Sand Thickness Exploration Model – Kailua Bay

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Introduction

The purpose of the geostatistical analysis is to analyze the spatial distribution of the thickness of sand deposits in the reef-front area of Kailua Bay, Oahu, and provide probabilistic maps that can be used as the basis for an exploration model for sand resources in the study area. The data used in the study consisted of seismic reflection measurements of the thickness of carbonate sand draped over a Pleistocene lowstand terrace.

The reef-front in Kailua Bay curves from a generally north-south orientation in the northern part of the study area, to an east-west orientation in the southern portion of the study area (Figure 1). This curvature posed a problem for the variogram analysis and geostatistical mapping because the sedimentation processes responsible for the distribution of sand were expected to be aligned parallel to the reef-front. In order to account for the curvature, we applied coordinate transformation techniques developed for braided fluvial aquifers (Deutsch and Wang 1996) to the data prior to the geostatistical analysis. The geostatistical analysis included variogram analysis (Isaaks and Srivastava 1989) to determine the spatial continuity of the sand thickness data. We then used the variogram models as input to a stochastic simulation algorithm and generated a suite of 100 simulations of the sand thickness. Probability methods were used to post-process the simulations and produce maps suitable for an exploration model for sand thickness in Kailua Bay. This model can be further developed with input on economic data relevant to developing the sand resource.

Exploratory Data Analysis

The locations of the measurements of sand thickness are shown on the contour map of the seafloor depth of Kailua Bay, see Figure 1. The twelve seismic sampling lines are approximately orthogonal to the contours of seafloor depth. The seafloor depth ranges from 40 to 100 m. The histogram of the sample measurements of sand thickness is given in Figure 2. The sand thickness data is positively skewed, and has a sample mean of 11m and a range of 0.0 to 40.5m.

The spatial distribution of sand thickness was first estimated and contoured using the commercial software SURFER, see Figure 3, which served as a preliminary conceptual model. As seen in the figure, the high estimates are located in the southeast corner of the map, where a channel cuts the reef flat and reef front. The sand thickness decreases gradually to the north and east of the channel along the reef-front. Figure 4 displays experimental variograms of the raw sand thickness measurements and the normal score transformed values. The normal score transform (Deutsch and Journel 1998) is often used to transform a skewed data set to a standard normal distribution, and is required for the Gaussian stochastic simulation technique used for the mapping and probabilistic analysis. Omnidirectional and directional variograms in the north-south and east-west directions are plotted in Figure 4, together with the spherical model for the omnidirectional variogram. The three variograms show similar spatial structure with two nested structures (Isaaks and Srivastava 1989) having a short-scale range of 500m and a long-scale range of 3000m.

Coordinate Transformation

Comparison of the seafloor bathymetry (Figure 1) and the preliminary contour map of sand thickness (Figure 3) suggests that the maximum spatial continuity might be found along the reef front parallel to the contour lines of seafloor depth, which are approximately parallel curves. Currently available geostatistical software is not able to infer the spatial continuity of a study area with this type of curved geometry. For example, the variogram models of the sand thickness shown in Figure 4 probably do not capture the anisotropy of the spatial continuity, because the direction of maximum continuity appears to follow the bathymetric contours (Figure 3), which are curved. In order to model properly the curvature of the spatial continuity of sand thickness, the coordinates of the original area were projected to a "straightened" rectangular computational space. The spatial continuity within the computational space then can be inferred followed by application of stochastic simulation to generate probability maps using the projected coordinates of the original curved area. The coordinate transformation is reversible, so that the final maps can be projected back into the original space.

Coordinate transformation is commonly used in modeling the distribution of porous sands in sinuous reservoirs and aquifers deposited by rivers. The approach used for this task is adopted from the hierarchical coordinate transformation developed by Clayton and Wang (1996) for fluvial reservoirs deposited by braided streams. The steps of the coordinate transformation include:

Step1: Translation and rotation

The original coordinates are in the UTM coordinate system denoted as an east x_1 and north y_1. In this study, the UTM coordinates of the origin of the original coordinate systems were set to an easting of 633013, and northing of 2368513, and the grid spacing used for the mapping was 25 m in the x_1 and y_1 direction. A transformed y_2 was determined by rotation of the coordinate system so that it was aligned as closely as possible with the expected direction of greatest spatial continuity. The direction perpendicular to y_2, the transformed x_2, is assumed to be the direction of least continuity. The algorithm of rotation of (x_1, y_1) to (x_2, y_2) is:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta - \sin\theta \\ \sin\theta - \cos\theta \end{bmatrix} \begin{bmatrix} x_1 - x_1^0 \\ y_1 - y_1^0 \end{bmatrix} \quad \text{...(1)}$$

where _ is the rotation angle defined as positive clockwise from the north direction (y_1), and (x_1^0, y_1^0) is the location of the origin of the stratigraphic coordinate system in the units of the original coordinate system. Figure 5 shows the rotated coordinate systems together with a contour map of the seafloor depth and the locations of seismic samples, which are marked by the blue crosses. The rotation angle was N35W degrees, and the origin (x_1^0, y_1^0) used for the rotation was (635300, 2368700).

Step2: Straightening

The 65m bathymetric contour, denoted by the red crosses in Figure 5, was chosen to be the centerline of the transformed coordinate system because it intersects all of the seismic lines. The boundary of the area of interest was determined by extending the region at least 920m from the centerline in either direction to cover the vast majority of the seismic samples (see lines marked by yellow crosses in Figure 5). If a sample would not be included within the predefined 920m, the boundary is extended to the sample location. Note that the samples to the left of x_2= -1500 (Figure 5), which are found on a single seismic line extending up the reef-flat channel, are not included in the analysis. The stratigraphic x_2 of the samples is "straightened" to a horizontal coordinate x_3 defined as the horizontal deviation from the centerline:

$$x_3 = x_2 - x_{2c} \quad [2]$$

where x_{2c} is the stratigraphic x_2 of the centerline with the same or closest y_2 as the sample. The y_3 coordinate stays unchanged during straightening of the area of interest, and has the same value as y_2.

Step3: Relative coordinate

The horizontal relative coordinate x_4 of the sample in the final rectangular computational grid is defined by standardizing the width from the y_3 centerline to the two boundaries $x_{3b}(y_3)$:

$$x_4 = \left(\frac{x_3}{x_{3b}|(y_3)|} + 1 \right) \cdot w \quad \text{...(3)}$$

where w is a factor that adjusts the final width to the rectangular computational grid, here w =500 is used. The final transformed coordinates of the seismic samples and the “straightened” contour map of the seafloor depth are shown in Figure 6. Note that the bathymetric contour lines are straightened parallel to the y_3 axis through the majority of the map where data are available; however, the upper and lower sections of the figure are slightly skewed due to lack of sample data. The variogram analysis and stochastic simulation described below were executed using the rectangular computational grid shown in Figure 6. The final transformed coordinates can be easily projected back to the original coordinates by reversing the steps described above in equations [3] to [1].

Variogram Modeling and Stochastic Simulation

Omnidirectional and directional variograms of the normal-score transformed sand thickness data are calculated and plotted in Figure 7. The corresponding spherical models are shown as the solid lines in Figure 7. The variogram model has a range of 750 m in the direction parallel to the reef front (N-S in Figure 6), and 270 m perpendicular to the reef front, which represent the directions of the greatest and least spatial continuity, respectively. Note that the spatial continuity of the omnidirectional variogram, with a model range of 600 m, is dominated by the greater continuity in the north-south direction. As discussed in the previous section, the anisotropy of the spatial continuity (i.e., the ratio of the variogram ranges in the directions of greatest and least continuity), could not be accurately measured in the original coordinate system. However, it can be properly delineated by directional variograms calculated using the transformed coordinate system.

