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**Probabilistic Method for Estimating Future Growth
of Oil and Gas Reserves**

by

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INTRODUCTION

In the United States, the estimated size (cumulative production plus remaining reserves) of oil and gas fields typically increases through time as fields are discovered, developed, and produced (e.g., Arrington, 1960; Attanasi and Root, 1994). This phenomenon is usually referred to as reserve growth or field growth; the term “reserve growth” is used here.

Reserve-growth patterns of individual fields are highly variable. Indeed, the sizes of some fields are observed to decrease through time. However, for United States fields as a whole, collective reserve growth is strongly positive and is a major component of remaining United States oil and natural-gas resources (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995; Schmoker and Attanasi, 1997).

International oil and gas fields also show clear evidence of reserve growth (e.g., Root and Attanasi, 1993; Oil & Gas Journal, 1996, p. 37; U.S. Geological Survey World Energy Project, private communication, 1999), even though criteria for estimating and reporting field sizes can be quite different from those of the United States. Worldwide, reserve growth has the potential to become very important in the future, especially for gas, as demand increases and opportunities for new large-field discoveries decrease. For these reasons, projections of the future reserve growth of known fields have become important, and in our view, necessary components of petroleum resource assessments.

Algorithms for estimating the future growth of known fields usually utilize the age of fields (years since discovery) as a predictive variable, on the assumption that age is a surrogate for the degree of the development activity that generates reserve growth. Two additional assumptions implicit in most reserve-growth models are: (1) reserve growth of a field is proportional to its field size, and (2) patterns of past reserve growth provide some basis for forecasting future reserve growth.

Schmoker and Crovelli (1998) presented an algorithm (a deterministic method) for estimating future reserve growth of oil and gas fields that incorporates the fundamental reserve-growth assumptions listed in the preceding paragraph, but which is programmed for a personal computer in the form of formulas for a Microsoft Excel spreadsheet. Major advantages of this spreadsheet program include its simplicity and ease of use. However, like all other published reserve-growth models of which we are aware, this program (Schmoker and Crovelli, 1998) generates single-value (point) estimates of future reserve growth, in contrast to estimates in the form of probability distributions.

The purpose of this report is to explain an analytic probabilistic method and spreadsheet software system called probabilistic reserve growth spreadsheet (PREGS). The probabilistic method herein is a probabilistic extension of the deterministic method of Schmoker and Crovelli (1998). The PREGS method is based upon mathematical equations derived from probability theory. The PREGS spreadsheet can be used to calculate probabilistic estimates of reserve growth of oil and gas reserves. First, the

deterministic method is reviewed; second, the probabilistic method is derived; and third, the spreadsheet system is described.

DETERMINISTIC METHOD

The spreadsheet that implements the deterministic method (Schmoker and Crovelli, 1998) for estimating future growth of oil and gas reserves is shown in figure 1. The data used to illustrate the spreadsheet system are based on successive annual estimates made between 1977 and 1991 of the sizes of lower 48 United States gas fields [these data were compiled by the U.S. Department of Energy and summarized in table 2 of Attanasi and Root (1994)].

Input Data

Column A of figure 1A and 1B identifies the age classes to which the input data of columns B and C apply, and is not used in the spreadsheet calculations. Age (column A) is defined as years since discovery.

In the deterministic method, a growth function is composed of a set of growth *factors*. A growth factor is a dimensionless parameter, that is, a multiplicative constant representing proportional growth. The growth function of column B is composed of seventeen 10-year growth factors (rows 2 through 18). Each 10-year growth factor applies to fields of a particular age or range of ages, as indicated by column A. For example, the 10-year growth factor of 1.529 in cell B7 applies to fields that are 5 years old. The size of fields 5 years of age would be multiplied by 1.529 to forecast the size of these same fields 10 years later, when they have become 15 years old.

Rows 2 through 18 of column C (fig. 1A and 1B) list the volumes of petroleum that will be “grown” by the 10-year growth factors of column B. In this illustration, the values in column C are the totals of the estimated sizes of lower 48 United States gas fields as of 1977 for each of the 17 field-age classes of column A. For example, the size as estimated in 1977 of lower 48 gas fields 5 years of age (those discovered in 1972) is 5,400 billion cubic feet of gas (bcfg) (cell C7).

Calculations

The formulas shown in columns D through L (fig. 1A) are used to calculate future growth of the petroleum volumes tabulated in column C. The headings of columns D through L (row 1) indicate that the spreadsheet program calculates reserve growth from 10 to 90 years beyond the date associated with the field-size estimates of column C, at 10-year increments.

