n}$, where ϕ_n is the phase shift produced by the conductive ground. Phase adjustment rotates the zero-phase axis of the measurement (receiver) reference frame to align with V_n. Therefore if a signal $S_0\, e^{j\phi_s}$ is measured in the phase shifted reference frame, the actual signal (i.e. in the transmitter reference frame) is given by $S_0\, e^{j(\phi_s+\phi_n)}$. The magnitude of ϕ_n can be determined by computing the total magnetic (primary plus secondary) field produced by the transmitter coil at the receiver coil location as a function of ground conductivity (see Table 5). In general the phase error for the horizontal coplanar coils increases with decreasing resistivity. As the frequency increases a maximum appears in the curve, being most pronounced for the 56 kHz coil pair. The vertical coaxial coil pairs show a slight decrease with resistivity, and then an increase. The phases are greater for the horizontal coplanar coils than they are for the vertical coaxial coils. The phase can be as large as 2-5 degrees at 56 kHz for resistivities of less than 50 ohm-m.

Q Adjustment

After phase adjustment the HEM receiver is used to adjust the Q of the calibration coil. It can been seen in (9) that the Q-coil-bird response is proportional to $\frac{Q^2 + jQ}{1+Q^2}$. Setting Q to 1 results in the Q-coil producing the same inphase and quadrature response, assuming there is no effect due to ground conductivity. In each survey the bird frequencies are often adjusted slightly to reduce noise. This adjustment requires a trim pot on the Q-coil to vary the loop resistance, which in turn varies the Q. The trim pot is adjusted until the presence of the Q-coil makes the same amplitude change in the measured inphase and quadrature signals. The actual amplitude is not of concern at this point, as that will be taken care of by the gain adjustment.

The presence of a conductive ground will introduce an additional phase shift into the Q-coil response, which is effectively nulled by the Q adjustment. Let us assume that the response of the Q-coil is the product of two terms; the first incorporates the ground conductivity, bird coil geometry, and bucking coil effects ($R_0\, e^{j\phi_r}$), while the second $\frac{Q^2 + jQ}{1+Q^2}$ takes into account Q adjustment. The total response is given by:

$$R_n = R_0\, e^{j\phi_r} \frac{Q^2 + jQ}{1+Q^2} \quad . \tag{18}$$

In the phased receiver reference frame, the total response is: $R_n\, e^{-j\phi_n}$. To produce equal inphase and quadrature responses, the phase of $e^{j(\phi_r-\phi_n)} \frac{Q^2 + jQ}{1+Q^2}$ must equal $\pi/4$. In other words, the Q is adjusted such that

$$\phi_r - \phi_n + \arctan 1/Q_a = \pi/4 \tag{19}$$

or

$$1/Q_a = \tan(\pi/4 - \phi_r + \phi_n), \quad (20)$$

where Q_a is the adjusted value.

Table 6 shows the misadjustment of Q caused by resistivity. In general, as the resistivity decreases the value of Q_a increases. However, as can be seen from the 7200-Hz HCP response, values of less than 1 are possible. The resistivity where the departure of Q_a from unity commences decreases as the frequency increases. The horizontal coplanar coils are affected at higher resistivities than the vertical coaxial coils.

Table 6 Q_a produced by the presence of a conductive half-space. Centers of coils are 1.23 m above the half-space.

Resistivity [ohm-m]	HCP 900 Hz Qa	HCP 7200 Hz Qa	HCP 56 kHz Qa	VCX 900 Hz Qa	VCX 7200 Hz Qa
10000	1.000	1.000	1.000	1.000	1.000
5000	1.000	1.000	1.000	1.000	1.000
2000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.001	1.001	1.000	1.000
500	1.000	1.001	1.001	1.000	1.000
200	1.000	1.003	1.003	1.000	1.000
100	1.001	1.006	1.006	1.000	1.000
50	1.002	1.011	1.010	1.000	1.000
20	1.004	1.021	1.016	1.000	1.002
10	1.007	1.028	1.020	1.000	1.008
5	1.012	1.027	1.022	1.000	1.024
2	1.023	0.986	1.027	1.005	1.083
1	1.029	0.918	1.036	1.016	1.162

If the phase of the Q-coil signal is position dependent, then mispositioning can introduce additional variations in Q_a. Fortunately this is not a significant effect, as a 5 cm positioning error of the Q-coil in any direction produces less than 0.1° of phase shift for all coil configurations at resistivities greater than 5 ohm-m. Even in the case of the 56-kHz coil pair, the variation becomes only 0.3°/5 cm when the resistivity drops below 2 ohm-m. These phase shifts are small compared to the values of ϕ_r, which are typically greater than 1-2°, and produce variations of Q_a of only 1 percent.

Gain Adjustment

The final adjustment of overall system gain is accomplished by placing the adjusted Q-coil at a specified distance and position from the receiver coil. From an analysis of the system parameters (operating frequency, intercoil distances, coil diameter and number of turns, and Q-coil inductance), the system is designed so that the Q-coil introduces a known value of inphase and quadrature signal (see Table 3), typically 200 ppm inphase and quadrature signal for the HCP coil pairs and 100 ppm of inphase and quadrature signal for the VCX coil pairs. (These values are given in true ppm. Divide the HCP values by 2 to get DIGHEM ppm values.)

To properly simulate the calibration sequence, we need to compute the Q-coil response taking into account the ground conductivity, the incorrect Q value, and the rotation of this signal into the phase adjusted receiver reference frame by multiplying by $e^{-j\phi_h}$

After these three adjustments, the system is calibrated. By "calibrated" I mean that the gain is properly set, though influenced by ground conductivity, in the receiver reference frame which is aligned with the phase of the magnetic field at the receiver coil. These results are shown in Table 7. The calibration error increases with decreasing half-space resistivity and increasing frequency. The error is larger for the horizontal coplanar coil pairs than it is for the vertical coaxial coil pairs at the same nominal frequency. The magnitude of the calibration signal is the same for the inphase and quadrature components because of the phasing adjustment. Calibration signals greater than the free space values result in system gains being set too low.

Table 7 Q-coil signal produced in the phase adjusted receiver-reference frame. Centers of coils are 1.23 m above the half-space.

