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Inferred oil and gas reserve estimates for the United States

by

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TABLE OF CONVERSIONS TO SI UNITS

| multiply unit | by | to obtain metric unit |
|---------------|---------|-----------------------|
| barrel | 0.159 | cubic meter |
| cubic foot | 0.02832 | cubic meter |
| foot | 0.3048 | meter |

UNIT ABBREVIATIONS

BBO Billions of barrels of oil

BBL Billions of barrels

TCF Trillions of cubic feet

MMBO.. Millions of barrels of oil

Inferred oil and gas reserve estimation for the United States

E. D. Attanasi

ABSTRACT: This report presents inferred oil and gas reserve estimates based on the 1998 Oil and Gas Integrated Field File compiled by the Energy Information Administration and computed with the same statistical procedures used in the US Geological Survey's 1995 National Assessment of Oil and Gas Resources. For the 30-year period from 1997 through 2026, additions to proved reserves from conventional oil and gas fields discovered prior to 1997 in onshore and state offshore areas are expected to amount to 34 billion barrels of oil (BBO), 195 trillion cubic feet of gas (TCF), and 9 billion barrels of natural gas liquids (BBL). Similarly for the 80-year period starting in 1997, about 52 BBO, 307 TCF, and 13 BBL billion barrels of natural gas liquids are expected to be added to reserves. Overall, the estimates presented here are consistent with the 1995 Assessment estimates of inferred reserves. Another National Oil and Gas Assessment is currently in progress, these estimates are provided at this time for the purpose of comparison with the 1995 estimates. Estimates of inferred reserves for the on-going Assessment have not been made.

INTRODUCTION

Inferred reserves are expected additions to proved reserves in fields already discovered. These additions are commonly called "reserves appreciation" or "field growth". This category of potential additions to reserves accounted for 65 percent of the total oil and 34 percent of the total gas assessed in the U.S. Geological Survey's (USGS) 1995 National Assessment of oil and gas in onshore and State offshore areas (Gautier and others, 1996, Root and others, 1997). Specifically, the Assessment predicted that the known recovery of pre-1992 conventional oil and gas discoveries in the United States would increase from the 1991 estimates by 60 billion barrels of oil (BBO) and 322 trillion cubic feet (TCF) of (dry) gas from 1992 through 2071.

Since the 1995 USGS Assessment, the Energy Information Administration (EIA) has updated and re-issued (EIA, 1998) the basic field database used in the calculations. EIA revised field definitions and added estimates of production and reserves for five additional years, that is, from 1992 through 1996. This report summarizes the inferred reserve estimates obtained when the statistical procedures used in the 1995 study are applied to these recent data. Overall, the estimates obtained are consistent with the 1995 Assessment estimates after adjustments that account for reserve appreciation that occurred from 1991 through 1996. The estimates presented in this report are provided at this time for the purpose of comparison with the 1995 estimates. Estimates of inferred reserves for the current and on-going National assessment have not been made.

ACKNOWLEDGMENTS

The Oil and Gas Integrated Field File used in this analysis was compiled by the late Bill Monroe of the Energy Information Administration. Robert Ryder of the Eastern Energy Resources Team, and Richard Mast and Chuck Spencer both retired from the US Geological Survey, provided assistance identifying unconventional fields not identified during the 1995 National Assessment. The technical reviews and comments by L. J. Drew and D. H. Root are also appreciated as well as editorial comments from Ted Dyman.

DEFINITIONS AND DATA

For this discussion, *known field recovery*, or *field size*, at a given date (time) is defined as the sum of the field's cumulative production and the field's *proved reserves*. Proved reserves are estimated quantities of hydrocarbons which geologic and engineering data demonstrate with reasonable certainty to be recoverable from known fields under existing economic and operating conditions (Society of Petroleum Engineers, 1987; Security and Exchange Commission, 1981). Estimates of *proved reserves* are reported in financial statements, commercial transactions, legal contracts, and in response to regulatory mandates by government. Its technical definition reflects a high degree of certainty about the economically recoverable volumes of hydrocarbons in identified fields.

Proved reserve estimates (and estimates of known recovery) calculated with this definition are conservative. This is true, because they increase with normal field development as boundaries of proved areas are extended by drilling, as new pay zones, pools, or reservoirs are found and confirmed by drilling, as new infill wells (vertical and horizontal) or well stimulation procedures contact previously inaccessible hydrocarbons, and with the introduction of a water-flood or other fluid injection programs.

There are very few case histories for which field size estimates from the year of discovery to the year of abandonment are available. Available data consist of short series of estimates of known recovery of fields grouped by year of discovery. EIA developed the Oil and Gas Integrated Field File (OGIFF) from required proprietary responses to Form EIA - 23 ("Annual Survey of Domestic Oil and Gas Reserves") to provide a basis for tracking field growth. This file consists of field records that include estimates of known recovery of crude oil and wet gas¹ made annually since 1977.