A	B	C	D	E	F	G	H	I	J	K	L
1	age, yr	growth func	ESF known	ESF +10yr	ESF +20yr	ESF +30yr	ESF +40yr	ESF +50yr	ESF +60yr	ESF +70yr	ESF +80yr
2	0	4.46	2024	=C2*B2	=D2*B10	=E2*B11	=F2*B12	=G2*B13	=H2*B14	=I2*B15	=J2*B16
3	1	2.484	2794	=C3*B3	=D3*B10	=E3*B11	=F3*B12	=G3*B13	=H3*B14	=I3*B15	=J3*B16
4	2	2.077	3890	=C4*B4	=D4*B10	=E4*B11	=F4*B12	=G4*B13	=H4*B14	=I4*B15	=J4*B16
5	3	1.809	7009	=C5*B5	=D5*B10	=E5*B11	=F5*B12	=G5*B13	=H5*B14	=I5*B15	=J5*B16
6	4	1.696	5603	=C6*B6	=D6*B10	=E6*B11	=F6*B12	=G6*B13	=H6*B14	=I6*B15	=J6*B16
7	5	1.529	5400	=C7*B7	=D7*B10	=E7*B11	=F7*B12	=G7*B13	=H7*B14	=I7*B15	=J7*B16
8	6	1.449	5334	=C8*B8	=D8*B10	=E8*B11	=F8*B12	=G8*B13	=H8*B14	=I8*B15	=J8*B16
9	7-9	1.304	11402	=C9*B9	=D9*B10	=E9*B11	=F9*B12	=G9*B13	=H9*B14	=I9*B15	=J9*B16
10	10-19	1.224	77058	=C10*B10	=D10*B11	=E10*B12	=F10*B13	=G10*B14	=H10*B15	=I10*B16	=J10*B17
11	20-29	1.167	105916	=C11*B11	=D11*B12	=E11*B13	=F11*B14	=G11*B15	=H11*B16	=I11*B17	=J11*B18
12	30-39	1.13	81283	=C12*B12	=D12*B13	=E12*B14	=F12*B15	=G12*B16	=H12*B17	=I12*B18	=J12*B19
13	40-49	1.1	47880	=C13*B13	=D13*B14	=E13*B15	=F13*B16	=G13*B17	=H13*B18	=I13*B19	=J13*B20
14	50-59	1.09	93233	=C14*B14	=D14*B15	=E14*B16	=F14*B17	=G14*B18	=H14*B19	=I14*B20	=J14*B21
15	60-69	1.08	14748	=C15*B15	=D15*B16	=E15*B17	=F15*B18	=G15*B19	=H15*B20	=I15*B21	=J15*B22
16	70-79	1.08	102	=C16*B16	=D16*B17	=E16*B18	=F16*B19	=G16*B20	=H16*B21	=I16*B22	=J16*B23
17	80-89	1.08	0	=C17*B17	=D17*B18	=E17*B19	=F17*B20	=G17*B21	=H17*B22	=I17*B23	=J17*B24
18	90+	1	0	=C18*B18	=D18*B19	=E18*B20	=F18*B21	=G18*B22	=H18*B23	=I18*B24	=J18*B25
19											
20	sum of columns	=SUM(C2:C18)	=SUM(D2:D18)	=SUM(E2:E18)	=SUM(F2:F18)	=SUM(G2:G18)	=SUM(H2:H18)	=SUM(I2:I18)	=SUM(J2:J18)	=SUM(K2:K18)	=SUM(L2:L18)
21											
22	reserve growth	=C20-C20	=D20-C20	=E20-C20	=F20-C20	=G20-C20	=H20-C20	=I20-C20	=J20-C20	=K20-C20	=L20-C20

A	B	C	D	E	F	G	H	I	J	K	L
1	age, yr	growth func	ESF known	ESF +10yr	ESF +20yr	ESF +30yr	ESF +40yr	ESF +50yr	ESF +60yr	ESF +70yr	ESF +80yr
2	0	4.4600	2024	9027	11049	12894	14571	16028	17470	18868	20377
3	1	2.484	2794	6940	8495	9914	11202	12323	13432	14506	15667
4	2	2.077	3690	8080	9889	11541	13041	14345	15636	16887	18238
5	3	1.809	7009	12679	15519	18111	20466	22512	24338	26501	28621
6	4	1.696	5603	9503	11631	13574	15338	16872	18391	19862	21451
7	5	1.529	5400	8257	10106	11794	13327	14660	15978	17257	18638
8	6	1.449	5334	7729	9460	11040	12475	13723	14958	16155	17447
9	7-9	1.304	11402	14868	18199	21236	23999	26398	28775	31077	33563
10	10-19	1.224	77058	94319	110070	124379	136817	149131	161061	173946	187862
11	20-29	1.167	105916	123604	139672	153640	167467	180865	195334	210961	210961
12	30-39	1.130	81283	91827	101010	110101	118909	128422	138695	138695	138695
13	40-49	1.100	47880	52668	57408	62001	66961	72318	72318	72318	72318
14	50-59	1.090	93233	101624	109754	118534	128017	128017	128017	128017	128017
15	60-69	1.080	14748	15928	17202	18578	18578	18578	18578	18578	18578
16	70-79	1.080	102	110	119	119	119	119	119	119	119
17	80-89	1.080	0	0	0	0	0	0	0	0	0
18	90+	1.000	0	0	0	0	0	0	0	0	0
19											
20	sum of columns	483656	557163	629585	697458	761288	814310	863301	903747	930552	944472
21											
22	reserve growth	0	93507	165929	233802	297632	350654	399645	440091	466896	480816

Figure 1. Spreadsheet program for estimating future oil- and gas-reserve growth: (A) calculation formulas and example of input data (in this case representing lower 48 United States gas fields); (B) results of calculations. See text for detailed explanation. growth funct = growth function (dimensionless); ESF = estimated size of fields (cumulative production plus remaining reserves); units for ESF and reserve growth in this example are billion cubic feet of gas.

(from Schmoker and Crovelli, 1998)

The spreadsheet algorithm can be explained by considering how reserve growth is calculated for the fields of a single age class. Consider the fields that were 5 years old (row 7) in 1977, when the field sizes of column C were estimated. To calculate the first growth increment of 10 years, the petroleum volume in cell C7 is multiplied by the 10-year growth factor that applies to 5-year-old fields (cell B7). This multiplication is done in cell D7. In the example used here, the total estimated size of 5-year-old fields of 5,400 bcfg grows to 8,257 bcfg (fig. 1B, cell D7), after the first 10-year growth period.

After 10 years of growth, fields that were 5 years old have become 15 years old. Therefore, to calculate the second growth increment of 10 years, the petroleum volume in cell D7 is multiplied by the 10-year growth factor that applies to 15-year-old fields, which is in cell B10. This multiplication is done in cell E7. The total estimated size of 5-year-old fields of 5,400 bcfg grows to 10,106 bcfg (fig. 1B, cell E7), after two 10-year growth periods.

Fields that were 5 years old have become 25 years old after two 10-year growth periods. The 10-year growth factor that applies to 25-year-old fields is in cell B11 and the multiplication process continues. The estimated size of 5-year-old fields grows from 5,400 bcfg to 20,129 bcfg (fig. 1B, cell L7) after nine 10-year growth periods. Reserve-growth calculations for the other age classes proceed similarly.

