Methods for determining topography in data sparse regions of East Antarctica

A. E. Smith,[1] R. E. Bell,[1] I. Velicogna,[2] and M. Studinger[1]

[1]Lamont-Doherty Earth Observatory of Columbia University, Palisades, NY 10964, USA (adrienne@ldeo.columbia.edu , robinb@ldeo.columbia.edu , mstuding@ldeo.columbia.edu)
[2]CIRES and Department of Physics, University of Colorado, Boulder, CO 80309, USA and Jet Propulsion Laboratory (NASA), 4800 Oak Grove Drive, Pasadena, CA, 91109, USA (isabella@colorado.edu)

Summary Large regions of East Antarctica lack a reasonable topographic model because, until recently, only a few observations of ice thickness have been available to constrain the bedrock elevation. The acquisition of GRACE satellite gravity data has created a new opportunity to model the sub-ice topography. Here we have applied two methods for predicting topography based on the satellite data. Gravity inversion is a classical geophysical technique that predicts topography based on the physics relating it to gravity. Cokriging is a statistical method that uses the spatial covariance between datasets to predict one in the absence of the other. The geophysical and statistical solutions are compared to the best-known topography model (BEDMAP) in an area that is relatively well constrained by the BEDMAP data coverage.

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Introduction

Although a fundamental line of evidence of the tectonic past and a control on the dynamics of continental ice sheets, the subglacial topography of Antarctica is largely unconstrained. In 2001, the BEDMAP consortium compiled Antarctic ice thickness data to construct an ice thickness model, removed these values from the then best available surface DEM (Liu et al., 1999) and produced a gridded topography model of the subglacial surface. The final BEDMAP topography solution relies on an inverse distance weighting (IDW) interpolation to predict ice sheet thickness in between measurements. While applying the IDW technique, this model accounted for variable data density and directional anisotropy by defining equally weighted octants around the point to be predicted and using 2 points in each octant to predict a given topographic value. Despite careful consideration by the BEDMAP team,

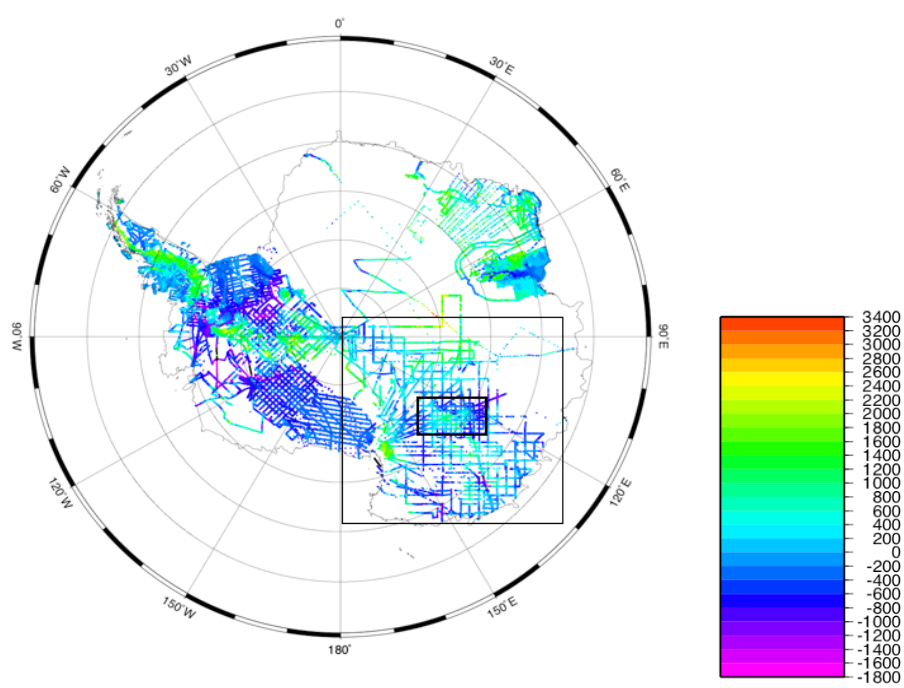

Figure 1. Area of detailed study. Inner box contains the best-constrained topography of East Antarctica. The larger box encloses the area in which topography models were generated with each method to compare quality of the solutions. Elevations are given in meters.

data scarcity prevented the calculation of a reasonable model over the entire Antarctic, often relying on *a priori* assumptions to fill in data gaps. The BEDMAP model is known to have nodes of thin and thick ice that cannot be confirmed. (Lythe et al., 2001) In addition to author-documented problems, recent radar surveys have shown that the model is locally inaccurate by up to 2km.

