



Methods To Achieve Accurate Projection of Regional and Global Raster Databases

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OBJECTIVES/JUSTIFICATION

Need

Modeling regional and global activities of climatic and human-induced change requires accurate geographic data from which we can develop mathematical and statistical tabulations of attributes and properties of the environment. Many of these models depend on data formatted as raster cells or matrices of pixel values. Recently, it has been demonstrated that regional and global raster datasets are subject to significant error from mathematical projection and that these errors are of such magnitude that model results may be jeopardized (Steinwand, *et al.*, 1995; Yang, *et al.*, 1996; Usery and Seong, 2001; Seong and Usery, 2001). There is a need to develop methods of projection that maintain the accuracy of these datasets to support regional and global analyses and modeling.

Objectives

Although recent research indicates that projection problems exist for raster databases at global and regional scales, there is little theoretical background for handling the relationships between the distortion that is due to projection methods and the raster representation of the distorted features. Also, little is theorized about reprojecting raster datasets. Currently, it is difficult to select the best projection in relation to raster pixel sizes, latitude, the spatial pattern of categories, and the number of categories. Furthermore, it is even more challenging to understand the latent errors in raster databases already-projected and to fix them without further information loss. This research aims at building a decision support system (DSS) for selecting an optimum projection considering various factors, such as pixel sizes, areal extent, number of categories, spatial pattern of categories, resampling methods, and error correction methods. Specifically, this project will investigate the following three goals theoretically and empirically and will use an existing empirical knowledge base with the results to develop an expert system for the map projection of raster data for regional and global database modeling. The three theoretical goals are as follows:

- 1) Developing a dynamic projection that adjusts projection formulas for latitude on the basis of raster cell size to maintain equal-sized cells.
- 2) Investigating the relationships between the raster representation and the distortion of features, number of categories, and spatial pattern.
- 3) Developing an error correction and resampling procedure based on error analysis of raster projection.

Hypotheses

Regarding the first goal, we hypothesize that regional and global raster data can be accurately projected with appropriate equations that account for raster cell size and latitudinal position. For

the second goal, we hypothesize that scale factors explain the impact of distortion on raster representation and that more categories and more complex spatial patterns cause more errors. Finally, we hypothesize for the third goal that error correction and resampling methods can be used for optimizing the projection accuracy of regional and global raster datasets. This proposed research potentially could affect all U.S. Geological Survey (USGS) programs involving the use of large regional and global raster data, such as Global Change Research and Place-Based Studies.

BACKGROUND

With the advent of digital computers and their application to map projection problems beginning in the early 1960s, one might think that all projection problems had been solved. It is true that geographic data for small areas at high resolution and large scale tend to have small projection effects compared with other sources of data error and inaccuracy. Renewed difficulties occurred in the late 1970s and 1980s with the introduction of a datum change in the United States from the North American Datum of 1927 (NAD 27) to the geocentric-based North American Datum of 1983 (NAD 83) (ACSM, 1983). In recent years this datum shift has plagued users of geographic information systems (GIS), and even with the current status of complete ellipsoid, datum, and projection conversions available in most commercial GIS software packages, the knowledge to use such conversions effectively is still lacking in the GIS user community (Welch and Homsey, 1997). Often approximations to projection equations are used, resulting in error, and comparing the results from various projections is difficult (Snyder, 1985; Tobler, 1986a; 1986b).

We are now entering a phase of GIS and digital cartographic use in which large high-resolution datasets are available for global and regional modeling applications. With these large areas and high resolution, data problems due to map projections again have become significant. Although excellent reference works exist on the theory of map projections and the distortion resulting from map projection (such as Pearson II, 1990; Maling, 1992; Bugayevskiy and Snyder, 1995; Yang, *et al.*, 2000), little research has been conducted and even less applied to the projection of regional and global raster GIS databases. In particular, raster datasets suffer accuracy problems directly attributable to projection transformation (Snyder, 1983; 1987; Steinwand, 1994; Steinwand *et al.*, 1995).