Rows 2 through 18 of each column (except columns A and B) are summed in row 20 (fig. 1). Thus, cell C20 contains the total initial volume of all the fields that are to be grown (463,656 bcfg in this example), cell D20 contains the total volume of these same fields increased by 10 years of reserve growth (557,163 bcfg), cell E20 contains the total volume of these same fields increased by 20 years of reserve growth (629,585 bcfg), and so on. The total volume of petroleum attributable to reserve growth following each 10-year growth increment is calculated in row 22 by subtracting cell C20 (the total initial volume) from cell D20 (the total volume after 10 years of reserve growth), from cell E20 (the total volume after 20 years of reserve growth), etc. In this example, total reserve growth in the 90 years between 1977 and 2067 is projected to be 480,816 bcfg (fig. 1B, cell L22).

PROBABILISTIC METHOD

In the probabilistic method, a growth function is composed of a set of growth *variables*. A growth variable is a dimensionless random variable having a probability distribution. In contrast, a growth factor (previous section) is a multiplicative constant. It is important to the understanding of this paper to keep these two definitions in mind. Each 10-year growth variable, like the corresponding 10-year growth factor, applies to fields of a particular age or range of ages. The growth variable in the probabilistic method is related to the growth factor in the deterministic method as follows: the mean value of a growth variable is set equal to its corresponding growth factor.

The mean-based left-triangular probability distribution (figure 2) is used here as a probability model for the random variable X : growth variable (dimensionless). The right-skewed shape of the left-triangular probability distribution is appropriate as a probability model for a growth variable, whereby high probability of low growth values tapers off to low probability of high growth values. The left-triangular probability distribution is described in Law and Kelton (1991, p. 516). The defining parameters of the mean-based left-triangular probability distribution are the minimum (a) and mean (μ). From the defining parameters, the standard characterizing parameters of the left-triangular probability distribution are obtained: minimum (a) and maximum (b). Namely, given the formula for the mean in terms of a and b

$$\mu_x = a + (b - a)/3$$

We solve for the maximum, b , and get

$$b = 3\mu_x - 2a$$

The probability density function for the left-triangular distribution is given by

$$f(x) = \frac{2(b - x)}{(b - a)^2} \quad a \leq x \leq b$$

Figure 2. Left-triangular probability distribution of growth variable (dimensionless) where a : minimum, b : maximum, μ : mean, and fractile $F100p$ for $0 \leq p \leq 1$.

The graph of the probability density function for the left-triangular distribution of the growth variable is displayed in figure 2.

The standard deviation is equal to

$$\sigma_x = (b - a) / (3\sqrt{2})$$

The fractiles can be computed from

$$F100p = b - \sqrt{p}(b - a) \quad 0 \leq p \leq 1$$

$$\text{where } P(X \geq F100p) = p$$

For example, the median (where $p = 0.50$) can be shown to be

$$F50 = a + 0.293(b - a)$$

Suppose we denote the parameter c : initial petroleum volume, that is, the volume of petroleum that will be “grown” (for example, column C of figure 1). Then the random variable cX represents the grown petroleum volume (total estimated size) after the first growth period. It can be proved that the random variable cX is also distributed as a left-triangular probability distribution with characterizing parameters: minimum (ca) and maximum (cb). The mean, standard deviation, and fractiles of cX are the following:

$$\mu_{cX} = c\mu_x$$

$$\sigma_{cX} = c\sigma_x$$

$$F100p = c[b - \sqrt{p}(b - a)] \quad 0 \leq p \leq 1$$

In general, the calculated future growth is based upon a series of multiplications by the appropriate growth variables, as in the case of the growth factors in the previous section and shown in figure 1A. Let us denote the random variable Y_i : grown petroleum volume (total estimated size) after the i th growth period. The growth process can be described as a stochastic process consisting of the set of random variables $\{Y_i\}$ where $i = 0, 1, 2, 3, \dots, n$. Note that Y_0 represents the initial petroleum volume c . Also, X_i represents the growth variable that is applied in the i th growth period, where $i = 1, 2, 3, \dots, n$. Therefore, we have the following sequence of grown petroleum volumes after i growth periods for $i = 0, 1, 2, 3, \dots, n$:

$$Y_0 = c$$

$$Y_1 = Y_0 X_1 = cX_1$$

$$Y_2 = Y_1 X_2 = cX_1 X_2$$

$$Y_3 = Y_2 X_3 = cX_1 X_2 X_3$$

...

$$Y_i = Y_{i-1} X_i = cX_1 X_2 X_3 \dots X_i$$

...

$$Y_n = Y_{n-1} X_n = cX_1 X_2 X_3 \dots X_n = c \prod_{i=1}^n X_i$$

This stochastic process can be viewed as an example of the *law of proportionate effect*. The law of proportionate effect as stated by Aitchison and Brown (1957) is:

A variate subject to a process of change is said to obey the law of proportionate effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate.

The importance of the law is embodied in the following theorem:

A variate subject to the law of proportionate effect tends, for large n , to be distributed as a two-parameter lognormal distribution, that is, asymptotically lognormally distributed in a two-parameter form.

The lognormal probability distribution is a good approximate distribution for a product of independent random variables – the approximation becoming better with more variables (multiplicative central limit theorem for independent random variables). Hence, the fractiles of Y_i where $i = 2, 3, \dots, n$ can be approximated by using the lognormal distribution. As derived in Crovelli (1992), the characterizing parameters of the lognormal distribution, namely mu (μ) and sigma (σ), can be calculated from the mean μ_Y and standard deviation σ_Y of a lognormal random variable Y as follows

$$\mu = \ln \left(\frac{\mu_Y^2}{\sqrt{\mu_Y^2 + \sigma_Y^2}} \right)$$

$$\sigma = \sqrt{\ln(\sigma_Y^2 / \mu_Y^2 + 1)}$$

Knowing the lognormal characterizing parameters, the lognormal fractiles can be calculated from the formula

$$F100p = e^{\mu + z_p \sigma} \quad 0 \leq p \leq 1$$

Where Z is a standard normal random variable and $P(Z > z_p) = p$.

