Sand Thickness Exploration Model – Leeward and North Shore

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Introduction

The purpose of the geostatistical analysis was to analyze the spatial distribution of the thickness of sand deposits in the reef-front areas on the Leeward and North Shores of the island of Oahu, and to provide probabilistic maps that can be used as the basis for an exploration model for sand resources in the two study areas. The data used in the study consisted of seismic reflection measurements of the thickness of carbonate sand draped over a Pleistocene lowstand terrace. The seismic measurements were made by researchers at the U.S. Geological Survey (USGS). The reef-fronts in the Leeward and North Shore areas have different orientations and degrees of curvature (Figures 1 and 11). The study area on the Leeward shore extends from the northwest to the southeast with a relatively narrow shelf area on which seismic data were recorded. The Leeward shore contains scattered areas with no sand deposits that were mapped by the USGS (Figure 1). The study area on the North Shore extends from the west and curves toward the northeast (Figure 11). The areas with zero sand deposits on the North Shore are more widespread than those on the Leeward shore, and are often continuous from line to line. The areas with no sand data were interpreted by the USGS as bare coral ridges on the shelf that protrude above the sand deposit.

The curvature of the two study areas posed a problem for the variogram analysis and geostatistical mapping because the sedimentation processes responsible for the distribution of sand deposit were expected to be aligned parallel to the reef-front. Traditional variogram analysis cannot adequately deal with data in which the spatial continuity exhibits a high degree of curvature. In order to account for the curvature, the commercial software package, Gridgen® (www.pointwise.com), was used to generate orthogonal grids for the variogram analysis and simulation. First, the simulation domain boundaries of each area were defined in the Arc/Info™ Geographic Information System (GIS), which were then exported into Gridgen. An orthogonal structured grid of the desired spatial resolution was defined in Gridgen® using the elliptic PDE solver in Gridgen® (based on the Steger - Sorenson condition). These orthogonal grid points were exported into Arc/Info ™ coverages, and the GIS was used to associate each of the seismic thickness data to the nearest grid node. Seismic data that ended up being associated with the same grid coordinates were averaged. Finally, the resulting data set with the corresponding orthogonal grid indices was used in the variogram analysis and stochastic simulation. Since the original UTM coordinates corresponding to the orthogonal grid node indices were known from the exported Arc/Info™ coverage, the results from the geostatistical simulation in the orthogonal grid space could be plotted directly in the original UTM coordinates.

The geostatistical analysis included variogram analysis (Isaaks and Srivastava 1989) to measure the spatial continuity of the sand thickness data. Sequential Gaussian simulation was used to simulate the sand thickness across each of the study areas. The variograms of the normal score values (Deutsch and Journel 1998) of the sand thickness data were modeled and input in the simulation routine to generate a suite of 100 realizations in each study areas. These realizations were post-processed to generate maps of statistics for an exploration model of sand deposit in Leeward and North Shore. In the areas mapped by the USGS as having no sand deposits, any simulated values were replaced with zeroes during the post-processing.

Leeward Shore

Exploratory data analysis and coordinate transformation

The locations of the measurements of sand thickness on the Leeward shore are shown in Figure 1, with the thick black line denoting the boundary of the study area. The polygons outlined by thin black lines within the study area highlight the areas with no sand deposits. A total of 2471 measurements are distributed along the seismic sampling lines that are mostly orthogonal or parallel to the seafloor depth contour lines. The histogram in Figure 2 shows the positively skewed distribution of sand thickness data, with a sample mean of 6 m, a sample median of 4.7 m and a range of 0.0 to 23.8 m.

A simulation domain of 30 by 400 grid nodes was defined using Gridgen, with an approximate spacing of 50 m as shown in the mesh map in Figure 3. The coordinate of the nearest grid node was assigned to each original measurement, and then if more than one data value was associated with the same grid node, those values were averaged. The resulting 864 data on the computational domain are shown in Figure 4. The distribution of the transformed data remains close to that of the original data, with a sample mean of 5.6 m, a sample median of 4.3 m and a range of 0.0 to 20.3 m. Note that this transformation process "straightened" the curvature of the original data and projected the data into an orthogonal computational domain on which the variogram modeling and simulation were performed.

Variogram modeling and stochastic simulation

The directional semivariograms of the normal score values for the transformed sand thickness data are plotted in Figure 5. Spherical variogram models (Isaaks and Srivastava 1989) were fit to the directional semivariograms and are plotted as the solid lines in Figure 5. The direction of the greatest spatial continuity is the direction parallel to the reef front (deg=0 in Figure 5) with a range of 13 distance units. The direction of minimum continuity is perpendicular to it, with a range of distance units. The maximum and minimum ranges of spatial continuity are approximately 650 m and 300 m, respectively, in the original coordinate system. The nugget effect in the spherical model is 0.06. These variogram model parameters were then used as input to the stochastic simulation process.

The conditional probability distribution of the sand thickness within the computational domain was estimated from a series of equiprobable realizations generated by stochastic simulation. The sequential Gaussian simulation routine SGSIM from GSLIB (Deutsch and Journel, 1998) was used to generate a set of 100 realizations of sand thickness on the 30 by 400 grid node mesh in the computational domain. Note that zero values were assigned to the nodes within the polygons with no sand deposit after the simulation. The 100 simulated values at each node form a conditional distribution of sand thickness, which can be used to calculate several statistical parameters as described below.

The median of the 100 simulated values at each node is plotted in the original UTM coordinates as Figure 6. Note that the seismic data are honored at nearby simulation nodes. The area of greatest median sand thickness occurs in the northwestern part of the map. That area appears to have a thickness of more than 16 meters of sand over a distance of approximately 1000 meters parallel to the shore. The upper quartiles of the 100 simulations are also plotted in Figure 7, showing that most of the upper quartiles of the simulated sand deposit values are less than 14 meters except for the thick sand deposit in the northwestern part of the map. The difference between the 25th and 75th percentiles, i.e. the interquartile range, of the simulated values at each node shows the spread of the conditional distributions at the simulation nodes. The spread of the distribution represents a measure of spatial uncertainty at each simulated node. The simulated nodes that are close to conditioning data generally have very small interquartile ranges, indicating a very low uncertainty. As shown in Figure 8, lines of low interquartile ranges appear where seismic lines are located. The location with high uncertainty in the northern part of the study area represents a gap in the seismic lines that is somewhat larger than the spacing for the rest of the seismic lines. It suggests that an additional seismic line may be justified to reduce the uncertainty in this area. This is especially true because that area of relatively high uncertainty is only 1-2 km north of the area of thickest sand development on the Leeward shore. The conditional distributions of simulated sand thickness values were also used to calculate and map the probability that the thickness of the sand deposit exceeds certain threshold values. We used two thickness thresholds for the mapping, 5-m and 10-m, shown in Figures 9 and 10. For example, Figure 9 could be used to identify areas that have a high probability of exceeding the 5-m threshold, and target them for resource development. Using the map, we calculated that only 9% of the study area has at least a 70% probability of exceeding the 5-m threshold. Similar calculations could be performed for the probability of exceeding the 10-m threshold (Figure 10), and in fact, the exercise could be repeated for any combination of thickness and/or probability thresholds.

North Shore

Exploratory data analysis and coordinate transformation

The lines of seismic reflection measurements of sand deposit thickness are shown in Figure 11. The thick black line represents the simulation domain within which the orthogonal grids were generated. The scattered polygons outlined by thin black lines within the simulation domain are the areas that have no sand deposits, as mapped by the USGS. As mentioned above, these areas were set to zero during post-processing of the simulations, and they are much more extensive and continuous in the North Shore area than they are in the Leeward shore area. The positively skewed histogram of 3063 sand thickness measurements is plotted in Figure 12, with a sample mean of 5.0 m, a sample median of 3.9 m, and a range from 0.0 to 20.8 m.

As with the Leeward shore area, the simulation domain for the North Shore was defined using Gridgen®, resulting in an orthogonal grid with 300 grid nodes in the east-west direction and 80 grid nodes in the north-south direction (Figure 13). The approximate spacing in the east-west direction is 50 m and 40 m in the north-south direction. The seismic sand thickness data were associated to the nearest orthogonal grid coordinates using the GIS system. As in the procedure described above, those data associated with the same grid coordinates were averaged and output as one conditioning value for that node. The resulting 1660 transformed conditioning data are plotted on the orthogonal simulation grid indices in Figure 14. The frequency distribution of these transformed data is very close to that of the original data, with a sample mean of 4.3 m, a sample median of 3.4, and a range from 0.0 to 20.6 m. Note that the relative orientations between seismic lines in the original location map (Figure 11) are well preserved in the projected computational domain (Figure 14).

Variogram modeling and stochastic simulation

The directional semivariograms of the normal score values of the transformed sand thickness data are shown in Figure 15. The semivariograms were fit with a spherical model with a nugget of 0.06. Like the model for the Leeward shore, the greatest spatial continuity appears to be in the direction parallel to the reef-front, which was projected in the transformed X direction, with a range of 33 distance units (deg=90). The least spatial continuity was in the directional perpendicular to the reef-front as projected as the transformed Y direction (deg=0) with a range of 23 distance units. Therefore, the ranges of greatest and least spatial continuity in terms of the original UTM coordinates are approximately 1600 m and 920 m. This is about 3 times greater than the range of continuity of sand thickness values at the Leeward Shore, indicating that the sediment transport processes responsible for the sand deposits on the North Shore operated more smoothly over a larger area than occurred on the Leeward shore.

The parameters of the variogram model for the normal score values were used as input in the sequential Gaussian simulation routine SGSIM from GSLIB (Deutsch and Journel, 1998). A total of 300 by 80 grid nodes were simulated using the transformed sand data in the computational grid space. A suite of 100 realizations was generated to establish the conditional distributions on the simulation grids. The statistical parameters and probability of exceedance for the 100 realizations were calculated as above, including the median, upper quartile, interquartile range, and probability of exceeding a sand thickness of 5 m and 10 m. The simulated grid nodes within the polygons with no sand deposits were set to zero using the GIS system during post-processing of the simulations. The resulting contour maps in the original UTM coordinates are presented in Figures 16 to 20.