Equal-area projections are generally better for raster datasets since preservation of the area characteristic yields pixel areas that are more correct and equivalent. The interrupted Goode Homolosine projection has been recommended for global-raster GIS databases, particularly for products generated from the National Oceanic and Atmospheric Administration (NOAA) Advanced Very High Resolution Radiometer (AVHRR) data (Steinwand, 1994; Steinwand *et al.*, 1995). However, even the Goode Homolosine projection results in replication of some pixel values and distortion of areas (Yang, *et al.*, 1996). If a global GIS database is built using the vector data structure, an equal-area projection will preserve most of original information, such as the size of area, but research indicates that even projections designed to preserve areas, that is, equivalent or equal-area projections, may distort original information when the database is built using the raster GIS data structure (Steinwand *et al.*, 1995; Seong, 1999; Usery and Seong, 2001).

As Steinwand *et al.* (1995) indicate, the loss and distortion of original information occur during image warping as well as during reprojection of raster data. In addition, the spatial resolution of a raster pixel can cause an inaccuracy along with the projection selected. Assuming

a projection with minimum area distortion and allowing maximum angular distortion, the projection will be appropriate only when the raster pixel size is small enough not to be significantly affected by the angular distortion (Nyerges and Jankowski, 1989). As the pixel size is increased, the information for an area is affected significantly owing to the distorted shape.

Seong (1999) and Usery and Seong (2001) investigated specific effects of raster cell size and latitudinal position on accuracy of thematic attributes. In both investigations, they examined the Goode Homolosine projection as well as three other equal-area projections, the Lambert Azimuthal Equal-Area, the Mollweide Pseudocylindrical Equal-Area, and the Lambert Cylindrical Equal-Area. The Robinson projection was included to determine the results with a nonequivalent projection. The results indicate that all of the equal-area projections yield adequate accuracies with pixels of 8 km or smaller. With 8- to 16-km pixels, the Mollweide yields greater accuracy, and with 50-km pixels, the Lambert Cylindrical Equal-Area projection gives better results. These results are averages for all latitudes and were verified mathematically by Seong (1999) and empirically by Usery and Seong (2001). Seong (1999) also examined the effects of latitude on the accuracy of projecting raster data and found that the Mollweide retained accuracy better at all latitudes although with various discrepancies.

Results of Previous Work

Previous results indicate significant variation in tabulated areal statistics for land cover on the basis of projection, raster cell size, and latitudinal position. Table 1 from Usery and Seong (2001) illustrates the variance in areas of land cover in Asia for specific categories at 8-km pixel sizes for several projections. The percentages in the table are computed from the values for the same data at 1-km pixel size on the Lambert Azimuthal Equal-Area projection. These results, with differences on the equal-area projections of more than 8.0 percent for a given category (see mixed tundra in the Table) are typical of the variances in tabulating areas for land cover categories found among projections designed to preserve area. Similar tables for 1, 4, 8, 16, 25, and 50 km have been generated for a variety of projections and indicate the extent of this problem, which increases with pixel size.

In addition to the mathematical results of Seong (1999) and the empirical results illustrated in Table 1 and those documented in Usery and Seong (2001), Seong and Usery (2001) have also developed a theoretical error model. Based on the concept of scale factor error in both horizontal and vertical directions, the model accounts for error resulting from pixel size distortion, pixel category replication, and skew of pixels. The model has been verified with the Sinusoidal, Mollweide, and Lambert Cylindrical Equal-Area projections of raster data and accounts for greater than 99 percent of the theoretical error.

The research discussed above shows problems of raster database projection at global and regional scales. However, no solution is provided for the problems. This proposed research will extensively investigate the problem theoretically and empirically considering various factors, and will suggest the best projection, depending on areal extent, pixel size, resampling, and category characteristics.

Table 1
Asia Land Cover by Projection at 8-Km Pixels
Percentages of Lambert Azimuthal Equal-Area with 1-Km Pixels
Projection*