Records in the OGIFF can represent fields or segments of fields depending on whether the field extends into more than one state. Although EIA used a universal field code on each record/segment that identifies the parent field, the segment discovery dates *do not* always agree. So, the discovery table will depend on the criterion used for aggregating field segments to a single field. The procedure used in the 1995 U.S. Geological Survey National Oil and Gas Assessment assigned a field the discovery date of the segment with the largest

¹ Wet gas is the volume of gas produced that contains natural gas liquids that are typically removed at the gas plant. Dry gas has is the volume of gas remaining after all natural gas liquids are removed.

volume of known recovery. Similarly, fields that straddled the state/Federal offshore boundary were also classified as either state or Federal based on the location of the segment with the largest volume of hydrocarbon resources.

Fields are classified as non-associated gas if the ratio of the estimates of gas recovery to oil recovery is at least 20,000 cubic feet of wet gas to 1 barrel of oil. Otherwise a field was considered an oil field. This ratio assured that fields classified as non-associated gas fields were developed as gas fields. The OGIFF data that were used for calculation of field growth were the estimates of known recovery of oil and wet gas. Adjustments were later made to convert the wet gas into dry gas and natural gas liquids.

The OGIFF file issued in the fall of 1998 includes data through 1996, so there are 20 estimates of known recovery for each of the fields discovered prior to 1978. While most of the fields have retained their unique identification codes, there are 178 fewer entries of fields (discovered prior to 1992) in the 1998 version than in the 1993 version of OGIFF. Field consolidations can affect field growth analysis if hydrocarbons in the eliminated fields re-appear as field growth in the retained fields. The magnitude of these effects is *uncertain* at this time.

OGIFF data does not have geologic information. Fields producing principally from unconventional continuous-type accumulations (that is, oil and gas accumulations where a well-defined hydrocarbon-water contact is absent) were identified by geologists and excluded from the conventional field growth analysis for the 1995 Assessment. Continuous-type hydrocarbon accumulations of unconventional oil and gas were assessed in the 1995 Assessment with another methodology (see Schmoker, 1996, Rice, Young and Paul, 1996). In addition to the previously identified records, additional records, representing new discoveries in continuous-type accumulations or fields only recently recognized as producing principally from continuous-type accumulations were also removed from the file. The inferred reserve estimates reported here are restricted to oil and gas resources in conventional fields in onshore and State offshore areas that were discovered before 1997. Fields discovered before 1901 and those without discovery dates were not used to calibrate the cumulative growth functions or to estimate inferred reserves.

MODELING FIELD GROWTH

If fields are grouped by their year of discovery, the sum of the estimates of known recovery for each group often increases more or less systematically over time; suggesting a statistical approach to modeling this change. Suppose the *annual growth factor for year (i+1)* after discovery is defined as the ratio of the estimate of field size at (i+1) years after discovery to the estimate of the field size (i) years after discovery. A *cumulative growth factor* is the ratio of the size of the field *n* years after discovery to the *initial* estimate of field size.

At year *j*, the cumulative growth function (represented by a set of yearly cumulative growth factors) can be used to project field size estimates for year *j+k* with the following relation

$$\hat{c}(d, j+k) = c(d, j) * [G(j+k-d)/G(j-d)] \quad 1.$$

where j = current year

k = the time elapsed between an early year estimate at year j , and the later estimate, at year $j+k$;

$c(d, j)$ = volume of oil or gas discovered in year d and estimated in year j ;

$\hat{c}(d, j+k)$ = projected estimate of volume in year $j+k$; discovered in year d .

$G(\)$ = is the cumulative growth function

Individual growth factor calibration approach

Arrington (1960) published the first study that developed and applied growth factors to adjust recent discovery sizes. Arrington was interested in tracking the results of recent company exploration results. The annual growth factor for a group of fields in the $j+1$ year after discovery (that is, age $j+1$) is calculated as the sum of estimates of known recovery of fields at age $j+1$ divided by the sum of the estimates of known recovery of the same fields at age j . In particular,

$$a(j, j+1) = (\sum_d c(d, d+j+1)) / (\sum_d c(d, d+j)) \quad 2.$$

Cumulative growth factors were computed by multiplying annual growth factors for successive years. Because of a short data series Arrington actually grouped data and calibrated the first year's annual growth coefficient by extrapolating the trend in coefficients from later years to year 1. Root (1981) provides an analytical explanation for this approach and applied it to derive estimates of inferred reserves for the US Geological Survey's 1981 National Oil and Gas Assessment.

