Workshop on EARTHQUAKE RECURRENCE: STATE OF THE ART AND DIRECTIONS FOR THE FUTURE
Istituto Nazionale de Geofisica, Rome, Italy, 22-25 February, 1999

A Physically-Based Earthquake Recurrence Model for Estimation of Long-Term Earthquake Probabilities

William L. Ellsworth (U.S. Geological Survey, Menlo Park, CA)
Mark V. Matthews (Walden Consulting, Wayland, MA)
Robert M. Nadeau (Lawrence Berkeley Laboratory, Berkeley, CA)
Stuart P. Nishenko (Federal Emergency Management Agency, Washington, D.C.)
Paul A. Reasenberg (U.S. Geological Survey, Menlo Park, CA)
Robert W. Simpson (U.S. Geological Survey, Menlo Park, CA)

ABSTRACT

A physically-motivated model for earthquake recurrence based on the Brownian relaxation oscillator is introduced. The renewal process defining this point process model can be described by the steady rise of a state variable from the ground state to failure threshold as modulated by Brownian motion. Failure times in this model follow the Brownian passage time (BPT) distribution, which is specified by the mean time to failure, μ, and the aperiodicity of the mean, α (equivalent to the familiar coefficient of variation). Analysis of 37 series of recurrent earthquakes, M -0.7 to 9.2, suggests a provisional generic value of $\alpha = 0.5$. For this value of α, the hazard function (instantaneous failure rate of survivors) exceeds the mean rate for times $> \mu/2$, and is $\approx 2/\mu$ for all times $> \mu$. Application of this model to the next M 6 earthquake on the San Andreas fault at Parkfield, California suggests that the annual probability of the earthquake is between 1:10 and 1:13.

INTRODUCTION

It has been one quarter century since Utsu (1972a, 1972b), Rikitake (1974) and Hagiwara (1974) proposed a probabilistic approach for forecasting the time of the next earthquake on a specific fault segment. Their proposals were based on a model of earthquake occurrence that assumed that the probability of an earthquake was initially low following a segment-rupturing earthquake, and increased gradually as tectonic processes reloaded the fault. This approach to earthquake forecasting has been widely applied, particularly in Japan and the United States, as one basis for long-term forecasts of future seismic activity.

The conceptual model proposed by Utsu, Rikitake and Hagiwara considers earthquakes as a renewal process, in which the elastic strain energy accumulates over a long period of time after the occurrence of one earthquake before the fault is prepared to release in the next earthquake. This hypothesis has its roots in the ideas set forth by Gilbert

(1884) and Reid (1910). In his formulation of the elastic rebound hypothesis, Reid (1910) wrote:

> "It seems probable that a very long time will elapse before another important earthquake occurs along the San Andreas fault which broke in 1906; for we have seen that the strains causing the slip were probably accumulating for 100 years... It seem that the most consistent results might be obtained regarding the periodicity of earthquakes if only the earthquakes occurring in exactly the same place were considered in the series."

To forecast the likelihood that a particular earthquake will occur at some time in the future under these restrictions requires the specification of a probability distribution of failure times. A number of candidate statistical models have been proposed for the computation of conditional probabilities of future earthquakes, including the Double Exponential (Utsu, 1972b), Gaussian (Rikitake, 1974), Weibull (Hagiwara, 1974), Log-normal (Nishenko and Buland, 1987) and Gamma (Utsu, 1984) distributions.

All of these distributions have been widely discussed as failure model for a broad range of reliability and time-to-failure problems (e.g. Mann et al., 1974), although none of them has any particular claim as a proper model for earthquake recurrence. They have been used in the past chiefly because they are mathematically well-developed functions with well-known statistical properties; in other words the usual suspects. At present, it is not possible to discriminate between such candidate models, given the limited and uncertain nature of earthquake recurrence data, although we can firmly reject a Poisson process or Exponential model (Ellsworth, 1995). Furthermore, because the predictions obtained from these specific models differ significantly from one another, particularly at times removed from the mean failure time, it is important to consider alternatives to these familiar (and possibly inappropriate) probability models.

In this paper, we introduce a new (to earthquake forecasting) probability model that is based on a simple physical model of the earthquake cycle. This model has many desirable statistical properties that make it a suitable candidate for describing the statistics of earthquake recurrence. We determine the maximum likelihood estimate of the parameters of this model for 37 series of recurrent earthquakes, M -0.7 to 9.2, and propose a provisional generic model for recurrent earthquakes.

BROWNIAN PASSAGE TIME MODEL

Matthews (submitted) has recently proposed a physically-based renewal model for the earthquake cycle. This model considers the earthquake as a realization of a point process in which earthquakes only occur when a state variable reaches a fixed threshold (Y_f), at which time the state variable returns to a fixed ground state (Y_0). In Matthews' model, the loading of the system has two components, a constant-rate loading component, λt, and a random component, $\varepsilon(t)$, that is defined as Brownian motion or a random walk (Figure 1). An event occurs when $Y(t) = \lambda t + \varepsilon(t) = Y_f$. Matthews defines the Brownian motion as $\delta W(t)$, where W is standard Brownian motion, and δ is a nonnegative scale parameter. So, $Y(t) = \lambda t + \delta W(t)$ defines the state of a *Brownian relaxation oscillator*.

In this model, the times between events are independent, identically distributed random variables. The distribution function of "passage times" across the failure threshold is known as the *Brownian passage time* (BPT) distribution in the physics literature, and the *inverse Gaussian* distribution in the statistics literature. The BPT distribution is defined by two parameters, the mean time or period between events, μ, and the aperiodicity of the mean time, α. The aperiodicity, α, is equivalent to the familiar *coefficient of variation*. The probability density for the BPT model (Figure 2) is given by:

$$\sqrt{\frac{\mu}{2\pi\alpha^2 t^3}}\ \exp[-\frac{(t-\mu)^2}{2\alpha^2\mu t}]$$

Several important properties of the model include: (1) The probability of failure is 0 at t=0. (2) As $t \rightarrow \infty$ the hazard function, or instantaneous failure rate of survivors is finite. (3) The asymptotic value of the hazard function for the model with $\alpha = 1/\sqrt{2}$ is equal to the failure rate for a Poisson process with the same mean rate. (4) The mean residual lifetime at $t \rightarrow \infty$ is $2\mu\alpha^2$. It is higher than the mean rate when $\alpha < 1/\sqrt{2}$ and lower when $\alpha > 1/\sqrt{2}$. (5) Properties of the time to failure may also be expressed in terms of the parameters of the Brownian relaxation oscillator: loading rate λ, failure threshold Y_f, and "noise" (diffusion) scale δ. For example, mean time to failure, $\mu = Y_f/\lambda$, and $\alpha = \delta/\sqrt{Y_f \lambda}$.

It is sometimes useful to identify the state variable, Y, with stress, since the saw-toothed path that $Y(t)$ follows in the Brownian relaxation oscillator resembles the stress cycle in a simple block-and-slider model. In this interpretation, the random component represents perturbations to the stress state, and perturbations to the stress state due to external causes, such as a nearby earthquake, may be explicitly calculated (Matthews, submitted). The rate at which strain is observed to accumulate along faults, however, tends to be both monotonic and uniform. So, an objection to this interpretation of the state variable might be raised on the ground that the stress loading path is smoother than a random walk. For example, it seems unlikely that the mean load path for a great earthquake occurring on a major plate boundary could be as irregular as the state histories of Figure 1.

It should be emphasized, however, that the state variable is a formal parameter of a point process model. The resulting BPT captures not only the stochastic nature of the accumulating tectonic stress but also the spatial irregularities in the stress and the strength of the fault. Many factors contribute to the irregularity of earthquake times, including the details of the stress state of the fault after the last earthquake, time dependent changes in fault strength caused by healing of the gouge and other chemical processes, evolution of the pore pressure due to compaction and/or fluid migration to name just a few. Indeed, it may well be that such factors play the dominant role in the irregularity of earthquake series. It is also noteworthy that the BPT distribution also describes a Brownian ratchet, in which the random perturbations, $\varepsilon(t)$ are non negative and the resulting growth of the state toward failure is strictly monotonic.

COMPARISON OF THE BROWNIAN PASSAGE TIME MODEL WITH OTHER PROPOSED EARTHQUAKE FAILURE MODELS

It is useful to compare the properties of the BPT model with other candidate models. The most elementary model is the Exponential distribution, in which there is no memory of the past. This model has been widely applied in probabilistic seismic hazard modeling, and is, for instance, the basis of the new U.S. National Earthquake Hazard Maps. The Gamma and Weibul distributions are generalizations of the Exponential model. The Lognormal model is obtained by taking the exponential transformation of a Gaussian distribution. All of these distributions, except for the Exponential distribution, have probability densities that are generally similar (Figure 3). Conditional probabilities are also similar up to the mean recurrence time, at which point the distributions diverge.