Resistivity	HCP	900 Hz	HCP	7200 Hz	VCX	56 kHz	VCX	900 Hz	VCX	7200 Hz
[ohm-m]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]
10000	225.2	225.2	212.6	212.6	231.3	231.3	112.3	112.3	106.0	106.0
5000	225.2	225.2	212.6	212.6	231.3	231.3	112.3	112.3	106.0	106.0
2000	225.2	225.2	212.6	212.6	231.4	231.4	112.3	112.3	106.0	106.0
1000	225.3	225.3	212.7	212.7	231.5	231.5	112.3	112.3	106.0	106.0
500	225.3	225.3	212.8	212.8	231.7	231.7	112.3	112.3	106.0	106.0
200	225.3	225.3	213.1	213.1	232.6	232.6	112.3	112.3	106.0	106.0
100	225.4	225.4	213.6	213.6	234.3	234.3	112.3	112.3	105.9	105.9
50	225.5	225.5	214.7	214.7	238.1	238.1	112.3	112.3	105.9	105.9
20	225.9	225.9	218.1	218.1	249.4	249.4	112.3	112.3	105.7	105.7
10	226.6	226.6	223.5	223.5	265.1	265.1	112.2	112.2	105.5	105.5
5	228.0	228.0	232.8	232.8	285.8	285.8	112.1	112.1	105.6	105.6
2	232.3	232.3	249.6	249.6	306.3	306.3	111.9	111.9	107.7	107.7
1	239.0	239.0	258.3	258.3	298.8	298.8	111.8	111.8	114.2	114.2

Table 8 presents the data from Table 7 as a percentage error in gain, representing how the data must be compensated to remove the calibration error. These values assume that the correct calibration values are those given in Table 3, which are the same as the 10000-ohm-m results here.

Table 8 Percentage error in the calibration signal in the receiver reference frame.

Resistivity [ohm-m]	HCP 900 Hz [%]	HPC 7200 Hz [%]	HCP 56 kHz [%]	VCX 900 Hz [%]	VCX 7200 Hz [%]
10000	0.00	0.00	0.01	0.00	0.00
5000	0.00	0.01	0.01	0.00	0.00
2000	0.00	0.02	0.04	0.00	0.00
1000	0.00	0.04	0.08	0.00	-0.01
500	0.01	0.09	0.19	0.00	-0.01
200	0.03	0.23	0.58	0.00	-0.03
100	0.05	0.48	1.32	-0.01	-0.06
50	0.11	1.00	2.96	-0.02	-0.12
20	0.28	2.61	7.84	-0.05	-0.27
10	0.58	5.15	14.62	-0.09	-0.42
5	1.22	9.49	23.59	-0.18	-0.37
2	3.13	17.42	32.43	-0.37	1.65
1	6.11	21.52	29.18	-0.45	7.79

Table 9 Q-coil signal produced in the transmitter reference frame. Centers of coils are 1.23 m above the half-space.

Resistivity	HCP	900 Hz	HCP	7200 Hz	HCP	56 kHz	VCX	900 Hz	VCX	7200 Hz
[ohm-m]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]	I [ppm]	Q [ppm]
10000	225.2	225.2	212.6	212.6	231.2	231.4	112.3	112.3	106.0	106.0
5000	225.2	225.3	212.6	212.6	231.1	231.5	112.3	112.3	106.0	106.0
2000	225.2	225.3	212.5	212.7	230.9	231.8	112.3	112.3	106.0	106.0
1000	225.2	225.3	212.5	212.9	230.6	232.3	112.3	112.3	106.0	106.0
500	225.2	225.3	212.4	213.1	230.1	233.4	112.3	112.3	106.0	106.0
200	225.2	225.4	212.2	213.9	228.8	236.3	112.3	112.3	106.0	105.9
100	225.1	225.6	212.0	215.2	227.6	240.9	112.3	112.3	106.0	105.9
50	225.0	225.9	211.7	217.7	226.5	249.2	112.3	112.3	106.0	105.8
20	224.8	226.9	211.6	224.5	228.9	268.4	112.3	112.2	105.9	105.5
10	224.6	228.5	212.3	234.3	239.2	288.7	112.3	112.1	105.7	105.4
5	224.3	231.6	215.0	249.2	263.7	306.4	112.3	111.9	105.3	105.8
2	224.2	240.1	225.8	271.4	315.7	296.5	112.1	111.7	105.2	110.2
1	225.4	251.9	241.7	273.9	342.3	247.7	111.7	111.9	106.3	121.7

To complete the analysis the Q-coil signal must be multiplied by $e^{j\phi_n}$ to rotate the signal into the transmitter reference frame. This is done because we interpret data in the transmitter reference frame. These result are shown in Table 9. Notice that the inphase and quadrature responses are different, however, the behavior noted above for the receiver reference frame results still apply. Basically, the effect of the half-space resistivity becomes larger as the resistivity decreases, and as the frequency increases. The vertical coaxial coil pairs are affected less than the horizontal coplanar coils. Table 10 shows these results as a percentage error in the calibration signal.

Table 10 Percentage error in the calibration signal in the transmitter reference frame.

Resistivity	HCP	900 Hz	HCP	7200 Hz	HCP	56 kHz	VCX	900 Hz	VCX	7200 Hz
[ohm-m]	I [%]	Q [%]	I [%]	Q [%]	I [%]	Q [%]	I [%]	Q [%]	I [%]	Q [%]
10000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5000	0.00	0.04	0.00	0.00	-0.04	0.04	0.00	0.00	0.00	0.00
2000	0.00	0.04	-0.05	0.05	-0.13	0.17	0.00	0.00	0.00	0.00
1000	0.00	0.04	-0.05	0.14	-0.26	0.39	0.00	0.00	0.00	0.00
500	0.00	0.04	-0.09	0.24	-0.48	0.86	0.00	0.00	0.00	0.00
200	0.00	0.09	-0.19	0.61	-1.04	2.12	0.00	0.00	0.00	-0.09
100	-0.04	0.18	-0.28	1.22	-1.56	4.11	0.00	0.00	0.00	-0.09
50	-0.09	0.31	-0.42	2.40	-2.03	7.69	0.00	0.00	0.00	-0.19
20	-0.18	0.75	-0.47	5.60	-0.99	15.99	0.00	-0.09	-0.09	-0.47
10	-0.27	1.47	-0.14	10.21	3.46	24.76	0.00	-0.18	-0.28	-0.57
5	-0.40	2.84	1.13	17.22	14.06	32.41	0.00	-0.36	-0.66	-0.19
2	-0.44	6.62	6.21	27.66	36.55	28.13	-0.18	-0.53	-0.75	3.96
1	0.09	11.86	13.69	28.83	48.05	7.04	-0.53	-0.36	0.28	14.81

Conclusions

Errors in calibration of helicopter electromagnetic data arise from several sources. The most significant of these is due to mispositioning of the Q-coil with respect to the bird. Errors in positioning in-line with the bird have the greatest effect when the Q-coil is located along the axis of the bird (in-line geometry). When the Q-coil is located to the side of the bird (offset geometry), distance from the bird axis has the greatest effect, however, in-line errors are still significant. Ground conductivity becomes a significant factor when resistivities decrease below 100 ohm-m. These effects become more significant and start at higher resistivities as the transmitter frequency increases. The horizontal coplanar geometry is more susceptible to these errors than in the vertical coaxial geometry. Knowing the resistivity as a function of depth at the calibration site, the procedures described here can be used to calculate the calibration errors for a particular HEM survey. Correction factors can then be applied to the data before interpretation.