The two fractiles of most interest in this report are

$$F95 = e^{\mu - 1.645\sigma} \quad \text{and} \quad F5 = e^{\mu + 1.645\sigma}$$

At the core of the above stochastic process is the product of independent random variables in the form of YX , where Y represents a petroleum volume and X represents a growth variable. The mean, standard deviation, minimum, and maximum of YX are as follows:

$$\mu_{YX} = \mu_Y \mu_X$$

$$\sigma_{YX} = \sqrt{\sigma_Y^2 \sigma_X^2 + \sigma_Y^2 \mu_X^2 + \sigma_X^2 \mu_Y^2}$$

$$\text{Min}(YX) = \text{Min}(Y) \text{Min}(X)$$

$$\text{Max}(YX) = \text{Max}(Y) \text{Max}(X)$$

Suppose there are m initial petroleum volumes, that is, we are now interested in m volumes of petroleum that will be “grown.” For example in the deterministic case, figure 1 (column C) lists $m = 17$ initial petroleum volumes, two of which are zero. Let us denote the random variable Y_{ij} : j th grown petroleum volume (total estimated size) after the i th growth period, where $j = 1, 2, 3, \dots, m$. Note that Y_{0j} represents the j th initial petroleum volume c_j . Then the total grown petroleum volume after the i th growth period (S_i) would be the sum

$$S_i = \sum_{j=1}^m Y_{ij} \quad \text{for } i = 0, 1, 2, 3, \dots, n$$

Notice that the total initial petroleum volume is

$$S_0 = \sum_{j=1}^m Y_{0j} = \sum_{j=1}^m c_j$$

For example in the deterministic case, S_0 corresponds to cell C20 of figure 1.

Finally, the reserve growth after the i th growth period = (total grown petroleum volume after the i th growth period) – (total initial petroleum volume). That is,

$$\text{reserve growth}(i) = R_i = S_i - S_0 = \sum_{j=1}^m Y_{ij} - \sum_{j=1}^m c_j \quad \text{for } i = 0, 1, 2, 3, \dots, n$$

Also, we have total reserve growth after the n th growth period = reserve growth(n).

A simplifying assumption for purposes of mathematical tractability will be made concerning the nature of the Y_{ij} series for $j = 1, 2, 3, \dots, m$. The assumption of perfect positive correlation is made for these random variables. Under the assumption of perfect positive correlation, the standard deviations and fractiles are additive. (The means are always additive.) That is, the mean, standard deviation, and fractiles of S_i are

$$\mu_i = \sum_{j=1}^m \mu_{ij}$$

$$\sigma_i = \sum_{j=1}^m \sigma_{ij}$$

$$(F100p)_i = \sum_{j=1}^m (F100p)_{ij} \quad 0 \leq p \leq 1$$

Because of the simplifying assumption, we have

$$P\{(F95)_i \leq S_i \leq (F5)_i\} \geq 0.90$$

The probability of S_i being within the range from $(F95)_i$ to $(F5)_i$ is at least 0.90.

Also under the assumption of perfect positive correlation, the mean, standard deviation, and fractiles of R_i are

$$\mu_i = \sum_{j=1}^m \mu_{ij} - \sum_{j=1}^m c_j$$

$$\sigma_i = \sum_{j=1}^m \sigma_{ij}$$

$$(F100p)_i = \sum_{j=1}^m (F100p)_{ij} - \sum_{j=1}^m c_j \quad 0 \leq p \leq 1$$

Because of the simplifying assumption, we have

$$P\{(F95)_i \leq R_i \leq (F5)_i\} \geq 0.90$$

The probability of R_i being within the range from $(F95)_i$ to $(F5)_i$ is at least 0.90.

SPREADSHEET SYSTEM

The analytic probabilistic method described in the previous section is incorporated into a spreadsheet software system called probabilistic reserve growth spreadsheet (PREGS). PREGS consists of a series of ten panels in the spreadsheet. A panel is a set of approximately 11 columns of related calculations.

Note that column A of Panel 1 in the probabilistic spreadsheet is the same as column A in the deterministic spreadsheet (figure 1). Panel 1 includes the input of the defining parameters of the mean-based left-triangular probability distribution for each growth variable X in the growth function: mean (column B) and minimum (column D). Note that column B in the probabilistic spreadsheet is the same as column B in the deterministic spreadsheet. From the defining parameters, the maximum (column J) is computed, and then the other descriptive parameters: standard deviation (column C) and fractiles F95, F75, F50, F25, and F5 (columns E through I). For example, the 10-year growth variable with parameters in cell B9 (mean of 1.529) through cell J9 applies to fields that are 5 years old. The known estimated size of fields to be grown is also entered (column K). Note that column K in the probabilistic spreadsheet is the same as column C in the deterministic spreadsheet. For example, the size as estimated in 1977 of lower 48 gas fields 5 years of age (those discovered in 1972) is 5,400 billion cubic feet of gas (bcfg) (cell K9). The total initial petroleum volume S_0 is 463,656 bcfg (cell K22).

Panel 2 comprises the computed descriptive parameters of the left-triangular probability distribution for the grown petroleum volumes after the first growth period Y_{1j} , and the corresponding estimates of reserve growth R_1 (row 24): mean, standard deviation, and fractiles. Note that column L in the probabilistic spreadsheet is the same as column D in the deterministic spreadsheet (figure 1). For example, the total estimated initial size of 5-year-old fields of 5,400 bcfg grows to 8,257 bcfg (mean estimate in cell L9) with a range from 5,617 bcfg (F95 estimate in cell O9) to 12,054 bcfg (F5 estimate in cell S9), after the first 10-year growth period. The reserve growth is projected to be 93,507 bcfg (mean estimate in cell L24) with a range from 7,103 bcfg (F95 estimate in cell O24) to 217,794 bcfg (F5 estimate in cell S24), after the first 10-year growth period.