Some of the analytic properties of these distributions are listed in Table 1. The columns labeled "Shape", "$r(0)$", and "r_∞/r_{ave}" refer to properties of the hazard function, or instantaneous failure rate of survivors. In the case of the BPT model, the failure rate is 0 immediately after an event. It grows to a peak and then declines to a finite asymptotic rate at times long compared to the mean time. These are unique properties among this suite of models. Compare this behavior with that of the Lognormal model, for which the asymptotic failure rate is always 0. The Gamma distribution, like the BPT model, also has a finite asymptotic failure rate. However, when $\theta>1$, the failure rate is always smaller than the mean rate. Failure rate functions of Weibull variates either start at zero and increase to inifinity or start at a finite value and decrease to zero

We assert that the properties of the the BPT model not only satisfy the spirit of the hypothesis proposed by Reid (1910), Utsu (1972a), Rikitake (1974) and Hagiwara (1994), but also provide more realistic asymptotic behavior of the failure rate than the alternative models. In one sense, the behavior of the BPT model is similar to a delayed Poisson process, for which the failure rate is zero for a finite time following an event and then steps up to an approximately constant failure rate at all succeeding times.

PARAMETER ESTIMATION FOR THE BPT MODEL

The technically challenging problem of estimating the parameters for earthquake renewal models has been the focus of recent work in both Japan (Ogata, 1998; Working Group on Assessment Methods of Long-Term Earthquake Probability, 1998) and the U.S. (Matthews, submitted). Past workers, such as Nishenko and Buland (1987), attempted to study the shape of the recurrence distribution by normalizing individual earthquake series by their sample mean and then stacking the results. Unfortunately, this process alters the shape of the sampling distribution, thereby invalidating the use of series so normalized for discrimination between candidate models.

An important technical advance in the recent work of both Ogata and Matthews has been the incorporation of open intervals into the estimation procedure. Here, we apply adjusted maximum likelihood estimators for μ and α developed by Matthews. These estimators can be used with both closed and open recurrence interval data. The inclusion of open intervals typically becomes important when the open interval approaches the length of the sample mean. In general, both $\hat{\mu}$ and $\hat{\alpha}$ increase once the length of an open interval exceeds the mean when compared to estimates made without open intervals.

We face a particular problem in applying these estimators to data because the number of events in any earthquake series is typically very small. For a series with two events and no open intervals, we can estimate μ but not α. With three or more events, we can estimate both parameters. But how reliable are these estimates, since by hypothesis they are realizations of a random process?

To estimate how short a series can be and still have value for estimation purposed, we have used a bootstrap method to assess the parametric uncertainties in estimated values, $\hat{\mu}$ and $\hat{\alpha}$ as a function of the length of the earthquake series. Figure 4 illustrates bootstrap result for the case of μ=1 and α=0.5. The upper two plots show five representative synthetic series drawn from the BPT distribution for earthquake "catalogs" with lengths of 3.5 μ (left) and 10 μ (right). We have arbitrarily set the time of the most recent event at time 3 and 10, respectively, creating an open interval of 0.5 in the former case. The lower plots show bootstrap estimates of $\hat{\mu}$ (left) and $\hat{\alpha}$ (right), and their 68% confidence intervals. Neither quantity is estimated with much confidence for catalogs containing 3 - 10 events. It is particularly sobering to see how uncertain $\hat{\alpha}$ is, particularly for small samples. The large uncertainties in parameters estimated from short catalogs is not unique to either the BPT model or the estimators developed by Matthews. This problem exists for any probability distribution whose parameters must be estimated from short data samples, and in particular all of the alternative models of Table 1.

In addition to the parametric uncertainty created by short series, we must confront another issue when analyzing paleoseismically recognized earthquakes: the dates of the events themselves are uncertain. Our approach to this problem has been to perform a Monte Carlo analysis on each paleoseismic earthquake sequence. We use the probability distributions that constrain the dates of each individual event to construct 1000 realizations of the earthquake dates drawn at random from those distributions. Each of the realizations represents one possible set of dates for all of the earthquakes in that series, and it is analyzed individually to determine $\hat{\mu}$ and $\hat{\alpha}$. We report the mean values of $\hat{\mu}$ and $\hat{\alpha}$ as the preferred parameter estimates.

This process is illustrated in Figure 5 for the Brigham City segment of the Wasatch fault. McCalpin and Nishenko (1996) reported C^{14}-constrained dates of paleoearthquakes along this fault segment. They determined a mean recurrence interval of 1282±138 years for events younger than 5600 years. The upper part of Figure 5 shows the probability distributions of the six most recent events, and five examples of earthquake series drawn at random from those distributions. The lower plots show histograms of $\hat{\mu}$ and $\hat{\alpha}$ determined from 1000 realizations. Their mean values are 1476 years and 0.31, respectively. Note that not only is $\hat{\mu}$ greater than the mean recurrence interval determined by McCalpin and Nishenko (1996), but the aperiodicity is substantially greater than their coefficient of variation of 138/1282 = 0.11. The increase in both μ and α is directly attributable to the length of the current open interval.

TOWARDS A GENERIC BROWNIAN PASSAGE TIME MODEL: PROVISIONAL ESTIMATION OF APERIODICITY FROM GLOBAL EARTHQUAKE RECURRENCE DATA

A global data set of recurrent earthquake sequences has been assembled from the published literature to explore the range of α as a function of earthquake magnitude and tectonic style (Table 2). The data include repeating microearthquakes (M -0.6 to 1.6) (Nadeau and Johnson, 1998); repeating moderate earthquakes (M 3.7 to 5.1) (Ellsworth, 1995); historically documented sequences of large-magnitude events (e.g. Nankai Trough, Ishibashi and Satake, 1998), and sequences inferred from paleoseismic investigations (e.g. Cascadia megathrust events, Atwater and Hemphill-Haley, 1997). The bootstrap analysis of uncertainties strongly implies that sequences with just 2 or 3 recurrence intervals will be of little value for estimation of α. By restricting the sequences to those that have at least 4 closed intervals (5 events), a total of 35 sequences have been identified that are suitable for analysis. We also analyze two sequences, with 3 and 4 closed intervals, respectively, that have long open intervals on both ends of the catalog, giving us a total of 37 sequences.

The parameters of the BPT model have been estimated for each sequence using the adjusted maximum likelihood method of Matthews as described above. The current open interval was used in the estimation procedure in the case of those sequences where the open interval is longer than the mean of the closed intervals.

DISCUSSION

The estimated values of $\hat{\alpha}$ of Table 2 range from 0.11 for a Parkfield microearthquake to 0.93 for ground rupturing events at Pitman Canyon on the San Andreas fault. The mean value of $\hat{\alpha}$ is 0.44 and 68% confidence interval is ±0.22. We found no systematic difference between the values of $\hat{\alpha}$ when the data were grouped by tectonic style, or when they were examined as a function of either the number of events in the estimate or event magnitude (Figure 6). However, we caution that the total number of sequences is very limited, and the data are dominated by strike slip earthquakes of the San Andreas fault system.

The variability in $\hat{\alpha}$ is consistent with what would be expected if α=0.5, and 37 sequences were drawn at random from a suite of earthquake sequences with the same population statistics as the actual data. The lower part of Figure 6 compares the measured distribution of $\hat{\alpha}$ with those created using a bootstrap procedure with $\alpha = 0.5$. On this basis, we suggest that α=0.5 can serve as a working estimate of the aperiodicity for recurrent earthquake sequences of all sizes and in all tectonic environments.

Some properties of the BPT distribution for $\alpha = 0.5$ are shown in Figure 7. As with all BPT distributions, the probability density peaks before the mean time is reached. The 95% range of recurrence intervals is from 0.36μ to 2.26μ. There is less than 1 chance in 400 of failure occurring within $\mu/4$ of the preceding event, and the probability of failure in the first 0.1μ is vanishingly small. The asymptotic mean residual lifetime is 2μ. Both the failure rate and conditional probability approach their asymptotic values at times only slightly longer than μ.

Adopting our provisional value of $\alpha = 0.5$, we have estimated the mean recurrence time, $\mu_{0.5}$ for each of the 37 sequences (Table 2). In general, $\mu_{0.5} < \mu$ when $\alpha > 0.5$

and $\mu_{0.5} > \mu$ when $\alpha < 0.5$. The value of $\mu_{0.5}$ for Brigham City, Utah is exceptional, due to the long open interval. The very low expectation of failure at times $< \mu_{0.5}/4$ suggests that we should have some statistical leverage to reject this model in some cases. For the 27 sequences in Table 2 that have accurately know dates, none of the 147 recurrence intervals fall within this "forbidden" zone.