From the map of median simulated sand thickness (Figure 16), two major areas with high values of sand thickness can be seen. One area with sand thickness greater than 16 m occurs in the southwestern part of the map, and an area with sand thickness greater than 14 m occurs in the northeastern part of the map. The sand thickness in the area just east of the center of the map is somewhat higher than other areas, with thickness values mostly ranging from 6 m to 14 m with a few small areas having values greater than 14 m. The sand thickness values of the rest of the study area appear to be very low by comparison. The same trends in the distribution of the thickest sand deposits can be seen in the map of upper quartiles (Figure 17). The areas with highest uncertainty seen on the map of interquartile ranges (Figure 18) occur in a large gap between two seismic reflection lines just east of the area of greatest curvature (X ~ 591000, Y~ 2390000). Another area of higher uncertainty occurs in an area where high sand thickness data values are close to the polygons with no sand deposits in the northeastern part of the map (Figure 18). Those two areas are potential locations for additional seismic lines for reducing the uncertainty. Two threshold values of 5 m and 10 m were used to calculate the probability of exceedance for the 100 simulations, and those probability maps are shown in Figures 19 and 20. The areas with a high probability of exceeding the thickness thresholds occur in the three thick sand deposit areas found in the west, east central and northeastern areas of the map.

Conclusions

The exploration model for the thickness of sand deposits on the Leeward and North Shores of Oahu was processed by the following procedures. First, Gridgen was used to generate the desired orthogonal grids within the defined simulation domains, to account for the curvature of the study areas. The orthogonal grid indexes were used as the computational coordinates for variogram analysis and stochastic simulation. Variogram models fit to the transformed data suggest that the ranges parallel to and perpendicular to the reef front are approximately 650 m and 300 m for the Leeward shore area, and 1650 m and 920 m for North Shore. The resulting anisotropy ratios are similar to one another, at about 2.2 and 1.8, respectively, but the difference in the magnitude of the variogram ranges suggests that the thickness of the sand deposits is about 3 times as continuous at the North Shore area as it is on the Leeward shore. The 100 equiprobable realizations generated by sequential Gaussian simulation provide an estimate of the conditional distributions of the sand thickness throughout the study area. Statistical parameters calculated from the conditional distributions provide useful information for an exploration model of sand deposits in the study areas. Maps were produced based on these statistics, including the median simulated values, interquartile ranges as an indication of uncertainty, and the probability of exceeding selected sand thickness thresholds. These maps could be used to identify areas with reasonable sand resources that could be developed for beach replenishment, and to estimate the volume of sand present in those areas.

References:

Deutsch, C. V. and Journel, A. G., 1998. GSLIB: Geostatistical Software Library and User's Guide, 2^{nd} ed. Oxford University Press, New York, 368 pp.

Isaaks, E. H. and Srivastava, R. M., 1989. An Introduction to Applied Geostatistics. Oxford University Press, New York, 561 pp.

Tecplot, version 8.0, Amtec Engineering, Inc., Bellevue, Washington, U.S.A.

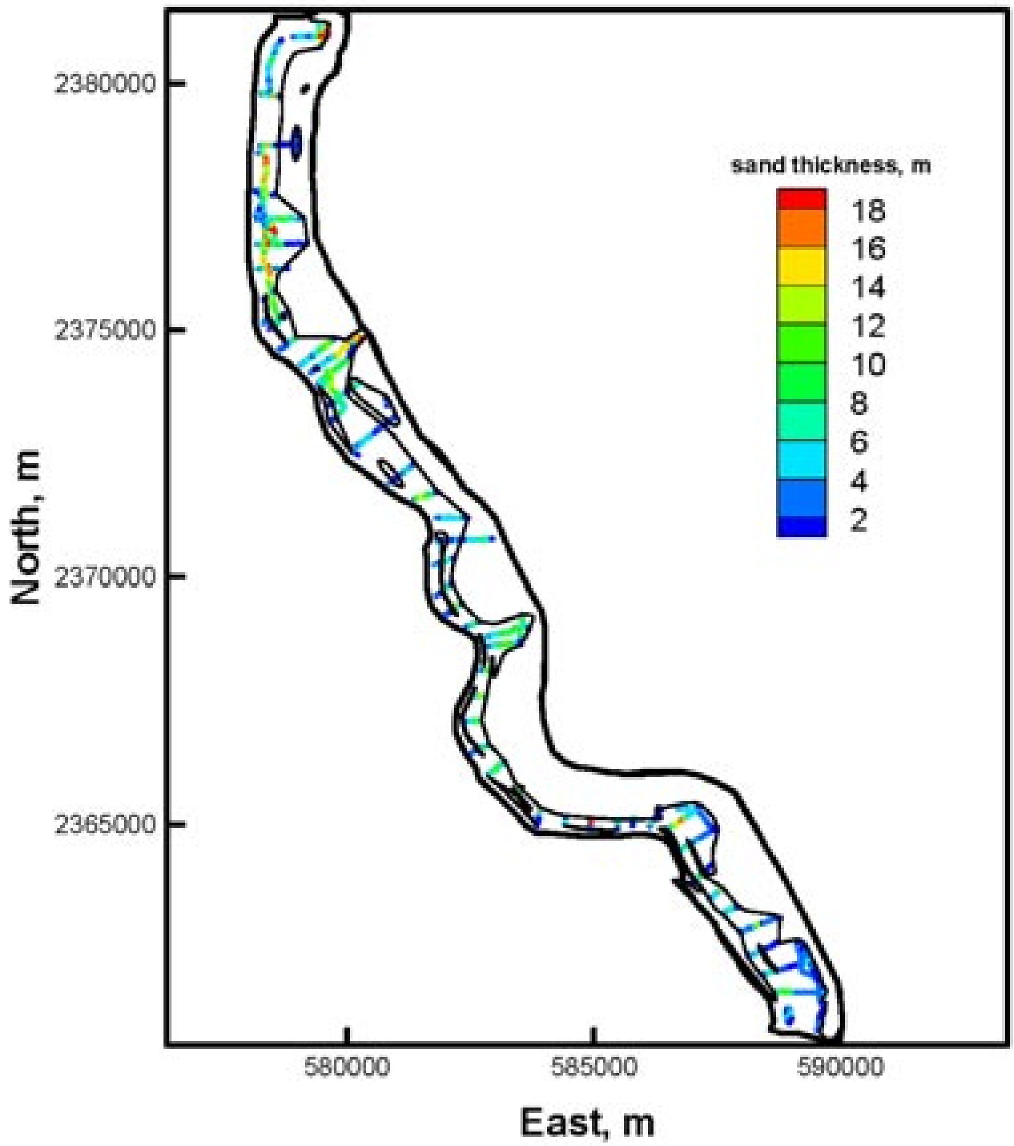

sand thickness, m
18
16
14
12
10
8
6
4
2
North, m
2380000
2375000
2370000
2365000
580000
585000
590000
East, m