Joint growth factor calibration approach

Coefficients of the cumulative growth function can be estimated simultaneously by choosing a set of coefficients that minimizes the sum of square errors (residuals), sse ,

$$e(d, j, k) = [(\hat{c}(d, j+k) - c(d, j+k))]^2 \quad 3a.$$

$$sse = \sum_{d,j,k} e(d, j, k) \quad 3b.$$

where $e(d, j, k)$ is the square of the residual for a single within sample forecast and sse is the sum of squared residuals. The cumulative growth function that results from minimizing the sum of squared deviations for the entire set of coefficients is called the least squares growth function. The properties of the growth functions were further restricted or required to satisfy the following inequality conditions:

$$1 \leq G(n+1)/G(n) \leq G(n)/G(n-1) \quad n \geq 1 \quad 4.$$

The left inequality imposes the condition that fields cannot shrink in size and the right inequality imposes the condition that older fields cannot grow by a larger factor than younger fields. Because the graph of the resulting cumulative growth function is continuous and non-decreasing, it is called a monotone function here. The factors of the monotone cumulative growth function are computed by minimizing the error function (equation 3a) subject to the graph of the function having a continuous non-decreasing shape, that is, equation 4 (Attanasi and Root, 1994).

GROWTH FUNCTIONS AND INFERRED RESERVE ESTIMATES

Cumulative growth functions

Cumulative growth functions were calibrated using annual field size estimates (from 1977 through 1996) for conventional fields grouped and summed by discovery year from 1901 through 1996. Figures 1 and 2 show cumulative growth functions for oil in oil fields and gas in gas fields, respectively. Each figure presents the cumulative growth function calculated with the Arrington-type annual factor approach, the least squares algorithm, and the least squares with a monotone restriction. Visual inspection shows great similarity among functions. Following the 1995 USGS Assessment procedure the monotone cumulative growth functions were used to project future field growth. The coefficients of the cumulative growth function are presented in appendix Table A-1.

Inferred Reserve Estimation Procedure

Inferred reserve estimates were based on field growth projected for 30 and 80 years beyond 1996. Cumulative monotone growth functions (Figures 1 and 2) for the primary commodities were used to project both the primary and by-product additions to reserves (inferred reserves) from pre-1997 fields for the lower-48 onshore and state offshore areas. In particular, the function used to project oil in oil fields was also used to project gas in oil fields and the function used to project gas in gas fields projected oil and condensate in gas fields. Wet gas was used in all the computations but was later converted to dry gas and natural gas liquids (NGLs).

Alaska was analyzed separately from the lower 48 states. Special operating conditions influence the development of Alaskan fields and inferred reserve estimates implicitly assume future operation of the Trans-Alaska Pipeline during the projection period. Statistically estimated cumulative growth functions were not calculated for Alaska because of the very small number of fields. The growth functions calibrated for the Lower 48 States were applied to the 1996 estimates of discovered fields in Alaska to compute crude oil inferred reserves. Except for associated gas on the North Slope, by-product commodities were assumed to grow proportionately to the primary commodities.

Gas resources (mostly associated gas) in identified North Slope fields amount to about 30.9 TCFG (Alaska Department of Natural Resources, 2000). However, because there is no ready market for this gas, the industry carries only

about 20 percent of this volume as a commercial asset. Some of the gas is used as lease fuel and for the maintenance of reservoir pressure in operating oil fields. Following the 1995 Assessment procedure, it was assumed that these resources would become fully valued reserves some time during the next 80 years and the realization of reserve additions would be the same as the profile for oil.

Inferred Reserve Estimates

Inferred reserves estimates for fields discovered prior to 1997 for the 30 and 80-year periods starting in 1997 are shown in Table 1. For the Lower 48 onshore and state offshore areas, 39.7 billion barrels of oil (BBO) and 293.1 trillion cubic feet of wet gas (TCFG) were estimated. The wet gas breaks down into 278 TCFG of dry gas and 13.1 BBL of gas liquids².

For Alaska growth in known recovery of oil through 2076 was calculated to be 12.1 BBO. The estimated inferred reserves that result from application of the growth function, calibrated from the lower 48 data, to the North Slope are quite uncertain but this uncertainty could not be quantified at this time. No additional inferred oil reserves were added to account for the known heavy oil resources of the West Sak and Ugnu accumulations in Northern Alaska. The estimated oil in-place of West Sak and Ugnu are 20 BBO and 15 BBO, respectively (Bird, 1991). Although applying the Lower 48 growth function to Alaska is not entirely satisfactory, there is little question about the existence of large volumes of oil in identified accumulations that could be added to reserves in the future.

Inferred gas reserves (for the 80 year period starting in 1997) from Southern Alaska's non-associated gas fields and oil fields were estimated at 4.4 TCFG with 0.2 BBL in natural gas liquids (NGL). The remaining additional gas in resources of discovered fields in Northern Alaska could add an additional 25 TCFG to the reserves in the OGIFF file to total 29.4 TCFG for the entire state.