Land Cover Category	Lam	EqCyl	Mw	Rob
1 Urban & Built-Up Land	101.68	101.21	103.12	96.75
2 Dryland Cropland & Pasture	100.11	100.21	100.55	94.49
3 Irrigated Cropland & Pasture	100.31	100.17	99.64	88.72
4 Cropland/Grassland Mosaic	100.04	100.38	99.77	95.16
5 Cropland/Woodland Mosaic	99.87	99.40	100.40	95.71
6 Grassland	99.99	100.34	99.97	99.48
7 Shrubland	99.77	99.86	99.94	95.94
8 Mixed Shrubland/Grassland	100.02	99.66	100.61	92.40
9 Savanna	99.02	99.74	100.29	98.09
10 Deciduous Broadleaf Forest	99.48	99.74	100.12	94.81
11 Deciduous Needleleaf Forest	98.63	98.96	98.94	111.06
12 Evergreen Broadleaf Forest	100.03	100.23	100.69	84.92
13 Evergreen Needleleaf Forest	98.71	99.23	99.20	93.56
14 Mixed Forest	100.23	99.99	100.04	102.78
15 Herbaceous Wetland	96.54	97.07	101.27	103.37
16 Wooded Wetland	104.95	94.95	99.04	107.67
17 Barren or Sparsely Vegetated	100.22	99.88	99.99	96.32
18 Herbaceous Tundra	99.47	95.30	100.70	112.73
19 Wooded Tundra	98.98	100.68	99.37	100.89
20 Mixed Tundra	91.81	108.50	92.79	106.54
21 Snow or Ice	98.13	100.22	98.23	91.37
Total Area	99.93	100.02	100.03	96.18

*Lam = Lambert Azimuthal Equal-Area, EqCyl = Lambert Cylindrical Equal-Area, Mw = Mollweide, Rob = Robinson

PROCEDURES/METHODS

Building on the empirical knowledge base and results with commercial vendor projection software from previous work, we will develop a projection selection system. This DSS will help users select the best projection for raster data on the basis of data type, raster cell size, and specific region or global area. Additionally, we will use three specific approaches to develop methods to correct the errors resulting from the projection of raster datasets. One or more of these approaches may result in a complete correction system that can be added to the DSS. The approaches correspond to the three hypotheses listed in the objectives above; that is, (1) a dynamic projection that adjusts projection formulas for latitude on the basis of raster cell size, (2) an error analysis between the raster representation and the distortion of features, number of categories, and spatial pattern of categories, and (3) a resampling approach and an error correction procedure that adjust the resampling method to account for the error which occurs in a

given cell location. Each of these approaches relies on previous projections research, such as the empirical analysis to determine which projections are best at given latitudes and cell sizes (Usery and Seong, 2001), error analysis of raster data projection (Seong and Usery, 2001), and the development of the Goode projection for raster data (Steinwand, 1994). The procedures for each approach are detailed below.

Projection Selection System

Since certain projections work better for specific cell sizes (for example, of several equal-area projections, the Mollweide has minimum error for raster datasets with cell sizes between 8 and 25 km (Usery and Seong, 2001)) and specific latitudes, a DSS tool can be developed to help users select the appropriate projection that is based on these characteristics. Such a tool can be used with commercial off-the-shelf (COTS) software to guide users toward selecting a projection that will minimize the error for a regional or global database. The system can also provide the user with the exact extent of error in specific locations on the globe. This tool can be developed primarily from the empirical knowledge base developed previously and briefly described above. The results of this project are now being formulated for publication and include analysis of a range of databases, such as global vegetation, climate, land cover, and elevation at a range of resolutions from 1-kilometer to 1-degree raster cells. The project evaluated empirical errors resulting from 10 different projections. This empirical base can be used to establish the essential rules of the DSS to help users select the best projection available in a commercial software package for a given dataset.

Dynamic Projection

The concept behind dynamic projection is to use the most appropriate mathematical formulation for the projection of a raster cell the basis of the cell size and the latitude of the cell. The development of a guidance tool or projection decision support system only allows users to select the best of currently available projection methods, all of which result in significant error. To obtain a solution without error, or at least a solution in which the error is significantly reduced from current projection methods for raster data, requires new developments. In a dynamic projection approach, a mathematical formulation will be developed on the basis of empirical knowledge and theoretical considerations, such as Tissot's scale factor analysis of distortion effects. For example, a formulation similar to the Mollweide (Snyder, 1987, p. 251) may be used for cells in the 8- to 16-km range since the Mollweide projection produces minimal distortion at these resolutions. The cylindrical equal-area (Snyder, 1987, p. 77) works better for 50-km cells, thus the dynamic projection may use formulas similar to those of the equal-area cylindrical for 50-km cell size raster datasets. Since the best projection changes with latitude, adjustments of the formulation must be made on a dynamic basis to account for latitude changes. The key to this dynamic projection is to mesh the formulas that achieve high projection accuracies for specific cell sizes at given latitudes to form a continuous image. The Goode Homolosine is a simple example of the application of this concept, since it uses the Mollweide formulation for high latitudes and the sinusoidal formulation for lower latitudes (Steinwand, 1994). However, with dynamic projection, the potential to have a series of discrete transformations that do not join in a continuous image is great and must be avoided. Thus, one task of this research will be to develop a mathematical formulation that is dynamic in achieving accuracy of raster cell area but continuous as latitude changes.

