A Program for Partitioning Shifted Truncated Lognormal Distributions into Size-Class Bins

By Emil D. Attanasi and Ronald R. Charpentier

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Conversion Factors

Multiply	By	To obtain
Volume		
barrel (bbl), (petroleum, 1 barrel=42 gal)	0.1590	cubic meter (m^3)
cubic foot (ft^3)	0.02832	cubic meter (m^3)

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Abstract

In recent years, oil and gas accumulation-size frequency distributions have become a standard way to characterize undiscovered conventional oil and gas resources that have been postulated by geologic assessments. The preparation of such distributions requires the assessment geologists to explicitly choose parameters for the probability distribution for the sizes of undiscovered accumulations. The purpose of this report is to present a computational scheme for obtaining a binned size frequency distribution of undiscovered accumulations when the undiscovered accumulation size distribution is shifted truncated lognormal.

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Introduction

Many national, international, and regional assessments of undiscovered oil and natural gas have often presented their primary results as aggregate estimates of volumes of resources that are recoverable using current technology but without reference to specific economic costs. The determination of economic cost is, however, very much dependent on the size distribution of the "packages" in which the producible resource occurs.

To the economist or policy analyst the most useful way to convey the potential commercial value of the undiscovered resource volume is to characterize the undiscovered accumulations by the size frequency distribution. Not only is commercial value commonly determined by size, that is, the volume of recoverable resources in a postulated accumulation, but the size also affects how quickly and economically oil and gas accumulations are discovered. In recent years the size frequency distribution paradigm has become the standard way to characterize the undiscovered oil and gas resources, so that the assessment requires the geologist to explicitly choose parameters for the probability distribution for the sizes of undiscovered accumulations.

The purpose of this report is to present a computational scheme for obtaining a binned size frequency distribution of undiscovered accumulations when the undiscovered accumulation size distribution is a shifted truncated lognormal distribution. First, the analytical form of the shifted truncated lognormal distribution is presented. Then the computational scheme to calculate the binned undiscovered accumulation-size frequency distribution and the associated volume of contained resources is described. Programs to calculate this binned distribution are presented in two forms, a Fortran[1] program and a Microsoft Excel® workbook. An additional Microsoft Excel® Workbook is included to calculate the similar cell-size distribution for assessments of continuous resources.

Shifted Truncated Lognormal Accumulation-Size Distribution

The shifted, right-truncated lognormal distribution used to model undiscovered accumulation sizes (figure 1) has a probability density function given by:

$$f(y;\mu_Y,\sigma_Y)=\frac{1}{F(T)\sigma_Y\sqrt{2\pi}(y-\gamma)}\exp\left[-\frac{1}{2}\left(\frac{\ln(y-\gamma)-\mu_Y}{\sigma_Y}\right)^2\right],\ \gamma\le y\le T$$

where *F(T)* is the cumulative probability evaluated at the truncation point. The random variable X = $ln(Y - \gamma)$ is normally distributed with mean μ_X and standard deviation σ_X. In figure 1, the shift parameter, γ, is the distance that the generic lognormal distribution which starts at zero is shifted right. To take advantage of the normal distribution for computations μ_X and σ_X must be calculated from parameters of the shifted truncated lognormal distribution. Following Aitchison and Brown (1957), then $\mu_X=\ln(f50-\gamma)$, where *f*50 is the median of the accumulation size distribution and $\sigma_X=(\ln(f001-\gamma)-\mu_X)/\Phi^{-1}(0.999)$, where *f*001 is the maximum accumulation size specified by the assessment geologist and the shift parameter, γ, is the minimum accumulation size specified by the geologist and the denominator of the expression for σ_X is the inverse normal probability function. It is assumed that $1 - F^{-1}(f001) = 0.001$, where $F^{-1}(.)$ is the inverse of the cumulative probability. The mean and variance of the truncated distribution can be obtained by numerical integration. Three sizes, namely *f*100 (minimum), *f*50 (median), and *f*001 (maximum) are used to obtain the normal distribution that correspond to the un-truncated lognormal distribution.