We estimated the conditional probability distribution of the sand thickness at each location in the map area by simulating a series of equiprobable realizations using the geostatistical technique known as stochastic simulation. For this purpose, a set of 100 realizations of sand thickness was generated using the sequential Gaussian simulation routine sgsim from GSLIB (Deutsch and Journel, 1998) on the 101 by 181 nodes of the computational grid. The normal score variogram models required for the simulation algorithm are shown in Figure 7. The 100 simulated values at each node were used to calculate several statistics, which are discussed below. The maps of those statistics were all plotted after back-transformation to the original (i.e., real-world) UTM coordinate system.

E-type estimate

The arithmetic average of the 100 simulated values at each grid node in the map area is the expected value of the conditional distribution at that location, and is called the E-type estimate (Deutsch and Journel 1992). E-type estimates provide a mean spatial distribution of the sand thickness in the study area. Figure 8 shows the E-type estimate map of sand thickness, with the sample locations plotted as black crosses. The area with the highest

E-type estimate of sand thickness is located in the southeast corner of the map near the reef-flat channel. A broad area of relatively thick sand deposits stretches for about a kilometer to the east of the channel, along the reef-front. The thickness of the sand to the north along the reef front is not as broad or extensive.

Percentiles of the conditional distributions and uncertainty mapping

Maps of the 10th and 90th percentiles of the 100 simulated values at each grid node in the map area are shown in Figures 9 and 10. Figure 9, the map of the 10th percentiles, can be used to identify areas with high values of sand thickness, because 90 percent of the simulated values in those locations are higher than the values indicated on the map. If the map shows areas of high thickness values, as it does in the area of the reef-flat channel, and east of the channel along the reef front, then those areas have a very high probability of containing thick sand deposits. On the contrary, Figure 10, the map of the 90th percentiles of the conditional distributions, can be used to identify the limits of areas that are almost certain to contain thin sand deposits. For example, north along the reef front, the northing coordinate of the maximum extent of thick sand deposits appears to be between 2370000 and 2370500.

The difference between the 10th and 90th percentiles of the 100 simulated values at each node is a measurement of the spread of the conditional distribution at the location. This indicates the spatial uncertainty at a location on the map, because if the uncertainty is high, then there will be a greater spread between symmetric percentiles of the conditional distribution of simulated values. This will also be true of other measures of uncertainty, such as the variance of the simulated values. The map of the difference between the 10th and 90th percentiles of the simulated values of sand thickness is given in Figure 11. The map shows that the uncertainty of the grid locations near conditioning samples (i.e., where the seismic lines are located) is generally low. Areas of high uncertainty are located where few samples are available or the transition from high to low values occurs within short distances. These locations can be considered as potential locations for additional sampling. The areas with the highest uncertainty are located in the area of the reef-flat channel (Figure 11).

Probability map of exceeding threshold values 10 and 15m

We also used the conditional distributions generated using stochastic simulation to assess the probability of exceeding threshold values for the sand thickness. Maps depicting the probability of exceeding two threshold values for sand thickness, 10 m and 15 m, are presented in Figures 12 and 13. For example, if 15 m is known to be the lower economic limit for developing the sand resource, one could identify locations for which the probability of exceeding that thickness was greater than 50% (or any other probability level desired). One could then calculate the total area in Kailua Bay meeting that criterion.

Conclusions

After transformation to a coordinate space that is elongate parallel to the Kailua Bay reef front, we determined that the sand thickness in the Kailua Bay area has well-developed spatial continuity. That spatial continuity appears to be controlled by the geologic processes responsible for deposition and alongshore redistribution of the sand. Variogram models fit to the data indicate that the sand thickness has a range of 750 m in the direction parallel to the reef front (N-S in Figure 6), and 270 m perpendicular to the reef front, yielding an anisotropy ratio of about 3:1. The variogram models were used in a stochastic simulation program to generate 100 equiprobable maps of the sand thickness. The 100 maps provide an estimate of the conditional distribution of the sand thickness at each point of the map. Those conditional distributions can be summarized by several statistics useful for an exploration model for sand resources, including the expected value of the sand thickness, as well as measures of uncertainty, percentile maps, and probability of exceedance maps. In order to develop a more complete exploration model, we would need to incorporate data on the value of the resource (e.g., in dollars/ton of sand) and on the costs of resource development. Such an exploration model could be used to derive an estimate of the sand thickness that would maximize the economic potential of the resource.

References:

Deutsch, C. V. and Journel, A. G., 1998. GSLIB: Geostatistical Software Library and User's Guide, 2nd ed. Oxford University Press, New York, 368 pp.

Deutsch, C. V. and Wang, L., 1996. Hierarchical object-based stochastic modeling of fluvial reservoirs: Mathematical Geology, v. 28, no. 7, p. 857-880.

Isaaks, E. H. and Srivastava, R. M., 1989. An Introduction to Applied Geostatistics. Oxford University Press, New York, 561 pp.