Table 2 shows 1995 Assessment estimates for inferred reserves from fields discovered prior to 1992. Data from the OGIFF field file indicated that during the period from 1991 through 1996, additions to reserves from the pre-1992 discoveries amounted to 4.5 BBO and 50.3 TCFG (47.7 TCFG, dry gas). Comparison of actual additions and predicted additions for the lower 48 States based on the 1995 Assessment growth functions for that 5 year period showed that actual additions for oil fell short of predicted additions for oil but actual gas additions exceeded predicted additions. Economic conditions and oil and gas prices affect the speed at which operators develop discoveries; the depressed oil prices and robust gas prices may explain some difference between prediction and realization.

² The conversion was based on the following relationship: 175 TCFG wet gas = 166 TCFG dry gas + 7.8 BBL gas liquids. This relationship is based on the 1996 estimates of US wet natural gas reserves, dry gas reserves, and natural gas liquids published in the US Crude Oil, Natural Gas, and Natural Gas Liquids Reserves: Annual Report 1996, (EIA, 1997).

SUMMARY AND IMPLICATIONS

Overall, estimates of inferred reserves presented in Table 1 are comparable to the adjusted estimates from the 1995 Assessment of inferred reserves. The absence of significant new conventional discoveries in recent years in the onshore areas of the lower 48 states suggests these new estimates of inferred reserves are indicative of an aging set of fields with reduced future yields.

In recent years, the growth of known fields had been the dominant source of annual additions to proved reserves. Trends in proved reserve levels are leading indicators of the sustainability of oil and gas production. Because no more than 10 to 15 percent of the proved reserves of conventional fields can be extracted annually without risking reservoir damage and reducing ultimate field recovery, proved reserve levels limit annual production to an amount well below recoverable resource volumes. Reserve additions typically fail to keep up with production because of a declining discovery rate (hydrocarbons discovered per well). Low drilling and development activity in identified fields slows the rate of proved reserve additions from inferred reserves. Because of this, over short time periods, the accuracy of projections may suffer from the vagaries of the industry's boom and bust cycles but the projections will be more accurate over the longer time periods.

The estimates of inferred reserves reported here are only based on the monotone growth functions and the 1996 Integrated Oil and Gas Field File. Estimates of inferred reserves for the ongoing National Oil and Gas Assessment have not been made.

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Table 1. Estimated inferred reserves for fields discovered as of December 31, 1996 (onshore and state water areas)

80 year period

| <u>Region</u> | <u>Crude oil</u> | <u>Gas</u> | <u>NG</u> |
|---------------|-------------------|------------------------|--------------------|
| | (Billion barrels) | (trillions cubic feet) | (billions barrels) |
| Alaska | 12.1 | 29.4 | 1.4 |
| Lower 48 | 39.7 | 278.0 | 13.1 |
| TOTAL | 51.8 | 307.4 | 14.5 |

30 year period

| | | | |
|----------|------|-------|-----|
| Alaska | 6.3 | 15.6 | .8 |
| Lower 48 | 27.4 | 179.0 | 8.4 |
| TOTAL | 33.7 | 194.6 | 9.2 |

Table 2. Estimated inferred reserves for fields discovered as of December 31, 1991 (onshore and state water areas) (from Root and others, 1997)

80 year period

| <u>Region</u> | <u>Crude oil</u> | <u>Gas</u> | <u>NGL</u> |
|---------------|-------------------|------------------------|--------------------|
| | (Billion barrels) | (trillions cubic feet) | (billions barrels) |
| Alaska | 13.0 | 32.0 | 0.5 |
| Lower 48 | 47.0 | 290.0 | 12.9 |
| <u>TOTAL</u> | <u>60.0</u> | <u>322.0</u> | <u>13.4</u> |

Table A-1 Coefficients* of cumulative field growth functions based on the 1996 Integrated Oil and Gas Field File compiled by the Energy Information Administration and issued 1998.

