



Shallow Shear-Wave Velocity Measurements in the Santa Clara Valley; Comparison of Spatial Autocorrelation (SPAC) and Frequency Wavenumber (FK) Methods

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INTRODUCTION

Microtremor noise measurements were made in the William Street Park near the Coyote Creek borehole in the Santa Clara Valley, California, (37.337N, 121.868W) as part of an effort by the U.S. Geological Survey to evaluate and to compare noninvasive methods of measuring shallow shear-wave velocities in urban areas. This study compares the spatial autocorrelation method (SPAC) and the frequency wavenumber spectrum analysis method (FK). Both of these approaches estimate Rayleigh wave phase velocities from microtremor noise and, therefore, do not need an active source.

DATA ACQUISITION

The data collection took place during the afternoon and evening of September 23, 2002. A ten-element array was used with a 100 meter aperture (fig. 1) that could be configured as either concentric rings of instruments or as a set of three nested triangles, similar to the configuration used by Liu and others (2000). This moveable instrument deployment was chosen because it allowed for the acquisition of data that lends itself to either SPAC or FK analysis. Each station utilized a separate K2 recorder with internal episensor accelerometer and external L4 vertical seismometer with a natural frequency of 1 Hz. Simultaneous recording was accomplished by individual global positioning system (GPS) clocks and timers to initiate and end recording on each K2. Having separate recorders for each site increased the installation time of the array. The same station configuration could be recorded by just two 6-channel K2s and running cable to external sensors. However, not every field deployment, particularly one in an urban environment, lends itself to using long lengths of cable. Table 1 shows the data sets that were collected for the different station configurations. In the analysis of the data, the accelerometer channels proved to have poor coherency among sites because of a low gain setting. All analysis, therefore, uses the L4 velocity channels. However, use of the L4 velocity channels limits the low-frequency response. Other studies have employed accelerometers with success, but they have used additional amplification (Kudo and others, 2002).

Table 1. Microtremor data sets.

Recording Window	Stations	Time Step (seconds)	Number of 40.96sec Record Lengths
22:45:FK	0,1,2,3,4,5,6,7,8,9	0.005	20
23:30:SPAC:29m	0,1,2,3,4,5,6,10,11,12	0.005	20
00:00:SPAC:29m	0,1,2,3,4,5,6,10,11,12	0.005	23
01:15:SPAC:58m	0,1,2,3,4,5,6,13,14,15	0.005	23
04:45:SPAC:58m	0,1,2,3,4,5,6,13,14,15	0.005	26
05:45:SPAC:29m	0,1,2,3,4,5,6,10,11,12	0.005	26
06:45:FK	0,1,2,3,4,5,6,7,8	0.005	27

Because of instrumental problems, only one record section (22:45:FK) recorded the full 10-element array of three nested triangles considered optimal for FK analysis. The next best record section with this configuration did not recover station number 9. However, FK analysis was still applied to the other SPAC concentric ring arrays to increase the data sample.

DATA ANALYSIS

The records are first bandpass filtered from 0.2 to 10.0 Hz with a zero-phase-shift Butterworth filter to remove noncoherent arrivals outside the frequency band of interest. Because the sensors were selected to have matched responses, no instrument correction is made. Each recording interval in table 1 then is divided into equal lengths of 8192 sample points or 40.96 sec ($\Delta t = 0.005$ sec). Depending on the total length of the recording interval, this produced between 20 and 27 separate samples of microtremor across the array (see table 1). The calculated Rayleigh wave phase velocities are averages over these 40.96 sec units. But each recording window in table 1 leads to an independent estimate of the phase velocity curve, resulting in six separate phase velocity curves based on the FK method. Additional phase velocity curves were obtained by using the SPAC method because the station configurations allowed for the calculation of spatial coherence over several subsets of the stations.