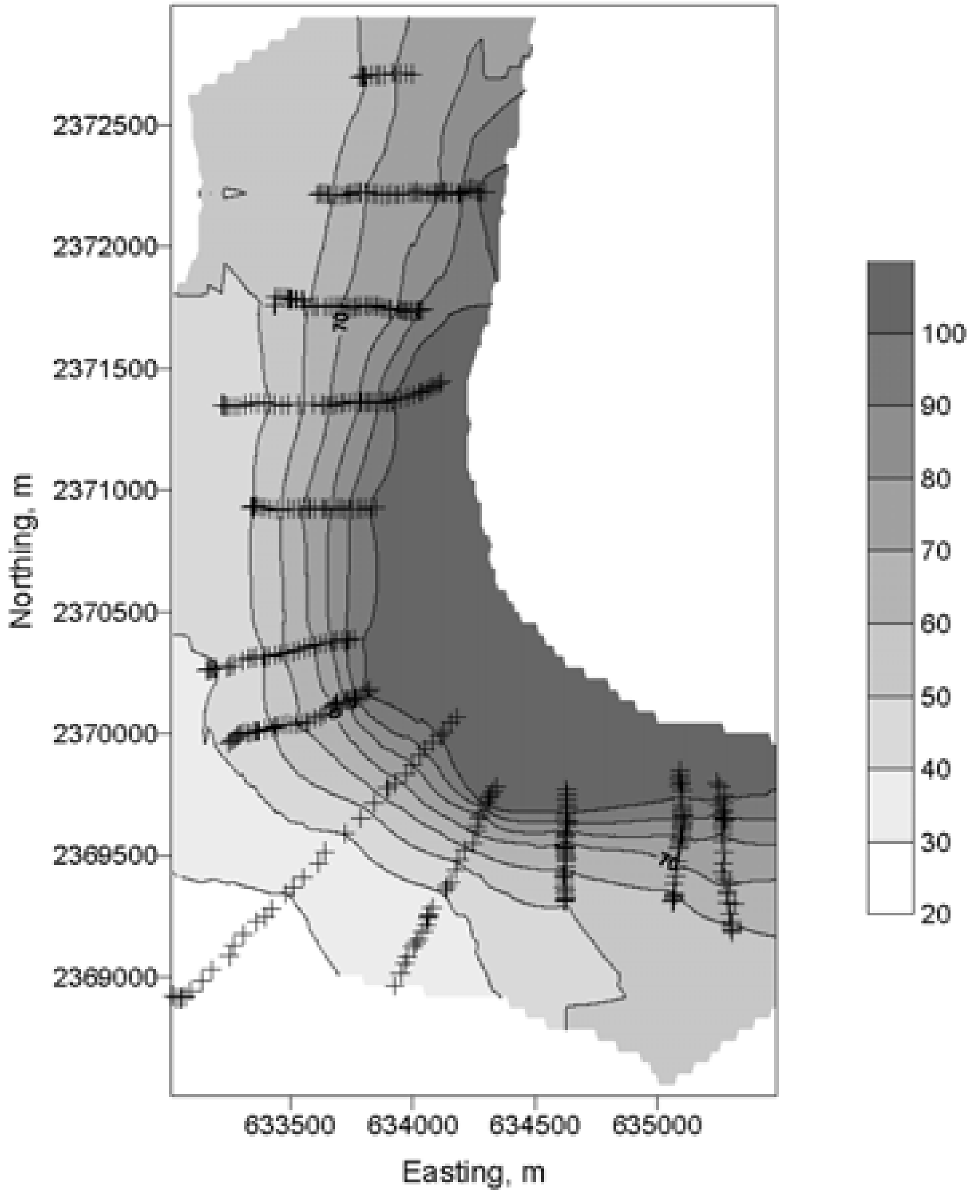

Figure 1: Seafloor depth (in meters) of Kailua Bay. Sample measurement locations are labeled with crosses.

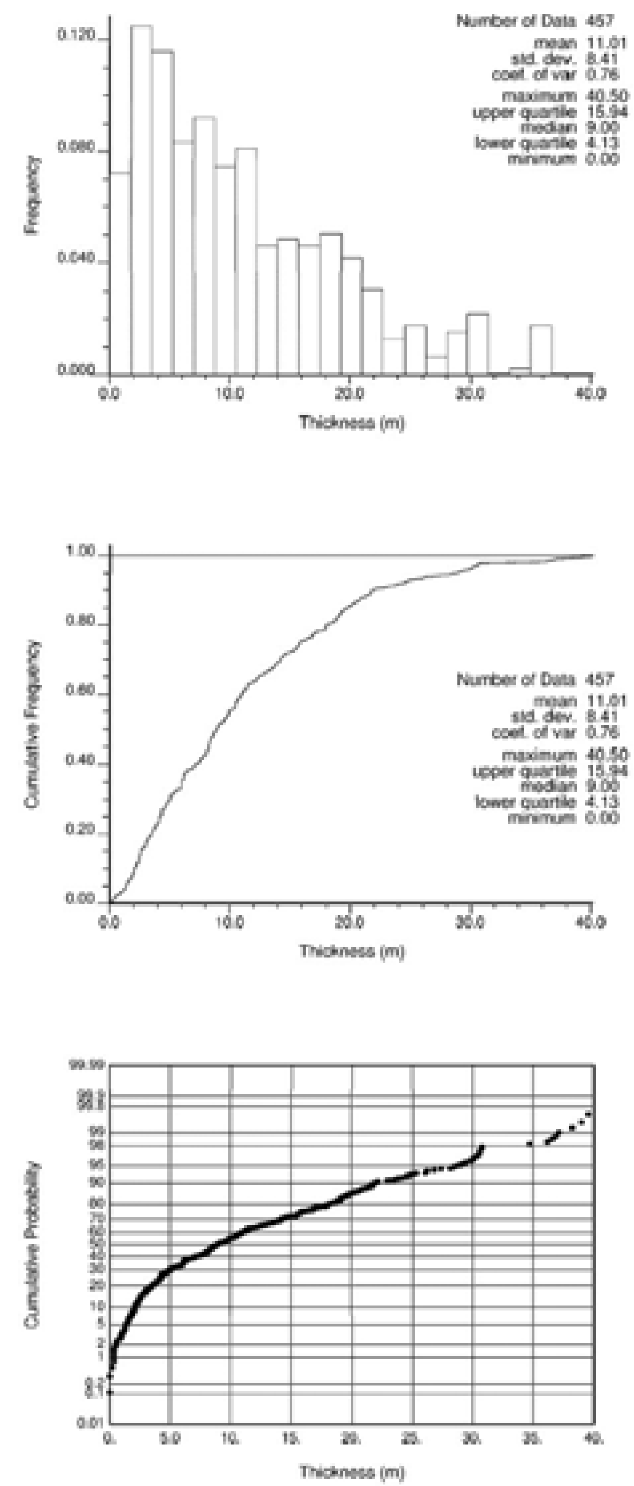

Figure 2: Histogram, cumulative density curve, and probability plot of sand thickness data in Kailua Bay

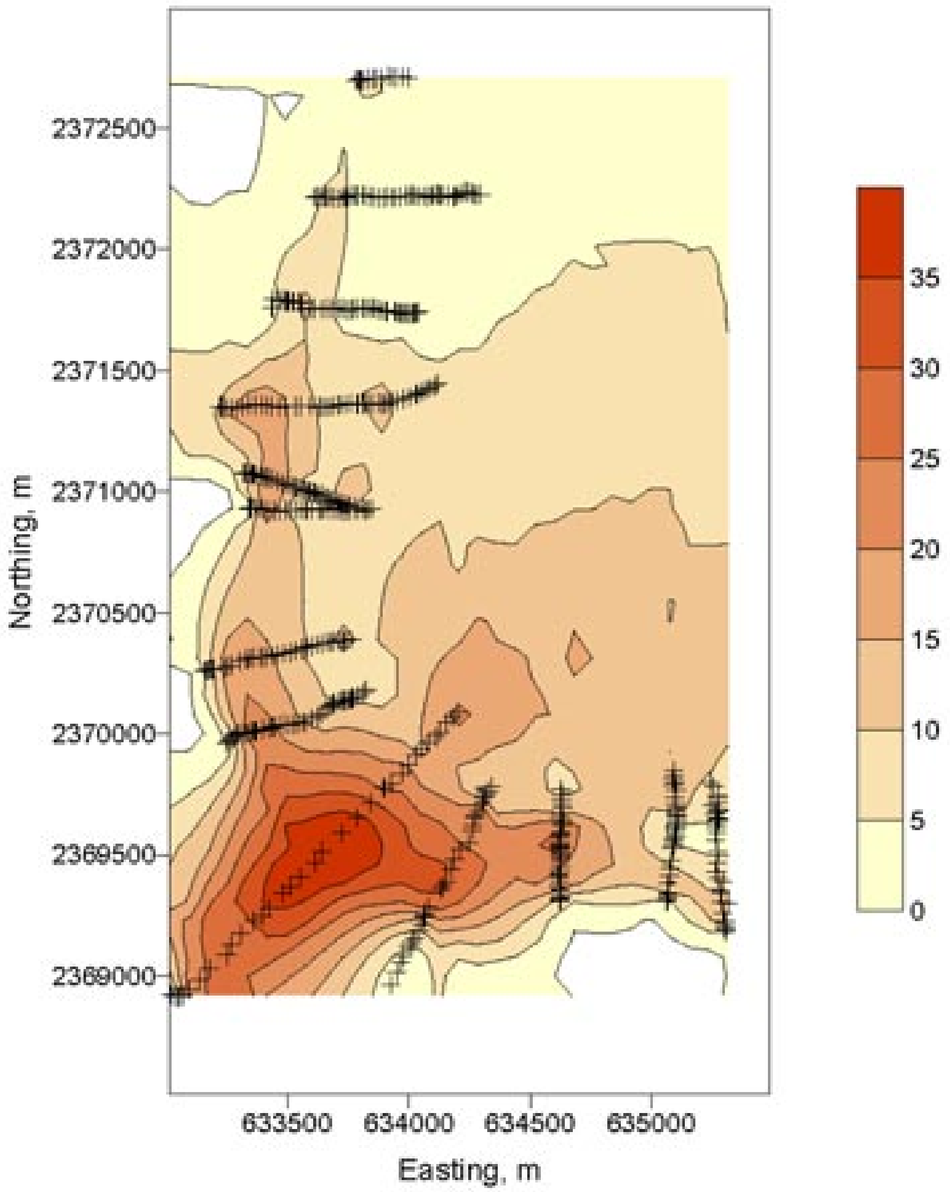

Figure 3: Preliminary contour of sand deposit thickness (in meters) in Kailua Bay