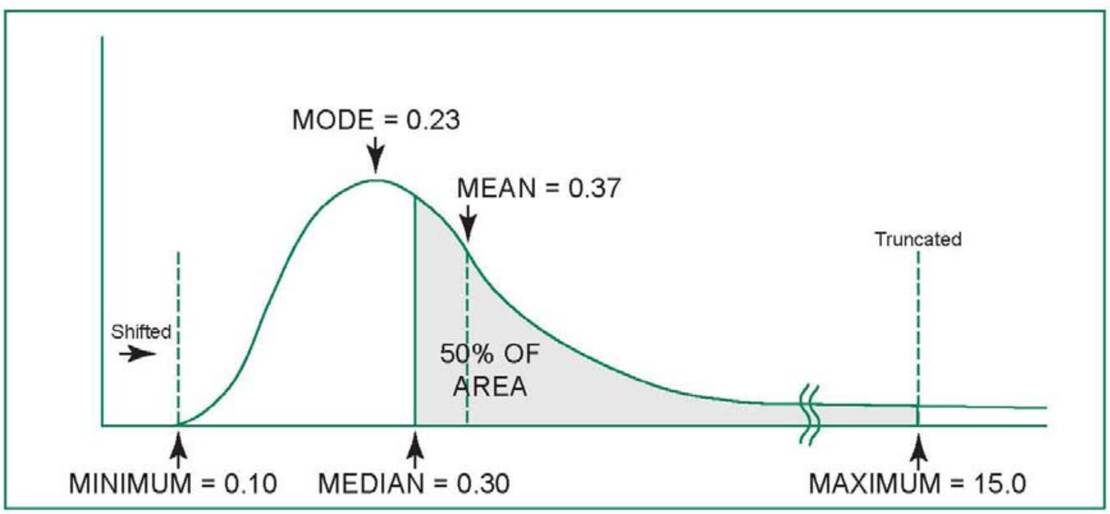

Figure 1 —Shifted truncated lognormal distribution showing truncation at the *f*001(maximum size) as well as minimum, mode, median, and mean values (from Klett and Charpentier, 2003).

[1] Fortran is a general purpose, procedural programming language that is especially suited to numeric scientific computation.

In recent oil and gas assessments prepared by the U. S. Geological Survey (USGS), the assessment geologists have been required to pick the minimum, median, and maximum sizes thought to describe the distribution of undiscovered accumulations. The right tail of the lognormal distribution is asymptotic to the horizontal axis, representing accumulation size. The finite maximum accumulation size specified by the geologist, or truncation point, was associated with the value at the 99.9 percentile of the parent lognormal distribution.

Size Frequency Distribution and Expected Volume of Resources in Undiscovered Accumulations

Size class probabilities were calculated using the estimated parameters (μ_x and σ_x) in the cumulative normal distribution (in log space) that corresponded to the lognormal distribution for sizes in real space. The shifted truncated lognormal is transformed into a two parameter truncated lognormal by subtracting the minimum size value from the median and maximum accumulation values. In particular the value of the mean of the corresponding normal distribution in log space is the (natural) log of the median of this two-parameter lognormal distribution. The standard deviation of the corresponding normal is computed using the natural log of the distribution value at the 99.9 percentile and the natural log of the value at the median of the two parameter distribution as shown above.

The assessment geologists estimate a distribution for the number of undiscovered accumulations, conditional on the occurrence of at least one undiscovered accumulation of a defined minimum size. The mean of the conditional distribution for the number of undiscovered accumulations is risked by multiplying it by the play probability, defined as the occurrence probability of at least one undiscovered accumulation of the minimum size. The accumulation-size frequencies are computed as the product of the class probabilities and the risked expected number of undiscovered accumulations.

The expected volume of oil or gas in undiscovered accumulations can be computed as the product of the (risked) expected number of undiscovered accumulations and the expected value of the size distribution of undiscovered accumulations. Although the expected value of the <u>un-truncated</u> distribution can be calculated analytically, the expected value of the shifted <u>truncated</u> lognormal distribution had to be done by numerical integration. The numerical integration algorithm was checked for accuracy by comparing results of selected cases with results obtained using the Maple V® (Waterloo Maple, 1998) mathematical analysis software package. Computations in Maple V® are carried out in integer arithmetic and thus minimize round-off and truncation error.