Based upon this analysis, several recommendations can be made concerning the calibration of helicopter electromagnetic data.

1. Calibration should be done at sites which have resistivities greater than 100 ohm-m.

2. If the resistivities are less than 100 ohm-m, then the influence of the conductive ground needs to be computed and appropriate corrections applied to the airborne data.

3. The Q-coil should be positioned with respect to the bird by means of a rigid jig so as to stabilize its position and eliminate geometrical errors.

The problem of accurate calibration becomes more of an issue as users of helicopter electromagnetic data try to perform data inversions as this requires greater data accuracy than the production of apparent resistivity maps and pseudo-sections. Similarly, trends toward higher bird frequencies will make these issues more important. Many calibration related problems may be reduced or eliminated by designing birds which have stable internal calibration coils, allowing accurate calibration while the bird is several hundreds of meters above the ground.

Acknowledgments

Completion of this work would have not been possible without the cooperation of Doug Fraser and Greg Hodges of Geoterrex-Dighem and Gary Black of A-Cubed Inc. Discussions with Maryla Deszcz-Pan, Vic Labson, and Karl Ellefsen sharpened my vision and clarified several points.

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Appendix: Mutual Impedance Formulation

The formulation of the mutual impedance response of loops over a layered earth is given by several authors (Frischknecht, 1967; Wait, 1982). We consider a half-space composed of M layers with conductivities σ_i for i=1,...,M and thicknesses d_i for i=1,..,$M-1$. The following expressions differ from those of Wait in the following ways: The x-axis is directed from the transmitter dipole toward the receiver coil, the y-axis is transverse to this direction, and the z-axis is vertical thereby forming a right-handed system. The horizontal dipole is directed along the x-axis, whereas Wait uses a dipole pointing along the y-axis.

Vertical Magnetic Dipole

The vertical component of primary magnetic field due to a vertical magnetic dipole source is given by

$$H_z^p = \frac{I\,S}{4\,\pi}\left[\frac{3\,(z-h)^2}{R^5} - \frac{1}{R^3}\right], \tag{A1}$$

where I is the transmitter current, S is the transmitter loop turn-area product ($n\,\pi\,a^2$), $r^2 = x^2 + y^2$, $R^2 = r^2 + (z-h)^2$, and h and z are the heights above the ground of the source dipole and receiver coil respectively. The vertical secondary magnetic field is given by

$$H_z^s = -\frac{I\,S}{4\,\pi}\frac{T_0}{\delta^3} \tag{A2}$$

where $\delta = \left(\frac{2}{\sigma_1\,\mu_0\,\omega}\right)^{1/2}$, μ_0 is the magnetic permeability, and ω is the angular frequency. T_0 is given by the familiar Hankel transform

$$T_0 = \int_0^{\infty} \bar{R}(g)\,g^2\,e^{-gA}\,J_0(gB)\,dg\,. \tag{A3}$$

$\bar{R}(g)$ is defined by the following recursion relation (Wait, 1982)

$$\bar{R}(g) = \frac{Y_1 - N_0}{Y_1 + N_0} \tag{A4}$$

where

$$Y_i = N_i\,\frac{Y_{i+1} + N_i\,tanh\,\left[(g/2)^2 + j/2\,\sigma_i/\sigma_1\right]^{1/2} D_i}{N_i + Y_{i+1}\,tanh\,\left[(g/2)^2 + j/2\,\sigma_i/\sigma_1\right]^{1/2} D_i}, \tag{A5}$$

$$Y_{M-1} = N_{M-1} \frac{N_M + N_{M-1} \tanh \left[(g/2)^2 + j/2\, \sigma_{M-1}/\sigma_1 \right]^{1/2} D_{M-1}}{N_{M-1} + N_M \tanh \left[(g/2)^2 + j/2\, \sigma_{M-1}/\sigma_1 \right]^{1/2} D_{M-1}} , \tag{A6}$$

$$N_0 = \frac{g}{j \mu_0 \omega \delta} , \tag{A7}$$

and

$$N_i = \frac{\left(g^2 + 2j\, \sigma_i/\sigma_1 \right)^{1/2}}{j \mu_0 \omega \delta} . \tag{A8}$$

A is the normalized sum of the transmitter dipole and receiver coil heights $A = \frac{(h+z)}{\delta}$, B is the induction number r/δ, and D_i is the normalized layer thickness $D_i = \frac{2 d_i}{\delta}$.

Horizontal Magnetic Dipole

The horizontal (in-line) primary and secondary magnetic fields due to a horizontal magnetic dipole pointing in the x-direction are given by

$$H_x^p = \frac{I S}{4 \pi} \left[\frac{3 x^2}{R^5} - \frac{1}{R^3} \right] \tag{A9}$$

and

$$H_x^s = \frac{I S}{4 \pi} \frac{x^2}{\delta^3 r^2} \left[\left(1 - \frac{y^2}{x^2} \right) \frac{T_2}{B} - T_0 \right] \tag{A10}$$

where

$$T_2 = \int_0^\infty \bar{R}(g)\, g\, e^{-gA} J_1(gB)\, dg . \tag{A11}$$

Mutual Impedance for Vertical Dipoles

We define the mutual impedance Z_{ba} as the ratio of the voltage in coil B due to the current in coil A

$$Z_{ba} = V_b/I_a = -j \omega \mu_0 H S_b/I_a \tag{A12}$$

where H is the component of magnetic field threading through coil B. For horizontal transmitter and receiver coils (vertical dipoles) at heights h_a and z_b respectively above the ground, (A12) becomes

$$Z_{ba} = \frac{-j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\left[\frac{3\,(z_b - h_a)^2}{R_{ba}^5} - \frac{1}{R_{ba}^3} - \frac{T_0}{\delta^3}\right]. \tag{A13}$$

It is customary to normalize this impedance by the mutual impedance of the coil system over free space to obtain the mutual coupling ratio $(Z/Z_0)_{ba}$. For the general case where the coils are not coplanar the free-space mutual impedance is

$$Z_{0\,ba} = \frac{j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\left[\frac{3\,(z_b - h_a)^2}{R_{ba}^5} - \frac{1}{R_{ba}^3}\right], \tag{A14}$$

and the mutual coupling ratio is

$$(Z/Z_0)_{ba} = 1 + \frac{R_{ba}^3}{\delta^3}\,\frac{T_0}{\left[1 - 3(z_b - h_a)^2/R_{ba}^2\right]}. \tag{A15}$$