Area of Study

The density of data coverage available when the BEDMAP model was constructed varies throughout East Antarctica. The subglacial topography in some regions is relatively well constrained by a grid of airborne radar surveys at ~50km spacing as shown in Figure 1. Based on data availability, this region should be one of the most accurately represented in the BEDMAP model of East Antarctica and was therefore chosen as a standard against which to judge the other topography determination methods described below.

GRACE

The Gravity Recovery and Climate Experiment (GRACE) was launched by NASA in 2002. Since then, monthly solutions for a spherical harmonic field have been published by the GRACE team. Periodically, these monthly solutions are spatially stacked and time-averaged to determine a static gravity field of higher resolution than the monthly solutions. (Bettapur, 2006) Grace Gravity Model 2C (GGM02C, Figure 2) is a geopotential model produced by the Center for Space Research (CSR) that is based on a weighted combination of 14 months of GRACE satellite data (GGM02S) and EGM96 coefficients to spherical harmonic degree 200. To achieve higher resolution, GGM02C can be expanded to degree 360, using the higher degree coefficients from EGM96 that were determined from terrestrial gravity surveys. The gravity anomalies are provided at .25 decimal degree intervals. (Tapley et al., 2005)

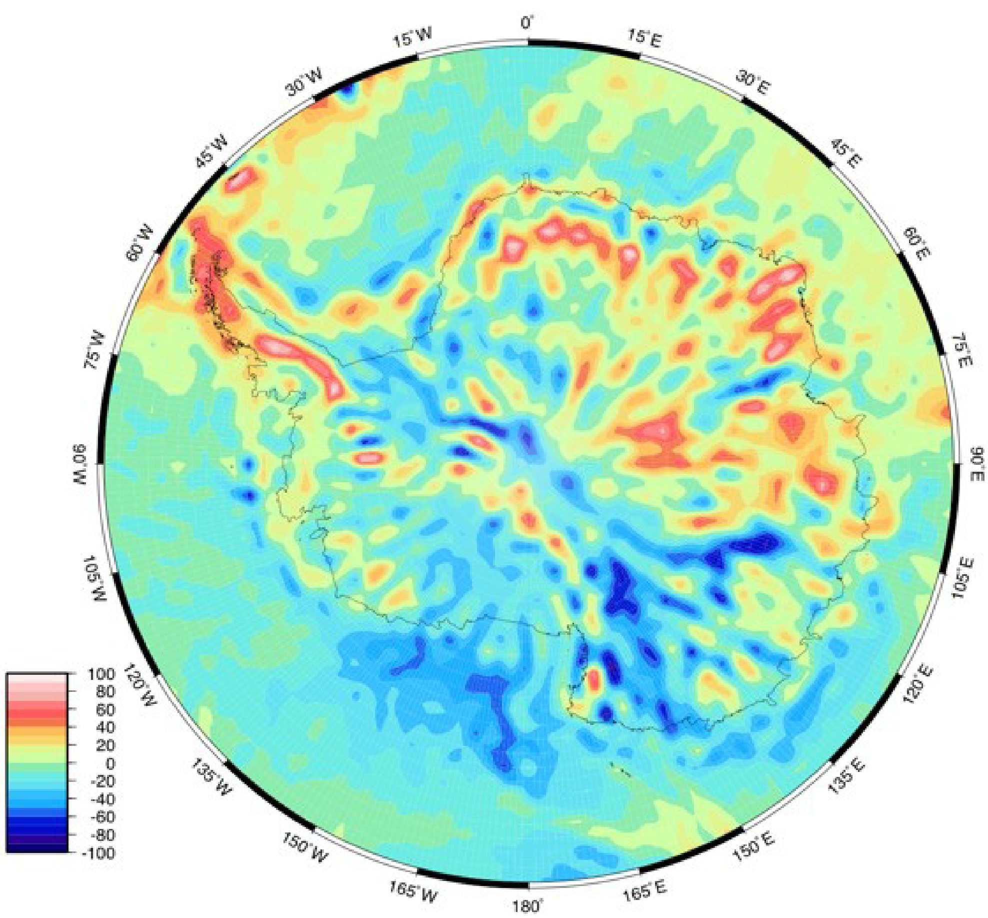

Figure 2. GGM02C free-air gravity anomalies in mGal.

Methods

Ice-surface correction

The topography of the ice-air interface is present as long-wavelength noise in this application of GRACE free-air gravity anomalies. To remove the ice surface effect, Parker's algorithm for the forward calculation of gravity anomalies due to an undulating surface was applied. Although more accurate DEM models exist (Liu et al., 2001; Bamber et al., 2005), the surface elevation model of the BEDMAP consortium was used (Liu et al., 1999) to ensure that differences in the topography models were not due to differences in the input ice sheet surface topography.

Topography determination

To determine subglacial topography, two independent methods were used. The Parker inversion uses only the GRACE gravity data and relies on the physics that relate gravity and topography whereas cokriging, which is purely statistical, relies on the special covariance of GRACE gravity anomalies and available topography measurements.

