

PREFACE

This report presents analytical solutions to the ground-water-flow equation for hydraulic interaction of stream-aquifer systems. The solutions are applicable to confined, leaky, and water-table aquifers. Two computer programs are described that were written to evaluate these analytical solutions. The programs have been successfully tested for a variety of applications. Users are requested to notify the USGS if errors are found in this report or the programs.

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CONVERSION FACTORS AND DEFINITION OF SYMBOLS

CONVERSION FACTORS

	Multiply	By	To Obtain
	cubic foot per day (ft ³ /d)	0.02832	cubic meter per day
cubic foot per foot of stream length (ft ²)		0.09290	cubic meter per meter of stream length
	foot (ft)	0.3048	meter
	foot per day (ft/d)	0.3048	meter per day
	square foot per day (ft ² /d)	0.09290	square meter per day

DEFINITION OF SYMBOLS

[L, length; T, time; --, dimensionless]

Symbol	Dimension	Definition
a	L	Streambank leakance
A	--	Dimensionless streambank leakance
b	L	Thickness of aquifer (or saturated thickness for water-table aquifer)
b'	L	Thickness of aquitard (or saturated thickness for water-table aquitard)
c	L	Instantaneous step change in water level of stream
cos	--	Cosine function
coth	--	Hyperbolic cotangent
d	L	Width of semipervious streambank material
erfc	--	Complementary error function
exp	--	Exponential function
F	L	System input
F'	L/T	Time rate of change of system input
h	L	Head in aquifer
h'	L	Head in aquitard
h_D	--	Dimensionless unit-step response solution for head in aquifer
$\bar{h}_D$	--	Dimensionless Laplace transform unit-step response solution for head in aquifer
$\bar{h}_D^*$	--	Dimensionless average head in a partially penetrating observation well
$\hat{\bar{h}}_D$	--	Dimensionless average head in a fully penetrating observation well
h'_D	--	Dimensionless unit-step response solution for head in aquitard

Symbol	Dimension	Definition
$\bar{h}_D'$	--	Dimensionless Laplace transform unit-step response solution for head in aquitard
h_i	L	Initial water level (or potentiometric surface) in stream-aquifer system
h_0	L	Water level in stream after step change
j	--	Upper limit of time integration
k	--	Time variable of integration (time step)
K_D	--	Dimensionless ratio of vertical to horizontal hydraulic conductivity
K_s	L/T	Hydraulic conductivity of semipervious streambank material
K_x, K_z	L/T	Horizontal and vertical hydraulic conductivity of aquifer, respectively
K'	L/T	Vertical hydraulic conductivity of aquitard
l_s	L	Length of stream reach
m	--	Dimensionless aquifer-aquitard Laplace transform variable
n	--	Integer counter in infinite summations
p	--	Laplace transform variable
q_n	--	Terms in the Laplace transform solutions for water-table aquifers
q'	1/T	Volumetric flow rate to or from aquifer per unit volume of aquifer
$\bar{q}_D$	--	Dimensionless Laplace transform leakage between aquifer and aquitard
Q	L ² /T	Seepage rate per unit length of stream
Q_D	--	Dimensionless seepage between stream and aquifer
$\bar{Q}_D$	--	Dimensionless Laplace transform seepage between stream and aquifer
Q_T	L ³ /T	Total seepage rate
R	L	Total recharge
S	--	Storativity (storage coefficient) of aquifer
S_s	1/L	Specific storage of aquifer
S_s'	1/L	Specific storage of aquitard
S_y	--	Specific yield of aquifer
S_y'	--	Specific yield of aquitard

Symbol	Dimension	Definition
$\sin$	--	Sine function
t	T	Time
t_D	--	Dimensionless time
t_{Dy}	--	Dimensionless time with respect to specific yield
t_e	T	Time of end of recharge
t_i	T	Time at start of simulation
t_p	T	Time of flood peak
t_s	T	Time of start of flood wave or recharge
T	L^2/T	Transmissivity of aquifer
$\tan$	--	Tangent function
$\tanh$	--	Hyperbolic tangent
Δt	T	Time-step size
V	L^2	Bank-storage volume per unit length of stream
V_T	L^3	Total volume of bank storage
W	--	Term for aquifer width in Laplace transform solutions for confined and leaky aquifers
W_n	--	Term for aquifer width in Laplace transform solutions for water-table aquifers
x	L	Horizontal coordinate
x_D	--	Dimensionless horizontal coordinate
x_L	L	Width of aquifer
x_{LD}	--	Dimensionless width of aquifer
x_0	L	Distance from middle of stream to stream-aquifer boundary (half-width of stream)
x_{0D}	--	Dimensionless distance to streambank
z	L	Vertical coordinate
z_1	L	Vertical coordinate of bottom of screened interval of observation well
z_2	L	Vertical coordinate of top of screened interval of observation well
z_D	--	Dimensionless vertical coordinate in aquifer

Symbol	Dimension	Definition
z_{D1}	--	Dimensionless vertical coordinate of bottom of screened interval of observation well
z_{D2}	--	Dimensionless vertical coordinate of top of screened interval of observation well
z'_D	--	Dimensionless vertical coordinate in aquitard
z_p	L	Vertical coordinate of observation piezometer opening
β_0	--	Dimensionless product of anisotropic ratio of vertical to horizontal hydraulic conductivity and square of dimensionless distance to streambank
$\varepsilon, \varepsilon_n$	--	Roots of equations in Laplace transform solutions for water-table aquifers
γ_1	--	Dimensionless ratio of aquitard to aquifer hydraulic conductivity
π	--	Pi (3.141592654)
τ	T	Time variable of integration (delay time)
Σ	--	Summation
σ	--	Dimensionless ratio of aquifer storativity to aquifer specific yield
σ_1	--	Dimensionless ratio of aquitard to aquifer storativity
σ'	--	Dimensionless ratio of aquifer storativity to aquitard specific yield
$\int$	--	Integral
∞	--	Infinity
d	--	Derivative
∂	--	Partial derivative