For coplanar coils ($z_b = h_a$, $R_{ba} = r_{ab}$), (A14) and (A15) reduce to the familiar expressions

$$Z_{0\,ba} = \frac{j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\,\frac{1}{r_{ba}^3} \tag{A16}$$

and

$$(Z/Z_0)_{ba} = 1 + B^3\,T_0\,. \tag{A17}$$

<u>Mutual Impedance for Horizontal Dipoles</u>

We now consider the case of horizontal magnetic dipoles which are parallel the positive x-axis, but not necessarily coaxial. The mutual impedance is given by

$$Z_{ba} = \frac{-j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\left\{\frac{3\,x^2}{R_{ba}^5} - \frac{1}{R_{ba}^3} + \frac{x^2}{\delta^3\,r_{ba}^2}\left[\left(1 - \frac{y^2}{x^2}\right)\frac{T_2}{B} - T_0\right]\right\} \tag{A18}$$

and the free-space mutual impedance is

$$Z_{0\,ba} = \frac{-j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\left[\frac{3\,x^2}{R_{ba}^5} - \frac{1}{R_{ba}^3}\right] \tag{A19}$$

When the dipoles are coaxial $z_b = h_a$, $y = 0$, and $R_{ba} = r_{ba} = x$, (A18) becomes

$$Z_{ba} = \frac{-j\,\omega\,\mu_0}{4\,\pi}\,S_b\,S_a\left\{\frac{2}{r_{ba}^3} + \frac{1}{\delta^3}\left[\frac{T_2}{B} - T_0\right]\right\}, \tag{A20}$$

and for free space (A20) simplifies to

$$Z_{0\,ba} = \frac{-j\,\omega\,\mu_0}{2\,\pi}\,S_b\,S_a\,\frac{1}{r_{ba}^3}\ . \tag{A21}$$

Normalizing (A18) by (A19) gives the mutual coupling ratio

$$(Z/Z_0)_{ba} = 1\ +\frac{R_{ba}^3}{\delta^3}\,\frac{x^2}{r^2}\,\frac{\left(1-y^2/x^2\right)\frac{T_2}{B}\ -T_0}{3x^2/R_{ba}^2-1}\ . \tag{A22}$$

For the case of coaxial loops ($z_b = h_a$, $y = 0$, and $R_{ba} = r_{ba} = x$) the mutual coupling ratio becomes the familiar

$$(Z/Z_0)_{ba} = 1\ +\frac{B^2}{2}\left[T_2\ - B\,T_0\right]. \tag{A23}$$

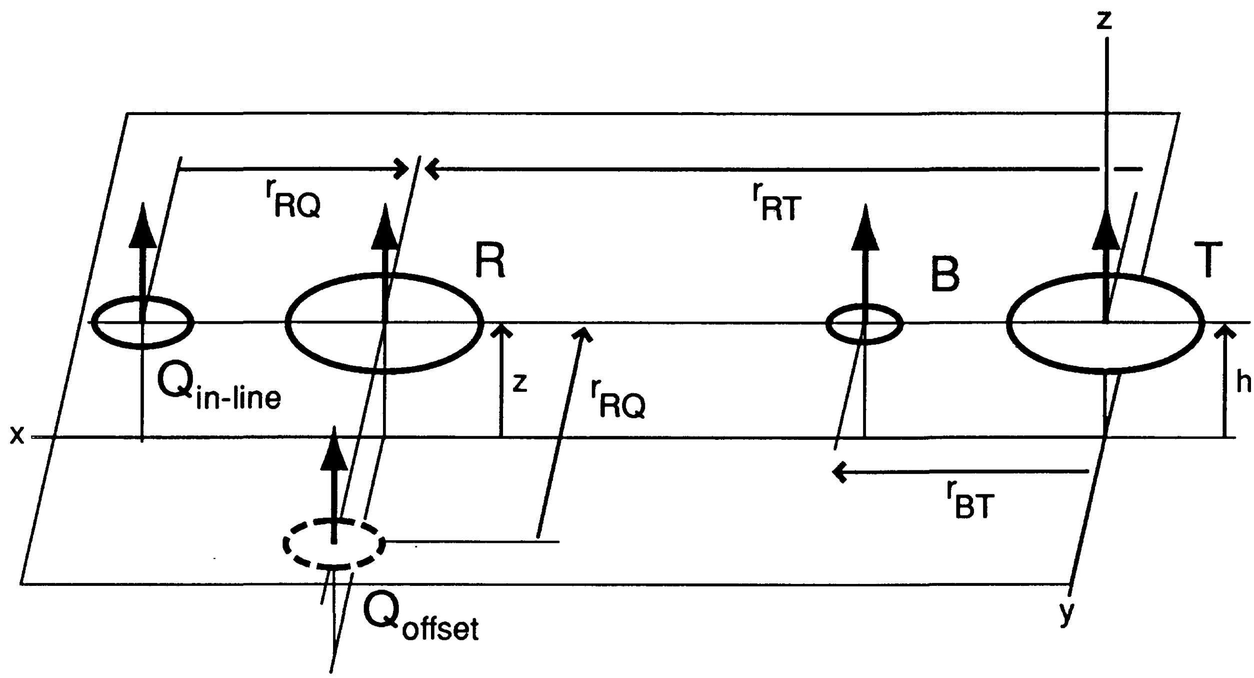

Figure 1 Location of transmitter (T) coil, receiver (R) coil, bucking (B) coil, and Q coil (Q) in a typical HEM bird. The Q-coil location will be either along (in-line) or perpendicular to (offset) the transmitter-receiver axis. The HCP geometry is shown schematically. For the VCX geometry, all coils are rotated so that they are parallel the y-z plane. Only the in-line Q-coil configuration is considered for the VCX geometry.