| Field age | Oil in oil fields | | | Gas in gas fields | | |
|-----------|-------------------|---------------|-----------|-------------------|---------------|-----------|
| | Monotone | Least squares | Arrington | Monotone | Least squares | Arrington |
| 0 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1 | 1.98172 | 1.98431 | 1.97943 | 1.73034 | 1.73011 | 1.79253 |
| 2 | 2.25754 | 2.25389 | 2.28777 | 2.36589 | 2.36542 | 2.23944 |
| 3 | 2.57176 | 2.57427 | 2.56190 | 2.74995 | 2.74694 | 2.41949 |
| 4 | 2.71298 | 2.75812 | 2.72207 | 2.96789 | 2.96413 | 2.62381 |
| 5 | 2.84490 | 2.80408 | 2.78244 | 3.16715 | 3.17632 | 2.82645 |
| 6 | 2.98323 | 2.95119 | 2.92794 | 3.33982 | 3.30801 | 2.98386 |
| 7 | 3.12830 | 3.15117 | 3.08840 | 3.52191 | 3.52900 | 3.17747 |
| 8 | 3.27433 | 3.28779 | 3.22365 | 3.69021 | 3.68157 | 3.25799 |
| 9 | 3.39626 | 3.37181 | 3.31189 | 3.80586 | 3.79866 | 3.37288 |
| 10 | 3.52274 | 3.52666 | 3.44349 | 3.91901 | 3.90366 | 3.46846 |
| 11 | 3.65392 | 3.65585 | 3.54009 | 4.03553 | 4.02773 | 3.62160 |
| 12 | 3.73758 | 3.73619 | 3.62699 | 4.14141 | 4.12985 | 3.73144 |
| 13 | 3.82009 | 3.81628 | 3.69754 | 4.23697 | 4.22465 | 3.81838 |
| 14 | 3.90442 | 3.92077 | 3.80514 | 4.33101 | 4.35000 | 3.93764 |
| 15 | 3.97731 | 3.97195 | 3.85899 | 4.40265 | 4.42325 | 4.02459 |
| 16 | 4.05156 | 4.02152 | 3.92091 | 4.47548 | 4.54081 | 4.15634 |
| 17 | 4.12719 | 4.11464 | 3.99623 | 4.54951 | 4.56509 | 4.22445 |
| 18 | 4.20424 | 4.21534 | 4.11307 | 4.62477 | 4.59048 | 4.29321 |
| 19 | 4.27855 | 4.37531 | 4.14402 | 4.70127 | 4.72068 | 4.38484 |
| 20 | 4.33710 | 4.40893 | 4.22523 | 4.77903 | 4.74972 | 4.44805 |
| 21 | 4.39646 | 4.31961 | 4.26837 | 4.85809 | 4.77474 | 4.53085 |
| 22 | 4.45664 | 4.37232 | 4.30896 | 4.93845 | 4.83867 | 4.61986 |
| 23 | 4.51763 | 4.49914 | 4.39464 | 5.02014 | 4.95700 | 4.68677 |
| 24 | 4.57946 | 4.55289 | 4.45399 | 5.10318 | 5.04865 | 4.79153 |
| 25 | 4.64214 | 4.65185 | 4.51409 | 5.18759 | 5.12265 | 4.85395 |
| 26 | 4.70568 | 4.71119 | 4.57013 | 5.27340 | 5.20932 | 4.91985 |
| 27 | 4.77008 | 4.70687 | 4.64057 | 5.36063 | 5.29756 | 5.00714 |
| 28 | 4.83537 | 4.80806 | 4.71473 | 5.44931 | 5.43167 | 5.06468 |
| 29 | 4.90155 | 4.91564 | 4.76432 | 5.53945 | 5.51871 | 5.16881 |
| 30 | 4.96669 | 4.96805 | 4.81143 | 5.63108 | 5.63735 | 5.29196 |
| 31 | 5.03270 | 5.02651 | 4.86988 | 5.72422 | 5.69534 | 5.37547 |
| 32 | 5.09959 | 5.04688 | 4.91015 | 5.80234 | 5.75854 | 5.45596 |
| 33 | 5.16737 | 5.13407 | 4.98572 | 5.86833 | 5.82904 | 5.53698 |
| 34 | 5.23604 | 5.21675 | 5.05239 | 5.92499 | 5.88509 | 5.61229 |
| 35 | 5.30564 | 5.28733 | 5.11978 | 5.98220 | 5.90409 | 5.63357 |
| 36 | 5.37615 | 5.36755 | 5.18495 | 6.03997 | 6.04008 | 5.75977 |