The standard size-bin classifications are given in Appendices 1 and 2. The accumulation-size classification in Appendix 1 is for conventional accumulations. The cell-size classification in Appendix 2 is for assessment of continuous (unconventional) resources using the cell-based method. The Fortran version of the algorithm was originally summarized in Attanasi, Schuenemeyer, and Charpentier (2000).

Using the Fortran Program

Compile the Fortran program. Call the program with the compiled program name. The program default setting is the USGS accumulation size categories. The program will ask the user whether the distribution is of oil or gas accumulations. Then the program will require the user to enter the mean number of accumulations and the play probability. Computational results are provided as a size frequency distribution and the expected total volume of the primary product contained in the accumulation-size frequency distribution.

Example of input query, with program queries in bold letters:

Enter play id; up to 8 letters & numbers
Campos
Are the accumulations gas accumulations? Y OR N
N
Check accumulation type = oil
Enter minimum accumulation size in millions of barrels or billions of cubic feet
6.
Enter median accumulation size in millions of barrels of billions of cubic feet
60.
Enter maximum accumulation size in millions of barrels or billions of cubic feet
4200.
Enter expected number of accumulations
75.
Enter play probability
1.

Example of output

Output file is dfq.prn

```
Play/Area type   class lower &upper   # accumulations  total oil or gas
  Campos   oil        0.5-      1.0     0.0000     10875.
  Campos   oil        1.0-      2.0     0.0000     10875.
  Campos   oil        2.0-      4.0     0.0000     10875.
  Campos   oil        4.0-      8.0     0.7238     10875.
  Campos   oil        8.0-     16.0     7.9538     10875.
  Campos   oil       16.0-     32.0    13.9879     10875.
  Campos   oil       32.0-     64.0    16.3910     10875.
  Campos   oil       64.0-    128.0    14.8922     10875.
  Campos   oil      128.0-    256.0    10.7447     10875.
  Campos   oil      256.0-    512.0     6.1724     10875.
  Campos   oil      512.0-   1024.0     2.8178     10875.
  Campos   oil     1024.0-   2048.0     1.0198     10875.
  Campos   oil     2048.0-   4096.0     0.2920     10875.
  Campos   oil     4096.0-   8192.0     0.0046     10875.
  Campos   oil     8192.0-  16384.0     0.0000     10875.
  Campos   oil    16384.0-  32768.0     0.0000     10875.
  Campos   oil    32768.0-  65536.0     0.0000     10875.
  Campos   oil    65536.0- 131072.0     0.0000     10875.
  Campos   oil   expected # accumulations    75.0  expected accumulation size
144.994
```

Using the Microsoft Excel Programs

Workbook lognormal.xls

This workbook is appropriate for use with conventional accumulations. The USGS methodology using these distributions for assessing conventional accumulations can be found in Schmoker and Klett (2000a, 2000b) and Klett, Schmoker, and Charpentier (2003).

All cells colored yellow are user-input cells. Units are in millions of barrels for liquids and billions of cubic feet for gas. On the Input worksheet, these specify whether the accumulations are oil or gas, the size distribution of undiscovered accumulations (minimum, median, and maximum), and the number of undiscovered accumulations. This number can be the mean number of undiscovered accumulations or any fractile of the distribution for number of accumulations. Also on the Input worksheet is an input cell (K14) specifying a cutoff value for a separate calculation of number of accumulations larger than that size. After entering the input, the user should click the Fit Distribution button to perform the calculations.

The number of accumulations larger than the cutoff size is given on the Input worksheet in cell K16. The rest of the output is given on worksheet Size Classes. The bins are specified and the frequencies of undiscovered accumulations are given in column G. The mean size of undiscovered accumulations is given in cell D25. Column F allows the user to specify the number of discovered accumulations in each bin. Discovered and undiscovered accumulations are plotted as a stacked bar chart.

Workbook lognormal_EUR.xls

This workbook is appropriate for use with continuous accumulations. The USGS methodology using these distributions for assessing continuous accumulations can be found in Klett and Charpentier (2003) and Klett and Schmoker (2003).