One of the objectives of this work is to compare SPAC and FK methods of analysis. Both of these methods are applied here assuming that the microtremors are mainly surface waves of primarily fundamental mode Rayleigh waves. The SPAC or spatial auto-correlation method proposed by Aki (1957) is implemented by using a code supplied by Okada (1998, 2001). Aki (1957) showed that the spatially-averaged, normalized coherence of microtremor, or SPAC coefficient, at frequency f can be equated to the phase velocity $c(f)$ through the Bessel function of the first kind and order zero, J_0 . The SPAC coefficient, $\rho(f, r)$, is given by $J_0(2\pi fr/c(f))$. Concerning the spatial averaging, Okada (2001) concluded that there was no significant advantage to using more than three stations equally spaced on a circle and one in the center. Most of our SPAC analysis is based on three-station-pair averages (Kudo and others, 2002). However, with this configuration, it is important to use the phase velocities only in the region where the SPAC coefficients are well correlated and only in the frequency range from the first maximum of the function $J_0(2\pi fr/c(f))$ to its first minimum (Okada, 2001). Therefore, to obtain the phase velocity curve over a broad frequency range, it is necessary to use rings of stations with several different radial distances.

The phase velocity curve is estimated by the FK method by using the high-resolution (HR) method of Capon (1969, 1973) and the multiple signal characterization (MUSIC) method of Schmidt (1986). The high-resolution method has been used by Liu and others (2000), among others, to estimate Rayleigh wave phase velocities for shallow deposits. The MUSIC method has not seen as wide an application in the determination of shallow shear-wave velocities but has been used by Hartzell and others (2003) to study surface wave propagation directions.

PHASE VELOCITIES

Figures 2 and 3 compare Rayleigh wave phase velocities from SPAC and FK MUSIC method and the FK high-resolution method, respectively. The agreement between SPAC and FK generally is favorable, except for a clear shift of about 50 m/sec in the FK curves toward higher phase velocities below about 2 Hz. The larger estimated phase velocities with the FK method, when applied to microtremor noise analysis, can be explained by the smearing of multiple sources in the wavenumber domain (Michael Asten, personal commun.). The SPAC method works best when microtremor noise sources are distributed evenly over a wide azimuthal range. In contrast, FK analysis functions best with a single, well-defined source. The SPAC curves are plotted as they are calculated with no postprocessing. The FK curves, because of their larger scatter, have been fit with a spline. We favor the phase velocities from the SPAC analysis because of their smaller scatter, the fact that the analysis is not biased by multiple sources, and because the standard deviation of the FK values increases significantly below 1.5 Hz. The larger

standard deviation is a reflection of the poorer resolution of the maximum peak in the frequency-wavenumber plane and gives a lower confidence to the phase velocities below 1.5 Hz. This result is affected by the nonoptimal station configuration for FK analysis.

Only one SPAC-determined phase velocity curve extends to frequencies above 4.75 Hz. This limitation is due to the fact that only one recording window (22:45:FK) in table 1 produced records from the smallest triangle on figure 1 (stations 7, 8, and 9). The FK data also suffer from this limitation. For this reason, we consider our velocity results to be more accurate below about 30 m than above this depth. Adequate SPAC data can be collected with only four stations, three equally spaced on a circle and one at the center, with associated moves of the circular pattern of stations to achieve different radial distances. Analysis with the FK method, although not requiring station movement, generally needs more stations for good resolution over the same frequency range.

INVERSION OF PHASE VELOCITY

The Rayleigh wave phase velocity curves are inverted for shear-wave velocity by using software from Herrmann (2002). The method uses an iterative, damped least-squares technique (Lawson and Hanson, 1974). Starting from an initial model where the layer thicknesses and velocities are specified, successive iterations adjust the layer velocities until the error between the phase velocities and the predictions of the model have reached a desired minimum. In general, the starting model can be one with constant velocities with depth. The algorithm iteratively inverts for the S-wave velocity, updating the P-wave velocity using the V_P/V_S ratio of the initial model. The new density is computed using the new V_P and the empirical Nafe-Drake relation. Figures 4 and 5 show the end result of this fitting process for the SPAC and high-resolution MUSIC phase velocity curves, respectively. Although there are several individual phase velocity curves, all the data are fit at once in a least-squares sense. Because of the higher phase velocities from the FK analysis, as noted above, the shear-wave velocities are greater for this model, particularly in the depth range from 100 to 350 m.