It is of some note that both the great earthquakes (M 7.8 - 9.2) and smallest microearthquakes (M < 2) display such a wide range of $\hat{\alpha}$. In the case of the microearthquakes, it is quite natural to expect that wide variability in recurrence time would be the norm when repeating earthquakes are influenced by elastic stress transfer from other nearby earthquakes, particularly those of larger magnitude. Stress transfer from nearby earthquakes may similarly be influencing the recurrence history, and thereby the aperiodicity, of the largest earthquakes. For example, it is well known that the Nankaido and Tonankai segments of the Nankai subduction zone in Japan sometimes fail in a single earthquake, or in closely spaced pairs of events. A similar phenomenon has been noted for the M 4-5 earthquakes on the San Andreas fault at Stone Canyon (Ellsworth, 1995). Thus, we should be optimistic that some portion of the aperiodicity can be understood, if not modeled, using elastic interactions of the kind envisioned by Matthews (submitted) in the formulation of the point process model.

A question arises as to when the estimated mean, $\hat{\mu}$, and aperiodicity, $\hat{\alpha}$, should be favored over the generic value of $\mu_{0.5}$ and $\alpha = 0.5$. We believe that Bayesian methods should be useful in this regard, but have not as yet tested this proposition. As always, independent estimates of the parameters can be of great value in deciding this question. For example, in the case of great earthquakes in the Cascadian subduction zone, our estimates come from one paleoseismic site of Atwater and Hemphill-Haley (1997) at Willipa Bay, where coseismic submergence provides the earthquake chronology. Adams (1990) also developed and earthquake chronology for Cascadia from the sedimentary records of turbidites in the Cascadia deep-sea channel. Although dates could not be determined for individual events, Adams established that 13 events occurred since the Mazama eruption with a mean interval of 590+-50 years. Accordingly the value of $\mu_{0.5} = 490$ obtained from the Willipa Bay is on the statistical edge of acceptability (p=0.05).

We conclude with an application of the BPT model to the next M 6 earthquake at Parkfield on the San Andreas fault in California. The prediction of the next earthquake is based upon the series of characteristic earthquakes that are hypothesized to have ruptured this segment of the fault in 1857, 1881, 1901, 1922, 1934 and 1966. Many studies have attempted to quantitatively estimate the occurrence time (or conditional probability) of the next earthquake in this series, including Lindh (1983), Sykes and Nishenko (1984), Bakun and Lindh (1985), Davis et al. (1989), the 1988 and 1990 Working Groups on California Earthquake Probabilities, and Jackson and Kagan 1998). Because each estimate is fundamentally tied to these same dates, with the exception of Jackson and Kagan who *a priori* reject the recurrence hypothesis, these studies differ in their choice of statistical model and treatment of the open interval.

The annual probability of the Parkfield earthquake as estimated with models advocated by most of these authors are compared in Figure 8, together with the results of this investigation. We computed a BPT model without an open interval to simulate results that would have been obtained in the early- to mid-1980's. Models with the current open interval were also computed with both α free and set to the generic value. The range of annual probabilities from all the models is noteworthy, and serves to underscore the critical role the underlying hypothesis plays in the estimate. Although all probability estimates (except for Jackson and Kagan) give similar estimates up to the mean time (~1988), they rapidly diverge at later times. If the generic BPT model had been used in the mid-1980's, that model would have given current probability of 1:10 for the Parkfield earthquake occurring this year. We believe that this un-updated model still provides a robust estimate of the hazard at Parkfield.

CONCLUSIONS

We have introduced a new failure time distribution for recurrent earthquake sequences, the Brownian passage time model. This model is based upon a simple physical model of the earthquake cycle, the Brownian relaxation oscillator, and is parameterized by the mean period between earthquakes, μ, and the aperiodicity about the mean, α. Both of these quantities can be estimated from data using adjusted maximum likelihood procedures.

Analysis of 37 recurrent earthquake sequences suggests a provisional value of α=0.5. This represents substantially greater aperiodicity than the value for the coefficient of variation of 0.21 obtained by Nishenko and Buland (1987) in their analysis of another global data set with the Lognormal model.

The paucity of reliably dated or otherwise constrained sequences of recurrent earthquakes, particularly those with at least 4 closed intervals, places severe limitations on the conclusions that may be drawn about the earthquake process from these data, as the measurement of α for individual sequences is dominated by small sample size effects. We clearly must develop longer sequences of events if we are to begin to understand both the scale and origin of the underlying "noise" processes that make earthquakes aperiodic.

REFERENCES

Adams, J., Paleoseismicity of the Cascadia subduction zone: evidence from turbidites off the Oregon-Washington margin: Tectonics, v. 9, p. 569-583, 1990.

Atwater, B. F., and Hemphill-Haley, E., Recurrence intervals for great earthquakes of the past 3,500 years at northeastern Willapa Bay, Washington: U. S. Geological Survey Professional Paper 1576, 1997. Bakun, W.H., and Lindh, A.G., The Parkfield, California earthquake prediction experiment: Science, v. 229, p. 619-624, 1985.

Biasi, G., and Weldon, R.J., Paleoseismic date refinement and implications for seismic hazard estimation: in Dating and Earthquakes: Review of Quaternary Geochronology and Its Application to Paleoseismology, Sowers, J.M., Noller, J.S., and Lettis, W.R., Eds., U.S. Nuclear Regulatory Commission, NURGE-5562, p. 3-61 - 3-66, 1998.

Davis, P.M., Jackson, D.D., and Kagan, Y.Y., The longer it has been since the last earthquake, the longer expected time till the next?: Bulletin of the Seismological Society of America, v. 79, p. 1439-1456, 1989.

Ellsworth, W. L., Characteristic earthquakes and long-term earthquake forecasts: implications of central California seismicity: in Urban Disaster Mitigation: the Role of Science and Technology, Cheng, F.Y., and Sheu, M.S., Eds., Elsevier Science Ltd., p. 1-14, 1995.

Gilbert, G. K., A theory of the earthquakes of the Great Basin, with a practical application: American Journal of Science, ser. 3, v. 27, no. 157, p. 49-53, 1884.

Hagiwara, Y., Probability of earthquake occurrence as obtained from a Weibull distribution analysis of crustal strain: Tectonophysics, v. 23, p. 323-318, 1974.

Ishibashi, K., and K. Satake, Problems on forecasting great earthquakes in the subduction zones around Japan by means of paleoseismology: J. Seismol. Soc. Japan, v. 50, Suppl., p. 1-21, 1998.

Jackson, D.D., and Kagan, Y.Y., Parkfield earthquake: not likely this year (abs.): Seismological Research Letters, v. 69, p. 151, 1998.

Lindh, A.G., Preliminary assessment of long-term probabilities for large earthquakes along selected segments of the San Andreas fault system in California: U.S. Geological Survey Open-File Report 83-63, 15 p., 1983

Mann, N.R., Schafer, R.E., and Singpurwalla, N.D., Methods for Statistical Analysis of Reliability and Life Data: John Wiley and Sons, New York, 564 p., 1974.

Matthews, M. V., A stochastic model model for recurrent earthquakes: Journal of Geophysical Research, 18 p., submitted 1998.

McCalpin, J.P., and Nishenko, S.P., Holocene paleoseismicity, temporal clustering, and probabilities of future large (M>7) earthquakes on the Wasatch fault zone, Utah: Journal of Geophysical Research, v. 101, p. 6233-6253, 1996.

McCalpin, J.P., and Slemmons, D.B., Statistics of paleoseismic data: Final Technical Report, Contract 1434-HQ-96-GR-02752, National Earthquake Hazards Reduction Program, U.S. Geological Survey, 62 p., 1998.

Nadeau, R. M., and Johnson, L. R., Seismological studies at Parkfield IV: moment release rates and estimates of source parameters of small repeating earthquakes: Bulletin of the Seismological Society of America, v. 88, p. 790-814., 1998.

Nishenko, S. P., and Buland, R., A generic recurrence interval distribution for earthquake forecasting: Bulletin of the Seismological Society of America, v. 77, p. 1382-1399, 1987.

Ogata, Y., Estimating the hazard of rupture using uncertain occurrence times of paleoearthquakes: Research Memorandum 702, The Institute of Statistical Mathematics, Tokyo, 32 p., 1998.

Pantosti, D., Schwartz, D.P., and Valensise, G., Paleoseismology along the 1980 surface rupture of the Irpinia fault: implications for earthquake recurrence in the southern Appenines, Italy: Journal of Geophysical Research, v. 98, p. 6561-6577, 1993.

Plafker, G., Geologic studies related to earthquake potential and recurrence in the "Yakataga Seismic Gap": U.S. Geological Survey Open-File Report 86-92, p. 136-143, 1986.