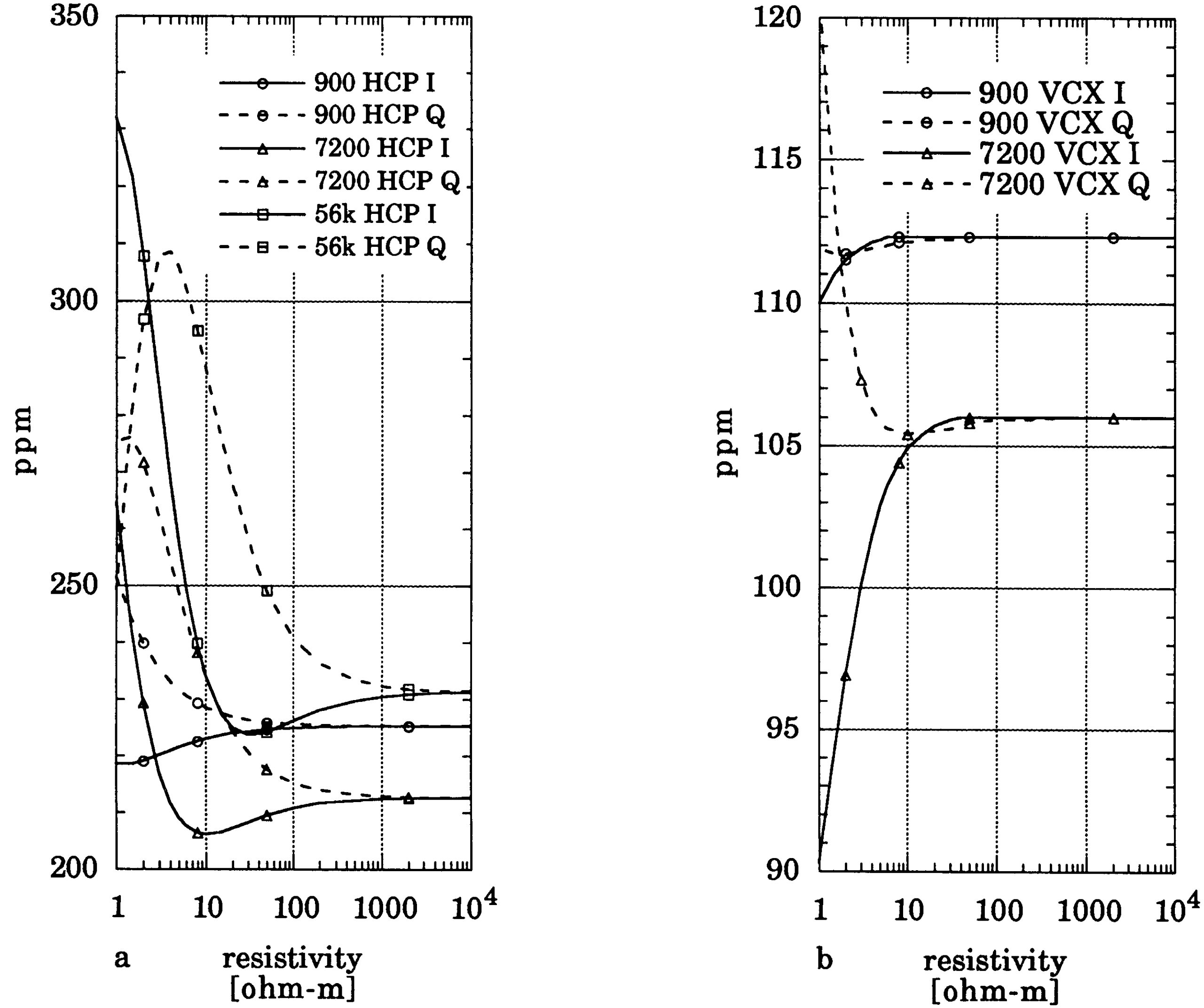

Figure 2 Effect of ground resistivity on Q-coil response for (a) HCP and (b) VCX geometries. Q is fixed at 1. Coil centers are 1.23 m above ground.

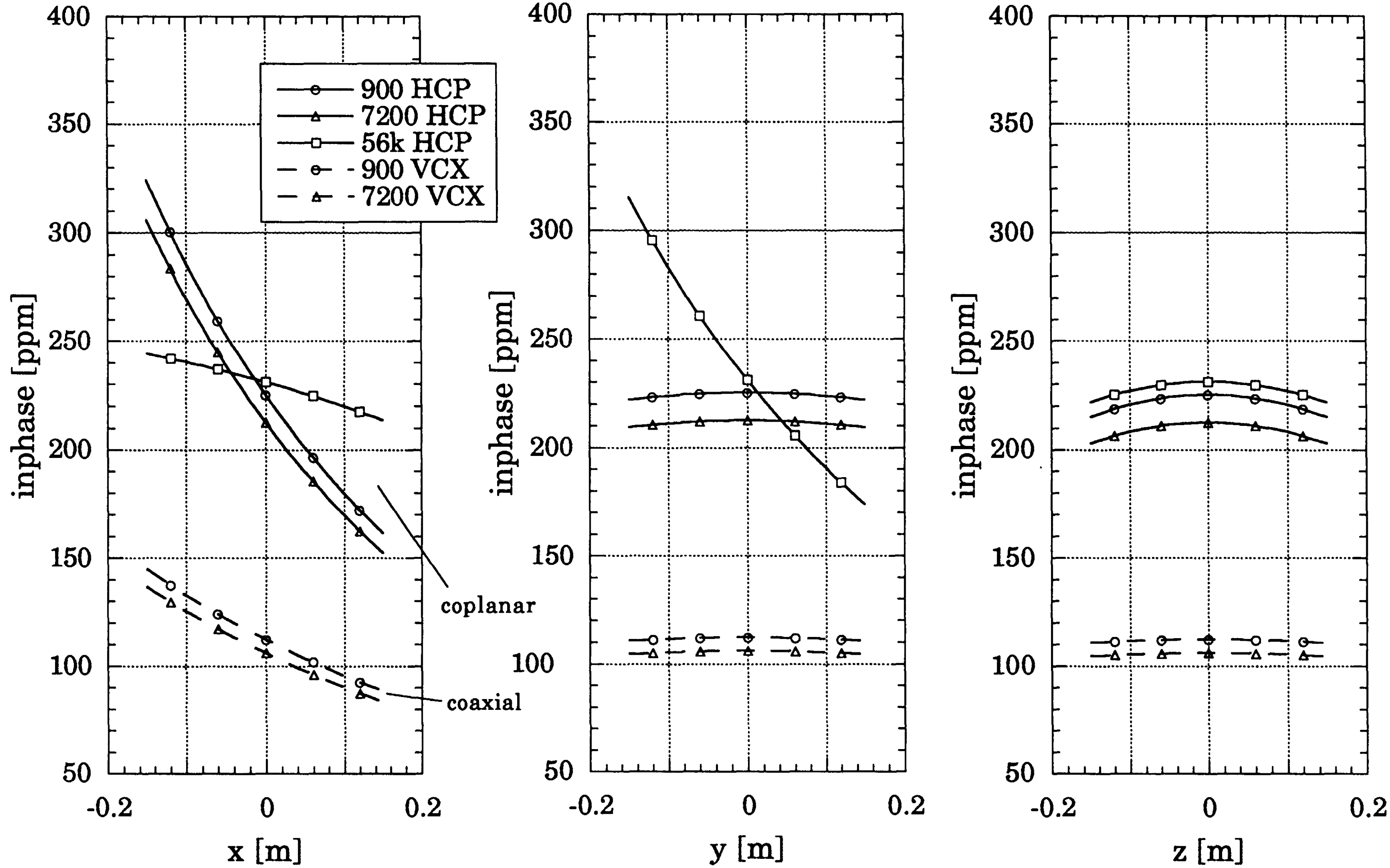

Figure 3 Effect of positioning error for x (in-line), y (perpendicular), and z (vertical) displacement of the Q-coil. Response is the same for inphase and quadrature (not shown) components of Q-coil signal. Half-space resistivity is 10k ohm-m. Coil heights are 1.23 m unless displaced.