The above conclusions, and the velocity models shown in Figures 4 and 5, were derived blind, ie prior to disclosure to the authors of the shear-wave suspension log in the Coyote Creek borehole. Figure 6, produced after interpretation of SPAC and FK data was complete, compares the two shear-wave velocity profiles with a smoothed version of the Coyote Creek suspension log. Because of our concerns about the poorer resolution and bias of the FK analysis below 1.5 Hz, we favor the velocity model obtained from the SPAC method. This model is presented in table 2.

Table 2. Preferred velocity profile from spatial autocorrelation (SPAC) method.

Layer	Thickness (meters)	VP (meters/second)	VS (meters/second)	Density (gm/cc)
1	10	342	171	1.68
2	10	520	260	1.83
3	10	756	378	1.96
4	10	792	396	1.98
5	10	661	331	1.92
6	50	866	432	2.01
7	50	1191	595	2.12
8	50	1174	587	2.11
9	50	1398	699	2.18
10	50	1515	758	2.20
11	50	1559	779	2.21

It is important to note that the inversion of Rayleigh wave phase velocities does not lead to a unique velocity model. The iterative inversion method used here, as well as other algorithms, can become trapped in a local minimum of the objective function. To get a feel for the range of applicable models, we started with different initial models. Some solutions lead to models with larger velocity jumps between layers but have a similar fit to the data. However, without any other additional information on the velocities, we have opted for a smoothly varying solution. The choice of the number of layers and their thicknesses is another subjective decision. Too many layers can lead to a large number of poorly constrained model parameters and too few layers to a poor fit to the data. Our layer thicknesses increase with depth reflecting the assumption of a larger velocity gradient near the surface than at depth and the expectation of decreased resolution with depth. We also have conducted tests by using different total model thicknesses to estimate the maximum depth resolution, which influenced our total model thickness of 350 m.

CONCLUSIONS

The microtremor noise measurements in this study can be done either with individual, independent recorders or by running cable to a few central recorders. If the field situation allows for running cables, the latter is preferred. However, this study has shown that independent recorders can be used for urban settings where cables are not feasible. Although our station configuration is not optimal for FK analysis because of instrument failure, we make the following general comparisons between the SPAC and FK methods. The SPAC method can be done with fewer stations. In general, SPAC method requires only four stations, allowing for moving them to different radial distances. FK analysis requires more stations for good resolution of wavenumber peaks. The different underlying assumptions about the distribution of noise sources in the SPAC and FK methods favor SPAC in the typical urban environment. SPAC works best when the noise sources are widely distributed in azimuth, a common occurrence in a noisy urban setting. FK analysis works best with a single, well-defined source. Multiple sources can lead to smearing of peaks in the wavenumber domain and an overestimation of phase velocities with the FK method.

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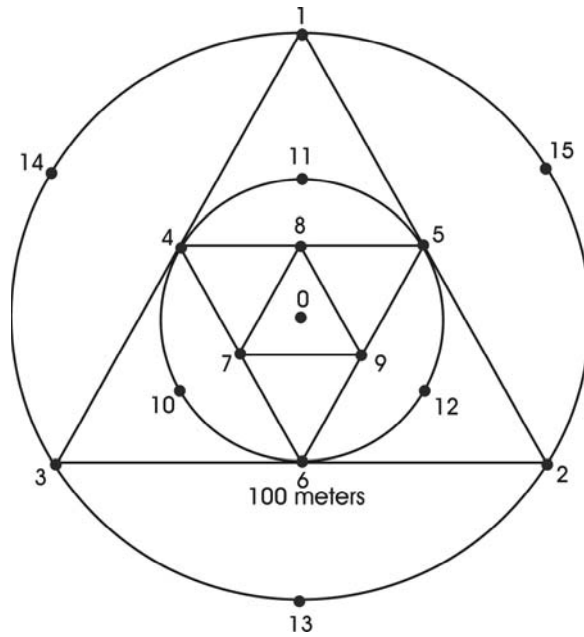


Figure 1. Station geometries used for the SPAC and FK field deployments. A maximum of ten stations were used at any given time arranged either on concentric circles or vertices of triangles. Station location 0 was always occupied. See table 1 for list of stations locations used.

