PREFACE

This report presents analytical solutions to the ground-water-flow equation for hydraulic interaction of stream-aquifer systems. The solutions are applicable to confined, leaky, and water-table aquifers. Two computer programs are described that were written to evaluate these analytical solutions. The programs have been successfully tested for a variety of applications. Users are requested to notify the USGS if errors are found in this report or the programs.

Although the computer programs have been used by the USGS, no warranty, expressed or implied, is made by the USGS or the United States Government as to the accuracy and functioning of the programs and related materials. Nor shall the fact of distribution constitute any such warranty, and no responsibility is assumed by the USGS in connection therewith.

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CONVERSION FACTORS AND DEFINITION OF SYMBOLS

CONVERSION FACTORS

Multiply	By	To Obtain
cubic foot per day (ft^3/d)	0.02832	cubic meter per day
cubic foot per foot of stream length (ft^2)	0.09290	cubic meter per meter of stream length
foot (ft)	0.3048	meter
foot per day (ft/d)	0.3048	meter per day
square foot per day (ft^2/d)	0.09290	square meter per day

DEFINITION OF SYMBOLS

[L, length; T, time; --, dimensionless]

Symbol	Dimension	Definition
a	L	Streambank leakance
A	--	Dimensionless streambank leakance
b	L	Thickness of aquifer (or saturated thickness for water-table aquifer)
b'	L	Thickness of aquitard (or saturated thickness for water-table aquitard)
c	L	Instantaneous step change in water level of stream
cos	--	Cosine function
coth	--	Hyperbolic cotangent
d	L	Width of semipervious streambank material
erfc	--	Complementary error function
exp	--	Exponential function
F	L	System input
F'	L/T	Time rate of change of system input
h	L	Head in aquifer
h'	L	Head in aquitard
h_D	--	Dimensionless unit-step response solution for head in aquifer
$\bar{h}_D$	--	Dimensionless Laplace transform unit-step response solution for head in aquifer
$\bar{h}_D^*$	--	Dimensionless average head in a partially penetrating observation well
$\hat{\bar{h}}_D$	--	Dimensionless average head in a fully penetrating observation well
h_D'	--	Dimensionless unit-step response solution for head in aquitard

Symbol	Dimension	Definition
$\bar{h}'_D$	--	Dimensionless Laplace transform unit-step response solution for head in aquitard
h_i	L	Initial water level (or potentiometric surface) in stream-aquifer system
h_0	L	Water level in stream after step change
j	--	Upper limit of time integration
k	--	Time variable of integration (time step)
K_D	--	Dimensionless ratio of vertical to horizontal hydraulic conductivity
K_s	L/T	Hydraulic conductivity of semipervious streambank material
K_x, K_z	L/T	Horizontal and vertical hydraulic conductivity of aquifer, respectively
K'	L/T	Vertical hydraulic conductivity of aquitard
l_s	L	Length of stream reach
m	--	Dimensionless aquifer-aquitard Laplace transform variable
n	--	Integer counter in infinite summations
p	--	Laplace transform variable
q_n	--	Terms in the Laplace transform solutions for water-table aquifers
q'	1/T	Volumetric flow rate to or from aquifer per unit volume of aquifer
$\bar{q}_D$	--	Dimensionless Laplace transform leakage between aquifer and aquitard
Q	L^2/T	Seepage rate per unit length of stream
Q_D	--	Dimensionless seepage between stream and aquifer
$\overline{Q}_D$	--	Dimensionless Laplace transform seepage between stream and aquifer
Q_T	L^3/T	Total seepage rate
R	L	Total recharge
S	--	Storativity (storage coefficient) of aquifer
S_s	1/L	Specific storage of aquifer
S_s'	1/L	Specific storage of aquitard
S_y	--	Specific yield of aquifer
S_y'	--	Specific yield of aquitard

Symbol	Dimension	Definition
sin	--	Sine function
t	T	Time
t_D	--	Dimensionless time
t_{Dy}	--	Dimensionless time with respect to specific yield
t_e	T	Time of end of recharge
t_i	T	Time at start of simulation
t_p	T	Time of flood peak
t_s	T	Time of start of flood wave or recharge
T	L^2/T	Transmissivity of aquifer
tan	--	Tangent function
tanh	--	Hyperbolic tangent
Δt	T	Time-step size
V	L^2	Bank-storage volume per unit length of stream
V_T	L^3	Total volume of bank storage
W	--	Term for aquifer width in Laplace transform solutions for confined and leaky aquifers
W_n	--	Term for aquifer width in Laplace transform solutions for water-table aquifers
x	L	Horizontal coordinate
x_D	--	Dimensionless horizontal coordinate
x_L	L	Width of aquifer
x_{LD}	--	Dimensionless width of aquifer
x_0	L	Distance from middle of stream to stream-aquifer boundary (half-width of stream)
x_{0D}	--	Dimensionless distance to streambank
z	L	Vertical coordinate
z_1	L	Vertical coordinate of bottom of screened interval of observation well
z_2	L	Vertical coordinate of top of screened interval of observation well
z_D	--	Dimensionless vertical coordinate in aquifer

Symbol	Dimension	Definition
z_{D1}	--	Dimensionless vertical coordinate of bottom of screened interval of observation well
z_{D2}	--	Dimensionless vertical coordinate of top of screened interval of observation well
z'_D	--	Dimensionless vertical coordinate in aquitard
z_p	L	Vertical coordinate of observation piezometer opening
β_0	--	Dimensionless product of anisotropic ratio of vertical to horizontal hydraulic conductivity and square of dimensionless distance to streambank
ε, ε_n	--	Roots of equations in Laplace transform solutions for water-table aquifers
γ_1	--	Dimensionless ratio of aquitard to aquifer hydraulic conductivity
π	--	Pi (3.141592654)
τ	T	Time variable of integration (delay time)
Σ	--	Summation
σ	--	Dimensionless ratio of aquifer storativity to aquifer specific yield
σ_1	--	Dimensionless ratio of aquitard to aquifer storativity
σ'	--	Dimensionless ratio of aquifer storativity to aquitard specific yield
$\int$	--	Integral
∞	--	Infinity
d	--	Derivative
∂	--	Partial derivative