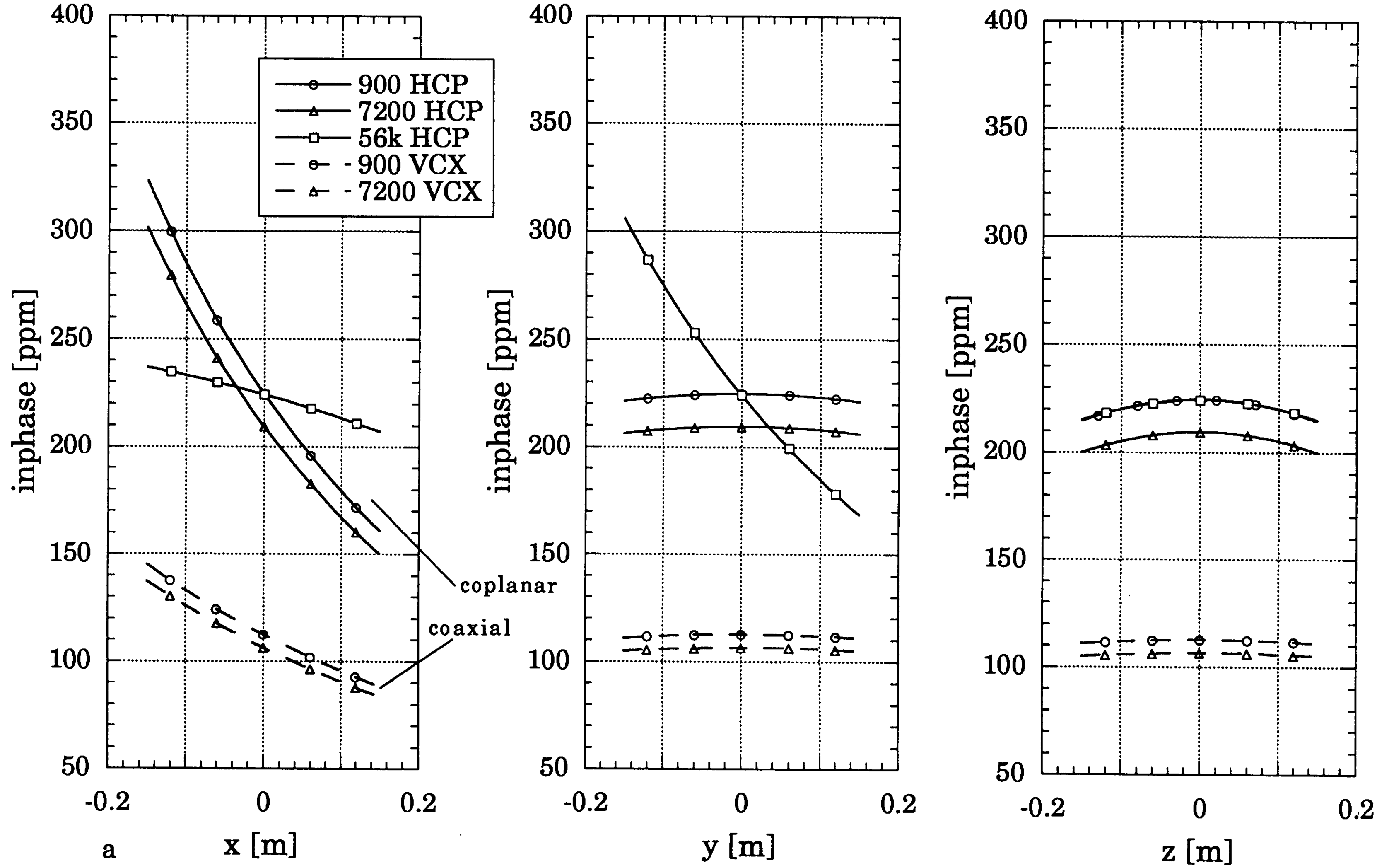

Figure 4 Effect of positioning error for x (in-line), y (perpendicular), and z (vertical) displacement of the Q-coil. Inphase (a) and quadrature (b) responses are different. Half-space resistivity is 50 ohm-m. Coil heights are 1.23 m unless displaced.