Again all cells colored yellow are user-input cells. Units are in millions of barrels for liquids and billions of cubic feet for gas. On the Input worksheet, these specify whether the cells are oil or gas and the size distribution of cells (minimum, median, and maximum) based on estimated ultimate recovery (EUR). No numbers of cells are input and the calculations on worksheet Size Classes are relative frequencies only. Also on the Input worksheet is an input cell (K14) specifying a cutoff value for a separate calculation of percent of cells larger than that size. After entering the input, the user should click the Fit Distribution button to perform the calculations.

The number of cells larger than the cutoff size is given on the Input worksheet in cell K16. The rest of the output is given on worksheet Size Classes. The bins are specified and the relative frequencies of cells are given in column F. The mean size of the cells is given in cell D25. Cell-size frequencies are plotted as a bar chart.

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Appendix 1. Conventional Accumulation-Size Class Definitions

Class	Oil accumulation size (millions of barrels)	Gas accumulation size (billions of cubic feet)
1	0.03125 – 0.0625	0.1875 – 0.375
2	0.0625 – 0.125	0.375 – 0.75
3	0.125 – 0.25	0.75 – 1.5
4	0.25 – 0.5	1.5 – 3
5	0.5 – 1	3 – 6
6	1 – 2	6 – 12
7	2 – 4	12 – 24
8	4 – 8	24 – 48
9	8 – 16	48 – 96
10	16 – 32	96 – 192
11	32 – 64	192 – 384
12	64 – 128	384 – 768
13	128 – 256	768 – 1,536
14	256 – 512	1,536 – 3,072
15	512 – 1,024	3,072 – 6,144
16	1,024 – 2,048	6,144 – 12,288
17	2,048 – 4,096	12,288 – 24,576
18	4,096 – 8,192	24,576 – 49,152
19	8,192 – 16,384	49,152 – 98,304
20	16,384 – 32,768	98,304 – 196,608
21	32,768 – 65,536	196,608 – 393,216
22	65,536 – 131,072	393,216 – 786,432
23	131,072 – 262,144	786,432 – 1,572,864

Accumulations that are exactly one of the boundary sizes belong to the higher class. For example, an accumulation of exactly 1 million barrels is in class 6.

Appendix 2. Continuous Cell-Size Class Definitions

Class	Oil cell size (millions of barrels)	Gas cell size (billions of cubic feet)
-1	0.0009765125 – 0.001953125	0.005859375 – 0.01171875
0	0.001953125 – 0.00390625	0.01171875 – 0.0234375
1	0.00390625 – 0.0078125	0.0234375 – 0.046875
2	0.0078125 – 0.015625	0.046875 – 0.09375
3	0.015625 – 0.03125	0.09375 – 0.1875
4	0.03125 – 0.0625	0.1875 – 0.375
5	0.0625 – 0.125	0.375 – 0.75
6	0.125 – 0.25	0.75 – 1.5
7	0.25 – 0.5	1.5 – 3
8	0.5 – 1	3 – 6
9	1 – 2	6 – 12
10	2 – 4	12 – 24
11	4 – 8	24 – 48
12	8 – 16	48 – 96
13	16 – 32	96 – 192
14	32 – 64	192 – 384
15	64 – 128	384 – 768
16	128 – 256	768 – 1,536

Cells that are exactly one of the boundary sizes belong to the higher class. For example, a cell of exactly 1 million barrels is in class 9. In order to apply the class size scale index, which was originally developed for conventional oil and non-associated gas accumulations, to well EUR values, size classes smaller than class one were defined. The two successively smaller size classes are denoted with indices 0 and -1.

