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Probabilistic Method for Subdividing Petroleum Resources into Depth Slices

by

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INTRODUCTION

The U.S. Geological Survey periodically assesses petroleum resources of the United States and the world. Understanding the distribution of these resources by depth interval is an important part of this activity. The purpose of this report is to explain the development of a spreadsheet software system called Deep Energy Estimated Percentages (DEEP). DEEP uses the median-based triangular probability distribution as a probability model for drilling depth to estimate the percentages of estimated petroleum resources below various depths or depth cutoffs and, also, between depth cutoffs, that is, subdividing resources into depth slices. The drilling depth represents the true vertical depth to potential undiscovered resources in an assessment unit or play. An assessment unit is a mappable volume of rock in a total petroleum system. First, the probabilistic method is derived, and second, the spreadsheet DEEP is described.

PROBABILISTIC METHOD

The median-based triangular probability distribution is used as a probability model for the random variable Z: drilling depth (meters). The defining parameters of the median-based triangular probability distribution are the minimum (a), maximum (b), and median (m). A necessary condition for a median-based triangular probability distribution to exist is the following:

$$0.707a + 0.293b \leq m \leq 0.293a + 0.707b$$

From the defining parameters, the standard characterizing parameters of the triangular probability distribution are obtained: minimum (a), maximum (b), and mode (c). There are three cases for calculating c, which are dependent upon the midpoint=$(a + b)/2$:

Case I: m = midpoint (symmetric)

$$c = m$$

Case II: m < midpoint (right skewed)

$$c = b - \frac{2(b-m)^2}{b-a}$$

Case III: m > midpoint (left skewed)

$$c = a + \frac{2(m-a)^2}{b-a}$$

The probability that the drilling depth is below, or equivalently greater than, a particular depth or depth cutoff (z) is denoted by $p(z)$ where

$$p(z) = P(Z > z)$$

Figure 1 displays the triangular probability distribution of drilling depth (meters) where a: minimum, b: maximum, c: mode, z: depth cutoff, and $p(z)$: probability of drilling depth below depth cutoff z. The probability function $p(z)$ is the complementary cumulative distribution function for the triangular probability distribution. The triangular cumulative distribution function can be found in Law and Kelton (1991, p. 341). It can be shown that

$$p(z) = \begin{cases} 1 & \text{if } z < a \\ 1 - \dfrac{(z-a)^2}{(b-a)(c-a)} & \text{if } a \le z \le c \\ \dfrac{(b-z)^2}{(b-a)(b-c)} & \text{if } c < z \le b \\ 0 & \text{if } z > b \end{cases}$$

The drilling depth distribution will be used to estimate the percentages of undiscovered petroleum resources below various depths or depth cutoffs, that is, the problem is to subdivide resources into depth slices. Thus, the primary interest is in the proportion of petroleum resources below a depth cutoff. The simplifying assumption will be made that the proportion or probability of petroleum resources below a depth cutoff can be adequately approximated by the probability of drilling depth below that depth cutoff. Therefore, $p(z)$ will be used to estimate the proportion of petroleum resources below depth cutoff z. An estimate of the proportion of petroleum resources between two depth cutoffs z_1 and z_2, where z_2 is greater than or equal to z_1, is given by

$$P(z_1 < Z < z_2) = P(Z > z_1) - P(Z > z_2) = p(z_1) - p(z_2)$$

SPREADSHEET

The above probabilistic method is incorporated into a spreadsheet software system called Deep Energy Estimated Percentages (DEEP). DEEP consists of a series of six panels or parts of a spreadsheet that can be expanded depending upon the number of depth cutoff values of interest. Panel 1 comprises the input of the defining parameters of the median-based triangular probability distribution for the drilling depth: minimum, median, and maximum. From the defining parameters, the midpoint and mode are computed. Panels 2 through 6 are similar in their composition. Each of these five panels allows for three depth cutoff values to be entered. From each depth cutoff value, the corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed. For example, in Panel 4 of Test 2 ($a = 3,000$ m, $b = 5,000$ m, $m = 4,000$ m, and $c = 4,000$ m) the percent below cutoff value of 3,800 m is equal to 68%, and the percent below cutoff value of 4,000 m is equal to 50%. The percent between cutoffs

of 3,800 m and 4,000 m is equal to 18%. Therefore, an estimate of the percentage of petroleum resources between depths of 3,800 m and 4,000 m is equal to 18%.