Plafker, G., and Rubin, M., Paleoseismic evidence for "Yo-Yo" tectonics above the eastern Aleutian subduction zone: coseismic uplift alternating with even larger interseismic submergence: Proceedings of the Workshop on Paleoseismology, U.S. Geological Survey Open-File Report 94-568, p. 155-157, 1994.

Reid, H. F., The mechanics of the earthquake, v. 2 of The California earthquake of April 18, 1906: Report of the State Earthquake Investigation Commission: Carnegie Institution of Washington Publication 87, 1910.

Rikitake, T., Probability of an earthquake occurrence as estimated from crustal strain: Tectonophysics, v. 23, p. 299-312, 1974.

Seitz, G., Weldon, R. II, and Biasi, G.P., The Pitman Canyon paleoseismicity record: a re-evaluation of the southern San Andreas fault segmentation: Journal of Geodynamics, v. 24, p. 129-138, 1997.

Sieh, K., Stuiver, M., and Brillinger, D., A more precise chronology of earthquakes produced by the San Andreas fault in southern California: Journal of Geophysical Research, v. 94, p. 603-623, 1989.

Shimazaki, K., and Nakata T., Time-predictable recurrence model for large earthquakes: Geophysical Research Letters, v. 7, p. 279-282, 1980.

Sykes, L.R., and Nishenko, S.P., Probabilities of occurrence of large plate rupturing earthquakes for the San Andreas, San Jacinto, and Imperial faults, California: Journal of Geophysical Research, v. 89, p. 5905-5927, 1984.

The Tanna Fault Trenching Research Group, Trenching study for Tanna fault, Izu, at Myoga, Shizuoka Prefecture, Japan: Bulletin of the Earthquake Research Institute, University of Tokyo, v. 58, p. 797-830, 1983.

Utsu, T., Large earthquakes near Hokkaido and the expectancy of the occurrence of a large earthquake of Nemuro: Report of the Coordinating Committee for Earthquake Prediction, v. 7, p. 7-13, 1972a.

Utsu, T., Aftershocks and earthquake statistics (IV): Journal of the Faculty of Science, Hokkaido University Series VII (Geophysics), v. 4, p. 1-42, 1972b.

Utsu, T., Estimation of parameters for recurrence models of earthquakes: Bulletin of the Earthquake Research Institute, University of Tokyo, v. 59, p. 53-66, 1984.

Van Dissen, R.J., and Berrymann, K.R., Surface rupture earthquakes over the last ~1000 years in the Wellington region, New Zealand, and implications for ground shaking hazard: Journal of Geophysical Research, v. 101, p. 5999-6019, 1996.

Working Group on Assessment Methods of Long-Term Earthquake Probability, Report on the Assessment Methods of Long Term Earthquake Probability and Some Applied Examples: The Committee on Earthquake Research, The Headquarters for Earthquake Research Promotion, The Prime Minister's Office, Japan, 1998.

Working Group on California Earthquake Probabilities, Probabilities of large earthquakes occurring in California on the San Andreas fault: U.S. Geological Survey Open-File Report 88-398, 62 p., 1988.

Working Group on California Earthquake Probabilities, Probabilities of large earthquakes in the San Francisco Bay region, California: U.S. Geological Circular 1053, 51 p., 1990.

FIGURE CAPTIONS

Figure 1. Three examples of a Brownian relaxation oscillator, showing the state variable, $Y(t)$, in the case of relatively weak (top), moderate (middle) and strong (bottom) aperiodicity.

Figure 2. Probability density of the Brownian passage time distribution for mean period = 1, and a range of aperiodicity values.

Figure 3. Comparison of probability density (left) and conditional probability for six different probability distributions. All distributions have a mean value of 1, and a coefficient of variation (α for the BPT distribution). Note the similarity of the density functions for all but the Poisson (Exponential) distribution, and the divergence in behavior of the conditional probability the same models for times greater than the mean time.

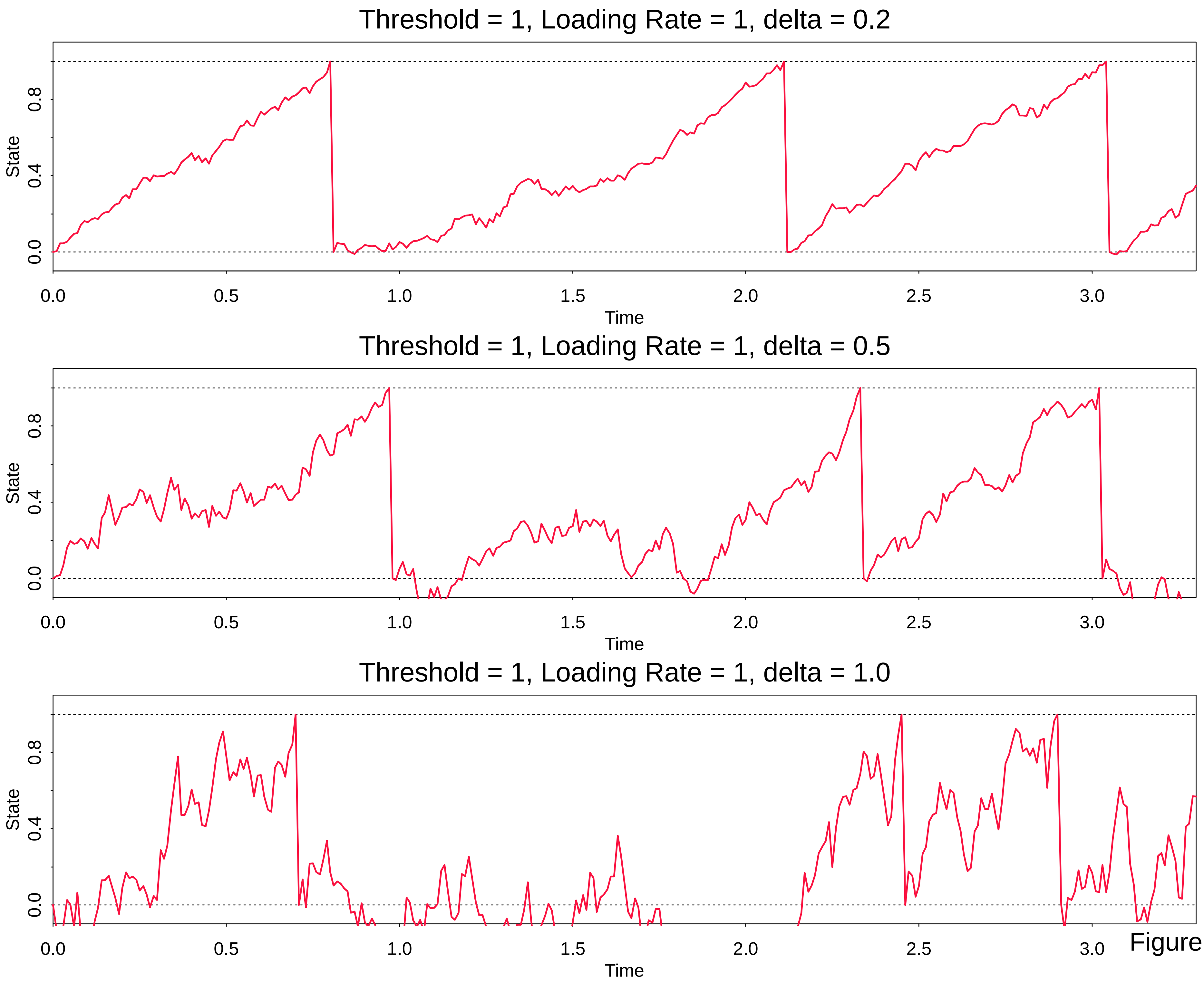
Figure 4. Bootstrap analysis of uncertainty in the adjusted maximum likelihood estimation of μ and α for the BPT distribution as a function of the observation period. Top figures show several realizations of sequences when the observation period is 3.5 (left) and 10 (right) mean periods. Bottom figures show the median and confidence intervals on the estimated mean period (left) and aperiodicity (right).

Figure 5. Illustration of Monte Carlo estimation procedure applied to paleoseismic earthquake sequences. Probability density distributions for dates of 6 most recent earthquakes on the Brigham City segment of the Wasatch fault, Utah (McCalpin and Nishenko, 1996) and 5 earthquake series drawn at random from those distributions (top). Histograms (bottom) of the mean recurrence time, $\hat{\mu}$, and aperiodicity, $\hat{\alpha}$ determined from 1000 realizations.

Figure 6. Properties of the estimated mean, $\hat{\mu}$, and aperiodicity, $\hat{\alpha}$ for the 37 earthquake series in Table 1. Tectonic style of faulting is indicated in a.) and b.) by "S" for strike slip, "N" for normal, and "T" for thrust cases. Histograms of the estimated aperiodicty in c.) compares favorably with d.) bootstrap estimate drawn from the BPT distribution with $\alpha = 0.5$.