Figure 1: Location map of sand thickness data on Leeward shore, Oahu

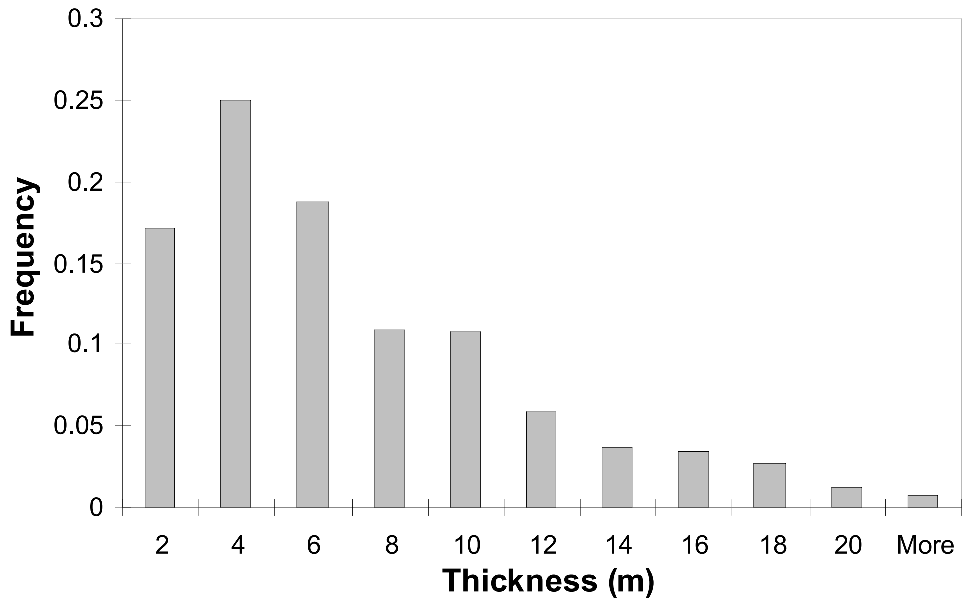


Figure 2: Histogram of sand thickness data of Leeward shore, Oahu

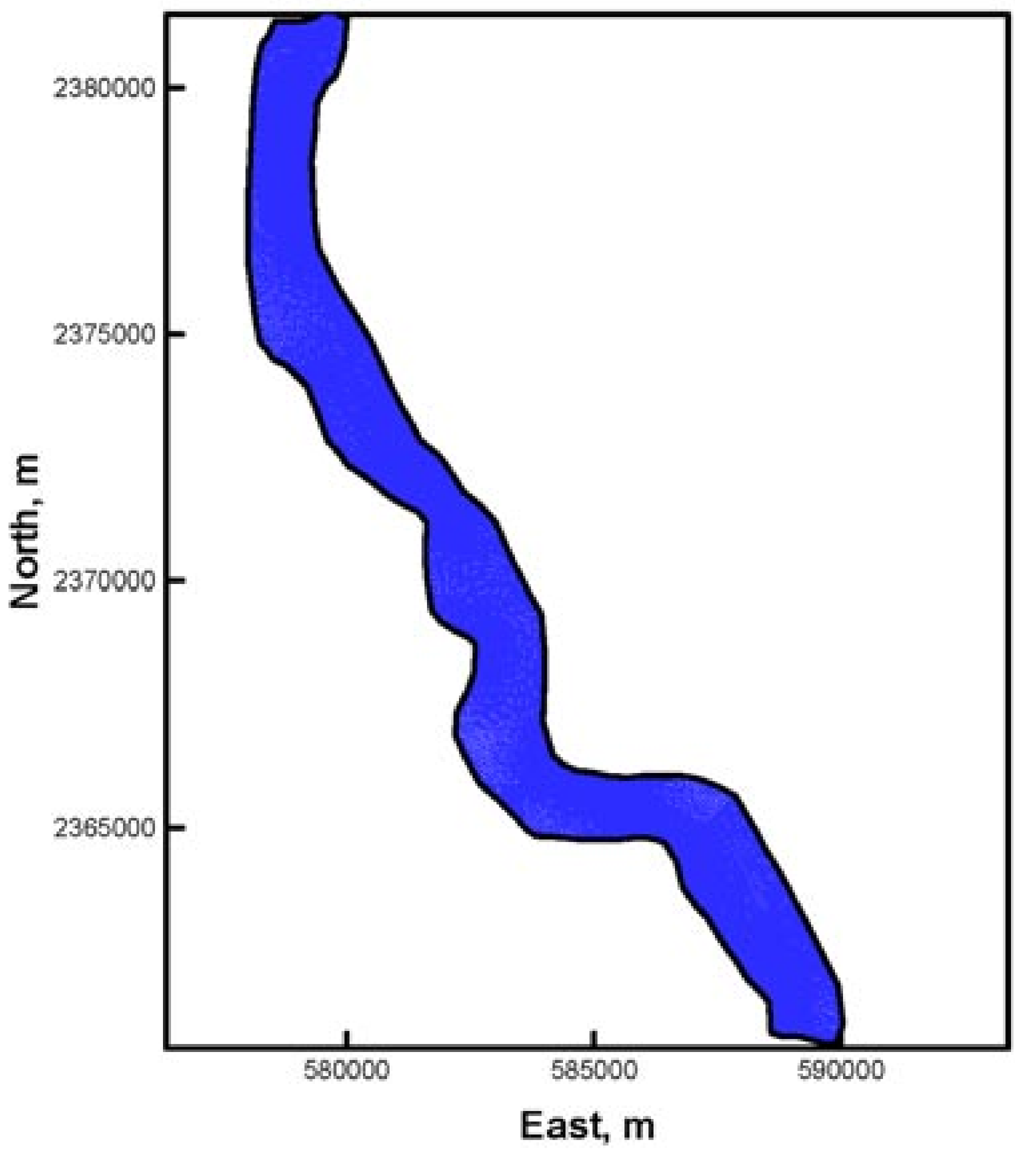
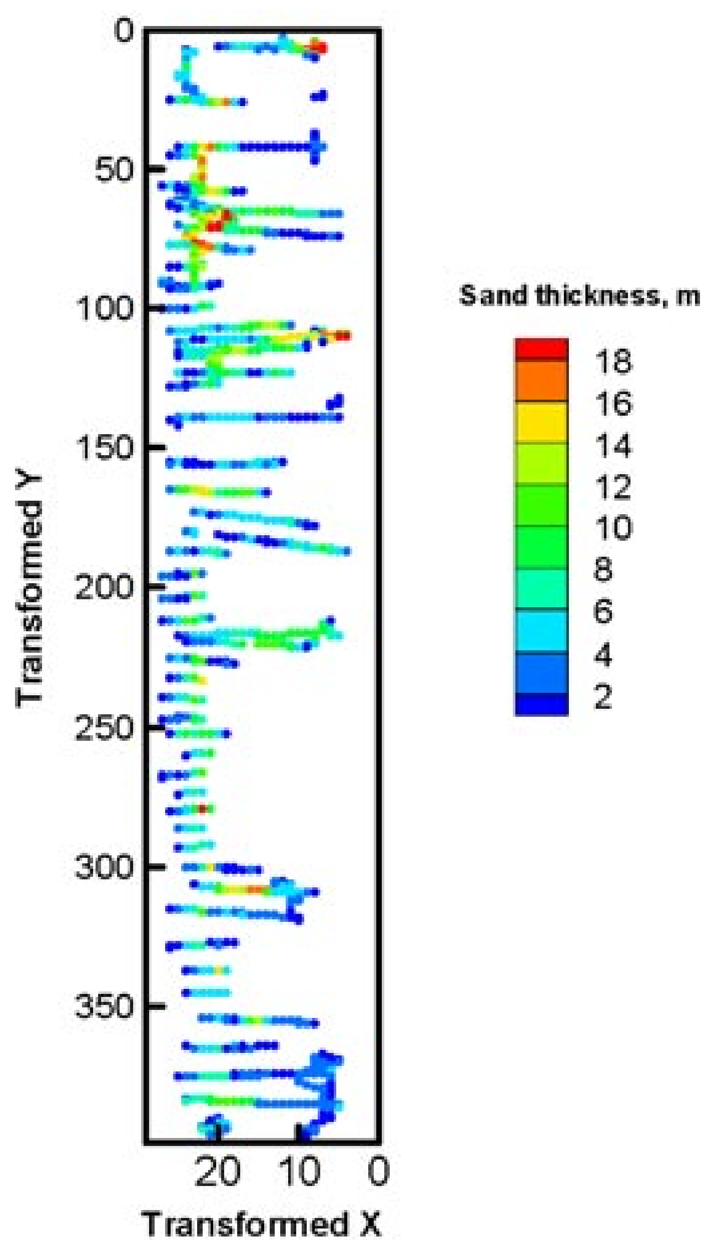

2380000
2375000
2370000
2365000
North, m
580000
585000
590000
East, m

Figure 3: Mesh map of the simulation domain of Leeward shore, Oahu



Sand thickness, m
18
16
14
12
10
8
6
4
2
Transfomed Y
0
50
100
150
200
250
300
350
20
10
0
Transformed X

Figure 4: Location map of transformed sand data of Leeward shore in the computational space

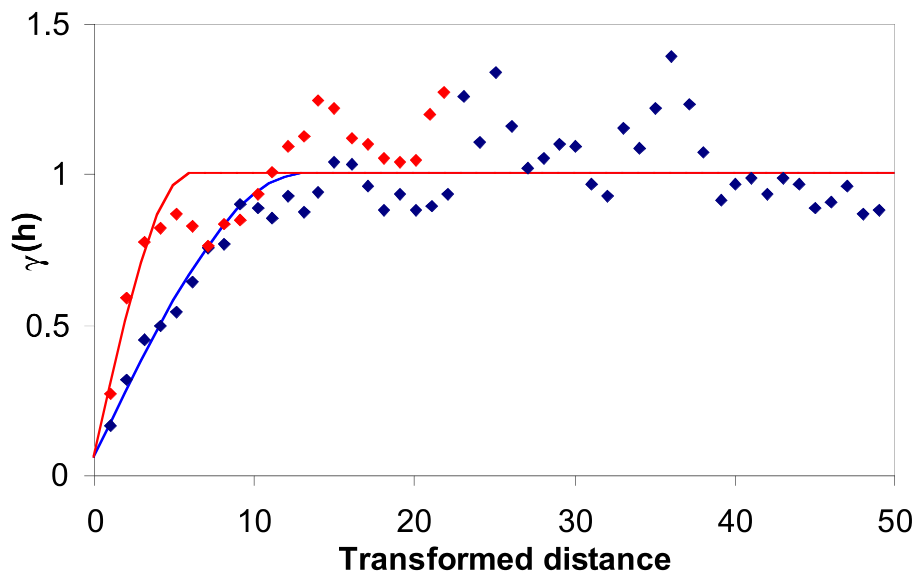


Figure 5: Directional semivariograms of normal score values of transformed sand thickness data of Leeward Shore in the computational space. The semivariograms in the direction of zero degree (N-S) and 90 degrees (E-W) in the computational space are labeled in blue and red, respectively.