| | | | | | | |
|----|---------|---------|---------|---------|----------|---------|
| 37 | 5.43145 | 5.49763 | 5.24388 | 6.09721 | 6.06621 | 5.82013 |
| 38 | 5.48390 | 5.54302 | 5.28516 | 6.15501 | 6.10591 | 5.87941 |
| 39 | 5.53685 | 5.58191 | 5.28543 | 6.21334 | 6.19626 | 5.93522 |
| 40 | 5.59031 | 5.51289 | 5.33872 | 6.27224 | 6.26856 | 5.96800 |
| 41 | 5.64429 | 5.54096 | 5.37844 | 6.33169 | 6.26110 | 6.01014 |
| 42 | 5.69879 | 5.60113 | 5.41475 | 6.39170 | 6.31156 | 6.07924 |
| 43 | 5.75382 | 5.68834 | 5.47209 | 6.45228 | 6.36902 | 6.14544 |
| 44 | 5.80937 | 5.74113 | 5.51516 | 6.51344 | 6.44498 | 6.21840 |
| 45 | 5.86547 | 5.82812 | 5.58593 | 6.57517 | 6.47867 | 6.26159 |
| 46 | 5.92210 | 5.86730 | 5.61238 | 6.63749 | 6.53178 | 6.32705 |
| 47 | 5.97928 | 6.00081 | 5.67877 | 6.70040 | 6.63620 | 6.41809 |
| 48 | 6.03702 | 6.05828 | 5.72640 | 6.76391 | 6.70368 | 6.45580 |
| 49 | 6.07048 | 6.06692 | 5.73277 | 6.82802 | 6.75177 | 6.51467 |
| 50 | 6.10413 | 6.09305 | 5.75127 | 6.89274 | 6.81909 | 6.57794 |
| 51 | 6.13796 | 6.12371 | 5.78065 | 6.95807 | 6.90433 | 6.66715 |
| 52 | 6.17198 | 6.14672 | 5.81333 | 7.02402 | 6.92661 | 6.72408 |
| 53 | 6.20619 | 6.18936 | 5.84359 | 7.09060 | 7.06282 | 6.82954 |
| 54 | 6.24059 | 6.22756 | 5.87818 | 7.15780 | 7.10515 | 6.88092 |
| 55 | 6.27518 | 6.25853 | 5.91384 | 7.22565 | 7.20113 | 6.93407 |
| 56 | 6.30996 | 6.26513 | 5.92255 | 7.29413 | 7.24487 | 6.99131 |
| 57 | 6.34493 | 6.26342 | 5.93680 | 7.36327 | 7.31796 | 7.05663 |
| 58 | 6.38010 | 6.29150 | 5.96061 | 7.43306 | 7.29784 | 7.06906 |
| 59 | 6.41546 | 6.33812 | 5.99495 | 7.50351 | 7.54363 | 7.19594 |
| 60 | 6.45102 | 6.34580 | 6.01930 | 7.55897 | 7.55572 | 7.22102 |
| 61 | 6.48678 | 6.33589 | 6.03516 | 7.61483 | 7.56177 | 7.27697 |
| 62 | 6.52273 | 6.35622 | 6.06270 | 7.67111 | 7.60250 | 7.31725 |
| 63 | 6.55888 | 6.38159 | 6.08766 | 7.72780 | 7.60339 | 7.33996 |
| 64 | 6.59524 | 6.41145 | 6.12533 | 7.78491 | 7.79374 | 7.50818 |
| 65 | 6.63179 | 6.42625 | 6.14124 | 7.84245 | 7.84965 | 7.56659 |
| 66 | 6.66855 | 6.42424 | 6.18843 | 7.90041 | 7.82806 | 7.56602 |
| 67 | 6.70551 | 6.53510 | 6.26607 | 7.95879 | 7.92343 | 7.65160 |
| 68 | 6.74268 | 6.58957 | 6.32823 | 8.01761 | 7.94047 | 7.67055 |
| 69 | 6.78005 | 6.63529 | 6.36705 | 8.07687 | 8.02278 | 7.74675 |
| 70 | 6.81763 | 6.67037 | 6.39767 | 8.13656 | 8.11925 | 7.83443 |
| 71 | 6.85542 | 6.71744 | 6.43586 | 8.19669 | 8.14483 | 7.87017 |
| 72 | 6.89341 | 6.80007 | 6.47149 | 8.25727 | 8.23569 | 7.98978 |
| 73 | 6.93162 | 6.82429 | 6.49781 | 8.28748 | 8.32398 | 8.13896 |
| 74 | 6.97004 | 6.87943 | 6.55234 | 8.31781 | 8.24502 | 8.08652 |
| 75 | 7.00867 | 6.94205 | 6.60238 | 8.34824 | 8.19173 | 8.15325 |
| 76 | 7.04752 | 6.90676 | 6.64558 | 8.37879 | 8.26550 | 8.26007 |
| 77 | 7.08658 | 6.99336 | 6.69839 | 8.40944 | 8.33016 | 8.34280 |
| 78 | 7.12586 | 7.17292 | 6.79500 | 8.44022 | 8.34531 | 8.43040 |
| 79 | 7.16536 | 7.24982 | 6.83861 | 8.47110 | 8.68588 | 8.40916 |
| 80 | 7.20507 | 7.36025 | 6.91734 | 8.50209 | 8.76907 | 8.52170 |
| 81 | 7.24501 | 7.39151 | 6.96449 | 8.53320 | 10.21231 | 8.58721 |