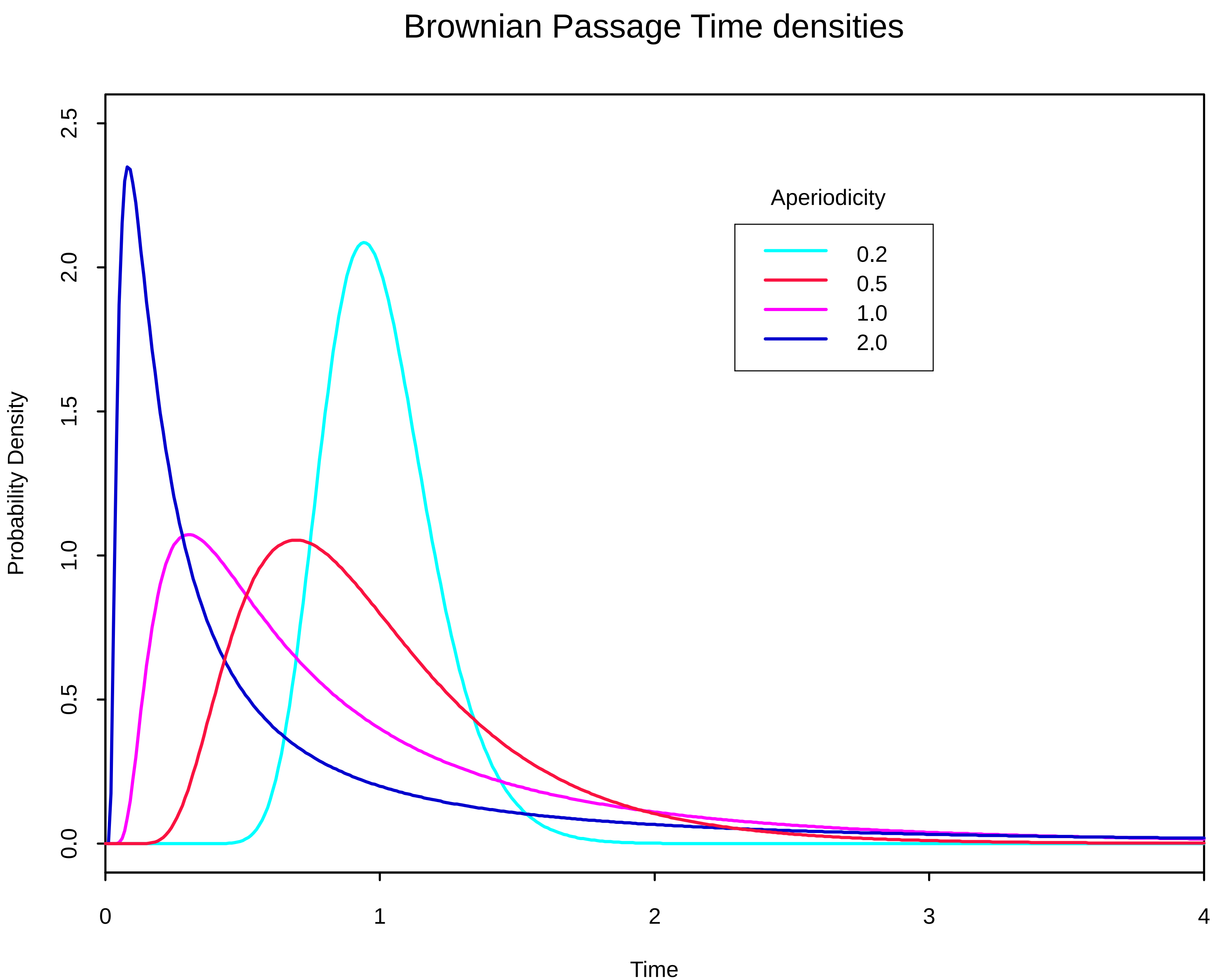
Figure 7. Properties of the Brownian passage time distribution for a mean period of μ=1 and aperiodicity of α=0.5.

Figure 8. Annual probability of the next characteristic M 6 earthquake at Parkfield, California as estimated by a number of studies using probability models as indicated. Probability estimates from the BPT were made with and without the open interval, and using the suggested generic aperiodicity of $\alpha = 0.5$.



Threshold = 1, Loading Rate = 1, delta = 0.2
State
0.0
0.4
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time
Threshold = 1, Loading Rate = 1, delta = 0.5
State
0.0
0.4
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time
Threshold = 1, Loading Rate = 1, delta = 1.0
State
0.0
0.4
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time

Figure 1



Brownian Passage Time densities
Aperiodicity
0.2
0.5
1.0
2.0
Probability Density
0.0
0.5
1.0
1.5
2.0
2.5
0
1
2
3
4
Time

Figure 2

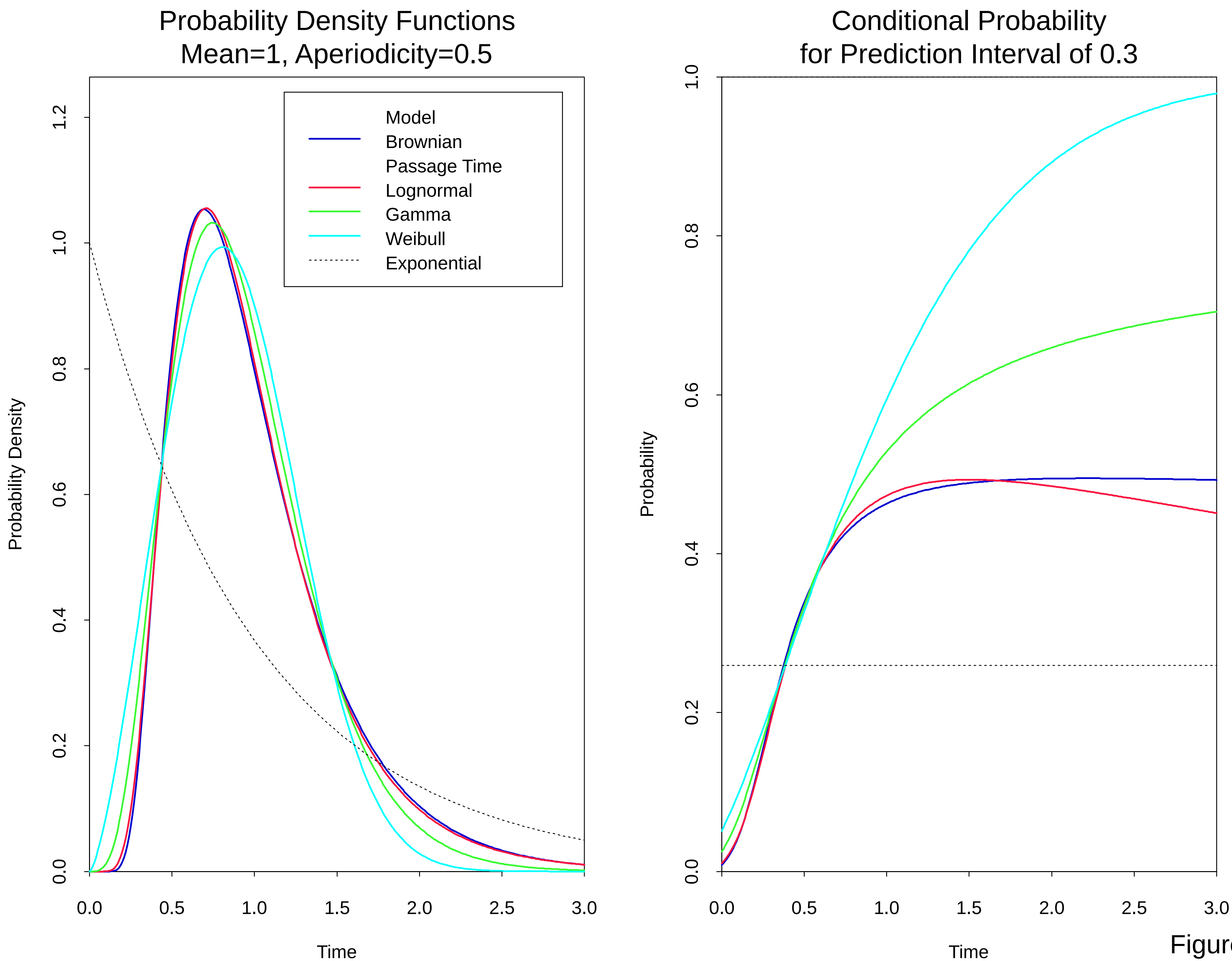

Probability Density Functions
Mean=1, Aperiodicity=0.5
Probability Density
Time
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Model
Brownian
Passage Time
Lognormal
Gamma
Weibull
Exponential
Conditional Probability
for Prediction Interval of 0.3
Probability
Time
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Figure 3