Gravity Inversion

The forward calculation of gravity anomalies due to topographic oscillations on a 2-dimensional surface of uniform density contrast was described in the frequency domain by Parker (1973):

$$F[\Delta g(x,y)] = -2\pi\gamma\Delta\rho e^{-\bar{k}z_0}\sum_{n=1}^{\infty}\frac{\bar{k}^{n-1}}{n!}F[h^n(x,y)]$$

Oldenburg (1974) demonstrated that the inverse calculation of Parker's (1973) algorithm is:

$$h(x,y) = F^{-1}\left[\frac{F[\Delta g(x,y)]e^{\bar{k}z_o}}{-2\pi\gamma(\rho_{basement}-\rho_{ice})} - \sum_{n=2}^{\infty}\frac{|k|^{n-1}}{n!}F[h^n(x,y)]\right]$$

3DINVER.M, a code for the Oldenburg-Parker formulation of this iterative inversion problem has been disseminated to the public by Gomez-Ortiz (2005) and was applied, with some modification, to the GGM02C free-air anomalies.

Rescaled ordinary cokriging

Kriging is a best linear, unbiased estimator based on the theory of regionalized variables that is widely used in hydrologic and mining applications (Bohling, 2005 and Davis, 1973). Many types of kriging exist including multivariate (i.e. cokriging) versions of: simple, ordinary and universal kriging. Each type of kriging requires building a weighting function, or variogram, which describes the influence of each known point as a function of the distance from the point to be estimated. (Davis, 1973)

In cokriging, the statistical relationship between primary and secondary variables is used to predict the primary variable when data of that type are not available. Unlike simple cokriging, ordinary cokriging does not require an assumption of second or even first order stationarity. Although the covariance between variables is defined universally, using all available data points, the mean is determined in the moving neighborhood around the point at which the primary variable is to be estimated. Ordinary cokriging traditionally follows the weighting constraint of simple cokriging. (Goovaerts, P., 1998) The work of Isaaks and Srivastava (1989) showed that the weights can be rescaled using a single constraint such that the weights of both the primary and secondary variable must sum to one. This maintains unbiased estimates without unduly reducing the impact of the secondary variable. In the case of using free-air gravity to predict topography, the predictive power of the secondary variable, gravity, should be maintained since free-air anomalies and topography are correlated at the 99% confidence level.

Variograms were constructed around randomly selected points in East Antarctica, which showed that an exponential model best fits the data over a 700km range. To simulate the data sparse conditions of much of East Antarctica in the area of study, the compiled topography data were reduced to a 1% random sample of the available data points. Then a cluster of data consisting of 18 points in a 50km x 50km area was added to the 1% sample to mimic the effect of a local base station, localized survey, or survey tie lines. Using these parameters, cokriging was accomplished with the MATLAB code COKRI.M written by David Marcotte (1991). The resolution of the cokriging topography depends on the topography data sample size, becoming longer wavelength with fewer input topographic data.

Discussion

The relative merit of each method for determining subglacial topography depends on the goal, and location, of a particular application. The primary difference between the solutions is that when many topography data points are available, cokriging preserves the short wavelength, high amplitude features while a GRACE-based inversion, which ignores available topography data, smoothes the result to the wavelength of GRACE. In regions lacking topography data, cokriging provides some improvement on the BEDMAP data by representing topographic structure manifested in the GRACE free-air anomalies, rather than predicting a region without topographic variance.

The GRACE inversion shows prominent highs and lows down to the scale of the basins in which Lake Vostok resides. The cokriging model, when constructed with a 1% sample, is longer wavelength, resolving only the pattern of subglacial highlands and basins characteristic of the BEDMAP model. Both models fail to reconstruct the minimum and maximum elevations in the area of study. A known weakness of kriging methods is that in order to produce a best estimate in the least squares sense (Davis, 1973), they generally under-predict the range of the primary variable being estimated (Salih et al., 2002). Furthermore, cokriging, since it is a moving average technique, will not extrapolate to predict values of larger amplitude than the input extremes (Davis, 1973). While the cokriging underestimation of topography has a statistical explanation, the causes of under-prediction in the inversion are more likely data based: the highest and lowest points in the topography occur at frequencies that are not present in the GRACE data. The assumptions of rock and ice density, chosen here to be consistent with values in BEDMAP, impact the model amplitude somewhat but are not large enough to account or the ~500m amplitude deficit in parts of the inversion model.

Where topography data is absent, a continental scale inversion of the GRACE data will reliably resolve some unknown topographic features at a shorter wavelength than the cokriging interpolation. In either case, the amplitude of the topography will be less than the true topographic variance.

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