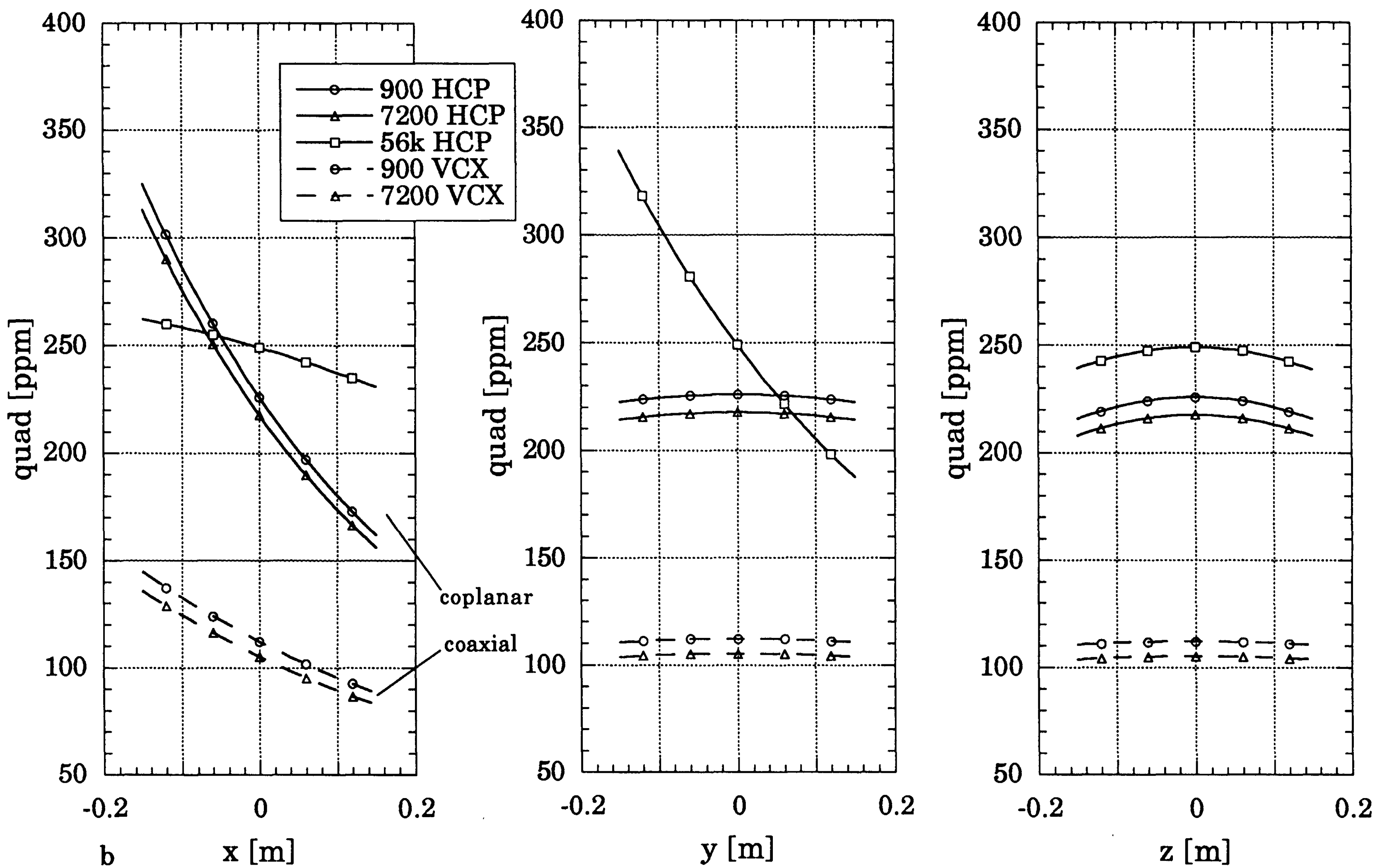

Figure 4 Effect of positioning error for x (in-line), y (perpendicular), and z (vertical) displacement of the Q-coil. Inphase (a) and quadrature (b) responses are different. Half-space resistivity is 50 ohm-m. Coil heights are 1.23 m unless displaced.

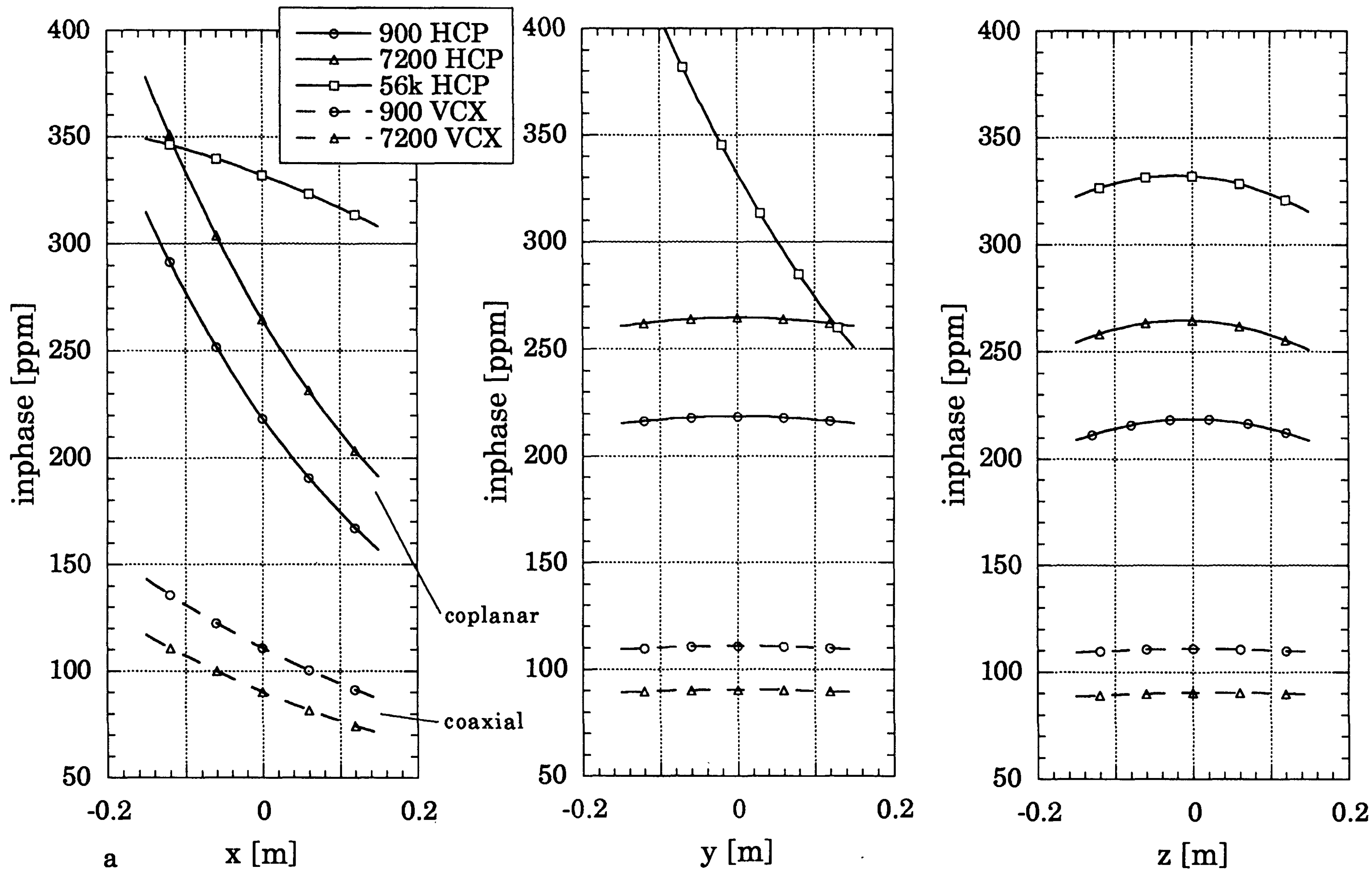

Figure 5 Effect of positioning error for x (in-line), y (perpendicular), and z (vertical) displacement of the Q-coil. Inphase (a) and quadrature (b) responses are different. Half-space resistivity is 1 ohm-m. Coil heights are 1.23 m unless displaced