Realizations of a Brownian Passage Time Point Process
with Mean Period = 1 and Aperiodicity = 0.5

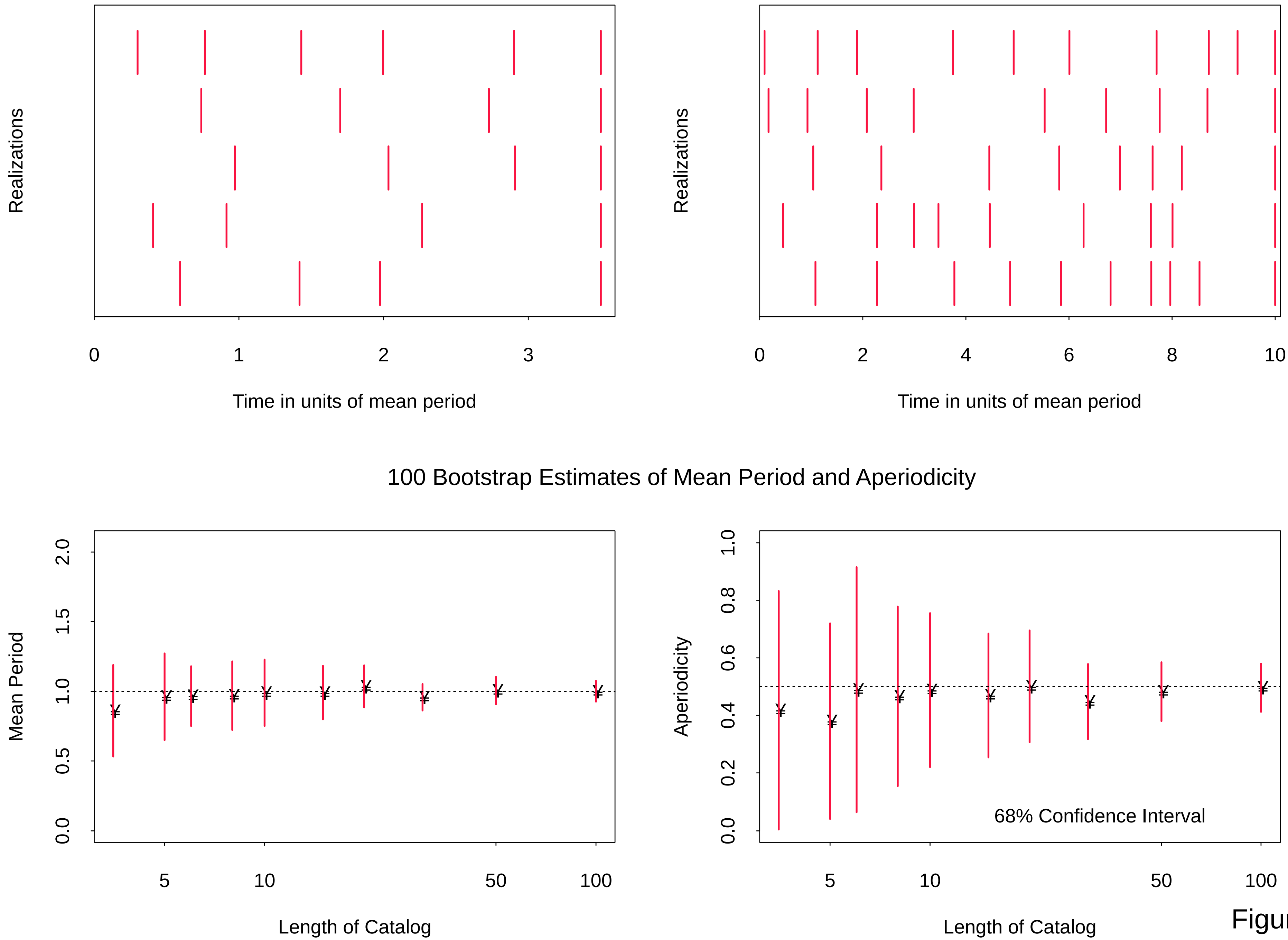


Figure 4

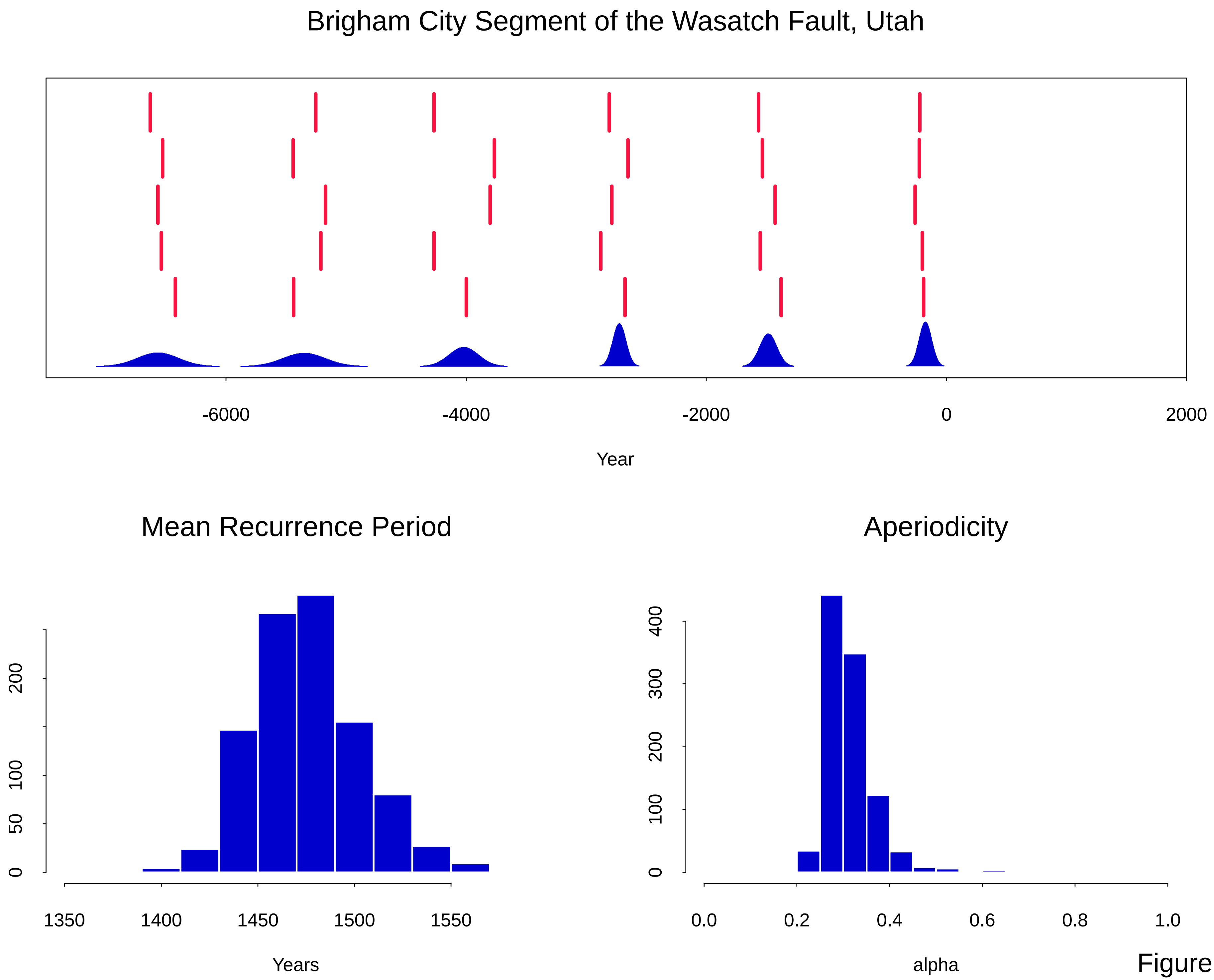

Brigham City Segment of the Wasatch Fault, Utah
-6000
-4000
-2000
0
2000
Year
Mean Recurrence Period
0
50
100
200
1350
1400
1450
1500
1550
Years
Aperiodicity
0
100
200
300
400
0.0
0.2
0.4
0.6
0.8
1.0
alpha