| | | | | | | |
|----|---------|----------|---------|---------|----------|----------|
| 82 | 7.28516 | 7.52994 | 7.06679 | 8.56443 | 10.58930 | 8.94890 |
| 83 | 7.32554 | 7.73837 | 7.20079 | 8.59576 | 11.92076 | 9.53450 |
| 84 | 7.36615 | 7.91786 | 7.33098 | 8.62722 | 12.11984 | 9.67950 |
| 85 | 7.40697 | 8.08097 | 7.43600 | 8.65878 | 12.56577 | 10.21657 |
| 86 | 7.44803 | 8.00932 | 7.47729 | 8.69047 | 11.10659 | 9.20259 |
| 87 | 7.48931 | 8.40189 | 7.72875 | 8.72226 | 15.11612 | 9.89398 |
| 88 | 7.53082 | 8.89412 | 7.87144 | 8.75418 | 14.10877 | 10.34645 |
| 89 | 7.57256 | 8.98877 | 7.93425 | 8.78621 | 12.43515 | 10.02058 |
| 90 | 7.61453 | 8.97369 | 7.91097 | 8.81836 | 12.77088 | 10.26698 |
| 91 | 7.65674 | 9.04266 | 7.89948 | 8.85063 | 12.62432 | 10.30822 |
| 92 | 7.69918 | 9.56364 | 7.99318 | 8.88301 | 12.56381 | 10.73594 |
| 93 | 7.74185 | 9.71880 | 8.05187 | 8.91551 | 17.33884 | 11.37121 |
| 94 | 7.78476 | 9.81610 | 8.11042 | 8.94814 | 21.51583 | 13.12062 |
| 95 | 7.82791 | 10.04574 | 8.18085 | 8.98088 | 24.89357 | 14.68939 |

* Calculation of the monotone growth function and least squares are discussed in text and the method used to calculate the Arrington function coefficients is presented by Root(1981).

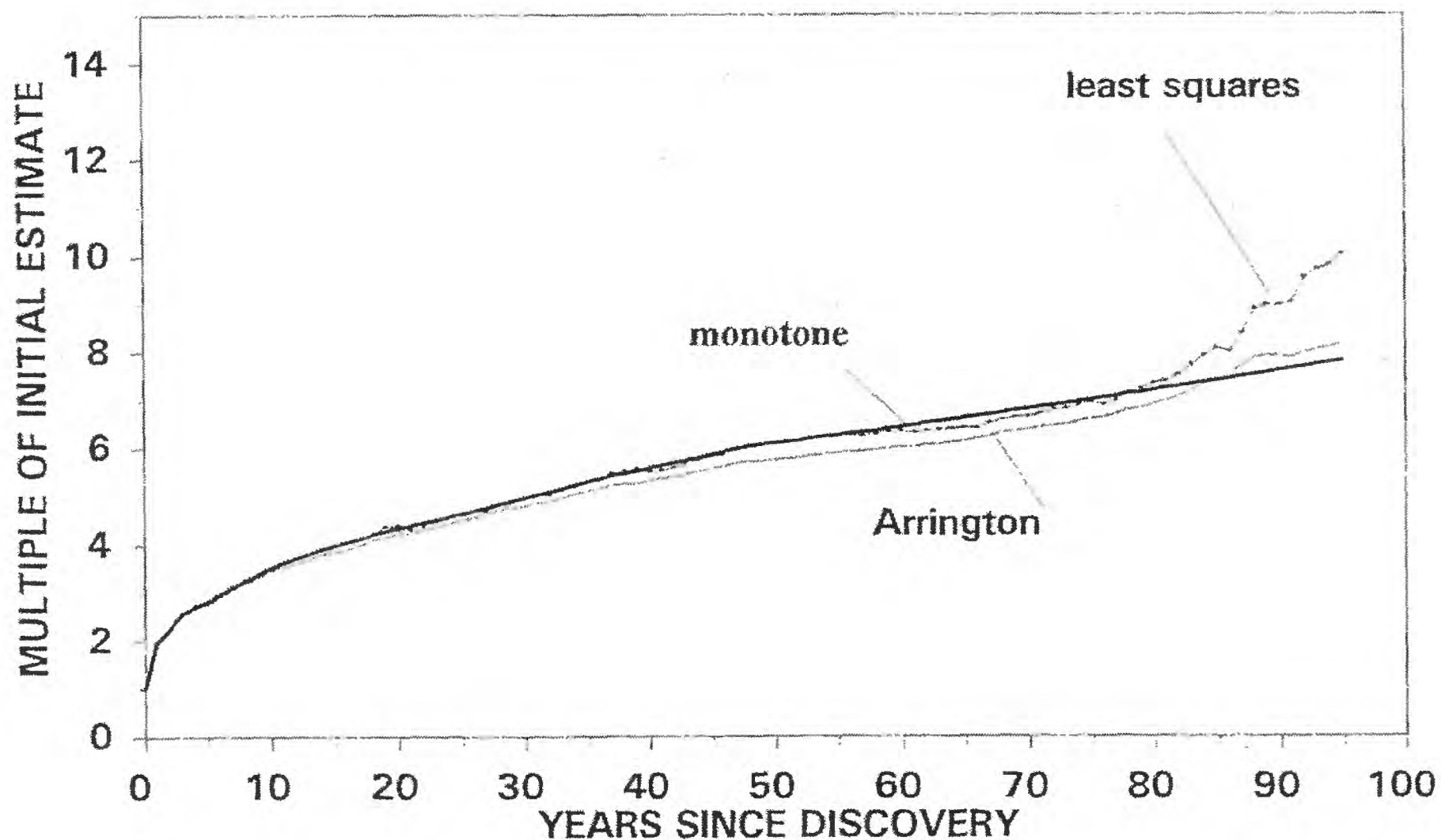


Figure 1. Cumulative growth functions for oil in conventional oil fields based on data from the 1998 version of the Oil and Gas Integrated Field File provided by the Energy Information Administration. The monotone (non-decreasing) function is used to estimate inferred reserves found in Table 1. The least squares function is calibrated by minimizing the error function (equations 3a and 3b) over the entire set of cumulative growth factors. The Arrington function is based on estimates of annual growth factors and calibrated using the method shown by Root (1981).