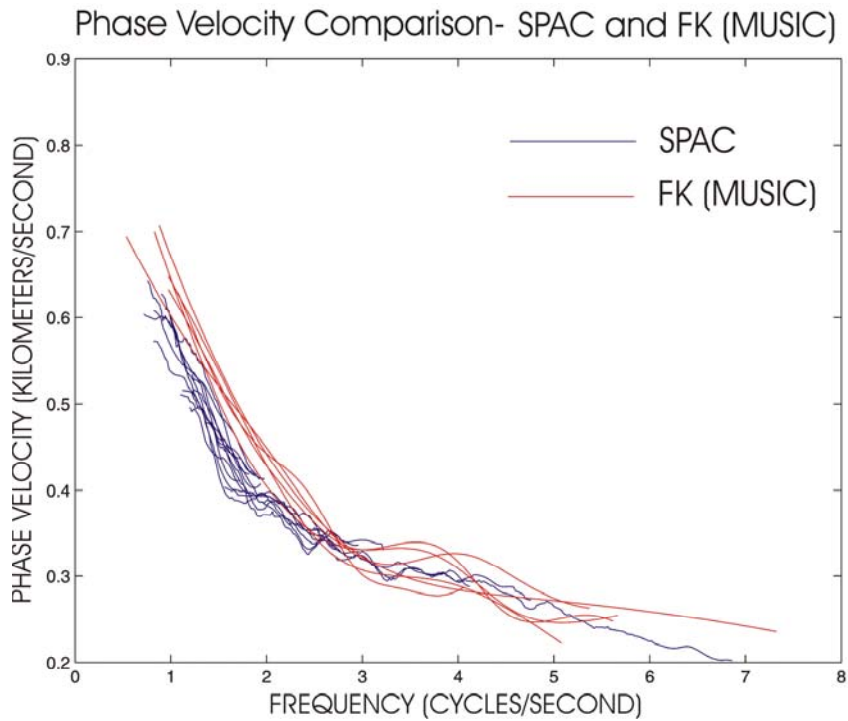


Figure 2. Comparison of surface wave phase velocity curves from SPAC and FK MUSIC analysis.