Appendix 3. Fortran Program Listing

```
Fortran program
c stlgfq.f = computed the expected size frequency distribution
c   from the distribution of the number of fields and field size distribution,
c   where (1) field/accumulation size distribution is the shifted truncated
c             lognormal distribution
c         (2) the mean number of undiscovered fields in input.
c  Subroutine yormx.f attributed to Rolf E. Bargmann, University of Georgia
c
c  subroutines:
c calprbs = computes probabilities in each size class for the shifted
c           truncated lognormal: class probabilities are based on yormx and
c           assume the values distribution is truncated at the 99.9 percentile
c           or the 0.1 percent fractile of the tail of the distribution
c sha = computes the expected sizes of fields; the mean of the shifted
c       truncated lognormal, based on truncation at the 0.1 percent fractile
c        based on truncation at the 0.1 percent fractile. The product of the
c        mean accumulation size and mean number of fields equals
c        the mean volume of assessed resource.
c yormx.f = normal distribution by Rolf E. Bargmann, University of Georgia
c  variable list:
c     Shifted truncated lognormal: qmin = minimum sizes, qmax = maximum size,
c     qmed = median or 50th fractiles size;
c     pr= play or assessment unit risk, for expected value distribuition
c     Multiply the expected conditional number of fields by the play risk.
c     The size distribution is assumed to be unaffected by play risk.
c     Calculate associated volumes of oil in oil fields, gas in gas fields
c     ic = 1 for oil fields, ic = 2 for gas fields
c
          implicit real*8 (b-h,o-z)
          character*1 a
          character*3 cmn(2)
          character*8 idu
          dimension dcls(19,3),xl(19),ul(19)
          data cmn/'oil','gas'/
          common/blk1/gx(100008),gd(100008)
          common/szpb/exp(19)
          common/loc/idu
          common/clss/xls(19)
          open(unit=12,file='dfq.prn')
          qtmx=0.001
          write(6,199)
  199     format(1x, 'Enter play id; up to 8 letters & numbers')
          read(5,*) idu
          write(6,200)
  200     format(1x,'Are the deposits gas deposits? Y OR N')
          read(5,*) a
          if(a.eq.'Y') ic=2
          if(a.eq.'N') ic=1
          write(6,198) cmn(ic)
  198     format(1x, 'Check deposit type = ',1x,a3)
          write (6,201)
  201     format(1x,'Enter minimum field size in millions of barrels or
     $billions of cubic feet')
          read(5,*) qmin
          write(6,202)
```

```
 202      format(1x,'Enter median field size in millions of barrels of bi
     $llions of cubic feet')
           read(5,*) qmed
           write(6,203)
 203      format(1x,'Enter maximum field size in millions of barrels or b
     $illions of cubic feet')
           read(5,*) qmx
           write(6,204)
 204       format(1x,'Enter expected number of fields')
           read(5,*) xmn
           write(6,205)
 205       format(1x, 'Enter play probability')
           read(5,*) pr
           call clprbs(qmin,qmed,qmx,qtmx,ic)
c
c *** Call subroutine sha for determining expected oil in assessment unit.
c
            call sha(qmin, qmed, qmx,qtmx,ic,szmn)
            oilmn = szmn*xmn
            write(12,209)
  209      format(1x, 'Play/Area',1x,'type',3x,'class lower &upper',
     $2x, ' # deposits' 2x, 'total oil or gas')
            do 10 k=1,18
            xl(k) = xls(k)
            ul(k) = xls(k+1)
            dcls(k,1)=xmn*exp(k)
c
c *** Compute & retain distribution associated with the expected values.