Panel 3 includes the computed descriptive parameters of the *lognormal* probability distribution for the grown petroleum volumes after the second growth period Y_{2j} , and the corresponding estimates of reserve growth R_2 : mean, standard deviation, and fractiles. The mean (column U) and standard deviation (column V) are used to compute the characterizing parameters of the lognormal distribution: μ (column W) and σ (column X) which, in turn, are used to compute the fractiles. Note that column U in the probabilistic spreadsheet is the same as column E in the deterministic spreadsheet. For example, the total estimated initial size of 5-year-old fields of 5,400 bcfg grows to 10,106 bcfg (mean estimate in cell U9) with a range from 6,209 bcfg (F95 estimate in cell Z9) to 15,264 bcfg (F5 estimate in cell AD9), after the second 10-year growth period. The reserve growth is projected to be 165,929 bcfg (mean estimate in cell U24) with a range from 29,855 bcfg (F95 estimate in cell Z24) to 332,862 bcfg (F5 estimate in cell AD24), after the second 10-year growth period.

Panels 4 through 10 are computationally similar in their composition to Panel 3. They compute the respective probabilistic estimates of reserve growth after the i th growth period for $i = 3, 4, \dots, 9$.

In the example used to illustrate PREGS, the system is applied to data based on successive annual estimates made between 1977 and 1991 of the sizes of lower 48 United States gas fields. The results of this application (Panels 1-10) are summarized in the graph displayed in figure 3. The probabilistic estimates of future reserve growth of lower 48 United States gas fields are in the form of a mean estimate with a range from a low F95 estimate to a high F5 estimate for nine growth periods (10-year increments).

SUMMARY

The major objective of this report is the development of a probabilistic method and spreadsheet system, called the PREGS system, for estimating future growth of oil and gas reserves. The primary advantages of the PREGS system are several-fold. The probabilistic method utilizes the same data as required by its deterministic predecessor in Schmoker and Crovelli (1998); most importantly, no additional data are needed. All of the advantages of the deterministic system carry over to the probabilistic system. The mean estimates computed by the probabilistic system equal the point estimates computed by the deterministic system. Additionally, the probabilistic system also computes a range of low and high estimates. Many operational benefits accrue from incorporating an analytic probabilistic method into a computer software spreadsheet.

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PREGS

	A	B	C	D	E	F	G	H	I	J	K	
1	PREGS: Probabilistic Reserve Growth Spreadsheet							R.A. Crovelli				(Panel 1)
2		Growth Function									ESF,known	
3	Age,yr	mean,E(X)	SD(X)	F100	F95	F75	F50	F25	F5	F0	(bcfg)	
4	0	4.46	2.446589	1	1.262827	2.390656	4.040232	6.19	9.058961	11.38	2024	
5	1	2.484	1.049346	1	1.112727	1.596455	2.303961	3.226	4.456503	5.452	2794	
6	2	2.077	0.761554	1	1.081811	1.432872	1.946338	2.6155	3.508526	4.231	3890	
7	3	1.809	0.572049	1	1.061453	1.325156	1.710852	2.2135	2.884306	3.427	7009	
8	4	1.696	0.492146	1	1.052869	1.279739	1.611561	2.044	2.621109	3.088	5603	
9	5	1.529	0.374059	1	1.040184	1.212618	1.464822	1.7935	2.232136	2.587	5400	
10	6	1.449	0.317491	1	1.034107	1.180464	1.394527	1.6735	2.045802	2.347	5334	
11	7--9	1.304	0.21496	1	1.023092	1.122185	1.267119	1.456	1.708071	1.912	11402	
12	10--19	1.224	0.158392	1	1.017015	1.090031	1.196824	1.336	1.521736	1.672	77058	
13	20--29	1.167	0.118087	1	1.012686	1.067121	1.14674	1.2505	1.388973	1.501	105916	
14	30--39	1.13	0.091924	1	1.009875	1.05225	1.114228	1.195	1.302793	1.39	81263	
15	40--49	1.1	0.070711	1	1.007596	1.040192	1.087868	1.15	1.232918	1.3	47880	
16	50--59	1.09	0.06364	1	1.006837	1.036173	1.079081	1.135	1.209626	1.27	93233	
17	60--69	1.08	0.056569	1	1.006077	1.032154	1.070294	1.12	1.186334	1.24	14748	
18	70--79	1.08	0.056569	1	1.006077	1.032154	1.070294	1.12	1.186334	1.24	102	
19	80--89	1.08	0.056569	1	1.006077	1.032154	1.070294	1.12	1.186334	1.24	0	
20	90+	1	0	1	1	1	1	1	1	1	0	
21												
22	Sum of columns										463656	
23												
24	Reserve growth										0	

Panel 1 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: example of input data (in this case the same as that used in figure 1 and representing lower 48 United States gas fields). ESF = estimated size of fields (cumulative production plus remaining reserves); units for ESF and reserve growth in this example are billion cubic feet of gas.

PREGS

	L	M	N	O	P	Q	R	S	T
1									(Panel 2)
2	Estimated Size of Fields, +10yr								
3	mean,E(Y)	SD(Y)	F100	F95	F75	F50	F25	F5	F0
4	9027.04	4951.897	2024	2555.9628	4838.68837	8177.429	12528.56	18335.34	23033.12
5	6940.296	2931.874	2794	3108.95968	4460.495	6437.266	9013.444	12451.47	15232.89
6	8079.53	2962.445	3890	4208.24381	5573.87177	7571.255	10174.3	13648.17	16458.59
7	12679.28	4009.494	7009	7439.72416	9288.02082	11991.36	15514.42	20216.1	24019.84
8	9502.688	2757.496	5603	5899.22692	7170.37738	9029.577	11452.53	14686.07	17302.06
9	8256.6	2019.921	5400	5616.99218	6548.13549	7910.036	9684.9	12053.53	13969.8
10	7728.966	1693.497	5334	5515.92568	6296.59381	7438.408	8926.449	10912.31	12518.9
11	14868.21	2450.979	11402	11665.299	12795.1515	14447.69	16601.31	19475.42	21800.62
12	94318.99	12205.36	77058	78369.1742	83995.6033	92224.88	102949.5	117262	128841
13	123604	12507.28	105916	107259.608	113025.217	121458.1	132448	147114.5	158979.9
14	91827.19	7470.01	81263	82065.4738	85508.9993	90545.54	97109.29	105868.9	112955.6
15	52668	3385.627	47880	48243.7046	49804.4111	52087.12	55062	59032.11	62244
16	101624	5933.312	93233	93870.3923	96605.5305	100606	105819.5	112777.1	118405.9
17	15927.84	834.2729	14748	14837.6226	15222.2058	15784.7	16517.76	17496.06	18287.52
18	110.16	5.769991	102	102.619847	105.279698	109.17	114.24	121.0061	126.48
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21									
22	557162.7	66119.24	463656	470758.93	501238.58	545818.5	603916.1	681450	744176.2
23									
24	93506.73	66119.24	0	7102.93008	37582.5804	82162.46	140260.1	217794	280520.2

Panel 2 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after one growth period (in this application, 10 years).