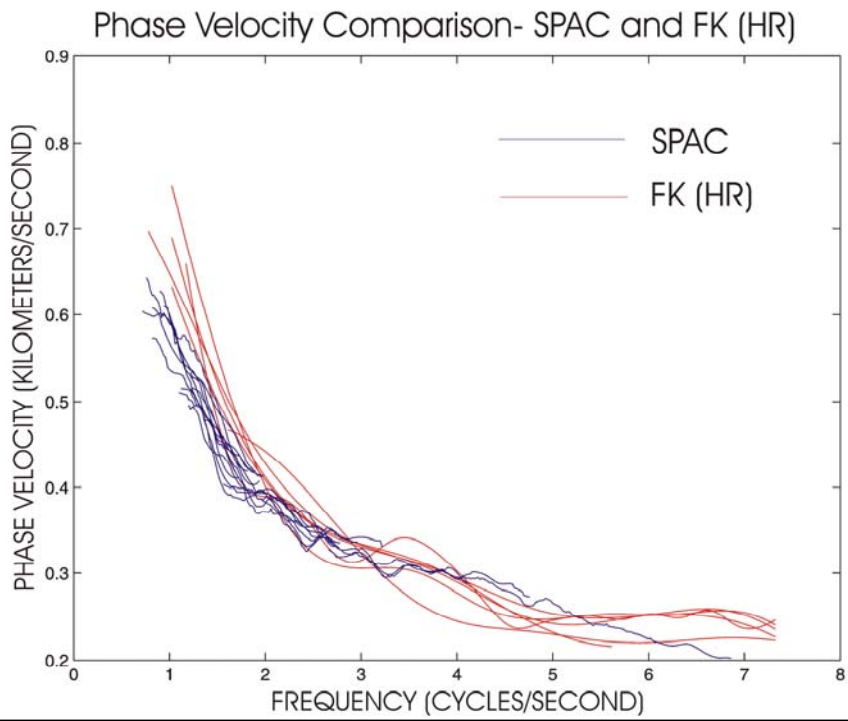


Figure 3. Comparison of surface wave phase velocity curves from SPAC and FK high-resolution analysis.

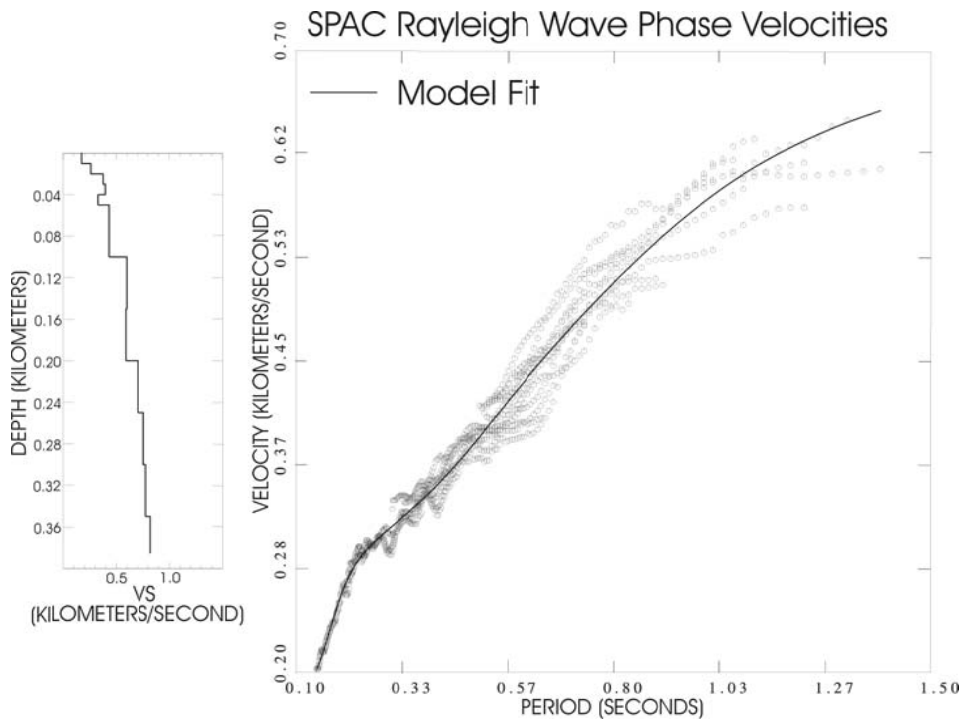


Figure 4. Result of damped least-squares inversion for the shear-wave velocity profile using SPAC phase velocities. The inversion performs a least-squares fit to the phase velocity curves from the different microtremor trials. The SPAC solution is preferred over the FK results. See text.