c
            write(12,212) idu, cmn(ic),xl(k),ul(k), dcls(k,1), oilmn
  212    format(2x,a8,1x,a3,2x,f9.1,'-',f9.1,2x,f9.4,1x,f10.0)
   10      continue
            write(12,210) idu,cmn(ic), xmn, szmn
  210   format(2x,a8,1x,a3,3x,'expected # deposits',1x,f7.1,2x,'expected
     $ deposit size',f12.3)
           close(9)
           endfile(12)
           close(12)
c          endfile(14)
c          close(14)
            stop
            end
c
c*******************
c
           subroutine clprbs(qmin,qmed,qmx,qtmx,ic)
c subroutine
c   assumes truncated shifted truncated lognormal distribution,
c     First, calculate the probability for each size class.e
c     Second, calculate expected sizes for bottom and top size classes.
c   qmin= minimum size of shifted truncated lognormal from assessment.  form
c   qmx = maximum size of shifted truncated lognormal from assessment form
c   qmed = median size of shifted truncated lognormal from assessment form
c   q50 = shifted median as given in world assessment form as
c           size of  lognormal shifted back to zero minimum
c   q001= shifted maximum size of lognormal shifted back to
c           zero minimum  as given in world assesmsent forms
          implicit real*8 (a-h,o-z)
          dimension dxp(19),cls(19),xlx(19)
c
c *** Size class started  at  0.5 mmbbl  oil in oil fields

c ***  & 3 bcf gas in gas fields
c
```

```
        data xlx/0.5,1. ,2., 4.,8.,16.,32.,64.,128.,256.,
    $512.,1024.,2048.,4096.,8192.,16384.,32768.,65536.,131072./
        common/szpb/exp(19)
        common/clss/xls(19)
c
c ** Determine the range of the data size classes:, nl lowest size class
c ** nu largest size class
c
        do 1 k=1,19
        if(ic.eq.1) xls(k)=xlx(k)*1.00
        if(ic.eq.2) xls(k)=xlx(k)*6.00
  1     continue
        do 2 k=1,19
        exp(k)=0.0
        dxp(k)=0.0
   2    continue
           nl=1
           nu=0
        do 3 k=1,19
        if(qmin.ge.xls(k)) nl=k
        if(qmx.le.xls(k)) nu=k
        if(qmx.le.xls(k)) go to 4
  3     continue
  4     continue
c
c *** Transform to shifted logrnormal       .
c
        q50=qmed-qmin
        q001= qmx - qmin
        do 5 k=nl,nu
        cls(k)=xls(k)-qmin
        if(k.eq.nl) cls(k) =0.0001
        if(k.eq.nu) cls(k) = q001
  5     continue
        xd=1.00-qtmx
        dv= yormp(xd)
         xmu=(alog(q50))
         xsig=(alog(q001)-alog(q50))/dv
c
c *** Correct probabilties for truncation by dividing by .999
c
        do 7 k= nl, nu-1
        ph1= (alog(cls(k))-xmu)/(xsig)
        ph2= (alog(cls(k+1))-xmu)/(xsig)
        exp(k)=(yormx(ph2)-yormx(ph1))/.999
  7     continue
        cb=0.0
        do 10 k=1,19
        cb=exp(k)+cb
 10     continue
        return
        end
c*****************************************************************
c
c*******************
c
      subroutine sha(qmin, qmed, qmx,qtmx,ic, szmn)
c   program to determine the expected size of fields; shifted truncated
c     lognormal distribution divided into 100,000 intervals
c
c ***  First, shift back lognormal.
c
       common/blk1/gx(100008),gd(100008)
        q50=qmed-qmin
```

```
         q001= qmx - qmin
         xmu=(alog(q50))
          xd =1.0-qtmx
           dv=yormp(xd)
              xsig= (alog(q001)-xmu)/dv
c        write(6,*) xmu,xsig
         fac=0.00001
         do 20 k=1,100001
         gx(k)=0.0
         gd(k)=0.0
 20      continue
         gx(1)=0.0
         do 30 k=2,100001
         tmp=((k-1.)*fac*.999)
         gx(k)=yormp(tmp)*xsig+xmu
         gx(k)= exp(gx(k))
 30      continue
         do 40 k=2,100001