ACKNOWLEDGMENTS

REFERENCE

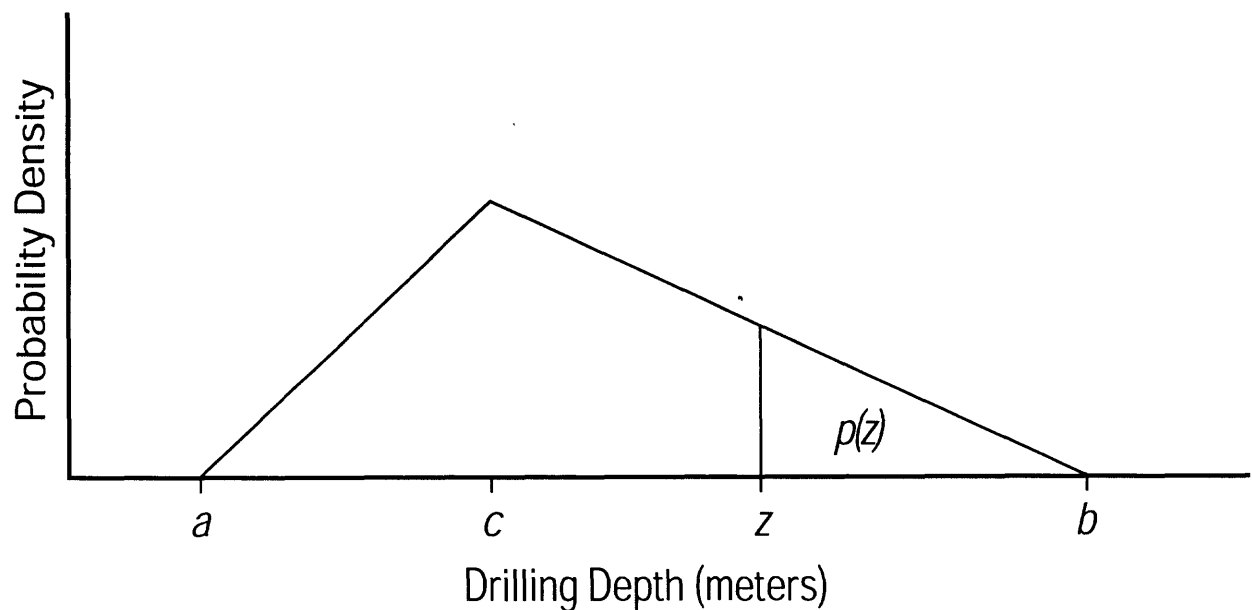Law, A.M. and Kelton, W.D., 1991, Simulation modeling and analysis: New York, McGraw-Hill, Inc., 2nd ed., 759 p.

Figure 1. Triangular probability distribution of drilling depth (meters) where a: minimum, b: maximum, c: mode, z: depth cutoff, and $p(z)$: probability of drilling depth below or greater than depth cutoff z.

DEEP: Deep Energy Estimated Percentages				R.A. Crovelli			(Panel 1)
							4/20/99
Assessment Unit			Drilling Depth (m) -- Median-Based Triangular D.				
Name	No.	Fields	Minimum	Median	Maximum	Midpoint	Mode
Test 1	12345671	Gas	3000	4000	5000	4000	4000
Test 2	12345672	Gas	3000	4000	5000	4000	4000
Test 3	12345673	Gas	3000	4300	5000	4000	4690

Panel 1 of spreadsheet DEEP. Parameters of the median-based triangular probability distribution for drilling depth: minimum, median, and maximum are input; whereas midpoint and mode are computed.

						(Panel 2)		
			Percent		Percent	Percent		Percent	Percent
		Depth	Below	Depth	Below	Between	Depth	Below	Between
No.	Flds	Cutoff(m)	Cutoff	Cutoff(m)	Cutoff	Cutoffs	Cutoff(m)	Cutoff	Cutoffs
12345671	Gas	4572	9.1592	4572	9.1592	0	4572	9.1592	0
12345672	Gas	2600	100	2800	100	0	3000	100	0
12345673	Gas	2600	100	2800	100	0	3000	100	0

Panel 2 of spreadsheet DEEP. Three depth cutoff values are allowed to be input; corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed.

No.	Flds	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	(Panel 3) Percent Below Cutoff	Percent Between Cutoffs
12345671	Gas	4572	9.1592	0	4572	9.1592	0	4572	9.1592	0
12345672	Gas	3200	98	2	3400	92	6	3600	82	10
12345673	Gas	3200	98.817	1.18343	3400	95.266	3.5503	3600	89.349	5.91716

Panel 3 of spreadsheet DEEP. Three depth cutoff values are allowed to be input; corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed.

No.	Flds	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	(Panel 4) Percent Below Cutoff	Percent Between Cutoffs
12345671	Gas	4572	9.1592	0	4572	9.1592	0	4572	9.1592	0
12345672	Gas	3800	68	14	4000	50	18	4200	32	18
12345673	Gas	3800	81.065	8.28402	4000	70.414	10.6509	4200	57.396	13.0178

Panel 4 of spreadsheet DEEP. Three depth cutoff values are allowed to be input; corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed.

No.	Flds	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	Percent Below Cutoff	Percent Between Cutoffs	Depth Cutoff(m)	Percent Below Cutoff	(Panel 5) Percent Between Cutoffs
12345671	Gas	4572	9.1592	0	4572	9.1592	0	4572	9.1592	0
12345672	Gas	4400	18	14	4600	8	10	4800	2	6
12345673	Gas	4400	42.012	15.3846	4600	24.26	17.7515	4800	6.4516	17.8087

Panel 5 of spreadsheet DEEP. Three depth cutoff values are allowed to be input; corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed.

				(Panel 6)						
			Percent	Percent		Percent	Percent		Percent	Percent
		Depth	Below	Between	Depth	Below	Between	Depth	Below	Between
No.	Flds	Cutoff(m)	Cutoff	Cutoffs	Cutoff(m)	Cutoff	Cutoffs	Cutoff(m)	Cutoff	Cutoffs
12345671	Gas	4572	9.1592	0	4572	9.1592	0	4572	9.1592	0
12345672	Gas	5000	0	2	5200	0	0	5400	0	0
12345673	Gas	5000	0	6.45161	5200	0	0	5400	0	0

Panel 6 of spreadsheet DEEP. Three depth cutoff values are allowed to be input; corresponding percent resource below cutoff and percent resource between (present and previous) cutoffs are computed.