Figure 5

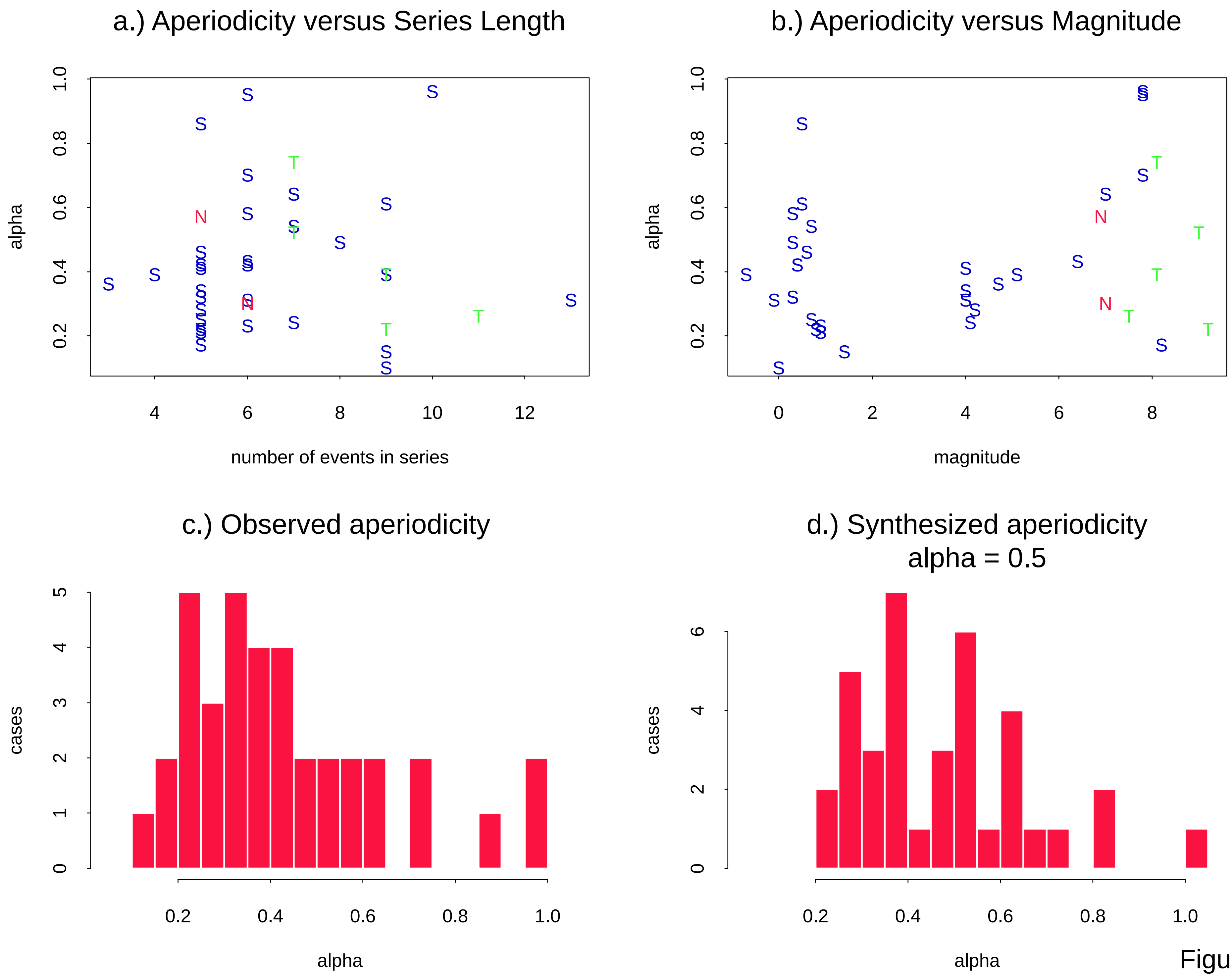
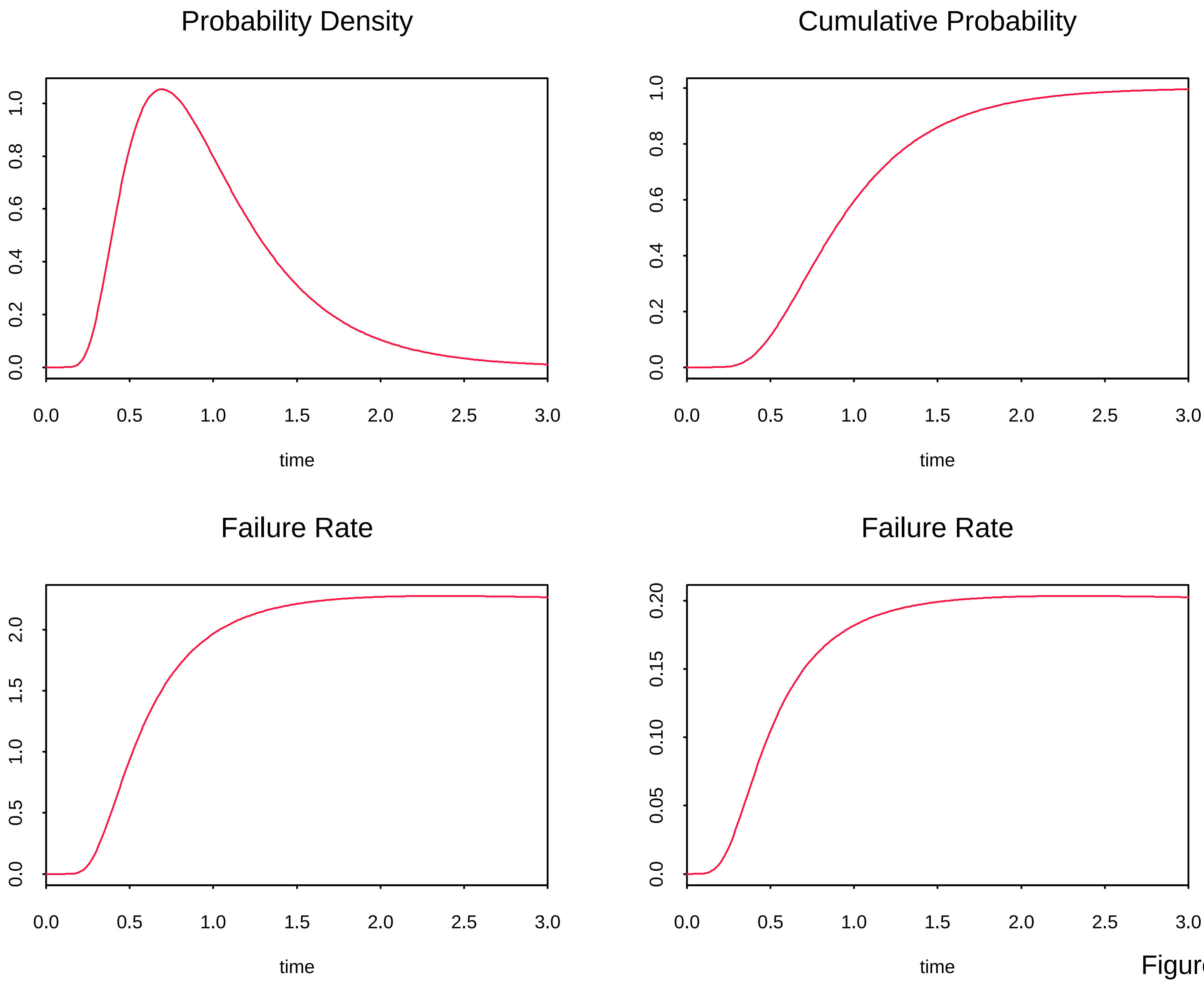
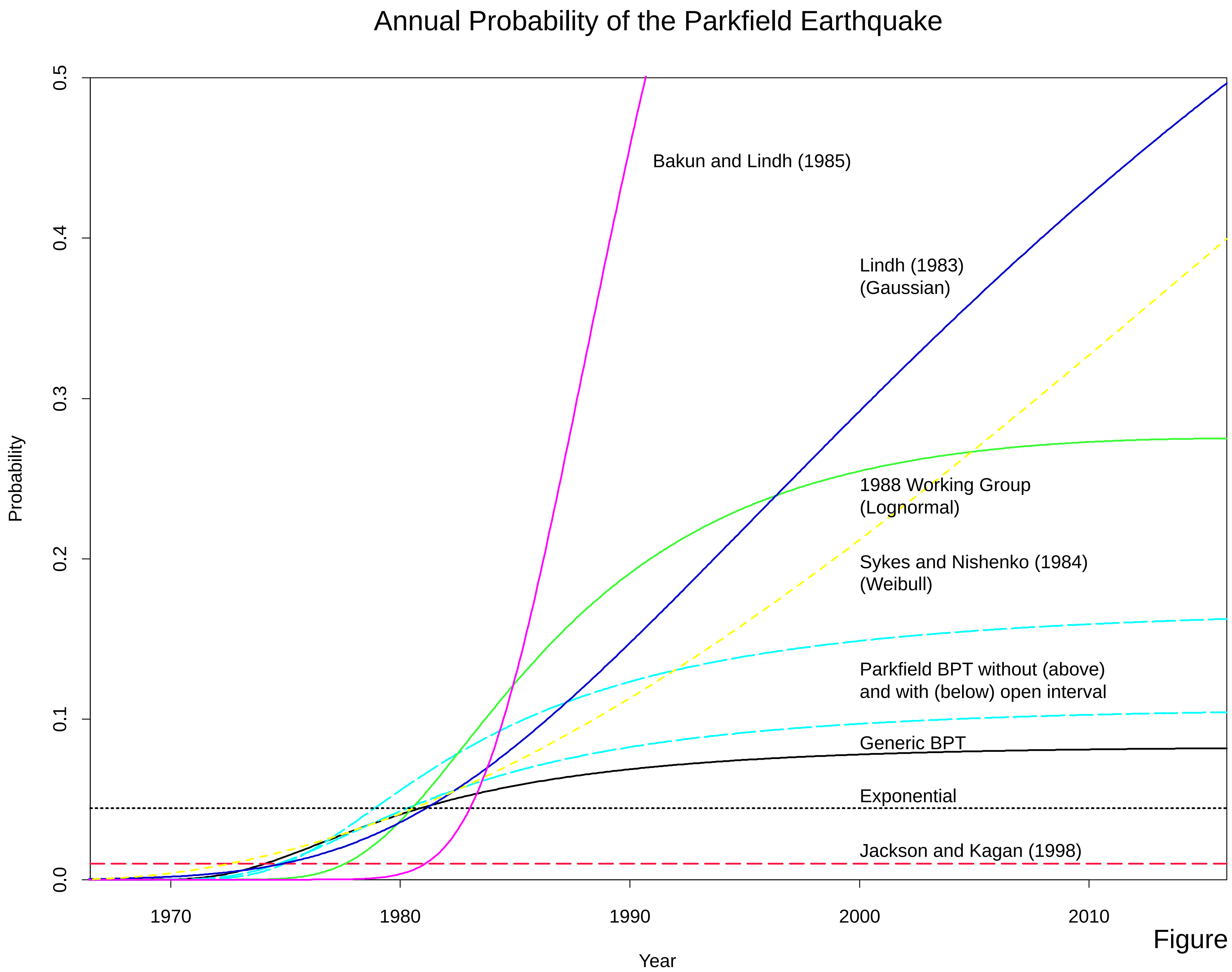