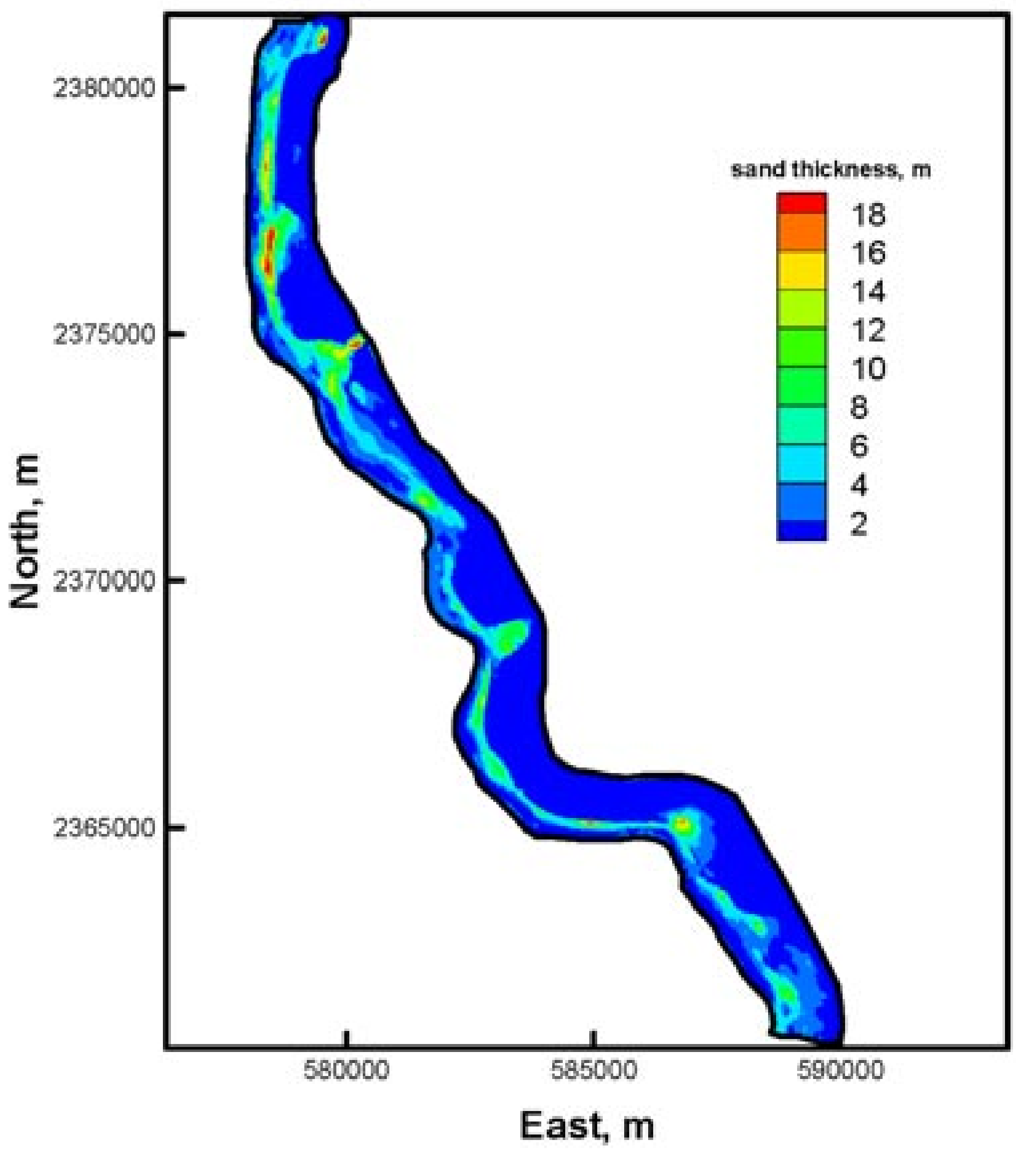

sand thickness, m
18
16
14
12
10
8
6
4
2
2380000
2375000
2370000
2365000
North, m
580000
585000
590000
East, m

Figure 6: Median of 100 SGSIM simulations of sand thickness in Leeward shore, Oahu

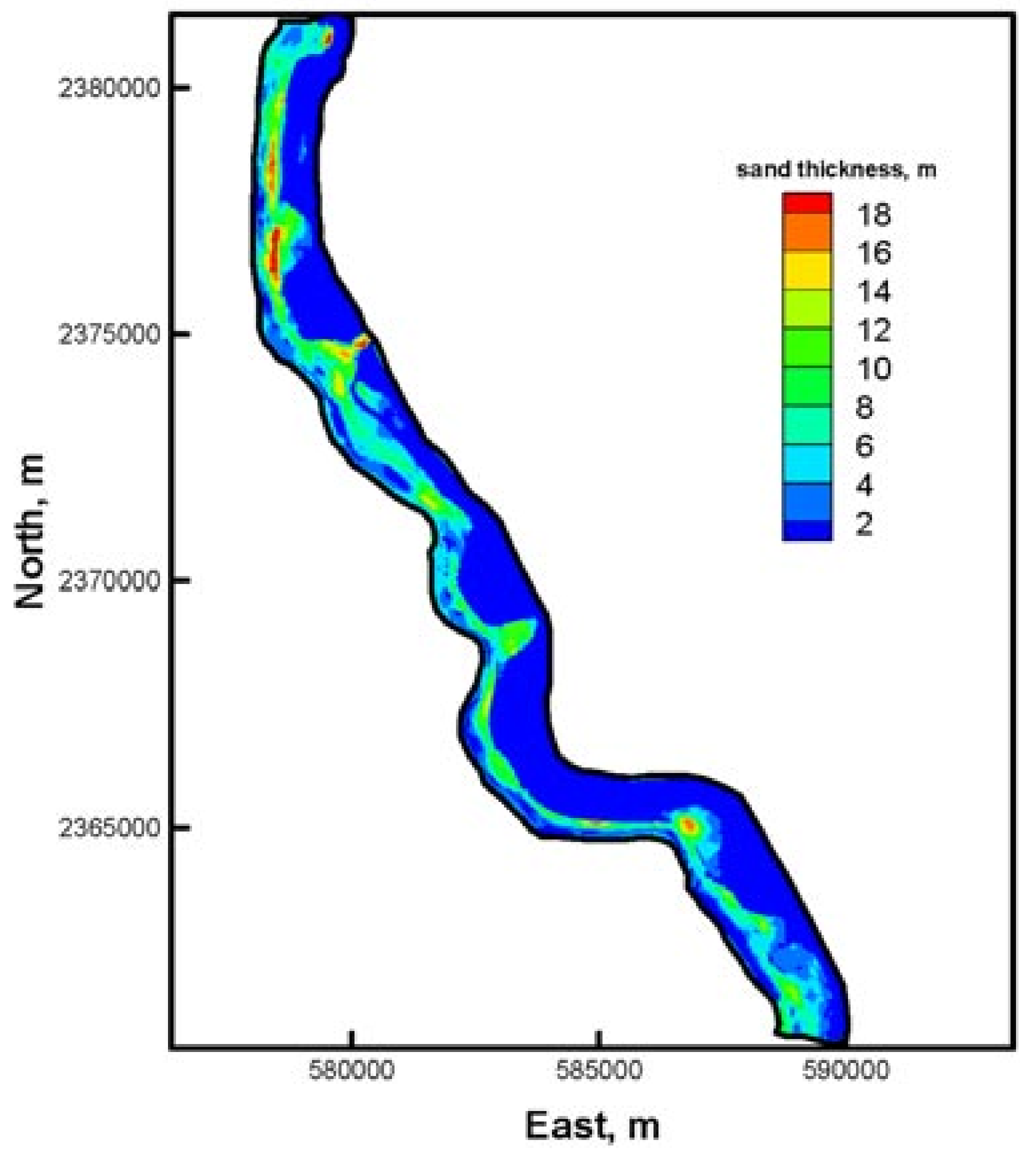
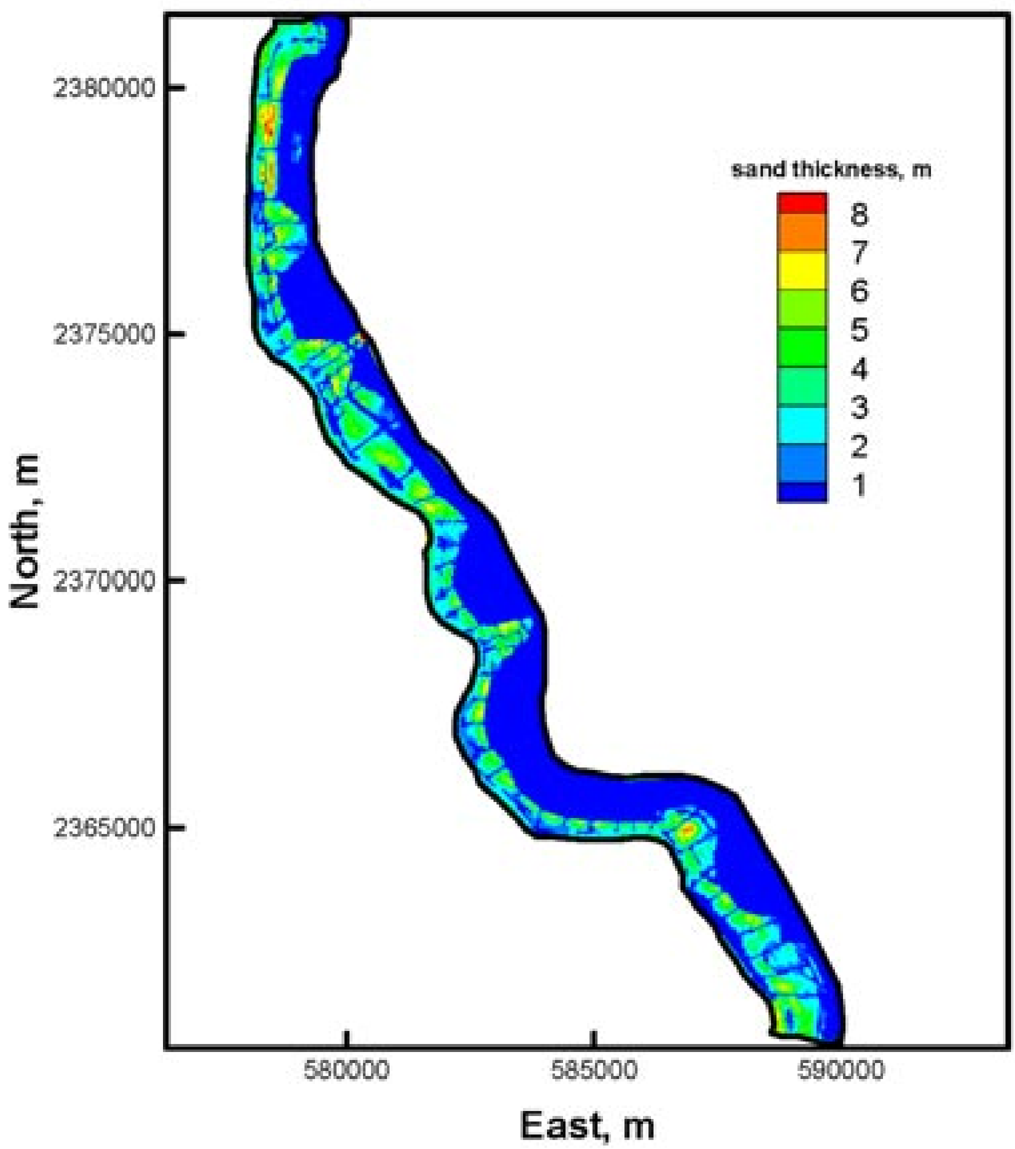

sand thickness, m
18
16
14
12
10
8
6
4
2
North, m
2380000
2375000
2370000
2365000
580000
585000
590000
East, m