PREGS

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1											(Panel 3)
2	Estimated Size of Fields, +20yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	11049.1	6276.683	9.1703	0.52885	2024	4025.121	6726.573	9607.163	13721.34	22930.38	38511.38
5	8494.922	3781.829	8.9568	0.42522	2794	3855.777	5826.769	7760.618	10336.29	15619.99	25469.39
6	9889.345	3873.758	9.1278	0.37782	3890	4945.979	7138.022	9208.112	11878.55	17143.08	27518.76
7	15519.44	5340.535	9.5939	0.33454	7009	8464.003	11712.51	14674.86	18386.45	25443.22	40161.18
8	11631.29	3721.296	9.3127	0.31218	5603	6628.924	8976.099	11078.12	13672.39	18513.52	28929.05
9	10106.08	2815.196	9.1835	0.27338	5400	6209.407	8097.165	9735.41	11705.11	15263.65	23357.51
10	9460.254	2422.25	9.1231	0.25199	5334	6054.635	7733.082	9164.612	10861.14	13872.04	20931.6
11	18198.69	3833.634	9.7874	0.20837	11402	12640.02	15474.56	17807.86	20492.99	25088.56	36450.64
12	110070.3	18138.65	11.595	0.16369	77058	82967.89	97260.76	108605.5	121273.5	142165.3	193390.3
13	139672.5	18170.55	11.839	0.12955	105916	111921.9	126924.6	138505.3	151142.7	171402.8	220982.1
14	101009.9	10486.15	11.518	0.10354	81263	84736.27	93697.94	100470	107731.4	119125.1	146842.2
15	57408.12	4989.925	10.954	0.08676	47880	49585.96	53944.1	57192.48	60636.46	65965.85	79049.88
16	109753.9	8615.254	11.603	0.07838	93233	96181.9	103787.3	109417.3	115352.7	124474	146823.3
17	17202.07	1275.101	9.75	0.07402	14748	15188.24	16320.11	17155	18032.61	19376.45	22676.52
18	118.9728	8.818842	4.7762	0.07402	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	629584.8	93749.62			463656	493511.1	563732.5	620501	685348.4	796517.9	1051251
23											
24	165928.8	93749.62			0	29855.11	100076.5	156845	221692.4	332861.9	587594.7

Panel 3 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after two growth periods (in this application, 20 years).

PREGS

	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP
1											(Panel 4)
2	Estimated Size of Fields, +30yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	12894.3	7477.014	9.3196	0.53839	2024	4600.643	7759.945	11154.6	16034.27	27045.15	57805.58
5	9913.574	4547.942	9.1062	0.43704	2794	4390.665	6711.635	9010.628	12097.11	18491.83	38229.55
6	11540.87	4691.431	9.2772	0.39107	3890	5618.827	8214.071	10691.27	13915.55	20342.91	41305.66
7	18111.19	6526.802	9.7432	0.34943	7009	9589.459	13463.23	17038.56	21563.35	30274.11	60281.93
8	13573.72	4575.927	9.4621	0.32809	5603	7497.82	10310.72	12862.48	16045.77	22065.55	43422.51
9	11793.79	3511.142	9.3329	0.29141	5400	6998.792	9287.784	11303.5	13756.69	18255.88	35059.62
10	11040.12	3052.934	9.2724	0.27145	5334	6808.369	8861.652	10640.76	12777.06	16630.39	31418.33
11	21237.87	4983.832	9.9367	0.23153	11402	14127.41	17688.8	20676.19	24168.1	30260.67	54712.42
12	124379.4	22918.75	11.714	0.18273	77058	90563.62	108146	122320.1	138352	165212.2	268812.5
13	153639.7	22331.52	11.932	0.14459	105916	119857.7	137924	152042.1	167605.2	192868.6	287276.7
14	110100.8	13130.51	11.602	0.11884	81263	89913.33	100910.9	109326.1	118443	132930.2	186489.6
15	62000.77	6298.293	11.03	0.10132	47880	52213.26	57611.46	61683.32	66042.97	72871	98021.85
16	118534.2	11196.32	11.679	0.09425	93233	101061	110745.8	118008.9	125748.3	137799	182060.9
17	18578.23	1687.764	9.8256	0.09066	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.07402	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	697457.5	116939			463656	529284.6	615154.3	685379.2	766342	906659.2	1413173
23											
24	233801.5	116939			0	65628.57	151498.3	221723.2	302686	443003.2	949517

Panel 4 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after three growth periods (in this application, 30 years).