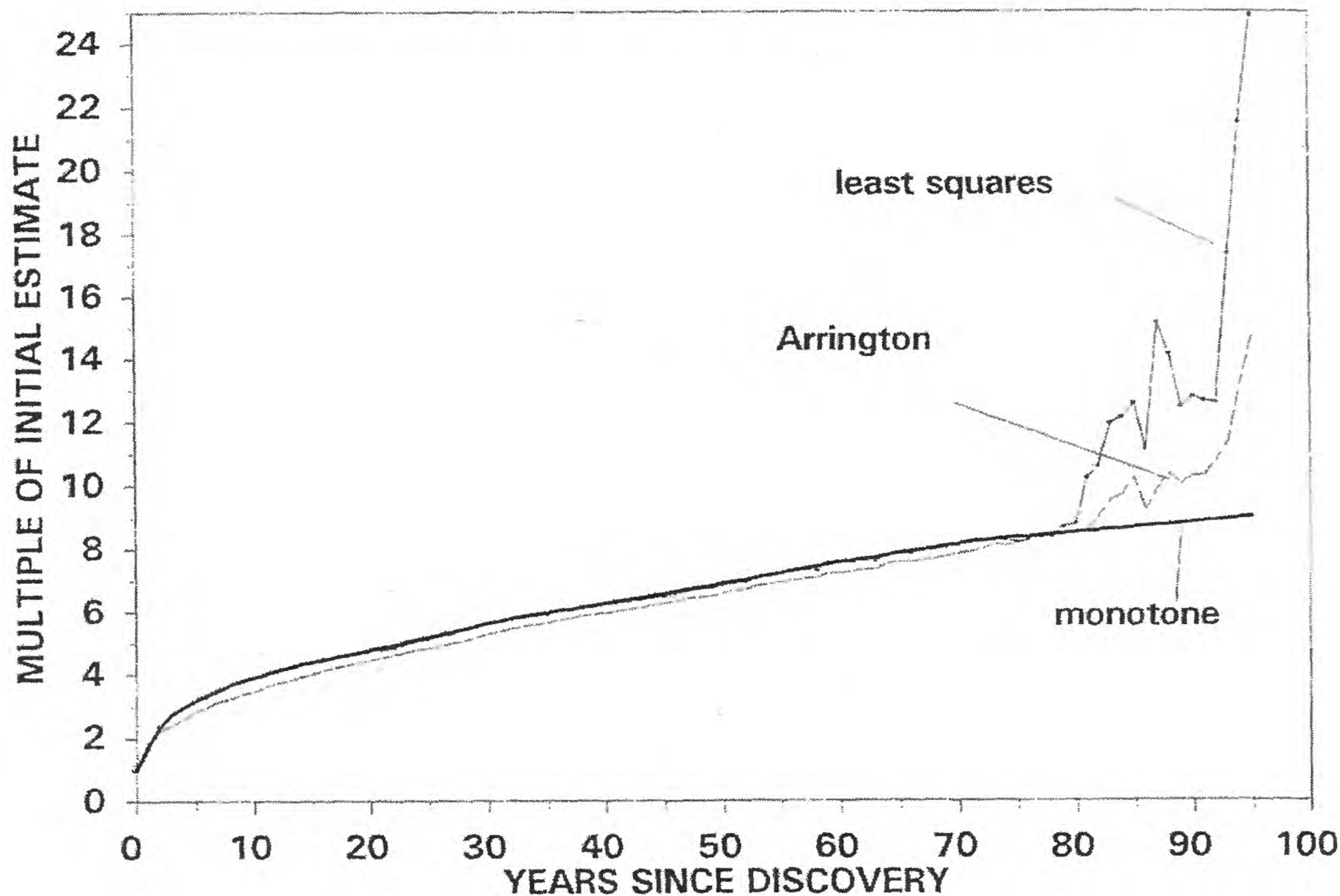


Figure 2. Cumulative growth functions for gas in conventional gas fields based on data from the 1998 version of the Integrated Oil and Gas Field File provided by the Energy Information Administration. The monotone (non-decreasing) function is used to estimate inferred reserves found in Table 1. The least squares function is calibrated by minimizing the error function (equations 3a and 3b) over the entire set of cumulative growth factors. The Arrington function is based on estimates of annual growth factors and calibrated using the method shown by Root (1981).