Error Analysis

The error correction approach is designed to use a single projection formulation, for example, the Mollweide or the sinusoidal, applied to the data with subsequent error correction procedures to adjust the raster cell area to the true value. Error analysis indicates that there are two types of errors for which adjustment is appropriate: (1) size errors, which cause pixels to be portrayed at sizes larger or smaller than the original, and (2) categorical error, which results from lost or gained pixels (Seong and Usery, 2001). Both types of error can be modeled with a scale factor model developed from the horizontal and vertical scale factors of the projection. An example of the error analysis approach is shown in Figure 1. With the model, corrections can be applied after projection to eliminate gained pixels.

An example analysis and application follows for the Mollweide projection, which has been used frequently for world maps, especially for mapping oceans, because it represents oval areas at midlatitude regions. In the Mollweide projection, all of the meridians are ellipses except the central meridian, which is a straight line, and the 90-degree meridians, which are circles. The main characteristics of the Mollweide projection are that the parallels are carefully spaced to maintain the equivalency so that the areal scale factor equals 1.0.

1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10
21	21	22	22	23	23	24	24	25	25	26	26	27	27	28	28	29	29	30	30
41	41	42	42	43	43	44	44	45	45	46	46	47	47	48	48	49	49	50	50
61	61	62	62	63	63	64	64	65	65	66	66	67	67	68	68	69	69	70	70
81	81	82	82	83	83	84	84	85	85	86	86	87	87	88	88	89	89	90	90

- a) X scale factor = 2.0. Y scale factor = 0.5
 Representation of original pixels = 50 pixels (50%)
 Loss and gain = 100% (50% loss + 50% gain)

21	21	21	21	21	22	22	22	22	22	23	23	23	23	23	24	24	24	24	24
71	71	71	71	71	72	72	72	72	72	73	73	73	73	73	74	74	74	74	74

- b) X scale factor = 5.0. Y scale factor = 0.2
 Representation of original pixels = 20 pixels (20%)
 Loss and gain = 160% (80% loss + 80% gain)

Figure 1. Scale factor error analysis.

Because the Mollweide projection uses straight parallels, the scale factor along the parallel can be used for examining raster representation accuracy. The horizontal scale factor in the Mollweide projection is calculated as follows (Bugayevskiy and Snyder, 1995):

$$n = (2 \sqrt{2} / \pi) \cos(\alpha) \sec(\phi) \tag{1}$$

where, $2\alpha + \sin(2\alpha) = \pi \sin(\phi)$, and ϕ is the latitude. Because the $2\alpha + \sin(2\alpha) = \pi \sin(\phi)$ is a transcendental equation, the ϕ was calculated using the Newton-Raphson method that is an iterative solution with an equation which has a rapid convergence if the initial guess for α_n is given the value of ϕ (Pearson, 1990):

$$\alpha_{n+1} = \alpha_n + [\pi \sin(\phi) - 2\alpha_n - \sin(2\alpha_n)] / [2 + 2 \cos(2\alpha_n)] \tag{2}$$

The horizontal local scale factor, n , becomes 1.0 at $\pm 40^\circ 44'$. It is smaller than 1.0 at the latitudes between $\pm 40^\circ 44'$ and larger than 1.0 beyond this latitude range, reaching around 3.0. This means that pixels will be duplicated vertically at the latitude ranges between $\pm 40^\circ 44'$, but horizontally beyond the range, which also implies that about 67 percent of the pixels may be duplicated in high-latitude areas. Considering the maximum possible error, the minimum accuracy of raster representation in the Mollweide projection can be calculated using the scale factor n and its reciprocal $[1/n]$.

Figure 2 shows experiment results of raster representation in the Mollweide projection. It shows that the accuracy is very high around the latitude of 40 degrees. Also, the accuracy is mostly the function of latitude as expected from the model. The average difference between model and experiment accuracies was 0.8 percent. This means the model, which uses horizontal and vertical scale factors, explains more than 99.2 percent of errors.