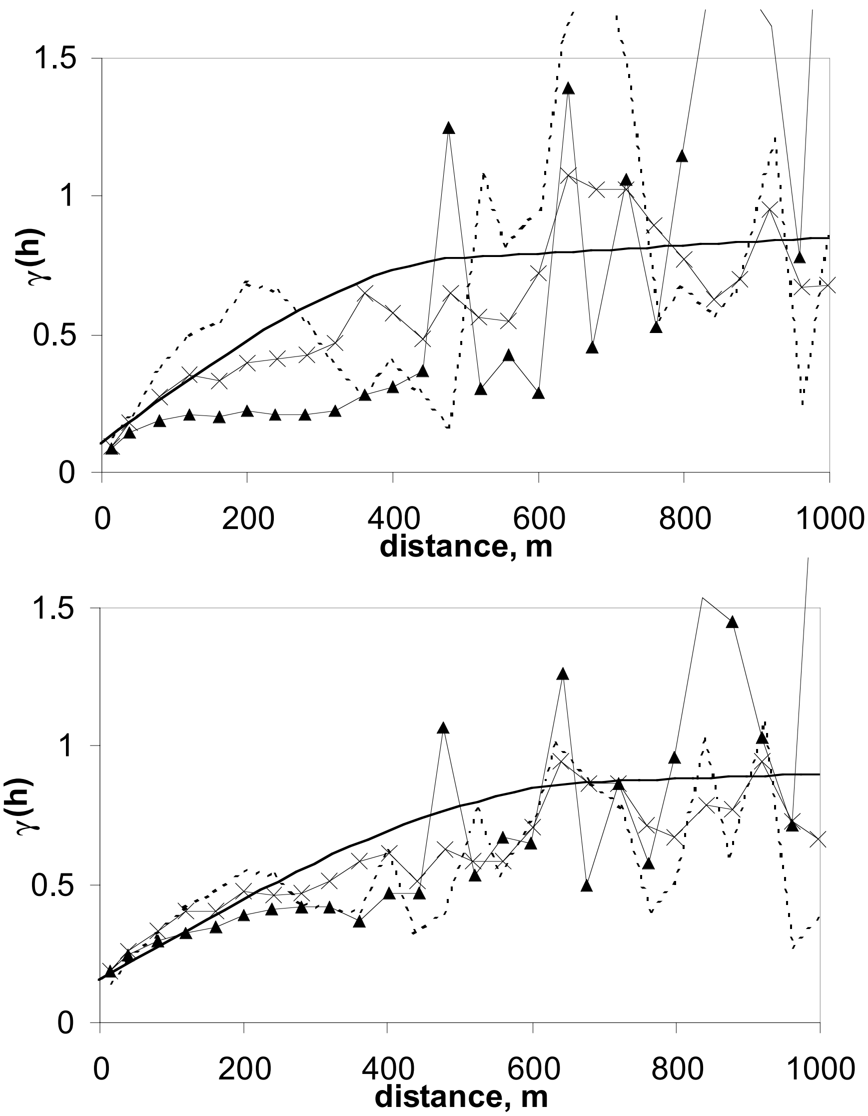

Figure 4: Semivariograms of original and normal score values of sand deposit thickness. The omnidirectional semivariogram is labeled with crosses, and the north-south and east-west semivariograms are labeled with squares and solid triangles, respectively. The black solid line represents the spherical model fit to the experimental variogram.

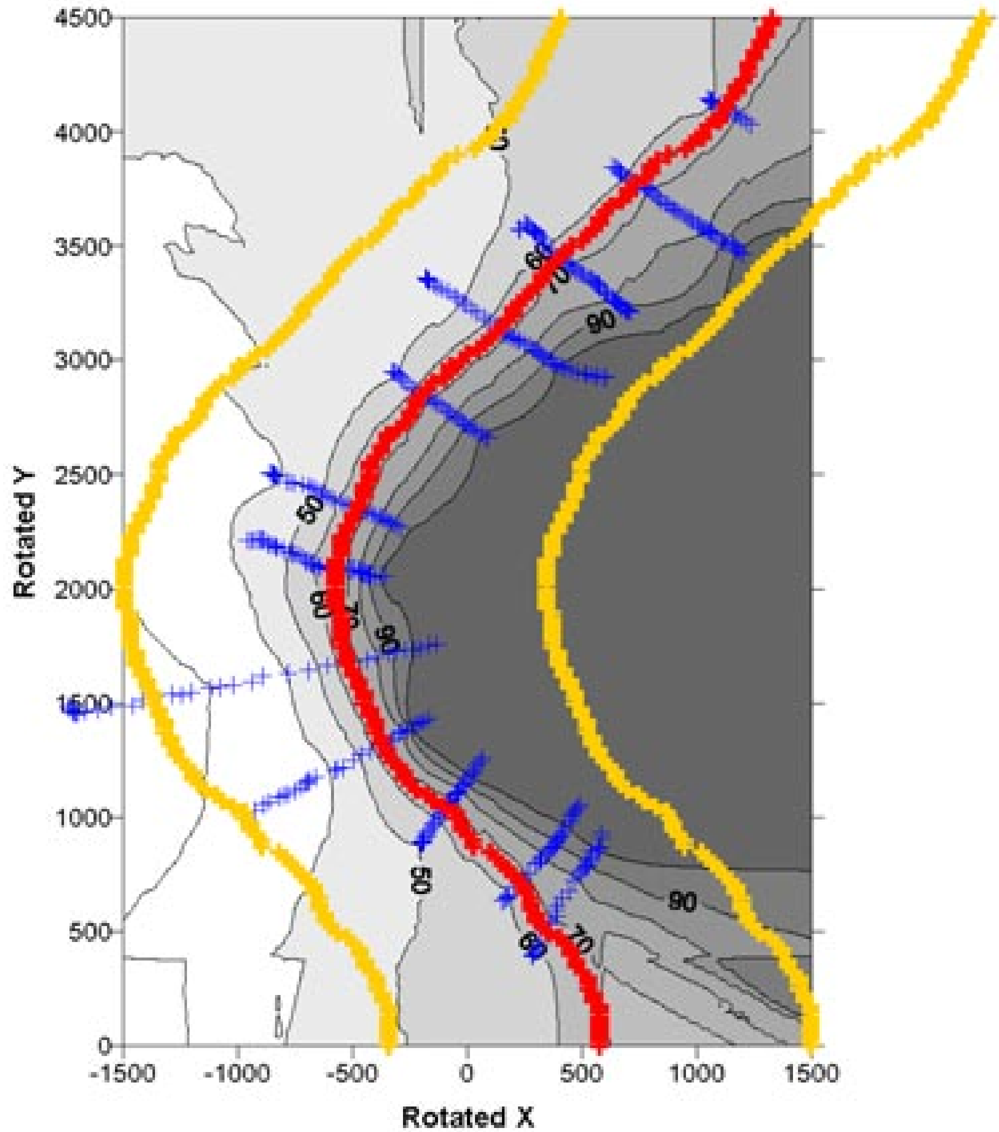

Figure 5: Rotated sample locations of sand deposit thickness (in blue cross) and contour of seafloor depth. The yellow and red crosses represent the boundaries of the transformation domain and the center line, respectively.