Figure 7: Upper quartile of 100 SGSIM simulations of sand thickness in Leeward shore, Oahu



sand thickness, m
8
7
6
5
4
3
2
1
2380000
2375000
2370000
2365000
North, m
580000
585000
590000
East, m

Figure 8: Interquartile range of 100 SGSIM simulations of sand thickness in Leeward shore, Oahu

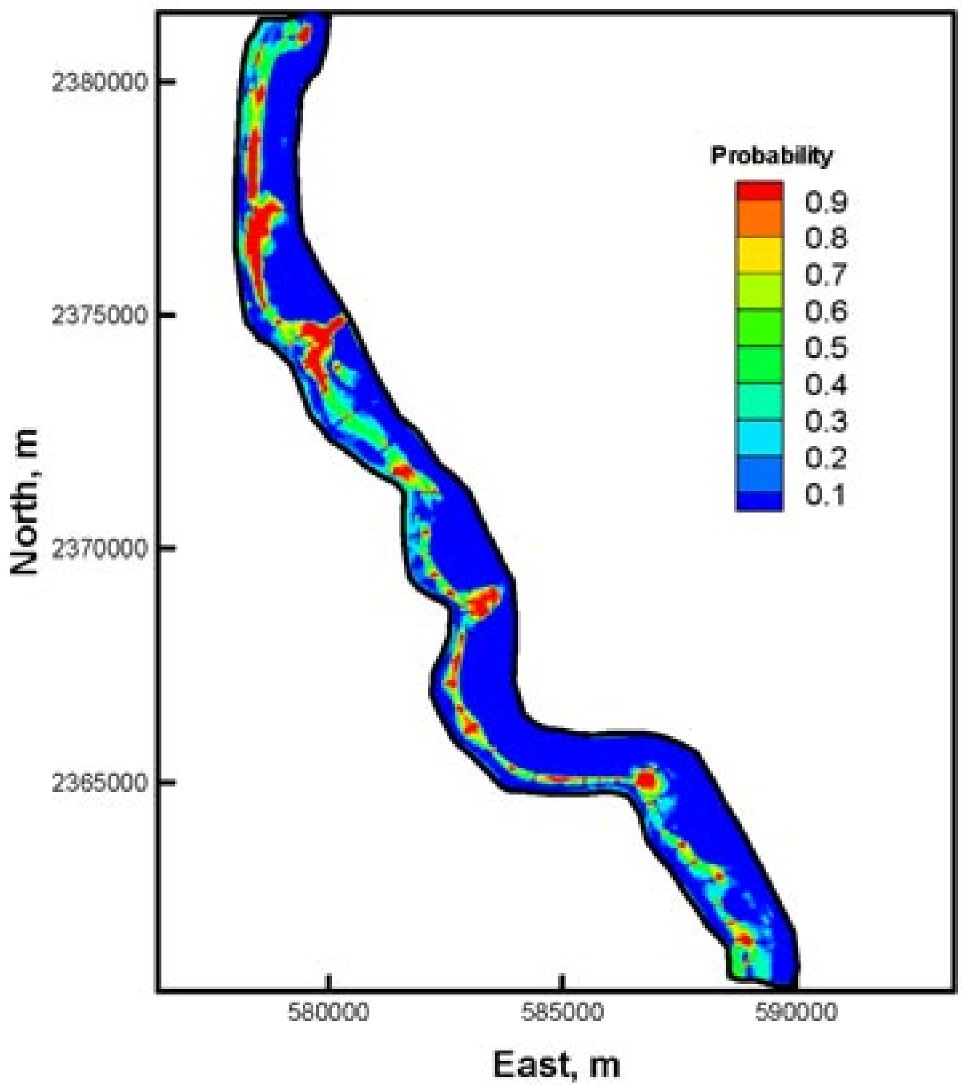
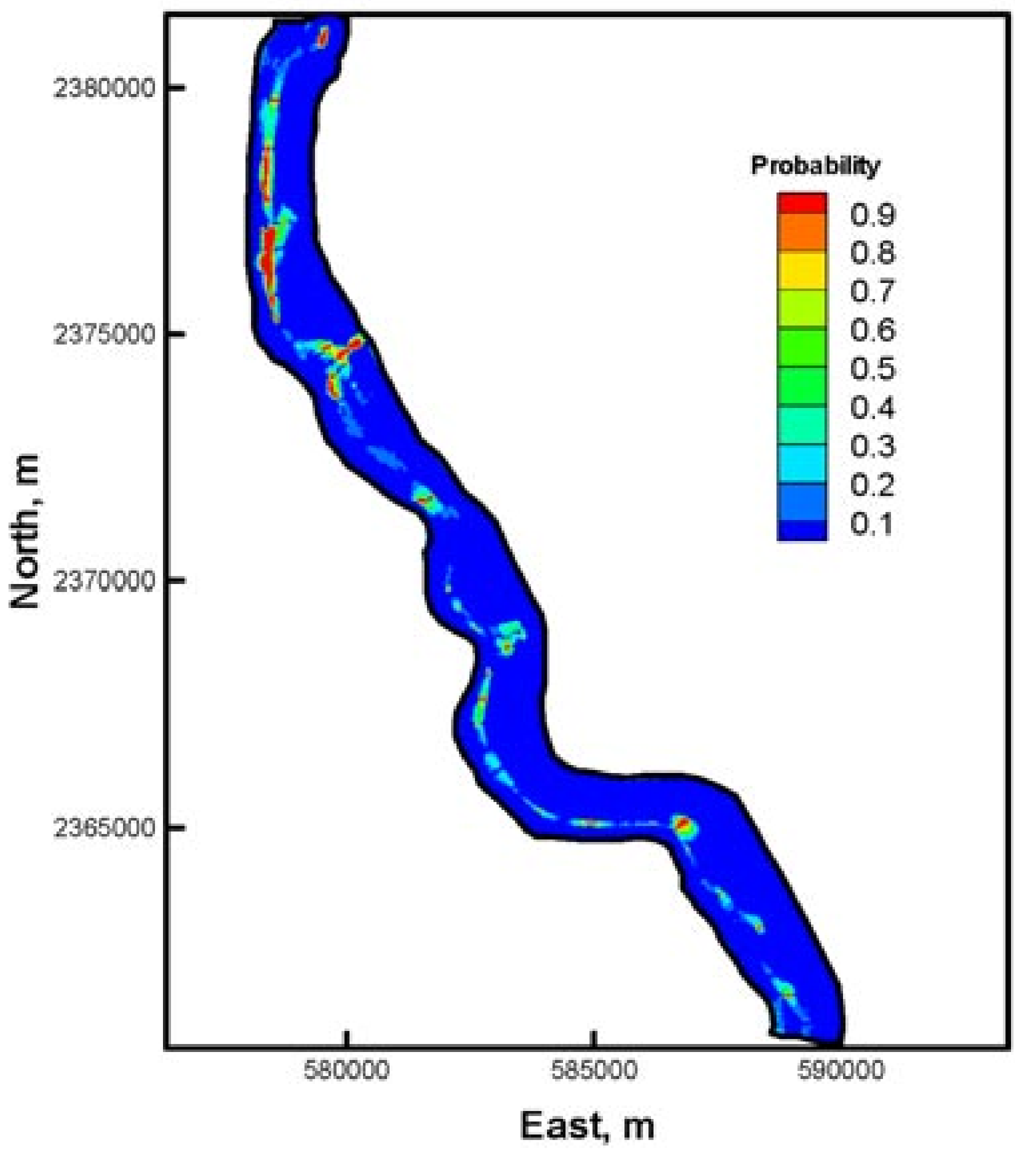

Probability
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2380000
2375000
2370000
2365000
North, m
580000
585000
590000
East, m