PREGS

	AQ	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA
1											(Panel 5)
2	Estimated Size of Fields, +40yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	14570.55	8559.402	9.4385	0.5445	2024	5129.951	8704.06	12563.2	18133.36	30767.13	80349.75
5	11202.34	5236.063	9.2251	0.4445	2794	4884.625	7521.153	10148.49	13693.61	21084.89	53139.08
6	13041.18	5423.597	9.3961	0.3994	3890	6242.1	9199.458	12041.36	15761.18	23228.46	57414.87
7	20465.64	7584.625	9.8622	0.3587	7009	10636.2	15068.47	19190.18	24439.3	34623.54	83791.88
8	15338.3	5335.819	9.581	0.338	5603	8308.196	11535.51	14486.75	18193.04	25260.11	60357.28
9	13326.99	4125.687	9.4518	0.3025	5400	7739.903	10382.64	12730.9	15610.28	20940.3	48732.87
10	12475.33	3606.924	9.3914	0.2833	5334	7519.619	9901.042	11984.48	14506.32	19100.39	43671.48
11	23998.79	5978.095	10.056	0.2454	11402	15553.49	19737.68	23287.17	27474.97	34866.29	76050.26
12	136817.3	26749.83	11.808	0.1937	77058	97639.24	117842.2	134275	152999.3	184657.1	349456.3
13	167467.3	26270.18	12.016	0.1559	105916	128015.7	148940.5	165444.1	183776.5	213815.6	364841.4
14	118908.9	15506.2	11.678	0.1299	81263	95231.93	108029.5	117910.5	128695.4	145989.9	231247.2
15	66960.83	7661.422	11.105	0.114	47880	55146.94	61604.73	66526.79	71842.11	80254.93	121547.1
16	128016.9	13841.22	11.754	0.1078	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.0907	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.074	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	761287.6	137575.6			463656	564683.8	664340.3	746484.8	841785.3	1008172	1824631
23											
24	297631.6	137575.6			0	101027.8	200684.3	282828.8	378129.3	544515.7	1360975

Panel 5 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after four growth periods (in this application, 40 years).

PREGS

	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL
1											(Panel 6)
2	Estimated Size of Fields, +50yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	16027.61	9490.864	9.5318	0.5483	2024	5596.473	9530.474	13791.05	19956.31	33984.45	104454.7
5	12322.57	5825.661	9.3183	0.4491	2794	5321.473	8230.589	11140.34	15078.77	23321.96	69080.8
6	14345.3	6048.974	9.4894	0.4045	3890	6794.594	10063.71	13218.21	17361.51	25714.73	74639.33
7	22512.2	8484.631	9.9554	0.3644	7009	11566.71	16477.73	21065.71	26931.16	38365.63	108929.4
8	16872.13	5980.68	9.6742	0.344	5603	9029.93	12611.42	15902.6	20052.68	28006.07	78464.47
9	14659.69	4644.234	9.545	0.3093	5400	8402.668	11345.71	13975.15	17213.97	23243.18	63352.73
10	13722.87	4072.493	9.4846	0.2905	5334	8157.544	10816.2	13155.77	16001.39	21216.47	56772.92
11	26398.67	6804.478	10.149	0.2536	11402	16843.06	21546.38	25563.13	30328.68	38797.79	98865.34
12	149130.9	30477.18	11.892	0.2023	77058	104754.4	127489	146110.9	167452.8	203794.9	443809.5
13	180864.7	29948.49	12.092	0.1645	105916	136139.1	159712.3	178435	199352.6	233871.6	452403.4
14	128421.6	18068.39	11.753	0.14	81263	101008.6	115717.6	127169.1	139753.8	160104.9	286746.5
15	72317.7	9110.459	11.181	0.1255	47880	58368.53	65931.78	71750.58	78082.91	88200.71	150718.4
16	128016.9	13841.22	11.754	0.1078	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.0907	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.074	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	814310	154494.4			463656	594618.9	705346.3	797173.4	904226.6	1092206	2242269
23											
24	350654	154494.4			0	130962.9	241690.3	333517.4	440570.6	628549.6	1778613

Panel 6 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after five growth periods (in this application, 50 years).

PREGS

	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV	BW
1											(Panel 7)
2	Estimated Size of Fields, +60yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	17470.1	10412.74	9.6163	0.5514	2024	6058.862	10348.95	15006.69	21760.74	37168.81	132657.4
5	13431.6	6408.944	9.4028	0.4529	2794	5754.72	8933.347	12122.33	16449.7	25535.7	87732.62
6	15636.37	6667.406	9.5738	0.4087	3890	7342.8	10919.95	14383.36	18945.23	28174.68	94791.95
7	24538.3	9374.123	10.04	0.3691	7009	12490.6	17874.21	22922.59	29396.84	42067.26	138340.4
8	18390.62	6617.731	9.7587	0.3489	5603	9746.836	13677.74	17304.37	21892.6	30721.89	99649.87
9	15979.06	5155.943	9.6295	0.3147	5400	9061.675	12300.5	15207.01	18800.31	25519.93	80457.96
10	14957.92	4531.527	9.5691	0.2963	5334	8792.335	11723.71	14315.41	17480.05	23307.91	72101.61
11	28774.55	7617.089	10.233	0.2602	11402	18129.15	23341.22	27816.44	33149.69	42680.12	125559
12	161061.4	34022.95	11.968	0.2089	77058	111748.4	136883.6	157583.8	181414.3	222219.4	550323.8
13	195333.9	33966.27	12.168	0.1726	105916	144878.8	171312.1	192446	216187.2	255630.8	560980.2
14	138695.3	20847.31	11.829	0.1495	81263	107257	124010.3	137154.6	151692	175386	355565.6
15	72317.7	9110.459	11.181	0.1255	47880	58368.53	65931.78	71750.58	78082.91	88200.71	150718.4
16	128016.9	13841.22	11.754	0.1078	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.0907	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.074	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	863300.9	170270.3			463656	622265.5	743130.8	843909	961911.6	1170196	2702910
23											
24	399644.9	170270.3			0	158609.5	279474.8	380253	498255.6	706540.4	2239254

Panel 7 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after six growth periods (in this application, 60 years).