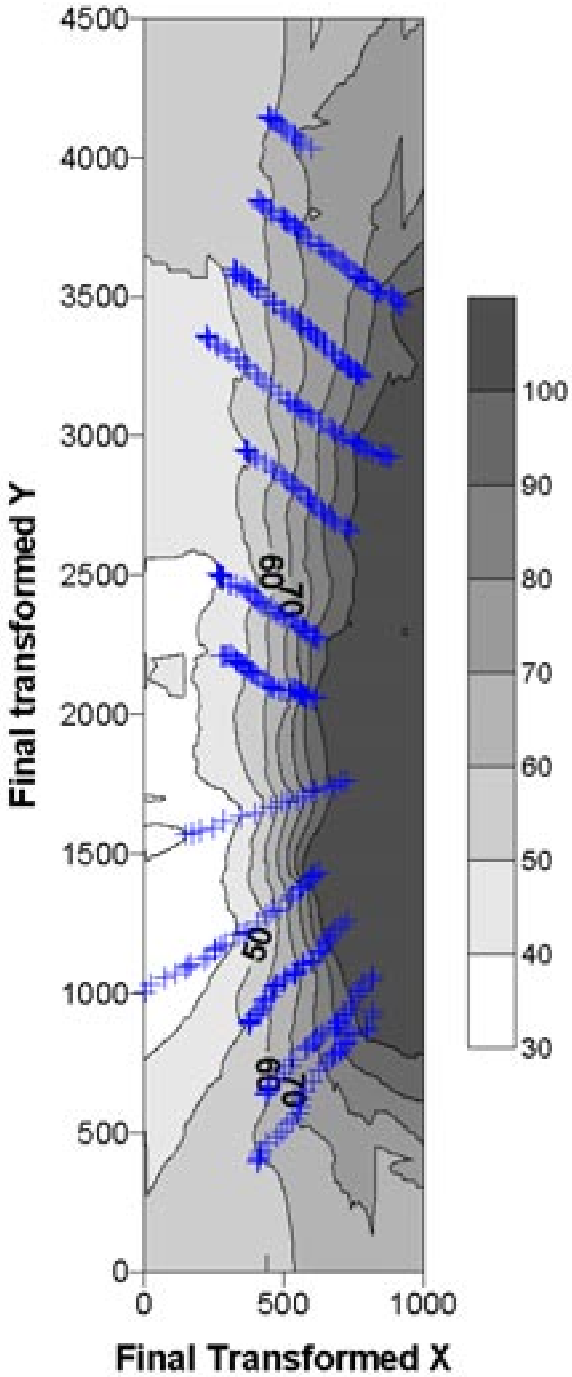

Figure 6: Seafloor depth of Kailua Bay in computational space

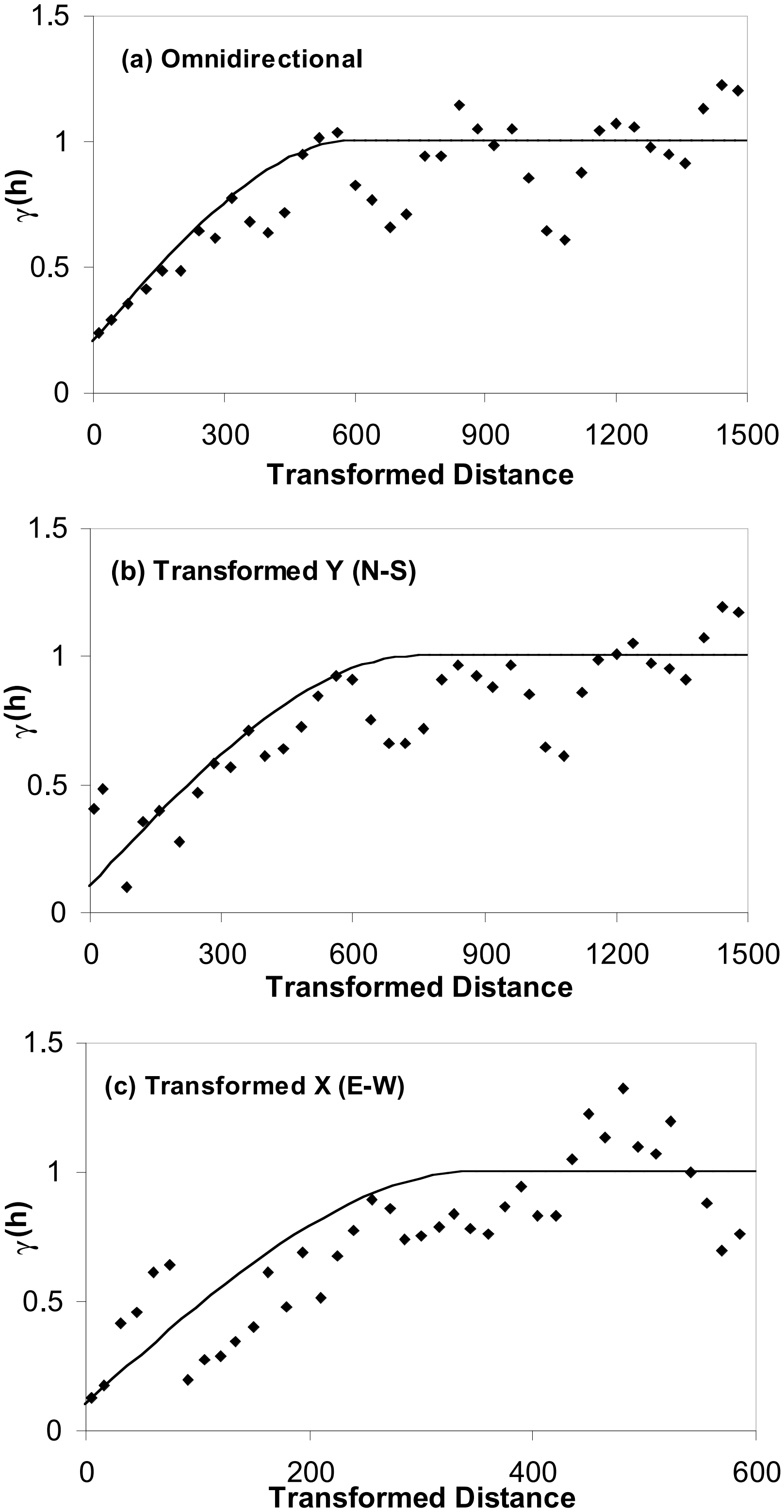

Figure 7: Semivariograms of the normal score values of sand deposit thickness in the computational space