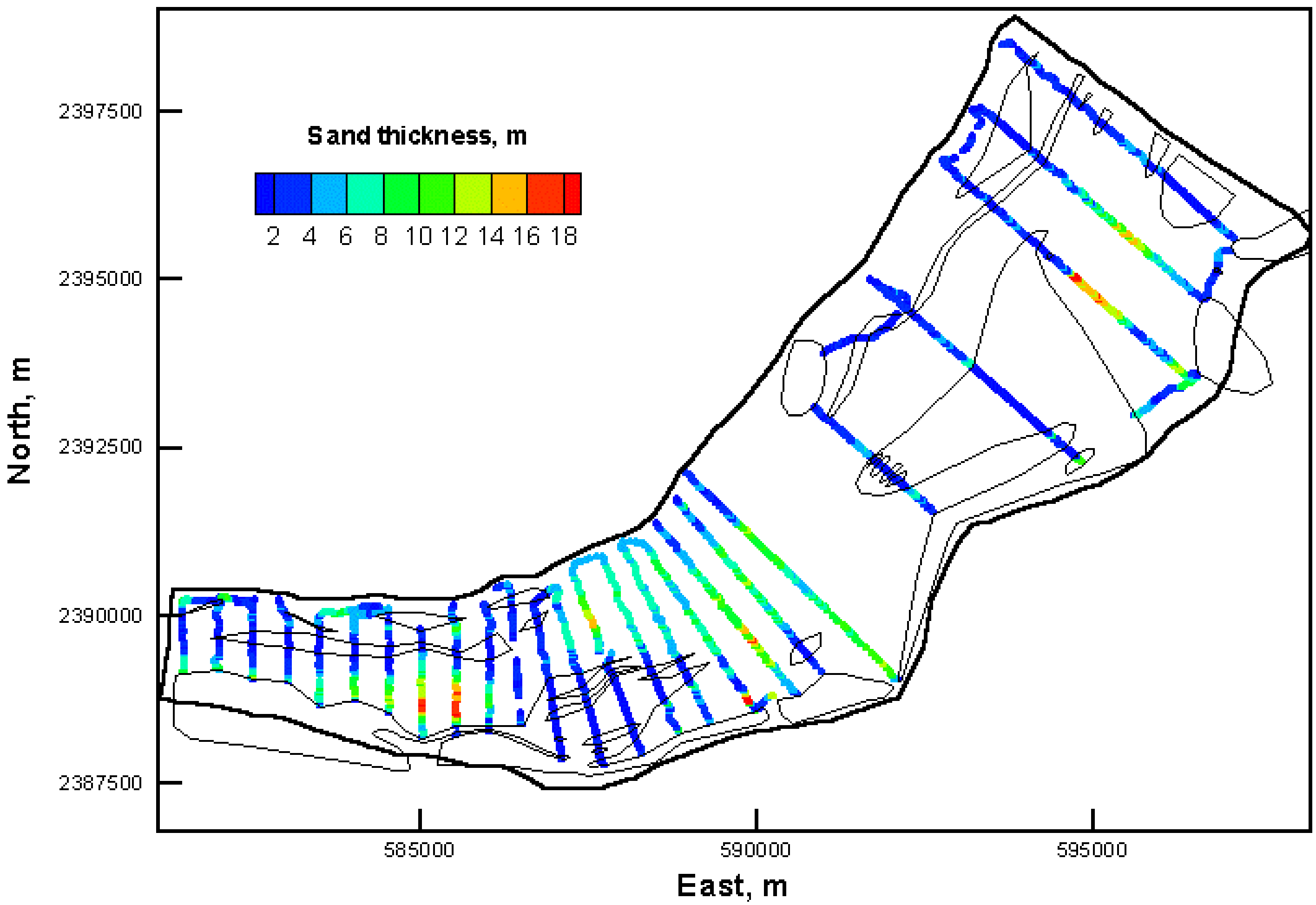
Figure 9: Probability of exceeding 5 m for 100 SGSIM simulations of sand thickness for Leeward shore, Oahu



Probability
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2380000
2375000
2370000
2365000
North, m
580000
585000
590000
East, m

Figure 10: Probability of exceeding 10 m for 100 SGSIM simulations of sand thickness in Leeward shore, Oahu

Figure 11: Location map of sand thickness samples in North Shore, Oahu



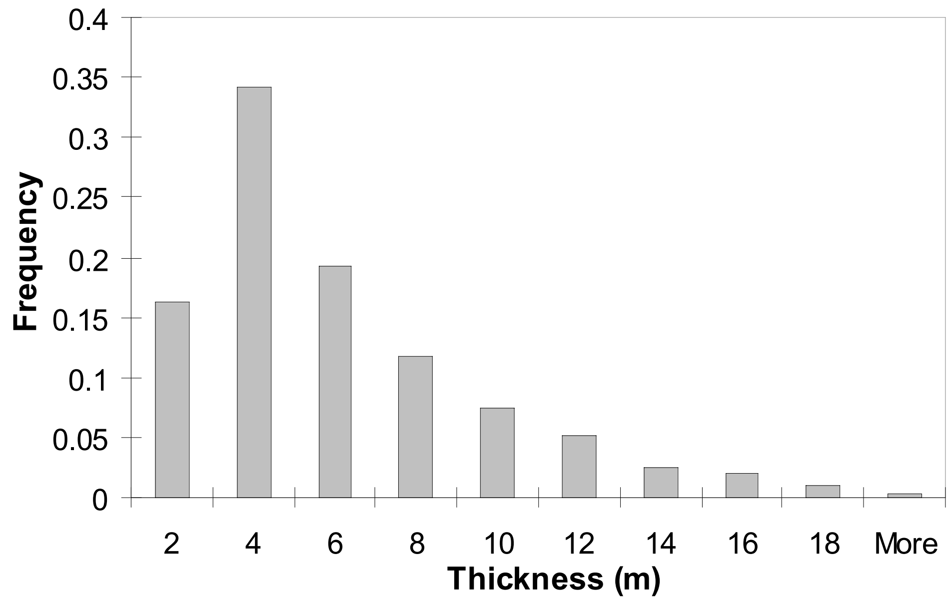


Figure 12: Histogram of original sand thickness data in North Shore, Oahu

Figure 13: Mesh map of simulation grids of sand thickness in North Shore, Oahu

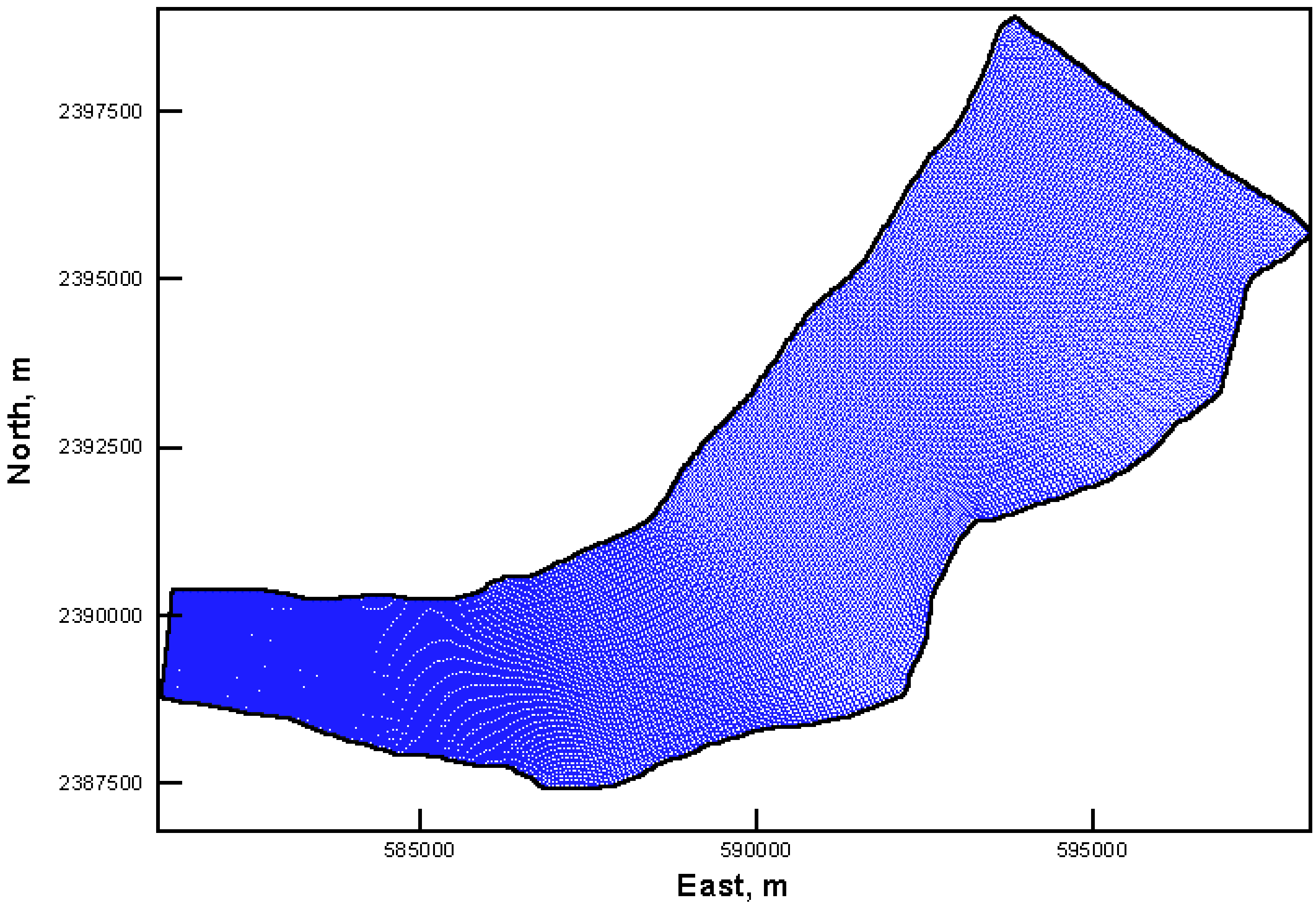
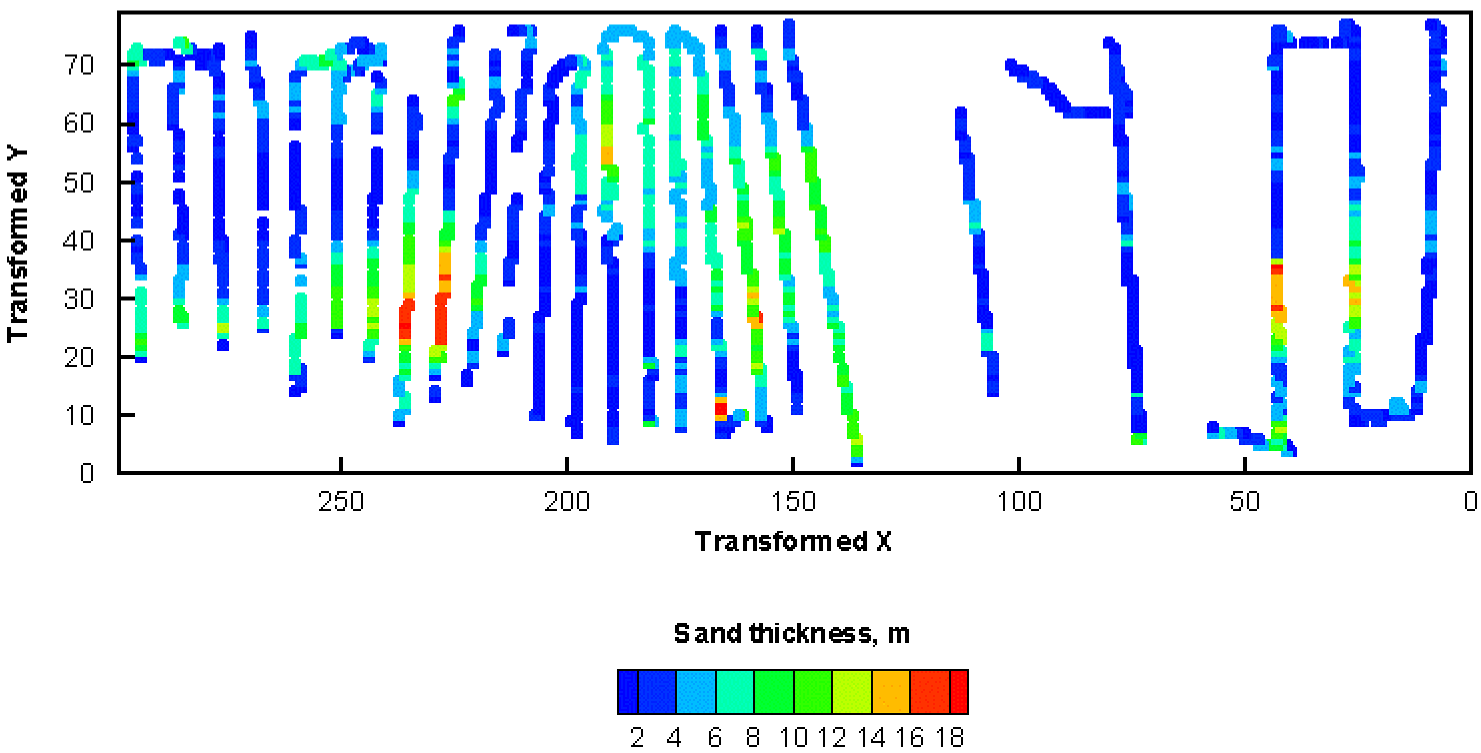