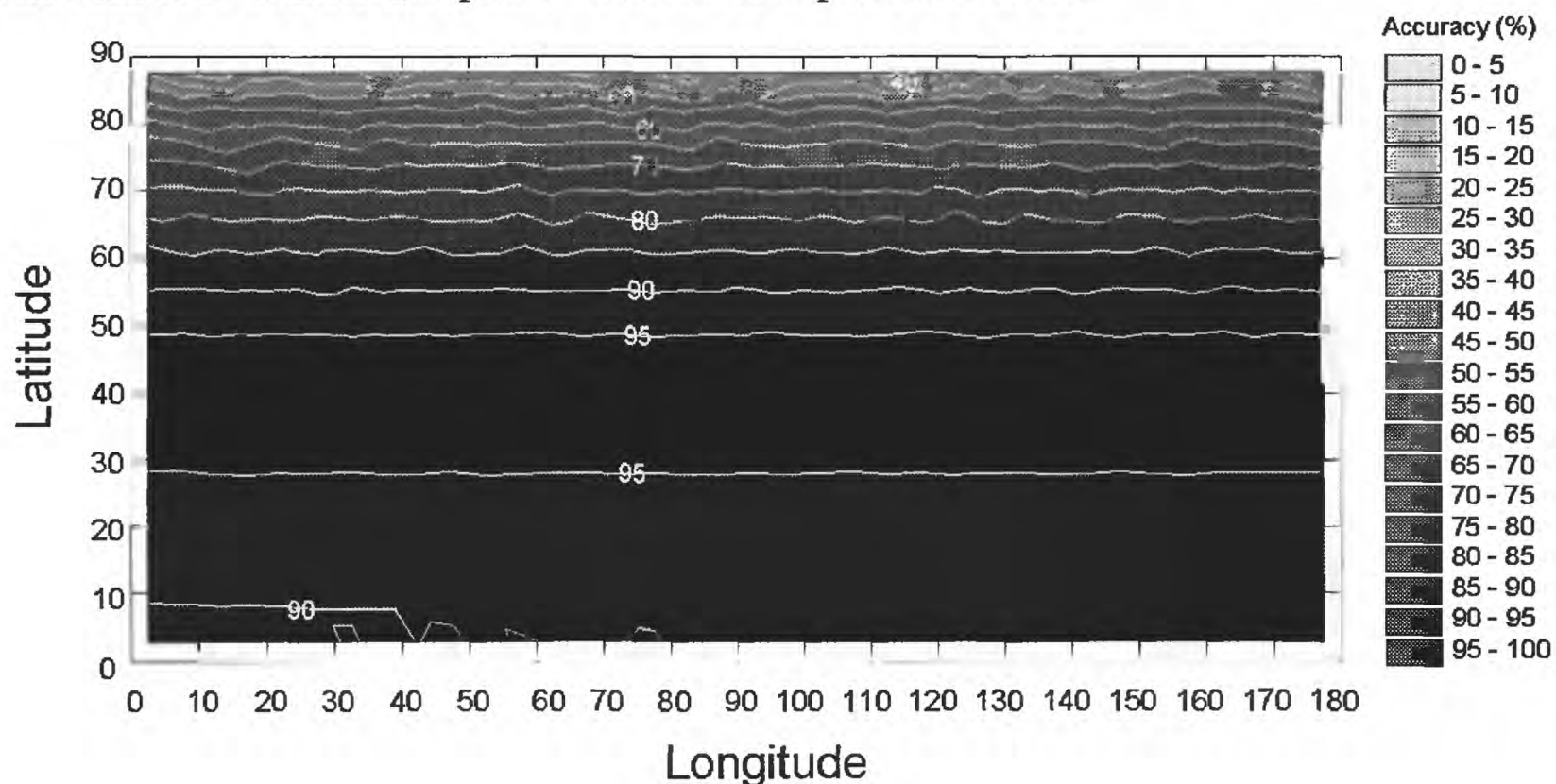


Figure 2. Accuracy results of scale factor error analysis applied to the Mollweide projection.

The scale factor model will be tested with various projections, including equal-area and conformal projections. In cylindrical and pseudocylindrical projections, it is relatively easy to calculate the horizontal and vertical scale factors with the assumption of parallelograms on hyperboloids. However, if parallels and meridians are curved, the accuracy of the raster representation of the scale factor model will be decreased. The extent of accuracy decrease will be modeled also.