PREGS

	BX	BY	BZ	CA	CB	CC	CD	CE	CF	CG	CH
1											(Panel 8)
2	Estimated Size of Fields, +70yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	18867.7	11304.45	9.6918	0.55383	2024	6508.02	11142.93	16185.04	23508.67	40251.18	164495.2
5	14506.13	6972.669	9.4784	0.45592	2794	6175.887	9615.251	13074.19	17777.44	27677.72	108788.4
6	16887.28	7264.719	9.6494	0.41206	3890	7876.002	11750.94	15512.76	20478.86	30554.31	117542
7	26501.37	10232.52	10.115	0.37278	7009	13389.79	19229.8	24722.51	31784.14	45646.91	171542.1
8	19861.87	7232.163	9.8343	0.35285	5603	10444.88	14712.97	18663.14	23673.85	33347.69	123565.8
9	17257.38	5648.841	9.7051	0.31904	5400	9703.957	13227.76	16401.09	20335.71	27720.22	99767.87
10	16154.56	4973.268	9.6447	0.30091	5334	9411.435	12605.24	15439.48	18910.99	25328.5	89405.99
11	31076.51	8397.009	10.309	0.26546	11402	19385.77	25085.73	30000.63	35878.46	46427.75	155693.1
12	173946.3	37906.39	12.043	0.2154	77058	119249.7	146990.9	169957.5	196512.6	242227.5	682401.5
13	210960.6	38359.78	12.243	0.18036	105916	154272.3	183799.6	207557.2	234385.6	279246.4	695615.4
14	138695.3	20847.31	11.829	0.14947	81263	107257	124010.3	137154.6	151692	175386	355565.6
15	72317.7	9110.459	11.181	0.12548	47880	58368.53	65931.78	71750.58	78082.91	88200.71	150718.4
16	128016.9	13841.22	11.754	0.10781	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.09066	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.07402	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	903746.8	183787.4			463656	644679.2	773976.6	882314.6	1009681	1235598	3169133
23											
24	440090.8	183787.4			0	181023.2	310320.6	418658.6	546025.2	771942	2705477

Panel 8 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after seven growth periods (in this application, 70 years).

PREGS

	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS
1											(Panel 9)
2	Estimated Size of Fields, +80yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	20377.12	12272.04	9.7674	0.5563	2024	6990.602	11997.92	17455.91	25396.82	43588.36	203974.1
5	15666.62	7585.322	9.554	0.4589	2794	6628.094	10349.34	14100.8	19212.09	29998.44	134897.7
6	18238.26	7914.515	9.725	0.4154	3890	8448.297	12645.39	16730.85	22136.24	33133.46	145752.1
7	28621.48	11167.35	10.191	0.3764	7009	14354.57	20688.7	26663.76	34364.47	49528.22	212712.2
8	21450.82	7901.731	9.9099	0.3567	5603	11193.71	15827.01	20128.59	25599.29	36195.37	153221.6
9	18637.97	6186.619	9.7807	0.3233	5400	10392.75	14225.47	17688.93	21995.64	30107.36	123712.2
10	17446.92	5455.573	9.7203	0.3054	5334	10075.27	13553.68	16651.81	20458.12	27521.13	110863.4
11	33562.63	9249.79	10.385	0.2706	11402	20732.91	26962.46	32356.32	38829.23	50496.11	193059.5
12	187862	42159.4	12.119	0.2217	77058	127294	157864.5	183302.8	212840.4	263955.4	846177.8
13	210960.6	38359.78	12.243	0.1804	105916	154272.3	183799.6	207557.2	234385.6	279246.4	695615.4
14	138695.3	20847.31	11.829	0.1495	81263	107257	124010.3	137154.6	151692	175386	355565.6
15	72317.7	9110.459	11.181	0.1255	47880	58368.53	65931.78	71750.58	78082.91	88200.71	150718.4
16	128016.9	13841.22	11.754	0.1078	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.0907	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.074	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	930551.5	193747.7			463656	658643.9	793729.6	907438	1041653	1280940	3580301
23											
24	466895.5	193747.7			0	194987.9	330073.6	443782	577996.8	817284.1	3116645

Panel 9 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after eight growth periods (in this application, 80 years).

PREGS

	CT	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD
1											(Panel 10)
2	Estimated Size of Fields, +90yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	22007.29	13321.94	9.843	0.5588	2024	7509.103	12918.6	18826.58	27436.41	47201.38	252927.9
5	16919.95	8251.11	9.6296	0.4619	2794	7113.64	11139.63	15208.01	20762.25	32512.71	167273.1
6	19697.33	8621.349	9.8006	0.4187	3890	9062.566	13608.16	18044.58	23927.33	35928.78	180732.6
7	30911.19	12185.32	10.267	0.3801	7009	15389.75	22258.8	28757.44	37153.4	53736.44	263763.1
8	23166.88	8631.29	9.9855	0.3605	5603	11997.03	17025.88	21709.12	27680.56	39283.56	189994.8
9	20129.01	6773.269	9.8563	0.3275	5400	11131.45	15299.01	19077.9	23790.18	32697.08	153403.1
10	18842.68	5982.073	9.7959	0.3099	5334	10787.09	14574.13	17959.34	22130.84	29900.34	137470.7
11	36247.64	10182.04	10.46	0.2756	11402	22177.13	28981.44	34896.99	42019.99	54912.42	239393.8
12	187862	42159.4	12.119	0.2217	77058	127294	157864.5	183302.8	212840.4	263955.4	846177.8
13	210960.6	38359.78	12.243	0.1804	105916	154272.3	183799.6	207557.2	234385.6	279246.4	695615.4
14	138695.3	20847.31	11.829	0.1495	81263	107257	124010.3	137154.6	151692	175386	355565.6
15	72317.7	9110.459	11.181	0.1255	47880	58368.53	65931.78	71750.58	78082.91	88200.71	150718.4
16	128016.9	13841.22	11.754	0.1078	93233	106592.2	118355.2	127275.2	136867.4	151971.5	225755.5
17	18578.23	1687.764	9.8256	0.0907	14748	15938.63	17405.33	18502.04	19667.86	21477.72	28118.89
18	118.9728	8.818842	4.7762	0.074	102	105.0448	112.873	118.6473	124.717	134.0112	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	944471.7	199963.1			463656	664995.5	803285.2	920141	1058562	1306544	3887068
23											
24	480815.7	199963.1			0	201339.5	339629.2	456485	594905.9	842888.3	3423412

Panel 10 of PREGS. Probabilistic spreadsheet system for estimating future oil- and gas-reserve growth: after nine growth periods (in this application, 90 years).

Figure 3. Graph of probabilistic estimates of future reserve growth of lower 48 United States gas fields in the form of a mean estimate together with a range from a low F95 estimate to a high F5 estimate for nine growth periods (10-year increments). Units for reserve growth are trillion cubic feet of gas. This example is derived from the results shown in Panels 1-10.