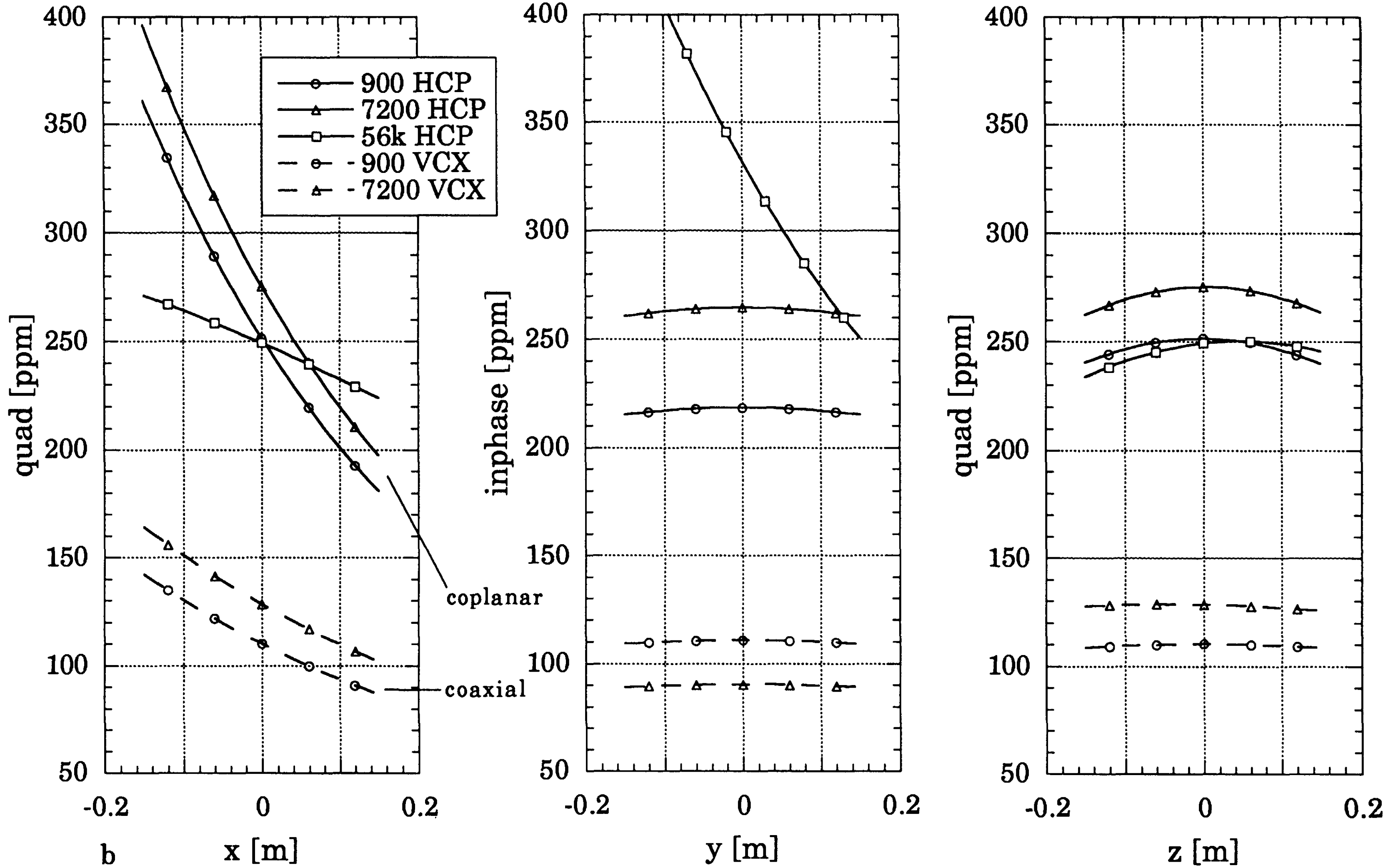

Figure 5 Effect of positioning error for x (in-line), y (perpendicular), and z (vertical) displacement of the Q-coil. Inphase (a) and quadrature (b) responses are different. Half-space resistivity is 1 ohm-m. Coil heights are 1.23 m unless displaced

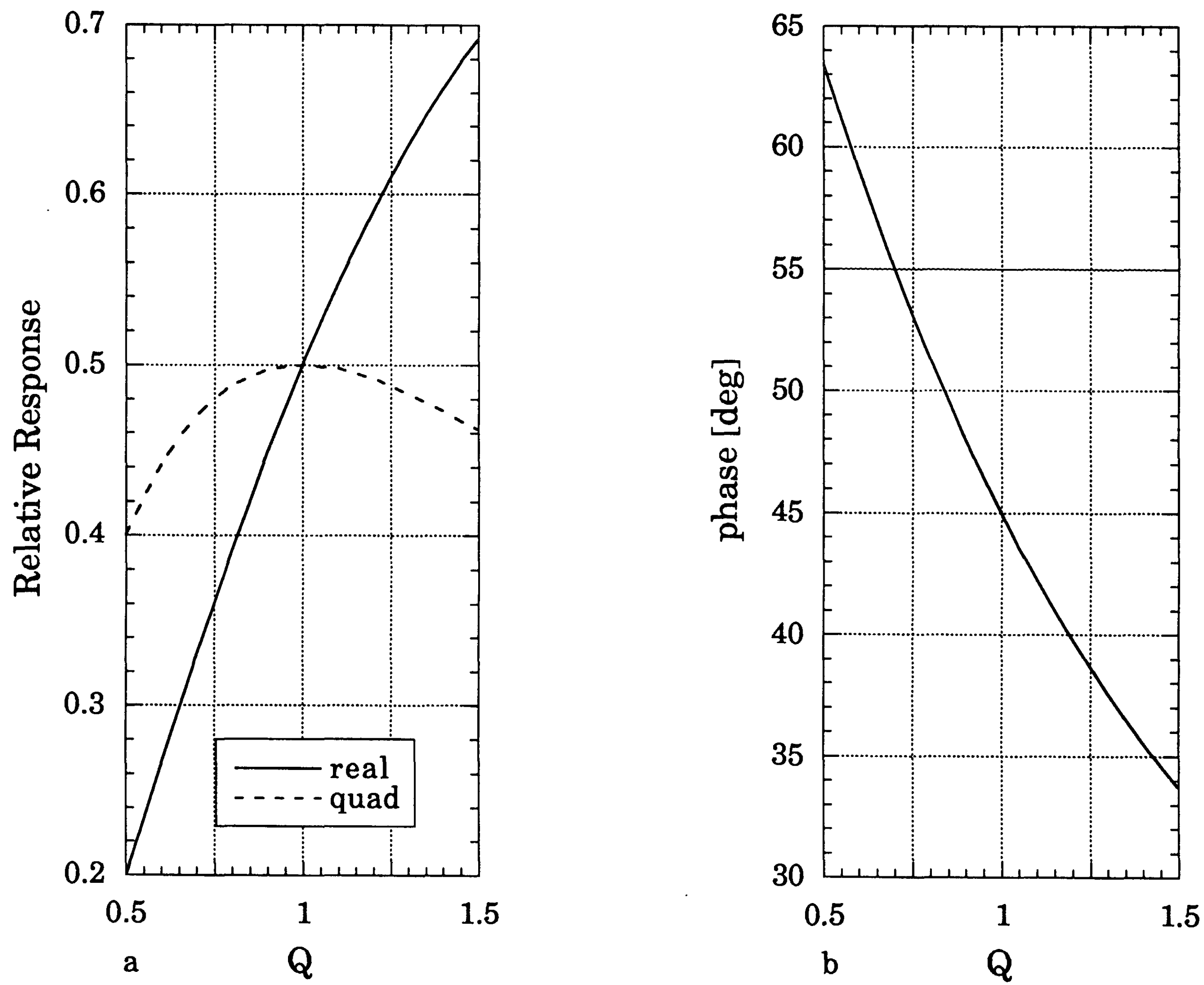

Figure 6 Effect of Q on relative response of inphase and quadrature signal (a) and phase (b).