a.) Aperiodicity versus Series Length
alpha
0.2
0.4
0.6
0.8
1.0
4
6
8
10
12
number of events in series
b.) Aperiodicity versus Magnitude
alpha
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
magnitude
c.) Observed aperiodicity
cases
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1.0
alpha
d.) Synthesized aperiodicity
alpha = 0.5
cases
0
2
4
6
0.2
0.4
0.6
0.8
1.0
alpha

Figure 6



Probability Density
Cumulative Probability
Failure Rate
Failure Rate
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5 2.0
0.0 0.05 0.10 0.15 0.20
0.0 0.5 1.0 1.5 2.0 2.5 3.0
time

Figure 7



Annual Probability of the Parkfield Earthquake
Probability
0.0
0.1
0.2
0.3
0.4
0.5
1970
1980
1990
2000
2010
Year
Bakun and Lindh (1985)
Lindh (1983)
(Gaussian)
1988 Working Group
(Lognormal)
Sykes and Nishenko (1984)
(Weibull)
Parkfield BPT without (above)
and with (below) open interval
Generic BPT
Exponential
Jackson and Kagan (1998)

Figure 8

Table 1. Properties of candidate interval distributions.. Columns are: 1. Name of parametric family (and indication, as necessary, of parameter range restriction for following columns). 2. Parametrized probability density function. 3. Shape of failure rate function: $\uparrow$= increasing, $\downarrow$= decreasing, $\rightarrow$= constant; combination of symbols indicates changing behavior with time, *e.g.*, the BPT failure increases for some time then decreases and flattens out to a constant asymptotic level. 4. Failure rate at $t = 0$. 5. Ratio of mean and quasi-stationary mean recurrence interval.

Name		Probability Density	Shape	$h(0)$	μ/m_∞
Brownian Passage Time		$\left(\frac{\mu}{2\pi\alpha^2 t^3}\right)^{1/2} \exp\left(-\frac{(t-\mu)^2}{2\alpha^2\mu t}\right)$	$\uparrow\downarrow\rightarrow$	0	$2\alpha^2$
Exponential		νe^{-nut}	$\rightarrow$	ν^{-1}	1
	$\theta < 1$		$\downarrow\rightarrow$	∞	
Gamma		$\frac{\nu^\theta t^{\theta-1} e^{-\nu t}}{\Gamma(\theta)}$			θ^{-1}
	$\theta > 1$		$\uparrow\rightarrow$	0	
	$\theta < 1$		$\downarrow$	∞	0
Weibull		$\theta\nu^\theta t^{\theta-1} \exp(-\nu^\theta t^\theta)$			
	$\theta > 1$		$\uparrow$	0	∞
Lognormal		$\left(\sqrt{2\pi}\sigma t\right)^{-1} \exp\left(-\frac{(\log t-\mu)^2}{2\sigma^2}\right)$	$\uparrow\downarrow$	0	0

Table 2. Recurrent Earthquake Sequences and Their Estimated Parameters for the Brownian Passage Time Model							
Location	M	Last	N	μ	α	$\mu_{0.5}$	Reference
Copper River Delta, USA	9.2	1964	9	683	0.23	753	Plafker and Rubin, 1994.
Willipa Bay, USA	9.0	1700	7	526	0.53	530	Atwater and Hemphill-Haley, 1997.
Wairarapa fault, N.Z.	8.2	1855	5	1551	0.18	1355	Van Dissen and Berryman, 1996.
Nankaido, Japan	8.1	1946	9	158	0.40	166	Ishibashi and Satake 1998.
Tonankai, Japan	8.1	1944	7	210	0.75	192	Ishibashi and Satake 1998.
Pallett Creek, USA	7.8	1857	10	146	0.97	115	Sieh et al., 1989.
Wrightwood, USA	7.8	1857	6	150	0.71	138	Biasi and Weldon, 1998.
Pitman Canyon, USA	7.8	1812	6	180	0.96	144	Seitz, Weldon and Biasi, 1997.
Miyagi-Oki, Japan	7.5	1978	11	36	0.27	40	Utsu, 1984.
Brigham City, USA	7	-130	6	1476	0.31	1645	McCalpin and Nishenko, 1996.
Tanna fault, Japan	7.0	1930	7	972	0.65	866	Tanna Fault Trenching Research Group, 1983.
Irpinia fault, Italy	6.9	1980	5	2058	0.58	2042	Pantosti et al., 1993.
Parkfield, USA	6.4	1966	6	25.0	0.44	26.5	Bakun and Lindh, 1995.
Stone Canyon (San Andreas fault)							
- Set 2	5.0	1995	4	14.6	0.40	18.6	Ellsworth, 1995.
- Set 3	4.7	1986	3	20.3	0.37	24.4	"
- Set 1	4.2	1995	5	14.7	0.29	16.2	"
- Set 10	4.1	1995	7	10.2	0.25	11.3	"
- Set 5	4.0	1990	6	10.6	0.32	12.4	"
- Set 8	4.0	1990	5	12.3	0.35	14.7	"
- Set 9	4.0	1990	5	13.0	0.42	14.7	"
Parkfield (San Andreas fault)							
- PK1	1.4	1994	9	1.12	0.16	1.25	Ellsworth, 1995.
- S46	0.9	1993	5	1.3	0.22	1.5	Nadeau and Johnson, 1998.
- S44	0.9	1995	6	1.7	0.24	1.9	"
- S40	0.8	1995	5	1.6	0.23	1.8	"
- S39	0.7	1993	7	0.99	0.55	0.99	"
- S35	0.7	1994	5	1.8	0.26	2.0	"
- S34	0.6	1993	5	1.6	0.47	1.7	"
- S33	0.5	1992	5	1.4	0.87	1.2	"
- S27	0.5	1992	9	0.54	0.62	0.52	"
- S25	0.4	1996	6	1.6	0.43	1.7	"
- S22	0.4	1992	5	0.83	0.43	0.87	"
- S21	0.3	1995	8	1.1	0.50	1.2	"
- S20	0.3	1995	6	1.5	0.59	1.5	"
- S18	0.3	1992	5	1.3	0.33	1.4	"
- S07	0.0	1992	9	0.64	0.11	0.72	"
- S05	-0.1	1995	13	0.73	0.32	0.78	"
- S01	-0.7	1995	9	0.95	0.40	1.0	"

Notes to Table 2. Location - geographic placename of paleoseismic site, name of fault, or fault segment. M - magnitude of typical event. Last - calendar year of last event; negative dates for B.C. N - number of events in earthquake series. μ - estimated mean recurrence interval in years. α - estimated aperiodicity of earthquake series. $\mu_{0.5}$ - estimated mean recurrence interval in years for $\alpha = 0.5$. Reference - primary data source for this earthquake series.