In the cases of cylindrical projections such as the Lambert Cylindrical Equal-Area, the James Gall, the Walter Behrmann, and the Trystan Edwards, which use standard parallels (ϕ_k) of 0° , 45° , 30° , and about 51° , respectively, the scale factor model can be applied with the horizontal scale factor and its reciprocal. Pseudocylindrical equal-area projections, such as the Mollweide, Sinusoidal, Eckert VI, Wagner I, Wagner IV, and Urmayev, will be investigated using the same approach.

For conic equal-area projections in their normal aspects, the local linear scale factor along a meridian will be used in calculating the vertical and horizontal scale factors, along with the difference in longitude. Albers equal-area conic projections and the polar azimuthal projections will be modeled with this approach. Also, the other conic and azimuthal projections will be modeled with the same approach, using the deviation of graticule intersection from a right angle on the map.

Some conformal, equidistance, and arbitrary projections will be tested using the scale factor model. In the nonequal-area projections, the error due to area change will be modeled. For those nonequal-area projections, it is necessary to calculate the vertical and horizontal scale factors independently, because one is not the reciprocal of the other.

The scale factor model will be used to monitor the raster representation errors due to reprojection. In this case, existing error in projected data and reprojection error will be modeled. The number of categories may affect the raster representation accuracy. Using experimental data that were labeled with various category numbers, we will model the effect of the number of categories. The total number of categories and the spatial pattern of categories will be analyzed in relation to the raster representation accuracy.

Finally, current global raster databases that are built on some projections will be used to empirically examine the scale factor model's validity. Also, the accuracy of raster representation of existing databases will be examined on the basis of the projections that are used, number of categories, and spatial pattern of categories.

Resampling

During the image warping and reprojection process, we determine pixel values in the final image by taking the original image pixel coordinate determined by the projection (mapping) transformation and rounding it to the nearest line/sample integer location; no interpolation of neighboring pixels is performed. This nearest neighbor resampling is often used by scientists who work with class data because it does not create new classes in the image-warping process.

This method of determining pixel values, when used in areas of high geometric distortion, can generate a data sampling that is not representative of the area in the original image, and it often results in blocky-looking data. An interpolating resampler, such as bilinear interpolation (Colwell, 1983), which uses the four neighboring pixels, or cubic convolution (Park and Schowengerdt, 1983), which uses the 16 neighboring pixels, produces a more geometrically accurate (but radiometrically smoother) result in large-scale, small-area studies (such as the scale

and extent of a Landsat scene). However, in large-area, small-scale studies, the use of these small neighborhood interpolators does little to improve the type of errors seen in previous research (Steinwand, 1995). In addition, these interpolating resamplers are not typically used with class data, for the reasons stated earlier.

New interpolation techniques that take into account resolution and scale changes for a given data point or area are therefore needed. These resamplers need to be adaptable. They need to be determined for each pixel in the final image because the extent of resolution and scale distortion is often not constant throughout the warped image space. They also need to be selectable for the type of data used: interpolative for signal-based data and binned for class-based data. A necessary byproduct of this new resampling technique is an error map (image) indicating data lost and created by the scale and resolution changes.

CONCLUSIONS

The USGS user community is rapidly advancing in the use of large regional and global raster databases for modeling. Those users are often unaware of the effects that projecting raster data can have on areal and positional accuracy. Since the USGS provides many of these databases, it needs methods to ensure accurate projection transformations. This project offers the potential to address this problem from several perspectives. First, the development of an expert guidance system in the form of a projection DSS from the large empirical base available will provide users the capability to use COTS software for projection transformations effectively and wisely. Second, the development of a correction system offers the potential to solve this problem for users. Each of the three proposed approaches--a dynamic mathematical projection specifically designed for raster data, error correction, and resampling--offers a possible solution. All three approaches draw on an experience base in both empirical and theoretical work. Each of the approaches will be implemented by the most experienced researcher, and the three approaches will be coordinated to achieve the best result for the USGS.

CONTRIBUTIONS AND SIGNIFICANCE

This research offers the potential for the USGS to make a major new contribution to the theory and development of map projections, an area of historical significance to the organization. Specifically, this project offers the following benefits:

- 1) A system to support users in the selection of appropriate map projections for large regional and global raster databases.
- 2) A method of estimating errors latent in existing global/continental raster databases.
- 3) Mathematical projection, error correction, and/or resampling methods appropriate for large regional and global raster databases.
- 4) More accurate environmental models resulting from more accurate data at global/continental extents.

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