Figure 14: Location map of transformed sand thickness data of North Shore, Oahu



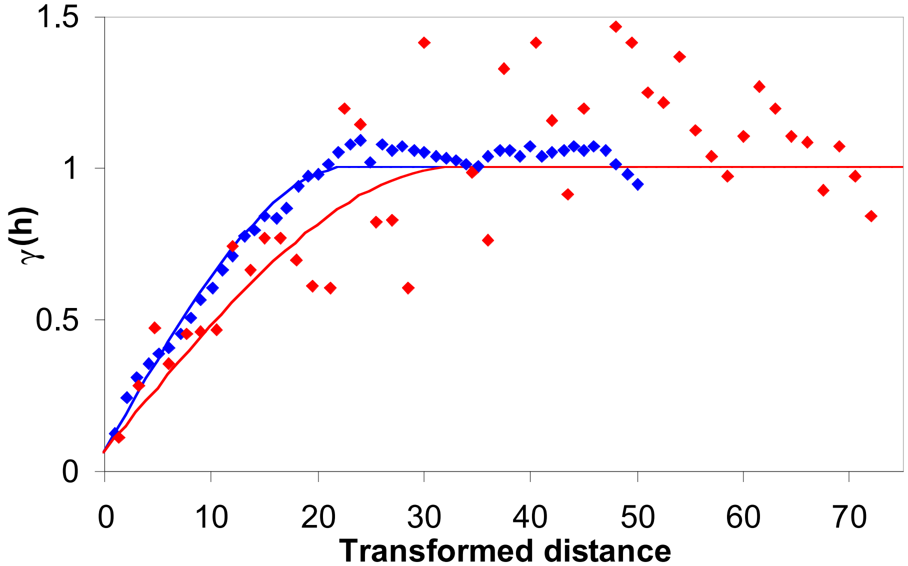


Figure 15: Directional semivariograms of normal score values of transformed sand thickness data of North Shore in the computational space. The semivariograms in the direction of zero degree (N-S) and 90 degrees (E-W) in the computational space are labeled in blue and red, respectively

Figure 16: Median of 100 SGSIM simulations of sand thickness in North Shore, Oahu

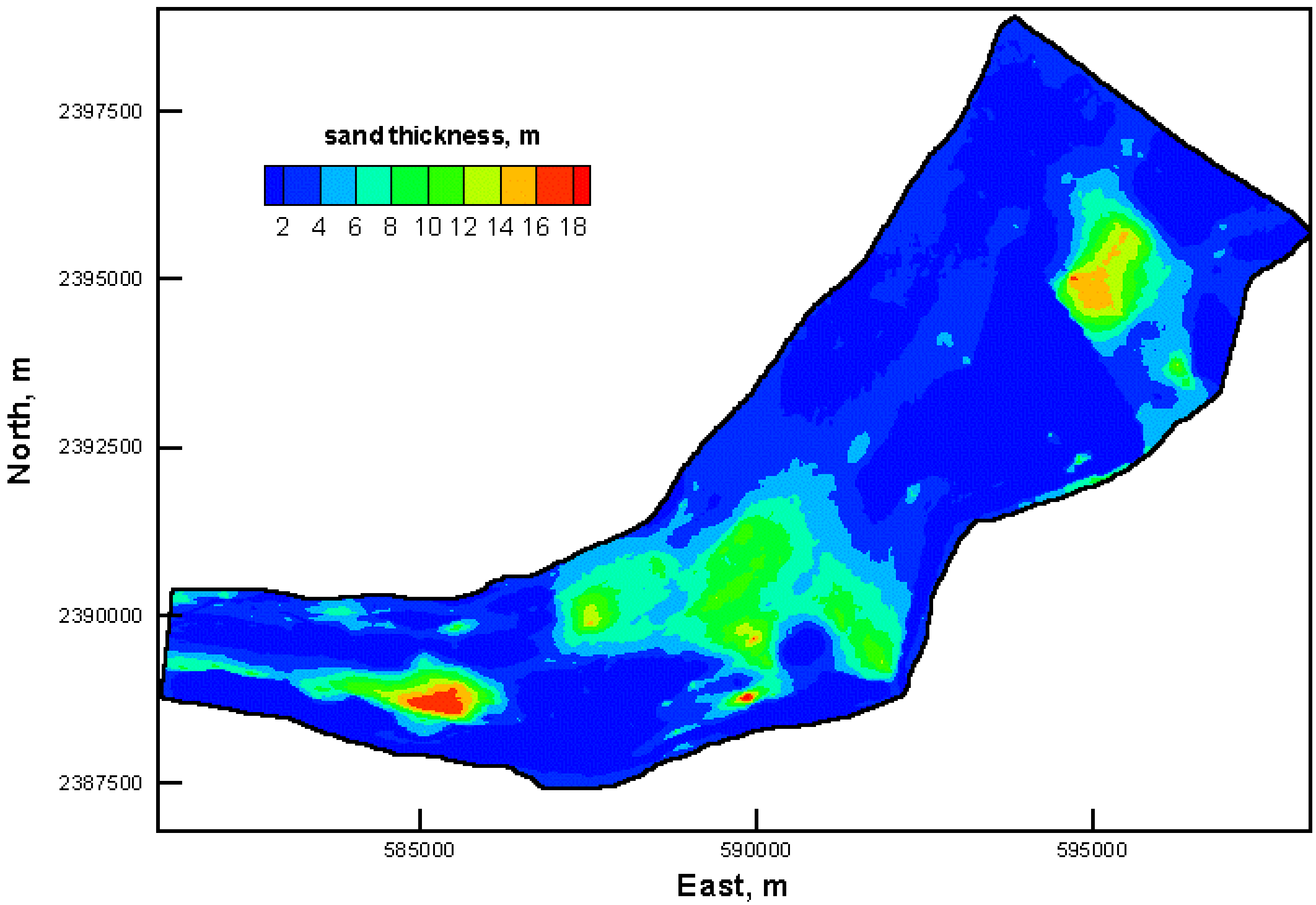


Figure 17: Upper quartile of 100 SGSIM simulations of sand thickness in North Shore, Oahu