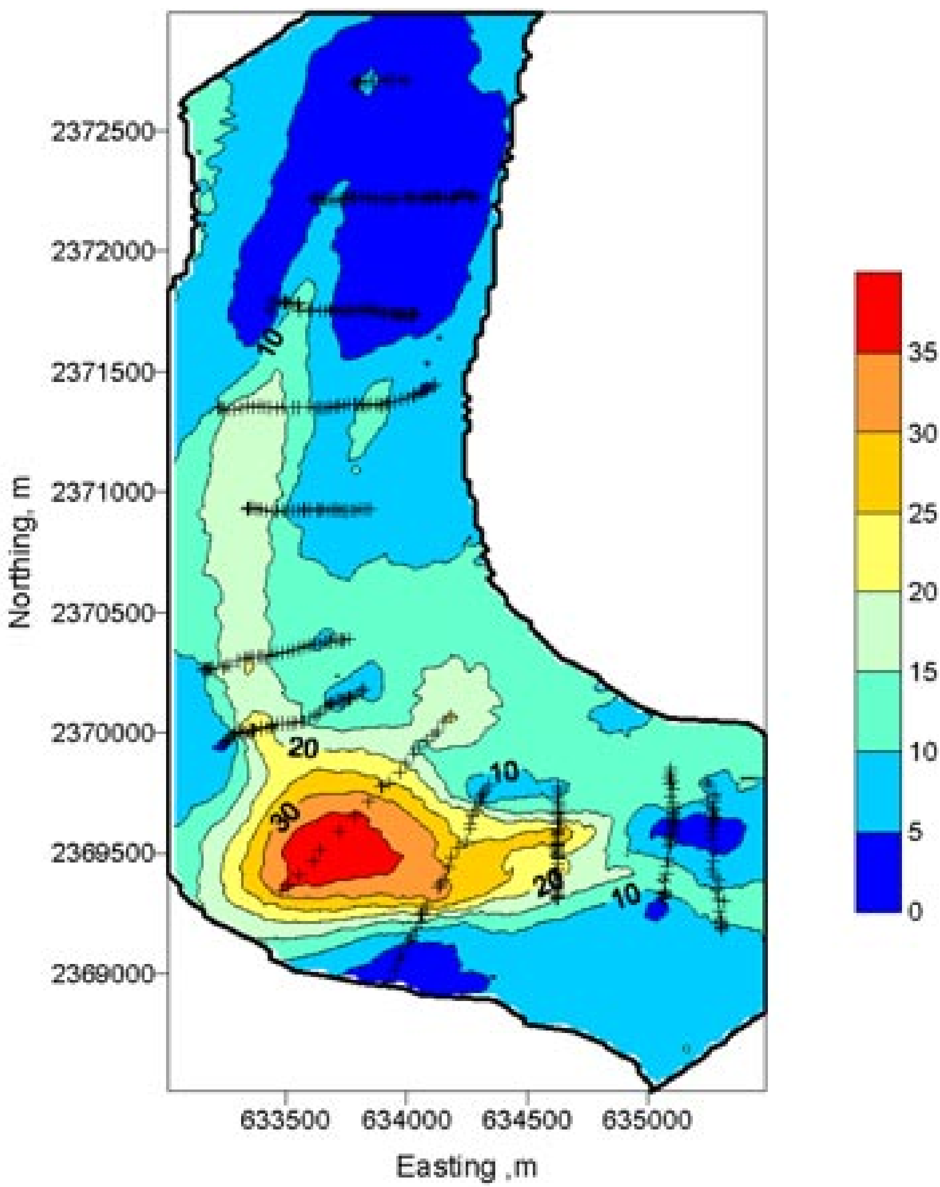

Figure 8: E-type estimates of simulated sand deposit thickness (in meters) in Kailua Bay. The data used in the simulation are labeled with black crosses.

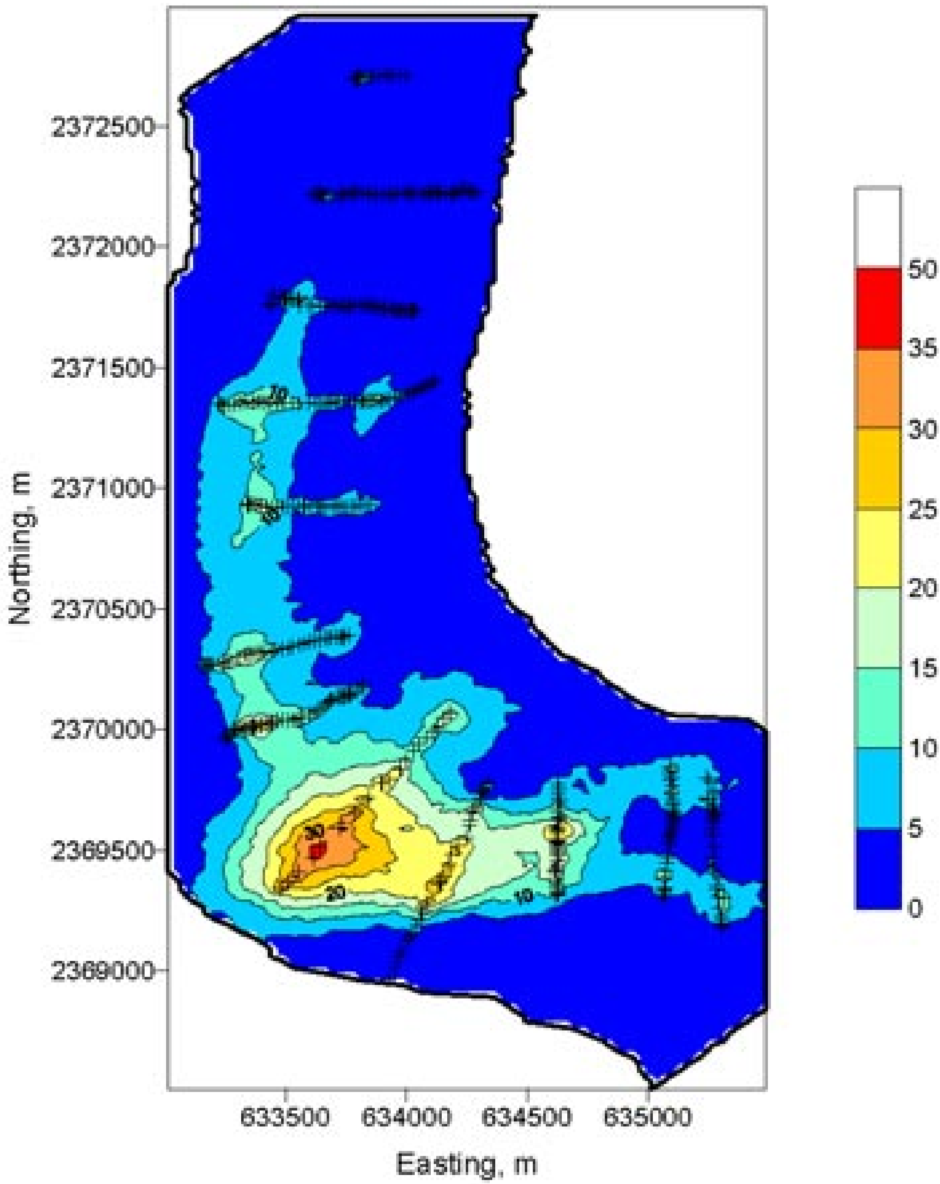

Figure 9: Map of the 10th percentile of the simulated sand deposit thickness (in meters) in Kailua Bay. The data used in the simulation are labeled with black crosses.