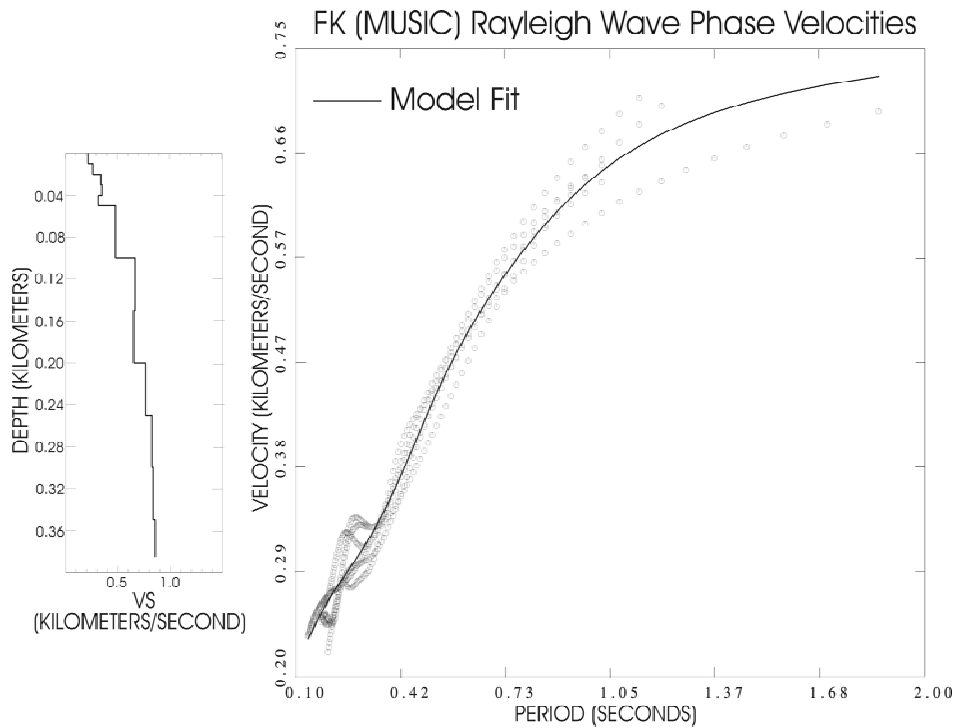


Figure 5. Result of damped least-squares inversion for the shear-wave velocity profile using FK MUSIC phase velocities. The inversion performs a least-squares fit to the phase velocity curves from the different microtremor trials

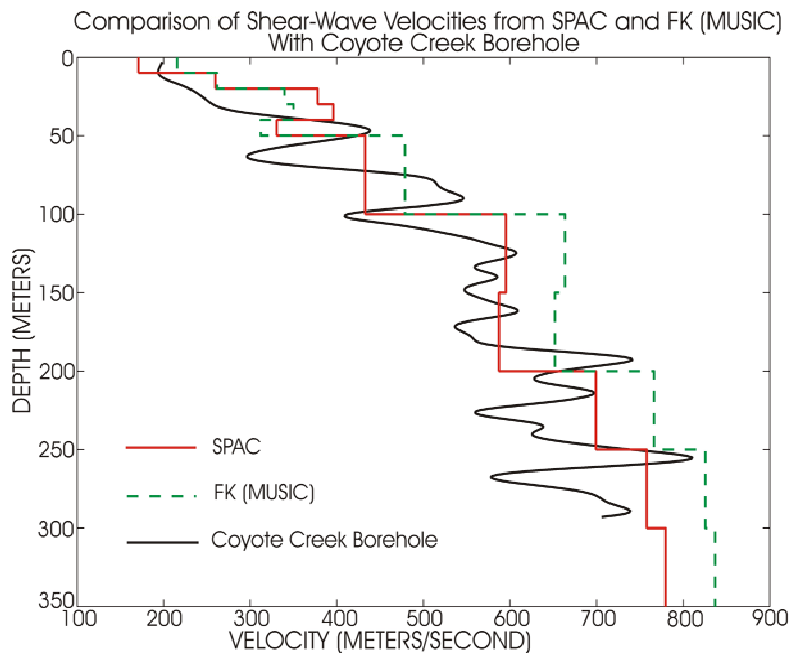


Figure 6. Comparison of shear-wave velocity profiles from the SPAC and FK MUSIC methods with a smoothed version of the Coyote Creek suspension log velocities. The former were obtained "blind" that is prior to supply of the latter