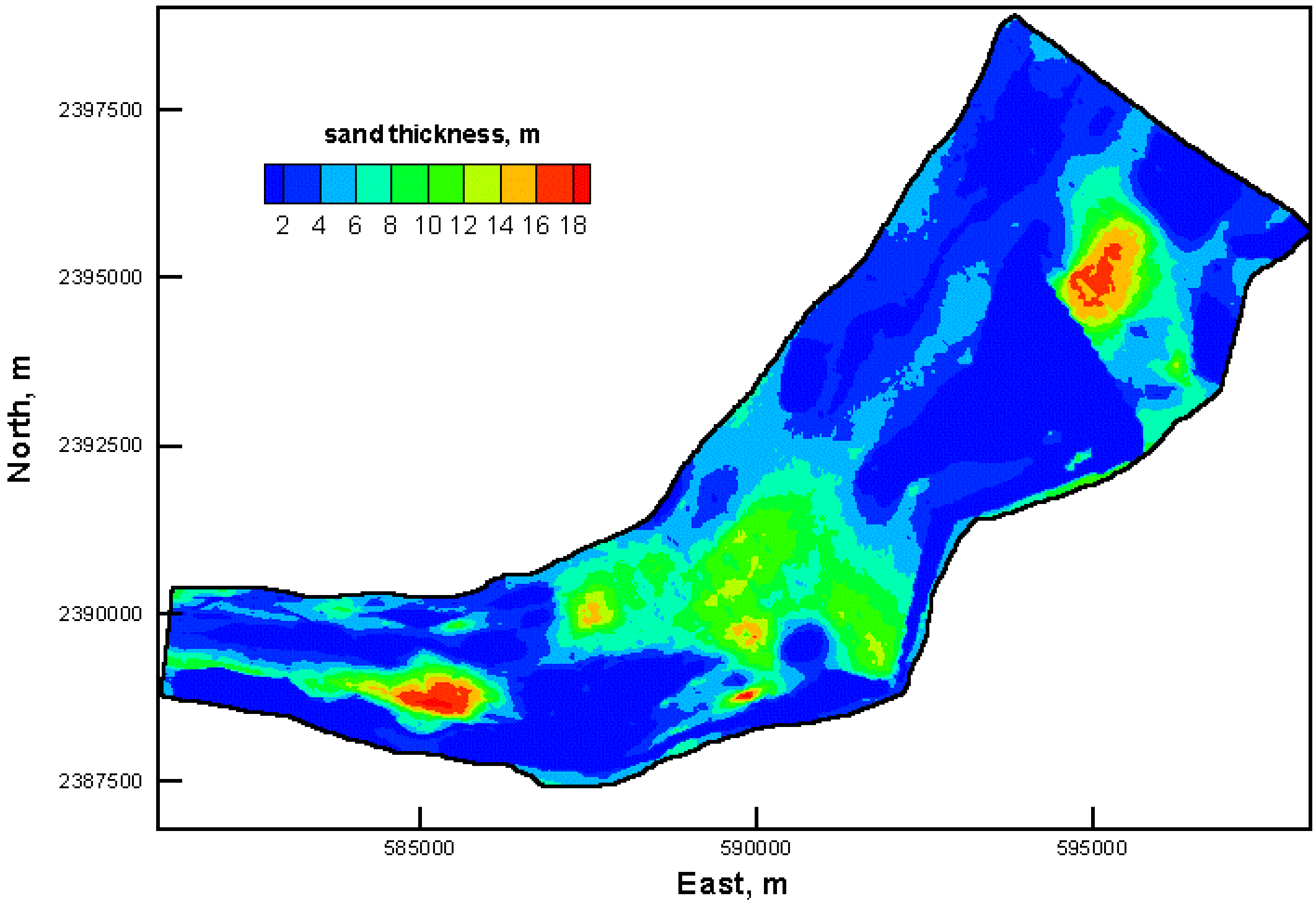


Figure 18: Interquartile range of 100 SGSIM simulations of sand thickness in North Shore, Oahu

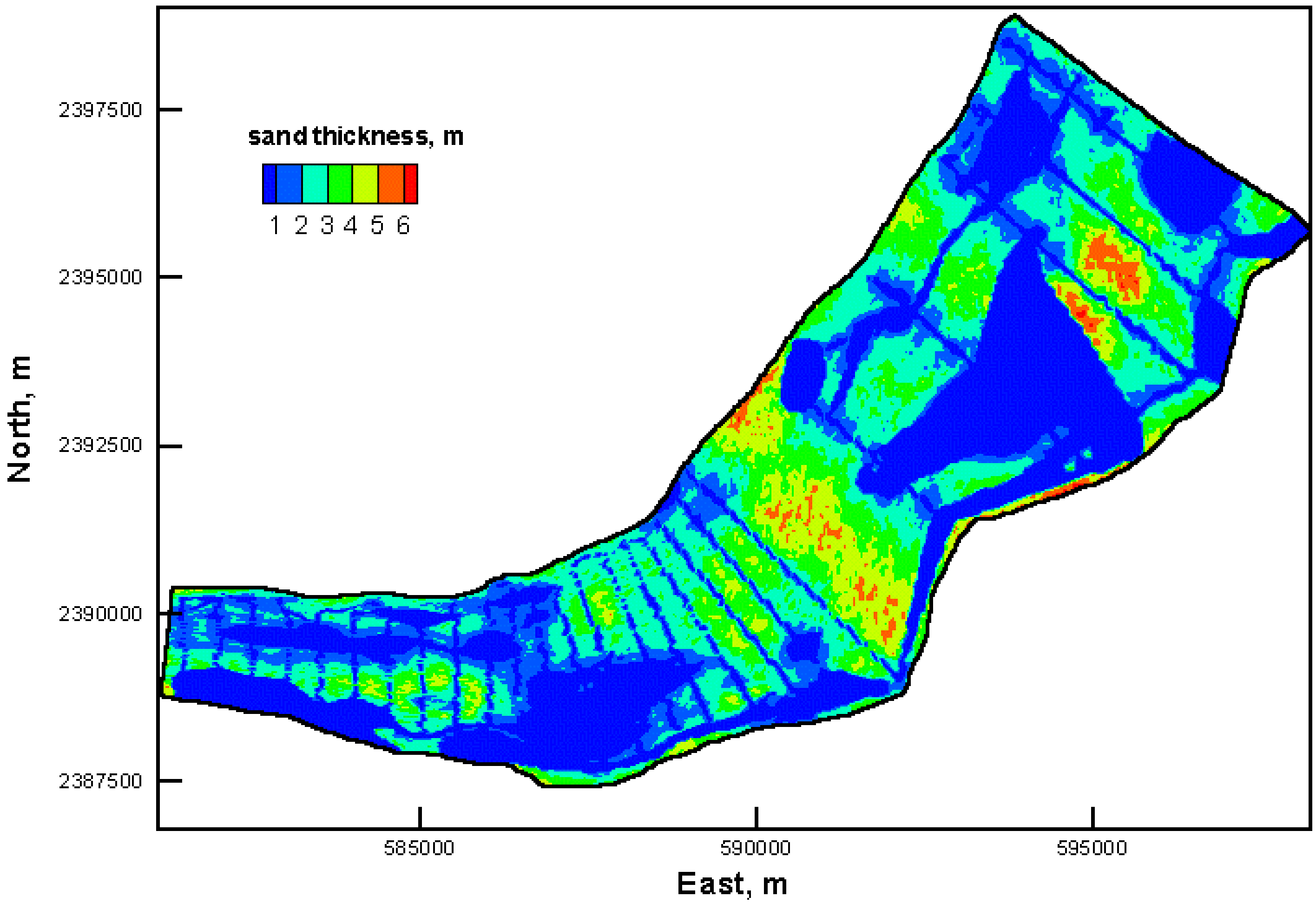


Figure 19: Probability exceeding 5 m for 100 SGSIM simulations of sand thickness in North Shore, Oahu

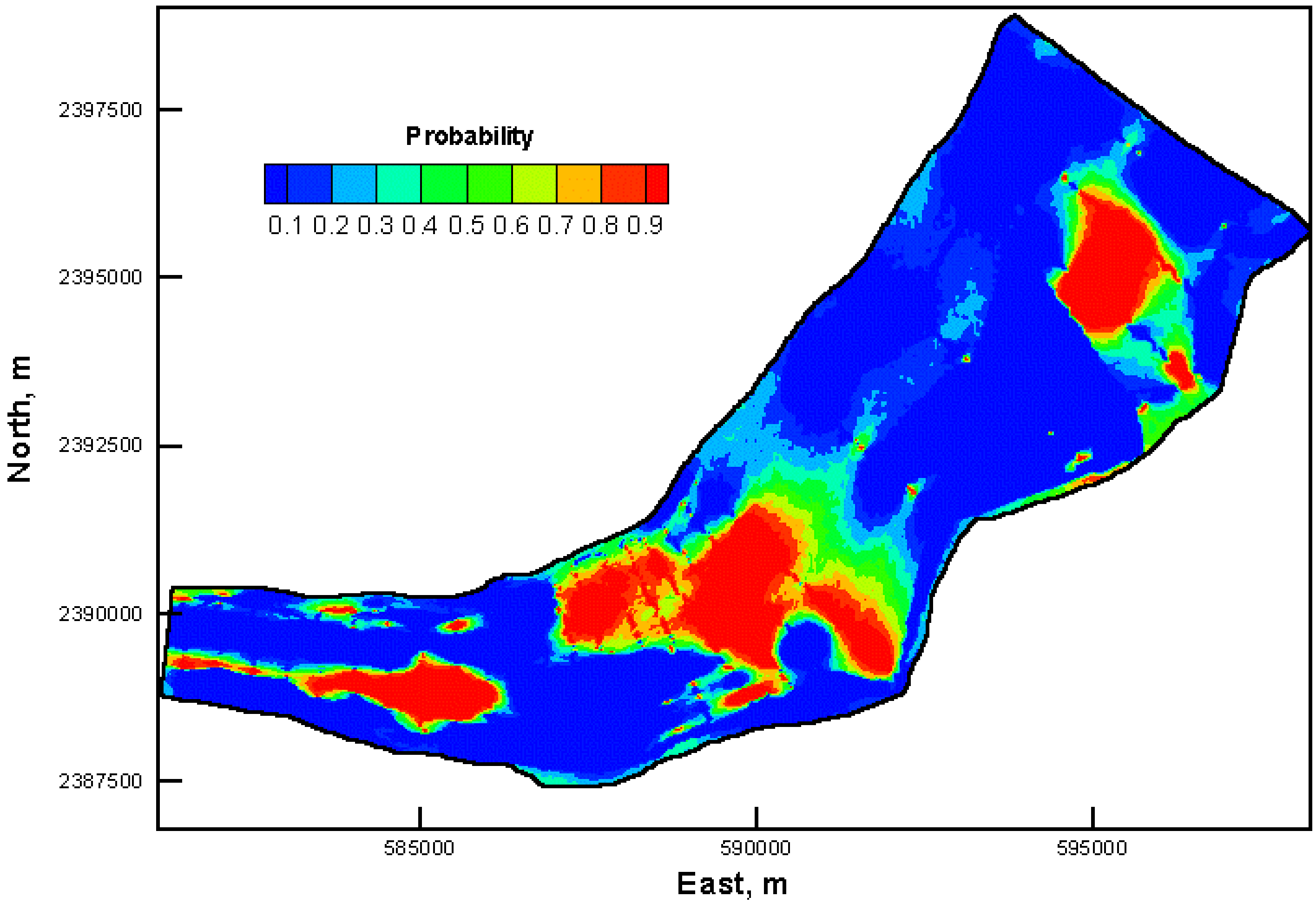


Figure 20: Probability exceeding 10 m for 100 SGSIM simulations of sand thickness in North Shore, Oahu