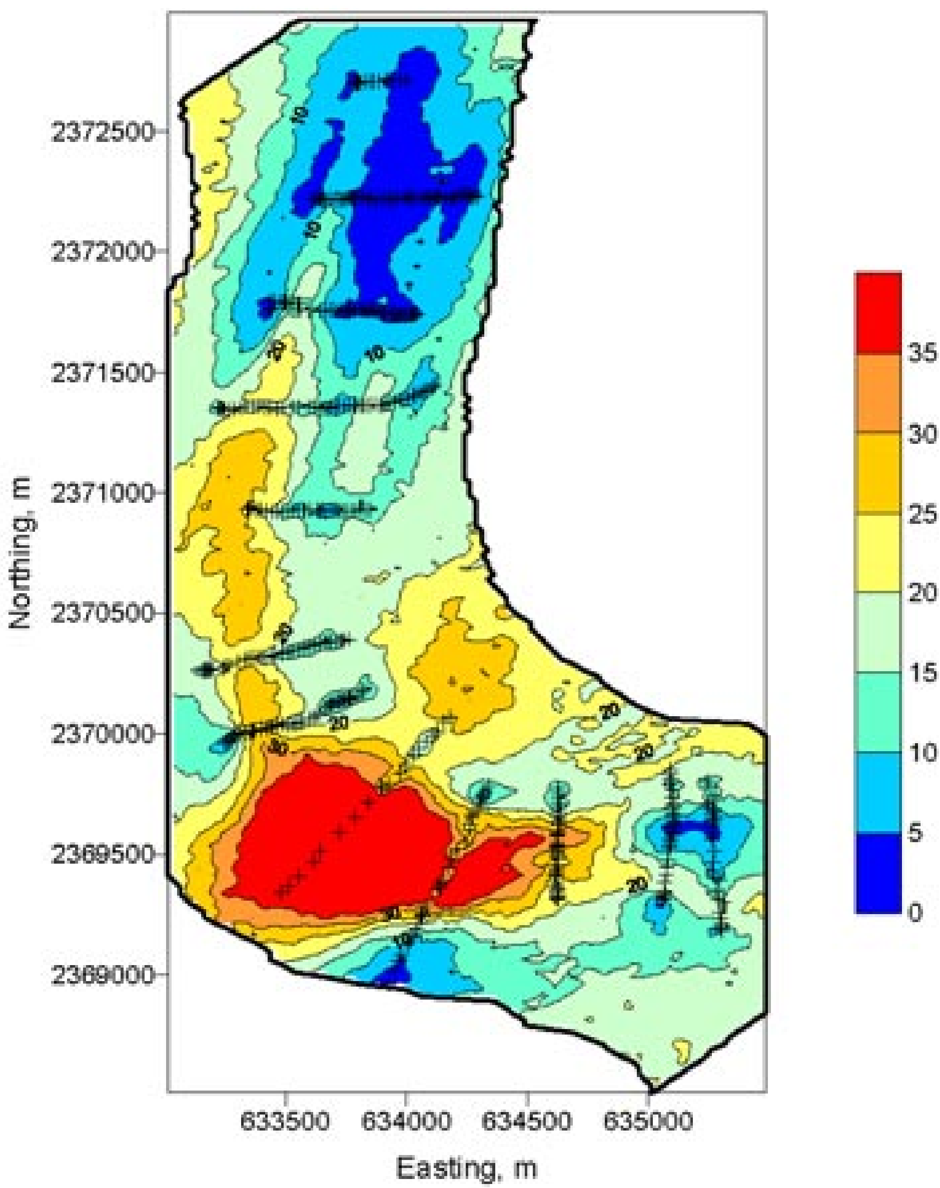

Figure 10: Map of the 90th percentile of the simulated sand deposit thickness (in meters) in Kailua Bay. The samples used in the simulation are labeled with black crosses.

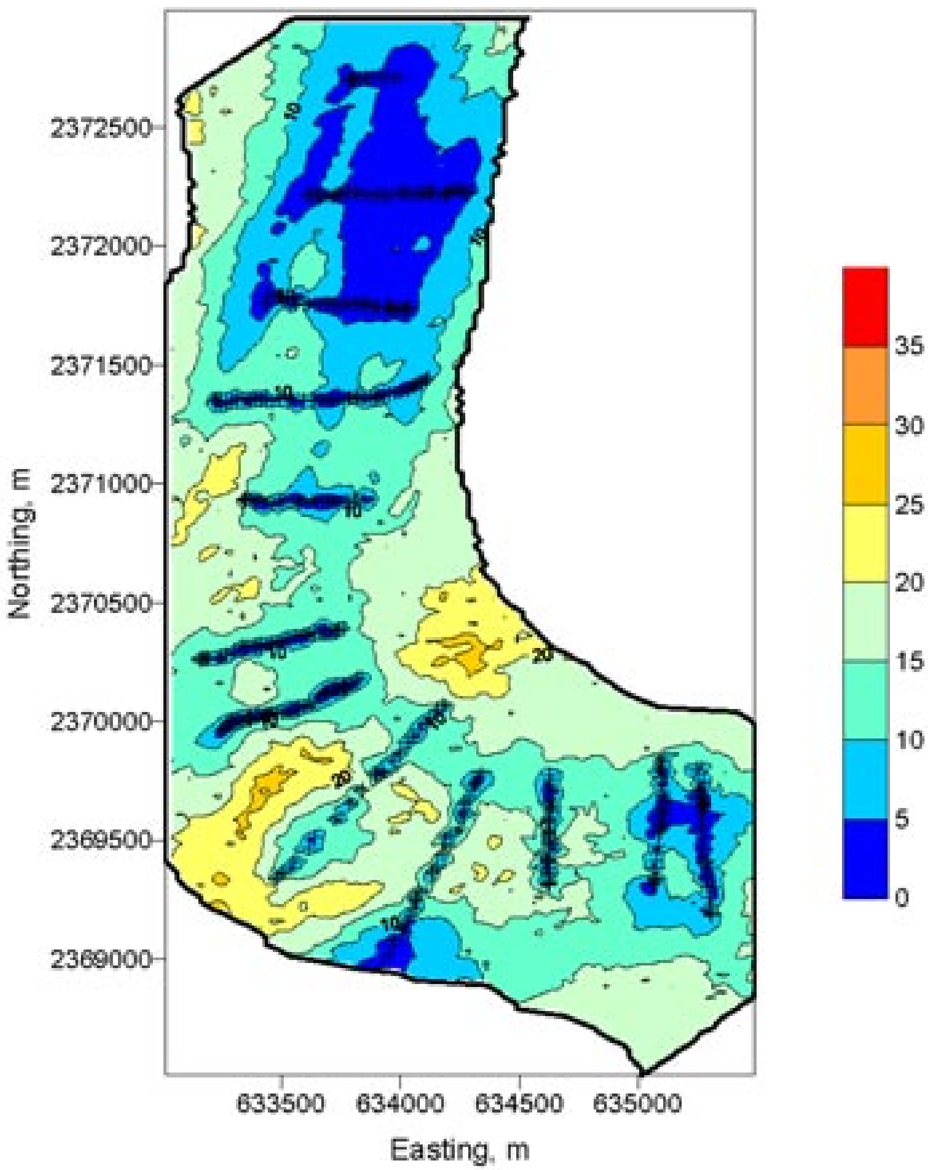

Figure 11: Map of the differences of the 90th and 10th percentile of the simulated sand deposit thickness (in meters) in Kailua Bay. The samples used in the simulation are labeled with black crosses.