         gd(k-1)=(gx(k-1)+gx(k))*.5*1.000
 40      continue
         cm=0.0
         do 45 k=1,100000
         cm= gd(k)+cm
 45      continue
         szmn=cm*.00001+qmin
         return
         end
c
c********************
c
      function yormp(p)
      implicit real*8(a-h,o-z)
c      Rolf E. Bbargmann, University. of Georgia
c      other options check
c     http://www.netlib.org/cgi-bin/search.pl
      dz = 0.d0
      de = 1.d0
      y=p
      domeg = 1.d76
      if(y)1,1,2
    1 yormp = -domeg
      go to 99
    2 if(y- de )3,4,4
    4 yormp = domeg
      go to 99
    3 if(y-.5d0)5,6,7
    6 yormp = dz
      go to 99
    5 dq = y
      go to 10
    7 dq =  de -y
 10   ncyl = 0
c
c *** obtains a first approximation.
c
      det = dsqrt(-2.d0*log(dq))
      dxn = det-((.010328d0*det+.802853d0)*det+2.515517d0)/(((.001308d0
     1*det+.189269d0)*det+1.432788d0)*det +  de )
      if(y-.5d0)12,12,13
   12 dxn = -dxn
   13 dpn = yormx(dxn)
      der = dpn -y
      if(abs(der/y) - 1.d-11) 14,14,15
   15 ncyl = ncyl +1
```

```
      if(ncyl-20) 17,17,14
   17 dxp = dxn
c
c *** uses the Newton-Raphson method to converge.
c
   29 dz = yormz(dxp)
      if ( dz ) 27,27,28
   27 dxp = dxp*0.5d0
      go to 29
   28 dxn = dxn-der/dz
      go to 13
   14 yormp = dxn
   99 return
      end
c
c********************
c
      function yormx (x)
      implicit real*8(a-h,o-z)
c
      data pi/0.398942280401432d0/
      abx=abs(x)
      x2=abx*abx
c
c *** checks for limitation of x.
c
      if(x.lt.-18.d0) go to 96
      if(x.gt.9.d0) go to 97
      if (abx.gt.3.1d0) go to 30
c
c *** evaluates p using the Shenton fixed length continued fraction.
c
      yormx=pi*exp(-x2*0.5d0)
      orm = 45.d0-23.d0*x2/47.d0
      is=1
      do 17 i=2,22
        ix=24-i
        dx = 2*ix-1
        di = is*ix
        orm = dx + di*x2/orm
        is = -is
 17   continue
      yorm = abx/(1.d0-x2/orm)
      yormx = yormx*orm
c      yormx=yormx*(abx/(1.d0-x2/(3.d0+2.d0*x2/(5.d0-3.d0*x2/(7.d0+
c     $ 4.d0*x2/(9.d0-5.d0*x2/(11.d0+6.d0*x2/(13.d0-7.d0*x2/(15.d0+
c     $ 8.d0*x2/(17.d0-9.d0*x2/(19.d0+10.d0*x2/(21.d0-11.d0*x2/(23.d0+
c     $ 12.d0*x2/(25.d0-13.d0*x2/(27.d0+14.d0*x2/(29.d0-15.d0*x2/(31.d0+
c     $ 16.d0*x2/(33.d0-17.d0*x2/(35.d0+18.d0*x2/(37.d0-19.d0*x2/(39.d0+
c     $ 20.d0*x2/(41.d0-21.d0*x2/(43.d0+22.d0*x2/(45.d0-23.d0*x2/47.d0
c     $ )))))))))))))))))))))))
      if(x.lt.0.d0) go to 20
      yormx=0.5d0+yormx
      go to 99
   20 yormx=0.5d0-yormx
      go to 99
   30 continue
      orm = abx+35.d0/abx
      do 19 i=1,34
       ix = 35-i
       dx = ix
       orm = abx+ix/orm
 19   continue
      yormx = pi*exp(-x2*0.5d0)/orm
```

```
c
c *** evaluates p using the Laplace fixed length continued fraction
c
c       yormx=pi*exp(-x2*.5d0)*(1.d0/(abx+1.d0/(abx+2.d0/(abx+3.d0/(abx+
c      $ 4.d0/(abx+5.d0/(abx+6.d0/(abx+7.d0/(abx+8.d0/(abx+9.d0/(abx+
c      $10.d0/(abx+11.d0/(abx+12.d0/(abx+13.d0/(abx+14.d0/(abx+15.d0/(abx+
c      $16.d0/(abx+17.d0/(abx+18.d0/(abx+19.d0/(abx+20.d0/(abx+21.d0/(abx+
c      $22.d0/(abx+23.d0/(abx+24.d0/(abx+25.d0/(abx+26.d0/(abx+27.d0/(abx+
c      $28.d0/(abx+29.d0/(abx+30.d0/(abx+31.d0/(abx+32.d0/(abx+33.d0/(abx+
c      $ 34.d0/(abx+35.d0/abx))))))))))))))))))))))))))))))))))))
      if(x.gt.0.d0) yormx=1.d0-yormx
      go to 99
   96 yormx=0.d0
      go to 99
   97 yormx=1.d0
   99 return
      end
c
c********************
c
      function yormz(x)
      implicit real*8(a-h,o-z)
c
      y=x
      ey = -y*y*0.5d0
      if(ey+170.d0)1,1,2
    1 yormz = 0.d0
      go to 99
    2 yormz = .3989422804014327d0*exp(ey)
   99 return
      end
```