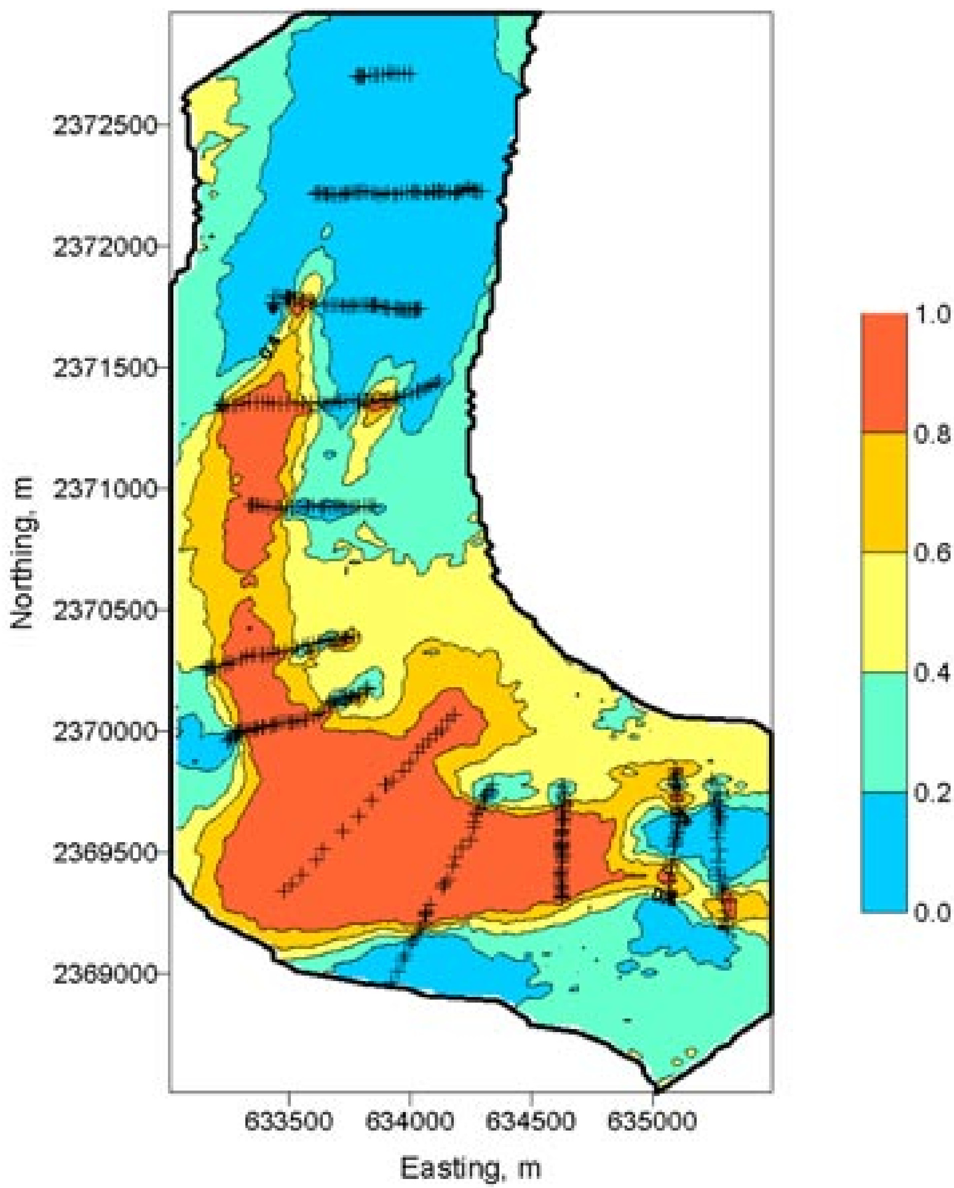

Figure 12: Probability of exceeding sand thickness of 10 m. The samples used in the simulation are labeled in black cross.

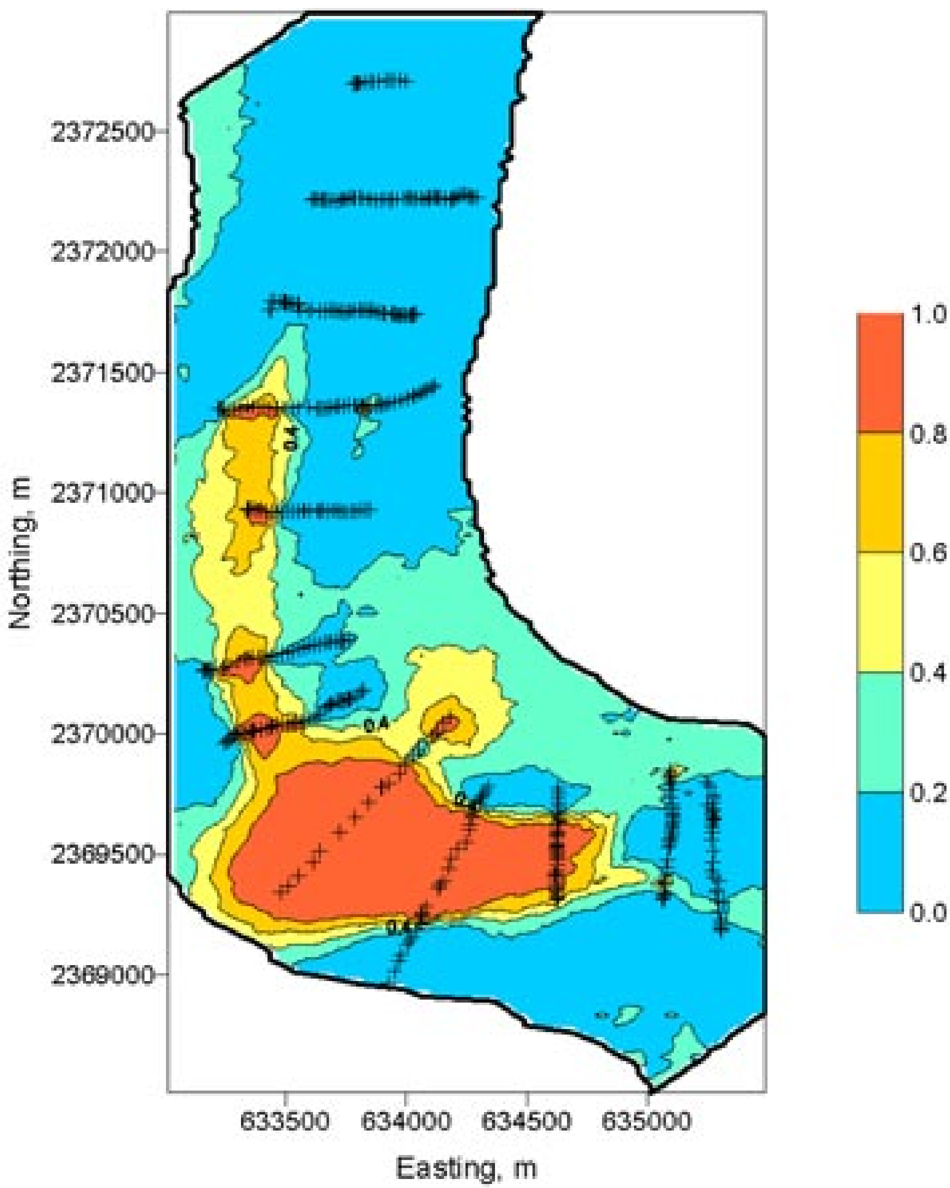

Figure 13: Probability of exceeding a sand thickness of 15 m. The samples used in the simulations are labeled with black crosses.